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by means of pay-as-you-go programs
- an investigation of alternative mechanisms**

by

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Intergenerational risk sharing by means of pay-as-you-go programs - an investigation of alternative mechanisms[*]

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Abstract

A pay-as-you-go (paygo) pension program may provide intergenerational pooling of risks to individuals' labor and capital income over the life cycle. By means of illuminating closed form solutions we demonstrate that the magnitude of the optimal paygo program and the nature of the underlying risk sharing effects are very sensitive to the chosen combination of risk concepts and stochastic specification of long run aggregate wage income growth. In an additive way we distinguish between the pooling of wage and capital risks within periods and two different intertemporal risk sharing mechanisms. For realistic parameter values, the magnitude of the optimal paygo program is largest when wage shocks are not permanent and individuals in any generation are considered from a pre-birth perspective, i.e. a "rawlsian risk sharing" perspective is adopted.

JEL Classification: H55; D91; E32; G11; E32

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1. Introduction

A growing literature which dates back to at least the contributions of Enders and Lapan (1982) and Merton (1983), considers the potential for intergenerational risk sharing by means of pay-as-you-go (paygo) social security programs. This issue reflects the insight that a paygo program is in effect a government created asset that permits one generation to trade in the human capital returns of the next. If the correlation between the growth of aggregate wage income (i.e. the implicit return of the paygo program) and alternative capital returns are less than perfect, the paygo program may consequently serve to correct for incomplete financial markets. Thus, one generation can by means of a mandatory paygo program pool its exposure to labor- and capital income risks over the life span with the succeeding generation's exposure to its labor income risk.

Looking closer at the various analyses of design and effects of paygo programs under uncertainty, it turns out that the nature of the risk sharing mechanisms varies along several dimensions. This paper considers in particular how the risk sharing effects and mechanisms vary in response to alternative stochastic specifications of wage income growth, which in our long-run intergenerational setting is equivalent to productivity growth. As discussed in some length below, empirical evidence hardly offers any clear guidance to the choice between alternative wage processes in our context. In the previous literature on social security under uncertainty, several papers assume that productivity and/or wage income follow a deterministic long run trend (i.e. shocks are temporary), see for example Enders and Lapan (1982, 1993), Gordon and Varian (1988), and De Menil et al. (2006). On the other hand, a series of other contributions to the same literature assume that wage income follows a random walk (i.e. shocks are permanent), see for example Bohn (1997), Smetters (2002) and Matsen and Thøgersen (2004). Hardly any paper in this area offers a justification of the chosen specification or an analysis of the comparative effects, however.

This paper also captures how the specification of risk sharing concept influences the intergenerational risk sharing effects of paygo programs.[1] Generally, the choice is between a "rawlsian" (or "ex-ante") perspective, which considers individuals in a pre-birth position, and an "interim" perspective, which considers individuals' position contingent on realized wages in their first part of life.[2] Some authors offer careful discussions of the two alternative risk concepts and also demonstrate that the risk sharing effects are sensitive to the choice between them, see in particular Rangel and Zeckhauser (2001), Ball and Mankiw (2001) and Wagener (2003). The interdependence

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[1] A third dimension which is also crucial for the risk sharing effects of paygo programs, is the choice between a fixed contribution rate and a fixed benefit rate, see Wagener (2003) and Thøgersen (1998).

[2] While the rawlsian risk concept is adopted more or less explicitly by, for example, Gordon and Varian (1988), Enders and Lapan (1982, 1993), Krueger and Kubler (2003) and Olovsson (2004), the interim perspective is captured by, for example, Hassler et al. (1997), Dutta et al. (2000), Rangel and Zeckhauser (2001) and Wagener (2003). We also observe that the term interim is synonymous with the terms "ex-post" and "true" as used by Hassler et al. and Wagener, respectively.

between the choice of risk concept and the stochastic specification of wage income is not addressed, however.

The contribution of this paper is to investigate how the intergenerational risk sharing effects vary with the chosen combination of risk sharing concept and stochastic specification of wage income. Utilizing that a paygo program can be characterized as a "quasi"-asset and employing a model that combines an overlapping generations framework with a portfolio choice model based on isoelastic utility and loglinear approximations (see Campbell and Viceira, 2002), we derive closed form solutions for the optimal size of the paygo program in the various cases. The simple and intuitive structure of these solutions allows us to disentangle the effects of alternative specifications in an additive and transparent way.

Having a two-period overlapping generations framework in mind and considering a given generation t, three different kinds of risk sharing effects turn out to be relevant for the design of optimal paygo programs: i) The pooling of generation t's capital income risk (in the second period of life) with the labor income risk of generation t +1. ii) The intertemporal pooling of generation t's capital income risk with its own labor income risk. iii) The intertemporal pooling of generation t's labor income risk with the labor income risk of the succeeding generation $t + 1$. Adopting an interim risk sharing concept, only the first effect is relevant regardless of the stochastic specification of wages. On the other hand, a rawlsian perspective implies that the first and second effect is relevant when wages follow a random walk (with drift), while the first and third effect is relevant when wage shocks are temporary. In addition to the risk sharing effects, there is also a standard income effect related to the expected gap between the return on capital income and the growth rate of aggregate wages.

A portfolio choice approach to social security design has recently been explored by Dutta et al. (2000), Persson (2002), Matsen and Thøgersen (2004) and De Menil et al. (2006). The essentially verbal study of Persson stresses the idea that a paygo program is a quasi-asset that should enter the optimal portfolio of the individual. Dutta et al. employs a simple static model with mean-variance preferences and derives closed form solutions for the "paygo-asset" in a case corresponding to interim risk sharing. De Menil et al. also studies the optimal mix between a public paygo program and private retirement saving. They present a rather general model which must be solved numerically.

Matsen and Thøgersen utilize the same model framework as the present paper and derive optimal paygo programs under the assumption that wage shocks are permanent. This paper extends the paper by Matsen and Thøgersen by adding an additional, crucial dimension, namely the possibility of temporary wage shocks, and by focusing on comparisons between different combinations of risk concepts and alternative stochastic properties for wage income.

A feature, which this paper shares with Dutta et al., De Menil et al. and Matsen and Thøgersen as well as for example Hassler and Lindbeck (1997) and Wagener (2003), is the assumptions that the relevant stochastic variables (i.e. capital returns, wage growth and population growth) are exogenous. This contrasts other papers that consider the risk sharing effects of paygo programs by means of

general equilibrium models. These contributions highlight how capital returns are endogenously determined by productivity, population growth and other variables, see Merton (1983) and the more recent papers by Storesletten et al. (1999), Krueger and Kubler(2003) and Olovsson (2004). On the one hand, such an approach is appealing theoretically and it seems particularly appropriate given that most of these papers focus on the trade-off between gains of risk sharing versus the costs caused by the paygo program's adverse effect on capital accumulation. On the other hand, these general equilibrium models tend to be rather complex and they do not permit derivation of closed form solutions for the optimal paygo program in the various cases considered in the present paper. It is also likely, as argued by Hassler and Lindbeck (1997), that the general equilibrium approach put overly strong restrictions on the strength of the interdependencies between the key variables mentioned above. In particular the correlation between wages and capital returns are significantly lower in the data than predicted by standard general equilibrium models.[3]

The next section presents our model and explains the two risk sharing concepts in more detail. Section 3 derives optimal paygo programs in the case of interim risk sharing and shows that a time inconsistency problem arises when wage income shocks are temporary. Section 4 adopts the rawlsian risk concept. Proceeding gradually, we first disregard capital income risk and focus on the paygo program's intertemporal pooling of wage income shocks only. In this case we are able to generalize the stochastic specification of wage income growth by introducing the possibility that the degree of persistence in the shocks may vary between zero (no persistence at all) to one (corresponding to permanent shocks). We then consider both wage and capital income shock and show how the optimal paygo program reflects quite different types of risk sharing effects depending on whether wage shocks are permanent or temporary. Section 5 presents some numerical examples and discusses key parameters and magnitudes involved in the analysis. We particularly comment on the empirical evidence regarding the correlation between wage growth and capital income defined as stock market returns. Finally, section 6 summarizes the findings and offers some concluding remarks. This includes a brief discussion of the empirical evidence related to the crucial stochastic specification of wage income growth.

2. Model framework

Following Matsen and Thøgersen (2004), we consider an overlapping generations economy where each generation has a two-period life span. Any generation t supplies inelastically one unit of labor in period t and enjoys retirement in period $t + 1$. The size of generation t is given by X_t and population

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[3] In many calibrated general equilibrium models used for social security analysis, the trick in order to match empirical plausible correlations between wages and capital income, is to introduce stochastic depreciation, see for example Olovsson (2004), Smetters (2002) and Krueger and Kubler (2002).

growth is deterministic and given by N, thus $X_{t+1} = (1+N)X_t$. We define W_t as the gross wage per unit of labor in period t.

Wages are subject to mean-zero stochastic shocks, Λ_t, which may be permanent or temporary. If wage shocks are permanent, we have

(1) $W_{t+1} = (1+\Theta)(1+\Lambda_{t+1})W_t$,

where $\Theta \geq 0$ is a deterministic drift. On the other hand, if wage shocks are temporary and wages tend to return to its deterministic long run trend, we have (when W_0 is given by history):

(1') $W_{t+1} = (1+\Theta)^{t+1}W_0(1+\Lambda_{t+1}) = (1+\Theta)\dfrac{1+\Lambda_{t+1}}{1+\Lambda_t}W_t$.

Abstracting from precautionary saving motives over the life cycle, we follow Gordon and Varian (1988), Ball and Mankiw (2001) and Matsen and Thøgersen (2004), and assume that individuals save their complete net wage income in the first period of life and, consequently, consume only in the second period. Savings is allocated between risk free saving ("bonds") with a return R^f, and risky saving ("stocks") with a stochastic return R_t, $E(R_t) > R^f > 0$.

The two stochastic variables, Λ_t and R_t, are lognormally, independently and identically distributed over time. We define $r_t \equiv \log(1+R_t)$, $\lambda_t \equiv \log(1+\Lambda_t)$, $r^f \equiv \log(1+R^f)$, $n \equiv \log(1+N)$ and $\theta \equiv \log(1+\Theta)$. Moreover, the expected excess returns are given by $\mu^\lambda \equiv E(\lambda_t - r^f)$ and $\mu^r \equiv E(r_t - r^f)$ where E denotes expectations, while variances are denoted σ_i^2 ($i = r, \lambda$) and the covariance $\sigma_{r\lambda}$. Below, a time subscript will be attached to the expectation-operator E when necessary.

We note that the growth rate of aggregate wage income, G_{t+1}, which defines the implicit return of the paygo program, is given by $1 + G_{t+1} = (1+N)\dfrac{W_{t+1}}{W_t}$.[4] Because products of lognormal variables are also lognormal, it follows from (1) and (1') that G_t is lognormal. Defining $g_t \equiv \log(1+G_t)$ and using (1) and (1'), it follows that

(2) $g_{t+1} = n + \theta + \lambda_{t+1}$,

when wage shocks are permanent, and

(2') $g_{t+1} = n + \theta + \lambda_{t+1} - \lambda_t$,

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[4] It follows that risks related to the return on the paygo program is attributed to wage (or productivity) fluctuations only, and not to demographic risks. An inclusion of demographic risks in a setting like the present one is clearly a topic for future research - and a challenge is then to disentangle between fertility- and longevity risks. For a paper on social security that focuses on demographic risks, see Bohn (2001).

when wage shocks are temporary. As we will explain below, $E_s(g_{t+1} - r^f)$ $(s = t - 1, t)$, the variance σ_g^2 and the covariance σ_{rg} will depend on the chosen risk concept and stochastic specification of wages. [5]

The period t expected utility of a representative individual in generation t is given by an isoelastic utility function

$$(3) \qquad E_t \left[\delta \frac{C_{t+1}^{1-\gamma}}{1-\gamma} \right],$$

where γ is the coefficient of relative risk aversion, δ is the time preference rate and C_{t+1} refers to consumption,

$$(4) \qquad C_{t+1} = (1-\tau)W_t \left(1 + R^f + \omega^P (R_{t+1} - R^f) \right) + \pi_{t+1} W_t .$$

Here τ is the fixed contribution rate of the pension system, ω^P is the portfolio share of net wage income invested in stocks and π_{t+1} is the replacement rate of the pension program. A useful and well known property of the specified utility function is that optimal portfolio shares will be independent of the wage level. Disregarding any initial public debt and tax-financing of other public expenditures, paygo-financing of the pension system implies that $\pi_{t+1} = \tau(1 + G_{t+1})$ when we assume that the paygo program is completely phased in.[6] Substituting this expression into (4) and rearranging yields

$$(5) \qquad C_{t+1} = W_t(1 + R_{t+1}^T), \ \ R_{t+1}^T = R^f + \tau(G_{t+1} - R^f) + (1-\tau)\omega^P (R_{t+1} - R^f) ,$$

where we refer to R_{t+1}^T as the effective return on the individual's total portfolio. It follows that τ and $(1-\tau)\omega^P$ can be interpreted as the effective portfolio shares in respectively "the paygo asset" and stocks. Below, we generally disregard the possibility of short positions in any asset.

The decision problem of the representative individual in generation t is to choose ω^P - after W_t has been revealed - in order to maximize (3) subject to (5) when the pension system is given. In order to solve this problem, we utilize the log-linear approximation method developed by John Campbell and Luis Viceira, see for example Campbell and Viceira (2002). Taking logs in (5) yields

$$(6) \qquad c_{t+1} = w_t + r_{t+1}^T \ \ ,$$

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[5] We assume that $\mu^r > E_s(g_{t+1} - r^f)$ and note that the possibility of $E_s(g_{t+1} - r^f) < 0$ does not necessarily imply dynamic inefficiency in a stochastic economy. We generally assume that the conditions for dynamic efficiency are fulfilled throughout this paper, see for example Abel et al. (1989) or Blanchard and Weil (1991) for a more detailed discussion of this issue.

[6] We might alternatively imagine a mixed public pension program, which is partly funded and partly paygo financed. If significant segments of individuals do not invest in stocks due, for example, to information asymmetries, there is a scope for such a mixed program, see Matsen and Thøgersen (2004).

where $r_{t+1}^T \equiv \log(1 + R_{t+1}^T)$, $c_{t+1} = \log C_{t+1}$ and $w_t = \log W_t$. In order to relate r_{t+1}^T to g_{t+1} and r_{t+1}, we follow Campbell and Viceira (2002: p. 27-29) and use a Taylor approximation of R_{t+1}^T, see (5).[7] This yields

$$r_{t+1}^T - r^f = \omega^p (1-\tau)\left[(r_{t+1} - r^f) + \tfrac{1}{2}\sigma_r^2 \right] + \tau\left[(g_{t+1} - r^f) + \tfrac{1}{2}\sigma_g^2 \right]$$

(7)

$$- \tfrac{1}{2}\left[(\omega^p)^2(1-\tau)^2\sigma_r^2 + \tau^2\sigma_g^2 + 2\omega^p\tau(1-\tau)\sigma_{rg} \right].$$

Maximization of (3) is equivalent to the maximization of the log of (3). Removing the unnecessary constant $\delta/(1-\gamma)$ and using that C_{t+1} is lognormal,[8] we rewrite the utility function as

(8) $\qquad \log E_t C_{t+1}^{1-\gamma} = (1-\gamma)E_t c_{t+1} + \tfrac{1}{2}(1-\gamma)^2\sigma_c^2$,

where σ_c^2 is the variance of c_{t+1}. Dividing by $1-\gamma$ and inserting from (6), we finally write the utility function of the representative individual as

(9) $\qquad E_t\left(w_t + (r_{t+1}^T - r^f) \right) + \tfrac{1}{2}(1-\gamma)\sigma_c^2$.

The representative individual in generation t chooses his optimal portfolio share ω^p for a given value of τ and after w_t has been revealed. It follows from (7) that

$$E_t\left(w_t + (r_{t+1}^T - r^f) \right) = w_t + \omega^p(1-\tau)\left[\mu^r + \tfrac{1}{2}\sigma_r^2 \right] + \tau\left[E_t(g_{t+1} - r^f) + \tfrac{1}{2}\sigma_g^2 \right]$$

(10)

$$- \tfrac{1}{2}\left[(\omega^p)^2(1-\tau)^2\sigma_r^2 + \tau^2\sigma_g^2 + 2\omega^p\tau(1-\tau)\sigma_{rg} \right]$$

and

(11) $\qquad \sigma_c^2 = (\omega^p)^2(1-\tau)^2\sigma_r^2 + \tau^2\sigma_g^2 + 2\omega^p(1-\tau)\tau\sigma_{rg}$.

Substituting (10) and (11) into (9), we derive the optimal value of ω^p by straightforward maximization,

(12) $\qquad (\omega^p)^* = \dfrac{\mu^r + \tfrac{1}{2}\sigma_r^2}{(1-\tau)\gamma\sigma_r^2} - \dfrac{\tau}{1-\tau}\dfrac{\sigma_{r\lambda}}{\sigma_r^2}$,

where the asterisk denotes an optimal value. Moreover, we have used that $\sigma_{rg} = \sigma_{r\lambda}$ from the decision perspective of the individual in both the case of permanent and temporary wage shocks, i.e. see (2) and

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[7] As discussed in more detail by Campbell and Viceira (2002: p. 32), this approximation is satisfactory even for our long run social security setting (see also Barberis, 2000, for details in this respect). We also note that we could have avoided the use of approximations simply by adopting mean-variance preferences or an exponential utility function with constant absolute risk aversion. As discussed in more detail by Matsen and Thøgersen (2004), there are at least two good reasons to resist these alternative options. First, it is well known that the specified isoelastic utility function with constant relative risk aversion captures far more realistic attitudes towards risks than these alternatives. Second, in order to design optimal pension systems, which are time consistent in a sense that will be discussed below, it is necessary that the derived portfolio shares are independent of wage fluctuations. This excludes both the alternatives.

[8] Here and throughout the paper, we repeatedly use the following result for a lognormal stochastic variable Z:

$\quad \log EZ_{t+1} = Ez_{t+1} + \tfrac{1}{2}\sigma_z^2$, $z_{t+1} \equiv \log Z_{t+1}$.

(2)' and recall that λ_t has been revealed when the individuals takes their portfolio decision. The first term on the RHS of (12) captures the trade-off between the log of the expected excess return on stocks, i.e. $\mu^r + \frac{1}{2}\sigma_r^2 = \log E_t((1+R_{t+1})/(1+R^f))$ by the result in footnote 8, and the related risks measured by the variance of the log returns. Lower risk aversion (γ) and a larger paygo program (τ) tilt this trade-off towards a higher $(\omega^p)^*$. The second term captures that stock market investments may serve as a hedge towards wage risk created by the paygo system when $\sigma_{r\lambda} \neq 0$.

Before we proceed to the derivation of optimal paygo programs, we will take a closer look at the two risk concepts. In the case of interim risk sharing, we derive the optimal paygo program contingent on realized wage uncertainty in the first period of life. Thus, for a representative individual in generation t, only the exposure to period $t + 1$ risks is relevant. Consequently, as observed from Figure 1, interim risk sharing in our context deals only with the pooling of generation t's own stock market exposure (r_{t+1}) with the labor income risk of generation $t + 1$ (λ_{t+1}). In the case of rawlsian risk sharing, we consider individuals in a pre-birth position, which means that the exposure to both period t and period $t + 1$ risks are relevant. As illustrated in Figure 1, the pooling of three risks must be taken into account in this case, namely generation t's own labor income risk (λ_t) in addition to r_{t+1} and λ_{t+1}.

*** Figure 1 ***

At this stage, we will also define our notion of time consistency.[9] Generally, our approach is to derive optimal paygo programs based on the feature that their implicit returns are given by the growth rate of aggregate wages. By the nature of paygo programs, i.e. transfers from the young and working generation to the old and retired generation, this assumes that succeeding generations must agree to the same paygo program, i.e. τ must not be altered. Thus, we define an optimal paygo program to be time consistent if the derived optimal τ^* for generation t is also optimal for any succeeding generation - given the adoption of the same risk concept. The above specification of an isoelastic utility function is crucial for time consistency. Still, as we will see in the next section, time inconsistency turns out to be a problem in the case of interim risk sharing and temporary wage shocks.

3. Interim risk sharing

In the case of interim risk sharing, the government derives the optimal paygo program for generation t by the maximization of (9) subject to (10) and (11) when w_t is given and we have substituted for ω^p

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[9] Time inconsistency problems of intergenerational tax-transfer schemes are discussed in more detail by Gordon and Varian (1988).

from (12). Using that $\sigma_g^2 = \sigma_\lambda^2$ and $\sigma_{rg} = \sigma_{r\lambda}$ from the interim time perspective in the cases of both permanent and temporary wage shock, see (2) and (2'), we derive (as shown in more details in the appendix)

$$(13) \qquad \tau^* = \frac{\left(E_t(g_{t+1} - r^f) + \frac{1}{2}\sigma_g^2\right) - \left(\mu^r + \frac{1}{2}\sigma_r^2\right)\dfrac{\sigma_{r\lambda}}{\sigma_r^2}}{\gamma\sigma_\lambda^2\left(1 - \rho_{r\lambda}^2\right)}.$$

Here $\rho_{r\lambda}$ denotes the coefficient of correlation between stock market returns and wage growth. We observe that the first term in the numerator captures that τ^* is increasing in the excess return of the paygo program, $E_t(g_{t+1} - r^f) + \frac{1}{2}\sigma_g^2 = \log E_t\left((1 + G_{t+1})/(1 + R^f)\right)$. The second term captures hedging demand in the sense that the paygo program contributes to a pooling of the wage income risk of generation $t + 1$ with the stock market risk of generation t, see Figure 1 above. Intuitively, the sign of $\sigma_{r\lambda}$ determines whether the hedging demand component increases or decreases τ^*.

It turns out that $E_t(g_{t+1} - r^f)$ is sensitive to the definition of the wage process, however. If wage shocks are permanent, (2) implies (when we use the result in footnote 8) that $E_t(g_{t+1} - r^f) + \frac{1}{2}\sigma_g^2 = n + \theta - r^f$ for all t. Thus, we can rewrite (13) as

$$(14) \qquad \tau^* = \frac{n + \theta - r^f - \left(\mu^r + \frac{1}{2}\sigma_r^2\right)\dfrac{\sigma_{r\lambda}}{\sigma_r^2}}{\gamma\sigma_\lambda^2\left(1 - \rho_{r\lambda}^2\right)},$$

and we observe that the optimal τ^* derived for generation t will also be optimal for any succeeding generations.

If wage shocks are temporary, (2') implies that $E_t(g_{t+1} - r^f) + \frac{1}{2}\sigma_g^2 = n + \theta - r^f - \lambda_t$, where λ_t is exogenous from the decision perspective of interim risk sharing. Rewriting (13) in this case yields

$$(14') \qquad \tau^* = \frac{n + \theta - r^f - \lambda_t - \left(\mu^r + \frac{1}{2}\sigma_r^2\right)\dfrac{\sigma_{r\lambda}}{\sigma_r^2}}{\gamma\sigma_\lambda^2\left(1 - \rho_{r\lambda}^2\right)},$$

and we immediately observe that the derived paygo program is time inconsistent because the revealed value of λ_t enters the formula, i.e. the optimal τ^* derived for generation t will not be optimal for succeeding generations. The intuition is simple. If the realized λ_t is non-zero, then - as long as generation $t + 1$ adheres to τ^* derived for generation t - the expected excess return on the paygo program will be tilted in the opposite direction of λ_t because the wage shock is temporary.

4. Rawlsian risk sharing

Turning to the rawlsian risk sharing concept, we first note that the government's derivation of the optimal paygo program for generation t is now based on the maximization of period $t-1$ expected utility. As we will observe below, the time-inconsistency problem identified in the case of temporary wage shocks and interim risk sharing, will not emerge in the rawlsian case as long as we resort to the stochastic wage processes given by (1) and (1').

The objective function of the government is

(15) $\qquad E_{t-1}\left(w_t + (r_{t+1}^T - r^f)\right) + \frac{1}{2}(1-\gamma)\sigma_c^2$,

where w_t is no longer exogenous and given by history as in (9) but subject to the shock λ_t. It follows from (1) and (1') that

(16) $\qquad w_t = \theta + \lambda_t + w_{t-1}$,

in the case of permanent shock and

(16') $\qquad w_t = \theta + \lambda_t - \lambda_{t-1} + w_{t-1}$,

in the case of temporary shock.

4.1. Pooling of wage income shocks

It is instructive to proceed gradually and to consider the pure intergenerational sharing of wage income shock as a point of departure.[10] Thus, we assume in this subsection that there is only one risk-free asset, i.e. $\omega^p = 0$ per definition.

We first consider the case of permanent wage shocks. Noting from (2) that $\sigma_g^2 = \sigma_\lambda^2$ and

$E_{t-1}(g_{t+1} - r^f) = n + \theta + \mu^\lambda = n + \theta - r^f - \frac{1}{2}\sigma_\lambda^2$, we use (6), (7) and (16) in order to derive

(17) $\qquad E_{t-1}(w_t + (r_{t+1}^T - r^f)) = \theta + w_{t-1} + \mu^\lambda + r^f + \tau(n + \theta - r^f - \frac{1}{2}\sigma_\lambda^2) + \frac{1}{2}\tau(1-\tau)\sigma_\lambda^2$,

and

(18) $\qquad \sigma_c^2 = (1+\tau^2)\sigma_\lambda^2$.

Deriving the optimal paygo program, we substitute (17) and (18) into (15) and maximize with respect to τ. This yields

(19) $\qquad \tau^* = \dfrac{n + \theta - r^f}{\gamma\sigma_\lambda^2}$.

Thus, the existence of a paygo program in this case can only be rationalized by an expected positive excess return on the paygo asset. As long as wage shocks are permanent, the paygo program can not contribute to intergenerational sharing of wage income shocks. This is evident from (18) and intuitive:

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[10] Taking into account that the majority of the population in most economies (even in the US according to Poterba, 2000) has rather small portfolio shares in stocks, disregarding stock market investments may not be that unrealistic.

Contributions to the paygo program in the first period of life "tax away" some part of the exposure to this period's wage shock - but this exposure returns by means of the paygo benefits in the second period of life because the shock is permanent - and hence reflected in the wage of the next generation.

Comparing (19) with (14), we also observe that the distinction between interim and rawlsian risk sharing disappears when we consider only wage shocks which are permanent, i.e. (14) is equivalent to (19) when we disregard stocks. This reflects the features of our isoelastic utility function. The shock λ_t, which is relevant in the rawlsian case - but not in the interim case, determines the endowment of generation t - and we know that the magnitude of the endowment does not influence optimal portfolio shares.

Turning to the case of temporary shocks, we first observe that (2') implies $\sigma_g^2 = 2\sigma_\lambda^2$ and $E_{t-1}(g_{t+1} - r^f) = n + \theta - r^f$. Using (6), (7) and (16'), we then derive

(17') $\quad E_{t-1}(w_t + (r_{t+1}^T - r^f)) = \theta + w_{t-1} - \lambda_{t-1} + \mu^\lambda + r^f + \tau(n + \theta - r^f) + \tau(1-\tau)\sigma_\lambda^2$

and

(18') $\quad \sigma_c^2 = \left((1-\tau)^2 + \tau^2\right)\sigma_\lambda^2$.

Substituting (17') and (18') into (15) and maximizing with respect to τ, we obtain

(19') $\quad \tau^* = \dfrac{n + \theta - r^f}{\gamma 2\sigma_\lambda^2} + \dfrac{1}{2}$.

Here, the last term on the RHS captures the optimal intergenerational risk sharing between succeeding generations facing independent shocks with zero persistence, i.e. generation t swaps half of its exposure to its own income shock, λ_t, with half of the exposure to the next generation's income shock, λ_{t+1} (confer Figure 1). This is the similar risk sharing mechanism as studied by Gordon and Varian (1988), Enders and Lapan (1982, 1993) and Thøgersen (1998). The first term on the RHS of (19') reflects the effect of the paygo program's excess return in the same way as in the case of permanent shocks. Comparing (19) and (19') and noting that $n + \theta - r^f$ is likely to be small or even negative, we can conclude that the scope for a paygo program in the case of rawlsian risk sharing is much larger in the case of temporary wage shocks - at least as long as risky stock market returns are not taken into account.

In the final part of this subsection, we also note that the narrow focus on only wage income shocks makes the derivation of closed form solutions possible under the generalized wage process

(1'') $\quad W_{t+1} = (1+\Theta)^{t+1} W_0 \left[\dfrac{W_t}{W_0(1+\Theta)^t} \right]^\beta (1 + \Lambda_{t+1}) = (1+\Theta)\dfrac{\left(W_0(1+\Theta)^{t-1}\right)^{\beta(1-\beta)}}{(W_{t-1})^{\beta(1-\beta)}} \dfrac{1+\Lambda_{t+1}}{(1+\Lambda_t)^{1-\beta}} W_t$,

where β, $0 \leq \beta \leq 1$, captures the degree of persistence in the wage income shocks. We observe that $\beta = 1$ implies that (1'') simplifies to (1), while $\beta = 0$ is equivalent to (1').

Using (1''), we may go through the same steps as above. Noting in particular that

(20) $E_{t-1}(g_{t+1} - r^f) = n + \theta + \beta E(\lambda) + \beta(1-\beta)\varphi_{t-1} - r^f$, $\varphi_{t-1} = w_0 + (t-1)\theta - w_{t-1}$,

and

(21) $\sigma_g^2 = \left(1 + (1-\beta)^2\right)\sigma_\lambda^2$,

we obtain, as shown in the Appendix,

(22) $\tau^* = \dfrac{n + \theta - r^f}{\gamma\left(1 + (1-\beta)^2\right)\sigma_\lambda^2} + \dfrac{\beta(1-\beta)\varphi_{t-1}}{\gamma\left(1 + (1-\beta)^2\right)\sigma_\lambda^2} + \dfrac{(1-\beta)(2\gamma - \beta)}{2\gamma\left(1 + (1-\beta)^2\right)}.$

Comparing with (19) and (19'), we first note that the two first terms on the RHS of (19'') captures the effect of the paygo program's excess return for generation $t + 1$ as observed from period $t - 1$. The first term is analogous to (19) and the first term on the RHS of (19'). The second term, which is non-zero for $\beta \in \langle 0,1 \rangle$, captures how the gradually declining weight of previous shocks influences

$E_{t-1}(g_{t+1} - r^f)$, see (20) and note the definition of φ_{t-1}. The last term captures the intergenerational sharing of wage shocks, and we easily see that this term simplifies to 0 or ½ for β equal to respectively 1 and 0. Moreover, the Appendix demonstrates that the magnitude of this term decreases monotonically as β increases (i.e. as the persistence of the wage shocks increases, the scope for intergenerational sharing of wage shocks declines) – given that $2\gamma - \beta > 0$. This is hardly a critical assumption because it can only be violated for unrealistically low degrees of relative risk aversion (γ must be below ½).

A more serious problem related to (22) is the presence of the same type of time-inconsistency problem as in the combination of interim risk sharing and temporary wage shock. When $\beta \in \langle 0,1 \rangle$, φ_{t-1} will fluctuate over time and τ^* derived for generation t will not be optimal for succeeding generations.

4.2 Wage and stock market shocks

Returning to the full model set-up, we first consider the case of permanent wage shocks. Then it follows from (2) that $E_{t-1}(g_{t+1} - r^f) = n + \theta + \mu^\lambda$, $\sigma_g^2 = \sigma_\lambda^2$ and $\sigma_{rg} = \sigma_{r\lambda}$ in the case of rawlsian risk sharing (just as in the case of interim risk sharing). Using (2), (6), (7) and (16), we derive

(23)

$E_{t-1}\left(w_t + (r_{t+1}^T - r^f)\right) = \theta + w_{t-1} + \mu^\lambda + r^f + \omega^p(1-\tau)\left[\mu^r + \tfrac{1}{2}\sigma_r^2\right] + \tau\left[n + \theta + \mu^\lambda + \tfrac{1}{2}\sigma_\lambda^2\right]$

$\qquad\qquad\qquad\qquad - \tfrac{1}{2}\left[(\omega^p)^2(1-\tau)^2\sigma_r^2 + \tau^2\sigma_\lambda^2 + 2\omega^p\tau(1-\tau)\sigma_{r\lambda}\right]$

and

(24) $\sigma_c^2 = \sigma_\lambda^2 + \tau^2\sigma_\lambda^2 + (\omega^p)^2(1-\tau)^2\sigma_r^2 + 2\omega^p(1-\tau)(1+\tau)\sigma_{r\lambda}.$

In order to derive corresponding expressions in the case of temporary shocks, we first note that (2') implies $E_{t-1}(g_{t+1} - r^f) = n + \theta - r^f$, $\sigma_g^2 = 2\sigma_\lambda^2$ and $\sigma_{rg} = 0$ from the time perspective of rawlsian risk sharing.[11] Using (2'), (6), (7) and (16'), we obtain

$$E_{t-1}\left(w_t + (r_{t+1}^T - r^f)\right) = \theta + w_{t-1} - \lambda_{t-1} + \mu^\lambda + r^f + \omega^p(1-\tau)\left[\mu^r + \tfrac{1}{2}\sigma_r^2\right]$$

(23')

$$+ \tau\left[n + \theta - r^f + \sigma_\lambda^2\right] - \tfrac{1}{2}\left[(\omega^p)^2(1-\tau)^2\sigma_r^2 + \tau^2 2\sigma_\lambda^2\right]$$

and

(24') $\sigma_c^2 = (1-\tau)^2\sigma_\lambda^2 + \tau^2\sigma_\lambda^2 + (\omega^p)^2(1-\tau)^2\sigma_r^2 + 2\omega^p(1-\tau)\sigma_{r\lambda}$.

Taking into account that the individuals still choose their own optimal portfolio share in stocks, we can substitute for ω^p from (12). In the case of permanent wage shocks we maximize (15) subject to (23), (24) and (12). This yields, as shown in the Appendix,

(25) $$\tau^* = \frac{1}{\gamma}\frac{(n+\theta-r^f)}{\sigma_\lambda^2\left(1-\rho_{r\lambda}^2\right)} + \left(1 - \frac{1}{\gamma}\right)\frac{\dfrac{(\sigma_{r\lambda})^2}{\sigma_r^2}}{\sigma_\lambda^2\left(1-\rho_{r\lambda}^2\right)} - \frac{\left(\mu^r + \tfrac{1}{2}\sigma_r^2\right)\dfrac{\sigma_{r\lambda}}{\sigma_r^2}}{\gamma\sigma_\lambda^2\left(1-\rho_{r\lambda}^2\right)}.$$

In the case of temporary wage shocks, the maximization of (15) is subject to (23'), (24') and (12). As demonstrated in the Appendix, we obtain

(25') $$\tau^* = \frac{1}{\gamma}\frac{(n+\theta-r^f)}{\sigma_\lambda^2\left(2+\rho_{r\lambda}^2\right)} + \left(1 - \frac{1}{\gamma}\right)\frac{\dfrac{(\sigma_{r\lambda})^2}{\sigma_r^2}}{\sigma_\lambda^2\left(2+\rho_{r\lambda}^2\right)} + \frac{1}{2+\rho_{r\lambda}^2}.$$

Comparing (25) to (25'), we first observe that the first term on the RHS of both equations reflects the expected excess return of the paygo programs. The remaining terms all reflect risk sharing effects, and we observe that the relevant mechanisms differ. In both cases there is an intertemporal risk sharing effect captured by the second term on the RHS of both equations.[12] This term, which is not present in the case of interim risk sharing, compare (25) and (25') to respectively (14) and (14'), reflects that the paygo program contributes to the sharing of generation t's stock market risk (in period $t + 1$) with its own labor income risk (in period t), see Figure 1.

As illustrated by Figure 1, there are two additional potential risk sharing effects. Firstly, the paygo program may pool generation t's exposure to stock market risk in period t + 1 with the labor income risk of the next generation. This effect turns up in the case of permanent wage shocks, see the third term on the RHS of (25), in the same way as in the case of interim risk sharing (both when wage

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[11] In order to verify that $\sigma_{rg} = 0$ in the case of rawlsian risk sharing, note that by definition

$\sigma_{rg} = E_{t-1}(r_{t+1}g_{t+1}) - E_{t-1}(r_{t+1})E_{t-1}(g_{t+1})$. Inserting for g_{t+1} from (2'), it is straightforward to verify that $\sigma_{rg} = 0$.

[12] We observe from (25) and (25') that the two first terms on the RHS of both equations are a weighted average of i) the paygo program's excess return (with weight $\frac{1}{\gamma}$) and ii) the intertemporal risk sharing effect (with weight $1 - \frac{1}{\gamma}$). An analogous weighted average (with similar weights) between an asset's expected risk premium and the associated intertemporal risk sharing effect is derived by Campbell and Viceira (2002, p. 56).

shocks are temporary and permanent), see the last term on the RHS of respectively (14) and (14'). As we observe from (25'), this effect is not present in the case of temporary wage shocks in combination with rawlsian risk sharing, however. This reflects that $\sigma_{rg} = 0$ in the latter case, while $\sigma_{rg} = \sigma_{r\lambda}$ in the other cases.

Finally, the paygo program may provide intertemporal pooling of generation t's own wage income shock with the succeeding generation's wage income shock. Intuitively, as elaborated in section 4.1 above, this effect is present only in the case of temporary wage shocks, see the last term of (25'). When $\rho_{r\lambda} = 0$, this effect is exactly ½, as in the restricted case of only wage income shocks, see the last term of (19'). Generally, because the last term of (25') must in any case be at least equal to $\frac{1}{3}$, we should expect that the magnitude of the optimal paygo program under rawlsian risk sharing is significantly larger in the case of temporary wage shocks than in the case of permanent wage shocks (compare (25) to (25')) - also when risky stock returns are taken into account.

5. A brief numerical example

In order to obtain a quantitative feeling of the magnitudes involved in our analysis, this section provides some illustrating numerical calculations. Table 1 presents the assumptions underlying the calculations. Broadly speaking, these are rough averages of the estimates derived by Matsen and Thøgersen (2004) for four countries (i.e. Norway, Sweden, the UK and the USA).[13] Defining one period in the model economy as twenty years, these estimates are annualized values reflecting historical observations of overlapping 20-years periods dating back to 1900.

As we observe from the table, $n + \theta - r^f = 1.5$ per cent, capturing that the paygo program has offered a positive but not very large excess return compared to the (approximated) risk free interest rate. Intuitively, the excess return on stock market investments is much higher, $\mu^r = 4.5$ per cent, capturing the historically high average returns on indices such as the total value World Equity Index of Dimson et al. (2002). The standard deviations illustrate that returns on stock market investments have been much more volatile than the implicit returns on the paygo program. The coefficient of relative risk aversion is specified equal to 3.

*** Table 1***

As we observe from the different formulas for the optimal paygo program, a key parameter is the correlation between stock market returns and the return on the paygo program. As discussed in more detail by Matsen and Thøgersen and the references therein, available estimates differ widely depending on (among other things) the specified time spans and the actual definition of capital

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[13] The calculations in Matsen and Thøgersen (2004) are mainly based on the long run data series of essentially Dimson et al. (2002) and Maddison (1991).

returns.[14] Matsen and Thøgersen's own calculations show that the correlations between the World Equity Index of Dimson et al. and GDP growth (used as a proxy for aggregate wage growth) in the four countries in their sample range from 0.28 to 0.63.[15] Even though this is a rather wide range, it still shows that the unity correlation predicted by straightforward general equilibrium model hardly fits the facts. Given the evident uncertainty related to the specification of the correlation between stock market returns and the return on the paygo asset, we choose to present calculations based on three different values capturing a rather wide range that should cover the relevant value for most economies, i.e. $\rho_{r\lambda} = 0.3$, $\rho_{r\lambda} = 0.5$ and $\rho_{r\lambda} = 0.7$.

Table 2 presents the calculated optimal paygo programs in the various cases. We immediately observe that the scope for paygo programs is generally much smaller under interim risk sharing than under rawlsian risk sharing, reflecting that only one of the risk sharing effects analyzed in this paper is relevant in the former case. Looking closer at (14), we see that the existence of a paygo program (i.e. $\tau^* > 0$) hinges on a positive sign and a sufficient magnitude of $n + \theta - r^f$ given the empirically plausible case of $\sigma_{r\lambda} > 0$. The low 1.5 per cent value of $n + \theta - r^f$, capturing the ageing of most OECD economies, therefore implies a rather small paygo program for a low $\rho_{r\lambda} = 0.2$ and no paygo program at all when a higher $\rho_{r\lambda}$ coefficient implies that the hedging demand component dominates, i.e. the last term in the numerator of (14).

*** Table 3***

Turning to the cases of rawlsian risk sharing, the optimal paygo programs are much larger. Generally, this reflects the inclusion of additional intertemporal risk sharing effects. We observe in particular that the cases that include intertemporal pooling of temporary wage shocks are characterized by large optimal paygo programs. Moreover, the difference between the cases of respectively permanent and temporary wage shocks is larger when stocks are excluded (the bottom part of Table 2) than included (the middle part of Table 2). Looking at the underlying formulas, this reflects that the hedging demand related to the pooling of contemporary wage and stock return shocks, i.e. the last term on the RHS of (25), contributes strongly to a smaller paygo program given that $\sigma_{r\lambda} > 0$.

As a last observation from the middle panel of Table 2, we note that the sensitivity to the magnitude of $\rho_{r\lambda}$ is much stronger when wage shocks are permanent rather than temporary. This reflects the combination of two effects. Firstly, the intertemporal pooling of temporary wage shocks is

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[14] Bottazzi et al. (1996) and Davis and Willen (2000) both present correlations between capital income and wage growth that are surprisingly close to zero, while Baxter and Jerman (1997) and Jerman (1999) present much higher correlations - particularly for longer time spans. Still, the calculations of Jerman show that the correlations drop when capital income is defined as stock market returns rather than alternative broader measures.

[15] Matsen and Thøgersen (2004) compare their calculations for the US with some of the calculations of Jerman (1999) and observe that the magnitude of the correlations does not seem to be very sensitive to whether aggregate wage growth is approximated as GDP growth when data reflect twenty years overlapping periods (at least not for the US).

not that sensitive to the magnitude of $\rho_{r\lambda}$, see the last term of (25'). Secondly, the hedging demand related to the pooling of wage and stock return shocks, is on the contrary very sensitive to $\rho_{r\lambda}$, see the last term on the RHS of (25).

Interpreting the figures of Table 2, we must have in mind that our model does not capture any distortionary effects of the paygo program on labor supply and retirement age. Assuming that such distortions tend to be significant in the cases of most real world pension programs, it seems clear that the calculated optimal paygo programs are clearly too large literally interpreted. Still, we conjecture that the calculations shed much light on how the magnitude of the optimal paygo program varies significantly in response to the actual wage process and the specified risk concept.

6. Final remarks

This paper has studied how the intergenerational risk sharing effects of paygo pension programs vary - in terms of magnitudes and mechanisms involved - in response to the actual combination of risk sharing concept and stochastic specification of aggregate wage growth. Our analytical framework allowed us to disentangle between three different risk sharing effects, capturing how any generation *t*'s exposure to its own capital and wage risk in various ways may be pooled with the succeeding generation's wage risk by means of a paygo program.

Adopting an interim perspective, only one risk sharing effect is relevant, namely the pooling of generation *t*'s capital income risk with the wage risk of the succeeding generation. Moreover, our analysis also shows that the optimal paygo program under interim risk sharing is time consistent when wage shocks are permanent - but not when they are temporary. Thus, the optimal paygo program is hardly sustainable in the latter case because generation *t* + 1 will not adhere to the optimal paygo program derived by generation *t*, implying that generation *t* can not rely on the derived risk-return features of the paygo program.

Adopting a pre-birth, rawlsian perspective, a richer set of risk sharing effects is present (i.e. recall Figure 1), and it also turns out that there are no time inconsistency problems as long as we specify wage shocks as either permanent or fully temporary within one period.[16] Regardless of the specified wage process, there is an intertemporal risk sharing effect that captures how a non-zero correlation between generation *t*'s own wage (in period *t*) and capital income (in period *t* + 1) does influence the optimal paygo program. In addition, there is a significant potential for pooling of generation *t*'s own wage shock with the wage shock of the succeeding generation. Intuitively, this second intertemporal risk sharing effect is only present when the wage shocks are temporary. As

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[16] Having in mind that one period in the model is (at least) 20 years, it follows that fully temporary wage shocks in this context do not exclude the possibility of a large degree of persistence over shorter time spans that might cover several years.

illustrated by the numerical calculations, this latter effect is, for reasonable parameter values, fairly large in magnitude compared to the other risk sharing effects. Finally, the pooling of generation t's capital income risk with the wage risk of generation $t + 1$ is relevant under rawlsian risk sharing only when wage shocks are permanent. This effect vanishes when wage shocks are temporary, reflecting that the implicit return of the paygo program, as seen from a pre-birth, rawlsian perspective, in this case captures two independent temporary wage shocks. This implies that the correlation between the return on the paygo program and capital income is zero.

While the relevant choice of risk concept in principle must reflect the preferences of the voters, or the politicians as designers of the actual pension programs, the relevant specification of the wage process is an empirical question. At the outset, the large empirical (and business cycle oriented) literature on the assessments of potential unit root properties of output, productivity, wages and other key macroeconomic variables should shed light on this latter issue. It turns out, however, that this literature, which flourished during the 1980s and early 1990s hardly offer conclusive evidence. While some early well known contributions presented evidence in favor of unit root, see for example Nelson and Plosser (1982), many subsequent papers found evidence of a deterministic trend, see for example Cochrane (1988).

A general conclusion from analyses that utilize quarterly or annual data seems to be that it is nearly impossible on statistical grounds to distinguish between a stochastic trend and a deterministic trend in combination with a cyclical component that features a high degree of persistence, see for example Christiano and Eichenbaum (1990) and Balke (1991). Data limitations, i.e. the lack of many independent 20-years periods, make it even more difficult to test alternative stochastic specifications for productivity in a long run social security context than in business cycle settings. We note, however, that the majority of more recent business cycle analyses which explicitly specify the stochastic properties of technology growth, seems to disregard a random walk and rather specifies a deterministic trend with a large persistence parameter (like the 0.95 parameter in Freeman and Kydland, 2000). Summing up, the impression is that available evidence is somewhat tilted towards a specification of a deterministic trend for real wage (or productivity) growth. As demonstrated in this paper, this implies a scope for quite significant paygo programs under the rawlsian risk sharing concept.

Appendix

Derivation of equation (13)

Substituting for ω^p in (10) and (11) from (12), we obtain

(A-1)
$$E_{t-1}\left(w_t + (r^T_{t+1} - r^f)\right) = w_t + \left(1 - \frac{1}{2\gamma}\right)\frac{\left[\mu^r + \frac{1}{2}\sigma^2_r\right]^2}{\gamma\sigma^2_r} - \frac{\tau\sigma_{r\lambda}}{\sigma^2_r}\left[\mu^r + \frac{1}{2}\sigma^2_r\right]$$
$$+ \tau\left[E_t(g_{t+1} - r^f) + \frac{1}{2}\sigma^2_g\right] + \frac{1}{2}\frac{\tau^2(\sigma_{r\lambda})^2}{\sigma^2_r} - \frac{1}{2}\tau^2\sigma^2_\lambda$$

and

(A-2) $\quad \sigma^2_c = \dfrac{\left[\mu^r + \frac{1}{2}\sigma^2_r\right]^2}{\gamma^2\sigma^2_r} + \tau^2\sigma^2_\lambda - \dfrac{\tau^2(\sigma_{r\lambda})^2}{\sigma^2_r}$.

Maximization of (9) with respect to τ and subject to (A-1) and (A-2), yields the first order condition

(A-3) $\quad -\dfrac{\sigma_{r\lambda}}{\sigma^2_r}\left[\mu^r + \frac{1}{2}\sigma^2_r\right] + \left[E_t(g_{t+1} - r^f) + \frac{1}{2}\sigma^2_g\right] + \dfrac{\tau(\sigma_{r\lambda})^2}{\sigma^2_r} - \tau\sigma^2_\lambda + \dfrac{1}{2}(1-\gamma)\left[2\tau\sigma^2_\lambda - \dfrac{2\tau(\sigma_{r\lambda})^2}{\sigma^2_r}\right] = 0$.

Solving for τ yields equation (13).

Derivation of equation (22)

It follows from (1'') that

(A-4) $\quad w_{t+1} = \theta + \beta(1-\beta)\varphi_{t-1} + \lambda_{t+1} - (1-\beta)\lambda_t + w_t$,

which is the generalized version of (16) and (16'). Using (A-4) and the fact that $g_{t+1} = n + w_{t+1} - w_t$

per definition, we obtain (20) and (21). Moreover, it follows from (A-4), (6) and (7) that

(A-5)

$$E_{t-1}(w_t + (r^T_{t+1} - r^f)) = \theta + \beta(1-\beta)\varphi_{t-2} + \mu^\lambda + r^f - (1-\beta)\lambda_{t-1} + w_{t-1} + \tau E_{t-1}(g_{t+1} - r^f) + \frac{1}{2}\tau(1-\tau)\sigma^2_g$$

and

(A-6) $\quad \sigma^2_c = \tau^2\sigma^2_\lambda + \left(1 - \tau(1-\beta)\right)^2\sigma^2_\lambda$.

Maximization of (15) subject to (A-5) and (A-6), yields the first order condition

(A-7) $\quad E_{t-1}(g_{t+1} - r^f) + \dfrac{1}{2}\sigma^2_g(1 - 2\tau) + (1-\gamma)\left(\tau - (1 - \tau(1-\beta))(1-\beta)\right)\sigma^2_\lambda = 0$.

Substituting from (20) and (21) into (A-7) yields after some manipulations

(22) $\quad \tau^* = \dfrac{n + \theta - r^f}{\gamma\left(1 + (1-\beta)^2\right)\sigma^2_\lambda} + \dfrac{\beta(1-\beta)\varphi_{t-1}}{\gamma\left(1 + (1-\beta)^2\right)\sigma^2_\lambda} + \dfrac{(1-\beta)(2\gamma - \beta)}{2\gamma\left(1 + (1-\beta)^2\right)}$.

Defining the last term on the RHS, which captures the intergenerational risk sharing effect, as

Ψ, $\Psi = \dfrac{(1-\beta)(2\gamma - \beta)}{2\gamma(1 + (1-\beta)^2)}$, we derive

$$(\text{A-8}) \quad \frac{d\Psi}{d\beta} = \frac{1}{2\gamma}\left(\frac{-(2\gamma-\beta)(1-(1-\beta)^2)-(1-\beta)(1+(1-\beta)^2)}{(1+(1-\beta)^2)^2}\right).$$

The denominator is strictly positive and the nominator is strictly negative as long as $2\gamma - \beta > 0$. Thus, given the latter plausible assumption, the magnitude of the intergenerational risk sharing effect decreases monotonically (from $\Psi = \frac{1}{2}$ to $\Psi = 0$) as β gradually increases from 0 to 1.

Derivation of equation (25)

Substituting for ω^p from (12) into (23) and (24), we obtain after some manipulations

$$E_{t-1}\left(w_t + (r_{t+1}^T - r^f)\right) = \theta + w_{t-1} + \mu^\lambda + r^f + \left(1-\frac{1}{2\gamma}\right)\frac{\left(\mu^r + \frac{1}{2}\sigma_r^2\right)^2}{\gamma\sigma_r^2} - \frac{\tau\sigma_{r\lambda}}{\sigma_r^2}\left[\mu^r + \frac{1}{2}\sigma_r^2\right]$$

(A-9)

$$+ \tau\left[n + \theta + \mu^\lambda + \frac{1}{2}\sigma_\lambda^2\right] + \frac{1}{2}\frac{\tau^2(\sigma_{r\lambda})^2}{\sigma_r^2} - \frac{1}{2}\tau^2\sigma_\lambda^2$$

and

$$(\text{A-10}) \quad \sigma_c^2 = \sigma_\lambda^2 + \frac{\left(\mu^r + \frac{1}{2}\sigma_r^2\right)^2}{\gamma^2\sigma_r^2} + \frac{2\left[\mu^r + \frac{1}{2}\sigma_r^2\right]\sigma_{r\lambda}}{\gamma\sigma_r^2} + \tau^2\sigma_\lambda^2 - \frac{\tau(\sigma_{r\lambda})^2}{\sigma_r^2}(2+\tau).$$

Substituting (A-9) and (A-10) into (15) and maximizing with respect to τ, we obtain the first order condition

(A-11)

$$-\frac{\sigma_{r\lambda}}{\sigma_r^2}\left[\mu^r + \frac{1}{2}\sigma_r^2\right] + (n + \theta - r^f) + \frac{\tau(\sigma_{r\lambda})^2}{\sigma_r^2} - \tau\sigma_\lambda^2 + \frac{1}{2}(1-\gamma)\left[2\tau\sigma_\lambda^2 - \frac{(\sigma_{r\lambda})^2}{\sigma_r^2}(2+\tau) - \frac{\tau(\sigma_{r\lambda})^2}{\sigma_r^2}\right] = 0.$$

Here we have used that $\mu^\lambda + \frac{1}{2}\sigma_\lambda^2 = -r^f$. Solving for τ yields equation (25).

Derivation of equation (25')

Substituting for ω^p from (12) into (23') and (24') in this case, we obtain after some manipulations

(A-12)

$$E_{t-1}\left(w_t + (r_{t+1}^T - r^f)\right) = \theta + w_{t-1} + \mu^\lambda + r^f - \lambda_{t-1} + \left(1-\frac{1}{2\gamma}\right)\frac{\left(\mu^r + \frac{1}{2}\sigma_r^2\right)^2}{\gamma\sigma_r^2}$$

$$-\left(1-\frac{1}{\gamma}\right)\frac{\tau\sigma_{r\lambda}}{\sigma_r^2}\left[\mu^r + \frac{1}{2}\sigma_r^2\right] + \tau\left[n + \theta - r^f + \sigma_\lambda^2\right] - \frac{1}{2}\frac{\tau^2(\sigma_{r\lambda})^2}{\sigma_r^2} - \tau^2\sigma_\lambda^2$$

and

$$(\text{A-13}) \quad \sigma_c^2 = (1-\tau)^2\sigma_\lambda^2 + \frac{\left(\mu^r + \frac{1}{2}\sigma_r^2\right)^2}{\gamma^2\sigma_r^2} + \frac{(1-\tau)2\left(\mu^r + \frac{1}{2}\sigma_r^2\right)\sigma_{r\lambda}}{\gamma\sigma_r^2} + \tau^2\sigma_\lambda^2 - \frac{\tau(\sigma_{r\lambda})^2}{\sigma_r^2}(2-\tau).$$

Substituting (A-12) and (A-13) into (15) and maximizing with respect to τ, we obtain the first order condition

(A-14)

$$
-\frac{\sigma_{r\lambda}}{\sigma_r^2}\left(\mu^r + \tfrac{1}{2}\sigma_r^2\right)\left(1 - \frac{1}{\gamma}\right) + (n + \theta - r^f + \sigma_\lambda^2) - \frac{\tau(\sigma_{r\lambda})^2}{\sigma_r^2} - \tau 2\sigma_\lambda^2
$$
$$
+ \frac{1}{2}(1-\gamma)\left[2\sigma_\lambda^2(2\tau - 1) - \frac{2(\mu^r + \tfrac{1}{2}\sigma_r^2)\sigma_{r\lambda}}{\gamma\sigma_r^2} - \frac{2(\sigma_{r\lambda})^2}{\sigma_r^2}(1 - \tau) \right] = 0 .
$$

Solving for τ yields equation (25').

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Figure 1: **Shocks to labor income and stock market returns in the model economy**
Generation *t* is considered from time "rawls" in the case of rawlsian risk sharing and from time "interim" in the case of interim risk sharing.
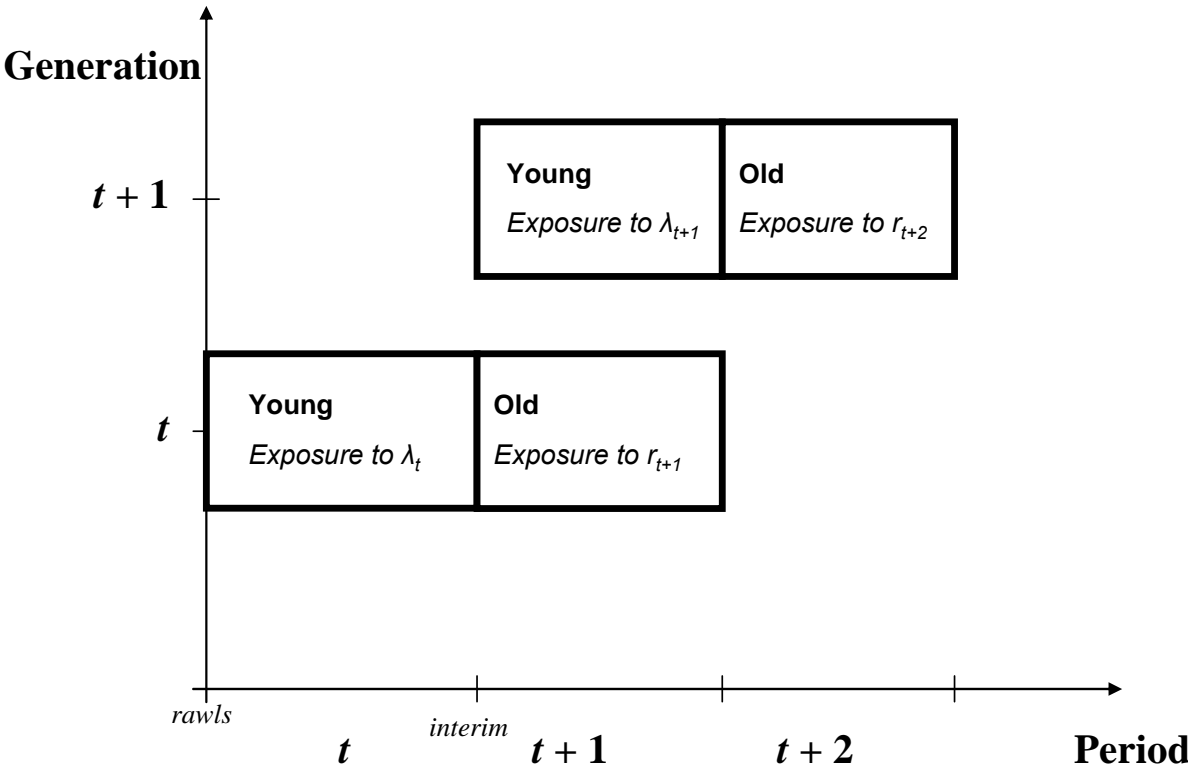
Table 1: Assumptions underlying the numerical calculations

$n + \theta - r^f$	1.5 per cent
μ^r	4.5 per cent
σ_λ	11 per cent
σ_r	20 per cent
$\rho_{r\lambda}$	0.3 - 0.5 - 0.7
γ	3

Table 2: Optimal paygo programs (τ^*), per cent

	$\rho_{r\lambda}$=0.3	$\rho_{r\lambda}$=0.5	$\rho_{r\lambda}$=0.7
Interim risk sharing			
- Permanent wage shocks, eq. (14)	12.9	0*	0*
Rawlsian risk sharing, stocks included			
- Permanent wage shocks, eq. (25)	19.5	11.7	9.9
- Temp. wage shocks, eq. (25')	70.5**	70.2**	69.9**
Rawlsian risk sharing, no stocks			
- Permanent wage shocks, eq. (19)	41.3		
- Temp. wage shocks, eq. (19')	70.7		

Note: In the cases denoted by "*", strict calculations imply that $\tau^* < 0$. Disregarding short positions in the paygo program as a meaningful characteristic of any public pension program, we have set $\tau^* = 0$ in these cases. In the cases denoted by "**", the positive and fairly large portfolio shares in the paygo program are accompanied by individual portfolio shares, which are positive and fairly large in the cases of stocks and negative (i.e. short positions) in the risk free asset.