

**Working Paper No. 26/05**

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And cournot Competition**

**by**

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SNF-Project No. 8300  
Nærings- og konkurransepolitikk

The project is financed by the Research Council of Norway

INSTITUTE FOR RESEARCH IN ECONOMICS AND BUSINESS ADMINISTRATION  
BERGEN, April 2005  
ISSN 1503-2140

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# Two-part Pricing, Consumer Heterogeneity and Cournot Competition\*

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April 2005

## Abstract

We analyze two-part tariffs in an oligopoly, where each firm commits to a quantity and a fixed fee prior to the determination of unit prices. In the case of homogeneous consumers, Harrison and Kline (2001) showed that the equilibrium involves marginal cost pricing and that increased competition affects industry profit and the tariff structure solely by reducing the fixed fee. We show that firms' pricing strategies may change when we allow for demand side heterogeneity. In particular, we find that the price per unit can be either above or below marginal cost, and that the fixed fee increases with increased competition. Finally, some numerical examples show that full market coverage may arise as an equilibrium feature in cases where a monopolist would exclude low-demand types. Hence, fostering competition may contribute to the fulfillment of the Universal Service requirement that is common in industries such as telecommunications, which applies nonlinear pricing on a normal basis.

JEL.CLASS.NO: L11, L13, D82

KEYWORDS: oligopoly, two-part tariff, Cournot

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\*We are grateful to Petter Osmundsen and Patrick Rey for their comments. In addition, we are indebted to participants at the Nordic Workshop on ICT-related research at the Norwegian School of Economics and Business Administration in June 2001, the EARIE meeting in Madrid in September 2002, and the doctoral seminar series at the Norwegian School of Economics and Business Administration for their helpful comments.

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# 1 Introduction

Nonlinear prices are common in many industries and have been studied extensively in the economic literature. As shown in Oi (1971), a single two-part tariff is more efficient when it comes to extracting consumer surplus in a monopoly than is a uniform price.<sup>1</sup> Harrison and Kline (2001) analyzed two-part tariffs in an oligopoly with strategic interaction and homogeneous consumers. In their paper, increased competition affects industry profit and tariff structure solely by reducing the fixed fee, and the tariffs are merely modifications of the tariffs described by Oi (1971) in a monopoly. However, most industries consist of heterogeneous consumers; for instance, in telecommunications there are distinct residential and business markets. This may have important implications for the firms' pricing strategies and for regulatory policy concerned with market coverage and universal service in the industry. The hypothesis that oligopolistic firms practice nonlinear pricing is empirically supported by Miravete and Röller (2003, 2004). In this paper, we build a more realistic model to study two-part tariffs than Harrison and Kline (2001) by assuming that there are two types of consumers, instead of only one. It turns out that such extension of their model has major impacts on the qualitative results obtained.

Our starting point is the framework developed in Harrison and Kline (2001). Each firm commits to a certain quantity, as in a traditional Cournot model. In addition, each firm sets its fixed fee, whereas the unit prices are determined by market forces endogenously. Without the latter assumption, it is not possible to escape a situation where a competing firm charges a uniform price and captures all consumers. Harrison and Kline (2001) offered some examples of industries where this treatment makes sense, such as home computer games and consumer clubs. Harrison and Kline (2001) extended the basic problem of charging a group of  $N$  identical consumers according to a two-part tariff, instead of a uniform unit price, to a setting with competition – that is, they extended the first part of the model in Oi (1971) to oligopoly. In equilibrium, they found that the unit price is set equal to marginal costs, and that the fixed fee is positive for a given number of firms, but that it approaches zero as the number of firms approaches infinity.

Our extension of the Harrison and Kline (2001) model to two instead of one type of consumer turns out to be important for the firms' pricing policy. In addition, it makes it possible to analyze how competition affects market coverage

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<sup>1</sup>The firm can achieve even greater profit by screening consumers whose willingness to pay differs via a menu of two-part tariffs, or by using a more general nonlinear outlay schedule. The firm takes into account the unobserved heterogeneity of individual demand by using a menu of two-part tariffs, or a nonlinear outlay schedule, thereby introducing self-selection. See Wilson (1993) on nonlinear pricing in a monopoly, Rochet and Stole (2003) for a guide to the screening literature, and Stole (2001) for a comprehensive guide to the literature on price discrimination in models with competition.

– whether both types of consumers are served. A particular aim in this paper is to explore whether full market coverage can arise as an equilibrium feature, and we assume that a regulator can impose a Universal Service Obligation (USO) on the firms. We explore whether a subgame with a Universal Service Obligation is robust against unilateral deviations. In the absence of such an obligation, we explore under what conditions firms are likely to cover the entire market.

First, let us assume full market coverage in the case of a USO, and pay attention to the firms pricing policy. We then find that the per unit price may be below marginal costs when as few as four firms exist, and that the fixed fee may be positive even with an infinite number of firms. Hence, we cannot extend the results in Oi (1971) to oligopoly with heterogeneous consumers. Also, introducing consumer heterogeneity makes the demand side of the model more complicated. In the homogeneous type case, consumers' behavior vis à vis each firm is described by a single participation constraint. A consumer is indifferent between two tariffs with different fixed fees and per unit prices, as long as they both satisfy his or her participation constraint. In the case of heterogeneous consumers, consumers' behavior vis à vis each firm is given by two different participation constraints. Whenever low-demand consumers are indifferent between two tariffs, high-demand consumers will strictly prefer the one with a low per unit price and a high fixed fee. On the other hand, if high-demand consumers are indifferent between two tariffs, low-demand consumers will strictly prefer the one with a low fixed fee and a high per unit price. Hence, if one consumer type is indifferent between two firms, the other one will strictly prefer one of the two firms. Consequently, unless they apply completely symmetric strategies, we cannot expect all firms to serve both types of subscribers.

The extension of the model from one to two types of consumers partly explains the change in the per unit price. Without demand-side heterogeneity firms practice cost-plus-fixed fee pricing, and a marginal increase in market share affects firm  $i$ 's profit via the fixed fee revenue. The per unit price is adjusted to maximize consumer surplus, whereas the fixed fee is adjusted to satisfy the participation constraint. It is well known from a monopoly model that a firm serving two types of consumers with a single two-part tariff should let the unit price exceed marginal costs. However, in our duopoly model with demand-side heterogeneity, firms cannot change the fixed fee on a unilateral basis without violating the Universal Service requirement. Hence, they are de facto restricted to compete in capacities. The isolated effect of this is that the per unit price will be above marginal cost. However, as each consumer pays a fixed fee, a marginal increase in the market share increases the fixed fee revenues, which induce oligopolistic firms to compete more aggressively than they would do otherwise. Our results show that if the fixed fee is sufficiently large, this effect may dominate and the firms price per unit below marginal cost.

Turning to the question of market coverage, we find that fostering competi-

tion can contribute to full market coverage and the fulfillment of the Universal Service requirement. Our results show that a subgame with a USO is robust against unilateral deviations. Moreover, in the absence of a USO, we find that a duopoly is more likely than a monopoly to cover the entire market. Hence, our model does not support theories predicting that the firm operating under a Universal Service Obligation serves low-demand segments of the market while other firms “skim the cream” by serving the more profitable high-demand segments. The monopolist’s decision to exclude low-demand segments is based on its ability to control consumers participation, and expected profit per consumer (fixed fee revenue plus variable sales revenue) is the same at the cut-off level whether the low demand segment is served or not. In a duopoly, if one firm deviates by changing its market coverage, the per unit price at the other firm tends to change at the same time. Hence, the participation constraints are changed, and the deviating firm is not able to sustain the same expected profit per consumer as before a deviation.

The empirical research in Miravete and Röller (2003, 2004) on the US cellular telephone industry support the hypothesis that nonlinear pricing prevails under oligopolistic competition in telecommunications. Ivaldi and Martimort (1994) showed that this is also the case for energy distribution.<sup>2</sup> Universal Service objectives have been given much attention in Telecommunications, especially after the introduction of competition.<sup>3</sup> In our case, the weakness involved in focusing on telecommunications is the assumption that firms commit to a fixed fee and a capacity, but do not commit to the per unit price at the first stage. However, one might argue that per unit prices are more flexible than fixed fees. For instance, many mobile phone firms change their fixed fees by replacing existing tariffs.<sup>4</sup> On the other hand, changes in the per unit prices are made within existing tariffs. Another practice that makes the per unit price more flexible is that the firms conduct continuous campaigns and promotional offers, such as offering con-

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<sup>2</sup>The predictions are more ambiguous within the theoretical literature, which shows that nonlinear pricing may not be sustainable in oligopoly. For example, Mandy (1992) found that in a traditional Bertrand oligopoly with homogeneous products – where we allow the firms to set nonlinear prices – all prices may collapse to a uniform price. This finding illustrates that some of the assumptions in the traditional oligopoly model have to be relaxed in order to make nonlinear prices sustainable in oligopoly. One extension is to introduce capacity constraints, as was done in Harrison and Kline (2001), Oren, Smith and Wilson (1983), Scotchmer (1985a,b), and Wilson (1993). Another extension of the traditional Bertrand model is to introduce product differentiation; see Calem and Spulber (1984), Castelli and Leporelli (1993), Economides and Wildman (1995), Shmanske (1991), and Young (1991). If future demand is stochastic to some extent, Hayes (1987) managed to sustain nonlinear pricing under competition by assuming that consumers are risk averse and have a preference for two-part tariffs. In the case of telephony, Miravete (2002) ruled out risk aversion as a possible source of consumers’ biased taste for flat rate tariffs.

<sup>3</sup>See for instance Riordan (2002) on Universal Service in Telecommunications.

<sup>4</sup>Some firms even list current and non-current tariffs on their web sites; see, for example, [www.vodafone.co.uk](http://www.vodafone.co.uk), and [netcom.no](http://netcom.no).

sumers free talk time, texts, or photo messages. In addition, telecommunications firms tend to lock-in consumers by offering subsidized handsets subject to a 12- or 18-month contracts. Within this period, the fixed fee is more or less the same, but the per unit price is more flexible.

This article is organized as follows. In section 2, we formulate our model using identical firms and characterize the outcome in two possible symmetric strategy subgames, where either both types of consumers are served or only one. In section 3, we explore the possibility that one, or both, of the symmetric strategy subgames described in section 2.1 can be equilibria for the whole game in a duopoly. We do so by asking first, whether one firm would earn a higher profit if it deviates and covers the entire market in the subgame where the other firm serves high-demand consumers only (section 3.1). Next, we check whether the firm would earn a higher profit if it deviates and serves high-demand consumers only in the subgame where the other firm serves both types of consumers (section 3.2). By using a numerical example, we show that there can be multiple equilibria. Moreover, we show that the situation where both firms serve both types of consumers can be an equilibrium duopoly outcome in cases where the monopolist would prefer to serve high demand types only. The driving force is that the rival, non-deviating firm supplies a given quantity that it is committed to selling, which acts as a constraint on the deviating firm's price setting. Our results are summarized in Figure 8. Finally, section 4 offers some concluding remarks.

## 2 Cournot competition with two-part tariffs

Consider a setup with  $k$  identical firms,  $k \geq 2$ , supplying a homogeneous product to  $N$  consumers, firm  $i \in K = \{1, 2, \dots, k\}$  serving  $n_i \leq N$  consumers. We consider the case where the number of firms is exogenous, leaving the question of entry outside the model. The cornerstones of the model with homogeneous demand are presented in Harrison and Kline (2001), so we provide only a brief presentation of the set-up.<sup>5</sup>

**Consumers.** We assume that there are two groups of consumers: low-demand and high-demand consumers. Demand-side heterogeneity is defined by a type parameter,  $\theta$ , in the utility function. A proportion of consumers,  $\lambda$ , have the taste parameter  $\theta_1$ , and a proportion,  $(1 - \lambda)$ , have the taste parameter  $\theta_2$ , where  $\theta_1 < \theta_2$ . We will refer to the first group as type 1 consumers, or low-demand consumers, and to the other group as type 2 consumers, or high-demand consumers.

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<sup>5</sup>It would be helpful for the reader to examine the seminal paper by Harrison and Kline that explores the model setup and provides a thorough treatment of the model and the assumptions that are needed in order to ensure that equilibrium exists. See also their article on entry in this model.

Consumers' preferences are defined by a quasi-linear utility function, as follows:

$$V = \begin{cases} u(q, \theta_\ell) - T & \text{if they pay } T \text{ and consume } q \text{ units,} \\ 0 & \text{if they do not buy,} \end{cases} \quad (1)$$

$$\begin{aligned} \theta_\ell &= \{\theta_1, \theta_2\}, \\ u(q, \theta_2) &\geq u(q, \theta_1) \text{ and } u_q(q, \theta_2) \geq u_q(q, \theta_1) \quad \forall q \end{aligned}$$

The utility function is assumed to be increasing and strictly concave in  $q$ ,  $u(0, \cdot) = 0$ ,  $\lim_{q \rightarrow 0} u_q(q, \cdot) \geq c$ ,  $\lim_{q \rightarrow \infty} u_q(q, \cdot) \leq 0$ . For any tariff  $T = A + pq$ , where  $A$  is a fixed fee that is paid up-front and  $p$  is a per unit price, utility maximization yields a downward-sloping demand curve for each individual that is independent of income and, therefore, of the fixed fee.

Then, the consumers' indirect utility, gross of the fixed fee, and the two consumer types' demand curves are given by:

$$\begin{aligned} q_\ell(p) &\equiv q(p, \theta_\ell) = \max_q u(q, \theta_\ell) - pq - A, \\ V(p, \theta_\ell) &= u(q_\ell(p), \theta_\ell) - pq_\ell(p), \\ V'_p &= -q_\ell(p), \\ \ell &= 1, 2. \end{aligned} \quad (2)$$

**Firms.** The cost function is characterized by constant returns to scale,  $C(Q) = cQ$ , where  $c > 0$  is the marginal cost and  $Q$  is output. For simplicity, fixed costs are omitted from all measures. If firm  $i$  serves  $n_i$  consumers, its profit is given by:

$$\Pi_i = n_i A_i + (p_i - c) Q_i. \quad (3)$$

As the fixed fee is a lump sum transfer from consumers to the firm, the unit price in firm  $i$ 's tariff is adjusted in such a way that aggregate demand for firm  $i$ 's product is equal to firm  $i$ 's supply. Hence, the unit price is independent of the fixed fee. Whenever the fixed fee is positive, consumers will make all or nothing purchases at firm  $i$ . When firm  $i$  serves a total of  $n_i$  consumers, the unit price is adjusted to satisfy the following market clearing condition:

$$Q_i = n_i q_2(p_i). \quad (4)$$

The unit prices charged by rival firms are adjusted to satisfy the market clearing condition and consumers' participation constraints. Let the aggregate supply by firms competing with firm  $i$  be given by:

$$Q_{\mathcal{J}} \equiv \sum_{j \neq i} Q_j. \quad (5)$$

In particular, if every other firm charges the same per unit price  $p_{-i}$ , this must satisfy the condition:

$$Q_{\mathcal{J}} = (N(1 - \lambda) - n_i) q_2(p_{-i}). \quad (6)$$

The firm cannot exclude any consumer from buying its product. Taking consumer behavior into account, firms act to maximize profit by choosing an appropriate strategy  $s_i = (Q_i, A_i)$ , with  $Q_i > 0$  for all  $i \in K$ . We assume that firms are able to commit to this strategy.

**Consumer equilibrium.** The allocation of consumers across firms  $(n_i, i \in K)$  is described by utility maximization. With quasilinear utility, we can measure the indirect utility in monetary terms. Consumers choose to buy if they obtain a nonnegative net surplus at some firm  $i$ , that is, if  $V(p_i, \theta_\ell) - A_i \geq 0$ , ( $i \in K, \ell = \{1, 2\}$ ). They buy from the firm that provides them with the highest surplus – that is, they buy from firm  $i$  if  $V(p_i, \theta_\ell) - A_i > V(p_j, \theta_\ell) - A_j$ , ( $i, j \in K, i \neq j, \ell = \{1, 2\}$ ). For a given strategy combination  $s = (Q_i, A_i)_{i \in K}$ , there exists a consumer equilibrium satisfying both utility maximization and market clearing, defining a consumer-price profile as  $(n, p) = ((n_1, \dots, n_k), (p_1, \dots, p_k))$ , with  $n_i$  being the number of consumers buying from firm  $i$ , and  $p_i$  being the unit price that clears the market at firm  $i$ . Although we define a firm’s strategy in terms of its capacity and the fixed fee, from a consumer’s point of view he choose the quantity that maximizes his or her utility for a given  $A_i$  and  $p_i$ , and obtain net utility  $V(p_i, \theta_\ell) - A_i$ . This is formally defined in Harrison and Kline (2001).

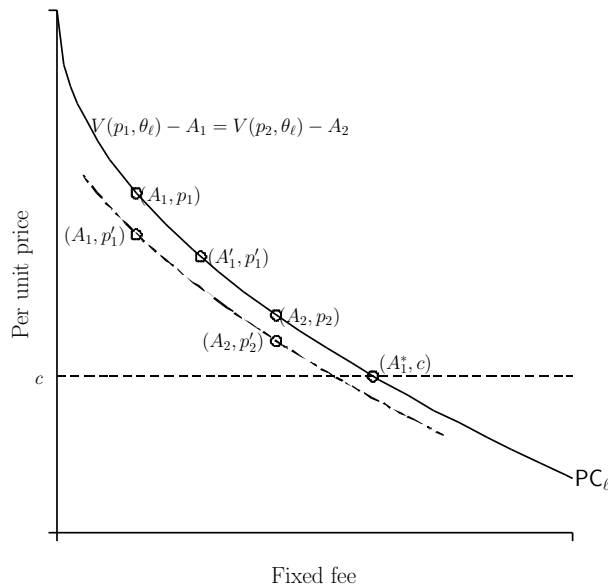


Figure 1: *Consumer behavior with identical consumers.*



Consider a situation with two firms, 1 and 2, serving consumers of type  $\ell$  only. As illustrated in figure 1, the consumer is indifferent between  $(A_1, p_1)$  and  $(A_2, p_2)$  because they are located on the same indifference curve,  $PC_\ell$ . The slope of the indifference curve is the consumer's marginal rate of substitution (MRS) between the per unit price and the fixed fee, which is given by  $dp/dA = -1/q_\ell(p)$ ; that is, the larger is the consumer's willingness to pay for quantity, the smaller is his or her MRS. Given that the market clears at both firms at prices  $p_1$  and  $p_2$ , strategies  $(Q_1, A_1)$  and  $(Q_2, A_2)$  map into a consumer equilibrium  $((n_1, n_2), (p_1, p_2))$ , with  $n_i$  determined by  $Q_i = n_i q_\ell(p_i)$ . If Firm 1 increases its output, the price at Firm 1 will fall, which will induce consumers to switch from Firm 2 to Firm 1, and consumer equilibrium is restored at  $(A_1, p'_1)$  and  $(A_2, p'_2)$ . If Firm 1 increases output and the fixed fee simultaneously, consumer equilibrium is restored at  $(A'_1, p'_1)$  and  $(A_2, p_2)$ . As is well known from Oi (1971), if a monopoly can appropriate the entire consumer surplus above some exogenous level  $\bar{U}$ , it will apply marginal cost pricing,  $(A_1^*, c)$ . Harrison and Kline (2001) showed that oligopoly firms will attract customers by lowering the access fee. In addition, they observed that per unit price equals marginal cost, as in a monopoly. Thus, established insights from monopoly two-part pricing could lead us to believe that the oligopoly results extend to demand-side heterogeneity, and that price will be set above marginal cost to adjust for the heterogeneity in consumers' willingness to pay. However, as we show in the following section, introducing demand-side heterogeneity makes the demand side of the model more complicated because consumer behavior vis à vis each firm is given by two different participation constraints, one for low-demand consumers and one for high-demand consumers.

Firms choose their strategies simultaneously and independently, giving a strategy combination  $s = (s_1, s_2, \dots, s_k)$ . If firms choose symmetric strategies,  $s_i = s = (Q, A)$ , there are two possible outcomes: (i) both types of consumers are served, or (ii) only high-demand consumers buy. Confining ourselves to symmetric strategy combinations, we explore whether it is likely that the entire market is covered in an oligopoly, and analyze the pricing strategy that may arise in equilibrium. Sections 2.1 and 2.2 characterize two possible subgames where the firms adopt symmetric strategies with respect to the capacity and fixed fee. Section 3 explores the firms' incentives to deviate from the two symmetric-strategy subgames in a duopoly.

## 2.1 Consumer equilibrium with heterogeneity

For a strategy combination to map into a consumer equilibrium where all firms serve both consumer types, all firms must charge identical fixed fees. In a consumer equilibrium, it is not possible for any consumer to obtain a larger surplus by switching to another supplier. If it is possible to obtain a positive surplus from at least one firm, there will be no excess demand. Further, demand will be

equal to supply at each firm.

Consider a strategy combination where firms  $i$  and  $j$  announce different fixed fees (as in figure 1) above). Low-demand consumers are indifferent between firms  $i$  and  $j$  when:

$$V(p_i, \theta_1) - A_i = V(p_j, \theta_1) - A_j. \quad (7)$$

Hence, if  $A_i > A_j$ , it must be the case that  $p_i < p_j$ . If this is the case between all pairs  $i, j$ ,  $i \neq j$ , each firm will serve  $(N/k)\lambda$  low demand types. If a high-demand consumer buys from firm  $i$ , he or she obtains:

$$V(p_i, \theta_2) - V(p_i, \theta_1) + V(p_j, \theta_1) - A_j, \quad (8)$$

whereas if he or she buys from firm  $j$ , the high-demand consumer obtains:

$$V(p_j, \theta_2) - A_j. \quad (9)$$

High-demand types are indifferent between the firms if:

$$0V(p_i, \theta_2) - V(p_i, \theta_1) + V(p_j, \theta_1) - A_j = V(p_j, \theta_2) - A_j. \quad (10)$$

Hence, when per unit prices are adjusted to satisfy (7), the high-demand consumers' participation constraint cannot bind simultaneously because (10) requires that  $p_i = p_j$ , as  $q_2(p) > q_1(p)$ .

This situation is illustrated in Figure 2. Low-demand consumers obtain a zero surplus along  $PC_1$ , whereas high-demand consumers obtain a zero surplus along  $PC_2$ , and a strictly positive surplus along  $PC'_2$  and  $PC''_2$ . At any point, low-demand consumers' indifference curves are steeper than are high-demand consumers' indifference curves because  $q_2(p) > q_1(p)$ . In other words, high-demand consumers are willing to accept a larger increase in the fixed fee against a decrease in the per unit price, relative to low-demand consumers. Moreover, the high-demand consumer's utility at B is larger than at A, whereas low-demand types are indifferent between A and B. Type 1 consumers are indifferent no matter what the sizes of  $p$  and  $A$  are at each firm, as long as they are adjusted according to (7). However, type 2 consumers will always prefer the tariff with the lowest per unit price. If low-demand types are indifferent between the two tariffs, high-demand types strictly prefer the tariff with the low per unit price. Consequently, a situation with different fixed fees is not a sustainable consumer equilibrium if we want this to involve all firms serving both types of consumers.

If the consumer equilibrium is the result of a symmetric strategy combination  $s_i = (Q, A)$ , ( $i = 1, \dots, k$ ), we assume that all firms serve an equal share of each consumer type, and the consumer equilibrium is given by the profile  $(n, p)$ . Hence, if both types are served by all firms, firm  $i$  serves a number of consumers given by  $N/k$ . If low-demand types are excluded, the number is  $(1 - \lambda)N/k$ .

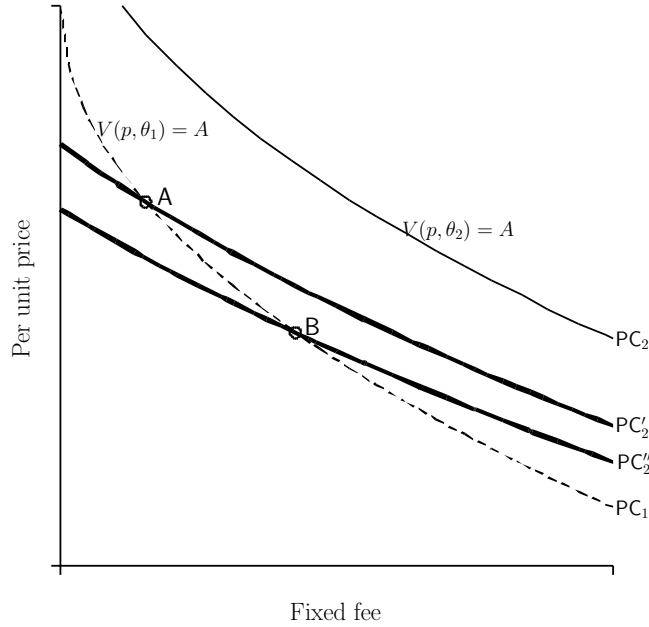


Figure 2: *High- and low-demand consumers' indifference curves over a fixed fee and per unit price. Low-demand types are indifferent between A and B, whereas high-demand consumers strictly prefer B.*

A strategy combination with identical fixed fees, but different capacities across firms, will result in a consumer equilibrium where the firms serve different numbers of consumers. Suppose, for instance, that there are two firms in the market, Firm 1 and Firm 2. Suppose that Firm 1 increases its output. This causes a fall in the per unit price at Firm 1 because there would be excess supply otherwise. Consumers are attracted by the lower price at Firm 1 and are tempted to leave Firm 2. Hence, the per unit price will fall at Firm 2 as well. As the fixed fee is identical across firms, unit prices must also be the same. Firm 1 will serve more customers than under a symmetric strategy combination, with both  $A$  and  $Q$  identical across firms. However, we will assume that the firms all serve the same share of high-demand and low-demand consumers because each consumer is indifferent between all firms. This is sufficient to ensure that there exists a consumer equilibrium defining a consumer-price profile  $(n, p)$  for a strategy combination with symmetric fixed fees, when there is demand-side heterogeneity.

## 2.2 Symmetric strategy subgames with linear demand

Given the discontinuities of demand faced by the firms, generally it is not possible to solve for the equilibria of the game analytically. Instead, we derive some

insights into two-part pricing in oligopoly by focusing on two possible symmetric strategy combination subgames. In the first subgame, low-demand types are excluded from making purchases, whereas in the second subgame, both types are served. Next, in section 3, we search for the equilibria for the duopoly version of the game, using a numerical example with linear demand and constant marginal cost.

Hereafter, we focus on a specific case where consumers' preferences are represented by a quadratic utility function. We let the reservation utility be zero for both consumers.  $V = \theta_\ell q - \frac{1}{2}q^2 - T$ ,  $\ell = 1, 2$ , if the consumers pay  $T$  and consume  $q$  units. Otherwise, the consumers obtain zero utility. Each consumer has a linear demand function  $q_\ell = \theta_\ell - p$ ,  $\ell = 1, 2$ . Letting  $\theta \equiv \lambda\theta_1 + (1 - \lambda)\theta_2 \geq \theta_1$ , expected demand is  $\lambda q_1 + (1 - \lambda)q_2 = \theta - p \equiv q(p, \theta)$ . For a consumer paying a unit price of  $p$ , the indirect utility, exclusive of the fixed fee, is  $V(p, \theta_\ell) = \frac{1}{2}(\theta_\ell - p)^2$ ,  $\ell = 1, 2$ . Demand elasticity is defined as  $\varepsilon(p, \theta) = -q'_p(p, \theta)p/q(p, \theta) = p/(\theta - p)$ .

Because we are interested in how equilibrium strategies are affected by heterogeneity in demand, the example is simplified by letting  $\theta_1 = 1$  and  $c = \frac{1}{2}$ . Increased demand-side heterogeneity is captured by variations in  $\lambda$  and  $\theta_2$ . Then, high heterogeneity can arise either from an increase in the number of type 2 consumers ( $\lambda$  decreases), or because a type 2 consumer has a higher willingness to pay relative to a type 1 consumer ( $\theta_2$  increases). Hence, increased demand-side heterogeneity is captured by an increase in  $\theta$ .

In this section, we are particularly interested in whether a symmetric strategy subgame where all firms serve both consumer types can be a stable equilibrium, or if it is more likely that a symmetric strategy subgame where all firms serve high-demand consumers only is an equilibrium for the whole game. We do not consider asymmetric strategy subgames as equilibrium candidates. Hence, we proceed by characterizing the firms' symmetric strategies and the corresponding consumer equilibrium in these two specific subgames. Then, we analyze the firms' incentives to deviate from a symmetric strategy. As will be demonstrated in section 3, there is no unique equilibrium with respect to market coverage in this game, and we may have multiple equilibria.

**Low-demand types excluded.** Consider first the subgame when the firms serve high-demand consumers only (S1), which is the case presented in Harrison and Kline (2001). Firms choose a capacity and a fixed fee at stage one, and per unit prices are determined at stage two. When rival firms charge their consumers according to the tariff  $T_{-i} = A_{-i} + p_{-i}q$ , a consumer is indifferent between buying from firm  $i$  and one of the other firms when his or her participation constraint is binding. If the firm leaves the consumer with additional surplus, it sacrifices profit. If there is at least one additional active firm where consumers would buy a strictly positive quantity, we expect that the participation constraint is binding.

Firm  $i$  maximizes profit subject to voluntary participation and market clear-

ing, as follows:

$$\Pi_i = n_i A_i + (p_i - c) Q_i. \quad (11)$$

$$\begin{aligned} A_i &= V(p_i, \theta_2) - V(p_{-i}, \theta_2) + A_{-i} \\ Q_{\mathcal{J}} &= (N(1 - \lambda) - n_i) q_2(p_{-i}) \\ Q_i &= n_i q_2(p_i). \end{aligned} \quad (12)$$

The outcome is stated in Lemma 1. The calculations leading to Lemma 1 are given in Appendix A.

**Lemma 1 (S1: Low-demand types are excluded)** *In a symmetric strategy subgame,  $s = s_i = (Q_{TT}^2, A_{TT}^2)_{i \in K}$ , where low-demand consumers are excluded, the firms' best response with respect to capacity and the fixed fee results in a consumer equilibrium where the per unit price is equal to marginal cost. As the number of firms ( $k$ ) approaches infinity, the fixed fee and the profit approach zero*

$$\begin{aligned} p_{TT}^2 &= c, \\ A_{TT}^2 &= \min \left\{ V(c, \theta_2), \frac{c q_2(c)}{(k-1) \varepsilon(c)} \right\}, \\ Q_{TT}^2 &\equiv Q = \frac{N}{k} (1 - \lambda) q_2(c). \end{aligned}$$

Lemma 1 is the result in Harrison and Kline (2001). These authors provided a thorough treatment of Cournot competition with two-part tariffs and a single consumer type and guided the reader through all proofs. In addition, they showed that the pricing described in Lemma 1 is a unique Nash equilibrium in pricing strategies in the single consumer-type game. In addition to the equilibrium with symmetric market shares, there also exist equilibria that are asymmetric in market shares.

According to this Lemma, the fixed fee in the single consumer-type case converges toward zero as the number of firms approaches infinity. The optimal tariff is a cost-plus-fixed-fee tariff. If firm  $i$  takes the number of consumers it serves as given, for any tariff charged by rival firms, the reservation utility is defined as a constant and will not affect the optimization with respect to unit price. Then, the problem resembles the monopoly problem, and the marginal price is identical to that in a monopoly. To attract additional consumers from rival firms, firm  $i$  has to adjust the fixed fee. Hence, a marginal increase in market share affects firm  $i$ 's profit via the fixed fee. Finding the profit maximizing strategy reduces to finding the optimal number of consumers to serve.

**Both consumer types are served.** Now, we turn to the symmetric strategy subgame where all firms are obliged to cover the market (S2). As prices are determined by market clearing conditions at the last stage, it is possible to determine

the best response capacity for some arbitrary fixed fee,  $A \geq 0$ . As described above, one firm can increase its market share by increasing its capacity and accepting a lower per unit price. For any fixed fee  $A$ , in the interval  $[0, V(0, 1)]$ , there exists a unique best response capacity  $Q = \frac{N}{k}q(p, \theta)$ . However, for each fixed fee there exists only one per unit price such that the participation constraint for low-demand consumers binds simultaneously. Then, of all the possible pairs  $(A, p)$  satisfying the firms' best response behavior, we focus on the pair that satisfies  $V(p, 1) = A$ . Lemma 2 presents the symmetric strategy subgame where these conditions are taken into account.

**Lemma 2 (S2: Both consumer types are served)** *In a symmetric strategy subgame  $s = s_i = (Q_{TT}^{12}, A_{TT}^{12})_{i \in K}$ , where low-demand consumers are served, the firms' best response with respect to capacity choice results in a consumer equilibrium with a per unit price between zero and the uniform Cournot price.*

*If the number of firms ( $k$ ) is not too large, and/or if demand-side heterogeneity ( $\theta$ ) is sufficiently large, the per unit price is above marginal cost. Otherwise, the market clearing price falls below marginal cost. Let  $\theta \in \langle 1, 2 \rangle$*

(a) *The per unit price and the fixed fee are determined jointly by conditions:*

$$\frac{p_{TT}^{12} - c}{p_{TT}^{12}} = \frac{1}{k\varepsilon(p_{TT}^{12}, \theta)} \left( 1 - (k-1)\varepsilon(p_{TT}^{12}, \theta) \frac{A_{TT}^{12}}{q(p_{TT}^{12}, \theta)p_{TT}^{12}} \right),$$

$$A_{TT}^{12} = V(p_{TT}^{12}, 1) > 0.$$

(c) *The firms' capacity is given by:*

$$Q_{TT}^{12} = \frac{N}{k} \left( \lambda q_1(p_{TT}^{12}) + (1 - \lambda) q_2(p_{TT}^{12}) \right) > 0.$$

(d) *For  $k \leq 3 + 8\theta(\theta - 1)$ , the per unit price is above marginal cost. Otherwise, the per unit price is below marginal cost. The fixed fee (the per unit price) increases (decreases) with  $k$ . For any finite number  $k$ , fixed fee revenues more than cover losses in variable profit. In the limit case where  $k \rightarrow \infty$ , the fixed fee revenues  $(N/k)A_{TT}^{12}$  exactly covers the aggregate per unit loss  $(p_{TT}^{12} - c)Q_{TT}^{12}$ , and profit becomes zero. Finally, profit is always larger in this subgame with two-part tariffs than it is in a uniform pricing game.*

Lemma 2 is proved in Appendix B. Lemma 2 shows that there are nontrivial differences between pricing in the homogeneous type case and the heterogeneous type case when the firm is exposed to competition. First, depending on the degree of competition and on the demand-side heterogeneity, the per unit price varies

between zero and the uniform Cournot price.<sup>6</sup> Hence, firms may charge per unit below marginal cost. Second, the size of the fixed fee increases with the number of firms, and firms are able to extract all surplus from low-demand consumers. In contrast to this, the per unit price with homogeneous demand is always equal to marginal cost and independent of the number of firms, and the fixed fee decreases with the number of firms.

The extension of the model from one to two types of consumers partly explains the change in the per unit price. Without demand-side heterogeneity firms practice cost-plus-fixed fee pricing, and a marginal increase in market share affects firm  $i$ 's profit via the fixed fee revenue. The per unit price is adjusted to maximize consumer surplus, whereas the fixed fee is adjusted to satisfy the participation constraint. It is well known from a monopoly model that a firm serving two types of consumers with a single two-part tariff should let the unit price exceed marginal costs; see Oi (1971).<sup>7</sup> However, in our duopoly model with demand-side heterogeneity, firms cannot change the fixed fee on a unilateral basis without violating the Universal Service requirement. Hence, they are de facto restricted to compete in capacities. The isolated effect of this is that the per unit price will be above marginal cost. However, as each consumer pays a fixed fee, a marginal increase in the market share increases the fixed fee revenues, which induce oligopolistic firms to compete more aggressively than they would do otherwise. The fixed fee is sheltered from competition, and the firms compete aggressively in order to capture the fixed fee revenues by lowering the per unit price. If the number of competitors is big, and demand-side heterogeneity is not too large, this effect may dominate and the firms prefer a high fixed fee and a per unit price below marginal cost. Hence, if the firms can coordinate (or collude) on the fixed fee, they prefer a high fixed fee and a per unit price below marginal cost. The regulated per subscriber line price of unbundled access to the local loop can be an instrument for such coordination (or collusion).

Figure 3 provides a graphical illustration of Lemma 2 in the duopoly case. We find the duopoly symmetric strategy subgame tariff by searching for a fixed fee along the path  $\hat{p}(\theta, A)$ , such that the low-demand consumers' participation constraint is binding (at  $D(\theta)$ ). The monopoly tariff is found by choosing a fixed fee such that the participation constraint binds at a per unit price  $p_M^{12}$  (at  $M(\theta)$ ). As  $\theta$  approaches one – that is, as demand-side heterogeneity vanishes – the duopoly tariff approaches  $(\bar{A}_{TT}^{12}, \bar{p}_{TT}^{12})$ , whereas the monopoly tariff approaches  $(\bar{A}_M^{12}, c)$ . However, as the demand-side heterogeneity ( $\theta$ ) increases,  $p_M^{12}$  increases

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<sup>6</sup>The per unit price in a uniform pricing  $k$ -firm Cournot oligopoly is given by:  $(p_{UC}^{12} - c)/p_{UC}^{12} = \frac{1}{k\varepsilon(p_{UC}^{12}, \theta)}$ .

<sup>7</sup>The mark-up in a two-part pricing monopoly with heterogeneous demand is given by:  $(p_M^{12} - c)/p_M^{12} = \frac{1-\lambda}{\varepsilon(p_M^{12}, \theta)} \left( \frac{q_2(p_M^{12}) - q_1(p_M^{12})}{q(p_M^{12}, \theta)} \right) = \frac{1}{\varepsilon(p_M^{12}, \theta)} \left( 1 - \frac{q_1(p_M^{12})}{q(p_M^{12}, \theta)} \right)$ , i.e.,  $p_M^{12} \geq c$ . The price-cost margin is higher the larger is the difference between the consumer types ( $\theta_1$  versus  $\theta_2$ ), and the larger is the proportion of high-demand consumers ( $\lambda$  approaches zero).

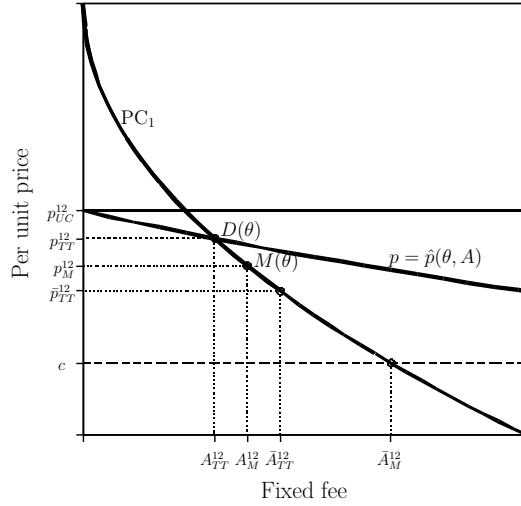


Figure 3: *Fixed fee and per unit price in a symmetric strategy subgame duopoly.*

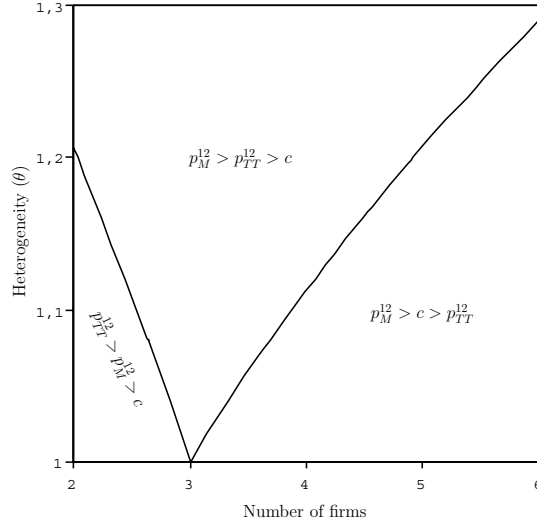


Figure 4: *Per unit prices in monopoly and oligopoly. Both  $p_M^{12}$  and  $p_{TT}^{12}$  increase in  $\theta$ , and  $p_{TT}^{12}$  decreases in  $k$ . The duopoly per unit price can exceed the monopoly per unit price, and  $p_{TT}^{12}$  can fall below marginal cost for  $k \geq 3$*

faster than  $p_{TT}^{12}$ , and for  $\theta > \frac{1}{2}(\sqrt{2} + 1)$  we find that  $p_M^{12} > p_{TT}^{12}$ , ( $M(\theta)$  lies north-west of  $D(\theta)$ ). If  $\theta > \frac{5}{4}$ , we find that  $p_M^{12} > p_{UC}^{12}$ , ( $M(\theta)$  lies north-west of  $p_{UC}^{12}$ ). When  $\theta$  is close to 1.5,  $p_M^{12}$  is close to one and low-demand consumers' purchases are close to zero,  $p_{TT}^{12}$  is close to one when  $\theta$  is close to two. The trade-off between competition and demand-side heterogeneity in per unit prices is illustrated in Figure 4. Figure 5 illustrates pricing and profit in the  $k$ -firm



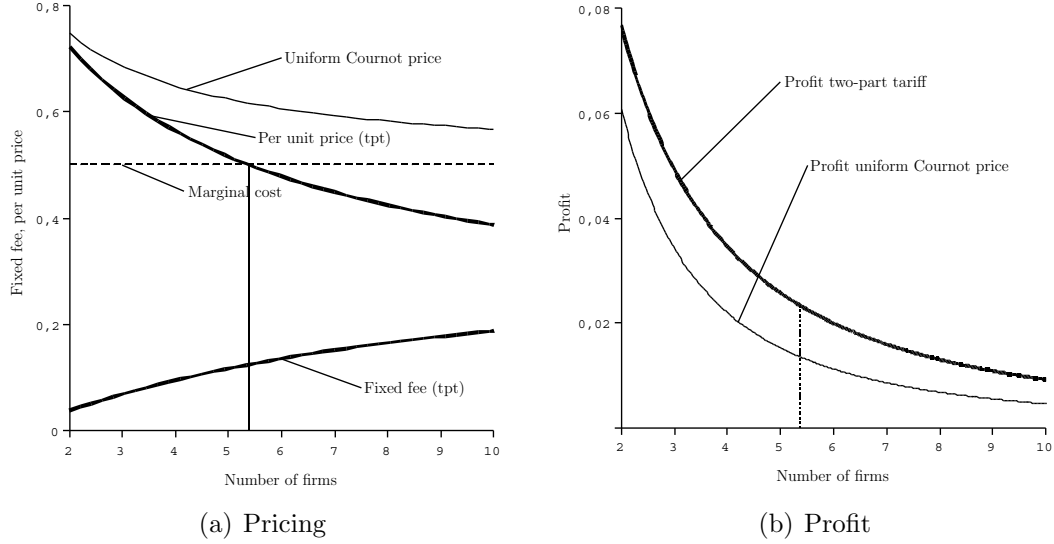


Figure 5: *Pricing and profit in a  $k$ -firm oligopoly,  $\theta_2 = 1.8$ ,  $\lambda = .7$ , ( $p_M^{12} = .74$ )*

symmetric strategy subgame described in Lemma 2. The fixed fee increases as competition becomes fiercer, whereas the per unit price decreases and falls below marginal cost if a sixth firm enters the market.

### 3 Symmetric strategy duopoly equilibrium

In this section, we ascertain whether one of the symmetric strategy combinations in section 2.2 can be an equilibrium for the whole game when firms make their decision about market coverage in a duopoly. In particular, we want to explore whether full market coverage can arise as an equilibrium feature. As is well known from studies on monopoly, it is sometimes beneficial for a firm to exclude low-demand segments in order to exploit the larger willingness to pay of high-demand segments. In other cases, it will be preferable to serve both high-demand and low-demand segments. It is important for a regulatory agency concerned about universal service in the industry to establish whether full market coverage arises in equilibrium. If the subgame S1 is robust against unilateral deviations, full market coverage is less likely to occur. It is likely that the regulator emphasizes market coverage and it may impose a Universal Service Obligation on one of the firms. If the subgame S2 is robust against unilateral deviations, fostering competition can be a way to achieve full market coverage and the Universal Service requirement. If the subgame S2 is not robust against deviations, we expect that the firms will try to avoid the obligation, for instance, by offering optional tariffs that are attractive to high-demand consumers only. Firms operating without such obligations will

aim at high-demand segments, in other words, “cream scimming”.

The monopolist’s cut-off value is found simply by comparing the monopoly profit in the two alternatives. Hence, if  $\Pi_M^2 > \Pi_M^1$  low-demand segments will be excluded. If  $\lambda < \lambda^*$ , the monopolist serves only type 2 consumers, whereas if  $\lambda > \lambda^*$ , it serves both segments.

$$\lambda \equiv \lambda^*(\theta) = \frac{1}{2} + \frac{3-4\theta_2 + \sqrt{(4\theta_2^2-3)(4\theta_2^2-8\theta_2+5)}}{8(\theta_2-1)^2}. \quad (13)$$

For any pair  $(\theta_2, \lambda)$  satisfying  $\lambda^*(\theta_2)$ , the per unit price is higher than the monopoly per unit price.<sup>8</sup> According to Lemma 2, the duopoly extracts all surplus from low-demand consumers in subgame S2. Thus, although a duopoly extracts all surplus from low-demand consumers, the joint duopoly profit is lower than the monopoly profit. Further, if the two firms were able to coordinate their decisions with respect to market coverage, they would choose to exclude low-demand consumers in some cases where these consumers would have been served in a monopoly. However, as we will demonstrate, without coordination it is difficult to sustain a duopoly outcome that excludes low-demand consumers from the market. A monopoly firm can extract surplus from high-demand segments by deliberately excluding low-demand segments, because a monopoly can choose to design a tariff that low-demand segments would never accept. However, a duopoly firm cannot prevent a competing firm from applying a tariff that low-demand segments will accept, and such a tariff will be strictly preferred by high-demand segments.

In the following two sections, we examine the incentive of one firm to deviate from the symmetric strategy combination subgames in Lemma 1 and 2. Section 3.1 examines one firm’s incentive to deviate when the low-demand segment is excluded (from S1). Section 3.2 examines the incentive to deviate when both consumer types are served (from S2).

### 3.1 Deviation if low-demand consumers are excluded

First, let us consider a symmetric strategy combination where both firms serve only high-demand consumers, and the tariff is given by  $(A_{TT}^2, c)$ ,  $A_{TT}^2 = V(c, \theta_2)$ , and  $n_a = n_b = \frac{N}{2}(1 - \lambda)$ .

One firm, say Firm  $a$ , could deviate by setting a tariff (or, properly speaking, choosing a strategy) that low-demand consumers are willing to accept, thus capturing all type 1 consumers,  $\lambda N$ . However, as low-demand consumers derive a nonnegative surplus, high-demand consumers will derive a strictly positive surplus by switching to Firm  $a$ . Then, the deviating firm will serve a mix of both

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<sup>8</sup>They are identical for  $\lambda = \frac{1}{2} + \frac{\theta_2 - \sqrt{2}}{2(\theta_2 - 1)}$

types. It will serve all type 1 consumers and more than half, but not all, of type 2 consumers because the other firm holds capacity  $n_b q_2(c)$ , which it will sell. Because Firm  $a$  increases its market size, this tends to make such a deviation profitable. However, profit per consumer is reduced.

Let the deviating firm choose a strategy  $(\tilde{Q}_{TT}^{12}, \tilde{A}_{TT}^{12})$ , or equivalently, charge a tariff  $(\tilde{A}_{TT}^{12}, \tilde{p}_{TT}^{12})$  in order to maximize profit, subject to voluntary participation and Firm  $b$ 's strategy  $(Q_{TT}^2, A_{TT}^2)$ . The problem is to maximize the following:

$$\begin{aligned} \tilde{\Pi}_{TT}^{12} = [N - \bar{n}_b] \tilde{A}_{TT}^{12} + \\ (\tilde{p}_{TT}^{12} - c) [N\lambda\tilde{q}_1 + (N(1 - \lambda) - n_b)\tilde{q}_2], \end{aligned} \quad (14)$$

subject to

$$V(\tilde{p}_{TT}^{12}, 1) \geq \tilde{A}_{TT}^{12} \quad (15)$$

$$V(\tilde{p}_{TT}^{12}, \theta_2) - V(\tilde{p}_{TT}^{12}, 1) = V(\bar{p}_{TT}^2, \theta_2) - V(c, \theta_2) \quad (16)$$

$$\frac{N}{2}(1 - \lambda)q_2 \geq \bar{n}_b\bar{q}_2, \quad (17)$$

where  $\tilde{q}_i = q_i(\tilde{p}_{TT}^{12})$ ,  $\ell = 1, 2$ ,  $\bar{q}_2 = q_2(\bar{p}_{TT}^2)$ , and  $q_2 = q_2(c)$ . In order to restore voluntary participation, the unit price at firm  $b$  falls to  $\bar{p}_{TT}^2 < c$ . As a unit price reduction leads in turn to an increase in type 2 consumers' demand,  $q_2(\bar{p}_{TT}^2) > q_2(c)$ , the capacity supplied by Firm  $b$  becomes insufficient to serve half the population of type 2 consumers, and  $\bar{n}_b < (1 - \lambda)N/2$  is adjusted to restore market clearing at Firm  $b$ . Formally, the participation constraint (16) and the market clearing condition (17) jointly determine Firm  $b$ 's share of type 2 consumers as a function of Firm  $a$ 's strategy,  $\bar{n}_b = \bar{n}_b(\bar{p}_2(\tilde{p}_{TT}^{12}))$ . One such deviation is illustrated in Figure 6. S1 is given by the tariff at A on PC<sub>2</sub>, where high-demand types are indifferent between buying and not buying – that is, the high-demand types' participation constraint is binding. PC<sub>2</sub> is the indifference curve, where the net utility of type 2 consumers is zero,  $V(p, \theta_2) - A = 0$ . The participation constraint of low-demand consumers is binding along the indifference curve through C (equation 15), and high-demand types are indifferent between B and C because they are on the same indifference curves (equation 16). As the per unit price is lower in B, Firm  $b$  serves less than  $N(1 - \lambda)/2$ .

Although Firm  $a$  obtains a lower profit per consumer when it deviates, it expands its market. When  $\lambda$  is low, or  $\theta_2$  is high, the market expansion effect is less likely to cover the loss of profit per consumer. In that case, there are few type 1 consumers to serve and expected profit per consumer is significantly lower when Firm  $a$  deviates. Conversely, we expect that a deviation is profitable when demand-side heterogeneity is low. For a value of  $\lambda$  close to  $\lambda^*$ , the expected revenue per consumer is identical. Therefore, we conjecture that it is profitable to deviate.

In our numerical example, we find it profitable to deviate if  $\lambda > \lambda_D^*(\theta_2)$ , where  $\lambda^* < \lambda_D^*$ . The results confirm our conjecture.

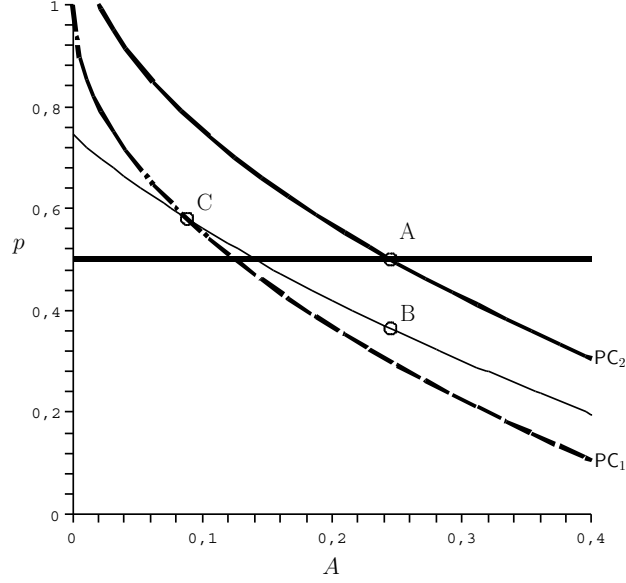


Figure 6: *Firm a deviates from  $A = (V(c, \theta_2), c)$  and offers a contract  $C = (V(\tilde{p}_{TT}^{12}, 1), \tilde{p}_{TT}^{12})$  along  $PC_1$ , and voluntary participation for type 2 is restored at  $B = (V(c, \theta_2), \tilde{p}_{TT}^2)$ , along  $PC_2$  through  $PC_1$ .*

### 3.2 Deviation if low-demand types are served

Second, let us consider the equilibrium candidate when both firms serve both types of consumers, and the firms' tariffs are given by  $(A_{TT}^{12}, p_{TT}^{12})$ . Type 2 consumers enjoy a positive surplus and type 1 consumers receive a zero surplus. Firms have equal market shares, serving  $N/2$  each, and they serve low-demand types and high-demand consumers in proportions  $\lambda$  and  $(1 - \lambda)$ , respectively. The consumer equilibrium per unit price will always be larger than the marginal cost because  $k = 2 < 3 + 8\theta(\theta - 1)$  for  $\theta > 1$ . Pricing in the subgame where both firms serve both consumer types is described in Appendix C. Let us now consider a unilateral deviation by Firm  $a$ ,  $(\tilde{A}_{TT}^2, \tilde{p}_{TT}^2)$ , when the strategy for Firm  $b$  is fixed,  $(Q_{TT}^{12}, A_{TT}^{12})$ .

In this case, Firm  $a$  can deviate by using one of two strategies: (i) Firm  $a$  aims to serve all type 2 consumers,  $n_a = N(1 - \lambda)$ , but to leave them a positive surplus. In that case, the firm sets  $\tilde{A}_{TT}^2 < V(c, \theta_2)$ ; (ii) knowing that Firm  $b$  has a limited capacity, Firm  $a$  could act as a monopoly on any residual demand. The firm will then serve less than the entire pool of type 2 consumers,  $n_a < N(1 - \lambda)$ , but it will extract all of the surplus  $\tilde{A}_{TT}^2 = V(c, \theta_2)$ .

Consider the first strategy. Firm  $a$  announces a tariff  $(\tilde{A}_{TT}^2, c)$  that is strictly preferred by type 2 consumers. The firm will extract as much as possible from

type 2 consumers via the fixed fee and will maximize the following:

$$\tilde{\Pi}_{TT}^2 = N(1 - \lambda) \tilde{A}_{TT}^2, \quad (18)$$

subject to:

$$V(c, \theta_2) - \tilde{A}_{TT}^2 \geq V(\bar{p}_{TT}^{12}, \theta_2) - A_{TT}^{12} \quad (19)$$

$$\frac{N}{2}(\lambda q_1 + (1 - \lambda), q_2) \geq N\lambda \bar{q}_1 \quad (20)$$

where  $q_i = q_i(p_{TT}^{12})$ , ( $\ell = 1, 2$ ), and  $\bar{q}_1 = q_1(\bar{p}_{TT}^{12})$ . The unit price  $\bar{p}_{TT}^{12}$  is adjusted to account for the fact that Firm  $b$  is now left with type 1 consumers only, instead of a mix of type 1 and type 2 consumers. Given that type 1 consumers receive exactly their reservation utility, the unit price that clears the market at Firm  $b$  cannot exceed  $p_{TT}^{12}$ . Instead, type 1 consumers are rationed at Firm  $b$ . Hence,  $0 < \bar{p}_{TT}^{12} < p_{TT}^{12}$ . This restricts the fixed fee in (19), which in turn will restrict the profits earned on type 2 consumers.

Looking at (18), it may appear that a deviation is profitable when  $\lambda$  is small. However, when  $\lambda$  is small,  $\bar{p}_{TT}^{12}$  is low to ensure market clearing at Firm  $b$ , which restricts the size of the fixed fee that Firm  $a$  can charge type 2 consumers. If  $\bar{p}_{TT}^{12} < c$ , we have  $\tilde{A}_{TT}^2 < A_{TT}^{12}$ . In our numerical example, we are not able to find any profitable deviations, except for  $(\theta_2, \lambda)$  close to  $(1, \frac{1}{2})$ , which is not economically interesting because it implies that the willingness to pay of the two groups is almost identical, and that the groups are of the same size. In that case, rather than charging a per unit price above marginal cost and extracting the entire consumer surplus, Firm  $a$  would rather serve one half of the market at a per unit price equal to marginal cost and extract almost all of the surplus. This suggests that a duopoly outcome where both firms serve both types of consumers can be an equilibrium outcome in situations where a monopolist would have preferred to serve only one type of consumer.

Figure 7(a)-7(c) illustrates possible deviations along the high demand consumers indifference curve through A, where high demand consumers are equal off as before. The bold, dashed line is low demand consumers' participation constraint (where  $V(p, 1) = A$ ). The other dashed lines represents indifference curves with positive net utility for type 1. The bold, solid line is high demand consumers' participation constraint (where  $V(p, \theta_2) = A$ ). The other solid lines represents indifference curves with positive net utility for type 2. Firm  $a$  aims to serve type 2 consumers at a tariff given by point B. If the per unit price at Firm  $b$  falls, for instance to point D, the fixed fee at the deviating firm must be lower than at point B. In figures 7(a) and 7(b), B is not a feasible tariff because type 2 consumers strictly prefer D over B. Type 2 consumers are indifferent between C and D. At best, therefore, the deviating firm can offer tariff C. In figure 7(c), low-demand consumers are rationed at Firm  $b$ , where they pay the per unit price before deviation. In addition, Firm  $a$  can actually apply the tariff at B, but it is left with few high-demand consumers.

The other possible deviation strategy is for Firm  $a$  to act as a monopoly on any residual demand from type 2 consumers, as illustrated by tariff B on PC<sub>2</sub> in figure 7(d). This time, consider a deviation where Firm  $a$  announces a tariff that extracts all surplus from type 2 consumers,  $B = (V(c, \theta_2), c)$ . Type 2 consumers enjoy a positive surplus by switching to Firm  $b$ 's tariff. Hence, because capacity at Firm  $b$  is insufficient to meet all demand, type 2 consumers will crowd out type 1 consumers at Firm  $b$ . Firm  $a$  earns monopoly profits on each type 2 consumer it serves, and aggregate profit is given by:

$$\tilde{\Pi}_{TT}^2 = [N(1 - \lambda) - \bar{n}_b] V(c, \theta_2), \quad (21)$$

where  $\bar{n}_b$  is the number of type 2 consumers that can be served by Firm  $b$ . Type 2 consumers are indifferent between the two firms' tariffs when they receive a zero surplus. Hence, the unit price in Firm  $b$ 's tariff must be adjusted in order to restore voluntary participation for type 2 consumers,  $\bar{p}_{TT}^2$ . This time, Firm  $b$  is left with type 2 consumers only, instead of a mix of type 1 and type 2 consumers, as follows:

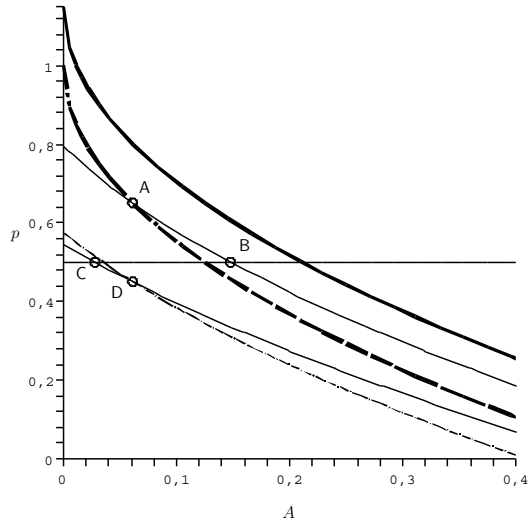
$$V(\bar{p}_{TT}^2, \theta_2) - A_{TT}^{12} \geq 0 \quad (22)$$

$$\frac{N}{2} (\lambda q_1 + (1 - \lambda) q_2) \geq \bar{n}_b \bar{q}_2 \quad (23)$$

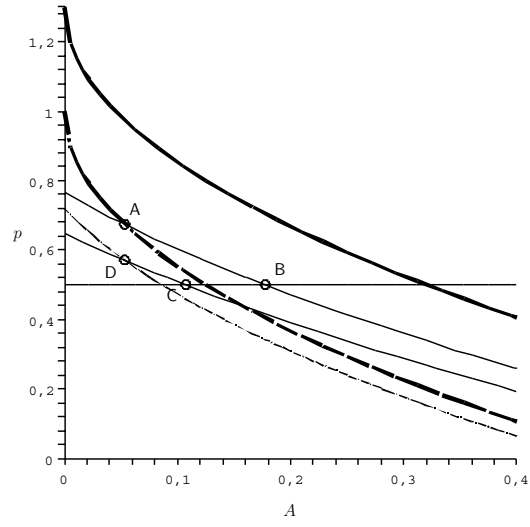
$$N(1 - \lambda) \geq \bar{n}_b. \quad (24)$$

We find no examples where such a deviation is profitable, except for  $(\theta_2, \lambda)$  close to  $(1, 0)$ , which is not very interesting because it means that there are very few type 1 consumers and that their willingness to pay is very close to that of type 2 consumers. Again, the fact that the non-deviating firm has committed itself to selling a certain quantity acts as a constraint on the deviating firm's behavior. If there are few type 2 consumers, the non-deviating firm would serve them all and the deviating firm would have no residual demand. If there are many type 2 consumers, the price per unit would be closer to marginal costs. If so, there is a more limited scope for the deviating firm to generate additional consumer surplus from type 2 consumers by setting price per unit equal to marginal costs.

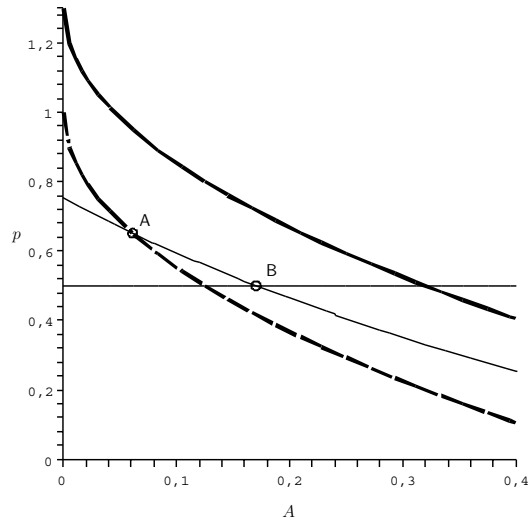
Figure 8 summarizes our conclusions. The north-west part of the diagram characterizes a case with low heterogeneity, whereas the south-east part characterizes a case with high heterogeneity. Using monopoly two-part pricing as a reference, we know that low-demand consumers will be excluded if the parameter values lie in regions B, B<sub>1</sub>, C, and C<sub>1</sub> (if  $\lambda < \lambda^*(\theta_2)$ ), whereas both consumers will be served in region A. Considering a duopoly with symmetric strategies only, we reach the following conclusions. In regions A, B, and B<sub>1</sub>, the entire market will be covered in equilibrium. All consumer types are served with a two-part tariff that extracts the entire surplus from low-demand consumers. The per unit price in the tariff is lower than the monopoly (two-part tariff) per unit price in regions A and B, whereas  $p_{TT}^{12} > p_M^{12}$  in region B<sub>1</sub>. In regions C and C<sub>1</sub>, we have



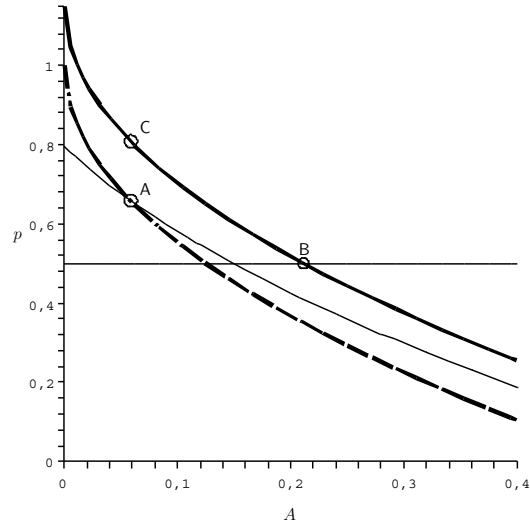
(a)  $\bar{p}_{TT}^{12} < c$ ,  $(\theta_2, \lambda) = (1.15, 0.4)$



(b)  $c < \bar{p}_{TT}^{12} < p_{TT}^{12}$ ,  $(\theta_2, \lambda) = (1.3, 0.54)$



(c)  $p_{TT}^{12} < \bar{p}_{TT}^{12}$ ,  $(\theta_2, \lambda) = (1.3, 0.7)$



(d) Residual demand,  $(\theta_2, \lambda) = (1.15, 0.3)$

Figure 7: Deviations from a symmetric strategy subgame in which both consumer types are served (A). Along  $PC_2$  through A (7(a)-7(c)) or along  $PC_2$  (7(d)). Solid lines represent the high-demand consumers' indifference curves, and dashed lines represent the indifference curves of low-demand consumers.

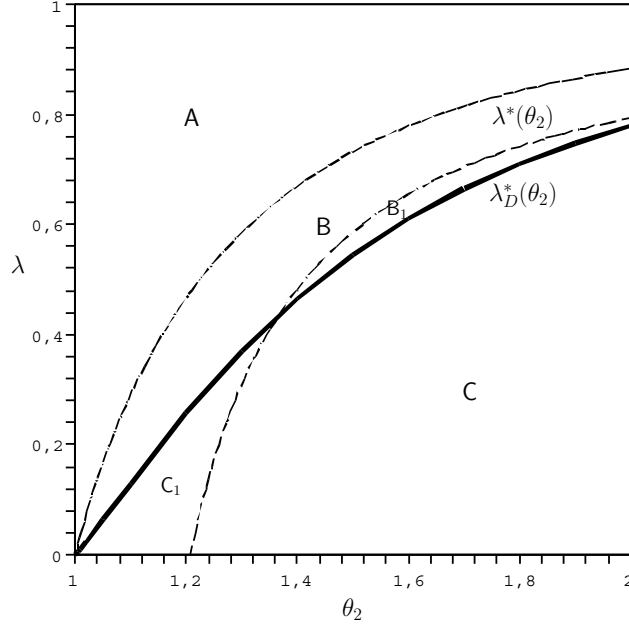


Figure 8: *Symmetric strategy Nash duopoly equilibria. Full market coverage (A, B, and B<sub>1</sub>), multiple equilibria (C<sub>1</sub> and C).*

multiple equilibria. For the duopoly game, a symmetric strategy subgame where both types are served, and a symmetric strategy subgame where low-demand types are excluded, can both be Nash equilibria. If both types are served, the per unit price  $p_{TT}^{12}$  is below  $p_M^{12}$  in region C<sub>1</sub>, whereas the opposite is the case in region C.

Therefore, this particular case suggests that, to a large extent, a Nash equilibrium in a duopoly where both firms serve only one type of consumer coincides with the case where a monopolist prefers to serve only one type of consumer. However, there are parameter values for which low-demand consumers would be served in a duopoly, whereas they would be excluded in a monopoly.

## 4 Concluding remarks

Harrison and Kline (2001) have shown a way to extend the traditional Cournot model to a setting with not only a per unit price, but also a fixed fee. They found that each firm sets a price per unit equal to marginal costs, and a positive fixed fee that approaches zero only when the number of firms becomes large. We extended their model to account for demand-side heterogeneity, which is a natural extension because most industries face heterogeneous demand. At the same time, we added the assumption that a regulatory agency can impose a Universal Service



Obligation on the firms. We found that the conclusions in Harrison and Kline (2001) are not robust to such an extension because firms’ pricing strategies may change when we allow for demand-side heterogeneity. In addition, we showed that fostering competition may contribute to the fulfillment of the Universal Service requirement that is common in industries such as telecommunications, which applies nonlinear pricing on a normal basis.

If the market is covered – that is, if both types of consumers are served – we found that the per unit price may be below marginal costs when as few as four firms exist. In addition, the fixed fee may be positive even with an infinite number of firms. Hence, the results in Oi (1971) cannot be extended to oligopoly with heterogeneous consumers.

Our results showed that multiple Nash equilibria may exist. First, a potential equilibrium outcome is that both firms serve only one type of consumer. Numerical examples suggest that, to a large extent, this equilibrium outcome coincides with the case where the monopolist chooses to serve only one type of consumer. Nevertheless, the duopoly is more likely than a monopoly to cover the entire market. Second, we found that both firms serving both types of consumers can be an equilibrium outcome. In fact, we found no numerical examples where the firms would deviate from such an outcome, beyond a very small set of exceptions that are economically uninteresting. Deviations from a symmetric strategy subgame where both consumer types are served are not profitable because the rival, non-deviating firm has a fixed quantity that acts as a constraint on the deviating firm’s behavior.

A particular aim of this paper was to explore whether full market coverage can arise as an equilibrium feature. Interestingly, we found that fostering competition can contribute to full market coverage and the fulfillment of the Universal Service requirement. Our results showed that a subgame with a Universal Service Obligation is robust against unilateral deviations. Hence, our model does not support the theory that the firm operating under a Universal Service Obligation serves low-demand segments of the market while other firms “skim the cream” by serving the more profitable high-demand segments.

## Appendix

### A Subgame 1. Only high-demand types served

We assume  $N(1 - \lambda)$  identical consumers. Given that firm  $i$  serves a number of consumers,  $n_i$ , when it charges a tariff  $(p_i, A_i)$ , and that all other firms charge identical tariffs  $(p_{-i}, A_{-i})$ , firm  $i$ ’s maximization problem reduces to maximizing

profit subject to voluntary participation and market clearing, as follows:

$$\Pi_i = n_i A_i + (p_i - c) Q_i. \quad (\text{A.1})$$

$$\begin{aligned} A_i &= V(p_i, \theta_2) - V(p_{-i}, \theta_2) + A_{-i} \\ Q_{\mathcal{J}} &= (N(1 - \lambda) - n_i) q_2(p_{-i}) \\ Q_i &= n_i q_2(p_i). \end{aligned} \quad (\text{A.2})$$

Choosing  $Q_i$ , knowing that the market clearing condition must be satisfied, amounts to maximizing profit with respect to  $p_i$ , holding  $A_{-i}$ ,  $Q_{\mathcal{J}}$ , and  $A_i$  constant. Hence:

$$(p_i - c) \frac{dq_2(p_i)}{dp} = 0 \Rightarrow p_i = c. \quad (\text{A.3})$$

Now, given that  $p_i = c$  is optimal, the firm determines the size of the fixed fee  $A_i$  by determining the optimal number of consumers to serve. Hence, under the constraints of voluntary participation and market clearing, firm  $i$  maximizes profit with respect to  $n_i$ , which yields:

$$\frac{d\Pi_i}{dn_i} = V(p_i, \theta_2) - V(p_{-i}, \theta_2) + A_{-i} + n_i \left[ -V'_p \frac{dp_{-i}}{dn_i} \right] = 0. \quad (\text{A.4})$$

Substituting from the participation constraint in (A.2), differentiating the market-clearing condition  $Q_{\mathcal{J}} = (N(1 - \lambda) - n_i)q(p_{-i})$ , and holding  $Q_{\mathcal{J}}$  fixed, enables us to state the following condition:

$$A_i = \frac{n_i}{N - n_i} \left[ q_2(p_{-i}) p_{-i} \left/ \frac{-p_{-i} \frac{dq_2(p_{-i})}{dp}}{q(p_{-i})} \right. \right] \quad (\text{A.5})$$

Further, let firms have equal market shares,  $n_i = N(1 - \lambda)/k$ . When all firms charge the same unit price,  $p_i = p_{-i} = c$ , and the participation constraint binds, we obtain  $A_i = A_{-i} \equiv A$ . Keeping in mind the fact that consumer surplus must be nonnegative, evaluating the condition in (A.5) for  $p = c$  yields the following expression for the fixed fee:

$$A = \min \left\{ V(c, \theta_2), \frac{cq_2(c)}{(k - 1)\varepsilon_2(c)} \right\}. \quad (\text{A.6})$$

This verifies the statements in Lemma 1.

## B Subgame 2. Both types are served

Now, let  $A$  be determined at stage zero. If all firms apply identical fixed fees, the market clearing price must be the same across firms. In the following calculations,

we impose the restriction that firm  $i$  chooses the same market coverage as all other firms. Hence, if every other firm serves both consumer types, firm  $i$  aims at designing a tariff that will maximize profit subject to the condition that both consumer types are willing to participate in the market. Given that every other firm holds its capacity fixed, if firm  $i$  changes its capacity, the market clearing price and the market share will change for every firm, including firm  $i$ .

Taking rival firms' capacity as given, firm  $i$  maximizes profit subject to market clearing, given that both consumer types find it individual rational to buy.

$$\Pi_i = n_i A + (p - c)Q_i. \quad (\text{B.1})$$

$$\begin{aligned} V(p, \theta_1) - A &\geq 0 \\ Q_{\mathcal{J}} &= (N - n_i)[\lambda q_1(p) + (1 - \lambda)q_2(p)] \\ Q_i &= n_i[\lambda q_1(p) + (1 - \lambda)q_2(p)]. \end{aligned} \quad (\text{B.2})$$

The optimality condition is given by:

$$\frac{d\Pi}{dp} = n_i(q(p, \theta) + (p - c)q'_p(p, \theta)) + \frac{dn_i}{dp}(A + (p - c)q(p, \theta)). \quad (\text{B.3})$$

First, differentiating the market clearing condition  $Q_{\mathcal{J}} = (N - n_i)q(p, \theta)$ , holding  $Q_{\mathcal{J}}$  fixed, and assuming  $n_i = N/k$ , we can write:

$$\frac{dn_i}{dp} = \frac{N}{k}(k - 1)\frac{q'_p(p, \theta)}{q(p, \theta)}. \quad (\text{B.4})$$

Now, we can reformulate the optimality condition as follows:

$$p - c = \frac{q(p, \theta)}{-kq'_p(p, \theta)} - \frac{k - 1}{k} \frac{A}{q(p, \theta)}. \quad (\text{B.5})$$

Finally, using the definition of the demand elasticity  $q'_p(p, \theta)p/q(p, \theta) = -\varepsilon(p, \theta)$ , we can write the mark-up as in Lemma 2.

$$\frac{p - c}{p} = \frac{1}{k\varepsilon(p, \theta)} \left( 1 - (k - 1)\varepsilon(p, \theta)\frac{A}{q(p, \theta)p} \right). \quad (\text{B.6})$$

Given the reduced form profit after the last stage of the game, firms will choose  $A$  such that the low-demand types' participation constraint is just binding if they can freely choose the fixed fee at stage zero. Hence:

$$A = V(p, \theta_1). \quad (\text{B.7})$$

This verifies the statements in Lemma 2

## C Tariff and profit in the numerical example

The following gives the firms' pricing in the two subgames of the duopoly, according to Lemmas 1 and 2. Superscript 12 is used when both consumer types are served and superscript 2 is used when type 1 consumers are excluded, while  $k$  denotes the number of active firms.

### C.1 Only type 2 consumers are served

Pricing is given by Lemma 1. The unit price is always equal to marginal cost,  $p_{TT}^2(2) = p_{TT}^2(3) = \dots = p_{TT}^2(k) = c$ , and the firms' profit per consumer is whatever they manage to capture via the fixed fee  $A_{TT}^2(k)$ . Hence,  $\Pi^2(k) = \frac{N(1-\lambda)}{k} A_{TT}^2(k)$ . With less than three active firms, we have:

$$A_{TT}^2(2) = A_{TT}^2(3) = \frac{1}{8} (2\theta_2 - 1)^2.$$

With more than three firms, we have:

$$A_{TT}^2(k) = \frac{(2\theta_2 - 1)^2}{4(k - 1)}$$

### C.2 Both consumer types are served

Pricing is given by Lemma 2. With two active firms, we have:

$$p_{TT}^{12} = \frac{4}{5}\theta - \frac{1}{5}\sqrt{\theta^2 + 5(\theta - 1)^2},$$

$$A_{TT}^{12} = \frac{1}{2} \left( 1 - \frac{4}{5}\theta + \frac{1}{5}\sqrt{\theta^2 + 5(\theta - 1)^2} \right)^2,$$

$$\Pi_{TT}^{12} = \frac{N}{100} \left( 20(\theta - 1)^2 + (5 - 2\theta) \left( \theta + \sqrt{\theta^2 + 5(\theta - 1)^2} \right) \right).$$

The per unit price in a corresponding monopoly is  $p_M^{12} = \theta - 1/2$ , whereas the per unit price in a corresponding capacity game with uniform pricing is  $p_{UC}^{12} = \frac{1}{3}(\theta + 1)$ . Hence,  $p_{TT}^{12} = p_M^{12}$  for  $\theta = (1/2)(\sqrt{2} + 1)$ , and  $p_{TT}^{12} < p_{UC}^{12}$  for  $\theta \in \langle 1, 2 \rangle$ .

With  $k$  active firms, the per unit price solves:

$$p_{TT}^{12}(k) = c + \frac{q(p, \theta)}{k} \left( 1 - (k - 1) \frac{q_1(p)^2}{2q(p, \theta)^2} \right),$$

and

$$A_{TT}^{12}(k) = \frac{1}{2} \left( q_1(p_{TT}^{12}) \right)^2, \quad \Pi_{TT}^{12}(k) = \frac{N}{k^2} \left( \frac{1}{2} q_1(p_{TT}^{12})^2 + q(p, \theta)^2 \right).$$

The per unit price is declining in  $k$ , and hence, the fixed fee is increasing in the number of firms. In addition, as  $k$  goes to infinity, profit becomes:

The cut-off value  $\lambda^*(\theta_2)$  in a monopoly solves the equation  $N(1 - \lambda)V(c, \theta_2) = N(V(p, \theta_1) + (p_M^{12} - c)q(p_M^{12}, \theta))$ .

When both consumer types are served in a  $k$ -firm oligopoly and all firms charge a uniform price, we have:

$$p_{UP}^{12}(k) = \frac{2\theta + k}{2(k + 1)}, \quad \text{and} \quad \Pi_{UP}^{12}(k) = \frac{N(2\theta - 1)^2}{k4(k + 1)^2}.$$

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