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Coordination of refinery production and sales planning

by

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Abstract

Coordination of activities in the supply chain is important in order improve the overall performance. The best approach is to use integrated planning of such activities. However, in many cases it is not possible to use integrated planning as the planning models themselves are to difficult to solve or manage. A second approach is to exchange information where some characteristics of the second activity is included in the planning of the first activity. In a pure push or pull logistic planning it is possible to only transfer information in one direction. However, in cases where there are problems to meet some specific demands it is necessary to transfer more information for the coordination. In this paper we study the performance of the coordination between refinery production planning and sales planning of finished products. We show limitations in existing approaches and propose a new approach where additional information is shared. We show with some illustrative numerical experiments that the performance is improved. In addition, the proposed approach is based on theoretical sound decomposition principles.

Contents

1 Introduction

There are many activities involved in the supply chain in the petroleum industry and one important part is the refinery production. The refinery process is linked up-stream with production units, e.g. oil-platforms, that produce crude oils of different qualities. The refinery converts crude oils to components that are blended into products. It is a divergent process as the number of products increases throughout the chain. The refinery is also linked down-stream with sales and distribution to different markets. Information systems, such as Enterprise Resource Planning (ERP) systems, are important tools for the management by providing information about the various parts of the chain. Having information available is, however, not sufficient for appropriate management. Managing, evaluating and controlling the supply chain involve a great deal of planning at different levels. Most ERP vendors provide some basic solutions for supply chain planning and management. These software are commonly called advanced planning and scheduling (APS). The APS-systems are structured into a number of different software modules. Each module is dedicated to solving a specific problem in the supply chain planning matrix. There are many cases when the standard APS-module has limitations in solving the actual problem at hand. This has often to do with the methods used in the modules being designed to deal with a generic planning problem formulation. In addition to the ERP vendors, there is a number of companies that have specialized in providing supply chain optimization solutions for specific industries or planning problems. When such tools are integrated into an ERP system we use the term Best-of-breed as the best planning tool is used.

There are several commercial software packages available to optimize the production planning. These systems include detailed information of the various processes that make up the refinery process. Information on available volume, quality and price is also needed of crude oils and components. For products to be sold information on value, demanded volume and quality requirements are needed. The general refinery planning problem is nonlinear and difficult to solve. The planning softwares available is typically based on solving a sequence of approximate Linear Programming (LP) problems as an approach to solve the underlying nonlinear problem. The objective in these approximate models is to maximize the profit margin (sales value - raw material and production cost) subject to demand and supply constraints. In addition there are many constraints describing the actual refinery process, quality requirements of products and components and to maintain flow conservation in each part of the process. There are also software modules available to support sales forecast and distribution planning.

In order to improve the supply chain performance it is important to integrate the planning over several steps or parts of the supply chain. In this way it is possible to avoid sub-optimal solutions that often appear when parts are optimized separately. However, this requires the development of larger and more detailed models together with methods that are capable to solve them. Some parts of the supply chain is not easily integrated. For example, it is difficult to integrate production planning and sales planning in the same optimization model. At the same time it is important that the planning is coordinated. Therefore it is a need to transfer information between the models and several alternatives are possible. In a pull based logistic system, only demanded volume of products are given to the refinery production planning. This planning then minimize the crude oil and production cost. In a push based system the crude oil available is used to maximize an expected sales value made up of estimated product prices. In practice a combination of the two is used depending on where the decoupling point is. The decoupling point defines where the system changes from being a push based system where production planning is based on forecast to a pull based system where the planning is based on confirmed orders. In refinery planning one approach is to use demanded product volumes with intervals together with actual market prices. At the same time the available crude oils is also given within some bounds. In this way the production planing

has a freedom to optimize the overall supply chain performance. At the same time, it is important that the sales modules that are supposed to find product market values and demand volumes has information how difficult or how costly it is to produce products. This information can be either direct product production cost with possible volumes or costs to produce single components that are later used to blend into products. There are several possibilities to find component prices. One commonly used approach is to use shadow prices or marginal values from the LP models used in the production planning.

In this paper, we study the coordination between production planing and demand or sales planning. There is no ASP planning module that cover both these problems at the same time. Instead, the there is a need for management to communicate relevant information between the two planning units. We study several alternatives in how and what information to be shared between the planning units. Some of the approaches are those traditionally used by refineries and some are new based on mathematical decomposition techniques. We study a simplified refinery and sales process and hence we can formulate an integrated model and find optimal solutions as comparison. We than then compare the information sharing alternatives with the optimal and study their overall efficiency.

The results show that traditional approaches for information sharing are limited and can provide information that leads no non-optimal solutions. Moreover, in order to converge to some reasonable solution there is a need to limit the models through restricted production or demand volumes. The results also show that if more information is shared, the better solution can be found.

The structure of the paper is as follows. In Section 2 we describe the overall production and sales planning and the different alternative to share information between the planning units. In Section 3 we describe a model that describes the integrated planing problem. In Section 4 we describe the models used for each of the production and sales planning. Here we base it on the fact that the sales model has blending decisions included. In Section 5 we give the models when there are no blending activities in the sales model. In Section 6 we give numerical results to show typical behaviour for the two standard alternatives for coordination. In Section 7 we describe how Lagrangian decomposition can be applied to the integrated planing model. This will result in two separate models, very similar to the production and sales models. However, the information used to coordinate the models are more detailed and used information on the blending decisions used in both models. In Section 8 we describe the behaviour of the proposed approach. In Section 9 we make some concluding remarks.

2   Problem description

Supply chain management can be divided into four main stages (Stadtler [2005]): *Procurement*, *Production*, *Distribution* and *Sales*. Procurement involves the operations directed towards providing for the raw material and resources necessary for the production. Production is the next process in the chain. In this process the raw materials are converted to intermediary and/or finished products. Thereafter, the distribution includes the logistics taking place to move the products either to companies processing the product further or to distribution centers, and finally to retailers. The sales process deals with all demand planning issues including customer or market selection, pricing strategy, forecasting and order promising policies. Depending on industry, organisation and information technology (IT) systems some stages may have integrated planning.

The planning horizon is in turn divided into *Strategic*, *Tactical* and *Operational* planning. Strategic decisions would relate to opening and closing of fields, location and size of new platform, products and markets development, financial and operational exposure, planning strategy (e.g. decoupling point) and inventory location. Tactical planning addresses allocation rules which defines which resource or group of resources should be responsible for realizing the different supply chain activities. It also addresses the definition of the usage rules defining production, distribution delays, lot sizing and inventory policies. An important contribution of the tactical planning is to define those rules through a global analysis of the supply chain. This planning serves as a bridge from the long-term strategic level to the detailed operative planning which has a direct influence on the actual operations in the chain. The tactical planning should ensure that the subsequent operative planning is not sub-optimized due to a shorter planning horizon, but rather that the direction which has been set out in the strategic planning is followed. The third level of planning is the operative planning, which is the planning that precedes and decides real-world operative actions. Because of that, there are very high demands on this planning to adequately reflect in detail the reality in which the operations take place. The operative planning is normally distributed to the different facilities or cells of the facilities because of the enormous quantity of data that needs to be manipulated at that level. Within the production process, scheduling of the different products are also typical operational planning tasks.

We consider the part of the SC that consists of the production planning and the sales planning. A simplified description is given in Figure 1.
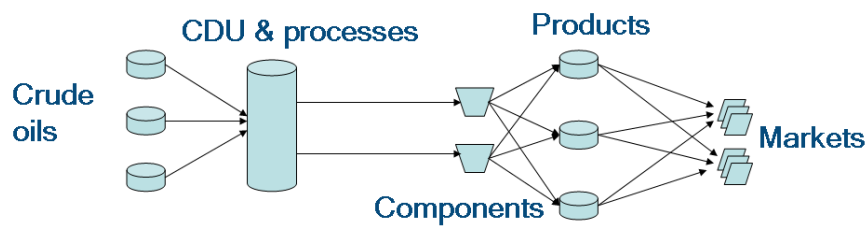

Figure 1: The integrated production and sales planning.

The first part of the refinery production planning is to determine which and what volumes to use in the refinery process. These volumes are then processed through a Crude Distiller Unit (CDU) where different components are produced. Normally there are a number of refining processes before the components are produced. Thereafter follows decisions on how to blend component to get the final products. Each product has specific quality restrictions on e.g. density, octane and sulphur contents and each components has given values in these qualities. Once the products are blended, they are transported to customers in different demand regions or markets. In this last step there are decisions describing the flow of products to a number of markets.

In practice it is not possible to solve an integrated model for both production and demand planning. Instead it is decomposed into two steps. As it is important to avoid suboptimal solution, there is a need to transfer necessary information in between the two planning modules. This is illustrated in Figure 2.

Depending on what type of process used, i.e. pull or push process, different type of information is provided. In a pure push system the production planning would provide the exact volumes produced and then let the sales planning making the best possible from this. In a pure pull system the sales planning would give the exact volumes and product required. However, such volume may not be possible to produce in the refinery. Hence, there is a need to use a combination of a push system and a pull system. Both planning units need to have use information from the other in order
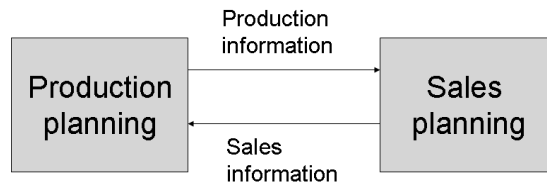
Figure 2: Information exchange between production planning and sales planning.

to optimize the supply chain. The production planning need to have access to real or estimated product values and bounds on the volumes of each product demanded. The sales planning need to have information on how expensive products or components are to produce. If only component prices are available, there is also a need for information on the actual blending and product quality requirements.

We will study two alternatives to make the coordination. The first alternative is to use component prices. Here, the production planning model provides information on the cost or value of each product. If it is a cost based version it states the cost to produce each of the products according to the optimal production plan. If it is a value based version it states the marginal values if additional volumes of the products are produced. This is typically based on the dual or shadow prices in the production planning. It also provide information on the volumes of each of the products produced. In this case the cost or value of the components are given to the sales planning. An important difference with the second alternative is that the blending must also be taken care of in the sales planning. Otherwise the quality requirements may not be satisfied. The production planning still use product values and demand volumes and the optimal blending is found in the production planning. This is illustrated in Figure 3.

Figure 3: Description of the information coordination between the Production planning and the demand planning using component prices.

The second alternative is based on product values only, see Figure 4. The sales planning in turn provides estimates on the product value (for example market value) and lower and upper bounds on the demanded products. This exchange of information can be done recursively a number of times until some convergence criteria is satisfied.
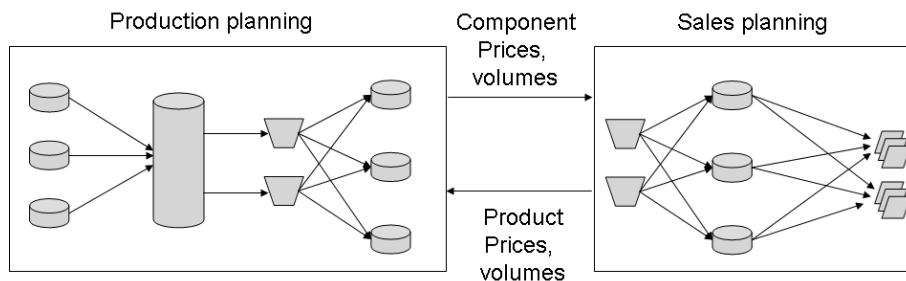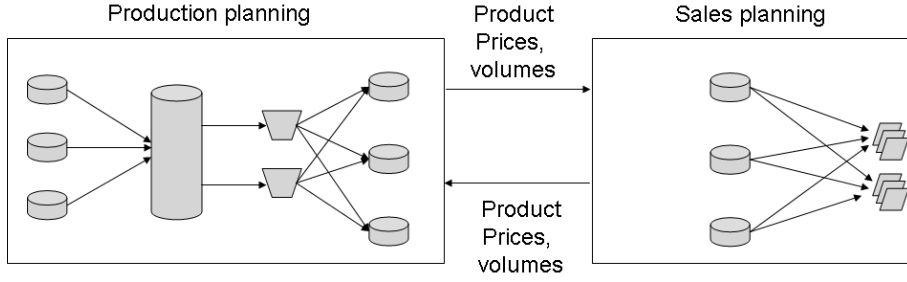
Figure 4: Description of the information coordination between the Production planning and the demand planning using product prices.

3   Integrated model

In this section we describe the integrated model. We have simplified the description in such a way that it will be more clear which parts that do effect the coordination issues. We will use only one CDU and no processes after the CDU. There are some sets and parameters that will be used throughout the paper. These are given below.

Sets:

O	:	set of crude oils
Q	:	set of qualities
C	:	set of components
P	:	set of products
M	:	set of markets

Parameters:

P	:	Production capacity
s_o	:	available volume of crude oil o
c_o	:	unit volume cost of oil o
a_{oc}	:	proportion of crude oil o going to component c
h_{qc}	:	value of quality q in component c
f_{pm}	:	unit volume cost to transport product p to market m
$\bar{d}_{pm}, \underline{d}_{pm}$	:	upper and lower demand of product p in market m
$\bar{e}_{pq}, \underline{e}_{pq}$	:	upper and lower value of quality q in product p
p^0_{pm}, p^1_{pmt}	:	coefficients to determine the value of product p in market m

The decision variables used in the integrated model are:

Variables:

x^I_o	=	volume of crude oil o used
y^I_{cp}	=	volume of component c used in product p
p^I_{pm}	=	unit volume value of product p in market m
z^I_{pm}	=	volume of product p transported to market m

The integrated optimization model [PI] can now be formulated as below.

$$[PI] \quad \max \quad z_I = \sum_{p \in P} \sum_{m \in M} \left((p^0_{pm} - f^S_{pm}) z^I_{pm} + p^1_{pm} (z^I_{pm})^2 \right) - \sum_{o \in O} c_o x^I_o$$

$$\text{subject to} \quad x^I_o \leq s_o, \ o \in O \qquad (I1)$$

$$\sum_{o \in O} x^I_o \leq P, \ o \in O \qquad (I2)$$

$$\sum_{p \in P} y^I_{cp} = \sum_{o \in O} a_{oc} x^I_o, \ c \in C \qquad (I3)$$

$$\sum_{p \in P} h_{qc} y^I_{cp} = \sum_{o \in O} f_{oqc} a_{oc} x^I_o, \ c \in C, q \in Q \qquad (I4)$$

$$\sum_{c \in C} h_{qc} y^I_{cp} \leq \sum_{c \in C} \overline{e}_{pqm} y^I_{cp}, \ p \in P, q \in Q \qquad (I5)$$

$$\sum_{c \in C} h_{qc} y^I_{cp} \geq \sum_{c \in C} \underline{e}_{pqm} y^I_{cp}, \ p \in P, q \in Q \qquad (I6)$$

$$\sum_{c \in C} y^I_{cp} = \sum_{m \in M} z^I_{pm}, \ p \in P \qquad (I7)$$

$$p^I_{pm} = p^0_{pm} + p^1_{pm} z^I_{pm}, \ p \in P, m \in M \qquad (I8)$$

$$z^I_{pm} \leq \overline{d}_{pm}, \ p \in P, m \in M \qquad (I9)$$

$$z^I_{pm} \geq \underline{d}_{pm}, \ p \in P, m \in M \qquad (I10)$$

$$\text{all variables} \geq 0, \qquad (I11)$$

The objective is to maximize the sales value minus crude oil prices and transportation costs. The demand description is based on a spatial equilibrium model where the product value depends on the demand of each product in the markets. The product price is hence unknown and is determined in the model. This is the reason why the objective function becomes quadratic. The constraint set (I1) gives the limited supply of crude oils and (I2) the production capacity. The set (I3) describes the process and links the crude oils with the components. Set (I4) describes the product qualities and sets (I5) and (I6) the minimum and maximum levels of the qualities. Set (I7) gives the flow conservation on products from the components to the markets. Set (I8) gives the market prices of the products, and the sets (I9) and (I10) the lower and upper bounds on the demand in the markets. Set (I11) gives the non-negativity restrictions.

4 Separate models - alternative 1

If we follow the separation into two different planning parts we get one production planning model and one demand planning model. We start with alternative 1 where the sales model include blending decisions.

4.1 Production model

The additional information needed in the production planning model is product prices. In order to be able to guarantee convergence we also need a way to limit the demanded product volume. The additional parameters needed are:

p_p^P :   the unit volume value of product p

$\overline{d}^P, \underline{d}^P$:   upper and lower accumulated demand volume of products

The decision variables used in the production planning model [PP] are:

x_o^P =   volume of crude oil o used

y_{cp}^P =   volume of component c used in product p

The production planning model can now be formulated as

$$[PP] \quad \max \qquad z_P = \sum_{c \in C} \sum_{p \in P} p_p^P y_{cp}^P - \sum_{o \in O} c_o x_o^P$$

$$\text{subject to} \qquad x_o^P \leq s_o, \ o \in O \qquad\qquad (P1)$$

$$\sum_{o \in O} x_o^P \leq P, \ o \in O \qquad\qquad (P2)$$

$$\sum_{p \in P} y_{cp}^P = \sum_{o \in O} a_{oc} x_o^P, \ c \in C \qquad\qquad (P3)$$

$$\sum_{p \in P} h_{qc} y_{cp}^P = \sum_{o \in O} f_{oqc} a_{oc} x_o^P, \ c \in C, q \in Q \quad (P4)$$

$$\sum_{c \in C} h_{qc} y_{cp}^P \leq \sum_{c \in C} \overline{e}_{pq} y_{cp}^P, \ p \in P, q \in Q \qquad (P5)$$

$$\sum_{c \in C} h_{qc} y_{cp}^P \geq \sum_{c \in C} \underline{e}_{pq} y_{cp}^P, \ p \in P, q \in Q \qquad (P6)$$

$$\sum_{c \in C} \sum_{p \in P} y_{cp}^P \leq \overline{d}^P, \qquad\qquad (P7)$$

$$\sum_{c \in C} \sum_{p \in P} y_{cp}^P \geq \underline{d}^P, \qquad\qquad (P8)$$

$$\text{all variables} \geq 0, \qquad\qquad (P9)$$

The objective is to maximize the product value minus the crude oil cost. The constraints are similar as in model [PI]. The only additional constraints are the lower and upper limits on the accumulated demands, i.e. (P7) and (P8).

4.2 Sales model

In the demand planning we need to have information on the component prices. In the same way as in the model [PP] we need to have limits in the component volumes. The additional parameters we need are:

$\overline{s}^S, \underline{s}^S$:   upper and lower accumulated supply of components

c_c^S :   unit volume cost of component c

The decision variables used in the model [PS-1] are:

y_{cp}^S =   volume of component c used in product p

p_{pm}^S =   unit volume value of product p in market m

z_{pm}^S =   volume of product p transported to market m

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The optimization model can now be formulated as

$$[\text{PS-1}] \quad \max \quad z_S = \sum_{p \in P} \sum_{m \in M} \left((p_{pm}^0 - f_{pm}) z_{pm}^S + p_{pm}^1 (z_{pm}^S)^2 \right) - \sum_{c \in C} \sum_{p \in P} c_c^S y_{cp}^S$$

$$\text{subject to} \quad \sum_{p \in P} \sum_{m \in M} y_{cp}^S \leq \overline{s}^S, \qquad\qquad (S1-1)$$

$$\sum_{p \in P} \sum_{m \in M} y_{cp}^S \geq \underline{s}^S, \qquad\qquad (S1-2)$$

$$\sum_{c \in C} y_{cp}^S = \sum_{m \in M} z_{pm}^S, \quad p \in P \qquad (S1-3)$$

$$\sum_{c \in C} h_{qc} y_{cp}^S \leq \sum_{c \in C} \overline{e}_{pqm} y_{cp}^S, \quad p \in P, q \in Q \qquad (S1-4)$$

$$\sum_{c \in C} h_{qc} y_{cp}^S \geq \sum_{c \in C} \underline{e}_{pqm} y_{cp}^S, \quad p \in P, q \in Q \qquad (S1-5)$$

$$p_{pm}^S = p_{pm}^0 + p_{pm}^1 z_{pm}^S, \quad p \in P, m \in M \quad (S1-6)$$

$$z_{pm}^S \leq \overline{d}_{pm}, \quad p \in P, m \in M \qquad (S1-7)$$

$$z_{pm}^S \geq \underline{d}_{pm}, \quad p \in P, m \in M \qquad (S1-8)$$

$$\text{all variables} \geq 0, \qquad\qquad (S1-9)$$

The objective is to maximize the product values sold in the markets minus transportation and component costs. The constraints are similar as in model [PI]. The only additional constraints are the lower and upper limits on the accumulated component volumes, i.e. (S1-7) and (S1-8).

5 Separate models - alternative 2

In this section we describe the second alternative when where the sales model does not include blending decisions. The models are otherwise similar. The production model is the same and will not be given again.

5.1 Sales model

In the demand planning we need to have information on the approximate product prices from the production. The only additional parameter we need is

c_{pm}^S : unit volume cost of product p

The decision variables used in the model [PS-2] are:

p_{pm}^S = unit volume value of product p in market m
z_{pm}^S = volume of product p transported to market m

The optimization model can now be formulated as

[PS-2]   max $$z_S = \sum_{p \in P} \sum_{m \in M} \left((p^0_{pm} - f_{pm}) z^S_{pm} + p^1_{pm} (z^S_{pm})^2 \right) - \sum_{p \in P} \sum_{m \in M} c^S_{pm} z^S_{pm}$$

$$\text{subject to} \quad p^S_{pm} = p^0_{pm} + p^1_{pm} z^S_{pm}, \quad p \in P, m \in M \quad (S2-1)$$

$$z^S_{pm} \leq \overline{d}_{pm}, \quad p \in P, m \in M \quad\quad\quad (S2-2)$$

$$z^S_{pm} \geq \underline{d}_{pm}, \quad p \in P, m \in M \quad\quad\quad (S2-3)$$

$$\text{all variables} \quad \geq \quad 0, \quad\quad\quad\quad\quad\quad\quad\quad (S2-4)$$

The objective is to maximize the product values sold in the markets minus transportation and component costs. The constraints are similar as in model [PS-1]. However, all constraints related to blending are removed.

6   Computational experiments

In order to test the planning process we need to compute product market prices and component/product prices. In the demand model we get market product prices directly as this is one of the variables. In the production model we do not have access directly to component prices. These need to be computed in alternative 1. One standard approach to establish these values is to use the shadow prices (or dual values) from constraint set (P3). These will provide the marginal values in an increase in the right hand side values. We can also use an estimate based on the crude oil costs. We know the crude oil usage to get the components and therefore we simple allocate this costs to the components. In this case it will not be a marginal price estimate but a full price allocation. For alternative 2 we use a similar approach.

6.1   Test case

We use an adapted version of the Aronofsky Aronofsky et al. [1978] case to illustrate the simulations described in this paper. The main differences are the estimated proportions of components available from the CDU. We have introduced two (small) markets with demand for the demand model. The AMPL model for the integrated model, [PI], and its data is found in appendix A.

The test case has two crude oils and one external resource of butane. There is one CDU and ten components. Five products are available in both markets, where one has a higher demand (seen as a higher accepted price function). The optimal solution to PI with profit $z_I = 96949.02$, with a crude allocation volume of 358.08 barrels. The optimal prices and quantities are

Market	Product	Quantity (z_{pm}^I)	Price (p_{pm}^I)
MA	premium	33.48	520.17
MA	regular	38.89	473.36
MA	distillate	70.25	349.00
MA	fueloil	25.28	247.79
MA	fuelgas	15.77	58.46
MB	premium	33.93	524.67
MB	regular	39.08	474.86
MB	distillate	60.70	371.50
MB	fueloil	23.30	255.29
MB	fuelgas	35.29	62.21

6.2 Tests with alternative 1

A solution to the production model [PP] supplies component volumes and estimated component prices for the demand model [PS-1]. Similarly, a solution to [PS] supplies demanded product volumes and their prices.

The component and product volumes are crucial for the models to give reasonable solutions. Indeed, if we were to exclude the volumes from the information exchange we would obtain results illustrated in Figure 5.
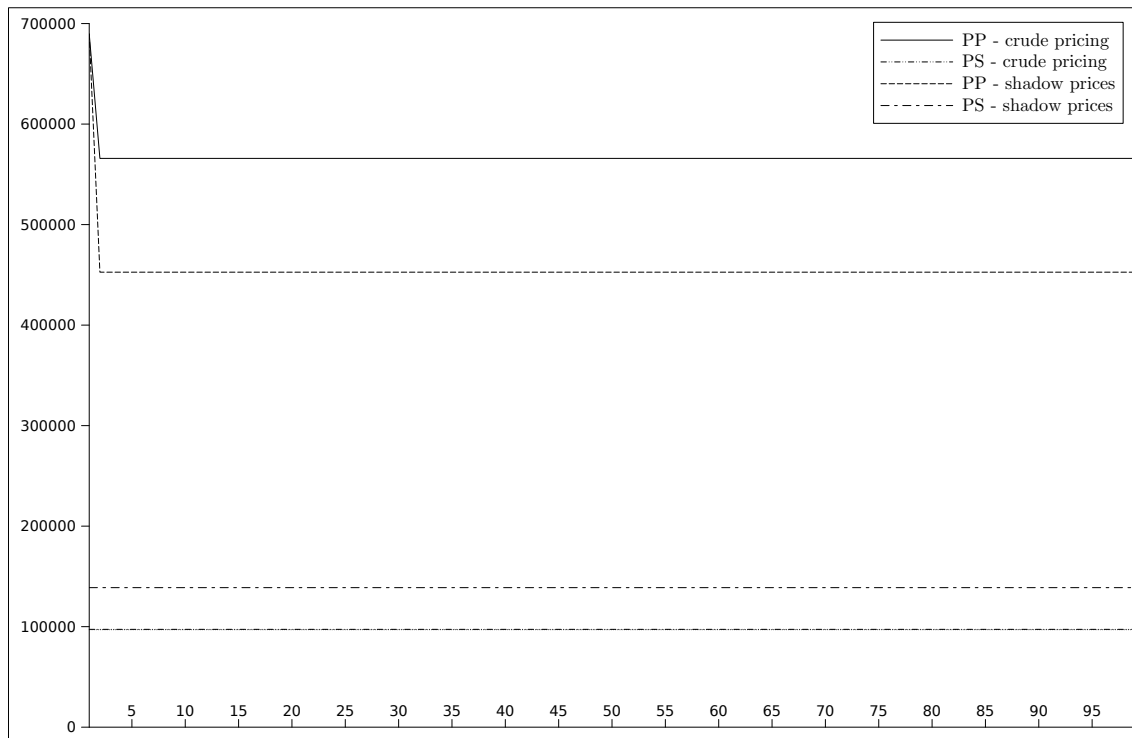

Figure 5: Illustration of problems with optimistic solutions for simulations without flow control. The graph plots the optimistic profit (y-axis), for each iteration (x-axis) in the simulation.

In the figure we see how the models [PP] gives optimistic solutions. The crude allocations obtained from [PP], and the prices from [PS-1], are not realistic and even gives very bad or infeasible solutions if they were inserted and fixed in the integrated model [PI]. To fix a price obtained with

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[PS-1] in [PI] is equivalent to setting fixed product demands and prices in [PP], and we can use the result from [PI] obtained with fixed prices to measure how well a set of prices are.

Similar problematic results are also obtained, when we do indeed exchange information about the product and component volumes, but with a too conservative method. In figure 6 we illustrate results from a simulation where we restrict the deviation in product and component volumes, and, although we start with near optimal prices, the models converge to a point far from optimal.
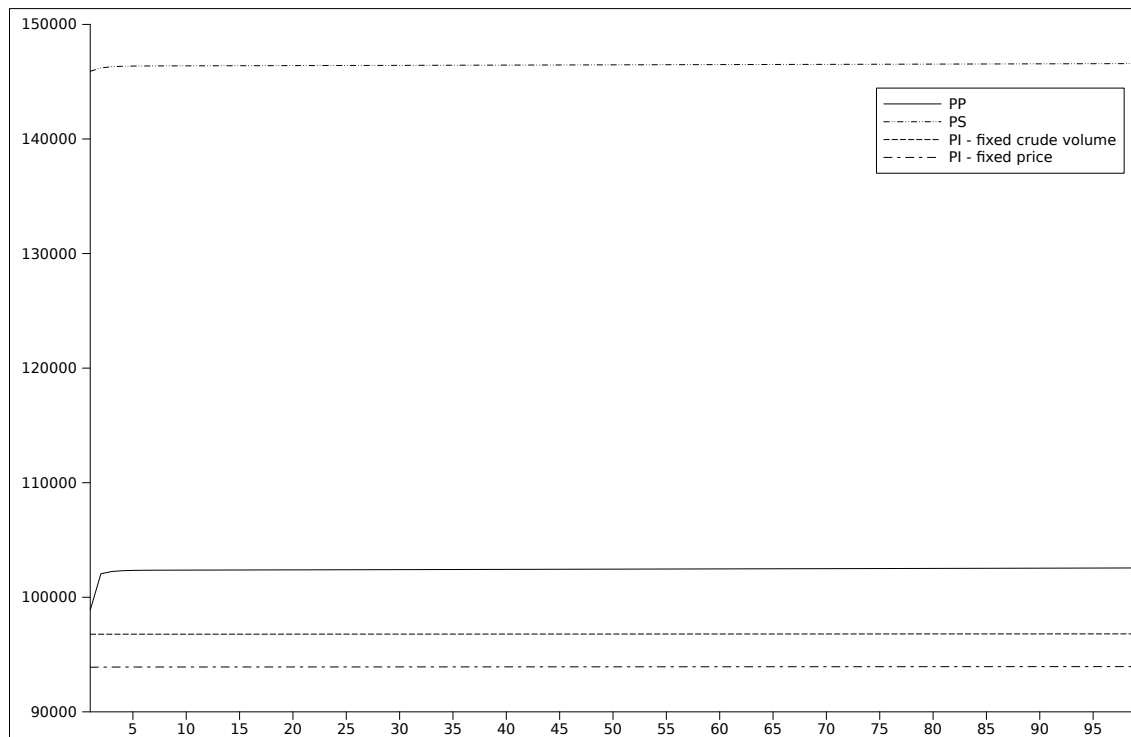

Figure 6: Simulation where very small deviations in flows are allowed and where we initially have close to optimal information. Component prices in the PS are the shadow prices from constraint P3. The PI series with fixed prices shows that the prices are indeed near optimal.

The reason to introduce flow control is that the the production model [PP] can, even with optimal prices, suggest production that is far from optimal from a sales perspective. If we know a good starting point and allow for only small deviations, we could make the production model accomodate to reasonable volumes suggested from the sales model. But in the figure we see that with only a guess of the initial volumes the flow control also has to allow for the right amount of deviation for a reasonable convergence to take place. In this example we are using a decreasing allowed deviation (defined by 2 / number of iterations). The constraints (S1-1), (S1-2) and (P8), (P9) are used for this purpose.

In the figures 7 and 8 we show how the results depend on the choice of method to estimate the component prices. The simulations are identical except in how the component prices are estimated.

In Figure 7 we show the results from a simulation where we use the crude oil allocation costs to set component prices. With the current algorithmic parameters (size of allowed deviation and starting values) and this pricing method we see that we can obtain a reasonable convergence. Still, in Figure 8 we can see that the algorithmic parameters are highly dependent on the component prices. This figure shows the results from the same simulation but with the only difference that the component prices are the shadow prices.
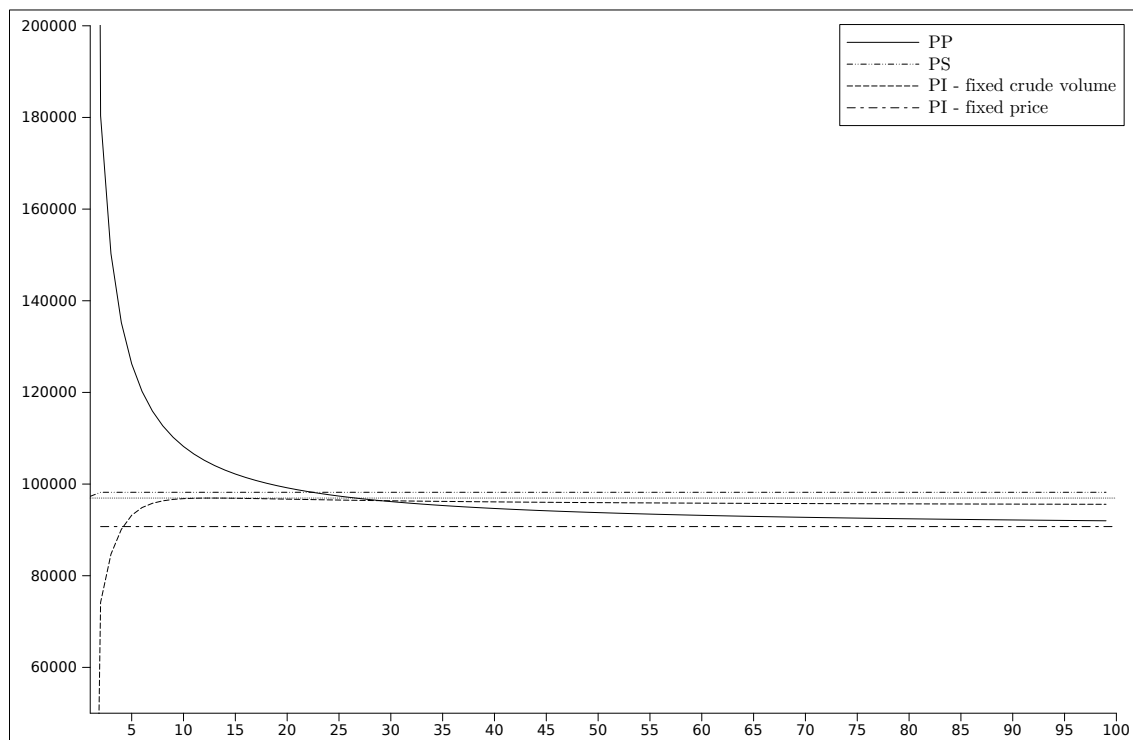
Figure 7: Simulation results with component price estimates based on crude oil allocation.

6.3   Tests with alternative 2

From initial experiments we have noticed that we obtain somewhat more stable results when we use the sales model [PS-2]. With this model the sales planning provides estimates on the product value (for example market value) and lower and upper bounds on the demanded products but does not consider any blending decisions.

Despite the more stable results as compared to alternative 1, it is still easy to find realistic examples where very small changes in parameters or start values significanlty have an effect on the results. For example, in the figures 9 and 10 we show the results from two settings with the only difference being the initial allowed deviation volumes. In both examples we use the shadow prices to estimate the component prices.

In the figures 11 and 12 we further illustrate how the allowed deviation affects the results. In the simulations we start by limiting the volumes with a maximal deviation from optimal of 10% in Figure 11 and 1% in Figure 12. Then, for each iteration we allow for 1,5,10 and 20 per cent deviation from the previously obtained volumes. With a 10% inital deviation we see from the figure that the prefered deviation limit in the succeding iterations is around 5%. On the other hand, when we start as close as 1% from optimal volumes, we see that the 5% limit is too large and the solution gets worse for each iteration. Therefore we note that we cannot set a reliable deviation limit unless we know the optimal solution, which is an unrealistic demand.
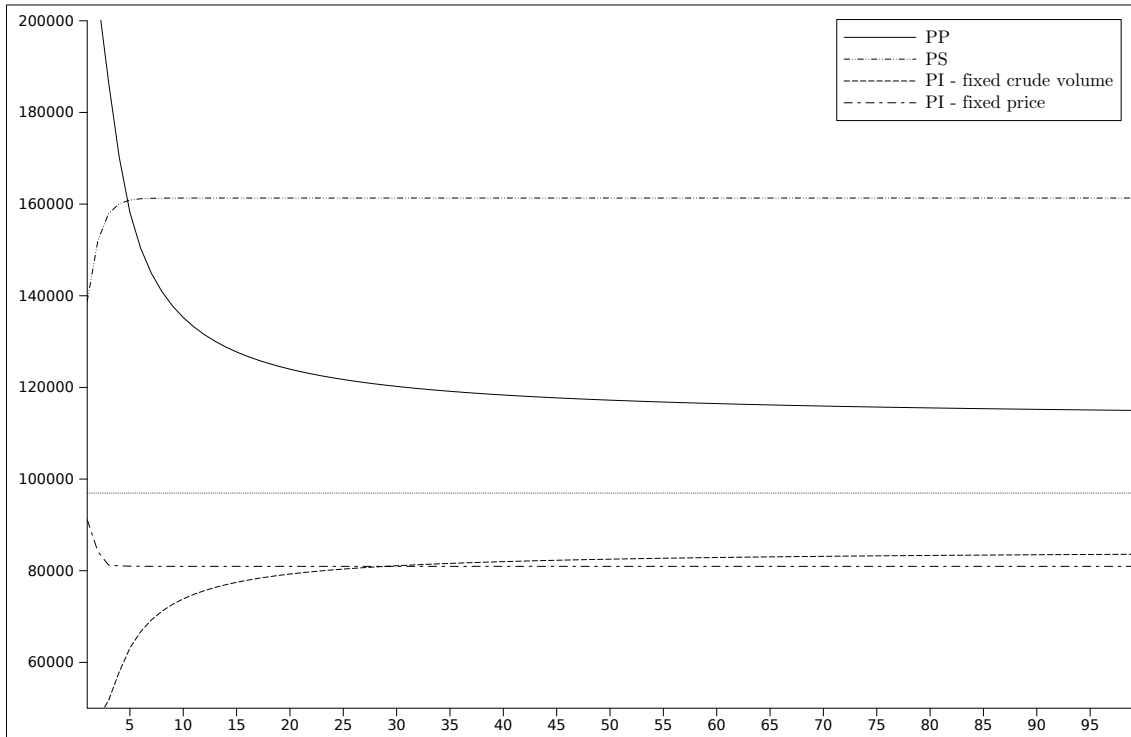
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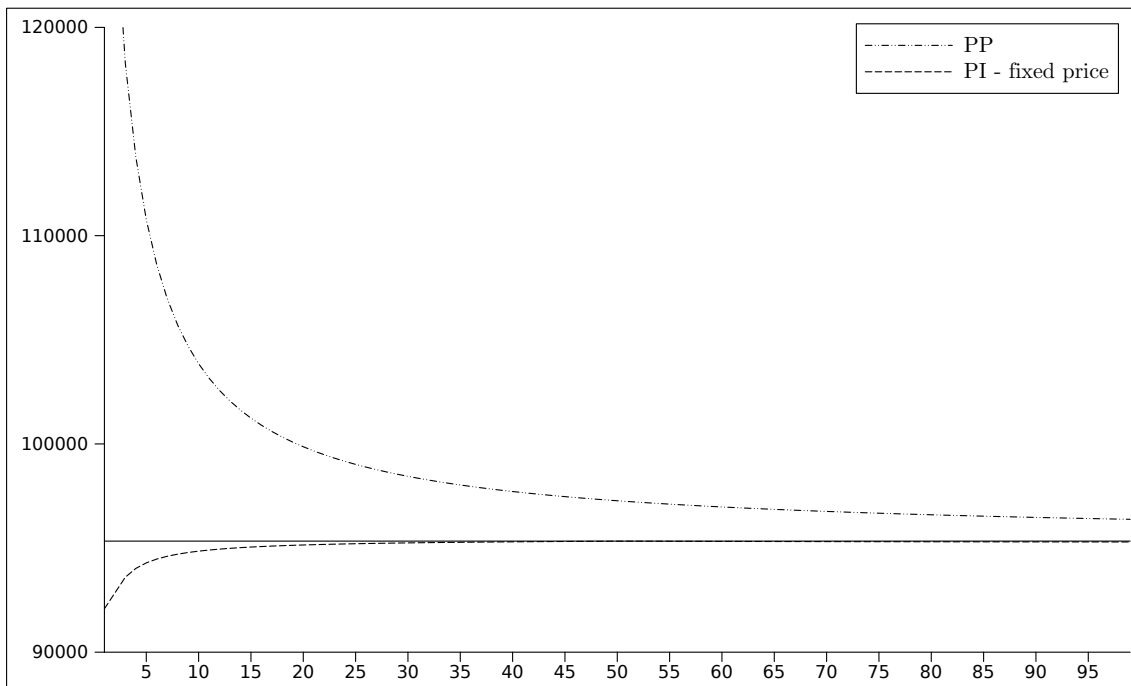Figure 8: Simulation results where the component prices are the shadow prices.

Figure 9: The profit (y-axis) is plotted against the iteration count (x-axis). The solid line is the optimal profit, the PP is the estimated profit in the production model, and, the PI is the actual profit if the prices from the sales model are fixed in the integrated model.
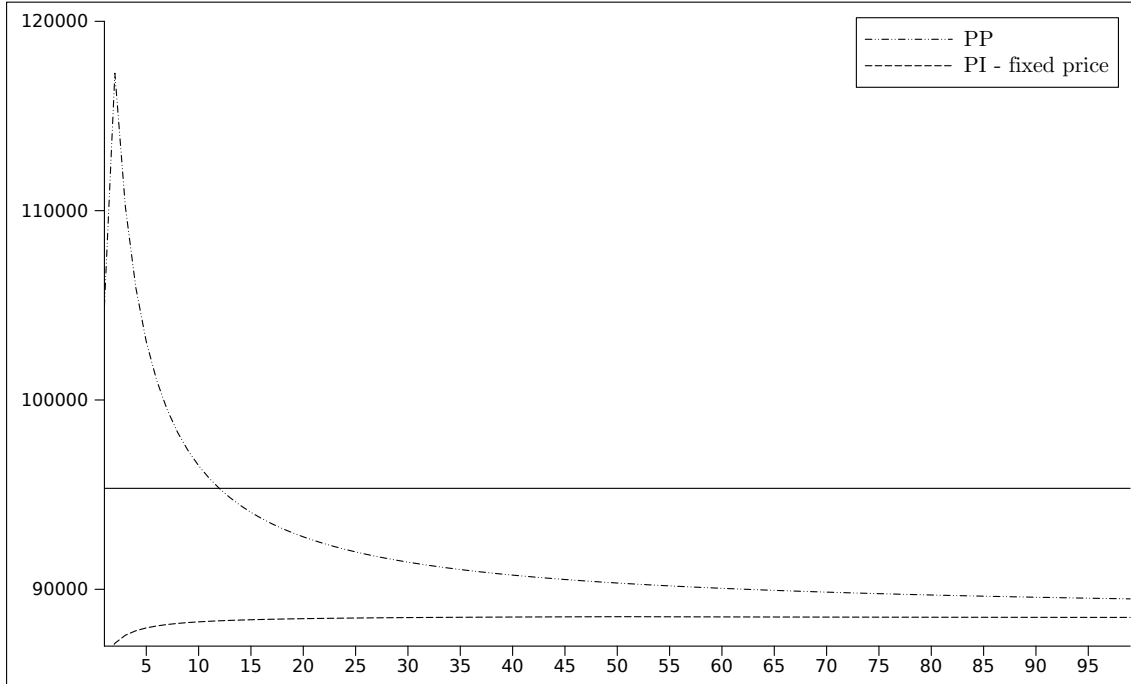
13

Figure 10: A simulation where the initially allowed deviation is too small.

Figure 11: This figure shows the profit obtained with prices from the sales model when we start with an allowed deviation of 10% from optimal volumes and allow at most 1,2,5, or 10 pec cent in succeding iterations.
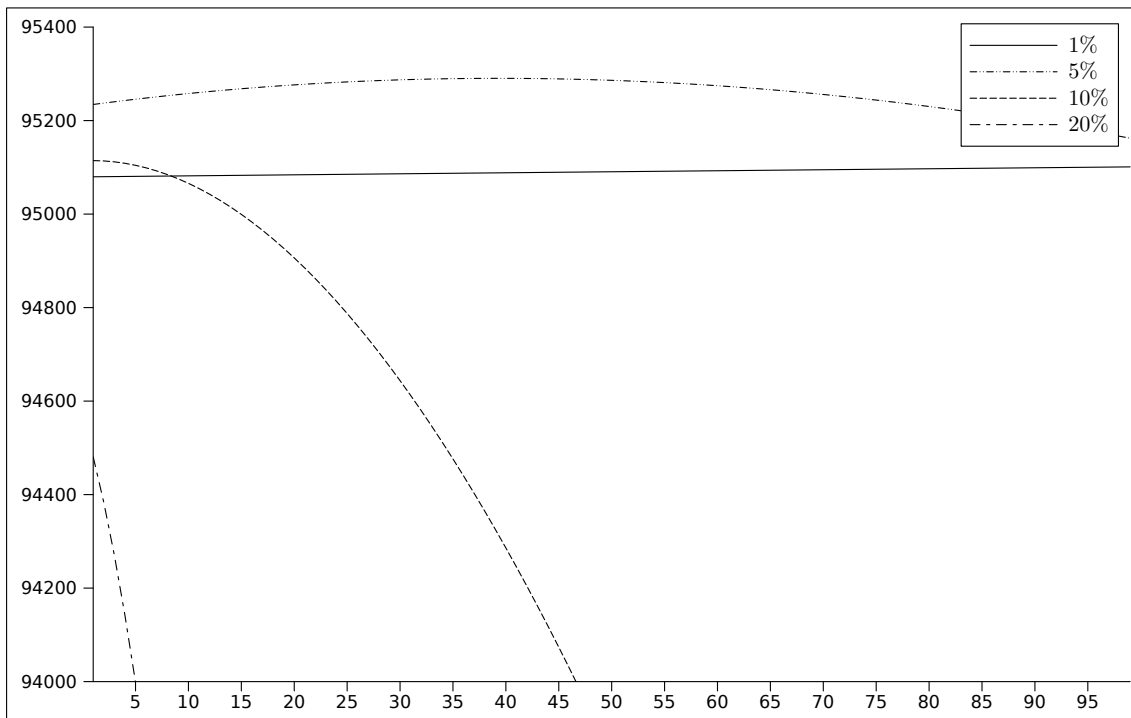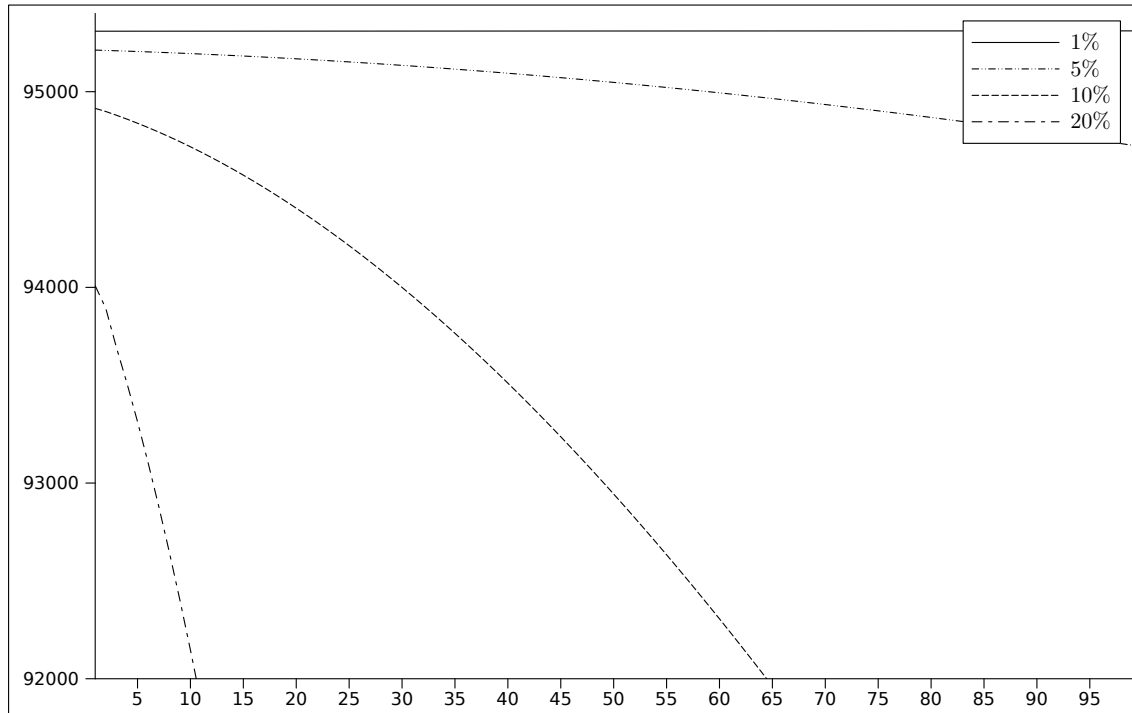
Figure 12: This figure shows the profit obtained with prices from the sales model when we start with an allowed deviation of 1% from optimal volumes and allow at most 1,2,5, or 10 pec cent in succeding iterations.

6.4 Comments on tests

From the tests done so far it is clear that it is difficult to simulate convergence to an optimal solution. The reason for this is that the underlying optimization problem is nonlinear and that making an *ad hoc* decomposition in a production planning and a sales planning problem is not enough. The information based on approximate component and product prices is not enough to guarantee convergence. Furthermore, in order to get some convergence there is a need to force the solution to be similar and one way is to make sure that the volumes produced in the production is similar or close to the volume used in the sales.

7 Lagrangian decomposition

In the previous sections we analysed two standard alternatives for coodination between the production and the sales models. One major drawback with these two alternatives is that the simulation progress relies on algorithmic parameters, values that in turn cannot be known without knowledge about the optimal solution. With small changes in the parameters, such as too small allowed changes between iterations, the simulation could in the worst case result in an infeasible solution, with far from optimal prices. In this section we will apply a decomposition scheme that implicitly takes care of some the problematic parameters.

One theoretical approach to decompose the model [PI] is to use Lagrangian decomposition Guignard and Kim [1987] or variable layering in Glover and Klingman [1988] or variable splitting in Näsberg et al. [1985]. The Lagrangian decomposition is based on duplicate one (or several) groups of variables and then relax the constraints that require the two variables to have the same value. In our case we introduce y_{cp}^{Ia}, y_{cp}^{Ib} and the constraint $y_{cp}^{Ia} = y_{cp}^{Ib}$ and then relax it with the Lagrangian multipliers λ_{cp}. Some constraints are copied in both y_{cp}^{Ia} and y_{cp}^{Ib}. The optimization model can now be formulated as

[PD]

$$\max \quad z_D = \sum_{p \in P} \sum_{m \in M} \left((p_{pm}^0 - f_{pm}^S) z_{pm}^I + p_{pm}^1 (z_{pm}^I)^2 \right) - \sum_{o \in O} c_o x_o^I + \sum_{c \in C} \sum_{p \in P} \lambda_{cp} (y_{cp}^{Ia} - y_{cp}^{Ib})$$

$$\text{subject to} \quad x_o^I \leq s_o, \ o \in O \qquad (D1)$$

$$\sum_{o \in O} x_o^I \leq P, \ o \in O \qquad (D2)$$

$$\sum_{p \in P} y_{cp}^{Ia} = \sum_{o \in O} a_{oc} x_o^I, \ c \in C \qquad (D3)$$

$$\sum_{p \in P} h_{qc} y_{cp}^{Ia} = \sum_{o \in O} f_{oqc} a_{oc} x_o^I, \ c \in C, q \in Q \qquad (D4)$$

$$\sum_{c \in C} h_{qc} y_{cp}^{Ia} \leq \sum_{c \in C} \overline{e}_{pqm} y_{cp}^{Ia}, \ p \in P, q \in Q \qquad (D5)$$

$$\sum_{c \in C} h_{qc} y_{cp}^{Ia} \geq \sum_{c \in C} \underline{e}_{pqm} y_{cp}^{Ia}, \ p \in P, q \in Q \qquad (D6)$$

$$\sum_{c \in C} y_{cp}^{Ib} = \sum_{m \in M} z_{pm}^I, \ p \in P \qquad (D7)$$

$$\sum_{c \in C} h_{qc} y_{cp}^{Ib} \leq \sum_{c \in C} \overline{e}_{pqm} y_{cp}^{Ib}, \ p \in P, q \in Q \qquad (D8)$$

$$\sum_{c \in C} h_{qc} y_{cp}^{Ib} \geq \sum_{c \in C} \underline{e}_{pqm} y_{cp}^{Ib}, \ p \in P, q \in Q \qquad (D9)$$

$$p_{pm}^I = p_{pm}^0 + p_{pm}^1 z_{pm}^I, \ p \in P, m \in M \qquad (D10)$$

$$z_{pm}^S \leq \overline{d}_{pm}, \ p \in P, m \in M \qquad (D11)$$

$$z_{pm}^S \geq \underline{d}_{pm}, \ p \in P, m \in M \qquad (D12)$$

$$\text{all variables} \geq 0, \qquad (D13)$$

This separates into two problems which are closely related to [PP] and [PS].

[PD-P] max
$$z_{DP} = -\sum_{o \in O} c_o x_o^I + \sum_{c \in C} \sum_{p \in P} \lambda_{cp} y_{cp}^{Ia}$$

subject to

$$x_o^I \leq s_o, \ o \in O \qquad\qquad (DP1)$$

$$\sum_{o \in O} x_o^I \leq P, \ o \in O \qquad\qquad (DP2)$$

$$\sum_{p \in P} y_{cp}^{Ia} = \sum_{o \in O} a_{oc} x_o^I, \ c \in C \qquad\qquad (DP3)$$

$$\sum_{p \in P} h_{qc} y_{cp}^{Ia} = \sum_{o \in O} f_{oqc} a_{oc} x_o^I, \ c \in C, q \in Q \quad (DP4)$$

$$\sum_{c \in C} h_{qc} y_{cp}^{Ia} \leq \sum_{c \in C} \overline{e}_{pqm} y_{cp}^{Ia}, \ p \in P, q \in Q \quad (DP5)$$

$$\sum_{c \in C} h_{qc} y_{cp}^{Ia} \geq \sum_{c \in C} \underline{e}_{pqm} y_{cp}^{Ia}, \ p \in P, q \in Q \quad (DP6)$$

all variables $\geq$ 0, $\qquad\qquad (DP7)$

[PD-S] max
$$z_{DS} = \sum_{p \in P} \sum_{m \in M} \left((p_{pm}^0 - f_{pm}^S) z_{pm}^I + p_{pm}^1 (z_{pm}^I)^2 \right) - \sum_{c \in C} \sum_{p \in P} \lambda_{cp} y_{cp}^{Ib}$$

subject to

$$\sum_{c \in C} y_{cp}^{Ib} = \sum_{m \in M} z_{pm}^I, \ p \in P \qquad\qquad (DS1)$$

$$\sum_{c \in C} h_{qc} y_{cp}^{Ib} \leq \sum_{c \in C} \overline{e}_{pqm} y_{cp}^{Ib}, \ p \in P, q \in Q \qquad (DS2)$$

$$\sum_{c \in C} h_{qc} y_{cp}^{Ib} \geq \sum_{c \in C} \underline{e}_{pqm} y_{cp}^{Ib}, \ p \in P, q \in Q \qquad (DS3)$$

$$p_{pm}^I = p_{pm}^0 + p_{pm}^1 z_{pm}^I, \ p \in P, m \in M \quad (DS4)$$

$$z_{pm}^S \leq \overline{d}_{pm}, \ p \in P, m \in M \qquad\qquad (DS5)$$

$$z_{pm}^S \geq \underline{d}_{pm}, \ p \in P, m \in M \qquad\qquad (DS6)$$

all variables $\geq$ 0, $\qquad\qquad (DS7)$

The constraints in [DP-P] is the same as in [PP] except for the constraints (P7) and (P8) which are the lower and upper limits on the aggregated demand of products. The main difference is that the objective coefficients are based on a value of blending component c to product p through the multipliers λ_{cp}. Problem [PD-S] is the same as [PS] except constraints (S7) and (S8). In the objective we have a cost not for components but for each blending decision. We note that the objective is more detailed when it comes to measure the cost or value of blending of components to products.

One standard approach to solve problem [PI] through the Lagrangian decomposition approach is to use subgradient optimization, see Shapiro [1979]. In this approach a sequence of [PD-P] and [PD-S] are solved with updated values of λ_{cp} through $\lambda_{cp} + t(y_{cp}^{Ia} - y_{cp}^{Ib})$ where t is a step length and $y_{cp}^{Ia} - y_{cp}^{Ib}$ the sub-gradient.

8   Numerical tests on the decomposition method

Decomposition results

The first observation we make is that we do not need any flow control to establish good prices. The product prices are not included at all in the model [PD-P], since λ_{cp} is a penalty parameter for deviation in the blending volumes and, in average, corresponds to component prices. It is worth noting that the this also means that the volumes obtained from model [PD-P] cannot be used for crude oil allocations. On the contrary, the prices in the sales model [PD-S] can be used to determine product volumes (with fixed prices in the integrated model [PI]).

In Figure 13 we show the results from a standard implementation of subgradient optimization. The upper bound (UBD) is the optimistic profit obtained from the optimal solutions to the decomposition models. As we can see in the figure, the UBD is closing the gap to the corresponding feasible solution (obtained with the sales model prices fixed in [PI]) when the parameters λ_{cp} are updated with the subgradient scheme.
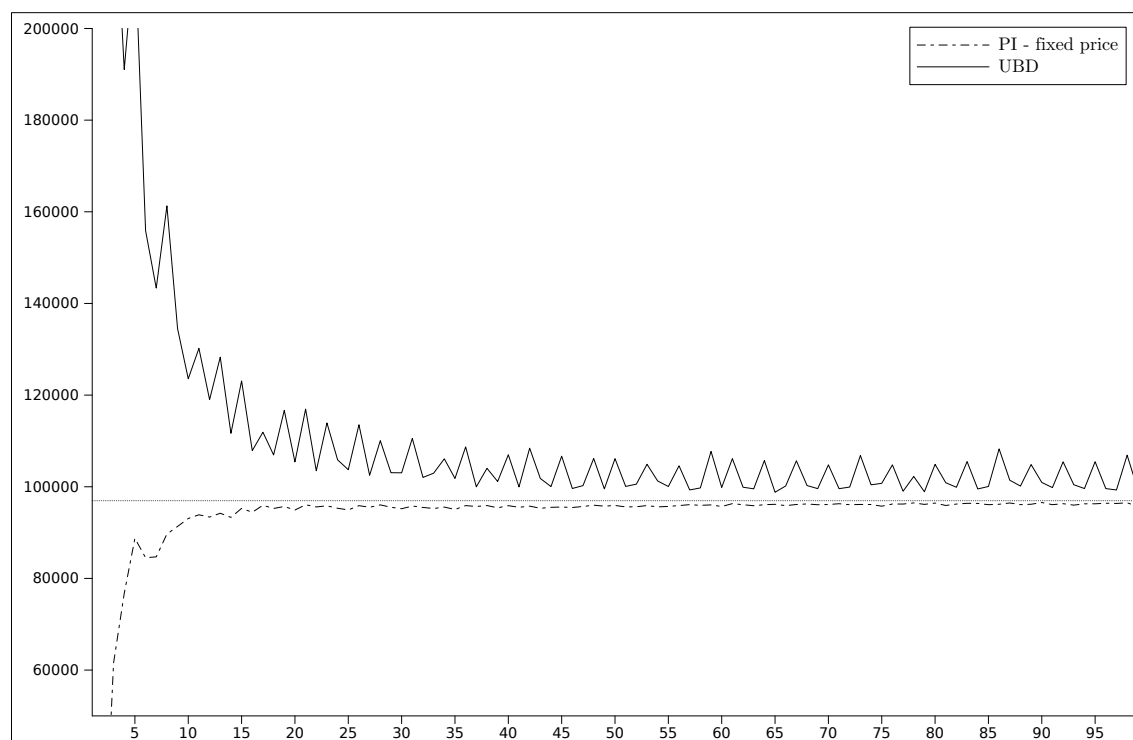
Figure 13: Simulation results from decomposition based pricing.

From the simulations presented in Section 6, we can note that the algorithmic instabilities obtained with those models are independent of a good initial approximation to the optimal volumes. In Figure 14 we show how the profit with prices from [PD-S] varies when we use different initial starting points for the λ_{cp} parameters in the subgradient algorithm. The results in this figure are based on a case where the optimum profit is 95333.07.

Figure 14: The series refer to simulations when the initial λ parameters where randomly set to values within a limit of 1,5,10, and, 20% to known near optimal parameters.

9   Conclusions

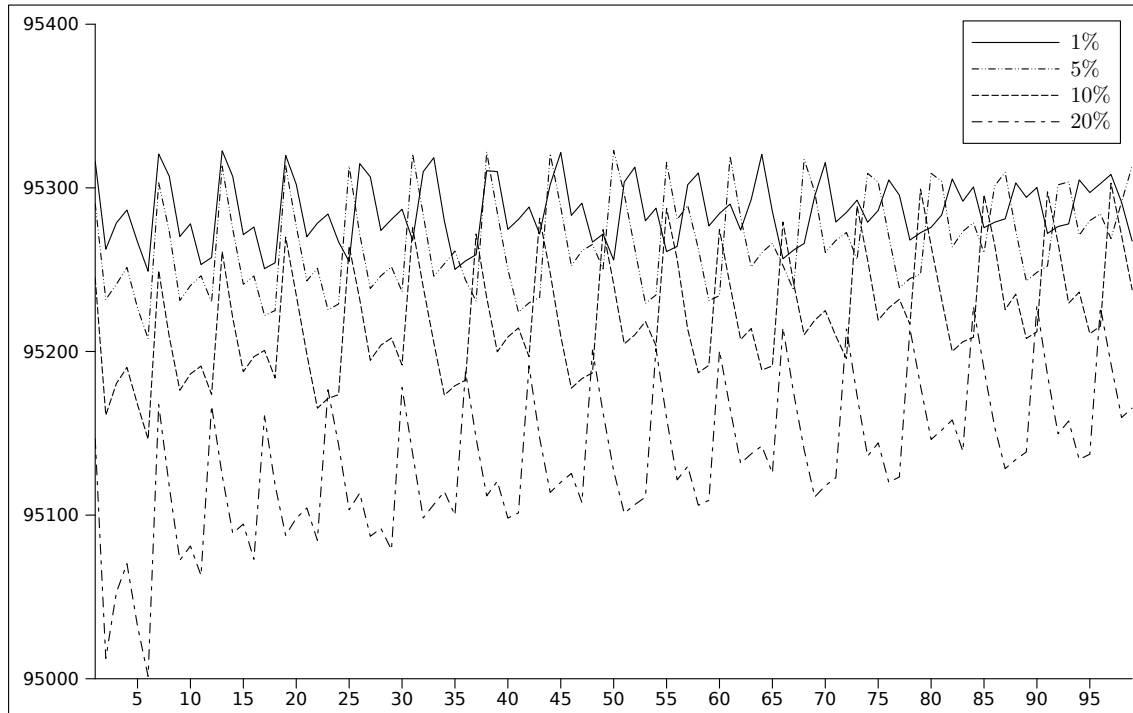The integrated production and sales planning problem arising in refinery planning is a nonlinear problem and difficult to solve in general. The standard approach in many industries and software packages is to divide the problem into one production problem and one sales problem. Then some information based on approximate component and product prices are used to co-ordinate the solutions. This is however not enough to converge to a solution and additional constraints to force the solutions from each problem closer to each other are enforced. Typically restrictions on the volumes are used. These constraints provide some convergence behaviour but not necessarily to an optimal solution. It is easy to get stuck on other less good solutions. In this paper we have proposed a decomposition scheme based on Lagrangian decomposition. It uses some additional information from the blending decisions to provide a methodology that is much more stable than the previous without requiring additional volume constraints.

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M. Guignard and S. Kim. Lagrangean decomposition: a model yielding stronger lagrangean bounds. *Mathematical Programming*, 39:215–228, 1987.

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A   AMPL model

```
set Crudes;%
set Qualities;%
set Components;%
set Products;%
set Markets;%

param Supply{Crudes};%
param Supply_cost{Crudes};%
param Proportion{Crudes, Components};%
param Quality_value{Crudes, Components, Qualities};%
param Quality_value_component{c in Components, q in Qualities};%
param Quality_value_product_up{Products, Markets, Qualities};%
param Quality_value_product_lo{Products, Markets, Qualities};%
param Transport_cost{Products, Markets} default 0.1;%
param Price_coeff{Products, Markets, c in 0..1};%

var crude_volume{c in Crudes} >= 0, <= Supply[c]; %
var blending{Components, Products} >= 0; %
var product_value{p in Products, m in Markets} >= 0; %
var transport_volume{p in Products, m in Markets} >= 0;%

maximize integrated_value :
   sum {p in Products, m in Markets}
     ( (Price_coeff[p,m,0] - Transport_cost[p,m]) * transport_volume[p,m]
       + Price_coeff[p,m,1] * transport_volume[p,m]^2 )
  -sum {c in Crudes} Supply_cost[c]*crude_volume[c];

subject to

I2_blend {c in Components} :
   sum {p in Products} blending[c, p]
  =sum {oil in Crudes} Proportion[oil, c] * crude_volume[oil];

I3_quality_proportions {c in Components, q in Qualities} :
   sum {p in Products} Quality_value_component[c, q] * blending[c, p]
  =sum {oil in Crudes} Quality_value[oil, c, q] * Proportion[oil, c]
    * crude_volume[oil];

I4_distribution {p in Products} :
   sum {c in Components} blending[c, p]
  =sum {m in Markets} transport_volume[p, m];

I5_product_quality_up {p in Products, m in Markets, q in Qualities} :
   sum {c in Components} Quality_value_component[c, q] * blending[c, p]
 <=sum {c in Components} Quality_value_product_up[p, m, q] * blending[c, p];

I6_product_quality_lo {p in Products, m in Markets, q in Qualities} :
   sum {c in Components} Quality_value_component[c, q] * blending[c, p]
 >=sum {c in Components} Quality_value_product_lo[p, m, q] * blending[c, p];

I7_price_def {p in Products, m in Markets} :
   product_value[p, m] = Price_coeff[p, m, 0]
      + Price_coeff[p, m, 1] * transport_volume[p, m];

data; %
set Crudes := CRA CRB BU; %
set Qualities := octane vapor density sulphur; %
set Components := sr_naphtha butane
```

```
                sr_gas sr_gas_oil sr_dist sr_res
                rf_gas cc_gas cc_gas_oil hydro_res;
set Products := premium regular distillate fuel_oil fuel_gas; %
set Markets := MA MB; %

param Supply_cost := CRA 117 CRB 105 BU 107;%

param Quality_value
[*,*, octane] (tr) :%
            CRA   CRB  BU := %
butane      91.8  91.8 92 %
sr_naphtha  65    65    0 %
sr_gas      78.5  78.5  0 %
sr_gas_oil   0     0    0 %
sr_dist      0     0    0 %
sr_res       0     0    0 %
rf_gas     104   104    0 %
cc_gas      93.7  93.7  0 %
cc_gas_oil   0     0    0 %
hydro_res    0     0    0 %

[*,*, vapor] (tr) :%
            CRA    CRB   BU := %
butane      199.2  199.2  199.2 %
sr_naphtha    6.54   6.54   0 %
sr_gas       18.4   18.4    0 %
sr_gas_oil    0      0      0 %
sr_dist       0      0      0 %
sr_res        0      0      0 %
rf_gas        2.57   2.57   0 %
cc_gas        6.9    6.9    0 %
cc_gas_oil    0      0      0 %
hydro_res     0      0      0 %

[*,*, sulphur] (tr) :%
            CRA   CRB   BU := %
butane      0     0     0 %
sr_naphtha  0.3   0.3   0 %
sr_gas      0     0     0 %
sr_gas_oil  1     1     0 %
sr_dist     2     2     0 %
sr_res      6     6     0 %
rf_gas      0     0     0 %
cc_gas      0     0     0 %
cc_gas_oil  0.4   0.4   0 %
hydro_res   6     6     0 %

[*,*, density] (tr) :%
            CRA   CRB   BU := %
butane        0     0    0 %
sr_naphtha  272   272    0 %
sr_gas        0     0    0 %
sr_gas_oil  298   298    0 %
sr_dist     295   295    0 %
sr_res      350   350    0 %
rf_gas        0     0    0 %
cc_gas        0     0    0 %
cc_gas_oil  295   295    0 %
hydro_res   365   365    0 %
;

param Proportion (tr) : CRA    CRB    BU :=
```

```
        butane      0.2    0.2    1
        sr_naphtha  0.23   0.13   0
        sr_gas      0.12   0.12   0
        sr_gas_oil  0.1    0.10   0
        sr_dist     0.08   0.03   0
        sr_res      0.05   0.1    0
        rf_gas      0.05   0.17   0
        cc_gas      0.07   0.05   0
        cc_gas_oil  0.04   0.03   0
        hydro_res   0.11   0.12   0
        ;

param Price_coeff %
[*, *, 0] %
(tr) : premium regular distillate fuel_oil fuel_gas := %
MA      855     784.5   630        450       90 %
MB      864     787.5   675        465       97.5

[*, *, 1] %
(tr) : premium regular distillate fuel_oil fuel_gas := %
MA      -10     -8      -4         -8        -2 %
MB      -10     -8      -5         -9        -1
;

param Quality_value_product_up default 0 %
[*, MA, *] %
(tr) :  premium regular distillate fuel_oil fuel_gas := %
octane    .       .       .          .          . %
vapor   12.7    12.7      .          .          . %
density   .       .     306        352          . %
sulphur   .       .       0.5        3.5        . %

[*, MB, *] %
(tr) :  premium regular distillate fuel_oil fuel_gas := %
octane    .       .       .          .          . %
vapor   10.7    12.7      .          .          . %
density   .       .     306        352          . %
sulphur   .       .       0.5        3.5        . %
;

param Quality_value_product_lo default 0 %
[*, MA, *] %
(tr) :  premium regular distillate fuel_oil fuel_gas := %
octane  92       86       .          .          . %
vapor     .       .       .          .          . %
density   .       .       .          .          . %
sulphur   .       .       .          .          . %

[*, MB, *] %
(tr) :  premium regular distillate fuel_oil fuel_gas := %
octane  90       87       .          .          . %
vapor     .       .       .          .          . %
density   .       .       .          .          . %
sulphur   .       .       .          .          . %
  ;
```