More is Less:
The Tax Effects of Ignoring Flow Externalities

Leif K. Sandal[*], Stein I. Steinshamn[†], R. Quentin Grafton[‡]

November 2000

Abstract

Using a model of nonlinear decay of the stock pollutant, and starting from the same initial conditions, the paper shows that a tax that only corrects for stock externalities can, at the steady state, be higher than a tax that corrects for both stock and flow externalities. The results indicate that the possibility exists that the optimal corrective tax (correcting for both externalities) may result in a steady state with fewer emissions and lower tax payments than a tax that only corrects for the stock externality. Thus, a failure to consider flow externalities may have important implications for the time path and steady states of production, emissions and taxes, and not just in terms of transitory consumption.

JEL Classification: H21, Q25

Key Words: corrective taxes, stock and flow externalities, nonlinear decay

[*]Norwegian School of Economics and Business Administration. Corresponding author.

[†]Foundation for Research in Economics and Business Administration

[‡]University of Ottawa

INTRODUCTION

Pigouvian taxes are widely used to mitigate the externalities which exist in production. Such taxes are favored when there exist many polluters and have been widely applied in Europe to address a large number of environmental externalities (Andersen 1994; OECD 1992). Worldwide, corrective taxes generate billions of dollars annually for governments. In theory, corrective charges should equal the costs imposed on society of a defined level of pollution. In practice, charges are often based on the notion that the current level or flow of pollution (such as the amount of phosphorous discharged into a river) represents the externality imposed on society. However, many pollutants impose both stock and flow externalities such that current and cumulative discharges affect the non-monetary variables of utility or production functions (Baumol and Oates 1988).

Much of the literature on Pigouvian taxes focuses either on flow externalities or stock externalities (Sinclair 1992; Ulph and Ulph 1994; Farzin and Tahvonen 1996; Hoel and Kverndokk 1996; Wirl and Dockner 1996). Moreover, many dynamic models do not specify the underlying demand parameters and private and social costs of production and, instead, optimize using an aggregate utility function. As a result, many dynamic models fail to consider flow externalities and set the corrective tax equal to the value of the co-state variable, commonly defined as the shadow price or shadow cost of pollution.

Where both stock and flow externalities exist and are explicitly considered (Sandal and Steinshamn, 1998), the optimal corrective tax will exceed the shadow cost of pollution, for a given level of pollution (Wirl, 1994; Farzin 1996). Thus, we would expect that a corrective tax which ignores flow externalities, when they are present, to result in more pollution and a lower tax payment. By contrast, our paper shows that different taxes result in different time paths and that the specification of the decay function of the stock of pollution has an important affect on transitory and steady-state production, taxes and emissions. Hence, the possibility exists that the corrective tax that corrects for both flow and stock externalities, may result in a steady state with higher emissions but lower accumulated pollution and lower tax payments than a tax that only corrects for the stock externality, or a case of more is less!

MODEL OF OPTIMAL PRODUCTION WITH STOCK AND FLOW EXTERNALITIES

A simple dynamic model can be used to incorporate both stock and flow externalities in production. The model assumes that the objective is to maximize welfare (W), defined as the discounted present value of social utility, which is a function of the stock of pollution (a) and the production of a good (x). Utility is defined as the sum of consumer and producer surplus, adjusted for flow externalities, less the effect of the stock externality, defined by $D(a)$ where D is increasing in a, i.e.,

$$U(a,x) = \int_0^x [P(z) - C^s(z)]\, dz - D(a) \equiv \Pi(x) - D(a)$$

where x is quantity produced, P is the inverse demand, and C^s is the social marginal cost of production.[1]

A dynamic constraint governs the change in the stock of pollution, $\dot{a}$, and is determined by the instantaneous increase in pollution γx, which is proportional to production x by a factor γ, and the decay of the stock of pollution, $d(a)$, which might be increasing or decreasing in a, depending upon the level of a.[2] The flow externality, τ_f, is the instantaneous externality which arises at the time the pollution is emitted. By definition, the marginal cost associated with the flow externality plus the private marginal cost of production, $C^p(x)$, equals the social marginal cost of production $C^s(x)$, thus $\tau_f \equiv C^s - C^p$.

Our analysis examines the case without discounting because, if the counterintuitive "more is less" occurs at a zero discount rate, it will also hold true with a positive discount rate. The dynamic problem is to maximize welfare defined as:

$$W = \int_0^T U(v(t))dt, \ \ v(t) \equiv (a(t), x(t)) \in R \times X$$

where $X = [0, B]$ is a given bounded interval, and W is maximized subject to the following dynamic constraint and initial condition:

$$\dot{a}(t) = x(t) - d(a(t)), \ \ a(0) = a_0, \tag{1}$$

where a_0 is the initial level of the stock pollutant.

We assume that a long-term steady state is desirable and thus solve for processes $v \in V$, where

$$V = V[R \times X], \quad \lim_{t \to \infty} a(t) = a^*. \tag{2}$$

Here V represents the set of $(a(t), x(t))$ such that a is continuously differentiable, and x is continuous and piecewise differentiable. To develop the model further, we define the following set of admissible processes.

Definition 1 *The set of admissible processes, A, is defined as all processes that satisfy (1) and (2).*

It is further assumed, unless otherwise explicitly stated, that:

1. Π, D and d are C^2-functions in their arguments whenever the arguments are positive.

2. $D : R \to R_+$ is strictly increasing and convex for positive arguments, and identically equal to zero for non-positive arguments. The state $a = 0$ is, by definition, a steady state without emissions and can be interpreted as the preindustrial level, that is $d(0) = 0$ and $D(0) = 0$. No damage is associated with the preindustrial level.

3. $\Pi : X \to R_+$ is strictly increasing and strictly concave.

4. $d : R \to R_+$ is strictly increasing for $0 < a < \widehat{a}$ and strictly decreasing for $a > \widehat{a}$. Further

$$\lim_{a \to 0^+} \left[\Pi'(d(a)) \cdot d'(a) - D'(a)\right] > 0.$$

Under these definitions, the optimal control problem is to determine the feedback rule $x(a)$ that can be written $\max\limits_{v \in A} W$.[3]

Our model represents either a single firm in a competitive world, or an entire competitive industry. In the absence of intervention, market equilibrium requires that $P = C^p$, where private marginal cost is strictly increasing in x, and an equilibrium price and quantity of x can be defined for any level a. If a is a constant, the solution collapses to the standard result of static models with flow externalities, namely that welfare is maximized when $P = C^s$.

To demonstrate the result of "more is less" we specify key variables through the following definitions.

Definition 2 *(i) Sustainable utility, S, is defined as the utility obtained when a is fixed at certain level:*

$$S(a) = \Pi(f(a)) - D(a).$$

Definition 3 *(ii)Total utility, K, is equal to the Hamiltonian in value, but it is a function of a and x only:*

$$K(a, x) = \Pi(x) - D(a) + \Pi'(x)\left[x - f(a)\right]$$

Using Definitions 1-3, it can be shown that[4]:

Theorem 1 *The OT-optimal production for the problem $\max\limits_{v \in A} W$ where $\frac{\partial^2}{\partial a^2} K(a, x) < 0$ on $R \times X$, is given by*

$$x(a) = \max(0, z(a)) \quad \textit{where } K(a, z(a)) = \max S(a) = S(a^*).$$

The long-term steady-state $(a^*, x^* = f(a^*))$ *is a saddle point for* $K(a,x)$ *and determined by* $S'(a^*) = 0$.

Moreover, the optimal production path is characterized by Proposition 1.[5]

Proposition 1 *The optimal steady state is to the left of* $\max d(a)$. *The separatrix part of the feedback solution is strictly decreasing to the right of* $\max d(a)$. *The steady state is the only critical point. If* $K(a,x)$ *is quasiconcave on*

$$L = \{(a,x) : a > a^*, 0 < x < x^*, x < f(a)\},$$

then the separatrix part of the feedback solution is concave below f.

Using these results, we can examine how this production level can be achieved through a system of corrective taxes.

AN OPTIMAL CORRECTIVE TAX

The result of Wirl (1994) and Farzin (1996) that the optimal corrective tax exceeds, in the presence of stock and flow externalities, the shadow price or cost of pollution can be derived by defining the current value Hamiltonian (H):

$$H(a,x,m) = U(a,x) + m[x - d(a)] \tag{3}$$

where m is defined as the current value co-state variable for the stock of pollution, a. Using Theorem 1, and Definition 3, yields

$$-m = U'(x).$$

The corrective tax, ignoring the flow externality, is thus

$$\sigma(x) = -m = \Pi'(x)$$

on the optimal path $x(a)$. Alternatively, we can rewrite the σ tax as

$$\sigma = P - C^s = P - C^p - (C^s - C^p) = \tau - \tau_f.$$

By contrast, the tax which corrects for both flow- and stock-externalities, defined as τ, is the difference between the consumer price and producer price, $\tau(x) = P(x) - C^p(x)$, and can be calculated at both the firm and industry level. As a result,

$$\tau(x) = \sigma(x) + \tau_f(x), \tag{4}$$

where (4) holds true at both the optimal steady state and on the path to the steady state.[6] Given these results, we can compare σ and τ corrective taxes and their effects on production, emissions and pollution.

COMPARISON OF τ AND σ TAXES

At any given pollution level, σ must be lower than τ because σ ignores the flow externality. As a result, we would expect that a σ-tax would be associated with more pollution, more production and a lower tax payment at all times. In fact, because emissions are different with the τ and σ taxes, the time path of pollution will be different and, thus, the possibility exists that σ may lead to a steady state with more aggregate pollution but, surprisingly, less production and a higher tax-level.

In order to find the development of a with the σ-tax we must first find the feedback rule for production that corresponds to σ. By ignoring the flow-externality and only using the stock-externality part of the optimal tax, which is $\sigma(x(a))$ for any given a-level, we can obtain a new market equilibrium characterized by

$$\tau(y) = \sigma(x(a)). \tag{5}$$

This relationship is illustrated in Figure 1. As (5) is an expression in y and a only, it can be used to solve for y as a function of a. Hence, we obtain new feedback rule,

$y(a) > x(a)$, and the development in a is given by

$$\dot{a} = y(a) - d(a).$$

This result can be stated in Proposition 2:

Proposition 2 *The σ-tax yields a production that is always higher than the optimal for a given level of a.*

The feedback rule based on σ leads to a different steady state with a higher a. This steady state, $a^{\#}$, can be found by substituting $y = d(a)$ into (5):

$$\tau(d(a)) = \sigma(x(a))$$

which eventually yields $a^{\#} > a^{*}$. At the steady state, $y^{\#} = d(a^{\#})$, which can be compared with $x^{*} = d(a^{*})$. To develop the result further, we define the following case.

Definition 4 *The counterintuitive case: The case where σ leads to a steady state where $x(a^{*}) > y(a^{\#})$, and hence $\sigma > \tau$, that is lower production and higher tax, is called the counterintuitive ("more is less") case.*

The counterintuitive case, however, can only occur if $d(a)$ is non-linear and non-monotone, as stated Proposition 3.

Proposition 3 *If the decay-function, $d(a)$, is monotonically increasing, σ will always lead to a steady state with higher production and lower tax than τ does.*

Proposition 3 provides an explanation why the counterintuitive case is hard to find in the literature as most of the literature uses monotone, and very often linear, decay functions.

Let the steady state corresponding to τ be (a^*, x^*) and let a^{**} be a solution of $x^* = d(a)$ for $a > a^*$ if it exists, and infinity otherwise. We are then able to provide necessary and sufficient conditions for the counterintuitive case, as per Proposition (4):

Proposition 4 *If $d(a)$ is quasiconcave, (a^*, x^*) is the steady state corresponding to τ and $(a^{\#}, y^{\#})$ is the steady state corresponding to σ, then $\sigma(y^{\#}) > \tau(x^*)$ iff $a^{\#} > a^{**}$.*

In Figure 2, it can be seen that discounting only shifts the x- and y-curves to the right and, thus, if the "more is less" result holds true at a zero discount rate, it will also hold true at a positive rate of discount. To better apply the result, we can derive sufficient conditions for the counterintuitive case to occur. First, we note that the values a^* and a^{**} can be found without solving the complete problem, or solving any differential equations. Thus, we can assume the following quantities are known:

$$\begin{aligned} b &= \max d(a),\ J = [0, b] \subset X,\ \ \Delta S^* = S(a^*) - S(a^{**}) \\ k_F &= \max_{X \in J} \left[\tau_f(x) \cdot x\right],\ \ \underline{M} = \min_{X \in J} \left[-\Pi''(x)\right], \\ & \quad \overline{M} = \max_{X \in J} \left[-\Pi''(x)\right] \end{aligned} \tag{6}$$

Under the above assumptions, sufficient conditions for the "more is less" result are provided by Propositions 5 and 6.

Proposition 5 *$\Delta S^* \geq k_F$ is a sufficient condition for the counterintuitive case not to occur for cases covered by Theorem 1.*

Proposition 6 *If there exists a z such that $0 < z < x^*$ and if $\underline{\tau_F} = \min_{x \in N} [\tau_f(x)]$ where $N = [z, x^*] \subseteq J \subseteq X$, then $2\overline{M}^2 \Delta S^* < \underline{M}\tau_f^2$ is sufficient for the counterintuitive case to occur for cases covered by Theorem 1.*

An illustration of the counterintuitive result, and how the optimal corrective taxes may be derived, is provided in the following section.

MORE IS LESS: AN EXAMPLE

The possibility that ignoring the flow externality may eventually reduce production and increase tax payments can be illustrated using a numerical simulation from the climate change literature. This shows that it is not only a theoretic possibility but it may occur in practice as well. We assume linear demand and linear marginal cost functions and a quadratic damage function.

The parameters used in the model, defined below, are stylized and provided to illustrate the theoretical results. Nevertheless, they are derived from the literature on climate change. For instance, current emissions of CO_2 are estimated at some 22 giga tonnes (Gt-CO_2), which is the private market equilibrium in our model when marginal costs are normalized to one, and production is measured as emissions. The cumulative anthropogenic emissions of CO_2, less decay, are estimated to be some 625 Gt-CO_2 above the pre-industrial level.

$$\begin{aligned}
P(x) &= 15 - 0.64 \cdot x, \\
C^p(x) &= 1 + 0.05 \cdot x, \\
C^s(x) &= 1 + 0.12 \cdot x, \\
\gamma x &= x, \qquad D(a) = 0.000005 \cdot a^2, \\
d(a) &= \max(0, 21 \cdot \exp\left(-(a-600)^2 \cdot 0.512 \times 10^{-5}\right) - 3.32)
\end{aligned}$$

The parameters above imply $\Pi(x) = 14 \cdot x - 0.38 \cdot x^2$, and, hence, the corrective taxes are

$$\begin{aligned}
\tau(x) &= 14 - 0.69 \cdot x, \\
\sigma(x) &= 14 - 0.76 \cdot x,
\end{aligned}$$

$$\tau_f = 0.07 \cdot x.$$

Under this numerical specification, we obtain the following quantities:

$$b = 17.68 \quad a^{**} = 638.5 \quad \Delta S^* = 0.462$$
$$\underline{M} = 0.76 \quad \overline{M} = 0.76$$

Thus, the steady states resulting from $\tau(x)$ versus $\sigma(x)$ are

$$a^* = 561.5 \quad x^* = 17.52 \quad \tau(x^*) = 1.91,$$
$$a^\# = 669.3 \quad y^\# = 17.17 \quad \sigma(y^\#) = 2.15.$$

The counterintuitive result corresponds with the sufficiency criterion given in Proposition 6. The criterion $2\overline{M}^2\Delta S^* < \underline{M}\tau_f^2$ is fulfilled in the region $11.97 < x < x^*$. In other words, starting at the same initial condition, a carbon tax that ignores flow externalities will initially be lower than a tax that accounts for both the stock and flow externalities. However, at the steady state, the optimal carbon tax rate will be lower, the output will be higher and the cumulative carbon emissions will be lower than with the tax that ignores the flow externalities.

Our model differs from the approaches of Sinclair (1992; 1994), Wirl (1994), Ulph and Ulph (1994), Farzin (1996) and Farzin and Tahvonen (1996) who have all examined the optimal paths of corrective taxes for GHG emissions. With the exception of Wirl (1994) and Farzin (1996), existing models do not explicity consider flow externalities as a function of the current level of emissions, and only Farzin and Tahnoven (1996) examine the effect of different rates of uptake of carbon in the atmosphere on the time paths of corrective taxes. Despite the fact that Farzin and Tahnoven (1996) ignore the flow externalities associated with GHG emissions, their approach is the most similar to our own because they explicitly consider the affect of the decay function on their results. Assuming multiple carbon stocks, each with different but constant rates of decay, they find that for carbon levels in excess of pre-industrial levels, the corrective tax may be decreasing or U-shaped over time. They conclude that the optimal carbon tax is "··· sensitive to the submodel describing the accumulation of atmospheric CO_2." (Farzin and Tahvonen 1996, p. 533).

By incorporating nonlinear decay in the pollution stock in a model of GHG emissions, which better represents the actual physical processes, we show the importance of accounting for both stock and flow externalities. Moreover, in contrast to the accepted view that flow externalities affect only transient consumption (Wirl 1994), we find that a failure to consider flow externalities in a model of GHG emissions may affect both the time paths and steady states of production, emissions, and taxes. This has important implications when examining the "no regrets" policies associated with climate change and pollution policies.

CONCLUDING REMARKS

Using a dynamic model with both flow and stock externalities, the paper shows that the possibility exists for an optimal corrective tax to result in less total emissions but lower tax payments than a corrective tax which ignores flow externalities. This counterintuitive result, which may arise if the decay of the stock pollutant is nonlinear, has important implications for corrective tax policies where there exist both stock and flow externalities.

The results emphasize that a failure to account for flow externalities, where there exist stock and flow effects, will affect both transitory and steady-state production, corrective taxes and emissions. Moreover, a failure to adequately model the rate of decay of a stock pollutant in models of stock and flow externalities, can result in higher steady-state levels of both the pollution and the corrective tax.

APPENDIX

Proof of Theorem 1

A feedback control must satisfy $K(a, x) = K_0$, that is, the Hamiltonian is constant for interior solutions of autonomous problems. The costate variable is $m = -U'(x) < 0$, and therefore $H(\cdot) = K(a, x) = K_0$. That we are heading for a steady state implies $K_0 = K(a, d(a)) = S(a)$. If the Hamiltonian is maximized, $a^* = \arg\max S(a)$. Our assumptions imply $S'(0^+) > 0$ and $S'(a) < 0$ to the right of $\max d(a)$. Thus, there exists a point where $S' = 0$ and this is the global maximum of S.

The optimal steady state is $S'(a^*) = 0$, and the feedback solution, defined as $K(a, x) = S(a^*)$, represents separatrix solutions of Hamilton's canonical equations. Note that $\frac{\partial K}{\partial x} = \Pi''(x) \cdot (d - x) \neq 0$ except at steady state. The "Implicit function theorem" guarantees that $K(a, x) = S^*$ defines a unique, continuously differentiable feedback, $x(a)$, outside to the steady state. The optimal feedback consists of the separatrices that leads to steady state and are positive in addition to parts where $x = 0$. Substitution into the Hessian matrix of K shows that the steady state is a saddle point.

The over taking OT-criterion of Seierstad and Sydsæter (1987) replaces the condition $\lim_{t\to\infty} m(t)\,[y(t) - x(t)] \geq 0$ for other admissible functions $y(t)$. The OT-criterion is fulfilled if $\exists\ t_0$ such that $\Lambda(t) \geq 0\ \forall t \geq t_0$ where

$$\Lambda(t) \equiv \int_0^t [\Pi(x) - D(a)]\, ds - \int_0^t [\Pi(y) - D(A)]\, ds.$$

In this case, (a, x) represents the separatrix solution and (A, y) represents other admissible solutions. These must satisfy $K(A, y) < K_0 \leq S^*$ in order to yield a steady state. From Definition 2:

$$\int_0^t [\Pi(y) - D(A)]\, ds = K_0 \cdot t - \int_{a_0}^A \Pi'(y(s)) ds.$$

Inserted, this yields

$$\begin{aligned}\Lambda(t) &= (S^* - K_0) \cdot t + \int_{a_0}^{A} \Pi'(y(s))ds - \int_{a_0}^{a} \Pi'(x(s))ds \\ &\geq (S^* - K_0) \cdot t - \Psi(|A - a_0| + |a - a_0|) \\ &\geq (S^* - K_0) \cdot t - \Psi(|A^* - a_0| + |a^* - a_0|)\end{aligned}$$

where $\Psi = \max_{x \in X} \Pi'$. Hence the OT-criterion is fulfilled for all other admissible solutions, and t_0 is obtained from setting the last expression equal to zero. Thus, we derive a unique OT-optimal solution as $D'' - d''\Pi' > 0$ guarantees that the Hamiltonian is strictly concave in (a, x). This solution is the separatrix solution wherever it is positive.■

Proof of Proposition 1

The steady state is a saddle point is shown in the proof to Theorem 1. We observe that $\frac{\partial K}{\partial a} = \frac{\partial K}{\partial x} = 0$ occurs only at steady state because Π is strictly concave. Theorem 1 implies $S'(a^*) = \Pi'(d(a^*))d'(a^*) - D'(a^*) = 0$, which implies that $d'(a^*) > 0$. Therefore the feedback intersects with d to the left of its maximum. Differentiating $K(a, x) = S^*$ implicitly yields $-\Pi'' \cdot (d - x)\, x' = D' - \Pi' d' > 0$ to the right of $\max d$ where $d - x > 0$ and $d' < 0$. Monotonicity follows from the assumptions about Π.

Concavity of the separatrix solution can be shown by differentiating $K(a, x) = S^*$ implicitly twice. This yields

$$x'' \frac{\partial K}{\partial x} = - \left[\frac{\partial^2 K}{\partial a^2} + 2x' \frac{\partial^2 K}{\partial a \partial x} \right]$$

Quasiconcavity implies that the right-hand side is non-negative and $\frac{\partial K}{\partial x} < 0$ on L.■

Proof of Proposition 2

As $\tau(y) = \sigma(y) + \tau_f(y) = \sigma(x) > \sigma(y)$, and as $\sigma = U_x$ is strictly decreasing, the proposition is fulfilled for all levels of a associated with non-negative production.■

Proof of Proposition 3

Steady states are, by definition, intersections between the d-curve and the production feedback-paths $x(a)$ and $y(a)$. Since y, as a feedback, always is higher than x, the intersection between y and an increasing d-curve will always imply that y is higher than x in steady state.$\blacksquare$

Proof of Proposition 4

The proof of this proposition can be derived by looking at Figure 2. This case occurs if and only if $x(a)$ intersects with d at a higher value than $y(a)$. Quasiconcavity of d implies that this occurs if and only if $a^{\#} > a^{**}$.$\blacksquare$

Proof of Proposition 5

The steady state resulting from a σ-tax is denoted $(a^{\#}, y^{\#})$. As S is concave, $S(a^{**}) \geq S(a^{\#})$ is a necessary and sufficient condition for the counterintuitive case to occur (see Proposition 4). It is easily verified that $K(a,x) = S^*$ is equivalent to

$$S^* - S(a) = \int_x^{d(a)} \left[\Pi'(x) - \Pi'(s)\right] ds.$$

Therefore

$$S(a^{**}) - S(a^{\#}) = \int_{x(a^{\#})}^{y^{\#}} \left[\Pi'(x(a^{\#})) - \Pi'(s)\right] ds - \Delta S^*. \tag{7}$$

From this result, it follows that

$$\Delta S^* \geq \int_{x(a^{\#})}^{y^{\#}} \left[\Pi'(x(a^{\#})) - \Pi'(s)\right] ds$$

is a necessary and sufficient condition for the counterintuitive case not to occur. The lower limit of integration is less than the upper limit according to Proposition 3, and Π' is decreasing. Thus,

$$\Delta S^* \geq \int_{x(a^{\#})}^{y^{\#}} \left[\Pi'(x(a^{\#})) - \Pi'(y^{\#})\right] ds = \tau_f(y^{\#}) \cdot \left[y^{\#} - x(a^{\#})\right]$$

is sufficient for the counterintuitive case not to occur. This result is reinforced when $\Delta S^* \geq \tau_f(y^{\#}) \cdot y^{\#}$, and which is guaranteed whenever $\Delta S^* \geq k_F \geq \tau_f(y^{\#}) \cdot y^{\#}$.■

Proof of Proposition 6

Let δ denote the left-hand side of equation (7). It will be shown that this proposition implies $\delta > 0$. Recalling $\sigma(x) = \Pi'(x)$ equations (4) and (7) imply

$$\begin{aligned}
\Delta S^* + \delta &= \int_{x(a^{\#})}^{y^{\#}} \left[\Pi'(x(a^{\#})) - \Pi'(s)\right] ds \\
&= \int_{x(a^{\#})}^{y^{\#}} \left[\Pi'(x(a^{\#})) - \Pi'(y^{\#})\right] ds - \int_{x(a^{\#})}^{y^{\#}} \left[\Pi'(s) - \Pi'(y^{\#})\right] ds \\
&= \tau_f(y^{\#}) \left[y^{\#} - x(a^{\#})\right] - \int_{x(a^{\#})}^{y^{\#}} \left[\Pi'(s) - \Pi'(y^{\#})\right] ds.
\end{aligned}$$

The first integral, together with (6), yield

$$\frac{1}{2}\underline{M}\left[y^{\#} - x(a^{\#})\right]^2 \leq \Delta S^* + \delta \leq \frac{1}{2}\overline{M}\left[y^{\#} - x(a^{\#})\right]^2.$$

This, together with last inequality, yield

$$\frac{\tau_f(y^{\#})}{\overline{M}} \leq y^{\#} - x(a^{\#}) \leq \frac{\tau_f(y^{\#})}{\underline{M}}.$$

It follows immediately that

$$\Delta S^* + \delta \geq \frac{1}{2}\underline{M}\left[y^{\#} - x(a^{\#})\right]^2 \geq \frac{1}{2}\underline{M}\left[\frac{\tau_f(y^{\#})}{\overline{M}}\right]^2.$$

If ΔS^* is smaller than the right-hand side, then this is sufficient for $\delta > 0$ and hence for the counterintuitive case to occur. The right-hand side requires that $(a^{\#}, y^{\#})$ has been solved and can be ensured by securing that ΔS^* is less than the smallest value that the right-hand side may take in the interval of interest.■

REFERENCES

Andersen, M.S. (1994) *Governance by Green Taxes, Making Pollution Prevention Pay.* Manchester: Manchester University Press.

Baumol, W.J. and W.E. Oates (1988) *The Theory of Environmental Quality.* New York: Cambridge University Press.

Farzin, Y.H. (1996) Optimal pricing of environmental and natural resource use with stock externalities. *Journal of Public Economics* **62**, 31-57.

Farzin, Y.H., and O. Tahvonen (1996) Global carbon cycle and the optimal time path of a carbon tax. *Oxford Economic Papers* **48**, 515-536.

Grafton, R.Q., L.K. Sandal, and S.I. Steinshamn (2000) How to improve the management of renewable resources: The case of Canada's northern cod fishery. *American Journal of Agricultural Economics* **82**, 570-580.

Hoel, M. and S. Kverndokk (1996) Depletion of fossil fuels and the impacts of global warming. *Resource and Energy Economics* **18**, 115-136.

Joos, F., M. Bruno, R. Fink, U. Siegenthaler, T.F. Stocker, C. Le Quere, and J.L. Sarmiento (1996) An efficient and accurate representation of complex oceanic and biospheric models of anthropogenic carbon uptake. *Tellus* **48B**, 397-417.

Organisation for Economic Co-operation and Development (1992) *Economic Instruments for Environmental Management in Developing Countries.* Paris: OECD.

Sandal, K. and S.I. Steinshamn (1998) Dynamic corrective taxes with flow and stock externalities: A feedback approach. *Natural Resource Modeling* **11**, 217-239.

Seierstad, A. and K. Sydsæter (1987) *Optimal Control Theory with Economic*

Applications. Amsterdam: North-Holland.

Sinclair, P. J. N. (1992) High does nothing and rising is worse: carbon taxes should keep declining to cut harmful emissions. *Manchester School* **60**, 41-52.

Sinclair, P. J. N. (1994) On the optimum trend of fossil fuel taxation. *Oxford Economic Papers* **46**, 869-877.

Ulph, A. and Ulph, D. (1994) The optimal time path of a carbon tax. *Oxford Economic Papers* **46**, 857-868.

Wirl, F. (1994) Pigouvian taxation of energy for flow and stock externalities and strategic, noncompetitive energy pricing. *Journal of Environmental Economics and Management* **26**, 1-18.

Wirl, F. and E. Dockner (1995) Leviathan governments and carbon taxes: Costs and potential benefits. *European Economic Review* **39**, 1215-1236.

Endnotes:

1. For conciseness, functional dependence of the variables is suppressed in the text.
2. We define $\gamma \equiv 1$ which is equivalent to measuring a and x in the same units.
3. The proof in the Appendix has a unique solution in the sense of "Over Taking" (OT) Optimal (Seierstad and Sydsæther 1987, p. 234).
4. The proof of Theorem 1 and all propositions are given in the Appendix.
5. An equivalent expression for the feedback solution has been applied by Grafton, Sandal and Steinshamn (2000).
6. This expression for the optimal tax as a feedback control law can also be found in Sandal and Steinshamn (1998).

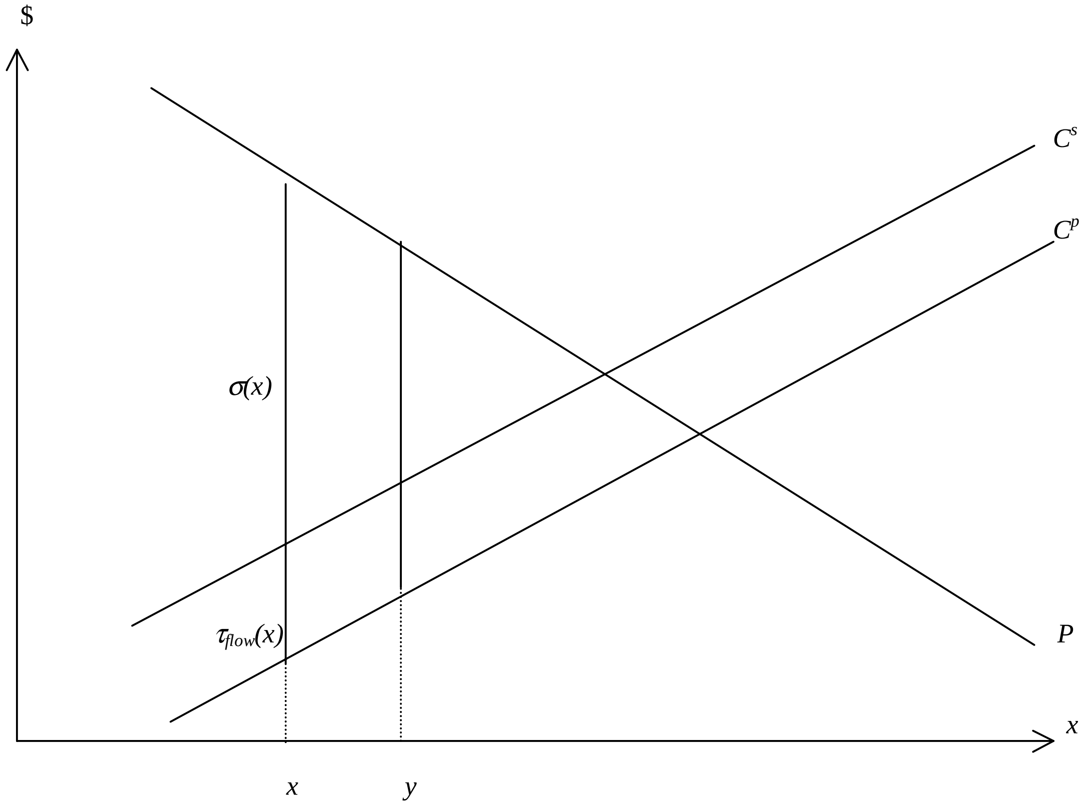

Figure 1. The tax yielding the market equilibrium y is equal to $\sigma(x)$, the stock part of the tax yielding market equilibrium x.

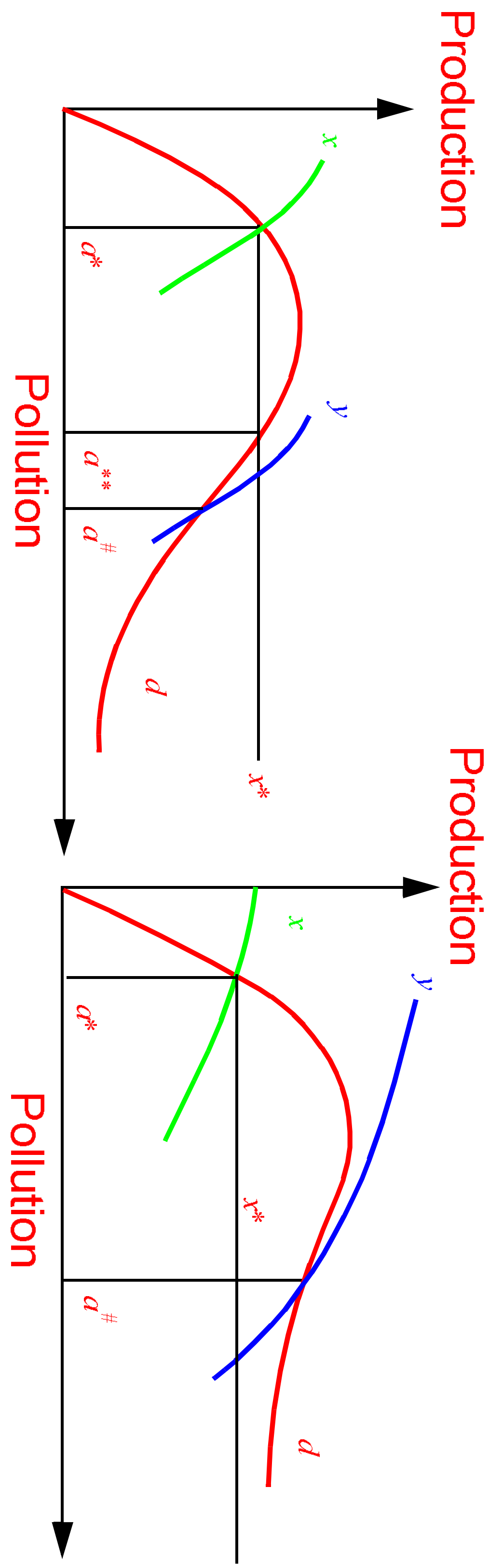

Figure 2. Quasiconcave decay function and correct steady state (a^*, x^*) and the flow neglected steady state $(a^{\#}, y^{\#})$ for a case of "more is less" ($a^{\#} > a^{**}$) and a case excluding the counterintuitive result due to $a^{**} \to \infty$.