Discussion paper

Do Species Interactions and Stochasticity Matter to Optimal Management of Multispecies Fisheries?

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DO SPECIES INTERACTIONS AND STOCHASTICITY MATTER TO OPTIMAL MANAGEMENT OF MULTISPECIES FISHERIES?

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Abstract

The multispecies fisheries management looks at a bigger picture in addressing the long-term consequences of present decisions. This implies an ecosystem management that includes a number of species and their physical, biological and economic interactions. These interactions make the growth of resources stochastic and increase complexity in understanding stock dynamics and optimal catch for such a stochastic and multiple stocks´ system. To address the issue of identifying optimal catch of stochastically growing multi stocks, we have formulated and applied a time-continuous stochastic model. The model contributes to multispecies bioeconomic management of marine ecosystems. An application of model in a predator-prey relationship in Barent Sea revealed that the optimal catch for stochastically growing stocks in a multispecies interaction model is different from the deterministic model.

Keywords

1. Introduction

Marine fisheries are vital resources for the ecology and economy. They also play a very crucial role in ensuring food security for the growing population (FAO, 2008). Rendering these fisheries as a sustainable source of food for the world, however, requires an effort of addressing overexploitation in the fishing industry and improving fisheries management (Speer, 1995). Among the majority of instruments available for marine fishery management, harvesting strategy is considered the best instrument (Agnarsson *et al.*, 2008). The strategy is considered optimal if the rent from the fishery is maximized over the planning horizon. There exists a large body of literature on bioeconomic modeling that serves the purpose of rent maximization in the public fishery management. These bioeconomic models date back to the early work of Gordon (1954) and Scott (1955). Among many others, Smith (1969), Clark (1973), Hannesson (1975), Clark *et al.* (1979), Grafton *et al.* (2000), Sandal & Steinshamn (2001a), and Arnason *et al.* (2004) suggested optimal management rules to maximize the economic rent from fisheries in a deterministic setting, primarily on single species modeling approach.

However, there are biological, physical and economic interactions among the species in the ecosystem. There exist management problems for the species that involve interactions between species at different trophic levels (May, *et al.*, 1979). The most common approach, single species management in multispecies fisheries ignores the ecological relationships among species as well as the technological and economic relationships between species (Kasperski, 2010). This may lead to misleading results and incorrect policy decisions causing an over or under exploitation of the stocks (Fleming & Alexander, 2003; Hoff *et al.*, 2010). Therefore, multispecies management is a key approach for sustainable management of such marine fisheries.

The importance of multispecies fishery management was realized during early 1970s when the world's major commercial fisheries collapsed (for example, see May, *et al.*, 1979). Multispecies fishery management is to look at the bigger picture addressing the long-term consequences of present decisions. It implies an analysis and management of marine ecological system that includes a number of species and their biological and physical interactions, rather than managing

different species individually. Needless to say - the economic interaction plays a crucial role in creating the overall harvesting pressure on commercially important species.

The earlier studies focused mainly on a predator- prey relationship on different trophic levels in their multispecies management model (for example Bogstad *et al.*, 1997; May*, et al.*, 1979; Yodzis, 1994). Those studies, however, focused on the biological yields without considering the economic aspects of harvesting. Later authors like Fleming and Alexander (2003) and Kar and Chaudhuri (2004) suggested a deterministic bioeconomic model with an optimal equilibrium solution. However, they also remarked that it is extremely difficult to find the optimal paths, even in the cases with linear objective functions. Hollowed *et al.* (2000) compared multispecies models with single species models and found that multispecies models provide distinct advantage over the single species models allowing the users to model natural mortality and growth rates more realistically. They also indicated that multispecies models improve the understanding of fish population dynamics.

Unfortunately, the multispecies bioeconomic models are very limited due to unavailability of the analytical solutions (Posch & Trimborn, 2010) and due to the computational difficulties (Singh *et al.*, 2006), particularly in solving non-linear dynamic models in higher dimensions. Some bioeconomic models by Clark (1990), Woodward and Bishop (1999), Iversen (2006), Agnarsson et. al. (2008) and Sandal and Steinshamn (2010) suggest optimal management of multispecies fishery in deterministic settings. Nevertheless, in reality, most of the decisions are required to take place in an uncertain environment (Charles & Munro, 1985). The physical interactions, such as growing environment, different external shocks and diseases in the ecosystem create stochastic growth of the marine resources. The stochastic process is central in explaining the uncertainty in growth and development of natural resources.

Although researchers such as Reed (1979), Charles (1983), Charles and Munro (1985), Clark and Kirkwood (1986), Hannesson (1987), Sandal and Steinshamn (1997a), Sethi *et al.* (2005), Singh *et al.* (2006), Kugarajh *et al.* (2006) and McGough *et al.* (2009)   include stochasticity in

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their single species models, stochastic multispecies models lack in the bioeconomic literature (Agnarsson, *et al.*, 2008).

To address the issue of optimal management of stochastically growing multi-stocks, we have formulated and applied a time-continuous stochastic model that contributes in multispecies bioeconomic management of marine ecosystems. Our study advances previous models by Agnarsson *et al.* (2008) and Sandal and Steinshamn (2010) and compares stochastic models with the deterministic models in single species and multispecies fisheries. We explore how the species interaction and stochasticity affect the optimal management of the multispecies fisheries. The main contribution of the work constitutes the identification of optimal policy for stochastically growing stocks in a multispecies interaction ecosystem.

This study employs a feedback approach (Sandal & Steinshamn, 1997b, 2001b), where the optimal control (harvest) is a direct function of the state variable (stock). In contrast to the commonly used time paths approach (optimal harvest as a function of time), the feedback approach is superior when faced with uncertainty (Agnarsson, *et al.*, 2008). The feedback models take prevailing stocks as inputs and therefore, these models automatically respond to the unexpected changes in the stock and thus adapt to new situations (Sandal & Steinshamn, 1997b). We apply a dynamic programing (DP) technique to obtain the optimal feedback solution. In the DP technique, value function iteration is carried out to solve for the optimal solution (Judd, 1998). DP is especially a useful method when considering multispecies management model under stochasticity (Sanchirico & Springborn, 2011).

2. The Bioeconomic Model

Bioeconomic models are dynamic and combine both economics and biology - an economic part that characterizes the optimal management policy and a biological part that defines the natural constraints for such optimal policy. The bioeconomic models can be a recruitment model (Ricker, 1954), a surplus production model (Schaefer, 1957) or a year-class model (Beverton & Holt, 1957). These models can be formulated in discrete or continuous time (Sandal & Steinshamn,

2010). Here we use a continuous time surplus production model.

Single species model: Following Clark (1990), Sandal and Steinshamn (1997b), McDonald et. al. (2002) and Agnarsson et. al. (2008), a general deterministic growth function for a single species stock can be given as:

(1) $\qquad dx = [f(x) - h]dt$

A stochastic growth function can be obtained by adding a stochastic term in equation (1)

(2) $\qquad dx = [f(x) - h]dt + \sigma_0(x)dB$

where $f(x) = rx^2(1 - \dfrac{x}{k})$ is a modified logistic growth function with $f(0) = f(k) = 0$, x is the stock biomass, h is the harvest rate, r denotes the intrinsic growth rate and k denotes the carrying capacity of the species.

The term $\sigma_x(x)dB_x$ represents the stochastic part of the stock incremental growth relationship. σ_0 is the diffusion term and represents volatility in the growth model. The term dt is time increment and dB_x is the Brownian motion which are iid with mean zero and variance dt. We assume the natural condition of non-negativity of x and h.

Multispecies model: The general biological interdependent deterministic growth functions for two interacting predator- prey species can be obtained from Agnarsson *et al.* (2008) and Sandal and Steinshamn (2010). Let x be prey species and y be the predator species. The deterministic growth increments of the species can be given as:

(3)
$$dx = [f(x, y) - h_x]dt$$
$$dy = [g(x, y) - h_y]dt$$

The function $f(x, y)$ and $g(x, y)$ are the biological growth functions of two fish species respectively. h_i represents the fishing mortality or the harvest rate of species $(i = x, y)$.

Furthermore, a two-species interaction model with stochastic dynamic can be formulated by adding stochastic terms in equation (3) as:

$$(4) \qquad \begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} f(x,y) - h_x \\ g(x,y) - h_y \end{pmatrix} dt + \begin{pmatrix} \sigma_{11}(x,y) & \sigma_{12}(x,y) \\ \sigma_{21}(x,y) & \sigma_{22}(x,y) \end{pmatrix} \begin{pmatrix} dB_1 \\ dB_2 \end{pmatrix}$$

or formally as $dZ = F(Z,h)dt + \sigma(Z)dB$.

Although it is likely that the stochastic events are correlated among the species, we assume it to be small enough to be neglected, i.e. $\sigma_{12}(x,y) = \sigma_{21}(x,y) = 0$. We further simplify by setting each species volatility as a linear function of its own stock level, $\sigma_{11}(x,y) = \sigma_1 x$ and

$\sigma_{22}(x,y) = \sigma_2 y$. This assumption is made for two reasons; for the simplicity in the numerical approximation and there is not much known about more complicated functional volatility dependence. Since the species are prey and predator species, the stochastic processes that affects prey directly, may not affect the predators in a direct sense or vice versa. The other species is still indirectly affected by such a process through the interaction in the drift (deterministic) part. Epidemics are typically of this kind. Therefore, we believe that inclusion of stochasticity improves the model compared to the deterministic case. The equation (4) can now be written as:

$$(5) \qquad \begin{aligned} dx &= (f(x,y) - h_x)dt + \sigma_1 x dB_1 \\ dy &= (g(x,y) - h_y)dt + \sigma_2 y dB_2 \end{aligned}$$

The biological growth functions are specified as:

$$f(x,y) = a_1 x^2 - a_2 x^3 - a_3 xy \quad \text{and} \quad g(x,y) = b_1 y^2 - b_2 y^4 + b_3 xy$$

a_1, a_2, a_3, b_1, b_2 and b_3 are parameters. The term xy is the interaction between the species where the predator feeds on the prey. In equations (5), the term $\sigma_{(.)}(.)dB_{(.)}$ represents the stochastic part of the stock growth relationship. $\sigma_{(.)}$ is the diffusion term and represents volatility in the growth models. The terms $dB_{(.)}$ are uncorrelated Brownian motions which are iid with mean zero and variance dt. We assume stocks and harvests rates to be non-negative.

Our basic biological model is now given by

$$dx = (a_1 x^2 - a_2 x^3 - a_3 xy - h_x)dt + \sigma_1 x dB_1$$

(6)

$$dy = (b_1 y^2 - b_2 y^4 + b_3 xy - h_y)dt + \sigma_2 y dB_2$$

Equation (6) shows that both species have stochastic growth and interactions among them. But what if only one of the species has stochastic growth? To understand the effect of stochastic growth of one species on optimal exploitation in multispecies ecosystem, equation (6) can be simplified by setting one of the stochastic parameters to zero. For example, the prey species stochastic growth in multispecies ecosystem is modeled by assuming $\sigma_{22} = 0$ and can be written as:

$$dx = (a_1 x^2 - a_2 x^3 - a_3 xy - h_x)dt + \sigma_1 x dB_1$$

(6.1)

$$dy = (b_1 y^2 - b_2 y^4 + b_3 xy - h_y)dt$$

While the predator species stochastic growth in multispecies ecosystem is modeled by setting $\sigma_{11} = 0$, equation (6) can be modified as:

$$dx = (a_1 x^2 - a_2 x^3 - a_3 xy - h_x)dt$$

(6.2)

$$dy = (b_1 y^2 - b_2 y^4 + b_3 xy - h_y)dt + \sigma_2 y dB_2$$

Although no stochasticity is assumed in one of the species in equation (6.1) and (6.2), both species become stochastic due to the existence of predator prey interaction term (xy) in the growth function.

The second part of the bioeconomic model consists of economic components, which are obtained from Agnarsson *et al.* (2008) and Sandal and Steinshamn (2010) as expressed below:

(7.1) $\pi_x(x, h_x) = p_x(h_x)h_x - c_x(x, h_x)$

(7.2) $\pi_y(y, h_y) = p_y(h_y)h_y - c_y(y, h_y)$

$\pi_x(x, h_x)$ and $\pi_y(y, h_y)$ are the net revenues from species x and y respectively. The net revenue from the harvesting of two species is the sum of the revenues from each species.

(7.3) $\pi(x,y,h_x,h_y) = \pi_x(x,h_x) + \pi_y(y,h_y) = p_x(h_x)h_x - c_x(x,h_x) + p_y(h_y)h_y - c_y(y,h_y)$

The function $\pi(x,y,h_x,h_y)$ is the net revenue. Functions $p(.)$ and $c(.)$ are inverse demand functions and cost functions respectively. The demand functions and cost functions are specified as:

$$p_x(h_x) = p_1$$

$$c_x(x,h_x) = q_1 h_x^{\alpha_1}$$

(8) $$p_y(h_y) = p_2 - p_3 h_y$$

$$c_y(y,h_y) = q_2 \frac{h_y^{\alpha_2}}{y}$$

After substituting the expression from equation (8) into equation (7.3), the profit function can be specified as:

(9) $$\pi(x,y,h_x,h_y) = p_1 h_x - q_1 h_x^{\alpha_1} + p_2 h_y - \frac{q_2}{y} h_y^{\alpha_2} - p_3 h_y^2$$

where $p_1, q_1, \alpha_1, p_2, q_2, \alpha_2$ and p_3 are economic parameters.

We assume that prey is a schooling species and, therefore, the unit cost of harvest is independent of stock size. Our revenue function depends only on the predator stock and harvest level of prey and predators and the revenue function equation (9) can simply be written as a function of 3 arguments $\pi(y,h_x,h_y)$.

Given the growth functions and profit function, the management objective is to maximize expected net present value of the return from the harvest schedule over an infinite time horizon.

This can be achieved by maximizing following function:

(10) $$J(y,h_x,h_y) = E\left[\int_0^\infty e^{-\delta t} \pi(y,h_x,h_y) dt \right]$$

The non-negative parameter δ is the discount rate and E is the expectation operator. Along with dynamic constraints and appropriate boundary conditions, the dynamic optimization problem can

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be written as:

$$V(x_0, y_0) = \max_{h_x, h_y \geq 0} J(y, h_x, h_y)$$

(11)
$$x(t = 0) = x_0$$

$$y(t = 0) = y_0$$

The optimal solution in predator-prey model can be obtained by solving following Hamilton-Jacobi-Bellman (HJB) equation (Kushner & Dupuis, 2001) along with appropriate boundary conditions. This is an equation for any feasible initial condition and hence we replace (x_0, y_0) with (x, y).

(12)
$$\delta V(x, y) = \max_{h_x, h_y \geq 0} \left\{ \pi(y, h_x, h_y) + V_a^T(x, y) F(x, y, h_x, h_y) \right.$$

$$\left. + \frac{1}{2} tr\left[\sigma(x, y) \sigma^T(x, y) V_{aa}(x, y) \right] \right\}$$

where, $V_a(x, y) = \begin{pmatrix} V_x(x, y) \\ V_y(x, y) \end{pmatrix}$, $F(x, y, h_x, h_y) = \begin{pmatrix} f(x, y) - h_x \\ g(x, y) - h_y \end{pmatrix} = \begin{pmatrix} a_1 x^2 - a_2 x^3 - a_3 xy - h_x \\ b_1 y^2 - b_2 y^4 + b_3 xy - h_y \end{pmatrix}$,

$\sigma(x, y) = \begin{pmatrix} \sigma_1 x & 0 \\ 0 & \sigma_2 y \end{pmatrix}$ and $V_{aa}(x, y) = \begin{pmatrix} V_{xx}(x, y) \\ V_{yy}(x, y) \end{pmatrix}$ are matrices.

The subscripts of V denote partial derivatives with respect to the index $i = x, y$. The HJB equation (12) can be rearranged as:

$$\delta V = \max_{h_x, h_y \geq 0} \left\{ \pi(y, h_x, h_y) + (f(x, y) - h_x) V_x + (g(x, y) - h_y) V_y \right.$$

(13)
$$\left. + \frac{1}{2}(\sigma_1 x)^2 V_{xx} + \frac{1}{2}(\sigma_2 y)^2 V_{yy} \right\}$$

After substituting $f(x, y) = a_1 x^2 - a_2 x^3 + a_3 xy$ and $g(x, y) = b_1 y^2 - b_2 y^4 + b_3 xy$ and

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$$\pi(y, h_x, h_y) = p_1 h_x - q_1 h_x{}^{\alpha_1} + p_2 h_y - \frac{q_2}{y} h_y{}^{\alpha_2} - p_3 h_y^2 \; , \text{ equation (13) yields:}$$

(14)
$$\delta V = \max_{h_x, h_y \geq 0} \left\{ p_1 h_x - q_1 h_x{}^{\alpha_1} + p_2 h_y - \frac{q_2}{y} h_y{}^{\alpha_2} - p_3 h_y^2 - h_x V_x - h_y V_y \right\}$$
$$+ (a_1 x^2 - a_2 x^3 - a_3 xy) V_x + (b_1 y^2 - b_2 y^4 + b_3 xy) V_y + \frac{1}{2}(\sigma_1 x)^2 V_{xx} + \frac{1}{2}(\sigma_2 y)^2 V_{yy}$$

Optimal solution can be derived by solving the HJB equation (14). The inner optimum with respect to controls (h_x and h_y) are given in appendix A. While it is difficult or impossible to solve analytically the HJB equation together with boundary conditions, we solve it using numerical approximation methods.

3. Numerical approximation approach

Our model is a two-dimensional model and strongly non-linear in control. Analytical solutions are usually not available to such problems and it is difficult to solve the Hamilton-Jacobi-Bellman (HJB) equation together with nonlinearity and given boundary conditions. Numerical approximation methods are the only viable alternatives. The Markov chain approximation approach, based on probability theory, is one of the most effective methods (Song, 2008). Numerical algorithms for optimal stochastic control problems of this kind can be found in Kushner and Dupis (2001). While dealing with the convergence of the numerical methods, it is shown that the value functions to which our approximations converge are the optimal value functions.

The numerical technique entails discretizing the state space for the HJB control problem (14), constructing transition probabilities for the controlled Markov chain by applying finite difference techniques and then iterating on the HJB equation with initial guess V_0 for the value function. The combined approximation in policy space and in value space is more powerful and faster as the

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value function is updated with the new policy at each step. The iteration is carried out until the value function converges to the optimal value functions (for details of the approximation refer to Kushner & Dupuis, 2001).

4. Application in cod and capelin species in Barents Sea

The Barents Sea is one of the most productive ocean areas in the world (O'Brien *et al.*, 2004), and it harbors two key species: capelin (*Mallotus villosus*), the plankton feeder and Northeast Arctic cod (*Gadus morhua*), the main predator of capelin. Cod is considered to be the main basis of the Norwegian commercial white fish industry (Kugarajh, *et al.*, 2006), while capelin is the largest pelagic fish species in the Barents Sea. Capelin is also potentially the largest stock in the world (Gjøsæter, 1998; Gjøsæter & Bogstad, 1998) and is of crucial importance as a prey for the growth of juvenile cod (Dalpadado & Bogstad, 2004; Hamre, 2003). The relationship between these two species is highly dynamic and is essential in the Barents Sea ecosystem (Bogstad, *et al.*, 1997).

Management of fisheries in the Barents Sea already includes species interactions to some degree. The importance of multispecies management in Barents Sea was realized after the capelin collapse in the mid-1980s and the subsequent dramatic effects on the cod (e.g. cannibalism) and other species, such as the mass migration of harp seals (*Phoca groenlandica*) to the coast of Norway where many of them drowned in fishermen's nets (Huag & Nilssen, 1995). Following this, a large multispecies research program was initiated. The predation by cod on mature capelin is included into the assessment of capelin, and cod cannibalism; and cod predation on haddock is included in the assessment of cod species (ICES, 2004). Furthermore, the cod recruitment and survivability is directly affected by climatic environment such as temperature, spawning season and the availability of food, such as prey stocks. Higher temperature at spawning time and more capelin have a positive effect on cod recruitment. However, high temperature results in decrease in capelin biomass through high herring recruitment, which affects the cod recruitment negatively through cannibalism (Hjermann *et al.*, 2007). Given these uncertainties, we choose a stochastic growth model consisting of capelin and cod as foundation for the decision making in the multispecies management.

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4.1 Specification of biological and economic parameters

Since the purpose of the paper is to extend and compare the finding with previous studies, both the biological and economic parameter values are obtained from Agnarsson *et al.* (2008) and Sandal and Steinshamn (2010) and they are specified below:

Growth for capelin in single species model: $r_1 x^2 (1 - \frac{x}{k_1}) = 0.00021781 x^2 (1 - \frac{x}{8293})$ $(10^6$ Kg /year)

Growth for cod in single species model: $r_2 y^2 (1 - \frac{y}{k_2}) = 0.000665 y^2 (1 - \frac{y}{2473})$ $(10^6$ Kg/year)

Growth for capelin species in multispecies model:

$$f(x, y) = a_1 x^2 - a_2 x^3 - a_3 xy = 0.00018 x^2 - 1.19 e^{-8} x^3 - 0.00021 xy \quad (10^6 \text{ Kg/year})$$

Growth for cod species in multispecies model:

$$g(x, y) = b_1 y^2 - b_2 y^4 + b_3 xy = 0.00022 y^2 - 3.49 e^{-11} y^4 + 1.82 e^{-5} xy \quad (10^6 \text{ Kg/year})$$

Demand for capelin: $p_x(h_x) h_x = p_1 h_x = 1 h_x$ (with price p_1=1 NOK/kg)

Demand for cod: $p_y(h_y) h_y = (p_2 - p_3 h_y) h_y = (12.65 - 0.00893 h_y) h_y$ (with price p_2=12.65 NOK/kg)

Cost for catch of capelin: $c_x(h_x) = q_1 h_x^{\alpha_1} = 0.07 h_x^{1.4}$ $(10^6$ NOK)

Cost for catch of cod: $c_y(y, h_y) = q_2 \frac{h_y^{\alpha_2}}{y} = 5848.1 \frac{h_y^{1.1}}{y}$ $(10^6$ NOK)

It is also worth mentioning that the price of cod (predator) is much higher compared to the capelin (prey) species. We also assume a constant cost of harvesting for capelin, which is independent of the stock size because of the schooling nature of the species (Aanestad *et al.*, 2007). Moreover, all the optimal feedback solutions are calculated with 5 percent discount rate (δ=0.05) and with different level of stochasticity, where the stochastic parameters are defined exogenously.

5. Results and Discussions

5.1 Optimal harvest in deterministic growth models

5.1.1 Single species model

The single species deterministic growth model specified in equation (1) is employed to obtain the results for each species individually. In a single species capelin model, it is suboptimal to harvest if the stock biomass level is below 1.2 million ton (figure 1a) because the future profit gain from conserving the stock will be higher. The harvest can be increased gradually if the stock biomass is between 1.2 to 2.5 million ton. It is optimal to harvest capelin as a 'bliss' or static optimum (myopic) if the stock reaches over 2.5 million ton.
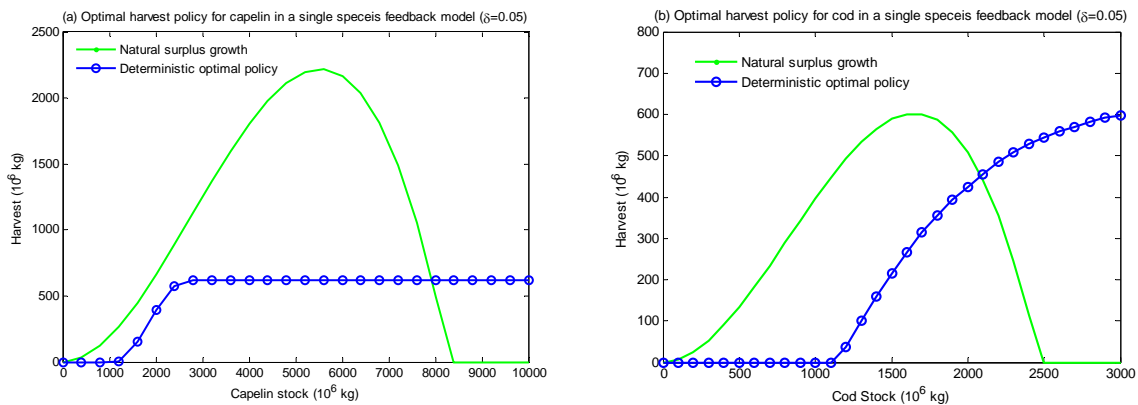

Figure 1: Single species deterministic optimal harvest policy (a) capelin (b) cod

The optimal harvest in cod species is more conservative than in capelin species. The moratorium is around 1.2 million ton of biomass near the maximum sustainable yield (1.5 million ton). There are two reasons for a higher moratorium level in cod. First, there could be low net revenue from the harvest due to stock dependent higher harvesting costs if harvested at a low stock level (no fixed costs are assumed in the model). Second, there is a possibility of higher future gain from conserving the stock. Although, we don't look at the risk aversion case, the downward sloping

demand curve has the same effect as risk aversion. The curve (figure 1b) shows that it is optimal to increase harvest gradually with the increase in the stock size up to two million ton.

5.1.2 Multispecies Model

The deterministic growth model specified in equation (3) is employed to find optimal solution for capelin and cod species simultaneously. The optimal feedback solution for capelin and cod harvest in the two-dimensional cod capelin state space is presented in figure 2.

The optimal harvest policy is considerably affected in capelin due to the interaction with cod except for zero cod stock. At zero cod level, the harvest policy is similar to the single species capelin model, as expected. The harvest pattern of capelin for large cod stocks level is less intuitive (see figure 2a). The pattern consists of considerable harvest at low capelin stock levels, then a moratorium over a certain range followed by a gradual approach to the static optimum. The intuitive explanation of harvest at low stock levels is that it would go extinct because of cod predation. Sandal and Steinshamn (2010) have clearly shown that the presence of cod in the model induces critical depensation along the optimal paths. In other words, if the capelin biomass level is below two million ton, it goes extinct even without harvest due to cod predation. Therefore, it is optimal to increase the harvest of capelin because if they are not harvested for human consumption they will be eaten by cod anyway. This occurs only if the cod stock is sufficiently abundant and therefore its commercial existence (profitability) does not depend too much on its present feeding on capelin. Hence, whether capelin is harvested or not, it does not reduce the value of the cod fishery as it approaches the single species fishery. As capelin stock increases over two million ton, cod processes capelin and cod can be harvested at lower level compared to the low capelin stock level so that profit will be increased due to the higher cod price. Sandal and Steinshamn (2010) also show that the 'valley region' in figure 2a means capelin stock could go extinct if it is roughly below 4 million ton and could rebuild if it is over 4 million ton. If the stock of capelin increase over 6 million ton, it can be harvested at 'bliss' or myopically as in the single species solution. The 'valley region' reflects at least two properties in the bioeconomic system. It represents an intrinsic precautionary policy trying to avoid capelin from

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going extinct and at those ranges of stocks it is more profitable to harvest the capelin through the cod. That is, the value created by letting cod eat it a unit of capelin is higher than the value created by landing that unit.

On the other hand, the species interaction has a minor effect on the harvest policy for cod species. Due to interaction, the cod moratorium shifts toward higher biomass level compared to single species model at low capelin stock level. For instance, the moratorium level is 1.4 million ton in the multispecies model compared to 1.2 million ton of biomass in single species model. This is because of low food availability for cod in the ecosystem. While the capelin stock becomes large (over 2 million ton), the cod moratorium level remains unaffected in the two species interaction model. This explains that the single species model suggests exploitation at lower level because it does not take account of the food availability of the cod stock. Furthermore, with increase in the cod biomass level, a higher exploitation is possible with more capelin in the ecosystem because of the increased food availability for cod.
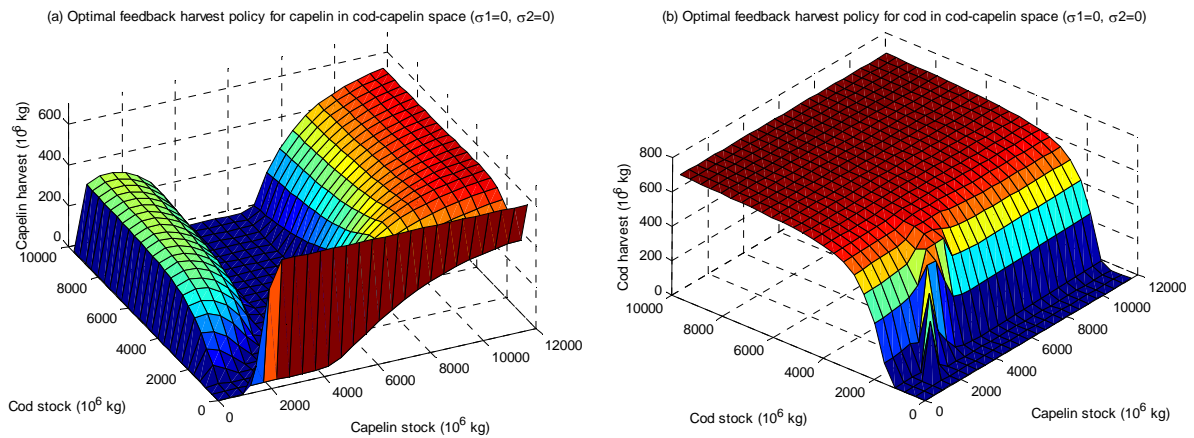

Figure 2: Multispecies deterministic optimal harvest policy (a) Capelin (b) Cod

In conclusion, the multispecies management approach makes it possible to increase the total profit by managing the stocks simultaneously. Because there is still a possibility of harvesting capelin at the same level as the single species optimal level if the capelin stock is maintained to a higher biomass level (>6 million ton), while we can observe that a sufficiently large catch of cod

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stock is possible in the cod - capelin model compared to the single species. Therefore, it can easily be generalized that management of ecosystem is more profitable than managing stock separately.

5.2 Optimal harvest in stochastic growth models

5.2.1 Single species model

The single species stochastic growth model specified in equation (2) is employed to obtain the optimal policy and is compared with the deterministic solution at various level of stochasticity. Figure 3 shows the optimal solutions for different levels of stochasticity for capelin and cod.

A high level of stochasticity in capelin leads to a more conservative harvest compared to the deterministic case. Compared to the sharp increase from moratorium level to 'bliss' in the deterministic case, the optimal solutions approach the static level more gradually in the stochastic case. At a high level of stochasticty (for example σ_0=0.7), it is optimal to harvest myopically at low stock levels because there is a significant possibility that the stock could go extinct due to stochasticty. But one should be more conservative if the stock becomes large (above 1.2 million ton) because there is only a small chance of extinction of the large stock even under high stochastic growth. With a fairly large stock (above 7 million ton), the optimal harvest policy for capelin remains more or less unaffected for all levels of stochasticity (figure 3a).

For stochastic levels below 0.5, there is a very small effect on optimal harvest of cod. For example, we see that the curves in the deterministic and stochastic cases stay close together and for practical purposes they can be regarded as identical. Due to the stock dependent harvest cost, the moratorium is large in cod and a large stock is relatively less influenced by stochasticity. For an increased level of stochasticity (σ_0 = 0.7), the harvesting strategy becomes myopic for smaller stock levels and conservative in the case of large stocks. But at fairly high levels of stochasticity (σ_0 =1.5), it is optimal to harvest myopically at all stock levels (see figure 3b).
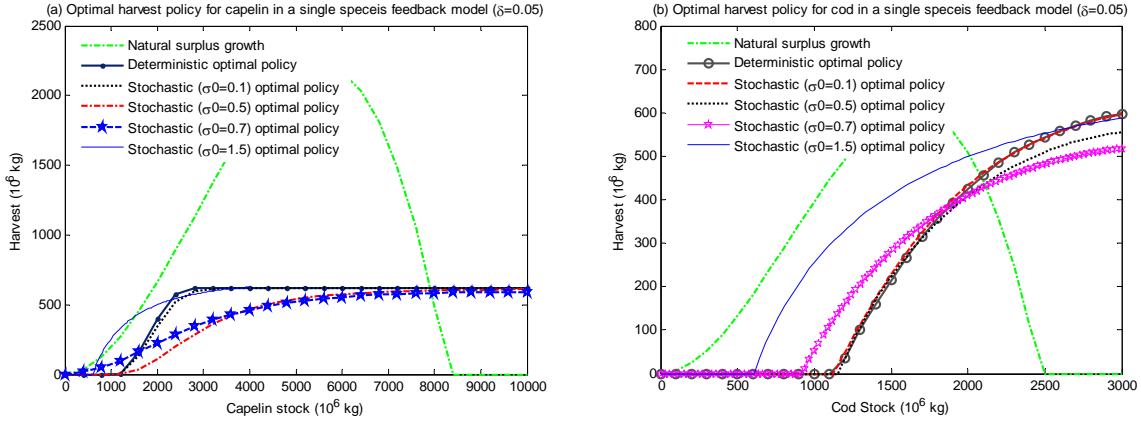
Figure 3: Effect of stochastic growth for optimal harvest policy in single species models (a) capelin (b) cod

5.2.2 Multispecies model

As in the single species model, we have employed different levels of stochasticity to study the effect on the optimal harvest policy. We have also observed the effects in three different cases - with prey stochasticity, with predator stochasticty and stochasticity in both species.

5.2.2a. Stochastic growth in capelin (prey) species

We have employed equation (6.1) to obtain the optimal feedback policies in the multispecies model where capelin has stochastic growth. The optimal solution for exploitation of capelin and cod in a two-dimensional state space with different levels of stochasticty in capelin is presented in figure 4. With an increased stochasticity in capelin, the optimal harvest of capelin becomes more conservative compared to the deterministic optimal policy. For low levels of stochasticity (for example, $\sigma_1=0.1$), the 'valley region' that appears in the deterministic case becomes larger suggesting no harvest of capelin in this particular region. Because by not harvesting capelin, we

can harvest the high priced cod which yields higher net revenue. But in a region with a very large capelin stock, it will be optimal to harvest capelin as 'bliss' along with harvest of cod. A further increase in stochasticity (for example, σ_1=0.3), makes the 'valley region' much wider suggesting that one should be conservative in the capelin harvest even for very large biomass levels.

There is no strong influence from the stochasticity on the optimal harvest of cod as long as the level of stochasticity is less than 0.3. The main reason is that the uncertainty is managed by reducing the harvest of capelin. At higher level of stochasticity (for example σ_1 >0.5), the 'valley region' in capelin gets narrower. This suggests that capelin should be harvested earlier compared to the low stochastic growth model because conserving the highly stochastic capelin stock may not contribute to the growth of cod and subsequently its harvest. However, one should still be conservative and save the capelin stock from extinction at large biomass level due to stochasticty and natural predation by the cod.

As stochasticity increases to one (σ_1 =1), the 'valley region' disappears but the optimal exploitation level is clearly different from the deterministic solution. The optimal harvest of capelin is less conservative when the stock is low and becomes conservative at large stock levels. The reason is that a small stock of capelin may not be utilized by the cod to a profitable level but a large stock of capelin could be utilized by the cod that increases net profits. Therefore, it is optimal to be conservative for the low priced capelin and let the cod prey on it when the capelin stock is high. But for very high stochasticity (σ_1>1.0), there is no gain of conserving a highly stochastic capelin stock for the purpose of cod feeding. Therefore, capelin should be managed as in a highly stochastic single species model, and the cod stock should also be managed as in the single species model (not shown).
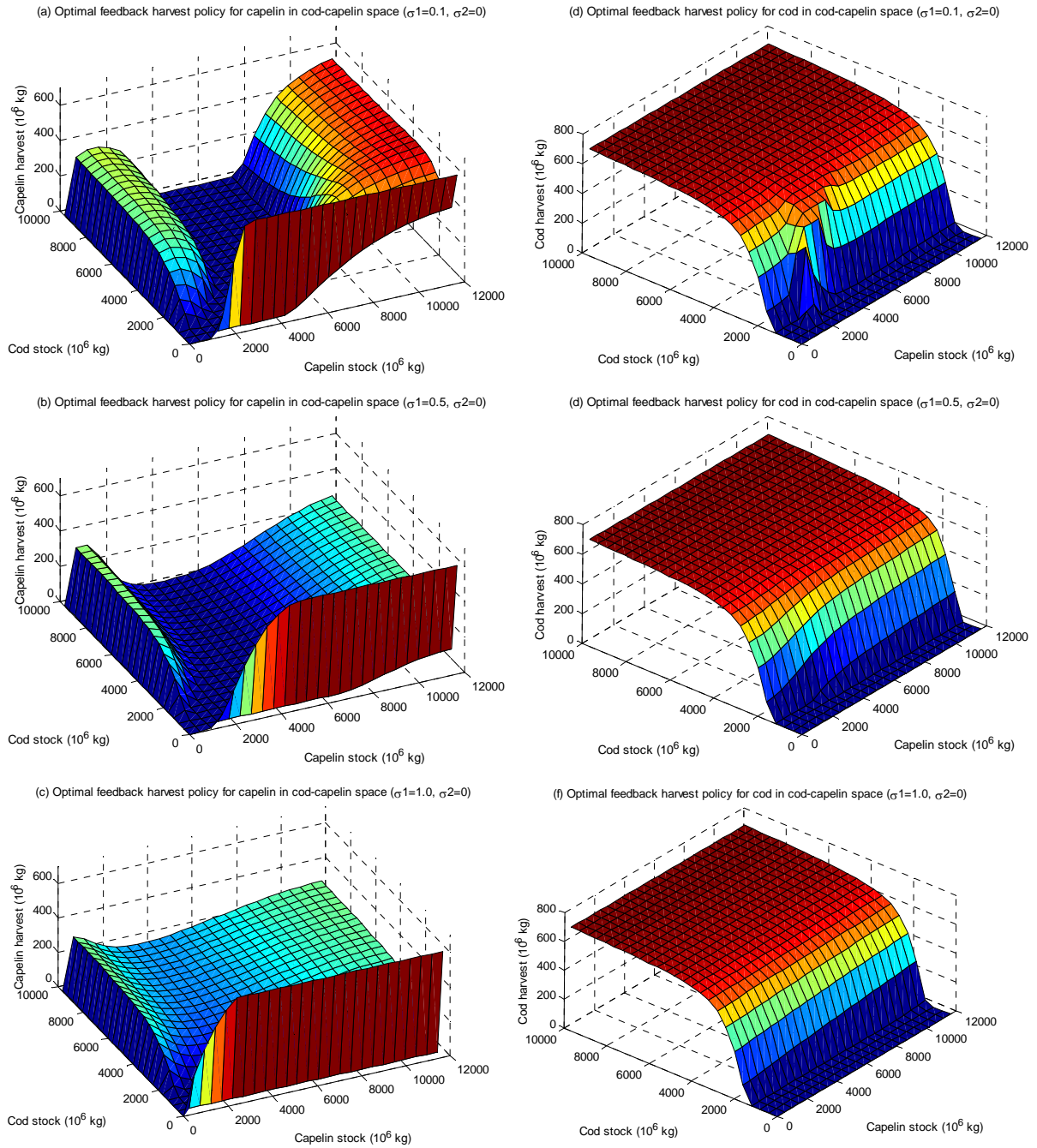
Figure 4: Effect of capelin stochasticity in optimal harvest (a) Capelin with σ_1=0.1 (b) Capelin harvest with σ_1=0.5 (c) Capelin harvest with σ_1 =1 (d) Cod harvest with σ_1=0.1, (e) Cod harvest with σ_1=0.5 and (f) Cod harvest with σ_1 =1

5.2.2b Stochastic growth in cod (predator) species

We have employed equation (6.2) to obtain the optimal harvesting rules in the multispecies model where cod has stochastic growth. Optimal solutions for exploitation of capelin and cod stocks in two dimensional state space for different levels of stochasticty (σ_2) are presented in figure 5.

It can be noted that a small level of stochasticty in cod (for example σ_2=0.1) affects the harvest of capelin stocks slightly by widening the 'valley region' in the deterministic solution. An increased level of stochasticity (for example σ_2=0.5) further widens the moratorium (the valley) with almost no harvest of the capelin stocks until it is above 7.5 million ton. This is very intuitive because the stochastically growing cod needs more food so that cod can be harvested unaffected. For stochasticity below 0.5, cod harvest is unaffected, but capelin harvest changes. At a substantially high stochasticty level (for example σ_2=1.0) in cod, the capelin should be harvested as in single species deterministic model because it does not pay off to feed a non-stochastic species to a highly stochastic species. Therefore, the highly stochastic cod should be harvested as in the single specie solution and at a myopic level.
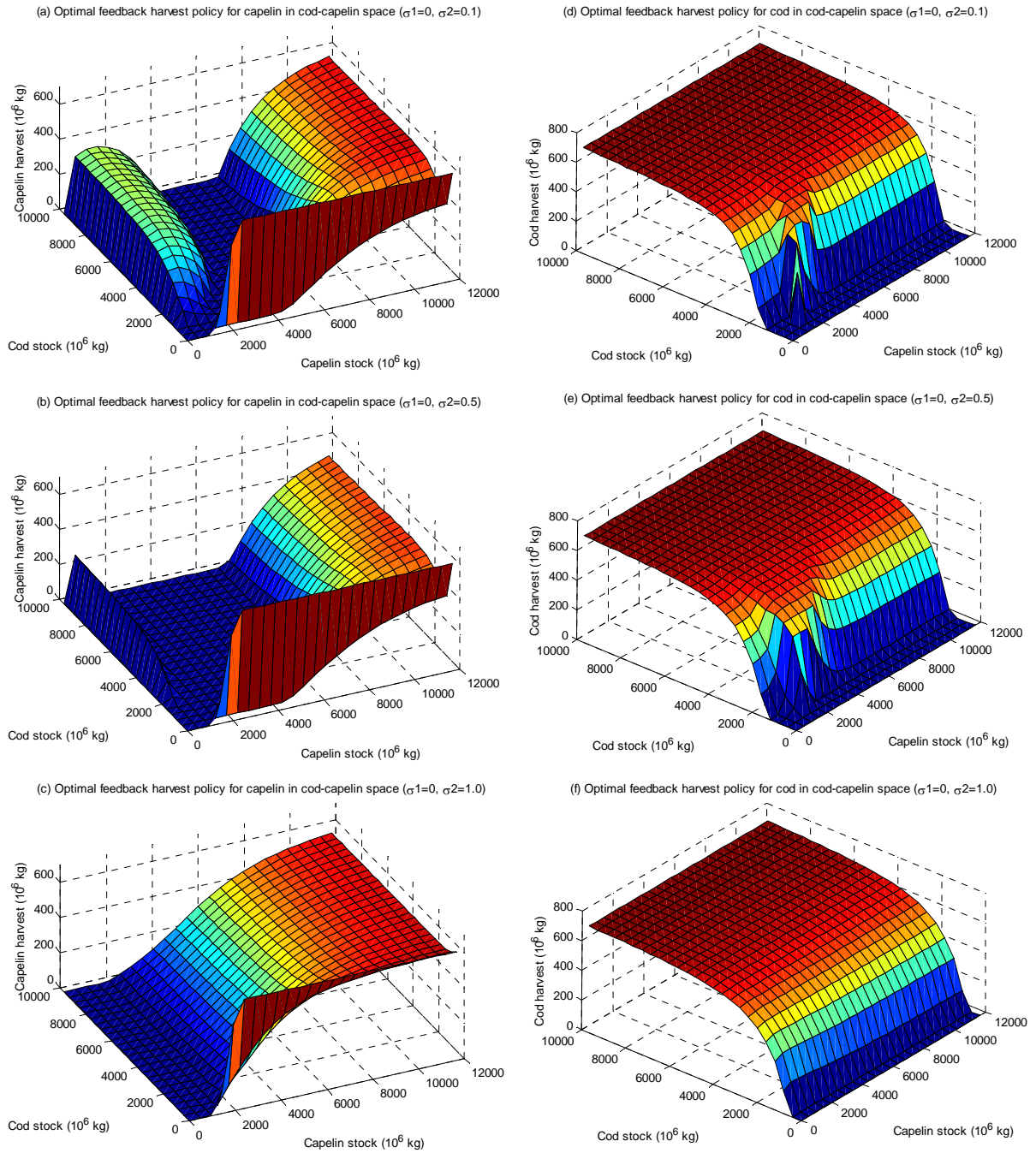
Figure 5: Effect of cod stochasticity in optimal harvest (a) Capelin harvest with σ_2=0.1 (b) Capelin harvest with σ_2=0.5 (c) Capelin harvest with σ_2 =1 (d) Cod harvest with σ_2=0.1, (e) Cod harvest with σ_2=0.5 and (f) Cod harvest with σ_2 =1

5.2.2c Stochastic growth in both species

The effect of stochastic growth of prey and predator species to optimal harvest in a multispecies model is obtained by using equation (6). Optimal exploitation policies with different level of stochasticty (σ) for capelin and cod in a two dimensional state space is presented in figure 6. To avoid many plots, we have only plotted the result for cases with $\sigma_1 = \sigma_2 = \sigma$ to capture the general features in the result.

At low levels of stochasticity (for example σ<0.3), the capelin species should be harvested in a conservative way in order to allow cod to feed on it so that cod can be harvested unaffected. On the other hand, with an increasing level of stochasticity (for example σ=0.5), the capelin harvest should be more conservative and cod harvest should also be more conservative to ensure better growth under this level of stochasticity.

However, at a high level of stochasticity (for example $\sigma = 0.7$), the capelin stock should be harvested conservatively. For a substantially higher level of stochasticity (for example σ >1), it should be harvested myopically at small stock levels and conservatively at large stock levels. This is similar to the single species stochastic solution. The cod stock should be harvested as in the single species stochastic solution and one should be myopic when the stock is relatively small. In both species the possibility of extinction of a small stock is high due stochasticity and the capelin extinction risk is further exacerbated by cod predation.

Looking at the three different cases - stochasticity in prey species, stochasticity in predator species and in both species - at different levels of stochasticity, we observe that there are various effects on the optimal harvests compared to the deterministic setting. Effects naturally depend on the stock size. If the stock is small, stochasticity has a more pronounced effect.
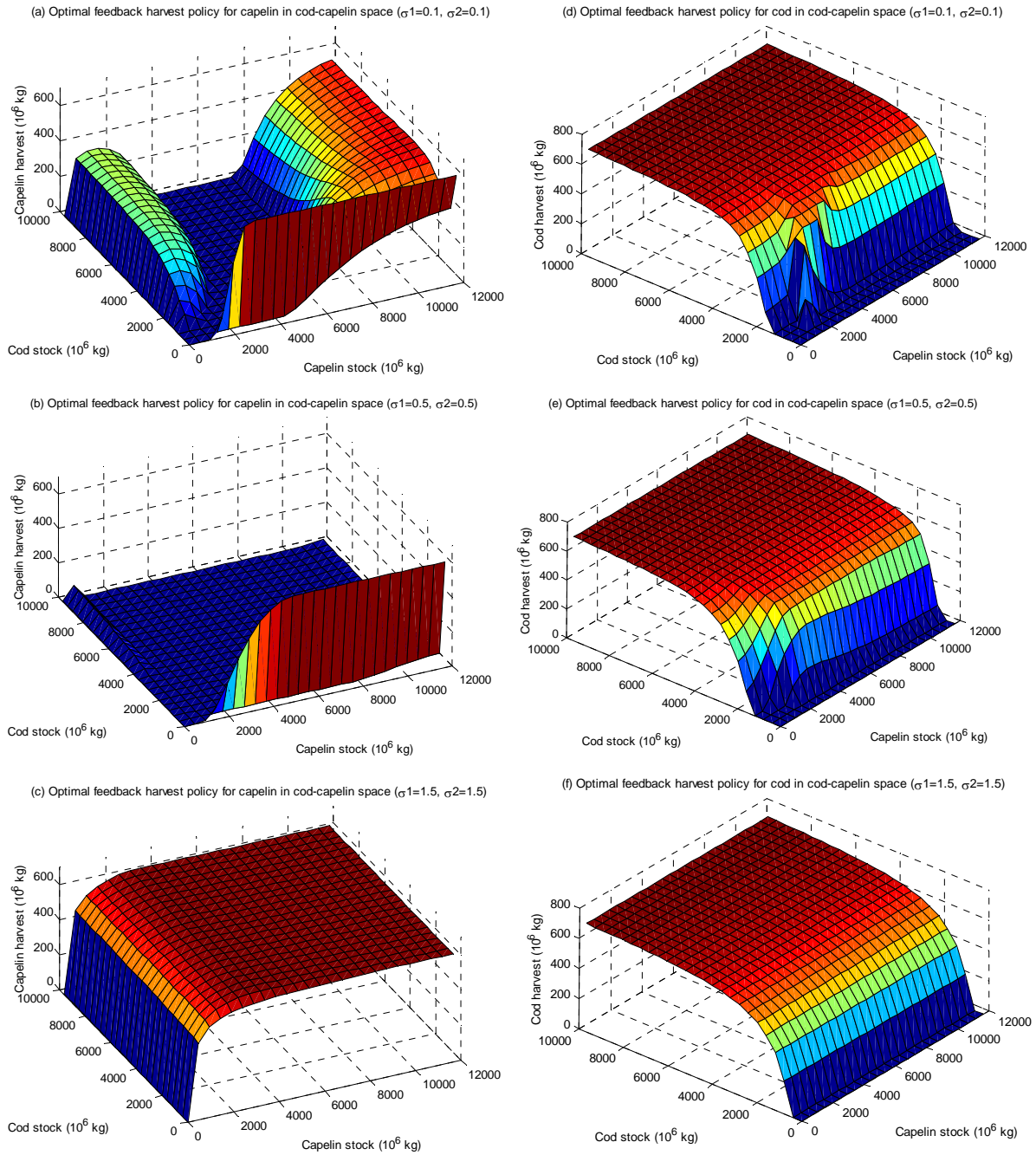
Figure 6: Effect of stochasticity in optimal harvest (a) Capelin harvest with σ=0.1 (b) Capelin harvest with σ=0.5 (c) Capelin harvest with σ=1 (d) Cod harvest with σ=0.1, (e) Cod harvest with σ=0.5 and (f) Cod harvest with σ=1.5

The stochastic growth either in prey species or in predator species or in both species affects only the prey species at low stochasticity levels ($\sigma < 0.3$). There are three possible reasons. First, by conserving the low priced prey species as a food for the higher priced predator, profits can be increased. Second, due to stock independent costs of harvest, there is a low moratorium level for the prey (capelin) in the deterministic solution and the stock becomes more sensitive with increasing stochasticity. Third, cod predation makes the prey more sensitive to optimal exploitation.

In the deterministic and small stochasticity case, we observed that it is possible to harvest capelin at exactly the same level as in the single species solution for large stock levels, and sufficiently large harvest of cod stock in a cod capelin multispecies model compared stochastic single species model. In other words, there is an advantage of multispecies management when the stock is large. Although we can observe a higher harvest of capelin in the single species solution compared to the multispecies solution, it is important to note that it is profitable to increase the harvest of cod by one kg while sacrificing up to 12 kg of capelin because of the price difference. Furthermore, our model suggests that capelin should be conserved at low to moderate levels of stochasticity in all three cases because the increase in harvest of cod is more profitable compared to harvesting both species conservatively in a single species model under the assumption of the stochasticity. The general impression is that ecosystem management is more profitable than managing individual stocks separately when there is limited uncertainty.

At moderate to high stochasticity levels (for example $0.3 < \sigma < 0.7$), the effect on optimal harvest strategy cannot be generalized as the effect depends on the stock size and the stochastic species. In general, effects are stronger at low stock levels.

Furthermore, at a substantially high stochasticity level (for example $\sigma > 1$), the stochastic species should be harvested myopically and the deterministic species should be harvested as in the single species model. If both species are highly stochastic, they should be harvested myopically. This means that with very high stochasticity, the effect of multispecies interaction cannot be observed. Although an assumption of a very high level of stochasticity may not be observed in real-world

fisheries, with the precautionary principle in mind, we still found it instructive to study such high levels of stochasticity in the model. However, we put emphasis on the effect of stochasticity at a low to moderate levels of stochasticity, which is important in real-world fisheries management.

5.3 Optimal harvest versus historical harvest

We compared the optimal solutions from four different models with the actual harvest data over the last 33 years. The optimal harvest and actual harvest are presented in figure 7 and 8. It can be noted that the actual harvest is fairly higher compared to the optimal harvest models during most of the periods in both the stocks.
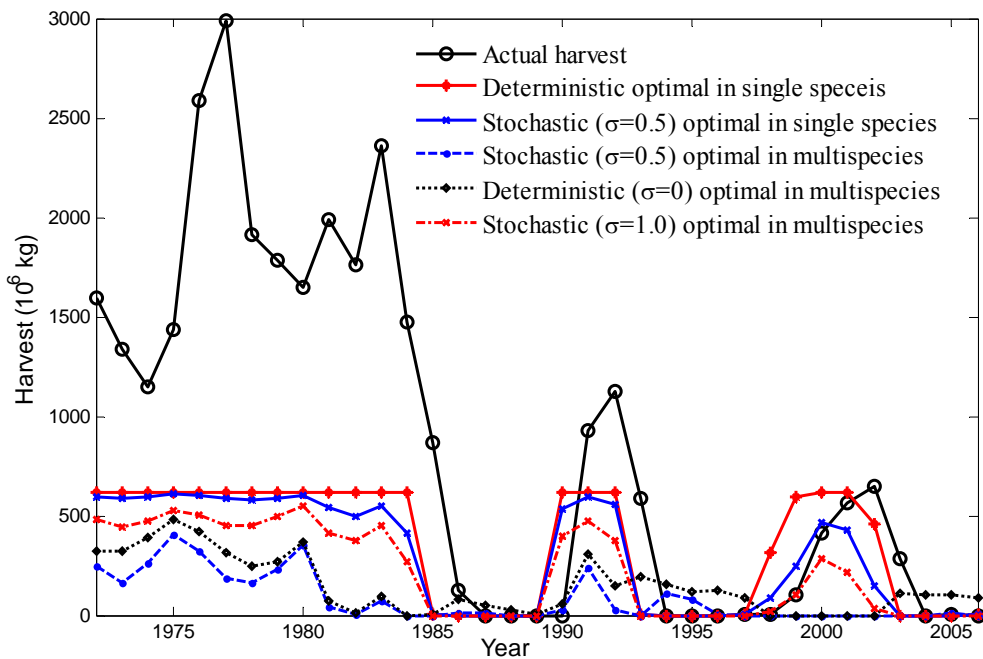

Figure 7: Actual versus optimal harvest of capelin species in different modeling approaches

The capelin curves reveal a dramatic overfishing of the capelin fishery until 1987 before the closure of the catch (Tereshchenko, 2002). In later period, it is close to the identified single species deterministic optimal catch level. Looking at the different multispecies optimal harvests,

it can be inferred that there was under-exploitation during 1994-1999 and after 2004. However, this is not pronounced compared to the case of overexploitation.

A similar overexploitation over the whole period is revealed in cod species compared to optimal catch identified in different models. However, the trends of historical actual catch and optimal catch are analogous over most of the periods. Moreover, it is very interesting to note that historical harvest nearly follow the very-high-level-stochastic model results which are close to a myopic policy. The model explains that the historical harvest was myopic or at the 'static optimum' for the cod harvest contrary to optimal harvest.
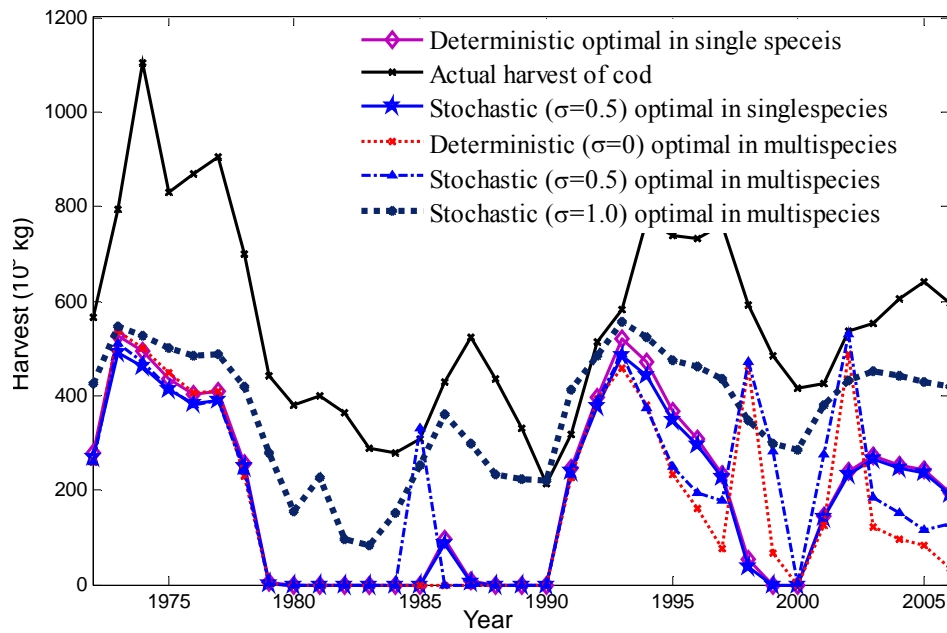

Figure 8: Actual versus optimal harvest of cod species in different modeling approaches

The general overexploitation of the fish stocks in Barents Sea could be the result of a policy that aims at a MSY or maximum sustainable yield (Agnarsson, et al., 2008) from biological point of view and the economic aspect might have been ignored. Moreover, MSY is a single species concept and it is normally infeasible to have both stocks at MSY simultaneously.

6. Concluding Remarks

We have demonstrated the applicability and usefulness of dynamic programming to multispecies management under stochasticity. While biological predator-prey interactions may have been well understood, much remains to be done for a comprehensive understanding of their economic consequences. Our approach sheds light on the interaction of economics with a complex biology.

Despite that the eco-biological system is relatively complex; the nature of our findings is intuitive. It is relatively more profitable to manage the ecosystem in stochastic multispecies fishery than individual stocks separately. Our results show fundamentally different conclusions with and without stochasticity. We conclude that a management strategy in a marine ecosystem that ignores biological interactions and stochastic variability cannot be expected to be close to close to an optimal strategy.

We believe that policy makers would benefit from an increased appreciation of the effects of stochasticity and the consequences of ignorance. The ignorance becomes apparent when we compare our solutions to historical catches. In periods, landings lie closer to the myopic policy than to the optimal dynamic feedback policy both in the stochastic and deterministic cases.

Although the general result might not reflect the best management due to the exogenous parameters adopted in the work, our paper is a breakthrough in complicated, stochastic multispecies modeling which combines both biology and economics and provides a basis to answer what the optimal response is for a bioeconomic manager of renewable resources.

The estimation of stochastic parameters in a multispecies model is a very difficult and complex task. But at the same time its accurate estimation is crucial for further improvement of the model. The stochastic parameter estimation, therefore, remains to be a future scope for the identification of optimal policy for stochastic multispecies fisheries. Similarly, the relaxation of the assumption of un-correlated stochasticity or the noise among the interacting species could improve the model considerably.

Finally, we submit that real world fisheries management is concerned with a limited number of fish stocks that are only a part of a larger ecosystem. To analyze the effect of stochasticity and to assess risk of stock collapse in an ecosystem framework is beyond our scope here. We are only on one of the first steps on a long ladder towards the ultimate goal of ecosystem management under uncertainty. But, it is an important step in the right direction.

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Appendix A: Inner optimum with respect to controls for multispecies model

The inner optimum can be obtained from HJB equation with respect to control (harvest) as follows.

(A1)
$$\delta V = \max_{h_x, h_y \geq 0} \left\{ p_1 h_x - q_1 h_x^{\alpha_1} + p_2 h_y - \frac{q_2}{y} h_y^{\alpha_2} - p_3 h_y^2 - h_x V_x - h_y V_y \right\}$$
$$+ (a_1 x^2 - a_2 x^3 - a_3 xy) V_x + (b_1 y^2 - b_2 y^4 + b_3 xy) V_y + \frac{1}{2}(\sigma_1 x)^2 V_{xx} + \frac{1}{2}(\sigma_2 y)^2 V_{yy}$$

The inner optimum for prey species,

$0 = p_1 - \alpha_1 q_1 h_x^{\alpha_1 - 1} - V_x$ or

(A2)
$$h_x = \left[\frac{p_1 - V_x}{\alpha_1 q_1} \right]^{\frac{1}{\alpha_1 - 1}}$$

The inner optimum for predator species;

The first order condition with respect to harvest (control)

$$p_2 - \frac{q_2 \alpha_2 h_y^{\alpha_2 - 1}}{y} - 2 p_3 h_y - V_y = 0$$

Which is solved numerically.