Pollution Decay, Consumer Awareness and Optimal Carbon Taxes

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Abstract

The effects of non-linear decay and consumer preferences are analyzed in a setting where optimal extraction of non-renewable resources is combined with stock externalities. The control is exercised via a corrective tax and the time horizon is divided into two periods: an initial phase with extraction and a terminal phase without extraction. The time horizon with extraction is determined endogenously. The model does not assume separability of the objective function. Sensitivity results indicate large differences in the optimal extraction period, the total level of extraction and cumulative emissions depending on the form of the decay function and the presence of consumers' awareness for the environment.

Keywords: Global warming, fossil fuel extraction, dynamic optimisation

JEL: C61, Q25, Q28, Q30

INTRODUCTION

Given the potential e¤ects of climate change, a great deal of attention has been focused on the derivation of optimal carbon taxes (Nordhaus 1982, 1991a, 1991b, Peck and Teisberg 1992, Sinclair 1994, Wirl 1994a, 1994b, 1995, Rubio and Escriche 2001, Pizer, 2002, van der Zwaan et al. 2002) to correct for the stock externality associated with greenhouse gas (GHG) emissions. Some papers have explicitly linked the corrective taxes to the optimal exploitation of non-renewable resources (Sinclair 1992, Falk and Mendelsohn 1993, Withagen 1994, Ulph and Ulph 1994, Farzin 1996, Farzin and Tahvonen 1996, Hoel and Kverndokk 1996, Tahvonen 1997), but no papers have evaluated the e¤ect of non-linear decay of GHG emissions on the optimal tax.[1] This de…ciency in the existing literature is important as the uptake of atmospheric carbon is non-linear in terms of cumulative carbon emissions (Joos et al. 1996).

To address the dynamic tax problem and assess the a¤ects on non-linear decay on the time path of corrective taxes and cumulative emissions, an optimal feedback control description is developed and some of its important features are derived. Unlike existing approaches that mitigate climate change, our approach makes it possible to determine an optimal corrective tax as a function of the level of cumulative emissions. The approach developed in this paper can be applied to maximize consumer surplus, producer surplus, or both. The application of the method to climate change is a genuine example of adaptive regulation. In each period, when new information on cumulative emissions is available (see also Sandal and Steinshamn, 1998), the corrective tax is adjusted. This approach provides insights for setting of taxes to address the potential problems of climate change. Sensitivity analysis suggests that the results are economically signi…cant for climate change as di¤erent pollution decay functions

[1] Farzin and Tahvonen (1996), however, use a more sophisticated linear formulation than the others.

yield quite different time horizons, total level of extraction and levels of pollution.

As the objective function in the model rests on the basic supply and demand functions, and as these may be general functions of the state variable, the model is particularly suited to investigate the effects of consumers' preferences on the optimal tax. By consumers' preferences is, e.g., meant that the demand for a polluting product may decrease with the aggregated level of pollution.

The article is structured as follows. The feedback model and some main properties of the optimal policy rule are derived in section two. To show the potential importance for climate change policy, the sensitivity of the results is assessed in terms of both the decay function for stock pollution and the dependence of consumer demand on cumulative emissions.

The paper concludes with an assessment of the approach and its insights in terms of mitigating the consequences of climate change and other environmental problems associated with a stock pollutant.

THE MODEL

The objective is to maximize accumulated welfare, defined by the function

$$W = \int_0^T e^{-\delta t} \{U(a(t), x(t)) - D(a(t))\}\, dt + \int_T^\infty e^{-\delta t} \left\{ \tilde{U}(t) - D(a(t)) \right\} dt, \qquad (1)$$

with respect to x. The variable U is the social benefit derived from production and consumption of the good, x. Here x represents extraction of fossil fuels, t is time and δ is the discount rate. The social benefit can also be affected by the aggregate level of pollution, a. In addition, we have the direct damage of a which is the stock externality D. Further, $\tilde{U}$ is an alternative technology that can replace fossil fuels, e.g. fuel cell technology for automobiles. There is positive extraction of the resource, $x > 0$, up to time T, and zero extraction after T. Unnecessary technicalities are avoided by assuming that we can not go back to the old technology after having switched to the new one. The switching time, T, is to be determined endogenously. To determine the optimal switching time, and investigate how it is affected by the decay, is an important aim in itself. In addition we want to determine a rather simple way to calculate the optimal corrective tax as a feedback control law. We will neglect the possibility of having a transition period where both types of technology are in place simultaneously. From a practical point of view this amounts to assuming that the transition period is short compared to the initial phase.

We strongly emphasize that the scope of this paper is to study how the time horizon, T, and the optimal feedback policy depend on the assumptions about non-separability in the objective function and about the decay function.

The functions U and D may, in principle, be fairly general in a. Social utility U may, e.g., represent the sum of consumers' and producers' surplus. The inclusion of a in U then describes how the level of GHG affects the demand and cost structure. For example, more pollution may increase consumers' concern for the environment and hence cause a downward shift in the demand curve for the polluting product. This case will be investigated later.

Denoting the remaining stock of fossil fuels s, equation (1) must be maximized subject to the constraints[2]

$$\begin{aligned} \dot{s} &= -x, \\ s(t) &= s_0 - \int_0^t x(u)du \geq 0, \quad s_0 = s(0) > 0, \\ x(t) &\geq 0, \quad \lim_{t \to \infty} a(t) = 0, \end{aligned} \tag{2}$$

and

$$\dot{a} = x - f(a) \tag{3}$$

where f is the decay function and $\dot{a}$ and x are measured in the same units. The condition on the aggregated level of pollution, a, in (2) ensures that we only consider

[2] Dots denote time derivatives.

policies that restore a clean environment in the long run and exclude policies that produce irreversibility.

Definition 1 (The Usual Assumptions) The following assumptions are made about the input functions if nothing else is specifically stated:

1. The damage function $D(a)$ is twice continuously differentiable, non-decreasing and convex on a fixed interval $A = (0, \bar{a})$ (sufficiently large) and $D(0) = 0$.

2. The decay function $f(a)$ is positive and twice differentiable on A, and $f(0) = 0$. Moreover, $\lim_{a \to 0} \int_a^{\alpha} \frac{ds}{f(s)} \to \infty$ for any $0 < \alpha \in A$. Possible convex parts of f are restricted by $U_x f'' \le D''$ on $A \times B$ where $B = (0, \bar{x})$. The marginal decay rate is limited by $\frac{f'(a)+\delta}{f(a)} \le \frac{D'(a)}{D(a)}$ for some $\delta > 0$.

3. The alternative utility function, $\theta(t) > 0$, is continuously differentiable on $(0, \infty)$ and non-decreasing.

4. The current utility function $U(a, x)$ is concave and twice continuously differentiable on $A \times B$. Further[3], $U_a \le 0$, $U_x > 0$, $U_{xx} < 0$ on $A \times B$.

The second item puts some constraints on possible convex parts of the decay function. It is, however, sufficient that it holds on the optimal path. Further, $f(0) = 0$ means that $a = 0$ is defined as the pre-industrial level of a which is a natural steady state. The last part of item 2 limits the relative change in natural cleaning versus the relative change in disutility associated with the stock of pollution.

A rather general decay function is useful as the decay of CO_2 through photosynthesis may be a very complex process (Joos et al., 1996). Global warming may affect the growth of forests and phytoplankton, which again affects the CO_2 level. Increased concentrations of GHG emissions may initially increase the assimilative capacity of

[3] Subscript denotes partial derivative

the environment to uptake carbon due to carbon fertilization. Further increases in GHG emissions, however, that lead to even higher GHG concentrations and higher surface temperatures, may eventually lead to plant die offs that could ultimately reduce carbon uptake. The fact that there is a saturation level for how much carbon the oceans can take, also calls for a non-monotone function. Obviously the decay of carbon is a complex process that can not be well represented by linear, or even monotone, functions.

The initial stock of fossil fuel, s_0, is given. There exists an exogenously given stock level below which the costs of extracting are so high that no extraction takes place. By rescaling units this level is defined as zero. Hence s_0 represents extractable reserves. The level $a = 0$ is defined as the natural, pre-industrial level of CO_2, which is a natural steady state and does not harm the global climate. Thus $f(0) = 0$ and $D(0) = 0$, and after extraction has terminated a will gradually approach zero.

By letting the benefit function represent the sum of producers' and consumers' surplus it can be written

$$U(a,x) = \int_0^x [P(a,y) - C(a,y)]\, dy$$

where P is the inverse demand function and C is the marginal cost of extraction, which is the market supply.

A more general formulation of the model would be to include s explicitly in U, but this would complicate the calculations considerably compared to the gain and blur our focus. Therefore the simplifying assumption is made that U is independent of s for $s \geq 0$, and that extraction costs increase to infinity ($U = -\infty$) for $s < 0$. Remember that s has been rescaled accordingly.

At any point in time market clearing is assumed, implying that the equilibrium level of x is given by $P = C$ without any policy measures. In other words, C is the market supply of x in a competitive economy. A competitive supply of fossil fuel is assumed throughout the paper.

In the literature it is quite common to choose objective functions that are quadratic both in the control variable and the state variable and constraints that are linear in both (so-called linear-quadratic models) for mathematical convenience. In the present model both the objective function and the dynamic constraint are fairly general in the state variable, a. In other words, it is assumed that demand can be affected in a rather general way by the level of CO_2, due to changes in environmental concern among consumers among other reasons.

The externality indicates that there is need for some policy instrument in the form of quotas or corrective taxes. In this paper we use an ad valorem tax defined by

$$\theta(a, x) = \frac{P(a, x) - C(a, x)}{C(a, x)}. \tag{4}$$

Here C is the producer price and P is the consumer price. Note that maximizing the sum of the consumers' surplus, the producers' surplus and the government's surplus, which is the tax revenue, is equivalent to maximizing $U - D$ (see Appendix 1).

It is important to keep in mind that the instrument is in effect only during the initial period with extraction. As the corrective tax is on extraction, x, it is not possible to levy any tax when $t > T$ even though the harmful effects, $D(a)$, persist into this period. An optimal tax in the initial period, therefore, also must take into account the stock externality in the terminal period. It does not matter whether θ or x is chosen as control variable in the mathematical model. The approach taken here is that the optimal extraction level, x, is found and substituted into (4) in order to find the optimal tax.

The time at which it is optimal to stop extraction, is determined by the value of the alternative technology, $\vartheta(t)$. The time-dependence in ϑ represents technological development, and it is therefore assumed that ϑ is non-decreasing.

Let H = H(t, a, s, x, m, n) denote the current value Hamiltonian and let m and

n denote the current value costate variables associated with pollution, a, and the remaining extractable resource, s, respectively. The necessary conditions are summarized in Table 1. A scrap value formulation of the problem is developed in appendix 2. The existence of an optimal policy using the classical Filippov-Cesari existence proof and the scrap value formulation is given in appendix 3. In this appendix it is also shown that an Arrow-type sufficient condition is satisfied.

Table 1

Description	Initial period	Terminal period
Time	$0 \leq t < T$	$t > T$
Production	$x > 0,\ x(T) = x_T \geq 0$	$x = 0$
Social welfare	$U(a,x) - D(a)$	$\tilde{U}(t) - D(a)$
Dynamic constraint 1	$\dot{a} = x - f(a)$	$\dot{a} = -f(a)$
Dynamic constraint 2	$\dot{s} = -x$	$\dot{s} = 0$
Hamiltonian	$\mathsf{H} = U(a,x) - D(a) +$	$\mathsf{H} = \tilde{U}(t) - D(a) +$
	$+\ m \cdot [x - f(a)] - n \cdot x$	$-\ m \cdot f(a)$
$x = \arg\max \mathsf{H},\ x \geq 0$	$m - n = -U_x,\ x > 0$	$m - n \leq 0,\ x = 0$
Costate equation 1	$\dot{m} = [\delta + f'(a)] \cdot m +$	$\dot{m} = [\delta + f'(a)] \cdot m +$
	$+\ D'(a) - U_a$	$+\ D'(a)$
Costate equation 2	$\dot{n} = \delta n$	$n \cdot s = 0,\ n \geq 0$

The interpretations of the costate variables are that m is the shadow cost of pollution (CO_2) whereas n is the scarcity rent of the resource.

In addition to this there is the requirement that the Hamiltonian and the state and costate variables are continuous at all times including T. The state variables in this maximization problem are a and s. As the stock of fossil fuel, s_0, is limited, the system will not settle on a non-trivial steady state.

Matching conditions

This section concentrates on the matching that takes place at the switching time T. By defining zero as the pre-industrial level of CO_2, which is the natural steady state, we have $f(0) = 0$. We introduce two important quantities Ψ and Ω that are key expressions in the matching conditions at the switching time T by

$$\Psi(a;\alpha) \equiv \int_a^\alpha \frac{ds}{f(s)} \qquad \text{and} \qquad \Omega(a,m;\alpha) = m \cdot f(a) + e^{\delta\Psi(a;\alpha)} \int_0^a e^{-\delta\Psi(s;\alpha)} D'(s)ds. \tag{5}$$

We suppress the dependence on the constant α and use for ease of notation $\Psi = \Psi(a)$ and $\Omega = \Omega(a,m)$.

The following proposition characterizes the shadow price on pollution in the second phase

Proposition 1 The quantities $t - \Psi$, $e^{-\delta t}\Omega$ and $e^{-\delta\Psi}\Omega$ are constants along the optimal path for $t > T$.

Totally differentiating the first two expressions with respect to time yields the result directly. The constancy of the third follows from the other two. The first follows from noticing that $\dot{\Psi} = 1$. The second results from:

$$\begin{aligned}
\dot{\Omega} &= \dot{m}f + m\dot{f} + \delta\dot{\Psi}(\Omega - mf) + e^{\delta\Psi} D' \cdot e^{-\delta\Psi} \cdot \dot{a} \\
&= [(\delta + f')m + D']f + mf' \cdot \dot{a} + \delta(\Omega - mf) + D' \cdot \dot{a} \\
&= \delta\Omega \Rightarrow \frac{d}{dt}\left[e^{-\delta t}\Omega\right] = 0.
\end{aligned}$$

Let us fix $\alpha = a(T) = a_T$ in the definition of Ψ. We then get the following corollary that will be useful later:

Corollary 1 Assuming that $mf \to 0$ when $a \to 0$ in the second phase ($t > T$), then the following relationships must hold:

$$t = T + \Psi(a) \text{ and } \Omega = 0 \text{ or } m \cdot f(a) = -e^{\delta\Psi(a)} \int_0^a e^{-\delta\Psi(s)} D'(s)ds. \tag{6}$$

Proof: The first is an immediate consequence of the definition of a_T. Letting $a \to 0$ in the expression for $-$ yields:

$$
\begin{aligned}
- &= \lim_{a \to 0} \left(e^{\delta \Psi(a)} \int_0^a e^{-\delta \Psi(s)} D'(s) ds \right) = \lim_{a \to 0} \frac{\int_0^a e^{-\delta \Psi(s)} D'(s) ds}{e^{-\delta \Psi(a)}} \\
&= \lim_{a \to 0} \frac{e^{-\delta \Psi(a)} D'(a)}{e^{-\delta \Psi(a)} (-\delta \Psi')} = \delta^{-1} \lim_{a \to 0} f(a) D'(a) = 0.
\end{aligned}
$$

L'Hospital's rule has been applied together with the fact that $\lim_{a \to 0} \Psi(a) \to \infty$ from the Usual Assumptions (or $t = T + \Psi \to \infty \Rightarrow a \to 0$).

Notice that in the limit of vanishing discount rate the above result implies $mf + D = 0$. This result must be interpreted carefully. In the case of zero discounting the optimality notion must be modified. A frequently used alternative is the notion of Catching-Up (CU) optimality (see e.g. page 232 in Seierstad and Sydsæther, 1987).

The following lemma can now be derived:

Lemma 1 The shadow price m is negative for all times $0 \le t < \infty$.

From the usual conditions and eq. (6) it is evident that $m < 0$ when $T \le t < \infty$. Let us therefore assume that there exists a last point in time $t_0 < T$ such that $m = 0$. The evolution equation for the shadow price at $t = t_0$ implies $\dot{m} = (\delta + f')m + D' - U_a = D' - U_a > 0$. This gives that $m > 0$ immediately after, contradicting the fact that $m < 0$ to the right of $t = t_0$. By continuity of the shadow price we have established Lemma 1.

Even though the result stated in lemma 1 is to be expected from an economic point of view, it serves the purpose of being a consistency check of our modelling approach. At the core of our approach lies the problem of determining when and how the switch will take place. The key result is given in the following proposition.

Proposition 2 (Matching Conditions) The pollution level a_T, the scarcity rent n_T and the consumption x_T immediately prior to the switch at time $t = T$, is determined by the following set of equations

$$U(a_T, x_T) = x_T \cdot U_x(a_T, x_T) + \tilde{U}(T) \tag{7}$$

$$n_T = m_T + U_x(a_T, x_T) \tag{8}$$

$$m_T \cdot f(a_T) = -\int_0^{a_T} e^{-\delta \tau(s;a_T)} D'(s) ds \tag{9}$$

where n_T and T satisfy

$$\begin{cases} n_T = 0 \quad \text{and} \quad s_0 > \int_0^T x(\tau) d\tau = \int_{a_0}^{a_T} \frac{X(s)}{X(s) - f(s)} ds \quad \text{or} \\ n_T > 0 \quad \text{and} \quad s_0 = \int_{a_0}^{a_T} \frac{X(s)}{X(s) - f(s)} ds \end{cases} \tag{10}$$

$$T = \int_{a_0}^{a_T} \frac{1}{X(s) - f(s)} ds \tag{11}$$

The function $x = X(a)$ represents the optimal feedback solution for the consumption which, at this point, is assumed to be known. In the next section we give the appropriate boundary value problem for the optimal feedback control law. The five relations in Proposition 2 determine in principle T, a_T, x_T, m_T and n_T when $X(a)$ is known. Because $X(a)$ will be known as a functional of these parameters it will lead to a non-trivial boundary value problem (BVP). Calculating actual values for these parameters is therefore a formidable task. This explains why much of the work in this field assume some of these parameters exogenously given in such away that the problem is reduced to a straightforward initial value problem. In the numeric section we will calculate all parameters for some particular cases.

Proposition 2 is derived as follows. Equation (7) follows from continuity of the Hamiltonian (see Table 1). The interpretation of this condition is that the difference in utility between the new and the old technology ($U - \tilde{U}$) shall equal the consumption at T valued by the marginal utility. At the switching time U is still greater than $\tilde{U}$, but the difference compensates for the future damage of the last produced units. That is, just prior to switching an additional polluting unit should account for the future cost of the associated pollution. Thus it is to be expected that $U - \tilde{U} > 0$ at the switching time $t = T$.

Equation (8) follows from continuity of the costates and $U_x = n - m$. which holds throughout the first phase as it follows from the condition that the production should maximize the Hamiltonian. The interpretation is simply that the marginal benefits and costs must balance each other.

Equation (9) follows directly from Equation(6). The relations (10) follow from the transversality condition on n_T and the boundary conditions on s. Equation (11) follows from $\dot{a} = X(a) - f(a)$.

The proposition also follows from standard transversality conditions for an associated finite horizon problem with a salvage value. This is shown in appendix 2.

Concavity of the maximized Hamiltonian

The Hamiltonian in the first phase, $0 \leq t \leq T$, is also the Hamiltonian for the alternative formulation with scrap value. Both formulations yield the same necessary conditions. Assume at this point that the solution to the necessary conditions has been found. It will be demonstrated that such a solution is optimal as it satisfies an Arrow-type sufficiency theorem. In order to do so, we must show that the maximized Hamiltonian is concave in the state space under consideration. Other details are given in appendix 3. The current value Hamiltonian is given in Table 1. There is an interior unique solution to $H_x = 0$ as $U_x > 0$:

$$H_x = U_x(a, x) + m - n = 0 \Rightarrow x = X(a, m - n) \text{ and } X_a = -\frac{U_{ax}}{U_{xx}}. \tag{12}$$

The maximized Hamiltonian is given by

$$H^0(a, m, n) = H(a, X(a, m - n), m, n).$$

Differentiating with respect to a and using (12):

$$\begin{aligned} H'_a &= H_a(a,X,m,n) + H_x(a,X,m,n)X_a(a,m-n) = H_a(a,X,m,n) \\ H'_{aa} &= H_{aa}(a,X,m,n) + H_{ax}(a,X,m,n)X_a \\ &= U_{aa}(a,X) - D''(a) - mf''(a) + U_{ax}(a,X)X_a, \end{aligned}$$

or in more suitable form:

$$H'_{aa} = \frac{U_{aa}U_{xx} - U_{ax}^2}{U_{xx}} + [-D'' - mf''].$$

It is now straightforward to prove that

$$H'_{aa} \leq \frac{U_{aa}U_{xx} - U_{ax}^2}{U_{xx}} \tag{13}$$

by using the fact that $m < 0$. This is seen by the following reasoning:

1) In any region where $f'' < 0$ it is trivially true as both terms in the square brackets are negative since $D'' > 0$ by the Usual Assumptions.

2) In the rest of the state space therefore $0 \leq -mf'' = (U_x - n)f'' \leq U_x f'' \leq D''$ or $-mf'' - D'' \leq 0$. The optimal condition $m - n = -U_x$ from (12) has been used in the equality, and the non-negativity of the scarcity rent has been used in the next inequality. The last inequality stems from Usual Assumption 2 that restricts the strength of the convexity of f.

The right-hand side of (13) is non-positive due to the convexity of U.

The optimal path

In order to study the optimal path of the control variable x, and the corresponding tax, it is useful to derive the optimal control as a function of the state variable; that is, as a feedback control law. A feedback control law represents an adaptive regulation as the optimal tax is directly affected by changes in the environment. The tax level is determined as soon as the level of pollution is estimated. Time, as such, is of no relevance.

The differential equation governing the optimal feedback rule for the control variable, x, is readily derived. Both from a mathematical and economic perspective it is useful to define the following scalar functions:

$$\begin{aligned} S(a) &\equiv U(a, f(a)) - D(a), \\ \mathcal{L}(a,x) &\equiv U(a,x) - U_x(a,x) \cdot (x - f(a)) - U(a, f(a)). \\ \mathcal{P}(a,x) &\equiv \mathcal{L}(a,x) + S(a) \end{aligned} \tag{14}$$

The economic interpretation of S is that it represents the level of social benefit that can be obtained at any time by fixing the level of CO_2 by producing $x = f$ such that $\dot{a} = 0$. $\mathcal{L}(a,x)$ holds a potential utility gain much the same way that a moving physical object holds a potential of doing work (its energy content) strictly associated with its movement. Its value is associated with the change in the pollution-level and, as such, can be viewed as a dynamic potential utility gain for changing the pollution state in the total asset.

Lemma 2 $\mathcal{L}(a,x)$ is positive semi-definite and it is zero on the curve $x = f(a)$.

The semi-definite property is a direct result of The Usual Assumptions, in particular the regularity and the concavity of U with respect to x.

Lemma 3 $\mathcal{P}$ is equal in value (but not as a function) with the quantity $\mathcal{H} + nf$, and it satisfies the relation $\dot{\mathcal{P}} = [nf' - \delta U_x]\,\dot{a}$ on any part of an optimal path in the first phase.

The first result follows from noticing that $\mathcal{H} + nf = U(a,x) - D(a) + (m - n) \cdot [x - f(a)]$. Inserting the first order condition for $m - n$ from Table 1 results in $\mathcal{H} + nf = U(a,x) - D(a) - U_x \cdot [x - f(a)] = \mathcal{L} + S$. The last part follows from differentiating the above result and applying the first order conditions:

$$
\begin{aligned}
\dot{P} &= \dot{H} + \dot{n} f + n\dot{f} = H_a \dot{a} + H_x \dot{x} + H_m \dot{m} + H_n \dot{n} + \dot{n} f + n\dot{f} \\
&= H_a H_m + H_m (\delta m - H_a) + H_n \delta n + \delta n f + n f^{0} \dot{a} \\
&= H_m \delta m - (x - f)\delta n + n f^{0} \dot{a} = \delta (m - n)\, \dot{a} + n f^{0} \dot{a} = [-\delta U_x + n f^{0}]\, \dot{a}.
\end{aligned}
$$

The quantity P can be interpreted as the total rent less the resource rent. This lemma gives us an important tool for producing the optimal feedback policy.

The problem initiated in this paper can now be stated as the following boundary value problem:

Proposition 3 (Boundary value problem) The first order condition for the control problem defined through equations (1, 2 & 3) implies the boundary value problem given by

$$
\begin{bmatrix} 1 & 0 & 0 \\ 0 & -U_{xx} & 0 \\ 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} \dot{a} \\ \dot{x} \\ \dot{n} \end{bmatrix}
=
\begin{bmatrix} x - f \\ -P_a - \delta U_x + n f^{0} \\ \delta n \end{bmatrix}, \tag{15}
$$

and the boundary conditions are given by the relations stated in the Matching Conditions together with the initial condition on a.

We only need to show the differential equation for the production, x. The other two are already given in table . Differentiating the equation stating that we have an inner optimum for $t < T$, and using the other first order conditions (see table), implies

$$
\begin{aligned}
\frac{d}{dt} U_x &= \frac{d}{dt}(n - m) = \dot{n} - \dot{m} = \delta n - [(\delta + f^{0}) m - U_a + D^{0}] = \delta (n - m) - f^{0} m + \\
&+ U_a - D^{0} = (\delta + f^{0})(n - m) - n f^{0} - U_a + D^{0} = (\delta + f^{0}) U_x + U_a - D^{0} - n f^{0} \\
&= \delta U_x - n f^{0} - D^{0} + U_a + f^{0} U_x = \delta U_x - n f^{0} + [P_a + (x - f) U_{xa}].
\end{aligned}
$$

Finally inserting $\frac{d}{dt} U_x = U_{xx}\dot{x} + U_{xa}\dot{a} = U_{xx}\dot{x} + U_{xa}(x - f)$ completes the derivation.

Notice that Lemma 3 yields

$$\dot{\mathcal{P}} = \mathcal{P}_a \dot{a} + \mathcal{P}_x \dot{x} = \mathcal{P}_a \dot{a} + \mathcal{L}_x \dot{x} = [-\delta U_x + nf']\dot{a}\ , \quad \mathcal{L}_x \dot{x} = [-\mathcal{P}_a - \delta U_x + nf']\dot{a}$$

Further notice that $\mathcal{L}_x = -(x - f)U_{xx} = -U_{xx}\dot{a}$. By dividing by $\dot{a}$, we get the result provided $\dot{a} \neq 0$. Alternatively, we can view the last derivation as a direct way to obtain the optimal feedback BVP problem as

$$-(x - f)U_{xx}\frac{dx}{da} = -\mathcal{P}_a - \delta U_x + nf' \quad \text{and} \quad (x - f)\frac{dn}{da} = \delta n.$$

together with the Matching Conditions.

The next proposition supply bounds on the optimal policy or the rate of change in pollution. This proposition covers the typical case where the level of pollution is non-decreasing in the period with the old polluting technology.

Proposition 4 (Lower bound on $\mathcal{P}$) The optimal feedback policy $x = X(a)$ satisfies in the limit $\delta \to 0$ the following relation:

$$M(a)[x - f(a)]^2 + S(a) \geq \mathcal{P}(a, x) = \mathcal{L}(a, x) + S(a) = \tilde{D}(T) + nf(a). \tag{16}$$

In the typical cases where $X(a) \geq f(a)$ and $\delta > 0$ for $t \leq T$ the optimal policy implies a lower bound on $\dot{a} = x - f(a)$ given by

$$M(a) \cdot \dot{a}^2 + S(a) \geq \mathcal{P}(a, x) \geq \tilde{D}(T) - \bar{D}(a_T) + nf(a) + \delta\eta(a, a_T), \tag{17}$$

where $M(a) = \max_{x=[0,\overline{x}]} |U_{xx}(a, x)|$, $\bar{D}(z) = D(z) - \int_0^z e^{-\delta\tau(a;z)} D'(a)da \geq 0$ and $\eta(a, a_T) = \int_a^{a_T} U_x(z, \overline{x})dz \geq 0$.

The properties of U imply

$$\begin{aligned} \mathcal{L}(a, x) &= U(x, a) - U(f, a) - U_x(x, a) \cdot (x - f) \leq (x - f) \cdot U_x(f, a) - (x - f) \cdot U_x(x, a) \\ &= (x - f) \cdot [U_x(f, a) - U_x(x, a)] = (x - f) \cdot U_{xx}(\theta, a) \cdot (f - x) \leq M(a) \cdot (x - f)^2. \end{aligned}$$

A utility function of the form $U(a,x) = \beta(a)x - \gamma(a)x^2$ results in the equality $P(a,x) = M(a)[x - f(a)]^2 + S(a)$.

Lemma 3 yields

$$
\begin{aligned}
P(a,x) &= P(a_T,x_T) + \int_T^t n\dot{f}\,dt - \delta \int_T^t U_x \dot{a}\,dt \\
&= P(a_T,x_T) + [nf]_T^t - \delta \left[\int_T^t nf\,dt + \int_T^t U_x \dot{a}\,dt \right] \\
&= \bar{U}(T) - \bar{D}(a_T) + nf(a) - \delta \left[\int_T^t nf\,dt + \int_T^t U_x \dot{a}\,dt \right] \\
&= \bar{U}(T) - \bar{D}(a_T) + nf(a) + \delta \left[\int_t^T nf\,dt + \int_t^T U_x \dot{a}\,dt \right] \\
&\approx \bar{U}(T) - \bar{D}(a_T) + nf(a) + \delta \int_t^T U_x \dot{a}\,dt\,.
\end{aligned}
$$

We used the matching condition $U(a_T,x_T) - x_T U_x(a_T,x_T) = \bar{U}(T)$ in the second line above. The limit $\delta \to 0$ imposes $\bar{D}(z) \to 0$ and the second line above implies relation (16). The third line yields $P(a,x) \approx \bar{U}(T) - \bar{D}(a_T) + nf(a) + \delta \int_a^{a_T} U_x(a,X(a))da \approx \bar{U}(T) - \bar{D}(a_T) + nf(a) + \delta\eta(a,a_T)$ when $\dot{a} \approx 0$.

The proposition assert that $X(a) = f(a) \pm \sqrt{\left(\bar{U}(T) + nf(a) - S(a)\right)/M(a)}$ is a very good approximation for an optimal policy if the discount rate is close to zero. It represents an explicit feedback expression in the case of a constant alternative utility with some of the non-renewable resource left unextracted ($n = 0$).

NUMERICAL EXAMPLES.

The boundary value problem determining the feedback rule described in the previous sections can be used to obtain the optimal extraction path both as a function of the aggregate level of CO_2 and as a pure function of time. The former seems, however, to be more useful from a regulators point of view as the tax is adaptive to the current CO_2-level, and no forecasting is required.

In this section the model is illustrated by a couple of numerical examples using quasi-realistic data. A quadratic damage function (stock externality) is assumed, and the effects of two different decay functions are investigated.

Numerical specification and results

The aggregate level as well as emissions of CO_2 are measured in giga-tons CO_2 (Gt-CO_2). One Gt-CO_2 corresponds to 7.81 parts per million (p.p.m., which is another common measure of carbon). The meteorological data are given in Table 2. The pre-industrial level, which is a natural steady state without extraction, is estimated to 2187 and the current level is 2812. Rescaling such that the pre-industrial level, by definition, is zero yields the present level $a(0) = a_0 = 625$.

Table 2. Meteorological data (Gt-CO_2)

Parameter	Value	Parameter	Value
$f(a_0)$	11.7	a_0	625
x_0	21.9	s_0	7000

The data in Table 2 have been provided by the Nansen Environmental and Remote Sensing Centre in Bergen, Norway. Also the stock of fossil fuels, s, are measured in the same units. The economic data given in Table 3 are based on short-term supply- and demand elasticities for fossil fuel equal to 2 and -0.15, respectively, when $a = a_0$ (Burniaux et al., 1992). As the model is adaptive, it is the short-term elasticities that are relevant. The inverse demand for fossil fuel is assumed to be linear:

$$P(x) = p_0 - p_1 \cdot x$$

where p_0 and p_1 are parameters. The marginal cost function is also assumed to be linear:

$$C(x) = c_0 + c_1 \cdot x,$$

implying that U is quadratic. All parameters in the demand and supply function can be made dependent on the state variable, a, if that is relevant. The next subsection looks at an example where the parameter p_0 has been made a-dependent in order to represent the consumers' concern for the environment.

The values of the economic parameters in this section are given in Table 3.

Table 3. Economic data (normalized)

Parameter	Value	Parameter	Value
p_0	15.3	p_1	0.64
c_0	1	c_1	0.05
$D(a)$	$9 \cdot 10^{-5} \cdot a^2$	$\tilde{\theta}(t)$	$101 + 0.01 \cdot t$

It is seen from Table 3 that the marginal cost of extraction at zero production has been normalized to one. Further, a price 15.3 times higher than this is assumed to choke all demand, and the market equilibrium level with these parameter values is equal to current emissions.

The stock externality is quadratic in a. The size of the externality is uncertain, but most studies indicate that it will be around two percent of the world's gross domestic product if current emissions continue (Schelling, 1997).

Two decay-functions have been used, namely the linear one $f(a) = \frac{11.7}{625}a$, and the nonlinear

$$f(a) = \frac{\left(5 + 958 \cdot e^{-1 \cdot 10^{-6} a^2}\right) a}{a + 34300}.$$

They yield quite different results as seen from Table 4.

Table 4. Key results

	Linear decay	Non-linear decay
$x(0)$	16.3	14.0
$x(T)$	19.1	14.6
$a(T)$	824	683
T	59.5	23.8
$\theta(0)$	169%	273%
$\theta(T)$	72%	245%

The panel in Figure 1 shows pollution, optimal production and the corresponding tax as functions of time. In addition, the corrective tax is also illustrated as a function of the pollution level, that is the feedback relationship. Notice that with linear decay the optimal tax is first slightly increasing and then decreasing.

The panel in Figure 2 shows the same relationships but with non-linear decay. In this case the optimal tax is strictly decreasing. The values of the key variables with linear and non-linear decay are listed in Table 4. It is seen from the table that with non-linear decay optimal production is lower and, hence, the corresponding tax is higher. The reason for this is that linear decay represents a too optimistic view on how much pollution nature itself can handle. The cleaning capability increases with pollution for all pollution levels. With a more realistic decay function it is realized that a more active policy is needed. Also the period with production is lower with non-linear decay. Total extraction therefore is smaller with non-linear decay. We note that the more conservative policy, that is the one with non-linear decay, is more stable with respect to the policy instrument. This is an advantage from the policy-makers point of view.

These examples are not meant to give a precise description of reality but to emphasize the importance of studying non-linear decay and that analyses based solely on linear decay functions are not sufficient. It emphasizes the need for both more empirical and theoretical studies on the decay function.

Consumer preferences

One of the advantages with the model presented in this paper is that the objective function is non-separable in a and x. This makes it possible to analyze the sensitivity of the results to, e.g., consumers' concern for the environment. Let us assume that the behavior of the consumers is such that the more pollution there is, the more the demand curve for the polluting product will shift downwards. This may be achieved by letting the parameter p_0 be a function of a instead of a constant. In this subsection it is shown that the optimal time period with extraction is quite sensitive to this assumption, and therefore this is an important property of the model.

The specification applied here represents a rather weak a-dependence of p_0:

$$p_0(a) = 16 - 0.00112 \cdot a.$$

At $a = a_0 = 625$ we have $p_0 = 15.3$; the same as earlier. Further, at $a = 1250$ we have $p_0 = 14.6$. In other words, increasing a by 22 % implies that the intercept of the linear demand curve is reduced by 5 % (remember that 625 is the rescaled value of a). Nevertheless, as can be seen from Table 5, such an a-dependence results in reduction of the optimal period of extraction from 23.8 to 21.7 with nonlinear decay and from

59.5 years to 42.1 with linear decay. Table 5 can be directly compared to Table 4.

Table 5. Key results when p_0 is a-dependent

	Linear decay	Non-linear decay
$x(0)$	15.8	13.6
$x(T)$	16.9	14.3
$a(T)$	754	671
T	42.1	21.7
$\theta(0)$	189%	292%
$\theta(T)$	134%	253%

By comparing Table 4 with Table 5 it is seen that both optimal production and the period with production are lower with consumer awareness. However, the relative change is smaller with non-linear decay than with linear decay. The reason for this is probably that with non-linear decay the optimal policy is already quite restrictive, and therefore the additional effect of consumer awareness is not that significant. Figures 3 and 4 correspond to figures 1 and 2, but now with a-dependence in the demand function.

Again this example is not meant to describe the reality but to emphasize the importance of including endogenous consumer behavior and also to stress the importance of more empirical research on these aspects. The bottom line in this section is that consumers' concern for the environment may partly compensate the decision makers failure to estimate the correct decay function, as long as the policy is adaptive and consumers' preferences are elaborated.

CONCLUSIONS

In this paper a feedback control law that can be used to control the production of fossil fuel products in the presence of stock externalities associated with emissions of greenhouse gases has been used to analyze the effects of non-linear decay and consumer awareness upon optimal carbon taxes. The main result is that the tax is quite sensitive to both these phenomena.

The total time horizon in the optimization problem is divided into two periods, one with extraction and emissions and one without extraction. The model has been used to analyze the time path of the corrective tax in the period with extraction by taking into account the stock externality both in this period and in the remaining period.

Special emphasis has been put on the effects of non-monotone decay of carbon in the atmosphere and the interaction with consumer preferences. If decay is, in fact, monotone the optimal period of extraction will be much longer than if it is non-monotone. This result, however, may again be affected if consumers have a strong concern for the environment such that the demand for a polluting product decreases with the level of pollution. In this case the period of extraction will be much shorter. The sensitivity to the shape of the decay function also stresses the importance of estimating the decay function. Assuming a linear decay function for mathematical convenience, as is often done, may represent a serious mistake if the actual decay is non-monotone.

Using current data on cumulative emissions, and starting from the same initial values, the time paths of the corrective taxes with non-linear and linear decay of the pollution stock are shown to be very different. Under non-linear decay, the optimal extraction period of the non-renewable resource is about 24 years while with linear decay the extraction period is almost 60 years.

The results in this paper confirms the results by Ulph and Ulph (1994) that the time path of the optimal path depends on the decay function, and that papers claiming that the tax should decline over time, e.g. Sinclair (1992), are not correct in general. However, as Ulph and Ulph look at linear decay, they conclude that positive decay makes the tax rise over time. In this paper it is shown that the optimal tax can increase or decrease over time depending on the shape of the decay function. The numerical results are somewhat different from what Ulph and Ulph found, especially as they assume an exogenous switching time.

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APPENDIX 1. THE OBJECTIVE FUNCTION

In this appendix it is shown that the sum of consumers' surplus, producers' surplus and the government's surplus is equal to $U - D$. If these surpluses are called CS, PS and GS, respectively, we have by definition

$$\begin{aligned} CS &\equiv \int_0^x P(a,s)\,ds - (C(a,x)+\tau)x, \\ PS &\equiv C(a,x)x - \int_0^x C(a,s)\,ds, \\ GS &\equiv \tau x - D(a). \end{aligned}$$

Summing these surpluses yields

$$\int_0^x [P(a,s) - C(a,s)]\,ds - D(a).$$

APPENDIX 2. A SCRAP VALUE FORMULATION

The purpose of this appendix is to show how the matching conditions (7) to (11) are derived using a salvage (or scrap) value approach. The present value of the last phase of the infinite time problem can alternatively be defined as the salvage value for a problem with a finite time horizon T where T is to be determined as part of the optimization.

In (5) the function $\Psi(a;\alpha)$ was defined. By setting $\alpha = a_T$, and applying Proposition 2, it is readily seen that

$$t = T + \tau = T + \tau(a, a_T) \text{ where } \frac{\partial \tau}{\partial a} = -\frac{1}{f(a)} \text{ and } \frac{\partial \tau}{\partial a_T} = \frac{1}{f(a_T)}.$$

Calling the present value of the last period φ, it can be defined and written as:

$$\varphi(T, a_T) = \int_T^{\infty} e^{-\delta t}\left[\bar{U}(t) - D(a)\right] dt = \int_T^{\infty} e^{-\delta t}\bar{U}(t)dt + \int_0^{a_T} e^{-\delta(T+\tau)}\frac{D(a)}{f(a)}da.$$

Partial integration of the last term yields

$$\varphi(T, a_T) = \int_T^{\infty} e^{-\delta t}\bar{U}(t)dt - \frac{e^{-\delta T}}{\delta}\left[D(a_T) - \int_0^{a_T} e^{-\delta\tau}D'(a)da\right].$$

The transversality conditions involve $\frac{\partial\varphi}{\partial T}$ and $\frac{\partial\varphi}{\partial a_T}$. Straightforward calculations yield

$$\begin{aligned} \frac{\partial\varphi}{\partial T} &= -e^{-\delta T}\left[\bar{U}(T) - D(a_T) + \int_0^{a_T} e^{-\delta\tau}D'(a)da\right] \\ \frac{\partial\varphi}{\partial a_T} &= -\frac{e^{-\delta T}}{f(a_T)}\int_0^{a_T} e^{-\delta\tau}D'(a)da. \end{aligned} \tag{18}$$

Notice that $\varphi(T, a_T) = -e^{-\delta T}\left[\bar{U}(T) - D(a_T)\right]$ from its definition. The Hamiltonian in this case is given by:

$$H(t, a, s, x, m, n) = e^{-\delta t}[U(a, x) - D(a)] + m \cdot (x - f(a)) - n \cdot x.$$

The transversality condition on the shadow price of pollution is $m_T = \frac{\partial\varphi}{\partial a_T}$ or

$$e^{\delta T} m_T f(a_T) = -\int_0^{a_T} e^{-\delta\tau}D'(a)da, \tag{19}$$

which is recognized as matching condition (9). The transversality condition associated with a free time horizon is $H + \frac{\partial\varphi}{\partial T} = 0$ at $t = T$. Before we apply this relationship, we use the fact that the optimal policy is, by definition, an interior solution which implies:

$$m - n = -e^{-\delta t}U_x(a, x). \tag{20}$$

Inserted into the Hamiltonian this yields $H = e^{-\delta t}[U - D - xU_x] - m \cdot f$. At the end of the first period, $t = T$, we get from (19):

$$
\begin{aligned}
H + \frac{\partial\varphi}{\partial T} &= e^{-\delta T}\left[U - D - xU_x + \int_0^{a_T} e^{-\delta a} D'(a)da\right] - e^{-\delta T}\left[\theta - D + \int_0^{a_T} e^{-\delta a} D'(a)da\right] \\
&= e^{-\delta T}\left[U - xU_x - \theta\right].
\end{aligned}
$$

The transversality condition on the Hamiltonian therefore implies that

$$U(a_T, x_T) - x_T U_x(a_T, x_T) - \theta(T) = 0,$$

which is matching condition (7) in the main text.

Continuity of the shadow values at the end point $t = T$ applied to (20) yields matching condition (8). Condition (10) is a direct consequence of the transversality condition on the costate variable n, and condition (11) follows directly from the dynamic equation for the pollution level (3).

APPENDIX 3. AN EXISTENCE PROOF AND AN ARROW-TYPE SUFFICIENCY RESULT.

In this section we show that our problem has a solution. We apply the Filippov-Cesari existence theorem as it is given in theorem 6.18 in Seierstad and Sydsæther, 1987. The time interval of interest is taken to be $[0, \hat{T}]$ for sufficiently large $\hat{T}$. All the conditions in the theorem is trivially satisfied except, possibly, for the convexity of the set $N(a, t, x)$ and the assumed upper constant bound on the state variables. The latter is straightforward as can be seen from the fact that $\dot{a} + \dot{s} = -f(a)$ implies $0 \le a + s \le a_0 + s_0$. The theorem assumes that the set

$$N(a, t, x) = \{(e^{-\delta t}[U(a, x) - D(a)] + \gamma, x - f(a), -x) : \gamma \le 0, x \in [0, \hat{x}]\}$$

is convex for all $(a, t) \in R \times [0, \hat{T}]$. We fix (a, t) and let $y_i = e^{-\delta t}[U(a, x_i) - D(a)] + \gamma_i$ and $\gamma_i \le 0$ for $i = 1, 2$ and let x_3 and y_3 be the convex combinations of x_1, x_2 and y_1, y_2.

From the concavity of U we have that the convex combination $\lambda(y_1, x_1 - f(a), -x_1) + (1 - \lambda)(y_2, x_2 - f(a), -x_2) = (\lambda y_1 + (1 - \lambda)y_2, x_3 - f(a), -x_3) = (y_3, x_3 - f(a), -x_3)$ and $y_3 = e^{-\delta t}[\lambda U(a, x_1) + (1 - \lambda)U(a, x) - D(a)] + \lambda\gamma_1 + (1 - \lambda)\gamma_2 \leq e^{-\delta t}[U(a, x_3) - D(a)] + \lambda\gamma_1 + (1 - \lambda)\gamma_2$ implying that $\gamma_3 \leq \lambda\gamma_1 + (1 - \lambda)\gamma_2 \leq 0$ and thereby $(y_3, x_3 - f(a), -x_3) \in N(a, t, x)$. Hence the set is convex.

The Arrow-type sufficiency result is based on note 6.20 in Seierstad and Sydsæther, 1987. This note deals with a more general problem than the present one. In addition to the concavity of the maximized Hamiltonian, we need to show that the scrap value is concave with respect to the state variables. All other conditions are trivially satisfied. The model presented in this paper has a simpler structure than the setting in the referenced note. We deal with simple state constraints in the form of non-negativity conditions and no combined constraints on state and policy.

The concavity of the scrap value $\varphi(T, a_T)$ is shown by differentiating (18). We obtain

$$\begin{aligned}\frac{\partial^2}{\partial a_T^2}\varphi(T, a_T) &= e^{-\delta T}\left[\frac{f'(a_T) + \delta}{f(a_T)^2}\int_0^{a_T} e^{-\delta\tau(a;a_T)}D'(a)da - \frac{D'(a_T)}{f(a_T)}\right] \\ &= \frac{e^{-\delta T}}{f(a_T)}\int_0^{a_T} e^{-\delta\tau(a;a_T)}D'(a)da \cdot \left[\frac{f'(a_T) + \delta}{f(a_T)} - \frac{D'(a_T)}{D(a_T)}\right] \leq 0.\end{aligned}$$

The last term in the last square brackets stems from noticing that $\int_0^{a_T} e^{-\delta\tau(a;a_T)}D'(a)da \leq D(a_T)$ and the inequality is a direct consequence of the constraining relation on the marginal decay function in The Usual Assumptions.

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