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Modeling and Computational Techniques in Bioeconomics

Four Articles on the North East Arctic Cod Stock (NEACS)

by

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Chapter 1

Preliminaries

1.1 Introduction

The main aim of this dissertation is to study two central issues in natural resource problems. First, mathematical models of dynamic resource exploitation are developed (Clark and Munro 1975; Clark 1990; Sandal and Steinshamn 1997). The models developed in this work are quite simple but still cover a wide range of issues in the area. They apply from the extreme case of no regulations, i.e., an open access fishery, to the case of the sole owner fishery, where full regulation is assumed. The bioeconomic analysis is based on the aggregated or lumped-parameter models[1]. Biological details such as age-, sex-structure are not taken into account when modeling the system. The biological function employed throughout the paper is the general production type[2]. Different growth functions are used in the analysis. While these models have been perpetually criticized, they remain the bases of most theoretical and empirical analyses. Several extensions have been proposed to include ecological, environmental and other effects (see Clark 1990). The consequence is an increase in the dimension and complexity of the problem. Throughout the dissertation, a single cohort or year-class model is employed. The pur-

[1] The reasons why we are using these models are that, the data available for the empirical tests in this thesis are aggregated and also the models are simple to analyze.

[2] These models have been applied in the theoretical and empirical studies of the Arctic cod stock (see Sandal and Steinshamn 1997(a,b))

pose is to demonstrate the potentials of the new and efficient techniques of the inverse methods. However, these techniques can efficiently be used to analyze more realistic and complex systems.

Appropriate models of real biological and economic systems are often differential or difference equations. These mathematical models are used to describe real phenomenon. In this work we formulate the original problems using differential equations and then use simple approximate difference schemes in the practical applications. Most real world fisheries may be better represented by discrete mathematical models (Clark 1990; Conrad and Clark 1987). The approximate discrete fisheries models are then employed to estimate physical quantities such as the growth rate in a way that makes the models more useful and pragmatic.

Second, computational techniques are developed in the context of the mathematical tools of the inverse methods and data assimilation (Bennett 1992; Evensen et al. 1998; Lawson et al. 1995). Inverse methods are employed to either estimate the variables of a dynamical system or to fit model dynamics to observed data. They can be used to improve the understanding of a system by analyzing historical data and/or to forecast future states of the biological and economic variables of a fishery. When used to fit dynamics to data it is simply referred to as model fitting (Spitz et al. 1997). The formulation is known as the variational adjoint method or "strong constraint formalism" (Sasaki 1970; Smedstad and O'Brien 1991)). The techniques can also be employed to perform sensitivity analysis of model parameters. A critical role of sensitivity analysis is to identify the most important input parameters whose changes affect the solution the most. The issue of sensitivity of input parameters is discussed later in the dissertation.

To estimate the dynamical variables, two approaches are commonly used; the sequential approach of Kalman filtering (Kalman 1960) and the variational approach known as the weak constraint formulation (Sasaki 1970; Talagrand and Courtier 1987). Weak constraint problems are formulated by admitting model errors, i.e., the model dynamics are assumed to hold approximately. This is a more general and realistic formulation because some of the neglected variables and uncertain inputs are accounted for as modeling errors which must be minimized. These techniques are novel and efficient. They present economists with boundless new opportunities and the potential to perform more realistic bioeconomic analysis. The increasing power of computers and the efforts in data acquisition by biologists and economists further strengthen our faith in the application of the inverse methods in resource modeling.

The computational part emphasizes optimal estimation methods (Gelb 1974). State and parameter estimation in dynamical systems have both been discussed in detail. The inverse problem is cast in a generalized least squares framework (Bennett 1992; Greene 1997). A criterion function is defined subject to the dynamical constraints and the control variables are automatically tuned until a solution which is as close as possible to the historical data is obtained.

An outline of this thesis is as follows. The main body of the dissertation is made up of four chapters. Chapter two contains a paper on the application of data assimilation to a simple bioeconomic fisheries model. The population dynamics here take the form of the surplus production model with the popular logistic growth function. In modeling the economics, the Cobb-Douglas production function is assumed. Additional assumptions were made which led to a linear proportional feedback relationship between the standing stock and the rate of exploitation of the stock. The variational assimilation method together with a Monte Carlo procedure was used to estimate the input parameters. The weak constraint formulation was then used to find improved estimates of the state variable.

In chapter three, the same biological and economic models were used as in chapter two. However, different growth models, i.e, the logistic and the Gompertz functions (Clark 1990) were used. The paper compared the estimates of these two functions when used to analyze the North East cod stock (NEACs) data (see Anon. 1998). The results suggested that it may be more appropriate to use the logistic growth function for the NEACs.

The remaining chapters are extensions of the previous ones. The dimension of the problems in these chapters is increased from the simple 1-dimensional (single) ordinary differential equation to more complex 2-dimensional problems of coupled nonlinear ordinary differential equations. In chapter four, a sole owner problem was posited. The manager is assumed to maximize a stream of discounted economic function subject to a natural constraint, e.g., the Schaefer dynamics equation (see Clark 1990). The mathematical objective function is assumed to be unknown and an attempt was made to uncover the form of the objective function of the sole owner. To avoid any inconsistencies, twin experiments were performed. That is, data were generated from the model itself and the model parameters were retrieved by the variational adjoint method.

Chapter five takes a quite different approach to modeling in fisheries management. The industry dynamics are modeled in a more general fashion. While the population is assumed to follow the Gordon-Schaefer dynamics, the industry dynamics are assumed to be driven by changes in an index of performance which can be positive, negative or zero. In an application to the NEACs, the industry was characterized as an open access fishery and the initial conditions as well as the model parameters entering the performance index and the biological function were estimated.

To apply the new techniques to real world fisheries data, the NEACs is used. Time series of observations of the actual catch and the estimated stock biomass were provided by the Norwegian fisheries directorate(See Anon. 1998). The data spans over the period since immediately after World War II (1946) until 1996.

To make the dissertation easily accessible and more appealing to readers we discuss and present a brief overview of the models and the computational methods. A summary of the work is also given.

1.2 Bioeconomics

Natural resource models (fisheries) have two principal components; a biological part which defines the natural constraints, e.g., ecological and environmental constraints, and an economic part which characterizes the operation of the industry (fishery). The latter defines the objective function of the management, the technological constraints, etc (see Homans and Wilen 1997). These two elements when suitably merged yield a bioeconomic model. In the remainder of the subsection, we describe the bioeconomic fisheries models studied in the dissertation. The models are formulated in continuous form mainly for mathematical convenience and tractability. However, discrete approximations of the models would be used in an application to the NEAC stock.

For the sake of generality, we present the population dynamics in a vector notation as

$$\frac{d\mathbf{x}}{dt} = F(\mathbf{x}, \mathbf{h}, t) \tag{1.1}$$

where $\mathbf{x}$ is a vector of biologically interacting species that may coexist in an ecosystem, $\mathbf{h}$ is a vector of controls and F a linear or nonlinear operator. The resource industry employs some inputs in the production process. These are called factors of production. They determine the rate at which the resource is exploited. The production function describes how the inputs are combined in the production process. In general, the rate of harvesting of the stock is related to the biomass and the inputs in a rather intricate manner. However, several useful approximations are made. It is a common practice in economics to assume a Cobb-Douglas specification (Clark 1990) in the analysis of resource problems. For purposes of exposition and simplicity, we define the vector form of the production function as

$$\mathbf{h} = \mathbf{h}(\mathbf{q}, \mathbf{e}, \mathbf{x}) \tag{1.2}$$

where $\mathbf{e}$ is a vector of fishing efforts, $\mathbf{q}$ is a constant vector and by definition, $h_j = q_j e_j^{b_j} x_k^{c_j}$ is the j^{th} output function, where b_j and c_j are constants. The output h_j depends on two important inputs, the stock biomass and the level of effort expended in fishing. x_k is the k^{th} stock biomass. In the fisheries economics literature it is often assumed that harvest is linear in effort and stock level, i.e., $b_j = c_j = 1$. The harvest function then reduces to $h_j = q_j e_j x_k$ where q_j is the j^{th} catchability coefficient. This results in the catch per unit effort which is proportional to the biomass. Several implicit assumptions underly the hypothesis including uniform distribution of fish, etc. The natural way to link the biology and economics of fishing is through the instantaneous mortality parameter f_j. Where $f_j = q_j e_j$ is generally a function of time. However, we shall assume that f_j is constant throughout our analysis. That is a constant fishing mortality rate or proportional removal rate of the standing stock policy is applied. This yields a simple harvest law which can be used by the management authorities to set total allowable catch quotas (TAC) if reliable stock estimates are available.

In fisheries management one often encounters the concept of a sole owner where a single firm or agent has full rights over the resource. A sole owner's problem is the maximization of stream of discounted net economic benefits. This can be mathematically expressed as

$$max \int_0^{T_f} \Pi(\mathbf{x}, \mathbf{h}, t)dt \tag{1.3}$$

subject to

$$\frac{d\mathbf{x}}{dt} = F(\mathbf{x}, \mathbf{h}, t) \tag{1.4}$$

$$\mathbf{x}(0) = \mathbf{x}_0 \tag{1.5}$$

where Π is the net economic benefits and T_f is the time horizon which can be finite or infinite. The problem stated above is in general a nonlinear control problem (Clark and Munro 1976). Solutions of the problem can be obtained by the use of the calculus of variations or optimal control theory (Kamien and Schwartz 1984). The degree of sophistication of the problem depends on the assumptions made in the analysis. For example, when Π is linear in the control variable(s), the optimal solution is the popular bang-bang solution (see Clark 1990). Under some fairly relaxed assumptions, Sandal and Steinshamn (1997b) derived an analytical feedback formula for a quadratic revenue function. In general however, closed-form solutions are unattainable and approximate numerical solutions are the only realistic alternatives (see Conrad and Clark 1987; Quentin et al. 2000).

Another approach to modeling the dynamics of the fishery was taken in the last part of the dissertation. This approach is rare in the literature and aims at restating the fact that more realistic assumptions can be incorporated in our economic models. The behavioral model is based on the assumption that the managers have an index of performance upon which their decision to invest or otherwise is hinged. The industry will make its investment decisions by, for instance, evaluating the average or marginal profits or even the profits and vary outputs accordingly (See Smith 1969). A simple example in which the industry is assumed to vary its output growth rate $\dot{h}/h$ in proportion to a given function ϕ is

$$\frac{dh}{dt} = \gamma h \phi(x, h) \tag{1.6}$$

where x is the capital stock and γ is a constant of proportionality. Notice that ϕ is a form of reaction function for the industry in question. One advantage of this model is that more realistic hypotheses about the industry can be made. These hypotheses can hence be tested using real data. The three classes of models discussed above are combined with data to demonstrate the new techniques of inverse methods and data assimilation.

1.3 Numerical Techniques

One of the main objectives of this research is to develop numerical methods for the analysis of bioeconomic fisheries models. This section will present the problem and discuss some of the numerical solution methods.

1.3.1 Inverse problem

The use of mathematical models to simulate a real process is an example of a forward problem (Hannesson 1975). Inputs of the model dynamics such as the initial conditions and the parameters of vital importance are prespecified in a forward problem. The real world fishery is then simulated using the model to study different scenarios.

An inverse problem can be formulated as follows. Given the model dynamics with as yet unknown or unspecified inputs and a set of observations, what are the values of the controls that give model forecasts that are as close as possible to the observed quantities. There are many ways of formulating the inverse problem. In the literature of inverse methods and data assimilation, different approaches have been taken. The common frameworks are the maximum likelihood formulation (Carrera and Neuman 1986(a,b,c)), the Bayesian formulation (Hamorn and Challenor 1997) and the least squares methods (Bennett 1990; Spitz et al. 1998; Yu and O'Brien 1992). The least squares method is the most used in empirical works (Bennett 1992; Lawson et al. 1995).

The inverse problem can be posed as follows. The dynamics, the initial conditions and the parameters are allowed to contain errors

$$\frac{d\mathbf{X}}{dt} = F(\mathbf{X}, \mathbf{Q}) + \hat{F}(t) \tag{1.7}$$

$$\mathbf{X}(0) = \mathbf{X}_0 + \hat{\mathbf{X}}_0 \tag{1.8}$$

$$\mathbf{Q} = \mathbf{Q}_0 + \hat{\mathbf{Q}} \tag{1.9}$$

$$\tag{1.10}$$

where $\mathbf{X}$ is the state vector, $\mathbf{X}_0$ is the best guess initial condition vector, $\hat{\mathbf{X}}_0$ is the vector of initial misfits, $\mathbf{Q}$ is a vector of parameters and $\hat{\mathbf{Q}}$ is the vector of parameter misfits. The term $\hat{F}$ is the dynamical residuals. The dynamics are assumed to approximately satisfy the constraints while the inputs, i.e., the initial conditions and the parameters, are uncertain.

In real life, measurements are often available. These observations may be sparse and noisy. The measurement equation is described as

$$\hat{\mathbf{X}} = \mathcal{H}[\mathbf{X}] + \epsilon \tag{1.11}$$

where $\hat{\mathbf{X}}$ is the measurement vector, ϵ is the observation error vector and $\mathcal{H}$ is a linear measurement operator. The misfits are assumed to be independent and identically distributed "iid" random deviates. The linear measurement operator may be defined as

$$\mathcal{H}_i[x] = \int_0^{T_f} x(t)\delta(t - T_i)dt = x(T_i) \tag{1.12}$$

where T_i is the measurement location in time, T_f is the time horizon, δ is the Dirac delta function and i denotes a component of the measurement functional which is a vector with dimension equal to the number of observations. It is crucial to note that the inverse problem is often ill-posed. That is, it is characterized by nonuniqueness and instability (Yeh 1986). To remedy this, smoothing or regularization is recommended (Navon 1997).

1.3.2 Some statistical assumptions

To describe the errors in the model, the data and the parameters, we require some statistical hypotheses. For our purpose in this paper the following hypotheses will suffice

$$\bar{\hat{F}}(t) = \mathbf{0}, \qquad \overline{\hat{F}^T(t_1)\hat{F}(t_2)} = \mathbf{W}_F^{-1}$$

$$\bar{\hat{X}}_0 = 0, \quad \overline{\hat{X}_0^T \hat{X}_0} = \mathbf{W}_{X_0}^{-1}$$
$$\bar{\epsilon} = 0, \quad \overline{\epsilon^T \epsilon} = \mathbf{w}^{-1}$$
$$\bar{\hat{\mathbf{Q}}} = 0, \quad \overline{\hat{\mathbf{Q}}^T \hat{\mathbf{Q}}} = \mathbf{W}_Q^{-1}$$
$$\overline{\hat{F}\hat{\mathbf{Q}}} = \mathbf{0}, \quad \overline{\hat{F}\epsilon} = \mathbf{0}$$
$$\overline{\hat{\mathbf{Q}}\epsilon} = \mathbf{0}$$

where the T denotes matrix transpose operator. The $\mathbf{W}$'s are the weighting matrices which are ideally the inverses of the error covariances of the observations. The overbar denotes the mathematical expectation operator.

1.3.3 The penalty function

The fitting criterion or the estimator is defined in a more general form as

$$\mathcal{J}[\mathbf{x}, \mathbf{Q}] = \frac{1}{2}\int_0^{T_f} \epsilon^T \mathbf{W} \epsilon dt + \frac{1}{2T_f}\int_0^{T_f} \hat{\mathbf{Q}}^T \mathbf{W}_\mathbf{Q} \hat{\mathbf{Q}} dt + \frac{1}{2}\int_0^{T_f} \hat{F}^T \mathbf{W}_F \hat{F} dt + J_s$$

where, the first term is the data misfits, the second term is the parameter misfit, the third term is the model residuals and J_s is a smoothing or regularization term (See Evensen 1994). Such a formulation results in an unconstrained minimization problem (Luenberger 1984). If the model dynamics were assumed to hold exactly, i.e, $\hat{F} \equiv 0$, the above problem reduces to a constrained minimization problem (Bennett 1992; Bertsekas 1982; Luenberger 1984). The problem is efficiently solved using the variational adjoint technique which provides an efficient way of calculating the gradient of the penalty functional through the use of the Lagrange multipliers. Other techniques of parameter optimization such as the simulated annealing and the Markov Chain Monte Carlo (MCMC) (Harmon and Charlenor 1997) are available. It is important to reiterate that the methods introduced in this work do not require analytical form of the functions to be estimated.

In data assimilation one can derive the adjoint equations in continuous form and then solve them. Alternatively, the adjoint code may be derived directly from the model code, i.e., the computer code of the model equations (Lawson et al. 1995). When solving the continuous problem with discrete data, the forcing term in the E-L equations comes in as impulse into the system thereby introducing periodic shocks which can create problems with convergence in the optimization part (See Lawson et al. 1995; Smedstad and O'Brien 1992). To deal with that, one may add a regularization or smoothing term in other to ensure stability of the inverse problem. For this dissertation, this approach is not taken because the chances of introducing errors in the process are high. By assuming that data is available at every grid point, the effect of the impulse or masking function is turned off. It is then possible to solve both the forward and backward equations at the same grid point.

1.3.4 Solution methods

Once an appropriate estimator is in place, the next task is to choose a suitable and efficient solution method. Several methods of solving the inverse problem exist. One approach is to derive the Euler Lagrange (E-L) equations and then solve them. The E-L equations are in general nonlinear and coupled and are often difficult to solve. The assumption of perfect dynamics decouples the system which is then solved by the forward and backward integration of the E-L equations (Smedstad and O'Brien 1991; Yu and O'Brien (1991,1992)).

The representer technique can be used to solve the E-L equations. It is an optimal technique for linear models (Bennett 1992; Evensen 1994). By expressing the solution of the coupled E-L equations as a first guess solution plus a linear combination of representers, the original two-point boundary value problem reduces to a sequence of initial value problems which are easier to solve (see Bennett 1992; Eknes and Evensen 1997). With nonlinear models, some numerical linearization is necessary.

The so called substitution algorithms avoid the integration of the forward and backward models. Examples are the gradient search methods and the statistical methods such as the simulated annealing (Kirkpatrick et al. 1983; Kruger 1992; Matear 1995).

Gradient search methods are the most popular methods of minimization of the penalty function. A variety of these methods ranging from the simplest (descent method) to the most advanced (Newton's method) have been used in the literature (Luenberger 1984; Gill et al. 1981). The quasi-Newton algorithm lies between the descent and the Newton's method. The method used in this research is a variable-storage or limited memory quasi-Newton method developed by Gilbert J. C. and Lemarechal of INRIA in France. The other substitution algorithm is the simulated annealing. It is a statistical and a derivative free algorithm. This algorithm is characterized by its up and down hill moves in order to find the global minimum (Goffe et al. 1992). More general objective functions can be used including discontinuous functions.

Sequential data assimilation algorithms such as the Kalman filter, the extended Kalman filter (Gelb 1974) and the newly developed Ensemble Kalman filter (Evensen 1997) have also been extensively used. In sequential assimilation, the model is integrated forward in time and the solution updated whenever measurement is available. For linear models the optimal sequential algorithm is the Kalman filter (see Gelb 1974; Evensen 1994). The extended Kalman filter has been used with nonlinear models but some linearization is required. Recently, the extended Kalman filter has been proposed for model parameter estimation (Navon 1997). Not much work has been reported so far on the success of the algorithm in parameter estimation.

1.3.5 Error analysis

The estimates of the parameters obtained using the data assimilation methods are often uncertain. In order for the solution of the model parameters to be complete, it must include estimates of the uncertainty in the optimal model parameters. If the errors in the observations are assumed to be normally distributed this information about the uncertainty is obtained by analyzing the Hessian matrix $\mathbf{H}$ (Tziperman and Thacker 1989; Matear 1995). The Hessian matrix is defined as

$$\mathbf{H} = \frac{\partial^2 \mathcal{J}}{\partial p_i \partial p_j} \tag{1.13}$$

where p's are the parameters for some i, j =1,...m, m is the number of parameters. By expanding the cost function about the optimal parameters ($\hat{p}$) using Taylor series and neglecting higher terms, we have

$$\mathcal{J} = \mathcal{J}_{min} + (p - \hat{p})^T \mathbf{H}(p - \hat{p}) \tag{1.14}$$

If the neglected terms were sufficiently small, then the uncertainties in the optimal parameters are normally distributed with mean zero and error-covariance matrix defined as the inverse of the Hessian (Matear 1995). The error-covariance matrix provides information about the distribution of the optimal parameters. The diagonal elements of the error-covariance matrix therefore provide a measure of the width of the distribution for the different parameters. These uncertainties can also be obtained by separately perturbing each model parameters and observing the effect on the output of the model or by using Monte Carlo methods.

1.3.6 Some numerical concerns

Our main concern in this subsection is to point out some of the potential problems that may be encountered when using simplified numerical schemes like the one (simple Euler) applied in this work. Throughout the rest of the thesis, a simple first order difference scheme was used to approximate the continuous models developed. This may be quite a simplified approximation in general but for the bioeconomic models used in this work this scheme seems to work fine. We have used a time step of 0.1 which corresponds to one year. For this particular analysis the time step used is small enough to ensure absolute stability of the scheme (see any introductory book on numerical analysis). It must be noted that in making a choice of the approximate scheme it is vital to performed stability analysis of the scheme one is using. Other higher order approximations such as the centered difference scheme, are available which may yield more accurate solution of the model equations. For simplified models the gains may not be that significant. This is however a digression since the focus is not numerical analysis of ODEs but the implementation of data assimilation methods in resource problems.

1.4 Conclusions

This chapter presents the preliminaries of the dissertation. It serves to put the remaining chapters together by unifying the main ideas. First, a broad and extended introduction was given. The objectives of the research were stated and the fundamental concepts and ideas presented.

Mathematical modeling and computational methods in bioeconomics have been the focal points of the work. Three different dynamic economic resource models have been developed. They are continuous time models with the Schaefer growth model as the biological foundation. The new and efficient techniques of inverse methods and data assimilation were also discussed. Both the weak and strong constraint problems have been studied. The remaining chapters are also briefly discussed in the order in which they appear. Some of the main results of the papers are stated.

A more general formulation for fitting numerical resource models to data in which the models can be considered as providing either weak or strong constraints is introduced with some success. The main strength of the work is that, this approach is a generalization of the statistical regression analysis. This dissertation has laid the foundation for exploring the advanced techniques of inverse methods and data assimilation in economics (resource). These techniques are novel and efficient. They present economists with extra ordinary opportunities and potentials in future. With increasing power of computers and an expansion in the volume of data available these methods we hope will become indispensable to economists. More research is however required in order to exploit all the efficient features of the techniques.

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Chapter 2

Assimilation of real time series data into a dynamic bioeconomic fisheries model: An application to the North East cod stock

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Assimilation of real time series data into a dynamic bioeconomic fisheries model:

An application to the North East Arctic cod stock

Abstract

This paper combines the new and elegant technique of inverse methods and a Monte Carlo procedure to analyze real data for the North East cod stock (NEACs). A simple nonlinear dynamic resource model is calibrated to real time series of observations using the variational adjoint parameter estimation method of data assimilation and the Monte Carlo technique. By exploring the efficient features of the variational adjoint technique coupled with the Monte Carlo method, optimal or best parameter estimates with their error statistics are obtained. Thereafter, the weak constraint formulation resulting in a stochastic ordinary differential equation (SODE) is used to find an improved estimate of the dynamical variable(s). Empirical results show that the average fishing mortality imposed on the NEACs is 16 % more than the intrinsic growth rate of the biological species.

2.1 Introduction

Two important sources of information to bioeconomists and other researchers are the model on one hand and the data on the other. The model is an embodiment of the scientific beliefs of the researcher. Mathematical or numerical models have been used extensively by economists to gain useful insights in the analysis of natural resource problems (Clark 1990; Hannesson 1993; Sandal and Steinshamn 1997). The other source is the observations obtained from field measurements. Unfortunately, this vital source has not been fully exploited thus far. Advanced and efficient techniques of combining these two sources of information need to be developed. This paper employs the technique of data assimilation and inverse methods (Bennett 1992) in which all the available information is used in the analysis of the North East Arctic Cod stock (NEACs).

Inverse methods are a set of methods employed to extract useful inferences about the real world from measurements. In other words inverse methods can be defined as a set of mathematical techniques for reducing data to acquire useful information about the physical world on the basis of inferences drawn from observations (Menke 1984). In data assimilation, observations are merged with a dynamical model in order to determine, as accurately as possible, a description of the state of the system. It can be used to estimate the variables of the dynamical model and/or the parameters of the model. It also leads to the resolution of mathematically ill-posed modeling problems (Bennett 1992).

In general, there are two forms of assimilation, sequential assimilation and variational assimilation. In sequential assimilation, the model is integrated forward in time and the model solution updated whenever measurements are available. A typical example is the Kalman filter (Kalman 1960; Gelb 1974) which is an optimal algorithm for linear dynamics. Variational assimilation, on the other hand aims at globally adjusting a model solution to observations available over the assimilation time interval. Two different formalisms exist in variational methods: the method of strong constraint popularly known as the variational adjoint method and the method of weak constraint which is related to the penalty methods (Smedstad and O'Brien 1991). The strong constraint formulation is shown to be the limiting case of the latter where the model is assumed to be perfect. In this paper, a variational inverse formulation will be employed to estimate the generalized inverse of the stock and the poorly known input parameter(s) of a bioeconomic fisheries model. This technique has been applied with success in parameter estimation in an Ekman flow model (See Eknes and Evensen 1997; Yu and O'Brien 1991).

During the last quarter of the century, many important developments have taken place which have affected the structural setup of fisheries management. An important example is the U.N. law of the sea in the late 1970s. The law resulted in the Extended Fisheries Jurisdiction (EFJ) from maximum of 12 to 200 nautical miles, for coastal states. It empowered, for example, Norway to manage the Barent sea cod together with Russia, Iceland and The Faroe Islands. This calls for annual quotas being determined a priori at the inception of each fishing season. This paper will aim at addressing the question of quota determination. The recent influx of data from both fisheries biologists and economists due to improved observational and measurement methods necessitate the development of techniques in which as much information as possible can be extracted.

Inverse methods and data assimilation methods have broad application and a wide range of advantages which is demonstrated by its extensive usage in operational meteorology, oceanography and other fields. These advantages can be explored in bioeconomics to a great extent. First, it can be used to analyze the incoming data to extract useful information which will lead to important policy implications about the operation of a fishery. This in effect will help answer some of the unanswered questions in this area.

Traditionally, data assimilation and inverse methods are used to estimate variables of dynamical models, using all the available information from the model and the information about the true state from the data. However, these techniques have also been proposed as a tool for parameter estimation in dynamical models (Evensen et al. 1998). The basic idea is that it should be possible to use mathematical tools to formulate inverse problems for parameter estimation given additional information in a form of measurement data. Thus, one may attempt to search for model parameters resulting in a model solution that is closest to the observations. Notice here that this technique is new and has obvious advantages compared with the traditional methods. The technique can be applied with equal force to both an open access fishery and the sole owner fishery. It is highly suitable for complex and high dimensional problems. Multidimensional fisheries models are more realistic as ecosystem effects may be incorporated.

Parameter estimation has been used extensively in economics and other fields. However, very few studies have so far been reported in which the techniques in this paper have been used. For all we know, no such study has been made in fisheries bioeconomics. The reason may be attributed to lack of data and computer power in the past.

The purpose of this study is threefold. First to introduce the powerful tool of data assimilation in the management of renewable resources such as fisheries. Second, to exploit the elegant and efficient properties of the inverse methods in order to extract the best information from the available measurements. Third, to estimate the parameters of the growth and production relations and thereby estimate the stock and harvest quotas under a dynamic constraint.

The remainder of the paper is structured as follows. Section two presents the general formulation of the inverse methods and discusses the null hypotheses. In section three, the estimator is defined and a comprehensive discussion of the solution method presented. The least squares method is used to define a scalar objective functional emphasizing the link between this technique and the theory of statistical estimation. In the fourth section, a simple bioeconomic model is defined. The biological base is the Schaefer growth function. It is tied to the economics by a catch per unit effort type of production function. Section five is a historical discussion of the NEACs and a sensitivity analysis of the parameters of the model. Finally, the results are presented in section six with an equilibrium analysis using the estimated parameters.

2.2 The Variational Inverse Formulation

A variational inverse problem can be formulated as either a strong constraint problem where the model is assumed to be perfect, i.e., the model holds exactly or the weak constraint formalism (Sasaki 1970) where the model is allowed to contain errors. In modeling a system, several assumptions are often made both for mathematical convenience and tractability. Several uncertain inputs are also used in the model resulting in a model that approximately represents the real system. Modeling errors are unavoidable in many situations. Thus, adding a term to the model that quantifies the errors makes the model more realistic. It is sometimes a common practice among some researchers to assume a model that is perfect then vary some of the free parameters such as the initial conditions of the model in order to find the solution which best fit the data (Yu and O'Brien 1992). Such a formulation is known as the strong constraint problem. It is shown that the strong constraint problem is a limiting case of the weak constraint problem (see Bennett 1992).

In this paper, the variational adjoint technique will be employed to fit the dynamic model to the observations. We then use the estimated parameters in an inverse calculation using the weak constraint formulation. In the first problem the control variables are the input parameters. Using the variational adjoint method the gradients of the cost functional with respect to the control variables are efficiently calculated through the use of the Lagrange multipliers. The gradients are then used to find the parameters of the model dynamics which best fit the data. In the second case however, the model variables are the control parameters. The gradients of the variables at each grid point are calculated and the values used to search for the minimum of the cost functional (see Bennett 1992).

2.2.1 The data and the model

To formulate the problem, a general nonlinear scalar dynamic model together with the initial condition is defined as

$$\frac{dx}{dt} = g(\beta; x) + q(t) \tag{2.1}$$

$$x(0) = u + a \tag{2.2}$$

$$\beta = \beta_0 + \hat{\beta} \tag{2.3}$$

where g is a nonlinear operator, β is a parameter(s) to be estimated and is assumed poorly known. The terms a, $\hat{\beta}$ and $q(t)$ are random white noise terms and are defined as the errors in the first or best guess of the initial condition (u), the parameter(s) (β_0) and the model formulation respectively. Such a formulation is referred to as the weak constraint general inverse problem (Evensen et al. 1998). The task involves solving for the optimal dynamical variables while updating the model parameters. The result is a solution of the model that is closest to the observations and simultaneously satisfies the model constraints approximately. If the errors in the initial condition and/or the model formulation are assumed to vanish identically, i.e., $a \equiv 0$ and $q \equiv 0$, then we retrieve the strong constraint parameter estimation problem.

The model is one source of information which in general is the physical laws governing the system, e.g., the population dynamic model of the Schaefer (1964) type. Additional available information is the set of observations given by

$$\mathbf{d} = \mathcal{H}[x] + \epsilon \tag{2.4}$$

where $\mathbf{d}$ is the measurement vector, $\mathcal{H}$ is a linear operator that relates the observations to its model counterpart and ϵ is the vector of measurement errors. The errors may be due to instrumental imprecision and from other sources.

2.2.2 Some statistical assumptions

To describe the errors in the model, the data and the parameters, we require some statistical hypotheses. For our purpose in this paper the following hypotheses will suffice

$$\begin{aligned} \bar{q}(t) &= 0, & \overline{q^T q} &= w_q^{-1} \\ \bar{a} &= 0, & \overline{a^2} &= w_a^{-1} \\ \bar{\epsilon} &= 0, & \overline{\epsilon^T \epsilon} &= w^{-1} \\ \bar{\hat{\beta}} &= 0, & \overline{\hat{\beta}^T \hat{\beta}} &= w_{\beta}^{-1} \end{aligned}$$

where the scalars $w's$ are the weights and the T denotes matrix transpose operator. That is, we are assuming that the errors are normally distributed with zero means and constant variances (homoscedastic) which are ideally the inverses of the optimal weights. The assumption of unbiasedness is very common in the literature (see Bennett 1992). The overbar denotes the mathematical expectation operator. It will be, however, more realistic to make the variances more general by allowing cross-variances, but this will not be used in this paper. The linear measurement operator may be defined as

$$\mathcal{H}_i[x] = \int_0^{T_f} x(t)\delta(t - T_i)dt = x(T_i) \tag{2.5}$$

where T_i is the measurement location in time, T_f is the time horizon, δ is the Dirac delta function and i denotes a component of the measurement functional which is a vector with dimension equal to the number of observations. In the subsequent sections, we shall present a simple but detailed discussion of the strong and the weak constraint formalisms.

2.3 The Least Squares Estimator

In data assimilation, the goal is to find a solution of the model which is as close as possible to the available observations. Several estimators exist for fitting models to data. In this paper, we seek residuals that result in model prediction that is in close agreement with the data. Hence the fitting criterion is the least squares loss function[1] which is the sum of the model, data, initial residuals and parameter misfits. This is given by

$$\mathcal{J} = w_q \int_0^{T_f} q(t)^2 dt + w\epsilon^T \epsilon + w_a(x(0) - u)^2 + w_\beta \hat{\beta}^T \hat{\beta} \tag{2.6}$$

where w_β, w_q, w_a and w are scalar constants. We have thus formulated a nonlinear unconstrained optimization problem. The last two terms in (2.6) are penalty terms on the dynamics and the initial condition respectively.

To derive the strong constraint problem as a special case, define $\lambda = w_q q$ and $\lambda_a = w_a a$ where $q \equiv 0$ and $a \equiv 0$, i.e., both the dynamics and the initial condition are perfect. This is equivalent to assigning infinitely large weights to the dynamics and the initial condition. The cost functional reduces to

$$\mathcal{J}_s = w\epsilon^T \epsilon + w_\beta \hat{\beta}^T \hat{\beta} \tag{2.7}$$

where $\mathcal{J}_s$ is the cost function for the strong constraint problem. Inserting λ and λ_a in (2.6) we obtain the Lagrange functional for the variational adjoint method. The necessary condition for an optimum (local) is that the first variations of the cost function

[1]loss is used here to avoid any confusion with the economic cost function. The terms loss, penalty or criterion are used to mean cost function as is normally used in the literature.

with respect to (wrt) the controls vanish $\partial\mathcal{J} = 0$.
There are many efficient algorithms for solving unconstrained optimization problems (Luenberger 1984). The once used most are the classical iterative methods such as the gradient descent, the quasi-Newton and the Newton methods. These methods require the derivatives of the cost functional and the Hessian for the case of the Newton methods. However, nonconventional methods could be used. For example, methods of optimization without derivatives and statistical methods such as simulated annealing could be used to find the minimum of the cost functional at a greater computational cost. Their advantage is that a more general cost functional including discontinuous functions could be used. The inherent problem of local solutions in the line search methods is said to be absent in simulated annealing (see Goffe et al. 1992; Matear 1995).
In order to make the paper accessible to more readers we avoid the mathematical and computational details but give a comprehensive verbal explanation of the methods.
One approach of solving the inverse problem is to derive the Euler-Lagrange (E-L) systems of equations and solve them. The E-L systems derived from calculus of variations or optimal control theory (see Kamien and Schwartz 1980) are generally coupled and nonlinear and require simultaneous integration of the forward and the adjoint equations. The task easily becomes arduous and very often impractical. Such a procedure is called the integrating algorithm. In the variational adjoint formulation, the assumption of a perfect model leads to the decoupling of the E-L equations. The forward model is then integrated followed by the backward integration of the backward equations. For the weak constraint inverse problem, the approach here avoids solving the forward and backward models but uses the gradient information to efficiently search for the control variables that minimize the loss function subject to the constraints. Given the cost functional, which is assumed to be continuous with respect to the controls, find the derivatives wrt the controls and then use the gradients to find the minimum of the cost function. The second procedure is referred to as the substituting algorithm and is generally efficient in finding the local minimum. In the case of the variational adjoint method, the algorithm is as follows:

- Choose the first guess for the control parameters.

- Integrate the forward model over the assimilation interval.
- Calculate the misfits and hence the cost function.
- Integrate the adjoint equation backward in time forced by the data misfits.
- Calculate the gradient of $\mathcal{J}$ with respect to the control variables.
- Use the gradient in a descent algorithm to find an improved estimate of the control parameters which make the cost function move towards a minimum.
- Check if the solution is found based on a certain criterion[2].
- If the criterion is not met, repeat the procedure until a satisfactory solution is found.

The solution algorithm for the weak constraint inverse problem is similar except that the gradients are not calculated from the backward integration of the adjoint equations but are obtained directly by substitution. The procedure is outlined below.

- Choose the first guess for the control variables.
- Calculate the misfits and hence the cost function.
- Calculate the gradient of $\mathcal{J}$ with respect to the control variables.
- Use the gradient in a descent algorithm to find an improved estimate of the control variables which make the cost function move towards a minimum.
- Check if the solution is found based on a certain criterion.
- If the criterion is not met repeat the procedure until a satisfactory solution is found.

[2]For example, $\mathcal{J} \leq \epsilon$, $||\Delta\mathcal{J}|| \leq \epsilon$ may be appropriate convergence criteria. Where ϵ is a small number.

2.4 The Bioeconomics

Fisheries management and bioeconomic analysis have been given considerable attention in the last two decades. Fisheries economists have for the past years combined biological and economic theory to understand and address management issues concerning the most important renewable resource stock, i.e., the fish. Questions about efficient exploitation and conservation measures are being raised both in the academic literature and in the media.

The mainstay of bioeconomic analysis is the mathematical models. In this paper we advance a little further by combining information both from the theoretical model of a fishery and the actual field observations. In formulating the bioeconomic model, we require a reasonable biological submodel as a basis. Following the tradition in the literature, we propose an aggregated growth model of the Schaefer (1964) type. Let x denote the total stock biomass and h denote the rate of harvesting from the stock. We represent the dynamics of the stock as

$$\frac{dx}{dt} = rx(1 - \frac{x}{K}) - h \tag{2.8}$$

where r, K are the intrinsic growth rate per unit time and the environmental carrying capacity in 10^3 tons respectively. The growth law for this fishery is assumed to follow the logistic law (Schaefer 1964). The dynamics of the stock depends on the interplay between terms on the right hand side of the equation. The stock will increase if h is less than the growth term and decreases if h is greater. If human predation ceases, i.e., $h = 0.0$ then the stock will increase at a rate equal to the natural growth of the stock. The stock biomass will increase towards the maximum population size K. This simple model describes a year-class model of the Gordon-Schaefer type. It basically describes the dynamics of an exploited fishery by linking the biological dynamics and the economics through the general production function $h(t)$.

2.4.1 The production function h

In this paper the general Cobb-Douglas production function $h(e, x)$ is defined as

$$h = qe^b x^c \tag{2.9}$$

where e is the fishing effort, q, b and c are constants. The production function quantifies the rate of production of the industry and describes how the inputs are combined in the production process. It depends on two important inputs, the stock biomass and the level of effort expended in fishing. In the fisheries economics literature it is often assumed that harvest is linear in effort and stock level, i.e., $b = c = 1$. The harvest function then reduces to $h = qex$ where q is the catchability coefficient. This results in the catch per unit effort which is proportional to the biomass. Several implicit assumptions underly the hypothesis including uniform distribution of fish, etc. The natural way to link the biology and economics of fishing is through the fishing footnote That is the instantaneous average fishing mortality mortality parameter rate f instantaneous average fishing mortality. Where $f = qe$ is generally a function of time. In this paper we will specialize a bit by assuming a nonvarying f over time. That is a constant fishing mortality rate or proportional removal rate of the standing stock policy is applied. This yields a simple harvest law which can be used by the management authorities to set total allowable catch quotas (TAC).

To understand the nature and kind of policy used in the management of the NEACs, we apply a simple feedback relation to analyze the data. The assumption may be unrealistic, but we still hope that much practical insight will be gained and will lead to better understanding of the fishery. Thus, the harvest function for the linear case is

$$h = fx \tag{2.10}$$

where f is the unknown, or poorly known economic parameter, to be estimated. This formulation appears quite simple but may be of immense contribution to our understanding of the practical management of the NEACs. It can be considered as a first order linear approximation of the true harvest function. The function proposed is by no means supposed to be the complete and absolute characterization of the feedback specifications but is considered as a useful and practical approximation of the true one. To reiterate, our purpose is to be simple and to construct a model that is tractable which will lead to some important policy implications.

Some remarks about the model

Linking the biology of the exploited species and the simple approximate harvesting or TAC rule above yields

$$\frac{dx}{dt} = rx(1 - \frac{x}{K}) - fx \tag{2.11}$$

put in another form gives

$$\frac{dx}{dt} = \gamma x - \frac{rx^2}{K} \tag{2.12}$$

where $\gamma = (r - f)$ is the difference between the intrinsic growth rate and the fishing mortality rate. Let us call this the residual growth rate of the species. The residual growth rate can be positive, zero or negative at least theoretically. If no fishing mortality is imposed on the stock ($\gamma = r, f = 0.0$) then it grows to its maximum population level K at a rate equal to the natural growth. If f is positive but less than r the population will settle at a level less than K. For the critical scenario where fishing mortality balances the intrinsic growth rate ($\gamma = 0.0$) the population is driven to extinction. This case can be seen mathematically as

$$\frac{dx}{dt} = -\frac{rx^2}{K} \tag{2.13}$$

It is also the case where f exceeds r and γ becomes negative. The population will be driven to zero even faster. The dynamics are shown as

$$\frac{dx}{dt} = \gamma x - \frac{rx^2}{K} \tag{2.14}$$

The predictions of this simple model are evident in the case of most commercial fisheries. Many important fisheries have collapsed in recent times. An example is the Norwegian spring spawning herring (Bjorndal and Munro 1998).

2.5 The North East Arctic Cod stock

The NEACs is the most important demersal species along the coast of Norway and Northern Russia. This fishery has played an important economic role within the coastal communities for the past thousand years. The NEAC stock has for the past half century experienced large variations which result in a corresponding variation in the annual harvest quantities. The stock size fell from its highest level in 1946 of 4.1 million tons to the lowest in 1981 of .75 million tons. However, the stock seems to be recovering from the depleted state in the 1990s due to improved management strategies. In this study, a time series of observations from 1946 to 1996 is used. The variables are the annual stock and harvest[3] measured in 10^3 tons. In what follows, we present a brief qualitative description of the data (see Anon. 1998).

Figure 2.1, is a plot of the stock divided by a factor of three and the harvest. The stock and the harvest have generally a downward trend with periodic oscillations. Apart from the first few years the directions of fluctuation in both the stock and the harvest are the same. It may be observed from the graph that there exists some proportional relationship between the harvest rate and the level of stock.

[3]Total international landings as reported in ICES 1998.

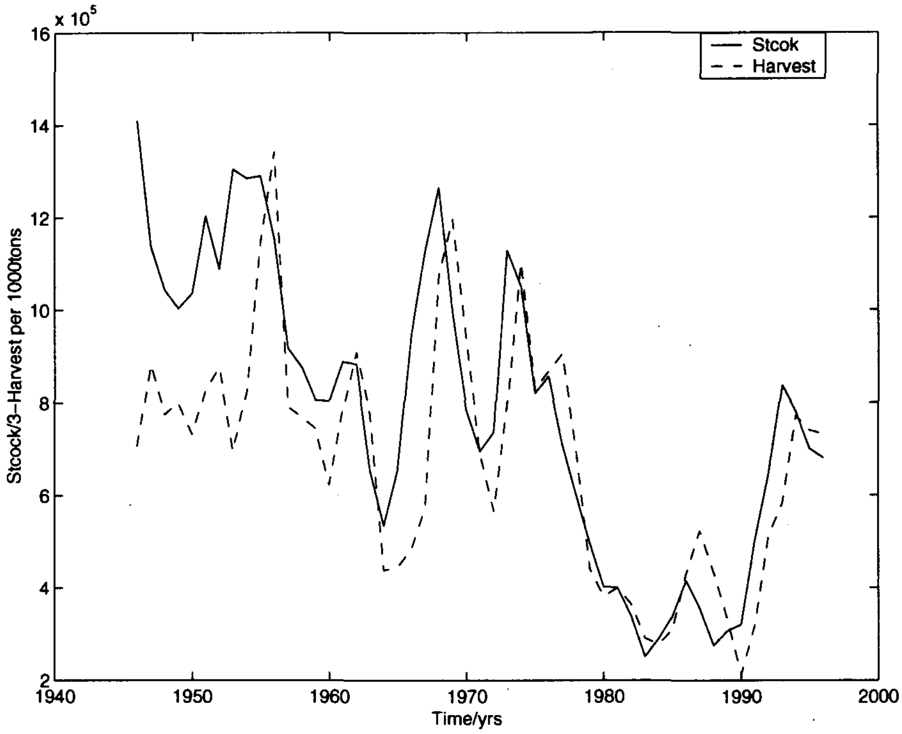

Figure 2.1: Time series plot of the stock and harvest rates of the NEACs. The stock size is scaled by a factor of one-third.

2.5.1 Sensitivity analysis

Input parameters of bioeconomic models are crucial in the analysis of the system. To provide good simulations, precise and reasonable parameters are required. Unfortunately, the values of these parameters are highly uncertain which translate into the output of the models. Sensitivity is a measure of the effect of changes in the given input parameter on a model solution. It quantifies the extent that uncertainties in parameters contribute to uncertainties in the model results (Navon 1997). Several analytical techniques of sensitivity analysis exist. To quantify the uncertainties of the k^{th} parameter, we define the following sensitivity index IS_k

$$IS_k = \frac{\sum_0^T (z_t - z_t^k)^2}{\sum_0^T z_t^2} \tag{2.15}$$

where z_t is the original model prediction and the z_t^k is the perturbed prediction. The results of the sensitivity of the biological and economic parameters are shown below. The parameters are each perturbed to 90 percent of their original values. These parameters are ranked in an increasing order of importance. Note that the units of r and f are (/year) and the unit of the carrying (average) capacity K is in kilo-tons. The sensitivity index is dimensionless.

Parameters	Original values	New values	IS_k
r	0.450	0.405	1.50
K	6000.0	5400.0	1.68
f	0.400	0.360	5.09

Table 2.1: Sensitivity index of model parameters.

The fishing mortality parameter is the most important and the growth rate is the least. The maximum population of the species is the more sensitive biological parameter which confirms the results of an earlier paper (Ussif et al. 1999a). The results indicate the fishing mortality rate is in fact very critical in the model. This outcome is used in the subsequent experiments to guide us in regard to which parameters to vary and which to give more attention.

2.6 Results

The empirical results of the research are discussed and shown in this section. All the results are based on actual observations of the NEAC stock for the period from 1946 to 1996. The results of the variational adjoint parameter estimation are presented. They are followed by the weak constraint inverse results and then a steady state equilibrium analysis is performed. Note that twin experiments were performed using both clean and noisy data to test the assimilation algorithm.

2.6.1 Estimation of the growth and yield functions

The combined variational adjoint-Monte Carlo technique was used to fit the bioeconomic model to the observations assuming that the fishery is exactly governed by the simple model. The model contains three input parameters: the intrinsic growth, the carrying capacity and the human predation coefficient. These are all important to a fisheries manager. Estimating all the parameters at the same time for this simplified model may pose a problem of identification. To obviate the bottleneck, the least sensitive parameter in the model is exogenously but randomly selected and then the other two, namely the carrying capacity and the fishing mortality rate, are optimally determined using variational adjoint methods. Relying on some physical information from experts, a range of r values between .25 and .45 is chosen. A subsample of 3005 was randomly drawned from the population. Using this sample, the variational adjoint method is used to find the optimal estimates of the parameters. The statistic of choice in this paper is the mean even though there are other estimators that are efficient. In table 2.2, we show the parameter estimates and their standard deviations.

Parameters	r (/yr)	K (1000 tons)	f (/yr)
Estimates	0.3499	5268.4	.4076
se	(0.0578)	(868.3)	(0.0579)

Table 2.2: Estimated parameters and their standard deviations.

These estimates are all reasonable and intuitively appealing. What is astounding is that the model has been able to capture the salient features of the NEACs. The estimated rate of capture of the stock exceeds the intrinsic growth rate of the species even when the population was highly vulnerable.

6.2 State estimation of the stock biomass

Inverse methods and data assimilation can be used to estimate the variables of a dynamical system or the parameters of a dynamical model, using all the available information from the model formulation and the set of observations. The former embodies all the beliefs of the modeler about the system she or he is interested in studying. They may use economic and biological theory as well as intuitive reasoning in order to construct a model that approximately represents the system. In the weak constraint formulation, the model dynamics are assumed to approximately hold. The fisheries model employed in this paper is quite oversimplified. Many important variables such as the environmental effects and predation from other species are disregarded. The harvest function is also a simple first order approximation. All these factors make the model quite unrealistic. To remedy this, we accept a certain unknown level of error in the model by adding a term that quantifies the errors and their uncertainties.

A cost functional measuring the disagreements between the data and the model predictions was defined, and a penalty term appended which penalizes the model misfits. A model prediction that is as close as possible to the data, is sought in a least squares sense. The optimization procedure used in this paper is the classical quasi-Newton method (Gilbert and Lemarechal 1991). The results are shown for two cases. The first case uses the solution of the variational adjoint method as the first guess solution. That is the parameter estimates from the first method were used to solve the model and the solution is taken as the best guess to start the optimization. To show that the algorithm is robust to the initial guess of the solution, a constant equal to the average of the first case is used as the first guess. The results are shown in the figures below. The circles denote actual data, the broken line is the first guess which is the solution of the strong constraint problem in case 1.

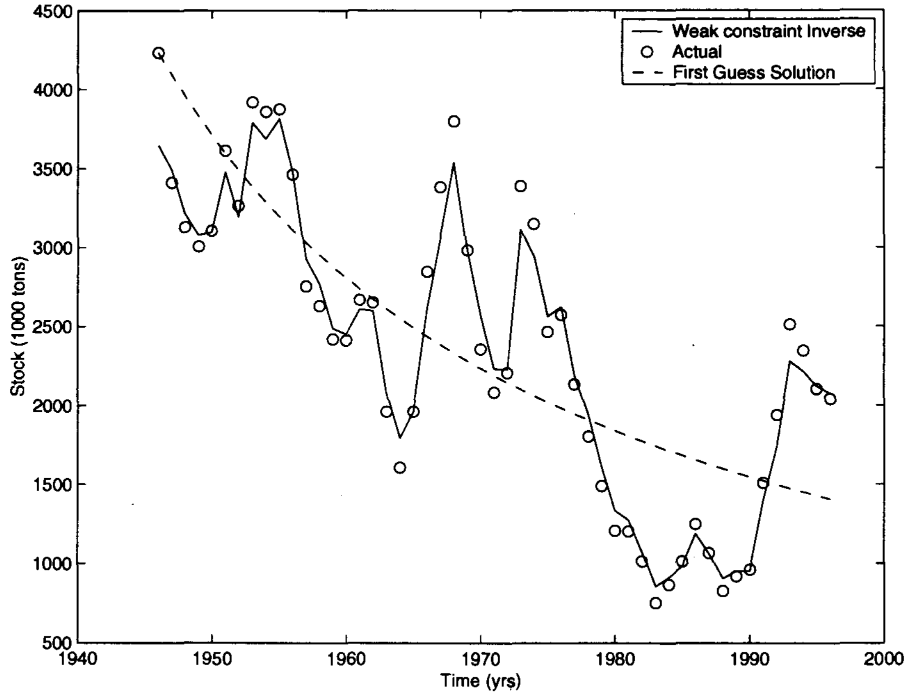

Figure 2.2: Graph of the stock biomass: case 1.

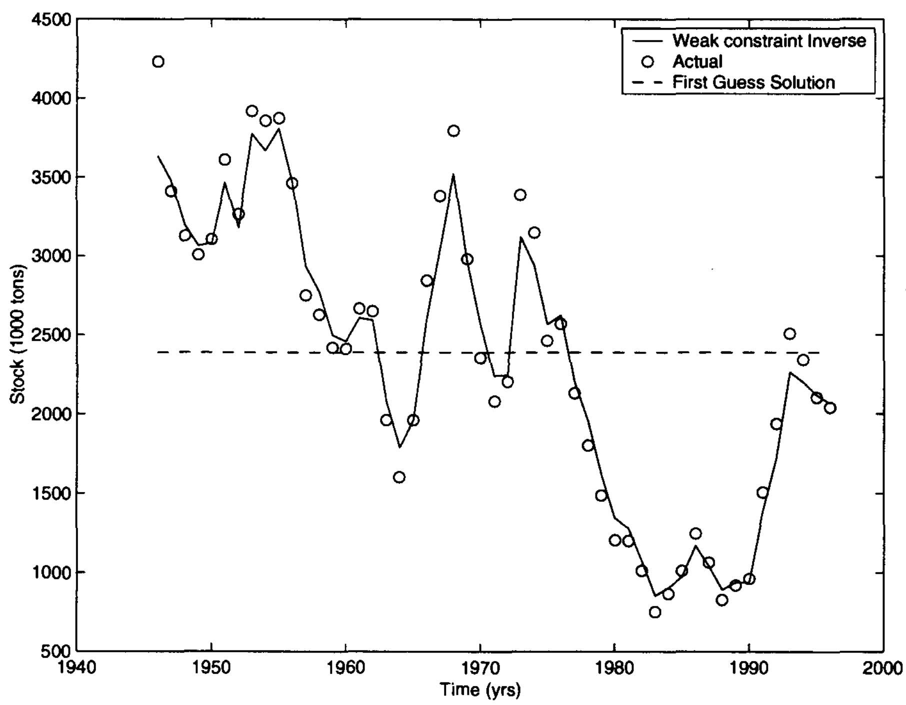

Figure 2.3: Graph of the stock biomass: case 2.

The fit of the model is good in general. The algorithm is also robust and did not depend very much on the initial guesses. However, the convergent rate is slightly affected by the choice of the initial guesses.

2.6.3 Equilibrium analysis using the deterministic model

The use of the population dynamic equation assumes the existence of equilibrium in the model. This section briefly discusses this concept in this application. At the steady state time is no longer important and the stock biomass becomes constant at a level x^*. This implies the time rate of change of the population is identically zero, i.e., the net growth of the stock balances the rate of harvesting

$$\frac{dx}{dt} = 0, \qquad h^* = rx^*(1 - \frac{x^*}{K})$$

It follows then that for the linear harvest function we have

$$fx^* = rx^*(1 - \frac{x^*}{K}) \tag{2.16}$$

where f is as defined previously. Hence the steady state biomass is

$$x^* = K(1 - \frac{f}{r}) \tag{2.17}$$

Ideally, the fishing mortality rate should not exceed the intrinsic growth rate of the biological species, i.e, $f < r$ for a fishery that is overexploited and is under rehabilitation. If the fishery is unexploited and initial stock is to the right of the maximum sustained biomass level then higher mortality rates may be applied in order to quickly adjust it to the desired optimal state. The equilibrium stock is a function of the biological and economic parameters. It is clear that if the carrying capacity (e.g., the aquatic environment) increases, x^* will increase and vice versa. The effect of small change in r is similar. However, increasing fishing mortality will result in a decline in the equilibrium biomass.

The concept of maximum sustainable yield has been the practical management objective for many fisheries (Clark 1990). The NEACs is not an exception to the rule. For the compensation model used in this paper, the $x_{msy} = K/2$, i.e., 2634.2 10^3 tons. The figure below shows the historical state of the NEACs from 1946 to 1996 and the x_{msy}. A careful study of the time series reveals some interesting observations. The fishery was in 1946 at a level of about 80 percent (4231.9 10^3 tons) of the carrying capacity. It was fished down to about 50 percent (x_{msy}) of K by 1958. It then remained at about that level forming a window until the late 1970s when the situation got completely out of control. However, due to the inherent stochastic nature of the biological species coupled with inadequate knowledge of the biology and economics of fishers/managers, the goal failed to yield results. This occurrence might also be attributed to the shortsightedness of the politicians and also the conflict of interest between the two major participants (Norway and Russia) in the exploitation of the stock.

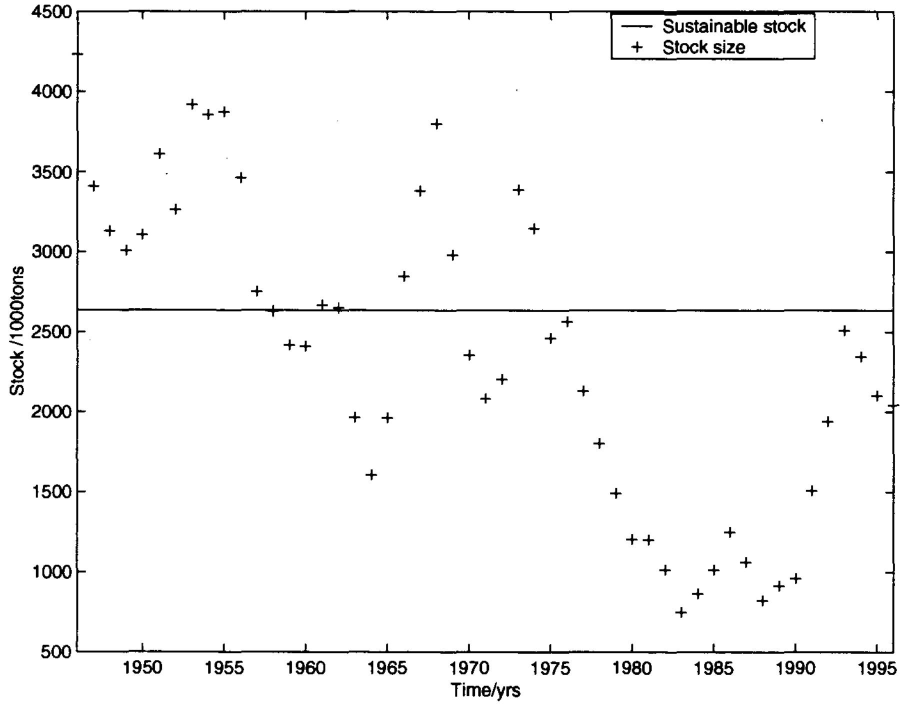

Figure 2.4: Plot of the stock and harvest rates (tons) vs. time (yrs).

The state of the stock continued to dilapidate and by 1983 it was at its worst level of less than 20 percent of the carrying capacity. The trend has, however, changed and the 1996 estimate of stock indicates a sign of recovery. Recent observations, however, indicate that the stock is again in deep trouble.

2.6.4 Conclusion

This paper uses a novel approach of data assimilation into dynamical models to analyze real data for the NEACs. Model parameters were estimated by the variational adjoint technique in combination with a Monte Carlo procedure. The variational adjoint technique provides an efficient way of calculating the gradients of the loss functional with respect to the control parameters. The estimates are as expected, the fit to the data is also good with the model amazingly capturing the trend in the data but failing to capture the oscillations. This is not surprising because the model is deterministic and does not have the ability to absorb the random events in the system. The estimated parameters are then used in an inverse calculation to find an improved estimate of the stock using the full information available in the form of observations and the model dynamics. The weak constraint model however does very well in capturing the stochasticity in the data. The key results of the paper are that for the NEACs the average intrinsic growth rate is about 0.35 per year and the maximum population that the environment can support is about 5.3 million tons. The fishing mortality rate is about 0.41 per year which is greater than the intrinsic growth rate. This implies the annual harvest or production from the fishery is consistently above the net growth curve. This is intuitively supported by the persistent decline of the stock since 1946. It is important to be reserved in generalizing the findings in this paper. The reason is that the model used in this paper is very simple and does not absolutely represent the fishery. Finally, the inverse and data assimilation methods have proven very efficient and can be very useful in analyzing, testing and improving resource models.

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Chapter 3

A new approach of fitting biomass dynamics models to real data based on a linear total allowable catch (TAC) rule: An optimal control approach

Submitted

A new approach of fitting biomass dynamics models to real data based on a linear total allowable catch (TAC) rule:

An optimal control approach

Abstract

A non-traditional approach of fitting dynamic resource biomass models to data is developed in this paper. A variational adjoint technique is used for dynamic parameter estimation. It provides a novel and computationally efficient procedure for combining all available information in the analysis of a resource system. Two alternative population growth models: the Schaefer logistic and the Gompertz model are used for estimating parameters of simple bioeconomic models by the method of constrained least squares. A simplified feedback rule is used to tie the biology and economics of fishing. The coefficient of determination (R^2 statistic) is used to evaluate the goodness of fit. Estimates of the parameters of the model dynamics are reasonable and can be accepted. The main inference from the work is that the average fishing mortality is found to be significantly above the maximum sustainable value.

3.1 Introduction

In spite of the growing criticisms of the biomass dynamics models or the surplus growth models (Clark 1990; Schaefer 1967), they remain the biological basis for most bioeconomic analysis. The trend in bioeconomic literature indicates that these models will continue to be in use for some time. Parameter estimation has been the greatest source of difficulty in applying the generalized biomass dynamics models in management schemes(Rivard and Bledsoe 1978). The bulk of the research in this area has been done by fishery biologists in the past. Several methods have been developed for fitting these models to observed data. Three approaches have been commonly used to fit surplus production models to observations: effort averaging methods, process-error estimators, and observation-error estimators (see Polacheck et al. 1993). Polacheck et al. (1993) used real and simulated data to compare the approaches and concluded that the methods yield different interpretations of productivity, e.g., the maximum growth rates. The method of effort-averaging, like many others, assumes that the stock is in equilibrium relative to effort. Ludwig et al. (1988) compared the method of total least squares (TLS) and the approximate likelihood (AL) method. They found the two methods to be consistent with some significant differences. For instance, the method of TLS involves more computations than the AL method. Least squares methods have also been used to estimate the Schaefer production model (e.g, the intrinsic growth rate) (Uhler 1979).

In bioeconomics, identification of model input parameters has not been accorded the attention it so deserves. Simulations of these models have mostly been performed using hypothetical values of the model parameters. Useful qualitative insights have been gained in a more general setting. However, issues of quantitative and operational nature have largely been ignored. Of interest to managers of resource stocks such as fish are questions about the size of the standing stock, the sustainable yield, the net growth, etc. To better advise managers on these important issues, bioeconomists ought to develop techniques of improving and efficiently estimating the existing bioeconomic models.

In view of the above arguments, we introduce a novel and advanced approach of fitting biomass dynamics models to measurements. The technique in this paper is an optimal control (variational adjoint) method of model parameter estimation (Lawson et al. 1995; Smedstad and O'Brien 1991). For recent applications of data assimilation techniques to biological and ecosystem models see (Lawson et al. 1995; Spitz et al. 1997; Matear 1995). The variational adjoint technique of data assimilation determines input parameters of a dynamical model using time series of observations of the state variables of the model dynamics. A least squares criterion function is defined subject to the natural dynamic constraints governed by the simple generalized population dynamics models. The variational adjoint technique is then used together with a quasi-Newton algorithm (Gilbert and Lemarechal 1991) to iteratively search for the minimum of the cost [1] functional. That is the difference between the data and the model solution. The method is very powerful and computationally efficient for parameter optimization. A major strength of the method is that it is highly suitable for high dimensional problems. It can also effectively handle nonlinear models. We also point out that this method does not require analytical forms of the functions estimated but only the numerical values which distinguishes it from existing methods.

Two functional forms of the existing biomass dynamics models (Clark 1990) in combination with a simple proportional exploitation rule will be used to estimate the biological and economic input parameters e.g, the fishing mortality rate, using real data for the North East Cod stock (NEACs). The bioeconomics employed in this analysis is quite simple. It combines simplified surplus growth models with a simple linear yield or harvest function to analyze the data. The biological functions contain parameters that are very crucial in determining certain important quantities of interest to fisheries management and researchers. Estimates of parameters such as the intrinsic growth rate and the environmental carrying capacity are rare for some important fish stocks around the world. Accurate measurement of these parameters are in fact very difficult if not impossible. As a consequence, quantities of considerable importance to management such as the maximum sustainable yield (MSY) are unreliable.

The goals of this paper are to demonstrate the potentials of the variational adjoint technique in the empirical analysis of natural resource systems, to apply the technique to

[1] We shall use loss, penalty or criterion in order to avoid confusion with costs in economics terms.

the NEACs for two different growth models and to make some inferences such is the MSY from the data. The paper is organized as follows. Section 2 is a discussion of the methodology used in the analysis. In section 3, we present the biological and economic submodels. The biology and economics are merged by the fishing mortality factor through a simplified yield function. In section 4 we present and discuss an empirical application of the model and conclude the paper.

3.2 Data Assimilation Methods

According to Sasaki (1970), a variational inverse problem can be cast as a weak constraint inverse problem where the model is allowed to contain modeling errors or the strong constraint problem (Bennett 1992; Evensen et al. 1998), where a perfect model is assumed. The weak constraint problem is a more general formulation with the strong constraint problem as a simple special case where the model weight is assumed to be infinitely large. It is a common practice among some researchers to assume a model that is perfect then vary some of the free parameters such as the initial conditions of the model in order to find the solution which best fit the data. Such a formulation is known as the strong constraint problem. In this paper, the variational adjoint technique will be employed to fit the dynamic resource models to the observations. Using the variational adjoint method the gradients of the cost functional with respect to the control variables are efficiently calculated through the use of the Lagrange multipliers. The gradients are then used to find the parameters of the model dynamics which best fit the data.

Data assimilation systems consist of three components: the forward model , the adjoint or backward model and an optimization procedure (Lawson et al. 1995). The forward model is our mathematical representation of the system we are interested in studying, e.g., an open access or a sole owner fishery model. The adjoint model consists of equations that provide a method of calculating the gradient of the cost function with respect to the control variables. The gradients are then used in a line search using standard optimization packages to find the minimum of the cost function. Most optimization routines are based on iterative schemes which require the correct computation of the gradient of the cost function with respect to the control variables. In the variational adjoint formulation, computation of the gradient is achieved by using the adjoint equations forced by the model-data misfits. The model equations are run forward in time while the adjoint equations are run backward in time which are then used to calculate the gradient of the cost function.

An important step in data assimilation is the choice of the criterion function for the goodness of fit. The commonly used fitting criterion is the generalized least squares criterion. It can be defined with no a priori information about the parameters or with prior information about the parameters incorporated as a penalty term in the criterion function. Some researchers argue that since some information about the parameters and their uncertainties are always available, adding the information is a plausible thing to do (Harmon and Challenor 1997; Evensen et al. 1998).

3.2.1 Perfect dynamics

In this paper we will assume perfect dynamics and initial condition(s). This implies that we are neglecting modeling errors. The model dynamics will be governed by a simple ordinary differential equation given by

$$\begin{aligned} \frac{dx}{dt} &= g(\mathbf{p}; x) \\ x(0) &= u \qquad (3.1) \\ \mathbf{p} &= \mathbf{p}_0 + \hat{\mathbf{p}} \qquad (3.2) \end{aligned}$$

where $g(\mathbf{p}; x)$ is a nonlinear operator, $\mathbf{p}$ is a vector of parameters to be estimated and is assumed poorly known and u is the first or best guess initial condition of the model. The vector $\mathbf{p}_0$ is the first guess of the parameters and $\hat{\mathbf{p}}$ is a vector of random white noise term. Assume that we also have a set of observations of the state variable(s) which are related to the true state in this simple linear fashion

$$\mathbf{x}^{\mathbf{obs}} = \mathbf{x} + \mathbf{v} \qquad (3.3)$$

where $\mathbf{x}^{\mathbf{obs}}$ and $\mathbf{x}$ are the observed and the model forecast vectors respectively, and $\mathbf{v}$ is the error vector in the observed values. The additive stochastic error term is quite general so far. In the subsections that follow, we will put some structure to the form of the noise term. Inverse methods combine the theoretical information contained in the model and the information about the true state from the data to optimally estimate the model parameters.

3.2.2 The estimator

One of the major components of data assimilation techniques is the choice of the estimator. Many estimators exist that are attractive in the literature. However, the least squares estimator has been the popular one among researchers partly because of its simplicity and mathematical convenience. The least squares fitting criterion is defined as

$$\mathcal{J} = (\mathbf{x} - \mathbf{x}^{obs})^T \mathbf{W} (\mathbf{x} - \mathbf{x}^{obs}) + (\mathbf{p} - \mathbf{p}_0)^T \mathbf{W}_p (\mathbf{p} - \mathbf{p}_0) \tag{3.4}$$

where $\mathbf{x}$ is the prediction of the model, $\mathbf{x}^{obs}$ is the observed or measured quantity. The $\mathbf{W}$ is the inverse measurement error covariance matrix, i.e., the weightng matrix and is assumed to be positive definite and symmetric and T denotes the transpose operator. Uncertainties in the parameters are represented by the symmetric positive definite covariance matrix $\mathbf{W}_p^{-1}$. The first term in the loss function is the sum of the squares of the model-data misfits $\mathbf{v} = (\mathbf{x} - \mathbf{x}^{obs})$ and the second term is a penalty on the parameters. If the model parameters are poorly known then greater penalty is imposed, i.e., they are given less weight and vice versa. To simplify the calculations, we make the following assumptions about the errors and their uncertainties. The model-data and the parameter misfits are assumed to be Gaussian mean zero and constant variances. That is we have

$$E\mathbf{v} = \mathbf{0}, \quad E\mathbf{v}\mathbf{v}^T = \mathbf{W}^{-1} = w^{-1}\mathbf{I} \tag{3.5}$$

$$E\hat{\mathbf{p}} = \mathbf{0}, \quad E\hat{\mathbf{p}}\hat{\mathbf{p}}^T = \mathbf{W}_p^{-1} = w_p^{-1}\mathbf{I}_p \tag{3.6}$$

where the capital letter E denotes mathematical expectation operator, $\mathbf{I}$'s are unit matrices and the scalar constants w^{-1} and w_p^{-1} are the variances of the random errors in the measurement and the parameters respectively. In view of the above assumptions, the cost function $\mathcal{J}$ can be identified with a normal probability distribution function. Thus, minimizing the cost function is equivalent to maximizing the likelihood, i.e., the best fit corresponds to the most likely outcome of the measurements. For the experiments in this paper, $w = 1.0$ and $w_p = 0.0$ will be used.

3.2.3 Minimization technique

Minimization of the cost functional $\mathcal{J}$ subject to the dynamics is a constrained optimization problem (Luenberger 1984; Bertsekas 1992). A computationally efficient[2] technique for the minimization of the cost functional is the variational adjoint method. It consists of transforming the constrained problem into an unconstrained optimization problem via the use of the undetermined Lagrange multipliers. It is then possible to use a gradient search method to find model parameters that yield predictions which are as close as possible to the observations. To illustrate the numerical procedure, we use the discrete equivalent of the continuous model dynamics

$$x_{n+1} = x_n + g(\mathbf{p}; x_n)dt, \tag{3.7}$$

$$x_0 = u, \quad 0 \le n \le N-1 \tag{3.8}$$

where N is the number of observations and dt is the time step. The discretization scheme used is a simple forward difference scheme. The discrete form of the Lagrange functional is constructed as follows

$$\begin{aligned} \mathcal{L} = {} & w\sum_{n=1}^{N}(x_n - x_n^{obs})^2 + w_p\sum_{i=1}^{N_p}(p_i - \hat{p}_i)^2 \\ & + \sum_{n=1}^{N-1} \lambda_n(x_{n+1} - \{x_n + g(\mathbf{p}; x_n)dt\}) \end{aligned} \tag{3.9}$$

where λ_n is the value of the multiplier at time step n and N_p is the number of model parameters which are the control variables of the problem. It is important to note here that we are assuming that data are available at every grid point which is the most ideal situation. The extrema conditions for the problem are

$$\frac{\partial \mathcal{L}}{\partial \lambda_n} = 0 \tag{3.10}$$

[2]This is compared with when calculating the gradients using finite difference approximation or when using techniques such as simulated annealing.

$$\frac{\partial \mathcal{L}}{\partial x_n} = 0 \tag{3.11}$$

$$\frac{\partial \mathcal{L}}{\partial p_i} = 0 \tag{3.12}$$

From these equations, we obtain

$$x_{n+1} - \{x_n + g(\mathbf{p}; x_n)dt\} = 0 \tag{3.13}$$

$$\frac{\partial \mathcal{J}}{\partial x_n} - \lambda_n(1.0 + dt\frac{\partial g}{\partial x_n}) + \lambda_{n-1} = 0 \tag{3.14}$$

$$\Delta_{p_i}\mathcal{L} = \Delta_{p_i}\mathcal{J} - \sum_{n=1}^{N-1} \lambda_n dt \frac{\partial g}{\partial p_i} = 0 \tag{3.15}$$

where $\Delta_{p_i}\mathcal{L}$ is the derivative with respect to the i^{th} parameter and $\frac{\partial g}{\partial x_n}$ is the tangent linear operator. It is immediately seen that equation (3.13) recovers the model dynamics, i.e., the forward model, equation (3.14) gives the backward model forced by the model-data misfits and equation (3.15) is the gradient with respect to the parameters. To find the model parameters that give model forecasts that are as close as possible to the observations using the classical search algorithms, correct values of the gradients are required. Methods of verifying the correctness of the gradient are available both numerically and analytically where possible (see, Spitz et al. 1997; Smedstad and O'Brien 1991). We have in this paper checked all gradient calculations to ensure reliable parameter estimates. The optimization procedure used for the minimization is the quasi-Newton procedure developed by Gilbert and Lemarechal (1991). Implementation of the variational adjoint parameter algorithm involves the following steps.

- Choose the first guess for the control parameters.
- Integrate the forward model over the assimilation interval.
- Calculate the misfits and hence the cost function.
- Integrate the adjoint equation backward in time forced by the data misfits.
- Calculate the gradient of $\mathcal{J}$ with respect to the control variables.
- Use the gradient in a descent algorithm to find an improved estimate of the control parameters which make the cost function move towards a minimum.

- Check if the solution is found based on a certain criterion (*e.g.*, $\mathcal{J} \leq \epsilon$) for some ϵ.
- If the criterion is not met, repeat the procedure until a satisfactory solution is found.

3.2.4 Goodness of fit measure

To examine the performance of the method we need a statistical measure of how the predicted and the observed variables covary in time. An appropriate parameter may be the correlation coefficient R. For the vectors $\mathbf{x}$ and $\mathbf{x}^{obs}$, the correlation coefficient is given by

$$R = \frac{\sum_{n=1}^{N}(x_n - \bar{x})(x_n^{obs} - \bar{x}^{obs})}{[\sum_{n=1}^{N}(x_n - \bar{x})^2 \sum_{n=1}(x_n^{obs} - \bar{x}^{obs})^2]^{1/2}} \tag{3.16}$$

where the bars denote the means or expected values of the random variables and N is the number of observations. Notice that R is a dimensionless quantity and lies between -1 and +1 inclusive. From the R relation, another important quantity called the coefficient of determination R^2 can be calculated. The coefficient of determination is defined as $R^2 = SSR/SST$, where SSR is the variance explained and SST is the total variance (see Greene 1997).

3.3 The Dynamics of the Biomass

Management of many fisheries have often been based on the simplified population dynamics models of the Schaefer type (Sandal and Steinshamn 1997; Clark 1990). It is apparent that these models will continue to be used for some time in the management of some of the important commercial species around the world. While efforts are underway in the development of more realistic models, it is appropriate to explore techniques of identifying the inputs of the existing models. A strong biological base is a key to good simulation and optimization analysis in renewable resource management. The surplus production models, though very simple, can be quite a good approximation of the complex dynamics. A continuous surplus biomass dynamics model is proposed for this analysis. The basic form of the mathematical equation is

$$\frac{dx}{dt} = g(x) - h \tag{3.17}$$

where $x(t)$ is the biomass at time t, $h(t)$ is the rate of depletion of the population due to human activities, e.g., commercial and recreational fishing, g is the natural additions to the biomass. Two functional forms of the net growth of the population will be investigated in this paper, i.e., the Schaefer logistic and the Gompertz functions will be used.

3.3.1 The net growth models

Two variants of the growth models are considered in this paper. Biological species grow by the gift of nature. The structure of their growth is quite complicated requiring sophisticated mathematical functions to adequately model them. Fortunately, there are simpler models that reasonably and approximately represent the intricate growth models. Two of the simplest parameterizations in fisheries management are

$$g(x) = \begin{cases} rx(1 - \frac{x}{K}) \\ rx\ln(\frac{K}{x}) \end{cases}$$

where x is as defined previously, r is the intrinsic growth rate per time (yr), K is the maximum population level of the biological species in kilo-tons. The first is the Schaefer logistic growth which is a special case of the modified logistic when the exponent is unity (Clark 1990, Haakon 1998) and the second is the Gompertz function. Note that the logistic function is a second order Taylor approximation of the Gompertz function.

The production function for a resource industry can be assumed to depend only on the stock biomass and the effort expended in fishing. The simplest form of the exploitation rate is the Gordon-Schaefer type of production function where the rate of removal of the stock is assumed to be linearly related to the effort and stock size. The coefficient of proportionality q in this case is called the catchability coefficient, i.e., $h = qex$, where e

is the fishing effort. For the present purpose, this simple linear model will be employed. That is, we apply a proportional fishing criterion in order to analyze the fishery.
Let $f = qe$ be the fishing mortality rate per uint time, then the simple rule takes the form

$$h(x) = fx \tag{3.18}$$

which implies that at any given level of the population a fraction f will be removed. The above formula (3.18) explicitly assumes exploitation of the species to the last fish. This is an oversimplification of the reality. However, it may serve as a good approximation of the complex system. For example, in the extreme situation where fishing becomes economically unprofitable or if on a purely ecological or social ground a moratorium is warranted, i.e., the fishery is closed which corresponds to f is set to zero. The fishing mortality parameter f is a policy instrument for the management authorities. It is a quite simple and an easy to use formula. Once accurate and reliable methods of stock assessments are available, the rule can be used to set quotas appropriate for the objective of the fishery.
Using the relation for h in (3.18) and (3.2), the biology of the stock is tied to the economics by the fishing mortality f. In Figure 3.1 below, we show plots of the growth functions using arbitrary values of the parameters. The values of the parameters r and K are the same for both the functions. A straight line curve with a slope equal to 0.407(see Ussif et al. 1999a) representing a linear in stock yield function is also shown (Note that the unit on the vertical axis is in kilo-tons).

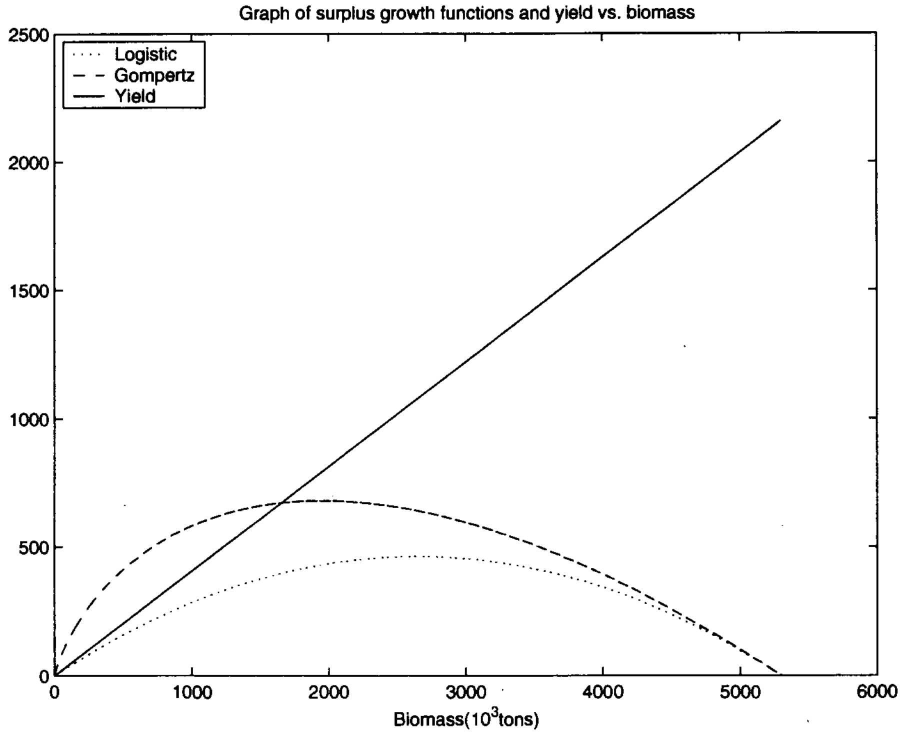

Figure 3.1: The growth models with r=.35, K = 5300. The vertical axis is measured in kilo-tons.

The graph of the logistic is symmetric about one half the carrying capacity (average)[3] while the Gompertz is asymmetric and is skewed towards the left. For the same K, the latter predicts lower MSY biomass (equal to (K/e), where $e \cong 2.71$ is the exponent operator and a corresponding higher MSY. In practical applications, the Gompertz function seems inappropriate for less resilient species. The combination of high MSY and low MSY biomass prescribed by this model can result in an unpardonable mistake on the side of management in case of recruitment failures.

3.4 An Application to NEACs

The NEACs is the most important demersal species along the coast of Norway and Northern Russia. This fishery has played an important economic role within the coastal

[3]The carrying capacity is assumed constant in this application which is another simplification in our models. It may be more realistic to allow it to vary with time.

communities for the past thousand years. The NEACs has for the past half century experienced large variations which result in a corresponding variation in the annual harvest quantities. The stock size fell from its highest level in 1946 of 4.1 million tons to the lowest in 1981 of 0.75 million tons. A time series plot of the history of the stock indicated a sign of recovery from its worst state in the mid 90's but recent reports show that the fishery is again in deep trouble (see Figure 3.2 below).

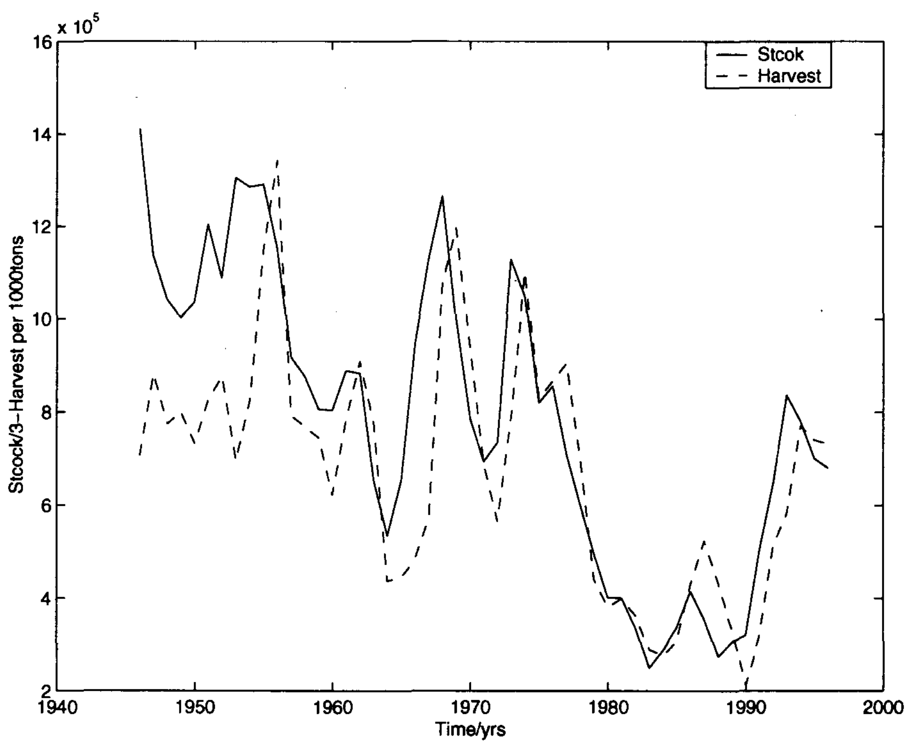

Figure 3.2: Graph of actual harvest and the stock biomass. The stock biomass is divided by a factor of three.

In this study, a time series of observations from 1946 to 1996 is used. The data was obtained from the International Commission for the Exploration of the Sea (ICES) report of the Arctic group (See Anon. 1998). The variational adjoint method is used to fit the hypothesized dynamics to the observations. The NEACs provides a good example to which the data assimilation method can be tested. To estimate the parameters, the intrinsic growth rate is assumed fairly known by fixing its value to 0.3499 (This value is obtained from Ussif et al. 1999b). One of the reasons why we are imposing such a

restriction is to reduce the number of the parameters. The justification is that the r is less sensitive of the two biological parameters. We then have two parameters i.e. K and f to estimate. The other parameters of the models are then estimated. The optimization was started by randomly generating reasonable initial guesses using a uniform random deviate intrinsic function. By seeding the generator, different initial guesses were used to check for the presence of local extrema. For all the experiments in this paper the convergence criterion for the optimization is $||\Delta\mathcal{J}||/||\Delta\mathcal{J}_1|| \leq 10^{-6}$ where $\Delta\mathcal{J}$ and $\Delta\mathcal{J}_1$ are the gradients of the current and initial points respectively and $||.||$ is the norm operator.
The performance of the algorithm is very impressive. Convergence was obtained in a few iterations in all the runs. The best fit parameters and the R^2 values are shown in the table 3.1 below.

Parameters	Logistic	Gompertz
r^*	0.3499	0.3499
K	5268.5	5499.99
f	0.4076	0.4964
R^2	0.550	0.529

Table 3.1: Model parameters for the biomass dynamics models. The units are (/year) for the r and f and kilo-tons for K.

The star in the table 3.1 means those values were restricted. The Schaefer logistic and the Gompertz functions tend to give plausible estimates. The fit to the data is quite good for both models with the logistic model explaining about 55.0% of the data while the Gompertz function explains about 53% of the data. It is observed that the estimates for the latter model are relatively higher than the former.
Plot of the actual biomass data and the solutions of the models using the estimated parameters shown below.

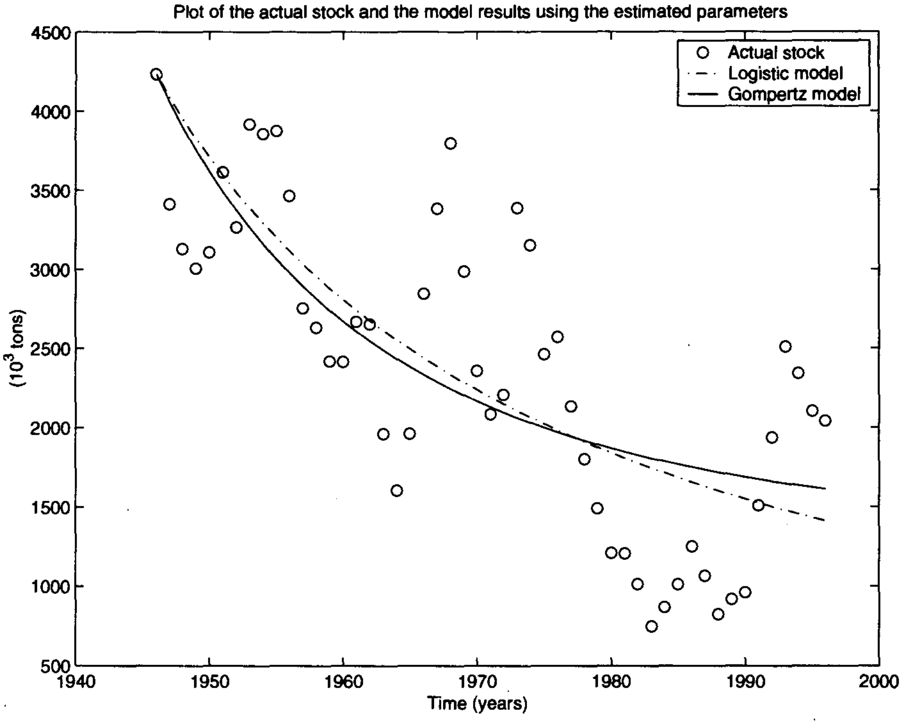

Figure 3.3: Plot of stock biomass vs. time

The two models have captured the general trend in the data. They fail to capture the stochastic component present in the data. Next, the growth functions are presented on the same graph with the actual harvest data. The goal is to show one of the findings of the paper. That is, the stock is exploited at an unsustainable rate leading to the alarming state of the fishery. Figures 3.4-3.5., show the plots of the actual harvest and growth curves against the biomass. The plus sign represents the actual harvest while the solid line represents the net growth curve. The logistic growth model predicts that the harvest rate has been persistently above the net growth curve see Figure 3.3 below. At the lower end of the graph, we notice that the actual harvest is close to the growth curve and is below it on a few occasions. One interesting observation is that several points tend to cluster around the maximum sustainable yield (MSY). This gives a more acceptable picture of the actual fishery.

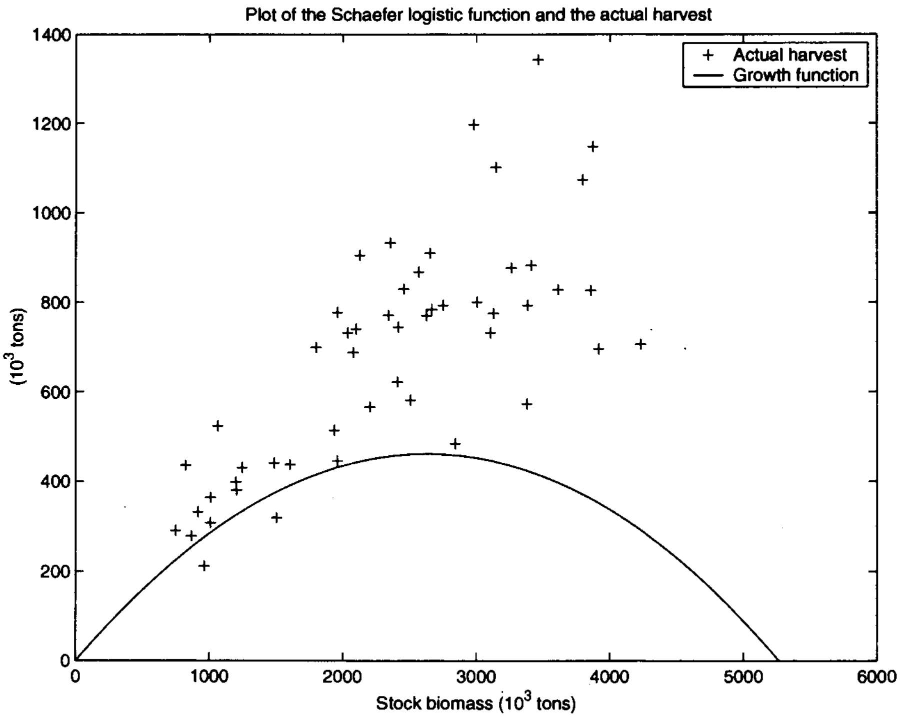

Figure 3.4: The logistic growth model.

The forecasts of the latter model, i.e., the Gompertz model, is quite similar to the predictions of the logistic model but appears to point to other factors for the recent troubles of the fishery rather than excessive harvesting of the stock (Figure 3.4). Several important fisheries have collapsed due to overexploitation (see Bjorndal and Munro 1998).

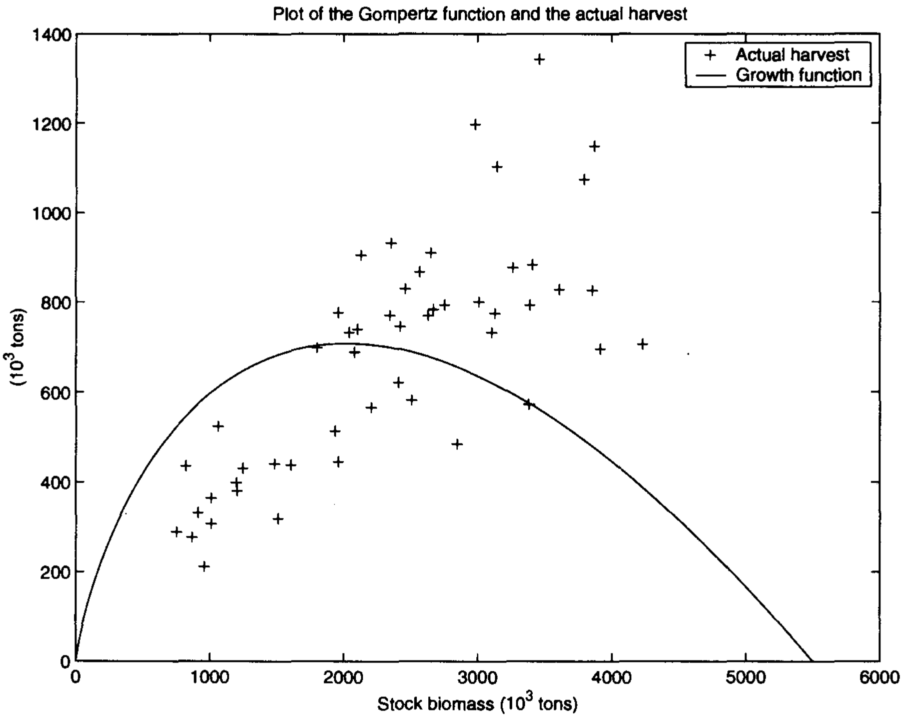

Figure 3.5: The Gompertz growth model.

To further discuss the results of the paper, we provide estimates that might be of considerable interest to managers of the NEACs. An important caveat however is that, while these values are quite reasonable, a direct translation of the results to the NEAC stock may not be advised.

The use of surplus growth functions implies there exist a certain level of biomass at which natural additions to the stock are greatest. This occurs at the extremum point of the concave growth functions. For each model an f exists that will direct the stock to the sustainable level. In the case of the Schaefer logistic, a simple algebra yields optimal fishing mortality rate for an MSY policy equal to one half the intrinsic growth rate ($f = r/2$) if the population is below the sustainable biomass level. The table 3.2 below shows some quantities of practical interest pertaining to the NEACs.

Parameters	Logistic	Gompertz
r^*	0.3499	0.3499
K	5268.5	5499.99
x_{MSY}	2634.25	2023.33
MSY	460.9	707.96

Table 3.2: Sustainable parameters for the two biomass dynamics models.

Estimates of x_{MSY} and MSY quantities are shown in rows 3 and 4 of table 3.2. The Schaefer logistic model seems to out perform its counterpart, i.e., the Gompertz model. It gave a lower MSY estimate but a higher value of optimum sustained biomass. These estimates are quite appealing and are more acceptable than the predictions of the Gompertz. The MSY for the Gompertz is around the values of TAC in the late 90's. The sustainable biomass level of around 2.0 million tons may be a bit low. However, it may not be advisable to completely discard the results from the Gompertz model since there are other important factors that may account for the troubles of the fishery. For instance, factors such as sea pollution and unfavorable weather conditions may be accountable for the recent sorry state of the NEACs stock.

3.4.1 Conclusions

The NEACs fishery is analyzed using an optimal control approach of dynamic model parameter estimation. Two alternative growth models are proposed and used in the analysis. The production relation for the fishery is assumed to be linear in the biomass and constitute a simple feedback rule. A quite restrictive assumption of constant fishing mortality is made which yields a proportional fishing policy. The model dynamic equation is nonlinear in the parameters and quadratic in the stock. A least squares criterion measuring the discrepancy between the data and its model equivalent was minimized subject to a dynamic constraint. The variational adjoint method is used to efficiently estimate the parameters. Parameter estimates from the Schaefer logistic and the Gompertz models are reasonably good. That is they are within acceptable range for the NEACs. Both models have about the same explanatory power $R^2 = .55$. This seems quite reasonable since the models were able to capture the trend in the data but failed to capture the periodic oscillations. It is obvious that the models are not sophisticated enough to explain the random events inherent in the system. Ecosystem effects and environmental variability are very important variables and ought to be included in the model. Predictions from these models are consistent with many recent experiences in fisheries and other natural resource stocks. Both the stock biomass and the amount harvested have been declining while fishing mortality is increasing due to technical innovations. More powerful boats are being developed and other advanced fishing equipments are available making the population more vulnerable to exploitation.

This paper has demonstrated the utility of the data assimilation methods in dynamic parameter estimation for two alternative resource models. It exposes the strengths and weaknesses of the simplified biomass dynamics models and provides model solutions that are in close agreement with the observations. The methods have numerous additional capabilities that are worth exploring in the future. Bioeconomists may find these methods indispensable if questions that interest managers most have to be answered and if more realistic models become readily available.

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Chapter 4

Estimation of biological and economic parameters of a bioeconomic fisheries model using dynamical data assimilation

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Estimation of biological and economic parameters of a bioeconomic fisheries model using dynamical data assimilation

Abstract

A new approach of model parameter estimation is used with simulated measurements to recover both biological and economic input parameters of a natural resource model. The procedure efficiently combines time series of observations with a simple bioeconomic fisheries model to optimally estimate the model parameters. Using identical twin experiments, it is shown that the parameters of the model can be retrieved. The procedure provides an efficient way of calculating poorly known model parameters by fitting model results to observations. In separate experiments with exact and noisy data, we have demonstrated that the variational adjoint technique of parameter estimation can be an efficient method of analyzing bioeconomic data. Results of sensitivity analysis of the model parameters show that the environmental carrying capacity (e.g., habitat) is the most important input parameter in the model.

4.1 Introduction

In this paper we use a very simple bioeconomic fisheries model to introduce a new technique of optimally estimating the parameters of a dynamic fisheries model and to demonstrate its potential usefulness. The approach is known as data assimilation. In data assimilation mathematical or numerical models are combined with available measurements in order to improve the model itself or to improve the model forecasts. The former application is known as model fitting (Smedstad and O'Brien 1991; Yu and O'Brien 1991; Lawson et al. 1995; Spitz et al. 1997). A variety of these techniques such as the Kalman Filter (Kalman 1960; Gelb 1974), the Extended Kalman Filter (Gelb 1974; Evensen 1994), Optimal Interpolation (Lorenc 1986) and the Variational Data Assimilation (Smedstad and O'Brien 1991) are already in common use. These techniques have extensively been used in areas such as groundwater hydrology and petroleum reservoirs (see for example Carrera and Neuman 1986(a,b,c); Yeh 1986) and more recently in ecosystem models (Spitz et al. 1997; Matear 1995). The technique in this paper is the so called variational adjoint parameter estimation. This method minimizes a preconstructed loss or criterion function which is defined by the differences between the data and the model forecasts. The optimal or best fit parameters are obtained by minimizing the loss function subject to the dynamical constraints via the so called adjoint equations which map the predefined loss function into the gradient with respect to the control parameters (Lawson et al. 1995; Spitz et al. 1997). The gradients are then used iteratively in a descent or Newton type of algorithm in order to search for the minimum of the loss function.

Application of these techniques is spreading very rapidly to many areas such as physical and biological systems (Spitz et al. 1997). Lawson et al. (1995) applied the technique of data assimilation to the well known and extensively studied predator-prey model in biology. In another application, Gauthier (1992) applied data assimilation with an adjoint model technique to the Lorenz model. Other techniques of data assimilation are the simulated annealing (Kirkpatrick et al. 1983; Kruger 1992) and the hybrid Monte Carlo techniques (Harmon and Challenor 1996). These are stochastic in nature and may be very costly to implement. Data assimilation is also widely used in meteorology and oceanography to estimate initial and boundary conditions. For reviews of these methods in meteorology and physical oceanography refer to (Bengtsson 1981; Navon (1986, 1997)).

Research in bioeconomics of fish stocks has been based on simple aggregated biological models (Schaefer 1954). Classical growth models such as the logistic and the exponential functions are the commonly employed models in the qualitative analysis of fish stocks (Sandal and Steinshamn 1997(a,b,c); Clark 1990).

The reality of these models has not yet been rigorously tested. Data assimilation gives us the opportunity to test these models using the available data which is assumed to be more realistic than the models themselves. More realistic bioeconomic models of renewable resource stock may be multidimensional and highly complex. They may contain many parameters such us the intrinsic growth rate, the catchability coefficient, and the environmental carrying capacity whose values are extremely difficult to measure. Parameterization of the models become mathematically untractable and impractical. As a consequence, biologists and bioeconomists have found it necessary to introduce simpler models. The issue of identification, i.e., the problem of estimating parameters so that the model predictions are more realistic and useful, has been raised by many natural resource economists (Clark 1990). However, adequate attention has not been given to the problem. Instead, economists have focused on the analytical considerations leading to a neglect of most of the vital questions resource managers/fishers are mostly concerned about, e.g., what is the safe biomass level (Deacon et al. 1998), etc.

Due to the progress in data collection and processing in recent times from both the fisheries biologists and economists and the advances in computer technology, techniques of data assimilation which are both data and computer intensive have good future prospects. The assimilation technique introduced in this paper has some advantages compared to the conventional techniques. First it is very attractive and computationally efficient to implement. Second, it can be used for more realistic and complex dynamical models of bioeconomic systems. Third, it can be used to adjust both the initial conditions and the parameters of the model. Fourth, it can also be used to estimate the variables of the dynamical model.

A variational adjoint formulation will be used in this paper as an alternative approach to the widely used regression analysis in economics. It is a more general formulation than the traditional regression. It can be used for linear and nonlinear models and is highly suitable for more realistic models where closed form solutions are unattainable. The methods can be used to simultaneously estimate a large number of parameters. It can also be used to perform sensitivity analysis of the input parameters of the model. Most of the optimization algorithms used in variational adjoint parameter estimation are iterative. Parameter estimation in dynamical systems is generally nonlinear even if the model is linear (Evensen et al. 1998). Consequently, the loss function may contain multiple extrema. Notice that as with all iterative techniques, one should expect problems with convergence to the global optimum due to flat regions in the domain of search. The technique introduced in this paper provides a more general and novel approach of incorporating information from field measurements into bioeconomic models. Primarily, the focus of the paper is to demonstrate the applicability of the methods in natural resource economics. The plan of this paper is as follows. First, the technique of data assimilation will be introduced and discussed. Second, the variational adjoint "strong constraint" (Sasaki 1970) method will be presented and the solution algorithm outlined. Third, the bioeconomic fisheries model will be presented and a sensitivity analysis performed to investigate the importance of the input parameters to the response. Fourth, identical twin experiments with clean and noisy data will be performed and the result discussed. Finally, we discuss and summarize the work. The mathematical details are relegated to the Appendix.

4.2 Data Assimilation

Variational adjoint technique determines an optimal solution by minimizing a loss function that measures the discrepancy between the model counterparts to the data and the available measurements. The method leads to the solution of the model equations that best fits the available measurements throughout the assimilation interval in a least squares sense. Data assimilation systems consist of three components: the forward model with a criterion function, the adjoint or backward model and an optimization procedure

(Lawson et al. 1995). The forward model is our mathematical representation of the system we are interested in studying, e.g., an open access, a regulated open access or a sole owner fishery.

The adjoint model consists of equations obtained by enforcing the dynamical constraints through Lagrange multipliers and provide a method of calculating the gradient of the loss function with respect to the control variables. The gradients are then used in a line search using standard optimization packages to find the minimum of the loss function. Most optimization routines are based on iterative schemes which require the correct computation of the gradient of the loss function with respect to the control variables. In the variational adjoint formulation, computation of the gradient is achieved through the adjoint equations forced by the model-data misfits. The model equations are run forward in time while the adjoint equations are run backward in time which is then used to calculate the gradient of the loss function.

An important step in data assimilation is the choice of the criterion function for the goodness of fit. The commonly used criterion is the generalized least squares criterion. It can be defined with no a priori information about the parameters or with prior information about the parameters incorporated as a penalty term in the criterion function. Some researchers argue that since some information about the parameters and their uncertainties are always available, adding the information is a plausible thing to do (Harmon and Challenor 1997; Evensen et al. 1998).

4.3 The Penalty Function

In variational adjoint parameter estimation a loss functional which measures the difference between the data and the model equivalent of the data is minimized by tuning the control variables of the dynamical system. The goal is to find the parameters of the model that lead to model predictions that are as close as possible to the data. A typical penalty functional takes the more general form

$$\mathcal{J}[\mathbf{X}, \alpha] = \frac{1}{2}\int_0^{T_f} (\mathbf{X} - \hat{\mathbf{X}})^T \mathbf{W} (\mathbf{X} - \hat{\mathbf{X}}) dt$$

$$+\frac{1}{2T_f}\int_0^{T_f}(\alpha-\alpha_0)^T\mathbf{W}_\alpha(\alpha-\alpha_0)dt \tag{4.1}$$

where $\hat{\mathbf{X}}$ is the observation vector, $\mathbf{X}$ is the model equivalent of the data, α_0 is first or best guess of the parameter vector and α is the parameter vector to be estimated. The period of assimilation is denoted by T_f and T is the transpose operator. The $\mathbf{W}$'s are the weight matrices which are optimally the inverses of the error covariances of the observations. They are assumed to be positive definite and symmetric. The second term in the penalty functional represents our prior knowledge of the parameters and ensures that the estimated values are not too far away from the first guess. It may also enhance the curvature of the criterion function by contributing a positive term $\mathbf{W}_\alpha$ to the Hessian (second order derivatives) of $\mathcal{J}$ (Smedstad and O'Brien 1991).

For this analysis, it will be convenient to assume that the errors are not serially correlated. This implies that the covariance matrices are now diagonal matrices with the variances along the diagonal. We further assume that the variances are constant. Then the weight matrices can be written as $\mathbf{W}=w\mathbf{I}$ and $\mathbf{W}_\alpha=w_\alpha\mathbf{I}_\alpha$ where the $\mathbf{I}'s$ denote unit matrices and the small lettered $w's$ are the weights. The discretized loss function becomes

$$\mathcal{J}=w\sum_{n=1}^{N}(\mathbf{X}_n-\hat{\mathbf{X}}_n)^T(\mathbf{X}_n-\hat{\mathbf{X}}_n)+w_\alpha\sum_{p=1}^{P}(\alpha_p-\alpha_{0p})^T(\alpha_p-\alpha_{0p}) \tag{4.2}$$

where P is the number of parameters and N is the number of observations. One special case is the choice of the weights equal to unity which leads to the least-squares procedure. The frequently made statistical assumption about the errors in the literature is that of normality. If this assumption is satisfied, then the optimal least squares estimators are the maximum likelihood estimators.

4.3.1 The variational adjoint method

Construction of the adjoint code is identified as the most difficult aspect of the data assimilation technique (Spitz et al. 1997). One approach consists of deriving the continuous adjoint equations and then discretizing them (Smedstad and O'Brien 1991). Another approach is to derive the adjoint code directly from the model code (Lawson et al. 1995; Spitz et al. 1997). For this analysis, it might be instructive to derive the adjoint equations using the first approach. Consider the forward model equation

$$\begin{aligned} \frac{\partial \mathbf{X}}{\partial t} &= F(\mathbf{X}, \alpha) \\ \mathbf{X}(0) &= \mathbf{X}_0 \\ \alpha &= \alpha_0 + \hat{\alpha} \end{aligned} \tag{4.3}$$

where $\mathbf{X}$ is a scalar or vector of model variables, $\hat{\alpha}$ represents vector of model parameter errors, F is a linear or nonlinear operator, and $\mathbf{X_0}$ is the vector of initial conditions. Formulating the Lagrange function by appending the model dynamics as a strong constraint

$$\mathcal{L}[\mathbf{X}, \alpha] = \mathcal{J} + \frac{1}{2}\int_0^{T_f} \mathbf{M}(\frac{\partial \mathbf{X}}{\partial t} - F(\mathbf{X}, \alpha))dt \tag{4.4}$$

where $\mathbf{M}$ is a vector of Lagrange multipliers which are computed in determining the best fit. The original constrained problem is thus reformulated as an unconstrained problem. At the unconstrained minimum the first order conditions are

$$\frac{\partial \mathcal{L}}{\partial \mathbf{X}} = 0 \tag{4.5}$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{M}} = 0 \tag{4.6}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha} = 0 \tag{4.7}$$

It is observed that equation (4.5) results in the adjoint or backward model, equation (4.6) recovers the model equations while (4.7) gives the gradients with respect to the control variables. Using calculus of variations or optimal control theory (see Appendix), the adjoint equation is

$$\begin{aligned} \frac{\partial \mathbf{M}}{\partial t} + [\frac{\partial F}{\partial \mathbf{X}}]^T \mathbf{M} &= \mathbf{W}(\mathbf{X} - \hat{\mathbf{X}}) \\ \mathbf{M}(T_f) &= \mathbf{0} \end{aligned} \tag{4.8}$$

and the gradient relation is

$$\Delta_{\alpha \mathcal{J}} = -\int_0^{T_f} [\frac{\partial F}{\partial \alpha}]^T \mathbf{M} dt + \mathbf{W}_\alpha(\alpha - \alpha_0) \tag{4.9}$$

The term on the RHS of (4.8) is the weighted misfit which acts as forcing term for the adjoint equations. It is worth noting here that, we have implicitly assumed that data is continuously available throughout the integration interval. Equations (4.3) and (4.8) above constitute the Euler-Lagrange (E-L) system and form a two-point boundary value problem. For details of the derivation see the Appendix.
The implementation of the variational adjoint technique on a computer is outlined below.

- Choose the first guess for the control parameters.
- Integrate the forward model over the assimilation interval.
- Calculate the misfits and hence the loss function.
- Integrate the adjoint equation backward in time forced by the data misfits.
- Calculate the gradient of $\mathcal{L}$ with respect to the control variables.
- Use the gradient in a descent algorithm to find an improved estimate of the control parameters which make the loss function move towards a minimum.
- Check if the solution is found based on a certain criterion ($e.g. \|\Delta\mathcal{J}\| \leq \epsilon$).
- If the criterion is not met repeat the procedure until a satisfactory solution is found.

The optimization step is performed using standard optimization procedures. In this paper, a limited memory quasi-Newton procedure (Gilbert and Lemarechal, 1991) is used. The success of the optimization depends crucially on the accuracy of the computed gradients. Any errors introduced while calculating the gradients can be detrimental and the results misleading. To avoid this incidence from occurring, it is always advisable to verify the correctness of the gradients (see, Smedstad and O'Brien 1991; Spitz et al. 1997). Verification of the adjoint code is performed by a simple Taylor series expansion of the penalty functional about a certain vector of the control variable $\mathbf{x}$, i.e.,

$$\mathcal{J}(\mathbf{x} + \gamma\mathbf{u}) = \mathcal{J}(\mathbf{x}) + \gamma\mathbf{u}^T\Delta_{\mathbf{x}}\mathcal{J}(\mathbf{x}) + O(\gamma^2) \tag{4.10}$$

where γ is a small scalar, $\mathbf{u}$ is an arbitrary vector (see, Smestad and O'Brien 1991; Lawson et al. 1995). Defining a function $\Phi(\gamma)$ and rewriting (4.10) we have

$$\Phi(\gamma) = \frac{\mathcal{J}(\mathbf{x} + \gamma\mathbf{u}) - \mathcal{J}(\mathbf{x})}{\gamma\mathbf{u}^T\Delta_{\mathbf{x}}\mathcal{J}(\mathbf{x})} = 1 + O(\gamma) \tag{4.11}$$

Hence, in the limit as γ approaches zero, the value of $\Phi(\gamma)$ approaches unity. If the scalar γ is small, the values of the function $\Phi(\gamma)$ will be approximately equal to one.

4.4 The Bioeconomic Model

This section presents the bioeconomic fisheries model. It is an aggregated or lumped parameter model of a single cohort or year-class (Clark 1990). The population dynamics of the fishery will be modeled as

$$\frac{dx}{dt} = f(x) - h \tag{4.12}$$

where $h(t)$ is the harvest rate from the stock, $x(t)$ is the stock biomass, $f(x)$ is the growth function which for this analysis we will use the simple logistic model. That is $f(x) = rx(1 - x/K)$ where r and K are positive constants called the intrinsic growth rate and the environmental carrying capacity respectively. The biological model specified above depends on two parameters (r, K). These parameters are little known in the scientific community for most fish stocks around the world. Accurate measurements of their values are difficult if not impossible. Statistical estimation methods were devised by (Schaefer 1967). This paper has similar goals of devising mathematical methods of estimating the parameters of the model equations (differential equations), testing the realism of the models against the observational data and subsequently improving the models.

The economics is also simple in this analysis. A hypothetical sole owner is envisaged. The goal of the management is to maximize a given discounted net economic function

$$max \int_0^{T_f} e^{-\delta t}\pi(h)dt \tag{4.13}$$

where $h(t)$ is the control variable, T_f is the time horizon which may be finite or infinite and $\pi(h)$ is a rescaled or normalized economic function given by

$$\pi = ah - h^2 \tag{4.14}$$

where a is an economic parameter to be estimated. The objective function is assumed to depend explicitly only on the harvest rate from the stock. This could be commercial, social or even political. For a commercially managed fishery, the objective may be maximization of discounted rents accruing from the exploitation of the biological species. A socially oriented manager may optimize the total utility or aim at conserving the stock. As oppose to these two objectives, a politically motivated goal may be to protect peoples jobs.

In the above formulation, δ is the discount rate which has been commonly set to the current interest rate. It may however be plausible to estimate the value of the discount rate from available data. That is instead of pre-setting the value, we allow the data to tell us what its value is for the fishery we are modeling. Theoretically, the optimum equilibrium conditions are met if the marginal productivity of the resource in question is equal to the discount factor or the social time-preference. However, this may not be the case in a real-world scenario. There are many convincing reasons why one will think that the managers time-preference is significantly different than the market interest rate. Another important input in the analysis of resource models is the time horizon. In this paper, the time horizon is a free parameter. It can be preset exogenously or it can be determined as an endogenous parameter in the problem. Hence, in spite of the quite restrictive form of the performance index in this paper the control problem is to some extent quite general. Various special cases can be formulated and discussed. The case considered here is a fixed horizon control problem.

Maximizing the above problem (4.13) subject to the natural constraint is a nonlinear optimal control problem (Clark and Munro 1970). The technique has been discussed by several authors (e.g., Kamien and Schwartz 1981)

Application of maximum principles to the above problem yields the following two system of coupled nonlinear ODEs (see Clark 1990; Appendix).

$$\frac{dx}{dt} = rx(1-\frac{x}{K})-h \tag{4.15}$$
$$\frac{dh}{dt} = -0.5(a-2h)(\delta-r(1-\frac{2x}{K})) \tag{4.16}$$

Analytical solutions of the system of nonlinear ODEs are in general unobtainable. However, approximate numerical solutions of these equations are easy to obtain if the necessary initial and boundary conditions are specified. In reality however, the initial conditions that lead to the separatrix solution are not precisely known a priori. Given a set of data, a solution could be found by either manually or automatically tuning the poorly known initial and/or boundary conditions as well as the free parameters of the model until a satisfactory solution is obtained. This could easily and efficiently be achieved by the use of the data assimilation technique introduced in this paper.

Identical twin experiments will be used throughout the analysis. That is data will be generated from the model itself using Monte Carlo simulations. This guarantees that the model and the data are consistent and serves as a good test of the varitional adjoint algorithm. A moderate objective of the paper is to demonstrate the usefulness of this approach in resource modeling, hence the use of this simple model which is fairly known and used in bioeconomic fisheries analysis. To carry out the experiments, reasonable parameter values will be chosen for the model and a variational adjoint parameter estimation technique will be used to recover the parameters.

4.4.1 Equilibrium analysis

The equilibrium behavior of the simple fishery model in this paper is studied. For an infinite horizon problem, the optimal path for the dynamics is the separatrix leading to the equilibrium point. At the optimal equilibrium, the following conditions hold: $\frac{dx}{dt}=0$ and $\frac{dh}{dt}=0$ which implies that $rx^*(1-\frac{x^*}{K})-h^*=0$ and $-0.5(a-2h^*)(\delta-r(1-\frac{2x^*}{K}))=0$. This gives rise to these equilibrium points: ($h^*=a/2$, $h^*=rx^*(1-x^*/K)$) and ($\delta=r(1-2x^*/K)$, $h^*=rx^*(1-x^*/K)$). Notice that the first equilibrium is a bliss point (i.e., point at which the benefits are maximum) and most rewarding from the economic viewpoint if achievable. It is the theoretical optimum for the quadratic benefit function defined above.

- Case 1. For the case of the bliss point the optimal equilibrium harvest and stock are $h^* = a/2$ and $x^* = \frac{2r \pm \sqrt{4r^2 - \frac{8ar}{K}}}{4r/K}$, where the term under the square root sign is restricted to be nonnegative. Many interesting observations are made here. Notice here that when the term under the square root sign is zero, we recover the maximum sustainable yield (MSY) optimum $x^* = K/2$. If the square root term is large compared to $2r$, we have a single equilibrium. However, if the square root term is small compared to $2r$ two different values exist for the equilibrium stock and the larger may be preferred. A larger standing equilibrium stock leads to higher sustained yield for the capital asset.

- Case 2. This case leads to the equilibrium biomass $x^* = K(1 - \delta/r)/2$. The term δ/r, i.e., the ratio of the discount factor to the growth rate, is what is referred to as the bionomic growth ratio (Clark 1990). Setting $\delta = 0$ yields the MSY policy as usual. The optimal equilibrium harvest h^* is obtained by simply substituting the x^* into the equilibrium stock-harvest relation above. The second case seems to be embedded in case one if $\delta = \sqrt{r^2 - 2ar/K}$.

To illustrate the technique, we use the population dynamic model with the popular logistic growth function as in Sandal and Steinshamn (1997) and Homans and Wilen (1997). We choose the biological parameters based on the views of some scientists for the North East Arctic Cod stock (NEACs). For this biological species a value between 0.2 and 0.5 per year for the growth rate and between 5000 and 7000 Kilo-tons for the carrying capacity are considered reasonable. Immediately after World War II, biologists have estimated the stock biomass for NEACs to be at about 4230 Kilo-tons which is believed to be greater than the x_{MSY}.
For the ensuing analysis, we assume that $r = .35$ per year and $K = 6000$ Kilo-tons. We also assume that the management objective is the maximum sustainable yield (MSY) policy in most parts of the paper. Using these values the $x_{MSY} = K/2 = 3000$ Kilo-tons and the corresponding $y_{MSY} = rK/4 = 525$ Kilo-tons. From case one, $a = 2y$ implies

$a = 1050$ Kilo-tons. To get simulation results which are of the same order of magnitude as in the case of NEACs values, we use the hypothetical stock values of 2300 Kilo-tons and 10% of the stock for the harvest rate as the initial conditions of our model.

4.4.2 Scaling

The performance of an optimization algorithm generally depends on the particular choice of the variables of the problem. Change of variables is usually recommended in practical applications (Luenberger 1984; Yu and O'Brien 1995). To study the effect of different choices of the parameter scales we rescale the variables by introducing the following transformation: Let $x = \rho z$ and substituting in

$$\begin{aligned} \frac{dx}{dt} &= rx(1 - \frac{x}{K}) - h \\ \frac{dh}{dt} &= -0.5(a - 2h)(\delta - r(1 - \frac{2x}{K})) \end{aligned}$$

yields :

$$\begin{aligned} \frac{dz}{dt} &= rz(1 - \frac{z}{k}) - y & (4.17) \\ \frac{dy}{dt} &= -0.5(a' - 2y)(\delta - r(1 - \frac{2z}{k})) & (4.18) \end{aligned}$$

where: $z = x/\rho$, $y = h/\rho$ and $a' = a/\rho$. The scaling factor ρ will be allowed to vary and the effect studied later in the paper. The small k is the normalized carrying capacity given by $k = K/\rho$. It must be noticed that the quantities are now nondimensional. It may easily be checked by dimensional analysis.

4.4.3 Sensitivity analysis

Models of physical, biological and economic systems have input parameters on which their predictions depend. Some of these parameters are more important than others. Sensitivity is a measure of the effect of changes in the given input parameter on a model solution. It quantifies the extent that uncertainties in parameters contribute to uncertainties in the model results (Navon 1997). Several analytical techniques of sensitivity analysis exist. The approach here is numerical method of calculating sensitivity of model response to input parameters. Let the quantity X depend on a parameter α. Then the absolute S_a and relative S_r sensitivities with respect to α can be defined respectively as

$$S_a = \frac{\partial X(\alpha)}{\partial \alpha} \tag{4.19}$$

and

$$S_r = \frac{\partial X(\alpha)}{\partial \alpha} \frac{\alpha}{X(\alpha)} \tag{4.20}$$

S_r is a nondimensional quantity which has the interpretation of percentage change in the output due to 1 percent change in the input parameter. To economists, equation (4.20) is an equivalent measure of elasticity. Input parameters will be perturbed from their true values and the model integrated over a given time horizon. Sensitivity of the output to the discount factor has not been studied in this paper. Its effect has previously been studied by, among others, (Sandal and Steinshamn 1997c). The results are graphed and discussed.

Figure 4.1. is a plot of the unperturbed solution and the solutions with each of the parameters reduced by 1% of their values. Results of the sensitivity analysis show that the model solution is sensitive to all the biological and economic parameters. The growth rate appears to have least effect on the model outputs while the carrying capacity has most influence (see Figure 4.1.).

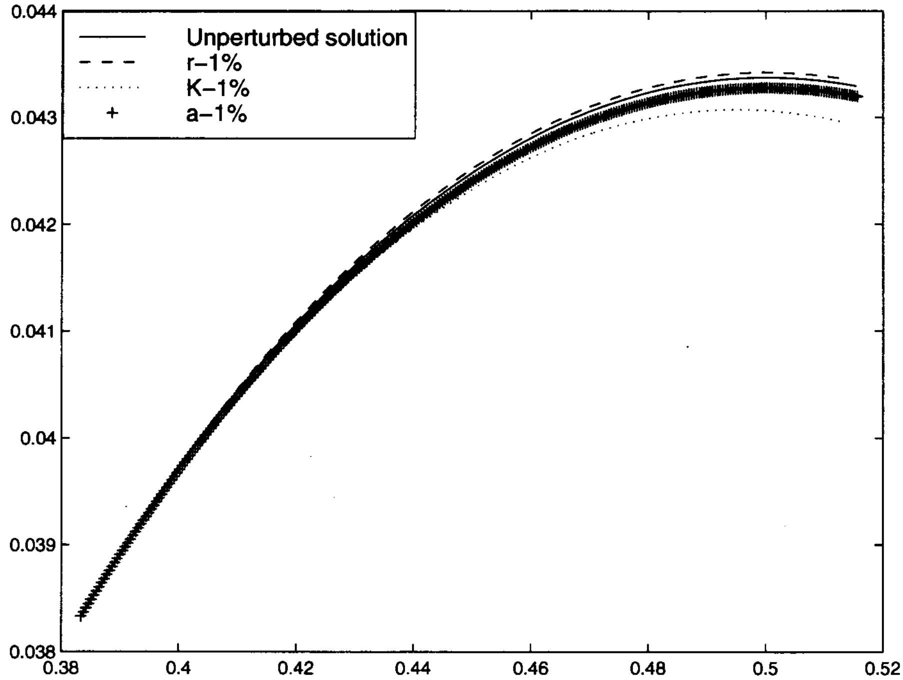

Figure 4.1: Phase plane plots of the sensitivity analysis.

It is observed that perturbing the parameter by larger values (10%) results in large distortion of the solution. Hence, care must be taken when choosing the initial guesses in a parameter estimation.

4.4 The Twin Experiments

As a first step we use artificially generated data (twin experiments) to test the performance of the data assimilation method. This will avoid any inconsistencies between the data and the model. First clean or exact data, i.e., data without measurement errors, will be simulated by integrating the model equations from known initial conditions and the parameters whose values are to be recovered. Second, stochastic errors will be introduced using random number generators from standard packages. The model will be solved and a certain level of noise added to the "true" solution. The data is assumed to be related to the model in a linear fashion, i.e, $\hat{x}_n = x_n + \epsilon_n$ where ϵ_n denotes the stochastic error term. In all the experiments to be carried out, we assume that data is available at every grid point. This might not be the case however in practice as data is sparse in time. For an actual fishery, biological and economic time series of observations may be available only on an annual basis.

4.4.1 Experiment with clean data

Several experiments were performed using data without any type of errors. That is, the exact solution of the model is used as the measurements. We investigate the effects of adding a priori knowledge of the parameters in the formulation of the penalty functional. Two results are shown here. Figure 4.2. is the plot of the parameters against the number of simulations for the case where we assume no a priori information, i.e., complete ignorance about the parameters. In Figure 4.3., we incorporated some information about the parameters and their uncertainties. Recovery of the model parameters was achieved in a few iterations depending on the initial guess. The convergence of the minimization procedure did not seem to depend very much on the initial guess of the parameters as long as they are fairly reasonable guesses. The graph of the normalized loss function shows a sharp decrease in its value. In about 2 iterations the value of the loss function fell to about 5% of the initial value. Not surprisingly, its value vanished at the optimum. This is expected in a twin experiment with a perfect data.

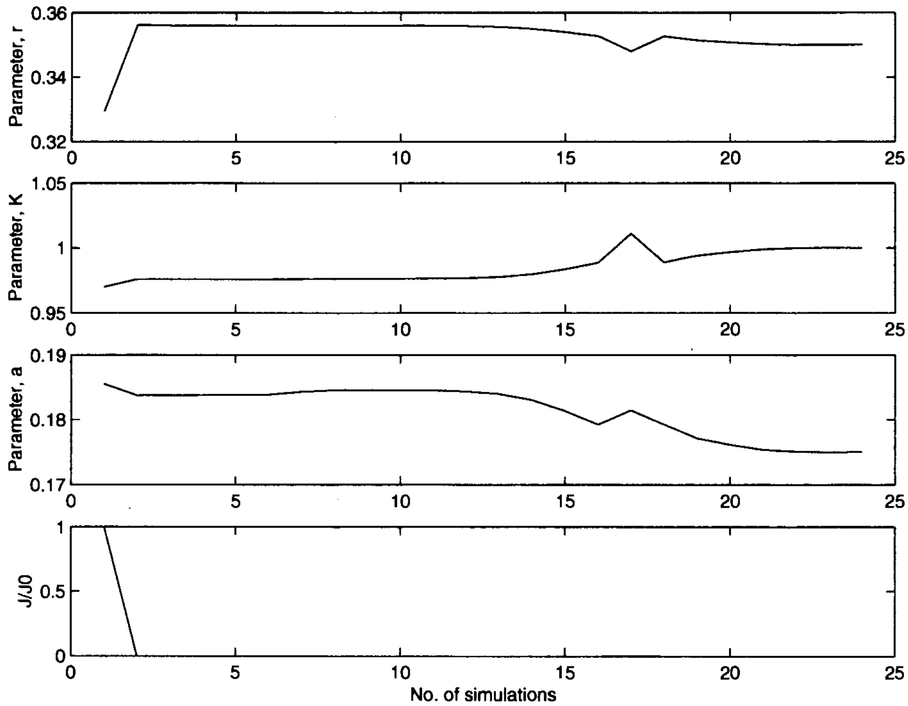

Figure 4.2: Plot of the parameters and the normalized loss function vs. no. of simulations: No penalty on parameters.

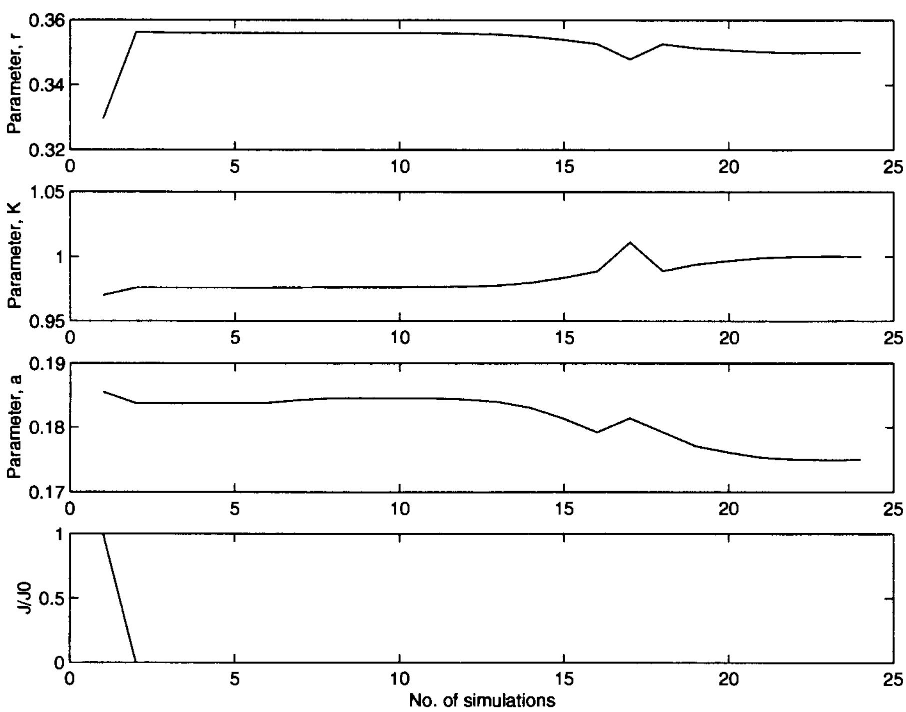

Figure 4.3: Plot of the parameters and the normalized loss function vs. no. of simulations: With penalty on parameters.

The conclusion drawn from the two cases with penalty and without is that, once the penalty is included, it becomes incumbent to have reasonably good prior (best) guesses of the parameters or their uncertainties or both. In fact, in this experiment, we encountered problems with recovering the parameters if our best guesses were a bit away and the weights relatively close to the data weights. If the weights were correctly chosen, the gains compared to the first case with no information on penalty were not very substantial. An initial best guess of 10% from the true parameters required data-to-parameter weight ratio of at least 10^6 for convergence.

4.4.2 Experiment with noisy data

For this experiment, the data is contaminated with normally or Gaussian mean zero and constant σ^2 variance $N(0, \sigma^2)$ distributed random errors. Similar experiments as in the case with perfect data were performed. We have made several runs to study various effects of adding different level of random noise to the solution of the model equations. With no penalty on the parameters, estimates were relatively further away from the "true" ones. Convergence to the "true" solution was quite difficult signifying the existence of local minima as the level of noise becomes significant. Very accurate initial guesses were required. This does not, however, indicate any serious setback in the variational adjoint parameter estimation. The aim of the assimilation is to adjust the free parameters such that the model predictions are as close as possible to the data. What happens is that the data is given large weight if there is no penalty on the parameters. Including prior information on the parameters results in better recovery of the parameters if the best guesses were quite good. The most troubling aspects of the experiment with a penalty on the parameter is the choice of the weights. Plots of the experiments are shown in the figures below. Figure 4.6. shows the simulated data and the best fit solutions. The level of noise added to the model solution to some extent affects the rate at which the parameters are recovered as well as the accuracy of the estimates.

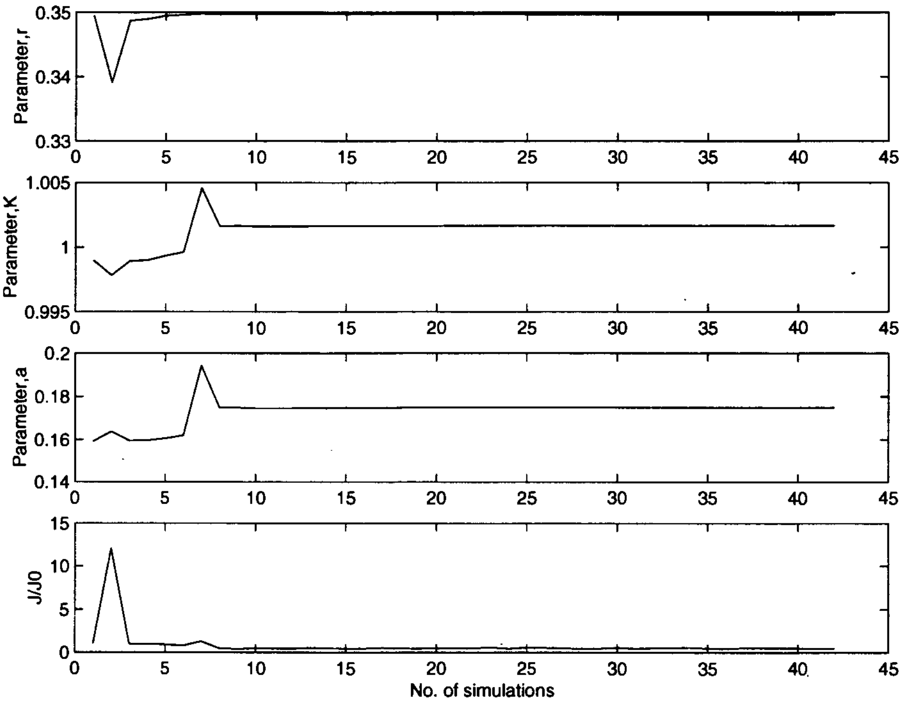

Figure 4.4: Plot of the parameters and the normalized loss function vs. no. of simulations: No penalty on parameters.

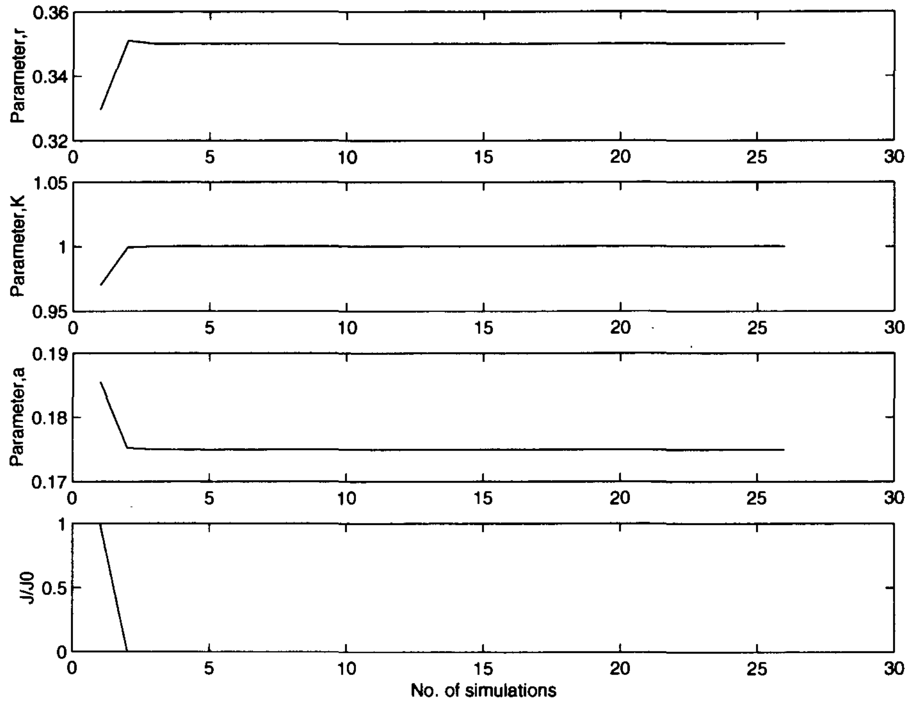

Figure 4.5: Plot of the parameters and the normalized loss function vs. no. of simulations: With penalty on parameters.

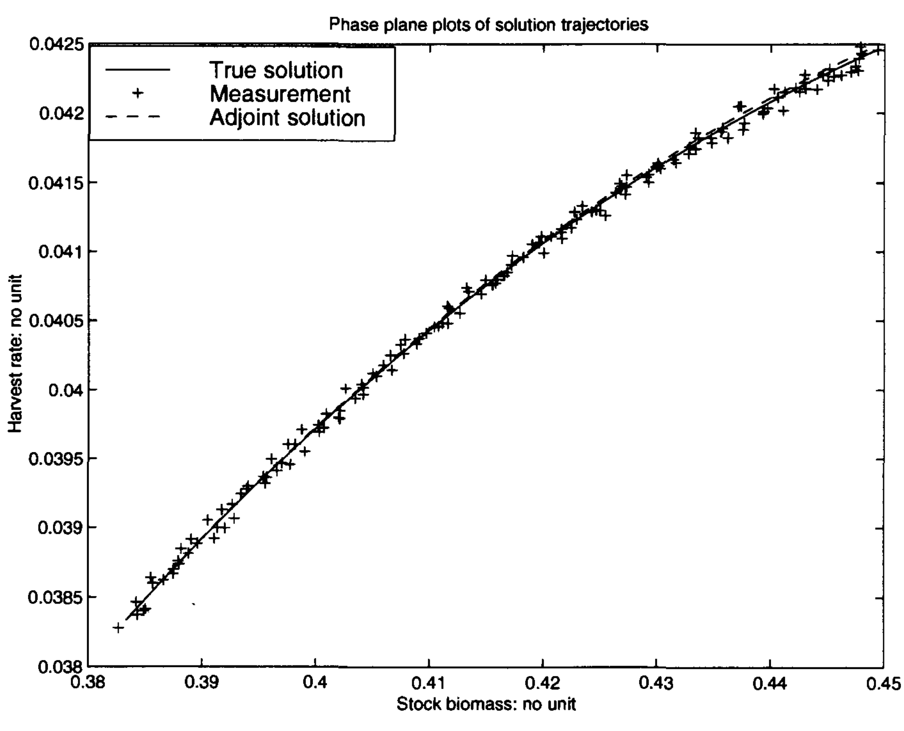

Figure 4.6: Plot of the harvest rate vs. biomass.

4.4 Summary and Conclusion

This work to our best knowledge represents the first attempt to explore the highly advanced and attractive method of parameter estimation in resource economics. It serves as a link between data and the model formulation. The basic idea is to find parameters of a dynamic fisheries model which yield model results that are as close as possible to the observed quantities. To achieve this goal a loss function measuring the distance between the model results and the data was predefined and the parameters estimated through an iterative process using the variational adjoint method. In the variational adjoint technique the model is assumed to hold exactly "strong constraint formalism" and the free parameters of the model are adjusted until the model predictions are as close as possible to the observations. A gradient search technique was used to iteratively explore the parameter space in order to find the minimum of the loss function.

In a few experiments we have demonstrated the utility of the variational adjoint technique in recovering model parameters such as the growth rate of a bioeconomic model. These parameters are vital in understanding the dynamics of the exploited species which can lead to a more accurate and realistic management policies. The results of the dual experiments show that model parameter sets can easily be estimated using both the information from measurements and the model formulation. In the experiment with clean data, parameters were recovered to within several orders of magnitudes in a few iterations.

The outcome of the sensitivity studies shows the relative importance of the input parameters. It is observed that the environmental carrying capacity is the most important parameter, i.e., small uncertainty in its value leads to large uncertainty in the output of the model. For an effective MSY policy, it is important to know very precisely the value of K. For most animal species around the world, the carrying capacity has been diminishing overtime. Implementation of MSY policy will require accurate and revised estimates of K regularly. It may thus be tempting to think that one of the reasons why MSY policy has not been very fruitful is due to lack of knowledge of the K, which means our estimate(s) of the sustainable stock biomass is not dependable and hence the MSY. From the little experience gathered, it may be advised that a sensitivity analysis be performed prior to parameter estimation.

To conclude, the paper has accomplished the following. A novel approach to dynamic model parameter estimation method has been introduced with reasonable degree of success using a simple bioeconomic model. The idea here is quite unique in a sense that both biological and economic theory were combined in the modeling of the resource system. In this case the parameter estimates are optimal compared to the situation where either a biological or an economic theory alone is used. Its potential in handling nonlinear models is also demonstrated and thus it is important to explore more of the capabilities of the method in future.

APPENDIX A

A.1 Mathematical Derivations

This section is reserved for the more technical aspects of the paper. First, it will present the mathematical formulation of the bioeconomic model pertaining to the optimal management of the stock in a sole owner context. Second, derivation of the adjoint model code will be presented in detail.

A.2 Derivation of the bioeconomic model

A more general bioeconomic model is formulated. The objective and the growth functions are assumed to depend on the stock biomass, the harvest and explicitly on time. This will allow us to derive several special cases as they apply to different fisheries. Consider the problem

$$max \int_0^{T_f} \pi(x,h,t)dt \tag{A.1}$$

subject to

$$\frac{dx}{dt} = F(x,h,t) \tag{A.2}$$

$$x(0) \;=\; x_0 \tag{A.3}$$

Defining the current value Hamiltonian function $H(x,h,m,t)$

$$H = \pi(x,h,t) + mF(x,h,t) \tag{A.4}$$

where $m(t)$ is the current value multiplier. Assuming an inner solution, the first order conditions are:

$$\begin{aligned} \frac{dH}{dh} &= \pi_h + mF_h = 0, \\ \frac{dm}{dt} &= \delta m - H_x \end{aligned} \tag{A.5}$$

From (A.3) we have

$$\frac{dm}{dt} = -\frac{\pi_{hh}\dot{h} + \pi_{hx}\dot{x} + \pi_{ht}}{F_h} + \frac{\pi_h(F_{hh}\dot{h} + F_{hx}\dot{x} + F_{ht})}{F_h^2} \tag{A.6}$$

equating (A.3) and (A.4), and rearranging yields

$$\begin{aligned} \frac{dh}{dt} &= \frac{\delta\pi_h F_h - (\pi_x + (-\frac{\pi_h}{F_h})F_x)F_h^2}{\pi_{hh}F_h - \pi_h F_{hh}} \\ &- \frac{(\pi_{hx}F_h - \pi_h F_{hx})(F-h) + (\pi_{ht}F_h - \pi_h F_{ht})}{\pi_{hh}F_h - \pi_h F_{hh}} \end{aligned} \tag{A.7}$$

Equations (A.2) and (A.7) constitute an n-dimensional dynamical systems.

APPENDIX B

B.1 Derivation of the adjoint model

The adjoint equations are derived by forming the Lagrange functional via the undetermined multipliers. The Lagrange function is

$$\mathcal{L}[\mathbf{X},\alpha] = \mathcal{J} + \int_0^{T_f} \mathbf{M}(\frac{\partial \mathbf{X}}{\partial t} - F(\mathbf{X},\alpha))dt \tag{B.1}$$

Perturbing the function $\mathcal{L}$

$$\begin{aligned}\mathcal{L}[\mathbf{X}+\delta\mathbf{X},\alpha] &= \mathcal{J}[\mathbf{X}+\delta\mathbf{X},\alpha] \\ &+ \int_0^{T_f} \mathbf{M}(\frac{\partial(\mathbf{X}+\delta\mathbf{X})}{\partial t} - F(\mathbf{X}+\delta\mathbf{X},\alpha))dt\end{aligned} \tag{B.2}$$

$$\begin{aligned}\mathcal{L}[\mathbf{X}+\delta\mathbf{X},\alpha] = \mathcal{J} + \Delta_{\mathbf{X}}\mathcal{J}\delta\mathbf{X}^T + \int_0^{T_f}\mathbf{M}(\frac{\partial\mathbf{X}}{\partial t} - F(\mathbf{X},\alpha))dt \\ -2\int_0^{T_f}\mathbf{M}(\frac{\partial\delta\mathbf{X}}{\partial t} - \frac{\partial F}{\partial\mathbf{X}}\delta\mathbf{X}^T)dt + O(\delta\mathbf{X}^2)\end{aligned} \tag{B.3}$$

Taking the difference $(\mathcal{L}[\mathbf{X}+\delta\mathbf{X},\alpha] - \mathcal{L}[\mathbf{X},\alpha])$

$$\delta\mathcal{L} = \Delta_{\mathbf{X}}\mathcal{J}\delta\mathbf{X}^T - 2\int_0^{T_f}\mathbf{M}(\frac{\partial\delta\mathbf{X}}{\partial t} - \frac{\partial F}{\partial\mathbf{X}}\delta\mathbf{X}^T)dt + O(\delta\mathbf{X}^2) \tag{B.4}$$

Requiring that $\delta\mathcal{L}$ be of order $O(\delta\mathbf{X}^2)$ implies

$$\Delta_{\mathbf{X}}\mathcal{J}\delta\mathbf{X}^T - 2\int_0^{T_f}\mathbf{M}(\frac{\partial\delta\mathbf{X}}{\partial t} - \frac{\partial F}{\partial\mathbf{X}}\delta\mathbf{X}^T)dt = 0 \tag{B.5}$$

By integrating the second term of the LHS by parts and rearranging, we have

$$\begin{aligned}\frac{\partial\mathbf{M}}{\partial t} + [\frac{\partial F}{\partial\mathbf{X}}]^T\mathbf{M} &= \mathbf{W}(\mathbf{X}-\hat{\mathbf{X}}) \\ \mathbf{M}(T_f) &= \mathbf{0}\end{aligned} \tag{B.6}$$

which are the adjoint equations together with the boundary conditions.

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Chapter 5

On the dynamics of commercial fishing and parameter identification

Submitted

On the dynamics of commercial fishing and parameter identification

Abstract

This paper has two main objectives. The first is to develop dynamic models of commercial fisheries different from the existing models. The industry is assumed to have a well defined index of performance based on which it either invests or otherwise. We do not however, assume that the industry or firm is efficient or optimal in its operations. The hypotheses in the models are quite general, making the models applicable to different management regimes. The second is that a new approach of fitting model dynamics to time series data is employed to simultaneously estimate the poorly known initial conditions and the parameters of the nonlinear fisheries dynamics. The approach is a data assimilation technique known as the variational adjoint method. Estimation of the poorly known initial conditions is one of the attractive features of the variational adjoint method. Unlike the conventional methods, the method employed in this paper, requires relatively less data. Economic parameters were reasonably estimated without cost and price data. The estimated equilibrium biomass is very close to the maximum sustainable biomass which means open access in this case led to economic overfishing but not biological overfishing.

Keywords: Data assimilation; variational adjoint method; index of performance; non-linear dynamics; open access

JEL classification:Q22,C51

5.1 Introduction

The most common approaches of modeling the dynamics of a natural resource system are by the routine application of the sophisticated techniques of the calculus of variations or optimal control theory and dynamic programming (Kamien and Schwartz 1984; Clark 1990). The economic theory of an optimally managed fishery has been advanced by many researchers. Clark (1990) discussed various models in some detail. Sandal and Steinshamn (1997a,b,c) made some of the most recent contributions in the area. These frameworks explicitly assume that agents are optimal and efficient. However, most real world fisheries have historically not been optimally managed.

The dynamics of single species models have extensively been studied in the literature of natural resource economics (Sandal and Steinshamn, op. cit.). Extensions have also been made to include ecological effects from other species. The simplest is the predator-prey model (see Clark 1990).

Commercial models of fisheries have previously been discussed by Crutchfield and Zellner (1962) and Smith (1969). The latter provided a model of theoretical nature which transforms specific patterns of assumptions about cost conditions, demand externalities and biomass growth technology into a pattern of exploitation of the stock. Smith also discussed the three main features of commercial fishing and mentioned the various types of external effects representing external diseconomies to the industry. In two earlier papers, Gordon (1954) and Scott (1955) noted that all of these externalities arise fundamentally because of the unappropriated "common property" character of most ocean fisheries (Smith 1969).

In this paper, we develop some commercial fishing models that do not necessarily assume optimal behavior of fishers. The goal is to develop models that are quite general and have much wider possible applications. Models of natural resource exploitation consist of two vital components. First, a sound biological base which defines the environmental and ecological constraints is required. Second, an economic submodel that incorporates the basic characteristics of the exploiting firms must be in place. For example, an industry or a firm may be assumed to vary levels of capital investments in proportion to some measurable quantities such as the total profits (Smith 1969; Clark 1979).

This paper also focuses on a very important aspect of fisheries management that has largely been ignored. Deacon et al. (1998) noted that much of the information managers need is empirical, i.e., measurements of vital relationships and judgments about various impacts. This area of the economics of fishing has not been adequately explored by economists probably due to lack of data and computational power in the past. Much of the research efforts were used in the search for qualitative answers to management problems.

This paper employs a new and efficient method of advanced data assimilation known as the variational adjoint technique (Smedstad and O'Brien 1991) to analyze real fisheries data. In data assimilation, mathematical or numerical models are merged with observational data in order to improve the model itself or to improve the model predictions. The former application is known as model fitting. Using the variational adjoint technique, in which the model dynamics are often assumed to be perfect, i.e., the dynamical constraints are satisfied exactly (Sasaki 1970), appropriate initial conditions and parameters of the nonlinear fisheries dynamics are estimated. Nonlinear fisheries dynamics are highly sensitive to the initial conditions and the parameters which are often exogenously given inputs to the system. These inputs are very crucial in simulation studies. Inverse methods and data assimilation are often ill-posed, i.e., they are characterized by nonuniqueness and instability of the identified parameters (Yeh 1986). It may thus be worthwhile to search for best initial and/or boundary conditions when using these models in analysis. The reader may have noticed that this approach has major advantages compared to conventional methods. It allows us to estimate initial conditions of the model dynamics as additional control variables on equal footing as the model parameters. Thus, treating the initial biomass level and the initial harvest rate as uncertain inputs in the system. Most recent models and traditional approaches consider the initial biomass and harvest amounts as known and deterministic. It also provides an efficient way of calculating the gradients of the loss function with respect to the control variables. Most importantly, data requirements are significantly reduced. In this paper, parameters entering the objective function of the industry have been reasonably estimated without having to use data on prices and costs. Large number of parameters could be estimated with observations on only a subset of the variables.

The structure of the remainder of the paper is as follows. Section 2 is a detailed discussion of the dynamics of the commercial fishing model. It presents a more general model without assuming any optimizing behavior. In section 3, we briefly discuss data assimilation and some basic concepts of the techniques are defined. All technical details are put in an Appendix. Section 4 is an application to the North East Arctic Cod stock (NEACs). It discusses the results and summarizes the work.

5.2 Dynamics of Commercial Fishing

The dynamics of the fishing industry are developed and discussed in detail in this section. A fishery resource has one unique characteristic, i.e., the ability to replenish by the laws of natural growth. The dynamics of the stock for a single species are formally described by the simple equation

$$\frac{dx}{dt} = f(x) - y \tag{5.1}$$

where x is the biomass in weight, dx/dt is the time rate of change of the stock and y[1] is the rate of exploitation by humans. The growth "or natural addition" to the existing stock is represented by the $f(.)$ operator and depend on the current stock. Several forms of the growth model exist. For some species, the empirical law of growth is asymmetric. In this paper, however, we will use the logistic growth law. The Schaefer logistic function takes the form $f = rx(1 - x/K)$, where r is the intrinsic growth rate and K is the maximum growth of the biological species if the population were not exploited. It is symmetric about $K/2$ and has the following properties, $f(0) = f(K) = 0$, $f(K/2)$=max f.
To model the fishing industry, we define the following relationship between the rate of increase or decrease of the exploitation of the fish biomass y and a function $\phi(x, y)$ such that

$$\frac{dy}{dt} = \gamma y \phi(x, y) \tag{5.2}$$

[1]Gordon (1954) assumed that harvest depends on stock and efforts while Smith (1969) assumed that harvest is a function of the number of identical firms and the catch rate. Here no such disaggregation has been made.

where $0 < \gamma$ is a constant of proportionality and ϕ is a certain well defined value function to be discussed shortly. The constant of proportionality reflects the rate at which capital is being put in or removed from the industry or firm. For instance, if ϕ is positive one may expect an increase in capital investment in the fishery and a decrease otherwise. The function(s) defined by ϕ can take different parametric forms reflecting our hypotheses about the operation of the industry. It may represent short or long run average costs of fishing vessels, the marginal or average net revenues of a firm, etc. Different forms of the ϕ functions will be discussed in detail. We will first model an industry that is perceived to be a price taker in the output market.

Let p be the unit exvessel price of fish and c be the per unit cost of harvesting. Assume for the first case that costs of fishing are linear in the harvest. Then, the average net revenue is given by

$$\phi(x, y) = p - \frac{c}{x} \tag{5.3}$$

The average cost of harvesting is assumed to depend explicitly on the size of the stock abundance. This takes into account the stock externalities, i.e., fishing costs decrease as the population of fish increases. The assumption that the total net revenue of the industry is linearly related to the harvest rate may be quite restrictive. We shall slack this assumption of price taking and introduce some relevant nonlinearities in the model. Next we discuss a model in which price depends on the rate of harvesting of the stock. We shall continue to assume that costs are linear in harvest and inversely related to the stock biomass. The average net revenue is defined by

$$\phi(x, y) \quad = \quad P(y) - \frac{c}{x} \tag{5.4}$$

where $P(y) = a - by$ is the inverse demand function which is assumed to be downward sloping and a, b are positive real constants. From the previous definitions of ϕ and the industry model, equation (5.2), it is obvious that the rate of harvesting from the stock for the industry is perceived to vary in proportion to the net revenue; that is the difference between total revenues and total costs. Put another way, the output growth rate $\dot{y}/y$ of the industry is proportional to the average or marginal net revenues.

Substituting these functions in equation (5.2) and combining with the population dynamics model, equation (5.1), the industry dynamics models are derived. This system of equations (5.1)-(5.2) constitute coupled nonlinear ODEs. For the empirical analysis, we will use the following models.

model 1

$$\begin{aligned} \frac{dx}{dt} &= f(x,r,K) - y \\ \frac{dy}{dt} &= \gamma(p - \frac{c}{x})y \end{aligned} \tag{5.5}$$

In the first model, the term $(py - cy/x)$ is the annual total profit (total revenues minus total costs). Owing to the linearity of the net revenue in the harvest, the average net revenue is equal to the marginal net revenue.

model 2

$$\begin{aligned} \frac{dx}{dt} &= f(x,r,K) - y \\ \frac{dy}{dt} &= \gamma(P(y) - \frac{c}{x})y \end{aligned} \tag{5.6}$$

In model 2, the demand function is downward sloping, i.e., the output of the industry affects its market price and costs are linear in harvest and inversely related to the stock biomass (Sandal and Steinshamn 1997b). Hence, the profit function is nonlinear both in the harvest and the biomass. Incorporated in these models are the hypotheses about the costs and the revenues. If the firms were optimizers, they should at least operate at a level where average or marginal profits are positive. In the construction of such behavioral models, an implicit assumption about the harvest rate being proportinal to the number of firms or fishing vessels is made (see Smith 1969).

The system of equations contains these input parameters, the biological parameters (r, K) and the economic parameters (γ, p, a, b, c). It is possible to estimate all of the parameters in the models but additional data may be required. To obviate the data problem, we reduce the dimension of the problem by redefining the parameters: $\theta = \gamma p$, $\alpha = \gamma a$, $\beta = \gamma b$, and $\tau = \gamma c$. That is, we now have these parameters $(r, K, \theta, \alpha, \beta, \tau)$ to estimate. Notice here that no data on prices and costs are necessary in order to fit the models. The method enables us to fit the bioeconomic models without using data on economic variables which are often unavailable. These mathematical models of the commercial fishing will be used to analyze real fishery data for the (NEACs).

5.3 Data Assimilation Methods

Data assimilation methods have been used extensively in meteorology and oceanography to estimate the variables of model dynamics and/or the initial and boundary conditions. These methods include the sequential techniques of Kalman filtering (Kalman 1960) and the variational inverse approach (Bennett 1992). The variational adjoint method has been proposed as a tool for estimation of model parameters. It has since proven to be a powerful tool for fitting dynamic models to data (Smedstad and O'Brien 1991). The methods have recently been used to estimate parameters of the predator-prey equation (Lawson et al. 1995) and also some high dimensional ecosystem models (Spitz et al. 1997 and Matear 1995). The basic idea is that, given a numerical model and a set of observations, a solution of the model that is as close as possible to the observations is sought by adjusting model parameters such as the initial conditions. The variational adjoint method has three parts: the forward model and the data which are used to define the penalty function, the backward model derived via the Lagrange multipliers and an optimization procedure. These components and all of the mathematical derivations are discussed in an Appendix. An outline of the technique is also presented for those who may be interested in learning the new and efficient method of data analysis.

5.4 An Application

The commercial fishing models developed in this paper are used in an application to (NEACs). The fishery has a long history of supporting large part of the Norwegian and Russian coastal populations. Data on catches and estimated stock biomass have been collected since immediately after World War II. Different techniques of stock assessments exist in fisheries management. The data on the (NEACs) are measured using the statistical Virtual Population Analysis (VPA) method. Catch data and biomass estimates obtained by the VPA may be somewhat correlated. This issue will not be dealt with in this paper.

The history of this fishery is not dissimilar from other commercial fisheries elsewhere around the world. It has supposedly been managed based on the common policy of the maximum sustainable yield (MSY) which is the most employed for the most of the last century. The historical data show a decreasing trend for both the stock biomass and the yield. It is also observed that the data are highly fluctuatory which depicts the inherent stochastic feature of a fishery resource. The data available on (NEACs) dates back to 1946 until 1996 (see Anon 1998). It is however intuitive to divide the period into the pre-quota (1946-1977) and the quota (1978-1996) periods which represent different management regimes. The first period may be dubbed the open access period and the second the regulated open access (total allowable catch TAC) period. We will apply our models to analyze the data for the first period. To analyze the second period, additional constraints such as quota restrictions and minimum safe biomass levels Homans and Wilen (1997) which reflect the regulations imposed by the management authorities are required. We shall however concern ourselves about the first period.

In this study, we combine the nonlinear dynamics models developed in the preceding section and the time series of observations to analyze the (NEACs). The technique in this paper provides a novel and highly efficient procedure of data analysis. Model initial conditions as well as parameters of the dynamics are estimated using the variational adjoint method. First, artificial data generated from the model itself using known initial conditions and known parameters were used to test the performance of the adjoint code. All the parameters were recovered to within the accuracy of the machine precision. Both clean and noisy data were used to first study the models. The results are not shown in this paper. Next, real data were used to estimate the initial conditions and all the parameters of the model dynamics. Starting from the best guesses of the control variables, the optimization procedure uses the gradient information to find optimal initial conditions and parameters of the model which minimize the penalty function. The procedure is efficient and finds the optimum solution in a matter of a few seconds. The estimated initial conditions and parameters of the two different models are tabulated below. Note the definition of the units: r (/year), x_0, y_0, and K are in kilo-tons.

Parameters	Model 1	Model 2
r	0.3271	0.44305
K	5264.85	5257.55
θ	0.13039	
α		4.1368
β		.00213
τ	309.01	7070.63
$x0$	3902.77	3670.00
$y0$	716.15	770.33

Table 5.1: Model parameters for the two dynamic models. Blank space means the parameter is not present in that model.

All the estimated parameters are reasonable and as expected. From the table 5.1 above, the estimated r's are different for the two different models. Model 2 which is more complex than model 1 gives a bigger r value. The maximum population K is about the same for both models. The initial conditions have also been adjusted in both cases. Note that the observed initial values were taken as the best guesses. To further explain the performance of these models, we present some graphics of the time series of the actual observations and the estimated quantities. Figure 5.1 is a plot of the actual observations (Act. observations) and the models predictions (Est. model 1 and Est. model 2) of the stock biomass using the estimated parameters.

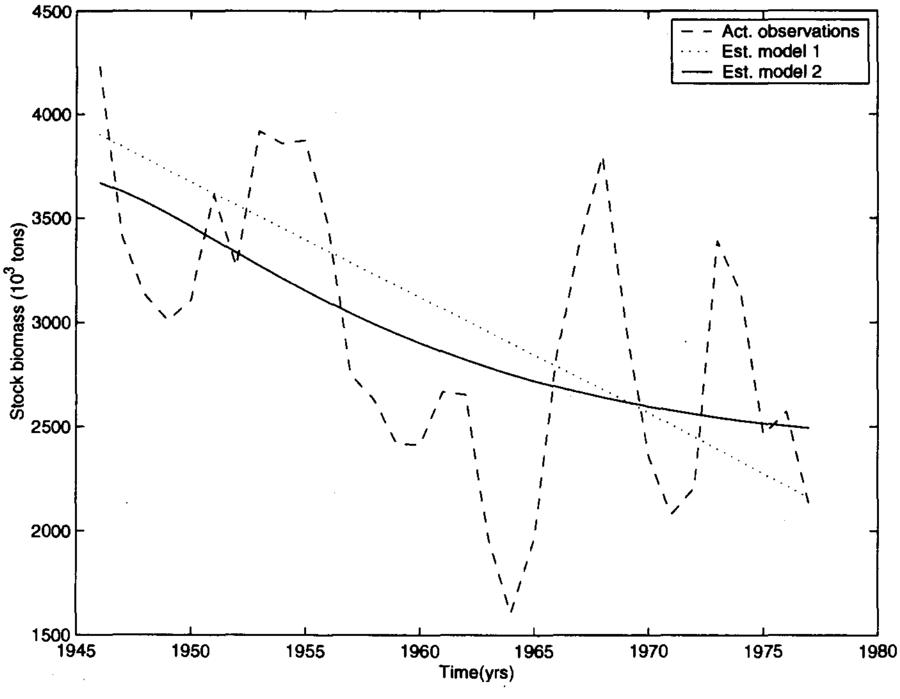

Figure 5.1: Graphs of the actual and the model estimated stock biomass for the two models

It is observed that, model 1 predicts higher biomass levels and is generally steeper than model 2. The models have both performed well in tracking the downward trend in the data. Model 2 seems to do a little bit better overall and at the tail end of the data. In Figure 5.2, we have the plot of the actual observations (Act. observations) and the model predictions (Est. model 1 and Est. model 2) of the rate of harvesting.

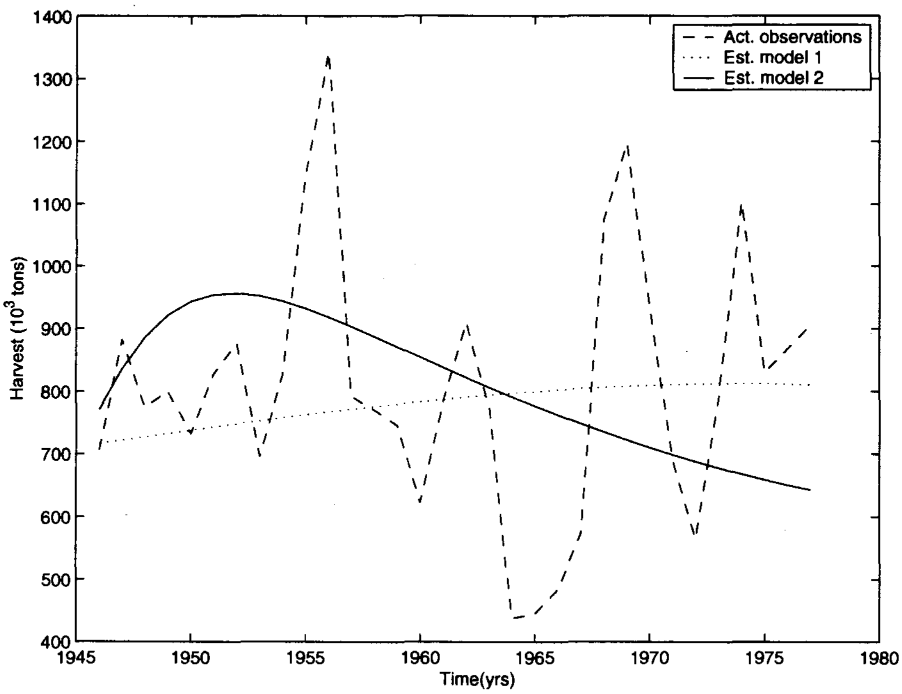

Figure 5.2: Graphs of the actual and the model estimated harvest for the two models

The fits are in general quite good for both models. Model 1 is more gentle overall. It gives lower estimates initially and then higher afterwards. Model 2 tries to correct for the occasional jumps in the data as shown in the figure. The models have generally performed as expected and have shown some reasonable degree of consistencies with the data. Note however that these data are highly random and may have large measurement errors.

The models we have developed measure the performance of the industry in question using the function $\phi(x, y)$. Industry equilibrium is attained when $\frac{dy}{dt} = 0$. That is, for an open access fishery, industry equilibrium is characterized by zero profits. The parameters of the ϕ function have been estimated using the variational adjoint method. For the (NEACs), it will be interesting to look at how the industry performed during the open access regime. To illustrate, we will plot the revenues and the costs versus the stock biomass for each of the two models. The revenue and cost functions are scaled by the parameter γ and the unit of currency is the Norwegian Kroner (NOK).

In figure 3 the total revenues and total costs are graphed. The difference between these represent the net profits. Costs were least when the stock size was largest but increased as the stock decreased. The profits were driven to zero when $x^* = c/p$, i.e., the industry is in a steady state. The industry equilibrium (point where total costs balance total revenues) was reached at the stock level of $x^* = 2370\ 10^3$tons which is the so called open access equilibrium. This is lower but very close to the $x_{MSY}{=}(K/2)$ level. A further reduction of the stock led to unprofitable investments. Costs exceeded revenues as the stock level fell beyond $x^* = c/p$.

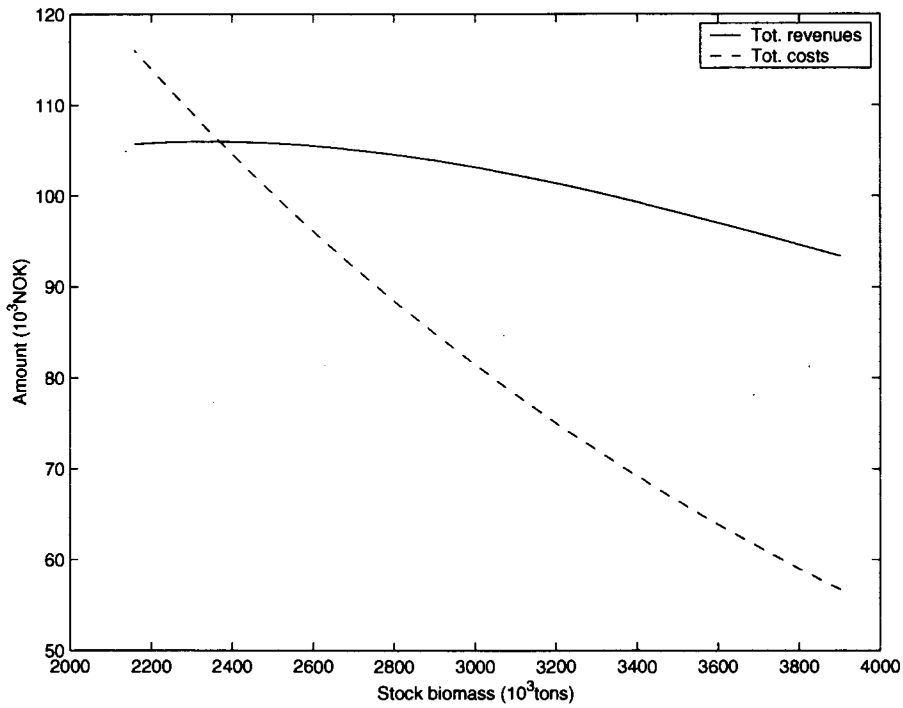

Figure 5.3: Graphs of the total revenues and the total costs vs. estimated stock biomass for model 1.

Figure 5.4 is a plot of the revenue and cost functions. The shapes of the functions indicate their level of complexities. The results of model 2 have some similar characteristics to model 1. However, the industry steady state occurred at a higher biomass level of about 3400 Kilo-tons. Extrapolation of the results of model 2 indicate another equilibrium $x^* = 2440\ 10^3$tons close to the one predicted by model 1. This point satisfies the equilibrium conditions $\dot{x} = \dot{y} = 0$. The hypothesis of a large industry whose output affects the market price resulted in a multiple industry equilibria. The first is quite unstable since only the industry reached equilibrium but not the biology. The biological and industry steady state occurred at the second point (extrapolation not shown).

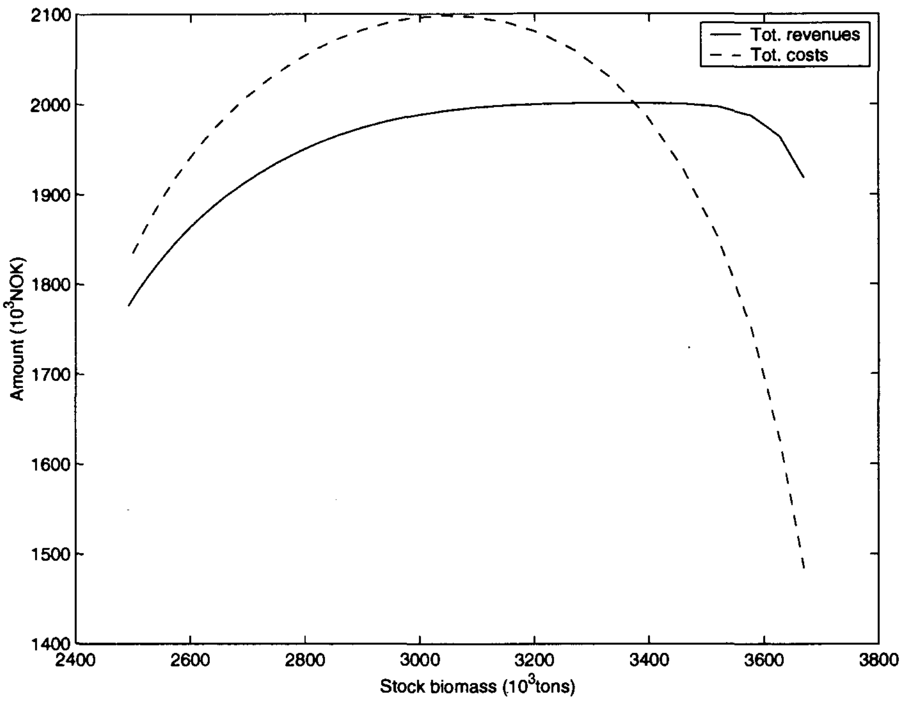

Figure 5.4: Graphs of the total revenues and the total costs vs. estimated stock biomass for model 2.

In both models, costs are assumed to be inversely related to the stock biomass. This underscores stock externalities in the models which appear to reasonably characterize the (NEACs). Note that the cod is a demersal species and does not exhibit the schooling characteristics of the species such as herring. Both models will attain bioeconomic steady state at about the same biomass level of little below the MSY biomass level. The question of which of these models is more appropriate for the (NEACs) is still immature to give a definite answer to. More research needs to be done. What is certain is that with more realistic models and data with less errors than the one available, it is possible to operationalize modern fisheries management.

5.4.1 Summary and conclusion

This paper, unlike most other papers, has addressed two major questions in bioeconomic analysis and fisheries management. It developed simple dynamic fisheries models in a way that is rare in the literature and employs a new and powerful approach of efficiently combining these models with available observations collected over a given time domain. The variational adjoint method is used to simultaneously estimate the initial conditions and the input parameters of the industry fishing models. An interesting finding of the paper, is that, the steady state without regulation is not too far away from the MSY. Which means that, open access in this case has meant economic overfishing but not necessarily biological overfishing. It is observed that the technique used in this paper has an added virtue compared to the conventional ones used in the literature. Initial conditions of the model dynamics are estimated on equal footing as the model parameters. It is highly versatile that, it enables researchers to include as much information as is available to them. The estimates were all reasonable and as expected for the (NEACs). The models have quite reasonable explanatory power. However caution must be exercised when interpreting the results due to the inadequacy of the models and the large measurement errors in the data.

It has been demonstrated here that, dynamic resource models can be combined with real data in order to obtain useful insights about real fisheries. Biological parameters such as the carrying capacity and economic parameters entering the objective functions of the industry are identified. These again can be used for dynamic optimization in order to improve the economic performance of the fishery. The variational adjoint method has proven to be very promising and deserves further research efforts not only in resource economics but economics in general.

APPENDIX A

A.1 Data Assimilation-A Background

This section formulates the parameter estimation problem and presents the mathematical aspects of the variational adjoint technique. Numerical issues have also been briefly discussed.

A.1.1 The model and the data

The model dynamics are assumed to hold exactly, i.e., the dynamics are perfect. The dynamics are described by the two models above. For the sake of mathematical convenience, we use the compact notation to represent the model dynamics as

$$\frac{d\mathbf{X}}{dt} = F(\mathbf{X}, \mathbf{Q}) \quad (A.1)$$

$$\mathbf{X}(\mathbf{0}) = \mathbf{X}_0 + \hat{\mathbf{X}}_0 \quad (A.2)$$

$$\mathbf{Q} = \mathbf{Q}_0 + \hat{\mathbf{Q}} \quad (A.3)$$

where $\mathbf{X} = (x, y)$ is the state vector, $\mathbf{X}_0$ is the best guess initial condition vector, $\hat{\mathbf{X}}_0$ is the vector of initial misfits, $\mathbf{Q}$ is a vector of parameters and $\hat{\mathbf{Q}}$ is the vector of parameter misfits. The dynamics are assumed to exactly satisfy the constraints while the inputs, i.e:, the initial conditions and the parameters are poorly known.

In real world situations, observations are often available for some variables such as the annual catches and fishing efforts. The set of observations are often sparse and noisy and are related to the model counterparts in some fashion. The measurement vector is defined by

$$\hat{\mathbf{X}} = \mathcal{H}[\mathbf{X}] + \epsilon \quad (A.4)$$

where $\hat{\mathbf{X}}$ is the measurement vector, ϵ is the observation error vector and $\mathcal{H}$ is a linear measurement operator. The misfits are assumed to be independent and identically distributed "iid" random deviates. To describe the errors in the initial conditions, the parameters and the data, we require some statistical hypotheses. For our purpose in this paper the following hypotheses will suffice

$$\bar{\hat{\mathbf{X}}}_0 = 0, \quad \overline{\hat{\mathbf{X}}_0\hat{\mathbf{X}}_0^T} = \mathbf{W}_{X_0}^{-1}$$

$$\bar{\epsilon} = 0, \quad \overline{\epsilon\epsilon^T} = \mathbf{W}^{-1}$$

$$\bar{\hat{\mathbf{Q}}} = 0, \quad \overline{\hat{\mathbf{Q}}\hat{\mathbf{Q}}^T} = W_{\mathbf{Q}}^{-1}$$

where the T denotes matrix transpose operator. That is, we are assuming that the errors are normally distributed with zero means and constant variances (homoscedastic) which are ideally the inverses of the optimal weights. For this paper, it will further be assumed that the errors are not serially correlated. This implies that the covariance matrices are now diagonal matrices with the variances along the diagonal. We further assume that the variances are constant.

A.1.2 The loss or penalty function

In variational adjoint parameter estimation, a loss functional which measures the difference between the data and the model equivalent of the data is minimized by tuning the control variables of the dynamical system. The goal is to find the parameters of the model that lead to model predictions that are as close as possible to the data. A typical penalty functional takes the more general form

$$
\begin{aligned}
\mathcal{J}[\mathbf{X},\mathbf{Q}] &= \frac{1}{2T_f}\int_0^{T_f}(\mathbf{Q}-\mathbf{Q}_0)^T\mathbf{W}_Q(\mathbf{Q}-\mathbf{Q}_0)dt \\
&+ \frac{1}{2T_f}\int_0^{T_f}(\mathbf{X}(0)-\mathbf{X}_0)^T\mathbf{W}(\mathbf{X}(0)-\mathbf{X}_0)dt \\
&+ \frac{1}{2}\int_0^{T_f}(\mathbf{X}-\hat{\mathbf{X}})^T\mathbf{W}(\mathbf{X}-\hat{\mathbf{X}})dt
\end{aligned}
\tag{A.5}
$$

where the period of assimilation is denoted by T_f and T is the matrix transpose operator. The $\mathbf{W}'s$ are the weight matrices which are optimally the inverses of the error covariances of the observations. They are assumed to be positive definite and symmetric. The first and second terms in the penalty functional represent our prior knowledge of the parameters and the initial conditions, and ensure that the estimated values are not too far away from the first guesses. They may also enhance the curvature of the loss function by contributing positive terms to the Hessian of $\mathcal{J}$ (Smedstad and O'Brien 1991). The variational adjoint technique determines an optimal solution by minimizing the loss function $\mathcal{J}$ which measures the discrepancy between the model predictions and the observations. The loss function is minimized subject to the dynamics. The constrained inverse problem above is efficiently solved by transforming the problem into an unconstrained optimization (Luenberger 1984). Several algorithms for solving the unconstrained nonlinear programming problem are available (Smedstad and O'Brien 1991). Statistical methods such as the simulated annealing (Matear, 1995; Kruger 1992) and the Markov Chain Monte Carlo (MCMC) (Harmon and Challenor 1997) have recently been proposed as tools for parameter estimation. The most widely used methods are the classical iterative methods such as the gradient descent and the Newton's methods (see Luenberger 1984).

A.1.3 The variational adjoint method

Construction of the adjoint code is identified as the most difficult aspect of the data assimilation technique (Spitz et al. 1997). One approach consists of deriving the continuous adjoint equation and then discretizing them (Smedstad and O'Brien 1991). Another approach is to derive the adjoint code directly from the model code (Lawson et al. 1995; Spitz et al. 1997). To illustrate the mathematical derivation, we use the first approach (see details in Appendix). Formulating the Lagrange function $\mathcal{L}$ by appending the model dynamics as strong constraints, we have

$$\mathcal{L}[\mathbf{X}, \mathbf{Q}] = \mathcal{J} + \frac{1}{2}\int_0^{T_f} \mathbf{M}\frac{dF}{d\mathbf{X}}dt \tag{A.6}$$

where $\mathbf{M}$ is a vector of Lagrange multipliers which are computed in determining the best fit. The original constrained problem is thus reformulated as an unconstrained problem. At the unconstrained minimum the first order conditions are

$$\frac{d\mathcal{L}}{d\mathbf{X}} = 0 \tag{A.7}$$

$$\frac{d\mathcal{L}}{d\mathbf{M}} = 0 \tag{A.8}$$

$$\frac{d\mathcal{L}}{d\mathbf{Q}} = 0. \tag{A.9}$$

It is observed that equation (A.7) results in the adjoint or backward model, equation (A.8) recovers the model equations while (A.9) gives the gradients with respect to the control variables. Using calculus of variations or optimal control theory, the adjoint equation is derived by forming the Lagrange functional via the undetermined multipliers $\mathbf{M}(t)$. The Lagrange function is

$$\mathcal{L} = \mathcal{J} + \int_0^{T_f} \mathbf{M}(\frac{\partial \mathbf{X}}{\partial t} - F(\mathbf{X}, \mathbf{Q}))\mathbf{dt} \tag{A.10}$$

Perturbing the function $\mathcal{L}$

$$
\begin{aligned}
\mathcal{L}[\mathbf{X}+\delta\mathbf{X},\mathbf{Q}] &= \mathcal{J}[\mathbf{X}+\delta\mathbf{X},\mathbf{Q}] \\
&+ \frac{1}{2}\int_0^{T_f}\mathbf{M}(\frac{\partial(\mathbf{X}+\delta\mathbf{X})}{\partial t} - F(\mathbf{X}+\delta\mathbf{X},\mathbf{Q}))\mathbf{dt}
\end{aligned}
$$

which implies

$$
\begin{aligned}
\mathcal{L}[\mathbf{X}+\delta\mathbf{X},\mathbf{Q}] &= \mathcal{J} + \Delta_{\mathbf{X}}\mathcal{J}\delta\mathbf{X}^T + \int_0^{T_f}\mathbf{M}(\frac{\partial\mathbf{X}}{\partial t} - F(\mathbf{X},\mathbf{Q}))\mathbf{dt} \\
&- 2\int_0^{T_f}\mathbf{M}(\frac{\partial\delta\mathbf{X}}{\partial t} - \frac{\partial F}{\partial\mathbf{X}}\delta\mathbf{X}^T)dt + O(\delta\mathbf{X}^2)
\end{aligned}
\tag{A.11}
$$

Taking the difference $(\mathcal{L}[\mathbf{X}+\delta\mathbf{X},\mathbf{Q}] - \mathcal{L}[\mathbf{X},\mathbf{Q}])$

$$
\begin{aligned}
\delta\mathcal{L} &= \Delta_{\mathbf{X}}\mathcal{J}\delta\mathbf{X}^T \\
&- 2\int_0^{T_f}\mathbf{M}(\frac{\partial\delta\mathbf{X}}{\partial t} - \frac{\partial F}{\partial\mathbf{X}}\delta\mathbf{X}^T)dt + O(\delta\mathbf{X}^2)
\end{aligned}
\tag{A.12}
$$

Requiring that $\delta\mathcal{L}$ be of order $O(\delta\mathbf{X}^2)$ implies

$$
\Delta_{\mathbf{X}}\mathcal{J}\delta\mathbf{X}^T - \int_0^{T_f}\mathbf{M}(\frac{\partial\delta\mathbf{X}}{\partial t} - \frac{\partial F}{\partial\mathbf{X}}\delta\mathbf{X}^T)dt = 0 \tag{A.13}
$$

By integrating the second term of the LHS by parts and rearranging, we have

$$
\begin{aligned}
&\frac{\partial\mathbf{M}}{\partial t} + [\frac{\partial F}{\partial\mathbf{X}}]^T\mathbf{M} = \mathbf{W}(\mathbf{X}-\hat{\mathbf{X}}) \\
&\qquad\qquad\quad \mathbf{M}(T_f) = \mathbf{0}
\end{aligned}
\tag{A.14}
$$

which is the adjoint equation together with the boundary conditions and from (A.8) the gradient relation is

$$
\Delta_Q\mathcal{J} = -\int_0^{T_f}\mathbf{M}\frac{dF}{d\mathbf{Q}}dt + \mathbf{W}_Q(\mathbf{Q}-\mathbf{Q}_0) \tag{A.15}
$$

The term on the RHS of (A.14) is the weighted misfit which acts as forcing term for the adjoint equation. It is worth noting here that we have implicitly assumed that data is continuously available throughout the integration interval. Equations (A.7) and (A.8) above constitute the Euler-Lagrange (E-L) system and form a two-point boundary value problem. The implementation of the variational adjoint technique on a computer is straightforward. The algorithm is outlined below.

- Choose the first guess for the control parameters.

- Integrate the forward model over the assimilation interval.
- Calculate the misfits and hence the loss function.
- Integrate the adjoint equation backward in time forced by the data misfits.
- Calculate the gradient of $\mathcal{L}$ with respect to the control variables.
- Use the gradient in a descent algorithm to find an improved estimate of the control parameters which make the loss function move towards a minimum.
- Check if the solution is found based on a certain criterion For example, $\mathcal{J} \leq \epsilon, ||\Delta\mathcal{J}|| \leq \epsilon$ may be appropriate convergence criteria. .
- If the criterion is not met repeat the procedure until a satisfactory solution is found.

The optimization step is performed using standard optimization procedures. In this paper, a limited memory quasi-Newton procedure (Gilbert and Lemarechal 1991) is used. The success of the optimization depends crucially on the accuracy of the computed gradients. Any errors introduced while calculating the gradients can be detrimental and the results misleading. To avoid this incidence from occurring, it is always advisable to verify the correctness of the gradients (see, Smedstad and O'Brien 1991; Spitz et al. 1997).

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