Discussion paper

Product quality, competition, and multi-purchasing

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Abstract: In a Hotelling duopoly model, we introduce quality that is more appreciated by closer consumers. Then higher common quality raises equilibrium prices, in contrast to the standard neutrality result. Furthermore, we allow consumers to buy one out of two goods (single-purchase) or both (multi-purchase). Prices are strategically independent when some consumers multi-purchase because suppliers price the incremental benefit to marginal consumers. In a multi-purchase regime, there is a hump-shaped relationship between equilibrium prices and quality when quality functions overlap. If quality is sufficiently good, it might be a dominant strategy for each supplier to price high and eliminate multi-purchase.

1 Introduction

Some consumers buy different variants of horizontally differentiated information goods (multi-purchasing) like newspapers and software programs, while others buy only one (single-purchasing). As an example, there are people who install Scientific Workplace as well as Mathematica on their computers, but others are not willing to pay for both – and installing two copies of the same software on a computer adds no benefit.

For hardware, compare smartphones (e.g., iPhone or HTC) and multi-purpose computer tablets (such as Samsung Galaxy or Apple iPad). Most smartphones and multi-purpose tablets have attributes like the ability to play games, make phone calls, read emails, watch videos, and listen to music. The main source of horizontal differentiation is the size. Compared to a smartphone, a tablet like Galaxy is superior for watching videos when sitting in the armchair at home, but a smartphone is significantly more convenient when travelling.[2] Many people thus end up buying both a smartphone and a tablet.[3] Likewise, a lot of consumers buy both an iPad and a Kindle. In a press release (December 27th, 2010) Amazon.com's founder and CEO Jeff Bezos said that *"We're seeing that many of the people who are buying Kindles also own an LCD tablet (e.g. an iPad). Customers report using their LCD tablets for games, movies, and web browsing and their Kindles for reading sessions. They report preferring Kindle for reading because it weighs less, eliminates battery anxiety with its month-long battery life, and has the advanced paper-like Pearl e-ink display that reduces eye-strain, doesn't interfere with sleep patterns at bedtime, and works outside in direct sunlight, an important consideration especially for vacation reading."* These devices thus have different attributes, so multi-purchasing provides incremental value over buying just one. But what will happen in the market place in the future if both these goods are improved and offer a larger number of attributes?[4] How might this

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[2]Due to their size, tablets are awkward for conventional voice telephony, but may be superior to smartphones for video conferences.

[3]See the discussion by Mies, PCWorld April 4th, 2010, " iPad Versus the iPhone". URL: http://www.pcworld.com/article/193420/ipad_versus_the_iphone_why_i_dont_need_bothyet.html.

[4]With the introduction of Kindle Fire (September, 2011) Amazon in fact introduced a version

affect prices and profits, and will multi-purchase become more or less likely? How does competition play out if attributes of the goods overlap substantially? These are among the questions we address in this paper.

Most economic analysis of discrete choice assumes that consumers buy one unit of at most one variant (see Anderson, de Palma, and Thisse, 1992). Yet the examples above suggest that there are many cases of purchase of (one unit of) several variants. For each variant, its attractiveness (quality) is increasing in its attributes: an attribute could be a particular mathematical tool in a software program, coverage of a certain news topic in a newspaper, and a web browsing opportunity or an e-ink display on a tablet. A consumer will buy several variants to the extent that this increases the number of different attributes she can access.[5] Thus, for any given consumer we will observe unit demand (at most) for any particular variant, but bringing both an iPad and a Kindle on a journey might be useful.

The Hotelling model is the workhorse for analyzing unit demand. We follow this line, but depart from most of the existing literature in two key ways. First, we model quality in a novel way. Specifically, we assume that the more satisfied a consumer is with the horizontal characteristics of a good, the greater will be her marginal utility of higher quality. On the tablet market, for instance, it is likely that when Amazon introduces a Kindle with color screen[6], then the increased willingness to pay for that device is more pronounced for a Kindle-lover than for an iPad-lover.[7] Second, we make single-purchase or multi-purchase an endogenous outcome which depends on consumer preferences, the qualities (the set of attributes) of the goods, and the strategic choices by the suppliers.

Our first innovation implies that the better the quality of variants, the higher

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of its e-book reader with more attributes. Hence, Kindle Fire is closer to a multi-purpose tablet like iPad, reducing the incremental value of having both.

[5]With due tribute to Lancaster (1966) for emphasizing the importance of the characteristics embodied in goods as the fundamental objects of desire.

[6]The Kindle Fire introduced in September 2011 has color screen; see footnote 4.

[7]To our knowledge, the only paper to use a similar formulation is Waterman (1989-90). He allows quality to interact with transportation costs in an extension of his analysis of the trade-off between quality and variety in a circle framework. He does not focus on the features of the formulation highlighted here.

their prices under single-purchase. This is in sharp contrast to standard results in symmetric Hotelling models, where prices are independent of whether suppliers provide high-quality or low-quality goods. This feature is also germane to more traditional circumstances with a single discrete choice between the variants offered. It seems plausible that prices should increase in product quality under single-purchasing. But is there reason to believe that the same holds under multi-purchasing? At the outset one might think so, but actually the opposite might be true. To see why, suppose that Apple improves iPad's ebook reading-quality and that Kindle becomes more of a multipurpose tablet. This will clearly increase the stand-alone value of each of the devices. However, it could also reduce the incremental value of having an iPad in addition to a Kindle (and vice versa). Put differently, while higher quality clearly increases the attractiveness of the goods, it may also make it less imperative for consumers to multi-purchase. Consistent with this, we show that there is a hump-shaped relationship between equilibrium prices and qualities under multi-purchase. One implication of this is that if the qualities are sufficiently high, a dominant strategy for each supplier might be to sacrifice some sales and set such high prices that no-one will buy both products.

The key property of the multi-purchasing equilibrium is that it is a special type of monopoly regime. Rival quality, but not rival price, determines demand. Prices are strategically independent even though they are determined by the quality levels of both goods. The starkly different properties of the purchase regimes are underscored by their comparative static properties. If the market is covered but consumers buy a single variant, equilibrium prices and profits are increasing in preference heterogeneity. By contrast, they are decreasing in preference heterogeneity under joint purchase.

These results imply that a market situation with multi-purchasing may be very poorly approximated by a traditional model of single purchases. To take multi-purchasing into account is fundamental for pricing, just as it is crucial to account for whether goods are substitutes or complements (see Gentzkow, 2007, who analyzes competition between print and online newspapers). An illustrative example is Amazon's pricing strategy in response to the introduction of iPad. When iPad

was introduced, Amazon could sacrifice sales for Kindle and set such a high price that they only attract the most passionate e-book-readers (single-purchase). The alternative was to follow a multi-purchasing pricing strategy and focus on consumer incremental value from having both tablets. The above quote from Amazon's CEO, and the fact that the price of Kindle (the 6" version) was reduced from \$260 to \$139 when iPad hit the market, suggest that Amazon's pricing strategy is based on the incremental value Kindle provides rather than on its stand-alone value.[8]

2 Related literature

This paper is related to several strands of the literature. Spatial differentiation à la Hotelling (1929) is a standard tool in product differentiation. It has recently played a prominent role in media economics, see e.g. Anderson and Coate (2005), Gabszewicz et al. (2004), Liu et al. (2004), Peitz and Valletti (2008), and Kind et al. (2012).[9] A limitation of the existing literature, though, is that it presupposes single-purchase ("single-homing").[10] To some extent the present paper can be considered as an extension of this literature, since we allow for multi-purchase ("multi-homing"). However, there is no advertising market in our model, and we thus abstract from many of the core issues in media economics. As accentuated in the Introduction, the application of the model is not restricted to media markets.

The quality formulation we use is somewhat reminiscent of the Mussa and Rosen (1978) formulation of vertical differentiation insofar as some consumers have higher

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[8]Note that the incremental price is below stand-alone monopoly price. The lower price is of course consistent with a competitive price under a single-purchase regime too. The presence of many individuals who own both suggests the incremental price may be the dominant force.

[9]For analysis of media market competition in non-Hotelling frameworks, see Godes et al. (2009) and Kind et al. (2008, 2009).

[10]The literature on two-sided markets (see Armstrong, 2006, and Rochet and Tirole, 2006), uses the terminology *single-homing* and *multi-homing* rather than single-purchase and multi-purchase. In a two-sided market context, Doganoglu and Wright (2006) analyze how multi-homing affects firms' incentives to achieve compatibility under network effects in a Hotelling framework. In a Cournot framework, Crémer, Rey, and Tirole (2000) likewise allow for multi-homing when analyzing firms' compatibility (interconnection) incentives.

willingness to pay for incremental quality. The present paper is also related to de Palma, Leruth, and Regibeau (1999), who analyze multi-purchase in a setting with Cournot competition and network effects, and to Gabszewicz and Wauthy (2003).[11] The latter extends the Mussa and Rosen (1978) framework by allowing for multi-purchasing. Two suppliers sell vertically differentiated goods, and consumers may buy both variants. As in the present paper, consumers do not buy two units of the same good, and the outcome depends on the incremental utility gained by consumers from buying both products. Kim and Serfes (2006) analyze multi-purchasing in a standard Hotelling framework, but in contrast to our model there is a linear relationship between equilibrium prices and quality.[12] Furthermore, in contrast to both Kim and Serfes (2006) and Gabszewicz and Wauthy (2003), we allow for quality to interact with the distance-based utility, and analyze the incentives to invest in quality.[13]

The equilibrium properties are also quite different from those in Gabszewicz and Wauthy. While they find no pure strategy equilibrium for some parameter values, we always have a pure strategy price equilibrium. In the Appendix we provide a detailed analysis of demand and reaction functions when we allow for both single-purchase and multi-purchase, and derive more general properties which apply to duopoly differentiated products pricing games. These results are useful for other applications, like spatial models, where kinks in demand are quite natural. We therefore give results for generalizations of our model, and then illustrate. For example, we find that local monopoly equilibrium cannot coexist with competitive equilibria, and there can be at most two (pure strategy) competitive equilibria.

The rest of the paper is organized as follows. In Section 3 we describe the basic set-up of the model. Section 4 provides a stand-alone analysis of single-purchase relevant to both the situation where multi-purchase is debarred by assumption and where single-purchase occurs in the wider equilibrium. Section 5 analyzes compe-

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[11]See also Ambrus and Reisinger (2006).

[12]Kim and Serfes (2006) implicitly assume no overlapping attributes or that the attributes are equally important independent of whether they are endowed in only one or both goods.

[13]Gabszewicz, Sonnac, and Wauthy (2001) analyze multi-purchase for complementary goods. They show how price equilibria depend on the degree of complementarity.

tition under multi-purchase, first for when it is an equilibrium outcome, and then for when the purchase regime is endogenously determined. Investment incentives are analyzed in Section 6. Section 7 concludes and discusses some routes for future research. Some of the proofs are relegated to the Appendix.

3 The model

Consider two suppliers, each producing one variant of a good. We normalize the universe of possible attributes (Q) a good can potentially deliver to 1, and denote the set of attributes that product i offers by $Q_i \subseteq Q$, where $i = 0,\ 1$. The larger the set Q_i, the more attractive is good i for the consumers. Each element (attribute) in Q is assumed to have the same intrinsic value for a consumer. Letting $q_i \in [0,1]$ denote the measure of Q_i, good 0 thus has a higher quality than good 1 if $q_0 > q_1$. The specifics of the multi-purchase scenario are further developed in Section 5.

The goods are horizontally differentiated, and are located at opposite ends of a "Hotelling line" with length 1. Good 0 is at the far left (point 0) and good 1 at the far right (point 1). Consumer tastes are uniformly distributed along the line. A consumer who is located at a distance x from point 0 receives utility equal to $R - tx$ from buying good 0 if it includes all possible attributes ($q_0 = 1$). Here R is interpreted as a reservation price, and t is the distance disutility parameter from not obtaining the most preferred type of product. When sold at a price p_0, consumer x's surplus from buying good 0 alone is given by

$$u_0 = (R - tx)\,q_0 - p_0. \tag{1}$$

The surplus from buying good 1 alone is similarly given by

$$u_1 = [R - t(1 - x)]\,q_1 - p_1. \tag{2}$$

The values of q_0 and q_1 are common knowledge.

The above describes preferences if consumers buy one product or the other, but we are also interested in the possibility of consuming both products. In Section 5 we

describe the utility in the case of multi-purchase, where consumers possibly enjoy greater benefit by buying both products.

The formulation in (1) and (2) is novel for the way the "quality" variable is introduced, as it interacts with the distance-based utility. In particular, the formulation implies that the smaller the difference between the horizontal location of a good and the preferences of a given consumer, the more valuable it is for her that new attributes are added to the good. Put differently, an increase in q_0 is more valuable for consumers located at the left-hand side of the Hotelling line than for those located on the right-hand side, and vice versa for an increase in q_1. Adding a color screen to Kindle, for instance, is perceived to be more valuable for a Kindle lover than for an iPad lover. As noted in the Introduction, this is reminiscent of the Mussa-Rosen (1978) formulation of vertical differentiation.

Aggregating the individual choices generates demands, $D_0(.)$ and $D_1(.)$. We assume away any marginal production costs of the goods. Let the profit function of supplier i be given by

$$\pi_i = p_i D_i - C(q_i), \quad i = 0, 1, \tag{3}$$

where $C(q_i) \geq 0$ is the cost of investing in quality, with $C'(q_i) > 0$ and $C''(q_i) > 0$. We assume that $C(q_i)$ is sufficiently convex to ensure the existence of a stable, symmetric equilibrium. For the first part of the analysis, however, we shall consider the sub-games induced for given q_i's, in order to elucidate the differences between the market outcomes at which each consumer buys a single good (single-purchase) or else some consumers buy both goods (multi-purchase).

4 Single-purchase

Assume for now that each consumer buys one and only one of the goods (single-purchase). This analysis covers the case when only single-purchase is feasible, and also the case when it is an equilibrium outcome in the broader formulation when multi-purchase is allowed but yet does not arise in equilibrium. We restrict attention to parameter values which guarantee that all consumers are served and that both

suppliers are operative (market coverage and market-sharing). Below, we show that there is such an equilibrium if and only if:[14]

Assumption 1: $R \geq \frac{3}{2}t$

Using (1) and (2) to solve $u_0 = u_1$ finds the indifferent consumer's location as

$$\hat{x} = \frac{tq_0 + (R - t)(q_0 - q_1) - (p_0 - p_1)}{t(q_0 + q_1)}. \tag{4}$$

Demand for good 0 is thus $D_0 = \hat{x}$, while demand for good 1 is $D_1 = 1 - \hat{x}$.

For given q_0 and q_1, the suppliers compete in prices. Because demand is linear, the best reply for supplier 0 is half the "choke-price" (the price p_0 for which $\hat{x} = 0$). Transposing subscripts generates the best-reply for supplier 1. Hence the price reaction function for supplier i is[15]

$$p_i = \frac{p_j + (R - t)(q_i - q_j) + tq_i}{2}, \quad i, j = 0, 1 \text{ and } i \neq j. \tag{5}$$

Equation (5) makes it clear that prices are strategic complements: $\partial p_i / \partial p_j > 0$. The linear reaction function has the standard fifty-cents-on-the-dollar property familiar from Hotelling models. The price-quality interaction is quite novel though, because the reaction depends positively on own quality in addition to the quality difference

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[14]For higher t values there is a continuum of constrained monopoly equilibria where the market is fully covered, yet each producer does not wish to cut price and directly compete with its rival. The consumer indifferent between the two products is also indifferent between buying and not. For still higher t values there is unconstrained local monopoly: recall $u_0 = (R - tx)q_0 - p_0$ so that 0's monopoly demand is $x = \left(R - \frac{p_0}{q_0}\right)\frac{1}{t}$. Its monopoly price, $Rq_0/2$, implies that equilibrium $x = \frac{R}{2t}$. Thus for $x < \frac{1}{2}$ (equivalently, $R < t$), there is local monopoly. We do not dwell on these parameter ranges in the text, though they are detailed in the Appendix. Demand functions comprise 2 linear segments, shallow in the high-price "monopoly" region, and steeper in the lower-price duopoly region. The kink begets a downward marginal revenue discontinuity which is at the heart of the multiplicity noted above, and discussed further in the Appendix. Because the discontinuity is downward, the monopoly segments in demand do not cause equilibrium existence problems in the price sub-games, whatever parameters.

[15]Already the symmetric equilibrium and the rationale for A1 can be seen here: under symmetry, $p = tq$. This is the heart of the result that the duopoly region covers the market: recall $u_0 = (R - tx)q_0 - p_0$ and so at $x = 1/2$ we have $u_0 = (R - t/2)q - tq$ which is positive iff $A1$ holds.

effect. This implies two important properties: that i's reaction function shifts up by more than j's shifts down following a quality change for i, and that equal increases in both qualities shift both reaction functions up. These two properties underlie the two Propositions below.

Solving the price reaction functions (for an interior solution, $\partial \pi_0 / \partial p_0 = \partial \pi_1 / \partial p_1 = 0$) implies that the outcome of the last stage is

$$p_i^* = \frac{R(q_i - q_j) + t(q_i + 2q_j)}{3}, \quad i, j = 0, 1 \text{ and } i \neq j. \tag{6}$$

From (6) we find, as expected, that the (sub-game) equilibrium price rises with own quality, which is consistent with the property noted above that the own reaction function shifts up more than that of the rival shifts back. Note also that the price charged for good i is increasing in the consumers' reservation price, R, if it has a higher quality level than its rival.

The relationship between p_i and q_j is less clear-cut; the "direct effect" of better quality of good j is to reduce p_i (see (5)). However, since prices are strategic complements, the fact that p_j rises with q_j provides a channel for p_i to increase with q_j. We thus find an ambiguous relationship between p_i and q_j because $\frac{dp_i^*}{dq_j} = \frac{2}{3}\left(t - \frac{1}{2}R\right)$:

Proposition 1: *Single-purchase. Prices are strategic complements. A supplier's equilibrium price rises with an increase in rival quality if and only if $t > \frac{1}{2}R$.*

This latter result contrasts with that for the standard Hotelling model (for which $u_i = q_i - t(|x - x_i|) - p_i$: see e.g. Ziss, 1993). In the standard model, a quality increase worth a dollar (to all consumers) leads to an own-price increase of only a third of a dollar. Another third is taken up by output expansion, while the rival's price increase eats up the last third. By contrast, in the present model, a quality increase is valued more by closer consumers. This effect renders demand more inelastic and so facilitates higher prices for both. Indeed, while a quality rise shifts the demand function up parallel in the standard approach, here demand pivots up around the quantity axis. The pivot (larger increase in willingness to pay for closer consumers) renders demand more inelastic, and so leads to upward price

pressure. The greater is t, the stronger is this effect because t represents consumer heterogeneity, and hence the price rises if t is large enough. The effect is seen through (6): the quality-difference term multiplying R delivers the "standard" one third mark-down effect of an increase in rival quality, but this is offset by the term in t, which dominates if t is large enough (although still satisfying A1).

Inserting (6) into (3) and (4) we obtain the sub-game equilibrium values:

$$D_i^* = \frac{R(q_i - q_j) + t(q_i + 2q_j)}{3t(q_i + q_j)} \text{ and} \tag{7}$$

$$\pi_i^* = \frac{[R(q_i - q_j) + t(q_i + 2q_j)]^2}{9t(q_i + q_j)} - C_i(q_i), \quad i, j = 0, 1 \text{ and } i \neq j. \tag{8}$$

From (6)-(8) it follows that if the goods differ in quality, then the good with the higher quality has the higher demand, price and operating profits. It can further be verified that a higher quality of product i always reduces its rival's output and profitability, although the prices of both goods might increase.

We now characterize the equilibrium if the quality levels are exogenously given by a common value q^S (we use superscript S for single-purchase). In this case the equilibrium common price (see (6)) is $p^S = q^S t$ and operating profits are $\pi^S = q^S t / 2$. In summary:

Proposition 2: (*Single-purchase, symmetry.*) *In a symmetric equilibrium with* $q_i = q^S$ *(i = 0, 1), the suppliers' operating profits are increasing in*

a) the heterogeneity of the consumers ($d\pi^S/dt > 0$)

b) in the quality levels ($d\pi^S/dq^S > 0$).

The result that equilibrium prices are increasing in t is standard (though it does not hold under multi-purchase, as we show below). However, the quality result in Proposition 2 is in sharp contrast to standard results in symmetric Hotelling models, where prices and profits are independent of the quality of the goods.[16] In the standard model, an equal increase in qualities leaves each consumer's demand

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[16] This is the obverse facet of the result that profits are independent of (common) marginal costs in Hotelling models. Basically, competition determines mark-ups independently of common costs: see the discussion in Armstrong (2006) for implications for two-sided markets.

the same because the extra quality simply cancels out in a comparison of goods. Prices are then unaffected: price neutrality comes from demand neutrality. In our formulation, from (1) and (2), the willingness to pay increases most for the consumers in each supplier's own turf. The increase in market power over captive consumers raises equilibrium prices.

5   Multi-purchase

We now open up the possibility that some of the consumers buy both goods. How much they gain from buying both goods rather than just one of them depends on the incremental value of buying a second variant. This in turn depends on the degree of overlap in the attributes embodied in the two goods. We proceed by first analyzing the situation conditional upon multi-purchase actually arising, and then showing when this is endogenously so.

   We assume that the set of attributes Q_i offered by good i is drawn randomly from Q. We can thus interpret $(1 - q_i)\, q_j$ as a measure of the attributes offered by good j but not by good i and $q_0 q_1$ as a measure of the overlap of the set of attributes. When we allow for multi-purchasing, we must further specify the consumers' valuation of having access to the same attributes in both goods. We think of consumers as accruing value from attributes first on the more preferred product, which we term the primary product (and will transpire below to be simply the closer one) and then accrue extra value on the secondary product. We let $(1 - \beta) \in [0, 1]$ represent the extra value per overlapping attribute, and specify the incremental benefit (gross of transportation costs) from consuming secondary good j in addition to primary good i as:

$$(1 - q_i)\, q_j + (1 - \beta)\, q_i q_j = (1 - \beta q_i)\, q_j. \tag{9}$$

As an example of what (9) tells us, look at the tablet market, where one of the attributes of Kindle is that it can read books aloud for you. If you have a Kindle, and consider buying an iPad in addition, would it then be important that the iPad can do the same? If it is, then β is low, while $\beta = 1$ if it is not. As another

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example, consider a left-wing and right-wing newspaper, and let a possible attribute be an analysis of a presidential scandal. If a consumer's value of reading about the scandal in a right-wing newspaper is independent of whether the scandal is also covered in the left-wing newspaper she reads, then $\beta = 0$. If, in contrast, she can see no additional value of reading about the scandal in both newspapers, then $\beta = 1$.

We must distinguish between the case where everyone buys both goods, and the case where only a fraction of the consumers do so. However, the former is quite trivially straightforward (as will become apparent from the analysis below: it involves pricing to make the most resistant consumer indifferent to adding the product, a form of monopoly pricing). We therefore deal with the latter case. Figure 1 illustrates one possible market outcome, where consumers located to the left of point A only buy product 0, and those to the right of C only buy product 1. Consumers located between A and C buy both.
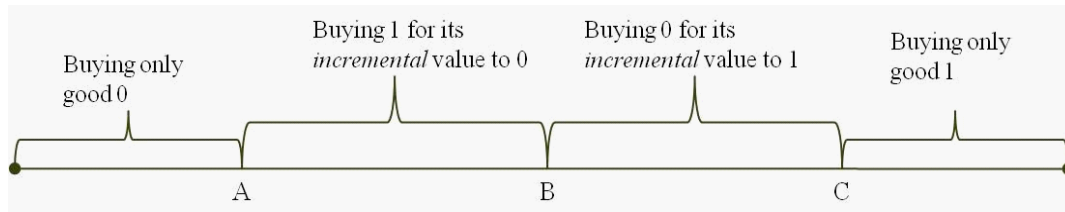
Figure 1: *Possible market outcome with multi-purchase.*

The utility of a consumer with primary product 0 who buys good 1 for its incremental value over good 0 is

$$u_{01} = u_0 + \left\{ [R - t(1 - x)](1 - \beta q_0) q_1 - p_1 \right\}. \tag{10}$$

The first term on the right-hand side of (10) is the utility that the consumer gets from buying good 0 (primary product). The second term is the additional utility the consumer obtains from also buying good 1 (the secondary product).

Analogous to equation (10), the utility of a consumer who buys good 0 for its incremental value over good 1 is

$$u_{10} = u_1 + \left\{ (R - tx)(1 - \beta q_1) q_0 - p_0 \right\}. \tag{11}$$

If $\beta > 0$ we find from (10) and (11) that $u_{01} > u_{10}$ for $x < 1/2$. This means that consumers located to the left of $x = 1/2$ have good 0 as their primary product and good 1 as their secondary product, and vice versa for consumers located to the right of $x = 1/2$.[17] The location of point B in Figure 1 is thus given by $x_B = 1/2$.

A consumer will buy both goods if the incremental value of his secondary product is positive.[18] This means that the location of the consumer who is indifferent between buying only good 1 and buying both goods is given by $u_{10} = u_1$ (location C in Figure 1). Clearly, for this consumer the price of good 1 is immaterial. Solving $u_{10} = u_1$ we thus find

$$x_C = \frac{1}{t}\left[R - \frac{p_0}{q_0\left(1 - \beta q_1\right)} \right],\tag{12}$$

so that demand for good 0 depends on own price and the qualities of the two goods, and not on the price charged by the rival. This key property of the multi-purchase regime is not an artefact of the uniform consumer distribution in the Hotelling model, but is a more fundamental property. It stems from the nature of recognizing the demand as the incremental value, and holds when infra-marginal consumers are not indifferent between buying and not buying, nor between switching brands.[19]

The above makes it clear that the multi-purchase equilibrium is a special type of monopoly regime. Rival quality – but not rival price – shapes demand. This property is what makes the regime particularly interesting – prices are strategically independent though they are determined by the quality of both goods. The strategic

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[17] The ranking of the goods as primary and secondary is subject to the consumer actually buying both goods. If p_0, say, is sufficiently high, not even consumers located close to point 0 will buy good 0. However, given that a consumer has bought both goods, his primary good is the one which is closer to his location.

[18] The arguments in the text use the incremental analysis approach for clarity. Note that the full consumer problem is to choose the option that maximizes $\{(R - tx)\, q_0 + (R - t\,(1 - x))\, q_1\,(1 - \beta q_0) - p_0 - p_1,\ (R - tx)\, q_0\,(1 - \beta q_1) + (R - t\,(1 - x))\, q_1 - p_0 - p_1,\ (R - tx)\, q_0 - p_0,\ (R - t\,(1 - x))\, q_1 - p_1\}$, where the 1st option treats 1 as secondary product, the 2nd treats 0 as secondary product, the 3rd is buying 0 alone, and the 4th is buying 1 alone.

[19] The property would not hold for example if the demand were specified as a "random choice" discrete utility model with i.i.d. idiosyncratic tastes, if choices were defined over all alternatives (including the joint one). However, it would seem more natural to define choices in the incremental manner done above, and then the property would hold still.

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independence here stems directly from profit independence of rival price.[20]

Inserting (12) into equation (3) and solving $\partial \pi_0 / \partial p_0 = 0$ we find $p_0 = \frac{Rq_0(1-\beta q_1)}{2}$ and $D_0 = \frac{R}{2t}$. For good 1 we likewise have $p_1 = \frac{Rq_1(1-\beta q_0)}{2}$ and $D_1 = \frac{R}{2t}$. Provided that $\frac{1}{2} < D_i = \frac{R}{2t} < 1$ (or $t < R < 2t$), the candidate equilibrium outcomes are thus given by:

$$p_i^* = \frac{Rq_i\left(1 - \beta q_j\right)}{2}; \quad D_i^* = \frac{R}{2t}; \quad \pi_i^* = \frac{R^2 q_i\left(1 - \beta q_j\right)}{4t} - C(q_i), \qquad i, j = 0, 1 \text{ and } i \neq j. \tag{13}$$

The restriction that $t < R < 2t$ ensures that each supplier's output lies between one half and one. This is a necessary condition for there to be an equilibrium where some - but not all - consumers buy both products.[21] The set of parameter values for this regime might seem rather narrow, in the sense that there is complete multi-purchase if $R > 2t$. However, this limited range is an artefact of our simplifying assumption that consumer preferences are uniformly distributed over the unit line. With an unbounded support, complete multi-purchase will not arise (and neither will there be full market coverage under single-purchase for that matter).[22]

It should also be noted that the clean condition which ensures that partial multi-purchase is independent of the individual q_i's (subject to no supplier wishing to deviate, as addressed below) holds for any preference distribution, since we cannot have multi-purchase of one good and not of the other.

Proposition 3: *Multi-purchase. Prices are strategically independent and extract the incremental benefit to the marginal consumer. Price (and profit) increases with own quality, and decreases in rival quality and in the overlap of attributes.*

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[20] Profit independence is sufficient but not necessary for strategic independence – consider the case of Cournot competition and exponential demands (and zero cost), where profits are not independent, but quantities are strategically independent.

[21] The outcome that a higher quality induces a higher price holds generally, while the equality of demands is a property of the uniform distribution in the Hotelling model. Suppose that the consumer density were $f(x)$. Then $\pi_0 = p_0 F(x_C)$ and $\frac{d\pi_0}{dp_0} = F(x_C) - p_0 f(x_C) t q_0 (1 - \beta q_0)$ and the candidate equilibrium price is $p_0 = \frac{F(x_C)}{f(x_C)} t q_0 (1 - \beta q_0)$. As long as $F(.)$ is log-concave, the RHS is decreasing in p_0 and the supplier with the higher quality again has the higher price.

[22] Complete multi-purchasing would also be less likely with a uni-modal consumer density (to the extent to which suppliers care less about a low density of consumers far away).

The results that $dp_i^*/d\beta < 0$ and $d\pi_i^*/d\beta < 0$ are self-evident because a higher β reduces the incremental value of the secondary good. This overlap effect is absent from single-purchase equilibria because the concept of overlap is irrelevant there.

Under single-purchase, we showed that the suppliers' operating profits are strictly increasing in their expected quality levels and in the heterogeneity of the consumers. From (13) we find that the opposite can be true under multi-purchase:

Proposition 4: *Multi-purchase. In a symmetric equilibrium with* $q_i = q^M$ *($i = 0, 1$), the suppliers' operating profits are*

a) decreasing in the heterogeneity of the consumers ($d\pi^M/dt < 0$), and

b) hump-shaped functions of the expected quality levels if $\beta > 1/2$ (with $d\pi^M/dq^M > 0$ for $q^M < \frac{1}{2\beta}$ and $d\pi^M/dq^M < 0$ for $q^M > \frac{1}{2\beta}$).

Under single-purchase, when consumers become more heterogenous, each supplier's market power over its own consumers increases, resulting in higher prices and higher profits ($d\pi^S/dt > 0$). Under multi-purchase, on the other hand, greater consumer heterogeneity implies that each supplier will have a smaller market ($dD_i/dt < 0$) and thus lower profits ($d\pi^M/dt < 0$). The intuition for this result is the fundamental property outlined above that prices are strategically independent under multi-purchase, which in turn implies that prices are independent of t. The effect of greater consumer heterogeneity is consequently only to reduce the share of the population which is willing to pay for both products.

At the outset, the second part of Proposition 4 might seem even more surprising. To see the intuition for this result, note that there are two opposing effects for the suppliers of an increase in q^M. The positive effect is that a higher quality level increases the consumers' willingness to pay for the products, as under single-purchase. The negative effect of a higher q is to make it less imperative for any of the consumers to buy both products, thereby tending to increase the competitive pressure between the suppliers. This negative effect dominates if $q^M > \frac{1}{2\beta}$. Only if $\beta < 1/2$, so that consumers have a strong value from consuming both products, will prices and profits be strictly increasing in q^M.

Finally, consider briefly the case of $\beta < 0$. This corresponds to there being

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complementarities among the common attributes offered by the competing products, so that enjoying an attribute on one good enhances its value on the other. The analysis differs from that above because a multi-purchasing consumer will now accrue the enhanced value of overlap on the primary product. In the earlier case of $\beta > 0$ (which corresponds to substitutability of attributes on different goods), it is the value of the attribute of the secondary product (the one further away in the characteristic space) which is diminished. To see this, note that the utility from joint purchase (ignoring prices) is

$$\max\{(R - tx)\,q_0 + (R - t(1 - x))\,q_1\,(1 - \beta q_0)\,,(R - tx)\,q_0\,(1 - \beta q_1) + (R - t(1 - x))\,q_1\},$$

where the former is greater for $x > 1/2$ if $\beta < 0$. This means that, for the case at hand, the joint-purchase utility for a consumer at $x > 1/2$ is $(R - tx)\,q_0 + (R - t(1 - x))\,q_1\,(1 - \beta q_0) - p_0 - p_1$ and so the incremental utility (over buying primary product 1 alone) is $(R - tx)\,q_0 - \beta(R - t(1 - x))\,q_1 q_0 - p_0$. Setting this to zero finds the critical $\hat{x}$ which is the demand for supplier 0's product under multi-purchase. The equilibrium price is found simply as half the (inverse) demand intercept, which is the demand price from setting $x = 0$, namely[23]

$$p_0 = \frac{q_0}{2}\left[R(1 - \beta q_1) + t\beta q_1\right].$$

This price is now increasing in both qualities, as expected when attributes are complementary. The expression differs from that for substitutes (see (13) for $\beta > 0$) through the addition of the extra term in t. In the rest of the analysis, we revert to the case of $\beta \in [0, 1]$.

5.1 Exogenous quality levels: single-purchase vs. multi-purchase

In this sub-section we compare the multi-purchase and single-purchase outcomes from the perspectives of the suppliers and the consumers, under the constraint that

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[23]To check this, note that inverse demand is $p_0 = (R - tx)\,q_0 - \beta(R - t(1 - x))\,q_1 q_0$, so the optimal output satisfies $(R - 2tx)\,q_0 - \beta(R - t(1 - 2x))\,q_1 q_0 = 0$, or $\frac{R - \beta(R - t)q_1}{2t(1 + \beta q_1)} = x$. Reinserting this in the demand, the price is readily verified.

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the goods have the same (exogenous) quality levels. We further determine under which conditions single-purchase and multi-purchase equilibria actually exist. To limit the number of cases to consider, we assume that $\frac{3}{2}t \leq R \leq 2t$. This ensures that there will be full market coverage under single-purchase (this requires that $\frac{3}{2}t \leq R$, cf. Assumption 1) and that there might exist an equilibrium with multi-purchase (as shown above, a necessary condition for an outcome where some, but not all, consumers buy both goods is that $t \leq R \leq 2t$).

In the Appendix we prove the following:

Proposition 5: *Assume that $\frac{3}{2}t \leq R \leq 2t$, and that the quality levels of both goods are equal to q. Compared to single-purchase, multi-purchase yields*
  a) lower prices ($p^M < p^S$) and higher consumer surplus ($CS^M > CS^S$) and
  b) higher profits if and only if $q < q^* \equiv \frac{R^2 - 2t^2}{\beta R^2}$.

Figure 2, where we have set $\beta = 1$, might be helpful to grasp the intuition for Proposition 5.[24] The left-hand side panel of the Figure shows that prices are strictly increasing in q under single-purchase; a higher expected quality unambiguously allows the suppliers to charge higher prices. This in turns implies that the suppliers' operating profits are increasing in q under single-purchase, as shown by the right-hand side panel of the Figure. Under multi-purchase, on the other hand, prices and profits are hump-shaped functions of q, as stated in Proposition 4. Note in particular that $p^M \to 0$ and $\pi^M \to 0$ as $q \to 1$. The intuition for this is that the additional benefit of buying a second product vanishes in this case. If prices do not approach zero, consumers to the left of $x = 1/2$ will thus buy only good 0 and those to the right of $x = 1/2$ will buy only good 1.[25] If $\beta < 1$, we always have $p^M > 0$ and $\pi^M > 0$. However, unless β is so small that $\frac{R^2 - 2t^2}{\beta R^2} > 1$, profits will necessarily be lower under multi-purchase than under single-purchase for sufficiently high values of q.

Despite the fact that prices are lower under multi-purchase than under single-purchase, the second part of Proposition 5 shows that $\pi^M > \pi^S$ if q is sufficiently

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[24]The other parameter values in Figure 2 are $t = 1$ and $R = 1.8$.

[25]This is straightforward to see from the term in the bracket of equations (10) and (11).

small ($q < q^*$). In the left-hand side panel of Figure 2 this is true if $q < 0.38$. The reason is simply that the price differences under the two regimes are then so small that the higher sales under multi-purchase ($D^M > D^S = 1/2$) more than outweigh the lower profit margins. It can further be shown that we might have $q^* > 1$ if $\beta << 1$, in which case multi-purchase always generates the higher operating profits.
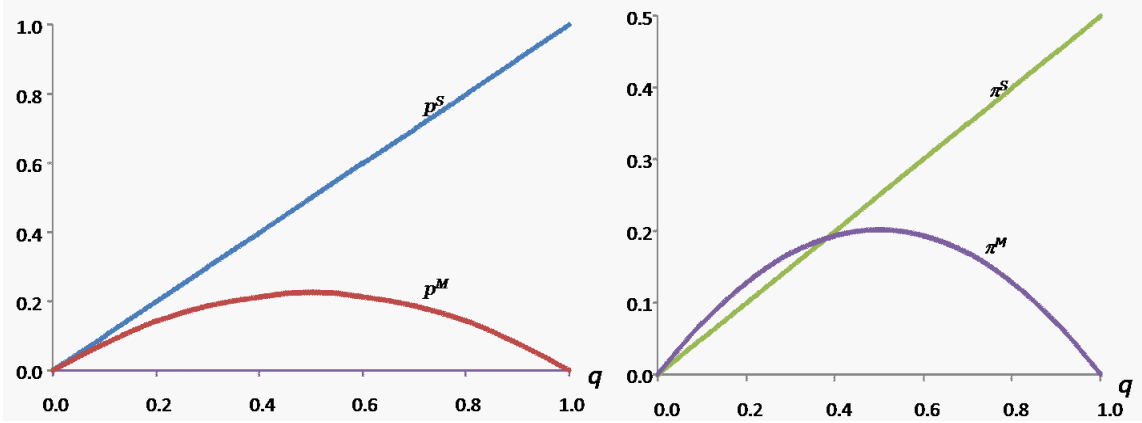
Figure 2: *Prices and profits under single-purchase and multi-purchase.*

Let us now analyze whether both single-purchase and multi-purchase constitute possible equilibria. For this purpose, let $q^{**} \equiv \left(4\sqrt{R(R-t)} + 2t - 3R \right) / (R\beta)$. It can be shown that $q^{**} > q^* \equiv \frac{R^2 - 2t^2}{\beta R^2}$,[26] and we have (see Appendix):

> **Proposition 6:** *Assume that $\frac{3}{2}t \le R \le 2t$ and $q^{**} < 1$. In this case there exist*
>
> *a) a unique equilibrium with multi-purchase for $q < q^*$,*
>
> *b) multiple equilibria for $q \in (q^*, q^{**})$; one with single-purchase and one with multi-purchase,*
>
> *c) a unique equilibrium with single-purchase for $q > q^{**}$.*

Proposition 6 is illustrated in Figure 3, where we have set $\beta = 0.9$ (so that both p^M and π^M are strictly positive for all values of q). The existence of an equilibrium is shown by a solid curve, and non-existence of the candidate by a dotted curve.

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[26] To see that $q^{**} > q^*$, define $z \equiv \frac{R}{t}$ (with $\frac{3}{2} \le z \le 2$). We then have $q^{**} - q^* = \frac{2}{\beta z^2}(A - B)$, where $A \equiv 2z\sqrt{z(z-1)}$ and $B \equiv (2z+1)(z-1)$. As both A and B are positive, it follows that $q^{**} - q^* > 0$ if $A > B$. This is true, since $A^2 - B^2 = 1 + 3z > 0$.

Note that multi-purchase cannot take place if both β and q are sufficiently close to one.

Consistent with Proposition 5, the left-hand side panel shows that consumer surplus is always higher with multi-purchase, while the right-hand side panel shows that profits might be higher under single-purchase. However, for $q < q^*$ the suppliers also prefer multi-purchase; a supplier which deviates from this equilibrium could charge a higher price and only sell to those consumers who do not buy the rival's product, but that would excessively reduce sales. The quality of the products is simply too low to allow for a sufficiently high single-purchase price. This is different for $q > q^{**}$; single-purchase prices are then so high that each supplier prefers to sell only to its most "loyal" consumers, even if the rival should set the relatively low multi-purchase price and thus capture the larger share of the market. The suppliers thereby unambiguously end up in the high price-high profit equilibrium. For $q \in (q^*, q^{**})$, though, it is unprofitable for either supplier to charge a high single-purchase price unless the rival does the same.
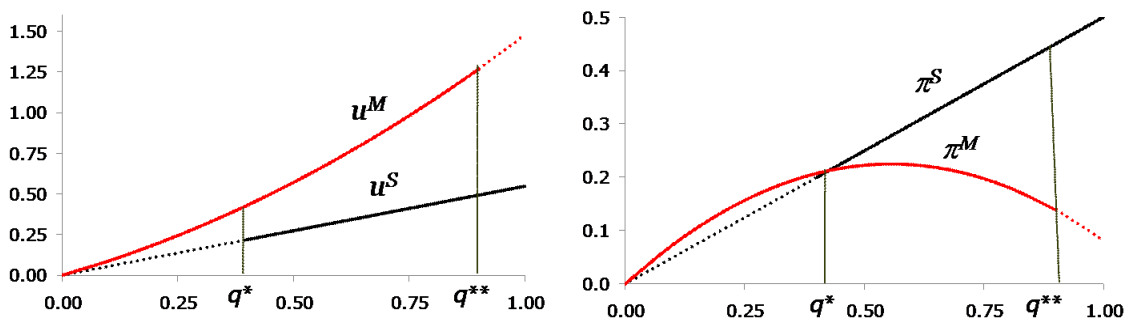
Figure 3: *Single-purchase vs. multi-purchase. Multiple equilibria.*

The discussion above provides an intuitive approach to finding the possible equilibria that may arise when we open up for multi-purchase. In the Appendix we offer a more formal and general analysis, and explain why we always have a pure strategy price equilibrium.

6 Investment incentives

In this final section we endogenize investments. We start out by deriving the first-order conditions for optimal investments conditional upon a single-purchase regime, and then do likewise for multi-purchase.

6.1 Investment incentives under single-purchase

The first-order condition for optimal investments in quality of product i under single-purchase is found by differentiating equation (8) with respect to q_i. This yields

$$\frac{\partial \pi_i^*}{\partial q_i} = p_i^* \frac{\partial D_i^*}{\partial q_i} + D_i^* \frac{\partial p_i^*}{\partial q_i} - C'(q_i) = 0, \qquad i = 0, 1, \tag{14}$$

where $\frac{\partial D_i^*}{\partial q_i} = \frac{(2R-t)q_j}{3t(q_0+q_1)^2} > 0$ and $\frac{\partial p_i^*}{\partial q_i} = \frac{R+t}{3} > 0$. By investing in quality, the supplier thus expects to be able to increase its equilibrium output and to charge a higher price. These positive market responses are clearly increasing in the consumers' reservation price R (which places an upper limit on the price that the suppliers can charge). We further find the comparative static result:

Proposition 7: *Single-purchase. In a symmetric equilibrium with $q_i = q^S$ ($i = 0, 1$), the suppliers invest more the more heterogenous are consumers ($dq^S/dt > 0$).*

Proof: Setting $q_0 = q_1 = q^S$ and inserting for (6) and (7) into (14) we find that the first order condition when evaluated at a symmetric solution is:

$$\frac{4R+t}{12} = C'(q^S), \tag{15}$$

and hence $dq^S/dt = \frac{1}{12C''(q^S)} > 0$.   **Q.E.D.**

The reason why $dq^S/dt > 0$, is simply that the more heterogenous the population of consumers, the higher is each supplier's market power on its own turf. An increase in t thus allows the suppliers to set higher prices, making it more profitable to invest in order to increase output. Of course, in equilibrium the suppliers still share the market equally, so that they do not actually gain any more output. But the higher

q^S induced from a higher t is not a zero-sum game, since the equilibrium price, tq^S, is increasing in the common quality level.

Note that the effect on profits of higher t is ambiguous. The direct effect is an increase in profit through the price effect noted above. However, rival quality rises too, and this brings profit down per se. To see the net effect, we use (15) to rewrite (8) as $\pi^S = q^S \left[6C'(q^S) - 2R\right] - C\left(q^S\right)$, where we know that $dq^S/dt = \frac{1}{12C''(q^S)}$. Hence, differentiating yields

$$\frac{d\pi^S}{dq^S} = 5C'(q^S) - 2R + 6q^S C''(q^S)$$
$$= \frac{5t - 4R}{12} + 6q^S C''(q^S)$$

where we have substituted back (15). By A1 $(R \geq \frac{3t}{2})$ the first term is negative, but the second is positive.[27] Hence, there exists a fundamental ambiguity in the tension between the beneficial effects of more inelastic demand with the deleterious effects of higher rival quality. Perhaps surprisingly, the multi-purchase case yields unambiguous results, despite similar tensions (see below).

6.2   Investment incentives under multi-purchase

To find optimal investments under multi-purchase, we use (13) to solve $\partial \pi_i / \partial q_i = 0$. This yields the first order condition:

$$R^2 \frac{1 - \beta q_j}{4t} = C'(q_i); \quad i \neq j, i, j = 0, 1. \tag{16}$$

From the comparative static properties of this expression at a symmetric situation (where $q_i = q^M$ for $i = 0, 1$), we can state:

Proposition 8: *Multi-purchase $(R < 2t)$. In a symmetric equilibrium with $q_i = q^M$, $i = 0, 1$, the suppliers invest less in quality*
   a) the more heterogenous the consumers $(dq^M/dt < 0)$

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[27] The second derivative of π_i with respect to q_i, when evaluated at a symmetric candidate solution, q^S, is $\frac{(2R-t)^2}{36tq} - C''(q^S)$, which places no further restriction (given that $C''(q^S) > 0$) on the derivative $\frac{d\pi^S}{dq^S}$ given in the text.

b) the weaker the consumer preferences for having the same attributes in both goods ($dq^M/d\beta < 0$).

Proof: $\frac{dq^M}{dt} = -\frac{(1-\beta q^M)R^2}{R^2\beta t + 4t^2 C''(q^M)} < 0$ and $\frac{dq^M}{d\beta} = -\frac{q^M R^2}{R^2\beta t + 4t C''(q^M)} < 0$. **Q.E.D.**

The relationship between consumer heterogeneity and investment incentives is the opposite in this case compared to single-purchase. The reason why $dq^M/dt < 0$ is that the larger is t, the smaller is the size of the market for each supplier (recall that $D_i = R/2t$). The gain from investing more in quality to increase the price is therefore strictly decreasing in t under multi-purchase.

Note that there cannot be asymmetric multi-purchase equilibria if $C(.)$ is convex enough: from (16), i's reaction function slope is $\frac{dq_i}{dq_j} = -\frac{\beta R^2}{4t C''(q_i)}$, which exceeds -1 for $C''(.) > \frac{\beta R^2}{4t}$. If the latter condition holds, the firms will consequently be symmetric in a multi-purchase equilibrium.

Consider now the effect on profits of higher t. By inserting the quality first-order condition (16) into (13) we have $\pi^M = \frac{R^2 q^M (1-\beta q^M)}{4t} - C(q^M)$. The derivative with respect to q^M is simply $q^M C''(q^M) > 0$ and so equilibrium profits necessarily fall with t. Similar logic indicates that equilibrium profits must fall with β. That is, a lower distinctive overlap value (i.e., more similarity in jointly provided qualities) hurts profit.

7 Conclusions

We have analyzed a Hotelling duopoly model where quality interacts positively with consumer closeness, where consumers are not restricted to buy only one good, and where multi-purchase benefits depend on the overlap of functions provided by products. Under single-purchase, prices and operating profits are strictly increasing in quality levels. In contrast, when some consumers multi-purchase, prices and profits can be hump-shaped functions of the quality levels. If the quality levels of both goods are sufficiently high, the additional benefit of buying the second variant might vanish. Other things equal, competition will then press prices down towards mar-

ginal costs. However, in this case it may be a dominant strategy for the suppliers to set such high prices that no-one will buy more than one of the varieties. Whether there is a hump-shaped relationship between price and quality depends on the consumers' preferences for having the same functionality for different variants of a good. This is likely to vary significantly from market to market. Some consumers clearly benefit from having smartphone and computer tablet with overlapping attributes. For instance, people may like to play the game Angry Birds on iPad at home and on iPhone when travelling. If we consider newspapers (or journalism more generally), consumers may have preferences for a second opinion. However, the number of people buying both The Times and The Guardian to get both Right-wing and Left-wing presentations of the state budget is rather small.

One topic for further research is to analyze multi-purchase in a two-sided market structure. Many information goods, such as online newspapers, are financed by advertising. Since these goods are offered for free in order to attract more customers (and thus increase advertising revenue), the degree of multi-purchasing (termed "multi-homing" in this context) is by its very nature high. It should also be noted that a scoop published by an online newspaper typically becomes available from rival outlets within minutes. As a consequence, the willingness to pay for a second online newspaper will presumably be small. This may help explain the observation that online newspapers rarely charge readers.

We have assumed a particular specification for how components interact. A more general set-up would specify multi-dimensional heterogeneity in all three dimensions: say, an individual-specific taste for a combination of the basic goods, their components, and how these interact under multi-purchase. And yet, generality tends to yield paucity of predictions that are restored by restrictions. Accordingly, we have strong predictions, albeit generated by restrictive assumptions (which we nonetheless believe constitute a reasonable starting point).

8 Appendix

8.1 Discussion of demand and reaction functions

Finding the equilibria for this model is quite elaborate because of the demand kinks. What we find is rather particular: there are either two equilibria or one (along with a possibility of a continuum of local monopoly equilibria that preclude any other equilibrium). Gabszewicz and Wauthy (2003) find for a vertical differentiation model with the option of multi-purchase that there is also the additional possibility of no equilibrium. This is not true in our set-up, and we here explain why. In doing so, we establish key reaction functions properties. The properties, and the techniques we use, pertain to other duopoly problems which exhibit kinks in demand, such as spatial models where kinks in demand arise naturally (e.g., Anderson, 1988, Anderson and Neven, 1991, Peitz and Valletti, 2008). We therefore detail how to find the reaction functions and the implications for equilibria. We exemplify the text model, but the techniques have a wider applicability.

8.1.1 Finding the reaction functions

The duopoly problem involves best-reply price choices where different price pairs correspond to different demand segments. Typically, price choices can be bounded below by constant marginal cost (here zero) and some maximum (reservation) price at which no consumer will buy. In the present case, the maximal price is Rq_i, $i = 1, 2$, which is the maximum the most dedicated consumer (the one located at the supplier location) will pay. The strategy space is then a rectangle (a compact and convex set).

Next, divide this strategy space into the constituent regimes corresponding to the demand regimes (e.g., local monopoly and single-purchase, etc.) We then find the *conditional reaction functions,* which are the profit maximizing prices conditional upon being in a particular demand regime. Assuming (as we do henceforth) that each demand regime entails a strictly (-1)-concave demand,[28] these conditional reaction

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[28]This means that the reciprocal of demand is convex in price.

functions are simply the solution to the first order condition, because profits are then quasi-concave over the demand regime.[29]
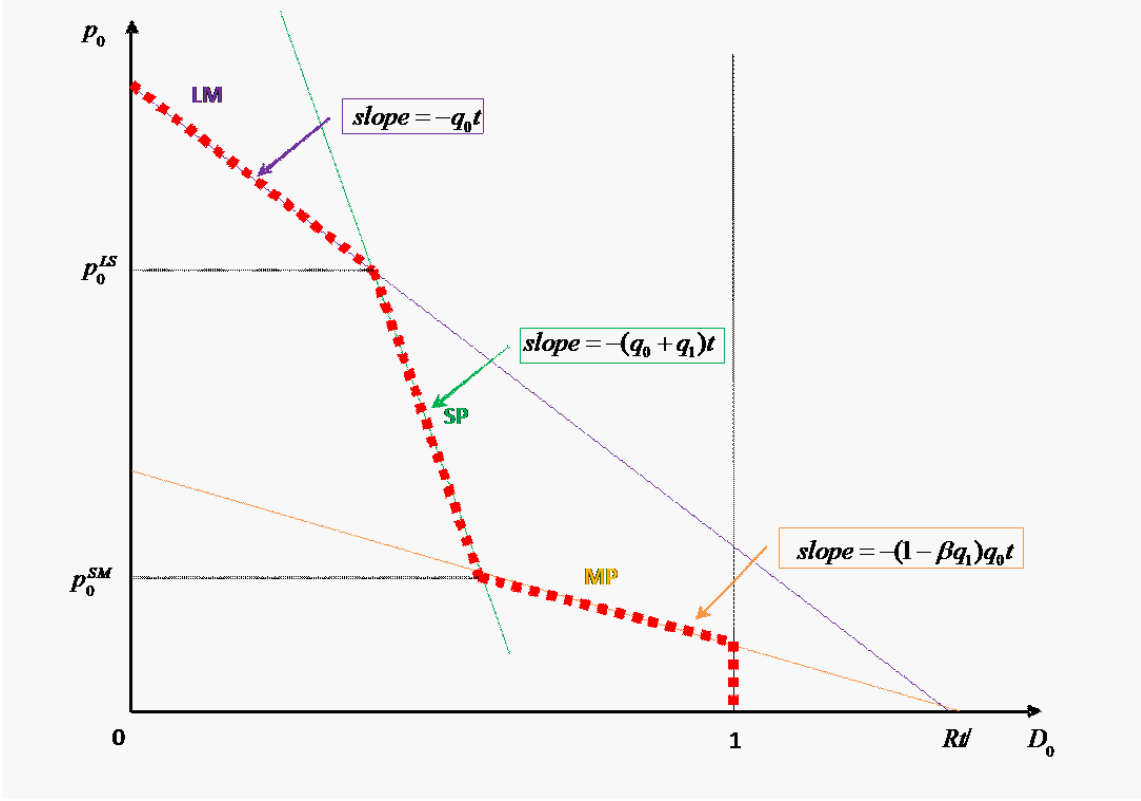
When the conditional reaction function lies within its corresponding regime in the joint price space, the conditional reaction function represents a local maximum in profit. If the conditional reaction function solution lies above the relevant regime in the price space (i.e., at a higher price), then profit is increasing in own price throughout the region. This follows from quasi-concavity of profit. Conversely, if the conditional reaction function lies below its price-space region, profits are falling throughout the regime.

We can now deal simply with the boundaries between regimes in the price space. First, if profits rise towards a boundary from both above and below, then the boundary is a local maximum to profit. This situation corresponds to a downward kink in demand (i.e., steeper demand for lower prices). Second, if profits rise in both directions away from the boundary, the boundary is ruled out as being part of the reaction function since it is a local minimum. This corresponds to an upward kink in the demand function (and a corresponding jump up from negative to positive marginal revenue). In this case, the full solution is either a higher or a lower price, and indicates that profits will need to be evaluated to find the solution. Last, if profits rise towards a boundary and continue rising once it is passed, the solution is not on the boundary. This can occur for both types of kink noted above. Either marginal revenue each side of the kink is negative, or it is positive. In the latter case, profits rise as price falls, while profits rise as price rises in the former case.

The upshot is that the conditional reaction functions indicate whether profits are increasing, decreasing, or locally maximized within a region. This is illustrated in Figure 5 below for the case at hand. Note that profits are always increasing from the boundary towards the interior of the price space, because pricing at marginal cost yields zero profit, and pricing at the reservation price yields zero profits as long as almost all consumers do not buy at that price (as is true here and most usually).

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[29]In the text model, demands are piecewise linear, so conditional profits are quadratic functions.

Figure 5: *Conditional reaction functions, general case.*

Local maxima are then determined by the direction of profit increases. A unique global maximum is indicated by profit increases toward it from all points below and above. This will be a boundary (corresponding to the second type of demand kink noted above) if there is no interior conditional reaction function crossed for the rival price considered. There remains the case of multiple local maxima, and these need to be directly compared (although there may still be short-cuts to choosing which is operative, as per the analysis below).

The reaction functions already enable us to give some characterizations of equilibrium. We focus here on the properties of the present game: these are nonetheless shared with several other contexts. First, if the reaction functions are continuous, there is at least one equilibrium (since they must cross). Second, if the only jumps are upward, then there always exists an equilibrium if suppliers are symmetric (in the present case, if $q_0 = q_1$). This is because the reaction function must then cross the 45-degree line ($p_0 = p_1$). However, notice that without symmetry, and if the

reaction function slopes down over some of its traverse (as it does here), it may *a priori* be possible that one reaction function goes through the discontinuity in the other, and so jeopardizes equilibrium existence. In the current problem, and others of its like, this cannot happen.

The reason is as follows (and this property is shared by other models with similar properties). For high enough (joint) prices, there is a natural monopoly regime. The boundary of this regime (in the joint price space) is downward-sloping, and occurs where prices are such that the market is fully covered and the indifferent consumer at the market boundary between suppliers is also indifferent between buying and not. Call this the Local Monopoly (LM) boundary. Below that regime, reaction functions slope up, and any discontinuities are upward jumps.

Then there are two cases. Either the reaction functions have already crossed (at least once) before reaching the local monopoly boundary, or they have not. If they have not, then they must cross on the boundary or above it. The reason is that the reaction function follows the boundary *down* after touching it, and is then independent of the rival's price (in the interior of the local monopoly regime). There is then either a continuum of local monopoly equilibria on the boundary, or else a single one in the interior of the local monopoly region (with some consumers not buying). This means there must be an equilibrium (involving local monopoly) if there is no "competitive" equilibrium. The converse is also true: if there is a competitive equilibrium then there is no local monopoly equilibrium. To see this, suppose then that the reaction functions have already crossed. When they reach the boundary, they move down it, and then strike out independently. This means that they cannot cross again.

There is a further property of note in the present problem (also shared with other problems). First, if the reaction functions have positive slope below one in the competitive regions, and no jumps, there is at most one competitive equilibrium, and, by the results above, there is only one equilibrium. Second, if there is a single jump up, and still the reaction functions have positive slope below one in the competitive regions, there are at most two equilibria in the competitive regions.[30]

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[30] With k such jumps, there can be at most $k+1$ competitive equilibria.

By the results above, there is no other equilibrium.

In summary, under the conditions given, there is always at least one equilibrium. If there is an equilibrium with each supplier a strict local monopoly, then there is no other equilibrium. There are at most two competitive equilibria, and if there are such, there can be no local monopoly equilibrium. Finally, there can be a continuum of "touching" local monopoly equilibria on the local monopoly boundary, in which case there is no other equilibrium.

8.1.2 Application to the text model

We now analyze the suppliers' demand and reaction functions in more detail. There are at most 3 interior segments to the individual demand functions.

There are two "monopoly" segments to demand. For high prices (of both suppliers), each supplier is a local monopoly. Then inverse demand for product 0 is given by setting the single product utility (1) to zero as

$$p_0 = q_0 \left(R - t\hat{x} \right), \tag{17}$$

where $\hat{x}$ is here and below the demand of product 0.

The other "monopoly" region is for low prices, when some consumers buy both products. They buy product 0 as long as its incremental value is positive; from (12), 0's inverse demand is

$$p_0 = q_0 \left(1 - \beta q_1 \right) \left(R - t\hat{x} \right). \tag{18}$$

Comparing to (17), (18) is lower, with flatter slope. Both demands emanate from the same horizontal intercept: when $p_0 = 0$, $\hat{x} = R/t$. We will suppose for the discussion below that this exceeds 1 (i.e., $R \geq t$), as is assumed in the paper. This implies that demand will be capped at 1 (everyone buys) at a price above zero.

The last segment is the competitive segment imposed by the single-purchase regime. From (4),

$$p_0 = tq_1 + R \left(q_0 - q_1 \right) + p_1 - t \left(q_0 + q_1 \right) \hat{x}, \tag{19}$$

which is steeper than both of the other monopoly segments above. This segment moves out parallel as rival price p_1 rises, while the other segments stay put.

Now superimpose the 3 segments on the same diagram along with the vertical segment at 1: see Figure 5. Where they intersect is where regimes shift. The critical values are calculated below, and are given on the Figure: the demand function is shown in red dots. The inverse demand function is thus given by the flattest segment, (17), until this hits (19) at a price

$$p_0^{LS} = \left(2R - t - \frac{p_1}{q_1} \right) q_0 \tag{20}$$

It then follows the steepest segment, (19), until it hits the flatter segment, (18), at

$$p_0^{SM} = \left(2R - t - \frac{p_1}{q_1} \right) \frac{q_0 \left(1 - \beta q_1 \right)}{\beta q_0 + 1}, \tag{21}$$

which it then follows till it reaches the market constraint (unit demand). Of course, depending on the value of p_1, the single-purchase segment may dominate one or both of the others over the relevant range. The two kinks in the demand, one up and one down, generate two different types of behavior in the reaction function.

The reaction function diagram is usefully broken up into 3 regions, corresponding to the 3 segments above. From (17) and the analogous condition for good 1, Local Monopoly for both transpires if 0's monopoly demand, $\left(R - \frac{p_0}{q_0} \right) \frac{1}{t}$ plus 1's demand, $\left(R - \frac{p_1}{q_1} \right) \frac{1}{t}$, sum to no more than 1. This means $\left(2R - \frac{p_0}{q_0} - \frac{p_1}{q_1} \right) \leq t$. When the inequality is weak, the market is not fully covered. On the boundary of this regime, the locus $\left(2R - \frac{p_0}{q_0} - \frac{p_1}{q_1} \right) = t$ (the Local Monopoly boundary), demands sum to 1 but there is a consumer with zero surplus. This is the top right region of Figure 5.

At the other extreme, there is joint purchase by some customers if demands for the two goods sum to more than 1. (If each is 1, there is joint purchase by all consumers.) From (18), demand faced by supplier 0 is $\left(R - \frac{p_0}{q_0(1-\beta q_1)} \right) \frac{1}{t}$, and similarly 1's demand is $\left(R - \frac{p_1}{q_1(1-\beta q_0)} \right) \frac{1}{t}$, so the condition is $\left(2R - \frac{p_0}{q_0(1-\beta q_1)} - \frac{p_1}{q_1(1-\beta q_0)} \right) > t$, which is the region in the bottom left around the origin in Figure 5. In between these regions lies the single-purchase region. Its boundaries correspond to the kinks in the demand curve.

We know from the earlier text what the conditional reaction functions must look like, conditional on being in a particular region. That is, we can find the reaction function corresponding to each demand segment, as if that linear demand constituted

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the actual demand, and intersect it with the region of applicability. As noted in the preceding sub-section, that is not sufficient to find the reaction function, since the suppliers may deviate to another conditional reaction function, or indeed to the higher boundary. This can only happen if another conditional reaction function (or boundary) lies vertically above or below.

The conditional reaction functions and the derivation of the reaction function are shown in Figure 6. Recall that a deviation from a region to its own boundary is not profitable since such point was already viable (and revealed not preferred) on the region's demand segment. Second, the lower boundary cannot constitute a most profitable deviation since the demand kink there is upward, corresponding to an upward jump in marginal revenue.
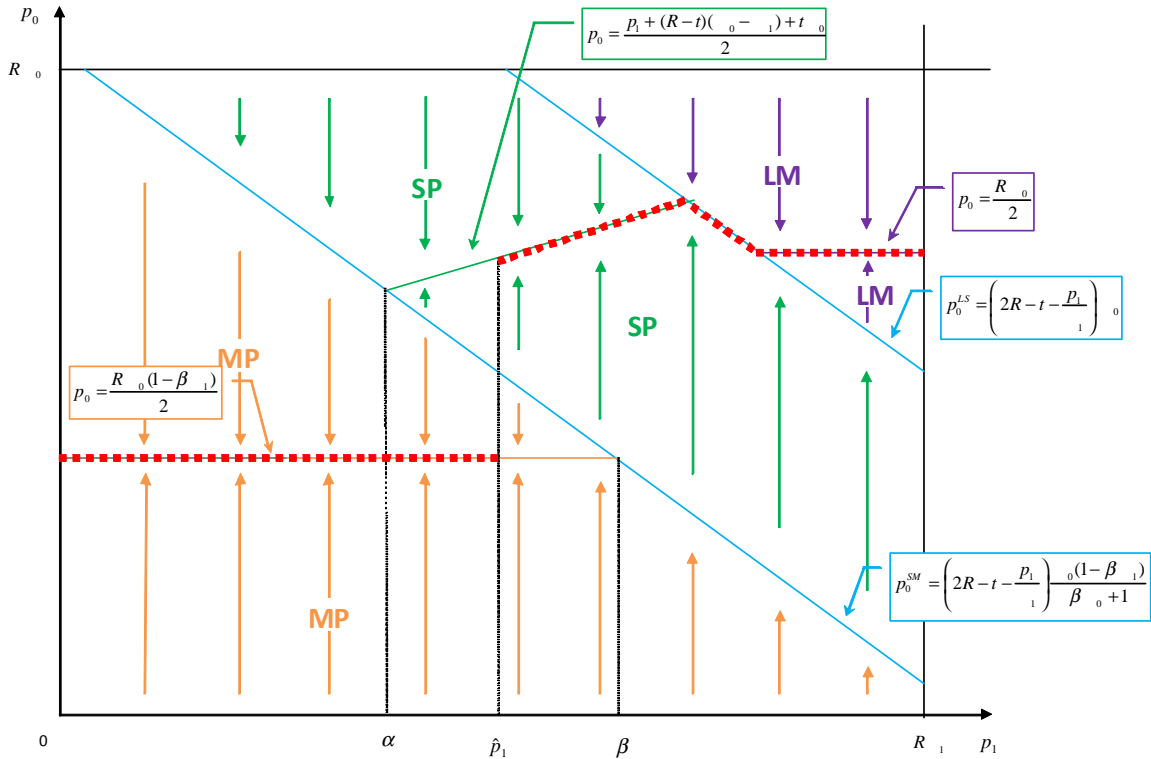
Figure 6: *Conditional reaction functions.*

The conditional reaction function for supplier 0 in the joint purchase region is flat right across the region. The next region out is single-purchase, which comprises

a stripe on top of the joint purchase region; the reaction function is upward sloping (slope 1/2) across this region. The final conditional reaction function is the flat one in the Local Monopoly region.

Any price p_1 left of the point α in Figure 6 entails a unique local maximum, which is therefore a global maximum, on the lowest conditional reaction function. For any price p_1 above the point β, there is again a unique local maximum, which is therefore global. It is on the middle conditional reaction function (the single-purchase one) until this conditional reaction function reaches the Local Monopoly boundary. The local maximum (hence the global maximum and the reaction function) then follows the Local Monopoly boundary down until it reaches the highest of the conditional reaction functions, the local monopoly one, which is then followed to the highest possible p_1.

Between the points α and β there are two conditional reaction functions operative, and so two local maxima. It is straightforward to argue that there is a jump up in the reaction function from the lower to the middle conditional reaction function at some point between α and β. Note that at point α the global maximum is on the lower conditional reaction function: the higher conditional reaction function, having just begun, represents an inflection point at α. Likewise, at point β the global maximum is on the higher conditional reaction function because the lower conditional reaction function represents an inflection point. By profit continuity along the conditional reaction functions, there is a switch between conditional reaction functions where they have equal profits. Notice that profit on the lower conditional reaction function is constant as a function of p_1. However, along the higher conditional reaction function, profit is increasing with p_1. Therefore there is a unique rival price, $\hat{p}_1$, where profits are equal, as shown in Figure 6, and the reaction function follows the single-purchase conditional reaction function beyond that.

We summarize this in Figure 7, where we illustrate the three types of competitive equilibria. In the first panel there are two equilibria (one single-purchase and one multi-purchase). In the second panel there is a unique multi-purchase equilibrium, while in the third panel there is a unique single-purchase equilibrium.
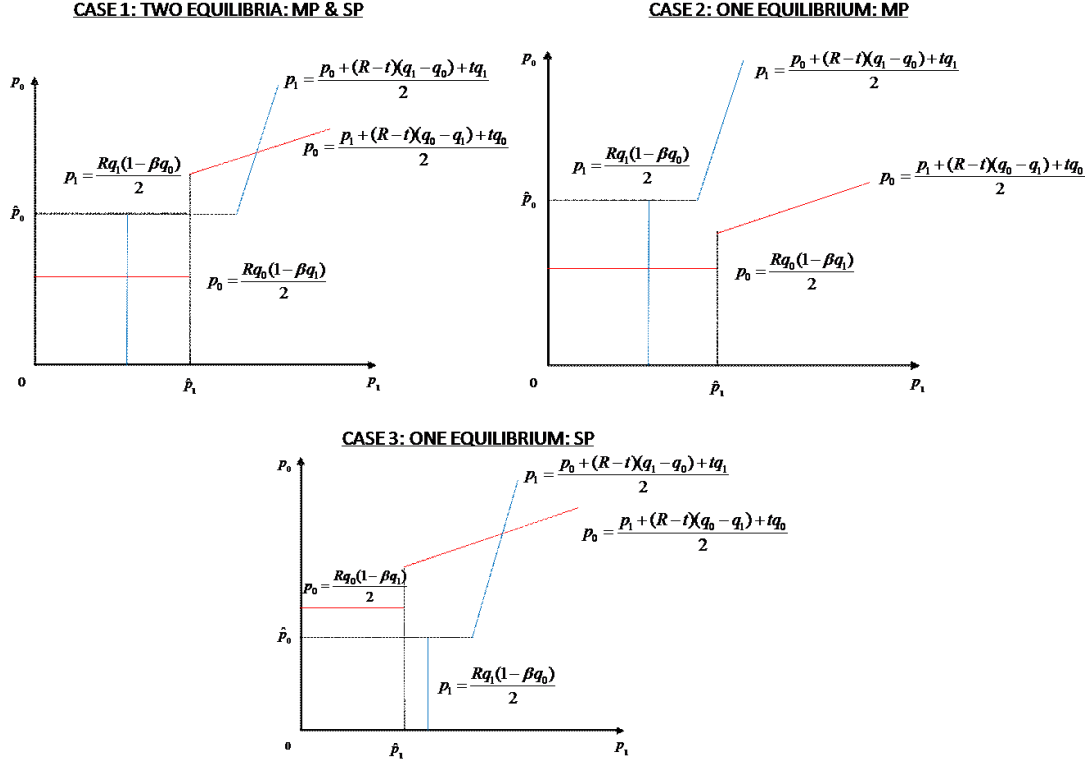
Figure 7: *Competitive equilibrium types.*

8.2 Proof of Proposition 5:

Inserting $q_i = q_j = q$ into (6) and (8) yields $p^S = qt$ and $\pi^S = qt/2 - C(q)$, while (13) yields $p^M = Rq(1 - \beta q)/2$ and $\pi^M = \frac{R^2 q(1-\beta q)}{4t} - C(q)$. This implies that

$$p^S - p^M = \frac{2t - R(1 - q\beta)}{2}q > 0$$

for all relevant values of β and q. We further have

$$\pi^S - \pi^M = q\frac{2t^2 - R^2(1 - q\beta)}{4t} > 0 \text{ for } q > q^* = \frac{R^2 - 2t^2}{\beta R^2}.$$

The consumers who buy only one good are clearly better off under multi-purchase, since $p^S > p^M$. To show that the same is true for those who consume both products, it suffices to show that the utility of the consumer located at $x = 1/2$ is higher buying both products under multi-purchase $(u_{ij}^M(x = 1/2))$ than by buying only one product under single-purchase $(u_i^S(x = 1/2))$. This is true because

$$u_{ij}^M(x = 1/2) - u_i^S(x = 1/2) = \frac{qt(1 + q\beta)}{2} > 0. \qquad Q.E.D.$$

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8.3 Proof of Proposition 6:

If both suppliers price according to single-purchase, then $\pi^S = qt/2$. Suppose that supplier i deviates (superscript D), and sets the price that maximizes profits if he also sells to some of the consumers who buy the rival's product. This optimal price is independent of the price charged by the rival - cf. the discussion leading to equation (12)) - such that $p_i^D = \frac{Rq(1-\beta q)}{2}$ and $\pi_i^D = \frac{R^2 q(1-\beta q)}{4t} - C(q)$. Since $\pi_i^D = \pi^M$, it follows that supplier i deviates from single-purchase if and only if $\pi^M > \pi^S$, in which case also the rival will do the same. This proves Proposition 6a).

To prove Propositions 6b) and 6c), suppose that supplier i believes that the rival sets the multi-purchase price; $p_j = Rq(1 - \beta q)/2$. Will it be optimal for supplier i to charge a higher price, and accept that he will not sell to any of the consumers who buy product j? The location of the consumer who is indifferent between the two products is then given by $u_0 = u_1$. Inserting for $p_j = Rq(1 - \beta q)/2$ this yields

$$D_i = \frac{2(qt - p_i) + qR(1 - q\beta)}{4qt}.$$

Solving $\partial \pi_i / \partial p_i = 0$ we find

$$p_i = \frac{R(1 - q\beta) + 2t}{4} q \text{ and } \pi_i = q\frac{(2t + R(1 - q\beta))^2}{32t} - C(q).$$

Because $\pi_i > \pi^M$ for $q > q^{**} = \frac{4\sqrt{R(R-t)} + 2t - 3R}{R\beta}$, it is thus optimal for supplier i to deviate from multi-purchase and sell only to those who do not buy the rival product if and only if $q > q^{**}$. If $q > q^{**}$ it follows that both the suppliers will have incentives to set single-purchase prices. However, for this to be an equilibrium, it must also be true that the consumers will actually not buy both products at these prices. To check out that this holds, we insert $p_i = p_j = p^S$ into equations (1) and (10) for $x = 1/2$ to find:

$$u_i^{p=p^S} = \frac{2R - 3t}{2} q$$

$$u_{ij}^{p=p^S} = \frac{2(2R - 3t) - q\beta(2R - t)}{2} q.$$

If $u_i^{p=p^S} > u_{ij}^{p=p^S}$ the consumer located at $x = 1/2$ will only buy one of the products at $p = p^S$. This requires that $q > q^{***} \equiv \frac{2R - 3t}{\beta(2R - t)}$ (such that the single-purchase prices

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are sufficiently high). Calculating the difference between q^{**} and q^{***} we obtain

$$q^{**} - q^{***} = 2\frac{2\sqrt{R(R-t)}\left(2R-t\right)-\left(R-t\right)\left(4R-t\right)}{R\beta\left(2R-t\right)}. \tag{22}$$

The denominator in (22) is always positive. It can further be shown that the numerator is positive if $R^2 t\left(R-t\right)\left(R\left(8R-5t\right)+t^2\right)>0$, which is always the case for $t<R<2t$. The consumers will consequently not buy both products if $p=p^S$ and $q>q^{**}$. Q.E.D.

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