Essays on Changing Volatility in Thinly Traded Equity Markets

by

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Chapter 1 Introduction

This dissertation studies mean and volatility effects from non-synchronous trading in the Norwegian equity market. Non-synchronous trading means that observed trade-to-trade prices do not correspond to true daily returns since securities do not trade every day at market close. Non-synchronous trading may therefore affect daily return and volatility characteristics for asset, portfolio, and index series due to the facts that the returns are not equally spaced at 24-hour intervals. Consequently, non-synchronous trading is relevant to asset price and risk evaluation in general, risk management calculations and valuations of derivatives. Moreover, due to changes in mean and volatility characteristics, non-synchronous trading may affect relevant risk measures in capital asset pricing models (mean-variance efficiency) and abnormal return calculations employing the market model in event studies (bivariate non-synchronous trading). Traders, fund managers, and professional corporate board members may therefore find non-synchronous trading effects to be relevant for their decision and advisory business making. Moreover, regulators and policy makers may find non-synchronous trading effects to be an important contributor to the important and ongoing liquidity discussions for international capital markets. The direction of the relationship between liquidity and risk premium is important to determine.

The dissertation's focus is on efficient mean and volatility specifications with the intention to obtain identical and independently distributed model residuals for univariate and bivariate return series. By applying lag specifications we define efficient conditional mean and volatility equations. Moreover, by applying elaborate test statistics on these models' residuals, systematic factors due to non-synchronous trading in both mean and volatility may be efficiently filtered from the adjusted raw returns. Therefore, sorting return series across non-synchronous trading measures, this dissertation's essays construct efficient conditional mean and volatility specifications and inspect liquidity effects in return series. Consequently, these investigations may bring new insight into the asset pricing processes and market dynamics in many equity markets globally.

The following steps are performed to obtain our objectives. Firstly, it will be important to relate mean drift and serial correlation characteristics to various degrees of non-synchronous trading effects. Does non-synchronous trading effects suggests changes to the conditional mean process relative to continuously trade series? Secondly, it will be important to relate volatility weight to long-run average volatility, and serial correlation characteristics to various degrees of non-synchronous trading effects. Does non-synchronous trading induces changes to the latent conditional volatility process relative to continuously traded series? Thirdly, it will be important to relate model misspecifications to various degrees of non-synchronous trading effects. Does non-synchronous trading suggests model misspecifications suggesting spurious parameters and systematic factors in contrast to continuously traded series? As every equity market shows assets that occasionally do not trade for long periods of time, these three important questions induce an increasing interest in equity markets for model specifications accounting for non-synchronous trading effects in the conditional mean and volatility.

To obtain a better understanding of non-synchronous trading, this dissertation, in contrast to much of the international empirical literature on equity markets, therefore concentrate its investigations on formal conditional mean and volatility specifications across a variety of liquid and illiquid return series. By definition, illiquid series possesses non-synchronous trading effects in mean and volatility. Formal conditional mean and volatility specifications across series may therefore show effects by changing parameter values and changing optimal and efficient lags in mean and volatility specification. Consequently, the dissertation aims at applying and extending a few econometric techniques solving some issues of interest in conditional mean and volatility specifications across return series showing varying degree of liquidity, which is often found in the world's equity markets. Below we introduce the dissertation's main categories for investigations and how we want to approach the various domains.

The dissertation will in seven essays measure non-synchronous trading effects in three major topics. The first three papers study mean and volatility effects in univariate returns series. Essay four and five study effects for one-factor equilibrium models (the Capital Asset Pricing Model). Finally, essay six and seven study effects on event studies employing the market model for abnormal return calculations. Across increasing non-synchronous trading effects, all essays hypothesise asset series mean and volatility parameter changes and in extreme cases, lag changes. The essays assume that the most frequently traded return series induce continuous trading while lower trading volume introduce an increasing non-synchronous trading effects into the mean and volatility of return series.

The first three essays study mean and volatility characteristics from univariate asset, portfolio, and index series. The first paper measures aggregated average mean and volatility for asset samples showing continuous versus one, two and three days of non-trading in an open (Monday to Friday) and closed (Saturday, Sunday and holidays) market. The aggregated mean and volatility processes contain important information for market participants and especially option traders. By assuming non-synchronous trading effects, the following information may be exploited by market participants: The predictability of asset returns may be exploited by investors and portfolio managers; policy makers may enhance/limit liquidity for equity markets; and option traders may find that the option pricing formula produces erroneous call and put prices for asset derivatives, due to a changing volatility process. Moreover, investors and portfolio managers would appreciate suggestions for mean and volatility process changes, asymmetric volatility and changes in return distribution characteristics for asset and derivative valuations and especially risk management (Value At Risk).

The dissertation's second essay investigates non-synchronous trading effects applying a formal conditional mean and volatility specification applying portfolio and index series. The main objective of the essay is to pursue mean and volatility predictability for return series exhibiting non-synchronous trading effects. The essay aims to show that a positive relationship between non-synchronous trading and asset return predictability may suggest highly profitable strategies applying advanced econometric mean and volatility models. Consistent and significant parameter value and lag changes across non-synchronous trading effects for the conditional mean, suggest predictability in mean rejecting the Martingale hypothesis. Secondly, consistent and significant parameter value and lag changes across non-synchronous trading effects for the conditional volatility, suggest predictability in volatility rejecting the Independence hypothesis.

The third essay investigates non-linear and data dependence in asset, portfolio, and index series. The main focus of the paper is (non-) linear mean and volatility model specifications applying elaborate tests for model misspecifications. The essay therefore specifies (non-) linearity models for the mean and volatility across asset, portfolio and index series exhibiting varying degrees of non-synchronous trading effects. Elaborate test statistics investigate all model residuals for significant data dependence. Suggestions of data dependence across asset series will be of outmost importance for traders, investors, and portfolio managers, as significant data dependence induces missing systematic factors and spurious model parameter values. Assuming that high non-synchronous trading effects suggest significant data-dependence, the applied modelling framework does not appropriately specify non-synchronous trading effects in the conditional mean and volatility equations. The result suggests a need for increased liquidity to eliminate predictability for these return series. Importantly, for regulators and policy makers, positive relationship between non-synchronous trading effects and data dependence, suggest that intuitive, analytical and linear reasoning in this equity market becomes extremely difficult.

The dissertation's essays number four and five investigates non-synchronous trading effects and the one-factor equilibrium models (CAPM. For market equilibrium models, non-synchronous trading may cause bias and spurious relationships in the moments and the co-moments of bivariate return series. Therefore, the essays investigate changes in conditional mean and volatility specifications across non-synchronous trading effects for univariate and bivariate dynamic CAPM specifications.
The fourth paper investigates the one-factor model employing four univariate index series; two equal-weighted and two value-weighted indices all deducted the daily NIBOR[1] interest rate representing the risk free rate, obtaining excess market return series. These four return series may show different effects from non-synchronous trading due to the fact that non-trading may have stronger bearings in the equal-weighted series than in the value-weighted series[2]. The investigation applies daily, weekly and monthly return series. If the conditional CAPM holds for daily, weekly and monthly index series, investors and portfolio managers may apply formal model specifications for asset pricing and relevant risk calculations. However, by rejecting the conditional CAPM we may have to reject the mean-variance relationship in formal asset pricing. Moreover, any signs of positive relationship between model misspecifications at daily, weekly and monthly return intervals and non-synchronous trading

[1] NIBOR = Norwegian Inter Bank Offered Rate.
[2] Due to the fact that trading volume and market value has high positive correlation as shown by Campbell et al. (1997) page 130; quote: "We use market capitalization to group securities because the relative thinness of the market for any given stock is highly correlated with the stock's total value; hence stocks with similar market values are likely to have similar non-trading probabilities".

effects suggest spurious mean and volatility characteristics inducing missing systematic factors. Factor models may therefore be more appropriate for these index series.

The fifth essay investigates the one-factor model applying a bivariate specification model across series showing various degrees of non-synchronous trading. In these bivariate models across varying non-synchronous trading effects, asset and portfolio series are specified together with a proxy for the market portfolio, which is assumed to be the all-market value-weighted index[3], obtaining efficient conditional mean and volatility specification for CAPM equilibrium. The model specification investigates daily dynamics in asset series in relation to the dynamics of the market proxy across non-synchronous trading effects. Irrespective of non-synchronous trading effects, an important question is whether the conditional CAPM and the relevant risk measure (beta) constitute an appropriate description of mean-variance pricing in equity markets. Therefore, if the conditional CAPM turns out to be true, the conditional volatility specification may give interesting information to all market participants. In contrast, if it is not true, the conditional CAPM model is not appropriate for the conditional mean and volatility specifications. Moreover, if the conditional model across non-synchronous trading effects shows model misspecification, investors, portfolio and fund managers should show extreme care when interpreting variance/covariance results from formal asset pricing models (the conditional CAPM). Several alternative forms of volatility specifications (variance/ covariance matrix) may be specified in the conditional mean equation hypothesising relevant risk (covariance), residual risk (variance), and one dynamic factor (market variance) risk. These volatility-in-Mean results may show new and interesting risk insights across non-synchronous trading effects. Finally, as the conditional beta frequency distribution is readily available from the model output, we may investigate whether the relevant risk measure (beta) shows any relationship to non-synchronous trading effects. If we find a relationship between beta and non-synchronous trading effects, the CAPM cannot be rejected, and we can reject model misspecifications, the results suggest a relation between the degree of non-synchronous trading effects and relevant risk. These model results may be applied to portfolio management for fund managers and investors establishing personal portfolios adjusting risk profiles. Moreover, traders, portfolio and fund managers may evaluate the conditional CAPM versus the residual risk model and the one dynamic factor model.

The last two essays of the dissertation investigate changes in conditional mean and volatility from non-synchronous trading effects in non-event and event periods[4]. Any changes in the conditional mean and volatility parameters from non-event to event periods suggest a need to control for non-synchronous trading effects, which are not usually corrected for in classical event studies. The sixth paper investigates therefore the relationship between non-synchronous-trading effects and event-induced conditional mean and volatility parameter changes. The motivation for the modelling efforts is that changes in non-synchronous trading effects (and information flows) may disrupt profoundly the

[3] The market index series location on or outside the efficient set will not be discussed in the thesis.
[4] Event periods are periods with important information announcements to the public. For example, for a merger announcement an event period may be defined as 40 days prior and 40 days after the announcement day. Non-event periods are periods outside such event periods.

classical event study's synchronous trading and constant volatility assumptions. Classical event studies normally assumes constant volatility and synchronous trading in both asset and market proxy series from estimation period to event period. The sixth paper aims therefore to show that non-synchronous trading effects changes conditional mean and volatility specifications from non-event to event periods. An alternative abnormal return market model specification may therefore show more appropriate model specifications suggesting a sounder basis for abnormal return calculations. The paper estimates mean and volatility equations for several non-event and event series restricting the event series model parameters to be inside the ranges of non-event parameters. If the restrictions show significance, a non-synchronous trading model specification is warranted.

The dissertation's last and seventh essay employ the suggested methodology from essay six and sets out to perform a mean and volatility specifications individual assets controlling for non-synchronous trading and changing volatility relative to a classical synchronous and constant volatility specification. To take the advantages of model specification a simultaneous estimation and event period estimation is performed. The essay test for the proportion for asset return series appropriately specified using non-synchronous trading and changing volatility relative to the alternative of synchronous trading and constant volatility. Finally, changes in model preferences are evaluated and discussed. The results are important for the shareholders for acquirers and sellers, corporate boards, regulators and policy makers owing to (1) do shareholders of acquirers (sellers) really obtain significant abnormal return in event periods and (2) how efficient does the market reacts to event announcements? The first question may signal a need for reduced merger and acquisition enthusiasm from especially acquirers and the market for corporate control. The second question may induce rejection of the semi-strong efficient market hypotheses suggesting weaker form market efficiency. Both questions are important for traders as well as portfolio managers applying event information in their daily trading and portfolio rebalancing. Finally, the test statistics may show the need for rework of classical event studies, which applied synchronous trading and constant volatility assumptions.

Consequently, the seven essays constitute a collection of essays investigating mean and volatility effects from non-synchronous trading. Each essay applies econometric techniques to solve conditional mean and volatility modelling to account for non-synchronous trading effects in observed return series for equity markets. The essays aim to show the importance of controlling for these effects in formal model specifications over several topics applying techniques in financial econometrics. The treatments are not comprehensive so new findings may be achieved in other areas of international empirical finance. For example, temporal aggregation for non-traded assets may be an interesting and extended investigation for illiquid equity markets. The essays are arbitrarily ordered and therefore cross-referenced. The research articles conform to the generally accepted standards of scientific inquiry and provide pragmatic interpretations of findings. As each essay is intended for international journal publication, each essay is complete and can be read independently of each other. Some of the dissertation's essays may therefore overlap in especially the data and data adjustment sections, as the data series are from the same equity market and period.

Chapter 2 Unifying theme and core hypotheses

The dissertation's investigations bring evidence on workings of financial markets showing non-synchronous trading effects. In particular, the dissertation investigates conditional mean and volatility characteristics due to non-synchronous trading effects in several financial domains. As non-synchronous trading suggests an unequally spaced return interval, it is of outmost importance to find appropriate model specifications for liquid as well as illiquid return series. The dissertation aims therefore to apply and extend financial econometric techniques solving some issues in non-synchronous trading effects. Importantly, unequal spaced return intervals are most likely observed in markets showing low trading volume. The essays aims therefore to construct model specifications that eliminate potential serious biases in the moments and co-moments of asset return in thinly traded equity markets. The investigations may therefore bring new insight into pricing mechanism and market dynamics for these asset series. The fact that many equity markets contain return series exhibiting non-synchronous trading, controlling for non-synchronous trading applying conditional mean and volatility specifications may find interest in many international equity markets.

The investigations are performed and implemented in three topics in financial econometrics. The first topic is univariate studies applying asset, portfolio and index return series. The second topic is one-factor asset pricing models and especially the conditional Capital Asset Pricing Model. The third topic in financial econometrics is abnormal return calculations employing the market model in classical event studies.

The investigation of non-synchronous trading effects is performed applying formal conditional mean and volatility lag specification using observed monthly, weekly and daily return series from the Norwegian equity market. The time-period we investigate is from 1983/84 to 1995, constituting approximately 125 months, 520 weeks and 2600 days of return observations. Firstly, we investigate the conditional mean equation assuming drift and serial correlation in observed return series. During periods of strong non-synchronous trading (non-trading) an asset's observed return is zero. When it does trade, its observed return reverts to the cumulated mean return. Therefore intuitively, illiquid series suggest moving average and negative serial correlation over time. Secondly, we investigate the latent conditional volatility equation assuming elements from the long-run average volatility, moving average and serial correlation. Again, during periods of strong non-synchronous trading (non-trading) an asset's observed return is zero. When it does trade, its observed return is the cumulated mean return, suggesting larger model error terms. These error terms are important ingredients for the changing volatility modelling. Intuitively and relatively constant volatility, non-synchronous trading suggests higher weight to the average long-run volatility, lower moving average and stronger serial correlation in the latent volatility. Thirdly, we investigate whether the conditional mean and volatility specifications show model misspecifications. Misspecifications suggest systematic factors in model residuals. Therefore, intuitively, if our model specification is rejected, the conditional mean and volatility specification does not appropriate model non-synchronous trading effects.

The core hypotheses over financial econometrics topics are mean and volatility parameter and lag changes due to non-synchronous trading effects. Moreover, non-synchronous trading in its extreme may cause formal model misspecifications (systematic factors), suggesting a need for more elaborate model specifications. To test for non-synchronous trading effects that our formal model specifications cannot control for, we use the filtered residual series and employ elaborate test statistics testing for model misspecifications. Hence, we hypothesise model misspecification for extreme non-synchronous trading. The investigations are performed to identify implications for classical economic theories and well-known empirical international results. As non-synchronous trading effects will most likely be observed in low-liquid markets showing low trading volume, the Norwegian equity market may be a good choice for time series selections and characteristics (see chapter 4). The unifying theme and core hypotheses in this dissertation's seven essays and three financial econometric topics are identified and discussed below.

The first essay from grand mean and variance ratios, proposes mean and volatility ratio hypotheses applying Brownian motions in asset specifications with grand mean drift (μ) and constant volatility diffusion (σ^2) for open (Monday-Friday) and closed markets (Saturday, Sunday and holidays). The essay's two core propositions hypothesise mean and volatility ratios, proportional to the number of non-trading days in both open and closed equity markets. Moreover, the essay investigates an adjustment to the Brownian Motions volatility assuming Poisson distributed trade arrivals. For investors, portfolio and fund managers and liquidity traders in equity markets, the results may explain observed increased volatility at close and open. Moreover, mean and volatility ratio changes across non-trading grand totals may suggest asset return predictability.

The second essay applies the Bayes Information Criterion (BIC) (Schwarz, 1978) modelling conditional mean and volatility specifications hypothesising changing mean and volatility across portfolio and index series. The essay's first proposition hypothesises serial correlation and moving average for the conditional mean equation (changing mean). The second proposition hypothesises average long-run volatility weight, moving average and serial correlation in the latent conditional volatility (changing volatility). The third main proposition hypothesises model misspecification due to inappropriate conditional mean and volatility specifications. Symptoms of systematic mean and volatility factor characteristics suggest spurious parameter results and systematic factors suggesting asset return predictability. Note that the predictability is conditional on identification of systematic factors contributing to insignificant model misspecification test statistics.

The third essay extends the model misspecification investigations in essay no. two, applying (non-) linear mean and volatility specifications across asset, portfolio and index series. The essay investigates three core hypotheses. The first core proposition hypothesises model misspecifications applying a linear mean model and constant volatility (OLS-specification). The second core proposition hypothesises model misspecification applying a non-linear mean and a constant volatility. The third core proposition hypothesises model misspecification applying a linear mean and a conditional heteroscedasticity specification for the volatility. Finally, the fourth core proposition hypothesises model misspecification applying a non-linear mean and a conditional heteroscedasticity specification for the volatility. Consequently, the model tests the Martingale (linear mean) and the Independence hypothesis (constant volatility). A non-linear mean will make intuitive, analytic and linear reasoning in equity markets extremely difficult. Non-linear volatility suggests a changing volatility specification.

The interesting and important issue of whether less liquid equity markets alters known facts and dynamics in one-factor models as for the capital asset pricing model (CAPM) are hypothesised in essay four and five. The fourth essay performs a univariate investigation of the conditional CAPM. The essay proposes three core hypotheses. The first proposition hypothesises a zero intercept (drift) parameter and an influential variance measured by a constant aggregate risk aversion coefficient for all three return intervals. These two is elements constitute the conditional CAPM proposition. The second hypothesis is a conditional versus an unconditional volatility specification for all three return intervals. Finally, the third proposition hypothesises model misspecifications for all return intervals and model specifications. Misspecification suggests missing systematic factors and an inappropriate conditional model specification.

A bivariate investigation of the one-factor model (CAPM) is performed in essay five. The essay proposes two core hypotheses due to the fact that the specification gives access to the whole variance-covariance matrix. The first proposition hypothesises mean equation effects from the conditional covariance series (a contemporaneous CAPM specification), the conditional variance series (residual risk specification) or the one-dynamic factor series (market variance specification) across all return series. The second proposition hypothesises model misspecification for all bivariate series. Model misspecification rejects the changing mean and volatility specification suggesting invalid and spurious mean effects from the various forms of the conditional variance equation.

The two last essays in the dissertation investigate non-synchronous trading effects applying the market model in event studies. Classical event studies in financial markets apply important assumptions about synchronous trading and constant volatility across event and non-event periods. The two essays hypothesise changing mean and volatility from non-event to event period in classical event studies. That is, the effects from changing non-synchronous trading and volatility suggest parameter differences from non-event to event periods. The result may reject earlier empirical abnormal return findings. Moreover, many international authors have warned about failures in classical event studies. An investigation hypothesising a rejection of synchronous trading and constant volatility from non-event to event period, suggests therefore a need for changing mean and volatility market model specifications in classical event studies.

The sixth essay hypothesises a need for changing mean and volatility specification from non-event to event period due to changes in non-synchronous trading and conditional heteroscedasticity. The essay applies a Norwegian sample of mergers and acquisitions. By grouping asset series for sellers, acquirers and both, the essay hypothesises effects from non-synchronous trading. The core proposition hypothesises conditional mean and volatility changes from non-event to event period. The proposition hypothesises changes in mean and volatility characteristics from non-event to event periods. Significantly changing non-synchronous trading and the volatility will suggest a need for a new conditional mean and volatility specification for event studies.

The seventh essay hypothesises a non-synchronous trading and changing volatility specification versus synchronous trading and constant volatility performing a classical event study for the merger and acquisition firm sample in Norway first reported in Eckbo and Solibakke (1992). BIC preferred lag specifications for mean and volatility due to non-synchronous trading are enforced. The first proposition hypothesises an unchanged number of misspecification over methodologies across all assets. The second proposition hypothesises no changes in parameter values and abnormal returns across the methodologies.

Methodologies for modelling non-synchronous trading and changing volatility with diagnostics for model misspecifications are presented in chapter 3.

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Chapter 3 Methodology

The dissertation's seven essays investigate non-synchronous trading effects in three different econometric topics in corporate finance. The first topic investigates mean and volatility characteristics in univariate time series. The second topic investigates conditional mean and volatility specifications for one-factor models (CAPM) and finally the third topic investigates conditional mean and volatility processes calculating abnormal returns applying the market model in event studies. All seven essays are empirical work using data series from the Norwegian equity market. The papers investigate mean and volatility characteristics from non-synchronous traded relative to continuously traded series. This chapter specifies empirical methodologies in modelling non-synchronous trading effects in mean and volatility equations for the Norwegian equity market. The chapter also briefly described test statistics for appropriate model specifications.

3.1 Introduction

The non-synchronous trading effect induces potentially serious biases in the moments and co-moments of asset returns. Consequently, the means, variances, covariance, betas and (cross-) serial correlation coefficients may become spurious. Consider, for example the daily closing prices of firms quoted on the Oslo Stock Exchange and reported daily by the financial press. Note that the closing price reported is the price at which the last transaction occurred on the previous day. In a thinly traded equity market the closing price will generally not occur at the same time each day. Hence, the asset may on one particular Monday quote its last reported trade at 14$^{\underline{05}}$, which will become the closing price reported by the financial press that particular Monday even though the Oslo Stock Exchange closes at 16$^{\underline{00}}$. Moreover, the following day Tuesday, the last quoted trade was reported at 15$^{\underline{15}}$. This example shows that referring to them as "daily" prices, we have implicitly and incorrectly assumed that they are equally spaced in 24-hour intervals. Moreover, many firms listed on Oslo Stock Exchange reported zero trading volume for several days in 1999, that is, several days of non-trading. The Norwegian equity market exhibits low trading volume relative to elaborate markets in US and UK and contains assets that show low trading volume relative to continuously traded assets (see chapter 4). The market may therefore exhibit strong non-synchronous trading effects. In the same vein as for the information flow, the characteristics of thinly traded assets may not be the same as that for actively traded assets (Gallant, Rossi and Tauchen, 1992).

Several methodologies may be applied modelling non-synchronous trading effects. Internationally, several theoretical and empirical studies have investigated univariate conditional mean and volatility effects from non-synchronous trading (Lo & MacKinlay, 1990). Moreover, several studies have investigated non-synchronous trading effects on the Capital Asset Pricing Model and the Arbitrage Pricing Theory (Sharpe & Williams, 1977). These studies focus on changes in the mean process. In contrast, this dissertation aims to model non-synchronous trading effects in both the mean and the latent volatility. Campbell et al. (1997) in their non-synchronous trading model, suggests low mean effects, variance changes and serial correlation in the mean equation.

The non-synchronous trading effects will in this dissertation apply ARMA-GARCH methodologies[1], which have its origin from Engle (1982) and Bollerslev (1986, 1987). The return process will in this methodology applies ARMA specification for the conditional mean and (G)ARCH for the conditional volatility. Therefore, the mean equation contains an explicit modelling of drift, autoregressive and moving average effects and the volatility equation contains modelling of weight to long-run average volatility, moving average and serial correlation[2]. For all the lag specifications all the essays employ the Bayes Information Criterion (BIC) for efficient lag specification for the conditional mean. The Schwarz Bayes information criterion is computed as $BIC = s_n(\hat{\theta}) + \frac{1}{2} \cdot \frac{p_\theta}{n} \cdot \log(n)$ with small values of the criterion preferred. The criterion reward good fits as represented by small $s_n(\hat{\theta})$ but uses the term $+\frac{1}{2} \cdot \frac{p_\theta}{n} \cdot \log(n)$ to penalise good fits obtained by means of excessively rich parameterisations. The criterion is conservative in that it selects sparser parameterisations than the Akaike AIC Information criterion (Akaike, 1969). Schwarz is also conservative in the sense that it is at the high end of permissible range of penalty terms in certain model selection settings (Potscher, 1989).

The conditional mean process apply an ARMA specification to control for non-synchronous trading effects. The conditional volatility process apply (G)ARCH (Generalised AutoRegressive and Conditional Heteroscedasticity) specification for the changing volatility specification[3]. As for the conditional mean, all papers employ the Bayes Information Criterion (BIC) for efficient lag specification for the conditional volatility. Engle (1982) shows that a test of the null hypothesis that $\varepsilon_{i,t}$ has a constant conditional variance against the alternative that the *ARMA* theory follows through. That is, employing the squared residual $\varepsilon_{i,t}^2$ we can identify *u* and *n* in an ARMA (*u,n*) specification for the conditional variance by applying the same methodology as the conditional mean ARMA (p,q) modelling in the previous section. Below we give an overview of the ARMA-GARCH methodologies employed in all the seven papers constituting this dissertation.

3.2 ARMA-(G)ARCH-in-Mean lag specifications with extensions

The history of ARMA-G)ARCH models is a short one but the literature has grown in a spectacular fashion. The model has been applied to numerous economic and financial series. However, it has seen relatively less theoretical advancement. The ARMA-(G)ARCH model has been applied to numerous and diverse areas. For example, it has been used to test the CAAPM, the I-CAPM, the

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[1] ARMA is an abbreviation for AutoRegressive and Moving Average returns and GARCH is an abbrebiation for Generalised AutoRegressive and Conditional Heteroscedasticity volatility.
[2] Note that the dissertation does not focus on stationary time series. Differencing of the time series is therefore not investigated.

CCAPM and the APT; to develop volatility tests for market efficiency and to estimate the time varying systematic risk in the context of the market model. In macroeconomics, it has been used to construct debt portfolios for developing countries, to measure inflationary uncertainty and to examine the relationship between exchange rate uncertainty and trade. For our purpose the ARMA-(G)ARCH model is useful because it may capture some non-synchronous trading effects in illiquid markets.
In univariate and simplest possible form we specify ARMA (p,q)-GARCH(m,n)-in-Mean models through the following equations (1)-(4):

$$y_t = \phi_0 + \sum_{i=1}^{p} \phi_i \cdot y_{t-i} + \rho_1 \cdot h_t^{\frac{1}{2}} + \upsilon_t \tag{1}$$

$$\upsilon_t = \varepsilon_t + \sum_{j=1}^{q} \theta_j \cdot \varepsilon_{t-j} \tag{2}$$

$$\varepsilon_t \sim N(0, h_t) \text{ og } D(0, h_t, \omega) \tag{3}$$

$$h_t = \alpha_0 + \sum_{i=1}^{m} \alpha_i \cdot \varepsilon_{t-i}^2 + \sum_{j=1}^{n} \beta_j \cdot h_{t-j} \tag{4}$$

Equation 1 is the structural mean specification (for both linear and non-linear models), where y is price change and y_{t-i} is lagged price changes. Equation 2 defines moving average, which is modelled by measuring lagged residuals effect on price changes. Equation 3 defines the distribution of the residuals (ε_t); that is a normal distribution $N()$ or a student-t distribution $D()$ with ω degrees of freedom. Finally, equation 4 specifies the structural form of the conditional volatility (h_t). ϕ is the vector for lagged price changes (the AR-process), θ is the vector for lagged residuals (the MA-process), α_0 is a parameter for the weight to the long-run average volatility, α_t is the vector for the weights of the lagged and squared residuals ε^2_{t-i} (the ARCH-process) and β is the weights for the lagged conditional volatility h_{t-j} (the GARCH-process). From these specifications we obtain three important features for our models for non-synchronous trading effects in illiquid markets. Firstly, modelling serial correlation in the mean. Secondly, modelling unconditional homoscedasticity[4] but conditional heteroscedasticity (changing conditional volatility). Thirdly, as ARMA-GARCH applies a maximum likelihood algorithm we are able to model the high kurtosis and skew (leptokurtosis[5]) often found in illiquid markets, by applying student-t distributed likelihood functions[6] with the degree of freedom estimated by the model.
Moreover, Ding et al. (1993) extends the symmetric GARCH model into asymmetric GARCH. Asymmetric GARCH (AGARCH) models the volatility as (5):

$$h_t = \alpha_0 + \sum_{i=1} \alpha_i \cdot (|\varepsilon_{t-i}| - \gamma_i \cdot \varepsilon_{t-i})^{\delta} + \sum_{j=1} \beta_j \cdot h_{t-j} \tag{5}$$

where α_t is the vector for the weights of the lagged residuals ε^{δ} (the ARCH-process). For the classical asymmetric model we define $\delta = 2$, while in "power" AGARCH model we also estimate δ. It is γ_i that measures asymmetry in the volatility. The mean equation is identical to (1) – (3). Especially one model

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[3] For the stochastic volatility specification see Solibakke (2001b).
[4] See Morgan, 1976.
[5] Departure from normally distributed price changes.
[6] See Gouriéroux, C., 1997 og Campbell et al., 1997.

has been applied many times in the international finance literature. The truncated GARCH (GJR) (Glosten et al, 1993) specifies the volatility as (6)-(7):

$$\lambda_{it} = \gamma_i \quad \textit{if and only if} \quad \varepsilon_{t-i} < 0 \tag{6}$$

$$h_t = \alpha_0 + \sum_{i=1} (\alpha_i + \lambda_{it}) \cdot \varepsilon_{t-i}^2 + \sum_{j=1} \beta_j \cdot h_{t-j} \tag{7}$$

If $\lambda_{it} > 0$, the GJR specification will generate higher values for h_t when $\varepsilon_t < 0$ then when $\varepsilon_t > 0$; otherwise equal in absolute size. The mean equation is identical to (1) – (3). The Exponential GARCH model (EGARCH) (Nelson, 1991) specifies the volatility by using the natural logarithm. The EGARCH model specifies the volatility as (8)-(9):

$$\varepsilon_t \sim \sqrt{h_t} \cdot \upsilon_t \tag{8}$$

$$\ln h_t = \beta_0 + \sum_{j=1} \beta_j \cdot \ln h_{t-j} + \sum_{i=1} \gamma_i \cdot (\theta_0 \cdot \upsilon_{t-i} + \gamma_0 \cdot \{| \upsilon_{t-i} | - E | \upsilon_t |\}) \tag{9}$$

Equation 8 shows the distribution of the residuals[7] and Equation 9 shows the structure in the conditional volatility. θ_0 in Equation 8 defines the asymmetric volatility and υ measures the thickness of tails in the distribution. Note that for all the GARCH specifications we require that $\sum_{i=1}^{m} \alpha_i + \sum_{j=1}^{n} \beta_j < 1$, with the exception of EGARCH. EGARCH requires that $\beta_j < 1$. Also for this model the mean equation is defined by (1) – (3). Finally, it is reasonable to expect that the mean and variance of a return move in the same direction. Hence, introducing $\sqrt{h_t}$ into the mean equation represents a measure of time varying risk premium especially applicable for financial models. M(aximum) L(ikelihood) estimates of the (G)ARCH-in-Mean model can be obtained by maximising the likelihood function using the BHHH[8] algorithm. Note however, that the information matrix is no longer block diagonal, so that all the parameters must be estimated simultaneously. This requires an iterative solution technique[9], also known as non-linear optimisation. For estimation details see Section 3.6 below.

3.3 Interpretations of ARMA specifications

The ARMA specification in Equation 1 and 2 represent a serially correlated time series; that is, a changing mean process. The ϕ_0 parameter represents a drift (predictable), ϕ_i represent autoregressive effects and θ_i represent moving average effects. Significant serial correlation in the mean equation suggests a rejection of the random walk hypothesis. One of the main contributors to serial correlation seems to be non-synchronous trading. A high frequency (e.g., fitted to daily data) ARMA process aggregates to a low frequency (fitted to say, weekly data) ARMA process. Hence, an efficient conditional mean equation may control for non-synchronous trading removing any forms of systematic

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[7] In the exponential GARCH model we assume a General Error Distribution, which contains the normal distribution as a special case ($\upsilon = 2$).
[8] the BHHH algorithm is described in: Berndt, Hall, Hall, Hausman (1974).
[9] the technique is available in GAUSS ver 3.2.1.

factors from the model residuals. Reported ARMA coefficients may therefore suggest predictability in asset returns assuming constant non-synchronous trading and information flow. Moreover, multivariate ARMA specifications may reveal cross serial correlation. Lead and lag structures between asset and portfolio series may give additional insight into the way non-synchronous trading contributes to serial correlation. However, if non-synchronicity is purposeful and informational motivated, then the serial correlation should consider genuine and purely statistical models of non-synchronous trading are inappropriate.

3.4 Interpretations of (G)ARCH specifications

The main reason for the success of ARCH models is that they take account of many observed features of the data. Features as thick tails of the distribution, clustering of large and small observations, non-linearity and changes in our ability to forecast future values are easily modelled giving a wide range of interpretation alternatives to the models. The first interpretation was based on the fact that econometricians' ability to predict the future varies from one period to another. Using a conditional mean model usually does predictions. Uncertainty about the conditional mean can be expressed by a random coefficient formulation[10]. The second interpretation is a conditional mixture model following the work of Clark (1973) and Tauchen and Pitts (1983). An interesting rationale for the presence of conditional heteroscedasticity and heterogeneity in the higher order moments of asset prices is presented in Gallant et al. (1991). Assuming that the observed return process yt can be written as $y_t = \mu_t + \sum_{i=1}^{I_t} \zeta_i$ where $\zeta_i \sim$ IID $N(0,\tau^2)$. The μ_t can be interpreted as a predictable component, the ζ_i's are the incremental changes and I_t is the number of times new information comes to the market in period t. I_t is a serially dependent unobservable random variable and is independent of $\{\zeta_i\}$. The randomness of I_t produce a non-normally distributed y_t; it is in fact a mixture of normal distributions. Note, we can view y_t as a subordinated stochastic process, where $y_t - \mu_t$ is subordinate to ζ_i, and I_t is the directing process. We can now write $y_t = \mu_t + \tau I_t^{\frac{1}{2}} v_t$ with $v_t \sim N(0,1)$. Hence, conditional on the information set Ω_{t-1} and I_t, the conditional heteroscedasticity normal distribution emerges $y_t \mid \Omega_{t-1}, I_t \sim N(\mu_t, v^2 I_t)$. In practice, since I_t is not observable we can only work with the conditional distribution $y_t \mid \Omega_{t-1}$. Following Gallant et al. (1991) the conditional variance is $E\left[(y_t - \mu_t)^2 \mid \Omega_{t-1}\right] = \tau^2 E\left[I_t \mid \Omega_{t-1}\right]$. Denoting $y_t - \mu_t = \tau I_t^{\frac{1}{2}} v_t$ as the error term ε_t, the covariance becomes $Cov(\varepsilon_t^2, \varepsilon_{t-j}^2) = \tau^4 Cov(I_t v_t^2, I_{t-j} v_{t-j}^2) = \tau^4 Cov(I_t, I_{t-j})$. Now assuming non-synchronous trading in a market, it is quite plausible that the I_t's is serially dependent introducing correlation into the squared errors. The (G)ARCH specification tries to capture this correlation.

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[10] For details see Bera et al. (1990, 1992) and Sentana (1991).

The third interpretation is a non-linear model specification. One of the essential features of ARCH models is $Cov(\varepsilon_t^2, \varepsilon_{t-j}^2) \neq 0$, although $Cov(\varepsilon_t, \varepsilon_{t-j}) = 0$, for $j \neq 0$. In other words, (G)ARCH postulates a non-linear relationship between et and its own past values. Moreover, note that changes in volatility are represented by changes in the conditional variance, linking volatility to a natural measure of risk. Two more important interpretations are found in the international literature. Mizrach (1990) developed a model of asset pricing and learning in which (G)ARCH disturbances evolve out of the decision problem of economic agents. Errors made by agents are very persistent and dependent on all past errors, leading the conditional variance to have an (G)ARCH like structure. Finally, Stock (1988) established the link between time deformation and (G)ARCH models. Any economic variable, in general, evolves on an "operational" time scale, while in practice it is measured on a "calendar" time scale. A time deformation model of a random variable et can be approximated by $\varepsilon_t = \rho_t \varepsilon_{t-1} + v_t$, where $v_t \mid \Omega_{t-1} \sim N(0,h_t)$, where $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2$. Hence, when a relatively long segment of operational time has elapsed during a unit of calendar time, ρ_t is small and h_t is large. A number of researchers have investigated the relationship between serial correlation and volatility[11]. The general result after introducing ARCH into the model[12], was that lower correlations were connected with periods of high volatility. Non-synchronous trading and accumulation of news are possible explanations. As some assets do not trade close to the end of the day and information arriving during that period is reflected on the next days trading, serial correlation may emerge in price changes. Furthermore, non-synchronous trading results in overall lower trade volume, which has a strong positive relationship with volatility. Note especially, that when new information reaches the market very slowly, for traders the optimal action is to do nothing until enough information is accumulated, leading to low trade volume and high correlation.

These interpretations of ARCH effects suggest considerable affects from non-synchronous trading and accumulation of information. This thesis may bring new knowledge into the international changing volatility literature suggesting origins of (G)ARCH effects.

3.5 Multivariate ARMA-(G)ARCH lag specifications

Inter-related economic variables are a well-known fact. Hence, extension from univariate to multivariate models is quite natural. Apart from possible gains in efficiency in parameter estimation, estimation of a number of financial time series such as the systematic risk (beta) and the hedge ratio requires sample values of covariance between relevant variables. Moreover, multivariate (G)ARCH may also stem from the fact that many economic variables react to the same information, and hence,

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[11] See for example Kim (1989), Sentana and Wadhwane(1990), Ødegård (1991) and LeBaron (1992).
[12] Ødegård (1991) found that autocorrelation decreased over time, which he attributed to new financial markets. However, introducing (G)ARCH into the models, the evidence of time varying autocorrelation became very weak.

have nonzero covariance conditional on the information set. Applying two variables and using the notation from 3.4 and the non-synchronous trading and aggregation of information interpretation, let $y_{1,t} = \mu_{1,t} + \tau I_t^{\frac{1}{2}} v_{1,t}$ and $y_{2,t} = \mu_{2,t} + \tau I_t^{\frac{1}{2}} v_{2,t}$, where $y_{1,t}$ and $y_{2,t}$ are two time series, driven by the same directing process I_t, and $\begin{pmatrix} v_{1,t} \\ v_{2,t} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & c_{12} \\ c_{12} & 1 \end{pmatrix}\right]$. The $\begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} | \Omega_{t-1}, I_t \sim$ $N\left[\begin{pmatrix} \mu_{1,t} \\ \mu_{2,t} \end{pmatrix}, I_t \begin{pmatrix} \tau_1^2 & c_{12}\tau_1\tau_2 \\ c_{12}\tau_1\tau_2 & \tau_2^2 \end{pmatrix}\right]$ is the bivariate counterpart of the univariate specification and provides a rationale behind higher dimensional processes. H_t can be expressed in either vector form (VECH) or classical square matrix operations (BEKK). As the BEKK representation is almost guaranteed to be positive definite, it will be easier to estimate. Moreover, the BEKK representation will normally require a lower number of parameters to be estimated. Several attractive ways of simplifying H_t has been proposed in the literature. Finally, Diebold and Nerlove (1989) introduced that only a few factors influence all variables $(y_1, \ldots, y_N)$ and their conditional variances. The suggested a one factor multivariate ARCH model represented by $y_t = \lambda F_t + \eta_t$, where $\eta_t = (\eta_{1,t}, \ldots\ldots, \eta_{N,t})$, $\eta_{i,t} \sim (0, \sigma_{i,i})$, $i=1, \ldots, N$ and the unobservable factor F_t is conditionally distributed as $F_t | \Omega_{t-1} \sim N(0, h_t)$. Then $Var(y_t | \Omega_{t-1}) = h_t \lambda\lambda' + diag(\sigma_{11}, \ldots\ldots, \sigma_{NN})$ and we can specify a univariate GARCH process for h_t. The effect of the common factor F_t on y_i is measured by λ_i $(i=1, \ldots\ldots, N)$.

3.6 Estimation

The estimation of ARMA-GARCH specifications from historical data is conducted by an approach known as the maximum likelihood method. It involves choosing values for the parameters that maximize the chance (or likelihood) for the data occurring. The analysis allows the return series (y_t) to follow a ARMA (P,Q) process, so that the ARMA model becomes $\phi(B)(y_t - \mu) = \theta(B)\varepsilon_t$, where B is the lag operator and the GARCH(M,N) model becomes

$h_t = E[\varepsilon_t^2 | \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots] = a_0 + \sum_{i=1}^{M} a_i \varepsilon_{t-i}^2 + \sum_{i=1}^{N} \beta_i h_{t-i}$. This latter equation can be written as

$h_t = z_t' \omega = z_{1t}' \omega_1 + z_{2t}' \omega_2$, where $z_t' = (z_{1t}' : z_{2t}') = (1, \varepsilon_{t-1}^2, \ldots, \varepsilon_{t-M}^2; h_{t-1}, \ldots, h_{t-N})$,

and $\omega' = (\omega_1' : \omega_2') = (a_0, a_1, \ldots, a_M; \beta_1, \ldots, \beta_N)$. Using this notation, maximum likelihood estimates of the ARMA-GARCH model can be obtained following this procedure. Define Θ as the vector of parameters in the model given by the mean equation and the volatility equation and partition it as $\Theta = (\omega' : \varphi')$; $\varphi' = (\phi_1, \ldots, \phi_M; \theta_1, \ldots, \theta_N, \mu)$ being a vector containing the parameters in the mean equation. We can also define $\Theta_0 = (\omega_0' : \varphi_0')$ as the true parameter vector. The log-likelihood function for a sample of T observations is, LT(Q) = $T^{-1}\sum_{t=1}^{T} l_t(\Theta)$, where $l_t(\Theta) = -.5*(ln(2*pi) + (\varepsilon./h)^\wedge 2 + 2*ln(h))$ in the case of normal distributed returns and $l_t(\Theta) = c - 0.5*ln(h) - ((\varpi+1)/2)*ln(1 + (\varepsilon^\wedge 2)./((\varpi-2)*h))$, in the case of student-t distributed returns, where $c = lnfact(0.5*(\varpi+1)-1) - 0.5*ln(pi*(\varpi-2)) - lnfact(0.5*\varpi-1)$ and ϖ is the estimated parameter for the degree of freedom in the student-t distribution. Precise details of maximum likelihood estimation may be found in, Engle (1982), Weiss (1986a, 1986b) and Bollerslev (1988). Several estimation algorithms are available for computation. Among others, the Berndt, Hall, Hall and Hausmann (BHHH, 1974) and the Broyden, Fletcher, Goldfarb, and Shanno (BFGS, 1980) algorithms are readily available. The maximum likelihood estimate $\hat{\Theta}_{ML}$ is strongly consistent for Θ_0 and asymptotically normal with mean Θ_0 and a covariance matrix ζ^{-1}, consistently estimated by $T^{-1}(\sum_{t=1}^{T}\frac{\partial l_t}{\partial\Theta}\frac{\partial l_t}{\partial\Theta'})^{-1}$, which may be obtained from a last BHHH iteration.

3.7 Diagnostics for model misspecifications

For all essays appropriate model specification is one of several important findings. Below we therefore describe the methodologies used to test for data dependence in model residuals; that is, diagnostics suggesting model misspecifications.

3.7.1 The ARCH test statistic

The ARCH test statistic (Engle, 1982) is a test for constant conditional variance against conditional heteroscedasticity, based on the Lagrange Multiplier principle. The test procedure is to run a regression of the squared residuals on a constant and p lagged squared residuals. Then test the test statistic $T \cdot R^2$ as a $\chi^2(p)$ variate, where T is the sample size and R^2 is the squared multiple correlation coefficient and p is the degree of freedom. The ARCH test is a test for H_0: constant conditional variance against the alternative H_a: a conditional variance that obey an ARCH(p) specification. In fact, if ARCH is present in the residuals, non-linear dependence in the time series cannot be rejected.

3.7.2 The RESET test statistic

The Regression Error Specification Test (RESET; Ramsey, 1969) is a test statistic of linearity against an unspecified alternative. It is a test against general model misspecification[13] and has certainly been one of the most popular tests against misspecification of functional form.
In this paper it is carried out in three stages as follows:

[13] See also Tsay (1986), Spanos (1986) and Lee et al. (1993).

(1) We assume the linear part of the model is

$$y_t = \beta' \cdot z_t + u_t, \qquad t = 1,\dots,T$$

where $z_t = (1, y_{t-1}, \dots y_{t-p}, x_{t1}, \dots, x_{tk})'$. We estimate β by OLS and compute $\hat{u}_t = y_t - \hat{y}_t$ where $\hat{y}_t = \hat{\beta}' z_t$, and $SSR_0 = \sum \hat{u}_t^2$.

(2) Then we estimate the parameters of $\hat{u}_t = \delta' z_t + \sum_{j=2}^{h} \varphi_j \cdot \tilde{z}_t^{(j)} + \upsilon_t$ by OLS and compute $SSR = \sum \hat{\upsilon}_t^2$, where $\tilde{z}_t^{(j)} = (y_{t-1}^j, \dots, y_{t-p}^j, x_{t1}^j, \dots, x_{tk}^j)$, $j = 2,\dots,h$.

(3) Finally, we compute the test statistic: $F = \dfrac{(SSR_0 - SSR)/(h-1)}{SSR/(T-m-h)}$

where $m = p+k$. k is in our case zero. As z_t contains lags of y_t, then $(h-1)F$ has an asymptotic χ^2 distribution under the null of linearity. h was suggested by Thursby and Schmidt (1977) to be given the value 4 for the best result. This test is an Lagrange Multiplier (LM) type test against an Logistic Smooth Transition Regression (LSTR) model in which only one 'linear parameter' changes but the investigator does not know which one. The RESET test is thus rather narrow in that if more than one variable has a 'changing linear parameter' the regression no longer covers that possibility. Note, however, that the constant in the first regression should not be involved in defining the z_t and $\tilde{z}_t$ in the auxiliary regression, since the inclusion of such regressors would lead to perfect collinearity.

3.7.3 The BDS test statistic

3.7.3.1 The correlation integral

The correlation integral proposed by Grassberger and Procaccia (1983) is a measure of spatial correlation in an m-dimensional space. Let $\{\mu_t\}$ be a real-valued scalar time-series process. Construct the m-history process $\mu_t^m \stackrel{def}{=} (\mu_t, \mu_{t+1}, \cdots, \mu_{t+m-1})$. For $\varepsilon > 0$, the correlation integral at *embedding dimension m* is given by[14] $C_{m,\varepsilon} = \iint \chi\varepsilon(x^m, y^m) dF(x^m) dF(y^m)$, where $\chi \cdot \varepsilon(\cdot,\cdot)$ is the symmetric indicator kernel with $\chi \cdot \varepsilon(x, y) = 1$ if $||x - y|| < \varepsilon$ and 0 otherwise (indicator function), $||\cdot||$ represents the max-norm, and $F()$ is the distribution function of μ_t^m. $C_{m,\varepsilon}$ gives the mean volume of a cube with diameter ε. An estimator of the correlation integral for a sample size T for the process $\{\mu_t\}$ is

[14] If $\{\mu_t\}$ is a strictly stationary, absolutely stochastic process, the integral defined below exists.

given by the following U-statistic— cf. BDS (1987), $C_{m,\varepsilon} = \frac{1}{\binom{\bar{T}}{2}} \sum \sum_{1 \le s < t \le \bar{T}} \chi \cdot \varepsilon(\mu_t^m, \mu_s^m)$, where

$\bar{T} = T - (m-1)$.

3.7.3.2 The test statistic

Brock et al. (1988), Deckert (1991) and Scheinkman (1990), henceforth BDS (Brock, Deckert and Scheinkman), developed a test based on concepts that arise in the theory of chaotic processes. The BDS test statistic is a test of the null hypothesis of i.i.d. for a univariate time series against an unspecified alternative. That is, if $\{\mu_t\}$ is an i.i.d. process, then $C_{m,\varepsilon} = C_{1,\varepsilon}^m$, almost surely, for all $\varepsilon > 0$, $m = 1, 2, \ldots$. The BDS test presents the following result $V_{m,\varepsilon} = \sqrt{T} \cdot \frac{C_{m,\varepsilon} - (C_{1,\varepsilon})^m}{s_{m,\varepsilon}} \xrightarrow{d} N(0,1)$, $\forall \varepsilon > 0$, $m=2,3,\ldots$, where $s_{m,\varepsilon}$ is an estimator of the asymptotic standard deviation—$\sigma_{m,\varepsilon}$—of $\sqrt{T} \cdot (C_{m,\varepsilon} - (C_{1,\varepsilon})^m)$ under the null of i.i.d. Brock et al. (1991) used Monte Carlo methods to evaluate the choice of m and ε on the asymptotic normality of $V_{m,\varepsilon}$. Their results suggest that asymptotic normality of $V_{m,\varepsilon}$ holds well for sample sizes of at least 1000 observations, and for value of ε between 0.5 and 2 standard deviations of the data. They warned against relying on asymptotic normality for values of T/m less than 200 observations.

The BDS test has been shown to be robust to the non-existence of fourth moments, which may characterize stock returns (Brock and de Lima, 1995 and Hsieh, 1991). Hsieh (1991) points out that the robustness of the BDS test to the non-existence of fourth moments is one of the advantages of the BDS test over other tests of non-linearity such as Tsay (1986) and Hinich and Patterson (1985). Moreover, the BDS test statistic has power against models that are non-linear in variance but not in mean, as well as models that are non-linear only in mean. That is, a BDS rejection does not necessarily mean that a time-series has a time-varying conditional mean; it could simply be evidence for a time-varying conditional variance (Hsieh, 1991).

One-way to test whether conditional heteroscedasticity is responsible for the rejection of the i.i.d. hypothesis is to apply the BDS test statistic to the residuals from a ARMA - GARCH model (Brock et al 1991, and Abhyankar et al. 1995). The trouble is that we cannot depend on asymptotic normality of the BDS statistic. Hsieh (1991) overcomes this problem by using critical values of the BDS statistic for simulated EGARCH process[15]. However, a paper by de Lima (1995) shows that the asymptotic distribution of the BDS statistic remains valid if the test is applied to the natural logarithm of the squared standardized residuals from a GARCH model. This is because the BDS statistic is valid if it is

[15] The simulation is based on 2000 replications, each with 1000 observations.

applied to a data generating process that is additive in the error term. The GARCH process models the error term in a multiplicative form, $\mu_t = \sigma_t z_t$, where μ_t is a random variable following the GARCH process, z_t is i.i.d. random variable, and σ_t is the conditional standard deviation. The standardized residuals from this model are $z_t = \mu_t / \sigma_t$ in the normal case and $z_t = \mu_t / \sqrt{\sigma_t^2 \cdot \left(\frac{\eta - 2}{\eta}\right)}$ in the student-t[16] density case, where η is the degree of freedom parameter. It follows that $ln(z^2_t) = ln(\mu^2_t) - ln(\sigma^2_t)$ in the normal case and $ln(z^2_t) = ln(\mu^2_t) - ln(\sigma^2_t(\eta\text{-}2/\eta))$ in the student-t density case. Therefore, the asymptotic distribution of the BDS statistic remains valid if it is applied to $ln(z^2_t)$ (adjusted residuals) in both the normal and student-t density case.

3.8 The methodologies and the dissertation's topics

3.8.1 Univariate Time Series applications

The univariate time series investigations consist of three essays. The main focus is conditional mean and volatility specifications hypothesising non-synchronous trading effects. Serial correlation in mean and volatility are carefully modelled applying BIC efficient lag specifications. The mean effects from contemporaneous volatility are parameter measured. Where relevant the essays focus on model misspecifications implying inappropriate model non-synchronous trading modelling. The test statistics are applied to test for data-dependence. Significant test statistics suggest lag specifications that do not appropriately model mean and volatility processes suggesting a need for more elaborate model specifications[17].

3.8.2 One-factor models (CAPM)

In one-factor models (CAPM) mean and volatility equations controlling for non-synchronous trading effects in moments and co-moments are clearly relevant. The first (second) essay tests the CAPM by applying univariate (bivariate) excess return series, controlling for non-synchronous trading effects in the conditional mean and volatility equations applying BIC efficient lag specifications. Test statistics investigate for remaining systematic factors suggesting inappropriate mean and volatility modelling. Appropriate mean and volatility modelling suggests sound testing of the one-factor model in the Norwegian equity market.

3.8.3 Event study applications using the market model

Controlling for mean and volatility effects due to non-synchronous trading effects are relevant in classical event studies applying the market model. The dissertation's last two essays focus on event-

[16] We have chosen a Student-t distribution as it has been found to suit Norwegian equity data well (Solibakke, 2001a).

induced changes in the conditional mean and volatility specifications due to increased trading frequency and information flows. The first essay investigates conditional mean and volatility specification and parameter changes from non-event to event periods applying a Norwegian firm sample of mergers and acquisitions. In particular, the essay firstly investigates lag specifications and parameter changes in the mean and volatility equations and secondly investigates level shifts in the conditional volatility. Significant parameter changes and shifts in the conditional volatility suggest a need for more elaborate event study methodologies in financial econometrics. The last essay performs a simultaneous time BIC efficient conditional mean and volatility specification versus an unconditional OLS investigation (synchronous trading and constant volatility). The investigation calculates abnormal returns and statistical significance from a firm sample of merger and acquisition in the Norwegian market of corporate control. Firstly, the essay focuses on appropriate bivariate asset and index specifications, obtaining conditional models showing a minimum of model misspecifications. Secondly, any changes in parameter values are investigated. Significant parameter changes suggest a need for change in event interpretations due to the BIC efficient conditional mean and volatility specifications. Moreover, changes in parameter values suggest a need for a rework of many classical and international event studies. Consequently, controlling for non-synchronous trading effects in conditional mean and volatility processes may bring new insights to the market of corporate control.

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[17] Temporal aggregation and continuous time ARMA-GARCH may therefore be warranted (Drost and Nijman, 1993 and Solibakke, 2001c).

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Chapter 4 Stylised facts on Liquidity at Oslo Stock Exchange

The dissertation performs empirical research on market series exhibiting non-synchronous trading. The modelling approach intends to build econometric models for mean and volatility equations trying to satisfy all the characteristics of market return series. Return series exhibiting non-synchronous trading seem to exist in the Norwegian equity market. This chapter will therefore describe in great detail the characteristics of the Norwegian equity market applying returns from asset, portfolio and index series. The propositions hypothesise an equity market containing return series showing low trading volume, lead and lag structures and a need for adjustments for ergodic and stationary time series. The forthcoming essays focus on non-synchronous trading effects, changing volatility and potential data-dependence in model residuals inducing model misspecifications. The next three sections show that the Norwegian equity market contains trading characteristics that make it a perfect choice for empirical investigations of illiquid equity markets.

Chapter 4 is organised as follows. Section 4.1 investigates raw return characteristics in the Norwegian equity market and focuses especially on trading volume and the relative non-trading frequency of Norwegian assets, which is prolonged into Norwegian portfolio and index series. Trading volume in Norwegian Kroner (NOK) proxies for asset trading frequencies. Section 4.2 investigates thin trading implications applying cross-autocorrelations and lead-lag relations, which are thoroughly discussed in Campbell et al. (1997). The section focuses on cross-autocorrelation matrices for trading-volume sorted markets and asset, portfolio and index returns. Finally, Section 4.3 describes a general adjustment procedure to obtain ergodic and stationary time series in the Norwegian equity market. The adjustment seems particularly important for markets showing low trade volume.

4.1 Characteristics of Norwegian equity market return series

The dissertation applies daily, weekly and monthly return and trading volume series from individual Norwegian stocks spanning the period from October 1983 to February 1994. Two sub-periods are defined; one before the crash in October 1987 and one after the crash. Daily return series are defined as $ln(p_{i,t}/p_{i,t-1})$, where $p_{i,t}$ is the daily closing price for asset i at time t. The daily series are aggregated to weekly and monthly return series. Trading volume is defined as the total transaction volume in NOK at day t for asset i including external trading (trading outside the organised market) for daily, weekly and monthly periods. We use fifteen individual asset, four portfolio and four index series from the Norwegian equity market. The fifteen time series are sorted from continuously traded (no. 1) to thinly traded series (no. 15) according to the daily ratio non-trading days divided by number of listed days for the entire period 1983 to 1994. The sorting is the identical in the two sub-periods. The individual asset series are grouped into portfolio series at period t based on trading volume in NOK available from the information set at $t-1$ (Ω_{t-1}). We rebalance the portfolios each month and to avoid a too frequent shift of component stocks among the asset portfolios, we employ the average daily trading volume for the last two years. Two years of daily trading volume is chosen to obtain a time overlap of 95% for each portfolio restructuring[1]. The assets are therefore grouped into portfolios based on changes in trading volume (NOK) over a considerable period. We emerge with four series; a thinly traded series that contains the most thinly traded assets (Portfolio 1), an intermediate thinly traded series (Portfolio 2), an intermediate frequently traded series (Portfolio 3), and finally a frequently traded series that contains continuously traded assets (Portfolio 4). We four series contains all assets in the Norwegian equity market, and on average, the four series contain at least 20-25 individual assets. Finally, we include four market wide index series consisting of all the assets in the Norwegian market with two equal-weighted and two value-weighted return series. The number of observations for the entire 10-year period from 1983 to 1994 is 2611 daily, 512 weekly and 126 monthly observations. The sub-period before the crash in October 1987 contains 1019 daily, 202 weekly and 50 monthly observations. The sub-period after the crash contains 1546 daily, 306 weekly and 74 monthly observations[2]. Note that a two-month crash period is excluded from the sub-samples but are included for the entire period 1983-94. Hence, we assume that market dumps are normal in equity markets. Table 1 reports the three data samples. Figure 1 and 2 reports the value-weighted and all-market index series and total market trading volume in Norwegian Kroner (NOK), respectively. This database is our main vehicle for investigations we perform in the dissertation.

{Insert TABLE 1 Panel A, Panel B and Panel C about here}

The three panels in columns "No. of observations" and "Proportion traded" (column 2 and 3) show the importance of the thin trading feature of the Norwegian equity market. For the sections "Individual assets" for all three panels, the proportion traded show numbers far below one (continuous trading) for many asset series. A proportion below one in column 3, suggests daily non-trading for individual assets in the equity market. For the section "Portfolios" in all three panels, the "Low Volume" portfolio also reports values below one, indicating non-trading even though the portfolio contains 20-25 assets. Hence, this portfolio suggests that in 9% of all trading days, ¼ of the Norwegian assets show non-trading. The non-trading feature seems therefore to be substantial in the Norwegian equity market. Moreover, note that the four index series are all-market series. Consequently, the index series will contain many assets that contain the non-trading feature. Moreover, note that since trading volume and market value are positively correlated[3], the equal-weighted index will show higher influence from non-trading than the value-weighted index.

Table 1 reports additional characteristics for the twenty-four hour interval return series in our sample for the three periods (column 4 to 9). The yearly mean column shows no particular pattern over the

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[1] Moreover, the first trading volume observation we have been able to collect for all assets in the Norwegian equity market is registered 01.09.81.
[2] The crash period in this study is defined as the two-month period October and November in 1987. The estimation and specification results for the two sub-periods are not reported due to space considerations. However, all results are available from the author upon request.
[3] Campbell et al., (1997), page 130: "We use market capitalization to group securities because the relative thinness of the market for any given stock is highly correlated with the stock's total market value; hence stocks with similar market values are likely to have similar non-trading probabilities".

trading volume sorted asset, portfolio and index series. However, the standard deviation shows a considerable growth from continuously (trading proportion close to one) to thinly traded series (trading proportion lower than one). This growth result in standard deviation suggests temporal aggregation in return series due to non-trading. When trades do occur, the price changes are higher in absolute values for non-traded relative to continuously traded assets. The daily maximum and minimum numbers seem to confirm the temporal aggregation results. The high kurtosis numbers for the thinly traded assets suggest an overrepresentation of zero returns. This is exactly what we would expect from non-traded assets due to registered zero returns, when trading volume is zero. The skew results seem to suggest an overrepresentation of negative extreme return observations for all series. Moreover, in contrast to individual assets the portfolio and index series show highest negative skew for the most frequently traded assets. For the individual assets in sub-period 1983 to 1987 (before the crash) the skew is small and even positive for many individual assets and relatively small for portfolio and index series. The crash in October 1987 may therefore have considerable influence on the skew results for the entire period.

Finally, in Table 1 Panel A, which report characteristics for the entire period 1983 to 1994, we also report some elaborate test statistics for non-normality (K-S Z-test[4]) data-dependence in the mean equation (RESET[5]), data-dependence in the volatility equation (ARCH[6]) and general non-linear dependence (BDS[7]). Almost all test statistics report non-normality, non-linearity in mean, non-linearity in volatility and general non-linear dependence. Consequently, elaborate model specifications need to be conducted for all the reported asset, portfolio and index series from the Norwegian equity market, which seem to report high non-synchronous trading in Table 1. We express this trading volume influence in this dissertation by applying the term: The Norwegian thinly traded equity market. Moreover, to elaborate the thin trading findings from Table 1, the next sub-section (4.2) will apply cross-autocorrelations and lead-lag relations thoroughly described in Campbell et al. (1997) chapter 2 and 3, showing lead-lag between international markets and between Norwegian assets.

4.2 Lead-lag relations between markets and assets

The dissertation sets out to perform several empirical investigations of the thinly traded Norwegian equity market. From the Norwegian characteristics found in Section 4.1 and Table 1, the dissertation hypothesises that the Norwegian equity market show thin trading relative to more developed markets (New York and London) and contains assets that show thin trading relative to continuous traded assets. The dissertation therefore hypothesises equity market characteristics that exhibit non-synchronous trading effects for asset, portfolio and index return series. Applying the definition employed in Campbell et al. (1997), these non-synchronous trading effects arise when time series from asset prices are recorded for our database series at time intervals of one length when they in fact

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[4] K-S is Kolmogorov-Smirnov. See Table 1 for a description of the test statistic.
[5] See Ramsey, 1969. See Table 1 for a description of the test statistic.
[6] See Engle, 1982. See Table 1 for a description of the test statistic.
[7] See Brock et al., 1988, 1991, Scheinkman, 1990). See Table 1 for a description of the test statistic.

are recorded at time intervals of other, possibly irregular, lengths. For example, assume that we possess the daily closing prices of a Norwegian firm quoted on the Oslo Stock Exchange and reported daily in the financial press. Note that the closing price reported in the financial press is the price at which the last transaction for the firm occurred on the previous day. In a thinly traded equity market the closing price will generally not occur at the same time each day. Hence, the firm may on one particular Monday quote its last reported trade at $14^{\underline{05}}$, which will become the closing price reported in the financial press that particular Monday even though the Oslo Stock Exchange closes at $16^{\underline{00}}$. Moreover, the following day Tuesday, the last quoted trade was reported at $15^{\underline{15}}$. This example shows that referring to them as "daily" prices, we have implicitly and incorrectly assumed that they are equally spaced in 24-hour intervals. Hence, non-synchronous trading may induce potentially biases in the moments and co-moments of assets returns. In particular, we focus on coefficients for means, variances, co-variances, betas, autocorrelations and cross-autocorrelations in return series.

The fact that individual assets, portfolio and index series show positive cross-autocorrelation across time can now be exploited to suggest trading differences between equity markets. We will apply cross-autocorrelations relationships (Campbell et al (1997) and Lo and MacKinlay (1990)) showing international equity markets lead-lag relations. Campbell et al. measures lead-lag relations using daily, weekly, and monthly return series. For this work, non-symmetric co-variances between international equity markets suggest lead and lag structures. To gauge the degree of asymmetry in the co-variances the difference between first-order autocorrelations and the transposed first-order autocorrelations is reported. Moreover, to show that the Norwegian equity market show lead-lag relations between assets, we apply the same methodology between individual Norwegian assets.

4.2.1 Lead-lag relations between international markets

The investigation applies daily value weighted indices from USA (US), United Kingdom (UK) and Norway (N) for the time period from 1984 to 1998. S&P500 (US) index shows highest liquidity, followed by FTSE350 (UK) and TOTX (N). Table 2 reports the first-order autocorrelation matrices $\hat{\Gamma}_1$ for the vector of three index series using daily (Panel A), weekly (Panel B) and monthly (Panel C) return intervals. An interesting pattern emerges from Table 2: The entries below the diagonals of $\hat{\Gamma}_1$ (1) are always larger than those above the diagonals (asymmetric matrices). To gauge the degree of asymmetry in these matrices, the difference $\hat{\Gamma}_1 - \hat{\Gamma}_1'$ is reported in the second column of the three panels of Table 2. This column of Table 2 more apparently shows the intriguing lead-lag pattern, where liquid index series lead less liquid index series. The result is valid for daily, weekly, and monthly return intervals. By now applying a formal non-synchronous trading model the non-trading probability and average time between trades may be calculated. These simple lead-lag relations are also confirmed in ARMA-GARCH lag specifications in Solibakke (1999). Our results therefore show that the thinnest traded market index (Totx) in Norway, is lead by the more liquid market (Ftse350) in UK, which is again lead by the most liquid market (S&P500) in USA.

{Insert Table 2 about here}

4.2.2 Lead-lag relations for individual assets

The first-order autocorrelation analysis for individual assets employs fifteen randomly picked assets from the Norwegian equity market spanning the whole range of trading volume. Thus, the section aims to show that liquidity for individual assets contribute to lead-lag relation insights. For brevity, Table 3 reports characteristics for six assets of these fifteen randomly selected assets. The first asset (*F1*) is a continuously and frequently traded asset while asset six (*F6*) is characterized by a high non-trading probability[8]. Table 3 shows a high divergence in trading frequency for individual assets. For example, asset F6 is only traded in approximately 30% of listed days on the exchange, while asset F1 is continuously traded. Average trading volume is a decreasing function of the non-trading probability and the standard deviation seem to increase the higher the non-trading. The mean return over assets shows no clear patterns.

{Insert Table 3 about here}

Table 4 reports first order autocorrelation matrices $\hat{\Gamma}_1$ for the vector of six individual asset series using daily (panel A), weekly (panel B) and monthly (Panel C) return intervals[9]. An interesting pattern emerges from Table 4: The entries below the diagonals of $\hat{\Gamma}_1$ are usually larger than those above the diagonals (asymmetric matrices). To gauge the degree of asymmetry in these matrices, the difference $\hat{\Gamma}_1 - \hat{\Gamma}_1'$ is reported in the second part of the three panels of Table 4. This part of Table 4 more apparently shows the intriguing lead-lag pattern, where liquid assets lead less liquid assets. The result is valid for daily, weekly, and monthly return intervals. By now applying a formal non-synchronous trading model, the non-trading probability and average time between trades may be calculated[10]. Our results show that the thinnest traded assets (F6 and F5), is lead by the more liquid assets (F5, F4 and F3), which again is lead by the most liquid assets (F2 and F1) in USA. Cross-autocorrelation and autocorrelation is therefore important factors to consider in formal multivariate mean and volatility specifications.

{Insert Table 4 about here}

4.3. Adjustment procedures for day and month anomalies, and location and scale effects

A financial time series investigation using formal statistical methods assumes the observed series as a particular realization $\{x_t\}_1^T$ of a stochastic process. The stochastic process will be a family of random

[8] See Solibakke (1997) for an unconditional and conditional probability of trading (duration models) using Norwegian individual asset trading volume information.
[9] As far as possible we have tried to exclude days where trading is halted for "news pending" or other stock exchange regulations.
[10] See for example Campbell et al. (1997) chapter 2, The Predictability of Asset Returns.

variables defined on an appropriate probability space and can be described by a T-dimensional probability distribution. The procedure of using a single realization to infer the unknown parameters of a joint probability distribution is only valid if the process is ergodic[11]. Moreover, stationary requires the process to be in a particular state of "statistical equilibrium" (Box and Jenkins, 1976, p. 26). A stochastic process is strictly stationary if its properties are unaffected by a change of time origin implying a constant mean and variance as long as $E|x_t|^2 < \infty$. Applying the conditions to only the first- and second-order moments of the process this is known as stationary in wide sense (weak form). Note especially that a process may possess weak stationary but not strict stationary conditions.

Figure 1 and 2 depicts the value-weighted all-market index series and the total market trading volume in Norway from 1983 to 1994. The value-weighted index shows an approximate yearly growth of 12% in this period. However, note the strong and erratic trend in trading volume for the Norwegian Market in the same period. On average, the growth in the trading volume in Norwegian Kroner (NOK) is approximately 32,9% per year.

{Insert Figure 1 and 2 about here}

The increase in trading volume suggests a need for testing for ergodicity and stationary in the Norwegian time series. Moreover, several authors have reported anomalies applying Norwegian equity market[12] return series. Hence, for all time series we perform procedures described by Gallant, Rossi and Tauchen (1992) to adjust for systematic location and scale effects in all return series[13]. The procedure gives us series that become more homogenous allowing us to focus on the day-to-day dynamic structure under an assumption of stationary series without any disturbance to mean and volatility characteristics. Moreover, the procedure validates inference of unknown parameters of a joint probability distribution from the single realisation (ergodicity). The log first difference of the price index is adjusted. Let ϖ denote the variable to be adjusted. Initially, the regression to the mean equation $\varpi = x \cdot \beta + u$ is fitted, where x consists of calendar variables that are most convenient for the time series and contains parameters for trends, week dummies, calendar and day separation variable, month and sub-periods. To the residuals, $\hat{u}$, the variance equation model $\hat{u}^2 = x \cdot \gamma + \varepsilon$ is estimated. Next $\frac{\hat{u}^2}{\sqrt{e^{x \cdot \hat{\gamma}}}}$ is formed, leaving a series with mean zero and (approximately) unit variance given x.

Lastly, the series $\hat{\varpi} = a + b \cdot (\frac{\hat{u}}{\sqrt{e^{x\hat{\gamma}}}})$ is taken as the adjusted series, where a and b are chosen so that $\frac{1}{T} \cdot \sum_{i=1}^{T} \hat{\varpi}_i = \frac{1}{T} \cdot \sum_{i=1}^{T} \varpi_i$ and $\frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{\varpi}_i - \overline{\varpi})^2 = \frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{u}_i - \overline{u})^2$. The purpose of the final location and scale transformation is to aid interpretation. In particular, the unit of measurement of the

[11] Ergodic roughly means that the sample moments for finite stretches of the realisation approach their population counterparts as the length of the realisation becomes infinite (Granger and Newbold, 1977)
[12] See Johnsen, 1995 and Gjølberg, 1987 among others.

adjusted series is the same as that of the original series. We report the result for the thinly and continuously traded portfolio series. Table 5 shows that for continuously traded assets, only the two periodic definitions GAP3 and January 1st to January 07th show significant influence on the price process. The volatility series report significant patterns for day of the week and for the month of November. No other factors seem to influence the return and volatility series. For the thinly traded series, Wednesdays and January 8th to January 15th show significant influence on the price process. The volatility series report significant patterns for day of the week and for the month of July. Note, the continuously traded assets show early January effects (1st to 7th), while thinly traded assets show mid-January effects (8th to 15th). It seems that the continuously traded asset series lead the thinly traded series. We plot the raw and adjusted time series for thinly traded and continuously traded assets in Figure 1, panel A and B, respectively.

{Insert Table 5, Figure 3 and 4 about here}

Due to significant parameters in the OLS model, the adjustments to both thinly and frequently traded assets make intuitive sense. Importantly, for both thinly and continuously traded assets, the raw and adjusted time series seem to show small differences over time. Even more important for our investigation, it seems there is no regime shifts in the time series.

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Table 1. Individual asset/Portfolio/Index series characteristics for the Norwegian Equity Market.

Panel A. Period 1983-1994. (crash October 1987 included).

Individual Assets:	No. of obs.	Prop. Traded	Yearly Mean	Yearly std.dev.	Daily Max.	Daily Min.	Kurtosis	Skew	K-S Z-stat	ARCH (6)	RESET (12;6)	BDS m=2;ε=1	m=3;ε=1
1. VP-505560	2611	1.00	15.196	32.417	13.04	-20.27	8.405	-0.311	3.150	288.37	7.4905	8.507	10.11
2. VP-532720	2606	1.00	12.278	39.964	16.00	-17.20	5.102	0.424	3.151	83.200	4.3965	7.706	10.18
3. VP-430640	2611	1.00	25.140	33.153	13.44	-23.43	9.163	-0.374	2.962	70.856	18.259	8.025	9.468
4. VP-468440	2607	0.99	19.297	33.175	12.91	-20.93	7.441	-0.317	3.207	125.37	13.159	12.24	13.06
5. VP-403130	2604	0.99	1.023	57.347	22.33	-29.60	6.013	-0.279	3.203	181.60	3.4225	12.86	15.55
6. VP-392001	2611	0.97	8.598	46.407	26.76	-46.98	30.219	-1.314	3.448	254.25	1.8484	11.96	13.24
7. VP-317220	2609	0.92	28.981	56.390	19.84	-25.65	4.382	0.161	3.462	143.43	18.515	12.30	16.19
8. VP-507221	2369	0.90	-10.479	116.66	32.26	-45.84	3.929	-0.142	3.477	246.22	3.0610	13.91	17.79
9. VP-413560	2577	0.83	12.645	57.56	24.41	-33.69	11.892	-0.623	5.352	276.38	5.4025	10.86	14.61
10.VP-562085	2540	0.78	-8.830	231.74	192.86	-278.6	84.487	-2.763	10.618	271.39	4.9790	11.84	18.24
11.VP-526960	2566	0.62	1.076	163.33	89.85	-85.27	16.153	-0.206	6.634	270.34	2.7892	8.955	12.87
12.VP-513100	2515	0.57	42.318	130.70	81.20	-167.9	95.667	-3.984	8.731	203.48	4.3437	12.07	16.53
13.VP-314690	2383	0.52	-6.432	119.78	79.82	-61.42	26.567	0.984	10.089	357.26	2.4446	16.93	23.93
14.VP-311170	2499	0.43	34.618	108.42	53.59	-56.57	17.440	-0.562	10.813	291.68	4.0270	19.40	26.38
15.VP-523040	2146	0.33	1.964	116.98	107.21	-74.52	41.590	0.883	11.464	235.61	1.0785	15.43	20.63
Portfolios:													
Low Volume	2611	0.91	21.456	32.553	10.80	-15.91	5.7203	-0.116	2.574	89.352	4.5614	7.472	8.143
Intermediate Low	2611	1.00	7.238	21.706	10.33	-14.53	11.180	-0.603	3.428	634.58	1.8810	8.379	10.53
Intermediate High	2611	1.00	3.405	22.047	11.55	-16.19	14.869	-0.987	3.676	638.75	0.1807	13.13	15.39
High Volume	2611	1.00	5.502	25.127	13.32	-23.06	26.146	-1.315	3.809	391.01	0.5791	16.19	19.10
Indices:													
Equal Weighted Oslo	2611	1.00	7.676	17.610	11.42	-16.66	29.844	-1.546	4.580	630.15	1.8598	13.29	14.76
NHH Equal Weighted	2527	1.00	18.151	19.115	10.07	-15.03	16.838	-0.877	3.794	548.89	10.631	11.71	14.25
Value Weighted Oslo	2611	1.00	13.278	20.581	10.48	-21.22	36.143	-2.004	3.800	288.89	5.1850	12.65	14.91
NHH Value Weighted	2527	1.00	13.552	21.077	11.27	-20.90	34.779	-1.968	3.796	296.23	5.8152	12.62	14.68

VP-505560=Norsk Hydro, VP-532720=Saga Petroleum, VP-430640=Hafslund-Nycomed, VP-468440=Kværner, VP-403130=Elkem, VP-392001=Det Norske Luftselskap, VP-317220=Ganger-Rolf, VP-507221=Kirkland, VP-413560=Norske Skog, VP-562085=Tandberg, VP-526960=Rena Karton, VP-513100=Nydalen Compagnie, VP-314690=Eidsiva, VP-311170=Brogestad, VP-523040=Porsgrund Porselænfabrikk. The VP-XXXXXXs are therefore individual assets sorted from high to low trading volume. Moreover, "Low Volume" is a portfolio containing the most thinly traded assets and "High Volume" contains the most frequently traded assets. Finally, two equal-weighted and two value-weighted index series are included from the Norwegian equity market.

Yearly mean is daily mean multiplied by 252 trading days and yearly standard deviation is daily standard deviation multiplied by the square root of 252 trading days. Skew is a measure of heavy tails and asymmetry of a distribution (normal) and kurtosis is measure of too many observations around the mean for a distribution (normal). K-S Z-test: Used to test the hypothesis that a sample comes from a normal distribution. The value of the Kolmogorov-Smirnov Z-test is based on the largest absolute difference between the observed and the theoretical cumulative distributions. ARCH (6) : ARCH (6) is a test for conditional heteroscedasticity in returns. Low {.} indicates significant values. We employ the OLS-regression $y^2_t = a_0 + a_1 * y^2_{t-1} + \ldots + a_6 * y^2_{t-6}$. $T*R^2$ is χ^2 distributed with 6 degrees of freedom. T is the number of observations, *y* is returns and R^2 is the explained over total variation. a_0, a_1 ... a_6 are parameters. RESET (12,6) : A sensitivity test for mainly linearity in the mean equation. 12 is number of lags and 6 is the number of moments that is chosen in our implementation of the test statistic. $T*R^2$ is χ^2 distributed with 12 degrees of freedom. Finally, BDS (m=2,ε=1): A test statistic for general non-linearity in a time series. The test statistic BDS $=T^{1/2}*[C_m(\sigma*\varepsilon) - C_1(\sigma*\varepsilon)^m]$, where *C* is based on the correlation-integral, m is the dimension and ε is the number of standard deviations. Under the null hypothesis of identically and independently distributed (i.i.d.) series, the BDS-test statistic is asymptotic normally distributed with a zero mean and with a known but complicated variance.

Table 1. Individual Asset/Portfolio/Index characteristics for the Norwegian Equity Market.

PANEL B: Period 1983-1987. (before the crash in October 1987).

Individual Assets:	No. of Observ.	Prop. Traded	Yearly Mean	Yearly deviation	Daily Maximum	Daily Minimum	Kurtosis	Skew
1. VP-505560	1021	1.00	20.2690	30.3308	9.4386	-9.9571	2.7096	0.1791
2. VP-532720	1017	0.99	9.4767	38.6201	15.8977	-10.0689	5.4959	0.7216
3. VP-430640	1022	0.98	46.5469	32.9545	10.5613	-9.7213	3.0290	0.3895
4. VP-468440	1018	0.98	23.8117	31.4618	8.3457	-8.4929	2.4350	-0.1161
5. VP-403130	1015	0.99	7.8879	34.7823	12.2605	-9.4246	2.5242	0.2826
6. VP-392001	1022	0.96	34.2618	35.3953	11.6754	-9.6149	2.3993	0.2023
7. VP-317220	1022	0.83	52.9822	62.2176	20.3172	-31.8317	7.3511	0.1902
8. VP-507221	1009	0.93	10.3353	90.8420	26.9697	-27.3424	2.5640	0.0376
9. VP-413560	1015	0.59	25.1306	42.3586	18.4158	-14.3036	5.9957	0.1170
10.VP-562085	990	0.54	-40.3946	114.863	40.5465	-40.5531	6.8991	-0.1975
11.VP-526960	1020	0.59	1.8113	64.0576	22.2964	-22.2964	4.5529	-0.0031
12.VP-513100	1002	0.43	38.7784	48.7684	16.1313	-14.2986	3.5116	0.0131
13.VP-314690	988	0.45	-15.0536	80.9840	37.9490	-29.7252	6.3330	0.3806
14.VP-311170	990	0.35	31.0354	85.3893	40.5586	-36.0130	14.721	0.3149
15.VP-523040	1007	0.27	24.1920	100.920	31.5910	-49.2386	10.710	-0.9456
Portfolios:								
Low Volume	1022	0.86	11.5332	22.4079	7.1418	-5.3251	1.0620	-0.0491
Intermediate low	1022	1.00	18.2834	13.6279	4.3342	-4.6447	2.7425	-0.3078
Intermediate High	1022	1.00	9.0217	17.2026	4.0898	-6.4375	3.5686	-0.6725
High Volume	1022	1.00	9.7217	17.3120	5.0685	-6.4698	2.8969	-0.5084
Indices:								
Equal Weighted	1022	1.00	11.3187	12.3212	3.7935	-5.1569	4.9717	-0.7277
NHH Equal Weighted	1022	1.00	9.9133	12.8491	3.2823	-4.1326	3.3566	-0.5843
Value Weighted	1022	1.00	8.6181	14.7039	4.4844	-4.3649	1.9451	-0.2986
NHH Value Weighted	1022	1.00	10.0816	14.4191	4.4770	-5.0366	2.7519	-0.4359

See Table 1, Panel A for a description series and test statistics.

Table 1. Individual asset/Portfolio/Index characteristics for the Norwegian Equity Market.

PANEL C: Period 1987-1994 (after crash period)

Individual Assets:	No. Of Observ.	Prop. Traded	Yearly Mean	Yearly deviation	Daily Maximum	Daily Minimum	Kurtosis	Skew
1. VP-505560	1546	1.00	22.3156	28.7122	10.9610	-9.8888	4.3009	0.1278
2. VP-532720	1547	1.00	24.2300	37.1896	12.9812	-12.6254	2.3282	0.2518
3. VP-430640	1547	1.00	20.2613	29.3544	7.8767	-10.6563	2.9705	-0.0086
4. VP-468440	1547	0.99	23.3745	32.0047	14.4500	-12.7143	5.5106	0.2209
5. VP-403130	1547	0.99	14.6289	65.8346	28.3066	-33.8782	8.7066	-0.2570
6. VP-392001	1547	0.97	0.8901	48.5731	18.0183	-23.7785	7.2061	-0.3481
7. VP-317220	1545	0.95	21.4935	49.2509	17.3093	-14.2614	2.6465	0.2918
8. VP-507221	1318	0.76	-14.8811	132.4724	44.8025	-55.9616	9.6744	-0.4798
9. VP-413560	1516	0.83	12.4658	59.5667	24.5122	-29.3761	8.9024	-0.3704
10.VP-562085	1503	0.65	13.4264	283.8298	161.1774	-230.4338	40.1676	-1.8342
11.VP-526960	1500	0.42	10.8942	205.9428	109.8612	-96.5081	16.9691	-0.3550
12.VP-513100	1449	0.42	51.2486	165.5793	91.7187	-174.9785	78.0626	-2.3078
13.VP-314690	1326	0.39	9.1281	141.6753	58.7787	-51.0826	9.7346	0.1376
14.VP-311170	1433	0.38	43.4562	123.7460	77.3190	-55.9616	20.2976	0.7081
15.VP-523040	1092	0.28	-11.5236	130.5414	59.7837	-55.9616	15.8117	-0.3334
Portfolios:								
Low Volume	1547	0.97	-2.2367	37.5609	10.7153	-15.9911	4.6601	-0.0610
Intermediate low	1547	1.00	-7.5019	24.2255	7.2915	-11.3647	4.8492	-0.1927
Intermediate High	1547	1.00	3.0173	21.7502	8.7645	-10.5545	7.4429	-0.3601
High Volume	1547	1.00	3.2342	25.6842	12.3118	-12.2878	9.4502	-0.2045
Indices:								
Equal Weighted	1547	1.00	-0.2280	17.5637	6.6717	-9.6965	7.9538	-0.5667
NHH Equal Weighted	1547	1.00	0.6475	19.9151	7.4230	-9.8052	5.8222	-0.1356
Value Weighted	1547	1.00	3.6568	19.8875	8.8818	-11.3701	9.8390	-0.4833
NHH Value Weighted	1547	1.00	2.6227	20.6236	9.2644	-12.5005	11.5817	-0.6560

See Table 1, Panel A for a description series and test statistics.

Table 2. First-order Autocorrelation Matrices for Three Differently Traded Indices

		$\hat{\Gamma}$				$\hat{\Gamma}-\hat{\Gamma}'$		
		Totx	Ftse350	S&P500		Totx	Ftse350	S&P500
Daily	Totx	0.0810	-0.0376	-0.0857	Totx	0.0000	-0.2027	-0.4949
	Ftse350	0.1651	0.0973	-0.0650	Ftse350	0.2027	0.0000	-0.3884
	S&P500	0.4092	0.3235	0.0182	S&P500	0.4949	0.3884	0.0000
		Totx	Ftse350	S&P500		Totx	Ftse350	S&P500
Weekly	Totx	0.1451	0.1021	-0.0627	Totx	0.0000	-0.0986	-0.2130
	Ftse350	0.2008	0.2522	0.0159	Ftse350	0.0986	0.0000	-0.2272
	S&P500	0.1503	0.2432	0.0051	S&P500	0.2130	0.2272	0.0000
		Totx	Ftse350	S&P500		Totx	Ftse350	S&P500
Monthly	Totx	0.1274	-0.0879	-0.1850	Totx	0.0000	-0.3003	-0.3425
	Ftse350	0.2124	0.1217	0.0619	Ftse350	0.3003	0.0000	-0.0903
	S&P500	0.1574	0.1522	0.0639	S&P500	0.3425	0.0903	0.0000

$\hat{\Gamma}$ = first order autocorrelation matrix, $\hat{\Gamma}'$ = transposed first-order autocorrelation matrix
Totx=Value-weighted index Oslo; Ftse350=Value-weighted index 350 assets London,
S&P500=Standard & Poor 500 asset index, New York

Table 3. Characteristics for differently Traded Assets at Oslo Stock Exchange

	F1	F2	F3	F4	F5	F6
Mean	0.0590	0.0992	0.1100	0.0550	-0.0104	0.0679
Standard deviation	2.0569	2.0924	3.3437	3.0252	3.5607	3.1889
Maximum	15.0227	11.1974	22.3144	30.2281	31.8454	35.6675
Minimum	-25.1853	-26.9221	-26.7702	-51.9875	-40.5465	-25.1314
Average daily trading volume	141165	84822	37913	24274	7655	6010
Number of Trading days	2619	2598	2179	1549	395	198
Number of non-trading days	1	11	185	326	370	334
Probability	100.0 %	99.6 %	92.2 %	82.6 %	51.6 %	37.2 %

F1=Continuous traded asset; F6=Thinly Traded Asset, F2-F5= Relative Intermediately Traded Assets

Table 4. First-order Autocorrelation matrices for trading volume sorted assets*

Panel A. Daily Returns

Γ_1	F6	F5	F4	F3	F2	F1
F6	0.01418	-0.00279	-0.01613	-0.02019	-0.00357	-0.00588
F5	0.01593	0.01172	-0.00482	-0.00979	-0.01401	-0.00781
F4	0.02423	0.07709	0.02870	0.02733	0.03772	-0.03266
F3	-0.00313	0.02936	0.09778	0.13284	0.06881	0.05602
F2	0.03214	0.03903	0.11099	0.07288	0.09087	-0.00758
F1	0.00058	0.01193	0.08101	0.06773	0.09621	0.06529

$\Gamma_1 - \Gamma'_1$	F6	F5	F4	F3	F2	F1
F6	0	-0.01871	-0.04036	-0.01706	-0.03571	-0.00646
F5	0.01871	0	-0.08190	-0.03916	-0.05304	-0.01974
F4	0.04036	0.08190	0	-0.07044	-0.07327	-0.11367
F3	0.01706	0.03916	0.07044	0	-0.00408	-0.01170
F2	0.03571	0.05304	0.07327	0.00408	0	-0.10379
F1	0.00646	0.01974	0.11367	0.01170	0.10379	0

Panel B: Weekly Returns

Γ_1	F6	F5	F4	F3	F2	F1
F6	-0.06863	-0.01927	0.00999	0.06219	-0.03259	0.01549
F5	0.06350	-0.08637	-0.01812	0.05678	0.05844	0.06020
F4	0.01865	0.03447	-0.09682	0.01139	0.01437	0.02013
F3	0.14394	0.06468	0.12186	0.16633	0.07035	0.09982
F2	0.05711	0.07142	0.10275	0.08550	0.09994	0.12855
F1	0.07184	0.16170	0.04637	0.10535	0.05944	0.01040

$\Gamma_1 - \Gamma'_1$	F6	F5	F4	F3	F2	F1
F6	0	-0.08277	-0.00866	-0.08174	-0.08970	-0.05635
F5	0.08277	0	-0.05259	-0.00789	-0.01298	-0.10150
F4	0.00866	0.05259	0	-0.11047	-0.08837	-0.02624
F3	0.08174	0.00789	0.11047	0	-0.01516	-0.00553
F2	0.08970	0.01298	0.08837	0.01516	0	-0.06911
F1	0.05635	0.10150	0.02624	0.00553	0.06911	0

Panel C: Monthly Returns

Γ_1	F6	F5	F4	F3	F2	F1
F6	0.16759	0.11201	-0.18271	0.06533	-0.09803	0.00883
F5	0.15534	-0.00932	0.06792	0.04604	-0.10011	0.09429
F4	0.04053	0.06061	-0.12812	0.00065	-0.10209	-0.04590
F3	0.30249	0.08938	0.08794	0.09880	0.00766	0.19024
F2	0.18417	0.14286	-0.01962	0.12261	0.06457	0.16282
F1	0.13538	0.04268	0.03892	0.11937	-0.06755	0.09231

$\Gamma_1 - \Gamma'_1$	F6	F5	F4	F3	F2	F1
F6	0	-0.04333	-0.22324	-0.23715	-0.28219	-0.12655
F5	0.04333	0	0.00731	-0.04334	-0.24297	0.05161
F4	0.22324	-0.00731	0	-0.08729	-0.08247	-0.08482
F3	0.23715	0.04334	0.08729	0	-0.11495	-0.07087
F2	0.28219	0.24297	0.08247	0.11495	0	-0.23037
F1	0.12655	-0.05161	0.08482	0.07087	0.23037	0

* See Table 2 for description of the columns F1 to F6

Table 5. Data adjustment Coefficients for Thinly and Continuously traded assets

	Thinly Traded Assets				Continuously Traded Assets			
	Return Series		ln(residual²)		Return Series		ln(residual²)	
	Coeff.	t-value	Coeff.	t-value	Coeff.	t-value	Coeff.	t-value
INTR	1.2712	{1.6234}	-2.2497	{2.5163}	0.6179	{1.0289}	0.3083	{0.3418}
THUS	-1.3991	{1.9100}	1.7195	{2.0361}	-0.6091	{1.0843}	-1.9881	{2.3783}
WEDN	-1.5432	{2.0922}	1.6990	{2.0803}	-0.4861	{0.8594}	-2.0890	{2.4819}
THUR	-1.4321	{1.9401}	1.7372	{2.2524}	-0.4395	{0.7763}	-2.1362	{2.5360}
FRID	-1.3520	{1.8278}	1.8850	{1.2358}	-0.3517	{0.6200}	-2.3382	{2.7699}
GAP1	-0.2806	{0.5526}	-0.7097	{0.2005}	0.4098	{1.0522}	0.3792	{0.6545}
GAP2	-0.3723	{0.3880}	0.2177	{0.9212}	0.5478	{0.7445}	-2.1334	{1.9482}
GAP3	0.2764	{0.2891}	-0.9960	{0.3965}	1.4975	{2.0423}	-0.2389	{0.2191}
GAP4	-0.2789	{0.3461}	-0.3615	{1.4978}	-0.4608	{0.7455}	-0.0730	{0.0794}
GAP5	-0.5926	{0.9694}	1.0357	{1.5292}	0.3157	{0.6733}	0.5589	{0.8012}
HOLID	-1.1985	{1.2459}	1.6636	{1.1924}	-1.1835	{1.6042}	0.2578	{0.2350}
Jan01-07	0.7566	{1.8124}	0.5629	{0.9912}	0.8645	{2.7003}	0.6253	{1.3130}
Jan08-15	0.8501	{2.1755}	-0.4380	{0.9260}	-0.1162	{0.3878}	0.5782	{1.2972}
Jan16-23	0.7136	{1.8262}	-0.4092	{0.1872}	0.0683	{0.2279}	0.4563	{1.0237}
Jan24-31	0.5226	{1.4522}	-0.0762	{1.5350}	0.0912	{0.3304}	0.5686	{1.3852}
FEBR	0.3913	{1.2557}	-0.5412	{1.2914}	-0.0279	{0.1168}	0.2285	{0.6424}
MAR	0.2580	{0.8315}	-0.4532	{0.8897}	0.0733	{0.3082}	0.2066	{0.5837}
APR	0.1917	{0.6096}	-0.3165	{1.4622}	-0.0773	{0.3204}	0.1599	{0.4457}
MAY	0.3321	{1.0453}	-0.5255	{0.6507}	-0.1358	{0.5575}	0.2971	{0.8196}
JUN	0.1999	{0.6432}	-0.2288	{1.7108}	-0.2774	{1.1639}	-0.1425	{0.4018}
JUL	0.1926	{0.6243}	-0.5971	{2.0604}	0.0410	{0.1735}	-0.1955	{0.5553}
AUG	0.3740	{1.2113}	-0.7198	{1.6557}	-0.1922	{0.8116}	0.4430	{1.2573}
SEPT	0.1316	{0.4289}	-0.5747	{0.7745}	-0.1879	{0.7982}	0.2768	{0.7906}
OCT	0.1892	{0.6187}	-0.2679	{0.7854}	-0.1930	{0.8229}	0.4293	{1.2305}
NOV	0.1348	{0.4392}	-0.2725	{1.5372}	-0.3830	{1.6275}	0.7462	{2.1322}
DEC2	0.5179	{1.3255}	-0.6792	{1.4531}	0.1905	{0.6357}	0.8168	{1.8326}
DEC3	-0.0352	{0.0901}	-0.6421	{0.1820}	-0.4338	{1.4477}	0.4373	{0.9811}
DEC4	0.6947	{1.6510}	-0.0866	{0.1806}	0.2140	{0.6632}	-0.0867	{0.1806}
TRD	----	----	1.0148	{1.6041}	----	----	0.9449	{1.4809}
TRD2	----	----	0.3112	{0.5080}	----	----	0.0513	{0.0831}

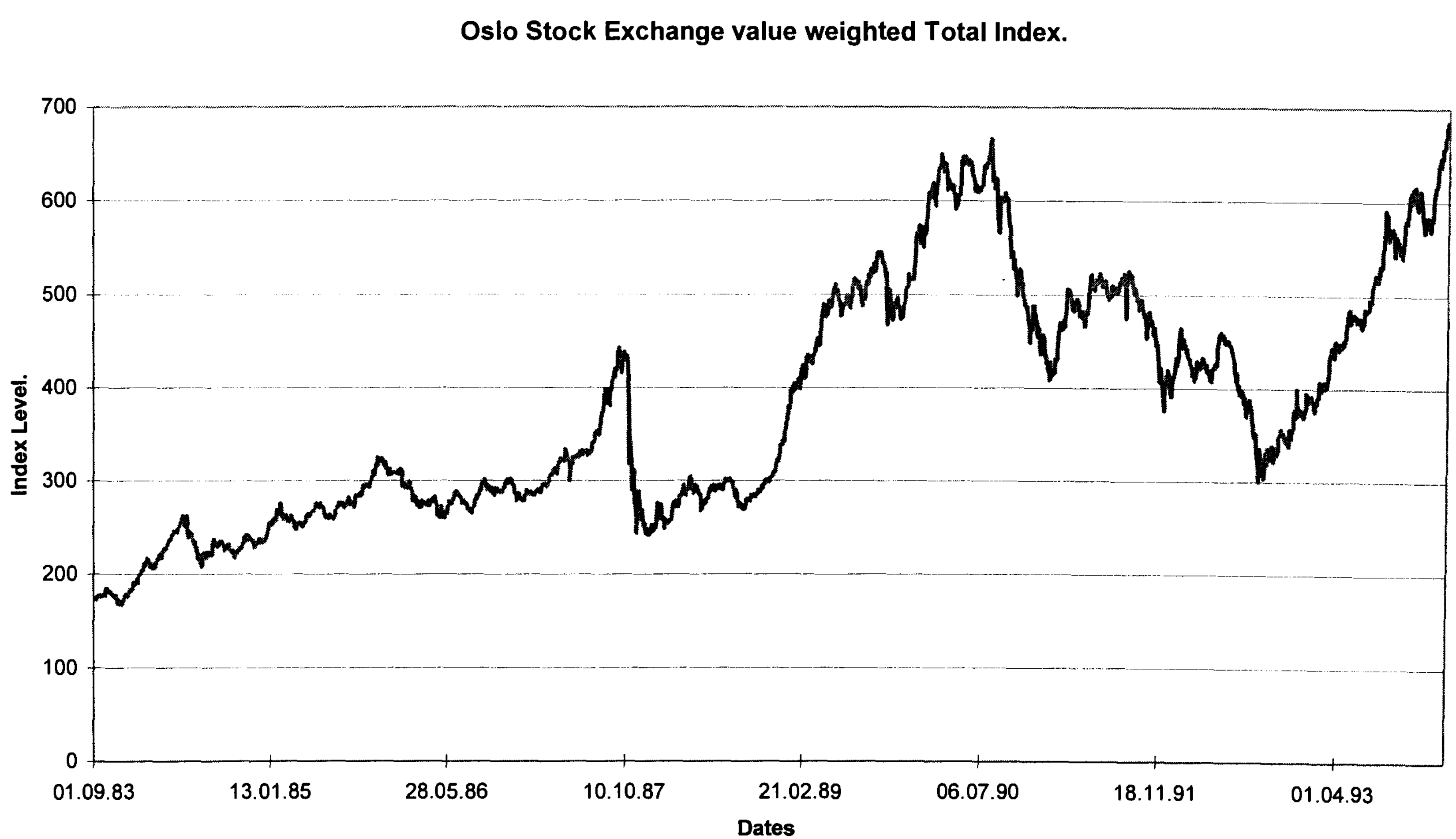

Figure 1. Oslo Stock Exchange all-market value-weighted Index (Totx)

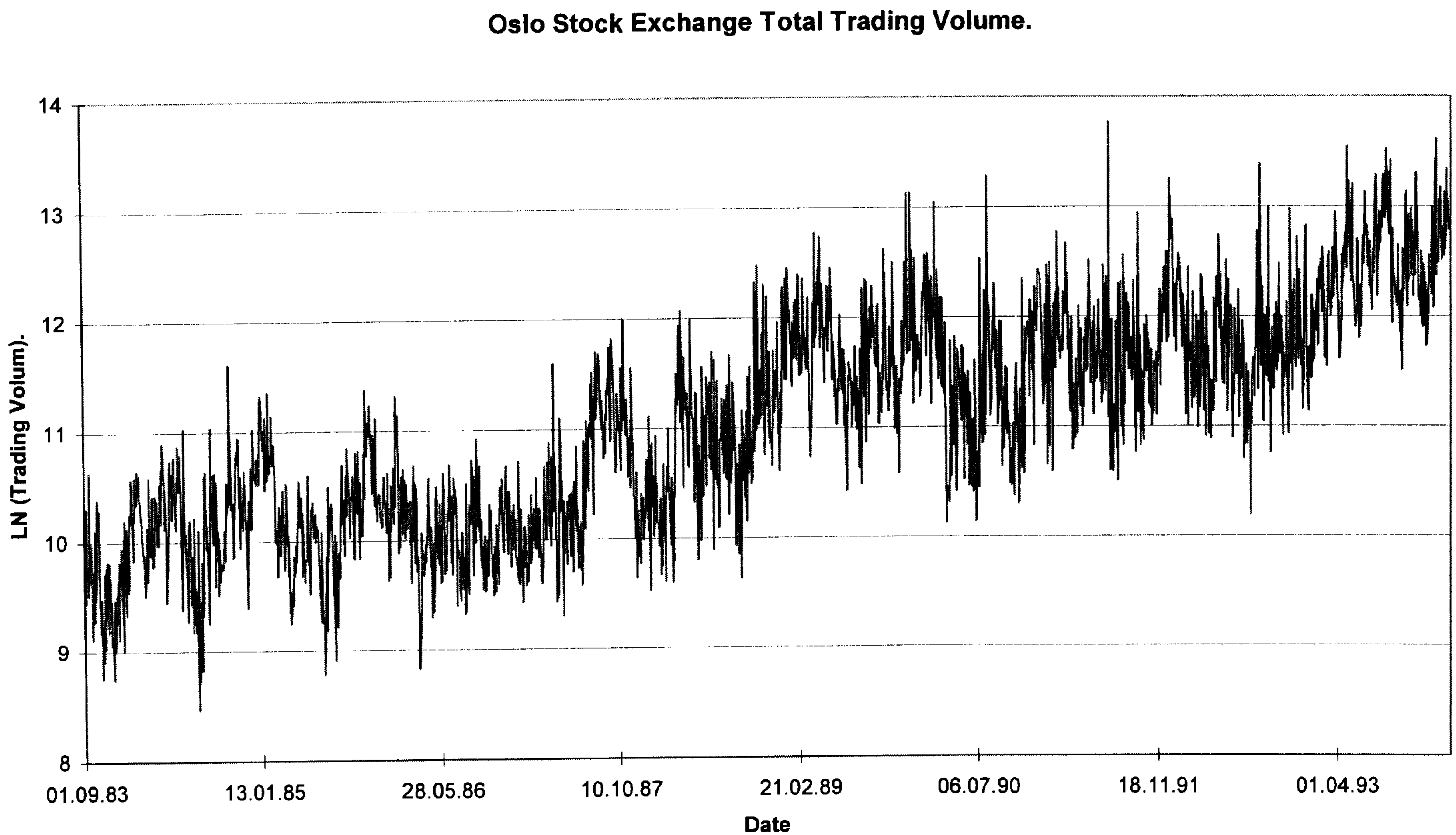

Figure 2. The Natural logarithm of Total Market Trading Volume (NOK)

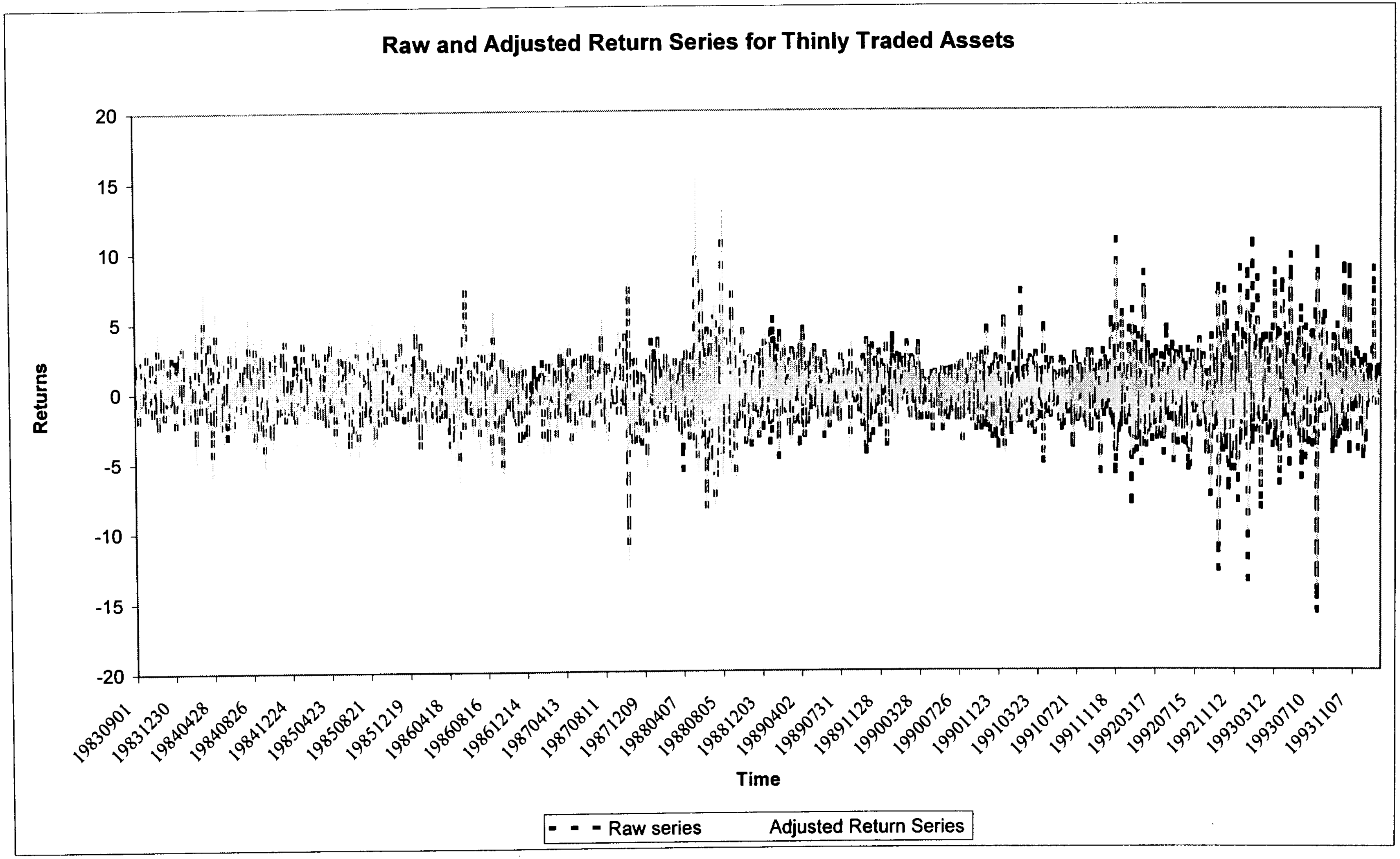

Figure 3. Raw and Adjusted Time Series for Thinly Traded Asset Portfolio

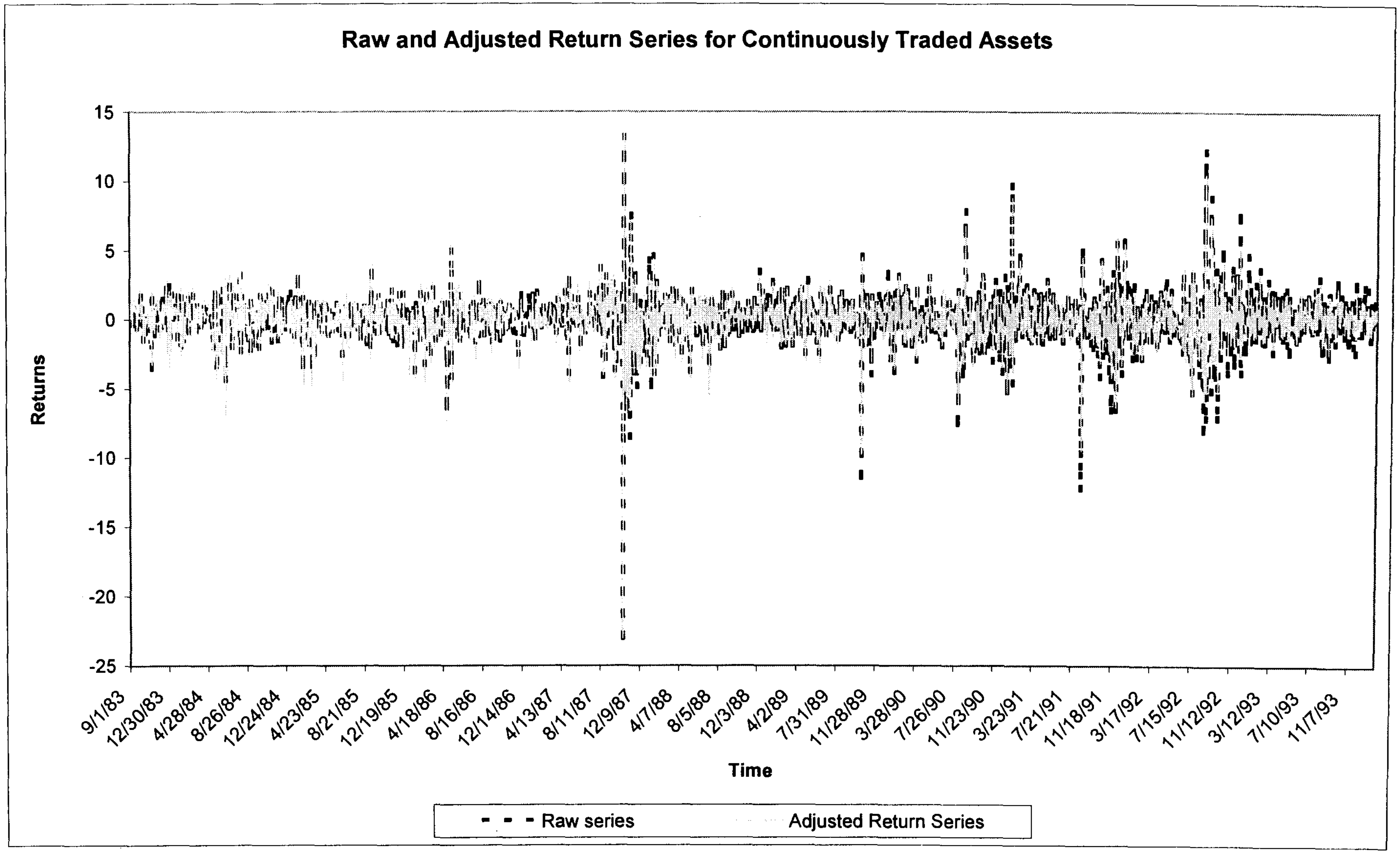

Figure 4. Raw and Adjusted Time Series for Continuously Traded Asset Portfolio

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Stock return volatility in thinly traded markets. An empirical analysis of trading and non-trading processes for individual stocks in the Norwegian thinly traded equity market

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This paper reports studies of the volatility of prices for individual stocks in the thinly traded Norwegian equity market during periods of trading and non-trading when the market is open for trading and closed. Building a model using Brownian motions, returns and variance ratios in trading and non-trading periods can be hypothesized. The model presents results that show an identical volatility in periods in which the market is open but no trades occur, and in periods of frequent trading. Furthermore, when the market is closed (weekends and holidays), the volatility is almost identical to consecutive days of trading. That is, the observed that on correspondence between return variance and transaction arrival is dependent on whether the market is open, and not simply on whether the stock is trading. This finding prevails after adjusting for non-synchronous trading using Poisson distributed trade arrivals.

I. INTRODUCTION

Bachelier (1900) first developed the random walk model (that uses a stochastic process called Brownian motions), that assumes that security prices from transaction to transaction are independent, identically distributed random variables. Bachelier's model, together with the central limit theorem, suggest that price changes are normally distributed and that their variances will be linearly related to the time interval. In the literature, one prominent explanation for the observed departure from Bachelier's model is the mixture of distributions hypothesis. This maintains that trade-to-trade asset returns exhibit leptokurtosis because they are really a combination of return distributions that are conditioned on information arrival. This means that periods of little or no information arrival result in observed return distributions different from periods when information arrives frequently. This means again that return distributions on thinly traded stocks should differ from the distributions of stocks that are traded often. A thinly traded stock might not be traded for days, and when it is traded, it is often traded on low volume. Non-trading pricing processes are therefore an important factor for the understanding of return distributions. Furthermore, non-trading pricing processes must be understood in both open and closed markets. In other words, are the non-trading return characteristics influenced by active and inactive markets?

This paper studies the returns and variance of thinly traded stocks in the Norwegian equity market. The Norwegian market has a sufficient number of thinly traded stocks (long periods of non-trading) that makes it possible to study both open and closed market processes. By developing a model for non-synchronous trading, return and variance ratios can be hypothesized for consecutive days of trading versus k numbers of non-trading days in open and closed markets. Any differences can help one to understand the processes behind the information arrival and return processes. To the author's knowledge a simultaneous comparative study of closed and open markets has so far not been performed.

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As the Norwegian market is a competitive dealer market, the results from this study should be applicable to a number of lowly traded and emerging markets in Europe, America and Asia. Moreover, as this study analyses non-trading processes in the Norwegian computerized trading market, it should also be applicable to the US OTC (over the counter) market, which is both a computerized and thin market.

The paper is organized as follows. Section II gives a literature review. Section III defines a model for the stochastic return process. Section IV gives the empirical data, the results/findings of the variance ratio analysis and, finally, Section V summarizes findings and draws conclusions.

II. LITERATURE REVIEW

In order to investigate the nature of the returns of differently traded stocks, a good starting point is the return generating model set forth in Scholes and Williams (1976, 1977). This formulation models returns and transaction arrivals and is discussed fully in Section III. Most studies in this literature review confirm that returns and volume are simultaneously jointly determined and are linked to information arrival. Therefore, if we can determine how returns and trades are distributed, this gives increased understanding of how information changes affect the market (including its micro-structure).

Returns in open and closed markets

When stock markets are closed, no trades occur. Thus although investor expectations about returns may have changed, or information arrives that would alter the expected return on a stock, price effects are not observable until the markets reopen. A common problem, therefore, in theoretical and empirical studies of financial markets is the identification of returns when the markets are closed or in non-trading periods. Many theoretical models of the return-generating process assume that price changes are independent of when and how often trades occur. That is, there exists a 'true' price whether or not a trade occurs. Thus, under this proposition a return is generated both over weekends and evenings when the market is closed and in a thin market like the Norwegian when the market is open but the asset is not frequently traded (Scholes and Williams, 1976, 1977; Lo and MacKinlay, 1990). This assumption draws theoretical justification from models of symmetrically informed traders, for example Marshall (1974) and Rubinstein (1975), in which prices can change without trading as investors' expectations change in unison.

An opposing hypothesis is that returns and transactions occur only when information arrives. With regard to returns, Ross (1989) assumes that information arrives through a Martingale process, and though no-arbitr conditions, demonstrates that return variance is direc related to the flow of information. Similarly, transacti arrivals are also likely related to the flow of information. this case, price changes when there is new information a the price changes are coincident with trades. Non-tradi periods could represent periods in which no informati arrives and hence price and return do not change.

The true relationships between information arriva transaction frequency and return probability lie somewhe between these two extremes. For instance, French and R (1986) find that prices are more volatile when markets a open than when they are closed. Their results suggest th there is a continuous component to the return, as well as component that is driven by the information arrival. If o assumes further that information arrival is more likely happen when markets are open, then one is likely to fi that trading frequency is positively related to the mean ar variance of returns.

In an early discussion of the distinction between tradir and non-trading periods, French (1980) tested two altern tive models of the process generating stock returns. Und the first, the calendar time model, returns are generat continuously in calendar time. The alternative tradir time model suggests that returns are only generated durir active trading. He found that neither model was support by US data.

French and Roll (1986) examined the difference betwee variances on trading and non-trading days. Using the dai returns they found that the hourly variance when the Ne York exchanges are open was roughly seventy times th hourly variance when they are closed. To explain th phenomenon they test several hypotheses. Their resul indicate that, on average, approximately 4 to 12% daily variance is driven by noise trading. The rest can b explained by differences in the flow of information durin trading and non-trading hours, most of which, they asser is private information.

Barclay *et al.* (1990) extend French and Roll (1986) an test the three hypotheses above and are in favour of th private information hypothesis. In particular, they find n evidence for either the public information or the nois trading hypothesis. Furthermore, their results suppo rational trading models in which private informatio revealed through trading causes variance. Booth an Chowdhury (1996) confirm that stock return variance are larger during trading hours than during non-tradin hours, and provide evidence consistent with the privat and public information hypothesis and against the noise trading explanations. Subrahmanyan (1991) shows tha when informed traders are risk averse, noise trading raise price volatility because these traders respond less aggres sively to an increase in noise trading than do risk-neutra informed traders. Further, De Long *et al.* (1990) sugges that the presence of a certain type of noise traders, 'positiv feedback traders', may lead to an increase in volatility. This occurs when informed speculators, rather than taking positions opposite the positive feedback traders, reinforce the market price movement (away from its fundamental value).

In other research in a similar vein, Lockwood and Linn (1990) examine the hourly variance of market returns during 1964–69. They find that variances during the trading day are from 2.34 to 4.37 times greater than the overnight period. McInish *et al.* (1985), Harris (1986) and McInish *et al.* (1990) examine intra-day stock returns over brief periods and find that market volatility is high near the open and close of the trading day and that volatility is greater the greater the time since the last trade.

Returns in thin markets

In a theoretical development of the role of thinness in securities markets, Cohen *et al.* (1978) use compound Poisson processes to model the discrete time arrival of transactions. They show that under heterogeneous expectations, variance is inversely related to the market value of a stock. Using total market value as an inverse proxy for thinness, they find that thinness is a significant determinant of variance. Silber (1975) investigates empirically the effect of thinness on stocks listed on the Tel Aviv Stock Exchange. He finds that two salient characteristics of thinness are a large bid–ask spread and a large variability in price per unit of excess demand. Moreover, he examines the relationship between price change volatility and the following variables: (1) the volume traded of each security, (2) the total supply outstanding of each security, (3) the number of stockholders, (4) the total asset of the firm and (5) the number of days on which no trading occurred in each security during a particular interval. His results show that the volume of trade is the best indicator of lack of thinness for the equity market. In the bond market, the number of days of no trading is the most consistent indicator of thinness followed by volume traded and size of the issue.

Because securities in thin markets often trade only once every several days, there exists a measurement problem for empirical studies that use daily returns. Observed trade-to-trade does not correspond to true daily returns since securities do not trade every day at market close. Therefore, any use of reported daily returns rather than true returns results in the econometric problem of errors in variables. As shown by Scholes and Williams (1976, 1977), failing to account for the non-synchronous trading problem results in overstated variances and spurious auto- and cross-correlations. In addition, ordinary least squared estimators for alphas and betas in the market model are biased and inconsistent.

Lo and MacKinlay (1990) develop a stochastic model of non-synchronous asset prices that accommodates the problem of non-trading. In particular, their model assumes more realistically that the time between trades is stochastic rather than limiting the model by forcing a trade per day, as Scholes and Williams did (1976, 1977). Lo and MacKinlay also derive closed-form expressions for the unconditional means, variances and covariance of observed returns as functions of the non-trading process. Among other results, they find that non-synchronous trading does not affect the means of individual returns, while, on the other hand, it increases the observed variance of these same security returns.

This study is slightly different from Lo and MacKinlay's in that multiple-day return variances including one or more non-trading days, will be compared to variances measured over consecutive trading days given that the number of non-trading days in each multiple-day return observation is known. Consequently, the relation of measured variances to true variances in the presence of non-synchronous trading and non-trading and conditional on a known number of non-trading periods must be developed.

III. A MODEL FOR A CONTINUOUS STOCHASTIC RETURN GENERATING PROCESS

Assume that the return is a continuously compounded rate R_t per trading period $\{t-1, t\}$. The price of the asset at time t is log-normally distributed and denoted P_t, so $R_t = \ln(P_t/P_{t-1})$ and normally distributed. We therefore assume further that P_t is a Brownian motion with parameters μ and σ^2 selected so that $E(R_t) = \mu$ and $\text{Var}(R_t) = \sigma^2$. The simplest representation of the (arithmetic) Brownian motion is:

$$R_t = \mu\, \mathrm{d}t + \sigma\, dz_t$$

where $\mathrm{d}z_t$ is the increment of a Wiener process, defined as $\mathrm{d}z_t = \varepsilon t \cdot \sqrt{\mathrm{d}t}$, where ε_t has zero mean and unit standard deviation, $E(\mathrm{d}z_t) = 0$ and $\text{Var}(\mathrm{d}z_t) = E((\mathrm{d}t)^2) = \mathrm{d}t$. μ is called the drift parameter and σ the variance parameter. Note that over any time interval $\mathrm{d}t$, R_t, is normally distributed, and has expected value $E(R_t) = \mu\,\mathrm{d}t$ and variance $\text{Var}(R_t) = \sigma^2 \cdot \mathrm{d}t$. Furthermore, note that a Wiener process has no time derivative in a conventional sense; $\mathrm{d}z_t/\mathrm{d}t = \varepsilon_t \cdot (\mathrm{d}t)^{-1/2}$, which become infinite as $\mathrm{d}t$ approaches zero.

In general, the return $R_{a,b}$ over a time period $\{a, b\}$ is given by

$$R_{a,b} = \ln(P_b/P_a)$$

where P_a and P_b are the observed prices at time a and b ($a < b$). When a and b is constantly changing among the component stocks, this may cause non-synchronous trading.

If we assume that the return generating process $R_{a,b}$ follows an arithmetic Brownian motion a model for $R_{a,b}$ is given by

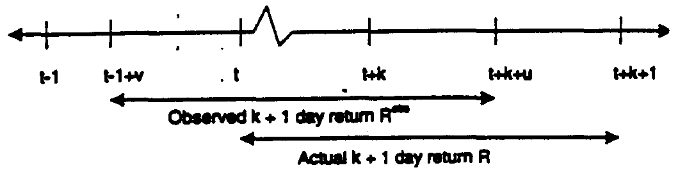

Fig. 1. *Observed versus actual return*

$$R_t = (b-a)\mu + \sigma(dz_a - dz_b) \quad (1)$$

where $dz_b - dz_a$ is normal with mean 0 and variance $b-a$.

Let $t-1+v$ be the time of the last trade during trading period $\{t-1, t\}$ and let k be the number ($k = 0, 1, 2, 3$) of trading periods after $\{t-1, t\}$ in which there are no trades. This time definition gives a definition of v that is the time until the last trade of the day t. This means that there is at least one trade during $\{t-1+k+1, t-1+k+2\}$. Let u be the time for the trading period $\{t+k, t+k+1\}$. We illustrate the notation in Fig. 1.

R^{obs} represents the observed $k+1$ period return based on the last trades in period $\{t-1, t\}$ and $\{t+k, t+k+1\}$. Assuming that k and v are independent and that u and v are identically, independently distributed, employing Equation 1 leads to

$$\begin{aligned} R^{obs} &= \mu((t+k+u) - (t-1+v)) + \sigma(dz_{(t+k+u)} \\ &\quad - dz_{(t-1+v)}) \\ &= \mu(k+1+u-v) + \sigma(dz_{(t+k+u)} - dz_{(t-1+v)}) \end{aligned}$$

Using the standard result from statistical calculus gives us the expected return $E(R^{obs})$ for a given day equals

$$\begin{aligned} E(R^{obs}) &= \mu((k+1+E(u) - E(v)) + \sigma(E(dz_{(t+k+u)} \mid k) \\ &\quad - E(dz_{(t-1+v)} \mid k)) \\ &= \mu(k+1) + \sigma(k+1)(0) \\ &= \mu(k+1) \end{aligned}$$

That is, the expectation of the observed return is equal to the true mean one-period return multiplied by $k+1$. This is consistent with both Scholes and Williams (1976, 1977) and Lo and MacKinlay (1990) who find that mean returns are unaffected by non-synchronous trading.

To compute the observed variance for R^{obs}, $\mathrm{Var}(R^{obs}/k)$, we can use the fact that

$$\mathrm{Var}(A) = \mathrm{Var}(E(A \mid B)) + E(\mathrm{Var}(A \mid B))$$

Thus,

$$\begin{aligned} \mathrm{Var}(R^{obs} \mid k) &= \mathrm{Var}(E(R^{obs} \mid k, u, v)) \\ &\quad + E(\mathrm{Var}(R^{obs} \mid k, u, v)) \end{aligned}$$

Some algebra leads to

$$\begin{aligned} &= \mathrm{Var}(\mu(k+1+u-v)) + E(\sigma^2(k+1+u-v)) \\ &= \mu^2 \mathrm{Var}(0+u-v) + \sigma^2 E(k+1+u-v) \end{aligned}$$

Now, since u and v are identically and independently d tributed

$$\mathrm{Var}(R^{obs} \mid k) = 2\mu^2 \mathrm{Var}(u) + \sigma^2(k+1) \quad ($$

which provides a general form of the relationship betwe observed and true variance.

Equation 2 allows for correction of the variance measured returns in thin markets. If we determine the va ance of the measured returns, their means, and the varian of the time interval between the beginning of a day and t last trade, then the true variance of returns can also determined. However, unless we assume a specific dist bution for the trading process, Var(u) cannot easily determined and a closed-form relationship between mea ured and actual returns cannot be obtained.

IV. EMPIRICAL DATA AND RESULTS

Data

The study uses daily and return series for Norwegian stoc spanning the period from October 1983 to February 199 This high frequency time series database gives at most 261 observations for each firm.

Daily trading volume data was obtained from the 'Osl Børs Informasjon' database. All stocks in the database a used in the analysis. We have divided the period into tw subperiods; one period from October 1983 to Septemb 1987 (1019 observations) and one period from Decemb 1987 to February 1994 (1546 observations). Daily stoc returns are calculated as the change in the logarithm successive closing prices.

Sample firms satisfied the following criteria:

(1) The firms are listed at the Norwegian Stoc Exchange and information of daily ask, bid an settlement prices including trading volume are avai able.
(2) The firms must have at least five observations of tw day returns including one non-trading day.

Return observations on consecutive days were mo numerous than two-days' returns that included one no trading day. Thus, consecutive-day variances are in gener estimated with more precision than variances measure over $k+1$ days where k is the number of non-tradin days. The intersection of these selection criteria yielde 227 sample firms spanning the whole period.

Table 1. *Summary statistics of trading and non-trading days in the three periods under examination are presented below for each firm in the sample*

Period	Statistic	Listed days	Nontrading days	Trading days	Percentage of trading days
1 June 1983	Mean	1531.27	611.93	918.75	60.05%
February 1994	Standard deviation	826.54	591.54	691.28	27.52%
Number of firms: 227	Minimum	52.00	12.00	33.00	4.54%
	Maximum	2681.00	2361.00	2660.00	99.25%
1 June 1983	Mean	914.56	342.85	571.36	63.87%
1 October 1987	Standard deviation	277.07	309.50	314.64	28.22%
Number of firms: 127	Minimum	65.00	11.00	23.00	6.68%
	Maximum	1092.00	1019.00	1081.00	98.99%
1 December 1987	Mean	1042.10	424.69	616.98	58.94%
1 February 1994	Standard deviation	464.37	366.08	420.94	27.03%
Number of firms: 183	Minimum	52.00	9.00	16.00	4.31%
	Maximum	1547.00	1418.00	1538.00	99.42%

Listed days are the total number of days in each period for which a firm was listed as a trading stock. Non-trading days are the number of listed days on which the market was open and volume was zero, while trading days are the listed days with non-zero volume. Trading days as a percentage of total listed days is presented in the last column.

Results

Table 1 presents summary data on frequency of trading and non-trading for the stocks in the sample. As can be seen from Table 1, not all stocks were listed for the entire 6-year period. In order to determine the percentage of trading days for a given stock, only days on which the stock is listed and a first settlement price is quoted are considered.

Among the stocks in the sample, there is quite a large range in the frequency of trading. For the entire period from 1 June 1983 to 1 February 1994, the mean percentage of trading days is 60.05%, while the minimum is 4.54% and the maximum 99.25%. Thus the sample contains both frequently and infrequently traded stocks. Finally, note especially that the periods have quite different numbers of listed days. Moreover, the main reason for dividing the periods as shown above is the crash in October 1987 and the fact that the Oslo Stock Exchange switched to an electronic trading system in March 1988.

Observed mean and variances over trading and non-trading periods when the market is both open and closed. If the return-generating process is, as is commonly assumed, geometric Brownian motion, then stock return means and variances are linearly related to the time interval, and thus they are constant across trading and non-trading periods of the same length. Therefore, the mean and variance of returns over a period in which there is no trading for n days should be n times the mean and variance respectively of a weekday return that is measured between consecutive days of trading. To test these hypotheses, for each firm continuously compounded returns are calculated for each day, returns are calculated for $k = 1$, 2 and 3 non-trading days (zero trading volume)[1] and returns are calculated for Mondays ($k = 2$) and holidays ($k = 1$). Figure 2 illustrates this calculation for a consecutive return ($k = 0$) and a three-day return ($k = 2$). Thus, for $k = 0$, the return is the calculated continuously compounded return using closing prices at $t = 1$ and $t = 2$. With two days of non-trading, also shown in Fig. 2, the 3-day return is calculated as the continuously compounded return over 3 days using closing prices at $t = 2$ and $t = 5$.

From these returns, means and variances are calculated at the firm level for each category ($k = 0$, 1, 2, 3, Mondays ($k = 2$) and holidays ($k = 1$)). Each firm has a mean and variance of returns for consecutive days of trading and for periods in which there are 1, 2, and 3 days of non-trading when the market is open, and for periods in which there are 1 and 2 days of non-trading when the market is closed. To be included in the comparison of $k = 0$ and $k = 1$ returns, a firm had to have five observations of each return. Similarly, to be included in the $k = 0$ versus $k = 2$ comparison, a firm had to have at least five observations of each. The same constraint was imposed for $k = 0$ versus $k = 3$, Mondays ($k = 2$) and holidays ($k = 1$) comparisons. This means that frequently traded firms which appeared in the $k = 0$ versus $k = 1$ comparison were not likely to appear in the $k = 0$ versus $k = 3$ comparison.

The means and variances calculated above are then, in turn, aggregated across firms as grand averages for each period. Thus each time period provides a mean and vari-

[1] Non-trading periods greater than 3 days implies that only observations within the 5 weekdays are accounted for.

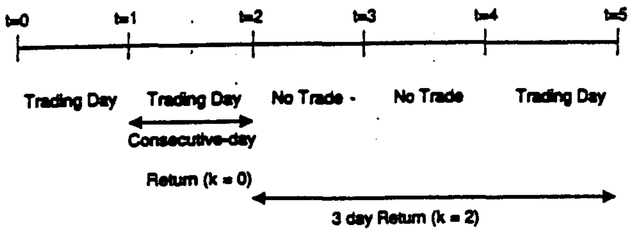

Fig. 2. *Trading and non-trading return calculations*

ance across firms for consecutive days of trading, each of 1 to 3 days of non-trading, Mondays ($k = 2$) and holidays ($k = 1$). Formally, the grand means and variances for a given k and period are calculated as

$$E(R^{\mathrm{Obs}} \mid k) = \frac{1}{M_k} \cdot \left[\sum_{i=1}^{M_k} \left(\frac{1}{N_k} \cdot \left(\sum_{n=1}^{N_k} R_{in}^{\mathrm{Obs}} \right) \right) \right] \quad (3)$$

and

$$\mathrm{Var}(R^{\mathrm{Obs}} \mid k) = \frac{1}{M_k} \cdot \left[\sum_{i=1}^{M_k} \left(\frac{1}{N_k} \cdot \sum_{n=1}^{N_k} (R_{in}^{\mathrm{Obs}} - E[R^{\mathrm{Obs}} \mid k])^2 \right) \right] \quad (4)$$

where

R_{in}^{Obs} = total returns for firm n in period i;
N_k = total number of observations for a given firm n and non-trading period k;
M_k = total number of firms in the sample non-trading periods.

Table 2 presents the mean returns for each period. Th null hypothesis is that the non-trading day mean retur $E(R^{\mathrm{obs}} \mid k)$ should be $k+1$ times the consecutive da returns. The alternative is that the non-trading day mea: return $E(R^{\mathrm{obs}} \mid k)$ is different from $k+1$ times the con secutive day returns.

Hypothesis 1:

$$\mathrm{H_0}: E(R^{\mathrm{obs}} \mid k) = (k+1)\mu, \qquad \text{for } k = 1, 2, 3,$$
$$\text{Mondays } (k = 2) \text{ and holidays } (k = 1)$$
$$\mathrm{H_A}: E(R^{\mathrm{obs}} \mid k) \neq (k+1)\mu, \qquad \text{for } k = 1, 2, 3,$$
$$\text{Mondays } (k = 2) \text{ and holidays } (k = 1)$$

Using a t-test,[2] when the market is open we find th probability that $|t|$ takes a value higher than the calculatec value for the degrees of freedom is very high (> 90%). Thi: suggests that when the market is open the null hypothesi:

Table 2. *Sample means*

Period:	Number of non-trading days k	Number of firms	Consecutive-day mean return $-k = 0$	Mean return $k = 1, 2, 3$
1 June 1983–	1	218	0.29074825	−0.15545964
February 1994	2	159	0.29927974	−0.50128789
	3	105	0.31726758	−0.64096307
	Mondays	224	0.25280417	0.08760102
	Holidays	173	0.17203636	0.65244010
1 June 1983–	1	133	0.30190492	0.08846285
1 October 1987	2	94	0.30755424	0.07781368
	3	72	0.37591182	−0.17648476
	Mondays	134	0.21119597	0.38462166
	Holidays	89	0.08411117	0.42821442
1 December 1987–	1	177	0.31982536	−0.11076306
1 February 1994	2	121	0.33456256	−0.71609132
	3	89	0.39928584	−0.65447482
	Mondays	195	0.29716997	−0.02975241
	Holidays	138	0.22903584	0.76729028

Returns over $k+1$ days are measured between the last transaction on a pair of trading days and include k non-trading days. The third column shows the number of firms in a given period that has at least 5 observations for a particular k. In each sample period, the mean returns are calculated at the firm level for both consecutive daily returns ($k = 0$) and for non-trading periods ($k = 1, 2, 3$, Mondays and holidays). These are then aggregated to find mean returns for consecutive days of trading and for non-trading and for non-trading periods.

cannot be rejected at 5% for any k non-trading days. That is, the mean returns may in fact be an integer multiple of the time interval. The result is consistent for all non-trading days when the market is open over all three periods. However, note the close-to-zero and negative returns for the non-trading days in all periods in contrast to positive returns for consecutive days of trading. Furthermore, when the market is closed, the results still show the same favourable attitude towards the null. That is, the mean returns are in fact an integer multiple of the time interval. For days following Mondays ($k = 2$) and holidays ($k = 1$) we find the probability that $|t|$ takes a value higher than the calculated value for the degrees of freedom is also higher than 90%. That is, one (two) day(s) of non-trading returns when the market is closed may in fact be equal to 1 (2) days of consecutive day returns. The results are consistent over all three periods. The first hypothesis shows therefore no significant results and is probably driven by the fact that variances are largely relative to the magnitude of mean returns. Therefore, to extend and further analyse the firms, a second hypothesis that mean $k+1$ day returns are equal to zero are tested. This hypothesis is set out below.

Hypothesis 2:

$$H_0: E(R^{obs} \mid k) = 0, \qquad \text{for } k = 1,2,3,$$

$$\text{Mondays } (k = 2) \text{ and holidays } (k = 1)$$

$$H_A: E(R^{obs} \mid k) \neq 0, \qquad \text{for } k = 1,2,3,$$

$$\text{Mondays } (k = 2) \text{ and holidays } (k = 1)$$

For all non-trading cases when the market is open and closed, using the same form of t-test as above, the null hypothesis cannot be rejected at 5%. That is, the mean returns for all non-trading categories may in fact be zero. The result is consistent over all three periods. This result is also probably driven by large variances relative to mean returns. Finally a third hypothesis that mean $k+1$ day-returns are equal to mean consecutive returns is tested. This hypothesis is set out below.

Hypothesis 3:

$$H_0: E(R^{obs} \mid k) = \mu, \qquad \text{for } k = 1,2,3,$$

$$\text{Mondays } (k = 2) \text{ and holidays } (k = 1)$$

$$H_A: E(R^{obs} \mid k) \neq \mu, \qquad \text{for } k = 1,2,3,$$

$$\text{Mondays } (k = 2) \text{ and holidays } (k = 1)$$

When the market is open and closed all the non-trading cases cannot reject the null hypothesis at 5%. That is, the mean returns are in fact equal to the consecutive day mean return. Therefore, all three hypotheses above cannot be rejected.

However, some observations are interesting and readily available. For the non-trading cases when the market is open, consistent positive mean consecutive day returns are found. The returns are remarkably stable showing results of about 0.3% to 0.4% for all three sample periods. The consecutive daily returns for all three periods are lowest for $k = 1$ and highest for $k = 3$. This suggests that frequently non-traded firms show positive returns when they are traded for consecutive days. A possible interpretation is that lowly traded firms (periods of non-trading) are rewarded with a highly daily return when they are traded for consecutive days. Therefore for lowly traded firms a possible interpretation of our results is that there is a trading effect in the market. The positive-trading return effect seems to increase the less the firm is traded in the market. Moreover, the non-trading periods k mean returns are mainly negative except for $k = 1$ and 2 in the first sub-period 1983–87. However, the returns are close to zero. Therefore, results suggest that firms experience higher negative returns the longer the non-trading period. In this case a possible interpretation is that there is a non-trading effect for long non-trading periods. The non-trading negative return effect seems to increase strongly from $k = 1$ to $k = 2$ and 3. Trading volume in the form of the number of trading and/or non-trading days is therefore a candidate for an independent variable in cross-sectional regressions of daily stock returns.[3]

Finally, results also suggest a shift in the well-known Monday effect. For the sub-period 1983–87 a high positive daily Monday effect is found beyond other days the average consecutive trading day returns. This result also dominates the whole period 1983–94. However, studying the period 1987–94 shows negative and close-to-zero Monday returns. This return is far below the other days' average consecutive trading day returns. Therefore, the well-known Monday effect seems to have moved from positive before the crash to negative after the crash. Probably more interesting is the consistently positive returns following holidays for all three time periods.

In summary, the result of the sample means suggest positive consecutive trading and negative non-trading mean return effects in the market. Moreover, it seems that the longer the non-trading period the higher the negative market means return effect. We now turn to the variance ratios. Finally, returns following holidays show consistently high positive returns and the Monday effect has moved from a positive to a negative anomaly.

Table 3 shows the results for the return variances. In the rightmost columns of this table, the average variance ratio

Table 3. *Variance ratios*

Period	Number of non-trading days k	Number of firms	Consecutive-day variance $-k=0$	Mean variance $k = 1, 2, 3$	Mean variance ratio $k = 1, 2, 3$	
1 June 1983– February 1994	1	218	16.383864	28.322080	1.728657	*
	2	159	16.688568	40.646385	2.435583	*
	3	105	16.866576	52.244640	3.097525	*
	Monday	224	16.124898	16.213218	1.005477	*
	Holiday	173	14.489125	14.053013	0.969901	*
1 June 1983– 1 October 1987	1	133	11.965577	18.290556	1.528598	*
	2	94	12.895828	26.237021	2.034536	*
	3	72	13.949368	42.770345	3.066114	*
	Monday	134	11.127184	10.154260	0.912563	*
	Holiday	89	9.586768	9.094071	0.948607	*
1 December 1987– 1 February 1994	1	177	19.891649	32.743578	1.646097	*
	2	121	18.947065	47.538090	2.508995	*
	3	89	20.477151	60.083678	2.934182	*
	Monday	195	19.087888	18.993608	0.995061	*
	Holiday	138	15.652051	15.760224	1.006911	*

* Statistical significance for the first hypothesis set forth above in the table header at a 5% confidence level.
** Statistical significance for the second hypothesis set forth above in the table header at a 5% confidence level.
Returns over $k+1$ days are measured between the last transaction on a pair of trading days and include k non-trading days on which th market is open but no trades occur. The third column shows the number of firms in a given period that has at least 10 observations for particular k. In each sample period, the variances are calculated at the firm level for both consecutive daily returns ($k = 0$) and for non trading periods ($k = 1, 2, 3$, Mondays and holidays). These are then aggregated to find mean variances for consecutive days of tradin and for non-trading periods. Dividing the non-trading period variance by the consecutive day variance results in the mean variance ratic

for each sample period and non-trading duration k are presented. Variance ratios are determined by dividing variance measured over $k+1$ days for $k = 1, 2, 3$, Mondays ($k = 2$) and holidays ($k = 1$), by the variance of consecutive day ($k = 0$) returns.[4]

If returns follow a random walk, the variance ratio of each trading day category should equal $k+1$. This hypothesis is set out below.

Hypothesis 4:

$$H_0: \frac{\mathrm{Var}\,(R^{obs} \mid k)}{\mathrm{Var}\,(R^{obs} \mid k=0)} = k+1 \qquad \text{for } k = 1,2,3,$$

Mondays ($k = 2$) and holidays ($k = 1$)

$$H_A: \frac{\mathrm{Var}(R^{obs} \mid k)}{\mathrm{Var}\,(R^{obs} \mid k=0)} \neq k+1 \qquad \text{for } k = 1,2,3,$$

Mondays ($k = 2$) and holidays ($k = 1$)

The conventional method of using the F-test[5] for testin variances across samples is to take their ratio, adjust fo degrees of freedom and compare this to one. In our case, t perform the F-test in this fashion, one must first multipl Var_0 by $k+1$ or divide $\mathrm{Var}\,k+1$ by $k+1$ and then pro ceed with the conventional method. For example, a vari ance ratio based on two-day returns with $k = 1$ day of non trading should equal two.

Employing an F-test for each period and for each cate gory k, the 5% level of the null from hypothesis 4 is show by a * to the right of each k for each sample. That is, for al three periods and all non-trading periods ($k = 1, 2, 3$) th random walk hypothesis cannot be rejected. Variances or periods that include one or more non-trading days appea to be equal to the prediction of the random walk model This suggests that the return variance occurs both on trad ing and non-trading days.[6]

Consider the variance ratio over the entire sample perioc

[2] The t-statistic use: $t = |(R_k/(k+1)) - \mu/\sigma|$, where R_k is k non-trading days return, m is the consecutive days of trading return, σ is the standard deviation, k is the number of non-trading days. The statistic gives the probability that $|t|$ takes a value greater than the calculated value for the stated degrees of freedom. This is thus a two-tailed test.

[3] A non-parametric integrated hazard function is therefore of considerable interest.

[4] Note that all variance ratio calculations are done within the same firm and the numbers reported are the average over all the firms in the sample.

[5] The F-test uses: $(\sigma_1^2/f_1)/(\sigma_2^2/f_2) =$, where σ_1^2 and σ_2^2 are independent χ^2 variables with f_1 and f_2 degrees of freedom respectively.

[6] Heinkel and Kraus (1988) suggest a model of non-trading stocks, which may fit the empirical data well. These authors assume that the return variance of individual stocks on days in which they do not trade is equal to the return variance of the market portfolio. Firm-specific information accumulated over non-trading days is then aggregated on to the first day of trade following a non-trading period. They then estimate betas through an iterative GLS procedure.

for $k = 1$, that is 1.7287. If return variance on the first day of trade after a one-day non-trading period is equal to consecutive day variances, then the non-trading day variance is 86.44% of the variance over one consecutive trading day. The same number for a two-day (three-day) non-trading period is 81.19% (77.44%) of the variance of a 2 (3) day period of consecutive trading.

To consider the situation when the market is closed we study the Monday and holiday results. The non-trading days will be days when the market is closed. That is, for all three periods and both Mondays and holidays the random walk hypothesis is rejected. Variances on periods that include 1 or 2 non-trading days when the market is closed do not follow the prediction of the random walk model. This suggests that the return variance occurs only when the market is open regardless of trading or non-trading periods.

We now explore the conjecture that the variance of returns occurs primarily on days when the market is open. Such a test is performed by using the hypothesis that the variances of consecutive days of trading when the market is open, are equal to the variances of those periods that include non-trading days when the market is open ($k = 1, 2, 3$) and closed (Mondays and holidays). This is equivalent to testing the hypothesis that the ratio of variances is one:

Hypothesis 5:

$$H_0: \frac{\mathrm{Var}(R^{obs} \mid k)}{\mathrm{Var}(R^{obs} \mid k = 0)} = 1 \qquad \text{for } k = 1, 2, 3, 4, 5, 6$$

Mondays ($k = 2$) and holidays ($k = 1$)

$$H_A: \frac{\mathrm{Var}(R^{obs} \mid k)}{\mathrm{Var}(R^{obs} \mid k = 0)} \neq 1 \qquad \text{for } k = 1, 2, 3, 4, 5, 6$$

Mondays ($k = 2$) and holidays ($k = 1$)

Table 3 shows that this null hypothesis can be rejected over all three periods for non-trading days when the market is open. However, the null hypothesis cannot be rejected over the two non-trading days when the market is closed. That is, for Mondays and holidays the variance is constant; for non-trading days when the market is open, the variance is $(k + 1)$ the variance of consecutive days of trading.

Variances and variance ratios – adjusted for non-synchronous trading. In this section it is assumed that the occurrence of trades follows a Poisson distribution. This means that trades occur as a Poisson process with parameter λ, where λ is the mean number of trades per period, and that k is known. Let $s = 1 - u$ represent the time remaining in a given trading period after the last trade. Then s is distributed exponentially $\lambda e^{-\lambda s} (\lambda > 0)$ on $0 \leq s < 1$ with the probability of no trade during any trading day of PROB $(s \geq 1) = \int_1^\infty \lambda \cdot e^{-\lambda s} ds = e^{-\lambda}$. If s is conditioned on at least one trade per trading day, the density function is

$$f(s) = \frac{\lambda e^{-\lambda \cdot s}}{1 - e^{-\lambda}} \qquad 0 \leq s < 1$$

Given the density function $f(s)$ above, and remembering that $u = 1 - s$, the conditional variance of u is calculated using integration by parts:

$$\mathrm{Var}(u) = \frac{\lambda^2 e^{\lambda} + e^{(-\lambda)} - 3 + 3 e^{\lambda} - \lambda^2 - e^{(2\lambda)}}{\lambda^2 (e^{\lambda} - 1)^2 (-1 + e^{-\lambda})} \tag{5}$$

Substituting Equation 5 into Equation 2 above provides the relationship between observed variance and true variance, given that the process describing trades is Poisson:

$$\mathrm{Var}(R^{obs} \mid k) = 2\mu^2 \times \left[\frac{\lambda^2 e^{\lambda} + e^{(-\lambda)} - 3 + 3e^{\lambda} - \lambda^2 - e^{(2 \cdot \lambda)}}{\lambda^2 (e^{\lambda} - 1)^2 (-1 + e^{(-\lambda)})}\right] + \sigma^2 (k + 1) \tag{6}$$

Equation 6 is identical to Scholes and Williams' result except that the k non-trading periods are not constrained to be zero.

Now all the inputs necessary to relate the measured variance to the true variance are at hand. If an empirical estimate is used for λ, the mean arrival rate of transactions, then this model provides an approximation of how much of the observed variance is due to non-synchronous trading and provides a means to correct measured variances for non-synchronous trading.

Equation 6 captures the general form of the relationship between observed and true variance. As in Scholes and Williams, the observed variance is dependent on the mean return and always overstates the true variance of returns. Increasing time between trades increases the observed variance. And as λ increases, the observed variance quickly approaches the true variance; in other words, as trading becomes more frequent, measurement problems diminish. This is illustrated in Fig. 3.

To estimate λ, the mean number of trades per period, the number of trading days (days on which trading volume > 0) is divided by the total number of trading days possible. If one trading day is taken to be one period, this provides an approximation of the empirical probability density of trading for one day. This estimate can then be used to estimate λ.

The Poisson process is given by

$$P(X = x) = \left(\frac{\lambda^x}{x!}\right) e^{-\lambda}$$

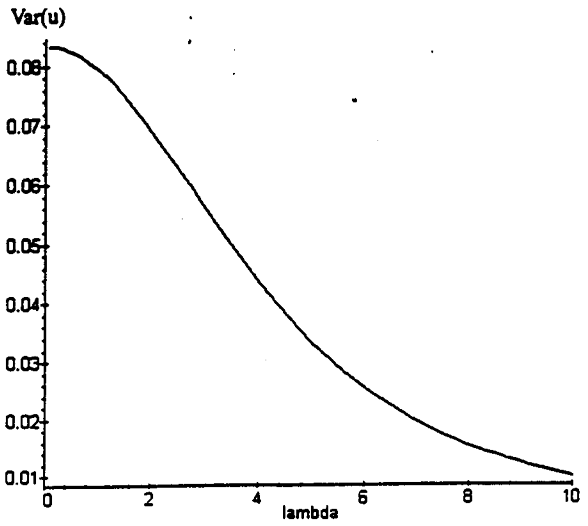

Fig. 3. *The variance of u with increasing λ (lambda)*

where x = the number of trades, and λ = mean number of trades per period. The probability of no trading is

$$P(X=0)=\left(\frac{\lambda^0}{0!}\right)e^{-\lambda}=e^{-\lambda}$$

Therefore,

$$\lambda = -\ln(P(X=0)) \quad (7)$$

For $P(X=0)$ we substitute the percentage number of days in the year on which there are no trades. Now using Equations 5 and 6 the true variance σ^2 can be estimated because all other variables are known empirically. Using observed means and variances from the results above and estimated λs using Equation 7, estimates of true variances are calculated according to Equations 5 and 6 and presented in Table 4. Rearranging Equation 6,

$$\sigma^2=\frac{\mathrm{Var}(R^{\mathrm{Obs}}\mid k)-2\mu^2[\mathrm{Var}(u)]}{k+1}$$

As the mean transaction arrival rate increases, the variance of u, the time between the beginning of the day and the last trade is reduced. Given k constant and $\mu_i=\mu\neq 0$ for all firms i, more thinly traded stocks will have bigger adjustments to measured variances. And, for a particular μ and λ, as the length of the trading period increases, the correction to measured variances diminishes at a rate $1/(k+1)$.

In Table 4, the significance levels in Table 3 prevail for each variance ratio test after adjusting for non-synchronous trading and non-trading when the market is open and closed. In fact, the magnitudes of the adjustments for non-synchronous trading do not materially affect the comparisons. Non-synchronous trading alone is unable to explain the difference in return variance between trading and non-trading periods when the market is open and closed.

Table 4. *Variance ratios, adjusted for non-synchronous trading*

Period	Number of non-trading days k	Number of firms	Consecutive-day variance $-k=0$	Mean variance $k=1, 2, 3$	Mean varaince ratio $k=1, 2, 3$	
1 June 1983– February 1994	1	218	16.681249	29.761413	1.784124	*
	2	159	16.929198	43.325007	2.559188	*
	3	105	16.839335	51.613798	3.065073	*
	Monday	224	16.355083	16.621279	1.016276	**
	Holiday	173	14.574597	15.400688	1.056680	**
1 June 1983– 1 October 1987	1	133	12.374857	19.080943	1.541912	*
	2	94	13.173797	27.784941	2.109106	*
	3	72	13.889328	42.295576	3.045185	*
	Monday	134	11.319234	10.576255	0.934361	**
	Holiday	89	9.643568	10.145162	1.052013	**
1 December 1987– 1 February 1994	1	177	20.309926	35.319905	1.739046	*
	2	121	19.236521	51.487278	2.676538	*
	3	89	20.433834	59.464833	2.910116	*
	Monday	195	19.362259	19.512647	1.007767	**
	Holiday	138	15.753671	17.366666	1.102388	**

* Statistical significance for the first hypothesis set forth above in the table header at a 5% confidence level.
** Statistical significance for the second hypothesis set forth above in the table header at a 5% confidence level.
Returns over $k+1$ days are measured between the last transaction on a pair of trading days and include k non-trading days on which the market is open but no trades occur. The third column shows the number of firms in a given period that has at least 10 observations for a particular k. In each sample period, the variances are calculated at the firm level for both consecutive daily returns ($k=0$) and for non-trading periods ($k=1, 2, 3$, Mondays and holidays). These are then aggregated to find mean variances for consecutive days of trading and for non-trading periods. Dividing the non-trading period variance by the consecutive day variance results in the mean variance ratio.

V. SUMMARY AND CONCLUSIONS

Empirical studies of returns and transaction arrivals typically reject simple assumptions as (1) return means are known to differ over weekends, holidays, and month of the year (French, 1980; Gibbons and Hess, 1981); (2) return variances are known to be lower during periods when the market is closed including weekends (Fama, 1965), exchange holidays (French and Roll, 1986) and overnight periods (Lockwood and Linn, 1990); (3) return variances exhibit season differences such as across days of the week (Lockwood and Linn, 1990) and near the open close of trading hours (McInish *et al.* 1985; Harris, 1986; McInish *et al.*, 1990). An increasing body of evidence following GARCH specifications indicates that return variances are also auto-regressive (French *et al.*, 1992; Solibakke, 1997).

Similarly, transaction arrivals do not appear to arrive independently over time. For example, Jain and Joh (1988) find that trading frequency is dependent on the time of day when the market is open, namely, trading is heavier at the beginning and end of the trading day and lighter in the middle. In a semi-non-parametric GARCH setting, Gallant *et al.*, (1991) find that return variances are serially, cross- and serially cross-dependent. That is, variance and volume are jointly determined both cross-sectional and over time.

The results in this study indicate that return variances are related to whether or not the market is open. In particular, return variances over non-trading periods from 1 to 3 days when the market is open appear to be significantly equal to $k + 1$ the return variances over consecutive periods of trading. However, return variances over non-trading periods of 1 to 2 days when the market is closed, appear to be equal to the return variances over one consecutive day of trading. Therefore, in all periods, (1) variances for all non-trading periods when the market is open did conform to the random walk model and (2) variances for all non-trading periods when the market is closed did not conform to the random walk model.

A model of non-synchronous trading and non-trading is developed to allow for correction of the measurement error inherent in periods of infrequent trading. Consistent with other findings (Scholes and Williams, 1976, 1977; Lo and MacKinlay, 1990), the model analytically shows that while observed mean returns are unbiased, observed variances consistently overstate true variances. The greater the non-trading period, the lesser the measurement error in the calculation of variances. Despite the correction for non-synchronous trading, the results remain unchanged. When the market is open variances are not affected of trading/non-trading and when the market is closed the variances are almost equal to consecutive days of trading.

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Efficiently ARMA–GARCH estimated trading volume characteristics in thinly traded markets

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ARMA–GARCH lag specification is employed to fit a model exhibiting nonsynchronous trading and volatility clustering for the Norwegian thinly traded equity market. In particular, characteristics of the conditional mean and conditional volatility inhibited in thinly traded equity markets are investigated. Trading volume is employed as a proxy measure for trading frequency. Low to no trading volume induces thin trading and non-trading effects while a relative higher trading frequency induces continuous trading. The main objective is to investigate trading frequency differences in serial correlation and cross-autocorrelation in the mean equation and volatility clustering in the volatility equation as well as any symptoms of data dependencies in the model residuals, which imply ARMA–GARCH model misspecification. BIC efficient ARMA–GARCH lag specifications are employed for the conditional mean and volatility and relevant mean and volatility parameter measures introduced that are well known from the changing volatility literature. The empirical results report consistent mean and volatility patterns over the increasing trading frequency series. Nonsynchronous trading and non-trading effects show a consistent pattern in serial correlation and cross-autocorrelation for the conditional mean and the latent volatility exhibits a consistent pattern in past shocks, past conditional volatility, persistence and weight to long-run average volatility. In contrast to the more relatively frequently traded asset series the most thinly traded series report insignificant asymmetric volatility. Moreover, for the most thinly traded series, specification tests suggest data dependence, which seems to be prolonged into the equal-weighted index series. Hence, due to serial correlation and data dependence in the model residuals the ARMA–GARCH lag specifications seem only appropriate for relatively frequently traded return series.

I. INTRODUCTION

ARMA–GARCH lag specification for the conditional mean and volatility is employed for a nonsynchronous trading and changing volatility model characterizing the thinly traded Norwegian equity market. Solibakke (2000a) shows that the Norwegian equity market is a relative thin market compared to more elaborate markets and that the Norwegian market contains several asset series that show very thin trading (non-trading) together with many frequently and continuously traded series.[1] The main motivation is therefore to investigate any differences in the conditional mean and volatility from thinly to continuously traded series. Trading volume is applied as a

[1] See Chapter 4 of dissertation (Solibakke, 2000a) for a definition and classification of thin trading for the Norwegian equity market.

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proxy measure for trading frequency. Return series is observed and non-trading and therefore zero returns, is characterized by zero trading volume in Norwegian Kroner (NOK). As the Norwegian market exhibits long return series with zero trading volume, trading volume proxies well for nonsynchronous trading and non-trading effects induced by the zero return series. To establish trading frequency series four portfolios are formed based on trading volume in NOK. These four series and an equal- and a value-weighted market index series become the main empirical investigation focus. The dynamics in return series are investigated that exhibit an increasing trading frequency, employing ARMA–GARCH methodology to model the conditional mean and volatility processes. ARMA–GARCH estimations from thinly to continuously traded series may give new and interesting information of nonsynchronous trading and non-trading effects as well as volatility clustering. The investigation is especially interested in effects from autocorrelation and cross-autocorrelation in the conditional mean and shocks, autocorrelation, persistence and asymmetry in the conditional volatility. Hence, this investigation studies the relationships between trading frequency and conditional mean and volatility dynamics in an estimation context that control for nonsynchronous trading and conditional heteroscedasticity. Finally, to complete the model features, asymmetric volatility is incorporated as well as a measure of residual risk from the conditional volatility to the mean (in-mean). To the authors knowledge the focus of trading frequency and Schwarz (1978) preferred lead and lag structures for the conditional mean and volatility specifications are new and are not previously been carried out in international studies.

The portfolio series are organized based on historic trading volume and are rebalanced monthly, where the thinnest traded portfolio captures very thin trading, the intermediate thinly traded portfolio captures dynamics for thinly traded series and the two frequently traded portfolios capture medium to frequent (continuous) trading. All time series are adjusted for systematic scale and location effects and a correct lag structure for the conditional mean and volatility are achieved by applying the Bayes Information Criterion (Schwarz, 1978) preferred ARMA–GARCH-in-mean lag specifications. Note that the univariate ARMA–GARCH-in-mean specification represents a departure from Brownian Motions (Bachelier, 1964) and random walk. The specification explicitly allows for predictability measures in both mean and volatility processes.

It is believed that the contribution of this paper is a higher understanding of the workings of mean and volatility processes in thinly traded markets, where nonsynchronous trading and non-trading effects as well as volatility clustering may contribute significantly to the dynamics of asset pricing. The specification contributes by the following model features across varying trading frequency. First, the specification seeks consistent coefficient differences in the conditional mean equation; that is, autocorrelation and cross-autocorrelation. Secondly, consistent and significant coefficient differences in the volatility equations may contribute to a higher understanding of lagged shocks effects, autocorrelated and asymmetric volatility and the weight to long-run average volatility. Thirdly, as the degree of leptokurtosis in residuals measures the departure from the normal distribution, any systematic and significant coefficient differences may contribute to a higher understanding of non-normal returns. Fourthly, as the Bayes information criterion (BIC) is employed for lag specification in both the mean and the volatility equations, efficient ARMA–GARCH specification is obtained in both mean and volatility. Any change in lag structures may offer new and higher understanding of mean and volatility dynamics for thinly traded markets. Fifthly, as elaborate specification test statistics are performed and single and joint tests for volatility prediction biases, any misspecifications will be reported.

An expansion path is followed starting from adjusted raw returns and eventually specify both ARMA (mean) and GARCH (volatility) specifications for all the employed data series. As these Norwegian equity series contain assets that show thin trading relative to continuous trading, this investigation may contribute substantially to the international nonsynchronous trading[2] and changing volatility literature. Consequently, the ARMA–GARCH specifications for the Norwegian equity market may characterize nonsynchronous trading and non-trading effects as well as volatility clustering across varying trading frequency series not earlier shown in international finance.

The remainder for this paper is therefore organized as follows. Section II gives a literature overview of changing volatility, nonsynchronous trading and volatility clustering. Section III defines the data and describes a general adjustment procedure for systematic location and scale effects in time series. Section IV specifies the ARMA lag specification for the conditional mean and the GARCH lag specification for the conditional volatility, employing the BIC methodology to ensure efficiency. Section V reports the empirical results and Section VI reports the findings from the analysis. Finally, Section VII summarizes and concludes our findings.

II. LITERATURE OVERVIEW

If a subordinated stochastic volatility model determines asset returns, then returns during periods of nonsynchro-

[2] See Solibakke (2000b) for non-trading characteristics for individual assets in the Norwegian equity market.

nous trading and non-trading would differ from returns during periods of synchronous and continuous trading. Assuming trading frequency is a proxy for (non)synchronous trading effects, it may be hypothesized that trading periods containing low or no trading volume is characterized by mean and volatility processes different from processes in trading periods containing high trading frequency. Clark (1973) develops a subordinated stochastic process model for speculative price series. He argues that observed daily price changes are driven by two components; (1) a subordinated (or conditional) price change process and (2) a driving (or operational time) process. Clark found that the variability of the observed price process differs from one chronological time period to another, depending on the volume of transactions. Hence, a mix of finite volatility processes may describe price change series. Tauchen and Pitts (1983) have later refined this research assuming a stochastic volatility process. Moreover, Gallant and Tauchen (1996) employ efficient method of moments to estimate stochastic volatility models with diagnostics. They find that stochastic volatility models describe market characteristics well allowing for autocorrelation in both the mean and volatility processes.

Other research supports the mixture of distribution's hypothesis by testing subordinated stochastic process models of price change series and trading volume series.[3,4] Harris (1989) argues that observed properties of daily data are a consequence of similar properties of transaction data. Because each transaction price change is leptokurtic, leptokurtosis is a result of daily price changes when transaction data are aggregated to obtain daily data. Using a mixture of distribution model for daily data that is conditioned on the arrival of information in a given day, Harris finds kurtosis, skews and heteroscedasticity in daily price changes. His results also suggest that the daily transactions count may be a useful instrumental variable of estimating unobserved realizations of stochastic price variances. However, the system is still incomplete, as the dynamic properties of the information arrival process, which is assumed to drive return, volatility and volume, remain unspecified. Hence, in recent years it is found that many analytical models of information arrival find that returns and trading-volume are co-determined. For example, Admati and Pfleiderer (1988) model the effects of private information on order flow and that the trades of several classes of investors (informed traders and discretionary liquidity traders) will tend to cluster. This clustering of trades causes return variance to be highest during periods of active trading. In an alternative approach to the relation of information arrival, volume, return and variances, Ross (1989) assumes that information arrives according to a martingale process and, though no arbitrage conditions, demonstrates that return variances are proportional to the rate of information flow. In this case, price change when there is new information is coincident with trades.

Return volatility and trading volume will be related if transaction arrivals are related to the flow of information in the model. Hence, a growing body of empirical evidence supports the joint determination of return variance and trading volume.[5] However, while international research focuses on short-term conditional heteroscedasticity in bivariate asset trading frequency and return estimation (see for example SNP[6] estimation in Gallant, Rossi and Tauchen, 1992), focus here is on systematic and consistent differences in lag structures and coefficient changes in conditional mean and volatility equations for thinly and frequently traded assets employing univariate ARMA–GARCH-in-mean lag specifications. Hence, in contrast to Lamoureux and Lastrapes (1990), trading volume series are not directly modelled in the conditional volatility process. In this univariate investigation the aim is to find consistent lag and coefficient differences in the conditional mean and volatility processes for series showing an increasing trading frequency.

The model specifications focus on changes in lag structures as well as changes in coefficients in the conditional mean and volatility across varying trading frequency series for the Norwegian market. The focus will be on differences in nonsynchronous trading and non-trading effects as well as conditional heteroscedasticity and volatility clustering. Intuitive thinking suggest that an asset that reports non-trading responds to new information with a time lag. These lagged responses may induce biases in the moments and co-moments of daily return series. The serial correlation may influence tests of predictability and nonlinearity as well as volatility risk and expected returns. The first to recognize the importance of nonsynchronous trading was Fisher (1966). Campbell *et al.* (1997) reviews and extends existing theory. They show that large stocks tend to lead those of smaller stocks, which suggest that nonsynchronous trading may be a source of correlation. However, they also find that the magnitudes for the autocorrelations imply an implausible level of non-trading and therefore leads them to the conclusion that non-trading is only responsible for some of the autocorrelation. Moreover, applying estimated

[3] Epps and Epps (1976), Morgan (1976), Westerfield (1976) and Tauchen and Pitts (1983).
[4] Theoretically, these processes can be derived as discrete time approximations to the solution of the option valuation problem when the volatility of the underlying asset price is stochastic. Research in this vein has been carried out by, for example, Scott (1987), Wiggins (1987), Chesney and Scott (1989) and Melino and Turnbull (1990).
[5] See Barclay *et al.* (1990), Gallant *et al.* (1992) and Andersen (1994).
[6] A Semi-Non-Parametric Score Generator (Gallant and Tauchen, 1989).

non-trading probabilities from daily autocorrelations (Campbell *et al.*, 1997) they find little support for nonsynchronous trading and non-trading effects as an important source of serial correlation in the returns for common stock over daily and longer frequencies.[7]

III. EMPIRICAL DATA AND METHODOLOGIES

Empirical data

The study employ daily returns and trading volume for individual Norwegian stocks spanning the period from October 1983 to February 1994. Daily return series are defined as $\ln(p_{i,t}/p_{i,t-1})$, where $p_{i,t}$ is the daily closing price for asset i at time t. Trading volume is defined as the total transaction volume in NOK at day t for asset i including external trading (trading outside the organized market). The individual shares are grouped into portfolios at period t based on trading volume series in the information set at $t-1$, Ω_{t-1}. The portfolios are rebalanced each month and to avoid a too frequent shift of component stocks among the asset portfolios the average daily trading volume for the last two years is employed. Two years of daily volume is chosen to obtain a time overlap of 95% for each portfolio restructuring.[8] Hence, assets are arranged into portfolios based on changes in trading volume over a considerable time period. The result of this exercise are four series; a thinly traded series that contains the most thinly traded assets (Portfolio 1), an intermediate thinly traded series (Portfolio 2), an intermediate frequently traded series (Portfolio 3), and finally a frequently traded series that contains the most frequently traded assets (Portfolio 4). In this exercise all assets in the Norwegian thinly traded market have been employed and on average all series therefore contain at least 25 assets. Moreover, the time periods are divided into two subsamples; (1) a time period before the crash in October 1987 (1019 daily observations), (2) a time period after the crash (1546 daily observations), and (3) a time period for the entire 10 years time period 1983–1994 (2611 daily observations).[9] Note that to keep the paper within reasonable limits, results for the entire period only are reported. Relevant subperiod results are described in footnotes. Note also that the crash is included for the entire period 1983–1994, which induce that market dumps are considered normal in equity market. Moreover, two market wide indices are included consisting of all the stocks in the Norwegian market with (1) equally weighted stocks and (2) market value weighted stocks. These indices are included with the aim to recognize patterns from trading volume portfolios on the index level. Therefore, this high frequency time series database gives potentially 2611 observations for each portfolio and index and is the main vehicle to achieve the objectives for the investigation of thinly traded markets.

For all time series the procedures described by Gallant *et al.* (1992) are employed to adjust for systematic location and scale effects in all six return series.[10] The procedure gives a series that becomes more homogenous allowing focus on the day-to-day dynamic structure under an assumption of stationary series without any disturbance to mean and volatility characteristics. To show these properties from the procedures the value-weighted index and the natural logarithm of the total trading volume are reported in Figs 1 and 2, respectively. The index shows an approximate yearly growth of 12%. In particular, note the strong and erratic trend in trading volume for the Norwegian Market. On average, the growth in the trading volume in NOK is approximately 32.9% per year.

The characteristics of the adjusted portfolio and market index series are reported in Table 1. Table 1 shows that the mean returns are highest for the thinnest traded series and are accompanied by the highest standard deviation. Hence, both expected return and total risk are at its peak in these series. The most frequently traded series show lower return and standard deviation relative to the most thinly traded series. The two intermediately traded series show characteristics between the thinnest and the continuously traded series. The most frequently traded series show lowest minimum and highest maximum daily return, which also induce high absolute skew[11] (negative tails) and high kurtosis.[12] The ARCH test statistic is a test of changing volatility in the return series. All series report highly significant changing volatility and suggests a need for ARCH/GARCH specification of the second moments. The RESET (Ramsey, 1969) test statistic suggests nonlinearity in the mean for all series. The BDS (Brock and Deckert (1988), Brock and Baek (1991) and Scheinkman (1990)) test statistic suggests general nonlinearity for all series at all dimensions (m) and for ε equal to one.[13] Finally, the K–S Z-test

[7] See also Boudoukh *et al.* (1995), Mech (1993) and Sias and Starks (1994). All three papers conclude that non-trading cannot completely account for the observed autocorrelations.
[8] Moreover, the first trading volume observation available for collection for all assets in the Norwegian equity market is registered 01.09.81.
[9] The crash period in this study is defined as the two months October and November in 1987. The estimations and specifications results for the two subperiods are not reported based on space considerations. However, all results are available from the author upon request.
[10] The scale and location results for all six series are not reported but are available from the author upon request.
[11] Skew: A measure of the thickness of the tails of a distribution.
[12] Kurtosis: A measure of the asymmetry of a distribution.
[13] Calculated as (ε standard deviation). ε equal to 0.5, 1.5 or 2 does not materially change the conclusions.

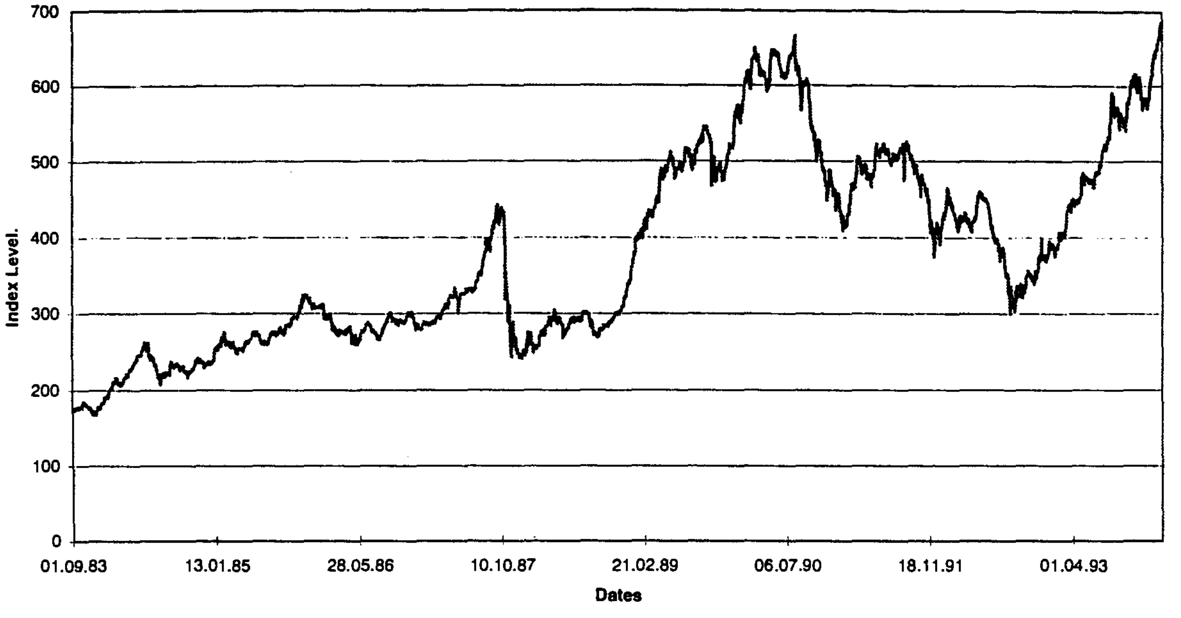

Fig. 1. *Oslo Stock Exchange all-market value-weighted Index (Totx)*

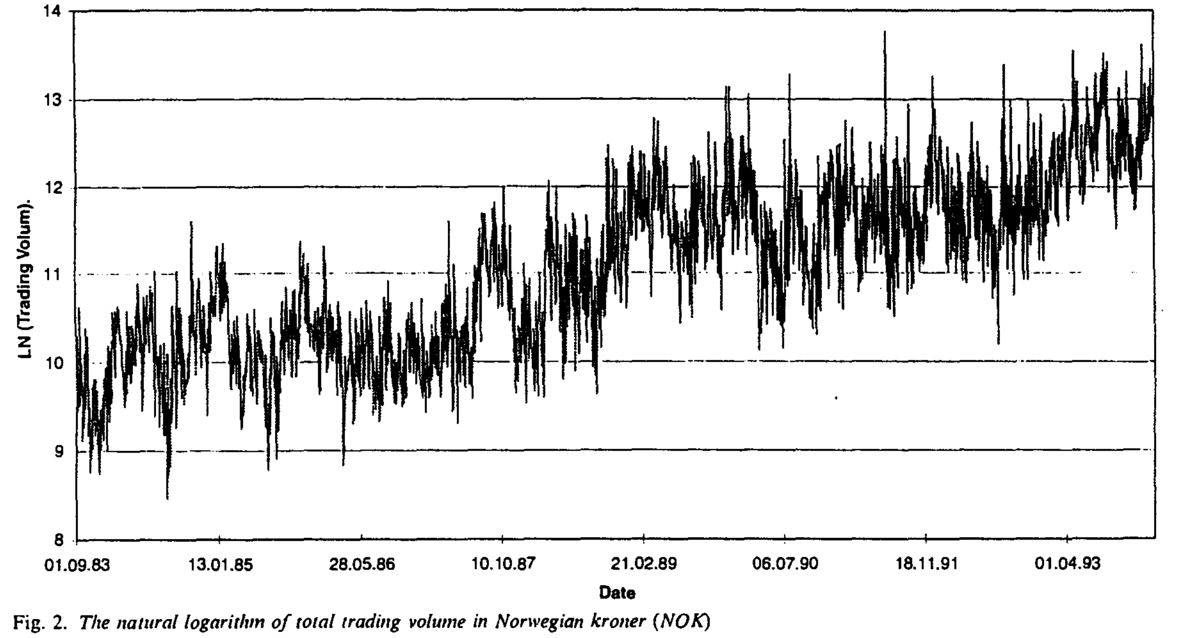

Fig. 2. *The natural logarithm of total trading volume in Norwegian kroner (NOK)*

suggests deviation form normally distributed adjusted returns for all series. The thinnest traded series show lowest deviation while the most frequently traded series show highest non-normal returns. The skew and kurtosis numbers confirms all these K–S Z-test results.

The market index series show as expected from portfolio literature, lower standard deviation. The numbers for the mean, maximum and minimum returns show no extraordinary pattern relative to the other portfolio series. As for the other series, the indices report changing volatility

(ARCH), nonlinearity in the mean (RESET) and general non-linear dependence (BDS).[14] The numbers for skew and kurtosis suggest considerable deviation from normality and are higher in absolute values than for the other trading frequency series. In fact, from the kurtosis and skew numbers the strongest deviation from normally distributed returns is found for the two index series, which is confirmed by the K–S Z-test statistic.

The ARMA–GARCH-in-mean methodology

Table 1 reports autocorrelation, changing volatility, nonlinearity and systematic leptokurtosis in the return distributions for all series. Hence, Table 1 suggests non-normal returns, ARMA effects in mean, ARCH/GARCH effects in volatility and a need to control for serial correlation and data dependence in model residuals (misspecification). Table 2 reports mean, standard deviation and autocorrelation for daily returns and squared returns in order to accomplish model specification. The ARMA–GARCH lag specification is strongly enhanced as the autocorrelation for returns and squared returns show clear patterns. In fact, the autocorrelation structure report the necessary underlying material for the conditional mean and volatility specification in the ARMA–GARCH methodology found in Engle (1982), Bollerslev (1986, 1987) and Engle and Bollerslev (1986). Moreover, Table 2 reports the mean and standard deviation for all series, which enhance the setting of starting values for serial correlation estimation in both mean and volatility. The ARMA lag specification models nonsynchronous trading and non-trading effects while the GARCH lag specification models conditional heteroscedasticity and volatility clustering. International literature applying ARMA–GARCH specifications have shown that these models are able to account for many of the lag structures found in observed mean and volatility processes.

To obtain the most efficient ARMA–GARCH lag specification for the conditional mean and volatility, a

Table 1. *Portfolio characteristics for the Norwegian Equity Market*

	Trading volume series				Market Index series	
	R_1	R_2	R_3	R_4	R_{EM}	R_{VM}
Daily mean	0.08514	0.02872	0.01351	0.02183	0.03046	0.05269
Yearly mean	21.4565	7.23848	3.40539	5.50204	7.67598	13.2784
Daily st. dev.	2.05062	1.36740	1.38880	1.58280	1.10930	1.29650
Yearly st. dev.	32.5525	21.7063	22.0466	25.1267	17.6101	20.5812
Max return	10.8004	10.3330	11.5550	13.3180	11.4230	10.4810
Min return	−15.9060	−14.5250	−16.1890	−23.0630	−16.6640	−21.2190
Skewness	−0.11584	−0.60257	−0.98671	−1.31470	−1.54580	−2.00398
Kurtosis	5.7203	11.1800	14.8691	26.1456	29.8444	36.1425
K–S Z-test	15.4010	13.8466	11.7075	14.2367	12.5083	10.5244
ARCH (6)	109.368	318.469	779.638	526.529	818.958	550.225
RESET (12;6)	25.7275	54.0216	80.0946	74.9514	76.9191	68.8097
BDS($m = 2; \varepsilon = 1$)	7.47176	8.37882	13.1261	16.1939	12.8661	12.6531
BDS($m = 3; \varepsilon = 1$)	8.14346	10.5286	15.3908	19.1019	15.1892	14.9082

Note: R_1 is the portfolio containing the most thinly traded, R_2 contains the intermediate thinly traded assets, R_3 contains the intermediate frequently traded assets and R_4 contains the most frequently traded assets. Yearly mean is daily mean multiplied by 252 trading days and yearly standard deviation is daily standard deviation multiplied by the square root of 252 trading days. Skew is a measure of heavy tails and asymmetry of a distribution (normal) and kurtosis is measure of too many observations around the mean for a distribution (normal) K–S Z-test; Used to test the hypothesis that a sample comes from a normal distribution. The value of the Kolmogorov–Smirnov Z-test is based on the largest absolute difference between the observed and the theoretical cumulative distributions. ARCH (6): ARCH (6) is a test for conditional heteroscedasticity in returns. Low {.} indicates significant values. The OLS-regression $y^2 = a_0 + a_1 * y_{t-1}^2 + \cdots + a_6 * y_{t-6}^2$. $T * R^2$ is used and is χ^2 distributed with 6 degrees of freedom. T is the number of observations, y is returns and R^2 is the explained over total variation. $a_0, a_1 \ldots a_6$ are parameters.
RESET (12,6): A sensitivity test for mainly linearity in the mean equation. 12 is number of lags and 6 is the number of moments that is chosen in implementation of the test statistic. $T \cdot R^2$ is χ^2 distributed with 12 degrees of freedom. BDS ($m = 2, \varepsilon = 1$): A test statistic for general nonlinearity in a time series. The test statistic BDS $= T^{1/2}.\ [C_m(\sigma * \varepsilon) - C_1[\sigma * \varepsilon)^m]$, where C is based on the correlation-integral, m is the dimension and ε is the number of standard deviations. Under the null hypothesis of identically and independently distributed (i.i.d.) series, the BDS-test statistic is asymptotic normally distributed with a zero mean and with a known but complicated variance.

[14] Subperiods report (1) a reduced mean return and (2) an increased volatility story after the crash in October 1987. The average reduction in mean return for the four trading volume portfolios and the two market indices is approximately 107% and 82.8%, respectively, and the average increases in volatility is 54.8% and 38.6%, respectively. The thinnest traded assets have the highest increase in volatility. Moreover, the last subperiod (1987–1994) produces higher positive Kurtosis, show higher negative skew and the nonlinear dependence seem to increase.

specification procedure is performed that accommodates the characteristics of the return series. The model's lag specifications are approach for the conditional mean and volatility for our return series below. Applying elaborate specification test statistics to the resulting lag structure residuals will determine whether the ARMA–GARCH model is able to accommodate the observed market characteristics.

The conditional mean specification. For the conditional mean specification, Table 2 reports the autocorrelation structure up to lag 6 for the adjusted daily return series. Negative serial correlation at lag one is found for the thinly traded series. In contrast, the frequently traded series report significant positive serial correlation. The general picture is therefore negative serial correlation for thinly traded series and positive serial correlation for frequently traded series. The correlation structure suggests that thinly traded series show mean reversion and therefore negative time dependence (Poterba and Summers, 1988; Fama and French, 1988). However, thin trading imply series of zero returns and may therefore induce biases to the moments of the return series, which may produce spurious autocorrelation. Table 2 reports that the magnitudes of the serial correlation coefficients decay very fast at higher orders. By applying the above reported correlation structure and applying a procedure described by Box and Jenkins (1976) a parsimonious representation of the conditional mean structure may be established. As it is required to establish the model specification of an ARMA (p,q) process BIC (Schwarz, 1978) is employed to determine p and q. The BIC criterion is computed as:

$$BIC(p,q) = \ln \sigma^2 + (p+q)T^{-1} \cdot \ln T$$

where σ^2 is the estimated error variance and T is the number of time periods employed. Small values of the criterion are preferred. The criterion reward good fits as represented by small $\ln \sigma^2$ and uses the term $(p+q)T^{-1} \cdot \ln T$ to penalize good fits that is got by means of excessively rich parameterizations. The criterion is conservative in that it selects sparser parameterizations than the Akaike information criterion (Akaike, 1969) (AIC), which uses the penalty term $2 \cdot (p+q)T^{-1}$ instead of $(p+q)T^{-1} \cdot \ln T$. BIC is also conservative in the sense that it is at the high end of the permissible range of penalty terms in certain model selection settings (Potscher, 1989). Between these two extremes is the Hannan and Quinn (Hannan, 1987) criterion. The usual suggestion is to use BIC to move along an upward expansion path until an adequate model is determined. Hence, the procedure $BIC(p_1,q_1) = \min BIC(p,q), p \in P$, $q \in Q$ is employed.

Table 3 reports the computed BIC, AIC and HQ criteria for three ARMA model specifications for all series. Table 3 shows that the three most frequently traded series and the value-weighted market index all prefer an ARMA (0,1) lag specification for the conditional mean. Hence, these four series prefer an autocorrelation specification that employs a one period lagged moving average specification (MA (1)). The thinnest traded portfolio BIC prefers an ARMA (0,2) lag specification for the conditional mean. This lag specification suggests severe non-trading effects modelled by a two periods lagged moving average specification (MA(2)). Finally, the equal-weighted index prefers an ARMA (1,0) lag specification for the conditional mean. This specification suggests that combined series from thinly and continuously traded series seem to prefer an autoregressive specification (AR(1)). Moreover, interestingly, thin trading and non-trading effects seem to affect the mean specification differently in combined series containing similar assets versus all equity equal-weighted index series. The preferred conditional mean specification for all series therefore becomes

Table 2. *Summary statistic for adjusted daily returns*

Series	$\mu_i(x)$	$\sigma_i(x)$	$\rho_i(1)$	$\rho_i(2)$	$\rho_i(3)$	$\rho_i(4)$	$\rho_i(5)$	$\rho_i(6)$	$Q_i(6)$
R_1	0.0851	2.0506	−0.185	−0.060	−0.029	0.030	0.006	0.012	103.752
R_2	0.0287	1.3674	−0.041	0.025	−0.051	0.024	−0.013	0.016	15.6070
R_3	0.0135	1.3888	0.178	−0.015	−0.064	−0.010	0.013	0.014	95.6570
R_4	0.0218	1.5828	0.129	−0.066	−0.073	−0.011	0.013	0.019	70.6710
R_{EM}	0.0305	1.1093	0.096	0.013	−0.030	0.016	0.010	0.031	30.4060
R_{VM}	0.0527	1.2965	0.140	−0.046	−0.053	−0.013	0.009	0.028	67.0390
R_1^2	4.2123	10.8679	0.142	0.094	0.056	0.098	0.018	0.044	115.374
R_3^2	1.8705	9.6138	0.460	0.074	0.026	0.039	0.042	0.069	589.449
R_3^2	1.9290	7.3750	0.485	0.188	0.083	0.062	0.106	0.110	796.782
R_4^2	2.5058	12.6401	0.375	0.074	0.044	0.047	0.033	0.056	403.173
R_{EM}^2	1.2315	8.0628	0.460	0.079	0.030	0.042	0.033	0.056	587.832
R_{VM}^2	1.6837	9.9076	0.320	0.048	0.032	0.053	0.023	0.056	292.262

Note: See Table 1 for a description of series and test statistics.
Returns (μ_i), volatility (σ_i) are reported and autocorrelation (ρ_i) and $Q(6)$ is the Ljung–Box (1978) joint test statistic for serial correlation at the first moment up to lag 6.

Table 3. *Optimized Likelihood and Model section BIC Criteria.*

	p	q	s_n	BIC	HQ	AIC	Likelihood
Thinly	1	0	10 630.916	11 083.495	11 080.561	11 077.627	−5537.814
traded	0	1	10 558.596	11 065.672	11 062.739	11 059.805	−5528.902
Portfolio	1	1	10 519.141	11 063.765	11 057.898	11 052.030	−5524.015
(R_1)	0	2	10 515.080	11 062.757	11 056.890	11 051.022	−5523.511*
Intermediate							
Thinly	1	0	4875.757	9048.244	9045.310	9042.376	−4520.188
traded	0	1	4876.109	9048.182	9042.315	9045.249	−4520.282*
Portfolio (R_2)	1	1	4865.670	9050.705	9044.837	9038.970	−4517.485
Intermediate							
Frequently	1	0	4876.061	9048.407	9045.473	9042.539	−4520.270
traded	0	1	4869.402	9044.838	9041.905	9038.971	−4518.486*
Portfolio (R_3)	1	1	4869.239	9052.619	9046.752	9040.884	−4518.442
Frequently	1	0	6433.942	9772.306	9769.372	9766.438	−4882.219
traded	0	1	6417.588	9765.660	9762.727	9759.793	−4878.896*
Portfolio (R_4)	1	1	6410.248	9770.541	2446.569	−4877.403	−4877.403
Equal Weighted	1	0	3185.294	7936.661	7933.727	7930.793	−3964.397*
Market	0	1	3185.995	7937.236	7934.302	7931.368	−3964.684
Index (R_{EM})	1	1	3185.262	7944.503	7938.635	7932.768	−3964.384
Value Weighted	1	0	4307.636	8724.777	8721.843	8718.910	−4358.455
Market	0	1	4298.363	8719.151	8716.217	8713.283	−4355.642*
Index (R_{VM})	1	1	4295.928	8725.539	8719.672	8713.804	−4354.902

Note: * BIC preferred model.

$$R_{i,t} = \mu_i + \phi_{i,1} \cdot R_{i,t-1} + \varepsilon_{i,t} - \theta_{i,1} \cdot \varepsilon_{i,t-1} - \theta_{i,2} \cdot \varepsilon_{i,t-2}$$
$$i = 1,2,3,4, EM, VM \qquad (1)$$

where R_i is return for portfolio series i, the ϕ_i coefficient is equal to zero for all i except $i = EM$, the $\theta_{i,1}$ is equal to zero for $i = EM$ and the $\theta_{i,2}$ is equal to zero for $i = 2,3,4, EM$ and VM. It is now possible to estimate the mean structure employing standard ARMA (p,q) methodology, where p and q are BIC preferred.

The estimated coefficients of the ARMA (p,q) models of the conditional mean are reported in Table 4. The autoregressive coefficient, $\phi_{i,1}$ and the moving average coefficients $\theta_{i,1}$ and $\theta_{i,2}$ captures the first and second order serial correlation for all the return series. Table 4 reports strong autocorrelation for the conditional mean in the Norwegian equity market and suggest considerable predictability in return series.

The reported autocorrelation and distribution characteristics for the residuals (ε) reported in Table 5, suggest that the ARMA (p,q) specification appropriately specify the conditional mean. Only the most frequently traded series may suggest model misspecification by the $Q_i(6)$ statistic (Box and Jenkins, 1976). None of the other series shows significant autocorrelation for the residuals up to lag 6. Hence, the BIC preferred ARMA (p,q) models seem to provide a well-specified form for the conditional mean process in the Norwegian market. Moreover, the numbers for skew and kurtosis are reduced relative to the same numbers for adjusted raw data series. However, the ARCH (6) test statistics rejects strongly conditional homoscedasticity, the RESET test statistic rejects linearity in the mean and the BDS test statistic rejects identically and independently (i.i.d.) distributed residuals. In fact, the ARCH, the RESET and the BDS test statistics are mostly maintained from the adjusted raw series at all dimensions. Hence, the data dependence reported in Table 1 and 5, may originate

Table 4. *ARMA (p,q) coefficients for six return series*

The model $R_{i,t} = \alpha_i + \phi_i R_{i,t-1} + \theta_{i,1}\varepsilon_{i,t-1} + \theta_{i,2}\varepsilon_{i,t-2} + \varepsilon_{i,t}$, is estimated where i is four asset series and two index series. R_{it} is the return series. α_i is a constant parameter, ϕ_i is the autoregressive parameter and θ_1 and $\theta_{i,2}$ is the moving average parameters. ε_i is model residuals.

		ARMA(p,q)		
Series*	Log-likelihood	ϕ_i	$\theta_{i,1}$	$\theta_{i,2}$
R_1	−5295.41	–	0.21640	0.05142
		–	{10.767}	{2.625}
R_2	−4004.53	–	0.13516	–
		–	{5.723}	–
R_3	−4000.65	–	−0.06211	–
		–	−{2.250}	–
R_4	−4325.09	–	−0.25168	–
		–	−{9.337}	–
R_{EM}	−3347.27	0.17198	–	–
		{8.340}	–	–
R_{VM}	−3842.35	–	−0.24194	–
		–	{11.459}	–

Note: See Table 1 for a definition of the series.

Table 5. *Summary characteristics from an ARMA (p,q) specification*

	$\rho_1(1)$	$\rho_1(2)$	$\rho_1(3)$	$\rho_1(4)$	$\rho_1(5)$	$\rho_1(6)$	$Q^i(6)$/ $Q^2(6)$	$\mu_i(x)$/ $\sigma_i(x)$	Kurtosis$_i$/ Skew$_i$	ARCH (6)	RESET (12;6)
ε_1	0.001	−0.001	−0.022	0.032	0.020	0.031	7.5320	0	4.3688	57.7784	25.2299
							{0.2740}	2.0034	−0.0629	{0.0000}	{0.0138}
ε_2	−0.001	0.023	−0.049	0.022	−0.011	0.019	10.3030	0	24.942	549.255	54.2867
							{0.1120}	1.3662	1.4041	{0.0000}	{0.0000}
ε_3	0.001	−0.004	−0.063	−0.001	0.013	0.000	10.8510	0	13.798	846.041	81.7624
							{0.0930}	1.3656	−1.1187	{0.0000}	{0.0000}
ε_4	−0.007	−0.056	−0.065	−0.003	0.013	0.007	19.8410	0	24.449	696.325	77.6147
							{0.0030}	1.5677	−0.7689	{0.0000}	{0.0000}
ε_{EM}	0.000	0.007	−0.034	0.018	0.005	0.019	4.8890	0	42.073	882.221	77.7210
							(0.5580}	1.1042	−1.3783	{0.0000}	{0.0000}
ε_{EM}	−0.005	−0.038	−0.046	−0.007	0.009	0.011	10.0350	0	32.205	588.766	71.6336
							{0.1230}	1.2822	−1.3448	{0.0000}	{0.0000}
ε_1^2	0.101	0.088	0.058	0.072	0.022	0.043	75.5630	4.0136	167.788	–	–
							{0.000}	10.12	10.646	–	–
ε_2^2	0.432	0.067	0.024	0.040	0.040	0.072	521.993	1.8666	1136.38	–	–
							{0.0000}	9.6799	30.075	–	–
ε_3^2	0.559	0.245	0.105	0.061	0.099	0.096	1061.53	1.8648	443.485	–	–
							{0.0000}	7.4054	18.204	–	–
ε_4^2	0.486	0.106	0.048	0.043	0.034	0.063	670.326	2.4575	945.661	–	–
							{0.0000}	12.627	27.522	–	–
ε_{EM}^2	0.529	0.104	0.034	0.037	0.032	0.053	775.287	1.2192	1129.37	–	–
							{0.0000}	8.0863	31.071	–	–
ε_{VM}^2	0.447	0.082	0.037	0.049	0.027	0.066	561.843	1.6442	1295.89	–	–
							{0.0000}	9.6070	32.682	–	–

Notes: Means (μ_i), volatility (σ_i), auto-correlation (ρ_i), and the distribution properties kurtosis and skew are reported. The Q is the Ljung–Box (1978) statistics and their p-values are in brackets {}.
See Table 1 for definition of the ARCH (6) and RESET (12;6) test statistics and their p-values are given in brackets.

from the conditional volatility process. The conditional volatility lag structure is modelled below.

The conditional volatility specification. Table 5 above reports the serial-correlation structure in the residuals and squared residuals from the ARMA (p,q) lag specification of return series, where p and q are BIC preferred. The standardised residuals in Table 5 show close to zero autocorrelation and suggest an appropriate conditional mean specification. However, Table 5 finds strong evidence of autocorrelation among the squared residuals (ε^2). This empirical finding lends strong support to an ARCH/GARCH specification for the conditional volatility process. To achieve a lag specification for the conditional volatility process the applied test for ARCH effects described in Table 1 are employed. Engle (1982) shows that a test of the null hypothesis that $\varepsilon_{i,t}$ has a constant conditional variance against the alternative that the ARMA theory follows through. This implies that by employing the squared residual $\varepsilon_{i,t}^2$, u and n can be identified in an ARMA (u,n) specification for the conditional variance by applying the same methodology as conditional mean ARMA (p,q) modelling in the previous section. Hence, Table 6 reports the BIC, HQ and AIC for ARMA (u,n) models of the squared residuals from BIC preferred ARMA (u,n) models of the conditional mean process for all series. For all series the autocorrelation lag structure in the squared ARMA (u,n) residuals is consistent with an ARMA (1,1) model. Hence, the conditional variance equation is used

$$h_{i,t} = m_{i,0} + a_{i,1} \cdot \varepsilon_{i,t-1}^2 + b_{i,1} \cdot h_{i,t-1} \quad i = 1,2,3,4,EM,VM \tag{2}$$

which is known as the GARCH (1,1) specification[15] for the conditional volatility.[16] In this model, the coefficient $a_{i,1}$ measures the tendency of the conditional variance to cluster, the $b_{i,1}$ measures the autocorrelation in conditional volatility, while the coefficients $a_{i,1}$ and $b_{i,1}$ together measures the degree of persistence in the conditional variance process. For a stable GARCH (1,1) process require that $a_{i,1} + b_{i,1} < 1$. Otherwise, the weight applied to the long-

[15] For applications see Bollerslev *et al.* (1992).
[16] The GARCH(1,1) specification was introduced by Bollerslev (1986, 1987) and seems to be the major specification for GARCH(m,n) models in international finance.

Table 6. *Optimized likelihood and model selection BIC criteria*

Portfolio:	u	n	s_n	BIC	HQ	AIC	Likelihood
Thinly	1	1	258 729	19 434	19 425	19 416	−9704.99*
traded	2	1	258 306	19 437	19 425	19 414	−9702.86
Portfolio (R_1)	1	2	258 333	19 437	19 426	19 414	−9702.99
Intermediate							
Thinly	1	1	194 169	18 684	18 675	18 666	−9330.24*
traded	2	1	194 155	18 692	18 680	18 668	−9330.15
Portfolio (R_2)	1	2	193 832	18 687	18 676	18 664	−9327.97
Intermediate							
Frequently	1	1	97 546	16 887	16 878	16 869	−8431.52*
traded	2	1	97 545	16 895	16 883	16 871	−8431.52
Portfolio (R_3)	1	2	97 545	16 895	16 883	16 871	−8431.52
Frequently	1	1	306 437	19 875	19 867	19 858	−9925.92*
traded	2	1	306 424	19 883	19 871	19 860	−9925.87
Portfolio (R_4)	1	2	306 398	19 883	19 871	19 860	−9925.76
Equal Weighted	1	1	113 346	17 279	17 270	17 261	−8627.51*
Market	2	1	113 323	17 286	17 274	17 263	−8627.25
Index (R_{EM})	1	2	113 292	17 285	18 274	17 262	−8626.89
Value Weighted	1	1	187 988	18 600	18 591	18 582	−9288.01*
Market	2	1	187 983	18 607	18 596	18 584	−9287.97
Index (R_{VM})	1	2	188 037	18 608	18 596	18 585	−9288.35

Note: * BIC preferred models.

term variance is negative. The weight is $\delta_{i,0} = 1 - (a_{i,1} + b_{i,1})$ and the long-term variance is $V_i = m_{i,0}/\delta_{i,0}$.

In-mean, cross portfolio effects and asymmetric volatility. $(h_{i,t})^{1/2}$ is included in the mean equation in an attempt to incorporate a measure of risk into the return generating process. Therefore a measure of residual risk is induced (Lehmann, 1990) into the model. $\gamma_{i,j}(i \neq j)$ is also included in the conditional mean to control for any cross series effects of the type identified by Lo and MacKinlay (1990). Finally, a coefficient for asymmetric volatility is included in the conditional variance equation (Nelson, 1991; Glosten *et al.*, 1993). The methodology of Glosten *et al.* (1993) is applied to model asymmetric volatility[17] in the conditional variance equation (λ_i). Finally, an innovation ε_t is assumed that follows a conditional student-t distribution[18] to accommodate leptokurtosis, which is observed in Table 1 and 4 for all the series' return distributions. The BHHH[19] algorithm is employed for estimation in GAUSS[20] for all series. The final iteration employs the Newton–Raphson[21] algorithm to extract all information from the Hessian matrix. Hence, the t-ratios are based on the sum of the $k \times k$ matrix of second differentials over n observations.

IV. EMPIRICAL RESULTS

Maximum likelihood estimates of the parameters for the BIC preferred ARMA–GARCH lag specifications for the conditional mean and volatility are reported in Table 6 for all series. Among the four trading frequency series, only the most frequently traded series report a significant and positive α_i coefficient in the mean equation. The index series report significant and positive α_i coefficients.

Autocorrelation is statistical significant in all series. The autocorrelation moves from significant positive coefficients and therefore negative serial correlation for the thinly traded series to significant negative coefficients and therefore positive serial correlation for the frequently traded series. The result suggests that assets in thinly and frequently traded portfolios exert different price adjustment mechanisms. The thinly traded assets seem to overreact from shocks at $t-1$ and therefore next period at t reverse this overreaction and move prices back to a new and now

[17] For reference purposes this asymmetric model for GARCH-GJR will be denoted. Note also that the GJR specification is Lagrange Ratio Test preferred for all series relative to an exponential GARCH lag specification (Nelson, 1991).
[18] The number of freedoms is estimated.
[19] The BHHH algorithm is described in Berndt *et al.* (1974).
[20] Gauss is a programming and estimation tool from Aptech Systems
[21] The Newton–Raphson algorithm estimates the Hessian matrix directly.

Table 7. *An ARMA–GARCH-in-mean specification for portfolio returns*
This table contans the estimated coefficients from the model mean

$$R_{i,t}\alpha_i + \phi_{i,1}\cdot R_{i,t-1} + \sum_{j=1|i\neq j}^{4}(R_{j,t-1}\cdot\gamma_{ij}) - \theta_{i,1}\cdot\varepsilon_{i,t-1} - \theta_{i,2}\cdot\varepsilon_{i,t-2} + \varepsilon_{i,t}$$

where $i = 1,2,3,4$, *EM*, *VM* and $E(\varepsilon_i, t\,|\,\Omega_{t-1}) \sim D(0, h_{i,t}, v_i)$, D is the student-t distribution with v degrees of freedom and for the model volatility $h_{i,t} = m_i + (a_{j,1} + \lambda_{i,1t}) * \varepsilon_{t-1}^2 + b_{i,1} * h_{i,t-1}$, where $\lambda_{i,1,t} = \varepsilon_{i,t-1}$ iff $\varepsilon_{i,t-1} < 0$. The γ_{ij} control for any corss effects of the type identified by Lo and MacKinaly (1990).

Return series	Log likelihood	α_i	$\phi_i/\theta_{i,2}$	β_i	$\theta_{i,1}$	γ_{i1}	γ_{i2}	γ_{i3}	v_i
R_1	−5295.41	0.19092	0.05142	−0.05059	0.21640	0.07240	0.05665	0.05541	6.08771
		{1.2311}	{2.6250}	−{0.5830}	{10.7668}	{1.8733}	{1.2543}	{1.9786}	{8.9069}
R_2	−4004.53	−0.02528	–	0.06091	0.13516	0.02754	0.17437	0.08076	5.69920
		−{0.2694}		{0.7107}	{5.7235}	{1.5661}	{6.2192}	{3.3065}	{9.9107}
R_3	−4000.65	−0.01620	–	0.04864	−0.06211	0.00188	0.00496	0.21152	5.79819
		−{0.6682}		{1.1800}	−{2.2501}	{0.1313}	{0.1822}	{9.5229}	{9.5610}
R_4	−4325.09	0.18529	–	−0.08558	−0.25168	0.01345	−0.02075	−0.00787	6.19008
		{2.4649}		−{1.3999}	−{9.3367}	{1.1840}	−{0.7997}	−{0.2422}	{9.2445}
R_{EM}	−3347.27	0.14601	0.17198	−0.09694	–	–	–	–	5.06988
		{2.7663}	{8.3396}	−{1.6100}					{10.8847}
R_{VM}	−3842.35	0.18345	–	−0.09792	−0.24194	–	–	–	6.45581
		{2.5595}		−{1.4167}	−{11.4592}				{8.9356}

Return series	m_i	a_i	b_{i1}	$a_i + b_{i1}$	λ_i	Skews	Kurtosis	Q(6)	$Q^2(6)$
R_1	0.05267	0.03564	0.95055	0.98619	−0.09518	−0.06293	3.45442	2.2540	21.079
	{2.6797}	{4.6851}	{90.0631}		−{0.6881}			{0.895}	{0.002}
R_2	0.10462	0.10706	0.81743	0.92449	−0.27330	−1.2698	13.9357	1.4960	7.7520
	{3.1839}	{4.9072}	{20.8524}		−{1.9887}			{0.960}	{0.257}
R_3	0.10836	0.15788	0.77044	0.92832	−0.26668	−0.94408	10.6133	2.8310	6.6010
	{3.7337}	{6.1384}	{21.2518}		−{2.9592}			{0.830}	{0.359}
R_4	0.19390	0.20347	0.69609	0.89956	−0.33569	−0.61607	5.76354	11.650	16.219
	{3.3634}	{4.9991}	{11.6728}		−{3.4819}			{0.070}	{0.013}
R_{EM}	0.06611	0.12991	0.79766	0.92758	−0.19598	−1.04078	13.1475	5.2770	13.631
	{3.9107}	{5.6598}	{23.8332}		−{2.5435}			{0.509}	{0.034}
R_{VM}	0.12940	0.14878	0.74098	0.88976	−0.33196	−0.72032	7.00691	10.153	14.650
	{3.5265}	{5.2451}	{14.8135}		−{3.3607}			0.118	{0.023}

Notes: See Tables 1 and 2 for definitions of the return series and test statistics. t-values are given in brackets, below each parameter coefficient. The Q is the Ljung–Box statistics. Their p-value is given in brackets, below each coefficient.

correct price (mean reversion). The positive autocorrelation (θ_1) coefficients for the frequently traded assets suggest an adjustment to new information at both t and $t+1$. In both cases prices do not adjust immediately to new information. However, depending on trading frequency, asset prices either overreact and reverse or adjust slowly over several days. However, be aware of the series of zero returns in thinly traded assets, which may induce spurious autocorrelation in theses series.

Cross-autocorrelation ($\gamma_{i,j}$) (Lo and MacKinlay, 1990) is found for all series except for the most frequently traded series. The pattern in the cross-autocorrelation implies mainly influence from series that show more frequent trading. However, also here thinly traded series induce spurious cross-autocorrelation. The estimated β_i coefficients on the GARCH-in-mean terms show no significant 'mean' effects. This is also true for the market indices. The degree of freedom coefficients (v_i) suggest thick distribution tails. For all series the coefficients are strongly significant and show values ranging between 5 and 6.5.

Among the estimated conditional variance coefficients, which are all strongly significant a clear pattern is found. The past squared errors (a_1) have more influence over the conditional variance of the frequently traded portfolios than they do over the conditional variance of the thinly traded portfolios. The two market indices seem to report shock effects in line with the effects of the two frequently traded portfolios. In contrast, the past conditional variance (b_1) exerts a greater influence over the current conditional variance in the case of the most thinly traded series. Also for the conditional variance the two market indices seem to follow the results from the two frequently traded portfolios for past conditional variance. Hence, the combination of these two features of the conditional variance suggest that although shocks to the volatility of thinly traded portfolios have less impact than shocks to the volatility of frequently traded portfolios, they are much more persistent. Finally, for all portfolios and indices a negative asymmetric volatility coefficient (λ_i) is found, which imply higher volatility from negative shocks in all series. The negative asymmetric volatility is insignificant for the most thinly traded series but increases in size and significance as trading frequency increases.

To test the validity of the results, several model specification tests are performed for all six return series. As a first specification test, the sixth order Ljung and Box (1978) statistic is calculated for the standardized residuals[22] (Q) and squared standardized residuals (Q^2) for all six series. For all series Table 7 shows no significant evidence of serial correlation in the standardized residuals ($Q(6)$) at 1%. However, for the squared residuals up to lag 6 ($Q^2(6)$) the thinly traded series report significant autocorrelation, while all other series report insignificant autocorrelation. The numbers for kurtosis and skews for the standardised residuals are lower in absolute values for all series. Hence, the ARMA–GARCH filter suggests clearly more normal residuals. This result is confirmed by the K–S Z-test statistic (not reported).

Table 8 report extended model specification tests. Table 8 reports the ARCH, RESET and BDS test statistic for the BIC preferred ARMA–GARCH standardized residuals (ε) and adjusted standardized residuals[23] ($\ln(\varepsilon^2)$). The ARCH test statistic reports conditional homoscedasticity for all series except the most thinly traded series. The RESET test statistic cannot reject linearity in the mean for any series. The BDS test statistic rejects identically and independently distributed residuals (i.i.d.) for the most thinly traded series and the equal-weighted market index. Hence, the model specification test statistics suggest that the ARMA–GARCH model seems to capture most of the market dynamics appropriately. However, for thinly traded series the ARCH and BDS test statistics suggest a wrongly specified model. The result induce that the ARMA–GARCH model does not appropriately describe thinly traded series and long series of zero returns. Furthermore, the inclusion of the nonlinear dependent ARMA–GARCH residual and thinly traded series into the index series seems to induce a wrongly specified model also for the equal-weighted market index.

Finally, Table 9 reports three simple bias tests and one joint test (Engle and Ng, 1993). Table 9 from column 2 to 6, report significant ($\varepsilon_{t-1}, S_{t-1}$) biases. Hence, bad news is not very well predicted by this model. Especially, the thinnest traded portfolio suggests that bad news is badly predicted. Moreover, the joint bias test statistic in column 7 and 8, reports significant prediction biases for the most thinly traded series.

V. FINDINGS FROM THE NORWEGIAN THINLY TRADED MARKET

The main focus of this investigation is characteristics in thinly traded markets, especially nonsynchronous trading and non-trading effects as well as conditional heteroscedasticity and volatility clustering. The Norwegian market exhibit characteristics in trading volume (NOK) that make it possible to establish asset portfolio series that contains the desired trading frequency characteristics. Hence, the investigation looks for trading volume characteristics in the mean and volatility equations as well as in the overall model specification and discuss implications for market dynamics in thinly traded markets.

Only the most frequently traded series and the two market indices report significant positive drift, while the three more thinly traded portfolios report nonsignificant drift. These results together with the significant and positive α_i coefficients for the two market indices, suggest that a significant and positive drift may solely originate from continuously traded series. Moreover, it seems to be the case that series exhibiting nonsynchronous trading and non-trading characteristics reject positive drift. For an investor in the Norwegian market the results imply that in a long hold strategy, thinly traded series should be avoided and should only involve continuously traded assets. Furthermore, as the most thinly traded assets also imply the high nonsynchronous trading and non-trading effects, the assets may induce high spurious autocorrelation and cross-autocorrelation. These nonsynchronous trading and non-trading effects may also influence the drift coefficients for these assets. The direction of the influence may be difficult to classify, but as zero return will be registered in a non-trading period, the drift will probably be influenced towards a zero drift coefficient.[24]

Autocorrelation is found in all series, which imply substantial predictability in asset returns for the thinly traded Norwegian market. The thinly traded series report strong negative autocorrelation while the frequently traded series and the indices exhibit strong positive autocorrelation.[25] However, returns for thinly traded series may contain

[22] Standarized residuals are calculated as $\varepsilon_t/\sqrt{(\mathbf{h}_i^* \upsilon_i/(\upsilon_i - 2))}$ where υ_i is the degree of freedom in the student-t distribution.
[23] See deLima and Pedro (1995a, 1995b).
[24] Solibakke (2000c) shows that the drift becomes more positive by applying virtual returns and a continuous time GARCH specifications.
[25] A negative dependence story can be found in Poberta and Summers (1988) and Fama and French (1988) and a positive dependence story can be found in Taylor (1986/2000).

Table 8. *BDS and ARCH test statistics for i.i.d. residuals*

				BDS-test statistic						
Series*	ARCH (6)	RESET (12;6)	Residual!	$m = 2; \varepsilon = 1$	$m = 3; \varepsilon = 1$	$m = 4; \varepsilon = 1$	$m = 5; \varepsilon = 1$	$m = 6; \varepsilon = 1$	$m = 7; \varepsilon = 1$	$m = 8; \varepsilon = 1$
R_1	20.628	11.779	ε	2.3420	1.5097	1.2023	0.9430	0.5436	0.3754	0.2718
	{0.002}	0.4636	$\ln(\varepsilon^2)$	−1.0434	−1.7225	−1.8179	−2.0027	−1.9317	−2.1212	−2.2424
R_2	7.686	13.499	ε	1.7247	1.7166	1.5743	1.2402	1.1240	1.1896	1.1962
	{0.262}	{0.334}	$\ln(\varepsilon^2)$	−0.4405	−1.2493	−0.7225	−0.5082	−0.0129	0.3434	0.7283
R_3	6.460	8.4523	ε	0.8069	0.7797	0.5099	0.2287	0.4103	0.4014	0.5942
	{0.374}	{0.749}	$\ln(\varepsilon^2)$	0.9080	1.6429	1.5374	1.2772	0.8960	0.1974	−0.4677
R_4	15.466	7.169	ε	1.8614	1.7593	1.5673	1.1419	1.1427	1.0600	0.8274
	{0.017}	{0.846}	$\ln(\varepsilon^2)$	−1.1988	−1.0702	−1.7677	−1.4437	−1.1363	−0.9045	0.1413
R_{EM}	13.369	8.1559	ε	2.2676	1.5451	1.1734	0.7718	0.7293	0.5214	0.4001
	{0.038}	{0.773}	$\ln(\varepsilon^2)$	1.3702	1.7538	2.3106	2.5541	2.7253	2.9374	3.1955
R_{VM}	8.333	9.6817	ε	1.1349	1.4170	1.0015	0.6551	0.8835	1.0128	1.1865
	{0.215}	{0.644}	$\ln(\varepsilon^2)$	−0.3541	−1.3481	−1.9556	−1.9230	−1.9302	−1.7664	−1.8072

Notes: See Table 1 for definition of return series and test statistics (ARCH, RESET and BDS).
! Residuals are standardized residuals (ε) or adjusted residuals ($\ln(\varepsilon^2)$) applying results from deLima (1995).

Table 9. *Simple and joint bias test for model misspecification*

Series	Simple bias tests S_{t-1}		$\varepsilon_{T-1}S_{t-1}$		$\varepsilon_{T-1}S^*_{t-1}$		Joint bias test $T\cdot R^2$	$\chi^2(3)$
R_1	0.20398	{1.4887}	−0.46684	−{5.1259}	−0.08382	−{0.8795}	25.026	{0.0000}
R_2	−0.05298	−{0.2066}	−0.41698	−{2.7551}	−0.17266	−{0.9381}	9.137	{0.0275}
R_3	−0.17886	−{0.7993}	−0.17150	−{1.2725}	0.11198	{0.6969}	5.254	{0.1541}
R_4	−0.23602	−{0.4770}	−0.21046	−{1.9979}	0.13106	{1.0857}	11.323	{0.0101}
R_{EM}	−0.17349	−{0.6472}	−0.43625	−{2.8916}	−0.09759	{0.5199}	9.608	{0.0222}
R_{VM}	−0.15252	−{0.8610}	−0.29516	−{2.5924}	0.13589	{1.0698}	9.752	{0.0208}

Notes: See Table 1 for definition of return series.
S = Dummy-variable equal to 1 when $\varepsilon_{t-1} \leq 0$, and S^* = Dummy-variable equal to 1 when $\varepsilon_{t-1} > 0$.
This simple bias test statistics are simple OLS parameters with associated t-statistics in brackets to the right.
The joint BIAS-test statistic tests the relation $\varepsilon_t^2 = a + a_1 \cdot S_{t-1} + a_2 \cdot \varepsilon_{t-1}^2 \cdot S_{t-1} + a_3 \cdot \varepsilon_{t-1}^2 \cdot (1 - S_{t-1})$. The statistic tests whether all the a-parameters are significantly different from zero. TR^2 is χ^2 distributed with 3 degrees of freedom.

characteristics of nonsynchronous trading and non-trading effects, which may cause spurious autocorrelation as discussed in Campbell *et al.* (1997). Hence, the results for thinly traded series implying overreaction and reversion, may originate from spurious autocorrelation, which stem from many zero return observation. The validity of autocorrelation results for thinly traded series may therefore be disputed. For the frequently traded series the positive serial-correlation coefficient suggests adjustment to new information that may take several days. It is well known from international literature that positive dependence (Taylor, 1986/2000) in assets returns is more often found than negative dependence. Hence, collectively, the serial-correlation coefficients imply substantial predictability among all the return series. This predictability results for frequently traded series are not disputed while thinly traded series may show spurious predictability.[26] Moreover and probably very important for investors in the Norwegian market, by employing frequently traded assets in portfolio construction and prediction, the results seem to suggest a consistent short run predictability of asset returns. Note that the reported negative autocorrelation for thinly traded series may be spurious and may distort the predictability in these series. Hence, also applying estimation results form 1987 to 1994, the overall predictability may therefore be illusionary for the Norwegian market.

Significant cross-autocorrelation is found among trading volume series. The cross portfolio results therefore strongly indicate that the thinly traded Norwegian market show return effects from more frequently traded series into more thinly traded series. That is, the result suggests that thinly traded series adjust to new information with a lag to more frequently traded series (γ_j). Hence, new information is incorporated into assets starting with the most frequently traded assets and then with a lag, moved into more thinly traded assets. Hence, investors may therefore follow the following procedure to obtain a long run profit. Study carefully the most frequently traded asset within an industry. When these assets move up or down take appropriate positions (long or short) in more thinly traded assets. The asset position must be constantly monitored and may be expensive owing to transaction costs. Moreover, an investor that builds a portfolio based on trading volume and combine highly and lowly traded assets within a industry into portfolios, he or she can adjust positions based solely on movements in the most frequently traded assets. In summary, the most frequently traded assets leads the market while more thinly traded assets copy these movement with a lag. However, thin trading implies spurious cross-autocorrelation. Hence, the lead and lag results for thinly traded assets may turn spurious. However, for more frequent traded assets the lead and lag structure may still be valid.

As the 'in-mean' coefficients are insignificant for all six series and the residual risk hypothesis is rejected (Lehmann, 1990) for the Norwegian market. The degree of freedom coefficient (ui) is strongly significant and induces deviation from normally distributed return series. The results seem therefore to indicate leptokurtosis in all six series independently of frequency of trading and non-trading effects.

The conditional variance equation report several interesting features from the thinly traded Norwegian market. First, the ARCH-coefficient (shock) increases the higher the trading frequency. Hence, past squared errors influence strongest today's volatility for the most frequently traded series. The two market indices show results close to the two most frequently traded series. The past squared error for thinly traded series show a low volatility influence rela-

[26] The positive autocorrelation results seem to disappear in the 1987–1994 subperiod. Hence, no obvious autocorrelation results after the crash in 1987 are found for the thinly Norwegian market.

tively to more frequently traded series. However, also for the conditional volatility spurious past squared error results may be found. In case of non-trading the observed return is zero and may produce artificial shocks for the volatility process. For thinly traded series a spurious and too low ARCH coefficients may therefore be observed. Secondly, the past conditional volatility influences strongest today's volatility for the thinnest traded series. Autocorrelation in the conditional volatility process seems therefore to be highest for the thinnest traded series. However, note that the nonsynchronous trading and non-trading effects may cause spurious autocorrelation into the conditional volatility process. This may distort any volatility patterns for non-trading series.

Thirdly, the persistence $(a_i + b_i)$ is strongest for the thinnest traded series. A clear picture of the persistence in the volatility process can be obtained by calculating the half-life of a shock to the process, that is, the time that it takes for half of the shock to have dissipated. Some algebra shows that the half-life in trading days for portfolio i may be calculated as[27] $\text{Half-life}_i = \ln(0.5)/\ln(a_{i,1} + b_{i,1})$ and for calendar days as (252 . Half-lifei)/365 = ln(0.5)/ $\ln(a_{i,1} + b_{i,1})$. Hence, $\text{Half-life}_i = (\ln(0.5)/\ \ln(a_{i,1} + b_{i,1}))$. (365/252). For the six return series the results for both formulas in Table 10 are reported. Table 10 suggests a significant difference in persistence length over the six series. Highest persistence is found for the thinly traded series, which report shock persistence for approximately 50 trading days. For the most frequently traded series the persistence is only 6.5 trading days. The information in the shock- and persistence-effects for series may be useful for investors building volatility strategies in an option market. One implication of an active option market that increases trading activity and therefore volume in the underlying asset may therefore be higher shock effects and lower persistence, that is a more erratic volatility. However, nonsynchronous trading and non-trading effects may distort the result. Series strongly influenced by zero return observations may emphasize the autocorrelation in conditional volatility too much, which may result in spurious persistence coefficients. Applying results from the more frequently traded series imply rather strong non-trading effects. The three more frequently traded series plus the indices show all quite similar results. Hence, nonsynchronous trading and non-trading effects may be severe for the conditional volatility in the thin Norwegian market. Fourthly, the constant $(m_{i,0})$ increases the higher the frequency of trading. Hence, as $m_{i,0} = \delta_{i,0} \cdot V_i$ and $\delta_{i,0} + a_{i,1} + b_{i,1} = 1$, the weight to the long-term average volatility seem to increase the higher the trading frequency. This feature implies that weight to the unconditional volatility is at its lowest for the most thinly traded series.

Table 10. *Number of days for half of a shock to have dissipated*

Series	Trading days	Calendar days
R_1	49.8516	72.2057
R_2	8.8283	12.7870
R_3	9.3194	13.4984
R_4	6.5484	9.4847
R_{EM}	9.2196	13.3538
R_{VM}	5.9345	8.5956

Note: See Table 1 for definition of return series.

Fifthly, asymmetric volatility (λ) is present in all series except for the most thinly traded series. The negative coefficients imply that it is the most frequently traded series that seem to show the highest asymmetry in the conditional volatility. The lack of asymmetry for the thinly traded series may also be attributed to the strong serial correlation. As both the weight to the long run average volatility and the shock effects is low in this series, the autocorrelation structure seem to be the dominant factor for the conditional volatility process. For all other series the asymmetric coefficient is negative and significant.

Turning now to the specification tests, several interesting features are found, which may originate from nonsynchronous trading and non-trading effects. First, the $Q^2(6)$ statistic report autocorrelation for the thinnest traded series. Neither market index nor more frequently traded asset series report autocorrelation in first and second moment residuals. Secondly, the ARCH test statistic reports conditional heteroscedasticity for the most thinly traded asset series. As for autocorrelation neither market index nor more frequently traded asset series report conditional heteroscedasticity. The RESET test statistic reports linearity in the conditional mean for all six series. Finally, the BDS test statistic reports general nonlinearity for the thinnest traded series and the equal-weighted market index at some dimension (m). The three more frequently traded series and the vale-weighted index series show a BDS test statistic that fail to reject i.i.d. at any dimension (m). It seems therefore to be the case that the inclusion of the thinly traded series seems to introduce nonlinearity into the equal-weighted market index. Hence, nonsynchronous trading and non-trading effects cause nonlinear dependence and model misspecification. Consequently, the ARMA–GARCH model specification seems not appropriate for thinly traded asset series.

The simple bias tests for volatility prediction show that especially bad news is not appropriately predicted in the GARCH–GJR model. However, only the most thinly traded series show biases when a joint bias test is performed. Moreover, the prediction bias is not strongly significant in the simple test statistic. When models are re-

[27] See Taylor (1986/2000) for details.

Table 11. *(Un-)conditional volatility characteristics*

Series	Mean	Standard deviation	Std.dev./ mean	Unconditional volatility**
R_1	3.9473	1.9880	0.5036	3.8145
R_2	1.6677	2.6646	1.5978	1.3855
R_3	1.8291	3.4213	1.8704	1.5118
R_4	2.3895	5.3139	2.2239	1.9304
R_{EM}	1.1476	2.7199	2.3700	0.9129
R_{VM}	1.5236	3.1349	2.0575	1.1739

Notes: See Table 1 for definition of return series.
** Unconditional volatility is calculated as $m_i/(1-(a_i+b_i))$ from the GARCH(1,1) process.

estimated for two subsamples; 1983–1987 and 1988–1994, the second time period show a small change in autocorrelation for the conditional mean. In particular, after the crash in 1987 the slow adjustment process for the frequently traded series has changed to immediate adjustment. That is, no autocorrelation in the residuals for these series. The general conditional variance results are maintained in the subperiods. However, later years (1988–1994) indicate an increased persistence in the variance process. For the thinnest traded series the conditional variance process show almost integrated GARCH.

Finally, Table 11 reports the first and second moments for the conditional volatility and the calculated unconditional volatility from the GARCH models.[28] The conditional volatility mean and the calculated unconditional volatility both report an U-shaped pattern as also reported for R_i^2 in Table 2. The calculated unconditional volatility is quite close to the conditional volatility mean. Moreover, the higher the trading frequency a strong and consistent increase in the standard deviation of the conditional volatility series is found. Hence, the highest mean but lowest standard deviation is found for the conditional volatility process of the thinnest traded series and lower mean but highest standard deviation for the most frequently traded series. The results are in accordance with the ARCH/GARCH parameters for the conditional volatility estimations. Now calculation of the index standard deviation divided by the mean is an index that measures the relative uncertainty/change in the volatility process. The result shows clearly that it is the most frequently traded series and the two market indices that show highest changing volatility around a mean. Hence, non-trading effects show high volatility but lower changes in volatility. However, as the model specification is disputed for the most thinly traded series this result may be spurious. For option markets on individual and index series the estimations may produce valuable information for strategists. As the Norwegian market quote options for only continuously traded series, investors should be aware of this changing volatility result in applying the Black and Scholes option pricing formula. Estimates of the underlying asset's volatility may be very important for correct option pricing in these assets. To forecast future volatility using GARCH (1,1) model results is a well-known and easy exercise.

VI. SUMMARIES AND CONCLUSIONS

Several ARMA–GARCH-in-mean model specifications have been modelled and estimated for the Norwegian thinly traded equity market. Trading volume has been applied as a proxy for trading frequency. As all the estimated ARMA–GARCH lag specifications for the series are BIC preferred, the model captures the autocorrelation and cross-autocorrelation structure in the conditional mean and the shocks, autocorrelation, persistence and asymmetry in the conditional volatility across varying trading frequencies. Moreover, the model measures the effect of 'thick distribution tails' (leptokurtosis) through the degree of freedom parameter in the student t-distribution and potential residual risk is measured applying the in-mean specification. The thinnest traded series and the equal-weighted index series report ARMA–GARCH lag structure misspecification. The results for these series may be spurious and must therefore be interpreted with great caution.

The study reports the following conclusions. ARMA–GARCH models seem to fit the Norwegian thinly traded market well, except for thinly traded series. The thinly traded series exhibit severe nonsynchronous trading and non-trading effects, which induce data dependence and misspecification in the BIC efficient ARMA–GARCH model. The equal-weighted index series seem to inherit these data dependence and misspecification results in the Norwegian market. For relatively frequently traded series a consistent pattern in autocorrelation, cross-autocorrelation, volatility clustering and asymmetric volatility are found. For all these series insignificant specification test statistics are found. Hence, the ARMA–GARCH model and its parameter results for relatively frequently traded series suggest that previous regression models in the Norwegian market may have been wrongly specified owing to four specification failures. First, a failure to efficiently incorporate the serial correlation structure in the conditional mean applying a BIC preferred lag specification. Secondly, a failure to incorporate the appropriate structure for measuring weight to long-run average volatility, shocks, autocorrelation, persistence and asymmetric volatility in the conditional variance equation applying a BIC preferred lag specification. Thirdly, a failure to specify thick-tailed distribution characteristics obtaining

[28] The unconditional variance in a GARCH(1,1) specification is the long-run average volatility.

close to normally distributed residuals. Fourthly, a failure to control for data dependence in the model residuals, which implies spurious parameter results for thinly traded asset series.

Consequently, nonsynchronous trading and non-trading seem to imply an extra challenge for modelling the dynamics in thinly traded markets. Classical regression models assuming conditional homoscedasticity seem obsolete. Moreover, for the applied ARMA–GARCH methodology, which uses the residuals for volatility specification, we find strongly significant misspecification is found for thinly traded series. Hence, the ARMA–GARCH methodology seems also to be a wrongly specified model in thin markets and thin series. As stochastic volatility models generate volatility processes independently of the conditional mean, the methodology may be an alternative model specification.[29] Alternatively, virtual returns may be applied (Campbell *et al.*, 1997) and continuous time GARCH models (Drost and Nieman, 1993) for thin series. However, for relatively frequently traded series the ARMA–GARCH model seems appropriate.

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[29] See the SNP methodology of Gallant *et al.* (1992).

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Essay no. 3*

Non-linear Dependence and Conditional Heteroscedasticity in Stock Returns Evidence from the Norwegian Thinly Traded Equity Market

Abstract

We investigate the presence of non-linear dependencies in stock returns for the Norwegian Equity Market as it is very difficult to interpret the unconditional distribution of stock returns and its economic implications if the i.i.d. assumption is violated. Standard tests of non-linear dependence give strong evidence for the presence of non-linearity in raw returns. Modelling non-linear dependence must distinguish between models that are non-linear in mean and hence depart from the Martingale hypothesis, and models that are non-linear in variance and hence depart from independence but not from the Martingale hypothesis. Therefore, we formulate three non-linear models of asset returns applying ARMA-GARCH specifications for the conditional mean and variance equations. We go on to answer which model that seems to have the necessary characteristics that are sufficient to account for most of the non-linear dependence. In the Norwegian equity market most of the non-linear dependence seems to be conditional heteroscedasticity. However, the most thinly traded assets still report significant non-linear dependence for all non-linear specifications. These results imply that we can reject the independence hypothesis for all assets, portfolios and indices. Moreover, for thinly traded assets we can also reject the Martingale hypothesis. The economic implications from the unconditional distributions of thinly traded assets are therefore very difficult to interpret and are unfamiliar territory for those who are accustomed to thinking analytically, intuitively and linearly.

Classification:

Keywords: Non-linearity, conditional heteroscedasticity, trading/non-trading, ARMA-ARCH/GARCH, Non-linear test statistic

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1 Introduction

Non-linear dependence in stock returns has recently attracted much attention. Examples are Abhyankar et al. (1995) from the UK market and de Lima (1995 a, b), Hsieh (1991) Brock et al. (1991), and Lee et al. (1993) from the US market[i]. Non-linear structure in univariate time series departs from the random walk model and will be unfamiliar territory to those who are accustomed to thinking analytically, intuitively, and linearly. The Random Walk model (Bachelier, 1964), which assumes that security prices from transaction to transaction are independent, identically distributed (i.i.d) random variables, together with the central limit theorem, suggests that price changes are normally distributed and that their variances will be linearly related to the time interval. However, as noted by Hsieh (1991), it is difficult to interpret the unconditional distribution of stock returns and its economic implications, if the i.i.d. assumption is violated. If stock returns are i.i.d. and follow fat tails distributions such as Cauchy (Mandelbrot, 1963) or the Student-t density (Blattberg and Gonedes, 1974) or Normal Inverse Gaussian (Eberlein and Keller, 1994 and Barndorff-Nielsen, 1994), then the probability of observing large absolute returns such as that on 19-22 October 1987 is small but non-zero. In this case market crashes such as that of the 1987 could happen at any time but with very low probability (Brown, Goetzman and Ross, 1995). The behavior of risk averse agents will consequently take this into account (Bollerslev et al., 1993). Our crucial point is that such an interpretation is so dependent on the i.i.d. assumption since the unconditional distribution will always have fatter tails than the conditional distribution if the data has some form of conditional dependence[ii].

One prominent explanation for the observed departure from Bachelier's (1964) model is the mixture of distributions hypothesis (Epps and Epps, 1976 and Tauchen and Pitts, 1983). This maintains that trade-to-trade asset returns exhibit leptokurtosis because they are really a combination of return distributions that are conditioned on information arrival. This means that periods of little or no information arrival result in different observed return distributions than in periods when information frequently arrive (Clark, 1973, Harris, 1989 and French and Roll, 1986). If thin or no trading volume is indicative of small or no arrival of information, than the characteristics of thinly traded assets are not the same as that for actively traded assets (Gallant, Rossi and Tauchen, 1992).

Therefore, this paper studies non-linear dependence in stock returns for a sample of differently traded assets, trading volume portfolios and differently value-weighted market-index series in the Norwegian thinly traded market[iii]. Earlier Norwegian results (Solibakke, 2000b, 2000c) suggest that a null hypothesis of i.i.d. for adjusted raw return series is strongly rejected. The main objective for this study is therefore to find any systematic difference in non-linear dependence over the whole trading volume range (including non-trading) as well as to find the origin of non-linear dependence. The origin of non-linear dependence in univariate time series is (1) non-linear dependence in mean, (2) non-linear dependence in variance or (3) non-linear dependence in both mean and variance. Therefore, this non-linear dependence study examines three different non-linear conditional mean and volatility ARMA-GARCH specifications in an attempt to find the nature of the observed non-linear dependence in Norwegian return series. Modelling non-linear dependence must distinguish between models that are non-linear in mean and hence depart from the Martingale hypothesis, and models that are non-linear in variance and hence depart from the assumption of independence but not from the Martingale hypothesis. We therefore employ an efficient auto-regressive moving average (ARMA) specification for the conditional mean and an efficient auto-regressive conditional heteroscedasticity (ARCH/GARCH) specification for the conditional variance. ARMA-GARCH models are conditional homoscedastic. Observed characteristics as leptokurtosis and asymmetric volatility are incorporated into the model specifications. Therefore, using these specifications for the conditional mean and variance equations, we can hypothesize non-linear dependence in the Norwegian equity market.

The study differs from other studies (Abhyankar et al., 1995) in several ways. Firstly, we apply three different test statistics for non-linear dependence. Secondly, we use data for individual frequently and thinly traded asset returns and four portfolios (we rebalance the portfolios each month) of differently traded component assets, to examine whether individual and portfolio asset returns behave similar to aggregate stock market series. Otherwise, any generalization of the findings from aggregate to individual series would be inaccurate. Moreover, non-trading effects on non-linear dependence can now explicitly be studied employing data series from the Norwegian thinly traded market. Thirdly, we employ both a normal and a student-t density log-likelihood function for individual, portfolio and index return-series. Student-t density functions may account for observed leptokurtosis in stock market returns. Fourthly, we apply the recent adjustment suggested by de Lima (1995b) to the residuals from the GARCH model before conducting the BDS test statistic (Brock and Dechert, 1988, Scheinkman, 1990). Fifthly, all raw return series are adjusted for systematic size and location effects as suggested by Gallant, Rossi and Tauchen (1992). Sixthly, and finally, the "leverage effect" are modelled in the conditional volatility equations (Nelson, 1991)[iv].

The rest of the paper is therefore organized as follows. Section 2 specifies three non-linear ARMA-GARCH models and describes the BDS, ARCH (Engle, 1982 and Engle and Bollerslev, 1986) and RESET (Ramsey, 1969) test statistics for non-linear dependence. Section 3 describes the Norwegian data and the Gallant, Rossi and Tauchen (1992) adjustment procedures. Section 4 reports the empirical findings and finally Section 5 summarizes and concludes our findings.

2 Specifications of non-linear relationships and test statistics

2.1 Non-linear ARMA-GARCH specifications

Many aspects of economic behaviour may not be linear. Most evidence and introspection suggest that investor's attitude towards expected return and risks are non-linear. Moreover, most derivative securities provide non-linear terms and the strategic interaction between market participants, the process by which information is incorporated into security prices and the dynamics of economy-wide fluctuations are all inherently non-linear. However, no economic theory or behaviour has so far distinguished between non-linear dependence in conditional mean and variance. Therefore, we have to distinguish between models that are non-linear in mean and hence depart from the Martingale hypothesis and models that are non-linear in variance and hence depart from the assumption of independence but not from the Martingale hypothesis.

In non-linear time-series analysis the underlying shocks are typically assumed to be i.i.d. However, we typically seek a possibly non-linear function relating the series x_t to the history of shocks. A general representation is $x_t = f(\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, ...)$ where the shocks are assumed to have mean zero and unit variance, and *f()* is some unknown function. The generality of the representation makes it very hard to work with—most models used in practice fall into a somewhat more restricted class that can be written as $x_t = g(\varepsilon_{t-1}, \varepsilon_{t-2}, ...) + \varepsilon_t h(\varepsilon_{t-1}, \varepsilon_{t-2}, ...)$. Here the function *g()* represents the mean of x_t conditional on past information, since $E_{t-1}\left[x_t\right] = g(\varepsilon_{t-1}, \varepsilon_{t-2}, \cdots)$. The innovation in x_t is proportional to the shock ε_t, where the coefficients of proportionality is the function *h()*. The square of this function is the variance of x_t conditional on past information, since $E_{t-1}\left[(x_t - E_{t-1}\left[x_t\right])^2\right] = h(\varepsilon_{t-1}, \varepsilon_{t-2}, \cdots)^2$. Models with non-linear *g()* are said to be non-linear in mean, whereas models with non-linear $h()^2$ are said to be non-linear in variance. The second equation leads to a natural division in the non-linear time-series literature between models of the conditional mean *g()* and models of the conditional variance *h()*. Most time-series models concentrate on one form of the non-linearity or the other. However, the (General) Auto-regressive Conditional Heteroscedasticity ((G)ARCH) model of Engle (1982) makes modelling of non-linear dependence in both mean and variance possible.

Three non-linear models will be analysed in this study. A linear ARMA model with a constant (drift) takes the form

$$x_t = \alpha_0 + \sum_{j=1}^{p} \varphi_j \cdot x_{t-j} + \varepsilon_t + \sum_{i=1}^{q} \theta_i \cdot \varepsilon_{t-i} \ ; \qquad h_t = a_0 \tag{1}$$

where p and q are the BIC (Schwarz, 1978) preferred respective lag lengths; ϕ_i is the auto-regressive parameters and θ_i is the moving average parameters. The first non-linear model is an extended ARMA model where non-linearity in the mean is introduced through the squared residual (ε_t^2). This simple non-linear ARMA model takes the form

$$x_t = \alpha_0 + \sum_{j=1}^{p} \varphi_j \cdot x_{t-j} + \varepsilon_t + \sum_{i=1}^{q} \theta_i \cdot \varepsilon_{t-i} + \gamma \cdot \varepsilon_{t-1}^2 \; ; \qquad h_t = a_0 \tag{2}$$

where p and q is based on the Schwarz Bayesian Criterion (BIC) (1978) from the original adjusted return time-series and a_0 is an estimated constant[v]. The third model analyses changing volatility and model non-linear dependence in only the variance equation (the mean is linear). The model takes the form

$$x_t = \alpha_0 + \sum_{j=1}^{p} \varphi_j \cdot x_{t-j} + \varepsilon_t + \sum_{i=1}^{q} \theta_i \cdot \varepsilon_{t-i} \; ; \; \lambda_{jt} = \varsigma_j \quad \text{if } \varepsilon_{t-j} < 0\text{; else } 0$$

$$h_t = a_0 + \sum_{j=1}^{m} (a_j + \lambda_{jt}) \cdot \varepsilon_{t-j}^2 + \sum_{i=1}^{n} b_i \cdot h_{t-i} \tag{3}$$

where p and q is based on the BIC Criterion (Schwarz, 1978) from the original data series. By analogy with ARMA models, the third equation in (3) is called a GARCH (m,n) model. The coefficient b_i measures the extent to which volatility today feeds through into next period's volatility, while $(\sum a_j + \sum b_i)$ measures the rate at which this effect dies out over time. The GARCH (m,n) model is an ARMA (u,m) model for squared innovations, where $u=max(m,n)$. Hence, using the BIC criterion for the squared innovation from an ARMA (u,m) model produces the necessary m and n lags. λ_{it} is the vector of parameters for the asymmetric process (leverage)[vi]. Finally, the fourth model combines model (2) and model (3) and takes the form

$$x_t = \alpha_0 + \sum_{j=1}^{p} \varphi_j \cdot x_{t-j} + \varepsilon_t + \sum_{i=1}^{q} \theta_i \cdot \varepsilon_{t-i} + \gamma_1 \cdot \varepsilon_{t-1}^2 + \gamma_2 \cdot h_t \; ;$$

$$\lambda_{it} = \varsigma_i \quad \text{if } \varepsilon_{t-j} < 0\text{; else 0; } \; h_t = a_0 + \sum_{j=1}^{m} (a_j + \lambda_{jt}) \cdot \varepsilon_{t-j}^2 + \sum_{i=1}^{n} b_i \cdot h_{t-i} \tag{4}$$

where p, q is based on the BIC criterion (Schwarz, 1978) from the raw data series and m and n is based on the BIC criterion from the squared residuals in a linear ARMA[vii] model. Model (4) is a specification for non-linearity in both conditional mean and conditional variance. All models may be estimated under both a normal-[viii] and a student-t[ix] density log-likelihood function.

2.2.1 The RESET test statistic (Ramsey, 1969)

The Regression Error Specification Test (RESET; Ramsey, 1969) is a test statistic of linearity against an unspecified alternative. It is a test against general model misspecification[x] and has certainly been one of the most popular tests against misspecification of functional form.
In this paper it is carried out in three stages as follows:

(1) We assume the linear part of the model is

$$y_t = \beta' \cdot z_t + u_t, \qquad t = 1,....,T$$

where $z_t = (1, y_{t-1},...y_{t-p}, x_{t1},...,x_{tk})'$. We estimate β by OLS and compute $\hat{u}_t = y_t - \hat{y}_t$ where $\hat{y}_t = \hat{\beta}' z_t$, and $SSR_0 = \sum \hat{u}_t^2$.

(2) Then we estimate the parameters of $\hat{u}_t = \delta' z_t + \sum_{j=2}^{h} \varphi_j \tilde{z}_t^{(j)} + \upsilon_t$

by OLS and compute $SSR = \sum \hat{\upsilon}_t^2$, where $\tilde{z}_t^{(j)} = (y_{t-1}^j,...,y_{t-p}^j, x_{t1}^j,...,x_{tk}^j)$, $j = 2,...,h$.

(3) Finally, we compute the test statistic: $F = \dfrac{(SSR_0 - SSR)/(h-1)}{SSR/(T-m-h)}$

where $m = p+k$. k is in our case zero. As z_t contains lags of y_t, then $(h-1)F$ has an asymptotic χ^2 distribution under the null of linearity. h was suggested by Thursby and Schmidt (1977) to be given the value 4 for the best result. This test is an Lagrange Multiplier (LM) type test against an Logistic Smooth Transition Regression (LSTR) model in which only one 'linear parameter' changes but the investigator does not know which one. The RESET test is thus rather narrow in that if more than one variable has a 'changing linear parameter' the regression no longer covers that possibility. Note, however, that the constant in the first regression should not be involved in defining the z_t and $\tilde{z}_t$ in the auxiliary regression, since the inclusion of such regressors would lead to perfect co linearity.

2.2.2 The ARCH test statistic for non-linear dependence (Engle, 1982)

The ARCH test statistic is a test for constant conditional variance against conditional heteroscedasticity, based on the Lagrange Multiplier principle. The test procedure is to run a regression of the squared residuals on a constant and p lagged squared residuals. Then test the test statistic $T \cdot R^2$ as a $\chi^2(p)$ variate, where T is the sample size and R^2 is the squared multiple correlation coefficient and p is the degree of freedom. The ARCH test is a test for H_0: constant conditional variance against the alternative H_a: a conditional variance that obey an ARCH(p) specification. In fact, if ARCH is present in the residuals, non-linear dependence in the time series cannot be rejected.

2.2.3 The correlation integral

The correlation integral proposed by Grassberger and Procaccia (1983) is a measure of spatial correlation in an *m*-dimensional space. Let $\{\mu_t\}$ be a real-valued scalar time-series process. Construct the *m*-history process $\mu_t^m \overset{def}{=} (\mu_t, \mu_{t+1}, \cdots, \mu_{t+m-1})$. For $\varepsilon > 0$, the correlation integral at *embedding dimension m* is given by[xi]

$C_{m,\varepsilon} = \iint \chi\varepsilon(x^m, y^m) dF(x^m) dF(y^m)$, where $\chi \cdot \varepsilon(\cdot,\cdot)$ is the symmetric indicator kernel with $\chi \cdot \varepsilon(x,y) = 1$ if $||x - y|| < \varepsilon$ and *0* otherwise (indicator function), $||\cdot||$ represents the max-norm, and *F()* is the distribution function of μ_t^m. $C_{m,\varepsilon}$ gives the mean volume of a cube with diameter ε. An estimator of the correlation integral for a sample size *T* for the process $\{\mu_t\}$ is given by the following *U*-statistic— cf. BDS (1987), $C_{m,\varepsilon} = \frac{1}{\binom{\overline{T}}{2}} \sum \sum_{1 \le s < t \le \overline{T}} \chi \cdot \varepsilon(\mu_t^m, \mu_s^m)$,

where $\overline{T} = T - (m-1)$.

2.2.4 The BDS test statistic

Brock et al. (1988,1990), henceforth BDS (Brock, Dechert and Scheinkman), developed a test based on concepts that arise in the theory of chaotic processes. The BDS test (1988,1990) is a test of the null hypothesis of i.i.d. for a univariate time series against an unspecified alternative. That is, if $\{\mu_t\}$ is an i.i.d. process, then $C_{m,\varepsilon} - C_{1,\varepsilon}^m$, almost surely, for all $\varepsilon > 0$, $m = 1, 2, \ldots$. BDS (1988, 1990) present the following result

$V_{m,\varepsilon} = \sqrt{T} \cdot \frac{C_{m,\varepsilon} - (C_{1,\varepsilon})^m}{s_{m,\varepsilon}} \xrightarrow{d} N(0,1)$, $\forall \varepsilon > 0$, $m=2,3,\ldots$, where $s_{m,\varepsilon}$ is an estimator of the asymptotic standard deviation—$\sigma_{m,\varepsilon}$—of $\sqrt{T} \cdot (C_{m,\varepsilon} - (C_{1,\varepsilon})^m)$ under the null of i.i.d.

Brock et al. (1991) used Monte Carlo methods to evaluate the choice of *m* and ε on the asymptotic normality of $V_{m,\varepsilon}$. Their results suggest that asymptotic normality of $V_{m,\varepsilon}$ holds well for sample sizes of at least 1000 observations, and for value of ε between 0.5 and 2 standard deviations of the data. They warned against relying on asymptotic normality for values of *T/m* less than 200 observations.

The BDS test has been shown to be robust to the non-existence of fourth moments, which may characterize stock returns (Brock and de Lima, 1995 and Hsieh, 1991). Hsieh (1991) points out that the robustness of the BDS test to the non-existence of fourth moments is one of the advantages of the BDS test over other tests of non-linearity such as Tsay (1986) and Hinich and Patterson (1985). Moreover, the BDS test statistic has power against models that are non-linear in variance but not in mean, as well as models that are non-linear only in mean. That is, a BDS rejection does not necessarily mean that a time-series has a time-varying conditional mean; it could simply be evidence for a time-varying conditional variance (Hsieh, 1991).

3 Data Definitions and Data Adjustment Procedures

The study uses daily returns of individual Norwegian stocks spanning the period from October 1983 to February 1994. The assets examined are assets in the Norwegian equity market. The assets are sorted from frequently traded (no. 1) to thinly traded assets (no. 15). Trading volume is the amount traded of the asset in Norwegian Kroner (NOK); that is, the number of stocks traded multiplied by settlement price at time of trading. Moreover, individual shares are grouped into portfolios at period *t* based on trading volume at *t-1*. Portfolio 1 consists of the thinnest traded assets; portfolio 2 and 3 consist of the intermediate traded assets and portfolio 4 consists of the most frequently traded assets. The portfolio rebalance is done each month using information at *t-1*. Moreover, assets traded throughout a month, is assigned to one of the four portfolios on basis of their average daily trading volumes in NOK for the last 2 years in the market. The two-year average avoids a too frequent shift of portfolio-assets. Finally, we employ four market wide indices consisting of all the stocks in the Norwegian market with 1) equally weighted stocks (Oslo and NHH[xii]) and 2) market value weighted stocks (Oslo and NHH). The crash in October 1987 is included. We therefore assume that a crash is normal in the equity market.

We adjust for systematic location and scale effects (Gallant and Tauchen, 1992) in all time series. The log first difference of the price index is adjusted. Let ϖ denote the variable to be adjusted. Initially, the regression to the mean equation $\varpi = x \cdot \beta + u$ is fitted, where x consists of calendar variables that are most convenient for the time series and contains parameters for trends, week dummies, calendar day separation variable, month and sub-periods. To the residuals, $\hat{u}$, the variance equation model $\hat{u}^2 = x \cdot \gamma + \varepsilon$ is estimated. Next $\frac{\hat{u}^2}{\sqrt{e^{x \cdot \hat{\gamma}}}}$ is formed, leaving a series with mean zero and (approximately) unit variance given x.

Lastly, the series $\hat{\varpi} = a + b \cdot (\frac{\hat{u}}{\sqrt{e^{x\hat{\gamma}}}})$ is taken as the adjusted series, where a and b are chosen so that $\frac{1}{T}\cdot\sum_{i=1}^{T}\hat{\varpi}_i = \frac{1}{T}\cdot\sum_{i=1}^{T}\varpi_i$ and $\frac{1}{T-1}\cdot\sum_{i=1}^{T}(\hat{\varpi}_i - \overline{\varpi})^2 = \frac{1}{T-1}\cdot\sum_{i=1}^{T}(\hat{u}_i - \overline{u})^2$. The purpose of the final location and scale transformation is to aid interpretation. In particular, the unit of measurement of the adjusted series is the same as that of the original series. We do not report the result of these raw data series adjustments[xiii].

The two Norwegian value-weighted indices are depicted in Figure 1. Both the indices show an approximate yearly growth of 12%. The natural logarithm of total trading volume is depicted in Figure 2. Note especially its strong but erratic trend in trading volume for the Norwegian thinly traded market. On average, the yearly growth in the trading volume is approximately 32,9%. The characteristics of the individual assets, the equal weighted trading volume portfolios and the market indices are reported in Table 1. Following immediate observations can be extracted. The standard deviation of returns seems to increase as trading volume decrease. The daily maximum and minimum returns for individual assets and portfolios seem also to suggest that highest absolute numbers are found for the thinnest traded assets. The mean returns show no clear pattern over assets, portfolios and indices. However, the variation in mean return among the thinly traded assets is high. We therefore find both highly positive and highly negative mean returns among the thinnest traded assets. For the portfolio, we find highest mean return accompanied by highest standard deviation, for the thinnest traded asset portfolio. The portfolios and indices show a surprisingly equal maximum and minimum daily return. The exception is possibly the frequently traded asset portfolio. A possible explanation is that it was the most frequently traded assets that experienced the highest price drop at the crash in October 1987. As should be expected from portfolio theory, all four indices show the lowest standard deviations.

{Insert Figure 1 and 2 here }

{Insert Table 1 here}

The numbers for kurtosis and skew for the stock returns suggest a substantial deviation from the normal distribution. The deviation is strongest for asset 10 and 12. Interestingly, the two value-weighted market indices also report high kurtosis. Moreover, from Table 1 it seems as especially the kurtosis increases as the number of combined assets in the portfolio increases[xiv]. All portfolios and market indices report negative skew. Together, the kurtosis and skews suggest too much probability mass around the mean, too little around 1-2 standard deviation from the mean and some extreme values on especially the negative side of the distribution. The results for both frequently and thinly traded individual assets report approximately the same kurtosis and skew results. The kurtosis and skewness indication of non-normality is strongly supported by the Kolmogorov-Smirnov Z-test statistic (K-S Z) for normality for all assets and portfolios. The ARCH (Engle, 1982) and BDS (Brock and Deckert, 1988 and Scheinkman, 1990) for $m = 2$ *and* 3 and $\varepsilon = 1$ test statistics both report non-linear dependence in all adjusted raw return data series. The ARCH test suggests changing conditional volatility and the BDS test statistic report a clear pattern for the degree of non-linear dependence. For individual assets the BDS test statistic for both $m=2$ *and* 3, increases as trading volume decreases. Moreover, especially where we find non-trading periods, the BDS statistic reports highly significant values. In contrast, the volume-portfolios report increased non-linear dependence when trading volume increases. However, non-trading is absent in these portfolios. Overall the BDS test statistic reports a surprisingly stable and strongly significant non-linear dependence in assets, portfolios and indices. Finally, the RESET (Ramsey, 1969) test statistic rejects the null of linearity in only a few assets, none of the portfolios and in the equal weighted index from Norwegian School of Economics and Business Administration (NHH). The difference in results between on the one side the BDS and ARCH tests and on the other the RESET test may be caused by the relative strength of the test statistics. While the BDS and ARCH test statistic focus on non-linearity in both the conditional mean and variance, the RESET test statistic focus more on non-linearity in the conditional mean and parameter changes therein.

Interestingly, and in contrast to the ARCH and BDS test statistics, it seems as the RESET test reports higher values and therefore rejects linearity the more frequently an asset is traded. Together the test statistics seem to report high ARCH (non-linearity in the conditional variance) effects for thinly traded assets in contrast to low RESET (non-linearity in the conditional mean) effects. The BDS test statistic reports results in line with the ARCH test statistic, which suggests that non-linear dependence is mostly found in the conditional variance equation. However, the results seem to suggest a need for a balancing of conditional mean and variance non-linear dependence. Therefore, a closer look at the origin of non-linear dependence is clearly warranted from the Norwegian raw data series. Moreover, the inspection to follow must include other characteristics of the Norwegian thinly traded equity market, which is clearly indicated from Table 1. Especially, non-normality and changing volatility are therefore all ingredients of our empirical results.

4 Empirical Results

Since my interest is in non-linear dependence in an ARMA-GARCH specification, the study first looks at filtering stock returns using a suitable *ARMA (p,q)* (auto-regressive and moving average) process for the conditional mean with the lag truncation lengths chosen according to the Schwarz BIC Criterion (Schwarz, 1978). For the BIC choice of *p* and *q*, the seven most frequently traded assets (1-7), asset number 12, the four trading volume portfolios and the two value weighted market indices, BIC prefer an ARMA (0,1)[xv] specification. The thinnest traded individual assets (asset no. 8-15) BIC prefer an ARMA (0,2) specification. The two equal-weighted market indices BIC prefer an ARMA (1,0) specification. The ARCH, RESET and

BDS test statistics are all applied to the residuals from the ARMA processes with a constant conditional variance. To stay inside the boundaries for asymptotic normality of the BDS test, the statistic is computed for m in the range from 2 to 8[xvi], and $\varepsilon = 1$. The results are presented in Table 2 under the category-line υ (linear; model (1)) and ω (non-linear; model (2)) for all individual assets in panel A and for portfolios and indices in panel B. The ARCH and BDS test statistics clearly suggest that the null hypothesis of i.i.d. asset returns is rejected at 1% for all series examined at all dimensions *(m)* for both linear and non-linear specifications. In contrast the RESET test show almost no significant rejections of the null. Moreover, the BDS results for $m = 2$ and 3 and $\varepsilon = 1$ is almost identical to the numbers from Table 1. Therefore, the ARCH and BDS test statistics strongly reject linearity for the residuals of both linear and non-linear ARMA specifications, in the same manner and magnitude as it rejects linearity of the adjusted raw return series for m equal to 2 and 3 in Table 1. These findings are consistent with the results of de Lima (1995a) from US. One more important implication from our results is worth mentioning. The extremely significant result for the thinly traded assets could be caused by non-trading and therefore series of zeroes or missing observations. The ARCH and BDS test statistics seem to report strong non-linear dependence in series that incorporate periods of zeroes. This is not so for the RESET test. Therefore, the results so far seem to suggest that the non-linear dependence in return series are strong but that non-linear dependence in mean is small. Lee et al. (1993) raised the issue of whether the detection of non-linear dependence in financial time series could be due to either neglected non-linear structure in the mean or ARCH/GARCH effects (conditional heteroscedasticity). We have above found small mean non-linear dependence. Therefore, the next step is to test for conditional variance non-linear dependence. We proceed therefore to model (3) and (4) defined in section 2. Note that these specifications are unfamiliar territory to those of us who are accustomed to thinking analytically, intuitively, and linearly. The model we now first approach is a specification that employs a linear conditional mean equation and a non-linear conditional variance equation.

One way to test whether conditional heteroscedasticity is responsible for the rejection of i.i.d. hypothesis is to apply the BDS test statistic to the residuals from a ARMA - GARCH model (Brock et al 1991, and Abhyankar et al. 1995). The trouble is that we cannot depend on asymptotic normality of the BDS statistic. Hsieh (1991) overcomes this problem by using critical values of the BDS statistic for simulated EGARCH process[xvii]. However, a recent paper by de Lima (1995 b) shows that the asymptotic distribution of the BDS statistic remains valid if the test is applied to the natural logarithm of the squared standardized residuals from a GARCH model. This is because the BDS statistic is valid if it is applied to a data generating process that is additive in the error term (de Lima, 1995b). The GARCH process models the error term in a multiplicative form, $\mu_t = \sigma_t z_t$, where μ_t is a random variable following the GARCH process, z_t is i.i.d. random variable, and σ_t is the conditional standard deviation.

{Insert Table 2 Panel A about here}

{Insert Table 2 Panel B about here}

The standardized residuals from this model are $z_t = \mu_t / \sigma_t$ in the normal case and

$z_t = \mu_t / \sqrt{\sigma_t^2 \cdot \left(\frac{\eta - 2}{\eta}\right)}$ in the student-t[xviii] density case, where η is the degree of freedom parameter. It follows that $ln(z^2_t) = ln(\mu^2_t) - ln(\sigma^2_t)$ in the normal case and $ln(z^2_t) = ln(\mu^2_t) - ln(\sigma^2_t(\eta\text{-}2/\eta))$ in the student-t density case. Therefore, the asymptotic distribution of the BDS statistic remains valid if it is applied to $ln(z^2_t)$ (adjusted residuals) in both the normal and student-t density case.

Therefore, we examine whether conditional heteroscedasticity is responsible for rejection of the i.i.d. hypothesis by applying the ARCH, BDS and RESET test statistics to the residuals from an BIC efficient ARMA *(p,q)* - GARCH *(m,n)* model. Moreover, as discussed above, the BDS test statistic is also applied for the adjusted standardized residuals from the same specifications. The GARCH *(m,n)* model for the conditional variance equation, the *m* and *n* lags are chosen based on the BIC criterion (Schwarz, 1978) of the squared residuals from an ARMA *(p,q)* process in the conditional mean equation. Moreover, in most cases a GARCH (1,1) is an appropriate and parsimonious representation of conditional variance equations (Bollerlev, 1986; Akigary, 1989; and Bollerslev et al., 1992). By use of the BIC criterion the highest lag representation is $m = 1$ and $n = 2$; that is, a GARCH (1,2) representation. All portfolios and indices BIC prefer a GARCH (1,1) specification. The individual assets number 2, 6 and 12 to 15 BIC prefer a GARCH (1,1) specification, while the assets number 1, 3 to 5 and 7 to 11 BIC prefers a GARCH (1,2) specification for the conditional variance equation[xix]. The results are reported in Table 2 for normal residuals and in Table 3 for student-t density log-likelihood function residuals for individual assets, portfolios, and indices under the category-lines ξ *and* $ln(\xi^2)$ for model (3).

In the normal case reported in Table 2, the results of applying the ARCH test statistics to the standardized residuals in line ξ, suggest that the null of constant conditional variance is rejected at 1% for all individual assets, portfolios and indices. Therefore, filtering series through an ARMA-GARCH specification report non-significant ARCH effects for the conditional variance equation. In contrast, for almost all the individual assets the RESET test fails to reject linearity against an unspecified alternative. The RESET statistic rejects linearity for only the most frequently traded portfolio. The result suggests a possible parameter shift in the conditional mean equation for the most frequently traded portfolio. Finally, the BDS test statistic for the standardized residuals fail to reject linearity for only assets that show continuous trading frequency (asset 1 to 3). For all portfolios and indices the BDS test statistic fail to reject linearity for the standardized residuals. For assets 4 to 15, all assets reject the null hypothesis of linearity at some dimension (*m*). As argued for above, applying the BDS test statistic to the adjusted standardized residuals reported in line ln(ξ^2), suggests that the null of i.i.d. is rejected for (i) assets no. 8 to 15 and (ii) the equal- and value-weighted indices from Oslo Stock Exchange at some dimension. The indices report significant test statistics only at higher dimensions, while the individual assets from 8 to 15 all report consistent non-linear dependence at all dimensions. All the assets no. 1 to 7, the four volume portfolios and the two NHH indices all fail to reject linearity of the residuals. This result suggests that non-linear dependence is fully accounted for by the conditional heteroscedasticity if the trading frequency is higher than 75% of total trading time.

The student-t density case is reported in Table 3. Applying the ARCH test statistics to the estimated degrees of freedom adjusted standardized residuals; fail to reject the null of constant conditional variance for all assets, except the two thinnest traded assets (asset no. 14 and 15). However, only the two intermediately traded portfolios and the equal-weighted Oslo Stock Exchange market index fail to reject constant conditional variance at 1%. The RESET statistic fails to reject linearity for all assets, portfolios and indices except assets no. 10, 14 and 15 and the most frequently traded portfolio.

Finally, the BDS statistic for dimension *m=2 to 8* is reported in Table 3. For almost all individual assets and trading volume portfolio, Table 3 reports non-linear dependence in the standardized residuals. Only the most frequently traded asset and the two value-weighted indices fail to reject linearity. Therefore, using the standardized residuals from a student-t log-likelihood function does reduce but does not remove the non-linear dependence in asset returns. However as above applying the test to the adjusted residuals (ln(ξ^2)) from the student-t density estimation, the results suggest that the null of i.i.d. is rejected for (1) assets no. 9 to15 at all dimensions and (2) the equal weighted index from Oslo Stock Exchange at dimension 3 to 8. All other series fail to reject i.i.d for the residuals.

Our first point in summary is that conditional heteroscedasticity is the major cause of non-linear dependence in our time-series. For all the reasonably frequent traded assets non-linear dependence seems to be removed in the residuals. Among portfolios and indices, only one of the equal-weighted indices, where non-trading contributes strongly from component stocks, fails to reject linearity.

Secondly, the time-series that rejects the null of linearity in all cases are the thinly traded individual assets from no. 9 to 15. Therefore, non-trading seems to be a major cause of high non-linear dependence in especially the ARCH and BDS test statistics. Long periods of non-trading and therefore long series of zero return are a major source for rejection of the null of i.i.d. in stock return series. Moreover it seems as a trading proportion below 75% report significant non-linear dependence. Finally, we introduce non-linear dependence in the mean to see if the remaining non-linearity can be removed from the data series. Note that this model departs from the Martingale hypothesis and is described in detail in Section 2, model (4). The results are reported in the lines ϖ and $ln(\varpi^2)$ in Table 2 and 3. For both Table 2 and 3 and therefore for both normal and student-t density estimations the non-linearity in mean seems to introduce increased non-linear dependence in the data-series that has been earlier removed using the non-linear conditional variance specification. Especially the thinnest traded assets seem to report symptoms of increased non-linear dependence. Moreover, the equal-weighted index includes these assets with an equal weight to more frequently traded assets.

{Insert Table 3 Panel A about here}

{Insert Table 3 Panel B about here}

This fact may explain the non-linear dependence in the Oslo index. Introduction of non-linear mean in a normal GARCH (*m,n*) estimation seems therefore not to be recommended for Norwegian data. Moreover and more importantly the result suggests that we do not have to explain dynamics in asset returns that depart from the Martingale hypothesis.

The results suggest that series from thinly traded assets with several and/or long periods of non-trading, must not be included in especially the BDS tests of i.i.d. Further, the results suggest that zero return series for the BDS test statistic immediately reject the i.i.d. proposition. Moreover, they suggest that the BDS test statistic can lead to different conclusion depending on whether it is applied to the standardized residuals or the adjusted standardized residuals.

If we ignore the results from the thinly traded asset series (no. 9 to 15) none of the BDS test statistics is significant for the student-t density estimation. From these results it seems that the conditional heteroscedasticity in fact, count for almost all non-linear dependence in the Norwegian equity market. Even more encouraging, introduction of non-linearity in the conditional mean equation does not materially change our test statistics and we do not have to depart from the Martingale hypothesis. The main finding of this research suggests that reasonable frequently traded assets (greater than 75%) in the Norwegian equity market produce linear conditional mean equations but conditional heteroscedasticity must be accounted for using simple GARCH(*m,n*) specification to secure conditional homoscedasticity.

5 Summaries and Conclusions

We have found strong evidence to reject the null hypothesis of i.i.d. for Norwegian asset raw returns. This finding supports the results of de Lima (1995a) that non-linear dependence cannot be ruled out as an explanation to the dynamics of the stock returns after the 1987 crash for US data. Moreover, the three test statistics suggest a need to control conditional heteroscedasticity (BDS and ARCH) rather than changing parameter values (RESET). The results suggest that the rejection of i.i.d. appears to be almost exclusively caused by conditional heteroscedasticity, when frequent non-trading assets are excluded from the sample. These thinly traded assets introduce non-linear dependence through non-trading. Non-trading and conditional heteroscedasticity are therefore the carriers of non-linear dependence. Moreover, my results highlight the importance of using the simple adjustment of de Lima (1995b) to the GARCH residuals before applying the BDS test since de Lima show that the asymptotic normality is valid only in this case. Finally, and an important findings of our analysis, is that if we are interested in modelling non-linearity in stock return series, our attention should be on conditional heteroscedasticity rather than conditional mean dependence. In fact, almost all significant non-linear dependence are ruled out using a simple BIC efficient GARCH (*m,n*) model for the conditional variance equation for frequently traded assets. Therefore, we don't have to depart from the Martingale hypothesis in the Norwegian thinly traded equity market.

Our findings also suggest a strong relationship between non-trading and non-linear dependence. This research has not solved the non-trading issue, which must be left to future research. However, one way to proceed is by applying temporal aggregation (Drost & Nijman, 1993) in ARMA-GARCH specifications. In the mean time, the non-trading phenomena in thinly traded markets, make economic implications very difficult to interpret and make analytical, intuitive and linear thinking almost impossible.

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[i] For an overview of Non-linear Dependence in Financial Data see Campbell, Lo and MacKinlay, 1997.

[ii] Conditional dependence will through conditional mean and volatility models of raw returns normally generate more normal residuals.

[iii] See Solibakke (2000a) for a definition of a thinly traded market.

[iv] The asymmetric GARCH specification (Glosten et. Al., 1993) is Lagrange Ratio preferred using both normal and student-t density maximum log- likelihood functions in the Norwegian thinly traded Market.

[v] a_0 is the estimated long run average volatility (constant). An alternative is to specify $h_t = 1$. However, the upcoming non-linear results in section 4 show very small changes, and do not affect the conclusions of our work.

[vi] See Glosten et al. (1993).

[vii] Employing the non-linear ARMA residuals don't change the BIC preferred values for m and n.

[viii] Normal log likelihood function: $-0.5 \cdot \ln(2 \cdot \pi) + \varepsilon / h)^2 + 2 \cdot \ln(h)$; where ε is the residuals and h is the conditional variance.

[ix] Student-t log likelihood function: $C - 0.5 \cdot \ln(h) - ((\eta + 1)/2 \cdot \ln(1 + \varepsilon^2 /(\eta - 2) \cdot h))$; where C is a constant, ε is the residuals, h is the conditional variance and η is the degree of freedom parameter.

[x] See also Tsay (1986), Spanos (1986) and Lee et al. (1993).

[xi] If $\{\mu_t\}$ is a strictly stationary, absolutely stochastic process, the integral defined below exists.

[xii] NHH is the abbreviation for Norwegian School of Economics and Business Administration.

[xiii] The results are readily available from the author upon request.

[xiv] Often named the mixture of distributions hypothesis, which maintains that asset returns exhibit leptokurtosis because they are really a combination of returns distributions.

[xv] ARMA (0,1) is found to model non-synchronous trading (Lo and MacKinlay, 1990).

[xvi] The maximum choice of m, 8, is chosen so that *T/m* is higher than 200 (Brock et al., 1991)

[xvii] The simulation is based on 2000 replications, each with 1000 observations

[xviii] We have chosen a Student-t distribution as it has been found to suit Norwegian equity data well (Solibakke, 2000c).

[xix] The result implies 4 different ARMA-GARCH models to estimate for assets and portfolios. The BHHH algorithm (Berndt et al., 1974) is employed for estimation.

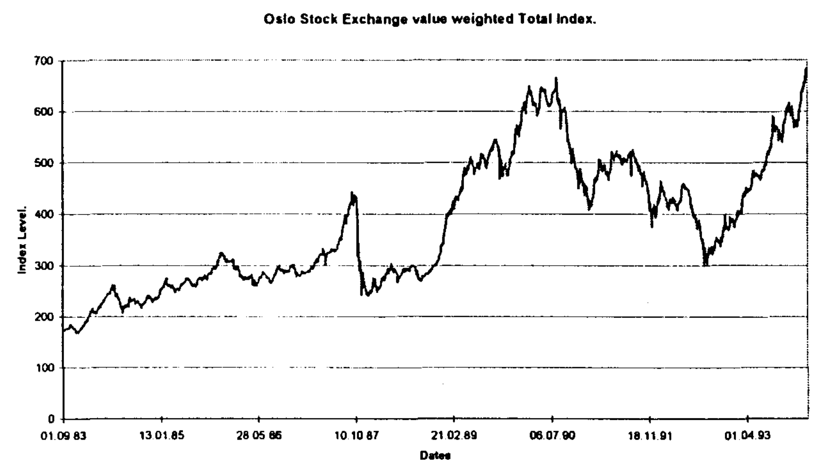

Figure 1. Oslo Stock Exchange Index (Totx).

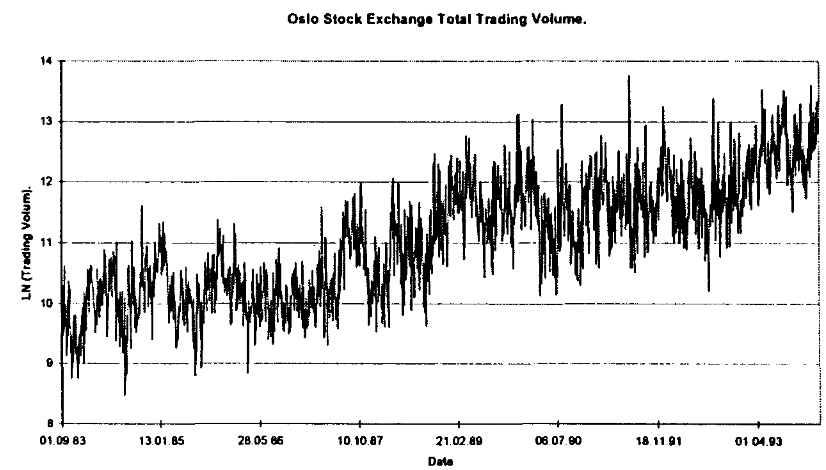

Figure 2. The Natural logarithm of Trading Volume (NOK).

Table 1. Series characteristics for the Norwegian Thinly Traded Equity Market.

Individual Series	Obs / Prop.	Yearly Mean	Yearly std.dev.	Max. Min.	Kurtosis Skew	K-S Z-stat	ARCH (6)	RESET (12;6)	BDS m=2	(ε = 1) m=3	m=4
1. VP-1	2611	15.196	32.417	13.04	8.4051	3.062	288.41	43.4515	1.920	2.837	3.251
	1.00			-20.27	-0.3109	{0.004}	{0.000}	{0.000}	{0.063}	{0.007}	{0.002}
2. VP-2	2611	25.140	33.153	13.44	9.1629	3.148	70.922	42.199	2.531	2.000	1.598
	1.00			-23.43	-0.3739	{0.003}	{0.000}	{0.000}	{0.016}	{0.054}	{0.111}
3. VP-3	2611	8.598	46.407	26.76	30.219	3.456	254.31	37.37	3.479	3.337	2.900
	0.97			-46.98	-1.3141	{0.001}	{0.000}	{0.000}	{0.001}	{0.002}	{0.006}
4. VP-4	2369	-10.479	116.66	32.26	3.9290	3.483	246.17	35.22	7.048	5.563	4.477
	0.90			-45.84	-0.1425	{0.001}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}
5. VP-5	2577	12.645	57.56	24.41	11.8919	5.236	276.42	73.33	9.391	10.152	10.350
	0.83			-33.69	-0.6232	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}
6.VP-6	2515	42.318	130.70	81.20	95.667	8.399	203.57	79.69	15.533	18.822	21.057
	0.57			-167.9	-3.9843	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}
7.VP-7	2499	34.618	108.42	53.59	17.440	8.915	291.73	38.45	20.608	23.900	30.789
	0.43			-56.57	-0.5615	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}
8.VP-TT	2611	21.456	32.553	10.80	5.7203	2.574	89.350	25.727	7.472	8.143	8.819
	0.91			-15.91	-0.1158	{0.015}	{0.000}	{0.012}	{0.000}	{0.000}	{0.000}
9.VP-FT	2611	5.502	25.127	13.32	26.146	3.809	390.988	74.951	16.194	19.102	21.166
	1.00			-23.06	-1.3147	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}
10.VP-EW	2611	7.676	17.610	11.42	29.844	4.580	630.085	76.919	13.294	14.757	15.850
	1.00			-16.66	-1.5458	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}
11.VP-VW	2611	13.278	20.581	10.48	36.143	3.800	288.878	68.810	12.653	14.908	15.746
	1.00			-21.22	-2.0040	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}

VP1, VP-2,...,VP-7 are individual assets sorted in ascending order of trading volume. VP-TT is the portfolio containing the most thinly traded and VP-FT contains the most frequently traded assets. VP-EW and VP-VW are an equal-weighted and a value-weighted index for the Norwegian market.

Yearly mean is daily mean multiplied by 252 trading days and yearly standard deviation is daily standard deviation multiplied by the square root of 252 trading days. Skew is a measure of heavy tails and asymmetry of a distribution (normal) and kurtosis is measure of too many observations around the mean for a distribution (normal). K-S Z-test: Used to test the hypothesis that a sample comes from a normal distribution. The value of the Kolmogorov-Smirnov Z-test is based on the largest absolute difference between the observed and the theoretical cumulative distributions. ARCH (6) : ARCH (6) is a test for conditional heteroscedasticity in returns. Low {.} indicates significant values. We employ the OLS-regression $y^2 = a_0 + a_1 . y^2_{t-1} + \ldots + a_6 . y^2_{t-6}$. $T \cdot R^2$ is χ^2 distributed with 6 degrees of freedom. T is the number of observations, *y* is returns and R^2 is the explained over total variation. $a_0, a_1 \ldots a_6$ are parameters.

RESET (12,6) : A sensitivity test for mainly linearity in the mean equation. 12 is number of lags and 6 is the number of moments that is chosen in our implementation of the test statistic. $T \cdot R^2$ is χ^2 distributed with 12 degrees of freedom.

BDS (m=2,ε=1): A test statistic for general non-linearity in a time series. The test statistic $BDS = T^{1/2} . [C_m(\sigma \cdot \varepsilon) - C_1(\sigma \cdot \varepsilon)^m]$, where *C* is based on the correlation-integral, m is the dimension and ε is the number of standard deviations. Under the null hypothesis of identically and independently distributed (i.i.d.) series, the BDS-test statistic is asymptotic normally distributed with a zero mean and with a known but complicated variance.

Table 2. Test statistic for ARMA and GARCH specifications with normal residuals.

Series:	Residual:	Test statistics: ARCH	$\chi^2(6)$	RESET	$\chi^2(12)$	BDS test statistic: (ε = 1.0) m = 2	m = 3	m = 4	m = 5	m = 6	m = 7	m = 8
1. VP-1	υ	369.21	{0.000}	42.971	{0.000}	8.5510	10.0162	11.2498	11.6067	12.6495	13.8086	14.9050
	ω	202.67	{0.000}	40.991	{0.000}	9.6084	10.8099	11.5134	11.7112	12.2171	12.7911	13.4521
	$Ln(\xi^2)$	6.7902	{0.341}	11.325	{0.501}	-0.8803	-0.3265	0.1130	0.2632	0.4571	1.0048	1.9318
	$Ln(\varpi^2)$	6.5125	{0.368}	11.506	{0.486}	-0.6208	-0.2309	0.0852	0.3161	0.6184	1.3509	2.2648
2. VP-2	υ	99.744	{0.000}	40.444	{0.000}	7.6281	8.9439	9.4337	9.4346	10.1117	10.8621	11.8171
	ω	35.168	{0.000}	35.012	{0.000}	7.7776	9.0362	9.2817	9.5616	10.0224	10.4880	10.8626
	$Ln(\xi^2)$	2.7913	{0.835}	14.770	{0.254}	-0.0463	-0.2422	-0.1760	0.1640	0.7018	0.3484	0.3665
	$Ln(\varpi^2)$	3.1260	{0.793}	14.159	{0.291}	0.2564	0.2813	0.2522	0.3815	0.7625	0.3229	0.6639
3. VP-3	υ	257.41	{0.000}	37.412	{0.000}	11.9842	13.2841	14.3353	16.9892	20.4752	25.2495	27.6834
	ω	8.0755	{0.233}	20.372	{0.060}	11.5542	12.5671	12.5220	13.0422	13.7803	14.3479	14.7436
	$Ln(\xi^2)$	2.3429	{0.886}	15.586	{0.211}	0.5611	0.2160	0.2681	0.7090	0.4295	0.1837	-1.1281
	$Ln(\varpi^2)$	2.3445	{0.885}	15.105	{0.236}	0.5535	0.1894	0.2004	0.5679	0.6592	0.7832	0.4842
4. VP-4	υ	175.39	{0.000}	32.861	{0.001}	13.6738	17.4681	22.1961	25.2432	34.4052	18.1240	-5.3455
	ω	154.54	{0.000}	30.727	{0.002}	15.1138	17.3960	18.8071	20.3668	22.4793	25.0904	28.0774
	$Ln(\xi^2)$	7.8973	{0.246}	12.235	{0.427}	1.8822	1.4112	0.8538	0.5931	0.4380	-0.8034	-1.2040
	$Ln(\varpi^2)$	9.2157	{0.162}	10.911	{0.537}	1.9673	1.5942	1.1864	1.2834	1.1320	0.9604	0.6299
5. VP-5	υ	207.98	{0.000}	67.390	{0.000}	11.6209	14.7820	17.8220	20.3445	22.2221	25.0255	31.9776
	ω	209.28	{0.000}	67.412	{0.000}	13.0195	15.2500	16.9169	18.3810	19.8070	21.4047	23.3438
	$Ln(\xi^2)$	2.4115	{0.878}	10.001	{0.616}	4.5311	5.2427	4.8054	5.0089	4.7085	3.7040	4.7137
	$Ln(\varpi^2)$	2.3139	{0.889}	10.204	{0.598}	4.7305	5.4736	5.0423	5.3908	4.5261	2.7400	3.9282
6.VP-6	υ	95.119	{0.000}	86.328	{0.000}	21.0558	27.9705	38.2024	59.7633	103.030	217.044	476.155
	ω	93.586	{0.000}	85.963	{0.000}	16.6573	18.0466	19.3285	20.0234	20.7093	21.4976	22.5865
	$Ln(\xi^2)$	4.4821	{0.612}	15.816	{0.200}	9.5848	8.3431	8.3270	8.2894	8.4489	6.9131	6.2213
	$Ln(\varpi^2)$	5.5198	{0.479}	16.547	{0.167}	9.3262	8.3420	8.1306	8.2592	8.9126	7.3368	8.4389
7.VP-7	υ	176.33	{0.000}	53.377	{0.000}	36.0967	55.2813	87.9003	148.915	277.593	557.858	1194.73
	ω	176.75	{0.000}	51.736	{0.000}	15.6134	15.9042	16.0970	16.9001	17.7405	18.6658	19.7833
	$Ln(\xi^2)$	7.7186	{0.259}	10.086	{0.608}	29.3216	27.2604	28.4112	31.2741	47.2066	79.3681	166.350
	$Ln(\varpi^2)$	8.2269	{0.222}	9.8907	{0.626}	29.2591	27.6074	28.4628	32.6828	48.3300	81.0214	168.112
8.VP-TT	υ	57.7683	{0.000}	25.231	{0.014}	6.1891	6.8691	7.2851	7.9796	8.3611	9.4105	10.3236
	ω	58.057	{0.000}	24.544	{0.017}	6.3154	7.3980	8.1286	8.5379	8.6721	9.0331	9.3971
	$Ln(\xi^2)$	13.230	{0.040}	11.598	{0.478}	-2.0211	-2.3023	-2.2785	-2.0012	-1.1215	-0.7245	0.3702
	$Ln(\varpi^2)$	13.606	{0.034}	10.408	{0.580}	-2.3309	-2.6923	-2.4845	-2.1558	-1.3843	-1.3634	-1.1203
9.VP-FT	υ	696.829	{0.000}	77.613	{0.000}	14.1415	16.8083	18.7161	20.6489	23.2936	26.1429	29.4136
	ω	107.228	{0.000}	61.943	{0.000}	14.8847	17.0585	18.4922	19.7159	21.1360	22.7302	24.5316
	$Ln(\xi^2)$	9.36171	{0.154}	8.9307	{0.709}	0.3242	0.3054	0.3079	0.3670	0.5047	0.4547	0.4390
	$Ln(\varpi^2)$	10.657	{0.100}	7.14585	{0.848}	-1.4945	-1.2455	-1.8494	-1.7013	-1.7168	-2.0197	-1.7640
10.VP-EW	υ	882.376	{0.000}	77.718	{0.000}	11.4712	13.1767	14.3758	15.5042	16.7248	17.9349	19.1912
	ω	91.195	{0.000}	48.696	{0.000}	11.8183	13.2399	14.3738	15.3158	16.4717	17.6254	18.7489
	$Ln(\xi^2)$	6.2967	{0.391}	9.6714	{0.645}	1.0641	1.1486	1.7411	1.9639	2.1802	2.2108	3.0017
	$Ln(\varpi^2)$	5.93344	{0.431}	9.8308	{0.631}	1.7199	1.7958	1.9718	1.9218	1.9135	2.3255	3.2857
11.VP-VW	υ	589.024	{0.000}	71.628	{0.000}	10.1122	12.4968	13.3401	13.9719	15.3633	16.6591	18.1129
	ω	64.0411	{0.000}	51.184	{0.000}	10.5922	13.1446	14.0522	14.7188	15.9064	16.9993	18.3095
	$Ln(\xi^2)$	8.73444	{0.189}	10.459	{0.576}	0.2326	0.1819	0.1300	0.0982	0.0710	0.1606	0.3480
	$Ln(\varpi^2)$	14.8494	{0.021}	11.498	{0.487}	-0.9033	-1.6247	-2.3502	-2.3511	-2.2995	-2.4163	-2.1255

Table 3. Test statistics for ARMA and GARCH specifications. Student-t density residuals.
(estimated degrees of freedom in parantheses in first column for GARCH models)

		Test statistics:				BDS test statistic: (ε = 1.0)						
Series:	Residual:	ARCH	$\chi^2(6)$	RESET	$\chi^2(12)$	m = 2	m = 3	m = 4	m = 5	m = 6	m = 7	m = 8
1. VP-1	υ	369.21	{0.000}	42.971	{0.000}	8.5510	10.0162	11.2498	11.6067	12.6495	13.8086	14.9050
	ω	202.67	{0.000}	40.991	{0.000}	9.6084	10.8099	11.5134	11.7112	12.2171	12.7911	13.4521
(η = 5.051181)	$Ln(\xi^2)$	7.8557	{0.249}	11.653	{0.474}	-0.8482	-0.2557	0.0178	0.4446	0.3850	0.9010	1.9451
(η = 5.036366)	$Ln(\varpi^2)$	7.0335	{0.318}	12.180	{0.431}	-0.4919	-0.1447	0.1211	0.5505	0.4599	0.8103	1.6872
2. VP-2	υ	99.744	{0.000}	40.444	{0.000}	7.6281	8.9439	9.4337	9.4346	10.1117	10.8621	11.8171
	ω	35.168	{0.000}	35.012	{0.000}	7.7776	9.0362	9.2817	9.5616	10.0224	10.4880	10.8626
(η = 4.533413)	$Ln(\xi^2)$	3.3378	{0.765}	13.246	{0.351}	0.3143	-0.1800	-0.0981	0.2406	0.4830	0.0122	0.2088
(η = 4.440638)	$Ln(\varpi^2)$	2.8655	{0.826}	12.374	{0.416}	0.5983	0.1708	0.2327	0.2951	0.6470	0.6555	-0.4521
3. VP-3	υ	257.41	{0.000}	37.412	{0.000}	11.9842	13.2841	14.3353	16.9892	20.4752	25.2495	27.6834
	ω	8.0755	{0.233}	20.372	{0.060}	11.5542	12.5671	12.5220	13.0422	13.7803	14.3479	14.7436
(η = 4.382588)	$Ln(\xi^2)$	3.5333	{0.740}	14.924	{0.246}	0.6305	0.2356	0.1635	0.5328	0.4559	0.0871	-0.3496
(η = 4.362314)	$Ln(\varpi^2)$	3.6470	{0.724}	14.362	{0.278}	0.8214	0.5941	0.5372	0.7771	0.8743	0.3831	0.0398
4. VP-4	υ	175.39	{0.000}	32.861	{0.001}	13.6738	17.4681	22.1961	25.2432	34.4052	18.1240	-5.3455
	ω	154.54	{0.000}	30.727	{0.002}	15.1138	17.3960	18.8071	20.3668	22.4793	25.0904	28.0774
(η = 4.20412)	$Ln(\xi^2)$	7.0298	{0.318}	11.282	{0.505}	1.4770	1.1442	0.7963	0.7493	1.0373	0.5344	1.0387
(η = 4.158861)	$Ln(\varpi^2)$	7.3253	{0.292}	10.043	{0.612}	1.4922	1.2922	1.0109	1.0110	1.2431	1.8013	3.4529
5. VP-5	υ	207.98	{0.000}	67.390	{0.000}	11.6209	14.7820	17.8220	20.3445	22.2221	25.0255	31.9776
	ω	209.28	{0.000}	67.412	{0.000}	13.0195	15.2500	16.9169	18.3810	19.8070	21.4047	23.3438
(η = 3.29325)	$Ln(\xi^2)$	2.2157	{0.899}	12.346	{0.418}	3.5223	4.1219	4.0231	3.4512	2.5847	1.4104	0.6104
(η = 3.295526)	$Ln(\varpi^2)$	2.0568	{0.914}	12.365	{0.417}	3.4622	4.1440	4.0970	3.6637	2.9759	2.6206	2.2552
6.VP-6	υ	95.119	{0.000}	86.328	{0.000}	21.0558	27.9705	38.2024	59.7633	103.030	217.044	476.155
	ω	93.586	{0.000}	85.963	{0.000}	16.6573	18.0466	19.3285	20.0234	20.7093	21.4976	22.5865
(η = 2.622771)	$Ln(\xi^2)$	0.2495	{1.000}	6.7053	{0.876}	5.0168	5.5153	5.3258	7.0001	8.4744	8.5042	16.9384
(η = 2.629503)	$Ln(\varpi^2)$	0.2509	{1.000}	6.5082	{0.888}	4.6847	5.3236	5.0033	6.3201	7.8735	8.4026	16.2952
7.VP-7	υ	176.33	{0.000}	53.377	{0.000}	36.0967	55.2813	87.9003	148.915	277.593	557.858	1194.73
	ω	176.75	{0.000}	51.736	{0.000}	15.6134	15.9042	16.0970	16.9001	17.7405	18.6658	19.7833
(η = 2.364853)	$Ln(\xi^2)$	14.657	{0.023}	14.943	{0.245}	9.4015	11.5595	17.2619	29.0393	55.2045	110.367	251.309
(η = 2.362968)	$Ln(\varpi^2)$	13.712	{0.033}	14.515	{0.269}	11.0482	13.6064	19.2758	32.1468	61.5073	126.285	290.464
8.VP-TT	υ	61.229	{0.000}	25.000	{0.015}	6.1891	6.8691	7.2851	7.9796	8.3611	9.4105	10.3236
	ω	61.159	{0.000}	24.649	{0.017}	6.3154	7.3980	8.1286	8.5379	8.6721	9.0331	9.3971
(η = 6.21166)	$Ln(\xi^2)$	15.113	{0.040}	11.858	{0.957}	-1.8410	-2.2261	-2.0752	-2.0572	-1.6335	0.0708	0.4348
(η = 6.256039)	$Ln(\varpi^2)$	15.367	{0.018}	10.841	{0.543}	-2.1215	-2.4885	-2.2916	-1.9996	-1.9722	-0.3804	0.0747
9.VP-FT	υ	888.06	{0.000}	79.968	{0.000}	14.1415	16.8083	18.7161	20.6489	23.2936	26.1429	29.4136
	ω	69.865	{0.000}	67.177	{0.000}	14.8847	17.0585	18.4922	19.7159	21.1360	22.7302	24.5316
(η = 6.220762)	$Ln(\xi^2)$	11.875	{0.065}	9.1982	{0.686}	-1.6278	-1.4376	-1.8309	-1.9083	-1.6891	-1.7654	-1.8384
(η = 6.268164)	$Ln(\varpi^2)$	18.120	{0.006}	9.0229	{0.701}	-1.2198	-1.0094	-1.7295	-1.7802	-1.8112	-1.9254	-1.6756
10.VP-EW	υ	1090.7	{0.000}	81.075	{0.000}	11.4712	13.1767	14.3758	15.5042	16.7248	17.9349	19.1912
	ω	838.42	{0.000}	97.718	{0.000}	11.8183	13.2399	14.3738	15.3158	16.4717	17.6254	18.7489
(η = 5.020918)	$Ln(\xi^2)$	12.151	{0.059}	8.0514	{0.781}	1.7452	2.0208	2.5350	2.5926	2.6177	3.1507	3.5963
(η = 5.110305)	$Ln(\varpi^2)$	11.517	{0.074}	10.847	{0.542}	1.7738	2.0061	2.2870	2.5049	2.7409	2.8546	3.6363
11.VP-VW	υ	751.92	{0.000}	74.180	{0.000}	10.1122	12.4968	13.3401	13.9719	15.3633	16.6591	18.1129
	ω	50.531	{0.000}	51.687	{0.000}	10.5922	13.1446	14.0522	14.7188	15.9064	16.9993	18.3095
(η = 6.512626)	$Ln(\xi^2)$	14.127	{0.028}	10.459	{0.576}	-0.5066	-1.4408	-1.8923	-1.8700	-1.9201	-1.7037	-1.6792
(η = 6.542097)	$Ln(\varpi^2)$	30.107	{0.000}	12.038	{0.443}	-0.4234	-1.4330	-1.8904	-1.4782	-1.2253	-1.0075	-0.6089

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Testing the univariate conditional CAPM in thinly traded markets

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Traditional tests of asset pricing undertaken within the CAPM framework have to control for nonsynchronous trading and non-trading as well as volatility clustering in especially thinly traded financial markets. This investigation therefore set out to control for nonsynchronous trading and non-trading effects and volatility clustering in the Norwegian equity market. The problem is approached by applying a linear ARMA–GARCH-in-mean lag specification. The ARMA lag specification controls for nonsynchronous trading and non-trading effects in the mean equation. The GARCH lag specification controls for conditional heteroscedasticity and volatility clustering in the latent conditional volatility equation. All lags are Schwarz efficient. The results suggest that the conditional CAPM cannot be rejected but the in-mean parameter in ARMA–GARCH-in-mean specifications show very low statistical significance except for daily data. The result therefore suggests a compensation for risk only for short time-horizons and the in-mean parameter in ARMA–GARCH-in-mean lag specifications is a poor proxy for risk in the conditional CAPM sense. Conditional heteroscedasticity and volatility clustering need to be controlled for in daily and weekly time intervals while nonsynchronous trading needs to be controlled for in daily, weekly and monthly time intervals.

I. INTRODUCTION AND LITERATURE REVIEW

From a theoretical perspective, the Capital Asset Pricing Model (CAPM) of Sharpe (1963), Lintner (1965), Mossin (1966) and Black *et al.* (1972) is a one-period equilibrium model and as such, it is not designed to account for temporal dependence and nonsynchronous trading. From a practical perspective, it is well known that the distributions of asset returns exhibit volatility clustering, which manifest itself as temporal dependence in variances, and nonsynchronous trading and non-trading effects, which manifest itself as autocorrelation in mean. Turtle (1994) has shown that serial correlation will be induced into model disturbances, when conditional variances are time varying. Campbell *et al.* (1997) has shown that spurious serial correlation will be induced into model disturbances. Consequently, tests of an unconditional CAPM in the thinly traded Norwegian market may be wrongly specified.

Researchers have attempted to test the CAPM in a conditional framework utilizing the generalized autoregressive conditional heteroscedastcity in-mean (GARCH-in-mean) model (Engle, 1982; Engle and Bollerslev, 1986) in which asset returns are modelled as a function of their conditional variance. Examples include Baillie and DeGennaro (1990), French *et al.* (1987) and Harris (1989). However, while models that explicitly allow for ARCH effects have been reasonably successful in modelling financial time series, the GARCH-in-mean model has typically been used as a pure statistical description of returns. Turtle (1994) has provided a theoretical asset pricing application of the GARCH-in-mean model. Turtle's results from the US market suggest that while the GARCH-in-mean model cannot be rejected, the conditional CAPM can be rejected. Moreover, the Brailsford and Faff (1997) results from the Australian market cannot reject the GARCH-in-mean model in daily and weekly series but can be rejected for monthly series while the conditional CAPM cannot be rejected for

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weekly and monthly series but can be rejected for daily series.

Hence, the purpose of this paper is to apply and extend Turtle's (1994) US market results and Brailford and Faff (1997) Australian analyses to the Norwegian thinly traded market. It is of international interest to see how robust these US and Australian equity market results are in the context of thinly traded markets. Thinly traded markets exhibit nonsynchronous trading and non-trading effects, which may create negative serial correlation in asset returns (Solibakke, 2000a, 2000b). To control for the serial correlation an ARMA lag specification is applied for the conditional mean. Moreover, as the GARCH specifications apply lagged residuals in the volatility specification, nonsynchronous trading and non-trading may create spurious conditional volatility effects (Solibakke, 2000a, 2000b). Equally important a second objective for this paper is the effect of sampling interval on the empirical performance of the conditional CAPM and the ARMA–GARCH-in-mean specification. The objective is especially important for thinly traded markets as market indices may contain assets not traded for days and possibly weeks. It is therefore possible to see how important it is to control for nonsynchronous trading over sampling intervals.

The Bayes Information Criterion (BIC) (Schwarz, 1978) is applied to measure the optimal lag structure both in the conditional mean and in the conditional variance equations. The optimal lag structure is able to be found for all series at all sampling intervals. Both value-weighted and equal-weighted market index series are employed to see whether the weighting of the market indices influences the findings. The equal-weighted indices will most likely show the highest influence from nonsynchronous trading and non-trading effects.

The study differs therefore from other similar studies in especially two ways. First, the use of several index series, equal- and value-weighted contains thinly traded market characteristics. Hence, nonsynchronous trading and non-trading effects need to be controlled for and the BIC criterion applied for serial correlation adjustment in the conditional mean equation. Secondly, the BIC criterion is applied to find the optimal GARCH specification for conditional heteroscedasticity in the conditional variance equation. Thirdly, specification tests are applied to find appropriate specifications for the Norwegian thinly traded market over daily, weekly and monthly sampling intervals.

The rest of the paper is organized as follows. Section II describes the CAPM model, the conditional, and the unconditional ARMA–GARCH-in-mean specifications. Section III describes the data and adjustments procedures. Section IV reports the empirical results. Section V reports Norwegian findings and finally Section VI summarizes and concludes.

II. THE CAPM AND ARMA–GARCH-IN-MEAN SPECIFICATIONS

The static CAPM

The Sharpe–Lintner–Mossin CAPM is a one-period model, which describes how assets are priced in equilibrium in terms of the relationship between relevant risk (beta) and expected return. Specifically, the CAPM states that a positive linear relationship should hold between risk and expected return, namely, $E(R_i) = R_F + \beta_i \cdot [E(R_M) - R_F]$, where $\beta_i = \sigma_{i,M}/\sigma_M^2$, where $\sigma_{i,m}$ denotes the covariance between asset i and the market and σ_M^2 denotes the market variance. Note also that $\beta_i = (\sigma_i \cdot \rho_{i,M}/\sigma_M)$, can be written, where $\rho_{i,m}$ denotes the correlation coefficient between asset i and the market and σ_i^2 denotes asset i variance. In their widely cited study, Fama and French (1992) empirically examine the CAPM given above and find that the estimated value of β_i is close to zero. They interpret the 'flat' relation between average return and beta as strong evidence against CAPM. The static CAPM is also tested in the Norwegian thinly traded market. However, Carlsen and Ruth (1991) fail to reject the null of μ_0 significant different from zero for both univariate and multivariate tests.[1] The result may therefore give evidence both for and against the static CAPM, but it is not necessarily evidence for and against the conditional CAPM. The CAPM was developed within the framework of a hypothetical single-period model economy. The real world, however, is dynamic and hence, expected returns and betas are likely to vary over time. Even when expected returns are linear in betas for every time period based on the information available at that time, the relation between the unconditional expected returns and the unconditional beta could be close to zero.[2] In the next section it is assumed that CAPM holds in a conditional sense, i.e., it holds at every point in time, based on whatever information is available at that instant.

The conditional CAPM

Assuming hedging motives are not sufficiently important following Merton (1980) and hence the CAPM will hold in conditional sense and if it is assumed that expectations in CAPM at time t are conditioned on the information set available to agents at time $t-1$, Ω_{t-1}, then the conditional CAPM[3] can be written as $E_t(R_{i,t}|\Omega_{t-1}) = R_{F,t-1} +$

[1] See also Carlsen and Ruth, 1990; Stange, 1989; Semmen, 1989 and Hatlen *et al.* 1988.
[2] Because an asset that is on the conditional mean-variance frontier need not be on the unconditional frontier (Dybvig and Ross, 1985 and Hansen and Richard, 1987).
[3] See Jagannathan and Wang, (1996).

$\delta_{1,t-1} \cdot \beta_{i,t-1}$, where $\beta_{i,t-1}$ is the conditional beta of asset i defined as $\beta_{i,t-1} = (\text{Cov}(R_{i,t}, R_{M,t}|\Omega_{t-1})/\text{Var}(R_{M,t}|\Omega_{t-1}))$. $\mathbf{R}_{F,t-1}$ is the risk free rate, and $\delta_{1,t-1}$ is the conditional market risk premium defined as $E_t(R_{M,t}|\Omega_{t-1}) - R_{F,t-1}$.

As the model is now stated, it is not operational because of the lack of an observed series for the expected market excess return. However, the conditional CAPM model assumes neither the beta nor the risk premium is to be constant over time. If the conditional CAPM is reformulated and written

$$E_t(R_{i,t}|\Omega_{t-1}) = R_{F,t-1} + \text{Cov}(R_{i,t}, R_{M,t}|\Omega_{t-1}) \cdot \frac{\delta_{1,t-1}}{\text{Var}(R_{M,t}|\Omega_{t-1})}$$

then the ratio between the conditional risk premium and the conditional variance of the market portfolio has been defined. This ratio, defined as the aggregate risk aversion coefficient λ, can be assumed constant over the sample time periods. Therefore, a testable version of the conditional CAPM is given by the specification

$$E_t(R_{i,t}|\Omega_{t-1}) = R_{F,t-1} + \text{Cov}(R_{i,t}, R_{M,t}|\Omega_{t-1}) \cdot \lambda_{i,t-1} \quad (1)$$

Alternatively, model (1) can be expressed as $E_t(r_{i,t}|\Omega_{t-1}) = \lambda_{i,t-1} \cdot \text{Cov}_t(t_{i,t}, r_{M,t}|\Omega_{t-1})$, where $\lambda_{i,t-1} = E_t(r_M|\Omega_{t-1})/\text{Var}_t(r_M|\Omega_{t-1})$, and $E_t(r_i) = E_t(R_i) - RF$ and $E_t(r_M) = E_t(R_M) - R_F$.

In a multivariate setting this model (Equation 1) requires the specification of the dynamics of $\text{cov}(R_{i,t}, R_{M,t}|\Omega_{t-1})$. However, if the model is analysed for the special case where $i = M$, then the model becomes $E_t(r_{M,t}|\Omega_{t-1}) = \lambda_{i,t-1} \cdot \text{Var}_t(r_{M,t}|\Omega_{t-1})$, where $\lambda_{i,t-1} = E_t(r_M|\Omega_{t-1})/\text{Var}_t(r_M|\Omega_{t-1})$ is assumed constant. This theoretical specification provides the central focus of the tests conducted in this paper.

The conditional and unconditional ARMA–GARCH-in-mean model

The empirical counterpart of the last equation in the previous section is given by

$$r_{M,t} = \mu_{M,t-1} + \sum_{i=1}^{p} \phi_{M,i} \cdot r_{M,t-i} + \lambda_{M,t-1} \cdot \sigma^2_{M,t} - \sum_{j=1}^{q} \theta_{M,j} \cdot \varepsilon_{M,t-j} + \varepsilon_{M,t} \quad (2)$$

where $r_{M,t}$ is excess return on the market and $\sigma^2_{M,t}$ is variance of excess returns on the market both in period t. $\mu_{M,t-1}$ and $\lambda_{M,t-1}$ are constant coefficients for intercept and slope, respectively. This ARMA (p,q) specification and as suggested by Turtle (1994), the model in Equation 1 and empirical counterpart in, Equation 2, can be estimated as an univariate ARMA-GARCH-in-mean model process or as a simple conditional mean model (ARMA). Assuming now that an ARMA (p,q)–GARCH (m,n)-in-mean model is appropriate, Equation 2 is supplemented by the conditional volatility equation given by

$$\sigma^2_{M,t} = m_0 + \sum_{r=1}^{m} a_r \cdot \varepsilon^2_{M,t-r} + \sum_{s=1}^{n} b_s \cdot \sigma^2_{M,t-s} \quad (3)$$

The null hypothesis that the conditional CAPM is 'true' in the context of Equations 1 and 2 is given by $H_0 : \mu = 0$. Hence, if $\mu = 0$ in Equation 2 then the excess market return is explained by only its conditional variance consistent with the conditional CAPM. Alternatively, the model can be estimated in its unconditional form:

$$r_{M,t} = E_t(r_M) + \nu_t \quad (4)$$

where $E_t(r_M)$ is the conditional expectation of the excess return of the market. A comparison of the conditional model in Equations 2 and 3 with the unconditional model in Equation 4, gives rise to a set of subsidiary tests. Specifically, the null hypothesis is given by $H_0 : \lambda = a_r = b_s = 0$. Hence, if these restrictions are imposed on Equations 2 and 3 the conditional model collapses to its unconditional form given by Equation 4.

Finally, it is interesting to compare the estimates of the unconditional mean and variance of excess market returns implied by the GARCH-M model to their counterparts from the unconditional model. A close similarity between these estimates would suggest that the conditional model is performing well empirically. For example, the unconditional mean of excess market returns implied by the (conditional) ARMA–GARCH-in-mean model is given by $\hat{\mu} + \hat{\lambda} \cdot \sigma^2_{M,t}$(Volatility). Moreover, the unconditional volatility of market excess returns implied by the ARMA–GARCH-in-mean model is given by $\hat{m}_0/[1 - (\sum \hat{a}_r + \sum \hat{b}_s)]$.

III. DATA AND DATA ADJUSTMENT PROCEDURES

The study uses daily return series from the Norwegian equity market spanning the period from October 1983 to February 1994. We employ four market wide indices. *Pindx* and *Pequl* are indices from Oslo Stock exchange, value-weighted and equal-weighted, respectively. *Nhhvw* and *Nhhew* are two indices from 'Børsdataprosjektet' at the Norwegian School of Economics and Business Administration (NHH), value-weighted and equal-weighted respectively. For the two indices from Oslo Stock Exchange, the entire 10 years' time period 1983–1994 give 2611 daily observations. The two indices from NHH support daily observations from 1 January 1984; that is, 2525 daily observations. The crash is not excluded from the sample. It is therefore assumed that crashes are 'normal' in equity markets. Moreover, as excess returns are necessary a proxy for the risk free rate of return is needed.

A 90-day bank accepted bill series is obtained from the Central Bank of Norway. Finally, systematic location and scale effects are adjusted for (Gallant *et al.*, 1992) in all series.

The log first difference of the price index is adjusted. Let ϖ denote the variable to be adjusted. Initially, the regression to the mean equation $\varpi = x \cdot \beta + u$ is fitted, where x consists of calendar variables as are most convenient for the time series and contains parameters for trends, week dummies, calendar-day separation variable, month and subperiods. To the residuals, $\hat{u}$, the variance equation model $\hat{u}^2 = x \cdot \gamma + \varepsilon$ is estimated. Next $\hat{u}^2/\sqrt{e^{x \cdot \hat{\gamma}}}$ is formed, leaving a series with mean zero and (approximately) unit variance given x. Lastly, the series $\hat{\varpi} = a + b \cdot (\hat{u}/\sqrt{e^{x\hat{\gamma}}})$ is taken as the adjusted series, where a and b are chosen so that

$$\frac{1}{T} \cdot \sum_{i=1}^{T} \hat{\varpi}_i = \frac{1}{T} \cdot \sum_{i=1}^{T} \varpi_i \quad \text{and} \quad \frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{\varpi}_i - \bar{\varpi})^2 = \frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{u}_i - \bar{u})^2$$

The purpose of the final location and scale transformation is to aid interpretation. In particular, the unit of measurement of the adjusted series is the same as that of the original series. The result of these raw data series adjustments are not reported on.[4] Characteristics for the excess return of the four indices and three sampling intervals are reported in Table 1.

Following immediate observations can be extracted. The mean and standard deviation shows no clear patterns over sampling intervals and indices. The maximum and minimum returns report remarkable similarities over sampling intervals. The numbers for kurtosis and skew for daily return series suggest a substantial deviation from the normal distribution, which is confirmed by the K–S Z-test. For weekly series, the K–S Z-test report close to normally distributed returns for the value-weighted indices, while the equal-weighted indices show deviation from normality. For monthly series, normality cannot be rejected. Moreover, the skew is significantly negative for all indices and all sampling intervals. The kurtosis and skews together, suggest too much probability mass around the mean, too little around 1–2 standard deviation from the mean and extreme values on especially the negative side of the distributions. The results suggest a need for heavy tails specification for especially daily but also weekly returns. Therefore a student-t distribution is employed (Blattberg and Gonedes, 1974) for daily and weekly returns while monthly returns employ normal distributions.[5]

The Ljung and Box statistics Q(6) and $Q^2(6)$, reports evidence of serial correlation up to lag 6 for return and squared return series, respectively, for daily and weekly sampling intervals. For monthly series, the value-weighted indices report no serial correlation, while the equal-weighted indices report serial correlation. These results suggest nonsynchronous trading and non-trading effects in almost all indices. Hence, the series need a linear ARMA lag specifications for the conditional mean in models (2) and (4). Gallant and Tauchen (1997), Solibakke (2000a, 2000b) among others, employ the BIC criterion for optimal lag structures in ARMA specifications. An ARMA (0,1) model is chosen for the two value-weighted daily indices and an ARMA (1,0) for the equal-weighted daily indices. For weekly {monthly} returns, ARMA (1,0) {ARMA (0,1)} are obtained for the value-weighted and ARMA (2,0) {ARMA (1,0)} for equal-weighted indices.

The $Q^2(6)$ and the ARCH test (Engle, 1982) report serial correlation and autoregressive and conditional heteroscedasticity (ARCH) in the squared return series for all series and sampling intervals except monthly intervals. Hence, volatility clustering and changing volatility is found in all daily and weekly series. Applying the BIC criterion on the squared residuals from the above-defined ARMA lag specifications, all indices, and sampling intervals prefers a GARCH (1,1) specification.[6] However, monthly series may show insignificant GARCH parameters.

Finally, the RESET (Ramsey, 1969) and the BDS (Brock and Deckert, 1988; Brock *et al.* 1991 and Scheinkman, 1990) for $m = 2$, 3 and 4 and $\varepsilon = 1$ specification test statistics both report data dependence in all adjusted raw data series. The RESET test suggests data dependence in the mean and the BDS test suggests general nonlinear dependence in all series at some dimension (m). The BDS test statistic reports a surprisingly stable and strongly significant nonlinear dependence in all market indices for all sampling intervals. Hence, these test statistics are employed to measure data-dependence after applying ARMA–GARCH filters for all series. Any significant test values induce specification errors in the BIC preferred lag model and will probably make economic implications difficult to interpret.

IV. EMPIRICAL RESULTS

The results of estimating[7] the ARMA–GARCH-in-mean model for excess market return series are presented in Table 2 for all four indices and all three sampling intervals.

[4] The results are readily available from the author upon request.
[5] See Berndt *et al.*, 1974, for a detailed description of the iterative optimization routines (BHHH).
[6] This result conforms to other international findings and is documented for the Norwegian market for daily returns in Solibakke (2000a).
[7] The BHHH algorithm of Berndt *et al.* (1974) is employed.

Table 1. *Properties for raw excess return time series in the Norwegian equity market*

Sampling int.	Mean st. dev.	Max./ Min.	Kurtosis skew	Q(6)	$Q^2(6)$	K-S Z-test	ARCH (6)	Reset (12;6)	BDS test statistic ($\varepsilon - 1$) $m = 2; \varepsilon = 1$	$m = 3; \varepsilon = 1$	$m = 4; \varepsilon = 1$
Pindx; Oslo Stock Exchange value weighted market index. Excess returns.											
Daily returns	0.0196	10.4861	33.124	67.107	291.276	3.7983	306.37	68.749	12.63204	14.890	15.729
	1.2965	−21.317	−1.8841	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}
Weekly returns	0.0200	8.1329	2.4430	21.331	42.518	1.1109	48.146	24.388	1.8861	0.9367	−0.2352
	2.9432	−17.203	−0.5763	{0.002}	{0.000}	{0.169}	{0.000}	{0.018}	{0.067}	{0.257}	{0.388}
Monthly returns	0.0931	14.0758	0.0253	4.6170	8.5650	1.0864	12.211	19.609	−0.2477	−4.0045	−3.6498
	7.1758	−20.087	−0.7032	{0.594}	{0.200}	{0.189}	{0.429}	{0.075}	{0.387}	{0.000}	{0.001}
Nhhvw; Norwegian School of Economics and business administration (NHH) values weighted market index.											
Daily returns	0.0208	10.4890	27.697	56.422	309.367	3.7913	314.06	66.139	12.60914	14.66196	15.5199
	1.3281	−20.451	−1.6194	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}
Weekly returns	0.0148	8.9623	2.3879	23.536	34.898	1.1484	38.107	23.080	2.6796	3.1652	2.4774
	2.9238	−15.356	−0.5758	{0.001}	{0.000}	{0.143}	{0.000}	{0.027}	{0.011}	{0.003}	{0.019}
Monthly returns	0.1608	12.8650	−0.2274	5.3190	10.6100	1.0541	19.581	22.989	5.1262	5.1676	21.1107
	7.0401	−19.555	−0.5940	{0.504}	{0.101}	{0.216}	{0.075}	{0.028}	{0.000}	{0.000}	{0.000}
Pequl; Oslo Stock Exchange equal-weighted market index. Excess returns.											
Daily returns	−0.0026	12.4700	41.198	30.841	585.581	4.5959	634.87	76.835	13.30145	14.753	14.831
	1.1096	−18.709	−1.9040	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}
Weekly returns	−0.0898	9.5343	2.8305	52.716	40.020	1.9073	42.239	23.649	3.6748	3.9147	4.4435
	2.6338	−11.771	−0.6027	{0.000}	{0.000}	{0.001}	{0.000}	{0.023}	{0.000}	{0.000}	{0.000}
Monthly returns	−0.3797	15.6494	0.0607	39.000	10.164	1.0201	17.211	21.877	2.9432	−0.4422	−3.7886
	6.9873	−21.936	−0.4013	{0.000}	{0.118}	{0.249}	{0.142}	{0.039}	{0.005}	{0.362}	{0.000}
Nhhew; Norwegian School of Economics and business administration (NHH) equal-weighted market index.											
Daily returns	0.0391	12.1854	34.627	31.149	527.169	3.8143	556.90	74.667	11.70766	14.246	15.594
	1.2045	−19.468	−1.6793	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}
Weekly returns	0.0590	8.0417	2.1063	39.342	58.082	1.4037	45.650	23.635	4.3383	4.8960	5.6090
	2.7036	−12.873	−0.5060	{0.00}	{0.00}	{0.039}	{0.000}	{0.023}	{0.000}	{0.000}	{0.000}
Monthly returns	0.5893	17.5260	0.1541	17.394	6.627	0.6961	10.682	25.013	0.2734	−1.0786	−4.4229
	7.1892	−21.275	−0.3455	{0.008}	{0.357}	{0.718}	{0.556}	{0.015}	{0.384}	{0.223}	{0.000}

Note: *Pindx* and *Nhhvw* is value-weighted index series and *Pequl* and *Nhhew* is equal-weighted index series from the Norwegian equity market, Skew is a measure of heavy tails and asymmetry of a distribution (normal) and kurtosis is measure of too many observations around the mean for a distribution (normal). K–S *Z*-test: Used to test the hypothesis that a sample comes from a normal distribution. The value of the Kolmogorov–Smirnov *Z*-test statistic is based on the largest absolute difference between the observed and the theoretical cumulative distributions. ARCH (6): ARCH (6) is a test for conditional heteroscedasticity in returns. Low {.} indicates significant values. The OLS-regression $y^2 = a_0 + a_1 * y_{t-1}^2 + \cdots + a_6 * y_{t-6}^2$ is employed. $T * R^2$ is χ^2 distributed with 6 degrees of freedom. T is the number of observations, y is returns and R^2 is the explained over total variation. $a_0, a_1 \ldots a_6$ are parameters.

RESET (12,6): A sensitivity test for mainly linearity in the mean equation. 12 is the number of lags and 6 is the number of moments that is chosen in implementation of the test statistic. $T * R^2$ is χ^2 distributed with 12 degrees of freedom. BDS ($m = 2, \varepsilon = 1$): A test statistic for general nonlinearity in a time series. The test statistic BDS $= T^{1/2} * [C_m(\sigma * \varepsilon) - C_1(\sigma * \varepsilon)^m]$, where C is based on the correlation-integral, m is the dimension and Y is the number of standard deviations. Under the null hypothesis of identically and independently distributed (i.i.d.) series, the BDS-test statistic is asymptotic normally distributed with a zero mean and with a known but complicated variance.

Table 2. *Estimated coefficients for indices in the Norwegian Thinly Traded market*

Sampling Interval	μ	λ	ϕ_1	θ_1/ϕ_2	m	a_1	b_1	v	Log-likelihood
Pindx; Oslo Stock Exchange value-weighted market index									
Daily returns	0.1138	−0.0465	–	−0.2444	0.1345	0.1527	0.7481	6.1971	−3848.1
(unrestricted)	{0.9592}	−{0.4041}	–	−{11.620}	{3.3876}	{4.8512}	{13.959}	{9.1516}	
Daily returns	–	0.0680	–	−0.2478	0.1316	0.1515	0.7471	6.4612	−3848.5
(restricted: $\mu = 0$)	–	{3.6885}	–	−{11.781}	{3.6370}	{5.2922}	{14.985}	{9.0749}	
Weekly returns	2.9417	−0.9714	0.0711	–	0.7950	0.0443	0.8618	7.3948	−1305.9
(unrestricted)	{1.9469}	−{1.8382}	{1.6339}	–	{1.3992}	{1.9205}	{11.189}	{3.0429}	
Weekly returns	–	0.0446	0.0738	–	0.7754	0.0550	0.8545	7.4130	−1308.7
(restricted: $\mu = 0$)	–	{1.0331}	{1.6773}	–	{1.6554}	{2.1798}	{12.743}	{3.3155}	
Monthly returns	2.7378	−0.1999	–	−0.0796	48.9908	0.2033	0.0010	–	−416.2
(unrestricted)	{0.4179}	−{0.2274}	–	−{0.6836}	{4.9282}	{0.8562}	{0.0059}	–	
Monthly returns	–	0.0451	–	−0.1141	45.0828	0.1045	0.0010	–	−415.1
(restricted: $\mu = 0$)	–	{0.4581}	–	−{1.1966}	{5.5305}	{0.5800}	{0.0053}	–	
Nhhvw; Norwegian School of Economics and business administration (NHH) value-weighted market									
Daily returns	0.1515	−0.0660	–	−0.2393	0.1624	0.1751	0.7145	5.7895	−3776.3
(unrestricted)	{0.7463}	−{0.3423}	–	−{11.085}	{3.0732}	{4.4916}	{10.514}	{9.3663}	
Daily returns	–	0.0762	–	−0.2397	0.1568	0.1693	0.7227	5.8865	−3777.1
(restrictred: $\mu = 0$)	–	{4.2181}	–	−{11.074}	{2.9206}	{4.2842}	{10.353}	{9.1732}	
Weekly returns	3.01993	−0.99274	0.10518	–	1.08395	0.06145	0.80977	6.99355	−1301.6
(unrestricted)	{1.9589}	−{1.8028}	{2.4075}	–	{1.7288}	{2.1391}	{9.5444}	{3.5395}	
Weekly returns	–	0.04356	0.11245	–	0.93772	0.07735	0.81243	7.29068	−1304.2
(restricted: $\mu = 0$)	–	{1.0566}	{2.5571}	–	{2.0320}	{2.3504}	{11.521}	{3.4917}	
Monthly returns	15.5924	−2.2517	–	−0.1347	42.1192	0.1252	0.0010	–	−411.6
(unrestricted)	{1.6608}	−{1.6473}	–	−{1.3060}	{5.1329}	{0.6706}	{0.0087}	–	
Monthly returns	–	0.0579	–	−0.1695	40.9416	0.1792	0.0010	–	−412.5
(restricted: $\mu = 0$)	–	{0.6252}	–	−{1.6683}	{5.3599}	{0.8282}	{0.0057}	–	

Pequl; Oslo Stock Exchange equal-weighted market index.									
Daily returns	0.0724	−0.0398	0.1704	–	'0.0646	0.1281	0.8080	4.9358	−3347.2
(unrestricted)	{1.7491}	−{1.1104}	{8.3681}	–	{3.8863}	{5.6104}	{24.813}	{11.112}	
Daily returns	–	0.0398	0.1728	–	0.0594	0.1293	0.8134	4.7798	−3347.9
(restricted: $\mu = 0$)	–	{2.3172}	{8.5055}	–	{4.1379}	{5.8318}	{27.250}	{12.264}	
Weekly returns	0.77214	−0.28237	0.13203	0.14599	1.02155	0.13291	0.74212	3.80610	−1212.3
(unrestricted)	{1.2356}	−{1.1498}	{3.1237}	{3.6284}	{2.2412}	{2.3730}	{8.7398}	{5.4179}	
Weekly returns	–	0.01592	0.13689	0.15294	0.98429	0.13126	0.74786	3.87108	−1213.2
(restricted: $\mu = 0$)	–	{0.4701}	{3.2344}	{3.8063}	{2.2807}	{2.3539}	{9.1450}	{5.2348}	
Monthly returns	3.38613	−0.39115	0.26954	–	41.36666	0.17247	0.00100	–	−405.17
(unrestricted)	{0.2074}	−{0.1625}	{2.3005}	–	{3.7903}	{0.3067}	{0.0031}	–	
Monthly returns	–	−0.00928	0.36567	–	36.87881	0.17722	0.00100	–	−403.15
(restricted: $\mu = 0$)	–	−{0.0890}	{3.8912}	–	{4.4761}	{0.9275}	{0.0115}	–	
Nhhew; Norwegian School of Economics and business administration (NHH) equal-weighted market									
Daily returns	0.0469	0.0204	0.1698	–	0.0552	0.1353	0.8255	5.4370	−3518.5
(unrestricted)	{1.0214}	{0.4479}	{7.9987}	–	{3.0378}	{5.5372}	{25.348}	{9.9381}	
Daily returns	–	0.0677	0.1708	–	0.0560	0.1363	0.8240	5.4282	−3518.8
(restricted: $\mu = 0$)	–	{3.8015}	{8.0929}	–	{3.1812}	{5.6098}	{25.867}	{10.002}	
Weekly returns	0.22971	−0.03270	0.13512	0.15770	1.00162	0.15088	0.73495	4.71361	−1254.6
(unrestricted)	{0.7214}	−{0.2824}	{3.0373}	{3.6944}	{2.3457}	{2.7249}	{9.4187}	{4.6829}	
Weekly returns	–	0.05403	0.13474	0.15847	0.98998	0.15205	0.73546	4.73468	−1254.8
(restricted: $\mu = 0$)	–	{1.4257}	{3.0332}	{3.7264}	{2.3952}	{2.7807}	{9.7099}	{4.6616}	
Monthly returns	6.46523	−0.86461	–	−0.27517	45.91235	0.09415	0.00100	–	−413.71
(unrestricted)	{0.4723}	−{0.4496}	–	−{2.7915}	{4.8832}	{0.4122}	{0.0052}	–	
Monthly returns	–	0.05262	–	−0.24301	48.61379	0.00116	0.00100	–	−413.52
(restrictred: $\mu = 0$)	–	{0.5730}	–	−{2.6850}	{15.661}	{0.0149}	{0.0038}	–	

Note: See Table 1 for series description.

For each series and sampling intervals, the first two lines report an unrestricted version of Equation 2 while line three and four report a restricted version ($\mu = 0$), which is consistent with the conditional CAPM.

The most notable result across all measurement intervals and indices is the general lack of significance of the in-mean parameter. This appears to indicate that the GARCH-in-mean model is an inadequate mean of capturing asset-pricing dynamics. However, this finding is somewhat offset by the insignificant coefficient estimates on μ in the unrestricted models for all return series. Hence, first, consider the first and second row for all indices and sampling intervals in Table 2. The first and second rows present the coefficient estimates and the t-statistics (in brackets), respectively, for the unrestricted model. All coefficient estimates for the in-mean parameter (λ) and the constant parameter (μ) are insignificant. The insignificant coefficient estimate of μ is indicative of acceptance of the conditional CAPM.

Secondly, consider the third and fourth row for all series and sampling intervals. The third and fourth rows present the coefficient estimates and the t-statistics (in brackets), respectively, for the restricted model ($\mu = 0$). Only for daily series, the estimation produces a positive and significant in-mean parameter (λ). It is this restricted version of the GARCH-in-mean model, which is implied by the conditional CAPM. The implication of this result is that investors are compensated for risk only over very short time horizons (i.e. daily).

Thirdly, for monthly return series the coefficient estimates show overall low significance. This result is consis-

Table 3. *Conditional and unconditional variance estimations*
The table reports the conditional mean and the conditional mean and the conditional variance implied by the GARCH-in-mean model in Equations 2 and 3 in contrast to the conditional mean and unconditional variance in Equation 4 from Section II. The differences are reported in column 4 (difference).

-	GARCH-M	Unconditional model	Difference
PINDX			
Mean daily series	0.05075	0.04394	0.00681
Variance daily series	1.35676	1.48817	−0.13141
Mean weekly series	−5.28179	0.07721	−5.35900
Variance weekly series	8.46532	8.57937	−0.11405
Mean monthly series	−9.57155	0.08568	−9.65722
Variance monthly series	61.5660	49.7685	11.7975
NHHvw			
Mean daily series	0.05432	0.05773	−0.00341
Variance daily series	1.47137	1.60493	−0.13356
Mean weekly series	−5.33573	0.08440	−5.42012
Variance weekly series	8.41681	8.47967	−0.06287
Mean monthly series	−92.9448	0.14726	−93.0921
Variance monthly series	48.2033	48.0679	0.13538
Pequl			
Mean daily series	0.03217	0.02643	0.00574
Variance daily series	1.01034	1.06857	−0.05823
Mean weekly series	−1.53598	0.00888	−1.54486
Variance weekly series	8.17405	7.21932	0.95473
Mean monthly series	−16.1902	−0.24586	−15.9443
Variance monthly series	50.0488	41.4614	8.58734
NHHew			
Mean daily series	0.07565	0.06390	0.01174
Variance daily series	1.40945	1.29761	0.11184
Mean weekly series	−0.05716	0.10590	−0.16306
Variance weekly series	8.77279	7.62397	1.14882
Mean monthly series	−37.4053	0.59653	−38.0018
Variance monthly series	50.7403	48.4568	2.28357

Note: See Table 1 for series description tent with international research (Bollerslev *et al.*, 1992). The monthly results suggest that the ARMA–GARCH-in-mean specification for the conditional CAPM should be interpreted with cautions. The ARMA–GARCH-in-mean model specification performs best for shorter time horizons, as suggested by several of the test statistics in Table 1.

Fourthly, the parameter v for the student-t distribution (degrees of freedom) is for all estimations lower than 7.5, suggesting a strongly significant parameter for daily and weekly return series. Consequently, the deviation from the normal distribution that was found in Table 1 seems to be confirmed in Table 2.

Table 3 reports the unconditional mean and variance implied by the ARMA–GARCH-in-mean model in Equations 2 and 3, where λ, a_1 and b_1 is restricted to zero, but μ and φ/θ are free to vary, in contrast to the mean and variance from the unconditional model of market excess return in Equations 4 and 3. In general, from these comparisons it appears that the conditional volatility models provide reasonable estimates of the unconditional variance for all sampling intervals. However, although most likely not significant (suggested by the ARMA–GARCH-in-mean estimation), the conditional mean show considerably lower absolute values for weekly and monthly estimations. For the daily series, the unconditional volatility model seems to provide reasonable estimates of the conditional mean. Consequently, the result suggests that the ARMA–GARCH-in-mean model perform best for the shortest time intervals.

Table 4, panel A reports the likelihood ratio test of the unrestricted ARMA–GARCH-in-mean model given by Equations 2 and 3 against the conditional CAPM which is implied by the restriction of $\mu = 0$ in Equation 2. The conditional CAPM cannot be rejected for any return series and all sampling intervals at 1%. However, for the value-weighted indices and weekly return intervals the conditional CAPM can be rejected at 5%. Moreover, the acceptance of the CAPM for especially the weekly and monthly return series need to be qualified by the fact that the in-mean parameter estimates (λ) were found to be insignificant in line 3 and 4 in Table 2.

Table 4, panel B reports the likelihood ratio tests of the unrestricted ARMA–GARCH-in-mean model given by Equations 2 and 3 against the unconditional model which is implied by the restrictions of $\lambda = a_1 = b_1 = 0$ in Equations 4 and 3. The ARMA–GARCH-in-mean specification cannot be rejected for daily and weekly return series, while monthly series reject the model specification.

Generally, the findings are consistent with the use of ARMA–GARCH-in-mean lagged processes to capture the dynamics of asset returns. The processes show improved fits at especially high frequencies. However, the results suggest caution in using such processes as part of formal asset pricing models.

Finally, the diagnostics tests are reported in Table 5 for daily and weekly series.[8] The kurtosis, skew, and K–S Z-test statistic report closer to normal standardized residuals. No serial correlation (Q(6)) is found in all series for both

Table 4. *Likelihood ratio tests*
Test of the conditional CAPM and the conditional GARCH-in-mean lag specification for Norwegian excess market returns for the period 1983–1994.

Panel A: Tests of the conditional CAPM

PINDX	Daily returns	Weekly returns	Monthly returns
χ^2 test statistic	0.70460	5.6306	2.1944
(prob. value) 1 df.	{0.40124}	{0.01765}	{0.13851}
NHHvw			
χ^2 test statistic	1.48130	5.924	1.7778
(prob. value) 1 df.	{0.22357}	{0.01494}	{0.18242}
Pequl			
χ^2 test statistic	1.29040	1.8224	3.498
(prob. value) 1 df.	{0.25597}	{0.17703}	{0.06144}
NHHew			
χ^2 test statistic	0.53140	0.14890	1.7778
(prob. value) 1 df.	{0.46602}	{0.69959}	{0.18242}

Panel B: Tests of the conditional GARCH-M model versus the unconditional model of excess market return.

PINDX	Daily returns	Weekly returns	Monthly returns
χ^2 test statistic	241.2546	18.9570	3.2852
(prob. value) 3 df.	{0.00000}	{0.00028}	{0.34971}
NHHvw			
χ^2 test statistic	238.7190	19.4999	3.5680
(prob. value) 3 df.	{0.00000}	{0.00022}	{0.31205}
Pequl			
χ^2 test statistic	241.5034	20.1152	1.2035
(prob. value) 3 df.	{0.00000}	{0.00016}	{0.75216}
NHHew			
χ^2 test statistic	253.7890	25.7793	0.2292
(prob. value) 3 df.	{0.00000}	{0.00001}	{0.97274}

Note: See table 1 for series description.
Panel A: This panel reports the results of testing the null hypothesis, $H_0 : \mu = 0$, in the model described in Equations 2 and 3 in Section II. The numbers in brackets are probabilities.
Panel B: This panel reports the results of testing the null hypothesis, $H_0 : \lambda = a_1 = b_1 = 0$, in the model described in Equations 2 and 3 in Section II. The numbers in brackets are probabilities.
Panel A and B: Estimation of the daily and weekly return series is obtained assuming ε_t has a student-t density distribution with v degrees of freedom, while estimation of the monthly return series are obtained assuming ε_t has a normal distribution. The test statistic is distributed as a chi-square variable with degrees of freedom referenced in the table.

[8] Monthly series report for all test statistics insignificant for all three models.

Table 5. *Specification test statistics for model misspecification*

Panel A. Daily series

Series model	Kurtosis Skew	Q(6)	$Q^2(6)$	*K–S* Z-test	ARCH (6)	RESET (12;6)	BDS test statistic $m = 2; \varepsilon = 1$	$m = 3; \varepsilon = 1$	$m = 4; \varepsilon = 1$	$m = 5; \varepsilon = 1$	$m = 6; \varepsilon = 1$
Unrestricted	7.2330	9.4870	13.973	1.7861	14.6335	9.7379	−0.5604	−1.3760	−1.8490	−1.8508	−1.9087
	−0.7365	{0.1480}	{0.0300}	{0.0034}	{0.2621}	{0.6389}	{0.3410}	{0.1548}	{0.0722}	{0.0720}	{0.0645}
PINDX Restricted	7.2703	9.2900	14.715	1.7988	15.3990	9.8057	−0.5846	−1.5273	−1.8533	−1.7628	−1.8221
	−0.7448	{0.1580}	{0.0230}	{0.0031}	{0.2203}	{0.6330}	{0.3363}	{0.1243}	{0.0716}	{0.0844}	{0.0758}
Unconditional	32.0594	20.705	704.02	4.0336	764.1373	74.119	2.4192	2.0843	2.0349	2.0990	1.6055
	−1.0676	{0.0020}	{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.0214}	{0.0455}	{0.0503}	{0.0441}	{0.1099}
Unrestricted	8.1863	7.6910	11.492	1.8002	12.7440	8.6679	0.1339	−0.1301	−0.4170	−0.7794	−0.4543
	−0.8536	{0.2620}	{0.0740}	{0.0031}	{0.3879}	{0.7310}	{0.3954}	{0.3956}	{0.3657}	{0.2944}	{0.3598}
Nhhvw Restricted	8.3007	7.1790	12.016	1.8036	13.2188	8.7731	0.0289	−0.4804	−0.7188	−1.1563	−1.1375
	−0.8664	{0.3050}	{0.0620}	{0.0030}	{0.3533}	{0.7222}	{0.3988}	{0.3555}	{0.3081}	{0.2044}	{0.2089}
Unconditional	34.9352	19.597	672.64	3.7763	699.8691	72.5054	3.5024	3.2064	2.8641	2.2933	2.5118
	−0.9217	{0.0030}	{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.0009}	{0.0023}	{0.0066}	{0.0288}	{0.0170}
Unrestricted	12.9455	5.0000	9.769	2.7298	10.62047	8.3253	1.3203	1.7060	2.2788	2.4318	2.6005
	−1.3257	{0.5440}	{0.1350}	{0.0000}	{0.5617}	{0.7592}	{0.1669}	{0.0931}	{0.0297}	{0.0207}	{0.0136}
Pequl Restricted	12.9916	7.5600	9.784	2.7600	10.6344	8.0452	1.4546	1.5986	2.1435	2.3414	2.1039
	−1.3365	{0.2720}	{0.1340}	{0.0000}	{0.5605}	{0.7816}	{0.1385}	{0.1112}	{0.0401}	{0.0257}	{0.0436}
Unconditional	43.9649	26.500	910.93	4.7087	1115.701	81.0180	3.9430	4.5567	5.1375	5.0343	4.8294
	−0.7861	{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.0002}	{0.0000}	{0.0000}	{0.0000}	{0.0000}
Unrestricted	8.1863	9.5040	12.300	−0.0442	12.764	14.6396	0.3743	0.7769	1.6166	1.9898	2.3481
	−0.8536	{0.1470}	{0.0560}	{2.2226}	{0.3864}	{0.2618}	{0.3719}	{0.2950}	{0.1080}	{0.0551}	{0.0253}
Nhhew Restricted	8.3007	10.2230	12.386	−0.0439	12.80494	14.6177	0.5758	1.0592	1.8209	2.0115	2.5354
	−0.8664	{0.1160}	{0.0540}	{2.2084}	{0.3834}	{0.2630}	{0.3380}	{0.2277}	{0.0760}	{0.0528}	{0.0160}
Unconditional	34.9352	23.565	819.56	−0.0779	947.1253	77.3009	2.9903	3.6081	3.2677	2.7204	2.4241
	−0.9217	{0.0010}	{0.0000}	{3.9152}	{0.000}	{0.000}	{0.0046}	{0.0006}	{0.0019}	{0.0099}	{0.0211}

Panel B. Weekly series

Series model	Kurtosis Skew	Q(6)	$Q^2(6)$	*K–S* *Z*-test	ARCH (6)	RESET (12;6)	BDS test statistic $m=2;\varepsilon=1$	$m=3;\varepsilon=1$	$m=4;\varepsilon=1$	$m=5;\varepsilon=1$	$m=6;\varepsilon=1$
Unrestricted	2.1660	11.769	7.0890	0.895056	12.81156	16.55774	−0.02243	−0.57439	−0.79336	0.0419	0.87843
	−0.4506	{0.0670}	{0.3130}	{0.3996}	{0.3829}	{0.1670}	{0.3988}	{0.3383}	{0.2912}	{0.3986}	{0.2712}
PINDX Restricted	2.1238	12.710	5.8910	0.867104	11.89402	15.00463	−0.2882	−0.75626	−0.23766	0.55752	2.2866
	−0.4454	{0.0480}	{0.4350}	{0.4397}	{0.4542}	{0.2412}	{0.3827}	{0.2997}	{0.3878}	{0.3415}	{0.0292}
Unconditional	2.3931	15.279	35.966	1.083966	41.1954	24.66029	−0.05721	−0.16283	−0.1596	0.48507	0.93746
	−0.5143	{0.0180}	{0.0000}	{0.1906}	{0.0000}	{0.0165}	{0.3983}	{0.3937}	{0.3939}	{0.3547}	{0.2571}
Unrestricted	1.8941	12.427	6.5080	0.822738	12.37239	16.51948	0.11464	−0.75301	−1.20179	−1.02622	−0.23455
	−0.4247	{0.0530}	{0.3690}	{0.5076}	{0.4163}	{0.1686}	{0.3963}	{0.3005}	{0.1938}	{0.2356}	{0.3881}
Nhhvw Restricted	1.8651	12.574	5.4650	0.824468	11.23573	14.56978	−0.74239	−1.25367	−1.40141	−1.13882	−0.98086
	−0.4345	{0.0500}	{0.4860}	{0.5049}	{0.5088}	{0.2658}	{0.3029}	{0.1818}	{0.1494}	{0.2086}	{0.2466}
Unconditional	1.9758	14.190	30.410	0.996694	33.73398	22.68778	−0.55778	−0.55778	−1.00402	−0.15434	−0.00118
	−0.4411	{0.0280}	{0.0000}	{0.2736}	{0.0007}	{0.0305}	{0.3415}	{0.3415}	{0.2410}	{0.3942}	{0.3989}
Unrestricted	3.6314	9.5330	3.3830	1.472923	7.421244	10.72877	−0.90786	−0.53804	−0.30904	−0.07875	−0.50058
	−0.6041	{0.1460}	{0.7590}	{0.0261}	{0.8286}	{0.5523}	{0.2642}	{0.3452}	{0.3986}	{0.3977}	{0.3520}
Pequl Restricted	3.5526	10.768	3.4190	1.585306	7.436968	10.20778	−1.20936	−0.92921	−0.62722	−1.37951	−1.784
	−0.5955	{0.0960}	{0.7550}	{0.0131}	{0.8274}	{0.5977}	{0.1920}	{0.2591}	{0.3277}	{0.1541}	{0.0812}
Unconditional	3.2596	8.9320	36.331	1.799762	39.61099	23.67701	−0.12454	−0.08459	−0.52341	−0.535	−3.71557
	−0.4459	{0.1770}	{0.0000}	{0.0031}	{0.0001}	{0.0225}	{0.3959}	{0.3975}	{0.3479}	{0.3457}	{0.0004}
Unrestricted	2.6698	5.5370	2.4110	1.377341	7.290087	10.8113	−0.27273	−0.15116	0.16636	0.23355	0.23899
	−0.3504	{0.4770}	{0.8780}	{0.0450}	{0.8379}	{0.5452}	{0.3844}	{0.3944}	{0.3935}	{0.3882}	{0.3877}
Nhhew Restricted	2.6662	5.6110	2.4240	1.392593	7.381656	11.02683	−0.293	−0.08083	0.23122	0.18205	−0.04202
	−0.3522	{0.4680}	{0.8770}	{0.0514}	{0.8314}	{0.4266}	{0.3822}	{0.3976}	{0.3884}	{0.3924}	{0.3986}
Unconditional	2.4712	4.1150	32.313	1.248025	29.99621	19.68623	1.58038	1.58038	1.93026	2.12691	1.55558
	−0.1815	{0.6610}	{0.0000}	{0.0887}	{0.0028}	{0.0733}	{0.1144}	{0.1144}	{0.0619}	{0.0416}	{0.1190}

Note: See Table 1 for series description and test statistics definitions.

the unrestricted and restricted GARCH-M model specifications. For the unconditional model, significant serial correlation is found. The reported pattern for serial correlation is also found for the Q^2, the ARCH, the RESET and the BDS test statistics. The test statistics report data dependence for the two equal weighted indices for daily sampling intervals. Hence, the ARMA–GARCH-in-mean specification cannot be rejected for the value-weighted index series. However, for the equal weighted index the model is rejected for daily series.

V. FINDINGS FROM THE THINLY TRADED NORWEGIAN MARKET

The main finding is that the conditional CAPM cannot be rejected. As indicated by the restricted ARMA–GARCH-in-mean specification, implied by the conditional CAPM, investors are compensated for risk over the shortest time interval. Such a conclusion is difficult to accept, as theory would propose that compensation for risk should occur irrespective of the return interval. A possible explanation is that the conditional variance proxy for risk in a generic sense such that it captures liquidity risk, data and measurement error and bid-ask bounce, all of which are greater at shorter return intervals. Hence, the rather crude measure of risk in the model captures market imperfections, which exert their greatest influence at the shortest return interval (daily).

Overall, the results suggest that nonsynchronous trading and non-trading effects as well as volatility clustering are high at daily series. The effects are strong in both the equal-weighted and the value-weighted indices. For weekly series, strong volatility clustering is still found while nonsynchronous trading and non-trading effects have decreased strongly for especially the value-weighted indices. In fact, one of the value-weighted indices (*Pindx*) reports insignificant serial correlation in the conditional mean. The equal-weighted indices still report serial correlation. Hence, an interpretation suggests that very thinly traded assets influence the aggregated returns at the weekly sampling interval. Moreover, for nonsynchronous trading and non-trading effects monthly series exhibits the same patterns as weekly series. For the value-weighted index series, insignificant serial correlation is found while the equal-weighted indices still report significant serial correlation. Hence, nonsynchronous trading and non-trading effects are present in the equal-weighted index series also for monthly sampling intervals. However, volatility clustering and changing volatility is not present in monthly series as suggested in Table 1.

Finally, the ARMA–GARCH-in-mean specification seems to be the preferred model specification for shorter time intervals. For monthly intervals, nonsynchronous trading is low and volatility clustering is no longer present in the series. Note however, that nonsynchronous trading and non-trading effects seem to be present for all sampling intervals for the equal-weighted indices while the value-weighted indices only report daily and weekly effects. Hence, The ARMA–GARCH-in-mean specification is rejected for the equal weighted indices for daily sampling intervals due to non-trading, while the unconditional model is rejected due to volatility clustering in daily and weekly series. The ARMA–GARCH-in-mean model for the value-weighted index series rejects misspecification. Nonsynchronous trading and non-trading as well as conditional heteroscedasticity and volatility clustering is therefore satisfactory modelled for these index series. Models that are more elaborate need to be developed to account for nonsynchronous trading and volatility clustering in high frequency (daily) equal-weighted indices.[9] Finally, it is not possible to reject specification errors for all the unconditional volatility models for monthly intervals. Unconditional models are therefore preferred to conditional models for these return intervals.

VI. SUMMARIES AND CONCLUSIONS

The ARMA–GARCH-in-mean lag specification has been applied to control for nonsynchronous trading and conditional heteroscedasticity in the Norwegian equity market. Two factors are found, which oppose the ARMA–GARCH specification differently. First, monthly series reject conditional heteroscedasticity. An unconditional model is therefore just as valid as a conditional model. Secondly, series strongly influenced from non-trading reject the ARMA–GARCH specification for daily series. Consequently, only the value weighted index series cannot reject the ARMA–GARCH-in-mean specifications for daily and weekly series and the equal-weighted index series cannot reject the specifications for weekly series. The inability of the in-mean parameter to achieve statistical significance is an empirical limitation for especially weekly and monthly data. Despite these results, the conditional CAPM cannot be rejected for any return series intervals. However, the result for the monthly series lacks power as the ARMA–GARCH model is rejected for all monthly series.

In summary, the ARMA–GARCH-in-mean model is a useful empirical tool for modelling equity return series at high frequencies such as daily and weekly return intervals

[9] Solibakke (2000b) models nonsynchronous trading and conditional heteroscedasticity employing virtual returns and a continuous time ARMA–GARCH lag specification.

for thinly traded markets. However, severe nonsynchronous trading and non-trading effects may cause ARMA–GARCH misspecification for high frequency series. Moreover, some care must be taken in placing economic significance to the ARMA–GARCH specification in an asset-pricing regime. As the return interval increases, the superiority of the model decreases, consistent with prior international empirical and theoretical work. The return interval appears not to influence the insignificant rejection result for the conditional CAPM. However, for weekly intervals and the value-weighted indices the conditional CAPM is rejected at 5% but not 1%. Therefore it may be induced that for thinly traded markets and for both daily and weekly return interval the ARMA–GARCH-in-mean specifications seem appropriate for asset-pricing tests. The problems of modelling virtually nonexisting time-varying dynamics for the monthly interval in these markets make monthly return intervals probably not appropriate for tests. Finally, thinly traded markets need more elaborate asset models for high frequency (daily) market dynamics, as indicated by specification test rejections of the ARMA–GARCH-in-mean lag specificiation.

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Essay no. 5*

Testing the Bivariate Conditional CAPM in Thinly Traded Markets

Abstract

This paper tests the conditional Capital Asset Pricing Model (CAPM) in the Norwegian equity market. By applying a bivariate ARMA-GARCH-in-Mean lag specification for the conditional mean and volatility, the full covariance matrix is estimated. The model results suggest that the in-Mean specifications are redundant, suggesting that we are unable to find any preference among conditional asset pricing models. Non-synchronous trading is found in all the bivariate estimations inducing index series with strong positive serial correlation and frequently (thinly) traded asset series show positive (negative) serial correlation. Positive cross-autocorrelation from index to return series is significant and seems to increase as thin trading increases. Volatility clustering is strong in all bivariate estimation suggesting a rejection of the independence hypothesis.

Classification: C14

Keywords: ARMA-GARCH-in-Mean, Risk Models, Non-synchronous trading, Changing Mean and Volatility

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1 Introduction

This paper makes use of a bivariate ARMA-GARCH-in-Mean specification for the purpose of studying thinly traded market characteristics on empirical applications of the Capital Asset Pricing Model (CAPM). The ARMA-GARCH-in-Mean specification estimates the full covariance matrix and makes it possible to carry through tests of among others the conditional CAPM, the residual risk model and the one dynamic factor model. Earlier studies of market volatility have shown that volatility moves together over time and across assets and markets. Recognising this commonality through a multivariate framework leads to obvious gains of efficiency. In this investigation we focus on trading frequency and therefore how asset mean and volatility moves together across thinly and frequently traded assets and the market. Trading volume in NOK is employed as a proxy for trading frequency. In fact, thin trading characteristics (including non-trading[1]) may induce non-synchronous trading effects as well as conditional heteroscedasticity. Therefore, our model specifications employ a bivariate ARMA-GARCH lag specification to account optimally of non-synchronous trading effects in the conditional mean and conditional heteroscedasticity in the conditional volatility, by employing the efficient lag specification from the Bayes Information Criterion (BIC) (Schwarz, 1978) in both the conditional mean and volatility. We therefore do not pursue a simultaneous return and trading volume specification as pursued in Clark (1973), Tauchen and Pitts (1983), Gallant et al. (1992) and Andersen, (1994), but rather employ individual return series to study characteristics over a wide variety of trading frequencies optimal ARMA-GARCH-in-Mean specifications. The advantage of such modelling is an explicit availability of the conditional mean and volatility series for individual series and the market index. Hence, the bivariate specification may give new insight to return and volatility characteristics of thinly traded markets. As the Norwegian market is a professional dealer market, it is ideal for this kind of market study as the market is a relatively thinly traded market and contains assets that exhibit relative thin trading frequency.

We design a BIC preferred bivariate ARMA-GARCH-in-Mean specification where we pair each individual return series with the index return series. The specification is a bivariate ARMA-GARCH approach (MGARCH) and will be able to capture temporal dependencies in the conditional mean, variances and covariance. The estimation is a one-stage procedure in which betas and risk premium are estimated simultaneously[2]. The bivariate ARMA-GARCH-in-Mean lag specification's error process will assume that the residuals of the regressions should be serially uncorrelated, conditional homoscedastic and normally distributed. Hence, we employ residual specification tests for the bivariate ARMA-GARCH-in-Mean model, to report any model misspecifications.

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[1] Solibakke (2000) show liquidity characteristics in the Norwegian equity market.

[2] Applying OLS estimation of CAPM in a time series context, the underlying theory requires a number of assumptions to hold. Specifically, we assume that the risk premiums are stationary, normally distributed and serially uncorrelated, which imply that the error process is i.i.d.

Our results show some interesting features. Firstly, we find a significant positive "zero-beta" coefficient for only the frequently traded series. This result suggests that it is the frequently traded assets that show a significant positive drift. Secondly, all series report significant autocorrelation and cross-autocorrelation. Thinly traded series report negative autocorrelation and frequently traded series report positive autocorrelation. Non-synchronous trading may induce the negative serial correlation in thinly traded series. All asset series report positive cross-autocorrelation to the market indices. Frequently traded assets report therefore slow adjustments at time t from its own and market past return $(t-1)$. In contrast, thinly traded assets report mean reversion from its own one period lagged returns and slow adjustment to market lagged returns. The mean reversion seem to increase as non-synchronous trading increases. These results induce return predictability for both frequently and thinly traded assets. Thirdly, all alternative in-Mean specifications are rejected, which imply rejection of the residual risk hypothesis, the one dynamic factor model and the conditional CAPM as well as no preference among alternative risk measures. This result suggests that the in-Mean model is not a very well specified volatility feedback methodology (risk). Fourthly, conditional heteroscedasticity is present in all asset and market index series and the univariate modelling approach of the conditional variances seem to be rejected due to significant bivariate GARCH coefficients. The market dynamics may therefore not be adequately controlled for in a univariate modelling approach. Fifthly, asymmetric volatility seems to be present in almost all return series. Sixthly, specification test statistics report data-dependence for thinly traded assets suggesting biases in the moments and co-moments. The autocorrelation and cross-autocorrelation result for thinly traded assets may therefore be spurious and the predictability may depend on non-synchronous trading effects rather than return predictability. Seventhly, the data dependence result for the thinly traded assets suggests a rejection of the ARMA-GARCH specification. Severe non-synchronous trading (non-trading) suggests therefore a need for rather elaborate modelling procedures. Finally, as we estimate the full co-variance matrix the conditional beta (β) measure is readily available. The cumulative frequency distribution of the estimated conditional CAPM's β series, classify series in nicely ascending order of trading frequency. Hence, trading frequency seems to classify the CAPM's relevant risk measure[3]. However, as co-moments are biased, the result for thinly traded series must be interpreted by extreme caution.

The article is organised as follows. Section 2 defines the methodology. Section 3 presents the data and adjustment procedures for stationery data series. Section 4 reports the results/findings of the analysis. Section 5 reports the conditional (co-) variance and beta characteristics and Section 6 summarises and concludes.

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[3] Due to high trading frequency correlation with market value, the relevant risk measure may also be classified also in accordance with size (Campbell et al., 1997, p 130).

2 Methodology

2.1 The Static CAPM (the Sharpe-Lintner-Black CAPM)

Let R_i denote the return on any asset *i* and R_M be the return on the market index (*M*) return of a value weighted asset index in the economy[4]. The Black (1972) version of the CAPM is $E[R_i] = \mu_0 + \delta_1 \cdot \beta_i$, where β_i is defined as $\beta_i = Cov(R_i, R_M) / Var[R_M]$, and *E[]* denotes the expectation, *Cov()* denotes the covariance and *Var[]* denotes the variance. Fama and French (1992) finds that the estimated value for δ_1 is close to zero and concludes therefore that the results suggest strong evidence against the CAPM. The static CAPM is also tested in the Norwegian market. However, Carlsen og Ruth (1991) fail to the reject the null of μ_0 significant different from zero for both univariate and multivariate tests[5]. The empirical results seem therefore to show no clear evidence for or against the static CAPM. However, this result does not necessarily imply evidence for or against the conditional CAPM. The CAPM was developed within the framework of a hypothetical single-period model economy. The real world, however, is dynamic and hence, expected returns and betas are likely to vary over time. Even when expected returns are linear in betas for every time period based on the information available at that time, the relation between the unconditional expected returns and the unconditional beta could be close to zero[6]. In the next section we assume that CAPM holds in a conditional sense, i.e., it holds at every point in time, based on whatever information is available at that instant.

2.2 The Conditional CAPM

If we assume that expectations in CAPM at time *t* are conditioned on the information set available to agents at time *t-1*, Ω_{t-1}, then the conditional CAPM[7] can be written as $E_t(R_{i,t} \mid \Omega_{t-1}) = \mu_{0,t-1} + \delta_{1,t-1} \cdot \beta_{i,t-1}$, where $\beta_{i,t-1}$ is the conditional beta of asset *i* defined as $\beta_{i,t-1} = \dfrac{Cov(R_{i,t}, R_{M,t} | \Omega_{t-1})}{Var(R_{M,t} | \Omega_{t-1})}$. $\mu_{0,t-1}$ is the conditional expected return on a "zero-beta" portfolio, and $\delta_{1,t-1}$ is the conditional market risk premium. Both the expected returns and the betas, will in general, be time varying in the conditional CAPM framework[8]. The model is stated in terms of conditional moments and assumes that investors use information at time *t-1* rationally and maximise their utility period by period.

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[4] We assume here that the market index is a good approximation for the market portfolio.
[5] See also Carlsen and Ruth, 1990, Stange , 1989, Semmen, 1989 and Hatlen et al. 1988.
[6] Because an asset that is on the conditional mean-variance frontier need not be on the unconditional frontier (Dybvig and Ross, 1985 and Hansen and Richard, 1987)
[7] See Jagannathan and Wang, 1996.
[8] See also Jaganathan and Wang, 1996.

As the model is now stated it is not operational because of the lack of an observed series for the expected market excess return. However the conditional CAPM model assumes neither the beta nor the risk premium is to be constant over time. Hence, if we now reformulate the conditional CAPM and write

$E_t(R_{i,t} \mid \Omega_{t-1}) = \mu_{0,t-1} + Cov(R_{i,t}, R_{M,t} \mid \Omega_{t-1}) \cdot \frac{\delta_{1,t-1}}{Var(R_{M,t} \mid \Omega_{t-1})}$, then we have defined the ratio between the conditional risk premium and the conditional variance of the market portfolio. This ratio, defined as the aggregate risk aversion coefficient λ, can be assumed constant over the sample time periods. Therefore, a testable version of the conditional CAPM is given by the specification

$$E_t(R_{i,t} \mid \Omega_{t-1}) = \mu^i_{0,t-1} + \lambda_{i,t-1} \cdot Cov(R_{i,t}, R_{M,t} \mid \Omega_{t-1}) \qquad (1)$$

where $\mu^i_{0,t-1}$ is the conditional expected return for asset *i* and $\lambda_{i,t-1} = \frac{\delta_{1,t-1}}{Var(R_{M,t} \mid \Omega_{t-1})}$, assumed constant. The model requires the specification of the dynamics of $Cov(R_{i,t}, R_{M,t} \mid \Omega_{t-1})$.

2.3 The ARMA-GARCH-in-Mean specification and the Conditional CAPM

Model (1) is possible to write as $R_{i,t} = \mu_{i,0} + \lambda_i \cdot Cov(R_{i,t}, R_{M,t} \mid \Omega_{t-1}) + u_{i,t}$ and

$R_{M,t} = \mu_{M,0} + \lambda_M \cdot Var(R_{M,t} \mid \Omega_{t-1}) + u_{M,t}$, where $\mu_{i,0}$ and $\mu_{M,0}$ is the drift in asset *i* and the market index (*M*), respectively, the $u_{i,t} = R_{i,t} - E_t(R_{i,t} \mid \Omega_{t-1})$ and $u_{M,t} = R_{M,t} - E_t(R_{M,t} \mid \Omega_{t-1})$ are the residual terms for asset *i* and the market index (*M*), respectively. We thus see that $Var_t(R_{i,t} \mid \Omega_{t-1}) = E_t(u^2_{i,t} \mid \Omega_{t-1}) = h_{i,t}$, and $Var_t(R_{M,t} \mid \Omega_{t-1}) = E_t(u^2_{M,t} \mid \Omega_{t-1}) = h_{M,t}$, and $Cov_t(R_{i,t}, R_{M,t} \mid \Omega_{t-1}) = E_t(u_{i,t} \cdot u_{Mt} \mid \Omega_{t-1}) = h_{i,M,t}$. Furthermore, as the Norwegian market is a thinly traded market showing non-synchronous trading and Campbell (1997) show that non-synchronous trading potentially induces serious biases in the moments and co-moments of return series, the non-synchronous trading is modelled using an ARMA mean specification where the lag lengths are determined by the BIC. Moreover, as non-synchronous trading may induce volatility clustering and changing volatility, the conditional heteroscedasticity is modelled using a GARCH conditional volatility specification, where the lag lengths are determined by the BIC. Thus, the time varying conditional CAPM can be put into a bivariate ARMA-GARCH-in-Mean form[9] [10], which induces the following specifications for our Norwegian CAPM tests.

$$R_{i,t} = \mu_{i,0} + \sum_{j=1}^{p_i} \phi_{i,j} \cdot R_{i,t-j} + \lambda_i \cdot Cov(R_{i,t}, R_{M,t} \mid \Omega_{t-1}) + \varepsilon_{i,t} - \sum_{j=1}^{q_i} \theta_{i,j} \cdot \varepsilon_{i,t-j} \qquad (2)$$

[9] See Hall et al., 1989, Bollerslev et al., 1988, Chan et al., 1992, Gonzales-Rivera, 1996.
[10] For applications see Bollerslev et al., 1992.

$$R_{M,t} = \mu_{M,0} + \sum_{j=1}^{p_M} \phi_{M,j} \cdot R_{M,t-j} + \lambda_M \cdot Var(R_{M,t} \mid \Omega_{t-1}) + \varepsilon_{M,t} - \sum_{j=1}^{q_M} \theta_{M,j} \cdot \varepsilon_{M,t-j} \quad (3)$$

$$\gamma_{k,j,t} = \delta_{k,j} \quad \text{if and only if} \quad \varepsilon_{k,t-j} < 0 \qquad k = i, M \quad (4)$$

$$h_{k,t} = m_{k,0} + \sum_{j=1}^{r_k} (a_{k,j} + \gamma_{k,j,t}) \cdot u_{k,t-j}^2 + \sum_{j=1}^{s_k} b_{k,i} \cdot h_{k,t-j} \qquad k = i, M \quad (5)$$

where $\lambda_{i,t-1} = \dfrac{E_t(r_M|\Omega_{t-1})}{Var_t(r_M|\Omega_{t-1})}$. The bivariate system of random vectors $R_t' = (R_{i,t}, R_{M,t})$ followed by the conditional variance-covariance matrix $h_{k,t}$, allows for a rich structure permitting interaction effects between the market index and the individual assets. The a_k are the vectors of the weights for the lagged ε^2 terms; this is the ARCH process. The b_k are the weights for the lagged h_k terms; this is the GARCH process. The m_k is a constant term for unexplained conditional variance. To determine the lag lengths in the conditional variance equation r_k and s_k, we apply the BIC on the squared residuals from the conditional mean ARMA specification. Solibakke (2001b) shows one more important feature from the Norwegian thinly traded market that needs to be incorporated into the bivariate ARMA-GARCH-in-Mean specification. Asymmetric volatility or the "leverage" effect (Nelson, 1991) is specified in the volatility equation (4) as suggested by the GJR model (Glosten et al., 1993). The model therefore apply $\gamma_{k,i,t} = \varepsilon_{k,t-i}$ if and only if $\varepsilon_{k,t-i} < 0$; else 0; in the conditional volatility equations. We allow the γ parameter to be less than zero. This theoretical specification of the conditional CAPM provides the central focus of the tests conducted in this paper.

3 Empirical data and adjustment procedures

The study applies daily returns of individual Norwegian stocks spanning the period from October 1983 to February 1994. As some of these assets exhibit non-synchronous trading characteristics, the assets are sorted from frequently traded assets (no. 1) to thinly traded assets (no. 7), where trading volume is employed as a proxy for trading frequency. Trading volume is the amount traded in the asset in NOK; that is, the number of stocks traded multiplied by settlement prices at the time of trading. Moreover, individual assets are grouped into portfolios at period *t* based on trading volume at *t-1*. Portfolio FT consists of the most frequently traded assets and Portfolio TT consists of the most thinly traded assets. The portfolio series are rebalanced each month using information at *t-1*. Moreover, assets traded throughout a month, is assigned to one of the two portfolios on basis of their average daily trading volumes in NOK for the last 2 years in the market. The two-year average avoids a too frequent shift of portfolio-assets. To proxy for the market portfolio we employ the value weighted market index[11] consisting of all stocks in the Norwegian market.

The crash in October 1987 is not excluded from the sample series. We therefore assume that a crash is normal in equity markets. Finally, we adjust for systematic location and scale effects (Gallant et al., 1992) in all time series. The log first difference of the price index is adjusted. Let ϖ denote the variable to be adjusted. Initially, the regression to the mean equation $\varpi = x \cdot \beta + u$ is fitted, where x consists of calendar variables as are most convenient for the time series and contains parameters for trends, week dummies, calendar day separation variable, month and sub-periods. To the residuals, $\hat{u}$, the variance equation model $\hat{u}^2 = x \cdot \gamma + \varepsilon$ is estimated. Next $\frac{\hat{u}^2}{\sqrt{e^{x \cdot \hat{\gamma}}}}$ is formed, leaving a series with mean zero and (approximately) unit variance given x. Lastly, the series $\hat{\varpi} = a + b \cdot (\frac{\hat{u}}{\sqrt{e^{x\hat{\gamma}}}})$ is taken as the adjusted series, where a and b are chosen so that $\frac{1}{T} \cdot \sum_{i=1}^{T} \hat{\varpi}_i = \frac{1}{T} \cdot \sum_{i=1}^{T} \varpi_i$ and $\frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{\varpi}_i - \overline{\varpi})^2 = \frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{u}_i - \overline{u})^2$. The purpose of the final location and scale transformation is to aid interpretation. In particular, the unit of measurement of the adjusted series is the same as that of the original series. We do not report the result of these raw data series adjustments[12].

{Insert Table 1 about here}

The characteristics of the assets, the equal weighted trading frequency portfolios and the value weighted market index are reported in Table 1. The following immediate observations can be extracted. The standard deviation of returns seems to increase proportionally with the level of thin trading. The daily maximum and minimum return series seem to suggest that highest absolute numbers are found for the thinly traded series. This variation in mean return among the thinly traded assets produces consequently the highest standard deviation. For the portfolio series the highest absolute minimum is found for the frequently traded series. The portfolio results suggest that thinly traded series containing zero asset returns outweighs high individual absolute returns. Finally, as expected from the portfolio theory, the market index produces the lowest standard deviation.

The calculated numbers for kurtosis and skew from stock returns, suggest a substantial deviation from the normal distribution. The kurtosis and skew indication of non-normality is strongly supported by the Kolmogorov-Smirnov Z-test statistic[13] (K-S Z-test) for normality for all series. The kurtosis and skew and the K-S Z-test suggest too much probability mass around the mean, too little around 1-2 standard deviation from the mean and some extreme

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[11] Note that about 20% of the assets in the Norwegian market count for 60% of the market value of the Oslo Stock Exchange.

[12] The results are readily available from the author upon request.

values on especially the negative side of the mean. The results induce that it is the thinly traded series that show the highest deviation from the normal distribution. However, the value-weighted market index reports high kurtosis and a high negative skew. From Table 1 it also seems as especially the kurtosis increases as the number of combined assets increases[14].

The ARCH (Engle, 1982), the RESET (Ramsey, 1969) and the BDS (Brock and Deckert, 1988, 1991 and Scheinkman, 1990) test statistics, suggest data dependence in all adjusted return series. The ARCH test suggests changing conditional volatility, which induce conditional heteroscedasticity. The RESET test suggests non-linear dependence in the mean and the BDS test statistic suggests strong general non-linear dependence. Especially where the non-synchronous trading seem strong (non-trading periods), the BDS statistic reports highly significant values. In contrast, the portfolio series report increased non-linear dependence when trading frequency increases, which may stem from a more erratic conditional volatility. Overall the ARCH, RESET and BDS test statistics report surprisingly stable and strongly significant data dependence for all series. Note that a non-linear conditional volatility imply a rejection of the independence hypothesis while a non-linear mean imply a rejection of the Martingale hypothesis.

4 Empirical results

Maximum likelihood estimates[15] of the parameters for the bivariate ARMA-GARCH-in-Mean specification[16] are given in Table 2 for all bivariate asset and market index daily return series. The two intercepts $(\mu_{i,0}, \mu_{M,0})$ in the mean equations of the bivariate system of equations are positive for all series. The market index reports significant positive mean drift for all estimations. It also seems that the positive drift is more significant for frequently traded assets. The GARCH-in-Mean parameters (λ) can as outlined above, be specified for several alternative outlines of the conditional means. As we estimate the full variance-covariance matrix in the conditional variance process, the conditional standard deviation, the conditional covariance (with the market portfolio) and the conditional market standard deviation, we can specify several alternative outlines of the conditional means. Firstly, we introduce the conditional variance series (h_i and h_M) in the conditional means for asset and market series, respectively. The conditional variance (h_i) can then be interpreted as residual risk and the

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[13] The K-S Z test statistic is a procedure to test the null that a sample comes from a population in which the variable is distributed according to a normal distribution.

[14] Often named the mixture of distributions hypothesis, which maintains that asset returns exhibit leptokurtosis because they are really a combination of returns distributions.

[15] We assume conditional bi-normality of the residuals. We also employ the BHHH (Berndt et al., 1974) algorithm for maximum likelihood estimation of parameters.

[16] The univariate ARMA lags, determined by the BIC criterion (Schwarz, 1978) are ARMA (0,2) for the thinly traded portfolio and assets no. 4 to 7; ARMA (0,1) for the most frequently traded portfolios and assets no. 1 to 3. All assets and portfolios employ a GARCH (1,1) lag specification applying the BIC criterion on the squared residuals from the ARMA lag

accompanying coefficient (λ_i) measures residual risk sensitivity, which is the sensitivity to total risks. The specification may be considered as a proxy for omitted risk factors (Lehmann, 1990). Secondly, we introduce the market variance (h_M) in both conditional means. The introduction of the market variance in the asset mean may be interpreted as a one dynamic-factor model, which implies that the dynamics and variation in the overall market index, guide all the return series. The $\lambda_{i,M}$ coefficient measure the sensitivity to total market dynamics. Thirdly and finally, we run the bivariate estimations with the specification in (2) to (4) in Section 2, which is the conditional CAPM specification.

The results in Table 2 suggest that none of the series report significant in-Mean coefficients (λ). The residual risks specification (λ_i) is rejected. The one dynamic factor hypothesis $(\lambda_{i,M})$ is rejected. Finally, the conditional CAPM specification $(\lambda_{i,i,M})$ is rejected. Moreover, the market index produces insignificant mean coefficient (λ_M) from its own conditional variance process.

Our results report a consistent positive coefficient (θ_i) for thinly traded series and a consistent negative coefficient (θ_i) for frequently traded series. We find significant positive cross-autocorrelation from market index to asset returns $(\phi_{M,i})$. The market index, which is employed as a proxy for the market portfolio, reports strongly significant autocorrelation (θ_M) for all bivariate estimations.

{Insert Table 2 about here}

Panel B of the Tables 2 report the conditional variance equations from the bivariate estimations. The t-statistics indicate that the parameters $m_{i,i}$, $m_{i,M}$, $m_{M,M}$, $a_{i,i}$, $a_{i,M}$, $a_{M,i}$ $a_{M,M}$, $b_{i,i}$, $b_{i,M}$, $b_{M,i}$, $b_{M,M}$ are almost all statistical significant at conventional levels. Interestingly, the cross-series GARCH parameters show strongly significant values. Asymmetric volatility or the "leverage effect" (γ_i and γ_M) seems to be present in almost all series.

As an overall specification test of the bivariate model, we calculate several elaborate test statistics in Table 3. Firstly, we calculate the sixth order Ljung and Box (1978) statistic for the standardised residuals (Q(6)) and squared residuals ($Q^2(6)$) from each bivariate estimation for all series and the accompanying market index series. We find no evidence of serial correlation in neither residuals nor squared residuals up to lag 6. Secondly, all the bivariate estimations show no significant cross-correlation at any lags (not reported). Thirdly, the numbers for kurtosis and skews for the standardised residuals report excess kurtosis, but importantly, the numbers for kurtosis and skew are strongly reduced relative to the adjusted raw data series. Our results therefore suggest that the bivariate ARMA-GARCH-in-Mean filter specification produce more normal time series residuals. These results are confirmed by strongly reduced

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specifications. Finally, the cross return series specifications for the conditional mean (ϕ_{15}) are determined by a likelihood ratio test among competing specifications.

K-S Z-test statistics from Table 1. However, the K-S Z-test statistic still disputes normality for all series.

{Insert Table 3 about here}

Fourthly, the ARCH test statistic reports insignificant test statistics for conditional heteroscedasticity for all series. The RESET test statistic report an insignificant test statistics implying no data-dependence in the conditional mean. Finally the Brock and Deckert (1988, 1991) and Scheinkmann (1990) (BDS) test statistic report general non-linear dependence at some dimension (*m*) 2 to 6, for $\varepsilon = 1$, for all thinly traded series, while frequently traded series report no general non-linear dependence.

5 Findings and Characteristics from the Norwegian thinly traded market

The findings from these bivariate ARMA-GARCH-in-Mean estimations may bring some new insights to thinly traded market dynamics. Firstly, the estimations suggest a clear pattern in the "zero-beta" return. The zero-beta return is clearly more significant for frequently traded assets. This result suggests that the drift show a lower daily variance for frequently traded assets than thinly traded assets. Hence, frequently traded assets seem to report a more regular daily positive return.

Secondly, none of the series report significant in-Mean coefficients (λ). Hence, the residual risks specification is rejected, which also suggests rejection of conditional multifactor models. The one dynamic factor hypothesis is rejected and the conditional CAPM specification is rejected. Consequently, all alternative conditional mean series specifications in bivariate ARMA-GARCH-in-Mean lag form do not add extra information to the conditional mean and our specification is not able to distinguish between alternative asset pricing hypotheses. The result suggests that the market show no short-term risk compensations. Moreover, the market index produces insignificant mean coefficients from its own conditional variance process. As all the coefficients are negative for the market, the result suggests lower returns during high volatility regimes, which seem to fit well with observed facts and the volatility-feedback hypothesis (Campbell and Hentschell, 1992).

The autocorrelation results in the conditional means suggest non-synchronous trading effects. For frequently traded asset series and the market index our results imply a significant negative MA(1) coefficient. The negative coefficient induces positive autocorrelation, which imply slow adjustment to shocks. However, non-synchronous trading may produce spurious positive autocorrelation[17]. For thinly traded assets our results suggest significant negative autocorrelation. The thinly traded series therefore seem to report overreaction and mean

[17] For the sub-period 1987-1994 the *MA(1)* coefficient turns insignificant.

reversion. The results are clearly influenced by non-synchronous trading effects and therefore long series of zero returns suggesting spurious serial correlation in the conditional mean.

Conditional heteroscedasticity is present in all series. Moreover, the market index influence in the conditional variance of asset series induces that the variance process for assets may not be modelled as an univariate processes. In our bivariate specifications the series own past conditional variance, the past conditional market index variance and the past conditional covariance significantly explain the volatility of the return series. Hence, the results strongly induce a preference for a bivariate relative to univariate specifications. It is therefore naturally to assume that an univariate representation does not adequately capture all temporal dependencies in the Norwegian equity market. Moreover, the GARCH coefficients also seem to induce that only past shocks and past conditional variance from frequently traded assets significantly influence the conditional variance of the market.

Finally, applying the specification test results induce several interesting findings. Firstly, for all bivariate series the ARCH test reports insignificant statistics. Hence, the result suggests that all conditional heteroscedasticity is removed form the series. For frequently traded assets the RESET and the BDS test statistics report insignificant statistics. Hence, for these bivariate ARMA-GARCH filter residuals neither the independence nor the Martingale hypotheses can be rejected. Hence, the filter implies that the lag specification adequately models the market dynamics for frequently traded assets. In contrast, for thinly traded assets, the RESET test statistics report insignificant values wile the BDS test statistics report significant statistics at some dimension (m). Hence, these assets report no conditional heteroscedasticity, no data-dependence in mean but general non-linear dependence. Hence, non-synchronous trading suggest data-dependence not possible to model in classical bivariate ARMA-GARCH lag specification models. More elaborate models need to be developed, which may apply virtual returns and explicitly account for return intervals (Campbell et al., 1997, Drost and Niemann, 1993)[18]. Hence, our results suggest that non-linearity in frequently stock returns originates from conditional heteroscedasticity, while thinly traded stocks seem to exhibit non-synchronous trading effects that a linear ARMA specification of the conditional mean cannot adequately model. However, for assets not subject to strong non-synchronous trading the bivariate ARMA-GARCH specification seems robust. Note that for thinly traded assets our results suggest that intuitive, analytical and linear reasoning may turn extremely difficult. Economic implications may be even more difficult to interpret.

5.1 Co-variance Characteristics

[18] An univariate version of temporal aggregation and continous time ARMA-GARCH non-synchronous trading model is work out in detail (Solibakke, 2001a)

The ARMA-GARCH lag specification can be used to create and analyse the conditional variance and covariance matrix. We report the volatility characteristics in Tables 4. The conditional variance means and fluctuations are strongly higher for thinly traded series relative to frequently traded series. The conditional covariance mean seems to be higher for frequently traded series while the fluctuations in the covariance seems to be higher for thinly traded series. These results suggest higher market sensitivity (β) for highly traded series. The mean of the conditional beta measure is highest for the frequently traded series while the standard deviation of the beta is clearly higher for the thinly traded series. A closer examination of the time varying covariance series also reveals negative covariance in the bivariate estimation for the thinly traded series. Moreover, both the mean and the standard deviation for the conditional covariance seem to increase in ascending order of trading volume portfolios. Hence, the betas should increase as the trading volume increases.

{Insert Table 4 about here}

To see if we find any relation between trading frequency and beta, we study the cumulative frequency distribution of the conditional time varying beta measure. We start by finding the frequency of the $\beta_{i,t-1}$ observations in a interval between -1 and 3 (bin-interval). We move on to accumulate the observations and define an empirical cumulative distribution function of the $\beta_{i,t-1}$ observations for each series. The cumulative distributions are plotted in Figure 1, with dotted lines for individual assets and lines for portfolios. The ordering of both assets and portfolios is of obvious interest. Figure 1 shows a cumulative distribution of the $\beta_{i,t-1}$ observations, that sorts the portfolios nicely in ascending order of trading frequency. Hence, the result seems to imply that the highest relevant risk will be found for the frequently traded series and lowest relevant risk will be found for the thinly traded series. Note especially that applying portfolio theory, the close to zero and negative beta series for the thinly traded assets may be of considerable interest for portfolio managers. Negative betas are usually very desirable in building asset portfolios. However, the specifications tests above suggest that these beta (β) results may originate from serious biases in the co-moments of the return series.

{Insert Figure 1 about here}

6 Summaries and Conclusions

This paper has estimated a bivariate ARMA-GARCH-in-Mean specification of the conditional CAPM in the Norwegian thinly traded equity market, controlling for non-synchronous trading and conditional heteroscedasticity. The bivariate conditional CAPM specification captures non-synchronous trading and conditional heteroscedasticity in asset series. Moreover, our specification captures the "leverage" effect (Nelson, 1991) in the bivariate conditional variance equations. The estimations focus on moment and co-moments characteristics in the Norwegian thinly traded market.

The in-Mean specification is redundant as all series report insignificant variance and covariance parameters in the conditional means. As a consequence, the dominance test of the conditional CAPM model versus the residual risk and the one dynamic factor model is left unsettled. Non-synchronous trading effects are present in the Norwegian thinly traded market. The thinly traded series report strong mean reversion while frequently traded assets as well as the market index report significant slow adjustment.

The ARCH- and GARCH-coefficients in the bivariate system of equations are for almost all coefficients strongly significant. The results imply firstly, conditional heteroscedasticity and secondly, a univariate specification may not capture enough market dynamics. Specification tests report rejection of thinly traded asset specification while frequently traded assets show adequate model specification. Hence, the data dependence in thinly traded assets induce a wrongly specified model for these assets and suggest a need for more elaborate models for daily return observations in thin markets. Finally, we find that the cumulative frequency distributions of the risk measure β, can be sorted according to an ascending order of trading frequency. The frequently traded assets and portfolio is the most risky measured by the conditional β series. However, the specification tests failures for thinly traded assets induce spurious moments and co-moments characteristics. The moment and co-moments results for thinly traded assets must therefore be treated by considerable scepticism. Consequently, analytical, intuitive and linear reasoning and economic implications become very difficult. The non-synchronous trading issue in thinly traded markets must therefore be left to future research.

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Table 1. Characteristics of Return series for the Norwegian Equity Market

Series	Obs. Prop	Mean / Std.dev.	Daily Max.	Kurtosis Skew	K-S Z-stat	Q(6)	$Q^2(6)$	ARCH (6)	RESET (12;6)	BDS test statistic m=2;ε=1	m=3;ε=1	m=4;ε=1
VP-1	2611	15.196	13.041	8.405	3.150	50.241	347.021	303.73	43.452	1.920	2.837	3.251
	1.00	32.417	-20.270	-0.311	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.063}	{0.007}	{0.002}
VP-2	2611	25.140	13.436	9.163	2.962	29.844	79.235	144.41	42.199	2.531	2.000	1.598
	1.00	33.153	-23.435	-0.374	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.016}	{0.054}	{0.111}
VP-3	2611	8.598	26.756	30.219	3.448	12.561	240.207	255.07	37.374	3.479	3.337	2.900
	0.97	46.407	-46.978	-1.314	{0.000}	{0.049}	{0.000}	{0.000}	{0.000}	{0.001}	{0.002}	{0.006}
VP-4	2577	12.645	24.413	11.892	5.352	54.31	449.57	292.88	73.327	9.391	10.152	10.350
	0.83	57.556	-33.691	-0.623	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}
VP-5	2515	42.318	81.202	95.67	8.731	264.1	240.478	205.66	79.693	15.533	18.822	21.057
	0.57	130.70	-167.94	-3.984	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}
VP-6	2499	34.618	53.591	17.440	10.813	233.91	374.174	350.16	38.454	20.608	23.900	30.789
	0.43	108.42	-56.571	-0.562	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}
VP-FT	2611	5.502	13.318	26.146	3.809	70.671	403.173	406.65	74.951	16.194	19.102	21.166
	1.00	25.127	-23.063	-1.315	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}
VP-TT	2611	21.456	10.800	5.7203	2.574	103.75	115.374	112.46	25.727	7.472	8.143	8.819
	0.91	32.553	-15.906	-0.116	{0.000}	{0.000}	{0.000}	{0.000}	{0.012}	{0.000}	{0.000}	{0.000}
VP-VW	2611	13.278	10.481	36.143	3.800	67.039	292.262	308.05	68.810	12.653	14.908	15.746
	1.00	20.581	-21,2188	-2.004	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}

Series: VP-1 is the series containing the most frequently traded series (Prop.=100%) and VP-6 is the series containing the most thinly traded assets (Prop.=43%). VP-FT (VP-TT) is a portfolio series containing only equally weighted frequently (thinly) traded assets. VP-VW is the value-weighted all assets market index. Obs. is the number of observations for that series.

Mean is daily mean multiplied by 252 trading days and standard deviation is daily standard deviation multiplied by the square root of 252 trading days. Skew is a measure of heavy tails and asymmetry of a distribution (normal) and kurtosis is measure of too many observations around the mean for a distribution (normal). K-S Z-test: Used to test the hypothesis that a sample comes from a normal distribution. The value of the Kolmogorov-Smirnov Z-test is based on the largest absolute difference between the observed and the theoretical cumulative distributions. ARCH (6) : ARCH (6) is a test for conditional heteroscedasticity in returns. Low {·} indicates significant values. We employ the OLS-regression $y^2 = a_0 + a_1 \cdot y^2_{t-1} + \ldots + a_6 \cdot y^2_{t-6}$. $T \cdot R^2$ is χ^2 distributed with 6 degrees of freedom. T is the number of observations, *y* is returns and R^2 is the explained over total variation. a_0, a_1 ... a_6 are parameters. RESET (12,6) : A sensitivity test for mainly linearity in the mean equation. 12 is the number of lags and 6 is the number of moments that is chosen in our implementation of the test statistic. $T \cdot R^2$ is χ^2 distributed with 12 degrees of freedom. BDS (m=2,ε=1): A test statistic for general non-linearity in a time series. The test statistic BDS $= T^{1/2} \cdot [C_m(\sigma \cdot \varepsilon) - C_1(\sigma \cdot \varepsilon)^m]$, where *C* is based on the correlation-integral, m is the dimension and ε is the number of standard deviations. Under the null hypothesis of identically and independently distributed (i.i.d.) series, the BDS-test statistic is asymptotic normally distributed with a zero mean and with a known but complicated variance.

Table 2. Bivariate Estimation for Norwegian Series

Panel A: Mean equation

	VP-1		VP-2		VP-3		VP-4		VP-5		VP-6		VP-FT		VP-TT	
LL/PP	-8391		-8808.6		-9966		-10197		-11729		-11556		-6068		-9315	
	Coeff	t-value	Coeff	t-value	Coeff	t-value	Coeff	t-value	Coeff	t-value	Coeff	t-value	Coeff	t-value	Coeff	t-value
μ_i	0.0983	{2.79}	0.1415	{3.84}	0.0364	{0.73}	0.0578	{1.06}	0.1672	{2.04}	0.2253	{2.38}	0.0766	{3.32}	0.0909	{2.45}
μ_M	0.1010	{5.28}	0.1026	{5.21}	0.0757	{3.64}	0.0787	{3.76}	0.0567	{2.58}	0.0602	{2.70}	0.1157	{6.07}	0.0612	{2.86}
λ_i	-0.0041	-{0.05}	-0.0912	-{1.00}	0.0763	{0.75}	0.0507	{1.21}	-0.0920	-{1.83}	-0.0043	-{0.10}	0.0068	{0.11}	-0.1126	-{1.27}
$\lambda_{i,i,M}$	0.0098	{0.57}	-0.0110	-{1.77}	-0.0015	-{0.06}	0.0240	{0.82}	0.0007	{0.01}	-0.0969	-{0.88}	0.0054	{1.03}	0.0694	{0.97}
$\lambda_{i,M}$	-0.0096	-{0.08}	-0.0366	-{0.32}	-0.2771	-{1.70}	-0.2019	-{1.01}	-0.3817	-{2.51}	-0.0868	-{0.41}	-0.0088	{0.11}	-0.0590	-{0.54}
λ_M	-0.0590	-{0.73}	0.0127	{0.14}	-0.1013	-{1.13}	-0.1403	-{1.56}	-0.1303	-{1.34}	-0.2504	-{2.33}	-0.0430	-{0.46}	-0.1029	-{1.04}
θ_i	-0.1120	-{5.89}	-0.0928	-{5.09}	-0.0237	-{1.14}	0.0782	{3.88}	0.3073	{14.9}	0.3604	{22.0}	-0.0612	-{3.23}	0.2361	{11.6}
θ_M	-0.2645	-{11.7}	-0.2371	-{10.4}	-0.2425	-{10.8}	-0.2595	-{11.6}	-0.2624	-{11.2}	-0.2596	-{11.5}	-0.2259	-{11.5}	-0.2665	-{11.2}
$\phi_{M,i}$	0.0691	{1.95}	0.1323	{3.32}	0.1979	{4.08}	0.3042	{6.22}	0.3452	{4.56}	0.1279	{2.29}	0.2122	{8.86}	0.1939	{6.64}

Panel B: Volatility equation

	VP-1		VP-2		VP-3		VP-4		VP-5		VP-6		VP-FT		VP-TT	
	Coeff	t-value	Coeff	t-value	Coeff	t-value	Coeff	t-value	Coeff	t-value	Coeff	t-value	Coeff	t-value	Coeff	t-value
$m_{i,i}$	0.5649	{14.9}	0.6500	{12.8}	1.9341	{32.2}	0.8127	{23.2}	0.8167	{34.7}	0.7388	{26.2}	0.5381	{27.5}	0.4112	{13.8}
$m_{i,M}$	0.3476	{12.8}	0.4058	{17.2}	0.1369	{3.59}	0.4208	{20.2}	0.2276	{6.95}	0.1245	{2.45}	0.4422	{23.5}	0.1290	{2.14}
$m_{M,M}$	0.3143	{17.5}	0.2560	{12.7}	0.2704	{4.32}	0.2089	{10.5}	0.3780	{15.1}	0.4197	{20.2}	0.1142	{20.0}	0.4900	{17.7}
$a_{i,i}$	0.2053	{10.2}	0.1705	{8.92}	0.4361	{14.5}	0.3435	{22.7}	0.2143	{30.6}	0.3707	{47.8}	0.4235	{9.89}	0.2038	{13.5}
$a_{i,M}$	-0.0438	-{3.11}	0.0007	{0.05}	0.0644	{8.30}	0.0056	{1.00}	-0.0020	-{0.79}	0.0060	{2.42}	0.0923	{2.64}	0.0026	{0.15}
$a_{M,i}$	0.2780	{13.7}	0.3884	{19.2}	0.3322	{7.66}	0.2219	{5.47}	0.0070	{0.13}	0.0535	{2.39}	0.0928	{1.70}	0.1216	{6.07}
$a_{M,M}$	0.5368	{30.6}	0.4958	{21.8}	0.2971	{11.9}	0.3917	{25.1}	0.2942	{13.4}	0.1398	{5.44}	0.3916	{8.78}	0.3605	{19.3}
$b_{i,i}$	0.9593	{67.1}	0.9485	{63.2}	0.4222	{7.66}	0.9252	{184}	0.9666	{1142}	0.9531	{765}	0.8669	{31.8}	0.9487	{169}
$b_{i,M}$	0.0415	{3.44}	-0.0051	-{0.44}	-0.1030	-{5.06}	-0.0055	-{1.94}	0.0001	{0.09}	-0.0020	-{1.90}	-0.0366	-{1.50}	0.0008	{0.09}
$b_{M,i}$	-0.1554	-{5.80}	-0.2118	-{8.32}	0.3572	{4.04}	-0.2385	-{9.11}	-0.0777	-{2.55}	-0.0203	-{0.74}	-0.0993	-{2.92}	-0.0456	-{1.81}
$b_{M,M}$	0.7345	{37.2}	0.7900	{42.0}	0.9415	{36.0}	0.8204	{74.9}	0.8330	{65.9}	0.8538	{80.8}	0.8352	{29.2}	0.7822	{47.7}
γ_i	0.0079	{1.10}	0.0189	{3.98}	0.0383	{1.43}	-0.0097	-{0.78}	0.0233	{5.85}	-0.0834	-{11.8}	0.0160	{3.64}	0.0259	{2.71}
γ_M	0.0190	{1.76}	-0.0117	-{2.33}	0.0993	{5.06}	0.0594	{4.69}	0.1678	{6.62}	0.2319	{11.6}	-0.0104	-{2.49}	0.1678	{5.62}

Table 3. Specification Tests for Model mis-specification

	VP-1		VP-2		VP-3		VP-4		VP-5		VP-6		VP-FT		VP-TT	
	Coeff	C-p/t	Coeff	C-p/t	Coeff	C-p/t	Coeff	C-p/t	Coeff	C-p/t	Coeff	C-p/t	Coeff	C-p/t	Coeff	C-p/t
$Q_i(6)$	7.343	{0.29}	9.041	{0.17}	8.359	{0.21}	4.1020	{0.66}	0.9010	{0.99}	4.853	{0.56}	12.9820	{0.04}	3.4550	{0.75}
$Q_M(6)$	9.626	{0.14}	9.779	{0.13}	10.823	{0.09}	11.4720	{0.07}	7.6100	{0.27}	13.01	{0.04}	12.3260	{0.06}	7.2110	{0.30}
$Q_i^2(6)$	7.236	{0.30}	8.860	{0.18}	1.083	{0.98}	2.0590	{0.91}	8.7310	{0.19}	4.452	{0.62}	14.2070	{0.03}	9.2610	{0.16}
$Q_M^2(6)$	10.501	{0.11}	12.623	{0.05}	11.645	{0.07}	13.8720	{0.03}	9.4070	{0.15}	5.953	{0.43}	14.6910	{0.02}	11.7410	{0.07}
Kurt /Skew(i)	3.64451	0.015	3.532	0.196	4.563	-0.195	4.6525	-0.215	20.3089	-1.559	14.0319	-0.241	5.5997	-0.577	2.6863	-0.089
Kurt /Skew(M)	5.8093	-0.595	5.84226	-0.615	5.30441	-0.585	6.4837	-0.678	7.1310	-0.774	7.6444	-0.803	5.8348	-0.609	6.3221	-0.643
K-S Z-test(i;M)	2.1942	1.8866	2.5094	1.9069	2.8411	1.8703	3.1301	1.8938	5.1550	1.9198	7.8543	1.9899	1.8235	1.8377	1.8596	1.8671
$ARCH_i$ (6)	10.506	{0.57}	12.500	{0.05}	3.239	{0.78}	18.3556	{0.01}	7.0697	{0.31}	11.6715	{0.07}	13.9844	{0.03}	10.0873	{0.12}
$ARCH_M$ (6)	11.918	{0.45}	13.456	{0.04}	16.735	{0.01}	9.4614	{0.15}	9.7748	{0.13}	7.1013	{0.31}	15.5346	{0.02}	12.4258	{0.05}
$Reset_i$ (12;6)	12.719	{0.39}	12.921	{0.37}	15.205	{0.23}	11.0530	{0.52}	13.4778	{0.34}	8.4312	{0.75}	7.23637	{0.84}	10.299	{0.59}
$Reset_M$ (12;6)	12.027	{0.44}	11.435	{0.49}	12.489	{0.41}	12.7593	{0.39}	10.9170	{0.54}	14.4687	{0.27}	10.1111	{0.61}	10.728	{0.55}
BDS m=2; (i;M)	-0.3597	-0.863	-0.6827	0.035	-0.7723	0.963	3.3077	-0.786	8.59662	-1.001	23.3621	-0.910	0.6693	-1.137	-0.8400	-0.712
BDS m=3; (i;M)	0.1733	-1.644	-1.5157	-0.012	-1.6658	0.530	3.9467	-1.672	7.8729	-2.001	21.8691	-1.840	0.0906	-1.823	-1.8335	-1.676
BDS m=4; (i;M)	0.3663	-1.951	-1.9400	0.064	-1.9121	0.555	3.9801	-2.226	7.9742	-2.469	21.7570	-2.349	-1.0181	-1.554	-2.0969	-2.228
BDS m=5; (i;M)	0.4998	-1.915	-1.8140	0.429	-1.9026	0.530	4.5807	-2.273	8.1282	-2.111	21.3724	-2.027	-1.4027	-1.701	-2.1950	-2.065
BDS m=6; (i;M)	0.9338	-1.950	-1.8767	1.033	-1.9505	0.360	4.4237	-2.418	9.4077	-2.269	21.6228	-1.795	-1.4696	-1.831	-2.3355	-2.159

* See Table 1 for a definition of test statistics.

Table 4. Conditional Variance, Covariance and Beta Characteristics

		VP-1	VP-2	VP-3	VP-4	VP-5	VP-6	VP-FT	VP-TT
Variance	Asset	1.2605	1.5829	8.4951	13.0723	65.8657	46.4205	2.4640	4.0125
Mean	Market	1.4437	1.4359	1.6115	1.6004	1.5973	1.5625	1.6439	1.5994
Standard	Asset	4.7952	5.4085	19.8076	17.9694	167.493	55.1550	5.8109	2.3111
deviation	Market	4.0999	3.9084	3.9745	3.4825	3.7676	3.7159	3.9452	3.9084
Covariance	Mean	1.5055	1.0581	1.7499	1.4938	0.7045	0.6928	1.8573	0.4159
	St.dev.	4.0065	4.4745	6.7545	5.0872	1.5727	3.3183	4.7222	1.4056
Beta	Mean	1.2003	1.0459	0.9596	0.8958	0.5985	0.3810	1.0929	0.2342
	St.dev.	0.2409	0.1674	0.3546	0.6079	0.7067	1.1209	0.1304	0.1743

* See Table 1 for the definition of the series. St.dev.= Standard Deviation.

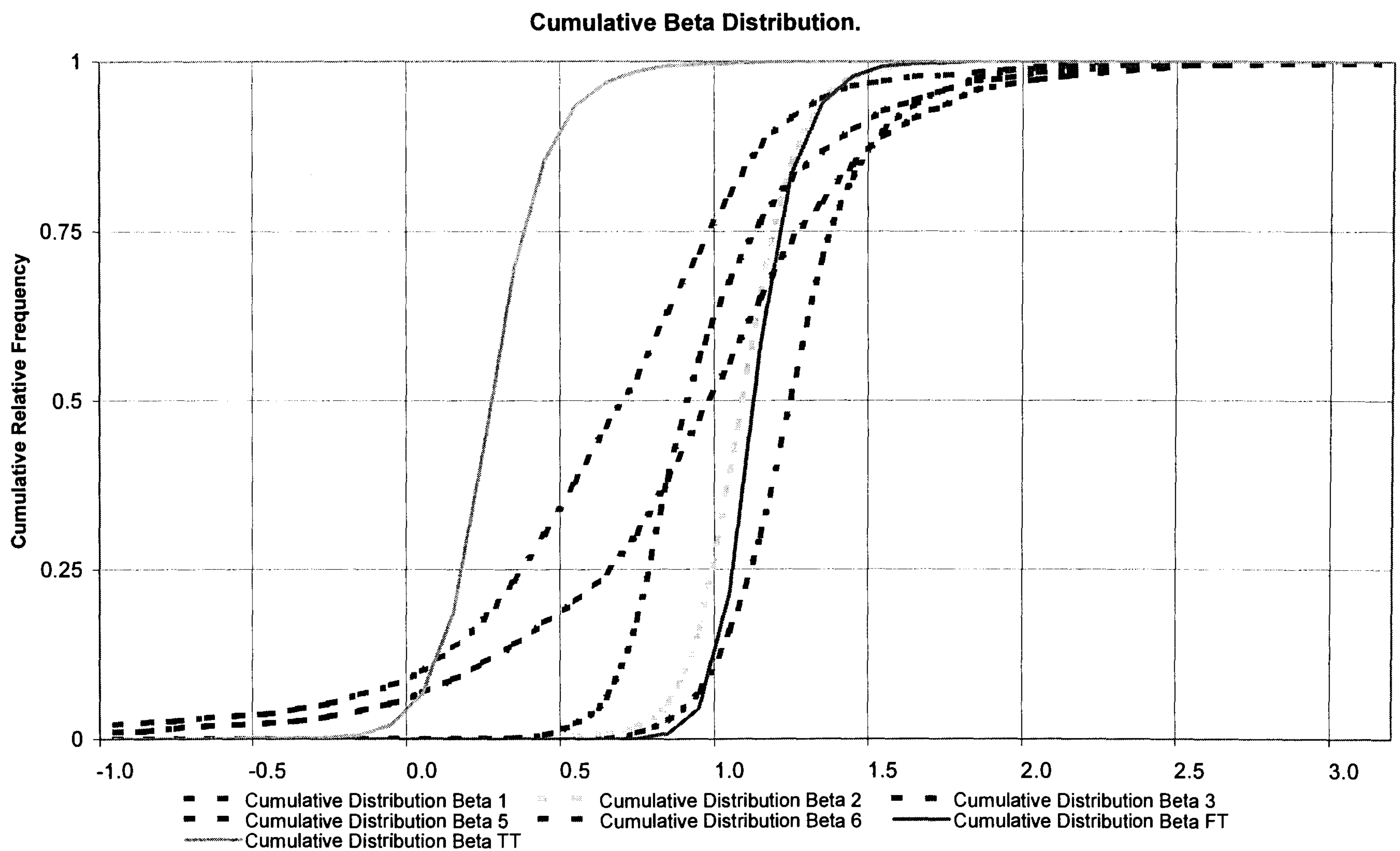


Figure 1. Frequency Distribution for the Conditional Beta series.

Essay no. 6*

Event-induced Volatility in Thinly Traded Markets

Abstract

This paper investigates non-synchronous trading effects as well as volatility clustering in asset returns during event and non-event periods in the Norwegian equity market. The main objective is to find any periodic differences from non-event to event periods in the conditional mean and volatility characteristics. The empirical specifications show significant serial correlation in the mean equation suggesting changing mean. The volatility specification shows significant clustering suggesting changing volatility. The empirical results for univariate and bivariate specifications suggest (1) that the conditional mean show changes and (2) that the conditional volatility increases strongly. Our results suggest that non-synchronous trading influences considerably mean and volatility parameters and in extreme cases, lag specifications. Hence, inferences may change classical event studies findings. The paper proposes an abnormal return model controlling for non-synchronous trading and volatility clustering.

Classification: C14

Keywords: Event studies, ARMA-GARCH specifications, Non-synchronous trading, Volatility Clustering

* Forskpub., Molde University College

1 Introduction

The main purpose of the paper is to show the need to control for event-induced changes in mean and volatility suggesting possible changes in inferences from classical event studies. Non-synchronous trading changes due to a shift in trading frequency may suggest a changing mean. Moreover, many authors have identified the hazards of ignoring event-induced volatility. Employing ARMA - (G)ARCH methodology[1] for the conditional mean and volatility processes we can control for both a changing mean equation and a changing volatility equation. This paper aims to show that firm samples, from non-event to event periods, reports changes in the conditional mean and volatility characteristics and report strong overall increase in the conditional volatility series. Boehmer et al. (1991) warn of event induced volatility in event studies and suggests a simple adjustment to the OLS test statistics. For the ARMA-GARCH maximum-likelihood estimation the model specification produces residuals that are unconditional homoscedastic and therefore suggest unadjusted abnormal return test statistics. Hence, finding mean and volatility changes or a strong increase in event volatility relative to non-event periods, justify strongly the ARMA-GARCH methodology.

This paper does not investigate the cause of event-induced volatility[2] but rather show the need to control for changing volatility. We employ a sample of mergers and acquisitions in the Norwegian thinly traded equity market. To determine the level of the volatility in event and non-event periods we form return series employing the event and non-event period firm samples. Event series are formed from event period firms in three different event windows. Non-event series are formed from non-event period firms where the return series are all collected from periods outside the largest event window. Our objective is to establish the mean and volatility characteristics for all these event and non-event series. The time series models must contain several elaborate features to avoid misspecification.

Firstly, in thinly traded markets non-synchronous trading may produce serious biases in the moments and co-moments and therefore may produce spurious relationships (Campbell, 1997 and Solibakke, 2001a, 2001b). To control for non-synchronous trading we employ an ARMA(p,q) lag specification in the conditional mean. The lag specifications for p and q are the BIC preferred (Schwarz, 1978) model. Secondly, the volatility of all event and non-event portfolios is specified employing (G)ARCH formulations, to control for volatility clustering and changing volatility. The conditional volatility are modelled applying a BIC preferred ARMA (m,n) lag specification for squared residuals from the conditional mean specification. The volatility series are therefore readily available from the estimations. Thirdly, asymmetric volatility is modelled as shown by Glosten et al. (1993) and Nelson (1991). Finally, leptokurtosis is found in the Norwegian equity market as in all other international equity

[1] An alternative methodology is Semi-Non-Parametric (Gallant & Tauchen, 1989, 1991).
[2] Brown et al. (1988,1989)

markets. Hence, we employ a univariate ARMA-GARCH lag specification with student t-density distributions (Bollerslev 1986/87) and bivariate GARCH-in-Mean models (Engle & Kroner, 1995) with multi-normal distributions.

Our specification tests show that both the univariate and bivariate ARMA-GARCH models filters all non-synchronous trading and volatility clustering in event and non-event periods. Hence, as our results suggest no data dependence in the return series, we can report no model misspecification. Moreover, if this result maintains its validity into the market model we have obtained a sounder basis for abnormal return calculations in event-studies employing unadjusted test statistics.

The investigation reports a strongly higher conditional volatility in event periods relative to non-event periods for both selling and acquiring firms. Therefore the investigation proposes new models for classical event studies in the future.

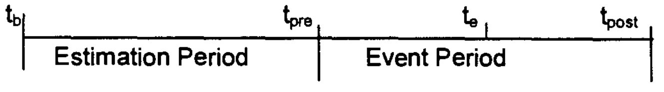
This paper extends previous works in several areas. Firstly, the increase in conditional volatility from non-event to event periods is measured employing both univariate and bivariate subordinated stochastic volatility specifications (Clark, 1973, Epps and Epps, 1976, Tauchen and Pitts, 1983). The bivariate ARMA-GARCH model is employed to better specify market dynamics and cross-autocorrelation in mean and volatility. Secondly, the leptokurtosis in distributions often found in stock markets are considered using student-t density log-likelihood functions. Thirdly, we employ asymmetric conditional volatility parameters for all estimations. Fourthly, elaborate specification tests are employed for model misspecification. Finally, we propose a new event-study methodology controlling for non-synchronous trading and volatility clustering employing the market model in classical event studies.

The remainder of the article is organised as follows. Section 2 defines the event periods, describes the equal-weighted series approach in event and non-event periods and defines the conditional mean and volatility equations from the family of ARMA-GARCH lag specification models. Section 3 describes the empirical data and the time series adjustment procedures. Section 4 reports the univariate results from the analysis. Section 5 reports the bivariate estimation results. Section 6 investigates the significance of the conditional volatility increase applying likelihood ratio tests. As Section 6 report significant changes, Section 7 suggests two time-series specifications for classical event studies. Finally Section 8 summarises our findings.

2. Definitions and Methodology

2.1 The Event Period

In event studies, the objective is to examine the market's response through the observation of security prices around such events. For merger and acquisitions[3] it is related to the release of information to market participants through the financial press. Normal or predicted returns for an asset are those returns that are expected if no event occurs. The time line for a typical event study for a mergers and acquisitions case may be represented as follows



where t_b is the first period used in the estimation of a normal security return; t_{pre} is the first period used in the calculation of abnormal returns; t_e is the event date; and t_{post} is the last period used in the calculation of abnormal returns. In the literature we usually find a selection of t_{pre} equal to -40 days and t_{post} equal to + 40 days relative to t_e (day 0). Hence, the event period will in this case consist of 80 days. Our study applies also narrower event periods. We define event periods of t_{pre} equal to -20 (-10) days and t_{post} equal to + 5 (+1) days relative to t_e (day 0). Note that the length of the estimation period is not relevant for this portfolio study. However, in a classical event study the length of the estimation period is an important decision to make.

2.2 Event and Non-Event Return Series

To study any change in return and volatility characteristics from non-event to event series we form equally weighted portfolios from firm return series classified in event periods and non-event periods. The classification of an event period follows the definitions of t_{pre} and t_{post} in section 2.1. All firms that by definition are categorized into a specific event period are included in the sample and the returns are averaged over the whole sample for each day relative to t_e.

These calculations for portfolio returns becomes $PR_{c,t} = \frac{1}{N_{c,t}} \cdot \sum_{i=1}^{N_{c,t}} R_{c,t,i}$ where $R_{c,t,i}$ is the continuously compounded return for portfolio c, day t, asset i. $PR_{c,t}$ is portfolio c's return at date t. $N_{c,t}$ is the number of assets in portfolio c at date t.

Note that the number of firms $N_{c,t}$ may change over time in especially the event series. Therefore, a possible and permissible value for $N_{c,t}$ is zero. The time series will set these dates to missing observations.

The definition of non-event series follows this procedure. Firstly, we find the average number of firms over all event dates for the widest event window {-40,+40} days relative to t_e and employ this number of assets for the non-event series. The returns are calculated as above using a random sample of firms for the non-event sample. Return series characteristics are reported in Section 4 below.

Finally, note that model specification assumes an ergodic and stationary return series. An ergodic return series suggest that the sample moments for finite stretches of the realisation approach their population counterparts as the length of the realisation becomes infinite. A stationary time series mean that the process is in a particular state of "statistical equilibrium" (Box and Jenkins, 1976). Strict stationary is obtained if its properties are unaffected by a change in time origin. In Section 3 we will apply an adjustment procedures to secure ergodic and stationary return series.

2.3 The Conditional Mean and Volatility Specifications

We will apply the ARMA-GARCH specification for estimation of the mean and volatility equations. The methodology applies conditional models where non-synchronous trading may be modelled in the conditional means and volatility clustering may be modelled in the conditional volatility. The ARMA methodology may be studied in detail in Mills (1990), while (G)ARCH specifications may be studied in Engle (1982) and Bollerslev (1986, 1987). In the international finance literature we find a high number of papers with origin from these pioneer works. For a small sample we refer to Bollerslev et al. (1987,1992), Engle et al. (1986, 1995), Nelson (1991) og deLima (1995a, 1995b). Moreover, Glosten et al. (1993) extended the GARCH model to truncated GARCH to account for the leverage effects. The ARMA-GARCH methodology may be univariate og multivariate. As event studies apply the market model to specify normal returns the multivariate model may be more relevant than the univariate model.

2.3.1 The univariate and asymmetric ARMA-GARCH-in-Mean specification

The general asymmetric ARMA(p,q) - GARCH(m,n) -in-Mean specification of the conditional mean and volatility can be defined as follows:

$$R_{j,t} = \phi_{j,0} + \sum_{i=1}^{p} \phi_{j,i} \cdot R_{j,t-i} + \delta_j \cdot h_{j,t}^{\frac{1}{2}} + \varepsilon_{j,t} - \sum_{i=1}^{q} \theta_{j,i} \cdot \varepsilon_{j,t-i} \tag{1}$$

$$\lambda_{j,i,t} = \gamma_{j,i} \quad \text{if and only if} \quad \varepsilon_{j,t-i} < 0 \tag{2}$$

$$E(\varepsilon_{j,t}^2 \mid \Phi_{j,t-i}) = h_{j,t} = m_{j,0} + \sum_{i=1}^{m} (a_{j,i} + \lambda_{j,i,t}) \cdot \varepsilon_{j,t-i}^2 + \sum_{i=1}^{n} b_{j,i} \cdot h_{j,t-i} \tag{3}$$

---

[3] For an OLS study of abnormal returns in Norway see Eckbo and Solibakke, 1992. For an international review Eckbo, 1987.

where $R_{j,t}$ is the portfolio *j's* return in period *t*; $\varepsilon_{j,t}$ is a random variable (residual) distributed as either normal $N(0,\sigma^2)$ or student-t $D(0,\sigma^2,\omega)$ where ω is the degree of freedom; $\theta_{i,j}$ is lag *i* for the moving average or non-synchronous trading parameters of portfolio *j* in the conditional mean equation (1); $\lambda_{j,i,t}$ measures the leverage effect, $m_{j,0}$ is the constant term for portfolio *j* in the conditional volatility equation; $a_{j,i}$ is lag *i* for auto-regressive parameters for shocks of portfolio *j*; and $b_{j,i}$ is lag *i* for the conditional volatility parameters of portfolio *j*. The lag lengths *p, q, m* and *n* are determined by the BIC criterion (Schwarz, 1978) for all series.

Linear models have constant conditional volatility whatever the information of observed returns. In our approach the conditional volatility may vary but the unconditional volatility is constant. Hence, the equations above lead naturally to the consideration of non-linear stochastic processes and the (G)ARCH-in-Mean model[4] (Engle, Lillien and Robbins (1987)) show a departure from white noise. Specifically, in our model we allow the serially correlated errors to be modelled as a moving average (*MA(q)*) process to capture the effect of non-synchronous trading, while the innovations $\varepsilon_{j,t}$ can be assumed to follow either a conditional normal - or a conditional student-t distribution. The conditional volatility enters the mean equation (in-Mean). Estimation normally applies the BHHH (1974) algorithm.

2.3.2 The bivariate and asymmetric ARMA-GARCH-in-Mean specification

As event studies apply the market model and therefore an overall market index to calculate normal returns, the univariate ARMA-GARCH models may not count for total market dynamics. Moreover, the index series may also contain non-synchronous trading and volatility clustering. Hence, to control for these market structure effects we employ a bivariate specification between return series and the overall index series. We employ a value-weighted index as a proxy for the market portfolio.

To model this bivariate specification we apply the MGARCH model. The Multivariate GARCH-in-Mean model (BEKK-formulation)[5] is defined as (in vector format)

$$\boldsymbol{R_t} = \phi_0 + \phi_1 \cdot \boldsymbol{R_{t-1}} + \Delta \cdot vech(\boldsymbol{H_t}) + \varepsilon_t - \theta_1 \cdot \varepsilon_{t-1} \quad (5)$$

$$\boldsymbol{H_t} = \boldsymbol{m'} \cdot \boldsymbol{m} + \boldsymbol{A'_1} \cdot \varepsilon_{t-1} \cdot \varepsilon'_{t-1} \cdot \boldsymbol{A_1} + \boldsymbol{B'_1} \cdot H_{t-1} \cdot \boldsymbol{B_1} \quad (6)$$

where $\varepsilon_t \,|\Omega_{t-1} \sim N(0,\boldsymbol{H_t})$ and *vech* $(\boldsymbol{H_t})$ is the column stacking operator of the lower portion of a symmetric matrix, $R_t = \begin{pmatrix} R_{i,t} \\ R_{M,t} \end{pmatrix}, \varepsilon_t = \begin{pmatrix} \varepsilon_{it} \\ \varepsilon_{Mt} \end{pmatrix}, \Delta = \begin{pmatrix} \delta_{11} & \delta_{1M} & \delta_{13} \\ \delta_{21} & 0 & \delta_{MM} \end{pmatrix}$, where μ is 2 x 1 vector of

[4] For applications see Bollerslev, Chou, Kroner, 1992.
[5] Engle and Kroner (1995); BEKK is named after an earlier working paper of Bollerslev, Engle, Kraft and Kroner). Moreover, a VEC or VECH formulation is also readily available.

constants in the conditional mean. $\boldsymbol{m}$, $\boldsymbol{A_1}$, $\boldsymbol{B_1}$ are 2 x 2 parameter matrices, and the elements of the conditional volatility matrix $\boldsymbol{H_t}$ are $h_{i,t} = var_t(R_{i,t})$, $h_{i,M,t} = cov_t(R_{i,t}, R_{M,t})$, and $h_{M,t} = var_t(R_{M,t})$. The θ_1 parameter specifies non-synchronous trading in the bivariate system of mean equations. The δ_{11} is the ARCH-in-Mean parameter in the equation of $R_{i,t}$ that corresponds to $h_{i,t}$, δ_{1M} is the ARCH-in-Mean parameter in the equation of $R_{i,t}$ that corresponds to $h_{i,M,t}$ and δ_{13} is the ARCH-in-Mean parameter in the equation of $R_{i,t}$ that corresponds to $h_{M,t}$. δ_{21} is the ARCH-in-Mean parameter in the equation of $R_{M,t}$ that corresponds to $h_{i,t}$, and δ_{MM} is the ARCH-in-Mean parameter in the equation of $R_{M,t}$ that corresponds to $h_{M,t}$. Note that the conditional volatility specification for $\boldsymbol{H_t}$ in (6) guarantees the positive definiteness of $\boldsymbol{H_t}$ and allows feedback between the volatility of the individual portfolio and the market. **m** is a lower triangular matrix. Finally, we extend this model to measure asymmetric volatility applying the GJR methodology (Glosten et al., 1993) in the bivariate estimation. Hence, we extend the above model by the parameters γ_1 for asymmetry in the portfolio and γ_2 for asymmetry in the market index. Note that this bivariate ARMA(1,1)-GARCH(1,1) can be extended to any lag lengths *p, q, m* and *n* as specified for univariate specifications. The Bayes Information Criterion (BIC) is applied for both ARMA (mean) and GARCH (volatility) lag specifications. As for univariate GARCH estimations, the BHHH (1974) algorithm is applied for estimation.

3 Empirical Data sources and Time Series Adjustments

The study uses daily continuously compounded returns ($\ln \frac{P_t}{P_{t-1}}$) of individual Norwegian stocks spanning the period from October 1983 to February 1994. The logarithmic returns are scaled by one hundred to avoid any scaling problems during estimation. Data are obtained from Oslo Stock Exchange Information A/S. The data includes the crash period of October 1987. There is no reason to exclude these outliers since they reflect the nature of the market. The raw data series for individual assets are grouped into portfolios as described in section 2.2. The dataset is therefore composed of event portfolios, non-event portfolios and one market index. The event and non-event portfolios are divided into seller (S), acquirer (A) and both seller and acquirer (B) portfolios. We define three different event period windows; (1) from 10 days before to 1 days after an announcement (PE{-10,+1}); (2) from 20 days before to 5 days after the announcement (PE{-20,+5}); and finally (3) from 40 days before to 40 days after the announcement (PE{-40,+40}). The non-event portfolios are formed by a random selection of event firms consisting of selling (PSNE), acquiring (PANE) and both selling and acquiring firms (PBNE), respectively. All firms in the non-event portfolios exclude event periods of -40 to +40 days relative to announcements. In case of several announcements for an individual firm all periods –40 to +40 days relative to announcements are excluded.

{Insert Table 1 about here}

Therefore, this daily time series database gives us potentially 2611 observations for each portfolio and index. This number of observations provides enough degrees of freedom to permit use of asymptotic tests. However, the event portfolios will most likely consist of a varying number of assets over dates and especially the shortest event period window will consist of a number of missing observations. Hence, all the sample sizes will be reported. The characteristics of the raw data from event and non-event equally weighted asset portfolios and the value weighted market index, are reported in Table 1.

The following immediate observations can be made. The mean returns are highest for the seller firm event portfolios in the two narrowest announcement period windows. The longest time period window for the selling companies show a considerably lower daily mean return. Moreover, compared to all other portfolios, the daily return standard deviations for selling firm portfolios are the highest for all three event period portfolios. For the shortest event period the mean return is 5 times and the standard deviation 3 times as high as the market index values. The same numbers for the acquiring portfolios are considerably smaller. That is, both expected return and standard deviation are highest for the event portfolios formed from selling firms. The non-event portfolios show results close to the market index. Hence, Table 1 suggests event-induced price and return turbulence. Figure 1, panel A, plots the raw value weighted market index. From this time series it seems to exist several periods of high volatility followed by periods of lower volatility. However, any pattern is not readily observable from the plots.

{Insert Figure 1 Panel A and B about here}

Following Gallant et al. (1992) many authors have noted systematic calendar effects in both mean and volatility of price movements. Hence, we adjust all portfolio and index time series by regressing the scaled returns on the set of adjustments variables: $\omega = x'\beta + u$ (mean equation). The adjustment variables consist of dummy and time-trend variables. The least square residuals are taken from the mean equation to construct a volatility equation: $ln(u^2) = x'\gamma + \varepsilon$. Finally, a linear transformation is performed to calculate adjusted return series (ω): $\omega_{adj} = a + b*(u/exp(x'\gamma/2))$, where a and b are chosen so that the sample means and volatility of ω and ω_{adj} are identical. This adjustment procedure for all portfolios and the value-weighted index allow us to focus on the day-to-day dynamic structure under an assumption of stationary series. We plot the adjusted value weighted index in Figure 1 Panel B. As for the index, all event and non-event portfolios show an adjusted time series that become more homogenous over time. Further discussion of the effects from the adjustment procedure is found in Gallant et al. (1992). Owing to space requirements we do not report details from the adjustment results[6].

To get an idea of the return distributions of the portfolio and index series, we have also reported the kurtosis and skew in Table 1. The numbers report leptokurtosis in all series. We find too much probability mass around the mean and too low probability mass around 1 and 2 standard deviation from the mean. The numbers for the skew is strongly negative for the market index and strongly positive for the shortest event firm portfolios. Hence, the event portfolios show more positive extreme return values than negatives in contrast to the market index. The kurtosis and skew suggest that the returns are not normally distributed. Hence, to accomplish this deviation from normality we employ a student t-density distribution in the log-likelihood function for the GARCH estimation. For the bivariate GARCH estimation (MGARCH) we assume a multi-normal distribution.

Finally, the portfolios and the index all report significant ARCH test statistics. The test statistic suggest volatility clustering and make the (G)ARCH methodology employable for all our sample series.

4 Empirical Results for Univariate Time Series

4.1 The univariate and asymmetric ARMA-GARCH-in-Mean[7] specification

Maximum likelihood estimates of the parameters in equation (1), (2) are given in Table 2 for a student-t density log-likelihood function. The constant ϕ_0 in our model is expected to be positive showing a positive drift. All the ϕ_0's are insignificant, which suggest that all the series cannot report a non-zero drift.

Non-synchronous trading or serial correlation is negative and statistical significant for the market index and all non-event portfolios except selling firms. This result suggests that the selling firms are thinly traded assets. The event series show all negatives or close to zero autocorrelation coefficients. However, for the shortest event period window, the series show insignificant coefficients. Hence, the market is reasonable information efficient in expectation of announcements (immediate adjustment). Our results therefore suggest that non-synchronous trading may be important to control for in classical event studies.

{Insert Table 2 about here}

[6] The results are readily available from the author upon request.
[7] All series and the market index BIC prefers p=0 and q=m=n=1.

The parameter for residual risk and contemporaneous conditional volatility (β) is negative but insignificant for all portfolios. The result suggests an insignificant negative relationship between return and volatility, which suggest lower returns in high volatility regimes. The insignificance may also suggest higher relevance for systematic risk (market risk) as suggested by several asset-pricing models. The in-Mean formulation seems therefore to be redundant in these ARMA-GARCH models.

Among the estimated conditional volatility ARCH/GARCH coefficients for the GARCH specification reported in Table 2, which are all strongly significant, we find clear patterns. Firstly, the constant coefficient m_0 in the conditional volatility process in the GARCH model is small but significant for the index, non-event series and the longest event periods. The result suggests a significant coefficient for unexplained conditional volatility. For the two narrowest event windows we find a strong increase in the m_0 coefficient, which suggests a strong increase in unexplained conditional volatility, which most likely is attributable to the events. The increase is especially strong for series consisting of selling firm. Secondly, the past squared errors have more influence over the conditional volatility of the two narrowest event portfolios than they do over the conditional volatility of the non-event portfolios and the index. The result suggests more sensitivity to past shocks for event portfolios relative to non-event portfolios. Thirdly, in contrast to the squared past error, the past conditional volatility exerts a greater influence over the current conditional volatility for non-event portfolios than event portfolios. Hence, the autocorrelation in the conditional volatility process is lower for event portfolios. For especially the event series most centred on the announcement day, we find low coefficients for the past conditional volatility. The parameter for asymmetric volatility is significant and negative for the index and all non-event portfolios. None of the event portfolios report significant asymmetric volatility. Our results therefore suggest that asymmetry may be redundant in event periods but is required in on-event periods.

{Insert Table 3 about here}

Summary values for the conditional volatility process ($h_{i,t}$) are reported in Table 3. Table 3 clearly indicates event-induced volatility. Highest relative conditional volatility increase is found for selling firm series. Our results indicate a 3 to 4 times mean increase in the conditional volatility. Also the acquiring firm series report an increase in the conditional volatility, but clearly smaller than selling firms. Hence, results suggest a need to control for the increased volatility in the event periods, for especially selling firms.

{Insert Table 4 about here}

Finally, as a specification test of our ARMA-GARCH models, we calculate the sixth order Ljung-Box (1978) statistic for the standardised residuals and squared residuals of each of the portfolios and the market index's expected returns in Table 4. In each portfolio there are no significant evidence of serial correlation (1%) in the residuals and squared residuals up to lag 6. The kurtosis is strongly reduced and all portfolios show lower absolute skews for the standardised residuals. In comparison to the adjusted raw returns the K-S Z-test confirms the more normal distributed residuals. The two features, close to normal residuals and the highly significant student-t density parameter υ_t seem to emphasis the importance of thick tails estimations. Our results therefore suggest that student-t densities in the log-likelihood function are preferred in classical event studies. The ARCH tests report no conditional heteroscedasticity in the standardised residuals. Hence, all conditional heteroscedasticity is captured by the GARCH specification of the conditional volatility. The BDS (Brock et al., 1991, 1995) test statistic shows insignificant values at all dimensions (m) and $\varepsilon = 1$ standard deviations. Hence, no data dependence and non-linearity is found in the standardised residuals. However, the joint bias tests (Engle and Ng, 1993) report some significant test statistics. Hence, we will find some bias in the conditional volatility prediction. However, overall our specification test results indicate that the current univariate and asymmetric ARMA-GARCH models are appropriate models for stock returns in event studies. Moreover, analytical, intuitive and linear reasoning may be conducted as we find insignificant test statistics for data dependence in all series.

5 The Bivariate and Asymmetric ARMA (p,q)-GARCH (m,n)-in-Mean[8] specification

Maximum likelihood estimates of the parameters for the bivariate GARCH-in-Mean model are presented in Table 5A and 5B for all portfolios. The bivariate estimation controls for market dynamics by incorporating a value-weighted market index into the estimation. The two intercepts (ϕ_0 and ϕ_M) in the mean equations from our bivariate system in Table 5A are positive indicating a positive drift.

Autocorrelation is present in all bivariate estimations except for selling firms. The market index shows negative and significant autocorrelation coefficients for all estimations. The event series show all significant negative coefficients except for the two selling firm series most centered on the announcement day. Our results therefore suggest that coefficients for non-synchronous trading are needed in almost all the bivariate event estimations. Cross-autocorrelation from index to event series is significant for all series.

The GARCH-in-Mean parameters can be reported for several alternative outlines of the mean equation. However, we estimate and report only the diagonal volatility matrix in the mean equation (only variances). The event series volatility may be interpreted as residual risk and can be considered as a proxy for omitted risk factors (Lehmann, 1990). None of the portfolios

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[8] All portfolios and the market index BIC prefer q=m=n=1.

and the market index report significant "in-Mean" coefficients. Hence, the "in-Mean" specification seems therefore redundant.

{Insert Table 5A and 5B about here}

From Table 5B the t-statistics indicate that all the conditional volatility parameters are almost all statistical significant. This result cast doubt on the validity of the univariate model specification. The constant coefficients in the conditional volatility equations report the same effects as for univariate estimations. The constant term m_{11} show considerable increases in especially the selling firm event portfolios compared to the non-event firm portfolios. This result implies that there is an increase in conditional volatility that is not possible to explain by the ARCH/GARCH coefficients alone (unexplained increase). The increase is also found for the constant term m_{22}. However, the increase is considerably smaller than for m_{11}. Moreover, the increase is higher the narrower the event period and therefore shows the highest non-explainable conditional volatility. For the past squared errors we find that the event series most centred on the announcement day are considerably more sensitive to past shocks than the non-event portfolios. In contrast the past conditional volatility exerts a greater influence over the current conditional volatility in the case of the non-event series. Moreover, as for the univariate estimations, the past conditional volatility coefficients show a decrease the shorter the event period for selling and acquiring firm series. We report values for the conditional volatility process ($h_{i,t}$) in Table 6. The results for the $h_{i,t}$ processes are very similar to that obtained in the univariate estimations. Moreover, $h_{i,t}$ produces mainly the same time series plots. Table 6, as Table 4 for the univariate case, clearly indicates event-induced volatility. Highest relative conditional volatility is found for the selling firm series. Hence, also our bivariate results indicate a 3 to 4 times mean increase in the conditional volatility for selling firms. Moreover, we find that the acquiring firm portfolios show an increase in the conditional volatility, but clearly smaller than selling firms. Therefore, as for the univariate case, our bivariate results suggest a need to control for volatility clustering in event studies.

{Insert Table 6 about here}

As for univariate models, specification tests of the bivariate model are performed and reported in Table 7. We find no significant serial correlation in the residuals and squared residuals up to lag 6. Furthermore, the bivariate cross-correlation series for -10 and 10 lags are calculated and checked (not reported). The result suggests very low to no significant cross-correlation in any lag for all bivariate ARMA-GARCH-in-Mean estimations. All portfolios and the market index show excess kurtosis for the standardised residuals. Almost all portfolio residuals show negative skews, except for the narrowest event portfolios. Moreover, the bivariate estimations, report lower kurtosis and skews than the univariate estimations. However, the K-S Z-test still reports non-normal standardised residuals for almost all portfolios and the index. The ARCH test statistic reports no volatility clustering in the standardised residuals. The BDS test statistic for i.i.d. reports that none of the portfolios show significant non-linear dependence at any dimension. Finally, the joint bias test reports no prediction bias for the conditional volatility. Hence, our bivariate model survives the specification tests and is at the same time a parsimonious model, which is able to capture dynamic structure.

{Insert Table 7 about here}

Since the market seems to play an important role, a univariate representation of the conditional volatility of stock returns will be disputed. Moreover, the specification tests unambiguously prefer a bivariate estimation technique.

6 Changing conditional volatility

To test for the changing volatility hypothesis from non-event to event firm samples, we perform a Likelihood Ratio Test (LRT). This test is a general test for testing the restrictions imposed on a model. The model is first estimated without any restrictions. The model is then re-estimated with the restrictions in place. Under the null hypothesis, LRT is distributed as χ^2 with number of restrictions as degrees of freedom. For our analysis we restrict the event samples GARCH parameters to be within the intervals obtained from non-event samples GARCH parameters. As we employ a GARCH (1,1) lag specification we introduce 6 restrictions on the event-period sample GARCH estimations. We report the LRT values with corresponding test statistics in Table 8.

{Insert Table 8 about here}

Table 8 report a significant change in parameter values for both univariate and bivariate estimations from non-event to event samples. The LRT test statistics rejects unchanged parameter values for all event series. Hence, our results suggest a significant increase in conditional volatility. The increase suggests a need for new event methodologies controlling for this increase as well as the significant non-synchronous trading effects in all our samples.

Therefore in Section 7, we suggest two alternative events study techniques, a univariate and a bivariate specification that models non-synchronous trading and volatility clustering. Note that we in the univariate specification do not control for non-synchronous trading and volatility clustering in the market index.

7 ARMA-GARCH Specifications for event study methodology

Our results suggest that we find different mean and volatility effects during event and non-event periods. Our findings therefore suggest a need for more advanced techniques for calculation of abnormal returns in classical event studies. In this paper we have shown that both univariate and bivariate GARCH-in-Mean models indicate higher conditional volatility for event portfolios relative to non-event portfolios. Hence, our results suggest event study models that put emphasis on non-synchronous trading and volatility clustering. ARMA mean equations emphasis non-synchronous trading and GARCH volatility equations emphasis volatility clustering.

Hence, our results suggest either a univariate or a bivariate ARMA-GARCH specification for estimation of abnormal return during event periods. Below we define these models for use in classical event studies employing the market model.

The first model is the univariate ARMA(p,q)-GARCH(m,n)-in-Mean model for individual assets. Based on our results from Section 4, this model becomes

$$R_{j,t} = \phi_{j,0} + \sum_{i=1}^{p} \phi_i \cdot R_{j,t-i} + \beta_{j,1} \cdot R_{M,t} + \xi_k \cdot D_{j,i,t} + \varepsilon_{j,t} - \sum_{i=1}^{q} \theta_i \cdot \varepsilon_{j,t-i}$$

where $j = 1, ..., N$ event firms, $R_{M,t}$ is the appropriate raw exogenous market index, $D_{j,i,t}$ is a dummy variable with value 0 outside the event period and 1 inside the event period. The definition of $D_{j,i,t}$ decides the length of the event period. $E(\varepsilon_{i,t} | \Omega_{t-1}) \sim D(0, h_{i,t}, \upsilon_i)$ is the student t-density distribution with υ degrees of freedom. Finally we define the $h_{i,t}$ employing the asymmetric GARCH(m,n) formulation for the conditional volatility process

$$\lambda_{j,i,t} = \gamma_{j,i} \quad \text{if and only if} \quad \varepsilon_{j,t-i} < 0$$

$$E(\varepsilon_{j,t} | \Omega_{t-1}) = h_{j,t} = m_{j,0} + \sum_{s=1}^{m} (a_{j,s} + \lambda_{j,s,t}) \cdot \varepsilon^2_{j,t-s} + \sum_{u=1}^{n} b_{j,u} \cdot h_{j,t-u} + c_{j,t} \cdot D_{j,t,t}$$

where $\varepsilon_{j,t}$ is the return for firm j day t; $\gamma_{j,t}$ is leverage effects (asymmetry), $m_{j,0} > 0$, $a_{j,1}, b_{j,1} \geq 0$, $a_{j,1} + b_{j,1} < 1$, and Ω_{t-1} is the set of all available information at time $t-1$. The model captures non-synchronous trading and volatility clustering for every asset j. However, the $R_{M,t}$ is the adjusted raw market returns.

The second proposed model is the bivariate ARMA(p,q)-GARCH(m,n)-in-Mean model. This model becomes[9]:

$$R_{j,t} = \phi_{j,0} + \sum_{i=1}^{p_j} \phi_i \cdot R_{j,t-i} + \beta_{j,1} \cdot \varepsilon_{M,t} + \delta_{j,k} \cdot D_{j,k,t} + \varepsilon_{j,t} - \sum_{i=1}^{q_j} \theta_i \cdot \varepsilon_{j,t-i}$$

$$R_{M,t} = \phi_{M,0} + \sum_{i=1}^{p_M} \phi_{M,i} \cdot R_{M,t-i} + \varepsilon_{M,t} - \sum_{i=1}^{q_M} \theta_{M,i} \cdot \varepsilon_{M,t-i} \quad and$$

$$H_{s,t} = m_{s,0}' \cdot m_{s,0} + \sum_{u=1}^{m} A'_{s,t-u} \cdot \varepsilon_{s,t-u} \cdot \varepsilon'_{s,t-u} \cdot A_{s,t-u} + \sum_{u=1}^{n} B'_{s,t-u} \cdot H_{s,t-u} \cdot B_{s,t-u} \; ; \; s=j,M,$$

9 You may incorporate cross-autocorrelation in the bivariate estimation.

where $\varepsilon_t \mid \Omega_{t-1} \sim N(0,H_t)$. ϕ_0 is 2 x 1 vector of constants, $m_{s,0}$, A_s, B_s are 2 x 2 parameter matrices, and the elements of $H_{s,t}$ are $h_{j,t} = var_t(r_{j,t})$, $h_{j,M,t} = cov_t(r_{j,t}, r_{M,t})$, and $h_{M,t} = var_t(r_{M,t})$. To allow for the leverage effect and asymmetric volatility we apply the GJR methodology $(\lambda_{j,1}, \lambda_{M,1})$ (Glosten et al., 1993) modeled as for univariate specifications. Note, that only the asset series will employ an event period dummy $(D_{j,k,t})$ for the bivariate estimation.

Among the two model specifications above for the market model in event studies we prefer the bivariate GARCH specification. The main reason for this choice is that we are able to control for non-synchronous trading (autocorrelation and cross-autocorrelation) in the conditional mean and volatility clustering and asymmetric volatility in the conditional volatility for both the asset series and the market index series. As we have shown above and in Solibakke (2001a, 2001b) these effects are important to control in especially thinly traded markets. Moreover, the index and asset series need to be flexibly modelled to allow for market dynamics. The bivariate model controls the co-moments of asset and index series. Hence, unadjusted statistics for the significance of abnormal return may appropriately be applied.

8 Summaries

This paper has estimated a univariate and a bivariate ARMA-GARCH-in-Mean specification for the conditional mean and volatility equations for event and non-event series in the Norwegian thinly traded equity market. The univariate model assumes a student-t density log likelihood function. Both models report strongly higher conditional volatility in event periods. Specification tests suggest that both models capture both non-synchronous trading and volatility clustering in return series. Moreover, specification tests suggest that both univariate and bivariate specifications reject data dependence but some bias in conditional volatility predictions exists. Formally we test for changing volatility in event periods applying a likelihood ratio test statistic for parameter restrictions obtained from non-event periods. All LRT test rejects unchanged parameter estimates.

Finally, the observed coefficient significances of the conditional mean and volatility equations, the strong increase in volatility for observed event series, suggest that event studies should be conducted within bivariate ARMA-GARCH lag specifications. Moreover, owing to removed biases in the moments and co-moments, abnormal returns calculations can apply unadjusted test statistics.

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Table 1. Portfolio Characteristics for event and non-event series

	Sample Size	Yearly Mean	Yearly st.deviation	Max. Return	Min. Return	Kurtosis	Skew	ARCH(6)	RESET(12;6)
Value-weighted market index	2611	13.2784	20.5812	10.4810	-21.2188	34.6329	-1.9681	286.737	68.791
Event portfolios:									
PSE(-10,+01)	1018	61.5848	61.3107	37.4693	-19.0620	4.9965	0.3385	38.813	25.343
PSE(-20,+05)	1483	52.6168	59.6772	37.4693	-36.7730	11.0238	-0.2823	62.460	32.974
PSE(-40,+40)	2289	8.5938	56.2096	26.0378	-69.5375	10.3822	-0.4455	171.043	74.091
PAE(-10,+01)	1506	32.9816	37.4190	18.2322	-14.3100	4.3302	0.2320	58.679	31.492
PAE(-20,+05)	1924	20.1455	37.5664	18.2322	-19.4160	6.6359	-0.3664	181.377	84.392
PAE(-40,+40)	2406	14.6016	31.4098	14.6603	-16.0062	15.3247	-0.3628	275.035	118.120
PBE(-10,+01)	1769	52.4124	46.0854	37.4693	-15.4150	6.7529	0.5695	69.063	29.875
PBE(-20,+05)	2153	41.2075	44.6394	37.4693	-19.4160	11.5863	-0.0267	209.777	85.252
PBE(-40,+40)	2491	12.7470	41.0366	17.1478	-69.5375	18.3244	-0.9328	252.394	104.325
Non-event Portf.									
PNERS1	2611	19.7281	30.8518	21.9211	-19.8416	19.9246	-0.4441	239.190	68.219
PNERS2	2611	4.7328	35.8128	43.4944	-16.5540	30.6905	-0.7461	123.704	54.121
PNERA1	2611	10.9136	25.5279	15.3072	-20.3526	13.3871	-0.7792	320.751	85.342
PNERA2	2611	-1.3003	27.1862	11.5826	-19.2288	18.0938	-1.1904	305.247	74.697
PNERB1	2611	6.0372	26.6409	9.3207	-14.9983	6.96239	-0.6732	300.484	63.918
PNERB2	2611	14.8498	31.1914	16.5265	-15.1988	6.34623	-0.0713	165.803	52.461

PSE(-10,+01) = Portfolio for selling firms in event period from -10 to +1 days relative to announcement.
PSE(-20,+05) = Portfolio for selling firms in event period from -20 to +5 days relative to announcement.
PSE(-40,+40) = Portfolio for selling firms in event period from -20 to +5 days relative to announcement.
PAE = PSE for acquiring firm portfolio and PBE = PSE for both selling and acquiring firm portfolio.
PSNER1 = Portfolio for selling firms in non-event periods; randomly selected sample no. 1.
PANER1 = Portfolio for acquiring firms in non-event periods; randomly selected sample no. 1.
PBNER1 = Portfolio for both acquiring and selling firms in non-event periods; randomly selected.
Daily mean is the average daily return over the period. Yearly mean is the average daily return multiplied by 252 trading days. Daily standard deviation is the square root of the daily return variance. Yearly standard deviation is the daily standard deviation multiplied by the square root of 252 days. Maximum return is the maximum return in the sample period. Minimum return is the minimum return in the sample period. ARCH (6) is a test for conditional heteroscedasticity in returns. Low {.} indicates significant values. We employ the OLS-regression $y^2 = a_0 + a_1 . y^2_{t-1} + \ldots + a_6 . y^2_{t-6}$. $T \cdot R^2$ is χ^2 distributed with 6 degrees of freedom. T is the number of observations, *y* is returns and R^2 is the explained over total variation. $a_0, a_1 \ldots a_6$ are parameters. All ARCH tests are significant at the 1% level. RESET (12,6) : A sensitivity test for mainly linearity in the mean equation. 12 is number of lags and 6 is the number of moments that is chosen in our implementation of the test statistic. $T \cdot R^2$ is χ^2 distributed with 12 degrees of freedom. All RESET tests are significant at the 1% level.

Table 2. An ARMA(0,1)-GARCH(1,1)-M process model for portfolio returns

This table contains the estimated coefficients from the model

$$R_{j,t} = \phi_{j,0} + \sum_{i=1}^{p} \phi_{j,i} \cdot R_{j,t-i} + \delta_j \cdot h_{j,t}^{\frac{1}{2}} + \varepsilon_{j,t} - \sum_{i=1}^{q} \theta_{j,i} \cdot \varepsilon_{j,t-i}$$

where $E(\varepsilon_{i,t}|\Omega_{t-1}) \sim D(0,h_{i,t},\upsilon_i)$ and $\lambda_{j,i,t} = \gamma_{j,i}$ if and only if $\varepsilon_{j,t-i} < 0$,

$$h_{j,t} = m_{j,0} + \sum_{i=1}^{m} (a_{j,i} + \lambda_{j,i,t}) \cdot \varepsilon_{j,t-i}^{2} + \sum_{i=1}^{n} b_{j,i} \cdot h_{j,t-i}$$

where $R_{j,t}$ are the daily returns on asset portfolio series and the market index. The model assumes a student t-density ($D(0,h_{j},t,\upsilon_j)$) log-likelihood function for parameter estimation, where the number of freedom's parameter υ is estimated. All parameters are estimated and the numbers in brackets below the estimated coefficients are t-statistics.

Port-folio(j):	Log like-lihood	υ	ϕ_0	β	θ_1	m_0	a_1	b_1	γ_1
Market Index	-3833.31	6.35849	0.15634	-0.07595	-0.25094	0.14560	0.07115	0.74287	0.15103
		{8.5805}	{1.4487}	-{0.720}	-{12.509}	{3.930}	{3.067}	{15.367}	{3.682}
PES{-10,+01}	-2665.33	3.15269	0.15551	0.01727	-0.03532	6.45523	0.35107	0.32589	0.12657
		{9.5002}	{0.3736}	{0.1559}	-{1.0602}	{4.9215}	{2.7342}	{3.5956}	{0.7966}
PES{-20,+05}	-3726.27	2.78265	0.12195	0.01702	-0.02938	2.43567	0.34912	0.65088	-0.09344
		{11.5924}	{0.5035}	{0.2515}	-{1.1574}	{2.4356}	{3.1730}	{7.3979}	-{0.9175}
PES{-40,+40}	-5558.29	3.25724	0.16303	-0.04185	-0.07392	1.06182	0.21178	0.73027	0.02676
		{12.8283}	{0.9122}	-{0.6882}	-{3.4821}	{3.8738}	{4.2491}	{16.754}	{0.4900}
PEA{-10,+01}	-3258.60	3.59845	-0.20356	0.12735	-0.07252	0.92958	0.10203	0.71193	0.10664
		{9.8003}	-{0.7174}	{1.0254}	-{2.7555}	{2.4683}	{2.1448}	{7.8634}	{1.6598}
PEA{-20,+05}	-4120.08	4.25198	0.28121	-0.08139	-0.07509	0.37062	0.06846	0.83961	0.05247
		{9.9854}	{1.2217}	-{0.7508}	-{3.2379}	{3.4184}	{2.8118}	{25.023}	{1.5497}
PEA{-40,+40}	-4607.73	4.78380	0.08046	-0.00115	-0.15775	0.29718	0.07966	0.78474	0.10377
		{10.8758}	{0.5152}	-{0.0121}	-{7.1755}	{1.8792}	{2.4250}	{8.8623}	{1.9425}
PEB{-10,+01}	-4137.59	3.57772	-0.02652	0.06342	-0.06950	2.44604	0.23229	0.47486	0.15001
		{10.6247}	-{0.1006}	{0.6489}	-{2.7594}	{3.1541}	{2.9793}	{4.0710}	{1.6037}
PEB{-20,+05}	-4803.24	3.50152	0.11059	0.02540	-0.06770	0.63284	0.15447	0.76963	0.02615
		{12.2185}	{0.6299}	{0.3450}	-{3.2112}	{3.9035}	{3.9293}	{19.502}	{0.5894}
PEB{-40,+40}	-5335.53	5.51141	0.28191	-0.09064	-0.13862	0.15549	0.07186	0.87631	0.04852
		{8.9224}	{1.9302}	-{1.2240}	-{6.7538}	{3.4628}	{3.8853}	{38.866}	{1.9585}
PNERS1	-5023.10	4.85994	-0.03180	0.08488	0.02764	0.22300	0.04305	0.84004	0.10160
		{10.480}	-{0.241}	{1.069}	{1.305}	{3.984}	{2.290}	{30.084}	{3.401}
PNERS2	-5368.52	4.45475	0.16758	-0.05500	0.01770	0.19732	0.03725	0.88204	0.07237
		{11.991}	{1.162}	-{0.733}	{0.885}	{3.199}	{2.265}	{33.360}	{3.143}
PNERA1	-4470.93	6.03981	0.19235	-0.06800	-0.18264	0.09999	0.06282	0.84104	0.10168
		{9.118}	{1.899}	-{0.857}	-{8.866}	{3.642}	{2.954}	{28.563}	{3.452}
PNERA2	-4657.61	5.78547	0.05879	-0.01247	-0.11299	0.17725	0.07053	0.79228	0.13977
		{10.380}	{0.541}	-{0.160}	-{5.315}	{3.942}	{2.972}	{21.769}	{4.010}
PNERB1	-4697.04	5.03013	0.15849	-0.11923	-0.00108	0.06870	0.06564	0.89082	0.05242
		{10.192}	{1.911}	-{1.784}	-{0.053}	{2.981}	{3.656}	{40.409}	{1.973}
PNERB2	-5178.98	5.10484	0.24179	-0.08229	0.05650	0.05952	0.02913	0.93985	0.02922
		{11.008}	{1.752}	-{1.049}	{2.765}	{2.274}	{3.286}	{62.805}	{1.980}

* See Table 1 for a desciption of the asset series

Table 3. Conditional Variance for ARMA-GARCH Specification

Student-t density log-likelihood function.

Panel A. Conditional variance for Selling Firm Portfolios

	PES{-10,+01}	PES{-20,+05}	PES{-40,+40}	PNERS1	PNERS2
Mean	18.70195	19.19703	14.44024	3.73851	4.82450
St.deviation	17.51336	23.25715	19.69196	5.28487	4.34063
Maximum	211.51874	431.78156	294.55571	108.70855	59.06402
Minimum	9.63035	7.13991	4.21650	1.56747	1.96091

Panel B. Conditional variance for Acquiring Firm Portfolios

	PEA{-10,+01}	PEA{-20,+05}	PEA{-40,+40}	PNERA1	PNERA2
Mean	6.21994	5.66700	3.72974	2.55937	2.92827
St.deviation	3.70463	4.53759	4.25134	3.83901	4.64144
Maximum	43.66252	58.84618	63.42697	81.48028	110.68654
Minimum	3.55286	2.61130	1.39184	0.71902	0.91348

Panel C. Conditional variance for Selling and Acquiring Firm Portfolios

	PEB{-10,+01}	PEB{-20,+05}	PEB{-40,+40}	PNERB1	PNERB2
Mean	9.50555	8.46261	6.52168	2.91710	3.83812
St.deviation	9.67752	10.71240	9.46285	2.68531	2.29449
Maximum	171.62060	133.99998	115.02129	38.09907	21.52474
Minimum	-5.84126	2.93724	1.50400	0.80627	1.27446

* See Table 1 for description of the series.

Table 4. Specification tests for An ARMA (p,q)-GARCH(m,n)-M process model

This table contains the specification tests for the portfolios and the market index.

Port folio (j):	Q (6)	$Q^2(6)$	Kurtosis / Skew	K-S Z-test	ARCH (12)	RESET (12;6)	BDS m=2;ε=1.	BDS m=3;ε=1.	Bias
Market	10.5030	11.1220	7.7681	2.0411	14.7699	6.4350	-0.1127	-0.8809	8.9373
Index	{0.105}	{0.085}	-0.77292	{0.001}	{0.254}	{0.376}	{0.396}	{0.271}	{0.030}
PES{-10,+01}	1.4750	16.6000	5.3392	2.32489	24.0767	9.0870	-0.54479	-0.30297	1.01484
	{0.961}	{0.011}	0.39462	{0.000}	{0.020}	{0.169}	{0.344}	{0.381}	{0.798}
PES{-20,+05}	2.1460	2.3990	7.5580	3.2737	12.6512	5.2359	1.1746	0.9327	2.1594
	{0.906}	{0.880}	0.77751	{0.000}	{0.395}	{0.514}	{0.200}	{0.258}	{0.540}
PES{-40,+40}	6.358	2.852	6.78896	3.25625	8.4179	4.1325	1.95748	2.01692	0.45789
	{0.499}	{0.827}	-0.29074	{0.000}	{0.027}	{0.659}	{0.059}	{0.052}	{0.928}
PEA{-10,+01}	10.67	2.484	4.46047	2.35	4.98127	2.4356	0.61521	0.31962	1.05138
	{0.099}	{0.870}	0.24976	{0.000}	{0.959}	{0.876}	{0.330}	{0.379}	{0.789}
PEA{-20,+05}	13.151	5.054	6.78025	2.30391	9.63868	5.5324	-1.05318	-1.05163	3.69917
	{0.041}	{0.537}	-0.51205	{0.000}	{0.648}	{0.478}	{0.229}	{0.229}	{0.296}
PEA{-40,+40}	6.099	7.417	8.88042	2.38431	10.3041	6.2123	-1.13472	-0.51932	6.81339
	{0.412}	{0.284}	-0.06102	{0.000}	{0.589}	{0.400}	{0.210}	{0.349}	{0.078}
PEB{-10,+01}	4.228	5.258	4.07124	2.64013	5.55637	3.1345	1.47908	1.72287	1.8465
	{0.646}	{0.511}	0.37666	{0.000}	{0.937}	{0.792}	{0.134}	{0.090}	{0.605}
PEB{-20,+05}	6.856	9.545	4.63928	2.83914	9.85222	7.2543	-0.93036	0.02778	5.91768
	{0.334}	{0.145}	-0.00765	{0.000}	{0.629}	{0.298}	{0.259}	{0.399}	{0.116}
PEB{-40,+40}	4.897	16.687	3.68873	1.95091	15.5967	10.2354	1.25869	1.18877	27.7879
	{0.557}	{0.011}	-0.35706	{0.001}	{0.021}	{0.115}	{0.181}	{0.197}	{0.000}
PNERS1	8.987	16.187	4.66169	2.63369	19.4327	12.3245	-0.20681	-0.53553	25.6189
	{0.174}	{0.012}	-0.40244	{0.000}	{0.079}	{0.055}	{0.391}	{0.346}	{0.000}
PNERS2	4.242	15.763	8.63806	2.55031	10.6054	7.1395	1.14193	0.88291	6.3593
	{0.644}	{0.015}	-0.24548	{0.000}	{0.563}	{0.308}	{0.208}	{0.270}	{0.095}
PNERA1	14.015	15.196	7.19188	2.13189	10.2771	6.8756	0.29656	-0.0075	5.64074
	{0.029}	{0.019}	-0.88133	{0.000}	{0.592}	{0.333}	{0.382}	{0.399}	{0.130}
PNERA2	8.775	13.364	3.95680	2.06189	19.6756	11.5534	-0.78678	-1.13493	13.8863
	{0.187}	{0.038}	-0.57688	{0.000}	{0.073}	{0.073}	{0.293}	{0.210}	{0.003}
PNERB1	6.573	14.074	2.27322	2.55567	20.5588	11.9567	0.32506	-0.10349	26.5459
	{0.362}	{0.029}	-0.31989	{0.000}	{0.057}	{0.063}	{0.378}	{0.397}	{0.000}
PNERB2	2.075	11.164	5.24054	2.18029	18.0673	10.8723	-0.40875	-0.4257	15.5956
	{0.913}	{0.083}	-0.23405	{0.000}	{0.114}	{0.092}	{0.367}	{0.364}	{0.001}

Q(6) and $Q^2(6)$ is the Ljung and Box (1976) test of serial correlation up to 6 lags. K-S Z-test: Used to test the hypothesis that a sample comes from a normal distribution. The value of the Kolmogorov-Smirnov Z-test is based on the largest absolute difference between the observed and the theoretical cumulative distributions. ARCH and RESET: see Table 1. BDS ($m=2, \varepsilon=1$): A test statistic for general non-linearity in a time series. The test statistic BDS $= T^{1/2} \cdot [C_m(\sigma\varepsilon) - C_1(\sigma\varepsilon)^m]$, where C is based on the correlation-integral, m is the dimension and ε is the number of standard deviations. Under the null hypothesis of identically and independently distributed (i.i.d.) series, the BDS-test statistic is asymptotic normally distributed with a zero mean and with a known but complicated variance.

Table 5A. A bivariate ARMA(0,1)-GARCH(1,1)-M model

This table contains the estimated coefficients from the model

$$R_t = \phi_0 + f(x_t, \beta) + \delta \cdot vech(H_t) + \varepsilon_t - \theta \cdot \varepsilon_{t-1}$$

$$H_t = m_0' \cdot m_0 + A_1' \cdot \varepsilon_{t-1} \cdot \varepsilon_{t-1}' \cdot A_1 + B_1' \cdot H_{t-1} \cdot B_1$$

where $\varepsilon_t \,|\, \Omega_{t-1} \sim N(0, H_t)$.

Portfolio (j)	Log-likelihood	ϕ_0	ϕ_M	δ_{11}	δ_{MM}	θ_1	θ_M
PES{-10,+01}	-4250.7	0.16178	0.02765	-0.10016	-0.09744	-0.01611	-0.27032
		{1.493}	{0.702}	-{0.636}	-{0.569}	-{0.454}	-{7.187}
PES{-20,+05}	-6018.2	0.20385	0.06213	-0.08404	-0.12865	-0.03518	-0.21495
		{0.088}	{1.206}	-{0.917}	-{0.895}	-{0.712}	-{6.264}
PES{-40,+40}	-9023.1	0.07929	0.06801	0.14167	-0.12777	-0.08795	-0.23081
		{1.270}	{2.748}	{1.949}	-{1.436}	-{3.754}	-{10.408}
PEA{-10,+01}	-5437.0	0.11279	0.04679	0.29356	0.17083	-0.06307	-0.21924
		{1.890}	{1.511}	{1.543}	{1.098}	-{2.253}	-{8.162}
PEA{-20,+05}	-6855.9	0.08253	0.07279	-0.05987	-0.03542	-0.08770	-0.20124
		{1.623}	{2.711}	-{0.592}	-{0.355}	-{3.780}	-{8.366}
PEA{-40,+40}	-7837.0	0.14128	0.09153	-0.01716	-0.09825	-0.14082	-0.16911
		{3.806}	{3.931}	-{0.218}	-{1.338}	-{7.096}	-{8.420}
PEB{-10,+01}	-6760.4	0.14064	0.07249	0.21477	0.05020	-0.06032	-0.22959
		{2.294}	{2.428}	{1.774}	{0.345}	-{2.350}	-{9.682}
PEB{-20,+05}	-7984.8	0.15355	0.08836	0.05682	-0.09235	-0.07143	-0.21784
		{3.031}	{3.251}	{0.749}	-{0.765}	-{3.200}	-{10.094}
PEB{-40,+40}	-8612.1	0.13273	0.08641	-0.06884	-0.08657	-0.15357	-0.18341
		{2.839}	{3.671}	-{0.998}	-{1.291}	-{7.818}	-{9.135}
PNERS1	-8703.5	0.08133	0.08228	0.00476	-0.02364	0.02412	-0.15069
		{2.5597}	{3.3730}	{0.0527}	-{0.2384}	{1.2166}	-{7.4599}
PNERS2	-9247.4	0.06001	0.09389	0.04982	-0.02558	0.00159	-0.15764
		{1.6394}	{3.9290}	{0.7207}	-{0.2561}	{0.0749}	-{7.4040}
PNERA1	-7193.8	0.12788	0.13422	-0.13819	-0.24987	-0.10414	-0.13395
		{4.4322}	{5.6642}	-{1.5715}	-{2.9939}	-{6.5911}	-{8.2239}
PNERA2	-7997.9	0.06227	0.11369	-0.04192	-0.03439	-0.06920	-0.11689
		{1.3048}	{3.0086}	-{0.6694}	-{0.4501}	-{3.3764}	-{4.3093}
PNERB1	-8289.9	0.09333	0.09512	-0.12848	-0.09727	-0.00661	-0.15740
		{3.5085}	{4.0283}	-{1.9348}	-{1.0480}	-{0.3658}	-{7.6919}
PNERB2	-8893.5	0.12070	0.11083	-0.00965	0.00350	-0.04042	-0.16814
		{3.7197}	{3.9930}	-{0.1230}	{0.0417}	-{2.1387}	-{6.9874}

* See Table 1 for a description of the asset series and all parameters and variables are defined in Section 2.3.3.

Table 5B. A bivariate ARMA(0,1)-GARCH(1,1)-M model (continued)

Port-folio (j):	m_0	$m_{0,M}$	m_M	a_{11}	a_{1M}	a_{M1}	a_{MM}	b_{11}	b_{M1}^*	b_{MM}	γ_1	γ_2
PES{-10,+01}	2.142	0.294	0.526	0.459	0.056	0.309	0.422	0.692	-0.194	0.668	0.067	0.279
	{5.517}	{2.677}	{7.271}	{5.263}	{3.647}	{1.585}	{6.977}	{5.663}	-{0.73}	{0.000}	{1.029}	{0.000}
PES{-20,+05}	0.718	0.355	0.149	0.358	0.008	0.364	0.311	0.937	-0.271	0.877	0.000	0.064
	{1.853}	{1.385}	{0.702}	{1.032}	{0.206}	{0.317}	{0.537}	{3.315}	-{0.33}	{61.68}	{0.00}	{0.11}
PES{-40,+40}	0.747	0.204	0.361	0.332	0.005	0.073	0.000	0.924	-0.093	0.809	0.015	0.093
	{12.10}	{3.61}	{11.22}	{12.44}	{0.66}	{0.99}	{10.31}	{87.55}	-{1.78}	{26.95}	{0.75}	{2.53}
PEA{-10,+01}	1.482	-0.036	0.353	-0.305	-0.035	0.415	-0.223	0.536	0.455	0.843	0.208	0.155
	{6.99}	-{0.60}	{6.95}	-{5.68}	-{1.79}	{4.51}	-{3.63}	{1.14}	{3.34}	{23.15}	{3.53}	{3.15}
PEA{-20,+05}	0.582	0.101	0.309	0.289	0.035	0.053	0.315	0.915	0.001	0.875	0.033	0.066
	{8.34}	{1.65}	{8.94}	{8.46}	{1.66}	{0.59}	{7.17}	{0.22}	{0.02}	{35.63}	{1.42}	{2.04}
PEA{-40,+40}	0.488	0.331	-0.196	0.324	0.038	0.195	0.437	0.940	-0.150	0.832	0.000	0.022
	{9.49}	{8.24}	-{11.4}	{11.11}	{2.03}	{3.93}	{12.68}	-{0.35}	-{5.25}	{39.44}	{0.01}	{1.88}
PEB{-10,+01}	1.423	-0.043	0.440	0.449	-0.011	-0.932	0.153	0.659	0.203	0.748	0.126	0.226
	{12.70}	-{0.72}	{6.58}	{10.00}	-{0.56}	-{11.7}	{3.63}	{3.36}	{1.42}	{13.70}	{3.24}	{4.67}
PEB{-20,+05}	0.766	0.221	0.260	0.462	0.017	-0.161	0.206	0.862	0.021	0.913	0.000	0.090
	{11.24}	{5.28}	{10.28}	{16.99}	{1.56}	-{2.18}	{6.41}	-{1.74}	{0.54}	{65.13}	{0.00}	{4.75}
PEB{-40,+40}	0.522	0.317	0.221	0.381	0.034	0.231	0.459	0.917	-0.127	0.823	0.000	0.068
	{10.72}	{8.33}	{12.0}	{16.74}	{2.87}	{4.00}	{13.66}	-{1.75}	-{3.44}	{38.60}	{0.01}	{2.79}
PNERS1	0.5580	0.2890	0.3853	-0.082	0.0564	0.4411	0.3710	0.9045	-0.064	0.7886	0.1255	0.0651
	{9.785}	{4.250}	{13.11}	-{2.00}	{2.885}	{7.860}	{8.026}	{54.118}	-{1.13}	{21.23}	{6.399}	{2.577}
PNERS2	0.5522	0.3756	0.3776	0.269	-0.028	0.2145	0.4555	0.9374	-0.183	0.7524	0.0460	0.0458
	{9.879}	{4.793}	{8.60}	{9.49}	-{1.78}	{3.698}	{9.732}	{67.762}	-{3.03}	{17.31}	{2.592}	{2.017}
PNERA1	0.5341	0.5144	0.1383	0.007	0.2209	0.5050	0.2527	0.9976	-0.249	0.7082	0.0099	0.0001
	{9.710}	{9.918}	{7.38}	{0.17}	{6.348}	{9.967}	{5.159}	{53.875}	-{5.05}	{15.03}	{2.686}	{0.021}
PNERA2	0.4111	0.3458	0.2816	0.263	0.0900	0.2085	0.3715	0.9166	-0.068	0.8387	0.0601	0.0001
	{12.65}	{9.125}	{6.57}	{9.22}	{3.238}	{4.478}	{9.377}	{53.966}	-{1.68}	{36.16}	{2.082}	{0.002}
PNERB1	0.3516	0.3619	0.3380	0.315	0.1465	0.0739	0.3354	0.9395	-0.084	0.7849	0.0308	0.0047
	{8.273}	{5.304}	{8.73}	{11.36}	{6.448}	{1.841}	{9.438}	{64.019}	-{1.88}	{22.23}	{2.893}	{0.248}
PNERB2	0.4154	0.3530	0.2530	0.207	0.0268	0.1545	0.4147	0.9638	-0.092	0.8333	0.0138	0.0001
	{7.666}	{5.029}	{6.97}	{7.38}	{0.965}	{1.544}	{5.430}	{117.86}	-{1.48}	{23.51}	{1.100}	{0.002}

! See Table 1 for a desciption of the asset series

*b_{1M} is not significant in any bivariate estimation and are therefore excluded from the table above due to space requirements

Table 6. Conditional Variance Series for Multivariate Time Series

Multi-Normal density GARCH specification.

Panel A. Conditional variance for Selling Firm Portfolios

	PES{-10,+01}	PES{-20,+05}	PES{-40,+40}	PNERS1	PNERS2
Mean Portfolio	15.94530	18.14798	12.89608	3.66679	5.28875
Mean Market	1.48276	1.37276	1.47878	1.60550	1.59491
St.dev. Portfolio	12.41234	19.76305	15.30360	5.59073	7.15208
St.dev. Market	2.00412	1.03545	2.24548	3.96847	3.41900
Maximum P.	142.72486	271.69155	172.98752	145.49595	134.18385
Maximum M.	38.39453	15.38199	39.81361	119.52256	108.28927
Minimum P.	5.23529	3.54822	0.57991	0.50203	0.69420
Minimum M.	0.37311	0.15723	0.28372	0.48178	0.32480

Panel B. Conditional variance for Acquiring Firm Portfolios

	PEA{-10,+01}	PEA{-20,+05}	PEA{-40,+40}	PNERA1	PNERA2
Mean Portfolio	5.61138	6.54770	3.97344	2.52930	3.01299
Mean Market	1.27385	1.36917	1.70819	1.60098	1.62191
St.dev. Portfolio	2.71816	7.50203	5.73072	3.62445	5.55674
St.dev. Market	0.98278	1.49781	4.50530	3.44173	3.47951
Maximum P.	45.62943	154.09945	143.29059	102.89281	162.48308
Maximum M.	12.49216	20.69934	119.72834	102.75415	116.08961
Minimum P.	3.16866	0.61568	0.34418	0.32414	0.34677
Minimum M.	0.10916	0.15425	0.17270	0.29587	0.24315

Panel C. Conditional variance for Selling and Acquiring Firm Portfolios

	PEB{-10,+01}	PEB{-20,+05}	PEB{-40,+40}	PNERB1	PNERB2
Mean Portfolio	8.63431	8.31635	7.08226	2.81985	5.28168
Mean Market	1.33609	1.33883	1.77241	1.57153	1.56266
St.dev. Portfolio	7.89637	11.72502	12.03868	2.67364	6.99355
St.dev. Market	0.62796	0.86673	4.77122	3.36400	3.36986
Maximum P.	96.84432	154.09945	207.93048	58.30610	132.87462
Maximum M.	6.11594	10.98339	116.75448	103.11406	104.83034
Minimum P.	2.52265	0.68603	0.22566	0.13821	1.11655
Minimum M.	0.15523	0.10681	0.10742	0.27968	0.50186

* See Table 1 for a description of the asset series

Table 7. Specification tests for MGARCH (1,1)-M process model

This table contains the specification tests for the portfolios and the market index.

Port folio (j):	Q (6)	$Q^2(6)$	Kurtosis / Skew	K-S Z-test	ARCH (12)	RESET (12;6)	BDS m=2;ε=1.	BDS m=3;ε=1.	Bias
PES{-10,+01}	1.9600	19.2340	4.9976	2.3089	20.5199	11.3321	-0.2743	0.1235	2.0635
	{0.923}	{0.004}	0.3403	{0.000}	{0.058}	{0.079}	{0.384}	{0.396}	{0.559}
Market	5.5270	2.0730	7.1715	1.7041	4.2089	1.6854	1.4654	0.0943	0.5852
Index	{0.478}	{0.956}	-0.2393	{0.006}	{0.979}	{0.946}	{0.136}	{0.397}	{0.900}
PES{-20,+05}	2.0050	2.3440	10.8111	3.0907	9.1256	3.4325	1.86905	1.216585	5.2886
	{0.919}	{0.885}	-0.2634	{0.000}	{0.692}	{0.753}	{0.070}	{0.190}	{0.152}
Market	2.5680	0.7460	6.3717	2.0236	1.3117	1.4512	0.56266	-0.56375	1.3327
Index	{0.861}	{0.993}	-0.3641	{0.001}	{1.000}	{0.963}	{0.341}	{0.340}	{0.721}
PES{-40,+40}	7.3410	2.8720	10.58408	3.2399	8.4408	4.3786	4.19067	4.100778	2.3795
	{0.290}	{0.825}	-0.40131	{0.000}	{0.750}	{0.626}	{0.000}	{0.000}	{0.497}
Market	15.3100	1.2560	11.84835	2.1817	1.7175	1.1816	1.49073	0.826279	0.8134
Index	{0.018}	{0.974}	-0.73273	{0.000}	{1.000}	{0.978}	{0.131}	{0.284}	{0.846}
PEA{-10,+01}	11.672	1.631	3.731813	2.2856	4.7003	1.7654	0.7978	0.6518	0.9945
	{0.070}	{0.950}	0.176449	{0.000}	{0.967}	{0.940}	{0.290}	{0.323}	{0.803}
Market	9.135	7.277	4.580735	1.5743	11.9616	1.0817	1.2245	0.0226	3.8035
Index	{0.166}	{0.296}	-0.18505	{0.014}	{0.449}	{0.982}	{0.189}	{0.399}	{0.283}
PEA{-20,+05}	12.2	5.256	6.44867	2.2075	9.3264	4.6759	-1.1265	-1.1293	3.7643
	{0.058}	{0.511}	-0.47747	{0.000}	{0.675}	{0.586}	{0.212}	{0.211}	{0.288}
Market	16.118	1.249	6.976373	1.9303	2.4476	1.5534	1.7236	1.1055	2.5158
Index	{0.013}	{0.974}	-0.73394	{0.001}	{0.998}	{0.956}	{0.090}	{0.217}	{0.472}
PEA{-40,+40}	8.116	9.325	7.087791	2.2417	11.8563	6.1829	-0.9790	-0.6323	6.5191
	{0.230}	{0.156}	-0.05905	{0.000}	{0.457}	{0.403}	{0.247}	{0.327}	{0.089}
Market	15.242	7.169	11.41769	2.2727	7.6763	2.1823	0.7047	0.3293	5.9318
Index	{0.018}	{0.305}	-1.06812	{0.000}	{0.810}	{0.902}	{0.311}	{0.378}	{0.115}
PEB{-10,+01}	3.752	6.232	2.618439	2.3836	8.0282	5.2169	1.7754	2.1156	3.1410
	{0.710}	{0.398}	0.144248	{0.000}	{0.783}	{0.516}	{0.082}	{0.043}	{0.370}
Market	11.265	5.431	2.982591	1.5392	12.7950	3.4512	1.1727	0.2322	2.5742
Index	{0.081}	{0.490}	-0.15756	{0.018}	{0.384}	{0.750}	{0.201}	{0.388}	{0.462}
PEB{-20,+05}	6.878	7.481	4.33677	2.7998	8.2039	5.5218	-1.0881	-0.0726	1.1817
	{0.332}	{0.279}	0.036636	{0.000}	{0.769}	{0.479}	{0.221}	{0.398}	{0.757}
Market	12.149	6.877	3.235842	1.5132	7.5304	2.4528	1.4024	0.7946	4.6260
Index	{0.059}	{0.332}	-0.34495	{0.021}	{0.821}	{0.874}	{0.149}	{0.291}	{0.201}
PEB{-40,+40}	6.388	13.592	2.981925	1.9258	17.0922	10.8691	1.1043	1.0970	7.4926
	{0.381}	{0.035}	-0.33714	{0.001}	{0.146}	{0.093}	{0.217}	{0.219}	{0.058}
Market	16.63	4.685	11.29445	2.2121	5.2376	2.0815	0.9439	0.6377	2.7707
Index	{0.011}	{0.585}	-1.03167	{0.000}	{0.000}	{0.912}	{0.256}	{0.326}	{0.428}
PNERS1	8.987	15.187	4.25152	2.6389	20.614907	12.3425	-0.6292	-0.55719	11.1453
	{0.174}	{0.019}	-0.30475	{0.000}	{0.056}	{0.055}	{0.327}	{0.342}	{0.011}
Market	15.896	12.459	6.56274	2.0415	12.916444	4.3428	0.27454	-0.34576	7.3957
Index	{0.014}	{0.086}	-0.67875	{0.000}	{0.375}	{0.630}	{0.384}	{0.376}	{0.060}
PNERS2	4.242	14.763	7.881765	2.6097	8.405008	4.2319	0.77751	0.374474	1.3818
	{0.644}	{0.022}	-0.04503	{0.000}	{0.753}	{0.645}	{0.295}	{0.372}	{0.710}
Market	2.067	17.135	5.94216	1.9944	18.184139	6.8321	0.15298	-0.33397	11.3614
Index	{0.913}	{0.017}	-0.62726	{0.001}	{0.110}	{0.337}	{0.394}	{0.377}	{0.010}
PNERA1	15.015	15.196	5.035987	2.3060	11.457897	7.8123	0.0825	-0.14219	9.1288
	{0.020}	{0.019}	-0.74307	{0.000}	{0.490}	{0.252}	{0.398}	{0.395}	{0.028}
Market	16.464	9.662	6.62387	2.0988	17.20038	7.5218	0.10906	-0.3976	9.7842
Index	{0.011}	{0.209}	-0.65099	{0.000}	{0.142}	{0.275}	{0.397}	{0.369}	{0.020}
PNERA2	8.775	18.364	3.684811	1.8763	16.197043	10.8451	-0.8163	-0.82145	11.7975
	{0.187}	{0.000}	-0.52111	{0.002}	{0.182}	{0.093}	{0.286}	{0.285}	{0.010}
Market	14.893	19.403	6.61759	2.0585	20.066719	8.9218	0.44082	-0.11149	11.7205
Index	{0.021}	{0.012}	-0.67863	{0.000}	{0.066}	{0.178}	{0.362}	{0.396}	{0.010}
PNERB1	6.573	17.074	2.199467	2.4500	20.5108	11.8923	0.39813	0.011667	11.0150
	{0.362}	{0.017}	-0.3051	{0.000}	{0.058}	{0.064}	{0.369}	{0.399}	{0.010}
Market	14.959	14.621	5.25272	1.8902	14.956496	6.4512	0.42287	-0.0902	8.6598
Index	{0.021}	{0.041}	-0.52124	{0.002}	{0.244}	{0.375}	{0.365}	{0.397}	{0.034}
PNERB2	2.075	16.164	4.672887	2.2313	9.155559	4.5129	-0.5923	-0.4118	6.2822
	{0.913}	{0.020}	-0.13246	{0.000}	{0.690}	{0.608}	{0.335}	{0.367}	{0.099}
Market	6.276	17.555	6.83036	2.0281	21.029395	9.8562	0.07892	-0.31816	11.3494
Index	{0.393}	{0.014}	-0.71207	{0.001}	{0.050}	{0.131}	{0.398}	{0.379}	{0.010}

Table 8. Likelihood Ratio Test for model restrictions.

Panel A. Univariate Specifications.

	Sellers		Acquirers		Both	
	LRT-Value	$\chi^2(6)$	LRT-Value	$\chi^2(6)$	LRT-Value	$\chi^2(6)$
PE{-10,+01}	30.5902	{0.0000}	23.241	{0.0007}	26.451	{0.0002}
PE{-20,+05}	28.2318	{0.0001}	22.852	{0.0008}	25.983	{0.0002}
PE{-40,+40}	27.6051	{0.0001}	21.438	{0.0015}	25.481	{0.0003}

Panel B. Bivariate Specifications.

	Sellers		Acquirers		Both	
	LRT-Value	$\chi^2(6)$	LRT-Value	$\chi^2(6)$	LRT-Value	$\chi^2(6)$
PE{-10,+01}	54.1216	{0.0000}	36.4518	{0.0000}	41.2431	{0.0000}
PE{-20,+05}	52.3451	{0.0000}	34.5673	{0.0000}	40.1219	{0.0000}
PE{-40,+40}	49.8781	{0.0000}	32.1879	{0.0000}	38.5791	{0.0000}

* See Table 1 for a description of the asset series. LRT – Likelihood Ratio Test statistic.

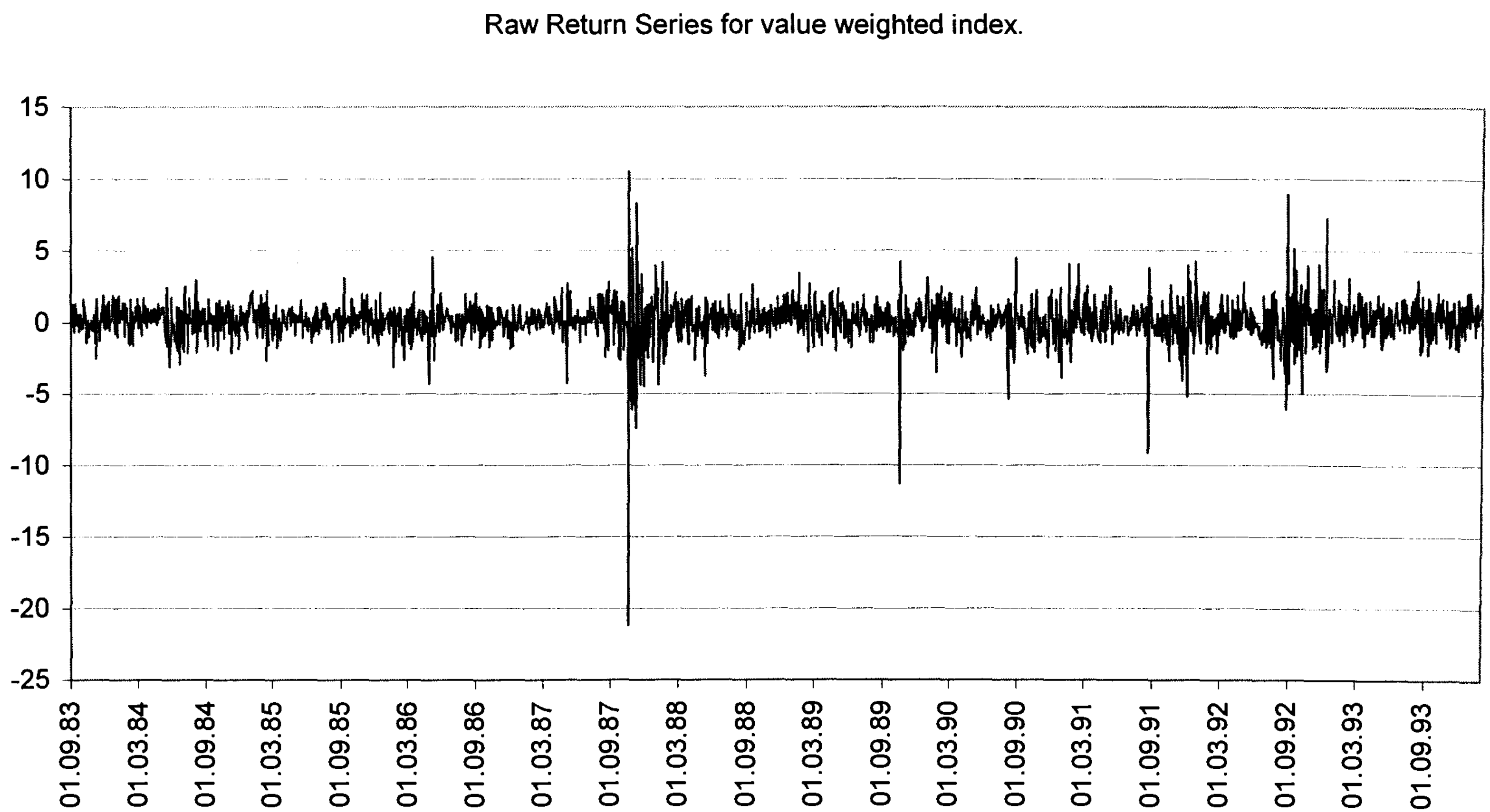

Figure 1. Panel A. Time series plots of raw value weighted index.

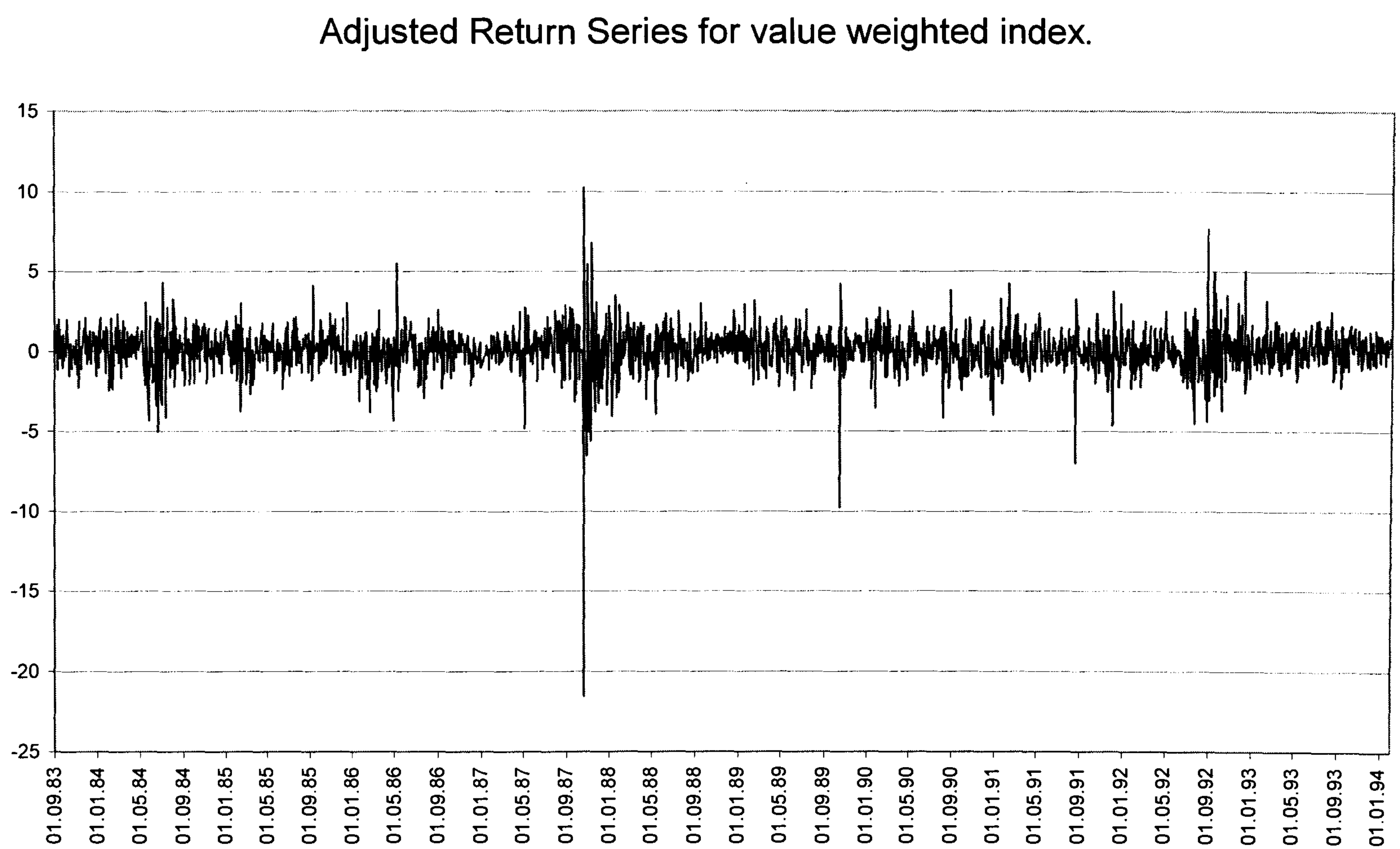

Figure 1. Panel B. Time series plots of adjusted value weighted index.

Essay no. 7*

Calculating Abnormal Returns in Event Studies:

Controlling for Non-synchronous Trading and Volatility Clustering in

Thinly Traded Markets

Abstract

This paper performs a classical market-model event study controlling for non-synchronous trading and volatility clustering. In contrast to many international event studies, this paper focuses on differences created by non-synchronous trading and conditional heteroscedasticity applying OLS and ARMA-GARCH market model specifications. The results suggest that non-synchronous trading and volatility clustering induce new market insight. We find no significant prior announcement effects, sustained higher post announcement abnormal returns for selling firms and no overall significant abnormal returns for acquiring firms. These results induce changes in event study inferences suggesting a need for a rework of many classical event studies.

Classification: c32, c52

Keywords: Event-studies, Mergers and Acquisitions, Non-synchronous trading, Volatility Clustering.

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1 Introduction and literature review

In economics and finance an important measurement is the effects of an economic event on the value of a firm. Such a measure can be constructed using an event study. Using financial market data, an event study measures the impact of a specific event on the value of the firm. The usefulness of such a study comes from the fact that, given rationality in the marketplace, the effects of an event will be reflected immediately in the security prices. Thus a measure of the event's economic impact can be constructed using security prices over a relatively short time period. In contrast, direct productivity related measures might require many months or even years of observations. The event study has many applications and especially in economics and financial research, event studies have been applied to a variety of firm specific and economic wide events. In this paper we intend to apply the event study methodology for a sample of mergers and acquisitions in the thinly traded Norwegian equity market[1].

Event study methodology has a long history, which perhaps started by James Dolley's (1933) stock split study. The level of sophistication of event studies increased from the early 1930s until the late 1960s. Examples are John H. Myers and Archie Bakay (1948), C. Austin Barker (1956, 1957, 1958), and John Ashley (1962). The improvements included removing general stock market price movements and separating out confounding events. Ray Ball and Phillip Brown (1968) and Eugene Fama et al. (1969) conducted seminal studies in the late 1960s. These introduced the methodology that is essentially the same as that in use today. Ball and Brown (1968) considered information content in earnings and Fama et al. (1969) studied the effects of stock splits after removing the effects of simultaneous dividend increases. In the years since the pioneering studies, the work by Stephen Brown and Jerold Warner (1980, 1985) summarizes further modifications. Furthermore, the work of R. Thompson in Jarrow et al. (1995) presented the most recent empirical methods in event studies. However, most studies assume ideal experiments. Hence, econometric problems are assumed to cancel out. Moreover, as Thompson points out; *"to incorporate increased variance during event periods into the inference problem is an interesting issue that is not completely resolved in the literature"* (Thompson 1995, p. 979).

This paper will review the event study methodologies under the hypothesis of non-synchronous trading and volatility clustering in individual asset returns. The economic implication is that events may influence the return generating process other than through a shift in the level of security prices.

Firstly, event periods may change trading frequency due to a higher information flow to the market and consequently generally higher financial press coverage. The change in trading frequency may change non-synchronous trading effects. Non-synchronous trading suggest that individual asset prices are taken to be recorded at time intervals of one length when in fact, they are recorded at time intervals of other, possibly irregular, lengths. Generally, especially in thinly traded markets, reported closing prices for individual assets do not occur at the same time each day because of non-trading. This non-trading effect induces potentially serious biases in the moments and co-moments of asset returns as shown in Campbell et al. (1997) and Solibakke (2001a, 2001b).

Secondly, theory might also imply an increase of residual risk during an event period[2]. Homoscedasticity of the residuals, i.e. their distribution show constant variance, may therefore be strongly disputed. Giaccoto and Ali (1982) and Boehmer et al. (1991) have shown that if homoscedasticity is not the case then standard methodology for measuring the effect of a specific event on security prices, have to be adjusted to take into account the presence of heteroscedasticity. More recently, a number of studies, for example Akgiray (1989) and especially Corhay and Tourani Rad (1994), show that the presence of time dependence in stock return series which, if not explicitly treated, will lead to inefficient parameter estimates and inconsistent test statistics. Solibakke (2001a, 2001b) show these effects in thinly traded markets. Applying discrete time ARMA-GARCH lag specifications for a variety of trading frequencies, Solibakke finds that thinly traded assets, which show high non-synchronous trading, report model misspecification. Moreover, Bera, Bubnys and Park (1988) show that market model estimates under ARCH processes are more efficient. Furthermore, Diebold, Im and Lee (1988) observed that residuals obtained using the standard market model exhibit strong ARCH properties.

Thirdly, asymmetric volatility controls for the 'leverage effect' (Nelson, 1991, Glosten et al., 1993). The asymmetry may change in periods where the information flow is high relative to more normal periods. The effect may be more severe in event periods due to higher sensitivity to negative news as for example announcement from the authorities that they will oppose the merger or acquisition. Consequently, we examine the impact of correcting the market model applying ARMA-GARCH lag specifications for bivariate time series estimations. While Boehmer et al. (1991) employ OLS and adjust test statistic, we enforce synchronous trading, conditional homoscedasticity and symmetric volatility in our model specification and therefore apply unadjusted test statistics.

We believe this study extends previous works[3] in several ways. Firstly, we employ a simultaneous dummy variable specification. Hence, the estimation and event period is studied simultaneously and the investigation controls for non-synchronous trading, volatility clustering and asymmetric volatility over both the estimation and the event period. Secondly, we employ a bivariate model. Hence, cross-correlation effects in conditional mean and volatility can be controlled for in the estimation. The parameters for the conditional means and the conditional variances are estimated simultaneously for firm and the market index series. We obtain synchronous trading, homoscedastic and symmetric volatility for both asset and market index. Moreover, we obtain contemporaneous market dynamics in both conditional mean and variance equations. Thirdly, using maximum likelihood, the bivariate ARMA-GARCH model, in contrast to OLS, has shown lower leptokurtosis in return distributions, making the residuals more normally distributed. Importantly, close to normal residuals indicate applying unadjusted test statistics. To our knowledge, this event study is so far the most comprehensive study of mergers and acquisitions in thinly traded markets. Moreover, our bivariate ARMA-GARCH lag specification approach applied to classical event studies and the market model is again to our knowledge not previously found in the international event literature.

The paper is organised as follows. Section 2 gives the details of the empirical model for classical event studies. Section 3 discusses the market model properties, criticizes the classical assumptions and shows the necessary adjustments to control for non-synchronous trading and changing and asymmetric volatility. Section 4 discusses some issues in event-study methodology and relates a two-step and a simultaneous event study. Section 5 describes the data, adjustment procedures and empirical test statistics. Section 6 presents the empirical results for the samples and model specifications. Section 7 summarizes and concludes our findings.

2 The empirical model, residual risk and a measure of variability

We focus on common stock returns. We structure the hypothesis in terms of the event's impact on the rate of return process for the corporation's securities. This hypothesis translates into the hypothesis that the rate of return earned on that security over an interval spanning the first public announcement of the event is more positive than normal. The classical event study methodology sets out to measure this abnormal return. For each security i, let returns follow a stationary stochastic process in the absence of the event of interest. When the event occurs, the market participants revise their value of the security, causing a shift in the return generating process. The conditional return generating process then becomes

$$R_t = x_t \cdot B + \varepsilon_t \tag{1}$$

for non-event periods and

$$R_t = x_t \cdot B + F \cdot G + \varepsilon_t \tag{2}$$

in an event period, where R_t is the return to a security in period t; x_t is a vector of independent variables not related to the event of interest; B is a vector of parameters; F is a row vector of asset characteristics or market conditions hypothesised to influence the impact of the event on the return; G is a vector of parameters measuring the influence of F on the impact of the event; and finally ε_t is a mean zero disturbance with no serial correlation and conditional heteroscedasticity. Hypotheses usually centre on G (Thompson, 1995).

Most event studies use a non-event period to estimate a forecast model and estimate the event's impact from forecast errors in the event period. An alternative characterization of the conditional return generating process under the same assumptions combines the event and non-event periods into a single model for security i. This model becomes

$$R_t = x_t \cdot B + D_j \otimes F \cdot G + \varepsilon_t \tag{3}$$

where now the vectors R, ε and x contain both event and non-event data while D_j represents j columns of dummies having zeros for non-event periods and ones in the event periods. This characterization of an event study is in a convenient econometric format. Moreover, the characterization makes it possible to perform event studies for simultaneous event and non-event periods. Note that the hypothesised event-induced variance obstructs conditional homoscedasticity in classical OLS studies. Hence, applying ARMA-GARCH specifications make changing conditional volatility a part of the model specification, while constant unconditional volatility is not violated.

Thompson points out: *"In an ideal experiment, created in the laboratory, it would be natural for firms to have constant residual variance across event and non-event periods"* (Thompson 1995, p. 979). Most researchers seem to accept this ideal experiment approach. However, as shown by Solibakke (2000) and noted by Christie (1993), forecast errors seem to have higher variance in event periods than in non-event periods. Solibakke (2000) shows that changing residual risk, measured by volatility, is especially strong for selling firm portfolios around the announcement date. Moreover, Beaver (1968) shows that a change in variance between event and non-event periods is a test whether or not an event reports new information to the market. Hence, volatility estimation procedures may affect inferences. Hence, our paper proposes a market model event study applying a bilateral ARMA-GARCH methodology controlling for non-synchronous trading, changing and asymmetric volatility for asset and market series. Hence, we obtain constant unconditional variance and changing conditional volatility across event and non-event periods. Finally, following Thompson; *"if we assume that omitted variables are drawn independently across the sample from a common population, then the increased variance in the event periods captures the noise added by the sampling variability of the omitted variables"* (Thompson 1995, p. 980).

The same story can be obtained from time series analysis. The time-series analysis hypothesises increased volatility in event periods caused by increased information flow. Hence, the volatility process distribution changes from non-event to event periods. From economic intuition the increased information flow makes sense. Market microstructure phenomena as rumours, insiders and coincident observers may obtain information details that make information asymmetric in the market. Trading on for example asymmetric information may produce price changes and will probably increase the volatility of the asset. Therefore, Collins & Dent (1984) suggested scaling the covariance matrix estimated in non-event periods by a factor strongly influenced by the estimated residuals from the event period.

By employing ARMA-GARCH methodology we are able to study changes in mean parameter estimates from the classical market model OLS estimates, by modelling the latent volatility process. Any change in inferences and different interpretation of the economic significance of events together with changes in mean parameter suggest a need for a rework of many previously performed classical event studies.

3 The Market model

3.1 The OLS market model specification with assumptions

Empirical researchers in financial economics widely use the market model for measuring the impact of an event on the shareholders wealth or testing market efficiency. This model relates the returns of an asset, $R_{i,t}$, to the returns of a market portfolio, $R_{M,t}$, through a slope coefficient, β_i, which is the asset's market and relevant risk

$$R_{it} = \alpha_i + \beta_i \cdot R_{Mt} + \varepsilon_i \qquad \textit{for } i = 1, 2, \ldots, N \tag{4}$$

where α_i is the intercept, N is the number of assets in the sample and t represents time. Hence, event studies include the contemporaneous rate of return on the market index as x in (1). In the ordinary least square (OLS) model, returns on a given asset i, are regressed against concurrent returns of the market. The announcement effect, $F \cdot G$, is estimated by the market model forecast error cumulated over the event period(s). Fama, Fisher, Jensen, and Roll (1969) suggested this OLS returns model in their study of stock splits. However, x may include the return on a similar firm or portfolio of similar firms that do not have the event of interest[4].

Certain assumptions have, however, to be satisfied to have efficient parameter estimates and consistent test statistics based on them. The first assumption is constant coefficients for the market model over time. Iqbal and Dheeriya (1991) resist this assumption and employ a random coefficient regression model allowing betas (β) to vary over time. They argued that the differences in abnormal returns obtained using the market model and their model can be attributed to the randomness in the beta coefficients. Secondly, Scholes and Williams, 1977, have recognized the potential for bias in the OLS β estimates due to non-synchronous trading. For securities traded with trading delays different than those of the market, OLS β estimates are biased. Likewise, for assets with trading frequencies different than those of the market index, OLS β estimates are biased. For actively traded stocks, the adjustments are generally small and unimportant. However, for thinly traded assets, trading frequency in isolation and in contrast to the market index is a real threat to the abnormal return results. Thirdly, classical studies assume homoscedasticity of the OLS residuals (constant variance). Giaccoto and Ali (1982) and Boehmer et al. (1991) have shown that if this is not the case, the standard tests to measure the effect of a specific event on security prices have to be adjusted to take into account the presence of heteroscedasticity.

All three cases suggest a rejection of the simple OLS market model. Below we propose an alternative model specification adjusting for non-synchronous trading and changing and asymmetric volatility.

3.2 The Bivariate ARMA-ARCH/GARCH specification[5] of the market model

Non-synchronous trading and temporal time dependence in stock return series can be handled by an ARMA -ARCH/GARCH methodology employing a market model event study. The ARMA model is applied for the conditional mean (Mills, 1990) and the GARCH model is applied for the conditional volatility (Bollerslev, 1986). ARCH/GARCH methodology was first introduced by Engle in 1982 and refined and extended by Bollerslev in 1986 and 1987. Engle and Kroner extended the models to the multivariate case in 1995[6].

The diagonal bivariate ARMA (p,q)–GARCH (m,n) market model, adjusting for non-synchronous trading *(θ_i, θ_M)*, asymmetric volatility *(γ_i, γ_M)* and conditional heteroscedasticity are defined as

$$R_{i,t} = \alpha_i + \sum_{j=1}^{p_i} \phi_{i,j} \cdot R_{i,t-j} + \beta_{i,1} \cdot \varepsilon_{M,t} + \gamma^{e}_{i,j} \cdot \mathrm{D}^{e}_{i,j,t} + \varepsilon_{i,t} - \sum_{j=1}^{q_i} \theta_{i,j} \cdot \varepsilon_{i,t-j} \quad (5)$$

$$E(\varepsilon^2_{i,t} \mid \Phi_{i,t-1}) = h_{i,t} = m_i + \sum_{j=1}^{m} a_{i,t-j} \cdot \varepsilon^2_{i,t-j} + \sum_{j=1}^{n} b_{i,t-j} \cdot h_{i,t-j} + \gamma^{v}_{i,1} \cdot D^{v}_{i,t} \cdot \varepsilon^2_{i,t-1} \quad (6)$$

$$R_{M,t} = \alpha_M + \sum_{j=1}^{p_M} \phi_{M,j} \cdot R_{M,t-j} + \varepsilon_{M,t} - \sum_{j=1}^{q_M} \theta_{M,j} \cdot \varepsilon_{M,t-j} \quad (7)$$

$$E(\varepsilon_{M,t}^2 \mid \Phi_{M,t-1}) = h_{M,t} = m_M + \sum_{j=1}^{m_M} a_{M,t-j} \cdot \varepsilon_{M,t-j}^2 + \sum_{j=1}^{n_M} b_{M,t-j} \cdot h_{M,t-j} + \gamma_{M,1}^v \cdot D_{M,t}^v \cdot \varepsilon_{M,t-1}^2 \quad (8)$$

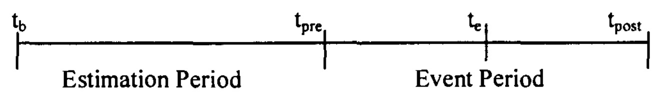
for $i=1,...,N$, where $R_{i,t}$ is the asset and $R_{M,t}$ the index return in period t; $\varepsilon_{i,t}$ and $\varepsilon_{M,t}$ are the error terms for the two mean equations (5) and (7) in period t, $\phi_{i,j}$, $\phi_{M,j}$, $\theta_{i,j}$ and $\theta_{M,j}$ are the non-synchronous trading parameters at lag j and $\gamma_{i,j}^e$ is the event window j's abnormal return for firm i. For asset (i) and index (M), respectively, the conditional variances $h_{i,t}$ and $h_{M,t}$ (conditional on the information set at time $t-1$, Φ_{t-1}) depend themselves upon the following parameters: m_i and m_M are the constant terms; $a_{i,j}$ and $a_{M,j}$ are the parameters for the lagged squared error at lag j; $b_{i,j}$ and $b_{M,j}$ are the parameters for the lagged conditional variance at lag j; $\gamma_{i,1}$ and $\gamma_{M,1}$ are the parameters for asymmetric volatility where $D^v_{i,t}$ ($D^v_{M,t}$) is a dummy variable taking the value one when $\varepsilon_{i,t-1}$ ($\varepsilon_{M,t-1}$) is less or equal to zero[7]. The ARMA (p,q) specification in (5) and (7) for the bivariate conditional mean is a serial correlation specification that is able to model non-synchronous trading. The GARCH (m,n) specification in (6) and (8) for the bivariate conditional volatility specifies time-varying and symmetric conditional volatility (h_t). Note, as shown in Solibakke (2000) the in-Mean specification is redundant for bivariate specifications. The bivariate ARMA-GARCH model allow for non-linear intertemporal dependence in the residual series (Solibakke, 2001b). Bera, Bubnys and Park (1988) showed market model estimates under ARCH processes are more efficient. Moreover, Diebold, Im and Lee (1988) observed that residuals obtained using the standard market model exhibit strong ARCH properties. Solibakke (2000 and 2001a) shows that employing ARCH (5) or GARCH (1,1) specification removes all ARCH-effects in residuals applying Norwegian individual asset, portfolio and index series.

Many authors before us have identified the hazards ignoring non-synchronous trading and event-induced variance in event studies[8]. As we already noted in Section 2, non-synchronous trading and changing and asymmetric volatility may lead to inefficient parameter estimates and inconsistent test statistics. As we here employ ARMA-GARCH methodology we may obtain synchronous trading, conditional homoscedasticity and symmetric volatility. Hence, as our specification removes several OLS assumptions, we may obtain a sounder basis for event-studies.

4 Event Study Methodology

4.1 Event and Non-Event periods; issues in methodology

In event studies, the objective is to examine the market's response through the observation of security prices around such events. For merger and acquisitions it is related to the release of information to market participants through the financial press (e.g., Børskurslisten, Aftenposten, Dagens Næringsliv). Normal or predicted returns for an asset are those returns expected if no event occurs. For mergers and acquisitions most event studies measure normal returns by estimating normal market model parameters over a time period prior to the period immediately surrounding the event date. The time line for a typical event study for merger and acquisitions may therefore be represented as follows



where t_b is the first period used in the estimation of a normal security return; t_{pre} is the first period used in the calculation of abnormal returns; t_e is the event date; and t_{post} is the last period used in the calculation of abnormal returns. Post event-period data will not be employed for obvious reasons.

In the literature we usually find a selection of t_{pre} equal to -40 days to t_e and t_{post} equal to + 40 days of t_e. The length of the estimation period is a weight of benefits of a longer period and the cost of a longer period. Usually, we find a choice from 12 to 14 months prior to the event announcement (t_e). Hence, there are from 230 to 270 daily return observations. This event study like most events studies, uses the non-event period to estimate a forecast model and estimates the event's impact from forecast errors in the event period. In addition, we employ the alternative characterization of the conditional return generating process under the same assumptions. Hence, we combine the event and non-event periods into a single model for security i. The two models are referenced as (2) and (3), respectively, in section 2 above. This approach maintains an algebraic equality between forecast errors from a two-step approach and the individual event period multiple regression event parameters. Finally, if an asset is involved in merger and acquisitions in the estimation period, we exclude a 10-day price period for this asset around the earlier event day.

4.2 Abnormal returns and statistical significance

4.2.1 Estimation in an Estimation Period and Forecasting Abnormal Returns

The abnormal returns (also referred to as the excess stock return or the prediction error) for an individual security for a given period is the difference between the observed return for that period and the expected or predicted return for that period: $AR_{i,t} = R_{i,t} - R^{*}_{i,t}$, where $AR_{i,t}$ is the abnormal security return for security i in period t; $R_{i,t}$ is the return on security i in period t; and $R^{*}_{i,t}$ is the expected return on security i in period t. The market model suggested by Fama, Fisher, Jensen, and Roll (1969) is employed for $R^{*}_{i,t}$. Aggregation of the individual security abnormal returns requires examining the cross section of abnormal returns for each period, where each period is relative to t_e, and t_e may be a different calendar time period for each security; thus, abnormal returns are aligned in event time. The mean abnormal return on a given day t for a portfolio of securities, $AR_{N,t}$, is the arithmetic mean of $AR_{i,t}$ for the particular day t: $AR_{N,t} = \frac{1}{N} \cdot \sum_{i=1}^{N} AR_{i,t}$. Now to calculate the cumulative effect, the individual $AR_{N,t}$ are accumulated over a number of periods to produce a cumulative abnormal return (CAR): $CAR_{N,t} = \sum_{t=T_1}^{T_2} AR_{N,t}$, where $CAR_{N,t}$ is the cumulative abnormal return for N securities for a period of length n; T_1 and T_2 is the first and last period in which the $AR_{N,t}$ are accumulated. The statistical tests related to abnormal stock returns require the use of the standard error of the forecast. For the market model, the standard error of the forecast for period t is:

$$s_{i,f,t} = s_{i,e} \cdot \left\{ 1 + \frac{1}{T} + \left[\frac{(R_{Mt} - R_M)^2}{\sum_{j=1}^{T} (R_{Mj} - R_M)^2} \right] \right\}^{\frac{1}{2}}$$

, where $s_{i,f,t}$ is the standard error of the forecast for security i in period t in the event period; T is the number of periods employed in the regression equation for parameter estimation; $R_{M,j}$ is the market return for period j within the estimation period; $R_{M,t}$ is the market return for period t within the estimation period; R_M is the mean return on the market over the estimation period; and $s_{i,e}$ is the standard error of the estimate for security i over T periods in the estimation period. Dividing the abnormal return by its estimated standard error yields a standardized abnormal return for a particular security on a given day: $SAR_{i,t} = \frac{AR_{i,t}}{s_{i,f,t}}$, where $SAR_{i,t}$ is the standardized abnormal return for security i in period t. The portfolio or sample standardized abnormal returns for a given day t are summed and divided by the square root of the number of securities in the portfolio: $SAR_{N,t} = \frac{1}{\sqrt{N}} \cdot \sum_{i=1}^{N} SAR_{i,t}$, where $SAR_{N,t}$ is the standardized abnormal return for a group of N securities on day t. If $SAR_{i,t}$ are independent and identically distributed with a finite variance, the $SAR_{N,t}$ are distributed unit normal for large N[9]. Finally the standardized cumulative abnormal return for a group of N securities, $SCAR_{N,n}$, can be calculated as: $SCAR_{N,n} = \frac{1}{\sqrt{N}} \cdot \sum_{t=1}^{n} SAR_{N,t}$, where $SCAR_{N,n}$ is the standardized cumulative abnormal return for a group of N securities over n periods. $SCAR_{N,n}$ is assumed to be distributed unit normal in the absence of abnormal security performance.

However, this specification of the abnormal return is a two-step analysis where the event induced variance is measured through the $s_{i,f,t}$ term. As shown by Solibakke (2000) the $s_{i,e}$ term is significantly lower during non-event periods than in event periods. Adjusting by employing the $s_{i,f,t}$ term will only measure volatility changes from the total market level. This methodology seems not adequate relative to a simultaneous analysis[10].

4.2.2 Simultaneous Estimation and Abnormal Return Calculation

As shown in Section 2, a convenient alternative characterization of the conditional return generating process under the same assumptions, we combine the event and non-event periods into a single model for security i applying the form

$$R_i = \hat{\alpha} + \hat{\beta}_i \cdot R_M + \gamma_i \cdot D_i + \varepsilon_i \qquad (9)$$

where the vectors R_i, ε_i, R_M and ε_M contain both the event and non-event data while D_i can be viewed as a matrix of zero-one variables, letting each column indicate a single event period. γ_i can be interpreted as the sum of the individual event period effects. Time series estimates of variability can be combined with estimates of individual asset effects in a number of ways. However, as we wish an unbiased estimate of the mean effect and a test of significance, this study tests the significance of the average event effect by using the time series estimates of variability to construct an estimate of variability for the arithmetic mean. To test for significance of the mean event effect, we compute the statistic

$$\frac{\sum_{i=1}^{I} \gamma_i}{\left(\sum_{i=1}^{I} \sigma_i^2\right)^{\frac{1}{2}}}$$

, where γ_i is the event effect for asset i and σ_i is the estimate of the standard error of γ_i around the true event effect for asset i. This statistic is equivalent to an OLS regression of forecast errors on a column of ones giving us a classical t-statistic. Here, the numerator and denominator both apply reference to asset i. However, note that the standard errors from OLS cross-sectional regressions are often ignored because they fail to account for neither non-synchronous trading nor conditional heteroscedasticity.

5 Data

The study uses daily continuously compounded returns ($\ln\left[(\frac{S_t}{S_{t-1}})\right]$) of individual Norwegian stocks and indices spanning the period from April 1st 1983 to April 1st 1994. These daily returns are scaled to avoid possible scaling problems in estimation. Let w denote the variable to be adjusted. Initially, the regression to the mean equation $w = x \cdot \beta + u$ is fitted, where x consists of calendar variables, which are most convenient for the time series and contains parameters for trends, week dummies, calendar day separation variable, month and sub-periods. To the residuals, $\hat{u}$, the variance equation model $\hat{u}^2 = x \cdot \gamma + \varepsilon$ is estimated. Next $\frac{\hat{u}^2}{\sqrt{e^{x \cdot \hat{\gamma}}}}$ is formed, leaving a series with mean zero and (approximately) unit variance given x. Lastly, the series $\hat{w} = a + b \cdot (\frac{\hat{u}}{\sqrt{e^{x\hat{\gamma}}}})$ is taken as the adjusted series, where a and b are chosen so that $\frac{1}{T} \cdot \sum_{i=1}^{T} \hat{w}_i = \frac{1}{T} \cdot \sum_{i=1}^{T} w_i$ and $\frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{w}_i - \overline{w})^2 = \frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{u}_i - \overline{u})^2$. The unit of measurement of the adjusted series is the same as that of the original series. The data set is obtained from Oslo Stock Exchange Information A/S.

To proxy for the market we employ the value weighted market index from Oslo Stock Exchange (TOTX) that is also scaled according to the above described procedures.

The sample period includes the crash of October 19, 1987. There is no reason to exclude these outliers since they reflect the nature of the market. This high frequency time-series database gives us potentially 2725 observations for each firm and index. The merger and acquisition selling firm sample consists of 126 Norwegian and foreign (listed on Oslo Stock Exchange) firms. The acquiring firm sample consists of 282 Norwegian and foreign (listed on Oslo Stock Exchange) firms. For the selling firms the sample is approximately 50% of the population in the period. For the acquiring firms the sample is approximately 65% of the population. The information is retrieved manually from the "Børskurslisten"[11] published by the Oslo Stock Exchange. All forms of mergers and acquisitions are included in the sample[12]. Finally, to secure ergodic and stationary time series we adjust all time series for systematic location and scale effects (Gallant, Rossi and Tauchen, 1992).

Moreover, we apply the BIC (Schwarz, 1978) for p_i, q_i, m_i and n_i and $i = i, M$, lag sizes in ARMA (p,q)-GARCH(m,n) specification. For the value weighted Norwegian market index, the ARMA (0,1) model is preferred (Solibakke, 2001a). Moreover, using the BIC criterion (Schwarz, 1978) on the squared residuals, produce an ARMA (1,1) specification for the index. This result implies a GARCH (1,1) specification for the conditional variance process. Individual assets in the sample prefer almost exclusively an ARMA (0,1)-GARCH (1,1) specification. However, some assets prefer $p \geq 0$ and $q = 0$, $p=0$ and $q \geq 1$. None of the assets prefer higher $p=q=n \geq 2$. All assets prefer $m=1$. Hence, the most elaborate specification for individual assets are ARMA (2,2)-GARCH (1,2)[13].

6 Empirical Results from a bivariate ARMA-GARCH specification

The OLS market model is denoted LS_{OLS} and the multivariate GARCH model, is denoted ML_{MGRCH}. In the LS_{OLS} market model the residuals are assumed to have a mean of zero and a constant variance (homoscedastic residuals), while in the alternative ML_{MGRCH} model, residuals can be controlled for non-synchronous trading and conditionally heteroscedasticity. Asymmetric volatility is adjusted for throughout the terms γ_i and γ_M for asset i and the market index (M), respectively, in the conditional volatility equation. We therefore apply the simultaneous estimation methodology from Section 4.2.2, to fully exploit all the advantages of the ARMA-GARCH lag specifications and to be able to apply unadjusted test statistics.

The average mean event effects for several event periods are reported in line 1 of Table 1 for the OLS regressions and in Table 2 for the bivariate GARCH regressions. Percent negative observations are reported in line 2 and statistical significant average event effects using the defined test statistic in section 4.2.2 are reported in line 3. Standard t-tests are employed using classical significance levels. A Z-statistic is reported in line 4 and is defined as $\frac{G - M \cdot p}{\sqrt{M \cdot p \cdot (1-p)}}$, where G is the number of negative parameter estimates, M is the total number of parameter estimates, and p is the probability of a negative parameter estimate. A null hypothesis of zero event effects set the probability p equal to 0.5. Table 1 and 2 report the main results.

{Insert Table 1 and 2 about here}

We approach the selling firm's sample in Panel A of Table 1 and 2. Firstly, we find no significant prior anticipation in ML_{MGRCH} in contrast to LS_{OLS}. For the event period –10 days to +1 day (*E-10+1*) relative to announcement day (t_e) shows a t-value of 2.00 for the OLS estimation and 1.02 for the bivariate ARMA-GARCH estimation. Even though the average parameter effect has increased in the ML_{MGRCH} specification relative to LS_{OLS}, the standard error of the parameter has increased relatively higher so that the t-ratio becomes insignificant for the MGARCH specification. Our results suggest no abnormal return for selling firms prior to announcement in MGARCH in contrast to OLS. Secondly, the MGARCH model suggests higher significance of abnormal returns in the post announcement day (t_e) period. For post period –1 to +20 relative to announcement day (E–1+20) and –1 to +40 relative to announcement day (E-1+40), the results suggest that the abnormal returns accrue to the shareholders of selling firms the first 20 days of the post event period. The significance of the {E-1+20} calculations is 4.36 in contrast to 4.30, even though the coefficient is lower in the MGARCH estimation; that is 0.3 to 0.26. However, both techniques suggest a substantial abnormal return to shareholders of selling firms. Moreover, we find no reversal effect from day +20 to +40 relative to announcement day in ML_{MGRCH} in contrast to LS_{OLS}.

For the acquiring firm's sample in Panel B of Table 1 and 2, our main finding is that we find no significant abnormal return for any of our pre-defined event periods in ML_{MGRCH}. In the LS_{OLS} specification our results suggest significant abnormal return close to announcement day (E-1+2). Hence, for the ML_{MLGRCH} estimation we find an overall insignificant event effect for acquiring firms. The ML_{MGRCH} results seem to suggest higher information flow and consequently higher volatility (higher standard errors) and hence, insignificant abnormal returns. The observation is interesting and is probably a result of an increase in volatility as documented in Solibakke (2000) for event periods.

Specification tests are summarized in Table 3. The null hypothesis (H_0), the proportion of OLS misspecifications is the same or less than the proportion of ARMA-GARCH misspecifications are strongly rejected. Hence, overall the ARMA-GARCH model specifies lower degree of misspecifications than the OLS model, which suggests fewer violations of model assumptions. Our results therefore seem to emphasize the finding that our ARMA-GARCH specification results may calculate abnormal return more adequately than OLS.

{Insert Table 3 about here}

The differences observed between these two models are due to the magnitude and dispersion of the α and β estimates over the samples. The properties of α, β and γ[14] from our specifications are reported in Figure 5, 6, and 7, respectively. The distributions of coefficients for LS_{OLS} and ML_{MGARCH} are slightly different over the samples for the three coefficients. The intercept coefficient, α, has a higher positive mean for the LS_{OLS} than the ML_{MGARCH} market model and the standard error is lower for both selling and acquiring firms. The slope coefficient, β, has a slightly lower mean and a slightly higher standard error for the LS_{OLS} than the ML_{MGARCH} specification for acquiring firms. For selling firms the LS_{OLS} mean and standard error for beta (β) are strongly higher than for ML_{MGARCH}. Hence, we find that the sellers are more responsive to market movements, due to non-synchronous trading and conditional heteroscedasticity. The event coefficient, γ, is reported for {E-1+20} for selling firms and {E-1+2} for acquiring firms. For the selling firms we find lower coefficient (0.04) and higher standard deviation (0.09) for ML_{MGRCH} than for LS_{OLS}. Hence, the growth in standard deviation out weighs the reduction in the coefficient. For the acquiring firms and event period minus one day to plus two days relative to announcement, we find both a higher coefficient and higher standard deviation. Also here we find a stronger increase in the standard deviation relative to the increase in the coefficient. Hence, for acquirers event coefficients show lower significance in ML_{MGRCH} relative to LS_{OLS}. In contrast sellers show lower significance in ML_{MGRCH} for pre-announcement periods and close to higher significance in post-announcement event periods.

{Insert Figure 1, 2 and 3 about here}

Our results suggest that the ML_{MGRCH} approach for event period estimation seems to be worth the extra effort for the calculation of abnormal returns. The relative large change in variance relative to mean seems indeed to change inferences in event studies. Moreover, for both assets and index series, the specification Z-tests suggest a strong preference for ARMA-GARCH lag specifications relative to OLS. The event-study results are therefore more adequately modelled in the ARMA-GARCH specification accounting for non-synchronous trading and changing volatility.

7 Summaries and Conclusions

The main purpose for this paper is to estimate market model parameters in classical market model event studies adjusted for non-synchronous trading and changing and asymmetric volatility. Even though there is no intrinsic interest in estimating the conditional variance, the market model should be estimated by maximum likelihood in order to obtain more efficient estimators of the regression parameters. The lack of efficiency of the least square estimator may result in such a poor estimate that the wrong conclusion may be drawn. Applying ARMA-GARCH methodology and a simultaneous estimation and event period specification, incorporates synchronous trading, constant and symmetric volatility in event studies. The results suggest that the calculated effects may lead to different interpretation of the economic significance from event announcements controlling for non-synchronous trading and volatility clustering in thinly traded markets. The presence of GARCH does not violate the assumptions of the second order properties of the least square estimator. However, the differences in abnormal returns obtained in our study are due to the fact that the coefficient estimates of α and β using the OLS market model are inefficient since they are not adjusted for GARCH (conditional heteroscedasticity). When the OLS market model residuals are tested for the presence of GARCH using the Lagrange Multiplier approach of Engle (1982), a strong evidence of ARCH properties is revealed. The GARCH models resolve this problem, and the estimators are more efficient. The economic significant changes in results from the OLS to the ARMA-GARCH specifications are summarised below.

The main results from the multivariate ARMA-GARCH specification suggest that there is no prior anticipation in ML_{MGRCH} for either selling or acquiring firm samples. The "close to the announcement day" effect for the acquiring firm's sample in OLS estimation is rejected in the ML_{MGRCH} specification. In fact, for acquiring firms we find no significant event effects at all in the ML_{MGRCH} specification. For selling firms the abnormal return in the post announcement period show increase in both level and significance relative to LS_{OLS}. We find no reversal to zero of abnormal return from day +20 to +40 in ML_{MGRCH} relative to LS_{OLS}. We find no prior anticipation for selling firms in ML_{MGRCH} in contrast to LS_{OLS}. Moreover, specification tests report significantly lower model misspecifications for the ARMA-GARCH than for the OLS specification. Hence, our findings suggest that ML_{MGRCH} estimation for event studies do indeed change inferences applying a simultaneous estimation and event period investigation. Therefore, applying the simultaneous ML_{MGRCH} methodology, our results suggest that classical studies should be replicated to control for non-synchronous trading and changing and asymmetric volatility often found in classical LS_{OLS} studies. In fact, our results suggest that the ARMA-GARCH methodology claims a more efficient market with no anticipation and reversal.

[1] We employ a sample of mergers and acquisitions in Norway from April 1st 1983 to April 1st 1994, a total sample of 512 firms.

[2] We do not assume a change in systematic risk (beta). The firm specific (unsystematic) risk may change due to higher information flow and higher financial press coverage.

[3] See for example the mergers and acquisition study in Eckbo and Solibakke, 1992.

[4] See Solibakke, 2000 for portfolios in event and non-event periods.

[5] For application, see Bollerslev et al. 1992.

[6] Two formulations are available: BEKK formulation (Bollerlev, Engle, Kraft and Kroner) and VEC(H) formulation. VEC(H) formulation allows non-positive conditional variance (H_t).

[7] The asymmetric specification is the well-known GJR-specification (Glosten et al., 1993). An alternative asymmetric specification is Exponential GARCH (EGARCH). However, the GJR specification is Lagrange Ratio preferred to EGARCH in over 96% of the bivariate estimations.

[8] See Boehmer et al, 1991, Brown, 1988, 1989.

[9] Applying the central limit theorem.

[10] Two-step ARMA-GARCH lag specifications are available from author upon request. The results seem to confirm that a two-step procedure does not adequately control for non-synchronous trading and volatility clustering relative to a simultaneous procedure.

[11] Also "Dagens Næringsliv" and "Aftenposten" are used for collection of information.

[12] The whole list of acquiring and selling firms in the two samples are available from the author upon request.

[13] The BIC preferred lag structure for individual asset must be considered in each estimation while the marked index always prefer the same specification, defined above.

[14] Minus 1 day to plus 20 days relative to announcement day for selling firms and minus 1 day to plus 2 days relative to announcement days for acquiring firms.

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Table 1. **OLS market model; LS_{OLS}**

Average and standard deviation; % negatives; t-ratios and a Z-statistic

PANEL A: Selling firm sample

	LS_{OLS} Alfa	Beta	E-10+10	E-1+20	E-10+1	E-1+2	E-40+0	E-1+40	E-40+40
Average	0.0051	0.7161	0.1921	0.3024	0.1819	0.0856	-0.0007	0.0725	0.0341
Std. error	0.0157	0.0210	0.0663	0.0703	0.0911	0.1164	0.0537	0.0492	0.0399
Negatives	49.59 %	9.51 %	33.70 %	30.43 %	41.30 %	45.65 %	46.74 %	48.91 %	47.83 %
T-ratio	0.3264	34.1584	2.8992	4.3047	1.9959	0.7358	-0.0122	1.4735	0.8560
Z-statistic	-0.0782	-7.7672	-3.1277	-3.7533	-2.0851	-0.8341	-0.6255	-0.2085	-0.4170

PANEL B: Acquiring firm sample

	LS_{OLS} Alfa	Beta	E-10+10	E-1+20	E-10+1	E-1+2	E-40+0	E-1+40	E-40+40
Average	0.0345	0.8803	0.0617	0.0297	0.0454	0.1644	-0.0178	0.0005	-0.0146
Std. error	0.1822	0.3881	0.4747	0.4494	0.7958	1.3515	0.3780	0.3627	0.3009
Negatives	40.8 %	1.77 %	47.5 %	50.7 %	50.7 %	45.0 %	53.9 %	50.0 %	51.4 %
T-ratio	2.5335	82.1495	1.4916	0.6978	0.7954	1.9868	-0.6326	0.0160	-0.6221
Z-statistic	-3.0966	-16.197	-0.8337	0.2382	0.2382	-1.9674	1.3101	0.0000	0.4764

E-10+10 = Event period dummy from day –10 to day +10 relative to announcement date
E-1+20 = Event period dummy from day –1 to day +20 relative to announcement date
E-10+1 = Event period dummy from day –10 to day +1 relative to announcement date
E-1+2 = Event period dummy for day –1 to day +2 relative to announcement date
E-40+0 = Event period dummy for day –40 to day +0 relative to announcement date
E-1+40 = Event period dummy for day –1 to day +40 relative to announcement date
E-40+40 = Event period dummy for day –40 to day +40 relative to announcement date

Table 2. Bivariate ARMA (p,q) - GARCH (m,n) model; ML_{MGRCH}**
Average and standard deviation; % negatives; t-ratios and a Z-statistic

PANEL A: Selling firm sample*

	ML_{MGRCH} Alfa	Beta	E-10+10	E-1+20	E-10+1	E-1+2	E-40+0	E-1+40	E-40+40
Average	-0.06940	0.96153	0.17451	0.26276	0.22254	0.12254	0.02254	0.22420	0.02542
Std. error	0.01992	0.04324	0.09045	0.06033	0.21833	0.13046	0.03049	0.05416	0.01062
Negatives	65.94 %	9.42 %	46.74 %	34.78 %	50.00 %	51.50 %	51.00 %	37.50 %	43.50 %
T-ratio	-3.48342	22.2373	1.92935	4.35576	1.01928	0.93928	0.73928	4.13928	2.39277
Z-statistic	3.05821	-7.78454	-0.62554	-2.91920	0.00000	-0.42515	-0.25150	-2.51500	-1.95234

PANEL B: Acquiring firm sample*

	ML_{MGRCH} Alfa	Beta	E-10+10	E-1+20	E-10+1	E-1+2	E-40+0	E-1+40	E-40+40
Average	-0.0032	0.8368	0.0614	0.0750	0.0794	0.2088	-0.0164	-0.0036	0.0025
Std. error	0.1880	0.4180	0.6350	0.5965	0.7303	1.8050	0.4353	0.5277	0.4066
Negatives	50.00 %	1.77 %	45.04 %	46.81 %	47.52 %	46.10 %	52.84 %	49.29 %	53.90 %
T-ratio	-0.2082	80.7296	1.6193	1.5504	1.4382	1.5850	-0.5341	-0.1096	0.0736
Z-statistic	0.0000	-16.1974	-1.6674	-1.0719	-0.8337	-1.3101	0.9528	-0.2382	1.3101

* See Table 1 for event period specification.
** The ARMA(p,q) and GARCH(m,n) are based on the BIC (Schwarz et al., 1978) for the raw returns and the squared residuals, of individual assets and index, respectively.

Table 3. Propotion misspecification

Panel A. ARMA-GARCH proportion model misspecifications

	Q(6)	$Q^2(6)$	ARCH(6)	RESET(12;6)	BDS m=2	BDS m=3
Selling Firms	0.075269	0.075269	0.075269	0.0752688	0.075269	0.075269
Acquiring Firms	0.032051	0.044872	0.044872	0.025641	0.076923	0.089744

Panel B. OLS proportion model misspecifications

	Q(6)	$Q^2(6)$	ARCH(6)	RESET(12;6)	BDS m=2	BDS m=3
Selling Firms	0.763441	0.817204	0.817204	0.5376344	0.913978	0.924731
Acquiring Firms	0.717949	0.801282	0.801282	0.5128205	0.865385	0.884615

Panel C. Z-test (5%) for Proportion misspecification in OLS <= ARMA-GARCH

Z-test Selling Firms	9.509929	10.17766	10.17766	6.8390006	11.43914	11.58512
Z-test Acquiring Firms	12.51269	13.52185	13.52185	9.70027	13.95024	14.04485

$$Z_{5\%} = \frac{\begin{pmatrix} difference \quad in \\ observed \quad proportions \end{pmatrix} - \begin{pmatrix} difference \quad between \quad proportions \\ under \quad the \quad null \quad hypothesis \end{pmatrix}}{Estimated \quad s\tan dard \quad error \quad of \quad the \quad differences}$$

Q(6) : Ljung and Box (1976) statistic for serial correlation up to lag 6; $Q^2(6)$: serial correlation for squared series up to lag 6. ARCH (6) : a test for conditional heteroscedasticity in returns. Low {.} indicates significant values. RESET (12,6) : A sensitivity test for mainly linearity in the mean equation. 12 is number of lags and 6 is the number of moments that is chosen in our implementation of the test statistic. $T \cdot R^2$ is χ^2 distributed with 12 degrees of freedom. BDS (m=2,ε=1): A test statistic for general non-linearity in a time series. The test statistic BDS $= T^{1/2}.[C_m(\sigma \cdot \varepsilon) - C_1(\sigma \cdot \varepsilon)^m]$, where C is based on the correlation-integral, m is the dimension and ε is the number of standard deviations. Under the null hypothesis of identically and independently distributed (i.i.d.) series, the BDS-test statistic is asymptotic normally distributed with a zero mean and with a known but complicated variance.

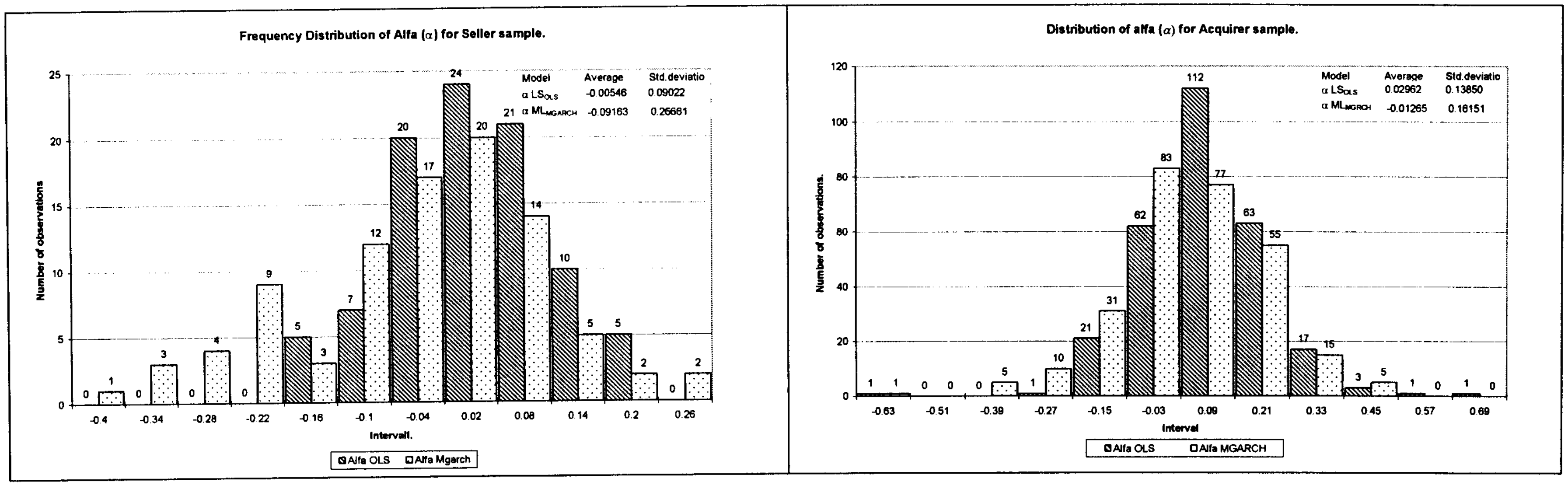


Figure 1. Histogram of Intercept (α) for LS_{OLS} and ML_{MGRCH} Simultaneous Market Model Study

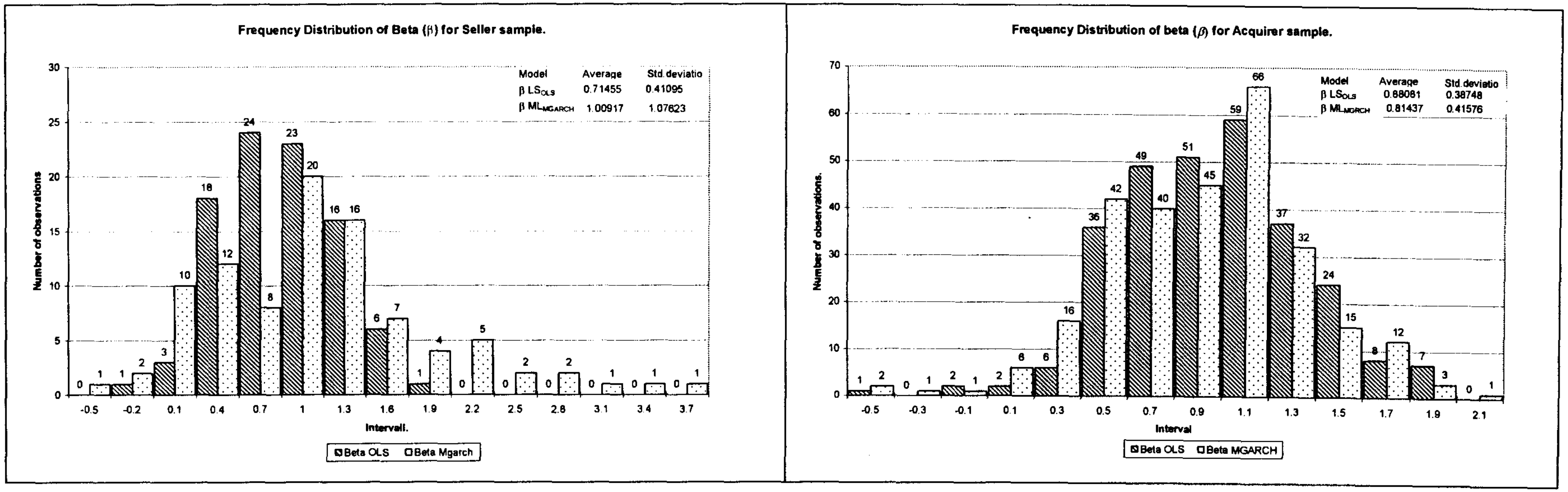


Figure 2. Histogram of Beta (β) for LS_{OLS} and ML_{MGRCH} Simultaneous Market Model Study

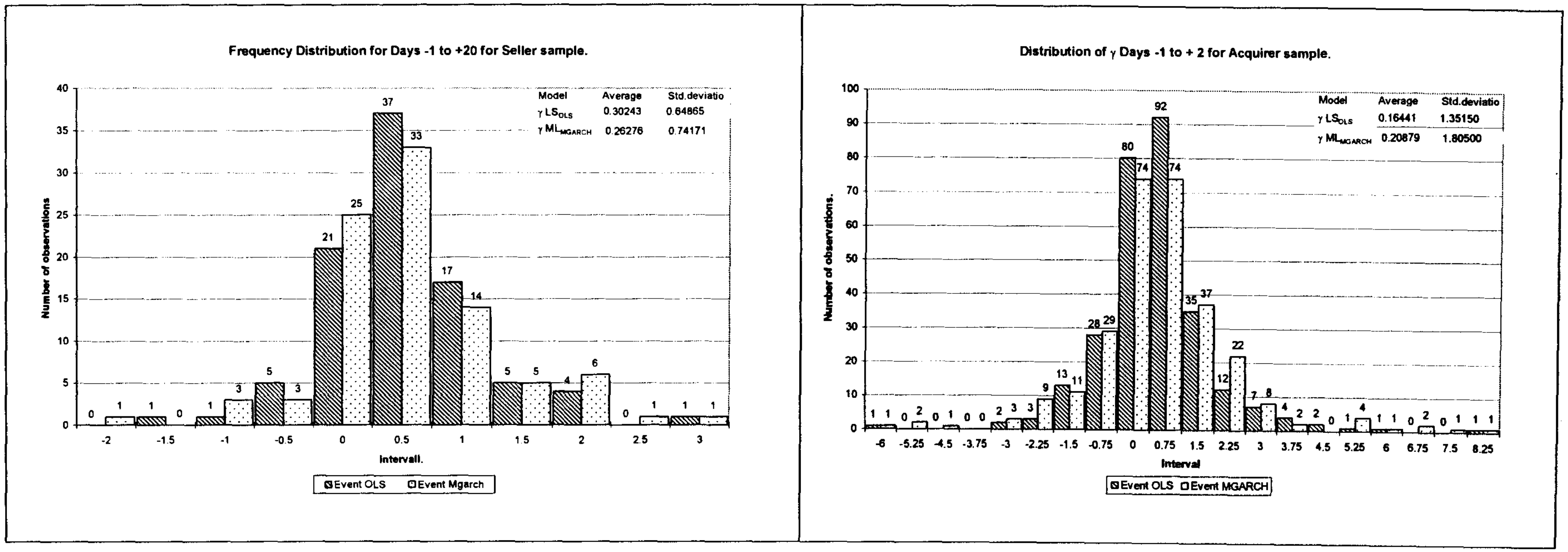


Figure 3. Histogram of Event (γ) for LS_{OLS} and ML_{MGRCH} Simultaneous Market Model Study

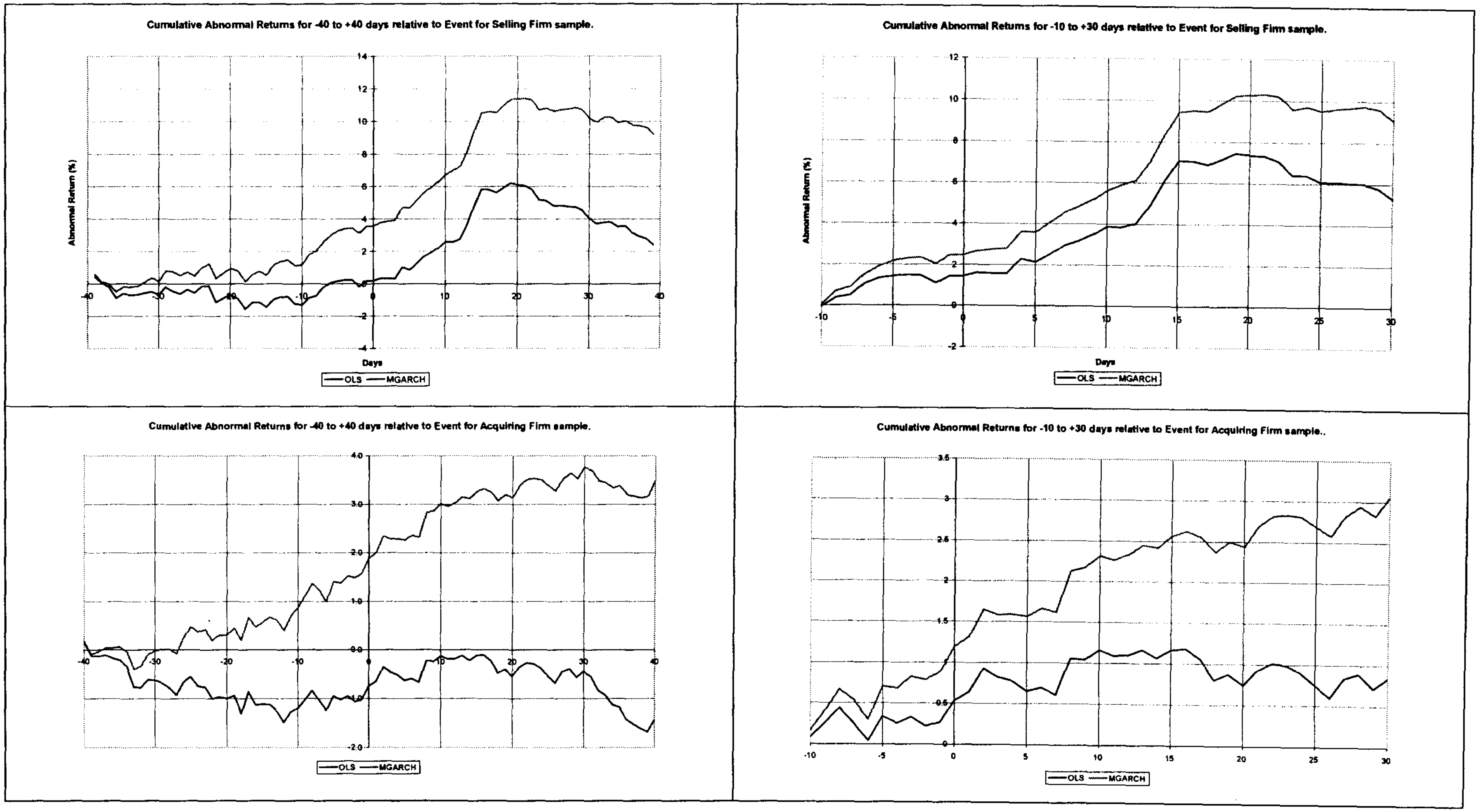


Figure 4. Cumulative Abnormal Return for Selling and Acquiring Firm Samples (Separate Estimation Period)

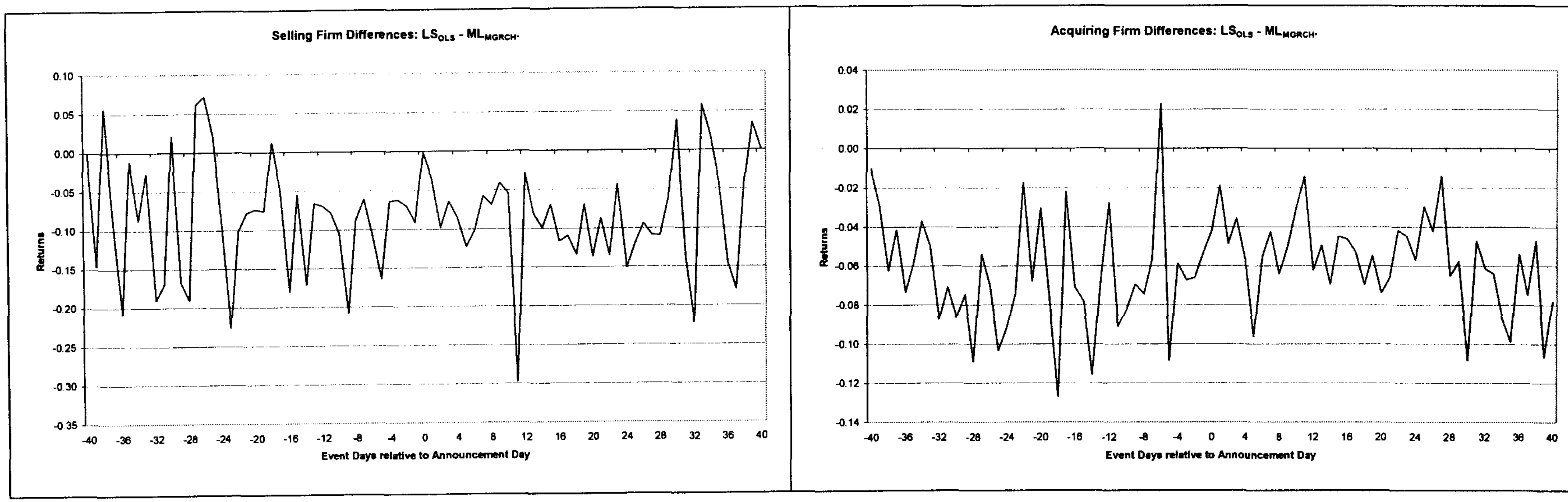
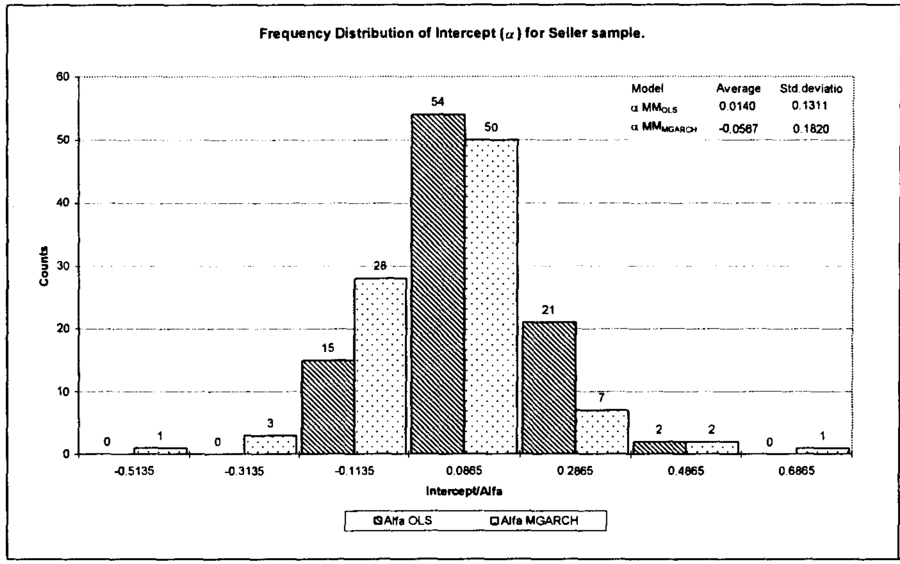


Figure 5. LS_{OLS}-ML_{MGRCH} differences for Selling and Acquiring Firm Samples (Separate Estimation Period)



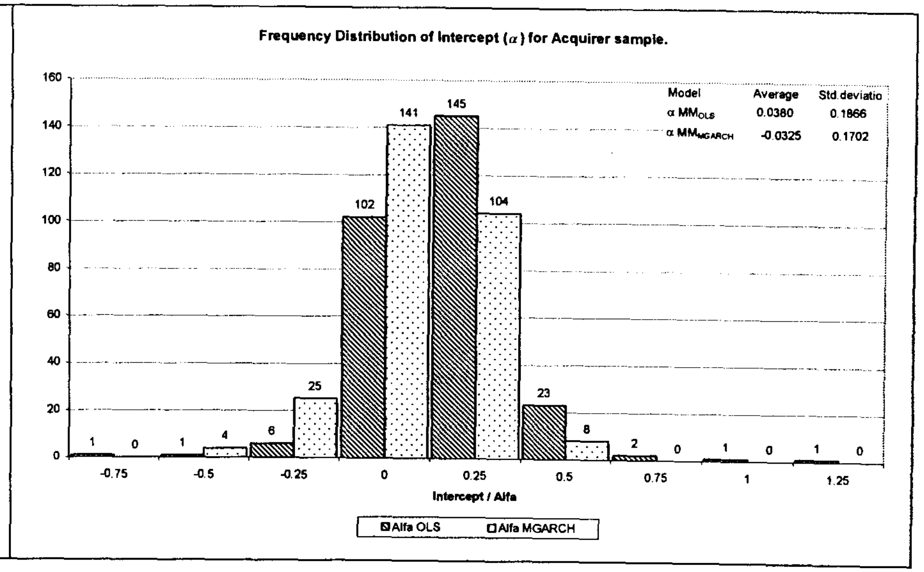
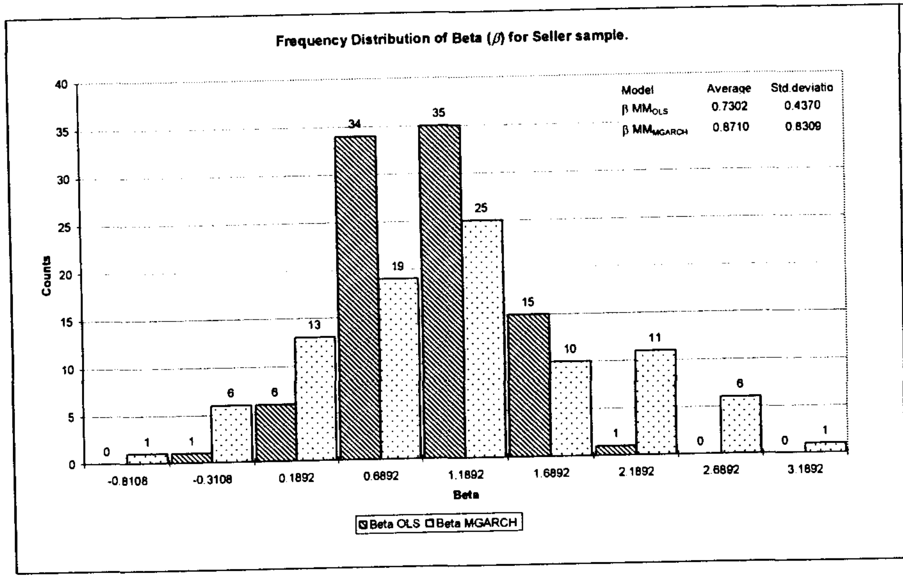


Figure 6. Histogram of Intercept (α) for LS_{OLS} and ML_{MGRCH} Market Models (Separate Estimation Period)



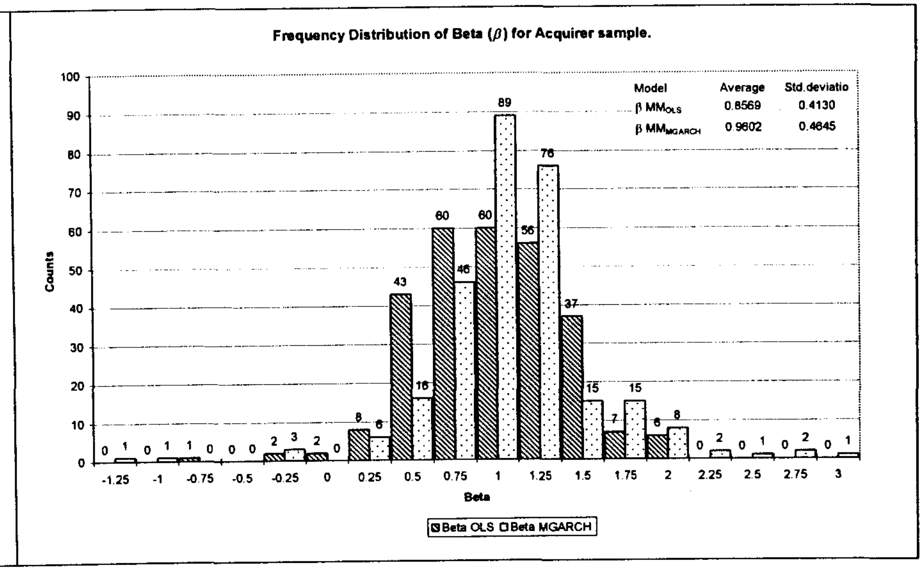


Figure 7. Histogram of Beta (β) for for LS_{OLS} and ML_{MGRCH} Market Models (Separate Estimation Period)