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Simulation of production and sales planning in refinery operations

by

David Bredström
Patrik Flisberg
Mikael Rönnqvist

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Abstract

In this paper we discuss some computational experiments on simulating refinery operations. We compare an integrated approach with recursively solving a production planning and sales planning problem. We test two different descriptions of the demand behaviour. The first is based on a fixed lower and upper limit and the second on a demand that varies with the product price. We also test the impact of different number of time periods in the planning horizon. We simulate the behaviour when detailed information of the demand is available.

Keywords: Co-ordination, Refinery operations, Demand planning, Sales planning, Production planning, Multiple time periods

Contents

1 Introduction

We consider the part of the SC that consists of the production planning and the sales planning. A simplified description is given in Figure 1.

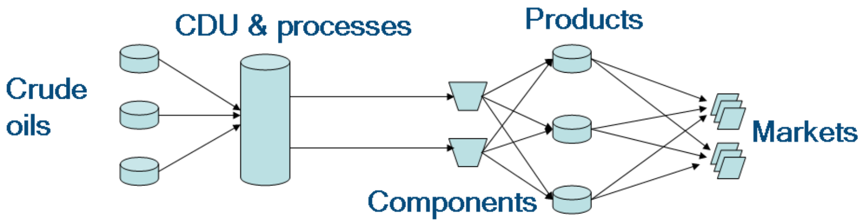

Figure 1: The integrated production and sales planning.

The first part of the refinery production planning is to determine which and what volumes to use in the refinery process. These volumes are then processed through a Crude Distiller Unit (CDU) where different components are produced. Normally there are a number of refining processes before the components are produced. Thereafter follows decisions on how to blend component to get the final products. Each product has specific quality restrictions on e.g. density, octane and sulphur contents and each component has given values in these qualities. Once the products are blended, they are transported to customers in different demand regions or markets. In this last step there are decisions describing the flow of products to a number of markets.

In practice it is not possible to solve an integrated model for both production and demand planning. Instead it is decomposed into two steps. As it is important to avoid a suboptimal solution, there is a need to transfer necessary information in between the two planning modules. This is illustrated in Figure 2.

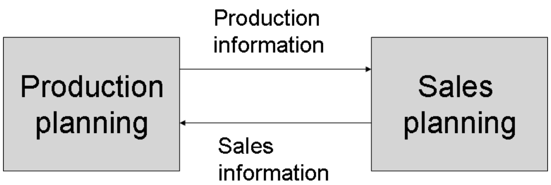

Figure 2: Information exchange between production planning and sales planning.

Depending on what type of process used, i.e. pull or push process, different type of information is provided. In a pure push system the production planning would provide the exact volumes produced and then let the sales planning making the best possible from this. In a pure pull system the sales planning would give the exact volumes of products required. However, such volumes may not be possible to produce in the refinery. Hence, there is a need to use a combination of a push system and a pull system. Both planning units need to have information from the other in order to optimize the supply chain. The production planning need to have access to real or estimated product values and bounds on the volumes of each product demanded. The sales planning need to have information on how expensive products or components are to produce.

The refinery process we are interested in is also studied in Bredström *et al.* [1]. In this report a general testing platform is described and evaluated. The underlying model and solution approach is described in Bredström *et al.* [2]. Here a set of commercial nonlinear solvers are tested. The coordination between production planning and sales planning is studied in Bredström and Rönnqvist [3]. The main purpose is to study how convergence can be achieved by sharing information on each problems' estimate on component and product prices. It was shown that the process is very sensitive to which type of restrictions that are enforced.

In this report we want to use a model similar to the one used in Bredström and Rönnqvist [3] but focusing on some other research questions. The first is to study how different solvers and description of demand behaviour effect the solution behaviour. The second question is to study how the number of time periods effect the solution.

The outline of the report is as follows. First we describe the integrated model in Section 2. In Section 3 we describe how to solve the integrated problem by recursively solve a production planning and sales planning problem. We also describe different ways to model the demand behaviour. In addition we discuss how to evaluate using different number of time periods. In Section 4 we describe the computational experiments. In Section 5 we provide some concluding remarks.

2 Mathematical model

The model is based on the system description in Figure 1. The optimization model and its components are given below.

Sets:

O	:	Set of all oil inputs to the planning process, index i
G	:	Set of processes with several inputs and several outputs, index c
C	:	Set of components, index j.
K	:	Set of products, index k.
Q	:	Set of qualities, index q.
T	:	Set of time periods, index t $(1, 2, \ldots, N_t)$.

Parameters:

$\overline{S}_{ot}$	:	Max supply of oil o during time period t.
$\overline{D}_{kt}, \underline{D}_{kt}$	:	Max and min demand of product k during time period t.
$C0_j$	:	Initial storage of component j.
F^{frac}_{cij}	:	Fraction of component j given input i in CDU c.
S^{frac}_{cijq}	:	Quality fraction of quality q from CDU c from input i giving component j.
$c^{K,vol}_{kt}$	:	Volume value of product k in time period t.
c^{O}_{it}	:	Cost of oil i in time period t.
$\overline{D}^{Q,prod}_{kq}, \underline{D}^{Q,prod}_{kq}$	:	Max and min Quality requirement of product q for quality q in time period t.
$U0^{Q,com}_{jq}$	:	Initial quality concentration for storage of component j and quality q.
$\overline{C}^{cdu}_{ct}$	:	Max volume generated in CDU c in time period t.
$\overline{C}^{all,cdu}_{t}$	:	Max volume generated in all CDUs together in time period t.
c^{cdu}_{ct}	:	CDU operating costs at CDU c in time period t.
$c^{C,stor}_{j}$	:	Storage cost for component j.
$\overline{D}^{com,vol}_{j}, \underline{D}^{com,vol}_{j}$	:	Max and min supply/demand of component j.
$\tilde{c}^{com,vol}_{j}$	:	value of component j (used in the production planning problem).
$\hat{c}^{com,vol}_{j}$	:	value of component j (used in the sales planning problem).
$\hat{u}^{Q,com}_{jq}$	:	value of quality q for component j.
a^{blend}_{jk}	:	volume of product k when blending one volume unit of component j.

Variables:

z^{avail}_{pt}	=	Volume of component p available in time period t (help variable).
z^{blend}_{jkt}	=	Volume blended from component j to product k in time period t.
$x^{oil,buy}_{it}$	=	Volume of oil i bought in time period t.
$x^{oil,cdu}_{ict}$	=	Flow of oil i to CDU c in time period t.
$u^{Q,com}_{jqt}$	=	Quality concentration after component j and quality q in time period t.
$u^{Q,prod}_{kqt}$	=	Quality concentration in product k and quality q in time period t.
$v^{com,stor}_{jt}$	=	End storage of component j in time period t.
$v^{prod,vol}_{kt}$	=	Volume of product k in time period t.

The integrated model can be formulated as

Problem $[Integrated]$

$$
\begin{aligned}
\max \quad z = & \sum_{k\in K}\sum_{t\in T} c_{kt}^{K,vol} v_{kt}^{prod,vol} - \sum_{i\in O}\sum_{t\in T} c_{it}^{O} x_{it}^{oil,buy} - \\
& \sum_{c\in G}\sum_{t\in T}(c_{ct}^{cdu} * \sum_{i\in O} x_{ict}^{oil,cdu}) - \sum_{j\in C}\sum_{t\in T} c_{j}^{C,stor} v_{jt}^{com,stor}
\end{aligned}
$$

subject to

$$
\begin{aligned}
\sum_{c\in G} x_{ict}^{oil,cdu} - x_{it}^{oil,buy} &= 0, && \forall i,t && (1)\\
\sum_{i\in O} x_{ict}^{oil,cdu} - \overline{C}_{ct}^{cdu} &\leq 0, && \forall c,t && (2)\\
\sum_{c\in G}\sum_{i\in O} x_{ict}^{oil,cdu} - \overline{C}_{t}^{all,cdu} &\leq 0, && \forall t && (3)\\
z_{jt}^{avail} - \sum_{c\in G}\sum_{i\in O} F_{cij}^{frac} x_{ict}^{oil,cdu} - v_{j(t-1)}^{com,stor} &= 0, && \forall j,t && (4)\\
v_{j0}^{com,stor} - C0_j &= 0, && \forall j && (5)\\
z_{jt}^{avail} - v_{jt}^{com,stor} - \sum_{k\in K} z_{jkt}^{blend} &= 0, && \forall j,t && (6)\\
u_{jqt}^{Q,com} z_{jt}^{avail} - \sum_{c\in G}\sum_{i\in O} S_{cijq}^{frac} x_{ict}^{oil,cdu} - u_{jq(t-1)} v_{j(t-1)}^{com,stor} &= 0, && \forall j,q,t && (7)\\
u_{jq0}^{Q,com} - U0_{jq}^{Q,com} &= 0, && \forall j,q && (8)\\
\sum_{j\in C} a_{jk}^{blend} z_{jkt}^{blend} - v_{kt}^{prod,vol} &= 0, && \forall k,t && (9)\\
u_{kqt}^{Q,prod} v_{kt}^{prod,vol} - \sum_{j\in C} a_{jk}^{blend} u_{jqt}^{Q,com} z_{jkt}^{blend} &= 0, && \forall k,q,t && (10)\\
x_{it}^{oil,buy} - \overline{S}_{it} &\leq 0, && \forall i,t && (11)\\
\underline{D}_{jq}^{Q,com} \leq u_{jqt}^{Q,com} &\leq \overline{D}_{jq}^{Q,com} && \forall j,q,t && (12)\\
\underline{D}_{kt} \leq v_{kt}^{prod,vol} &\leq \overline{D}_{kt} && \forall k,t && (13)\\
\underline{D}_{kq}^{Q,prod} \leq u_{kqt}^{Q,prod} &\leq \overline{D}_{kq}^{Q,prod} && \forall k,q,t && (14)\\
\text{all variables} \geq 0.
\end{aligned}
$$

The objective is to maximize profit described as the value of sold products minus crude oil cost, production costs in the CDUs and storage costs of components. The constraints provide limits on supply and demand, quality requirements, flow conservation, and computations of quality values. A summary and explanation of all constraints are found in Table 1.

Constraint set	Description
(1)	crude oil balance each time period
(2)	capacity restriction in each CDU each time period
(3)	capacity restriction for all CDUs together each time period
(4)	available components each time period
(5)	initial storage of components
(6)	flow balance of components blended each time period
(7)	quality balance for components each time period
(8)	initial quality of stored components
(9)	flow balance of blended products
(10)	quality balance for products each time period
(11)	upper limit on available crude oil
(12)	lower and upper limits on component quality
(13)	lower and upper limits on product volume
(14)	lower and upper limits on product quality

Table 1: Description of the constraints in Problem [*Integrated*].

3 Iterative solution method

Small problem instances of [*Integrated*] can be solved directly with nonlinear solvers like MINOS ([5]) or NPSOL ([4]). However, when the problem size increases, the nonlinear constraints (quality constraints for the blending) make the problem difficult to solve. When the integrated problem has one planning period and cannot be solved (i.e. when MINOS and NPSOL do not find good solutions) an approximate solution approach can be used where two different linear programming problems are iteratively solved. The first problem is a production problem and the second problem is a sales problem. When solving these recursively, information from the solutions are sent between the two problems as described in Figure 2.

When formulating the production and sales problems we have omitted the index t for clarity. The production problem is formulated as

Problem $[Prod]$

$$\max \quad z = \sum_{j \in C} \tilde{c}_j^{com,vol} z_j^{avail} - \sum_{i \in O} c_i^O x_i^{oil,buy} - \sum_{c \in G} (c_c^{cdu} * \sum_{i \in O} x_{ic}^{oil,cdu})$$

subject to

$$\sum_{c \in G} x_{ic}^{oil,cdu} - x_i^{oil,buy} = 0, \quad \forall i \quad (15)$$

$$\sum_{i \in O} x_{ic}^{oil,cdu} - \overline{C}_c^{cdu} \leq 0, \quad \forall c \quad (16)$$

$$\sum_{c \in G} \sum_{i \in O} x_{ic}^{oil,cdu} - \overline{C}^{all,cdu} \leq 0, \quad (17)$$

$$z_j^{avail} - \sum_{c \in G} \sum_{i \in O} F_{cij}^{frac} x_{ic}^{oil,cdu} - c_j^{C,T0} = 0, \quad \forall j \quad (18)$$

$$\sum_{c \in G} \sum_{i \in O} S_{cijq}^{frac} x_{ic}^{oil,cdu} + U0_{jq}^{Q,com} c_j^{C,T0} - \underline{D}_{jq}^{Q,com} z_j^{avail} \geq 0, \quad \forall j,q \quad (19)$$

$$\sum_{c \in G} \sum_{i \in O} S_{cijq}^{frac} x_{ic}^{oil,cdu} + U0_{jq}^{Q,com} c_j^{C,T0} - \overline{D}_{jq}^{Q,com} z_j^{avail} \leq 0, \quad \forall j,q \quad (20)$$

$$\underline{D}_j^{com,vol} \leq z_j^{avail} \leq \overline{D}_j^{com,vol} \quad \forall j \quad (21)$$

$$x_i^{oil,buy} - \overline{S}_i \leq 0, \quad \forall i \quad (22)$$

$$\text{all variables} \geq 0.$$

The objective is to maximize the production profit described by the value of produced components minus the bought oil and the production cost of using the CDUs. The difference in constraints compared to Problem [*Integrated*] is that in Problem [*Prod*] there are constraints on demand of components but no constraints for blending and products. A summary and explanation of all constraints are found in Table 2.

Constraint set	Description
(15)	crude oil balance
(16)	capacity restriction in each CDU
(17)	capacity restriction for all CDUs together
(18)	available components
(19)	lower limit on component volume
(20)	upper limit on component volume
(21)	lower and upper limits on component quality
(22)	upper limit on available crude oil

Table 2: Description of the constraints in Problem [*Prod*].

The parameters $\underline{D}_j^{com,vol}$, $\overline{D}_j^{com,vol}$ and $\tilde{c}_j^{com,vol}$ are updated in each iteration by using information from the sales problem.

The sales problem is formulated as

Problem $[Sales]$

$$
\begin{aligned}
\max \quad z = & \sum_{k \in K} c_k^{K,vol} v_k^{prod,vol} - \sum_{j \in C} \hat{c}_j^{com,vol} * \sum_{k \in K} z_{jk}^{blend} \\
\text{subject to} &
\end{aligned}
$$

$$
\begin{aligned}
\sum_{j \in C} a_{jk}^{blend} z_{jk}^{blend} - v_k^{prod,vol} &\leq 0, && \forall k && (23) \\
\underline{D}_j^{com,vol} \leq \sum_{k \in K} z_{jk}^{blend} &\leq \overline{D}_j^{com,vol}, && \forall j && (24) \\
\underline{D}_{kq}^{Q,prod} v_k^{prod,vol} \leq \sum_{j \in C} \hat{u}_{jq}^{Q,com} a_{jk}^{blend} z_{jk}^{blend} &\leq \overline{D}_{kq}^{Q,prod} v_k^{prod,vol}, && \forall k, q && (25) \\
\underline{D}_k \leq v_k^{prod,vol} &\leq \overline{D}_k && \forall k && (26) \\
\text{all variables} \geq 0. &
\end{aligned}
$$

The objective is to maximize the sales profit described as sold product value minus used component value. The constraint in this problem only include the blending and products demand. A summary and explanation of all constraints are found in Table 3.

Constraint set	Description
(23)	blending products
(24)	lower and upper limits on available components
(25)	lower and upper limits on product quality
(26)	lower and upper limits on product volume

Table 3: Description of the constraints in Problem [*Sales*].

The parameters $\underline{D}_j^{com,vol}$, $\overline{D}_j^{com,vol}$, $\hat{u}_{jq}^{Q,com}$ and $\hat{c}_j^{com,vol}$ are updated in each iteration by using information from the solution to Problem $[Prod]$.

3.1 Iterative solution approach

For Problem $[Prod]$, the parameters $\underline{D}_j^{com,vol}$ and $\overline{D}_j^{com,vol}$ are generated as 5% under and 5% over the volumes of used components from the solution to Problem $[Sales]$. With the dual value of Constraint (24) denoted SalesDmax.dual, the parameter $\tilde{c}_j^{com,vol}$ is updated as
$\tilde{c}_j^{com,vol} = 0.5\tilde{c}_j^{com,vol} + 0.5$SalesDmax.dual.

For Problem $[Sales]$, the parameter $\underline{D}_j^{com,vol}$ is set as 90% of the volume of produced components from the solution to Problem $[Prod]$ and the parameter $\overline{D}_j^{com,vol}$ is set as the volume of produced components. The parameter $\hat{u}_{jq}^{Q,com}$ is set as the quality of components from the solution to Problem $[Prod]$. We try two different alternative when updating the parameter $\hat{c}_j^{com,vol}$. In the first alternative dual information from Problem $[Prod]$ is used. With the dual value of Constraint (21) denoted ProdDmin.dual and ProdDmax.dual for the lower and upper limits the parameter is updated as
$\hat{c}_j^{com,vol} = 0.5\hat{c}_j^{com,vol} + 0.5$(Prodmin.dual - Prodmax.dual)/ProdDscale
where ProdDscale is a scaling factor to make sure the parameter value is not changed too much between two consecutive iterations. The second alternative is to set $\hat{c}_j^{com,vol}$ as the production cost of each component which includes the cost of used oil and the cost of running the CDUs for the production of that particular component.

In each iteration, both the two problems $[Prod]$ and $[Sales]$ are solved and the limits and costs are updated in each iteration.

3.2 Product demand vary of price

In many applications we want to be able to express the demand as a function of the price. The maximum volume of products that can be sold for a specific price are
$v_k^{prod,vol} \leq \overline{D}_k - p_k^K$,
where p_k^K is a variable describing the selling price of product k. Note that we omit the index t for clarity.

The objective of Problem $[Integrated]$ is changed to
$$\max \quad z = \sum_{k \in K} p_k^K v_k^{prod,vol} - \sum_{i \in O} c_i^O x_i^{oil,buy} - \sum_{c \in G} (c_c^{cdu} * \sum_{i \in O} x_{ic}^{oil,cdu}) - \sum_{j \in C} c_j^{C,stor} v_j^{com,stor}$$

The constraint (13) is changed to

$$\underline{D}_k \leq v_k^{prod,vol} \quad \leq \overline{D}_k - p_k^K \quad \forall k. \quad (13b)$$

Problem $[Prod]$ does not change but in Problem $[Sales]$ we change the objective and Constraint (26). The objective is changed to
$$\max \quad z = \sum_{k \in K} p_k^K v_k^{prod,vol} - \sum_{j \in C} \hat{c}_j^{com,vol} * \sum_{k \in K} z_{jk}^{blend},$$

and Constraint (26) is changed to

$$\underline{D}_k \leq v_k^{prod,vol} \quad \leq \overline{D}_k - p_k^K \quad \forall k. \quad (26b)$$

3.3 Aggregate time periods

A refinery production planning problem is often solved with a planning period of one month. However, the solution is often used for about a week and then the planning problem is again solved with the most recent data giving a new solution. We study possible advantages of dividing the planning period into more time periods, i.e. to use a week as one time period instead of using a month. This is interesting when the demand information is more detailed than on a monthly level.

4 Numerical results

4.1 Cases

To compare the methods described we have developed four different cases of refinery process problems, see Table 4. The cases Case1 and Case2 are nonlinear and are used to test the iterative solution method. Case1 is defined with constant demand of products and Case2 with variable demand depending on the selling price. Case3 is also nonlinear and is used to show that it is difficult to find the global optimum. Case4 has no quality restrictions and hence is a linear programming problem. The demand structure differs over the weeks for Case4. The case is used to study effects of using the solution to a problem with aggregated time periods.

Case properties	Case1	Case2	Case3	Case4
Crude oils	2	2	2	2
CDUs	1	1	1	1
Components	6	6	6	6
Products	5	5	5	5
Qualities	4	4	4	0
Time periods	1	1	4	4

Table 4: Description of the cases.

4.2 Iterative solution method

We solve both the integrated model and compare that solution with the solution to the iterative process for both alternatives of updating parameter $\hat{c}_j^{com,vol}$.

The result is presented in Table 5. For Case1 we performed 20 iterations with both the iterative alternatives. Alternative one finds the best solution after seven iterations and then it gets stuck since the demand constraints on components for the solution to the production problem are not active and the solution to Problem $[Sales]$ is the same from iteration seven onwards. The quality constraints are limiting the solution instead of the demand constraints for Problem $[Prod]$ (and hence dual prices equals 0 for the demand constraints). Alternative two finds the best solution after eleven iterations.

For Case2 we performed 100 iterations with both iterative alternatives. Alternative one finds the best solution after 74 iterations and alternative two finds the best solution after 61 iterations.

Problem	Case1	Case2
Integrated planning problem	640.3	1740.68
Iterative process - Alternative one	591.2	1740.39
Iterative process - Alternative two	638.1	1731.89

Table 5: Objective function value for different solution methods.

4.3 Solution quality

It is difficult to find the global optimal solution to the refinery planning problem even for small instances. This is illustrated in Table 6. We solve Case3 (basic problem) which is a planning problem over four time periods. We try two different solvers, NPSOL and MINOS. We then solve a restricted version of this problem (restricted problem) where a lower limit on the demand of one product in time period 4 is increased from 20 to 40.

Solver	basic problem	restricted problem
NPSOL	2542	3527
MINOS	infeasible	3513.3

Table 6: Objective function values for Case3.

A much better solution is found to the restricted problem. In the same time this restricted problem has a global optimal solution with an objective which is less than or equal to the objective of the global optimal solution to the basic problem. This implies that we have not found the global optimal solution to the basic problem.

4.4 Aggregated time periods

To analyze the effects of multiple time periods we have created Case4 which is a linear programming problem. This to be able to guarantee solutions which are global optimum. We solve the following three different instances of the problem

- P1period, where the four time periods are aggregated to one time period
- P2periods, where the four time periods are aggregated to two time periods
- P4periods, where the four time periods are used.

We assume the production is constant during each aggregated time period. Results are presented in Table 7. We present two different results for the aggregated problems. One where the blending is constant during each aggregated time period ('Fix blend') and one where the blending is done in an optimal way over the four time periods given the component volumes from the solution to the aggregated problem ('Optimal blend').

Problem	Blend type	Unfulfilled orders (ton)	Objective function value
P1period	Fix blend	336.09	2013.8
P1period	Optimal blend	5.73	1611.4
P2periods	Fix blend	56.3	1654
P2periods	Optimal blend	2.47	1543.7
P4periods	Optimal blend	0	1559.6

Table 7: Result from solving the refinery planning problem with different aggregate levels of time periods.

With different demand structure different weeks some orders cannot be fulfilled the first time periods when the time periods are aggregated, not even when the blending is done in an optimal way. The profit is of course higher with aggregated time periods but instead some orders are not fulfilled.

5 Concluding remarks

The integrated refinery planning problem is complex and it is only possible to solve small instances. By using an iterative process where we have decomposed the planning problem in two stages we can improve the solvability. The disadvantage is that we cannot guarantee convergence to an optimal solution. Using several time periods has an advantage as we better can meet detailed demands during each time period. Using only one aggregated time period means that we may have a shortage with respect to the required demand.

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