Valuation of Irreversible Investments: Private Information about the Investment Cost

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Abstract

This article examines dynamic investment decisions when there is an agency problem. A principal delegates the decision of an investment strategy of a project to an agent. The agent has private information about the investment cost, whereas the principal only knows the probability distribution of the cost. The principal's problem is how to compensate the agent in order to optimize the value of the principal's investment opportunity. Owing to the asymmetric information, it may be optimal for the principal to leave the agent some "information rent". An optimal compensation function dependent on the observable output from the investment is found.

1 Introduction

In the literature on real options, the option value resulting from the interaction of uncertainty, flexibility and (partly) irreversibility is recognized. The uncertainty taken into account is mainly "symmetric" uncertainty, i.e., the uncertainty in future income is common knowledge. However, in many situations there is also asymmetric information. An example of such a situation is when a manager (an agent) of an investment project has better information than the investor (the principal) about the investment costs, and the manager also has diverging interests from those of the investor. The situation is known from the principal-agent and the regulatory literature.

I sketch a model where a principal delegates the investment strategy of a project to an agent. The agent has private information about the exact investment cost, whereas the principal only knows the probability distribution of the cost. One reason for an owner of an investment possibility to delegate the management of a project to an agent, may be that the management requires expertise that the principal does not possess, or that is too costly for him to obtain. In other cases it may be impossible for the principal to make the decisions himself, but it may be possible for him to commit to a delegation contract.

The information asymmetry creates a situation where adverse selection may occur. The agent is compensated according to a contract. The principal observes the output from the investment project, and the contracted compensation is a function of this variable. Both the principal and the agent aim to maximize the value of the project. An implementable compensation function, with value equal to the sum of the value of the agent's private information and the true investment cost, is found. The value of private information depends upon the value of the output relative to the true investment cost, and upon the uncertainty in both these variables.

The model applies to situations where the production from the project is sold in perfect markets, whereas there are imperfections due to the costs of projects.

An application of the model is the case where a government owns natural resources. Production of natural resources involves large and (partly) irreversible investments, and uncertainty due to future output prices. A feature of production of natural resources is that uncertainty in output prices usually is common knowledge, whereas investment and production costs may be private information for those investing in and operating such projects. To exploit the resources, the government delegates the production of the resources to companies. The companies may have incentives to signal higher cost than the true cost in order to obtain a larger profit within the companies. The model presented in this paper gives the government a method of how to find the most efficient contract between the government and the companies. The contract can be in the form where the companies are paid a compensation for the management of the resources, or it can be in the form of a taxation system.

Shareholders versus corporate management is another example where the model may apply. The problem is then how to compensate the management given their private information about some costs. As in the example above, the management may want to signal higher costs than the true ones. An alternative interpretation is that the companies may have incentives to maximize slack in the organization, thereby increasing the realized investment cost compared to the necessary cost.

Bjerksund and Stensland (1999) have formulated an adverse selection model, somewhat similar to the model described in this paper, where an owner of some resource may exploit the resource in two ways: (i) Sell the resource in a competitive spot market at a constant price, or (ii) ship the resource to an agent for processing and sell the processed resource in a competitive market where the price of the processed resource is stochastic. Bjerksund and Stensland assume that the processing may be switched on and off at no cost (i.e. they formulate a "switching option", similar to Brennan and Schwartz 1985). In alternative (ii), the owner of the resource ("the regulator") must compensate the agent for the cost of processing the resource. The cost of processing is perfect, private information to the agent, whereas the regulator knows the probability distribution of the costs. The stochastic income process used in Bjerksund and Stensland (1999) is more general than the diffusion process presented in the model in this article.

The interaction between options and diverging incentives between a principal and an agent is also analyzed in Antle, Bogetoft and Stark (1996). They show how timing and incentive effects interact to affect investment strategies in a twoperiod model. At each of the two points in time where investment is possible, the manager (the agent) knows the investment cost, whereas the owner (the principal) does not. Before the time of an investment possibility, neither the owner nor the manager know the investment cost. However, they both agree on the distribution of future costs. Antle et al. find that incentive effects, as timing effects, lower the target costs. Incentive problems also have the effect of pushing investment towards periods of lower uncertainty, i.e., the target cost at time zero (today) may be increased by incentive effects, so much that the overall probability of investment can increase with incentive problems.

The article is organized as follows: In section 2 the problem is formulated, and model assumptions are given. In section 3 future cash flows in the model are evaluated using the market-based valuation approach (assuming dynamically complete markets). Section 4 presents the agent's optimization problem and his value of private information. The optimal investment strategies are given in sections 5 and 6 for the cases where the information about the investment cost is symmetric and asymmetric, respectively. In section 7 the optimal compensation function is found. The results are illustrated in section 8, using the uniform distribution for the investment cost, and the geometric Brownian motion for the income process. Section 9 concludes the article.

2 Model assumptions

A principal has an opportunity to invest in a project. The investment decision of the project is undertaken by an agent, and the principal compensates the agent based on the output from the project. The output is observable by both parties, whereas the agent has private information about the investment cost. In order to keep a larger part of the profit from the project, the agent has incentives to base his investment strategy on signaling a higher investment cost than the true cost. Thus, the problem for the principal is how to compensate the agent to maximize the value of the principal's investment opportunity.

The agent has perfect knowledge of the true investment cost θ of the project, whereas the principal knows only the probability density, $f(\tilde{\theta})$, of an assessed stochastic cost $\tilde{\theta}$. The cumulative distribution is denoted by $F(\tilde{\theta})$, and upper and lower levels of the investment cost are $\overline{\theta}$ and $\underline{\theta}$, respectively.

It is assumed that the option to invest is perpetual, and that the value of the output follows a stochastic process where the uncertainty is common knowledge. The value of the output at time t is denoted S_t. The stochastic process is defined by a complete, filtered probability space $(\Omega, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{F}, P)$ where the filtration satisfies the *usual conditions* (see e.g. Borodin and Salminen (1996), ch. I.3). Under the equivalent martingale measure Q (see e.g. Duffie (1996), ch. 6.H) the stochastic income process is given by

$$dS_t = (rS_t - \delta(S_t))dt + \sigma(S_t)dw_t, \quad S_0 \equiv s, \tag{1}$$

where r is a constant risk free rate, $\delta(S_t)$ reduces the drift in the stochastic process because of the convenience yield, and w_t is a standard Brownian motion with respect to the equivalent martingale measure. It is assumed that $\delta(S_t)$ and $\sigma(S_t)$ are continuous, and that 0 is an unattainable lower boundary for S_t.

The transfer function from the principal to the agent must be based on some observable variable. In the model, it is assumed that the value of the output, s, is observable. Also, recall that the information with respect to the process S_t is assumed to be symmetric. To avoid the agent from behaving opportunistically, the value of the compensation must not be paid before the time of investment.

Below the principal's optimization problem is formulated:

The principal optimizes his value function with respect to the compensation function $G(\cdot)$,

$$\begin{aligned} W(s, G(S_{\tau_K})) &= \sup_{G(S_{\tau_K})} E^{\tilde{\theta}} \left\{ E_0 \left[e^{-r\tau_K} \left(S_{\tau_K} - G(S_{\tau_K}) \right)^+ \right] \right\} \\ &= \sup_{G(S_{\tau_K})} \int_{\underline{\theta}}^{\overline{\theta}} E_0 \left[e^{-r\tau_K} \left(S_{\tau_K} - G(S_{\tau_K}) \right)^+ \right] f(\tilde{\theta}) d\tilde{\theta}, \end{aligned} \tag{2}$$

subject to the agent's optimization problem,

$$V(s, K; \theta) = \sup_{\tau_K} E_0 \left[e^{-r\tau_K} \left(G(S_{\tau_K}) - \theta \right)^+ \right], \tag{3}$$

and the agent's participation constraint,

$$V(s, K; \theta) \geq 0. \tag{4}$$

The expectation with respect to the cost level θ is denoted $E^{\tilde{\theta}}$. It is assumed that the uncertainty in the investment cost is the same under the P and the Q measure. The expectation operator $E_t[\cdot]$ denotes the expectation, conditioned on the time t information, with respect to the equivalent martingale measure Q. The stopping time with respect to the filtration $\mathcal{F}_t$, is denoted τ_K, and is a function of K, where K is the "cost" upon which the agent bases his investment strategy. The signaled cost, $K \equiv K(\tilde{\theta})$, is higher than or equal to θ, since the agent profits on signaling a higher cost than the true one. The exercise value of output is denoted S_{τ_K}, and $G(S_{\tau_K})$ is the agent's compensation, transferred at the investment time.

3 Valuation of future cash flows

We assume that the option to invest is perpetual. This implies that the optimal investment strategy is time homogeneous. Thus, we know that the optimal stopping time τ_K will be of the form

$$\tau_K = \inf\{t \geq 0 | S_t \geq \hat{S}(K)\}.$$

The "trigger value of income" $\hat{S}(K)$ is independent of time. We can therefore rewrite the principal's and the agent's value functions as, respectively,

$$W(s, G(S_{\tau_K})) = \sup_{G(S_{\tau_K})} \int_{\underline{\theta}}^{\overline{\theta}} E_0[e^{-r\tau_K}] \left(\hat{S}(K) - G(S_{\tau_K}) \right)^+ f(\tilde{\theta}) d\tilde{\theta},$$

and,

$$V(s, K; \theta) = \sup_{S_{\tau_K}} E_0[e^{-r\tau_K}] \left(G(S_{\tau_K}) - \theta \right)^+,$$

where the expected value of the discount factor is written independently of the value of the output and the compensation function. This independence simplifies the problem of finding the optimal investment strategy, since we will be able to optimize with respect to a "deterministic" trigger level $\hat{S}(K)$, instead of the stochastic trigger S_{τ_K}.

Using results from the classical theory of diffusions, the expected value of the discount factor can be formulated as a function of the trigger level $\hat{S}(K)$, and the time 0 value of the output, s (Borodin and Salminen [4], ch. II.10 and Ito and McKean [8], sect. 4.6),

$$E_0[e^{-r\tau_K}] = \begin{cases} \frac{\phi(s)}{\phi(\hat{S}(K))} & \text{if } s < \hat{S}(K) \\ 1 & \text{if } s \geq \hat{S}(K). \end{cases} \tag{5}$$

Defining $u(s) = E_0[e^{-r\tau_K}]$, the function $\phi(\cdot)$ is the strictly positive and increasing, unique solution to the ordinary differential equation,

$$\frac{1}{2}(\sigma(s))^2 u_{ss}(s) + (rs - \delta(s))u_s(s) - ru(s) = 0, \tag{6}$$

with boundary $\lim_{s\uparrow\hat{S}(K)} u(s) = 1$.

Using equation (5) the principal's and the agent's value functions can be reformulated. The principal's value function will be,

$$\begin{aligned} W(s, G(\hat{S}(K))) \quad = \quad & \sup_{G(\cdot)} \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \frac{\phi(s)}{\phi(\hat{S}(K))} \left(\hat{S}(K) - G(\hat{S}(K)) \right) \mathrm{I}(s < \hat{S}(K)) \right. \\ & \left. + (s - G(s))\mathrm{I}(s \geq \hat{S}(K)) \right\} f(\tilde{\theta})d\tilde{\theta} \end{aligned} \tag{7}$$

where $\mathrm{I}(A)$ is the indicator function of the event A. The agent's value function is formulated as,

$$V(s, K; \theta) = \sup_{\hat{S}(K)} \begin{cases} \frac{\phi(s)}{\phi(\hat{S}(K))} \left(G(\hat{S}(K)) - \theta \right) & \text{if } s < \hat{S}(K) \\ G(s) - \theta & \text{if } s \geq \hat{S}(K), \end{cases} \tag{8}$$

respectively. Note that the value functions now are functions of the "deterministic" trigger level $\hat{S}(\cdot)$ and the time zero value of the output process s, only.

4 The agent's optimization problem and his value of private information

The agent optimizes his value of the investment opportunity given by equation (8) with respect to investment strategy $\hat{S}(K)$. The first-order condition with respect to the investment strategy is

$$\frac{\partial V(s, K; \theta)}{\partial \hat{S}(K)} = G_{\hat{S}}(\hat{S}(K)) - \frac{\phi_{\hat{S}}(\hat{S}(K))}{\phi(\hat{S}(K))} \left(G(\hat{S}(K)) - \theta \right) = 0, \tag{9}$$

where $G_{\hat{S}}(\hat{S}(K))$ and $\phi_{\hat{S}}(\hat{S}(K))$ denote the first-order partial derivatives of the functions G and ϕ respectively, with respect to $\hat{S}(K)$.

For the investment strategy $\hat{S}(K)$ to be optimal, the second-order condition must be non-positive, i.e.,

$$\frac{\partial^2 V(s,K;\theta)}{\partial \hat{S}(K)^2} = \frac{\phi(s)}{\phi(\hat{S}(K))}\left\{\left[\frac{\phi_{\hat{S}\hat{S}}(\hat{S}(K))}{\phi(\hat{S}(K))} - 2\left(\frac{\phi_{\hat{S}}(\hat{S}(K))}{\phi(\hat{S}(K))}\right)^2\right]\left(G(\hat{S}(K)) - \theta\right) + G_{\hat{S}\hat{S}}(\hat{S}(K))\right\} \leq 0. \tag{10}$$

Equations (9) and (10) lead to the agent's optimal investment strategy given a compensation function $G(\cdot)$.

One approach which simplifies the task of finding the optimal compensation function is to use the *revelation principle* (see e.g. Baron and Myerson (1982) and Laffont and Tirole (1993)). By the revelation principle, the agent's value of private information can be found.

Under a revelation mechanism, the agent reports his private information to the principal, and the decision in question is then made according to a decision rule to which the principal has committed himself. Loosely speaking, the revelation principle makes use of the fact that for every contract between the principal and the agent that leads the agent to lie, there is another contract with the same outcome, but with no incentive for lying. This reduces the principal's optimization problem to optimizing over the set of truthful mechanisms.

In the model the investment decision is delegated to the agent. Consequently, the revelation principle does not apply directly here: there is no decision to be made by the principal, and therefore the agent does not have to report his private information. However, Melumad and Reichelstein (1987) have found that under certain conditions, the performance of an optimal revelation mechanism can be replicated by a delegation scheme which does not involve communication. This situation is valid in the presented model.

Implementation of the revelation principle requires that the agent's first order condition is satisfied for all $K \in \left[\underline{\theta}, \overline{\theta}\right]$. Using the envelope theorem, the first-order condition for optimization[1] is,

$$\left.\frac{dV(s,K(\theta);\theta)}{d\theta}\right|_{K(\theta)=\theta} = \frac{\partial V(s,K(\theta);\theta)}{\partial\theta} = -\frac{\phi(s)}{\phi(\hat{S}(\theta))} \quad \forall K(\theta), \theta \in [\underline{\theta}, \overline{\theta}]. \tag{11}$$

Incentive compatibility implies $V(s,\theta;\theta) = V(s,K;\theta)$. In order to simplify the notation, I define $V(s,\theta) \equiv V(s,\theta;\theta)$.

In addition the second-order condition for K must be satisfied at $K(\theta) = \theta$. The second-order condition is shown in the appendix, section A.

Integrating the condition in (11) gives an equivalent condition on the reward function (when $s < \hat{S}(\theta)$):

$$V(s,\theta) = \int_{\theta}^{\overline{\theta}} \frac{\phi(s)}{\phi(\hat{S}(u))} du + V(s,\overline{\theta}). \tag{12}$$

Equation (12) gives the agent's value of accepting the contract. The first term on the right-hand side of equation (12) is the agent's value of private

[1] $\left.\frac{dV(s,K(\theta);\theta)}{d\theta}\right|_{K(\theta)=\theta} = \frac{\partial V(s,K(\theta);\theta)}{\partial K(\theta)}\frac{dK(\theta)}{d\theta} + \frac{\partial V(s,K(\theta);\theta)}{\partial\theta}$. The agent optimizes $K(\theta)$, given his cost level θ. The first term on the right-hand side is zero when $K(\theta)$ is optimal.

information. In accordance with intuition, we see that the value of private information is decreasing in the true cost level.

The last term on the right-hand side, $V(s, \overline{\theta})$, is the value of the reservation utility. From the participation constraint (4) we know that the agent at least must earn his reservation utility in order to accept the contract. Also in the case where the agent's true cost is at the highest possible cost level, $\overline{\theta}$, the agent must earn his reservation utility. In this model the reservation utility is assumed to be zero, i.e., $V(s, \overline{\theta}) = 0$. Hence equation (12) represents the agent's value of accepting the contract that the principal offers.

5 Benchmark: Symmetric information

As a benchmark, we first study the case where the information about the investment cost θ is symmetric. When the agent has no private information, there is no need for the principal to compensate the agent with more than his true cost. Thus, the agent is compensated for his capital cost only, i.e.,

$$G(s) = \begin{cases} 0 & \text{if } s < \hat{S}(\theta) \\ \theta & \text{if } s \geq \hat{S}(\theta). \end{cases} \tag{13}$$

Inserting $G(\hat{S}(\theta)) = \theta$ into the agent's value function in equation (8), we find $V_{sym}(s, \theta) = 0$, where the subscript sym indicates that this is the value under symmetric information. The agent has no private information, and therefore the term, $\int_{\theta}^{\overline{\theta}} \phi(s)/\phi(\hat{S}(u))du$, of equation (12) is zero.

Deterministic θ and substitution of $G(\hat{S}(\theta))$ with θ into the principal's value function in equation (7), leads to

$$W_{sym}(s, \theta) = \sup_{\hat{S}(\theta)} \begin{cases} \frac{\phi(s)}{\phi(\hat{S}(\theta))} \left(\hat{S}(\theta) - \theta \right) & \text{if } s < \hat{S}(\theta) \\ s - \theta & \text{if } s \geq \hat{S}(\theta). \end{cases} \tag{14}$$

Equation (14) shows that when we have no asymmetric information, we have an optimization problem similar to the "standard" real option problem of exercising an infinite (American) option with exercise price θ, and $\hat{S}(\theta)$ as the critical level of exercising the option.

The optimal trigger value of income is given by the first-order condition,

$$\frac{\partial W_{sym}(s, \theta)}{\partial \hat{S}(\theta)} = 1 - \frac{\phi_{\hat{S}}(\hat{S}(\theta))}{\phi(\hat{S}(\theta))} \left(\hat{S}(\theta) - \theta \right) = 0. \tag{15}$$

For the trigger value in equation (15) to be optimal, the second-order condition has to be non-positive,

$$\frac{\partial^2 W_{sym}(s, \theta)}{\partial \hat{S}(\theta)^2} = -\frac{\phi(s)}{\phi(\hat{S}(\theta))} \frac{\phi_{\hat{S}\hat{S}}(\hat{S}(\theta))}{\phi(\hat{S}(\theta))} \left(\hat{S}(\theta)) - \theta \right) \leq 0,$$

The first-order condition (15) can be written as

$$S^*_{sym}(\theta) - \theta = \frac{\phi(S^*_{sym}(\theta))}{\phi_{S^*_{sym}}(S^*_{sym}(\theta))}, \tag{16}$$

where $S^*_{sym}(\theta)$ is the optimal critical value for investment. The last term on the right-hand side can be interpreted as the opportunity cost of exercising the option with payoff $S^*_{sym}(\theta) - \theta$. The fraction captures the wedge between the critical value S^*_{sym} and the investment cost θ.

By (14) and (16) the value of the investment opportunity is

$$W_{sym}(s,\theta) = \begin{cases} \frac{\phi(s)}{\phi(S^*_{sym}(\theta))}\left(S^*_{sym}(\theta) - \theta\right) & \text{if } s < S^*_{sym}(\theta) \\ s - \theta & \text{if } s \geq S^*_{sym}(\theta). \end{cases} \tag{17}$$

6 Asymmetric information: The optimal exercise strategy

In this section we solve the principal's problem of finding the optimal investment strategy, given the agent's private information.

In order to simplify the problem of finding an optimal strategy, we substitute the unknown function $G(\cdot)$ in the principal's value function in equation (7), with an expression of known functions of $\hat{S}(\theta)$. Using equations (8) and (12), the value of the compensation function may be written as the sum of the value of the true investment cost and the value of the agent's private information,

$$\begin{aligned} \frac{\phi(s)}{\phi(\hat{S}(\theta))}G(\hat{S}(\theta)) &= \frac{\phi(s)}{\phi(\hat{S}(\theta))}\theta + V(s,\theta) \\ &= \frac{\phi(s)}{\phi(\hat{S}(\theta))}\theta + \int_\theta^{\overline{\theta}} \frac{\phi(s)}{\phi(\hat{S}(u))}du. \end{aligned} \tag{18}$$

The right-hand side of the equation gives an representation of the value of the compensation, which contains known functions and θ, only.

Substituting the expression for $\frac{\phi(s)}{\phi(\hat{S}(\theta))}G(\hat{S}(\theta))$ in equation (18) into the principal's optimization problem in equation (7) leads to

$$\begin{aligned} W(s,\theta) = \ & \sup_{\hat{S}(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \left[\frac{\phi(s)}{\phi(\hat{S}(\theta))}\left(\hat{S}(\theta) - \theta\right) - \int_\theta^{\overline{\theta}} \frac{\phi(s)}{\phi(\hat{S}(u))}du \right] \mathrm{I}(s < S^*(\theta)) \right. \\ & \left. + (s - G(s))\mathrm{I}(s \geq S^*(\theta)) \right\} f(\theta)d\theta. \end{aligned} \tag{19}$$

From equation (19) we see that the substitution of $G(\hat{S}(\theta))$ implies that the principal's problem is reduced to finding an optimal trigger income $S^*(\theta)$.

A further simplification of the optimization problem in equation (19) can be done by partial integration of the term $\int_{\underline{\theta}}^{\overline{\theta}} \int_\theta^{\overline{\theta}} \phi(s)/\phi(\hat{S}(u))du f(\theta)d\theta$. Integration leads to (see appendix B for a derivation of equation (20)),

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_\theta^{\overline{\theta}} \frac{\phi(s)}{\phi(\hat{S}(u))}du f(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\phi(s)}{\phi(\hat{S}(\theta))}F(\theta)d\theta. \tag{20}$$

Inserting the right-hand side of (20) into the objective function (19), we find

$$\begin{aligned} W(s,\theta) = \ & \sup_{\hat{S}(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \left[\frac{\phi(s)}{\phi(\hat{S}(\theta))}\left(\hat{S}(\theta) - \theta - \frac{F(\theta)}{f(\theta)}\right) \right] I(s < S^*(\theta)) \right. \\ & \left. + (s - G(s))I(s \geq S^*(\theta)) \right\} f(\theta)d\theta. \end{aligned} \tag{21}$$

From the last term in equation (21) we see that the principal's optimization problem is now similar to the problem of optimally exercising an American call option, with optimal exercise price $\theta + F(\theta)/f(\theta)$. The term $F(\theta)/f(\theta)$ can be interpreted as the inefficiency due to the agent's private information.

Pointwise differentiation gives the first- and second-order conditions for the optimal "exercise value" $S^*(\theta)$,

$$\frac{\partial W}{\partial \hat{S}(\theta)} = 1 - \frac{\phi_{\hat{S}}(\hat{S}(\theta))}{\phi(\hat{S}(\theta))} \left(\hat{S}(\theta) - \theta - \frac{F(\theta)}{f(\theta)} \right) = 0, \tag{22}$$

The conditions for the trigger value are satisfied as long as the second-order condition

$$\frac{\partial^2 W(s, G(\cdot))}{\partial \hat{S}(\theta)^2} = -\frac{\phi(s)}{\phi(\hat{S}(\theta))} \frac{\phi_{\hat{S}\hat{S}}(\hat{S}(\theta)}{\phi(\hat{S}(\theta))} \left(\hat{S}(\theta)) - \theta - \frac{F(\theta)}{f(\theta)} \right) f(\theta) \leq 0,$$

holds. Hence, the optimal trigger value for the principal is given by

$$S^*(\theta) - \theta - \frac{F(\theta)}{f(\theta)} = \frac{\phi(S^*(\theta))}{\phi_{S^*}(S^*(\theta))}. \tag{23}$$

Given the compensation function (to be evaluated in the next section), the trigger value in equation (23) is also the optimal exercise strategy for the agent. Equation (23) shows that the trigger value is based on the principal's total cost of exercising the investment option, i.e., it is based on $\theta + F(\theta)/f(\theta)$. As in equation (16), the right-hand side represents the opportunity cost of exercising the option. Compared to the optimal investment strategy under symmetric information (equation (16)), the critical value for investment has increased due to the asymmetric information. This inefficiency leads to underinvestment because of the longer "waiting time" of investment.

7 Implementation of the optimal compensation function

We are now left with the problem of finding an implementable compensation function that leads to the optimal investment strategy. Considering equations (18) and (23), the time zero value of the optimal compensation function when $s < S^*(\theta)$ is given by

$$\begin{aligned} &\frac{\phi(s)}{\phi(S^*(\theta))} G(S^*(\theta)) \\ &= \tfrac{\phi(s)}{\phi(S^*(\theta))}\theta + \textstyle\int_\theta^{\overline{\theta}} \tfrac{\phi(s)}{\phi(S^*(u))} du \\ &= \tfrac{\phi(s)}{\phi(S^*(\theta))}\theta + \left[u \tfrac{\phi(s)}{\phi(S^*(u))} \right]_\theta^{\overline{\theta}} - \textstyle\int_\theta^{\overline{\theta}} u \left(-\tfrac{\phi(s)\phi_{S^*}(S^*(u))}{(\phi(S^*(u)))^2} \right) S_u^* du. \end{aligned} \tag{24}$$

The first right-hand side equality in (24) states that the compensation function must cover the agent's true cost (the first term), and the agent's value of private information (the last term). Notice that the compensation function in equation (24) is not written in a contractable form, as it is a function of the unobservable variable θ as well. The right-hand side of the equation must therefore be found as a function of observable variables only. From Melumad and Reichelstein (1987) we know that a compensation function $G(\theta, S^*(\theta))$ under a communication-based centralized contract (by the revelation principle) is compatible with the compensation function $G(S^*(\theta))$ under a direct delegation contract if for all $\theta \in [\underline{\theta}, \overline{\theta}]$, $G(\theta, S^*(\theta)) = G(S^*(\theta))$. This restriction is satisfied when the function $S^*(\theta)$ is one-to-one. Assuming that this is valid for $S^*(\theta)$,[2] we denote $\theta \equiv \vartheta(S^*(\theta))$. This leads to

$$\begin{aligned}
&\frac{\phi(s)}{\phi(S^*(\theta))} G(S^*(\theta)) \\
&= \tfrac{\phi(s)}{\phi(\overline{S}^*(\theta))}\overline{\theta} - \int_\theta^{\overline{\theta}} \vartheta(S^*(u)) \left(-\tfrac{\phi(s)\phi_{S^*}(S^*(u))}{(\phi(S^*(u)))^2}\right) S^*_u du \\
&= \tfrac{\phi(s)}{\phi(S^*(\overline{\theta}))}\overline{\theta} - \int_{S^*(\theta)}^{S^*(\overline{\theta})} \vartheta(S^*(u)) \left(-\tfrac{\phi(s)\phi_{S^*}(S^*(u))}{(\phi(S^*(u)))^2}\right) dS^*(u) \\
&= \vartheta(S^*(\theta))\tfrac{\phi(s)}{\phi(S^*(\theta))} + \int_{S^*(\theta)}^{S^*(\overline{\theta})} \vartheta'(S^*(u))\tfrac{\phi(s)}{\phi(S^*(u))} dS^*(u)
\end{aligned} \tag{25}$$

Thus, from equation (25), and the assumption that only the output of the investment is observable, we find that the contracted, optimal compensation function is given by

$$G(s) = \begin{cases} 0 & \text{if } s < S^*(\theta) \\ \vartheta(s) + \int_s^{S^*(\overline{\theta})} \vartheta'(S^*(u))\frac{\phi(s)}{\phi(S^*(u))} dS^*(u) & \text{if } S^*(\theta) \leq s < S^*(\overline{\theta}) \\ \overline{\theta} & \text{if } s \geq S^*(\overline{\theta}) \end{cases} \tag{26}$$

The above expression represents an implementable compensation function dependent upon the observable variables s and $S^*(\cdot)$, only. When $s < S^*(\theta)$ the compensation is zero, as the investment has not taken place in this range of the value of s. As long as $s < S^*(\theta)$, the agent will wait with exercising the option until the point in time where the time zero value of the output, s, reaches $S^*(\theta)$. When $S^*(\theta) < s \leq S^*(\overline{\theta})$ the compensation is dependent on s, only. The compensation is increasing in s. However, note that the compensation never can be higher than $\overline{\theta}$. The reason is that the principal knows that the investment cost is not higher than the upper level $\overline{\theta}$.

Equation (26) shows that the agent's private information results in a loss (relative to no private information) for the principal as long as $s < S^*(\overline{\theta})$. However, the compensation function leads to a second-best solution only for a part of the interval where $s < S^*(\overline{\theta})$. For some values of s the compensation function gives a sharing rule between the parties without leading to an inefficient investment strategy.

We can find the loss due to a second-best investment strategy by defining $L(s, \theta) = W_{sym}(s, \theta) + V_{sym}(s, \theta) - (\tilde{W}(s, \theta) + V(s, \theta))$. The notation $\tilde{W}(\cdot)$ is used about the principal's value from the project, for a given cost θ.

The agent's value of the project, $V(s, \theta)$, derived from equations (8), (25)

[2] $S^*(\theta)$ is a one-to-one function as long as it is continuous and strictly increasing in the interval $S^*(\theta) \in [S^*(\underline{\theta}), S^*(\overline{\theta})]$.

and (26), is

$$V(s,\theta)=\begin{cases}\int_{S^*(\theta)}^{S^*(\overline{\theta})}\vartheta'(S^*(u))\frac{\phi(s)}{\phi(S^*(u))}dS^*(u) & \text{if } s<S^*(\theta)\\ \vartheta(s)-\theta \\ +\int_{s}^{S^*(\overline{\theta})}\vartheta'(S^*(u))\frac{\phi(s)}{\phi(S^*(u))}dS^*(u) & \text{if } S^*(\theta)\le s<S^*(\overline{\theta})\\ \overline{\theta}-\theta & \text{if } s\ge S^*(\overline{\theta})\end{cases} \tag{27}$$

In section 5 it was shown that the agent's value from the investment is zero under symmetric information about the investment cost. Equation (27) states that the agent's value from the investment when he has private information about the cost, is positive as long as his investment cost is below $\overline{\theta}$. The agent's share of the total value of the investment, is larger the larger s is. However, the agent's value from the project will never exceed $\overline{\theta}-\theta$.

The principal's value of the investment option for a given θ is represented by

$$\tilde{W}(s,\theta)=\begin{cases}\frac{\phi(s)}{\phi(S^*(\theta))}\left(S^*(\theta)-\theta\right) \\ -\int_{S^*(\theta)}^{S^*(\overline{\theta})}\vartheta'(S^*(u))\frac{\phi(s)}{\phi(S^*(u))}dS^*(u) & \text{if } s<S^*(\theta)\\ s-\vartheta(s) \\ -\int_{s}^{S^*(\overline{\theta})}\vartheta'(S^*(u))\frac{\phi(s)}{\phi(S^*(u))}dS^*(u) & \text{if } S^*(\theta)\le s<S^*(\overline{\theta})\\ s-\overline{\theta} & \text{if } s\ge S^*(\overline{\theta}).\end{cases} \tag{28}$$

As is to be expected, the principal's time zero value is lower under asymmetric information than under the symmetric information case (compare (28) and (17)). The reason is that the investment occurs at a later time, and at a higher cost (as the compensation is higher than the true investment cost), thereby lowering the value of the investment. The principal's loss will, however, never be higher than the loss in the interval $s>S^*(\overline{\theta})$, i.e., it will not exceed $W_{sym}(s,\theta)-\tilde{W}(s,\theta)=\overline{\theta}-\theta$.

Below it is shown that the dead-weight loss is larger than zero when $s>S^*(\theta)$:

$$L(s,\theta)=\begin{cases}\frac{\phi(s)}{\phi(S^*_{sym}(\theta))}\left(S^*_{sym}(\theta)-\theta\right) \\ -\frac{\phi(s)}{\phi(S^*(\theta))}\left(S^*(\theta)-\theta\right)) \\ & \text{if } s<S^*_{sym}(\theta)\\ s-\theta-\frac{\phi(s)}{\phi(S^*(\theta))}\left(S^*(\theta)-\theta\right) & \text{if } S^*_{sym}(\theta)\le s<S^*(\theta)\\ 0 & \text{if } s\ge S^*(\theta).\end{cases} \tag{29}$$

The total dead-weight loss is 0 when $s\ge S^*(\theta)$ because in this range the agent's investment strategy leads to the same decision as in the full information case, and the contracted compensation function only gives a sharing rule between the principal and the agent. The agent's gain exactly equals the principal's loss because of the asymmetric information.

8 Illustration of the results

The preceding sections used a general diffusion (equation (1)) for the output process S_t, and an unspecified probability density $f(\tilde{\theta})$ for the assessed investment cost $\tilde{\theta}$. To illustrate the results the simple uniform distribution and the geometric Brownian motion are assumed for the investment cost and the income process, respectively.

A uniform distribution implies that $F(\theta)/f(\theta) = \theta - \underline{\theta}$. The geometric Brownian motion process of the value of the income is represented by

$$dS_t = (r-\delta)S_t dt + \sigma S_t dw_t, \quad S_0 = s, \tag{30}$$

under the equivalent martingale measure Q. The strictly positive and increasing solution $\phi(\xi)$ to the ordinary differential equation (compare equations (5) and (6)),

$$\frac{1}{2}\sigma^2\xi^2 u_{\xi\xi}(\xi) + (r-\delta)\xi u_\xi(\xi) - ru(\xi) = 0$$

is then found to equal $\phi(\xi) = \xi^\beta$, where

$$\beta = \frac{1}{\sigma^2}\left[\frac{1}{2}\sigma^2 - (r-\delta) + \sqrt{\left((r-\delta)-\frac{1}{2}\sigma^2\right)^2 + 2r\sigma^2}\right] > 1.$$

Hence, the solution to the expectation $E_0[e^{-r\tau_K}]$ is (using equation (5)),

$$E_0[e^{-r\tau_K}] = \begin{cases} \frac{\phi(s)}{\phi(S^*(\theta))} = \left(\frac{s}{S^*(\theta)}\right)^\beta & \text{if } s < S^*(\theta) \\ 1 & \text{if } s \geq S^*(\theta). \end{cases} \tag{31}$$

For the benchmark symmetric information case, the right-hand side of equation (16) becomes S^*_{sym}/β, and hence the optimal critical value for investment is $S^*_{sym}(\theta) = \theta\beta/(\beta-1) > \theta$, as $\beta > 1$. From equation (17), the corresponding value of the investment opportunity is $W_{sym}(s,\theta) = (s/S^*_{sym})^\beta \left(S^*_{sym}(\theta) - \theta\right) = \theta/(\beta-1)(s/S^*_{sym})^\beta$ for $s < S^*_{sym}(\theta)$. Recall that the agent obtains no profit under symmetric information, i.e., $V_{sym}(s,\theta) = 0$.

For the asymmetric information case, however, the optimal "trigger income" is found by equation (23), to be

$$S^*(\theta) = (2\theta - \underline{\theta})\frac{\beta}{\beta-1}, \tag{32}$$

which (when $\theta > \underline{\theta}$) is higher than the trigger under symmetric information, $S^*_{sym}(\theta) = \theta\beta/(\beta-1)$. The fraction $\beta/(\beta-1) > 1$ causes a wedge between the critical value for exercising the investment opportunity and the principal's cost of the investment, even in the case of symmetric information. The difference $(\theta - \underline{\theta})\beta/(\beta-1)$ is the increase in the trigger income caused by asymmetric information.

The variable $\vartheta(S^*(\theta)) \equiv \theta$, equals by equation (32),

$$\vartheta(S^*(\theta)) = \frac{1}{2}\left(S^*(\theta)\frac{\beta-1}{\beta} + \underline{\theta}\right).$$

In order to find the expression for the compensation function $G(s)$, we first insert the above variables into the integration in the second equality in (26). This leads to

$$\int_s^{S^*(\overline{\theta})} \frac{1}{2}\frac{\beta}{\beta-1}\left(\frac{s}{S^*(u)}\right)^\beta dS^*(u) = \frac{1}{2}\left[\frac{s}{\beta} - \left(\frac{s}{S^*(\overline{\theta})}\right)^\beta \frac{\beta}{S^*(\overline{\theta})}\right].$$

In addition, observe that $\vartheta(s)$ in (26) equals $\vartheta(s) = 1/2(s(\beta-1)/\beta + \underline{\theta})$. This gives

$$G(s) = \begin{cases} 0 & \text{if } s < S^*(\theta) \\ \frac{1}{2}\left[s + \underline{\theta} \right. \\ \left. - \left(\frac{s}{S^*(\overline{\theta})}\right)^\beta \left(S^*(\overline{\theta}) - (2\overline{\theta} - \underline{\theta})\right)\right] & \text{if } S^*(\theta) \le s < S^*(\overline{\theta}) \\ \overline{\theta} & \text{if } s \ge S^*(\overline{\theta}), \end{cases} \tag{33}$$

Further, we find that the time zero value of the agent's and the principal's value functions (equations (27) and (28)), are

$$V(s,\theta) = \begin{cases} \left(\frac{s}{S^*(\theta))}\right)^\beta \frac{1}{2}\left[S^*(\theta) - (2\theta - \underline{\theta}) \right. \\ \left. - \left(\frac{S^*(\theta)}{S^*(\overline{\theta})}\right)^\beta \left(S^*(\overline{\theta}) - (2\overline{\theta} - \underline{\theta})\right)\right] & \text{if } s < S^*(\theta) \\ \\ \frac{1}{2}\left[s - (2\theta - \underline{\theta}) \right. \\ \left. - \left(\frac{s}{S^*(\overline{\theta})}\right)^\beta \left(S^*(\overline{\theta}) - (2\overline{\theta} - \underline{\theta})\right)\right] & \text{if } S^*(\theta) \le s < S^*(\overline{\theta}) \\ \\ \overline{\theta} - \theta & \text{if } s \ge S^*(\overline{\theta}), \end{cases} \tag{34}$$

and

$$\tilde{W}(s,\theta) = \begin{cases} \left(\frac{s}{S^*(\theta)}\right)^\beta \frac{1}{2}\left[S^*(\theta) - \underline{\theta} \right. \\ \left. + \left(\frac{S^*(\theta)}{S^*(\overline{\theta})}\right)^\beta \left(S^*(\overline{\theta}) - (2\overline{\theta} - \underline{\theta})\right)\right] & \text{if } s < S^*(\theta) \\ \\ \frac{1}{2}\left[s - \underline{\theta} \right. \\ \left. + \left(\frac{s}{S^*(\overline{\theta})}\right)^\beta \left(S^*(\overline{\theta}) - (2\overline{\theta} - \underline{\theta})\right)\right] & \text{if } S^*(\theta) \le s < S^*(\overline{\theta}) \\ \\ s - \overline{\theta} & \text{if } s \ge S^*(\overline{\theta}), \end{cases} \tag{35}$$

respectively.

Observe that the total combined value for the principal and the agent is

$$\tilde{W}(s,\theta) + V(s,\theta) = \begin{cases} \left(\frac{s}{S^*(\theta)}\right)^\beta (S^*(\theta) - \theta) & \text{if } s \le S^*(\theta) \\ s - \theta & \text{if } s > S^*(\theta) \end{cases} \tag{36}$$

in the case of asymmetric information. Similar expressions held for the symmetric information case as well, but with $S^*(\theta)$ replaced by $S^*_{sym} < S^*(\theta)$.

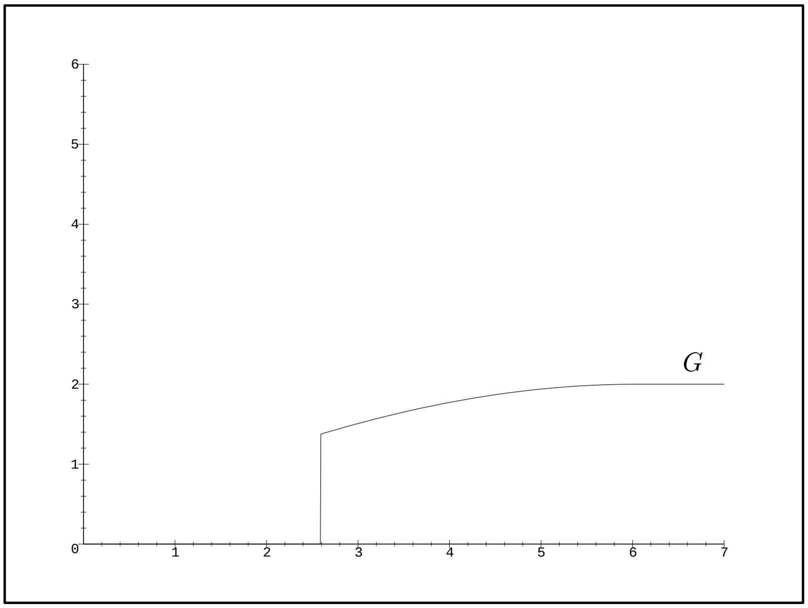

Figure 1: The compensation G as a function of s.

These relations are consistent with (29), which in the case the assumptions of a geometric Brownian motion and a uniform density, equals

$$L(s,\theta) = \begin{cases} \left(\frac{s}{S^*_{sym}(\theta)}\right)^{\beta}\left(S^*_{sym}(\theta)-\theta\right) \\ -\left(\frac{s}{S^*(\theta)}\right)^{\beta}\left(S^*(\theta)-\theta\right)) & \text{if } s < S^*_{sym}(\theta) \\ s-\theta-\left(\frac{s}{S^*(\theta)}\right)^{\beta}\left(S^*(\theta)-\theta\right) & \text{if } S^*_{sym}(\theta) \le s < S^*(\theta) \\ 0 & \text{if } s \ge S^*(\theta). \end{cases} \quad (37)$$

The results are illustrated graphically. In the base case the investment cost θ is set to 1, the lower level cost $\underline{\theta} = 0.5$, and the upper level cost $\overline{\theta} = 2$. For the parameters of the output process we set the risk-free rate $r = 0.04$, the convenience yield $\delta = 0.03$, and the volatility $\sigma = 0.1$. With a uniformly distributed investment cost, and an output process that follows a geometric Brownian motion, these parameters lead to $\beta = 2.37$, $S^*_{sym}(\theta) = 1.73$, $S^*(\theta) = 2.59$, and $S^*(\overline{\theta}) = 6.05$.

In figure 1 the compensation is plotted as a function of s. The compensation is zero when s is lower than the critical value of investment, $S^*(\theta) = 2.59$, as the compensation is not paid prior to the investment time. Therefore, at $S^*(\theta)$ the function jumps to the amount paid when $s \ge S^*(\theta)$, and it is increasing from

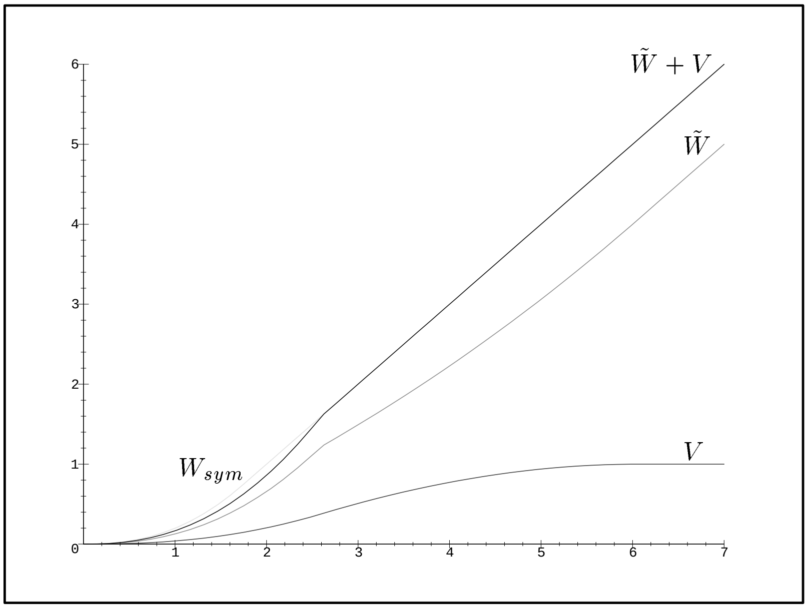

Figure 2: W_{sym}, $\tilde{W}+V$, $\tilde{W}$ and V as functions of s.

this point until $s = S^*(\overline{\theta}) = 6.05$. For $s \geq 6.05$ the compensation is constant at its maximum level $\overline{\theta} = 2$.

Both within regulation and corporate finance we often find that compensation functions are linear in the observable output from a project. In the numerical example given here the compensation function is concave. The reason is that the upper level for the cost has a significant effect. If the upper level the cost had been very high, the compensation function would have approached a linear function of s.

In figure 2 the principal's and the agent's value functions are shown as functions of s. The principal's value function under symmetric information is convex when $s < S^*(\theta) = 2.59$, and it is linear in the interval where the optimal decision is to invest immediately. This corresponds to the value of a "standard" real option as a function of the output price. Under asymmetric information, it is also the case that the principal and the agent have convex value functions in the interval where it is ex ante profitable to postpone the investment. This is for the same reason as under symmetric information: a volatility higher than zero implies a possibility of higher profitability in the future.

In the interval $S^*(\theta) \leq s < S^*(\overline{\theta})$ the agent's value is concave for the same reason as for the concavity in the compensation function: the upside potential for future profit is limited. For $s \geq S^*(\overline{\theta})$ the principal alone benefits from higher s, and the agent's value of the contract is constant at $\overline{\theta} - \theta = 1$.

Since the agent's value of information leaves less profit to the principal, and the agent's value function is concave in the interval $[S^*(\theta), S^*(\overline{\theta}))$, the

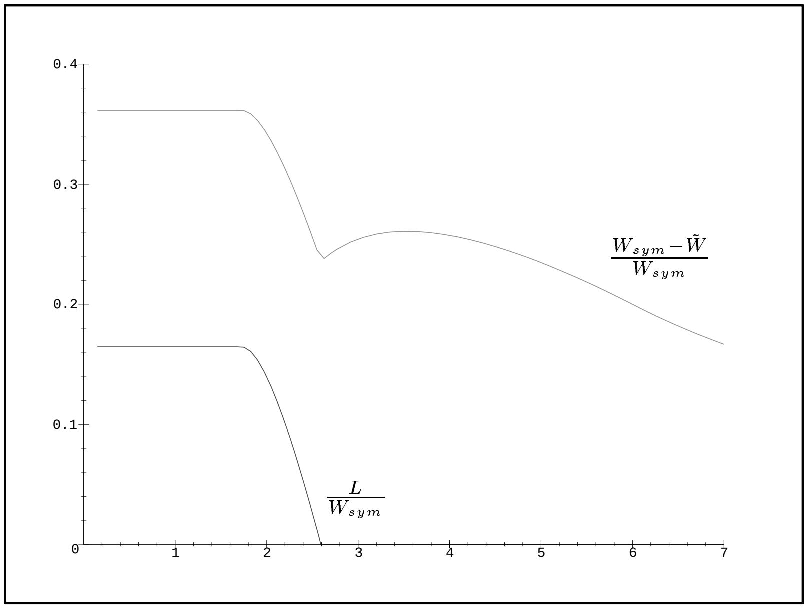

Figure 3: Principal's loss and dead-weight loss as functions of s.

principal's value is convex in the same interval. When $s \geq S^*(\overline{\theta})$, the principal's value under asymmetric information increases linearly, as the agent's value of information is zero in this interval.

Figure 2 also shows the sum of the principal's and the agent's value functions under asymmetric information, $\tilde{W}(s, \theta) + V(s, \theta)$. As long as s is higher than or equal to $S^*(\theta) = 2.59$, this curve is identical with the principal's value under symmetric information, $W_{sym}(s, \theta)$. The reason is that in this interval the contract between the principal and the agent gives a sharing rule without having any effect on the investment strategy compared to the situation of full information. In the interval $(0, S(\theta))$, $\tilde{W}(s, \theta) + V(s, \theta)$ is lower than $W_{sym}(s, \theta)$ due to an inefficient investment strategy. This fact is also illustrated in figure 3, where the relative dead-weight loss as a function of s is plotted in the lower curve. The relative dead-weight loss is defined as $(W_{sym} - \tilde{W} - V)/W_{sym})$.[3] The figure shows that dead-weight loss is positive when $s < S^*(\theta) = 2.59$. In addition, we see that in our example the dead-weight loss is about 16 per cent of the value in the case of no private information, and when $s < S^*_{sym} = 1.73$.

In figure 3 the principal's relative loss, $(W_{sym} - \tilde{W})/W_{sym}$, is plotted in the upper curve. Both the principal's relative loss, and the relative dead-weight loss is constant as long as the best decision under both asymmetric and symmetric information is to postpone the investment, i.e., when $s < S^*_{sym}(\theta) = 1.73$.

The losses are decreasing in the interval $[S^*_{sym}(\theta), S^*(\theta))$, since the inefficiency in the second-best investment strategy is decreasing as s approaches

[3]In the figures, the notation V_{sym} is not included as $V_{sym} = 0$.

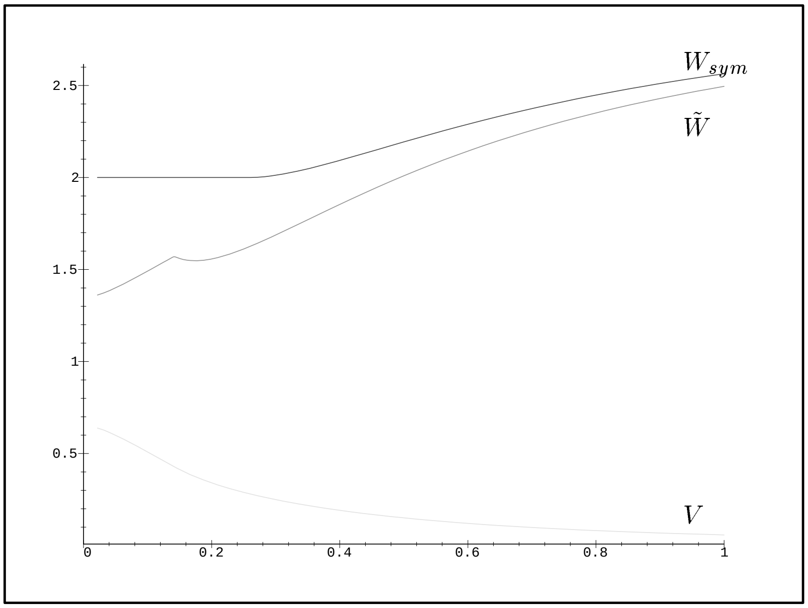

Figure 4: W_{sym}, $\tilde{W}$, and V as functions of σ, $s = 1$.

$S^*(\theta) = 2.59$. For all s higher than this point the investment strategy is the same for the symmetric and the asymmetric information case, i.e., there is no dead-weight loss.

In the interval $[S^*(\theta), S^*(\overline{\theta}))$, the principal's relative loss first increases and then decreases. The reason is that two effects pull in opposite directions: higher s leads to higher difference between the principal's values under symmetric and asymmetric information, which increases the relative loss, whereas an upper limit for the investment cost tends to decrease the agent's value of information as s gets closer to $S^*(\overline{\theta})$.

Figure 4 plots the parties' functions of σ when $s = 3$. In the "standard" real option problem of valuing an investment possibility, corresponding to the value of $W_{sym}(s, \theta)$, the value is increasing with respect to σ in the interval where the best decision is to postpone the investment. The reason is that as long as the option is not exercised, higher volatility increases the possibility of a higher future profit.

The principal's value function under asymmetric information depends on σ also in the interval where the optimal decision is to invest immediately, i.e., the interval $s \geq S^*(\theta)$, corresponding to $\sigma \leq 0.14$. The reason is connected to the agent's value of information: as σ increases, the agent's value of information decreases, and therefore the share of the profit left to the principal is increasing. The agent's value is decreasing in σ because of the upper limit on the agent's compensation.

For $s < S^*(\theta)$, corresponding to $\sigma > 0.14$, there is an additional effect on

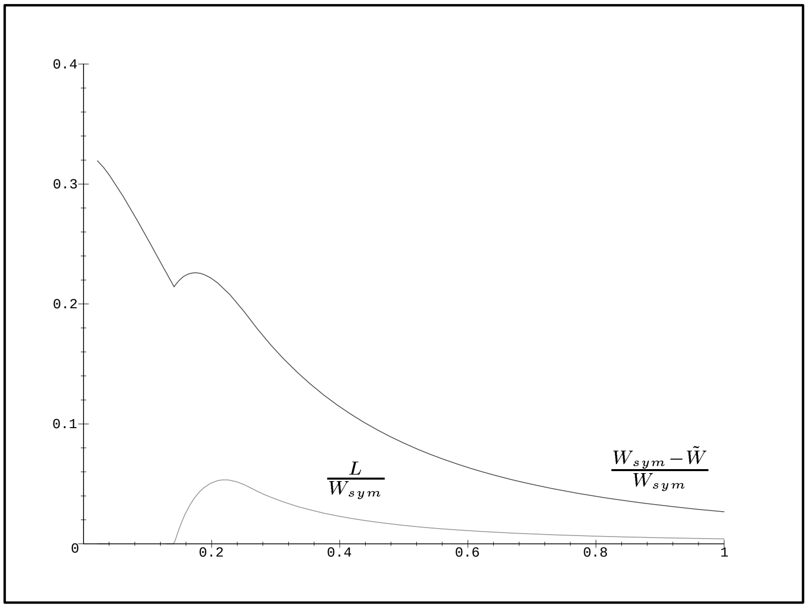

Figure 5: Principal's loss and dead-weight loss as a function of σ, $s = 1$.

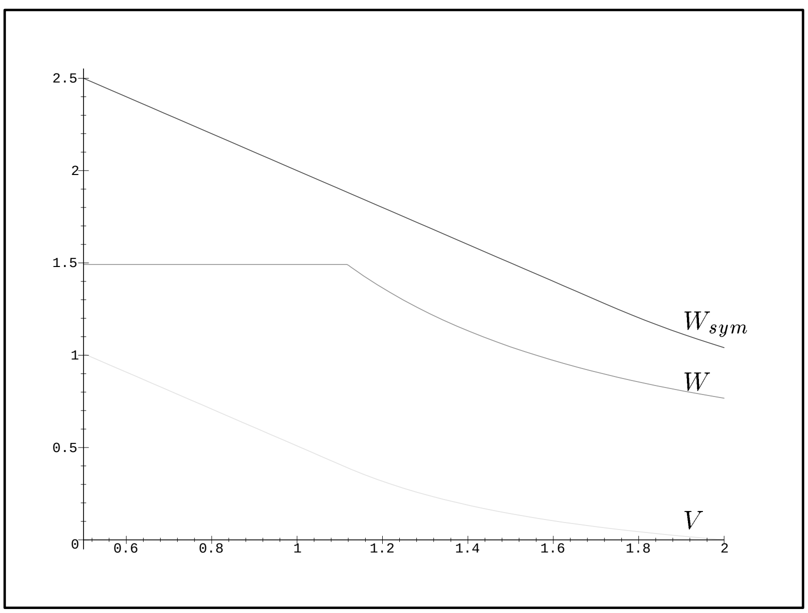

Figure 6: W_{sym}, $\tilde{W}$, and V as functions of θ, $s = 1$.

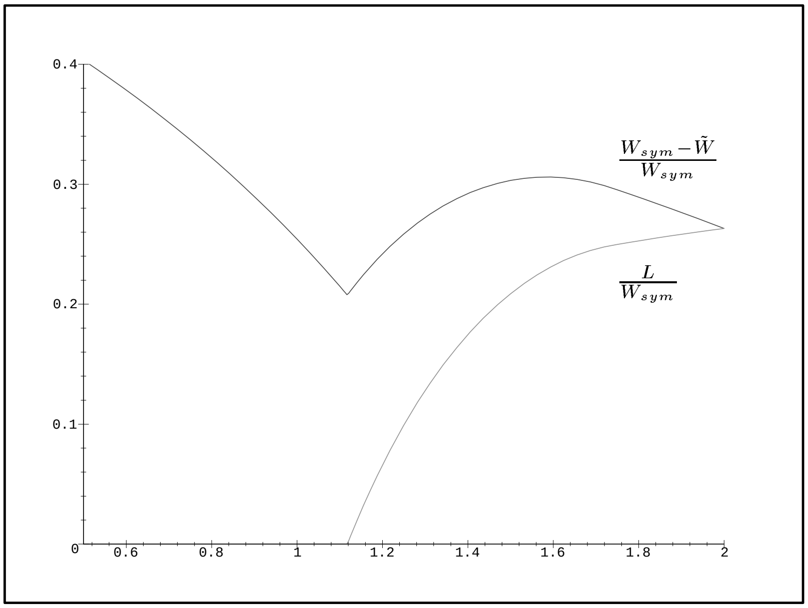

Figure 7: Principal's loss and dead-weight loss as a function of θ, $s = 1$.

the principal's value under asymmetric information, which tends to depress the principal's value: the loss in value because of an inefficient investment strategy. This effect is dominating when σ is between 0.14 and 0.18.

The same effects are reflected in figure 5. At the σ corresponding to $S^*(\theta) = 3$ the relative dead-weight loss gets positive, because then it reaches the interval $s < S^*(\theta)$, in which we know that the loss is positive. Both the relative dead-weight loss and the relative principal's loss increase in this interval as long as the effect of a second-best investment strategy dominates the effect that the agent's value of information decreases with an increasing volatility.

The principal's loss when $s \geq S^*(\theta)$ decreases because of the agent's decreasing value of information as σ increases.

Figure 6 plots the principal's and the agent's values as functions of the investment cost, θ. Both the principal's and the agent's value functions are nonincreasing with respect to θ, as a higher cost lowers the value of the investment for both. For $\theta \leq 1.1$, corresponding to $s \geq S^*(\theta)$, the principal's value is independent of the agent's investment cost. The reason is that the compensation paid to the agent cannot be a function of the unobservable variable θ.

Figure 7 shows that the relative dead-weight loss is increasing in θ. This is because higher costs lead to higher critical values for exercising the option, and thereby larger inefficiency in the investment decision.

The principal's relative loss is decreasing in θ for θ lower than or equal to 1.1, corresponding to $s \geq S^*(\theta)$. Once again the reason is connected to the fact that when $s \geq S^*(\theta)$, $\tilde{W}$ is independent of θ, and therefore an increase in θ

results in a corresponding increase in the principal's loss. For θ corresponding to $s < S^*(\theta)$, the dominating effect is the same as in the dead-weight loss as long as θ is lower than 1.6. For θs higher than 1.6, the dominating effect is the agent's value of information getting lower the closer to the upper level cost the true investment cost is. This tends to decrease the loss.

At $\theta = \overline{\theta}$ the principal's loss and the dead-weight loss coincide, as the value of the agent's information is zero at this point.

9 Conclusion

In this article, we study effects of asymmetric information on an optimal stopping problem. A principal owns an investment opportunity and delegates the investment strategy of the project to an agent. The agent has private information about the investment cost, whereas the stochastic output is common knowledge.

This setting may apply to a number of situations, both within regulation (the principal is a regulator, and the agent is a company) and corporate finance (shareholders represent the principal, and managers represent the agent).

The agent's private information about the cost implies that it is optimal for the principal to compensate the agent according to his value of private information. Thus, the compensation will be higher than the true investment cost in most cases, thereby increasing the principal's cost of his investment opportunity. A higher cost leads to a higher critical value for investment. Thus, it is found that the agent's private information about the investment cost may lead to underinvestment.

The agent's value of private information will, however, not always lead to an inefficient investment strategy. Inefficient decisions will occur only in the interval where the critical value of investment, given asymmetric information, is higher than the time zero value of the output from the investment. If the time zero value of the output is higher than the critical value of investment, the compensation function only gives a rule for sharing the profit between the principal and the agent, without having any inefficiency effects.

In the same way as asymmetric information about investment may depress activity, an agent's private information about the costs of shutting down an activity, may lead to higher activity than when there is no private information.

More generally, in an model where one can switch between options, private information about switching costs lead to higher costs and therefore fewer switches. For instance, in Dixit (1989) entry and exit decisions of production are discussed. In this model Dixit finds that entering and exiting an activity leads to a "hysteresis band" due to the uncertainty of future outcome and to the irreversible entry and exit costs. If an agent has private information about the costs of switching between activity and no activity, the hysteresis band will be larger than in Dixit's model. Thus, the costs of switching between the two options may lead to both too much and too little activity. Thus, on a macroeconomic level, even though the level of activity when there is private information should happen to be not far from the aggregate level when we have no private information, the activity may not necessarily take place in the activities where the profit is highest.

A switching option model can also be applied for financial investments. An example is the holder of a fund who delegates the trading strategy of the financial portfolio to an agent, and where there are some transaction costs. If the agent has private information about some fixed transaction costs, the investor can use a variant of the method described in this article to design the compensation to the agent in such a way as to optimize the agent's risk management.

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Appendix

A The second-order condition for incentive compatibility

The second-order condition for K must be satisfied at $K(\theta) = \theta$, i.e., the function $V(s,\theta)$ must be more convex than $V(K(\theta);\theta)$ and

$$\left.\frac{\partial V(s,K(\theta);\theta)}{\partial K(\theta)}\right|_{K(\theta)=\theta} \leq \frac{\partial V(s,\theta)}{\partial \theta}. \tag{38}$$

The first-order condition of $V(s,K)$ with respect to K is given by

$$\frac{\partial V(s,K;\theta)}{\partial K} = G_{\hat{S}}(\hat{S}(K))\hat{S}'(K) - \frac{\phi_{\hat{S}}(\hat{S}(K))}{\phi(\hat{S}(K))}\left(G(\hat{S}(K)) - \theta\right)\hat{S}'(K) = 0 \tag{39}$$

Differentiating the first-order condition in equation (39) when $K(\theta) = \theta$, with respect to θ yields,

$$\begin{aligned} &\frac{\partial^2 V(s;\theta)}{\partial \theta^2} \\ &= \frac{\phi(s)}{\phi(\hat{S}(\theta))}\Big\{2\left(\frac{\phi_{\hat{S}}(\hat{S}(\theta))}{\phi(\hat{S})}\right)^2(\hat{S}'(\theta))^2(G(\hat{S}(\theta)-\theta) \\ &\quad -\frac{\phi_S(\hat{S}(\theta))}{\phi(\hat{S})}\left(\hat{S}''(\theta)(G(\hat{S}(\theta))-\theta)+2(\hat{S}(\theta))^2 G_{\hat{S}}(\hat{S}(\theta)) - \hat{S}'(\theta)\right) \\ &\quad -\frac{\phi_{\hat{S}\hat{S}}(\hat{S}(\theta))}{\phi(\hat{S}(\theta))}(\hat{S}'(\theta))^2(G(\hat{S}(\theta)-\theta) + G_{\hat{S}\hat{S}}(\hat{S}(\theta))(\hat{S}'(\theta))^2 \\ &\quad +G_{\hat{S}}(\hat{S}(\theta))\hat{S}''(\theta)\Big\} \leq 0. \end{aligned} \tag{40}$$

This leads to the second-order condition (using the restriction in (38)),

$$\frac{\partial V(s,\theta)}{\partial \theta} - \left.\frac{\partial V(s,K(\theta);\theta)}{\partial K(\theta)}\right|_{K(\theta)=\theta} = \frac{\phi(s)}{\phi(\hat{S}(\theta))}\frac{\phi_{\hat{S}}(\hat{S}(\theta))}{\phi(\hat{S}(\theta))}\hat{S}'(\theta) \geq 0.$$

B Deriving equality (20)

$$\int_{\underline{\theta}}^{\overline{\theta}} \left(\int_{\theta}^{\overline{\theta}} \frac{\phi(s)}{\phi(\hat{S}(u))} du \right) f(\theta) d\theta = \left[\int_{\theta}^{\overline{\theta}} \frac{\phi(s)}{\phi(\hat{S}(u))} du F(\theta) \right]_{\underline{\theta}}^{\overline{\theta}} - (-) \int_{\underline{\theta}}^{\overline{\theta}} \frac{\phi(s)}{\phi(\hat{S}(u))} F(\theta) d\theta.$$

By inserting the bounds $\overline{\theta}$ and $\underline{\theta}$ in the first term on the right-hand side, we see that this term is zero: substituting θ with $\overline{\theta}$ yields $\int_{\underline{\theta}}^{\overline{\theta}} \phi(s, \hat{S}(u)) du = 0$, and substituting θ with $\underline{\theta}$ yields $F(\underline{\theta}) = 0$. Thus, we are left with the right-hand side term of equation (20).

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