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by

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***Abstract*—This paper focuses on the combined effects of loss factors and zonal pricing in a system that takes as a starting point the procedures of transmission pricing in the Norwegian power system. It interprets the notion of loss factors in an AC power flow model, and develops a method for finding zonal prices consistent with a restricted AC optimal power flow. The effects of decomposing the nodal prices to find a loss factor to cover the marginal cost of losses, and letting the congestion management be resolved by zonal prices based on a DC approximation of the flows, are evaluated by comparing the resulting net prices to those of an optimal AC power flow (AC-OPF).**

***Index Terms*—Congestion management, deregulated electricity markets, optimal power flow, loss factors, nodal pricing, Nord Pool, Norwegian power market, pool model, transmission pricing, zonal pricing.**

I. Introduction

The pool model has been realized in different ways in the numerous countries that have deregulated their power sectors. Thus, there are marketplaces where locational marginal prices are used, while the operation mode of other markets adheres to a uniform market clearing price. In addition, the choice between DC and AC power flow leads to quite different market performances. In this paper an operation mode parallel to that of the Norwegian electricity market is investigated.

The Norwegian power market is part of a larger international deregulated marketplace, the well-known Nord Pool [1], [2]. After the eastern part of Denmark joined Nord Pool in 2000, the common marketplace comprises all the Nordic countries except Iceland. The Nordic marketplace has five system operators and a pool operator whose responsibility is the operation of the Nord Pool Power Exchange. The system operators in each region are shown in Fig. 1. Only Denmark has two grid operators. The five regional system operators cooperate through Nordel, an organization which was established as early as in the 1960s in order to support the power trade between the Nordic countries.

In [3] the congestion management mechanisms of the Nordic countries are compared with the ones of other well-known liberalized markets using a unified framework. The generation scheduling issue in the Norwegian market is investigated in [4], while the electricity market reform in some countries, including Norway, is the topic discussed in [5]. Further economic investigations are given in [6], where the performance of energy prices in England and Norway is discussed considering bilateral contracts and the spot market.

The rest of this paper is structured as follows. In section II the transmission pricing methods of the Nordic market are described, with emphasis on the Norwegian pricing rules. Section III presents a novel method for the assessment of an operation mode like the Norwegian. Numerical examples are provided in section IV, pointing out the effects on the pricing by using zonal pricing, DC power flows and loss factors. Finally, conclusions are drawn in section V.

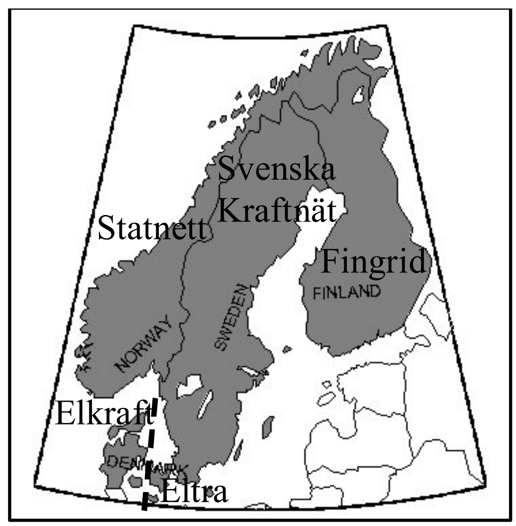

Fig. 1. The Nordic market

II. Transmission Pricing in the Nordic Market

At the Nordic level, zonal pricing is used for inter-zonal congestion while counter-trading [7] is adopted to resolve intra-zonal constraints. Norway may be split into two or more zones, whereas the areas of the other system operators have uniform prices.

The other part of the Norwegian transmission pricing is the loss factors. The Norwegian high voltage network spans 166 nodes. Periodically, state-owned Statnett, the Norwegian system operator, announces loss factors for each system node for a time interval of 6-10 weeks, with different prices for day and night/weekend. The objective of these factors is to charge for marginal losses. The factors are given as a percentage of

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the load at each system node, and each participant has to pay, for each traded MWh, to the central grid, an amount equal to the loss factor of the node where the participant is located, times the system price. Agents that reduce losses will be compensated. Of course, this will affect the bids that the players submit to the Pool, as shown in Fig. 2, where linear function bids of market participants are depicted.

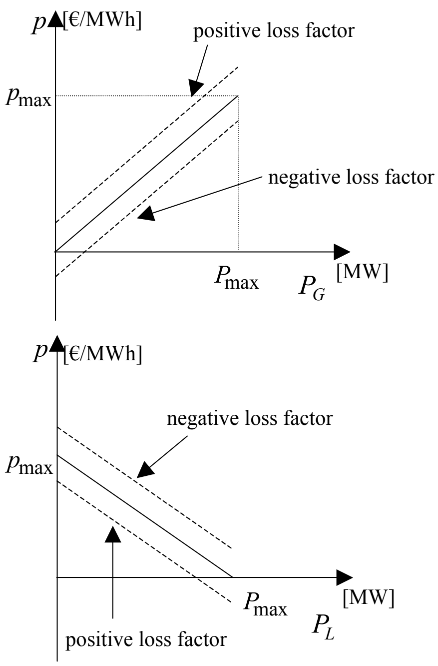

Fig. 2. The loss factors for suppliers (upper) and consumers (lower)

The upper diagram of Fig. 2 shows the bid curves of a supplier. The marginal cost curve is given by the solid line, and assuming no strategic bidding, the supplier will inject power into the network up to a maximum of $P_{\max}$ when the price is $p_{\max}$. When facing a positive loss factor, i.e. additional cost, the supplier will increase the price and so the new bid curve is the upper dashed line. If the loss factor is negative, then the supplier reduces the price, and so the new bid curve is the lower dashed line. The lower diagram of Fig. 2 presents the bid curves of a consumer. In this case, a positive loss factor results in a decrease of power consumed for a given price, i.e. the lower dashed line, while a negative loss factor has the opposite result, which is shown by the upper dashed line. In any case, for both suppliers and consumers, the shift of the original bid curve along the y-axis is equal to the charge for the marginal losses.

While losses are accounted for ex ante of bidding, congestion is taken care of by means of zonal pricing in the (day-ahead) spot market. In zonal pricing the system operator divides the grid into two or more zones and then runs an optimal power flow, taking into account the transfer limits between zones and requiring uniform prices for all the buses belonging to a given zone. Through this mechanism, the system operator reduces the power flow over the lines where the original market equilibrium led to a violation of transfer limits. Any remaining (intra-zonal) constraints are resolved by counter trading. Thus, for the Norwegian transmission pricing, a sequential pricing rule is adopted, starting with loss factor announcements, continuing with zonal pricing in the day-ahead market, and finally counter trading if necessary.

It is clear that different ways of dividing the grid into zones, and also to determine transfer limits between zones, lead to quite different market outcomes [8], [9]. However, in this paper we will focus on the decomposition of losses and congestion, and the effect of introducing DC power flows into this procedure.

III. Formulation of Loss Factors and Zonal Pricing in AC-OPF

The investigation of the operation mode of the Norwegian electricity market has been made using a complete AC-OPF. Both the use of loss factors and zonal pricing have been analyzed and interesting conclusions can been drawn. This section presents the necessary theoretical background in order to carry out this investigation.

The following steps describe the procedure, which leads to the assessment of an operation mode similar to the Norwegian.

Step 1: A complete AC-OPF is used. Nodal prices are calculated for each node.
Step 2: The loss component of the nodal prices is computed.
Step 3: This loss component is used as the nodal loss factor.
Step 4: The participants' bids are modified according to the loss component.
Step 5: The modified bids are given as input in a DC-OPF.
Step 6: New nodal/zonal prices are calculated.
Step 7: The net price of each participant is defined by adding/subtracting the loss factor from the new nodal/zonal prices.
Step 8: Comparisons are made between the original nodal prices, obtained by AC-OPF, and the new net nodal prices.

A. Loss Component

The procedure described above indicates the need for computing the loss components of the nodal prices. In [11] the nodal prices are decomposed into two components, one which is due to generation and losses, and another one corresponding to system congestions. Further work on nodal price analysis can also be found in [12]. For the scope of this paper it is necessary to identify the part of the generation/loss component that is caused by the system power losses. In this it is assumed that the nodal price at the reference bus, which is used for the nodal price analysis, is due only to generation. The assumption of the nodal price analysis shown in [11] is that, at the reference bus a marginal increase of demand can be locally covered by the bus generator. If this hypothesis holds then the marginal increase of demand at the reference bus does not cause additional power losses. Thus, it is so that:

$$\frac{\partial P_{Losses}}{\partial P_{L,r}} = 0 \tag{1}$$

where $P_{L,r}$, P_{Losses} are the power demand at reference bus r and the system power losses respectively. Assuming that (1) is satisfied, one may write for the reference bus that:

$$\lambda_{GL,r} = \lambda_{G,r} \tag{2.a}$$

$$\lambda_{Los,r} = 0 \tag{2.b}$$

where λ_G is the nodal price component due to generation (equal for all the system buses), λ_{Los} is the nodal price component due to losses, and λ_{GL} is the component due to generation and losses. Consequently, the loss component of the nodal price, at any system bus *i*, is given from the following equation:

$$\lambda_{Los,i} = \lambda_{GL,i} - \lambda_{G,r} \tag{3}$$

B. Zonal Pricing

A major characteristic of the Norwegian market operation mode is the use of zonal pricing as congestion management tool. The same tool is used in the Nordic market for inter-zonal congestions. In the case that zonal pricing is active all the nodes that belong to a zone face the same price. An objective of this paper is to simulate a zonal pricing situation within a complete AC-OPF (nodal price mechanism). The zonal pricing situation requires the same nodal prices for all nodes that belong to a zone.

Usually, the nodal price at a bus is equal to the marginal cost of this bus which is given by its bid curve. That is:

$$\lambda_i = -\frac{\partial K(\mathbf{P}_G)}{\partial P_{G,i}} \tag{4}$$

where $K(\mathbf{P}_G)$ is the objective function of the optimization problem. This function consists of the total cost, i.e. cost for generation and cost for not covering the demand. Since the demand can be simulated through fictitious generators, as it is proposed in [11], the vector $\mathbf{P}_G$ includes the power generation of real and fictitious generators as well. Through the optimization procedure this objective function has to be minimized. $P_{G,i}$ is the power generation at bus *i*.

Thus, if the aim is to obtain equal nodal prices for all the nodes that participate in a zone, an additional restriction must be put in the OPF in order to achieve this goal. This restriction demands that the generators of the buses, which participate in a zone, should operate with equal marginal cost.

The market operator may treat the supplier bid curves, shown in Fig. 2, as marginal cost curves resulting from polynomial cost functions. Moreover, the consumer bid curves may be simulated through the bid curve of a fictitious generator. The production of this fictitious generator would represent the uncovered part of demand. Thus, the bid curves of any participant may be deduced from a polynomial cost function. If the bid curves of Fig. 2 are treated as marginal cost curves then the corresponding cost function has the following form:

$$K(P_G) = aP_G^2 + c \tag{5}$$

where *a*, and *c* are constants. It should be underlined, that the possible existence of the first-degree term in (5) would not have distorted the generality of the following analysis. Hence, the analysis is also valid for the bid curves that are given by the dashed lines in Fig. 2. For any generator *i* the marginal cost, resulting from (5), is:

$$\text{marginal cost}\,(P_{G,i}) = 2a_i P_{G,i} \tag{6}$$

From Fig. 2 one may find out that the factor a_i is given by the relationship:

$$a_i = \frac{p_i}{2P_i} \tag{7}$$

where (p_i, P_i) is any corresponding pair of the bid curve. Consider now that in case of zonal pricing, there is a zone consisting of bus 1 and bus 2. Both of them are generation buses. If the generation limit at these two buses is not reached then the two generators should operate with equal marginal cost in order to face the same nodal price. Thus, it is:

$$\begin{gathered}(\text{marginal cost})_1 = (\text{marginal cost})_2 \Rightarrow \\ \Rightarrow 2a_1 P_{G,1} = 2a_2 P_{G,2} \Rightarrow P_{G,1} - \frac{a_2}{a_1} P_{G,2} = 0\end{gathered} \tag{8}$$

Equation (8) is the additional restriction which has to be incorporated in the OPF so as to obtain the same nodal prices for the buses 1 and 2. Consequently, the OPF is now:

$$\begin{aligned} \min\ & K(\mathbf{P}_G) \\ s.t\ \ & \mathbf{f}(\mathbf{x}) = \mathbf{0} \\ & \mathbf{g}(\mathbf{x}) < \mathbf{0} \\ & z(P_{G,1}, P_{G,2}) = 0 \end{aligned} \tag{9}$$

where **f** are the equality restrictions for nodal power balance, **g** are the inequality restrictions of the power system and *z* is the zonal pricing restriction for the buses 1 and 2. The vector $\mathbf{x} = (\mathbf{P}_G, \mathbf{Q}_G, \mathbf{V}, \boldsymbol{\theta})$ consists of the active and reactive power generation, the voltage and the phase angle of each node. The Lagrange function, which corresponds to (9), is as follows:

$$L(\mathbf{x}) = K(\mathbf{P}_G) + \boldsymbol{\lambda}\mathbf{f}(\mathbf{x}) + \boldsymbol{\mu}\mathbf{g}(\mathbf{x}) + \xi\, z(P_{G,1}, P_{G,2}) \tag{10}$$

where **λ**, **μ** *and* ξ are the Lagrange multipliers of the corresponding restrictions. At the optimal point, according to the Kuhn-Tucker theorem, it is:

$$0 = \nabla_{\mathbf{x}} K(\mathbf{P}_G) + \boldsymbol{\lambda}\nabla_{\mathbf{x}}\mathbf{f}(\mathbf{x}) + \boldsymbol{\mu}\nabla_{\mathbf{x}}\mathbf{g}(\mathbf{x}) + \xi\,\nabla_{\mathbf{x}} z(P_{G,1}, P_{G,2}) \tag{11}$$

Equation (11) facilitates the determination of a market equilibrium given the restriction of equal nodal prices for all generators participating in a zone. This common nodal price can be found by choosing from all the derivatives shown in (11) the ones with respect to P_{G1} and P_{G2}. The derivatives of **g** with respect to these two variables will be either zero (for the voltage, reactive production and power flow limits) or the corresponding multiplier μ will be zero for the active production limit since it is assumed that this limit is not reached for these two generators. Thus, it is so that:

$$\begin{gathered} 0 = \frac{\partial K(\mathbf{P}_G)}{\partial P_{G,1}} - \boldsymbol{\lambda}\frac{\partial \mathbf{f}(\mathbf{x})}{\partial P_{G,1}} - \xi\frac{\partial z(P_{G,1}, P_{G,2})}{\partial P_{G,1}} \Rightarrow \\ \Rightarrow \frac{\partial K(\mathbf{P})}{\partial P_{G,1}} = \lambda_1 + \xi \Rightarrow \lambda_{\text{expected}} = \lambda_1 + \xi \end{gathered} \tag{12a}$$

$$\begin{gathered} 0 = \frac{\partial K(\mathbf{P}_G)}{\partial P_{G,2}} - \boldsymbol{\lambda}\frac{\partial \mathbf{f}(\mathbf{x})}{\partial P_{G,2}} - \xi\frac{\partial z(P_{G,1}, P_{G,2})}{\partial P_{G,2}} \Rightarrow \\ \Rightarrow \frac{\partial K(\mathbf{P})}{\partial P_{G,2}} = \lambda_2 - \xi\frac{a_2}{a_1} \Rightarrow \lambda_{\text{expected}} = \lambda_2 - \xi\frac{a_2}{a_1} \end{gathered} \tag{12b}$$

From both (12a) and (12b) it is obvious that the price which has to be adopted as the common nodal price for the buses 1 and 2 is $\lambda_{\text{expected}}$. This price is equal to their common marginal cost and so it will be accepted by both participants.

It is important to underline that now the Lagrange multipliers λ_1 and λ_2 cannot be used as nodal prices for the buses 1 and 2. Both (12a) and (12b) point out that λ_1, λ_2 are different than the corresponding bid of the two producers, which is equal to the $\lambda_{\text{expected}}$. However, it can be shown that there is a fixed relationship between λ_1, λ_2 and $\lambda_{\text{expected}}$. By multiplying (12a) by a_2/a_1 and then adding it into the (12b) it is:

$$\frac{a_2}{a_1}\lambda_{\text{expected}} + \lambda_{\text{expected}} = \frac{a_2}{a_1}\lambda_1 + \frac{a_2}{a_1}\xi + \lambda_2 - \frac{a_2}{a_1}\xi \Rightarrow$$

$$(\frac{a_2}{a_1}+1)\lambda_{\text{expected}} = \frac{a_2}{a_1}\lambda_1 + \lambda_2 \Rightarrow \lambda_{\text{expected}} = \frac{\frac{a_2}{a_1}\lambda_1 + \lambda_2}{\frac{a_2}{a_1}+1} \Rightarrow$$

$$\lambda_{\text{expected}} = \frac{\frac{a_2}{a_1}\lambda_1 + \frac{a_2}{a_2}\lambda_2}{\frac{a_2}{a_1} + \frac{a_2}{a_2}} \quad (13)$$

Equation (13) can be generalized for *n* generators participating in a zone as follows:

$$\lambda_{\text{expected}} = \frac{\frac{a_n}{a_1}\lambda_1 + \frac{a_n}{a_2}\lambda_2 + .. + \frac{a_n}{a_n}\lambda_n}{\frac{a_n}{a_1} + \frac{a_n}{a_2} + .. + \frac{a_n}{a_n}} \quad (14)$$

Consequently, the common marginal cost $\lambda_{\text{expected}}$ is the weighted average of the Lagrange multipliers λ_i. At this point it should be underlined that (14) holds if the production limits of generators participating in a zone are not reached. If such limits are reached then (14) is affected by the corresponding Lagrange multiplier μ. However, it is necessary to mention that (14) is not needed in order to calculate the common marginal cost. This aim is served by the additional restriction introduced into the OPF. Once the power output of a generator participating into a zone is obtained, as co-product of OPF, the marginal cost can be estimated from the bid curve of this generator. The usefulness of (14) is in showing that there is a standard relationship between the Lagrange multipliers of the buses belonging to a zone, and their common marginal cost, given that the production constraints are not active.

IV. Numerical Examples

The analysis which is presented in Section III provides the necessary methods in order to assess a stepwise market operation mode like the Norwegian. The use of these methods will be highlighted using a 10-bus test system which is illustrated in Figure 3. The market, which is represented by this system, consists of four suppliers and four consumers. The participants' bids have the form which is shown in Fig. 2. The network data as well as the necessary data for the market players' marginal cost and benefit functions are given in the Appendix, Tables A1 and A2. Four different cases will be studied through subsections A to D.

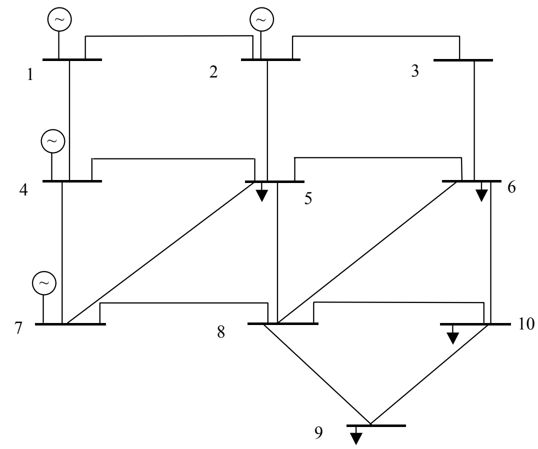

Fig. 3. 10-bus test system.

The assessment of the operation mode consists of calculating the deviations between the nodal prices obtained by AC-OPF and the prices that would be obtained if the system operates similar to the Norwegian market, with a sequential approach to transmission pricing, relying on ex ante announcement of loss factors and computing zonal prices based on adjusted bids and a DC computation of the flows. The loss factors are given directly in ct/kWh and not in % of the nodal price. The latter is also given when randomly chosen factors are used. All computations have been carried out with MATPOWER [13].

A. Unconstrained System

In case A, it is assumed that there are no binding transmission constraints in the system. Following the procedure of section III, nodal prices are calculated, assuming bids are provided to a market operator according to the marginal cost data of Table A2, and that the market operator runs an AC-OPF.

TABLE I
CASE A – UNCONSTRAINED WITH LOSS FACTORS FROM AC
(all prices in ct/kWh)

Bus	Nodal price AC	Loss factor	Nodal price DC	Net price DC	Difference: Net price DC- Nodal price AC
1	2.9296	-0.0234	2.9174	2.9408	0.0112
2	2.9527	-0.0003	2.9174	2.9177	-0.0350
3	2.9427	-0.0103	2.9174	2.9277	-0.0150
4	2.9565	0.0035	2.9174	2.9139	-0.0426
5	2.9633	0.0103	2.9174	2.9277	-0.0356
6	2.9526	-0.0004	2.9174	2.9170	-0.0356
7	2.9530	0.0000	2.9174	2.9174	-0.0356
8	2.9609	0.0079	2.9174	2.9408	-0.0201
9	2.9683	0.0153	2.9174	2.9177	-0.0506
10	2.9680	0.0150	2.9174	2.9277	-0.0403

In Table I, second column, the calculated nodal prices (in ct/kWh) are presented. To find the outcome of the stepwise pricing procedure, it is necessary to define a set of loss factors, and it is assumed that the market operator adopts the loss components of the nodal prices as defined in section III, as loss factors. Here, the loss factor is given directly as real number and not as % of nodal price. Bus 7 is used as reference bus since the nodal price of this bus is at the middle of the price spectrum, implying positive as well as negative loss components. Moreover, at bus 7 a generator with large capacity is located.

TABLE II
CASE A – UNCONSTRAINED WITH RANDOM LOSS FACTORS
(all prices in ct/kWh)

Bus	Nodal price AC	Loss factor	Loss factor as % of nodal price	Nodal price DC	Net price DC	Difference: Net price DC- Nodal price AC
1	2.9296	0.1465	+5	2.9707	2.8242	-0.1054
2	2.9527	0.0591	+2	2.9707	2.9116	-0.0411
3	2.9427	0	0	2.9707	2.9707	0.0280
4	2.9565	-0.0887	-3	2.9707	3.0594	0.1029
5	2.9633	0.0593	+2	2.9707	3.0300	0.0667
6	2.9526	0.1181	+4	2.9707	3.0888	0.1362
7	2.9530	0	0	2.9707	2.9707	0.0177
8	2.9609	0	0	2.9707	2.9707	0.0098
9	2.9683	-0.1781	-6	2.9707	2.7926	-0.1757
10	2.9680	0.1187	+4	2.9707	3.0894	0.1214

The calculated loss components are included in the third column of Table I. The next stage is the modification of the original bid curves of the players according to the loss factors, refer Fig. 2. The market operator then receives the adjusted bid curves and runs a DC-OPF.

The resulting nodal prices are shown in Table I in the fourth column. Since there is no congestion, these prices are equal. However, the final prices that the market participants face result from subtracting the loss factors from the nodal DC prices, i.e. the net DC prices of column 5 in Table I. The deviations of these prices from the nodal prices of the AC-OPF are given in the last column of Table I. It is interesting to note that even if there is no congestion and loss factors are based on the real AC situation, the DC approximation leads to different nodal prices than the AC-OPF, and that the differences in this case are of the same magnitude as the loss factors.

In the Norwegian system, loss factors are computed and announced for periods of 6-10 weeks, and may consequently apply for hundreds of hourly market clearing situations. Thus, it is reasonable to conclude that, in most cases, the constant loss factors are not associated with the changing real AC situation. Therefore, we repeat the pricing procedure with a set of randomly generated loss factors, in the interval ± 10% of the nodal price, as those are the administratively determined limits in the Norwegian system. Table II illustrates, in the third column, the randomly chosen factors. Again the DC-OPF is run with adjusted bids, and net nodal DC prices are calculated. As can be seen from the last column of Table II, the deviations from the AC nodal prices are now larger. This case study reveals that even in the absence of congestion and zonal pricing, deviations from the AC nodal prices may be expected when applying a sequential pricing procedure.

B. *Constrained System without Zonal Pricing*

If the power transfer limit of line 7-8 is equal to 94 MVA, the previous market equilibrium is not feasible, and the power flow restriction of line 7-8 is binding. In Norway, congestion may lead to a splitting in different zones, and zonal pricing will be investigated in cases C and D. However, for this research it is also interesting to compare the congested AC nodal prices with the prices obtained by the sequential operation mode, but without using zonal pricing, i.e. assuming that each bus is a zone.

The calculated AC nodal prices as well as their loss components are given in Table III. The market operator runs a DC-OPF on the loss-adjusted bids, calculates the DC nodal prices, and net nodal prices are found. The differences between AC nodal prices and the DC net prices are given in the last column of Table III. A comparison of these deviations to the ones given in Table I indicates that in case of congestion, the differences are larger.

Following the same procedure as in section A, instead of the AC nodal price loss components, randomly chosen loss factors may be used.

TABLE III
CASE B – CONSTRAINED WITH LOSS FACTORS FROM AC
(all prices in ct/kWh)

Bus	Nodal price AC	Loss factor	Nodal price DC	Net price DC	Difference: Net price DC- Nodal price AC
1	2.9340	-0.0242	2.9160	2.9402	0.0062
2	3.1140	-0.0098	3.0620	3.0718	-0.0422
3	3.2570	0.0026	3.1770	3.1796	-0.0774
4	2.7890	-0.0113	2.7700	2.7813	-0.0077
5	3.1680	0.0056	3.0920	3.0976	-0.0704
6	3.4030	0.0144	3.2930	3.3074	-0.0956
7	2.3800	0.0000	2.3950	2.3950	0.0150
8	3.6250	0.0120	3.4950	3.5070	-0.1180
9	3.5590	0.0200	3.4610	3.4810	-0.0780
10	3.5450	0.0198	3.4130	3.4328	-0.1122

TABLE IV
CASE B – CONSTRAINED WITH RANDOM LOSS FACTORS
(all prices in ct/kWh)

Bus	Nodal price AC	Loss factor	Loss factor as % of nodal price	Nodal price DC	Net price DC	Difference: Net price DC- Nodal price AC
1	2.9340	0.0880	+3	2.9391	2.8511	-0.0829
2	3.1140	-0.1246	-4	2.9909	3.1155	0.0015
3	3.2570	0	0	3.0318	3.0318	-0.2252
4	2.7890	0.0558	+2	2.8816	2.8258	0.0368
5	3.1680	-0.0950	-3	3.0025	2.9075	-0.2605
6	3.4030	0.1702	+5	3.0775	3.2477	-0.1553
7	2.3800	0	0	2.7327	2.7327	0.3527
8	3.6250	0	0	3.1579	3.1579	-0.4671
9	3.5590	0.2135	+6	3.1432	3.3567	-0.2023
10	3.5450	0.0355	+1	3.1247	3.1602	-0.3848

The corresponding loss factors as well as the comparison results are given in Table IV. It is evident that, in this case, the differences are essentially higher than in Table III, showing once more that differences are more likely to appear when loss factors are not associated with the real AC situation.

C. *Constrained System Considering Zonal Pricing*

The third case consists of introducing modest zonal pricing in the previous congested case. Assume that buses 1 and 2 form a zone, while all the other buses remain as single-bus zones. First, the AC nodal prices, shown in Table V, are calculated. The prices for buses 1 and 2 are now equal because the zonal pricing restriction has been incorporated in the AC-OPF. The loss components of these prices are given in the third column, and it is assumed that these components are announced as loss factors. Again the pool operator runs a DC-OPF on adjusted bids, now demanding the same nodal price for buses 1 and 2. The resulting prices as well as the net prices are also given in Table V. The deviations are, generally, higher than in the previous case where the zonal pricing was not incorporated.

By choosing loss factors different from the loss components, these deviations are increased, as it is indicated in Table VI.

TABLE V
CASE C – MODEST ZONAL PRICING WITH LOSS FACTORS FROM AC
(all prices in ct/kWh)

Bus	Nodal price AC	Loss factor	Nodal price DC	Net price DC	Difference: Net price DC-Nodal price AC
1	2.9956	0.0248	2.9670	2.9422	-0.0534
2	2.9956	-0.1010	2.9670	3.0680	0.0724
3	3.2620	0.0030	3.1740	3.1770	-0.0850
4	2.7870	-0.0115	2.7770	2.7885	0.0015
5	3.1720	0.0059	3.0920	3.0979	-0.0741
6	3.4100	0.0148	3.2860	3.3008	-0.1092
7	2.3720	0.0000	2.4090	2.4090	0.0370
8	3.6360	0.0124	3.4850	3.4974	-0.1386
9	3.6090	0.0204	3.4510	3.4714	-0.1376
10	3.5540	0.0202	3.4040	3.4242	-0.1298

TABLE VI
CASE C – MODEST ZONAL PRICING WITH RANDOM LOSS FACTORS
(all prices in ct/kWh)

Bus	Nodal price AC	Loss factor	Loss factor as % of nodal price	Nodal price DC	Net price DC	Difference: Net price DC-Nodal price AC
1	2.9956	0.2097	+7	3.0150	2.8053	-0.1903
2	2.9956	0.2396	+8	3.0924	2.8528	-0.1428
3	3.2620	0	0	3.1533	3.1533	-0.1087
4	2.7870	-0.0836	-3	2.9292	3.0128	0.2258
5	3.1720	0.1586	+5	3.1095	3.2681	0.0961
6	3.4100	0.2046	+6	3.2215	3.4261	0.0161
7	2.3720	0	0	2.7071	2.7071	0.3351
8	3.6360	0	0	3.3415	3.3415	-0.2945
9	3.6090	0.2526	+7	3.3211	3.5737	-0.0353
10	3.5540	0.0355	+1	3.2919	3.3274	-0.2266

D. *Constrained System with Intensive Zonal Pricing*

The last case describes a situation where a more intensive zonal pricing is applied, i.e. more buses participate in some zones. More specifically, it is assumed that the four generator buses form one zone, while a second zone consists of the four consumer buses. Buses 3 and 8 remain single-bus zones.

TABLE VII
CASE D – INTENSIVE ZONAL PRICING WITH LOSS FACTORS FROM AC
(all prices in ct/kWh)

Bus	Nodal price AC	Loss factor	Nodal price DC	Net price DC	Difference: Net price DC-Nodal price AC
1	2.7780	0.0026	2.8380	2.8354	0.0574
2	2.7780	0.0117	2.8380	2.8263	0.0483
3	3.5860	0.0221	3.2590	3.2811	-0.3049
4	2.7780	0.0059	2.8380	2.8321	0.0541
5	3.9320	0.0205	3.4480	3.4685	-0.4635
6	3.9320	0.0257	3.4480	3.4737	-0.4583
7	2.7780	0	2.8380	2.8380	0.0600
8	4.3500	0.0121	3.6900	3.7021	-0.6479
9	3.9320	0.0269	3.4480	3.4749	-0.4571
10	3.9320	0.0258	3.4480	3.4738	-0.4582

The results of this case are presented in Table VII. It is obvious, that the price differences for the great majority of the buses are essentially higher than in case C.

In case of more intensive zonal pricing, a set of randomly chosen loss factors may also be applied. Both loss factors and resulting price differences are given in Table VIII. Compared to Table VII, the price differences at some buses have increased while others are reduced.

TABLE VIII
CASE D – INTENSIVE ZONAL PRICING WITH RANDOM LOSS FACTORS
(all prices in ct/kWh)

Bus	Nodal price AC	Loss factor	Loss factor as % of nodal price	Nodal price DC	Net price DC	Difference: Net price DC-Nodal price AC
1	2.7780	0.1111	+4	2.8427	2.7316	-0.0464
2	2.7780	0.0556	+6	2.8427	2.7871	0.0091
3	3.5860	0	0	3.1960	3.1960	-0.3900
4	2.7780	-0.1389	-5	2.8427	2.9816	0.2036
5	3.9320	0.3146	+8	3.3554	3.6700	-0.2620
6	3.9320	0.2359	+6	3.3554	3.5913	-0.3407
7	2.7780	0	0	2.8427	2.8427	0.0647
8	4.3500	0	0	3.5570	3.5570	-0.7930
9	3.9320	0.0786	+2	3.3554	3.4340	-0.4980
10	3.9320	0.3146	+8	3.3554	3.6700	-0.2620

E. *Discussion of the Results*

Table IX shows the social surplus, i.e. the consumers' willingness to pay less the cost of production for the different cases considered, and also the surplus of the producer located in the reference bus [8], [14]. Column label AC refers to the prices based on AC-OPF, while DC(1) and DC(2) refer to the market solutions resulting from loss factors based on respectively, the AC loss components and randomly generated loss components. For each case and pricing method the upper number gives the social surplus, while the lower shows the surplus of the supplier of the reference bus. It is evident from the numbers that the social surpluses are fairly stable, the difference between the smallest and largest numbers for the congested cases being approximately 0.5%. For the supplier however, the surpluses vary considerably. Since the individual nodal prices vary considerably for the different solutions considered, see also the following Tables X and XI, this will be so also for other agents.

TABLE IX
SOCIAL SURPLUS AND PRODUCER SURPLUS AT REFERENCE BUS (€)

	AC	DC (1)	DC (2)
Case A	34067	34149	34139
	727	709	735
Case B	34020	34103	34128
	472	478	622
Case C	34011	34097	34113
	469	483	611
Case D	33952	34112	34088
	643	671	673

TABLE X
COMPARISON OF AC NODAL PRICES

Bus	Nodal prices, AC (ct/kWh)			
	Case A	Case B	Case C	Case D
1	2.9296	2.9340	2.9956	2.7780
2	2.9527	3.1140	2.9956	2.7780
3	2.9427	3.2570	3.2620	3.5860
4	2.9565	2.7890	2.7870	2.7780
5	2.9633	3.1680	3.1720	3.9320
6	2.9526	3.4030	3.4100	3.9320
7	2.9530	2.3800	2.3720	2.7780
8	2.9609	3.6250	3.6360	4.3500
9	2.9683	3.5590	3.6090	3.9320
10	2.9680	3.5450	3.5540	3.9320

TABLE XI
DEVIATIONS OF DC NET PRICES FROM AC NODAL PRICES

Bus	Difference: net price DC – nodal price AC (ct/kWh)		
	Case C loss components as loss factors	Case D loss components as loss factors	Case D randomly chosen loss components
1	-0.0534	0.0574	-0.0464
2	0.0724	0.0483	0.0091
3	-0.0850	-0.3049	-0.3900
4	0.0015	0.0541	0.2036
5	-0.0741	-0.4635	-0.2620
6	-0.1092	-0.4583	-0.3407
7	0.0370	0.0600	0.0647
8	-0.1386	-0.6479	-0.7930
9	-0.1376	-0.4571	-0.4980
10	-0.1298	-0.4582	-0.2620

Table X sums up the prices for the different congestion management methods, all based on AC-OPF, while Table XI shows the deviations of the net DC-prices from the nodal prices based on AC for the zonal pricing cases. For case D, both prices based on AC loss factors and randomly chosen loss components are exhibited. The price differences that can be found from Table X, and that are exhibited in Table XI, show that the introduction of zonal pricing have profound effects on prices, but they also show that the sequential pricing procedure, with announced loss factors and prices determined by DC computations, have similar effects, even if the AC loss factors are used. A general comparison between the four cases, when randomly chosen loss factors are used, is not proper. In this case, an average deviation resulting from a set of random loss factors would be a more appropriate approach.

The comparisons lead to the conclusion that the more administrative rules that are incorporated, such as loss factors and zonal pricing intensity, the larger are the deviations from the AC nodal prices. The analysis of the paper also highlights that it is possible to use alternative zonal pricing schemes, for instance based on AC-OPF, that are more in accordance with the real systems, but at the same time have only a few prices, and thus the same perceived simplicity for the market participants, as zonal prices based on the DC-approximation have.

V. CONCLUSIONS

In practical implementations of marginal cost transmission pricing, a number of approximations may be used in the process of finding prices for different locations taking into account the limits of the grid. This is so for the Nordic power market and, more specifically, the Norwegian market, where prices are influenced by the market clearing procedures at Nord Pool, as well as by the tariffs for marginal losses, that are determined by the Norwegian system operator. The focus of this paper has been to investigate a stepwise procedure inspired by the Norwegian transmission pricing rules, with ex ante announcement of loss factors and zonal pricing for congestion management.

In pursuing this goal the nodal prices are decomposed to find a loss factor for each node relative to a reference node. Moreover, a methodology for computing equal nodal prices using AC-OPF for nodes that belong to the same zone is developed. Based on loss factors from the AC model or, alternatively, randomly generated loss factors, a stepwise procedure is employed, highlighting the effects on the locational prices of the approximations for different cases of congestion and zonal pricing intensity. The approximations may have considerable effects on the final locational prices, and although the social surpluses have been computed, and show to be fairly stable, the individual nodal prices are not, thereby affecting to a great extent the surplus of the individual agents.

VI. Appendix

TABLE A1
DATA OF THE 10-BUS SYSTEM

Lines	r [p.u.]	x [p.u.]	b [p.u.]	Transfer capacity [MVA]
1-2,1-4,2-3, 2-5,3-6,4-5, 5-7,6-8	0.0034	0.0360	1.2696	800
4-7,5-6,8-10	0.0028	0.0288	1.0156	800
5-8,7-8,8-9	0.0017	0.0180	0.6348	800
6-10,9-10	0.0024	0.0252	0.8888	800

TABLE A2
BID OFFERS OF MARKET PARTICIPANTS FOR THE 10-BUS TEST SYSTEM

Bus	Art	P_{max} [MW]	p_{max} [ct/kWh]
1	Supplier	150	3
2,4	Supplier	150	6
7	Supplier	250	9
5,6,9,10	Consumer	100	20

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VIII. Biographies

Georgios C. Stamtsis (M'2002) was born in 1974. He received the Diploma and the Dr.-Ing. degree both in electrical engineering from the Aristotle University of Thessaloniki, Greece, and the University of Duisburg-Essen, Germany, in 1998 and 2003 respectively. His interests are focused in congestion management and transmission pricing in deregulated electricity markets including the application of game theory. He is member of IEEE, VDE and the Technical Chamber of Greece.

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István Erlich received his Dipl.-Ing. degree in electrical engineering from the University of Dresden, Germany in 1976. After his studies, he worked in Hungary in the field of electrical distribution networks. From 1979 to 1991, he joined the Department of Electrical Power Systems of the University of Dresden again, where he received his PhD degree in 1983. In the period of 1991 to 1998, he worked with the consulting company EAB in Berlin and the Fraunhofer-Institute IITB Dresden, respectively. During this time, he had also a teaching assignment at the University of Dresden. Since 1998, he is Professor and head of the Institute of Electrical Power Systems at the University of Duisburg, Germany. His major scientific interest is focused on power system stability and control, modelling and simulation of power system dynamics including intelligent system applications. He is member of IEEE and VDE.

Kurt Jörnsten was born in Sundbyberg, Sweden, in 1948. He received the degree of Doctor of Technology in Applied Mathematics (Optimization) from Linköping Institute of Technology, Linköping, Sweden. He is also Honorary Doctor at The Stockholm School of Economics, Stockholm, Sweden. Currently, he is Professor in Management Science at The Norwegian School of Economics and Business Administration, Bergen, Norway. His interests include the electricity deregulation, the cooperative and non-cooperative game theory as well as the application of optimization methods to various problems in economics and business.