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ECONOMIC PERSPECTIVES OF RISK DISTRIBUTION SYSTEM ASSET MANAGEMENT: PRINCIPLES OF RISK VALUATION OF GRID INVESTMENTS

by

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FOREWORD

A main challenge of the electricity grid company is to target the right level and choice of grid investment, maintenance and renewal. The objective of this report is to apply the insight from economic theories of risk valuation to the valuation of risky grid investments.

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Bergen, December 2010

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SUMMARY

A main challenge of the electricity grid company is to identify and target the right level and choice of different investments, maintenance and renewal actions in the grid. All these actions have implications for the future performance of the grid, and will in this report be termed as investments. The objective of this report is to apply the insight from economic theories of risk valuation to the valuation of risky grid investments. Our focus is on the economic valuation of investments, with a particular focus on the value of risk.

Economic Analysis in Decision Making

The estimated economic value of the investment shows the *profit* of the grid investment, where the investment's risk is priced as similar risk in the market. The grid investment is profitable if the value of investment benefits exceeds the value of the investment costs. If not, the grid investment is economically unprofitable.

The ultimate decision will however in many cases rest upon other criteria than company profits. These criteria may for example be such as safety, environmental, reputational, and quality conditions. These considerations may to some extent be incorporated in a socio-economic based cost-benefit analysis, where the cash flow, and thus also the net profit represents the society's costs and benefits, rather than pure company consequences. A socio-economic analysis or a multi-criteria based decision may indicate that an investment should be carried out. In this case a possible negative company profit, in effect indicates the company cost of pursuing other objectives.

The Nature of Grid Investments

In this report we define grid investments as an action characterized by initial cash expenditures, where the benefits of the investment are reaped in future years in terms of expected improvements in network performance. Thus our discussion on valuation methods is relevant for actions normally termed for example investments, maintenance, renewal, reinvestments, etc.

The network performance/reliability of a given grid may be described as a probability distribution of possible consequences and their probability of occurrence. Grid investments are assumed to improve network performance. Basically, they will either reduce the probability of failure, and/or reduce the consequences if failure occurs. The benefit of the investment thus follows from the *change* (improvement) in performance from the status quo scenario, to the after investment scenario.

Identifying Grid Investment Alternatives

A prerequisite for a good investment decision is that the best alternatives have been identified. The current report assumes that relevant investment alternatives are identified. The RISK DSAM methodology, see Nybø and Nordgård (2010), provides guidelines to develop risk-based maintenance and investment strategies, thus helping to identify grid investment alternatives. Important alternatives also include postponement and further information gathering.

Basic Input of Investment Analysis: The Investment Cash Flow

The economic investment analysis is based on the probability distribution of the future cash flow of the investment. In this, possible investment outcomes are valued in monetary terms. Using the CAPM-based valuation approach, the probability distribution may be summarized as the expected cash flow at future points of time. These values are often estimated based upon surveys, see e.g. Kjølle et al. (2008). If the cash flow reflects socio-economic consequences, the investment analysis will show the socio-economic value of the investment. If the cash flow reflects grid company consequences, the investment analysis will show the company value of the investment.

The Time Value of Money

Consequences of investments follow in future time periods. The same amounts of money received at different periods of time, do not have the same value. This difference in value is attributed to the cost of capital. In the case of certainty, the present value of a cash amount is found by discounting the future amount by the capital cost. The present value represents the current value of this amount. Also under uncertainty the time value of money has to be accounted for. A further problem here is to account for the value of risk, for example by adjusting the required return.

Expected Values ≠ Market Value Investment?

The expected value of a random (uncertain) variable is the sum of the probability-weighted outcomes. The expected value of the investment will not necessarily represent the true value of the investment. The reason for this is the value of risk. Many investors are risk averse. In general the market value of a risky asset often is lower than the expected value. The difference reflects the market valuation of risk.

Financial asset pricing theories provides theories on how risky assets, such as an investment, are valued. While the theories are fundamentally consistent, they reflect differences as to the use of market data, and in underlying assumptions, thus with different implications as to whether the theories can be adequately implemented.

Arrow-Debreu Pricing Theory

The classic Arrow-Debreu pricing theory is said to be the father of all asset pricing theories. It offers invaluable insight to understanding the value of risk, but is rather abstract and difficult to implement. The model explicitly prices state-contingent claims for each possible future scenario, showing the essences of the value of risk: Basically, the value of risk is related not only to the probability of occurrence, but also to the relative condition of scarcity in future scenarios. The theory cannot be directly implemented in valuing grid investments, but does remind us that the outcomes of a grid investment, such as avoided failure, potentially can be more valuable in some scenarios rather than in others.

Arbitrage Pricing

Arbitrage pricing approaches are basically based on the market prices of traded assets. In its simplest form, an arbitrage approach attempts to value a cash flow on the basis of the prices of components that make up the cash flow. For example, if the cash flow of e.g. a derivative can be duplicated by a portfolio of the underlying assets, the cash flow value should equal the portfolio value.

For other arbitrage pricing theories, such as the risk neutral valuation theory, risk valuation is deduced in more general terms. Based on the price processes of traded assets, the risk-neutral probability measure is deduced. This probability measure accounts for the value of risk. Cash flow expectations based on this probability measure can then be discounted by the risk free interest rate. This theory is applied in financial models of pricing stocks and derivatives, and is of particular interest as it is free of structural model assumptions.

For risk valuation in electricity markets in general, for example in evaluating derivatives or generation investments where the future electricity price is important, the method in many respects seems to be promising. For grid investments, the asset to be valued is somewhat different, as the focus is on the value of future avoided failure. It is thus not clear that an appropriate probability measure can be deduced from traded assets in the case of grid investments. The method may, however, shed light on special aspects of grid investments, but will probably not suffice as a sole method for the valuation of grid investments.

Equilibrium Pricing Models: The Capital Asset Pricing Model (CAPM)

Using the CAPM approach, the expected value of the cash flow at each point of time is discounted at a risk adjusted rate. This rate reflects the required market compensation for the risk of the grid investment. The risk adjusted required return may be represented as

$$r_i = r_f + \beta_i (r_M - r_f)\text{, where } \beta_i = \frac{\sigma_{iM}}{\sigma_M^2}.$$

Thus, the required return consists of the risk free interest rate r_f with the addition of a part β_i of the market premium $(r_M - r_f)$. The $(r_M - r_f)$ represents the market price of risk, while the β_i defines the risk contribution of the investment as defined by the market.

Within the CAPM framework, the risk of the investment is represented by its mean return and its variance. A central issue is that the relevant risk of an asset is its risk contribution with respect to a larger portfolio of assets; the market portfolio. Unless the assets of a portfolio are perfectly correlated, the variance of the portfolio will be smaller than the weighted average of the variances of the individual asset returns.

If a grid investment is imperfectly correlated with the existing activities of the company, the net risk contribution of the investment project to the company may therefore be less than the risk perceived in assessing the asset alone.

For a grid investment, the relevant risk to be compensated in terms of a higher required return is, however, neither the gross risk of the investment, nor the specific net contribution to the risk of the company. The relevant risk of the grid investment is the part of risk which is relevant for pricing risk in the market.

The relevant risk is more specifically the non-diversifiable (systematic) risk of the investment. This is the is represented by the beta of the project, i.e. $\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$, which in essence mirrors the extent to which the grid investment returns covary with the market return.

Studies on the β of grid companies indicate that the electric grid business is an activity of low systematic risk. In other words, the covariance between grid company returns and the market portfolio has been found to be low. This is partly attributed to the effect of regulation, which to some extent links the average grid company costs and revenue.

In addressing the specific grid investment, our interest is in the risk of the project itself, i.e. the project β. The question is thus to what extent grid investment returns are correlated with market portfolio returns. A qualified guess is that distribution grid investment outcomes, such as the avoidance of grid failure, may have a low correlation with the market portfolio, also indicating a low project beta. As such, the general grid company beta may represent a good starting point. Grid investments that indicate less (higher) systematic risk, then would call for a lower (higher) beta.

Conclusions

Grid investments are risky investments which pose several challenges to the grid company. Basic challenges are related to the identification of appropriate investment alternatives, cash flow construction which represents the main input to the economic investment analysis, and lastly the valuation of the risky cash flow.

Different asset pricing theories convey different insights as to the value of risky assets, and of grid investments in particular. In practice, each method, however, has its shortcomings. For practical implementation in grid investment analysis, normally the CAPM-based valuation method is used. The expected cash flow at each point of time is then discounted by a risk-adjusted return, which reflects the systematic risk of the grid investment. The net value of benefits and costs discounted at this value represent the profitability of the investment. The ultimate investment decision may, however, be based on further criteria.

1 Introduction

A main challenge of the electricity grid company in its Distribution System Asset Management (DSAM) is to identify and target the right level and choice of different (re)investments and maintenance/renewal actions to be performed in the grid in the short-, as well as in a medium- and long perspective. Available resources for investments are limited. This calls for ranking of grid investment alternatives based on an evaluation of the benefits and costs of alternative strategies.

The decision problem of the grid company thus implies a comparison of possible actions and prioritizing of relevant alternatives according to the objectives of the company. In general the company objectives comprise multiple, and even diverging considerations, covering e.g. economic, environmental, quality, safety, and reputational related standards and targets. All these aspects must be given due consideration in decision making. This report focuses on valuation of *economic* aspects of grid investments, with a particular focus on *risk.*

Different strategies for investment and maintenance affect the risk exposure of the distribution system and the company. Relevant risk exposure may comprise financial, safety, environmental as well as political risks. The ranking of different grid investment and maintenance strategies may be characterized as a choice between different alternatives with different risk implications. This makes the correct valuation of risk essential. Our focus is on the *valuation of the economic risk of grid investments*. Different investment alternatives may be viewed as different probability distributions of future costs and benefits. The objective of this report is to apply the insight from the main economic theories and techniques of risk valuation, to the problems encountered in the valuation of different grid investment strategies.

The report is balanced as follows: Chapter 2 discusses the concept of grid investments. Our definition of grid investments comprises all grid expenditures that increase the future quality of the grid network. In addition to the obvious investments of building new networks, we argue that many grid investments are often related to the improvement of grid quality in existing grids. Benefits of improved grid quality may for example be lower probabilities of grid failure, as well as less severe consequences of failure. This raises important questions related to the value of improved grid quality, as for example the valuation of reduced probabilities of failure.

The main inputs to economic analysis are the estimated cash flows of investments. In the main body of this report we will assume that the relevant cash flows are given, and focus on the evaluation of risk. The preparation of this underlying data for grid investments is, however, not straightforward. Chapter 3 briefly discusses issues of establishing the underlying cash flow of grid investments, covering e.g. aspects of quantifying the consequences of grid failure, and the importance of the viewpoint taken when quantifying consequences.

The remaining chapters of the report are dedicated to the valuation of risky grid investments. Chapter 4 gives a first introduction to the issue of the time value of money, and discusses the interpretation of net present values under certainty and uncertainty. The value of an investment may in general deviate from its expected value. Based on this, chapter 5 motivates the valuation of risk, looking into the underlying assumptions in financial theory as to rational behavior of decisions under uncertainty. The chapter concludes by giving an overview of different representations of risk valuation, and the main categories of underlying financial methods. Chapter 6 on arbitrage-based pricing models discusses arbitrage-based pricing in general, and the theory of Arrow-Debreu pricing and Risk-neutral evaluation in particular. In many respects these theories bring important insight to understanding the value of risk in grid investments. However, at the current state of research, we do not think that direct implementation of these theories for grid investment is feasible as the sole source of risk valuation. Chapter 7 turns to equilibrium-based pricing models, and in particular the theory of CAPM (Capital Asset Pricing Model), which currently is the most applicable model for valuing risky investments. The chapter discusses the aspect of diversifiable and non-diversifiable risk, and its implication for pricing grid investments. Chapter 8 concludes the report with a discussion on the implementation of the CAPM for valuing grid investments. Firstly, we review reports considering the beta of grid investment companies, and the recommended risk adjusted discount rate for grid companies. Secondly, we discuss the connection between grid condition analyses on one hand, and investment-relevant data input on the other hand.

2 Grid Investments

The grid company may undertake several actions to improve the quality and/or capacity of the distribution grid for future years. Several terms are used for such actions, e.g. investments, reinvestments, maintenance, renewal, etc. In this report we will refer to all such actions as *grid investments*, as further defined in section 2.1. For the purpose of evaluating grid investments, a basic first step is to understand the nature of benefits the grid investment characterized in terms of uncertainty. This is the topic in section 2.2.

2.1 Definition of Grid Investments

Let us first clarify our interpretation of the term *grid investments*, thus defining what part of the *grid* we focus on, as well as the meaning of the term *investments.*

The Grid

The main focus of the project, *Risk-Based Distribution System Asset Management,* is on risk exposure in the distribution sector. Our focus in this report is thus on investments in the *local distribution system.* This is the electric grid system administered by the electricity grid company in the local distribution system. It is defined partly by the area it covers, and partly by the voltage level of the grid covering lower-voltage lines (though pure voltage-based definitions are in general not sufficient).

As pointed out by Sand, Gjerde, and Nordgård (2007), it should also be noted that the local distribution system is an integrated part of the overall transmission and distribution system. Actions undertaken in other parts of the system may thus affect the value of potential investments the distribution system, (and vice versa).

They distinguish between the above main system level, the distribution level, and the level below, i.e. the loads, installations and sub-distribution systems.

- The **main system:** This represents the system level above the local distribution grid. Shortly put the main system feeds the local distribution system. The current condition as well as future plans for the main grid may be relevant information both in identifying alternative investment actions in the local distribution system, and may affect the consequences of investments in the local distribution system.

- The **local distribution system:** This is the distribution system of the local electricity grid company, which thus represents the planning area where actions such as maintenance, reinvestments, etc., are considered.

- The **loads, installations and sub-distribution systems:** This level represents the customers of the local distribution system. Investments in the local distribution system may have implications for the quantity (load levels, and type of load) and quality (e.g. probability distribution of interruptions, frequency, voltage) on this level.

Investments:

The term investment normally refers to the act of investing money in order to gain future profitable returns. There is normally an initial capital expenditure, while the benefits of this expenditure follow in future time. For example, a typical investment in other industries may be e.g. an investment in production capacity, where the future benefits are related to the profit of future sales. Another example is an investment in financial assets, where the future benefits are related to future interest, dividends or capital gains.

The local distribution system, hereafter termed *the grid*, consists of a large portfolio of different components, as for example lines, transformation stations, etc. Actions of installing entirely new components in the distribution system, the replacement of existing components, as well as upgrading existing components in the local distribution system, all will affect the performance of the grid in the mid- to long-term future. In practice, several of these actions are distinguished by different names, such as 'reinvestment', 'investment', 'maintenance', and 'renewal'. This differentiation may follow e.g. from different strategies for timing expenditure, custom, or accounting practices. All these actions, however, share the common features of the general term investments, that is, with an initial cash expenditure, and where the benefits of the expenditure follow in the years to come. The benefits of investments, reinvestments, maintenance, as well as renewals, are all related to expected improvements in network performance. All involve an initial expenditure and improve the quality of the grid in future years. With limited resources at its disposal, the grid company has to prioritize and weigh all these actions against one-another, choosing the actions which to the greatest extent fulfill the objectives of the grid company. Our focus in this report is on evaluation of risk, and in this respect all these different actions are treated equally[1]. We will therefore, as a common denominator term such actions with a common terminology as *grid investments*. To summarize, we thus define grid investments as follows:

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[1] In principle, as the pool of company resources is limited, these resources should preferably be channeled to different uses based on comparisons of the resulting value, regardless of accounting terms. A further implication is that if accounting and budget procedures and definitions actually set restrictions on the allocation of actions, the company should follow up as to whether these restrictions are rooted in real-economic values and consequences or not. This is however not a topic of this report.

> *A grid investment is an action characterized by initial cash expenditures, where the benefits are reaped in future years in terms of expected improvements in network performance.*

2.2 The Product of a Grid Investment

In evaluating an investment, the value of benefits is compared to investment costs. While investment costs to a large extent are clearly defined and valued, the benefits of a grid investment may warrant a further explanation and definition. Common benefits of grid investments are normally related to improvements in the existing grid or the construction of completely new lines. For example, possible products/benefits of the grid investment may be related to:

- *Changes in the reliability of grid delivery:* The reliability of delivery may be characterized as a probability distribution of interruptions. Note that this not only refers to the probability and expected frequency of when interruptions occur, but also to the distribution of different types of interruptions that may occur, e.g. the duration of interruptions, the scale of interruptions, the timing of interruptions (e.g. day versus night, winter versus summer, etc.). Improvements in grid reliability may in principle be lower probabilities for interruptions, and/or a shift towards interruptions with less serious consequences.

- *Other issues of improved performance:* Grid investments in an existing grid may also affect other quality aspects of the network, as for example voltage stability.

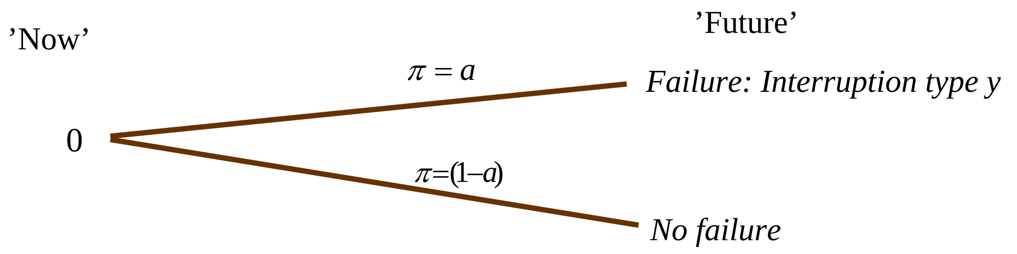
- *Expansion of capacity for distribution of electricity:* This is the ability to transfer a larger amount of electricity, and thus includes supply to customers that previously have not received electricity. Note that new capacity also may be characterized by the above quality aspects of grid performance.

Basically, the benefits of grid investments are related to *changes* in the future performance of the grid. Due to the uncertainty of future performance, grid investments are risky investments. More specifically, the benefits of grid investments may be visualized as a probability distribution of the possible benefits. A probability distribution may be characterized by the possible events/outcomes and their probability of occurrence. We will thus distinguish between two main effects: Possible benefits of grid investments on one hand may imply *improvements in possible outcomes*. This may for example be a shift towards the occurrence of grid failures with less serious consequences than before. On the other hand, given the same scenario of possible incomes, grid investments may *improve the probability distribution*, for example reducing the probability of severe outcomes. In this setting alternative investment strategies may essentially be regarded as the choice between different future probability distributions with different outcomes.

To illustrate these issues of the basic nature of the grid investment product, let us consider an extremely simplified network scenario where there are two time periods; 'now' and the 'future'.

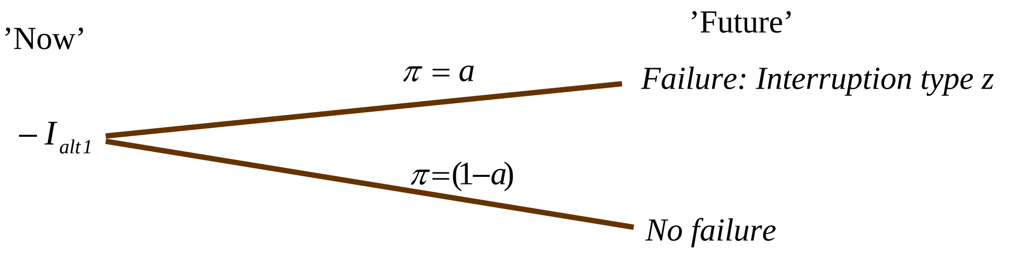
The current status *Status Quo* without any investment is illustrated in Figure 1. At the current time of 'Now', there are no investment costs in the Status Quo setting. In our simple illustration, the future situation is characterized by a binominal probability distribution with two possible outcomes; failure or no failure. If failure occurs, the outcome is an interruption of type y. Let π be the probability of the outcome in question. There is a probability $\pi = a \quad (0 \leq a \leq 1)$ in which interruption of type *y* occurs. The probability that the interruption does not occur is then $\pi = (1 - a)$.

Figure 1 Scenario: Status Quo



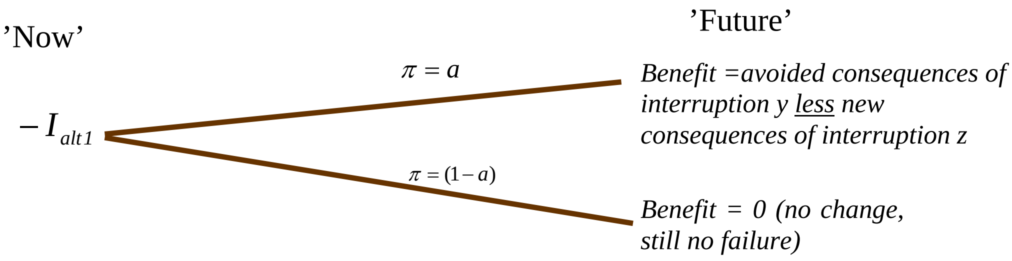
Now consider *investment alternative 1* which mainly *reduces the consequences of failure*. The investment cost of this alternative is $-I_{alt1}$, paid at the current time 'Now'. After having invested, the probability of interruption will still be the same as in Status Quo. However, if failure occurs, any failure will now result in an interruption of type *z*, which may be categorized as a less severe interruption, for example less severe in terms of damage, inconvenience, repairs, etc. Figure 2a shows the failure scenarios of investment *1*.

Figure 2a Scenario: Investment Alternative 1



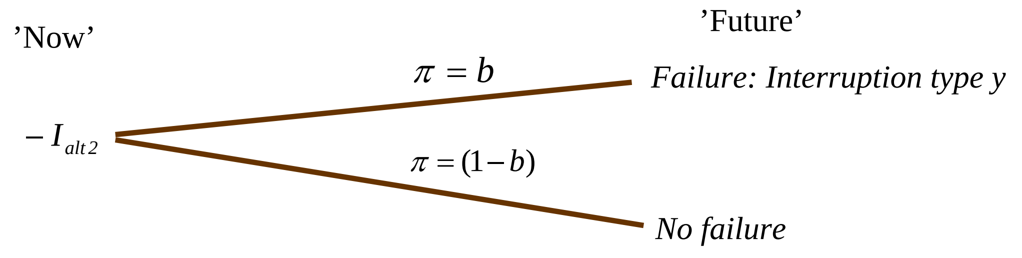
To evaluate the investment, however, our focus must be on the *change* brought about by the investment. The consequences of the investment thus follow from the *differences* between the Status Quo scenario and the investment 1 scenario. This is illustrated in figure 2b: At the current time 'Now' the investment cost $-I_{alt1}$ occurs, while the benefits are derived by the advantage of the occurrence of a less serious interruption, given the event that interruption occurs. It is the *value* of these benefits that must be weighed against investment costs.

Figure 2b Net Consequences of Investment Alternative 1



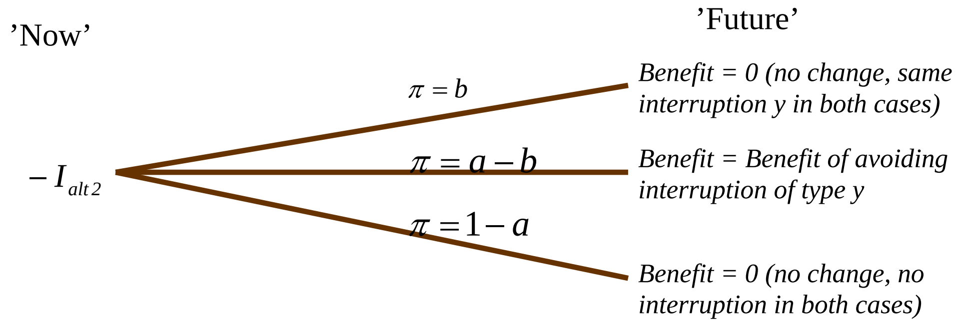
Investment alternative 2 illustrates an investment that mainly *reduces the probability of failure*. At the current time 'Now' the investment cost of this alternative is $-I_{alt2}$. The benefit of this investment follows from a change in probability distribution: If grid failure occurs, the severity of the interruption is still as in the base case of Status Quo, i.e. an interruption of type *y*. Due to the investment, however, there is a reduced probability of interruption, i.e. now with the probability $\pi = b$ $(0 \leq b \leq 1)$ that an interruption of type *y* occurs, where *b*<*a*. The benefit of the investment thus lies in the achievement of a more favorable probability distribution for grid failure. Figure 3a shows the new scenario following from the implementation of investment 2.

Figure 3 Scenario: Investment 2



The net consequences of the investment, which thus constitute the basis for evaluation, will then be the *differences* between the Status Quo scenario and the investment *ii* scenario, as illustrated in figure 3b. The cost is the given investment cost. The benefits of the investment follow from the fact that there now is a *(a-b)* lower probability of failure, i.e. of avoiding an interruption of type *y*. It is the *value* of these possible benefits that must be weighed against investment costs.

Figure 3b Net Consequences of Investment



Though the examples are in a stylistic setting, they illustrate basic features related to the product of grid investments in existing grids; benefits in the terms *more favorable consequences* of failure on one hand, and benefits in terms of an *improved* *probability distribution* on the other hand. Effects of real-case grid investments will display a considerably more complex pattern, however, it is reasonable to believe that the products of the grid investments most likely will be a combination of these basic features. To sum up:

> *The product or benefits of grid investments in existing grids may to a large extent be interpreted as a combination of lower probabilities of grid failure, together with a possible shift towards less serious types of consequences of grid failure. The net consequences of investment may conceptually be envisioned as a probability distribution of net benefits at different future points of time.*

Whether the project should be carried out or not, depends upon whether the decision maker values the benefits of the project higher than the associated investment costs. In this project our focus is on issues of estimating the economic *profitability* of grid investments, with a particular focus on risk evaluation. The above discussion indicates that due to the uncertain nature of the benefits, correct valuation of risk may be important in the valuation of grid improvement projects. For example, we face the challenge of finding the value of reduced probabilities of interruption, or the value of reducing potential (i.e. not necessarily realized) consequences of potential grid failure. Our main focus will be on theory and methods of risk evaluation. A first step for the analyst is, however, to prepare the underlying data for analysis of the grid investment project. Important inputs in the profit evaluation of investments are the estimated *cash flows* of the projects. Before proceeding to the specifics of risk evaluation in chapters 4-8, the following chapter briefly discusses special challenges of establishing cash flows for grid investment projects.

3 Establishing Cash Flows for Grid Investments

In this report our focus is on the techniques of risk evaluation applied to grid investments. We will therefore in the following chapters assume that cash flow estimates are given. Though outside the scope of the report, cash flow estimation is nevertheless a crucial part of the grid investment decision process. This chapter will briefly comment upon a few issues of establishing the cash flow. Section 3.1 first distinguishes between the task of risk evaluation and that of estimating the value (cost) of realized outcomes/incidents. The first aspect, risk evaluation, is a main topic of this report. The second aspect, which involves the estimation of the cash flow and valuation of possible outcomes, is not a topic of the report. Section 3.2 briefly looks into some of the challenges of incident valuation. Section 3.3 discusses issues as to the viewpoint taken by the grid investor in estimating the cash flow. Section 3.4 concludes this section by discussing the role of the benchmark scenario, and stressing that the relevant cash flow is the cash flow which states the change brought about by the investment.

3.1 About Risk Valuation and Estimated Values of Outcomes

The main products of grid investments often relate to improved grid performance, either by lower probabilities of failure, and/or by a shift towards less serious types of consequences. As to the value and profitability of investments, we face several challenges. For example, in terms of investment alternative 1, the main effect of the investment is that more favorable consequences occur in the event of an interruption (interruption type z is less costly than interruption type y). The question here is what is the value of such a shift in consequences, given that there is probability of $a<1$ that this interruption occurs. In terms of alternative 2, we face the question of what is the value of reducing the probability from $a<1$ to $b<a$ for an

interruption of type y. The question in this case is what is the value of reducing the probability of failure.

Valuing future improved grid performance, in terms of more favorable outcomes, and/or reduced probabilities of failure poses two basic challenges in evaluation:

> **i) Value of realized outcome:** The value of future grid performance is directly associated with the value associated with the realization of outcomes that may occur. Basically these outcomes may be described by their physical consequences, such as interruptions of various durations, voltage dips, etc. To establish a cash flow, i.e. the consequences in *monetary* terms, a value has to be attached to these outcomes given that they occur. In the case of grid investments, outcomes of failure are normally associated with a negative value, i.e. the cost of damage, inconvenience and repair costs that arises if the failure were to occur. The cash flow of an investment however should represent the *net change* in cash flow due to the investment. As we shall discuss in the following chapter, a main challenge for grid investments is to assess the corresponding cash flow. Unlike investments in the majority of commercial investments, there is not a clearly defined cash flow associated with these incidents, and this poses a special challenge in grid investment analysis.
>
> **ii) Value of uncertain outcomes:** Given a probability distribution of the possible monetary-valued incidents that may or may not occur, the second challenge is related to the valuation of risk. For example, what is the value of achieving a lower probability of grid failure? Several questions arise in this evaluation, for example: Can the value of improved grid performance be estimated by the expected value of grid improvement? And, if not, how should risk be evaluated? Here there

are a large range of questions arise as to e.g. relevant versus irrelevant risk, the market value of risk, co-variation and portfolio considerations, etc.

As for the first issue, i.e. establishing the *value of realized outcomes,* we will basically in this report take their probability distribution and the economic value of possible realized outcomes as given as well. The main focus in valuing grid investments will be on the second issue, the *value of uncertain outcomes,* where we focus on methods for the evaluation of risk. Before proceeding to this issue, let us briefly comment upon the first step, i.e. the estimation of the economic value of realized outcomes and the source of such information.

3.2 Establishing Grid Investment Cash Flows: Value of Outcome

The basic point of the first question above is about what is the value attached to given outcomes of grid failure (e.g. an interruption, a voltage disturbances, etc.) The answer to this question on one hand depends upon the viewpoint taken, i.e. whether we consider the potential damage and costs directly inflicted upon the decision making entity, i.e. the grid company, or whether our focus is on the aggregate loss suffered by users and other entities affected by the investment. In relation to the grid investment decision, the correct level depends upon the objective of the decision maker. This aspect is discussed in section 3.3.

Before approaching this discussion, we will in this section very briefly comment upon the general problem of attaching values to potential incidents of grid failure. For our purpose of estimating grid quality under different investment scenarios, the view taken is broad in that we focus on aggregate costs of grid failure for all entities affected. These entities may, on one hand, be direct users of the network, i.e. consumers, producers of electricity, as well as the network company itself. On the other hand, other parties may also be affected indirectly due to extended effects following from consequences inflicted upon the direct users.

For ordinary commodities, the market normally brings forth and visualizes the value or price of the products of the investment. The benefits of grid investments will, however, often be related to improved grid performance in terms of avoided grid failure. In cash terms this can be stated as the *avoided costs* of grid failure. With the exception of possible insurance contracts or risk-sharing contracts, the grid investment product of avoided grid failure is *non-marketable*, and the price, i.e. the value/cost of grid failure has to be *estimated*.

The quality of the grid investment analysis is thus highly dependent upon the quality of the underlying data and cash flow estimates. A basic challenge in preparing the cash flows of grid investment alternatives is to estimate the cost of grid failure, on an individual basis, as well as on an aggregate level. For the costs of interruption and other grid failures, this is normally done by means of direct surveys using e.g. estimation methods based on direct worth methods and willingness-to-pay, as well as various means of indirect estimation of the cost of grid failures. In this area there exists a large amount of literature, and surveys. As an example see Kjølle et al. (2008) for an overview of methods and main results in the 2001-2003 Norwegian survey. As such, these kinds of surveys may provide important information in estimating consequences of grid investments that are intended to provide better future grid performance.

3.3 Cash Flow Viewpoint: Company Value vs. Socio-Economic Value

In principle an investment is profitable, and should be carried out if the value of the benefits of the investment exceeds the investment cost. It is the grid company that in principle makes the decision whether to invest or not. This implies that the grid company assesses and weighs the benefits and costs as perceived by the company in relation to its objectives.

The electricity grid constitutes an important part of the infrastructure of modern society. The grid company is therefore subject to regulation by the regulatory authorities, which thus constitutes an important framework for the company. As such the objectives of the company, as well as company income and costs, are also affected by the regulatory regime. In principle, however, note that the grid economic company value of the investment may deviate from the socio-economic value of the investment:

- The *socio-economic value* of the investment follows from the *aggregate* '*real-value*' benefits and costs of the investment, i.e. extra investment cost and the increased benefits in use of the grid including reduced actual costs of grid failure. The socio-economic value in short comprises the real benefits and costs that in aggregate accrue to *all* parties affected by the investment: On one hand there are parties that are directly affected by the grid investment, such as electricity producers and consumers (e.g. reduced inconveniences of grid failure) and the grid company itself (e.g. lower maintenance and repair costs). On the other hand, there are secondary effects related to the general effects on society of a more stable electricity supply.

- The *grid company economic value* first and foremost follows from how the grid investment affects the profitability of the company. In cash terms grid investments may imply e.g. future reduced costs of maintenance and repairs, future reduced costs of compensations to customers, as well as changes in company income. Note that several of these terms may be affected by the regulatory regime.

The choice of perspective has important implications for cash flow construction and the identification and evaluation of outcomes in scenarios with and without investment. The question here is whether the cash flow should represent the broad socio-economic perspective or the more narrow company perspective. It is, as such, highly probable that the grid company economic value of investments may differ from the socio-economic value. The extent to which the grid company decision will diverge from the optimal socio-economic decision, however, depends upon several issues:

- *Regulatory corrections:* Regulatory corrections may contribute to internalize costs of grid failure. In this case, reduced costs of grid failure due to grid investments will materialize also in the grid company cash flow. An example of this is the Norwegian KILE arrangement. However, it is not probable that regulatory measures will capture the full socio-economic effects of grid failure.

- *Grid company objectives:* As mentioned above, for the ultimate decision, company objectives may comprise multiple other considerations in addition to economic profits. This includes e.g. environmental, quality standards, reputational, safety and other society related standards and targets. The extent to which the grid company decision will diverge from the optimal socio-economic decision will thus also depend upon the overall grid company objectives, and the degree to which they are based on company specific payoffs versus the incorporation of wider socio-economic objectives.

The degree to which grid company investment decisions support optimal socio-economic decisions thus depend on a number of factors. Important issues are related to the design of regulatory measures and their implied effect upon grid company objectives and economic incentives. An analysis of the regulatory mechanisms is, however, outside the scope of this report.

Our focus is on techniques of assessing the value of a risky investment. In this respect, it should be noted that these techniques of risk evaluation are the same whether the cash flow represents the socio-economic cash flow or the company specific cash flow. The techniques treated herein are important both from a company viewpoint, as well as from a socio-economic viewpoint.

3.4 Role of the Status Quo Scenario

Following standard investment theory, an investment is profitable if the net present value *after* the investment exceeds the net present value *before* the investment. Illustrated on an annualized basis, reinvestment in an existing grid should thus occur when annual income less cost after investment exceeds annual income less costs before investment.

Considering this, the right investment is made when *both* the value *before and after* the investment is estimated correctly. For the grid the profitability of a grid investment follows directly from the implied improvements vis-à-vis the current state of the grid. To estimate the true profitability of the grid investment, a proper estimation of the current state of the network, i.e. *without* the investment, is just as crucial as the proper estimation of the future state of the network *with* the investment.

For example, consider the simplest scenario of a one-component network, where the investment is simply to replace the existing component with an improved edition which will result in a 1‰ annual probability of default. Even though this may be considered a true fact of the grid quality after the investment, the value of the investment in effect hinges upon the change/improvement compared to the existing network. For example, the value of this investment is much higher if the existing component has a 90 ‰ probability of annual default, than in the case where the starting point is a mere 5 ‰ probability of annual default.

It is crucial to note that the value of an investment depends upon what alterative we compare it with. Thus in many ways the grid company faces equal estimation and valuation problems as to the value before and after the grid investment. The main aspect is that it is the *change* in quality which lays the basis for the value of the investment. An important general principle in evaluating investments is to be clear as to what the benchmark scenario is:

> *The consequences of grid investments follow from the change brought about by the investment. Thus correct valuation of the Status Quo scenario is crucial.*

3.5 Valuation of Grid Investments: Further Approach

We have argued that the basis for *economic* evaluation of the grid investments is the estimated cash flow showing the change in company or society cash flow due to the investment, i.e. the change compared to the status quo scenario. Above we have employed stylistic models and cases that illustrate the main evaluation problems encountered. The overall performance of the grid however will follow from the aggregate performance of each and every component of the grid, and thus represents complex interactions. The main idea has been to highlight the essence of the problem, and to provide a manageable framework to understand the nature of economic grid investment evaluation, as well as the transition to the cash flow upon which economic valuation is based. Chapter 8 will discuss risk evaluation in relation to more practical approaches for distribution system asset management.

With the cash flow given, the decision maker now faces the problem of correctly assessing the current value of the risky investment. The remaining chapters of this report discuss techniques for evaluating future risky outcome of investments, taking into account time dimensions, co-variation of outcomes, market value of risk, etc. Our objective will be to convey an understanding of the main economic problems encountered in risk valuation, and discuss the degree to which economic theory offers useful tools for grid investment analysis.

4 Time Value of Money – Case of Certainty

There are two main dimensions in valuing investments; the time dimension and the risk dimension. This chapter briefly discusses the time value of money. The fact that consequences of investments follow in future time periods, makes issues of the time value of money important. This chapter discusses basic problems related to the time value of money, starting with the notion of *present value* in the case of certainty. We then discuss the meaning of the present value under uncertainty and motivate the need for risk evaluation.

4.1 Brief Comment on Time Value of Money: Present Value

To focus on the essence of the time value of money under uncertainty, we will use simple two-periodic examples. The essence of the time value of money is that the same amount of money received at different points of time, do not have the same value. The difference in value is attributed to the cost of capital. A standard method of comparing amounts of money across different time periods is to *discount* future amounts to the current time, that is, to calculate the *present value*. We assume that the term present value is known to the reader, however, for a brief repetition, consider the following example that illustrates the time value of money:

> Consider the value of the two alternatives, which is either to receive NOK 1000 today, or alternatively NOK 1000 in one year. For the sake of simple calculation, let us assume that the cost of capital is 10 % per year both for borrowing as well as for the placement of money.
>
> *Future value:* Let us assume that the investor has a preference of consumption one year from now. In this case, if the amount of 1000 is received today, and placed at an interest of 10%, the amount available

> one year from now will be 1100. This is the future value of the NOK 1000 received today.
>
> *Present value:* Alternatively, if the investor has a preference of consumption now, on the certain payment of 1000 one year from now, he may borrow 909.09. This amount may be consumed now. In one year he receives 1000, which thus suffices to pay back the loan of 909.09 and the interest of 90.91. That is, the present value of NOK 1000 received in one year is 909.09.

In either case, we see that the value of receiving 1000 today is higher than that of receiving the same amount in one year. This is due to cost of capital which thus is the main explanation for the time value of money. By specifying the cost of capital to be 10% per year, the current value of 1000 received in one year, may be calculated to 909.09. This is termed the *present value* of the amount.

4.2 Interpretation of Net Present Value under Certainty

Under certainty, the interpretation of the present value is straightforward. Consider the following simplified two-period grid investment example in a scenario of certainty:

> *Cash Flow Status Quo:* No costs occur at the current time 0. At time 1 and at time 2 an interruption will occur causing damage and repair costs totalling 50 at time 1 and 100 at time 2. We assume that the income and other cash flows will *not* be affected by the investment. Cash flows that do not change are irrelevant in relation to the investment decision, and need therefore not be included. Thus the relevant aggregate cash flow facing the company in the Status Quo scenario may be represented as (0, -50, -100), i.e. showing the cash flow at times 0, 1 and 2 respectively.

> *Cash Flow after Investment:* At time 0 the grid company invests in an upgrading of the network, at the cost of 100. Now, no interruptions will occur at time 1 or 2. The cash flow facing the grid company after the investment is thus only the investment cost, represented by (-100, 0, 0) at times 0, 1 and 2 respectively.

A first issue is to determine the cash flow *resulting* from the investment, which we will term the *Investment Cash Flow.* This is the *change* in cash flow brought about by the investment:

> *Investment Cash Flow:* The change in cash flow that is *due to* the grid investment is (-100, +50, +100), i.e. an investment payment of 100 at time 0, and the advantage of not paying 50 and 100 at times 1 and 2.

To evaluate the profitability of this investment, a first issue is to find the cost of capital. Let us assume that the grid company free of risk may borrow or place money at a 10 % interest rate. This is the cost of capital. The net present value of this investment is thus:

$$\text{Net present value} = -100 + \frac{50}{1.1} + \frac{100}{1.1^2} = 28.10$$

In exchange for the cash investment of 100, the grid company avoids the costs of grid failure, i.e. 50 and 100 respectively at time 1 and 2. By taking into account the capital costs, i.e. by discounting the amounts, we found that the present value of these benefits is 128.10. With an investment cost of 100, this investment is profitable, and the profitability of the investment in terms of present value is 28.10.

Let us briefly elaborate on the meaning of this number. In this scenario of certainty, the amount of 28.10 is interpreted as a profit of 28.10. This is the extra amount available due to the investment, where the profit of 28.10 is available for free disposal, e.g. for investment in other projects, or payments to the owners. To see this, consider the following argumentation:

> Relative to the alternative of no investment, the company may borrow 128.10 at time 0. Of this, 100 is invested in the project, while the owner gets 28.10 to his disposal. The benefits of the investment will then suffice to pay interest and pay off the loan[2]. This is illustrated below, where we have stated the cash flows of the investment project together with this loan:

Cash flow at time:	**0**	**1**	**2**
Loan	+128.10		
Investment	-100.00		
Income (=avoided cost)		+50	+100
Interest on loan		-12.81 (=128.10*10%)	-9.09 (=(128.10-37.19)*10%)
Instalment on loan		-37.19 (= 50 - 12.81)	-90.91 (= 100 - 9.09)
Net cash flow	28.10	0	0

In this case of certainty, the net present value also represents the market value of the investment.

While this reasoning represents the basics of present value, the example illustrates an important issue with relation to grid investments: Note that the benefits of grid investments are normally related to improved network performance. A typical benefit is avoided grid failure, thus characterized by avoided costs of grid damage (e.g. repair costs, damage compensation to customers, etc.) In our simple case of

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[2] Note that extra cash flow in relation to the non-investment scenario in fact follows from the avoided costs.

certainty there is no risk associated with the benefits, and the state of nature of the grid is common knowledge, both in the status quo scenario and in the investment scenario. This special nature of the benefits of grid investments, however, poses special challenges:

- *In the case of certainty, the benefits of the investment are directly verifiable. Note, however, that in terms of accounting, only realized costs and incomes are registered. Hence, in relation to this simple example, accounting will in effect register no positive cash flows, thus only including the extra cash flows resulting from the costs of the investment. The grid company thus faces a challenge as to verify the benefits of the investment in accounting and company results, for example by visualising the improvements in grid failure.*
- *The challenge is even greater under uncertainty. In this case benefits are related to differences in the ex ante status quo probability distributions of e.g. failure, versus the ex post after-investment probability distributions of future consequences. Reports of grid quality are often based on realized grid failures. Even though the probability for grid failure in fact has improved, there is a chance that for example the number of grid failures increases. This poses a further challenge.*

4.3 Interpretation of Net Present Value under Uncertainty

The interpretation of the net present value of an investment under certainty is straightforward. As illustrated in the example above, the net present value is the extra cash amount that is available today, given that the company can borrow or place money at the risk free interest rate. As such, it may also be interpreted as the market value of the investment project.

Under uncertainty, the interpretation of the net present value of the project is, however, not straightforward. In this case, the cash flow is uncertain. There may be uncertainty about e.g. realized reductions in grid failure, uncertainty about the value of reduced damage, uncertainty about the time horizon, and uncertainty as to future capital costs. This implies that the net present value also will be a stochastic variable, and the direct interpretation as extra cash available at the present or future time is not possible.

To illustrate this, consider our simple example above, and let us assume that there is uncertainty as to the improvement in grid failure, either due to uncertainty in the occurrence of or uncertainty in costs of grid failure. For simplicity, let us assume that the upgrading of the network by the investment eliminates the risk of failure, and that the uncertainty lies in the Status Quo scenario[3]:

> *Status Quo Cash Flow:* No costs occur at the current time 0. At time 1 there is a 50% chance of no interruption, and a 50% chance of an interruption with damage and repair costs of 100. At time 2 there is similarly a 50/50 chance that either an interruption cost of 0 or 200 will occur. If we assume that the outcome of time 1 and time 2 are uncorrelated, there is a 25% chance for each of the following cash flow scenarios: i) (0, 0, 0), ii) (0, -100, 0), iii) (0, 0, -200), and iv) (0, -100, -200).
>
> *Cash Flow after Investment:* At time 0 the grid company invests in an upgrading of the network, at the cost of 100. Now, no interruptions

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[3] In our example, we have illustrated the uncertainty as related to the Status Quo scenario. The example could equivalently be constructed so that there was no uncertainty in the Status Quo, and that all uncertainty was attributed to the cash flow after the investment. Or, we could have constructed an example based on combinations of uncertainty in the Status Quo and in the results of the investment. The profitability of the grid investment is in all cases related to the resulting *differences* in the cash flow before and after the investment.

> will occur either at time 1 or 2. The cash flow facing the grid company after the investment is thus (-100, 0, 0) at times 0, 1 and 2 respectively.

The cash flow *resulting* from the investment is the *change* in cash flow brought about by the investment:

> *Investment Cash Flow:* This change in cash flow brought about the investment is thus also uncertain. There is a 25% chance for each of the following cash flow alternatives: i) (-100, 0, 0), ii) (-100, 100, 0), iii) (-100, 0, 200), and iv) (-100, 100, 200).

In this case the actual profit / net present value depends upon which scenario that will occur. For simplicity, let us still assume that we compute the net present value by an interest rate of 10%. In this case, the actual net present value that will occur depends upon which scenario that will occur:

Scenario	Cash flow	Net present value
i)	(-100, 0, 0)	-100
ii)	(-100, 100, 0)	-9,09
iii)	(-100, 0, 200)	65,29
iv)	(-100, 100, 200)	156,20

For example, if scenario i) occurs, it has ex post proven to be highly unprofitable, while it proves ex post to be highly profitable if scenario iv) is to occur. The net present value in each scenario shows the profit *given* that the scenario has occurred. We can for example calculate the *expected* net present value, which in our example is 28.10 (= (-100 - 9.09 + 65,29 + 156,20) * 25%). This is the same number as in our example of certainty. However, the *market value* of the project in the two cases is not necessarily the same. The difference is due to the valuation of uncertainty.

The decision of investment has to be taken on the basis of information available prior to investment, and thus has to be taken before the resulting scenario is revealed. In the case of certainty, the net present value could be interpreted as the market value of the investment project. The equivalent interpretation of the net present value under uncertainty is *not* possible. The net present value is in effect a *stochastic variable*. The analyst's job is to establish the value of the project *now*. In particular, the *market value* of a risky investment is *not* a stochastic variable.

The transition from a probability distribution of cash flows or net present values to a market value is not trivial. It involves an assessment of the value of *risk* associated with the investment.

Let us here shortly dwell on the meaning of risk: In normal everyday speech risk often refers to something negative, often referring to the possible downsides of any investment or project. When working with risk in general, note, however, that the term risk refers to uncertainty in a broader sense, referring to the variability of possible outcomes covering positive as well as negative outcomes. As such a risky investment is simply speaking an investment where the future outcome/cash flow is uncertain, i.e. where *the outcome is a random variable*. For grid investments, the uncertainty is often related to what the realized benefits in terms of reduced grid failure will be, thus the future cash flow is a random variable. The issue of valuing risk refers to how to translate the uncertain scenarios to a certain value on which to base the investment decision on. This will be the topic of the following chapters, in which we discuss how different methods and theories of risk evaluation apply to grid investments.

5 Value of Investment vs. Expected Values

We now draw our attention to the valuation of risk. We start by discussing the key figure of the *expected value*. The objective here is to prepare for the discussion on the relationship between the expected value and the value of the investment, and the different methodologies for valuing risk.

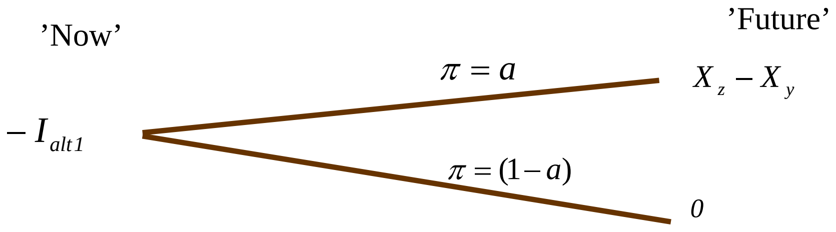
Section 5.1 first argues that uncertain grid investments may in general be summarized as probability distributions of the future cash flow. Typically, the cash flow of the investment at each point of time is characterized as several possible cash payments (positive or negative), each associated with a probability. The expected value of the cash flow is the probability-weighted average of the possible cash payments, as briefly commented upon in section 5.2. Section 5.3 discusses reasons for why the value of the investment may diverge from the expected value. Finally, section 5.4 gives an overview of different approaches to valuing risky cash flows. As such, this chapter gives a motivation for the following journey towards an understanding of which aspects that determine the value of risk for grid investments.

5.1 Cash Flow Probability Distribution

Consider the simple investment alternatives in chapter 2, investments *1* and *2*. The risky cash flow can be represented as a probability distribution of investment outcomes in *monetary* terms. Let X_y and X_z be the monetary-valued outcomes if respectively an interruption of type *y* or *z* occurs in the 'future'. Since the cash flow in the case of failure is likely to be a payment (e.g. repair costs or compensation), normally we will have $X_y < 0$ and $X_z < 0$. We also assume that the cash payment in the case of failure *y* is greater than for failure *z*, i.e. $|X_y| > |X_z|$.

For investment alternative *1*, the change in cash flow relative to the Status Quo scenario follows from the substitution of the type y interruption for the less serious type z interruption. This is a change in cash flow of $(X_z - X_y)$ in the case an interruption occurs, thus a positive effect on the cash flow[4]. There is a probability of a that this will happen. The investment cost equals a cash flow of $-I_{alt1}$. The probability distribution of the cash flow for investment 1 is shown in figure 2c.

Figure 2c Cash Flow Investment Alternative 1



For investment alternative *2*, we found that there was a probability of $\pi = a - b$ where the interruption will be avoided due to the investment. The change in cash flow follows from the avoided interruption, i.e. a cash flow of $-X_y$ with a probability of $\pi = a - b$. We also found that there was a probability of $\pi = 1-(a-b)$ that there was no change, i.e. no realized benefit from the investment[5]. The probability distribution of the cash flow for investment alternative *2* is shown in figure 3c.

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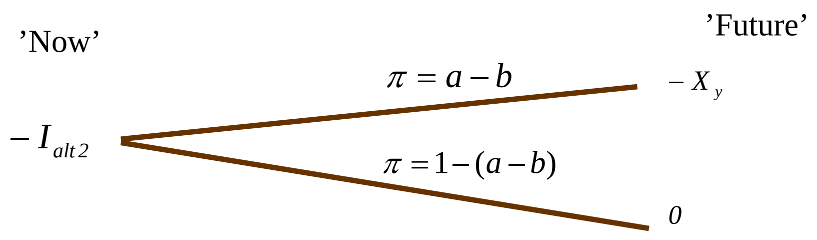
[4] If X_y=-100, and X_z=-10, note that $X_z - X_y$=(-10)-(-100)=90.

[5] The investment causes the probability of failure to drop from *a* to *b*, where *a>b*. Thus, for a probability of *a-b*, there will be no failure after the investment, while there would have been a failure without the investment. However, for the probability of *(1-a+b)* there will be no *change*

Figure 3c Cash Flow Investment Alternative 2



Note that, though the nature of benefits in the two stylized examples are different, (i.e. changes in type of interruption, and changes in probabilities respectively), we see that *the structure* of the resulting uncertain cash flow is the same, and can be represented by a probability distribution. The probability distribution conceptually consists of different outcomes and their probability. This applies to investments that improve outcomes (less severe consequences of failure), as well as investments that reduce the probability of adverse outcomes.

As such, for valuation purposes, the techniques of risk evaluation will be the same. The main question facing us in the following is; what is the value of the benefits of investment, given the uncertainty represented by their probability distribution.

5.2 Expected Profit of Investment

The *expected profit* of the investment summarizes the investment's outcomes and their probabilities. The calculation of the expected profit of each investment alternative is straightforward, where each cash flow is weighted by the probability that it will occur.

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relative to the situation before the investment (i.e. the *(1-a)* probability that there, as before, will be no interruption, *and* the *b* probability that there will, as before, be an interruption of type *y)*.

If we assume that the shown probability trees of investment alternatives *1* and *2* represent the full probability distribution of outcomes, the expected investment profit $E(IP)$ is, assuming no capital costs $E(IP) = -I_{alt1} + a(X_z - X_y)$ for investment alternative 1, and $E(IP) = -I_{alt2} + (a-b)(-X_y)$ for investment alternative 2.

In the simple multi-period example of section 4.3, we can likewise find the expected cash flow at each point of time. At times 0, 1 and 2 the expected cash flows are (-100, 50, 100). The expected present value of the investment depends upon our choice of interest rate. For example, using an arbitrary discount rate of 10%, we found the expected net present value to be 28.1.

The expected investment profit or expected present value will however not necessarily equal the investment *value*, denoted IV. Often we will have $IV < E(IP)$ due to the value of risk. Crucial questions here are in what instances the expected value will be a correct estimate of the value of the investment, and in what instances it is not, and furthermore how the correct value can be found. In the multi-period example, we face further questions as to how to account for the time value of money, as well as for the value of risk.

5.3 Expected Values versus Investment Value: Utility Function Perspectives

The expected value is obviously an important key number; it summarizes the value of realized outcomes and the probability of their occurrence. However, it does not necessarily represent the *value* of the investment. The reason for this is the value attached to risk. The first basic question is thus why the value of the investment would deviate from the expected value. Put differently, why does risk matter? To answer this question we will briefly look into the assumptions of rational behavior underlying risk evaluation in financial theory.

To convey an intuitive notion of the value of risk, let us first consider a simple situation example where the choice is between two alternative lotteries. The first alternative is to receive a certain amount, for example receiving 1.000.000 with certainty. The other is to choose a lottery where you either receive the amount 2.000.000 with a probability of 50%, or the amount 0 also with a probability of 50%. Both alternatives have the same expected value, that is 1.000.000. If this were the whole story, however, it is likely that many decision makers would choose the alternative with the certain amount. In fact, most people as well as investors are normally likely to attach a negative value to the presence of risk. In other words, the expected value of the investment will not necessarily represent the 'true' value of the investment. The investment value will often prove to be lower than the expected value, (but may under some circumstances equal the expected value).

Several theories of risk valuation are built upon the underlying assumptions of the economic rationality of the individual decision makers. Their choices are assumed to optimize their preferences subject to their constraints of e.g. income and wealth. As such their preferences must be characterized both with respect to choices under certainty, as well as under uncertainty. Central basic concepts for modeling consumer choice are the theoretical concepts of *utility functions* and the *expected utility theorem*:

> **Utility function:** The theory of choice under *certainty* is built upon three basic assumptions of economic rationality. These assumptions mainly imply that the investor is able to rank different choices

according to his preferences, and that these choices are consistent[6]. It is on this basis the concept of the basic *utility function* is constructed. The utility function in principle assigns values to different (certain) consumption bundles. The ranking of alternatives according to the utility function thus reflects the underlying preferences.

Expected utility theorem: Under *uncertainty* the investor, however, has to choose among alternatives with payoffs that depend upon which future scenario (state of the nature) that might occur. That is, the future payoff /cash flow is uncertain. The choices will then usually not be straightforward. For example, a given project may render the best outcome if one future scenario will occur, while it may render the worst outcome if another scenario is revealed. The *expected utility theorem* provides a set of hypotheses under which an investor's preference ranking over risky investments may be represented by a utility index. Unlike the basic utility function described above it is thus defined over lotteries / uncertain payoff structures. This utility index is the *expected utility function*, also called the von-Neumann-Morgenstern utility function. It is the probability weighted sum of the *utilities* of the individual outcomes (i.e. not of the expected monetary value[7]).

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[6] For interested readers we refer to for example Danthine and Donaldson (2005) or Pennacchi (2008).

[7] The essence of this hypothesis is that the *preference ranking* of alternatives with uncertain outcomes will be consistent with ranking based on expected utility. Thus, if a lottery $\tilde{y}$ is preferred to lottery $\tilde{z}$, the utility functions will give the same ranking based on expected utility, that is $E[U(\tilde{y})] > E[U(\tilde{z})]$. Note that this does *not* hold for expected monetary values. That is, $E[\tilde{y}] > E[\tilde{z}]$ does not necessarily imply that lottery $\tilde{y}$ is preferred to lottery $\tilde{z}$.

Utility functions are based on several assumptions that represent reasonability in our choices. Utility functions reflect the decision makers' attitude towards risk, and could as such be considered a yardstick for ranking different projects. However, the utility as measured by a specific utility function is *not* quoted in monetary values, and is as such *not a price/value* of the project. In practical use, the concept of utility at most gives us an understanding of the value of risk. We will therefore only quite briefly, mention some insights from the theory of utility which may provide us with intuitive insight which is useful in understanding the valuation of risky projects.

Many different forms of utility functions have been used in different models, where different functions may have different properties and implications with respect to the underlying preferences. Rather than commenting upon the attributes of specific functions, let us briefly comment upon some of the concepts that are relevant for our further treatment:

> ***Risk aversion:*** An assumption that is in conformity with most of financial theory is that decision makers are *risk averse*, thus reflecting a desire in some sense to *avoid risk*. For example, consider an investor with an initial wealth of W, i.e. with the corresponding utility of $U(W)$. Let us assume that this investor is offered a lottery of receiving an amount k with a probability of 50%, and loosing an amount k with a probability of 50%. The expected outcome of the lottery is then 0. A risk averse investor would, however, not wish to enter into such a contract. In terms of expected utility he is better off without this contract, i.e. $U(W) > \tfrac{1}{2}U(W+k) + \tfrac{1}{2}U(W-k)$. It can be shown that a

utility function which displays risk aversion has to be a concave function, i.e. with positive, but decreasing marginal utility[8].

Risk neutrality: While it is rather reasonable to assume that most investors are risk averse, however, there also may exist investors that are indifferent to risk. This is the class of investors who are *risk neutral*[9]. A risk neutral investor does not demand better than even odds when considering a risky investment. They are only concerned with the expected value of the investment. The existence of risk neutral investors may have a significant influence on the equilibrium in a financial market model.

The non-zero value attached to risk, thus can be said to originate from the existence of risk-averse investors in the market. The individual risk averse investor will always value a risky investment at something less than its expected value. Before proceeding to specific theories of risk evaluation in the following chapters, let us however, introduce two further terms:

Certainty equivalent: *The maximum certain amount an investor is willing to pay for a risky cash flow is defined as his certainty equivalent.* To explain the term more formally, let W be the current wealth of an investor, $\tilde{\mathbf{X}}$ the overall cash flow of the risky project, and $E(\tilde{\mathbf{X}})$ its expected value. The certainty equivalent of $\tilde{\mathbf{X}}$, denoted $CE(\tilde{\mathbf{X}})$, is

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[8] Utility functions all display a positive marginal utility, i.e. $U'(W) = \frac{dU(W)}{dW} > 0$, thus implying that the investor prefers more to less. The concept of risk aversion will technically be reflected in utility functions that are concave, i.e. functions where the marginal utility decreases with increasing wealth. For twice differentiable functions that is $U''(W) = \frac{d^2U(W)}{dW^2} < 0$. Essentially, the extra value attached to a given positive wealth increase is less than the extra value lost for the equivalent decrease in wealth.

[9] Their utility functions are linear, i.e. $U(W) = aW + b$, thus with $U'(W) = 0$.

then defined by the following relationship: $E(U(W+\tilde{\mathbf{X}}))=U(W+CE(\tilde{\mathbf{X}}))$. The investor is thus indifferent between receiving the risky cash flow of $\tilde{\mathbf{X}}$ or the certain amount of $CE(\tilde{\mathbf{X}})$.

Risk premium: *The risk premium,* $RP(\tilde{\mathbf{X}})$*, is defined as the difference between the project's expected value and its certainty equivalent,* $RP(\tilde{\mathbf{X}})=E(\tilde{\mathbf{X}})-CE(\tilde{\mathbf{X}})$.

We may find that different grid companies display different degrees of risk aversion. However, note that the valuation and compensation for risk in a market will reflect the *aggregate market valuation of risk*, and *not* that of the individual investor. It is this valuation which is relevant for estimating the value of grid investments. We now turn to financial models for risk evaluation and will in the following chapters discuss their relevance for the valuation of grid investments.

5.4 Different Approaches to Valuing Risky Cash Flows

Valuation of risky investments involves two main dimensions; the time value of money, and the value of risk. The time dimension in the case of certainty was commented upon in chapter 4. In this section we will look into different ways of representing both the time and the risk dimension in valuing risky investments, or more general, in valuing risky future cash flows.

Under *certainty*, the value of a future cash flow is given by the present value $PV=-I+\sum_t \frac{X_t}{(1+r_f)^t}$, where I is the initial cash investment, X_t is the cash flow at time t, and r_f is the risk free interest rate. On one hand, it should be noted that under uncertainty, as in the case of certainty, it can be shown that *the valuation* *process is additive*, meaning that the value of a portfolio of uncertain future cash flows will take the form of the sum of the values of the separate future cash flows.

On the other hand, however, there are differences. Under uncertainty, the cash flow at each point of time is a random variable $\tilde{X}_t$, implying that we do not know the value of X_t on beforehand. *Key reference points* here are the expected value $E(\tilde{X}_t)$ and the *risk free interest rate* for discounting. However, considering the underlying resistance and negative value attached to risk by risk averse investors, the corresponding value of '$-I+\sum_t \frac{E(\tilde{X}_t)}{(1+r_f)^t}$', (i.e. discounting the true expected cash flow by the risk-free interest rate), will clearly be misleading as to the value of the investment project. To account for the value of risk, this can be handled and represented in different ways:

i) Risk-adjusted interest rate:

A common strategy consists of *discounting the expected value at a rate that is higher than the risk-free rate, i.e. where the risk-free rate increased by a risk premium.* The cost of capital is thus increased to reflect the value of risk. The investment value of investment *i*, valued at time 0, denoted IV_i, may in these terms be represented as:

$$IV_i = -I + \sum_t \frac{E(\tilde{X}_t)}{(1+r_f+rp_i)^t} = -I + \sum_t \frac{E(\tilde{X}_t)}{(1+r_i)^t}$$

where r_f is the risk-free interest rate and rp_i represents the appropriate interest rate *risk premium* for this investment. The *risk-adjusted discount rate* for the investment is then $r_i = r_f + rp_i$.

ii) 'Certainty equivalent' of cash flow

Instead of adjusting the interest rate, these approaches seek *to correct the expected cash flow in such a way that one can continue discounting at the risk-free rate.* In other words, we substitute the expected cash flow with its 'certainty equivalent'. Note, however, that this is not the certainty equivalent in terms of the individual investor's preferences, but rather in a sense the equivalent certain cash flow as perceived by the market. There are two main ways to represent and derive this value.

> ***iia) Cash flow risk premium:*** One way is to *decrease the expected cash flow by a risk premium* $RP(\tilde{X}_t)$ reflecting the corresponding value of risk. The investment value can then be represented as:
>
> $$IV_i = -I + \sum_t \frac{E(\tilde{X}_t) - RP(\tilde{X}_t)}{(1+r_f)^t}$$
>
> ***iib) Risk adjusted probability measures:*** Another way is to calculate a risk-adjusted expected value. We saw that the true expected value of the cash flow was calculated by weighting each outcome by its true probability of occurrence. *Here we calculate the adjusted expected value by substituting the true probability measure, with a probability measure which reflects the value of risk*[10]. The expected cash flow with this modified probability distribution, $\hat{E}(\tilde{X}_t)$, represents the equivalent certain amount as viewed by the

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[10] See further explanations in chapter 6.3.

market. In this case we have that the investment value can be represented as:

$$IV_i = -I + \sum_t \frac{\hat{E}(\tilde{X}_t)}{(1+r_f)^t}$$

iii) State contingent claims

Let us now assume that all possible future scenarios or *states of nature* can be defined and identified. This valuation approach is based firstly on the idea that the future cash flow $\tilde{\mathbf{X}}$ can be decomposed into the cash flows that will occur in all these future states of nature. The resulting cash flow at a given time depends upon which state of nature that is realized. Denote $\tilde{X}(\theta_t^j)$ as the cash flow if state *j* occurs at time *t* (that is at the date-state θ_t^j). Secondly, we assume that there are assets which provide a unit payoff if and only if a given state *j* occurs. These assets are called *state contingent claims*. By using the prices of these state contingent claims, the investment value can then be calculated. Let $q_{\theta_t^j}$ be the price today of a state contingent claim which pays 1 if θ_t^j occurs, and 0 otherwise. The value of the project would then be:

$$IV_i = -I + \sum_t \sum_j q_{\theta_t^j} X(\theta_t^j)$$

We have reviewed different ways to represent the value of risky cash flows, and thus different ways to represent the value of risk. In terms of the discounted cash flow, the first approach alters the denominator by risk-adjusting the discount rate, while the second group of approaches alters the nominator by replacing the cash flow with its estimated ‘certainty equivalent’. In both these groups of approaches, we see that the time value of money is explicitly accounted for through the act of discounting. In the third approach, the cash flow is decomposed by state of nature, and the state-contingent prices reflect both the time value of money *and* risk valuation.

We have however, still not commented upon how to actually find the value of risk; that is how to estimate respectively the interest rate risk premium (cf. *i*), the cash flow risk premium (cf. *iia*), the risk adjusted probability measures (cf. *iib*), and the prices of state contingent claims (cf. *iii*).

Financial asset pricing theory provides us with theories on how *risky assets* are valued. *A risky asset is essentially the right to future cash flow:* Investments are risky assets, and so are financially traded assets such as stocks, bonds, options, etc. Regarding financial assets, the future cash flows may take the form of for example interest payments, dividend payments or resale of stocks. For investments the risky cash flows are the future income and payments that occur due to the investment. As such, asset pricing refers to the general task of valuing risky cash flows, including investments. When assessing the value of our investment project, we are, in effect, asking the question: *If this project's cash flow were traded as though it were a security, at what price would it sell given that it should pay the prevailing rates of return as for securities of the same relevant risk level.* Evaluating an investment project is thus a special case of evaluating a complex security.

Financial asset pricing theory comprises several different theories. Underlying the approaches outlined above are thus different theories of risk valuation: The Capital Asset Pricing Model (CAPM), the Consumption Capital Asset Pricing Model (CCAPM) and the Arbitrage Pricing Theory (APT) all are theories applicable to the estimation of risk premia (alternatives *i* and *iia*). The Risk Neutral Valuation theory (also termed the Martingale Approach) is the main theory underlying the risk-adjusted probability measures (alternative *iib*), and the theory of Arrow-Debreu pricing sets forth the contingent asset pricing model (alternative *iii*).

While the theories are fundamentally consistent and based on the same theoretical foundations, they partly reflect differences as to the use and existence of market data, and partly differences in underlying assumptions. There is as such, another even more fundamental way of classifying the alternative valuations theories, namely as to the basis for how the models derive the value of risk. All the known valuation theories are based on elements from either the equilibrium approach or the arbitrage approach:

> **Arbitrage pricing:** Arbitrage pricing approaches are basically built upon the prices of assets traded in the market. In its simplest form, an arbitrage approach attempts to value a cash flow on the basis of the explicit (or implicit) prices of components that make up the cash flow. Put simply, if the cash flow can be duplicated by a portfolio of traded assets, the value of the cash flow should reflect the value of this portfolio. More specifically, arbitrage pricing theories can deduce risk valuation in more general terms of e.g. state contingent claims, or risk-adjusted probability measures. Examples are the *Risk Neutral Valuation Theory* and the general *Arbitrage Pricing Theory* (APT). The classic *Arrow-Debreu* model bears important elements of the arbitrage approach, though the original model takes the form of a standard equilibrium approach.
>
> **Equilibrium pricing:** Central to the traditional equilibrium approach is an analysis of the factors determining the supply and demand underlying the cash flow in question, in particular with respect to preferences and attitudes towards risk of investors. Prices are then

> derived as equilibrium prices balancing supply and demand. Three main equilibrium theories are the *Capital Asset Pricing model* (CAPM), the *Consumption Capital Asset Pricing Model* (CCAPM), and also the basic *Arrow-Debreu* model.

The different theories mentioned all provide insight as to what determines the value of risk, thus helping to value risky assets. Financial theory is a field in continuous development. There is an ongoing development as to the theoretical development as well as appliance of theories to practical use.

In this report we do, however, not seek to give a full overview of the theories of pricing risky assets. *Our quest here is to apply financial theory to the evaluation of risky grid investments. To this objective, we will review aspects of financial theories which we consider convey relevant insight in assessing the value of risk in grid investment:* Chapter 6 intends to give a brief insight into the main ideas of arbitrage pricing theories, focusing on the general idea of arbitrage (Section 6.1), a brief look into the world of Arrow-Debreu state contingent claims (Section 6.2), and the Risk Neutral Valuation Theory using risk-adjusted probability measures (Section 6.3). All these theories represent important contributions to understanding the value of risk in grid investments. However, at the current state of research, we do not think that direct implementation of these theories on grid investments are feasible as the sole source of risk valuation.

Bearing this in mind, however, for the final estimation of grid investment values in practice, it will be necessary to resort to well-used methods, of which the risk-adjusted interest rate represents the status quo implemented method. Chapter 7 on the equilibrium approach is thus mainly dedicated to giving an overview of the CAPM model, discussing the basis for risk-adjusting the interest rate. Chapter 8 concludes this report discussing aspects in applying this method.

6 Arbitrage-Based Pricing Models

A highly simplified setting can reflect some of the basic ideas of arbitrage-based pricing models: If an asset with the same risky cash flow as the grid investment were competitively traded in the market, valuation would be easy; the investment should be the same as for this traded asset. Likewise, if the cash flow of the investment project can be duplicated by a portfolio of traded assets, the value of the investment project should equal the market value of this portfolio:

> For example, let us assume that $\tilde{\mathbf{X}}_{GI}$ represents the overall future cash flow of the project which we wish to evaluate, for example the grid investment project. Also assume that there are *N* traded assets. Let $\tilde{\mathbf{X}}_i$ represent the overall cash flow of a tradable asset *i*, which has the current market price of q_i. Let us then assume that we can duplicate the grid investment cash flow of $\tilde{\mathbf{X}}_{GI}$ by a portfolio of these *N* assets, with a share w_i of each asset *i*. It follows from the requirement of no arbitrage that the value of our grid investment then should be $IV_{GI} = \sum_{i=1}^{N} w_i q_i$.

In this simplified example we see that the value of an asset/investment is derived from the value of a suitable duplicating portfolio of traded assets. Arbitrage-based pricing models in general draw upon the actual pricing of risky assets in the market. An advantage is that the extent of structural and behavioral assumptions necessary to derive the theory is kept to a minimum.

Central to this class of theories is the notion of the *absence of a 'free lunch'*, i.e. no arbitrage. If the cash flow of a project or asset can be duplicated by a portfolio of traded assets, the price of our project should equal the value of this portfolio. If not, there would be profitable arbitrage possibilities, indicating that the pricing of the assets is not in balance.

The applicability of these models partly rests upon whether there are assets available which may reasonably duplicate the payoff of our investment object, and partly that their market price is established in a well-functioning market. For some assets, such as derivatives, their value can be derived directly from the value of a duplicating portfolio based on the underlying asset, together with the requirement of no arbitrage. Based on the main idea of no-arbitrage, however, more general arbitrage-based pricing models have also been developed. We will in the following take a brief tour through some of the theories of arbitrage-based pricing, focusing on issues that may be relevant for the valuation of risky grid investments.

6.1 Arbitrage Pricing: Pricing of Derivatives

Financial contracts where the future outcome depends upon the future movement of an underlying traded asset, are called *derivatives*. The value of derivates is often derived based on the requirement of no arbitrage, by constructing a duplicating strategy involving the underlying asset. Our objective in this section is on one hand to briefly look into the basic idea of arbitrage pricing in relation to derivatives, and on the other hand to discuss whether there are assets traded in the electricity market that can contribute to assessing the value of different aspects of grid investments.

The most common derivatives are forward, futures, and option contracts. All these contracts are written upon an underlying asset, for example a spot commodity, a stock, or an index of stocks. In the electricity market there are forward, futures and option contracts written upon the future spot price in a given period. The basic idea of pricing forward contracts and option contracts can, simplified, be described as follows:

> ***Forward contracts:*** A forward contract is a contract where the price for future delivery of e.g. a commodity is agreed upon now, and payment is made at delivery. In the simplest riskless case where the commodity in question can be bought today, and stored for delivery at the future time of delivery, the principle of no arbitrage indicates that the forward price should be the current spot price with the addition of storage costs. If not, there would be possibilities of a risk-free profit. For example, for a higher forward price, it would be profitable to sell forward contracts, buy the corresponding quantity in the spot market, store the commodity, and deliver it at maturity, thus reaping a non-risk profit. Even for more complex forward and futures contracts, similar reasoning can be used to deduce the value of these contracts.
>
> ***Options:*** An option gives the right, but not obligation to buy or sell an asset at a given future point of time. The payoff of an option can in principle be duplicated by a dynamic trading strategy in the underlying stock and a risk-free bond. This is for example the main idea underlying the famous Black-Scholes pricing formula for options.

These are two highly simplified examples where the no-arbitrage requirement is used in deriving the value of derivatives based on the value of underlying assets. For a thorough review of the pricing of options, futures and other derivatives, interested readers are referred to e.g. Hull (2008).

Turning to our field of interest, the question is then whether such insight can be helpful in assessing the value of grid investments. Note that most liquid traded assets of the electricity market such as spot, forward and futures contracts are related to the pricing of *energy* and load, thus implicitly assuming a *normal* deliverance mode of energy and load. For grid investments, however, the benefits underlying the cash flow are related to the avoidance of the rather *non-normal* situations of grid failure. These aspects are not directly priced in the market, though to some extent contracts pricing grid performance can be traded (e.g. interruptible energy tariffs). To the extent to which relevant assets are not traded in the market, we would thus argue that it is difficult to directly duplicate the grid investment cash flow by means of any existing traded securities.

However, indirectly even this simplest version of arbitrage-based pricing brings an important insight to the valuation of grid investments. *The example illustrates that the prices of other traded securities, as well as the prices of other alternatives may represent a relevant input to our investment analysis.* For example, traded forward and futures contracts represent the future value of delivered power. This is an aspect which may be important in evaluating investments in a new line. In other instances, relevant alternatives may represent the upper or lower bounds on the investment. For example, let us assume the main benefit of an investment is to avoid failure for a large customer. If it is possible for this customer to invest in facilities for emergency / standby power to prevent failure, it may be argued that the cost and operation of such an investment represents an upper bound on the investment value.

Still, the indicated conclusion above is that this simple use of no-arbitrage based arguments, though representing important insight, will not be sufficient as a sole tool for risk valuation of grid investments.

6.2 Arrow-Debreu Pricing

The classic Arrow-Debreu pricing theory based on the seminal work of Arrow (1951) and Debreu (1959) is in some sense the father of all asset pricing relationships. Conceptually it offers invaluable insight as to the understanding of risk evaluation. However, in practice this rather abstract theory is difficult to implement. We will give a brief and highly simplified outline of some of the main aspects of the theory, and point out some aspects which we believe will enrich our understanding of the value of grid investments. Let us first introduce a basic Arrow-Debreu setting:

> In a two-period setting ('now' at date 0, and 'future' at date 1), let us assume there are *J* possible *states of nature* (also called *states of the world*) at date 1. These states of nature can be envisaged as all possible scenarios that can occur at date 1. The probability that state *j* (denoted state θ^j) will occur at time 1 is π_j. In this simple Arrow-Debreu economy let us furthermore assume that there are traded securities of the following type: Security *j* pays one unit if state *j* occurs, but if state *j* does not occur, there is no payment. These primitive securities are called *Arrow-Debreu securities* or *state-contingent claims*, or simply *contingent claims*. The price of contingent claim *j* at time 0 is denoted q_{θ_j}. With minor modifications accommodating more periods, a fully general model can be constructed with state-contingent claims for all future states, i.e. contingent claims for all 'date-states' θ_t^j, and the corresponding prices at time 0 of $q_{\theta_t^j}$.

Before proceeding, note that the contingent claim is a *risky asset*, paying 1 if state *j* occurs at time *t,* and 0 otherwise. The price of this risky asset is $q_{\theta_t^j}$. It should here be noted that this price summarizes both the value of time and the value of risk. As to the valuation of risk let us further note that the price $q_{\theta_t^j}$ in fact depends upon the state of nature in which the payment is to be received: The value of receiving a payment of 1 if state *j* were to occur at time t, may be different from the value of receiving 1 if for example state *k* were to occur, (i.e. $q_{\theta_t^j} \neq q_{\theta_t^k}$). In this the essence of the value of risk lies: The value of risk is on one hand related to the *probability of occurrence*, and on the other hand to the *relative scarcity* in the future revealed scenario. In general the value of receiving a given amount will be greater in a state of scarcity than in a state of abundance.

As such, this basic model setup highlights an insight which may be of special importance in valuing risky grid investments: This is the insight that *the value of possibly receiving a given commodity, a given amount of money, or, as in our case, of avoiding a grid failure, will be greater in some states of nature than in others.* For example, the possibility to avoid grid failure, or to have extra generation capacity, could be extremely more valuable in the future state of a cold harsh winter, than in a mild winter. In principle these aspects should be reflected in the value of the grid investment.

While the insight itself is important, the question however remains whether this setup can be used for practical estimation of the investment value. If we were able to distinguish between the consequences of different future states of the world, and if we knew the prices of the state-contingent claims, the investment value could easily be calculated:

> Let us assume that we can decompose the investment cash flow $\tilde{\mathbf{X}}^{GI}$ into the cash flow that will occur in all future states of nature, where $\tilde{X}(\theta_t^j)$ is the cash flow at time *t* if state *j* occurs. The value of possibly receiving a cash flow in a future state *j* at time *t*, is then found by multiplying the cash amount by the price of the contingent claim. The investment value would then found by summing up for all future times and states of nature, i.e. $IV = -I + \sum_t \sum_j q_{\theta_t^j} X(\theta_t^j)$.

The Arrow-Debreu model was, however, derived as a theoretical general equilibrium theory, and under rather strict assumptions[11]. Unfortunately, its usefulness in practice is impaired by the difficulty of identifying individual states of nature, and by the fact that, even when a state can be identified, its realization cannot always be verified. As a result it is difficult to write the appropriate conditional contracts. These problems to a large extent explain why we do not see Arrow-Debreu securities being traded, and thus also why the theory is not directly applicable.

The Arrow-Debreu pricing theory has, however, also been interpreted in an *arbitrage* perspective. This perspective possibly represents a more fruitful approach to applying the theory to practical evaluation[12]. Very simplified, it can be argued that options written on a large market portfolio can be used to construct *synthesized state-contingent claims*, where the prices of these synthesized state contingent claims follow from no-arbitrage arguments[13]. Advantages here are that options in fact are traded and that the payoff pattern of an option is verifiable, thus making it possible to estimate state contingent prices.

While a large financial portfolio will, admittedly, not be able to represent *all* future states of nature, it can be argued that these synthesized state-contingent claims

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[11] For example, it was assumed that markets were complete. Simply speaking this means that there exists a market for a contingent claim for every state of nature, at every future point of time. In this case, any cash flow could be duplicated and valued by a portfolio of contingent claims. In practice, however, it may be argued that real world markets not necessarily are complete.

[12] For the sake of our further exposition, we have presented the Arrow-Debreu theory in this chapter on arbitrage-based pricing models, and not in the following chapter on equilibrium-based models.

[13] The idea is that if there are N states of nature, and $M=N$ linearly independent traded securities (not state contingent claims), markets are complete. Note that this applies even though the state contingent claims themselves are not traded. It is then possible to infer the prices of all the N hypothetically underlying contingent claims. These prices can then in principle be used to value other cash flows, that is, if the asset cash flow in each of the N states of nature can be identified. See for example Banz and Miller (1978) and Breeden and Litzenberger (1978).

may price the states of nature which are most relevant regarding most financial securities[14]. It still is, however, an open question whether contingent claims based on a financial portfolio would be able to price all states of nature that are relevant to grid investments. This is not obvious. On one hand, the fundamental factors driving the uncertainty of financial securities probably are quite different from fundamental factors causing e.g. grid failures. It is therefore not obvious that the (supposedly) derived synthesized contingent claims will suffice for evaluating grid investments. On the other hand, we will see that by the CAPM, the only relevant risk is the *non-diversifiable (also called systematic) risk*. (Diversifiable risk has zero market value). An interpretation is here that the only states of nature that are economically relevant are the states of nature that can be identified with different values of the market portfolio. In this view the synthesized contingent claims based on the market portfolio might possibly suffice. Still, even if we could identify the prices of relevant contingent claims, the problem still remains whether the outcome of a grid investment could be identified for different states of nature.

From the current state of research *we thus do not think that the direct appliance of the Arrow-Debreu pricing theory would be directly relevant in the practical risk evaluation of grid investments. When applying other methods one should, however, at least raise the question whether the special value of grid investments in specific future scenarios is adequately reflected in decision making.*

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[14] In principle, a necessary condition for the creation of a complete set of Arrow-Debreu securities, is that its payoff pattern distinguishes among *all* states of nature.

6.3 Theory of Risk-Neutral Valuation[15]

In the Arrow-Debreu pricing approach above, the time and risk dimension was jointly accounted for in the prices of state-contingent claims. Turning to the theory of risk-neutral valuation, we now return to a representation where the time value again is represented by discounting, while the risk dimension is handled separately, here by modifying the expected cash flows of the denominator.

Basically, in the risk-neutral valuation method a modified cash flow expectation is calculated - *not* by applying the *true* probability of occurrence, but rather by using *corrected* probabilities. This corrected probability measure is referred to as the *risk-neutral probability measure*. It is derived so that this new set of probabilities will account for the market value of risk. The modified cash flow expectations can then be discounted at the risk-free rate to find the value of the risky asset or investment[16]. The essential question is then how to find this new risk-neutral probability measure.

In section 5.4 we saw that all the known valuation theories are fundamentally based on elements from on one hand, either the arbitrage-based approaches which derive the value from the prices of other traded assets, or on the other hand from equilibrium approaches which derive values based on modeling the underlying supply and demand in the market. The theory of risk-neutral valuation is an arbitrage-based approach as the modified probabilities are derived from the prices of traded assets, under an assumption of no arbitrage opportunities in the market.

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[15] The theory of risk-neutral valuation (also called the Martingale pricing theory) was first developed by Harrison and Kreps (1979).

[16] Since the modified cash flow expectations can be discounted by the risk-free interest rate, they in a sense represent a ‘certainty equivalent’. The term certainty equivalent described in chapter 5, however, was defined to reflect the preferences of the individual investors. Here the modified expectation reflects the *market value* of risk.

As such, the model is free of any structural assumptions on preferences or expectations.

Risk-neutral probability distributions may be presented and implemented in a variety of forms, depending on the choice of setting. Several valuation procedures, for example used in valuing stocks, bonds, and even e.g. electricity futures, are normally derived in a continuous time setting of the stochastic price processes. Our objective here is to convey an underlying understanding of the principles of the method, and to discuss the method's contribution to understanding the value of grid investments. To do this, we will use a discrete, stylized, and rather non-formal example to provide insight and an intuition of the methodology. For a full review of the theory of risk-neutral valuation in discrete time, see Pliska (1997). Also see Danthine and Donaldson (2005).

> Let us start by considering the simplest possible setting with two dates, that is now *t=0*, and the future *t=1*. At date 1 there are *J* possible states of nature θ^j that can occur, each with the objective (true) probability of π_j. Consider a risky asset, for example an investment where the realized date 1 cash flow will depend upon which state of nature is realized. Let $\tilde{X}(\theta^j)$ be the realized cash flow which if state *j* occurs. The (true) *expected value* of the (time 1) cash flow calculated by the true probabilities of occurrence is $E^{\pi}(\tilde{X}) = \sum_j \pi_j X(\theta^j)$.

We have already seen that this true expected value does *not* represent the value of the asset at time 0. This is firstly because it is a time 1 expected cash flow, i.e. we have not discounted the cash flow. Secondly, the expression does not account for the value of risk: The expected value treats the outcome received in different states of nature proportional to their probability of occurrence. Drawing on our insight from the Arrow-Debreu setting, we thus see that the expected value does not account for differences in value attributed to differences in the relative scarcity in different scenarios.

Basically, the expectation calculated under the modified probability distribution will account for both the probability of occurrence, and the relative scarcity in different states of nature. By using the risk-adjusted probability measure, $\hat{\pi}_j$, it is the *value* of the risky cash flow that is calculated:

> The expected value calculated under the risk-neutral probability measure, i.e. $\hat{E}^{\hat{\pi}}(\tilde{X}) = \sum_j \hat{\pi}_j X(\theta^j)$, represents the time 1 risk-neutral value of the cash flow. Since it is risk-neutral, the cash flows can be discounted by the risk-free rate of return. The *market value* of the investment in this two-period setting is then: $IV = -I + \frac{\hat{E}^{\hat{\pi}}(\tilde{X})}{1+r_f}$.

This illustrates the usage of risk-neutral probabilities: If the risk-neutral probability measure is found (either discretely represented as in this example, or most often represented as a continuous process), the risk-neutral expected value can be calculated and discounted by the risk-free rent. The resulting value is a value which is in accordance with the market valuation of risk.

The main question is then how the risk-neutral probability measure is to be found. *Basically the risk-adjusted probability measure is derived from the prices of traded assets in a well-functioning competitive market, and under the requirement of no-arbitrage*. To illustrate this principle, consider the following simple setting:

Consider a competitive market where a risk-free bond and in addition *N* risky assets are traded. The market price at time 0 of the bond is q_b, while the market price of risky asset *i* is denoted q_{e_i}. These prices are observable in the market. The risk neutral probability measure is then derived from the prices of these traded assets. In the setting of this discrete example, a probability measure, i.e. a set of modified probabilities $\{\hat{\pi}_j\}_{j=1}^{J}$ defined on the set of states *j=1,…J,* is a *risk-neutral probability measure* if;

i) All probabilities are strictly positive, i.e. $\hat{\pi}_j > 0$ for all states *j*, and $\sum_j \hat{\pi}_j = 1$.

ii) The prices of all fundamental risky assets *i,* are priced in accordance with the risk-neutral probability measure.

So, if our market is competitive and free of arbitrage opportunities, the observed market prices must implicitly reflect the risk-neutral probability measure. In other words, the value/price of e.g. asset *i* has to equal the discounted modified cash flow, that is, $q_{e_i} = \frac{\hat{E}^{\hat{\pi}}(\tilde{X}_i)}{1+r_f} = \frac{\Sigma_j \hat{\pi}_j X_i(\theta^j)}{1+r_f}$. This applies to all assets. For our example this gives us the following set of equations for the *N* risky assets:

$$
\begin{aligned}
q_{e_1} &= \frac{\hat{E}^{\hat{\pi}}(\tilde{X}_1)}{1+r_f} = \frac{\Sigma_j \hat{\pi}_j X_1(\theta^j)}{1+r_f} \\
q_{e_2} &= \frac{\hat{E}^{\hat{\pi}}(\tilde{X}_2)}{1+r_f} = \frac{\Sigma_j \hat{\pi}_j X_2(\theta^j)}{1+r_f} \\
&\vdots \\
q_{e_N} &= \frac{\hat{E}^{\hat{\pi}}(\tilde{X}_N)}{1+r_f} = \frac{\Sigma_j \hat{\pi}_j X_N(\theta^j)}{1+r_f}
\end{aligned}
$$

> The risk-adjusted probabilities[17] are then found by solving these equations[18] with respect to the set of risk-adjusted probabilities $\{\hat{\pi}_j\}_{j=1}^{J}$.

Basically, the risk-neutral probability measure is derived from the prices of traded assets. Also note that the existence of a risk-neutral probability measure is highly related to the absence of arbitrage opportunities in the market: In a market with no arbitrage opportunities, it is not possible to conduct a trade which at the same time i) has no initial investments (e.g. a portfolio with long and short positions in assets), ii) has no possible losses in any state, and at the same time iii) earns a profit in at least one future state of nature. If such arbitrage possibilities exist, market prices are not in balance. The presence of an arbitrage opportunity is as such not consistent with a market in equilibrium. Another way to put it, is to say that the equations stated above below do not have a solution if market prices imply that there are arbitrage opportunities. In sum arbitrage opportunities are incompatible with the existence of a risk-neutral probability measure.

The theory of risk-neutral valuation is in fact applied in financial models of pricing stocks and derivatives. The method is especially interesting since it is free of structural assumptions, and thus directly reflects the actual risk valuation of the market. For risk valuation in electricity markets in general, for example in

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[17] Note the relationship between the risk-adjusted probabilities, and the Arrow-Debreu state contingent claim discussed above: In a two-period setting, the state contingent claim pays 1 at time 1 if state *j* were to occur, and 0 otherwise. Let q_{θ^j} be the current value/price of this state-contingent claim. From the theory of risk neutral valuation we saw that the price of a risky asset equals the expected cash flow calculated under the risk-adjusted probability measure, and then discounted by the risk free rent. Since the only non-zero cash flow only occurs in state j, the risk-adjusted cash flow expectation is $1^* \hat{\pi}_j$. Thus we have $q_{\theta^j} = \frac{\hat{\pi}_j}{1+r_f}$.

[18] If the market is incomplete, the equation system will be underdetermined, and there will be many solutions.

evaluating derivatives on one hand, and on the other hand in evaluating e.g. generation investments where the future electricity price is important, the method in many respects seems to be promising. The reason is that it draws on the stochastic valuation and price processes implied by spot prices, as well as by traded derivatives.

For grid investments, however, the asset which we are evaluating is somewhat different. The focus here is on the value of future avoided grid failure. These values may highly differ from prices of traded energy assets, which in many respects rather reflect energy prices in a normal setting. A main question when considering whether the methodology can be applied for evaluating grid investments (or specific aspects of grid investments), is thus whether there is any connection between e.g. the stochastic spot or future price processes and the occurrence of e.g. failure. For example: Is there a (stochastically proved) connection between high spot prices and grid failure – thus reflecting the possible underlying aspect that both high prices *and* grid failure occurs at similar times (e.g. peak-load in cold winters)?

We do believe that it might be fruitful to apply this methodology to some aspects of grid investment valuation to better understand the impact of market prices on the value of grid investment. *At this stage, however, though interesting, we believe that this is more a future research question for shedding light on special aspects of grid investments.* For practical use, where the value of grid investment depends on a large range of factors, as illustrated by the cash flow probability distribution, we believe that the method of the following chapter, using the CAPM methodology to adjust the discount rate, seems the most implementable method.

7 Equilibrium Pricing Models: The CAPM

In the equilibrium pricing approaches principles of risk valuation are derived from equilibrium conditions of market models. A frequently employed model is the Capital Asset Pricing Model (CAPM), which is based on the seminal work of Sharpe (1964), Lintner (1965), and Mossin (1966). Though even the CAPM cannot be said to give the 'true' value of risk, it does capture essential aspects, and will probably continue to be the most applicable model also for the evaluation of grid investments. We will thus confine our treatment of equilibrium pricing models to a discussion of the CAPM[19,20].

Using the CAPM approach, the expected value of the cash flow at each point of time is discounted at a risk-adjusted interest rate r_i. The risk-adjusted r_i basically reflects the required market compensation for the risk of the project or asset in question. The investment value of investment *i* valued at time 0, (denoted IV) may in these terms be represented as the expected cash flow of the grid investment discounted by a risk-adjusted rate, i.e. $IV = -I + \sum_t \frac{E(\tilde{X}_t)}{(1+r_i)^t}$.

A main question is thus how to derive the appropriate risk-adjusted return rate for grid investments. The basic idea is that the project's risk compensation should reflect the risk compensation of bearing similar risk in capital markets. The CAPM indicates two basic elements of risk compensation: i) the investment's risk

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[19] Note that the Arrow-Debreu pricing model covered in chapter 6 originally is an equilibrium pricing model, which however also may be interpreted in an arbitrage-based pricing setting. The Consumption Capital Asset Pricing Model (CCAPM) is another of the main equilibrium-based pricing models. Current research based on the CCAPM seeks a further insight in the link between financial markets and the real side of the economy, as well as gaining insight in dynamic features. In many respects, much of the basic insight is also conveyed by the CAPM. And, as its practical applicability is still limited, this report will not give a further treatment on the CCAPM.

[20] See for example Danthine and Donaldson (2005) or Pennacchi (2008) for a further treatment on the CAPM.

contribution as defined by the market, and ii) the general market price of risk. Here portfolio theory is an underlying prerequisite for understanding an investment's risk contribution. Section 7.1 first briefly accounts for the mean-variance orientation of portfolio theory and the CAPM. Section 7.2 discusses grid investment risk in a portfolio setting. In this, the discussion also sheds light upon how a grid investment affects grid company risk. However, the value of risk is, as mentioned, dependent upon how the market defines and prices risk. This is the topic of sections 7.3 and 7.4. Section 7.3 discusses the concept of the efficient frontier, thus defining the set of portfolios which in a mean-variance sense are superior to other portfolios. Section 7.4 finally reveals the main results of the CAPM, and their implications for risk compensation and the investment's required return.

7.1 Risk Representation in the CAPM

Investment risk as well as asset risk in general refers to the uncertainty of future cash flows. A basic question is how we choose to represent risk. A complete description of an asset's risk is given by its probability distribution. In working models for pricing risk, however, risk most often is summarized in terms of different key numbers. Modern portfolio theory and the CAPM explores the details of risk valuation and portfolio choice in a *mean-variance setting*. This means that risk valuation and asset/portfolio choice is based on the asset's/portfolio's *mean* return, together with the net contribution of the asset to the *variance* of the portfolio's return.

At the outset, it should be noted that the assumption that portfolio choice is based on mean-variance criteria is not trivial as it in fact implies several assumptions: The mean-variance-based choice criteria is only consistent if market participants either have a mean-variance based utility function (a quadratic utility function), or alternatively if asset returns are normally distributed. Strictly speaking, a quadratic utility function displays preference attributes which are fairly implausible. Furthermore, the normality hypothesis as to the rate of return processes will in general not be empirically satisfied[21].

Still, the mean-variance approximation is widely used. The main justification for using the mean-variance approximation is its tractability. Full probability distributions are cumbersome to represent as well as to estimate empirically. By summarizing risk by the first two moments of the probability distribution, i.e. mean and variance, the framework is not only easier to handle, but also facilitates empirical testing. As such the mean-variance representation of risk is used in the CAPM.

7.2 Portfolio Perspectives on Risk

Modern portfolio theory is an important building block of the CAPM. This section gives a brief discussion on risk in a portfolio perspective. A central issue of general portfolio theory is that the relevant risk of an asset/investment is not the risk of the asset alone, but rather its risk contribution with respect to a larger portfolio of assets. For financial assets, this would e.g. refer to a portfolio of securities. For a grid company, the term portfolio may for example refer to the aggregate investments of the company.

Portfolio choice by investors is based on the expected return and variance of the *portfolio*, rather than on the expected return and variance of the individual assets or investments. While the expected return of a portfolio is the weighted average of the expected returns of the assets of the portfolio, this is not generally true for the variance, where the aspect of *diversification* is important:

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[21] A more correct approximation is the specification of asset models in continuous time, and assuming that the continuously compounded rate of return is normally distributed.

> **Mean Return:** The expected/*mean return* of a portfolio is the share-weighted average of the expected returns of the assets composing the portfolio. For an investment, the parallel argument is that the expected return for the company is the share-weighted average of expected returns of the company's projects.
>
> **Variance**[22]**:** Unless the returns of the different assets/investments in the portfolio are perfectly correlated, the variance of the portfolio will be *smaller* than the weighted average of the variances of the individual asset returns. To some extent positive and negative returns of assets in the portfolio will outweigh each other, resulting in less variation in the total outcome. In this lie the gains from diversification.

To illustrate the aspect of diversification, consider a simple portfolio of only two assets or projects, with the shares of respectively w_1 and w_2 of the two assets, where $w_1 + w_2 = 1$. Let r_1 and r_2 be the expected returns of asset *1* and *2*, let σ_1^2 and σ_2^2 be the variance of returns for asset *1* and *2*, and let ρ_{12} be the correlation of the two assets. This portfolio thus has an expected return of $r_P = w_1 r_1 + w_2 r_2$ and a

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[22] Let us briefly refresh some of the general formulas of variance. Let $\tilde{x}$ and $\tilde{y}$ be two random variables with the means (expected values) of respectively $\bar{x}$ and $\bar{y}$. The *variance* of for example $\tilde{x}$ is defined as $\sigma_x^2 = E[(\tilde{x} - \bar{x})^2]$. The *standard deviation* is defined as $\sigma_x = \sqrt{\sigma_x^2}$. The *covariance* of $\tilde{x}$ and $\tilde{y}$ is $\sigma_{xy} = E[(\tilde{x} - \bar{x})(\tilde{y} - \bar{y})]$, and can thus be positive or negative. The *correlation coefficient* of $\tilde{x}$ and $\tilde{y}$ is $\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$, thus $-1 \le \rho_{xy} \le 1$. Also note that the *variance of the sum of two variables* is $\sigma_{x+y}^2 = \sigma_x^2 + \sigma_y^2 + 2\sigma_{xy} = \sigma_x^2 + \sigma_y^2 + 2\sigma_x \sigma_y \rho_{xy}$. For a weighted portfolio of w_x of $\tilde{x}$ and w_y of $\tilde{y}$, the portfolio variance is $\sigma_p^2 = w_x^2 \sigma_x^2 + w_y^2 \sigma_y^2 + 2 w_x w_y \sigma_{xy} = w_x^2 \sigma_x^2 + w_y^2 \sigma_y^2 + 2 w_x w_y \sigma_x \sigma_y \rho_{xy}$.

variance of $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{12}$. The importance of the portfolio perspective lies in the implied diversification of risk. The degree of diversification depends upon the correlation of asset returns. To see this, consider the following cases:

> **Perfectly correlated returns** $(\rho_{12} = 1)$**:** In this extreme case, there is *no gain from diversification*. Each asset perfectly matches the movements of the other. The portfolio's standard deviation will then be the weighted average of the standard deviations of each asset[23], i.e. $\sigma_p = w_1\sigma_1 + w_2\sigma_2$.
>
> **Imperfectly correlated returns** $(-1 < \rho_{12} < 1)$**:** In this case there will be *gains from diversification*, meaning that the standard deviation of the portfolio necessarily is smaller than the weighted average (which was the case in the scenario of perfectly correlated returns). More specifically, the lower the correlation ρ_{12} is, the lower the variance of the portfolio will be. To see this, note that the portfolio variance now is $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{12}$. By increasing the correlation of the assets, portfolio variance increases (since $\frac{\partial\sigma_p^2}{\partial\rho_{12}} = 2w_1w_2\sigma_1\sigma_2$>0). Conversely, a lower correlation implies a lower portfolio variance[24].

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[23] When the asset returns are perfectly correlated, that is $\rho_{12} = 1$, we have that $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2 = (w_1\sigma_1 + w_2\sigma_2)^2$, and thus $\sigma_p = w_1\sigma_1 + w_2\sigma_2$.

[24] In the extreme case where the two risky assets have returns that are perfectly negatively correlated, i.e. $\rho_{12} = -1$, it is in fact possible to construct a risk free portfolio by choosing the appropriate weights: For $\rho_{12} = -1$ we then have $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 - 2w_1w_2\sigma_1\sigma_2$ $= (w_1\sigma_1 - w_2\sigma_2)^2$. Since $w_2 = (1 - w_1)$ in the case of two assets, the weight of $w_1 = \frac{\sigma_2}{\sigma_1+\sigma_2}$ will

We find it reasonable to assume that the investor cares about the risk characteristics of his total holding of risky assets, that is, of his total portfolio. As such, a grid company cares about its total risk, and not the risk of the individual projects alone. The above insight is then important. Looking at the variance formulas above, we see that by simply putting together a portfolio of imperfectly correlated assets or projects, the investor may in fact reduce the risk as opposed to holding a risky asset alone. This aspect thus has important implications as to understanding the actual net contribution of the grid investment to company risk:

> *If a grid investment is imperfectly correlated with the existing activities of the company, the net risk contribution of the investment project to the company may be less than the risk perceived in assessing the investment project alone.*

The relevant risk of an asset or an investment is thus *not* the *gross* risk of the investment. By including the new grid investment in the portfolio of risky activities by the grid company, the net *company* risk contribution of the investment depends upon its correlation with the existing company projects[25].

Returning to the valuation of grid investments, our question was how the investment risk contribution should be *valued*. Basically, the underlying idea is that the risk of grid investments should be valued just like the market values the equivalent risk of other assets in the market. As such, the CAPM sees risk in an equilibrium setting for the whole market:

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give a zero variance portfolio, i.e. a risk-free portfolio. In this case all the upside outcomes of one asset will be exactly met by the downside outcomes of the other asset.

[25] In principle, a grid investment reduces consequences and the probability of grid failure, may even reduce the net company risk.

> *For an investment, the relevant risk to be compensated for in terms of a higher required return is neither the gross risk of the investment, nor the specific net contribution to the risk of the company. The relevant risk of the grid investment is the part of the risk which is relevant as to pricing risk in the market.*

In this, the CAPM takes an overall *portfolio perspective on risk* in the market. We will eventually see that risk that can be reduced by merely putting together an appropriate portfolio of risky assets, should not earn extra return in a market in equilibrium:

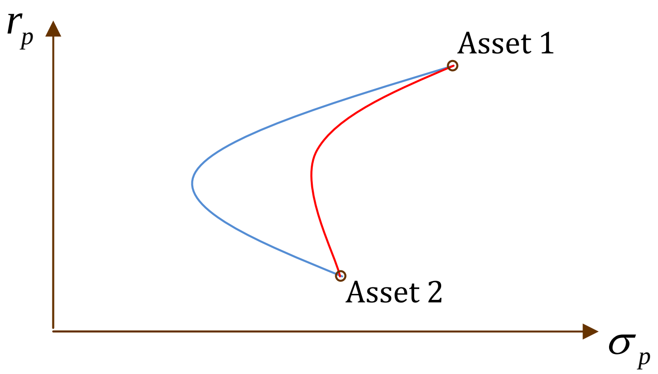
> *The relevant risk to be compensated is the risk that cannot be diversified away on a market basis.*

In the following section we will continue our tour of portfolio theory and the CAPM, now focusing on the pricing of risk in a market perspective.

7.3 The Efficient Frontier and the Market Portfolio

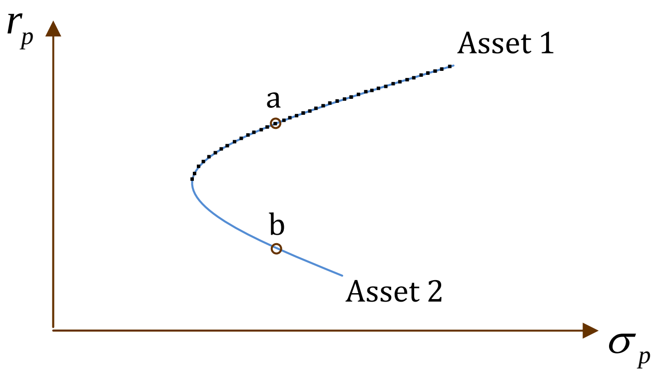
Let us now continue with our simple portfolio example to see further implications of diversification on portfolio choice. In our two-asset example, we can construct different portfolios with varying levels of expected return, simply by varying the weight of each asset. Higher (lower) weights of the asset with the highest (lowest) mean return give a higher (lower) portfolio mean return. In varying the weights, also the portfolio risk will be varied. The red and blue lines of figure 7.1 show typical relationships of the mean and standard deviation of portfolios of these two assets, though under two different assumptions of correlation: The red line assumes a higher correlation of the assets than the blue line. Thus we note that the diversification effect is greater in the indicated blue alternative where we have assumed that the assets are less correlated.

Figure 7.1 Diversification



In this mean-variance setting, the investor has a choice of portfolios with varying levels of mean return and variance. On one hand the typical investor 'likes' the expected return. This means that for a given variance, the investor prefers a larger return to a smaller return. On the other hand the investor 'dislikes' risk, here represented by the variance (or standard deviation). This means that for a given expected return, the investor prefers the portfolio with the lowest variance. Regardless of the investor's degree of risk aversion, we thus see that some portfolios will dominate other portfolios. To see this, consider figure 7.2, showing the mean and variance of possible portfolios of asset 1 and 2 in the alternative of the lowest correlation above (the blue line). This line shows the minimum variance possible for all alternative levels of expected return. We here see that portfolio *a* has the same variance as portfolio *b*, but has a higher expected return. Thus portfolio *a* clearly dominates portfolio *b*, which thus would not be chosen.

Figure 7.2 The Efficient Frontier: Two Risky Assets



For a given preferred expected return, the investor would only choose the portfolio with the lowest variance. Likewise, for a given preferred level of risk, the investor would only choose the portfolio with the highest return. This set of the non-dominated portfolios is called the efficient frontier, and is marked as the dotted line of figure 7.2.

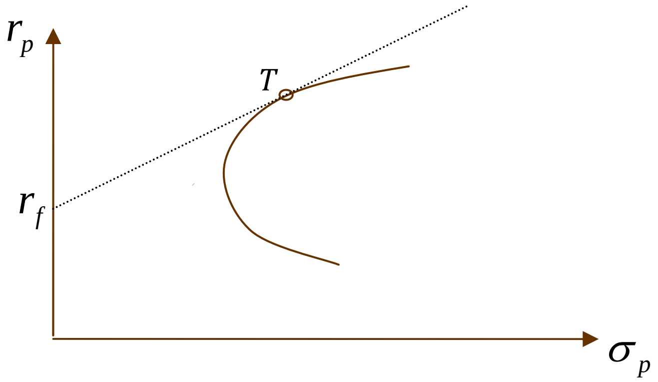
> **The Effcient Frontier:** Portfolio *1* is said to *mean-variance dominate* portfolio 2 if it has the same or a higher return, while its variance is lower, that is either in case i) $r_1 \geq r_2$ while $\sigma_1 < \sigma_2$, or case ii) $r_1 > r_2$ while $\sigma_1 \leq \sigma_2$. In this respect the *efficient frontier* is the set of all non-dominated portfolios.

The same argumentation may be made for portfolios of more than two assets. If there are *N* risky, imperfectly correlated assets, the efficient frontier will have a similar form as shown in figure 7.2. Each point on the efficient frontier described above represents the mean-variance characteristics of all portfolios of risky assets not dominated by other portfolios. By adding more assets to the two-asset framework of the example, the diversification possibilities will be improved, in principle displacing the efficient frontier to the left.

Note that normally a risk-free asset is also traded in the market. By only holding the risk-free asset, the investor has a risk-free portfolio with the expected return of the risk free rate. This thus offers further opportunities: An investor may then hold the risk-free asset in addition to a portfolio of risky assets. By successively holding a larger weight of the risky portfolio, the investor's portfolio will have a larger return, and a higher standard deviation. Furthermore, if it is possible to borrow at the risk-free rate to hold even higher shares of the risky asset, the investor may even construct a portfolio riskier than the riskiest portfolio of assets.

It can then be shown that the efficient frontier in the case of one risk-free and *N* risky assets is given by the straight line originating from the riskfree asset, and tangent to the risky asset frontier. Figure 7.3 shows the tangency portfolio *T*, the risk free asset, as well as the resulting efficient frontier which is given by the dotted line. Portfolios on this line will mean-variance dominate all other portfolios.

Figure 7.3 The Efficient Frontier: One risk-free and N risky assets



The investor's overall portfolio choice will thus lie on the efficient frontier. A highly risk averse investor will choose a high share of the risk-free asset, and a low share of risky assets, i.e. a low share of the risky portfolio *T*. A less risk-averse investor would however increase his share of the risky portfolio *T*. Note that all investors thus choose the same composition of the risky asset portfolio, i.e. the tangency portfolio *T*. Any differences in preferences merely imply different shares of the risk-free asset. The preferences do thus not affect which assets are included in the risky portfolio *T.* This is also the essence of the *two-fund theorem*, also known as the *separation theorem* because it implies that the optimal portfolio of risky assets can be identified separately from the investors' specific risk preferences. We will see that this result plays a significant role in the CAPM.

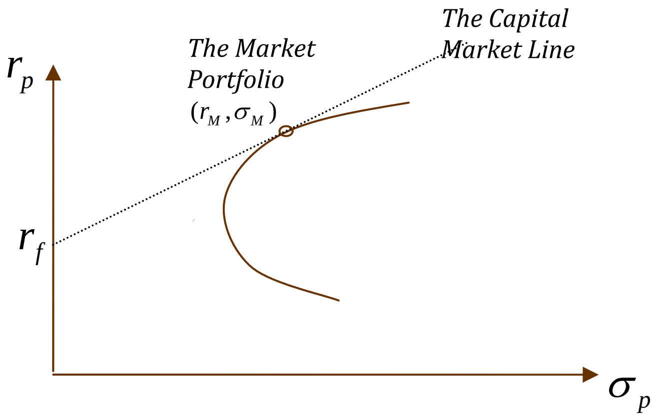
7.4 The Capital Asset Pricing Model

Till now, our discussion has been on investor choice, focusing on portfolio risk and the implications for portfolio choice. Based on modern portfolio theory, the CAPM derives the implications for the price of risk in an equilibrium market setting. Given that agents have homogeneous expectations and given the existence of a risk-free asset, the CAPM hypothesizes that the mean-variance efficient frontier will be the same for every investor. By the separation theorem, we saw that regardless of preferences, the investor chosen portfolio as to risky assets essentially will be the same portfolio, the tangency portfolio of figure 7.3. On this basis several characteristics of equilibrium asset and portfolio returns may be derived. Let us first review some basic concepts:

> **The Market Portfolio:** We argued that all investors would require shares in the same risky portfolio, i.e. the tangency portfolio of figure 7.3 and reproduced in figure 7.4. By definition of equilibrium, all existing assets must then belong to the portfolio (or else they would not be demanded, and thus not exist). This portfolio is termed the *market portfolio.* The market portfolio is then by definition efficient, since it is on the efficient frontier.

The Capital Market Line: We also argued that all optimal portfolios are located on the efficient frontier illustrated in figure 7.4. This line originating from $(r_f, 0)$, and going through the market portfolio (r_M, σ_M), is termed the Capital Market Line. The slope of the Capital Market Line is $\frac{r_M - r_f}{\sigma_M}$, and shows the compensation of the investor for a marginally riskier portfolio. As such it represents the market price of risk. The compensation or required return r_p for an efficient portfolio (i.e. a portfolio on the Capital Market Line) then is $r_p = r_f + \frac{r_M - r_f}{\sigma_M} \sigma_P$.

Figure 7.4 The Capital Market Line



Note that the Capital Market Line shown above, only applies to efficient (well-diversified) portfolios. All portfolios on the Capital Market Line are by definition efficient portfolios, i.e. portfolios in which all possible gains of diversification have been utilized. The efficient portfolio should thus compensated for its total risk. The risk compensation of an efficient portfolio can then be interpreted as the market price of risk $\frac{r_M - r_f}{\sigma_M}$, multiplied by the *total* risk σ_P of the (efficient) portfolio.

In assessing the value of an arbitrary asset or investment, we are, however, in essence interested in the risk compensation for assets *not* on the efficient frontier. In this case, the total risk of the portfolio consists of i) risk that cannot be diversified, and ii) risk that can be diversified. One of the major lessons of the CAPM is that only the non-diversifiable risk of an asset or portfolio is remunerated:

> *In equilibrium any asset/portfolio is compensated only for its non-diversifiable risk, also termed the systematic risk of the portfolio. This is the part of the portfolio risk which cannot be reduced /diversified by merely composing an alternative portfolio.*

The *non-diversifiable (systematic) risk* of an asset *i* is $\frac{\sigma_{iM}}{\sigma_M} = \rho_{iM}\sigma_i$. For any portfolio not fully correlated with the market, that is with $\rho_{iM} < 1$, we see that the risky asset is not compensated for its total risk since $\rho_{iM}\sigma_i < \sigma_i$. The remaining risk is a *diversifiable risk*, (also termed the *unsystematic* risk) and is thus not compensated. The risk compensation for an arbitrary asset (or portfolio) is thus still a product of the market price of risk $\frac{r_M - r_f}{\sigma_M}$, and the quantity of risk related to the portfolio, now represented by the systematic risk $\rho_{iM}\sigma_i$. This main result of CAPM may thus be summarized by the following relationship:

Required return risky asset i: $$r_i = r_f + \frac{r_M - r_f}{\sigma_M}\rho_{iM}\sigma_i$$

The intuition for why unsystematic risk is not remunerated may be more clear in a simple example of a portfolio of two assets *x* and *y* with the market prices of p_x and p_y. Let us then assume that the assets have negatively correlated returns. Now consider a portfolio *z* of these two stocks. Since the assets are negatively correlated, we know that the portfolio *z* is less risky than either of *x* or *y*. It then cannot be the case that p_z is high, because it corresponds to a less risky portfolio, while at the same time p_x and p_y are low because they are risky[26]. Rather, it is more reasonable that the price of *z* should reflect the prices of the two assets, so that the price of *z* is the share-weighted price of the assets in the portfolio, that is $p_z = w_x p_x + w_y p_y$. (This is also the essence of the value additivity theorem). The insight of CAPM shows why this relationship should hold; diversifiable risk which is eliminated in an efficient portfolio is *not* priced. It is only the non-diversifiable risk which is priced, thus, stating the cause for why the value additivity theorem holds. In this sense all prices reflect the value only of the non-diversifiable risk.

Finally, note that the standard CAPM equation is often represented in its 'beta'-form:

Expected return risky asset i: $r_i = r_f + (r_M - r_f)\beta_i$, where $\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$.

The β_i is thus the ratio of the covariance between the returns on asset *i* and the market portfolio returns, over the variance of the market returns[27]. The beta-formulation of the CAPM states, in other words, that the expected excess return or risk premium for asset *i* is proportional to its β_i. The β_i is then the sole specific

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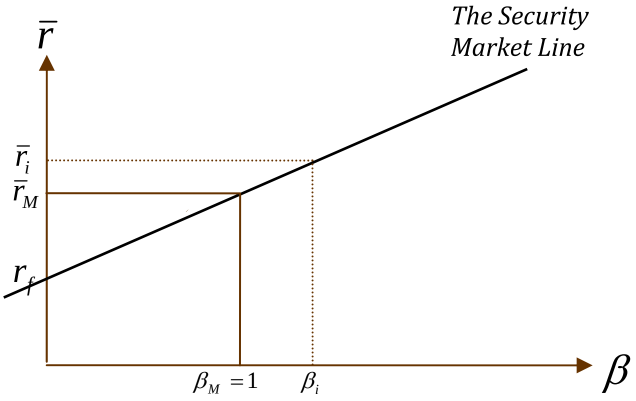
[26] The relationship between prices and the required return is close: A risky asset in principle requires a higher return to compensate for the higher risk. By discounting cash flows at the higher return rate, the resulting value or price will thus be lower.

[27] To sum up, note that $\rho_{iM} = \frac{\sigma_{iM}}{\sigma_i \sigma_M}$, and $\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$. Then there is the following connections between the different CAPM representations shown above: $r_i = r_f + \frac{r_M - r_f}{\sigma_M}\rho_{iM}\sigma_i = r_f + \frac{r_M - r_f}{\sigma_M}\frac{\sigma_{iM}}{\sigma_M}$ $= r_f + \frac{r_M - r_f}{\sigma_M}\beta_i\sigma_M = r_f + (r_M - r_f)\beta_i$. Note that since the covariance σ_{iM} in principle can be either positive or negative, also the β can in principle can be positive or negative.

determinant of excess return[28]. Figure 7.5 shows the relationship between expected return and the asset's β_i. This line is termed the *Security Market Line*.

Figure 7.5 The Security Market Line



Now we may return to our main focus, that is, to value a non-traded risky cash flow as for example a grid investment project. By the CAPM approach the value of the grid investment is found by discounting the expected cash flows by its appropriate risk-adjusted required return. The appropriate rate is thus the expected rate of return on other traded financial assets that represent cash flows of similar risk as the project in question. More specifically, the relevant risk of the investment project, i.e. the systematic risk, is given by the β of the project. The β in essence

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[28] An insight of previous chapters is that a higher payoff in states of scarcity is more worth than in states of abundance. This insight is reflected in the main results of the CAPM. Note that in a sense the value of the market portfolio represents the general scarcity of the society; when its value is high, there is a general net abundance; when its value is low, there is a general net scarcity. The CAPM then tells us that a security or investment is relatively undesirable the more it covaries positively with the market portfolio; This is because the cash flow of the investment is high when the market portfolio value is high, i.e. in times of abundance. Likewise, the cash flow of the investment is low, when the market portfolio value is low, i.e. in times of scarcity. In contrast, an asset or investment that covaries negatively would have its largest payoffs in times of scarcity. Thus, the intuition is that the more the asset covaries with the market portfolio, the higher return is commanded, and the lesser the value of its cash flow (since it is discounted by a higher required rate of return).

mirrors the extent to which the grid investment outcomes covary with the market outcome. The appropriate expected return is thus the rate $r_i = r_f + (r_M - r_f)\beta_i$, where the β_i is the project's β.

In applying the CAPM to grid investments the main input is thus the estimated β of the project. Since grid investment cash flows to a large extent may reflect aspects of grid failure outcomes, the question is to what extent these incidents reflect the ups and downs of the market portfolio, that is, how the grid investment cash flow covaries with the market portfolio. The following chapter discusses the β of grid companies, also referring to existing reports on the subject.

Before leaving this introduction to the CAPM, let us briefly make a few further comments upon the method. As mentioned at the outset, though widely used, even the CAPM does not provide a 'true' valuation. The CAPM is an equilibrium model that indicates the equilibrium expected return of a risky asset in an efficient capital market. Originally derived as a one-period model, subsequent research has extended its use to the multiperiod case. Empirical research does support the idea that risk compensation is related to the asset's non-diversifiable risk. However, empirical research also shows that the CAPM does not fully manage to explain the pricing of risk. A large range of theoretical and empirical research work has continuously explored and extended the boundaries of the original CAPM. We refer to Danthine and Donaldson (2005) or Pennacchi (2008) as a starting point for the interested reader to look into further aspects of the CAPM. See also e.g. Cochrane (2005) for a discussion on how different financial models can be applied to estimate the risk-adjusted discount factor. We also refer to Ekern (2006) which gives an extensive overview of different consistent CAPM-related methods.

8 Grid Investment Valuation in Practice

This report has given an introduction to economic aspects of valuing grid investments. We found that the different asset pricing theories convey different important insights as to the value of grid investments. Each method, however, has its shortcomings. Still, for practical implementation, the CAPM-based valuation will probably continue to be an essential basis for valuation of grid investments. We conclude this report with a few further comments on grid investment valuation using the CAPM approach. Two basic inputs to the CAPM-based grid investment valuation are the expected cash flow, and the appropriate risk-adjusted required return. Section 8.1 comments upon the required return for grid investments, here focusing on the β of grid investments. Section 8.2 concludes the report by revisiting cash flow construction, now commenting upon the link to the RISK-DSAM methodology.

8.1 CAPM-Based Valuation: The β of Grid Investment Projects

Using the CAPM-based valuation procedure, the expected cash flow is discounted by its appropriate required rate of return. More specifically, we recall that the required return of a risky asset by the CAPM was given by:

$$r_i = r_f + \beta_i (r_M - r_f), \text{ where } \beta_i = \frac{\sigma_{iM}}{\sigma_M^2}.$$

Thus, the required return is the risk free interest rate r_f with the addition of a part β_i of the market premium $(r_M - r_f)$. The risk free rate is often based on the estimated long-term risk free interest rate r_f. The market premium is often based on historically observed returns for the total market. Though it is not straightforward to estimate neither the risk free rate nor the market premium

$(r_M - r_f)$, these aspects are as such, general to all assets and will not be further commented upon here[29]. It is the project beta β_i that represents the systematic risk level of the investment project, and which will be the subject of the discussion in this section.

We recall that the systematic risk, represented by the project beta, is the ratio of the covariance of grid investment and market portfolio returns, over the market portfolio variance. A basic question is thus to what extent grid investment outcome varies with the market portfolio. Let us start by looking at the β of grid *companies*.

In general, the electric grid business has been considered an activity of low systematic risk[30]. In other words, the covariance between grid company returns and the market portfolio return, has been found to be low. Looking at grid company betas, defined as the beta of the company's employed capital[31], we find that the estimated company betas are considerably lower than 1. Dreber Lundkvist & PwC (2004) find beta estimates of 0.25-0.45, with a mid value of 0.35. NVE(2005b) estimates are 0.25-0.35, mid value of 0.3. Johnsen (2005) estimates a company beta of 0.4, while NVE(2006) has changed its estimates to 0.35. While the estimates consistently support the idea of a low grid company β, we also see there may be many different opinions as to the company beta, reflecting different views

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[29] See for example Johnsen (2005) for a further discussion.

[30] See e.g. NVE(2005a), NVE(2005b), Dreber Lundkvist & PwC (2004), Johnsen(2005). Note that the focus of these reports is to estimate the company beta in order to establish the allowed rate of return for the grid company. In this respect, a higher (lower) estimated beta will yield a higher (lower) rate of return, and thus a higher (lower) allowed level of revenues. *In addressing investments, note that for a given cash flow, a higher (lower) estimated beta, and thus a higher (lower) required rate of return, will yield a lower (higher) estimated value of a given investment project.*

[31] Johnsen (2005) gives a thorough discussion on the required return of grid companies, discussing the risk free rate, the market premium, weighted return, before and after tax rates, as well as the company beta β, the latter defined as a weighted equity beta, i.e. $\beta = \frac{E}{E+D}\beta_e$, where *E* is the company equity, *D* is the company debt, and β_e is the equity beta.

on the systematic risk of the grid company. We will not go into a closer discussion on company betas, and refer to the above papers. It is however interesting to point to explanatory aspects:

One of the reasons for a low β is attributed to the effect of regulation, where the current regulatory regime in effect establishes links between grid company costs and revenues: To some extent, changes in costs are matched by changes in revenue, thus contributing to a relatively more stable net return than for other industries.

Still, one might argue that the regulatory company-specific efficiency requirements in fact may add to the uncertainty faced by the company. For example, a company efficiency performance below the industry average, may result in lower allowed company income and returns. Variations in performance may thus add to the total risk of the company. Note, however, that this reasoning applies to company-specific cost changes relative to the industry average. As such it may be argued that this company-specific cost uncertainty in effect is unsystematic risk.

In contrast, it might be that the industry-wide variations in average costs levels covary with the market portfolio return. In principle such uncertainty thus represents systematic risk. However, if these cost variations due to regulatory provisions, are met by increased revenue, the effect of industry wide cost variations thus cancel out, thus also reducing risk. Together, these factors thus support the notion of a low grid company β.

In addressing the specific *grid investment*, we are however interested in the risk of the grid investment project itself, i.e. the project β. The question is thus to what extent grid investment returns are correlated with the market portfolio. For the grid investment, we note that a large degree of uncertainty often is related to future improved grid performance. A qualified guess would then be that investments outcomes, such as the expected avoidance of grid failures in distribution networks, are not likely to consistently follow the variations of the market portfolio, thus indicating low project beta. The grid company beta therefore represents a good starting point. If the project risk is similar to the general activities of the grid company, it is thus reasonable to base investment analysis on the grid company beta. Grid investment projects that for some reason indicate less (higher) systematic risk, would call for a lower (higher) beta.

Following this discussion on grid investment betas, and thus also the required return for grid investment projects, we see that there may be considerable uncertainty as to the ‘right’ beta, and the ‘right’ required return. As such, an important input to the investment decision, would be to perform a sensitivity analysis as to how the investment value depends upon the chosen rate of return.

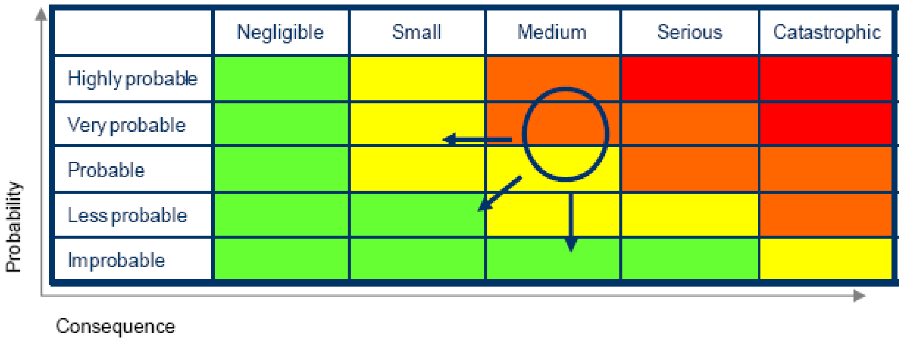
8.2 Grid Investment Cash Flow Revisited

The quality of the investment decision ultimately rests upon the quality of the underlying expected cash flow. The RISK-DSAM methodology as such provides important input to identifying grid investment alternatives, as well as a basis for cash flow estimation. Referring to Nybø and Nordgård (2010), we will in this concluding section briefly comment upon links between economic investment analysis and these guidelines for risk-based maintenance and reinvestment management of electricity distribution assets.

Products of Grid Investments

Investments in the distribution grid system are often related to improved performance, for example in terms of the reliability of the grid. Grid performance may be viewed as a probability distribution, represented by the possible consequences and their probability of occurrence. As discussed in chapter 2, grid investments may as such be viewed as measures for reducing the risk of the grid; i.e. by reducing the probability of occurrence, and/or the severity of possible consequences. This is alternatively illustrated as in figure 8.1.

Figure 8.1 Risk Matrix (Figure 1.4 Nybø and Nordgård(2010))



	Negligible	Small	Medium	Serious	Catastrophic
Highly probable	🟩	🟨	🟧	🟥	🟥
Very probable	🟩	🟨	🟧	🟧	🟥
Probable	🟩	🟨	🟨	🟧	🟧
Less probable	🟩	🟩	🟨	🟨	🟧
Improbable	🟩	🟩	🟩	🟩	🟨

Identifying Grid Investment Alternatives

A good investment decision rests not only upon a correct investment analysis. A basic prerequisite is that the right investment alternatives are identified. The distribution grid is as such a complex network of numerous components, where there also is uncertainty as to the technical condition of different parts of the network. As such, the RISK DSAM methodology provides guidelines to develop risk-based maintenance and investment strategies.

In chapter 2 we argued that the product of the investment in essence is the *change* (improvement) in the reliability (i.e. the risk factors) of the network. In this, a sorting of networks areas according to the above table of figure 8.1, would be an important tool to identifying alternatives that presumably are candidates for the most profitable gird investment alternatives. This is also illustrated in figure 8.2.

Figure 8.2 Risk Differentiation (Figure 1.5 Nybø and Nordgård(2010))

For example, investments that change the reliability from a 'red category' situation (with severe consequences, and high probability of failure), to a 'green category' situation (with negligible consequences, and low probability of failure), may potentially indicate a profitable area of investment.

Nybø and Nordgård (2010) further elaborate how assessment of risk (i.e. probability of failure, and severity of consequences) can be used to establish maintenance and (re)investment strategies. The main idea is to direct the grid company effort to where "*there is most to gain; spending fewer resources on low risk assets and more on high risk reinvestment resources*". In other words, this methodology contributes to indentifying areas for which maintenance and (re-) investments presumably are most profitable.

Important Alternatives: The Value of Information and of Waiting

The grid company faces several alternatives. We have above considered different alternatives of investments, such as comparing different areas of investments, or different methods of upgrading the network. An important acknowledgment here is that our knowledge of the network quality is imperfect. Important alternatives of action are thus:

- **Information gathering:** The value of an investment follows from the change from the status quo condition of the network, to the future post investment condition of the network. The investment analysis indicates whether the investment is profitable based on the current available information. In many instances, the value of gathering further information as to the network condition and/or possible lines of action may however be high. The value of information follows as the grid owner *now will have the option to choose other alternatives, or even not to invest, thus reflecting the best choice dependent upon the information received.*

- **Value of postponement:** Closely related is the value of waiting. The company alternatives are *not* restricted to either a current full scale investment, or no investment at all. A further alternative is to *postpone* the investment, for a shorter or longer period of time. This may be a full postponement with no current actions, or a semi-postponement represented by a smaller-scale investment or maintenance action. The costs of postponement/ costs of waiting follow from the costs of postponing an improvement in reliability. The benefits of postponement are related to the value of information received in the mean time. This may be information received as to the network condition, characteristics of demand, technological options, etc. On receiving this information, a better decision can potentially be made. *The value of waiting thus may be attributed to the value of the option of making a better decision at a later point of time.*

Cash Flow Construction

Using the CAPM-based valuation approach, the important investment data is the *expected cash flow*, that is the expected consequences of the investment valued in monetary terms. The risk matrix of the RISK DSAM methodology may here provide a useful framework for assessing the expected cash flow of the Status Quo, as well as the post investment scenarios. As such, the investor has to assess the expected value[32] of consequences in both scenarios, for example in the current 'red box' category, as compared to the target 'green box' category. The difference in expected values for each future point of time is thus the expected investment cash flow.

Investment Valuation

The final step is then to value the expected cash flow. By discounting the cash flow by the risk-adjusted discount rate $r_i = r_f + \beta_i (r_M - r_f)$, we account for the time value of money, as well as for the value of risk. The risk-adjusted present value represents the value of the investment alternative studied. The ultimate decision may rest upon further criteria, such as safety and environmental considerations. To consider the robustness of the investment decision, a sensitivity analysis should however also be carried out for crucial aspects of the estimated cash flow, as well as for the risk-adjusted discount factor.

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[32] We recall that the expected value is a probability weighted sum of the monetary valued consequences, that is including favorable (no failure) as well as unfavorable (failure) consequences.

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