Financial Distress and Idiosyncratic Volatility: An Empirical Investigation

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Abstract

We address the twin puzzles of anomalously low returns for high idiosyncratic volatility and high distress risk stocks, documented by Ang, Hodrick, Xing and Zhang (2006) and Campbell, Hilscher and Szilagyi (2005), respectively. We accomplish two objectives in this study. First, we investigate the link between idiosyncratic volatility and distress risk and find that the idiosyncratic volatility effect exists only *conditionally* on high distress risk. Second, using a corrected single-beta CAPM model, we provide a rational explanation for the twin puzzles. Joint statistical tests cannot reject the null hypothesis of zero abnormal returns across the idiosyncratic volatility and distress risk portfolios, for the corrected model.

Keywords: Distress risk, idiosyncratic volatility, single-beta CAPM

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1 Introduction

Distress, bankruptcy, default, and volatility–these terms are often synonymous with financial hardship. Yet studies of these topics reveal mixed messages and anomalies. It is crucial to understand these concepts, since they relate to the smooth functioning of financial markets, an issue that affects institutional investors, governments and individuals alike.

In this paper we examine distress and idiosyncratic volatility effects, and propose a rational explanation to two puzzles. Recent research on distress and idiosyncratic volatility has uncovered several fundamental puzzles, two in particular on which we focus. First, Ang, Hodrick, Xing and Zhang (2005) discover, unexpectedly, that stocks with high idiosyncratic volatility relative to the Fama-French (1993) model earn anomalously low returns. At the same time, there have been similar findings in another line of research focusing on the effect of bankruptcy (distress) risk on stock returns. For example, Dichev (1998) and Campbell, Hilscher, and Szilagyi (2005) document that stocks with high likelihood of distress receive anomalously low returns. This suggests that some risky (distressed) stocks do not receive compensatory returns.

There is an intuitive reason to believe that these two puzzles are related to each other. According to the Merton (1974) model, corporate debt is a risk-free bond less a put option on the value of the firm's assets, with strike price of the face value of the debt. Thus, a firm with more volatile equity is more likely to reach the boundary condition for default. Based on this argument, Campbell and Taksler (2003) show that idiosyncratic firm-level volatility can explain a significant part of cross-sectional variation in corporate bond yields. Given the evidence that stocks with high idiosyncratic volatility receive anomalously low returns, it may be possible that the idiosyncratic volatility puzzle is just an expression of the distress risk puzzle.

We first investigate the link between the idiosyncratic volatility effect and distress effect by sequential sorting. We proxy for firms' distress risk by Altman's (1968) Z-score and Ohlson's (1980) O-score. We control for the distress effect by first sorting stocks into quintiles according to their Z-score or O-score, then within each quintile, sorting again into portfolios based on firms' idiosyncratic volatility. After controlling for distress risk, stocks with high idiosyncratic volatility earn significantly lower returns than low idiosyncratic volatility stocks, only in the highest distress risk quintile. This result confirms our conjecture that the idiosyncratic volatility puzzle is closely related to the distress effect. In other words, the idiosyncratic volatility effect exists conditional on high distress risk. Our first contribution here is to establish a bridge between the idiosyncratic volatility and distress risk puzzles.

After we build this bridge, we move forward to investigate an appealing, rational explanation for the twin puzzles. For this purpose, we apply a methodology based on the work of Ferguson and Shockley (2003). Following the line of Roll's (1977) critique, Ferguson and Shockley (2003) observe that using an equity-only proxy for the market portfolio will understate equity betas, and that this understatement is an increasing and convex function of firm leverage. Due to this convexity, the beta estimation error should be more pronounced for more distressed firms. It then follows that in the cross section, the equity beta estimation errors will not be random. They will be systematically related to the relative leverage and relative distress of each firm in the sample. As a result, the model predicts that firm leverage and financial distress will capture the convex beta estimation errors induced by the use of an equity-only market proxy.

Our approach here is to apply a corrected single-beta CAPM model to address the issue of anomalously low returns on the most volatile and most distressed stocks. It turns out that when we use the corrected single-beta CAPM to adjust stock returns, the spread between high and low idiosyncratic volatility stocks, and spread between high and low distress risk stocks become insignificantly different from zero. Moreover, the test of Gibbons, Ross and Shanken (GRS, 1989) cannot jointly reject the null hypothesis of zero abnormal returns across the idiosyncratic volatility portfolios and distress risk portfolios. The second contribution in our paper, therefore, is to provide a rational explanation to the idiosyncratic volatility puzzle and distress risk puzzle.

The rest of this paper is organized as follows. Section 2 investigates the link between the idiosyncratic volatility and distress risk puzzles. We do sequential sorting to examine the idiosyncratic volatility effect, controlling for distress risk, and vice versa. We thereby forge a link between the idiosyncratic volatility and distress risk puzzles. In Section 3, we develop a simple, rational explanation for the twin puzzles. Specifically, we use a corrected single-beta CAPM model to adjust stock returns and implement joint statistical tests to determine whether the model can explain the abnormal returns across the idiosyncratic volatility portfolios and distress risk portfolios. Section 4 concludes.

2 Empirical relationship between the idiosyncratic volatility and distress risk effects

This section is devoted to studying the link between the idiosyncratic volatility and distress effects on the cross-section of stock returns. As mentioned above, Ang, Hodrick, Xing and Zhang (2005) discover that stocks with high idiosyncratic volatility earn low returns. Specifically, the return differential between low- and high-idiosyncratic volatility stocks is 1.31% per month. This result holds even after controlling for aggregate volatility risk, size, book-to-market, momentum, coskewness, dispersion in analysts' forecasts, and liquidity effects. A parallel finding exists in the distress risk literature. In particular, several papers on distress risk document that firms with high likelihood of distress also receive anomalously low returns (see for example, Dichev (1998) and Campbell, Hilscher and Szilagyi (2005)).

These puzzles are related to each other. Consider the Merton (1974) formalization of corporate debt as a risk-free bond less a put option on the value of firm assets, with strike price of the face value of the debt. Consequently, a firm with more volatile equity is more likely to reach the boundary for default. Developing this logic, Campbell and Taksler (2003) show that idiosyncratic firm-level volatility explains a significant part of cross-sectional variation in corporate bond yields.

Our insight is to develop this logic even further. In particular, we recognize that stocks with high idiosyncratic volatility have two empirical features and an important theoretical feature. The two empirical features are low returns (from Ang, Hodrick, Xing and Zhang (2003)), and a strong relation to the return on corporate bonds (from Campbell and Taksler (2003)). The theoretical feature, as mentioned above, is that high volatility corresponds to an increased likelihood of default or distress. Combining these theoretical and empirical features, it is a plausible conjecture that the idiosyncratic volatility puzzle might simply reflect distress risk.

2.1 Definition of idiosyncratic volatility and distress risk

Our measures of idiosyncratic volatility and distress risk are similar to those used in the literature. Regarding idiosyncratic volatility, we use the same approach as Ang, Hodrick, Xing and Zhang (2005). That is, we estimate idiosyncratic volatility relative to the Fama and French (1993) model as follows:

$$r_t^i = \alpha^i + \beta^i_{MKT} MKT_t + \beta^i_{SMB} SMB_t + \beta^i_{HML} HML_t + \epsilon_t^i \tag{1}$$

The idiosyncratic volatility is then $\sqrt{var(\epsilon_t^i)}$.

Regarding distress risk measures, we use Altman's (1968) Z-score and Ohlson's (1980) O-score. These two models are popular frameworks for bankruptcy prediction and have been widely used in empirical research and practice. For example, Griffin and Lemmon (2002) use O-score to examine the relationship between book-to-market equity, distress risk, and stock returns. Dichev (1998) investigates whether the risk of bankruptcy is a systematic risk using Z-score and O-score. Indeed, both Z-score and O-score are good for sensitivity analysis since they are derived in different time periods, using different independent variables and different predictive methodologies. The definitions are as follows. Z-score is given by:

$$Z = 1.2\frac{\text{working capital}}{\text{total assets}} + 1.4\frac{\text{retained earnings}}{\text{total assets}} + 3.3\frac{\text{earnings before interest and taxes}}{\text{total assets}}$$

$$+0.6\frac{\text{market value of equity}}{\text{book value of total liabilities}} + 1.0\frac{\text{sales}}{\text{total assets}}.$$

Ohlson'(1980) O score is defined as:

$$O = -1.32 - 0.407\log(\text{total assets}) + 6.03\frac{\text{total liabilities}}{\text{total assets}} - 1.43\frac{\text{working capital}}{\text{total assets}}$$

$$+0.076\frac{\text{current liabilities}}{\text{current assets}} - 1.72(1 \text{ if total liabilities} > \text{total assets, else } 0)$$

$$-2.37\frac{\text{net income}}{\text{total assets}} - 1.83\frac{\text{funds from operations}}{\text{total liabilities}} + 0.285(1 \text{ if net loss}$$

$$\text{for last two years, else } 0) - 0.521\frac{\text{net income}_t - \text{net income}_{t-1}}{|\text{net income}_t| + |\text{net income}_{t-1}|}.$$

Both of these models are quite accurate in predicting bankruptcy. From the definition of Z-score and O-score, we know that Z-score is a measure of financial strength, in the sense that higher Z means lower probability of bankruptcy, and O-score is a measure of financial distress(higher O means higher probability of bankruptcy).

Both Altman's and Ohlson's models are derived for industrials, so our sample consists of all industrial firms available simultaneously on the NYSE, AMEX CRSP tapes and the COMPUSTAT annual industrial and research tapes for the period of 1981 to 2005. Since COMPUSTAT discontinues reporting the data item of firms' funds from operations after 2000, we limit our O-score sample to the period from 1981 to 2000. The average number of firms per year in our study is 1036 firms for Z-score, and 1124 firms for O-score. The reason for restricting our sample to post-1980 years is that Ohlson's model becomes available in 1980, and the post-1980 years contain substantially more CRSP-COMPUSTAT matched firms for our study of distress risk. Data in Altman (1993) also show that the rate of insolvency and business failure has dramatically increased since about 1980.

Table 1: **Cross-sectional idiosyncratic volatility effect controlled for distress risk.** Panel A presents the idiosyncratic volatility effect controlling for the Z-score. In "Before Controlling" row, each June from 1981 to 2005, firms are sorted into five quintiles according to their idiosyncratic volatility from equation(1) using daily data in the past year. In "Controlling for Z-score" panel, in June of each year, stocks are first sorted into five quintiles according to their previous December Z-score. Within each quintile, stocks are then sorted into five groups on the basis of their idiosyncratic volatility for the past year. Portfolios are value-weighted. The abnormal returns(in percentage) adjusted by FF-3 model and associated robust Newey-West(1997) t-statistics are reported. The column "5-1" refers to the difference in monthly abnormal returns between portfolio 5(the portfolio of stocks with highest idiosyncratic volatility) and portfolio 1 (the lowest idiosyncratic volatility portfolio). Panel B repeats the same procedure using O-score. The sample period is from 1981 to 2000.

Panel A: Portfolios sorted on idiosyncratic volatility controlling for the Z-score

		Ranking on Idiosyncratic Volatility					
		Low 1	2	3	4	High 5	5-1
Before		0.16	-0.04	-0.47	-0.32	-0.44	-0.60
Controlling		[1.49]	[-0.29]	[-3.74]	[-1.69]	[-1.71]	[-2.05]
	Low Z1	-0.48	-0.94	-0.94	-1.03	-1.56	-1.08
Controlling		[-2.78]	[-4.14]	[-2.99]	[-3.10]	[-3.15]	[-2.07]
	2	-0.09	-0.08	-0.37	-0.22	-0.58	-0.49
for		[-0.58]	[-0.41]	[-1.81]	[-0.84]	[-1.50]	[-1.24]
	3	0.04	-0.20	-0.42	-0.59	-0.34	-0.38
Z-score		[0.29]	[-1.22]	[-2.16]	[-2.50]	[-1.07]	[-1.11]
	4	0.23	-0.04	-0.02	0.03	-0.44	-0.66
		[1.82]	[-0.21]	[-0.12]	[0.12]	[-1.37]	[-1.98]
	High Z5	0.39	0.17	-0.19	-0.41	-0.31	-0.70
		[2.53]	[0.91]	[-1.08]	[-1.61]	[-0.86]	[-1.61]

Panel B: Portfolios sorted on idiosyncratic volatility controlling for the O-score

		Ranking on Idiosyncratic Volatility					
		Low 1	2	3	4	High 5	5-1
Before		0.10	-0.13	-0.54	-0.22	-0.58	-0.67
Controlling		[0.92]	[-1.22]	[-3.91]	[-1.06]	[-2.24]	[-2.44]
	Low O1	0.16	0.03	-0.17	-0.09	-0.11	-0.27
Controlling		[1.29]	[0.18]	[-0.87]	[-0.41]	[-0.32]	[-0.68]
	2	0.21	-0.18	-0.29	-0.50	-0.08	-0.30
for		[1.46]	[-1.21]	[-1.50]	[-2.15]	[-0.26]	[-0.87]
	3	0.14	-0.11	-0.57	-0.67	-0.36	-0.50
O-score		[0.93]	[-0.72]	[-2.52]	[-2.70]	[-1.16]	[-1.49]
	4	-0.07	-0.49	-0.94	-0.85	-0.82	-0.75
		[-0.35]	[-2.26]	[-3.78]	[-2.92]	[-2.20]	[-1.92]
	High O5	-0.33	-0.92	-0.83	-1.44	-1.43	-1.09
		[-1.36]	[-3.09]	[-2.35]	[-3.44]	[-3.15]	[-2.29]

2.2 Analysis of the idiosyncratic volatility effect, controlling for distress risk

To examine whether the idiosyncratic volatility effect is different from the distress risk effect, we do sequential sorting according to firms' distress risk and idiosyncratic volatility. Specifically, we examine whether the idiosyncratic volatility puzzle exists in all distress risk quintiles, as well as in the whole sample. The bankruptcy scores are computed from COMPUSTAT as of the fiscal year-end of a given year t. To ensure that the accounting data are available to calculate the distress risk measures, we delay the bankruptcy scores by six months. In June of each year, stocks are first ranked into five distress quintiles according to their previous December Z-score/O-score. Within each distress quintile, firms are then sorted into five groups according to their idiosyncratic volatility from equation(1) using daily data in the past year. For July of t through June of $t+1$, the return on each portfolio is calculated as the value-weighted average return of the stocks in the portfolio. Table 1 presents the idiosyncratic volatility effect controlling for the distress risk effect. Panel A presents the sorting results using Z-score, while Panel B uses O-score sorting. The abnormal returns adjusted by FF-3 model are reported in percentage.

From Panel A of Table 1, there is clear evidence of a distress risk puzzle.[1] The lowest Z-score (highest distress risk) stocks earn significantly negative abnormal returns relative to FF-3 model. More importantly, the idiosyncratic volatility effect is significant only in the highest distress risk quintile, in the sense that only in that quintile (Z1), stocks with high idiosyncratic volatility earn lower returns than stocks with low idiosyncratic volatility.

[1] Recall that a firm with a lower Z-score has a higher probability of bankruptcy. Therefore, basic finance intuition should suggest that such firms will have higher returns to compensate for this elevated distress risk. A similar argument holds for the high O-score firms.

The difference in returns is of the order of -1.08% per month with a t-statistic of -2.07. In all other distress quintiles, the spread in returns is not significantly different from zero. The "Before Controlling" rows show that a significant spread in returns between high and low idiosyncratic volatility stocks is also found in the case of the whole sample, but the magnitude of the spread is only -0.6% per month. Using O-score, the result is same as in Panel A. In Panel B, before controlling firm's O-score, the spread between high and low idiosyncratic volatility stocks is significantly different from zero. The magnitude is -0.67% per month with a t-statistic of -2.44. However, after controlling for the O-score, the spread is only significantly different from zero in the highest O-score quintile.

The results in Table 1 clearly show that the idiosyncratic volatility effect is closely related to the distress effect. Indeed, it actually exists *conditional* on high distress risk. In the next subsection, we attempt to distinguish further the twin effects of idiosyncratic volatility and distress, by sorting stocks in the opposite manner.

2.3 Analysis of the distress risk effect, controlling for idiosyncratic volatility

By reversing the sorting order, we can assess whether the distress risk effect prevails in the presence of idiosyncratic volatility. For this experiment, in June of each year, we first sort stocks into five portfolios according to their idiosyncratic volatility in the past year. Within each quintile portfolio, we then sort stocks into five groups based on their previous December Z-score/O-score. Portfolios are held through next June and returns are value weighted.

Table 2 presents the results. From Panel A, we can see that high distress risk(low

Table 2: **Cross-sectional distress risk effect controlled for idiosyncratic volatility effect.** Panel A presents the distress risk(proxied by Z-score) effect controlling for idiosyncratic volatility effect. In "Before Controlling" row, each June from 1981 to 2005, firms are sorted into five quintiles according to their previous December Z-score. In "Controlling for idiosyncratic volatility" panel, stocks are first sorted into five quintiles each June based on their idiosyncratic volatility for the past year. Within each quintile, stocks are then sorted into five groups according to their previous December Z-score. Portfolios are value-weighted. The abnormal returns(in percengate) adjusted by FF3 model and associated robust Newey-West t-statistics are reported in percentage. The column "1-5" refers to the difference in monthly abnormal returns between portfolio 1(the portfolio of stocks with highest distress risk) and portfolio 5(the lowest distress risk portfolio). Panel B repeats the same procedure using O-score. The sample period is from 1981 to 2000.

Panel A: Portfolios sorted on Z-score controlling for idiosyncratic volatility

		Ranking on Z-score					
		Low 1	2	3	4	High 5	1-5
Before		-0.63	-0.11	-0.16	0.10	0.17	-0.80
Controlling		[-4.12]	[-0.84]	[-1.49]	[1.06]	[1.52]	[-4.39]
	Low Vol1	-0.19	0.11	0.10	0.35	0.31	-0.50
Controlling		[-1.20]	[0.72]	[0.69]	[2.35]	[1.86]	[-2.34]
	2	-0.50	-0.29	0.01	0.01	0.16	-0.66
for		[-2.62]	[-1.68]	[0.05]	[0.05]	[0.85]	[-2.44]
	3	-0.72	-0.49	-0.38	-0.30	-0.46	-0.26
Idiosyncratic		[-3.59]	[-2.42]	[-1.68]	[-1.34]	[-2.30]	[-0.92]
	4	-0.84	-0.18	-0.53	-0.51	0.07	-0.91
Volatility		[-2.93]	[-0.73]	[-2.05]	[-1.95]	[0.26]	[-2.60]
	High Vol5	-1.22	-0.69	-0.23	-0.54	-0.21	-1.00
		[-2.63]	[-1.90]	[-0.60]	[-1.39]	[-0.57]	[-1.81]

Panel B: Portfolios sorted on O-score controlling for idiosyncratic volatility

		Ranking on O-score					
		Low 1	2	3	4	High 5	5-1
Before		0.05	-0.01	-0.15	-0.41	-0.72	-0.77
Controlling		[0.49]	[-0.09]	[-1.24]	[-2.51]	[-3.31]	[-3.84]
	Low Vol1	0.15	-0.04	0.12	0.02	0.16	0.01
Controlling		[0.95]	[-0.25]	[0.84]	[0.14]	[0.84]	[0.03]
	2	0.06	-0.12	-0.45	-0.26	-0.23	-0.29
for		[0.30]	[-0.76]	[-2.84]	[-1.43]	[-1.19]	[-1.17]
	3	-0.29	-0.64	-0.58	-0.91	-0.58	-0.29
Idiosyncratic		[-1.39]	[-2.93]	[-2.70]	[-3.89]	[-2.24]	[-0.87]
	4	0.15	-0.04	-0.41	-1.02	-1.01	-1.16
Volatility		[0.48]	[-0.15]	[-1.45]	[-3.68]	[-3.00]	[-2.94]
	High Vol5	-0.27	-0.64	-0.53	-1.27	-1.50	-1.22
		[-0.78]	[-1.68]	[-1.43]	[-3.09]	[-3.64]	[-2.49]

Z-score) stocks generally earn lower returns than low distress risk stocks across idiosyncratic volatility quintiles. The spread ranges from -0.3% to -1.00% per month and it is generally statistically significant. Similar results are found when the whole sample is used, with a spread return of -0.80% per month between high and low distress risk stocks and a t-value of -4.39. This implies that not only does the distress risk effect exist at the whole sample level, but it is also not subsumed by the idiosyncratic volatility effect. When we use O-score in Panel B, the results are generally consistent with Panel A. The spread between high and low distress risk stocks in the whole sample is -0.77% per month with a t-statistic of -3.84. After controlling idiosyncratic volatility, the spread is significantly different from zero within the two highest volatility quintiles. This table confirms that idiosyncratic volatility effect cannot subsume the distress risk effect.

Taken together, the results in Table 1 and Table 2 suggest that the idiosyncratic volatility puzzle is closely related to the distress risk puzzle. In particular, the idiosyncratic volatility effect exists conditional on the existence of high distress risk.

2.4 Further details on volatility and distress portfolios

The preceding subsection has presented material on the return characteristics of the portfolios. It is also interesting to discern the volatility and distress properties of these portfolios. Such properties are presented in Tables 3 and 4.

Panel A of Table 3 presents the value-weighted Z-score and idiosyncratic volatility of our 5x5 sorted portfolios, where volatility is controlled for distress risk. From the table, distressed stocks (Z1) have much higher idiosyncratic volatility than healthy stocks(Z5).

Table 3: **Characteristics of 5×5 Idiosyncratic Volatility Portfolios Controlling for Distress Risk.** Panel A presents the characteristics of 5×5 Z-score and idiosyncratic volatility sorted portfolios. In June of each year from 1981 to 2005, stocks are first sorted into five quintiles according to their previous December Z-score. Within each quintile, stocks are then sorted into five groups on the basis of their idiosyncratic volatility for the past year. Value-weighted Z-score and idiosyncratic volatility for each portfolio are reported in the table. Panel B repeats the same procedure using O-score. The sample period is from 1981 to 2000.

Panel A: Portfolios sorted on idiosyncratic volatility controlling for the Z-score						
		Low 1	Idiosyncratic	Volatility		High 5
	Z-score					
	Low 1	2.03	1.73	1.59	1.15	-0.09
		3.11	3.04	3.05	3.00	2.98
Z-score		4.32	4.27	4.35	4.31	4.30
		6.59	6.58	6.62	6.65	6.65
	High 5	25.52	63.23	63.90	383.91	538.92
	Low 1	1.61	2.39	3.20	4.42	7.27
Idiosyncratic		1.37	1.81	2.26	2.87	4.16
		1.27	1.66	2.02	2.54	3.46
Volatility		1.21	1.61	1.95	2.41	3.33
	High 5	1.22	1.64	2.05	2.59	3.61
Panel B: Portfolios sorted on idiosyncratic volatility controlling for the O-score						
		Low 1	Idiosyncratic	Volatility		High 5
	O-score					
	Low 1	-3.05	-3.06	-3.16	-3.26	-3.48
		-1.97	-1.96	-1.94	-1.95	-1.92
O-score		-1.20	-1.15	-1.14	-1.13	-1.14
		-0.40	-0.33	-0.31	-0.26	-0.22
	High 5	0.97	1.26	1.23	1.95	3.25
	Low 1	1.12	1.50	1.84	2.28	3.08
Idiosyncratic		1.21	1.53	1.84	2.24	3.07
		1.30	1.65	1.97	2.42	3.33
Volatility		1.46	1.92	2.31	2.88	4.14
	High 5	1.79	2.65	3.44	4.64	7.66

The most striking finding is that the value weighted Z-score decreases monotonically from low to high idiosyncratic volatility stocks only within the lowest Z-score quintile, but increases monotonically from low to high idiosyncratic volatility stocks within the highest Z-score quintile. Recall that a higher Z-score connotes financial health. Hence this finding indicates that higher idiosyncratic volatility is associated with increased financial health, if the firms are the most healthy. However, for the least healthy (lowest Z-score) stocks, higher idiosyncratic volatility is associated with decreased financial health. In other words, before we can determine whether volatility is "good" or "bad", for a set of stocks in our universe, we must first condition on whether that set of stocks is among the most or least distressed. Remember in Panel A of Table 1, stocks with high idiosyncratic volatility earn significantly lower returns than low idiosyncratic volatility stocks only in the lowest Z-score quintile(distressed quintile). While Panel A of Table 3 shows that only in this distressed quintile(Z1), high idiosyncratic volatility stocks have much lower Z-score(i.e. higher distress risk). Given the fact that distressed stocks receive anomalously lower returns, we can see that this idiosyncratic volatility effect just reflects the distress risk effect.

Panel B presents similar results for the O-score. Same as results in Panel A, stocks with high O-score (distressed stocks) have much higher idiosyncratic volatility than stocks with low O-score (the most healthy stocks). The major pattern is that O-score increases monotonically from low to high idiosyncratic volatility stocks within the highest O-score firms, but decreases monotonically from low to high idiosyncratic volatility stocks within the lowest O-score firms. Since high O-scores correspond to financial distress, we can say that higher idiosyncratic volatility is associated with decreased financial health for the most distressed stocks, and associated with increasing financial health for the most healthy stocks. This accords with the results of Panel A. From Panel B of Table 1, stocks with high idiosyncratic volatility earn significantly lower returns than low idiosyncratic volatility stocks only in the highest O-score quintile(distressed quintile). While Panel B of Table 3 shows that only in this distressed quintile(O5), high idiosyncratic volatility stocks have much higher O-score(i.e. higher distress risk). Again, using O-score, we generate same conclusion: the idiosyncratic volatility effect just reflects the distress risk effect.

The next table, Table 4, presents the opposite sorting, that is, distress risk controlling for idiosyncratic volatility. In Panel A, we see that Z-score decreases monotonically (across volatility portfolios) for the first four Z-score portfolios, and increases monotonically for the fifth portfolio. At the same time, idiosyncratic volatility decreases monotonically from low to high Z-score stocks only within the highest volatility quintile. A similar pattern exists in Panel B: O-score increases monotonically for all except the lowest O-score portfolio. At the same time, idiosyncratic volatility increases monotonically from low to high O-score stocks only within the highest volatility quintile. This indicates that, except for the most healthy stocks (highest Z, lowest O), financial health tends to decrease with idiosyncratic volatility. Furthermore, idiosyncratic volatility tends to decrease with financial health only for the most volatile stocks. Taken together, the results of Tables 3 and 4 reinforce the idea from the previous subsection that volatility and distress effects interact at the extreme portfolios.

Table 4: **Characteristics of 5×5 Distress Risk Portfolios Controlling for Idiosyncratic Volatility.** Panel A presents the characteristics of 5×5 idiosyncratic volatility and Z-score sorted portfolios. In June of each year from 1981 to 2005, stocks are first sorted into five quintiles based on their idiosyncratic volatility for the past year. Within each quintile, stocks are then sorted into five groups according to their previous December Z-score. Value-weighted Z-score and idiosyncratic volatility for each portfolio are reported in the table. Panel B repeats the same procedure using O-score. The sample period is from 1981 to 2000.

Panel A: Portfolios sorted on Z-score controlling for idiosyncratic volatility						
		Low 1		Z-score		High 5
	Idiosyncratic Volatility					
	Low 1	2.72	3.96	5.30	8.06	29.13
		2.34	3.57	4.84	7.17	71.18
Z-score		1.99	3.24	4.45	6.72	127.62
		1.51	2.80	3.99	6.34	475.69
	High 5	-0.44	1.69	2.77	4.83	949.70
	Low 1	1.36	1.31	1.25	1.27	1.26
Idiosyncratic		1.77	1.76	1.76	1.75	1.75
		2.20	2.21	2.18	2.21	2.21
Volatility		2.90	2.87	2.83	2.84	2.84
	High 5	5.31	4.58	4.25	4.13	4.10
Panel B: Portfolios sorted on O-score controlling for idiosyncratic volatility						
		Low 1		O-score		High 5
	Idiosyncratic Volatility					
	Low 1	-3.40	-2.50	-1.88	-1.31	-0.44
		-3.22	-2.15	-1.54	-0.91	-0.02
O-score		-3.15	-1.94	-1.18	-0.49	0.69
		-3.33	-1.69	-0.81	-0.02	1.33
	High 5	-2.52	-0.57	0.43	1.47	4.91
	Low 1	1.05	1.05	1.10	1.10	1.15
Idiosyncratic		1.87	1.86	1.85	1.85	1.88
		2.05	2.05	2.10	2.05	2.10
Volatility		2.82	2.85	2.85	2.94	2.93
	High 5	3.98	4.16	4.17	4.66	5.79

3 Rational interpretation of idiosyncratic volatility and distress risk effects

We now investigate an intuitive, rational explanation for the twin puzzles. For simplicity, we will call this an "equity-bias argument". This section is closely based on the work of Ferguson and Shockley (2003). These researchers observe that using an equity-only proxy for the market portfolio will understate equity betas, with the error increasing with the firm's leverage. Thus, firm-specific variables that correlate with leverage (such as book-to-market and size) will appear to explain returns after controlling for proxy beta, simply because they capture the missing beta risk.[2]

3.1 A corrected single-beta CAPM model

Consider a simple continuous-time economy in which the single-beta CAPM prices all real assets, firms are allowed to finance their real assets with a simple capital structure. Under standard assumptions, equity claims will be priced as European calls on the underlying real assets. Let M be the market portfolio, which can be divided into two subportfolios: the economy's debt claims D, and the economy's equity claims E. Then the covariance between firm i's equity claim S_i and M is:

$$\sigma_{Si,M} = \frac{E}{M}\sigma_{Si,E} + \frac{D}{M}\sigma_{Si,D},$$

[2] In an empirical application, the authors document that their portfolios formed on leverage and distress can subsume the Fama-French (1993) factors SMB and HML.

where $\sigma_{Si,M}$, $\sigma_{Si,E}$, $\sigma_{Si,D}$ are the stock's covariances with the asset, equity, and debt markets, respectively. The true beta of firm i's equity claim can be written as:

$$\beta_{Si} = \frac{\sigma_{Si,M}}{\sigma_M^2} = \frac{E}{M}\frac{\sigma_{Si,E}}{\sigma_M^2} + \frac{D}{M}\frac{\sigma_{Si,D}}{\sigma_M^2},$$

where σ_M^2, σ_E^2, σ_D^2 are the return variances of the asset, equity, and debt markets, respectively. If we ignore the economy's debt claims in the construction of the market proxy, the proxy beta of firm i's equity will be

$$\hat{\beta_{Si}^E} = \frac{\sigma_{Si,E}}{\sigma_E^2},$$

where the superscript E is appended to denote the proxy with respect to the equity market. We can write the proxy equity beta as a transformation of the true equity beta:

$$\hat{\beta_{Si}^E} = \Phi^{-1}[\beta_{Si} - \Omega\hat{\beta_{Si}^D}], \tag{2}$$

where

$$\Phi = \frac{E}{M}\frac{\sigma_E^2}{\sigma_M^2}, \;\; \Omega = \frac{D}{M}\frac{\sigma_D^2}{\sigma_M^2},$$

and $\hat{\beta}_{Si}^D$ is the beta of the equity calculated against the economy's debt claims only (i.e., the assets omitted from the market proxy). From Equation (2), we can see that the error in the proxy equity beta ($\hat{\beta}_{Si}^E$) has two components: a scaling error Φ^{-1} that is common across all equities, and a firm-specific error $-\Omega\hat{\beta}_{Si}^D$ that reflects the stock's covariance with the (omitted) debt claims. It is the second component, the firm-specific term, that is critical for the researcher, because it is a function of the firm's financial leverage. Ferguson and

Shockley (2003) show that the second component $\Omega\hat{\beta}^D_{Si}$ is an increasing and convex function of the firm's leverage. Due to this convexity, the missed component effect should be more pronounced for more distressed firms, so we care as much about relative distress as about absolute leverage ratios. It then follows that in the cross section, the equity beta estimation errors will not be random. They will be systematically related to the relative leverage and relative distress of each firm in the sample. As a result, the model predicts that firm leverage and financial distress will capture the convex beta estimation errors induced by the use of an equity-only market proxy. This is what we refer to as an equity-bias. The following arguments illustrate a method of removing this bias.

According to the decomposition of the firm's equity beta in equation (2), the equilibrium model of excess returns is:

$$r_{Si} - r_F = [r_M - r_F]\beta_{Si} = \Phi[r_M - r_F]\hat{\beta}^E_{Si} + \Omega[r_M - r_F]\hat{\beta}^D_{Si}$$

Since $\Omega\hat{\beta}^D_{Si}$ is a function of firm's leverage and relative distress, the empirical solution for correcting this beta estimation error is to create portfolios based on relative leverage and relative distress. These factors formed on relative leverage and relative distress provide the best complements to the equity market index for explaining the cross section of stock returns. The corrected single beta capital asset pricing model can be expressed as:

$$R^e_i = \alpha_i + \beta^{MKT}_i R^{MKT}_t + \beta^{D/E}_i R^{D/E}_t + \beta^Z_i R^Z_t,\ t = 1, 2, \cdots, T, \tag{3}$$

where R^e_i is test asset's excess return, R^{MKT}_t is the return of stock market portfolio, $R^{D/E}_t$ and R^Z_t are returns of two portfolios to mimic the part of common return associated with relative leverage and the part of return associated with relative distress. Ferguson and Shockley (2003) use this corrected single-beta CAPM to examine whether sensitivities to returns on these portfolios help to explain the cross section of excess returns.

To construct portfolios for $R_t^{D/E}$ and R_t^Z, following Ferguson and Shockley (2003), in June of each year t, firms are assigned to one of three book debt-to-market equity(BD/ME) portfolios based on the one-third and two-third percentile breakpoints determined only from the NYSE firms in the sample. Independently and simultaneously, firms are assigned to one of two portfolios: $Z <= 2.675$ and $Z > 2.675$ according to their previous December Z-scores. Firms with $Z > 2.675$ are predicted to be in the healthy group, while firms with $Z <= 2.675$ are predicted to be in the distressed group. Only firms with ordinary common equity are used to form the leverage and distress portfolios. The intersection of the two independent sorts results in six debt-to-equity(D/E)/Z portfolios. Portfolios are valued weighted. $R_t^{D/E}$ is then calculated as the simple average return of the two Z portfolios within D/E portfolio 3(the highly levered firms) minus the simple average return of the two Z portfolios within D/E portfolio 1(the least levered firms). Similarly, R_t^Z is the simple average return of the three D/E portfolios within Z portfolio 2(high Z-firms) minus the simple average return of the three D/E portfolios within Z portfolio 1 (low Z-firms).

3.2 Testing idiosyncratic volatility and distress risk portfolios

Recall that in the previous section, we documented that the idiosyncratic volatility effect exists conditional on high distress risk. We now use tests that are motivated by the corrected-CAPM from above to investigate a simple rational explanation for both puzzles.

Specifically, if the model (3) can explain the idiosyncratic volatility and distress risk

Table 5: **GRS test on idiosyncratic volatility portfolios and distress risk portfolios.** The time series regression model is: $R_i^e = \alpha_i + \beta_i^{MKT} R_t^{MKT} + \beta_i^{D/E} R_t^{D/E} + \beta_i^Z R_t^Z$, where R_t^{MKT} is the stock market excess return, $R_t^{D/E}$ is the return spread between high and low D/E portfolios, and R_t^Z is the return spread between high and low Z portfolios. The construction of the idiosyncratic volatility portfolios and distress risk portfolios is described in Section 3.2. Panel A presents the GRS test on idiosyncratic portfolios. Regression coefficient estimates and associated t-value, Gibbons-Ross-Shanken test statistic(F-test and χ test) and associated p-value are reported. The sample period is from 1981 to 2005. Panel B presents same GRS test results on Z-score portfolios from 1981 to 2005. Panel C presents test results on O-score portfolios from 1981 to 2000. Construction of Z/O-score portfolios is described in Section 2.

Panel A: GRS test on idiosyncratic volatility portfolios								
	Ranking on idiosyncratic volatility							
	Low 1	2	3	4	High 5	5-1	GRS-χ	GRS-F
$FF-\alpha(\%)$	0.12	-0.04	-0.10	-0.35	-0.93	-1.05	13.33	2.15
	[2.06]	[-0.45]	[-0.93]	[-1.88]	[-3.16]	[-3.24]	(0.04)	(0.05)
$\alpha(\%)$	0.11	-0.06	-0.15	-0.30	-0.75	-0.86	6.40	1.03
	[1.16]	[-0.76]	[-0.90]	[-1.07]	[-2.05]	[-1.97]	(0.38)	(0.40)
β^{MKT}	0.86	1.07	1.32	1.53	1.53	0.68		
	[43.13]	[47.89]	[35.81]	[21.51]	[17.15]	[6.46]		
$\beta^{D/E}$	0.21	0.03	-0.28	-0.49	-0.77	-0.98		
	[4.24]	[0.95]	[-4.08]	[-3.82]	[-4.60]	[-4.77]		
β^Z	-0.12	0.15	0.27	0.16	0.18	0.31		
	[-2.67]	[3.61]	[3.14]	[0.95]	[0.78]	[1.13]		
Panel B: GRS test on Z-score portfolios								
	Ranking on Z-score							
	Low 1	2	3	4	High 5	1-5	GRS-χ	GRS-F
$FF-\alpha(\%)$	-0.60	-0.15	-0.15	0.10	0.18	-0.78	22.52	3.64
	[-3.92]	[-1.08]	[-1.39]	[1.02]	[1.57]	[-4.27]	(0.00)	(0.00)
$\alpha(\%)$	-0.35	-0.01	-0.10	0.05	-0.03	-0.32	9.90	1.60
	[-2.20]	[-0.10]	[-0.87]	[0.51]	[-0.29]	[-1.60]	(0.13)	(0.15)
β^{MKT}	1.15	1.01	0.96	0.90	0.88	0.27		
	[27.25]	[24.43]	[31.67]	[33.32]	[29.85]	[5.78]		
$\beta^{D/E}$	0.59	0.56	0.31	0.18	-0.09	0.68		
	[9.16]	[8.31]	[6.16]	[3.31]	[-1.56]	[8.69]		
β^Z	-0.65	-0.40	-0.17	0.03	0.34	-0.99		
	[-7.35]	[-4.78]	[-2.67]	[0.52]	[5.35]	[-9.43]		
Panel C: GRS test on O-score portfolios								
	Ranking on O-score							
	Low 1	2	3	4	High 5	5-1	GRS-χ	GRS-F
$FF-\alpha(\%)$	0.06	-0.02	-0.12	-0.43	-0.77	-0.83	19.69	3.15
	[0.58]	[-0.21]	[-1.00]	[-2.61]	[-3.46]	[-4.06]	(0.00)	(0.01)
$\alpha(\%)$	-0.00	-0.04	-0.09	-0.30	-0.62	-0.62	8.93	1.43
	[-0.01]	[-0.35]	[-0.71]	[-1.82]	[-2.70]	[-2.68]	(0.18)	(0.20)
β^{MKT}	0.91	0.96	1.05	1.04	1.17	0.26		
	[32.39]	[45.24]	[27.61]	[20.84]	[16.36]	[3.79]		
$\beta^{D/E}$	-0.10	0.13	0.22	0.32	0.21	0.31		
	[-1.45]	[2.11]	[3.69]	[3.63]	[1.93]	[2.85]		
β^Z	0.22	0.01	-0.10	-0.28	-0.20	-0.42		

puzzles, then the regression intercepts α_i should be zero if we run time-series regression tests on idiosyncratic volatility portfolios and distress risk portfolios. For this purpose, we use the (GRS) test of Gibbons, Ross and Shanken (1989). If the errors are i.i.d. over time, homoskedastic, and independent of the factors, the asymptotic joint distribution of the intercepts gives the test statstic,

$$T\left[1+\left(\frac{E_T(f)}{\hat{\sigma}(f)}\right)^2\right]^{-1}\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}\sim\chi^2_N.$$

We also use a finite-sample exact F test, which we now describe. When the errors are also normally distributed, a multivariate, finite-sample counterpart statistic is

$$\frac{T-N-K}{N}\left(1+E_T(f)'\hat{\Omega}^{-1}E_T(f)\right)^{-1}\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}\sim F_{N,T-N-K}.$$

Table 5 presents the GRS test results on the idiosyncratic volatility portfolios and distress risk portfolios. To form idiosyncratic volatility portfolios and distress risk portfolios, in June of each year, we sort stocks into five portfolios separately according to their previous December bankruptcy scores (Z-score/O-score) and their idiosyncratic volatility–computed from equation (1) using past year's daily data. From Panel A, there is clear evidence of the idiosyncratic volatility puzzle. High idiosyncratic volatility stocks earn significantly lower FF3-adjusted returns than low idiosyncratic volatility stocks. The spread is -1.05% per month with a t-statistic of -3.24. Moreover, the GRS tests jointly reject the null hypothesis of zero abnormal returns at 5% level. When we use the corrected single-beta capital asset pricing model (3) to adjust the returns, the spread between high and low idiosyncratic volatility stocks becomes not significantly different from zero. The model adjusted α is

-0.86% per month with a t-statistic of -1.97. Checking the loadings on the regressands, we can see that the key to interpret the idiosyncratic volatility puzzle is firm's leverage. Loadings on $R^{D/E}$ are significantly different from zero, and they are decreasing monotonically from low to high idiosyncratic volatility stocks. The pattern of loadings on R^{MKT} exacerbate the volatility effect. Loadings on R^{MKT} actually monotonically increase from low to high idiosyncratic volatility stocks. In support of the equity-bias arguments outlined above, the evidence shows that firm's leverage manages to capture the estimation errors induced by the use of an equity-only market proxy. Both the GRS F-test and χ-test show that we cannot jointly reject the null of zero abnormal returns across the idiosyncratic volatility portfolios. This implies that the corrected single-beta CAPM does a good job of explaining the idiosyncratic volatility effect.

In Panel B of Table 5, distress risk stocks (low Z-score stocks) have significantly lower FF3-adjusted returns. The spread between low and high Z-score stocks is -0.78% per month with a t-statistic of -4.27. Again, the joint GRS tests significantly reject the null hypothesis of zero abnormal returns at 1% level. However, when we use the corrected single-beta CAPM (3) to adjust stock returns, the spread between high (Z1) and low (Z5) distressed stocks is only -0.32% per month with a t-statistic of -1.60, which is not significantly different from zero. Moreover, the null hypothesis of zero abnormal returns across the Z-score portfolios is not rejected by the joint GRS F-test and χ tests.[3] Loadings on the R^Z are generally significantly different from zero and increase monotonically from distressed stocks (Z1) to healthy stocks (Z5). When we use O-score to form the distress risk

[3]Note that distressed stocks (Z1) have high loadings on the R^{MKT} and $R^{D/E}$, while healthy stocks (Z5) have low loadings on R^{MKT} and $R^{D/E}$. So they are not the reason to explain the puzzling low returns on distressed stocks.

portfolios, the results are consistent with results in Panel B. Although the spread between distressed stocks (O5) and healthy stocks (O1) is still significantly different from zero when we use the corrected single-beta CAPM model to adjust stock returns, the GRS test cannot reject the null hypothesis of jointly zero abnormal returns across the O-score quintiles.

The above evidence clearly shows that after we correct the beta errors in the CAPM, the model does a good job to explain cross-sectional idiosyncratic volatility effect and distress risk effect. And this provides a rational explanation to the idiosyncratic volatility and distress risk puzzles.

4 Conclusions

This paper investigates the link between the idiosyncratic volatility puzzle and the distress risk puzzle, and proposes a simple, rational explanation for both puzzles. We have two main contributions.

Our first contribution is to forge a link between the analysis of distress risk and volatility risk. Sequential sorting indicates that after controlling for distress risk, stocks with high idiosyncratic volatility earn significantly low returns only in the highest distress risk quintile. This implies that the idiosyncratic volatility effect exists conditional on the high distress risk. At the same time, distress risk seems to exist independently of idiosyncratic volatility effects. In other words, volatility and distress risk are linked, with the volatility effect being dependent on elevated levels of distress.

Our second contribution is to provide a simple explanation for both puzzles. We implement a corrected CAPM model, based on the analysis of Ferguson and Shockley (2003). In this framework, we document that we cannot reject the null hypothesis of zero abnormal returns across either idiosyncratic volatility or distress risk portfolios.

Why should we care about these results? From an asset pricing viewpoint, the reason is that we have made strides towards addressing the question of the importance of distress risk raised, for example, by Fama and French (1993). In this regard, an attractive feature of our research is that we test an empirical model of distress risk that is grounded in a transparent CAPM framework. From an investment perspective, the reason is that we have clarified a substantial part of the relation between volatility and distress risk, and thus provided an important insight into the mechanism of financial hardship.

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