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**Weight Restrictions in the DEA Benchmarking
Modul for Norwegian Electricity Distribution
Companies – Size and Structural Variables**

by

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i inntektsrammeregulering av landets nettselskaper

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Weight Restrictions in the DEA Benchmarking Model for Norwegian Electricity Distribution Companies – Size and Structural Variables

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Summary

In this report, referred to as Report 2, we investigate NVE's suggestions for relative weight restrictions on the structural and size dependent output variables, i.e. VR1-VR8 in NVE (2008). In the present DEA model these are variables with a large degree of slack, i.e. many companies have zero shadow prices on the variables, indicating some inefficiency that is not captured by the DEA analyses. We show that the two-sidedness of the weight restrictions and the large number of them, involving a large share of the output variables, may have unintended effects on the efficiency scores, and that the effects on the efficiency scores in many cases are determined almost completely by the exact limits of the relative weight restrictions. The latter implies that the specification of the limits is very important.

However, many of the output variables in the present DEA models are endogenous variables, in fact they are often input variables used to proxy or represent cost drivers on which there is not sufficient data, or that otherwise cannot be well represented by exogenous output variables. NVE (2008) uses to a great extent cost data, either annualized cost or the relationship between investment expenses, as a basis for determining relative weight restrictions. Yet, it is clear that since the output factors in question represent "more than themselves" it may be extremely difficult to settle on meaningful bounds on relative shadow prices. In order to illustrate this point, we also discuss the implicit assumptions about substitutability between output variables that relative weight restrictions represent. These interpretations that follow directly from linear programming theory, can serve as a test of the practical viability of the relative weight restrictions. For some of the variables in question, the substitution assumptions are at least questionable.

Thus, when it comes to the choice between different types of weight restrictions, i.e. absolute, relative or virtual, we recommend, as in Report 1, to pursue virtual weight restrictions on geography variables (maximum weight) and / or energy and customer variables (minimum weight). The proposed restrictions are formulated with respect to groups of variables rather than individual variables, thereby eliminating the need for detailed assumptions with respect to individual variables. In this report we give another justification for why maximum / minimum bounds of approximately 40 / 20, respectively, may be reasonable limits.

There are, however, other methods that could be used instead of weight restrictions, for instance multistage DEA. Thus, we show a two-stage DEA model, where the first stage consists of solving a DEA model without geography variables, and the second stage consists of regressing the efficiency results on the geography variables, using the regression results to adjust efficiency scores. Although in the two-stage DEA model, it is not necessary to specify any cost based bounds, there are still many implementation details that will influence the results. We show that the results from two-stage model are similar to those of the 40 / 20 max and min virtual weight restrictions, but that more companies are affected, and that efficiency scores can either increase or decrease.

Finally, we compare the results from the 40 / 20 virtual restrictions, as well as the results from the two-stage DEA procedure, to the method used by NVE to adjust the super efficiency scores. If the objective is to avoid very high efficiency scores, it is not obvious which of the three methods should be preferred. However, it can be argued that the alternative methods (i.e., the weight restricted DEA model or two-stage DEA) have better incentive properties than the NVE method for adjusting super efficiencies, since they do not put any explicit restrictions on the companies' measured super efficiencies.

1 Introduction

This is the second of two reports on possible weight restrictions in the DEA benchmarking model for Norwegian electricity distribution companies. It follows Bjørndal, Bjørndal and Camanho (2008), hereafter referred to as Report 1. In the first report we documented the need for weight restrictions by discussing the unrestricted weights, as illustrated by Table 1.1 and Figure 1.1 below. Table 1.1 shows the absolute values¹ of the output weights (shadow prices) for 2005 and 2006, and Figure 1.1 illustrates the virtual weights for 2006. These weights were discussed in Section 2.3 of Report 1, where we commented on the large variation in absolute weights, as well as unreasonable virtual weights for many companies. By “unreasonable” we mean that many companies have very high virtual weights for the geography variables, and/or very low weights for the energy/customer variables. We also showed that unreasonable weights are often accompanied by very high efficiency scores, indicating that weight restrictions may be appropriate.

	Average (NOK)		Max (NOK)		No. of zeros	
	2005	2006	2005	2006	2005	2006
Energy	21	32	93	92	68	48
Customers	605	510	2 343	2 671	73	82
Cottage customers	1 531	1 165	7 848	7 264	67	69
HV-lines	4 864	8 735	32 457	44 683	88	63
Network stations	15 979	12 896	45 769	52 548	50	59
Interface	1 174	1 300	7 032	7 701	69	51
Forest	29 284	28 184	222 056	215 491	44	57
Snow	18 445	24 193	109 824	123 595	73	58
Coast	22 847	22 700	148 469	165 919	82	81

Table 1.1: Output weights (shadow prices) for 2005 and 2006.

¹ Note that the geography variables have been rescaled according to our proposal in Section 4.2 in Report 1. The rescaling operation affects the shadow prices of the rescaled variables, but leaves other shadow prices unchanged.

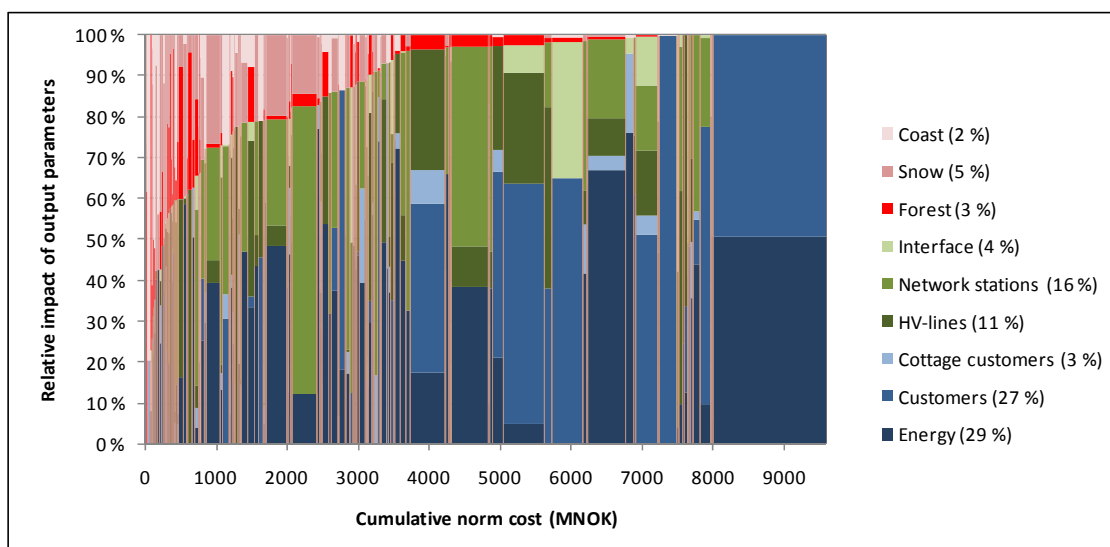


Figure 1.1: Virtual output weights for 2006.

In Report 1 we gave an overview of the various types of restrictions that are available (relative, absolute and virtual), and we evaluated the relative restrictions (VR9-VR11), with respect to the geography variables, that were proposed in NVE (2008). As an alternative to the relative restrictions we proposed virtual weight restrictions with respect to the aggregate share of, e.g., the geography variables. The main argument behind our alternative approach is that it requires less detailed knowledge about the companies' cost structure than the restrictions given in NVE (2008). If an output represents "more than itself", for instance by being a "proxy" measure on one or more cost drivers, or if it represents other correlated factors that are not included in the model, it is almost impossible to determine the effect of this particular output on the total cost of a company, and it therefore becomes difficult to determine reasonable cost ratios that can be used as a basis for relative (pair wise) weight restrictions. By focusing on groups of variables, we reduce the need for such detailed cost information.

In Report 2 we look at restrictions with respect to the remaining output variables in the model. We discuss the remaining restrictions VR1-VR8 proposed by NVE (2008), and we look at some alternatives. The structure of the report is as follows. In Section 2 we discuss the relative restrictions VR1-VR8 proposed by NVE (2008), and in Section 3 we present an alternative proposal based on the analysis and discussion from Report 1. In Section 4 we discuss alternative approaches for handling the geography effects, and we present an example of a two-stage DEA model. Section 5 compares the proposals from

Sections 3 and 4 to the NVE methods for super efficiency adjustment and efficiency score calibration.

2 Evaluation of weight restrictions proposed by NVE

Table 2.1 describes the weight restrictions proposed in NVE (2008). The overall motivation for the restrictions was discussed in Section 4 of Report 1, and the restrictions (VR9-VR11) with respect to the geography variables were also discussed there. In the following we consider the remaining restrictions VR1-VR8.

Restriction(s)	Involved variables	Mathematical formulation
VR1 / VR2	HV lines versus network stations	$0.952p_{NS} \leq p_{HV} \leq 8.572p_{NS}$
VR3 / VR4	Interface versus network stations	$0.02304p_{NS} \leq p_{Int} \leq 0.20738p_{NS}$
VR5 / VR6	Customers versus cottage customers	$1/3p_{Cust} \leq p_{CCust} \leq 3p_{Cust}$
VR7 / VR8	Network stations versus customers	$1.618p_{Cust} \leq p_{NS} \leq 58.252p_{Cust}$
VR9	Forest versus HV lines	$p_{Forest} \leq 0.04p_{HV}$
VR10	Snow versus HV lines	$p_{Snow} \leq 0.0053p_{HV}$
VR11	Coast versus HV lines	$p_{Coast} \leq 36.364p_{HV}$

Table 2.1: Weight restrictions in NVE (2008)²

2.1 Assumptions behind VR1-VR8

The limits in VR1 and VR2 are based on the ratio between investment expenses for HV lines and network stations, respectively. The “normal” investment expense is estimated at 419 000 NOK per kilometer of HV line, and 146 640 NOK per network station. NVE (2008) arrives at the ratio $419/146.64 = 2.8573$ by implicitly assuming that lines and network stations have the same life span. The lower and upper bounds shown in Table 2.1 are obtained by allowing the ratio between the prices of high voltage lines and network stations to vary between 1/3 and 3 times this number.

² See NVE (2008) for a more detailed description of the assumptions.

Restrictions VR3 and VR4 are based on similar reasoning. Since the interface variable is expressed in terms of the annual cost³ and the network station variable is expressed in terms of the number of network stations, we need to transform one of them in order to make comparisons. NVE (2008) chooses to compute the annual capital cost of a network station by assuming a life span of 35 years and an interest rate of 5.6 %, and by assuming that the operating costs are 50 % of the capital cost. By using an annuity formula, we get an annual cost of 14 446 NOK per network station, and the appropriate cost ratio is therefore 1 / 14.446. Again, lower and upper bounds are obtained by allowing the price ratio to vary between 1/3 and 3 times this number.

Restrictions VR5 and VR6 are based on the initial hypothesis that the marginal cost of an ordinary customer and a cottage customer are identical. NVE (2008) finds support for this hypothesis by comparing the average shadow prices of the two outputs. They therefore propose that the ratio between the corresponding shadow prices should be allowed to vary in the interval between 1/3 and 3. Note that the average unrestricted shadow prices in Table 1.1 indicate that the marginal cost of a cottage customer is more than twice as large as the marginal cost of an ordinary customer, contrary to the observation in NVE (2008). However, even if we accept the observed shadow prices in Table 1.1, we cannot conclude that cottage customers are more expensive than ordinary customers, since the two types of customers are structurally different. Cottage customers typically consume less energy than ordinary customers. We also know that the customer variables and the energy variable are highly correlated. Lower energy consumption will therefore cause some of the costs that, for ordinary customers, would be picked up by the energy variable, to be explained by the customer variable in the case of cottages. Thus, even if a cottage customer costs the same as an ordinary customer, we would expect the shadow price for cottage customers to be at least as high as for ordinary customers.

Finally, restrictions VR7 and VR8 are based on the computed cost of 14 446 NOK per network station, as well as an estimate of the cost per customer. The estimated cost per customer is based on an estimate of the annual operating and capital cost of low voltage lines, as well as the number of meters of low voltage lines per customer. By using these estimates, NVE arrive at an annual cost, related to the low voltage network, of 1 110 NOK per customer. The direct customer costs (metering, invoicing etc.) are estimated at 380 NOK, giving a total of 1 490 NOK per customer. This gives a cost ratio

³ See NVE (2006a). The interface variable is given as a weighted sum of components located in the interface between the distribution network and other network levels, where the weights are estimates of annual capital and operating costs.

of $14.446 / 1.490 = 9.7087$. The lower and upper limit in VR7 and VR8, respectively, correspond to $1/6$ and 6 times this cost ratio. NVE (2008) argues that the variation in the number of network stations is very large among the companies; hence they conclude that a factor of 6 should be used to set the limits in this case, instead of the factor of 3 that was used in the case of VR1-VR6.

2.2 How the optimal weights are affected by the restrictions

As seen from Table 1.1, many of the unrestricted weights are equal to zero, implying that the corresponding output variable is not considered when evaluating the efficiency of the company in question. The introduction of weight restrictions forces most of the weights to take positive values, as illustrated in Table 2.2 below. We see that many of the variables have positive weights for *all* the 127 companies in our dataset. Note that this is the case for all the variables that are included in one or more of the two-sided restrictions, i.e., all the variables except energy and the geography variables.

Variable	Before	After
Energy delivered	79	88
Customers	45	127
Cottage customers	58	127
HV lines	64	127
Network stations	68	127
Interface	76	127
Forest	70	78
Snow	69	65
Coast	46	45

Table 2.2: Number of companies with positive output weights before and after the introduction of weight restrictions, 2006 dataset.

Table 2.3 provides more insight into how the restricted weights are actually determined. For all the two-sided restrictions we see that a majority of the companies have optimal weights that are either at the lower or upper bound. For example, the weight on cottage customers is restricted to the interval between $1/3$ and 3 times the weight on ordinary customers. The table shows that the weight is at the lower bound for 72 companies, while it is at the upper bound for 33 companies. So there are 105 companies where either the lower or upper bound is binding. This phenomenon is not surprising, since the weights correspond to an optimal solution of an LP problem, and we know that LP problems will have optimal solutions at corner points of the feasible set.

Restriction(s)	Mathematical formulation	Lower binding	Upper binding	Sum
VR1 / VR2	$0.952p_{NS} \leq p_{HV} \leq 8.572p_{NS}$	45	33	78
VR3 / VR4	$0.02304p_{NS} \leq p_{Int} \leq 0.20738p_{NS}$	56	39	95
VR5 / VR6	$1/3p_{Cust} \leq p_{CCust} \leq 3p_{Cust}$	72	33	105
VR7 / VR8	$1.618p_{Cust} \leq p_{NS} \leq 58.252p_{Cust}$	13	51	64
VR9	$p_{Forest} \leq 0.04p_{HV}$	-	28	28
VR10	$p_{Snow} \leq 0.0053p_{HV}$	-	31	31
VR11	$p_{Coast} \leq 36.364p_{HV}$	-	34	34

Table 2.3: Binding weight restrictions, 2006 dataset.

Figure 2.1 illustrates how the various restrictions form a system that links the weights. Each node in the figure corresponds to an output variable, and the arrows correspond to weight restrictions. The direction of each arrow shows the direction of the corresponding restriction. For example, VR1 specifies a lower bound for the HV weight relative to the weight on network stations, and this is illustrated by an arrow from the HV-line node to the network station node. We see that all the variables are included in one or more weight restrictions, except the energy variable. We have also included the number of binding restrictions in the figure, i.e., the numbers from Table 2.3. The numbers suggest that some combinations of binding restrictions are more common than others. Specifically, there is a tendency for the network station weight to form a lower bound for the weights of the “surrounding” variables, i.e., HV lines, interface and customers. Also, we see that the weight on ordinary customers tend to form a lower bound for the cottage customer weight. In fact, the restrictions VR8 and VR5 are binding for 41 companies. So for about one third of the companies three of the output weights are determined by the simple equations $p_{NS} = 58.252p_{Cust}$ and $1/3p_{Cust} = p_{CCust}$.

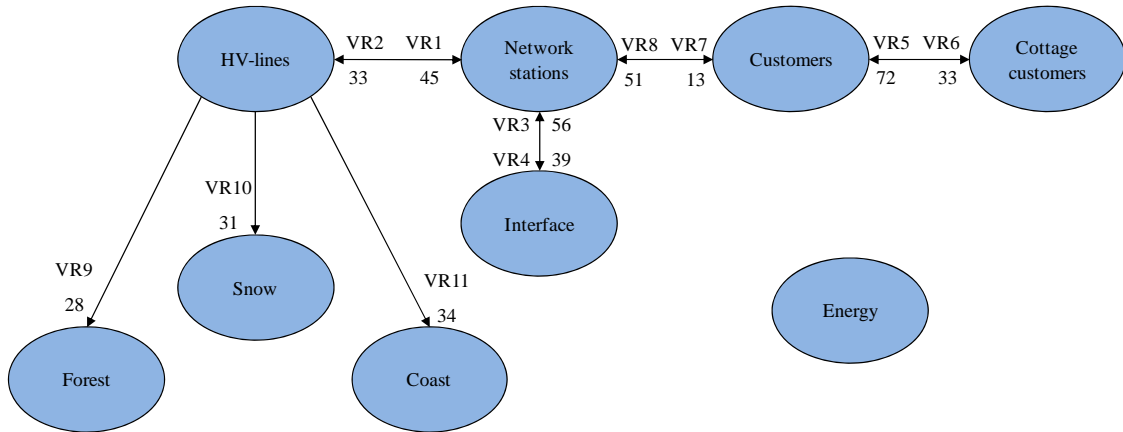


Figure 2.1: System of relative weight restrictions.

The histogram in Figure 2.2 further illustrates the extent to which the weight restrictions will be binding. With 4 two-sided restrictions and 3 one-sided restrictions, a maximum of 7 restrictions can be binding for a particular company, although this does not occur for any company in the dataset. The figure shows that there are two companies with 6 binding restrictions. Since each restriction links two weights, these two companies will have the relative values of 7 out of 9 output weights determined by an equation system consisting of the binding weight restrictions. A less extreme example is that 59 companies have at least 4 binding restrictions. This means that almost one half of the companies in the dataset have at least 5 out of 9 output weights determined by the binding weight restrictions.

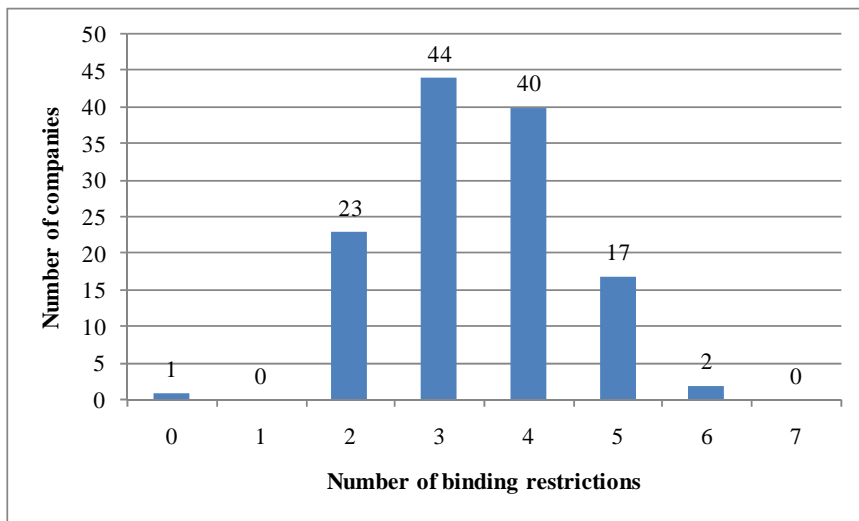


Figure 2.2: Binding weight restrictions, 2006 dataset.

The discussion in this section shows that the weight restrictions are not as innocent as they may seem, since they in practice to a large extent will be binding, and will determine the resulting weights. It is therefore very important that the underlying assumptions are well founded, and based on a good understanding of the output weights. In the next section we will show that this may represent a formidable challenge, given the properties of the present DEA model.

2.3 Weight restrictions on approximate cost drivers

The short presentation of VR1-VR8 in Section 2.1 shows that the restrictions are based on a number of more or less explicit assumptions. These assumptions can be determined by answering the following questions:

- 1) Which cost elements should be included when cost ratios are calculated, i.e., which cost elements are relevant for the output variables in question? Is it, for instance, relevant to use the cost of high voltage lines to represent the effect of the HV-variable, or should we also include other cost elements, e.g. part of the cost of low voltage lines?
- 2) What are the correct assumptions when converting investment expenses into annual capital costs, i.e., the relevant cost of capital and the equipment life span(s)? An example is the implicit assumption underlying VR1/VR2, namely that high voltage lines and network stations have the same life span.
- 3) How wide should the intervals be? In NVE (2008) a factor of 3 has been used, except in the case of VR7/VR8, where a factor of 6 was assumed.

It is very difficult to answer these questions, and in particular question 1. Some of the “input” variables on the output side, like HV lines and network stations, are included as proxies for factors that are difficult to measure, such as demographical and topological factors. Thus, we cannot necessarily compare the shadow price of one kilometer of HV lines to the observed cost of installing and operating the line, since this particular output may represent “more than itself”.

Even though we have a candidate variable that can be used to represent a particular output factor, the variable might be omitted from the model, and the cost effect of that variable will therefore be picked up by one or more of the included output variables. This is due to the way that the DEA model was constructed in the first place. As described in NVE (2006a/b), statistical tests were used to check whether variables should be included

in the model or not. Some cost drivers were excluded, either because they did not pass the statistical tests, or for other reasons. One example was low voltage lines, which was excluded, while the high voltage line variable was included. Table 2.4 below shows some examples of the so-called Banker test⁴ for these two variables. The Banker test was used to determine whether new output variables had a significant effect on the average efficiency score for the companies in the dataset. The second last column in Table 2.4 shows the t-ratio for the observed effect of adding the variable in question, and the last column shows the probability that the observed effect would happen accidentally. It is common to conclude that an observed effect is non-significant if the probability value is larger than 0.05. Based on this assumption, we see that the LV variable has a significant effect if it is added to a model where only the customer and energy variable is included initially, but the effect is non-significant if the variable is added to a model where also the HV variable is included. This is not surprising, since the LV and HV variables are highly correlated⁵.

New variable	Already included variables				t	p(T > t)
	Cust.	Energy	HV	LV		
LV	x	x			8.7	0.00
LV	x	x	x		1.4	0.08
HV	x	x			17.3	0.00
HV	x	x		x	9.4	0.00

Table 2.4: Test statistics for HV lines and LV lines, pooled dataset for 2001-2004.

NVE (2006a/b) finally concluded that the LV variable should be dropped from the model, and the final decision was based on the results of the statistical tests⁶ *as well as* the perceived low data quality of the LV variable. Since there is no doubt that LV lines represent a major cost driver for distribution companies, and because the LV and HV variables are highly correlated, we would expect part of the cost of low voltage lines to be picked up by the HV variable. However, since the LV variable is also highly correlated with other included variables, e.g. the network station variable⁷, it is impossible to specify how the LV cost effect is distributed among the included cost drivers.

⁴ See Banker & Natarajan (2004) and Kittelsen (1993).

⁵ The correlation coefficient value is 0.96 for the 2001-2004 dataset.

⁶ In NVE (2006a) the LV variable was only tested against a model that already included the HV variable. Table 2.4 shows that the HV variable would have a statistically significant effect even if it was added to a model that already contained the LV variable. This implies that arguments purely based on statistical tests could have led to a different outcome had the order of the tests in NVE (2006a) been reversed.

⁷ The correlation coefficient value is 0.97 for the 2001-2004 dataset.

The above discussion tells us that we should be very careful to interpret individual shadow prices, since we do not know what kind of cost effects they are explaining. This is our main reason for not recommending weight restrictions at a very detailed level, as given by VR1-VR11. We instead suggested that weight restrictions should be specified at a more aggregate level, i.e., with respect to groups of variables rather than for individual variables. We already discussed this in Report 1, and will repeat our suggestions in Section 3 below.

2.4 Implicit assumption - substitution possibilities

Weight restrictions can be given a mathematical interpretation in terms of the LP problems that are solved when computing the efficiency scores. Without weight restrictions the LP problem for company j^* is the following:

$$\begin{aligned}
 \text{(LP1)} \quad & \text{Min}_{\lambda} \sum_{j \neq j^*} \lambda_j x_j \\
 & \text{s.t.} \\
 & \sum_{j \neq j^*} \lambda_j y_{rj} \geq y_{rj^*} \quad r = 1, \dots, s \\
 & \lambda_j \geq 0 \quad j = 1, \dots, n
 \end{aligned}$$

There are n companies producing s different outputs. The total cost of company j is x_j while company j produces y_{rj} units of output r . The variable λ_j is the weight of company j in the reference set of the evaluated company j^* . The model is CRS (with constant returns to scale, $\lambda_j \geq 0$) and we assume super efficiency (sum over j except j^*). The interpretation of the linear program is that in the performance evaluation of company j^* we find the reference company, as a linear combination of the other companies in the industry, with minimum cost, such that it produces at least as much of each output as the evaluated company. Alternatively, we may formulate the dual problem of LP1:

$$\begin{aligned}
 \text{(LP2)} \quad & \text{Max}_p \sum_r y_{rj^*} p_{rj^*} \\
 & \text{s.t.} \\
 & \sum_r y_{rj} p_{rj^*} \leq x_j \quad j \neq j^* \\
 & p_{rj^*} \geq 0
 \end{aligned}$$

The decision variables are the prices p_{rj^*} for each output of the evaluated company, and the linear program can be interpreted so as to find prices for company j^* that maximize revenue, and at the same time assure that none of the other companies, as represented by their respective output vectors, exceed their total cost at these prices (they are within a budget limit). The prices p_{rj^*} in problem LP2 are the shadow prices of the output constraints in LP1, and consequently, p_{rj^*} gives the increase in minimum cost due to an increase in y_{rj^*} , and is a local per unit cost of output r .

As an example, the weight restrictions VR1 and VR2 will result in the following extra constraints (rows) in LP1:

$$-p_{HV,j^*} + 0.952p_{NS,j^*} \leq 0 \quad (\text{VR1})$$

$$p_{HV,j^*} - 8.572p_{NS,j^*} \leq 0 \quad (\text{VR2})$$

Since each row in the dual corresponds to a column in the primal, the primal (LP1) will be extended with two extra columns (decision variables), $\gamma_{HV,NS}^{UP}$ and $\gamma_{HV,NS}^{LO}$, and these variables will only appear in the output constraints for HV and NS:

$$\sum_{j \neq j^*} \lambda_j y_{HV,j} - \gamma_{HV,NS}^{LO} + \gamma_{HV,NS}^{UP} \geq y_{HV,j^*} \quad (\text{HV})$$

$$\sum_{j \neq j^*} \lambda_j y_{NS,j} + 0.952\gamma_{HV,NS}^{LO} - 8.572\gamma_{HV,NS}^{UP} \geq y_{NS,j^*} \quad (\text{NS})$$

The decision variable $\gamma_{HV,NS}^{LO}$ will only be allowed to take nonzero values if the lower price limit VR1 is binding⁸. In this case, company j^* will be allowed to “convert” HV lines into network stations in order to cover the output requirement for network stations. For every kilometer of HV line that the company gives up, it will gain 0.952 network stations. On the other hand, if the upper price limit VR2 is binding, the decision variable $\gamma_{HV,NS}^{UP}$ will be allowed to take nonzero values. In this case, company j^* will be allowed to “convert” network stations into HV lines in order to cover the output requirement for HV lines. For every kilometer of extra HV lines that the company adds, it will be forced to give up 8.572 network stations.

The interpretation of weight restrictions as extra substitution possibilities provides an additional test of whether the restrictions are reasonable or not. In the case of “similar” output variables like HV lines and network stations this may be reasonable. In other cases

⁸ This follows from the complementary slack conditions.

where the respective outputs represent very different kinds of objects, like network stations and customers, such implicit substitution assumptions may be more questionable.

3 Alternative weight restrictions

The main conclusion from Section 2 is that, since output variables may represent “more than themselves”, it is very difficult to interpret their shadow prices, and this makes it difficult to specify the detailed assumptions that are needed in order to formulate restrictions like VR1-VR8. The remaining restrictions VR9-VR11 were discussed in detail in Report 1, and we proposed a reformulation of the restrictions as well as the geography variables. However, the question of how to determine appropriate limits is no less difficult for these restrictions, and we will therefore not use them as part of our proposal. Instead we recommend the use of virtual weight restrictions formulated with respect to groups of variables rather than individual variables, thereby reducing the need for detailed cost information.

3.1 Virtual weight restrictions

The virtual output associated with an output variable is defined as the product of its optimal shadow price p_r and the physical output quantity y_{rj^*} , where j^* is the company that is being evaluated. Thus, virtual output can be interpreted as the “revenue” that company j^* can collect from output r , and if we sum all the virtual outputs we get the DEA cost norm (in NOK) for the evaluated company. Virtual weight restrictions are restrictions with respect to the absolute or relative “revenue” coming from one or more of the outputs.

It was documented in Report 1 that companies with very high efficiency scores also tend to weight the geography variables quite heavily. We therefore proposed, in Report 1, an upper bound on the relative virtual weight of these variables, given by:

$$\frac{P_{Forest} \cdot y_{Forest,j^*} + P_{Snow} \cdot y_{Snow,j^*} + P_{Coast} \cdot y_{Coast,j^*}}{\sum_r p_r \cdot y_{rj^*}} \leq \alpha \quad (1)$$

The number α has a value between 0 and 1, and represents the maximal share of the total cost norm (for the evaluated company j^*) that the geography variables may account for. We also proposed a lower bound for the virtual weight of the “essential” variables, namely energy delivered and customers:

$$\frac{P_{Energy} \cdot y_{Energy,j^*} + P_{Cust} \cdot y_{Cust,j^*} + P_{CCust} \cdot y_{CCust,j^*}}{\sum_r P_r \cdot y_{rj^*}} \geq \beta \quad (2)$$

The number β has a value between 0 and 1, and represents the minimal share of the total cost norm that the product variables may account for.

Restrictions (1) and (2) were discussed in detail in Report 1, and compared to a revised version of the relative restrictions VR9-VR11. We showed that the relative restrictions tend to affect a large number of companies, but the effect can be quite small for many companies. The virtual (group) restrictions, on the other hand, target companies with very high unrestricted efficiency scores, while leaving most other companies with very small or no changes in their efficiency scores.

Determination of the limits

In order to implement the virtual weight restrictions, we need to determine reasonable values for the upper and lower limits, i.e., α and β . This issue was also discussed in Report 1, and Table 3.1 below is related to Figure 6.8 in Report 1. We show efficiency scores from a model where the geography variables have been omitted, and the companies have been sorted according to a simple “geography index”⁹, where high index values indicate difficult geographic conditions. We see that the cost weighted average efficiency of the entire industry in 2007 was 85.1 %, indicating that the total cost of the industry can be reduced by 14.9 %. Note that this inefficiency can be caused by either bad management or external factors. Suppose that we give the companies the benefit of the doubt and assume that *all* of their inefficiency is caused by external factors, i.e., that all the inefficiency is caused by the geography factors. Hence, the geography factors cannot account for more than 14.9 % of the industry cost norm. Note that the effect can be much larger than this for individual companies, and restriction (1) needs to allow for this. For example, the company (Austevoll) with the highest index value had an efficiency score of 48 % in 2007, indicating that the geography cost share can be at most 52 %. Since there is some uncertainty with respect to the correctness of the DEA analysis, we should not base our estimate on data for only one (extreme) company, and we may e.g. use the average inefficiency score for the companies corresponding to the 5 highest index values, giving us an upper bound for the geography share of 37.7 %

⁹ The index is constructed by reformulating the geography variables as described in Section 4.2 in Report 1, and then dividing the variables by the number of kilometers of HV lines. The resulting index numbers can be interpreted as the share of a company’s network that is made up of “difficult” lines.

(100 % - 62.3 %). If we base the limit on the 10 companies with highest index values, we get a limit of 32.1 %. We choose a limit that is high enough to accommodate the 5 first companies in both years, and in our recommendation the upper limit for the geography share will be set equal to 40 %. This value is also consistent with the sensitivity analysis in Figure 5.4 in Report 1, where we concluded that a limit of 20-40 % will result in zero correlation between the geography variables and the efficiency scores.

Companies	2006	2007
Highest: Austevoll	48.9 %	48.0 %
5 highest: Austevoll, Tysnes, Nesset, Trollfjord, Rødøy-Lurøy	66.2 %	62.3 %
10 highest: Austevoll, Tysnes, Nesset, Trollfjord, Rødøy-Lurøy, Modalen, Stryn, Sunndal, Ørskog, Fusa (2006) / Stranda (2007)	73.6 %	67.9 %
20 highest	74.5 %	70.2 %
50 highest	77.4 %	75.1 %
All companies	89.8 %	85.1 %

Table 3.1: Cost weighted average efficiency scores from a DEA model without geography variables, where the companies have been sorted according to a “geography index”.

With respect to the lower limit for the energy / customer weights, this could be determined by looking at the cost structure of the companies, as in Table 3.2 below. We have estimated the cost shares for power losses and direct customer costs, respectively. The direct customer costs have been obtained from Note 6 (OpEx), and Note 17.400 and 17.410 (CapEx)¹⁰. Power losses are valued at area prices from NordPool spot that lie in the range 234-238 NOK per MWh. The companies have been sorted according to the combined share of energy/customer related costs. We see that the cost share for the entire dataset is 22.7 %, but that there are large variations for individual companies. For example, the 10 companies with lowest cost shares have an average cost share of only 14.1 %. Note that the cost shares are quite sensitive to the area prices used to compute the costs of power losses. Since the area prices were quite low in 2007, the cost shares reported in the table probably represents a low estimate. Considering that some of the “other” costs should also be related to energy / customers, a lower bound of 20 % for energy / customers seems reasonable.

¹⁰ There were 138 in the dataset, and we have removed 31 of these. We removed companies with no reported customers, no reported capital / depreciation for the customer-related activities, or extreme values with respect to the direct customer cost per customer. The remaining 107 companies correspond to 88 % of the industry cost (including network-related costs).

No. of companies	Costs of power losses	Direct customer costs	Total energy / customer related costs	Other costs
10	6.2 %	7.9 %	14.1 %	85.9 %
20	5.9 %	8.7 %	14.7 %	85.3 %
30	6.7 %	8.8 %	15.5 %	84.5 %
50	7.2 %	9.9 %	17.1 %	82.9 %
107	9.2 %	13.4 %	22.7 %	77.3 %

Table 3.2: Cumulative share of costs related to energy/customer (2007).

Consequences of the proposed restrictions

The consequences of the proposed restrictions are illustrated in Figure 3.1 (2006) and Figure 3.2 (2007), as well as parts of Table 3.3 and Table 3.4. The results are as expected from the analysis in Report 1, i.e., the restrictions have large effects for companies with high unrestricted efficiency scores, while the other companies are not affected to a large extent. This differs from the relative weight restrictions that we analyzed in Report 1 (i.e., the revised versions of VR9-VR11), where we saw considerable effects for a large number of companies.

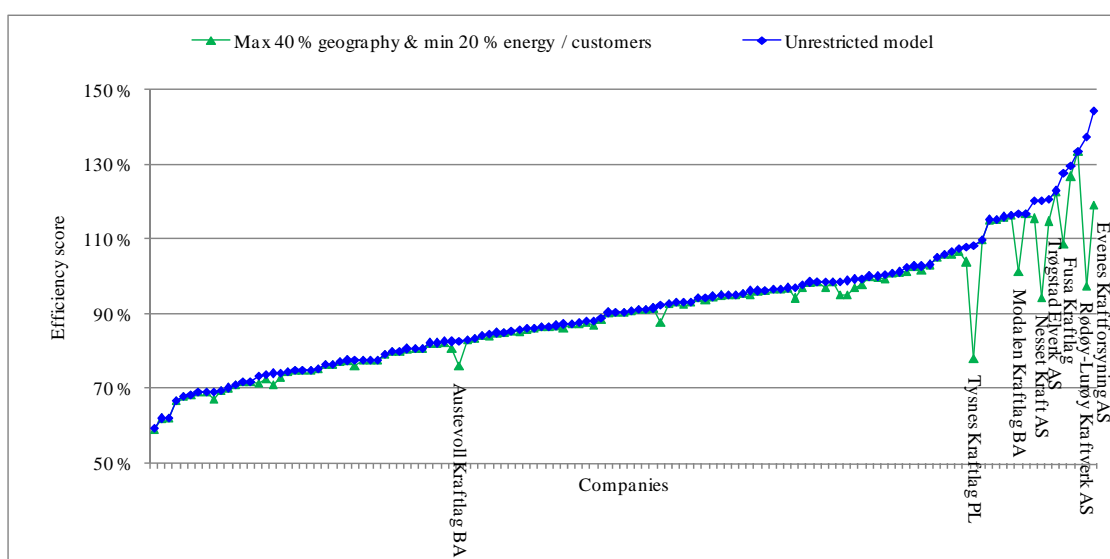


Figure 3.1: Effect of virtual restrictions with respect to geography and energy/customer (2006). Names indicate companies with a reduction of more than 5 %-points.

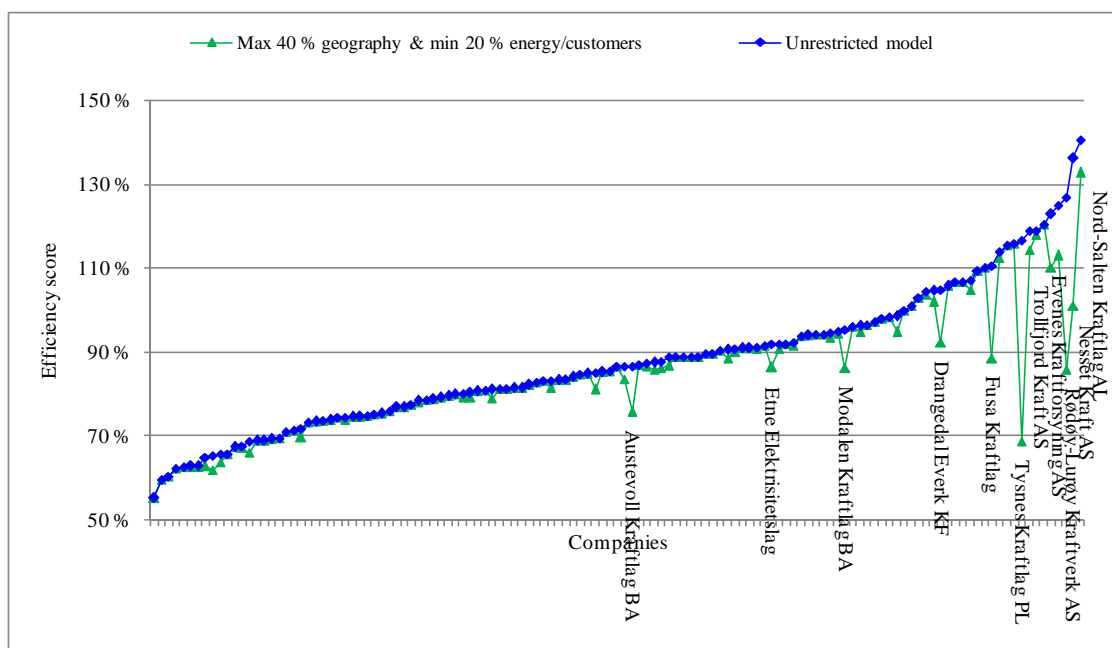


Figure 3.2: Effect of virtual restrictions with respect to geography and energy/customers (2007). Names indicate companies with a reduction of more than 5 %-points.

Model	2006	2007
Unrestricted model	93.6 %	89.1 %
40 / 20 virtual restriction	93.2 %	88.5 %
40 / 20 & customer restrictions	91.5 %	87.6 %

Table 3.3: Cost weighted average efficiency scores for the weight restrictions considered in Sections 3.1 (first two lines) and 3.2 (last line).

Reduction in %- points	40 / 20 virtual restriction		40 / 20 & customer restrictions	
	2006	2007	2006	2007
Over 50	0	0	0	0
25 - 50	4	3	4	3
10 - 25	2	5	2	5
5 - 10	2	3	10	6
0 - 5	43	39	98	100
No change	76	78	13	14

Table 3.4: Efficiency score reductions caused by the restrictions for the weight restrictions considered in Sections 3.1 (first two columns) and 3.2 (two last columns).

3.2 Restrictions on the customer weights

We consider the following restrictions with respect to the two customer variables:

- 1) A lower absolute bound for both variables. The direct customer costs for the 107 companies that we have analyzed in Table 3.2 have average direct customer costs of 503 NOK per customer. We argue that the shadow price for both of the customer variables should at least cover the direct cost, and therefore set a lower bound of 500 NOK for both the customer weights.
- 2) A relative restriction stating that the weight on the cottage customer variable should be at least as high as the weight on the customer variable. This reflects the motivation for including the cottage customer variable in the first place. I.e., that companies with a large share of cottage customers need to be allowed to weight these customers more heavily than other customers, since they consume less energy. The included restriction allows for this to take place, while weight schemes where $p_{Cust} > p_{CCust}$ will not be allowed.

The above restrictions can be summarized as:

$$p_{CCust} \geq p_{Cust} \geq 500 \quad (3)$$

The effects of these restrictions are illustrated in Figure 3.3 and Figure 3.4 below. We have analyzed the effect of adding the customer restrictions to the virtual restrictions for geography and energy / customers. We see that the additional effects are quite small, although a large number of companies are affected, as can also be seen from Table 3.4 above.

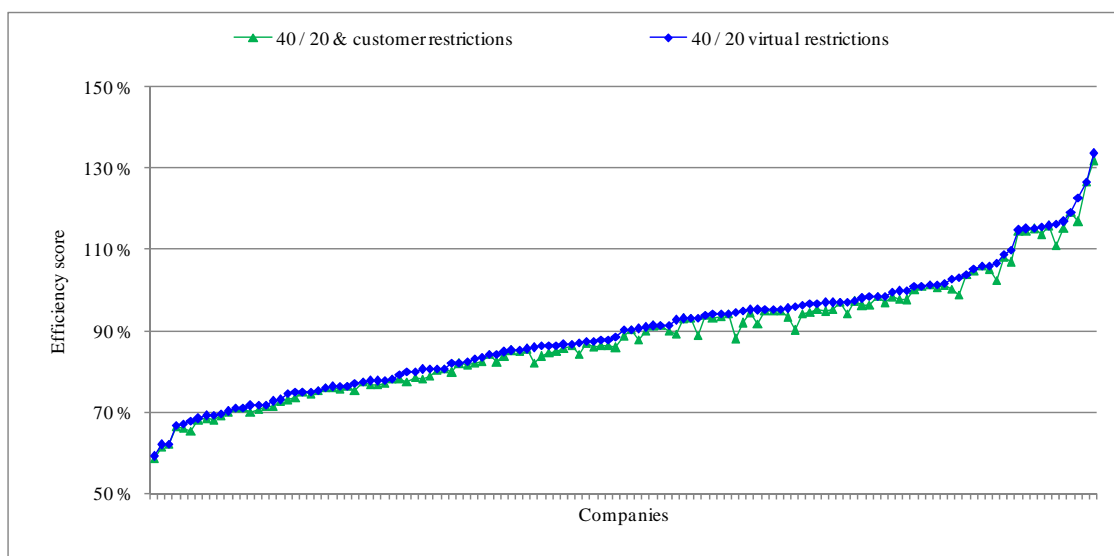


Figure 3.3: Additional effect of restrictions on customer weights (2006).

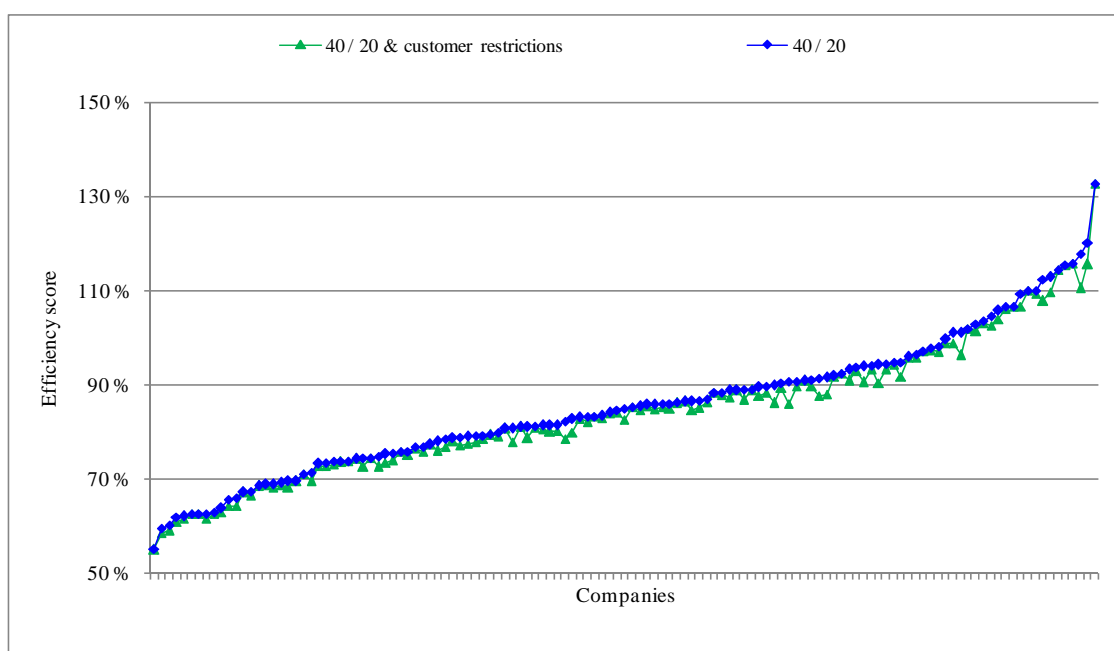


Figure 3.4: Additional effect of restrictions on customer weights (2007).

4 Alternative approaches to the geography effects

One of the main arguments for considering weight restrictions, as stated by NVE (2008), is to limit the effect of the geography variables on the efficiency measurement. An

alternative to weight restrictions is to look at the way that these variables are incorporated in the DEA efficiency analysis. As the DEA model is now, the geography variables are treated in the same manner as “ordinary” output variables. In the DEA literature several approaches to the consideration of environmental effects have been considered. These approaches can be categorized in the following manner (see Coelli et al., 2005):

- 1) Methods by which the companies (DMUs) are grouped into categories, and where a particular company is evaluated against a dataset consisting of companies with a less favorable environment than itself. For details, see Cooper et al. (2007).
- 2) Methods by which the environmental (geography) variables are used directly in the estimation of the efficiency scores, either as input / output variables in the DEA model, or they are used to adjust the initial efficiency scores. We distinguish between the following types of models¹¹:
 - a) One-stage models, where the environmental variables are included directly in the DEA model as either input or output variables. The approach used by NVE so far falls into this category.
 - b) Two-stage DEA models, where the first stage consists of running a DEA analysis *without* the environmental variables, and where these variables are used as explanatory variables in (e.g.) a regression analysis in the second stage. See Ray (1991) for a good introduction, whereas Barnum & Gleason (2008) discuss an important bias problem that may arise with this method.
 - c) Other multi-stage approaches. An example is given in Ruggiero (2004), where a third stage is added to a two-stage model, and where the third stage consists of running a new DEA analysis with an environmental (geography) index obtained from the regression analysis in the second stage. Other three- or four-stage approaches are discussed and compared in Cordero et al. (2009)¹² and Ruggiero (2004).

In the following we will compare the effect of the virtual restrictions on the geography variables and energy / customer variables, to the results from a two-stage DEA procedure. We have used the same approach as in Ray (1991), i.e.:

¹¹ A good overview of the different approaches can be found in Ruggiero (2004) and Cordero et al. (2009).

¹² See also Estelle et al. (2010) for some critical comments to this article.

Stage 1: A DEA analysis without the geography variables, i.e., with 6 output variables, is performed.

Stage 2: The (super) efficiency scores from the Stage 1 are used as the dependent variable in a linear regression analysis, where the geography variables¹³ are used as explanatory variables. As in Ray (1991) we have used adjusted¹⁴ residuals from the regression analysis as estimates of *managerial inefficiency*, i.e., inefficiency that is not caused by external factors. Then we compute the *efficiency* score of each company as 1 minus the inefficiency score.

The results from the two-stage procedure are compared to the results from the one-stage model in Figure 4.1. In order to make the results comparable, the efficiency scores have been calibrated¹⁵ such that the cost weighted average is equal to 100 %. The figure shows that the two methods are similar in that they give large reductions in efficiency scores for companies that appear as highly super efficient in the unrestricted DEA analysis. However, switching to a two-stage DEA model will have positive or negative effects also for inefficient companies. Table 4.1 shows that the two-stage DEA approach results in efficiency score increases of more than 2 %-points for 51 companies. The introduction of (virtual) weight restrictions can only lead to lower efficiency scores, although the calibration causes the large decreases for some companies to be offset by a small increase for the rest of the companies.

Change (%-points)	40 / 20 virtual restrictions	Two-stage DEA
> +2	0	51
-2 to +2	110	42
-10 to -2	10	26
-20 to -10	4	5
< -20	4	4
Sum	128	128

Table 4.1: Changes in calibrated efficiency scores, relative to unrestricted DEA (2007).

¹³ Each geography variable in the regression analysis is given by its index value, i.e., a number between 0 and 1, multiplied by the share of overhead high voltage lines for the respective companies. Cf. the reformulation described in Section 4.2 of Report 1.

¹⁴ The adjustment in Ray (1991) consists of adding the largest residual to the intercept term in the regression model, such that all the (adjusted) residuals become non-positive.

¹⁵ We have used the same calibration method that NVE applied when computing the revenue caps for 2008 and 2009, i.e., we have added a constant to all the efficiency scores. See Bjørndal & Bjørndal (2006b) and Bjørndal et al. (2008b) for a discussion of the various calibration methods that have been used.

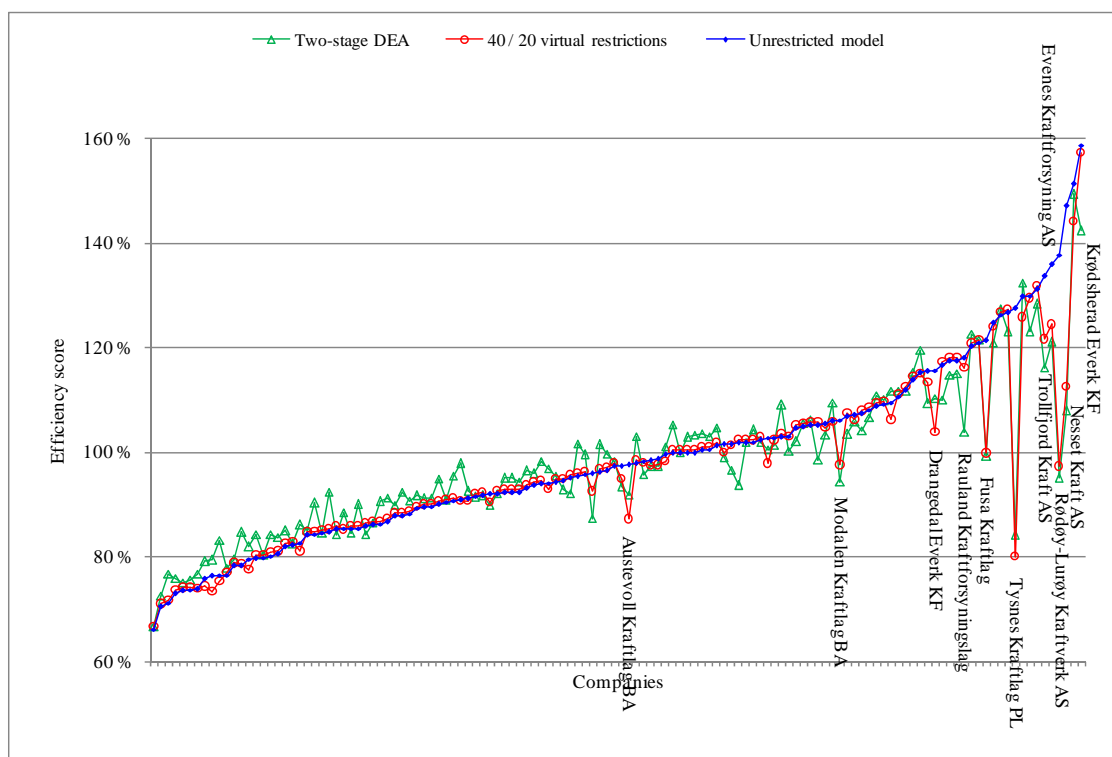


Figure 4.1: Virtual weight restrictions versus two-stage DEA (2007).

The next two tables focus on the companies for which the efficiency reductions are particularly large. Table 4.2 shows the companies with the largest efficiency score reductions if the virtual restrictions were adopted, and Table 4.3 shows similar results for the two-stage procedure. We see that the methods produce similar results relative to the unrestricted efficiency scores, i.e., they agree to a large extent on which companies should have the largest reductions. However, there are some exceptions, e.g. Drangedal and Austevoll in Table 4.2, and Krødsherad and Rauland in Table 4.3. These exceptions should be studied further, in order to find out why the two methods yield different results.

Company	Unrestricted efficiency score	Increase / decrease	
		40 / 20 restriction	2-stage DEA
Tysnes Kraftlag PL	127.6 %	-47.5	-43.4
Rødøy-Lurøy Kraftverk AS	137.7 %	-40.5	-42.7
Neset Kraft AS	147.1 %	-34.5	-39.2
Fusa Kraftlag	121.4 %	-21.6	-22.2
Trollfjord Kraft AS	133.7 %	-12.1	-17.5
Drangedal Everk KF	115.6 %	-11.9	-5.4
Evenes Kraftforsyning AS	135.9 %	-11.5	-14.8
Austevoll Kraftlag BA	97.5 %	-10.3	-5.7

Table 4.2: Companies most affected (< -10) by introduction of 40 / 20 virtual restrictions (2007).

Company	Unrestricted efficiency score	Increase / decrease	
		40 / 20 restriction	2-stage DEA
Tysnes Kraftlag PL	127.6 %	-47.5	-43.4
Rødøy-Lurøy Kraftverk AS	137.7 %	-40.5	-42.7
Neset Kraft AS	147.1 %	-34.5	-39.2
Fusa Kraftlag	121.4 %	-21.6	-22.2
Trollfjord Kraft AS	133.7 %	-12.1	-17.5
Krødsherad Everk KF	158.6 %	-1.4	-16.3
Evenes Kraftforsyning AS	135.9 %	-11.5	-14.8
Rauland Kraftforsyningslag	118.0 %	-1.9	-14.3
Modalen Kraftlag BA	106.1 %	-8.6	-11.9

Table 4.3: Companies most affected (< -10) by transition to two-stage DEA (2007).

5 NVE methods for super efficiency adjustment and calibration

Before computing the revenue caps, NVE makes the following adjustments of the efficiency results:

- 1) The quality costs (VOLL) in the DEA analysis are represented by average reported costs over the previous years. The efficiency results are adjusted for the difference between actual VOLL (in year $t - 2$) and average VOLL.
- 2) In order to prevent unreasonably high efficiency scores as a result of outliers in the dataset, super efficient companies are re-evaluated against a dataset from the year(s) preceding the year of the current dataset. The DEA model in the second

step includes data for the company itself, hence a company can only appear as super efficient if it has improved its performance relative to the previous year(s)¹⁶.

- 3) The efficiency scores are calibrated such that the cost weighted average becomes equal to 100 %. For 2008 and 2009, this calibration has been done by adding a constant to the efficiency score of each company¹⁷. The constant is equal to the difference between 100 % and the cost weighted average of the uncalibrated efficiency scores.

Adjustment 1) has very little effect for most companies, and we will therefore not consider it here, i.e., all the efficiency results presented below are unadjusted with respect to 1). Since weight restrictions are seen as an alternative method for limiting the super efficiency scores, it is interesting to compare the efficiency scores under weight restrictions to the adjusted super efficiencies computed by NVE. In order to make the two sets of efficiency scores comparable, we calibrate them using the method described in 3) above.

Figure 5.1 and Table 5.1 illustrate the difference between the NVE adjusted efficiency scores and the efficiency scores under the virtual weight restrictions. We see that the effect of going from the NVE adjusted efficiency results to weight restricted DEA can be positive or negative. There are two companies, Krødsherad and Nord-Salten, that are highly super efficient in the unrestricted model, and for which the weight restrictions have little or no effect. If the NVE super efficiency adjustment, as explained in 2) above, was replaced by virtual weight restrictions, these two companies would experience an increase in the (calibrated) efficiency score of 32 and 30 percentage points, respectively. In order to understand why these companies are left with such high efficiency scores even after the weight restrictions are imposed, we may look at the virtual weights of the various outputs, as shown in Figure 5.2. We see that Krødsherad weights cottage customers and forest heavily, while Nord-Salten puts the majority of its weight on HV lines and the coast parameter. However, it is not obvious that the weights of the two

¹⁶ For 2007, the initial DEA analyses were based on data from 2005, and the re-evaluation in the second step was against a dataset from 2004. For 2008 and 2009, the re-evaluation dataset was created by taking averages of the datasets for the periods 2004-2005 and 2004-2006, respectively.

¹⁷ For 2007 the calibration was done by dividing the individual efficiency scores by the uncalibrated cost weighted average. NVE (2006a) also describes a third calibration method, whereby the revenue shortfall of the industry is distributed among the companies in proportion their capital book values. This calibration method is used after the revenue caps have been calculated, in order to make the total industry revenue equal to the total industry costs. For 2007-2009 this calibration effect has been negative, and it has in fact served to withdraw the so-called compensation parameter. See Bjørndal et al. (2008b) for a discussion of the different calibration alternatives.

companies are unreasonable; hence we cannot conclude that the high efficiency scores are not representative of their true performance.

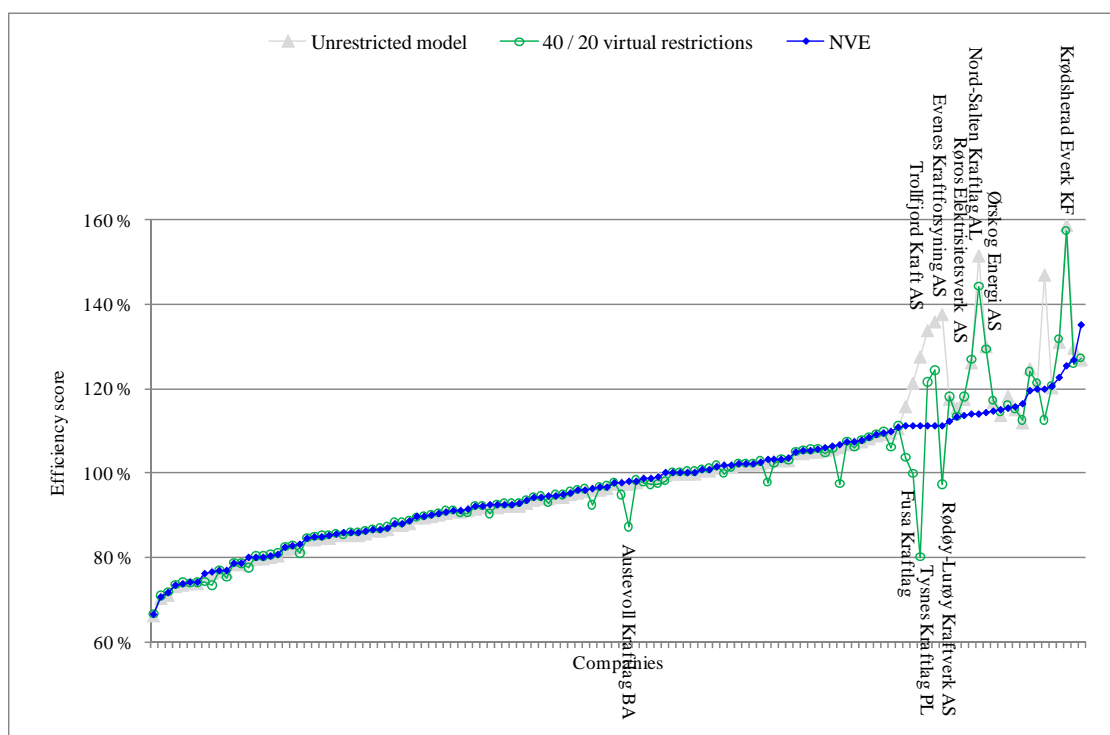


Figure 5.1: NVE adjusted efficiency versus efficiency under virtual weight restrictions (2007).

Company	NVE adjusted efficiency	DEA with 40 / 20 virtual restrictions	Increase / decrease
Krødsherad Everk KF	125 %	157 %	32
Nord-Salten Kraftlag AL	114 %	144 %	30
Ørskog Energi AS	114 %	129 %	15
Evenes Kraftforsyning AS	111 %	124 %	13
Røros Elektrisitetsverk AS	114 %	127 %	13
Trollfjord Kraft AS	111 %	122 %	10
Austevoll Kraftlag BA	98 %	87 %	-11
Fusa Kraftlag	111 %	100 %	-12
Rødøy-Lurøy Kraftverk AS	111 %	97 %	-14
Tysnes Kraftlag PL	111 %	80 %	-31

Table 5.1: NVE adjusted efficiency versus efficiency under virtual weight restrictions (2007).

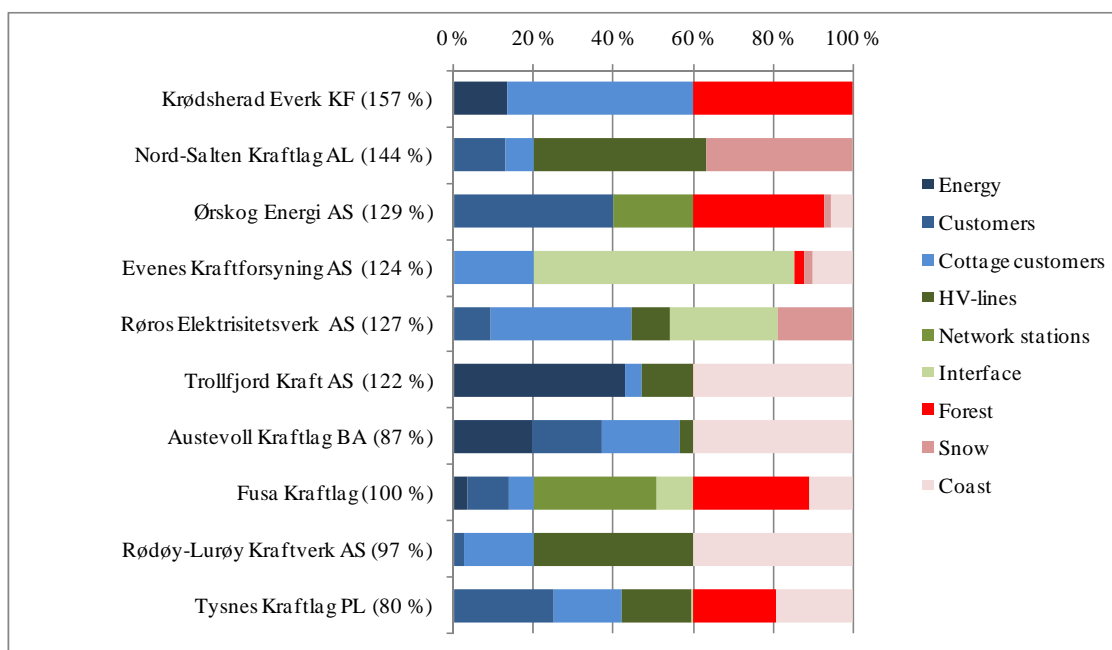


Figure 5.2: Virtual weights after the introduction of 40 / 20 weight restrictions (2007).

Figure 5.3 and Table 5.2 compare the results from the two-stage DEA procedure to the NVE adjusted results. We see that the two-stage DEA model yields efficiency results that are less similar to the NVE scheme than the weight restricted efficiency scores illustrated in Figure 5.1, since the former method causes changes for a large number of companies, while the latter method mostly affect the super efficient companies. For the super efficient companies the results from the two alternative methods are somewhat similar, as we noted in Section 4. Again Krødsherad and Nord-Salten stand out as the companies that get the largest increases in their efficiency scores, relative to the NVE efficiency scores.

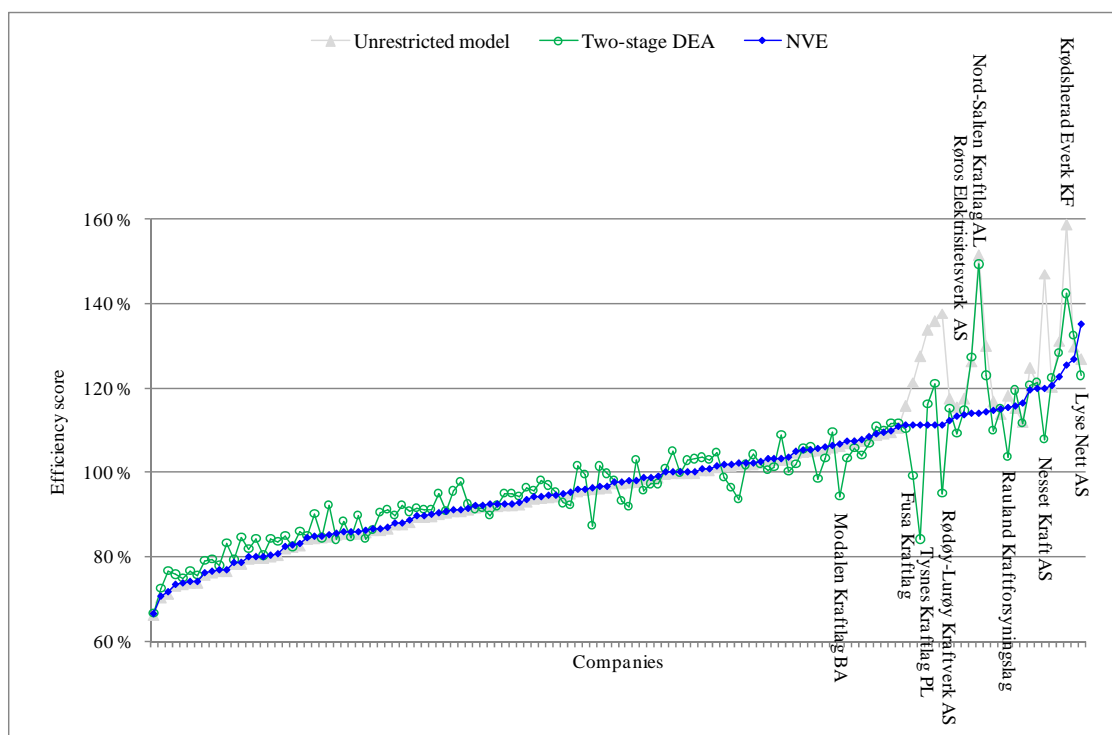


Figure 5.3: NVE adjusted efficiency versus two-stage DEA (2007).

Company	NVE adjusted efficiency	Two-stage DEA	Increase / decrease
Nord-Salten Kraftlag AL	114 %	149 %	35
Krødsherad Everk KF	125 %	142 %	17
Røros Elektrisitetsverk AS	114 %	127 %	13
Evenes Kraftforsyning AS	111 %	121 %	10
Rauland Kraftforsyningslag	115 %	104 %	-12
Neset Kraft AS	120 %	108 %	-12
Fusa Kraftlag	111 %	99 %	-12
Modalen Kraftlag BA	107 %	94 %	-12
Lyse Nett AS	135 %	123 %	-12
Rødøy-Lurøy Kraftverk AS	111 %	95 %	-16
Tysnes Kraftlag PL	111 %	84 %	-27

Table 5.2: NVE adjusted efficiency versus two-stage DEA (2007).

It is not possible to conclude from the above analysis which of the two methods (weight restrictions or two-stage DEA) is preferable. Neither method yields super efficiency scores that are dramatically higher than the NVE method (with perhaps one or two exceptions). An important argument against the NVE method is its questionable incentive properties for efficient companies. Because of the way that NVE limits the super

efficiency scores, these companies will, over time, tend to be their own reference, hence they will have weak incentives for “real” efficiency improvement¹⁸. Neither of the alternative methods, if implemented as in this report, puts any explicit restrictions on super efficiency scores. Thus, it can be argued that they both have better incentive properties than the NVE method.

6 Conclusions and recommendations

We investigate NVE’s suggestions for relative weight restrictions on the structural and size dependent output variables, i.e. VR1-VR8 in NVE (2008). The two-sidedness of the weight restrictions and the large number of them, involving a large share of the output variables, may have unintended effects on the efficiency scores, and the effects on the efficiency scores are in many cases determined almost completely by the exact limits of the relative weight restrictions. The latter implies that the specification of the limits is very important.

Since many of the output factors in question represent “more than themselves”, as discussed in Section 2.3, it may be extremely difficult to settle on meaningful bounds on relative shadow prices. In order to illustrate this point, we also discuss the implicit assumptions about substitutability between output variables that relative weight restrictions represent. These interpretations that follow directly from linear programming theory, can serve as a test of the practical viability of the relative weight restrictions. For some of the variables in question, the substitution assumptions are at least questionable.

Thus, when it comes to the choice between different types of weight restrictions, i.e. absolute, relative or virtual, we recommend, as in Report 1, to pursue virtual weight restrictions on geography variables (maximum weight) and / or energy and customer variables (minimum weight), thereby eliminating the need for the detailed assumptions with respect to individual variables that are required by VR1-VR11. In Section 3.1 we also give another justification for why maximum / minimum bounds of approximately 40 / 20, respectively, may be reasonable limits.

There are, however, other methods that could be used instead of weight restrictions, for instance multistage DEA. As an example we look at a two-stage method in Section 4. Although in the two-stage DEA model, it is not necessary to specify any cost based bounds, there are still many implementation details that will influence the results. We

¹⁸ See Bjørndal & Bjørndal (2006b) for a discussion.

show that the results from two-stage model are similar to those of the 40 / 20 max and min virtual weight restrictions, but that more companies are affected, and that efficiency scores can either increase or decrease.

Finally, in Section 5, we compare the (calibrated) efficiency results from the DEA model with weight restrictions and the two-stage DEA approach, to the NVE method of adjusting the super efficiency scores. If the objective is to avoid very high efficiency scores, it is not obvious which of the three methods should be preferred. It can be argued that the alternative methods (i.e., the weight restricted DEA model or two-stage DEA) have better incentive properties than the NVE method for adjusting super efficiencies, since they do not put any explicit restrictions on the companies' measured super efficiencies.

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