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with entry**

by

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Early information in auctions with entry

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Abstract

Consider an auction in which potential bidders must sink an entry investment before learning their values, but where the auction designer can release information so that the bidders learn their values *before* entry. Such early information will induce screening of high-value bidders, and it will give rise to information rents and thereby a difference between the socially optimal auction and the auctioneer's preferred mechanism. Therefore, the auction designer has too weak incentives to produce early information. Early information may increase or reduce equilibrium entry. If entry is sufficiently reduced, early information will harm the auction designer.

JEL classification numbers: D44, D82, L10, L51

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1 Introduction

Much of the received auction theory deals with how to sell an object to one out of a number of potential buyers, each of whom privately informed about his or her valuation of the object — independent private value (IPV) auctions.¹ While early contributions dealt with situations with a given information structure and a given number of bidders, more recent contributions have allowed the number of bidders to be endogenous — so-called “auctions with entry.” Participation in an auction often entails costs that do not depend on how much the buyer actually bids, or on whether he ends up with the object. A prospective bidder will participate in — or enter — such an auction if and only if the expected gains from participating cover the entry costs.² Consequently, the number of bidders is not fixed, but may depend on how the particular auction is designed.

Two earlier contributions make the starting point for the present paper. Levin and Smith (1994) study an auction in which potential bidders must sink an entry cost before they learn their values, while Samuelson (1985) study the opposite situation, in which potential bidders learn their values before they make their entry decisions.³ In the present paper I study a Levin-Smith type of model with the twist that the auction designer can release information that enables the potential bidders to learn their values before they enter. That is, I study situations in which the auction

¹Traditional auction theory is excellently surveyed by Milgrom (1987) and McAfee and McMillan (1987a).

²The entry costs may take different forms. Contenders for procurement contracts often have to sink relation-specific (or auction-specific) investments before they submit their bids (bid preparation costs; costs of establishing the necessary organization to carry out the project on time). People interested in buying a second-hand car at a car auction have to travel from their homes, and spend time at the auction site before they can submit bids for the car they wish to buy. In either case the costs do not depend on whether the bidder ends up winning the auction or not.

³In both cases, entry decisions are simultaneous and attention is restricted to symmetric equilibria, which in Levin and Smith’s case is a mixed-strategy equilibrium. Comparing the two models then involves comparing not only two different information structures, but also two different types of equilibria. An alternative is to restrict attention to pure-strategy equilibria. I expand on this point in Section 5 below.

designer can choose whether to transform a Levin and Smith (1994) framework into a Samuelson (1985) one. One example of a situation that might fit this description is a car auctioneer who may examine the cars and publish the result in the ads for the auction. Has he the right incentives to do so? Another example is a government that wants a private firm to build a new bridge. The government may spend resources to survey the available (e.g. technical) solutions before potential bidders establish the organizations necessary to be taken seriously as contenders for the contract. Has the government the right incentives to conduct such a survey and publish the results?

Attention is restricted to information that enables the bidders to learn their values before entry, without enabling the auction designer to learn anything about the bidders' values.⁴ In the car auction example, the information that can be produced may be about, say, the make, year and color of the cars to be sold. The auctioneer knows only the distribution of tastes for these attributes in the population. Each bidder, in contrast, knows his willingness-to-pay for a car of a given make, year and color, but (unless the auction designer produces the information) does not know which (i.e. the attributes of) cars will be for sale at a given auction.⁵ Similarly, in the procurement example the information may be about, say, which technology (e.g. steel vs. concrete construction) will be cost efficient for the actual project. The procurer knows the distribution of abilities to build different types of bridges among the population of construction firms. Each bidder, in contrast, knows his ability to build bridges using each given technology, but (unless the technical information is produced) does not know which (i.e. the attributes of) technology will be efficient for this particular bridge.⁶

⁴Often, information will inform not only the prospective bidders, but also the auction designer. This will surely add another reason to produce such information, but it is outside the scope of the present paper to assess how this effect blends with the effects examined here.

⁵Clearly, the list of attributes can be made very long. The important feature is that the considered attributes can be costlessly observed once a prospective bidder shows up at the auction site, and also, at some cost, communicated to each prospective bidder prior to his entry decision.

⁶This structure suggests that the auction in consideration is not a pure IPV (independent private values) auctions, but has elements of CV (common values) as well: The technological information is not firm-specific, but project-specific. However, the project-specific information is resolved before bidding takes place, and the auction can therefore be analyzed using the methods of IPV auctions.

Numerous scholars have contributed to our understanding of auctions with entry. French and McCormick's (1984) seminal contribution to our understanding of the rent dissipation effect of entry (costs) in auctions demonstrated, among other things, that with competitive entry, the winner's expected profit equals the sum of his competitors' sunk costs. Hausch and Li (1993) study information acquisition in common value auctions with entry. They also find that the seller pays indirectly for the information acquisition costs. Harstad (1990) was the first to point out the now well-known result that the expected price may be decreasing in the number of potential bidders.⁷

More closely related to the present paper, Menezes and Monteiro (2000) show that in the absence of a reserve price it is optimal to charge a positive entry fee. Moreover, they demonstrate that it does not matter whether first or second price mechanisms are used, or whether the number of entrants is revealed to the bidders before they submit bids. (These latter results are clearly due to risk neutrality.)⁸

To recapitulate the two starting points, Levin and Smith (1994) study situations in which the potential bidders do not learn their private information until after entry. With sufficiently many potential bidders, the only symmetric equilibrium is one in which each potential bidder enters with a common probability q^* . Levying entry fees or introducing reserve prices will reduce the equilibrium probability of entry, and this, they show, will be harmful to welfare as well as to expected revenue.

⁷Other relevant contributions to the theory of auctions with entry include McAfee and McMillan (1987b), which will be discussed later in this article, Tan (1992), Engelbrecht-Wiggans (1993), Kjerstad and Vagstad (2000) and Chakraborty and Kosmopoulou (2001).

⁸See also Chakraborty and Kosmopoulou (2001), who derive related results for common value auctions. An asymmetric auction is studied by Deltas and Engelbrecht-Wiggans (2001), whose common value auction features one potential bidder who is 'mildly irrational.' The outcome can be that all the rational bidders stay out while the irrational bidder comes out with positive profit. Also of interest is Engelbrecht-Wiggans (2001) who compares oral and sealed-bid auctions with entry; and Engelbrecht-Wiggans and Nonnenmacher (1999) who present a model with historical evidence of the importance of reserve prices (and, more generally, auction design) for economic development: taxing auctioned items, sold as well as unsold, created a pressure to sell and thereby to lower the reserve prices. This made the auctions more tempting for buyers, and "New York flourished," according to the authors.

In contrast, in Samuelson (1985) the bidders have perfect information before entry. Consequently, in equilibrium the potential bidders are screened such that only potential bidders with values above some common cut-off level actually enter and submit bids. In this case the bidders will earn information rents.

Early information may affect the outcome in different ways. First, early information will make low-value bidders stay out while high-value bidders enter. Clearly, this screening will affect the distribution of values among those who enter. Second, early information may affect both the expected and realized number of entrants. Both screening and changes in the number of entrants affect ex post competition. For a given number of bidders, early information implies harder competition due to reduced bidder heterogeneity, and a higher average quality of the bidders. However, if the entry is reduced, the seller's profit may decrease as a consequence of early information. The prospective bidders, on the other hand, always prefer early information, as early information is their only source of information rent.⁹

Since advertising is one type of information production that fits our description (cf. the car auction example above), it is worthwhile to compare my findings with those of the economic theory of advertising.¹⁰ One issue in that literature is whether the level of advertisement is appropriate from a social welfare point of view. The results are ambiguous; there might be underprovision of advertising because of non-appropriability of social surplus, and overprovision because of 'business stealing.' In our model there is no business stealing, hence we should expect to get – and we do get – underprovision.

The remainder of this paper is organized as follows. The model is introduced

⁹Intuitively, a higher average quality is good for each bidder's utility, while increased competition is bad. It turns out that the combined effects of the two can be either positive or negative, and this ambiguity also drives the other mixed results: early information may increase or decrease entry, welfare and revenues (but is always good for the bidders).

¹⁰See, e.g., Tirole (1988) for a discussion. Tirole divides the literature on advertising into two broad categories. The 'partial' view sees advertising as providing information to customers and thus enabling them to make rational choices. The 'adverse' view, in contrast, claims that advertising is meant to persuade and fool consumers. Clearly, the former view is the basis of our analysis.

in the next section, the main results are laid out in Section 3, an example with uniformly distributed values and only two prospective bidders is presented in Section 4, an extension to asymmetric pure-strategy entry is found in Section 5, while some concluding remarks are gathered in Section 6. Example details and some numerical simulations are relegated to the Appendix.

2 The model

Consider a seller who wishes to sell an object to one out of a number $N \geq 2$ of potential bidders, each of whom puts value v_i on the object. Values are drawn independently from a common distribution $F(\cdot)$ with support $[0, \bar{v}]$. The seller puts value 0 on the object. If a potential bidder decides to enter, he must sink an entry investment of size $k \geq 0$. The timing of events is as follows:

1. The seller decides whether or not to release all relevant information, and commits to an auction mechanism to be used.
2. If information has been released, each bidder learns his value v_i .
3. Each bidder decides simultaneously whether or not to enter the auction. If a bidder enters, he pays the entry cost k and possible (positive or negative) entry fees.
4. If information has not been released earlier, the bidders learn their values v_i .
5. Those who have entered are asked to submit bids.
6. A winner is selected and bidders are rewarded according to the mechanism chosen in step 2.

A rather broad range of mechanisms are allowed: The mechanism may involve entry fees or subsidies, and is else just a set of functions from submitted bids to i) selection of the winner, and ii) each bidder's reward (payoff). This set of feasible

mechanisms encompasses, but is not restricted to, the four commonly studied auctions (English, Dutch, 1st price sealed bid and Vickrey auctions), with or without reserve prices.

The information the seller can release is of a kind that transforms the framework from a Levin-Smith (1994) model into that of Samuelson (1985), cf. the introduction. We make the following assumptions:

- A1. The seller and all potential bidders are risk neutral.
- A2. The auction mechanism and the number of potential bidders (N) are common knowledge, and the number n of actual bidders is revealed prior to bidding.¹¹
- A3. Discrimination of bidders and coordination among bidders is infeasible.
- A4. The density of the value distribution, $f(\cdot)$, is log-concave.¹²
- A5. The environment is such that a unique symmetric Nash equilibrium bidding function exists, and that this bidding function is increasing.¹³

In what follows, I will characterize the equilibria in each of the polar cases: non-release of information and release of information prior to entry.

2.1 Non-release of information prior to entry

Suppose a plain auction without entry fees or reserve prices is used. Then, for entry costs in a certain range — $k \in (\underline{k}, \bar{k})$ — there is, roughly speaking, “room for” more than one bidder but not for all bidders. This is the case analyzed by Levin and Smith (1994). The only symmetric equilibrium in such a situation have each

¹¹This latter assumption is without loss of generality as long as all parties are risk neutral, cf. Menezes and Monteiro (2000).

¹²This corresponds to the routinely made assumption of monotonous hazard rate, e.g. that $\frac{1-F(v)}{f(v)}$ is decreasing in v . Bagnoli and Bergstrom (1988) demonstrate that many of the commonly used distributions share this property. This include the uniform and normal distributions, as well as truncated versions of these distributions.

¹³This is trivial for second-price auctions, in which it is a dominant strategy to bid one’s value. It also tend to hold for first-price auctions (see e.g. Menezes and Monteiro, 2000).

potential bidder enter with a common probability q^* .¹⁴ Levin and Smith show that if potential bidders conform to such a symmetric equilibrium, then for $k \in (\underline{k}, \bar{k})$ the optimal mechanism is indeed a plain auction without reserve prices or entry fees. The number of actual bidders follows a binomial distribution. The expected gross utility, denoted $E[u_i]$ of any potential bidder i is, if he enters, equal to the entry cost:

$$E[u_i] = \sum_{n=1}^N \binom{N-1}{n-1} [q^*]^{n-1} [1-q^*]^{N-n} E[u_i|n] = k, \quad (1)$$

where

$$E[u_i|n] = \int_0^{\bar{v}} \int_0^{v_i} (v_i - x) d[F(x)]^{n-1} dF(v_i) = \int_0^{\bar{v}} [1 - F(x)] F(x)^{n-1} dx \quad (2)$$

is the expected gross utility conditioned on there being n entrants.

Using (1) and (2), we can now characterize the range of entry costs giving rise to the mixed-strategy entry equilibrium: $\bar{k} = k_1$ and $\underline{k} = k_N$, where $k_n \equiv E[u_i|n] = \int_0^{\bar{v}} [1 - F(x)] F(x)^{N-1} dx$.¹⁵

With mixed strategy entry, potential bidders are indifferent between entering the auction and staying out, implying that there is no information rent on average. Therefore, the seller's surplus π^U equals the social surplus or welfare w^U , which is defined as the sum of all parties' surpluses and can be written as the expected maximum value minus the aggregate entry costs. (The superscript U denotes uninformed entry, as opposed to informed entry which is studied in the next subsection.) With n bidders the expected maximum value equals $\int_0^{\bar{v}} x d[F(x)^n] = \bar{v} - \int_0^{\bar{v}} F(x)^n dx$. Therefore

¹⁴For these intermediate entry costs there are also many asymmetric equilibria. Of particular interest among those are the pure-strategy entry equilibria, in which some potential bidders enter with certainty while the rest stay out with certainty. These are discussed in Section 5.

¹⁵Formally, as q^* approaches 1, $E[u_i]$ approaches $\sum_{n=1}^N \binom{N-1}{n-1} 1^{n-1} [\lim_{q^* \rightarrow 1} (1 - q^*)]^{N-n} E[u_i|n] = E[u_i|N]$. Similarly, as q^* approaches 0, $E[u_i]$ approaches $\sum_{n=1}^N \binom{N-1}{n-1} (\lim_{q^* \rightarrow 0} q^*)^{n-1} 1^{N-n} E[u_i|n] = E[u_i|1]$.

$$w^U = \pi^U = \sum_{n=1}^N \binom{N}{n} [q^*]^n [1 - q^*]^{N-n} \left[\bar{v} - \int_0^{\bar{v}} F(x)^n dx \right] - q^* Nk \quad (3)$$

What remains is to characterize equilibria when $k \notin (\underline{k}, \bar{k})$. First, if $k > \bar{k}$ then the entry cost exceeds the expected value of a single bidder, and it is optimal not to have entry at all, implying that $w^U = 0$. Second, if $k < \underline{k}$ then with a plain auction all N potential bidders will enter. Now it is no longer the case that the plain auction without entry fees or reserve prices is optimal, however, because each bidder will earn an information rent (equal to $\underline{k} - k$). Then the optimal mechanism consists of an entry fee of $\underline{k} - k$ followed by a plain auction.¹⁶ Then, in equilibrium, all N potential bidders enter, and there is no information rent. Therefore there is no difference between social surplus w^U and the seller's surplus π^U . They are both given by

$$w^U = \pi^U = \bar{v} - \int_0^{\bar{v}} F(x)^N dx - Nk \quad (4)$$

2.2 Release of information prior to entry

When the prospective bidders have information before entry they will in general earn information rent. This rent drives a wedge between social surplus and the payoff to the seller. We start with a private seller, and then study cases in which the seller maximizes social surplus (for instance because the seller is the government).

2.2.1 The seller maximizes profit

Now entry may be made contingent upon the value. In a symmetric equilibrium bidder i will enter if and only if $v_i \geq v_\pi$, where v_π is a cut-off estimate common for all prospective bidders (the subscript denotes which objective the auction is designed to maximize – profits). Again we start by studying interior equilibria, that is, situations in which $v_\pi \in (0, \bar{v})$. From Samuelson (1985) we know that in this case

¹⁶Note that entry fees and reserve prices are not equivalent in this case: while an appropriately set reserve price can extract all information rent, this does not come without an efficiency loss: since values are stochastic, there is a probability that all values will be lower than a positive reserve price. The proposed entry fee extracts all information rent without any associated efficiency losses.

the optimal mechanism, as seen from the seller's point of view, is a plain auction with reserve price $r = [1 - F(v_\pi)] / f(v_\pi)$.¹⁷

Consider a potential bidder who has learnt that his value equals the cut-off value v_π . If he enters and none of his competitors do, he will earn $v_\pi - r$. If he enters and at least one of his competitors also does, he cannot profit from entry (since with probability one his competitors have values exceeding v_π). Therefore, when the seller is maximizing profit, equilibrium entry must satisfy

$$\left[v_\pi - \frac{1 - F(v_\pi)}{f(v_\pi)} \right] F(v_\pi)^{N-1} = k. \quad (5)$$

Since $f(\cdot)$ is log-concave (by assumption A4), the left-hand side of (5) is strictly increasing in v_π . Consequently, the equilibrium cut-off value v_π is unique and strictly increasing in k . Inspection reveals that as k approaches 0, v_π approaches $\underline{v} \equiv \{v | v = [1 - F(v)] / f(v)\} > 0$. Moreover, as k approaches \bar{v} , also v_π approaches \bar{v} . From this we can conclude that entry will be interior (and described by equation (5)) for all $k \in (0, \bar{v})$.

I will now derive expressions for social surplus w_π^I , private surplus π_π^I and aggregate information rent, denoted u_π^I (superscript I denotes informed entry, as opposed to uninformed entry). Following the steps of Samuelson (1985), we now exploit the fact that we need not condition on the actual number of bidders n , as n follows from the realization of values (v_1, \dots, v_N) . Therefore, the social surplus can be written

$$w_\pi^1 = \int_{v_\pi}^{\bar{v}} v d [F(v)^N] - Nk [1 - F(v_\pi)]. \quad (6)$$

Moreover, as the individual expected utility equals $E [u_i] = \int_{v_\pi}^{\bar{v}} [1 - F(v)] F(v)^{N-1} dv$, the aggregate expected utility — aggregate information rent or u_π^I — can be written

¹⁷Samuelson does not consider the possibility of using entry fees. In contrast, Menezes and Monteiro (2000) have found that in the absence of a reserve price, the optimal auction features a positive entry fee. As long as the reserve price is lower than the cutoff value (which is always the case here), the two approaches are equivalent, as Menezes and Monteiro points out (cf. the discussion after their Theorem 1). In fact, if we allow for both entry fees and reserve prices, optimal entry can always be induced in a continuum of ways.

$$u_{\pi}^I = NE[u_i] = N \int_{v_{\pi}}^{\bar{v}} [1 - F(v)] F(v)^{N-1} dv. \quad (7)$$

Finally, as the social surplus equals the sum of the private surplus π_{π}^I and the aggregate information rent u_{π}^I , the private surplus can be written

$$\pi_{\pi}^I = \int_{v_{\pi}}^{\bar{v}} v d[F(v)^N] - N \int_{v_{\pi}}^{\bar{v}} [1 - F(v)] F(v)^{N-1} dv - Nk [1 - F(v_{\pi})]. \quad (8)$$

2.2.2 The seller maximizes social surplus

When the seller maximizes profit, we have seen that he imposes a strictly positive reserve price, inducing too little entry from a social welfare point of view. Samuelson (1985) has shown that social welfare is maximized by a plain auction without a reserve price. I will now study the ex post (i.e., after entry) effects of such a policy. (The before entry effects are discussed below.)

If $r = 0$ then, by the same logic as for the case of a profit-maximizing seller, equilibrium entry – denoted v_w – must satisfy

$$v_w F(v_w)^{N-1} = k. \quad (9)$$

Inspection reveals that as k approaches 0, v_w approaches $\underline{v} = 0$. Moreover, as k approaches \bar{v} , also v_w approaches \bar{v} . From this we can conclude that entry will be interior and described by equation (9) for all $k \in (0, \bar{v})$. Performing the same steps of calculus as in the preceding subsection, we get the following expressions for private and social surpluses as well as information rent in this case (the equations are equal to eqs. (6)-(8) except that the equilibrium cut-off differ – v_w instead of v_{π}):

$$w_w^I = \int_{v_w}^{\bar{v}} v d[F(v)^N] - Nk [1 - F(v_w)], \quad (10)$$

$$u_w^I = N \int_{v_w}^{\bar{v}} [1 - F(v)] F(v)^{N-1} dv, \quad (11)$$

$$\pi_w^U = \int_{v_w}^{\bar{v}} v d[F(v)^N] - N \int_{v_w}^{\bar{v}} [1 - F(v)] F(v)^{N-1} dv - Nk [1 - F(v_w)]. \quad (12)$$

3 Results

We are now ready to compare the outcomes in the three different cases characterized above. First we consider equilibrium entry.

Proposition 1 *Equilibrium entry may go up (for high levels of k) or down (for low values of k) as a consequence of early information.*

Proof: With no information before entry, the equilibrium probability of entry q^* equals 1 iff $k < \underline{k}$ and 0 if $k > \bar{k}$. In both cases with information before entry we have that the equilibrium entry probability $1 - F(v_j) \in (0, 1)$ for any $k \in (0, \bar{v})$, $j \in \{\pi, w\}$. This implies that early information increases entry for $k \in [\bar{k}, \bar{v})$ and reduces entry for $k \in (0, \underline{k})$. ■

Proposition 2 *For $N = 2$ there exists a number k^* such that early information increases entry if $k > k^*$ and reduces entry if $k < k^*$, while entry is unaffected if $k = k^*$.*

Proof: Entry in the two cases are described by

$$(v - r)p_1^I - k = 0 \quad (13)$$

$$p_1^U u_1 + p_2^U u_2 - k = 0 \quad (14)$$

where $p_n^U = \binom{N-1}{n-1} q^{n-1} (1-q)^{N-n}$ and $p_n^I = \binom{N-1}{n-1} (1-F(v))^{n-1} F(v)^{N-n}$. Since $p_2^U = 1 - p_1^U$, entry can be described in the following way:

$$(v - r)p_1^I = k = (u_1 - u_2)p_1^U + u_2 \quad (15)$$

Suppose entry is the same (that is, suppose $k = k^*$). Then $p_1^I = p_1^U$ and $u_2 > 0$ implies that $v - r > u_1 - u_2$. Then as k increases, p_1^U must increase more than p_1^I , simply because gross profit of the informed firm (the LHS of equation (15)) is more sensitive to changes in p_1^j ($j = I, U$) than is the gross profit of the uninformed firm (the RHS of equation (15)). Invoking continuity completes the proof. ■

Conjecture 1 *Also for $N \geq 3$ there exists a number $k^* \in (\underline{k}, \bar{k})$ such that early information increases entry for $k > k^*$ and reduces entry if $k < k^*$.*

I have not been able to derive an analytical proof of this conjecture (except for some special cases).¹⁸ However, no counterexample has been found either, and numerical simulations suggests that the result may be general. For instance, if v is uniformly distributed, Fig. 1 below demonstrates that if we plot the two entry probabilities (informed vs. uninformed entry) against k , the resulting two curves cross only once, whether $N = 2, 3, 5$ or 20 . (Needless to say, other values of N yield the same result.) In the appendix I report similar figures corresponding to exponentially and normally distributed values, confirming this pattern.

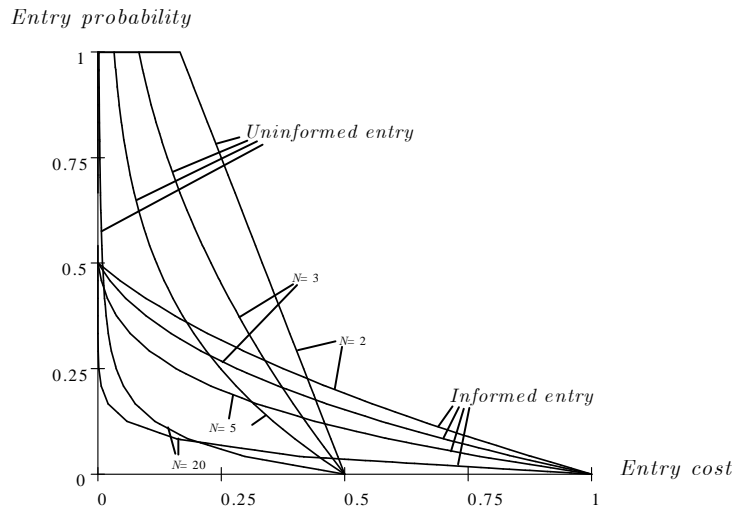


Figure 1: Entry behavior for different N

Next, note that when bidders have early information, then entry is higher when the seller maximizes social surplus than when the seller maximizes profits. The intuition is straightforward: the profit-maximizing seller sets a positive reserve, which reduces the bidders' expected rent and make them more reluctant to enter. Moreover, early information creates information rent on the hands of the bidders: $0 = u^U < u_\pi^I < u_w^I$. This rent drives a wedge between the social benefits of early information and the benefits that accrue to the principal. As a consequence, for any

¹⁸The technical problem is that while the proof of Proposition 2 relies on a simple competing risk argument (when p_1^j is to increase, $p_2^j = 1 - p_1^j$ must decrease), for $N \geq 3$ things are more complicated: it might be the case that both p_1^j and p_2^j increases.

$$k \in [0, \bar{v}), \pi_w^I < \pi_\pi^I < w_\pi^I < w_w^I.$$

In the extreme case of $k = 0$, all prospective bidders enter whether the bidders have early information or not, as long as the seller maximizes social surplus. After entry, the bidder with the highest value is always chosen. Consequently, in this case efficiency is not affected by early information (while distribution certainly is).

The following result establishes a link between equilibrium entry behavior and early information's effect on profits:

Proposition 3 *If entry increases as a consequence of early information when the seller maximizes social surplus, early information increases profits whether the seller maximizes profit or social surplus. Iff entry decreases, early information may lead to reduced profit.*

Proof: We start by noting that the seller prefers informed entrants – ceteris paribus – as they have their values drawn from a more favorable distribution than uninformed entrants (in a first order stochastic dominance sense). This implies that if the entry probability is unaffected by early information, the seller will be better off with informed entrants. The same holds of course if early information increases entry. Also note that since the mechanism under informed entry screens high-valuation bidders, the seller may benefit even if entry is somewhat reduced, hence the sufficiency but not necessity of increased entry.

Since increased entry is sufficient but not necessary for profits to increase, reduced entry must be necessary but not sufficient for profits to decrease. What remains is to point at a case in which profits actually go down as a result of early information. This is easily shown to be the case for $k = 0$. By continuity it will also be the case in a neighborhood of $k = 0$. ■

4 Example

I will now illustrate some of my findings in an example in which v_i is drawn from a uniform distribution on the interval $[0, 1]$ and $N = 2$.¹⁹

¹⁹Increasingly complicated closed-form solutions to the three auction games can also be found for $N = 3$ and $N = 4$, while no such solutions can be found for higher numbers of potential entrants.

First we plot the different entry probabilities against k to get a picture of entry behavior in the three regimes:

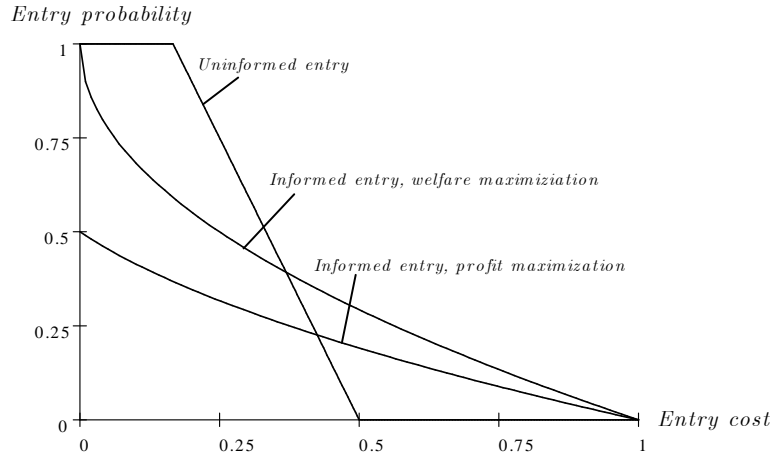


Figure 2: Entry behavior

Here we see clearly that early information reduces entry for low entry costs and increases entry for high entry cost (Propositions 1 and 2). Next we plot expected profit in the three cases:

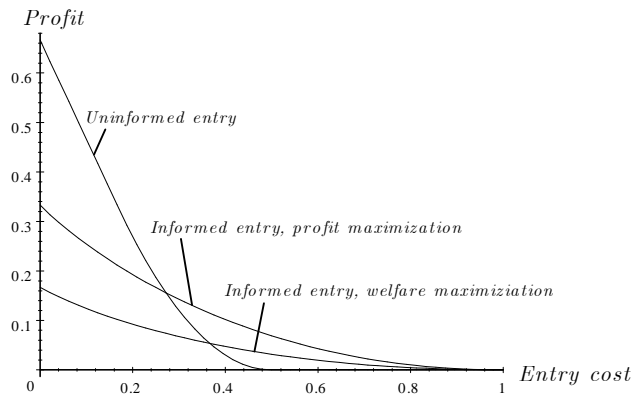


Figure 3: Profit

This picture essentially shows that early information is bad for the seller when entry costs are low, but good for the seller if the entry cost is high (cf. Proposition 3).

Numerical solutions are easily found also for higher numbers. However, since the essentials do not change as N increases, only the technically simpler case is reported.

Next we take a closer look at social surplus:

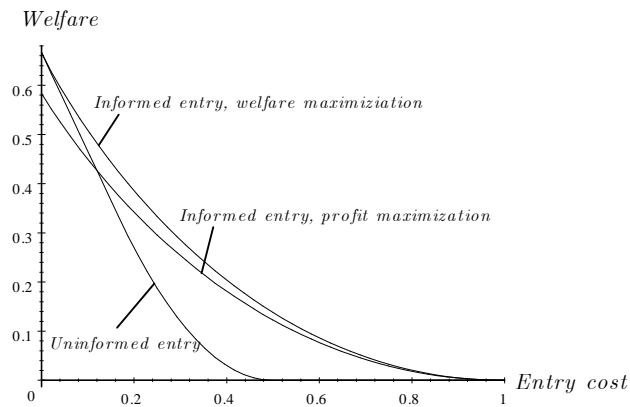


Figure 4: Social surplus

For obvious reasons, welfare is decreasing in the entry cost. What is perhaps more interesting is to get pictures of the *changes* in information rent, and private and social surpluses as early information is provided. The next figure shows these numbers for the case of a seller that maximizes profit:

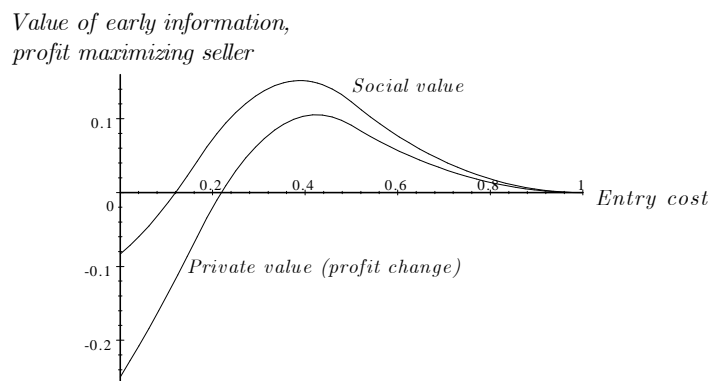


Figure 5: Value of early information, profit maximizing seller

In contrast, if the seller maximizes social surplus, we get the following picture:

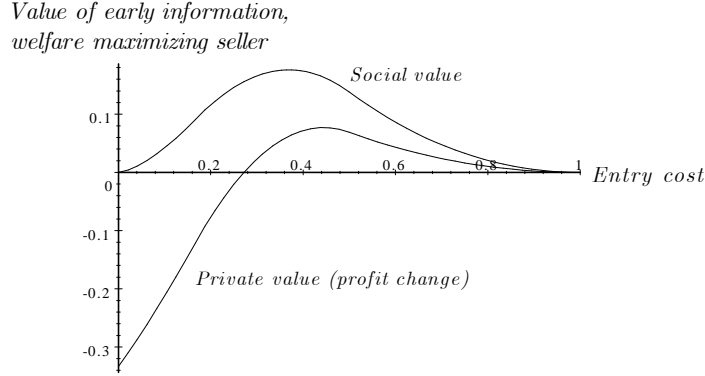


Figure 6: Value of early information, welfare maximizing seller

5 Asymmetric entry equilibria

In the preceding analysis I have compared symmetric equilibria with and without early information release. One might object that this comparison makes it difficult to see the pure effects of early information, since what it does is to compare a mixed-strategy equilibrium with a pure-strategy one, thereby making it difficult to disentangle the effects of early information from the effects of equilibrium change. This would not have been a problem if there had been only one equilibrium for each setup. Unfortunately, there are many equilibria of the non-release game.

Of particular interest for us, for intermediate values of the entry cost (in the sense that there is room for at least one bidder but not all of them), there are always asymmetric pure-strategy equilibria in which some potential bidders enter with certainty while others stay out, also with certainty.²⁰ While mixed-strategy entry is

²⁰Depending on N and k , there might also be asymmetric equilibria in which some bidders enter with certainty and/or some stay out with certainty, while at least two bidders randomize their entry decisions. For instance, if $N = 30$ and $k \in (k_5, k_4]$, there exists an equilibrium in which 2 bidders enter with certainty, 6 bidders enter with a common probability $q \in (0, 1)$, and the remaining 22 bidders stay out with certainty.

a plausible assumption if there are no coordination devices, pure-strategy entry is plausible if entry can be coordinated, e.g. if prospective bidders make their entry decisions sequentially and the decisions are observable or can be communicated to other prospective bidders before they decide. Moreover, to facilitate the comparison we will in what follows still assume that entry fees and reserve prices can be used whenever the seller sees fit.²¹ This is basically the framework of McAfee and McMillan (1987b).

In short, McAfee and McMillan demonstrate that first-best entry is achieved if neither entry fees nor reserve prices are used. Unlike the mixed-strategy case, however, now the information rent is not necessarily totally dissipated, but dissipated down to an integer approximation only. However, as long as the seller and the prospective bidders are symmetrically informed before entry, the seller can extract all surplus by combining a reserve-free auction with an entry fee set to extract all information rent, thus funneling all surplus into the seller's pocket.

Equilibrium entry, denoted n , is given by

$$E[u_i|n] \geq k > E[u_i|n+1]$$

This amounts to having exactly n bidders enter iff $k \in (k_{n+1}, k_n]$. With the entry fee set to extract all information rent, the seller's surplus π^U equals the social surplus w^U , which can be written as the expected maximum value minus the aggregate entry costs (provided at least one firm enters, that is, provided $k \leq k_1$). The expected maximum value equals $\int_0^{\bar{v}} x d[F(x)^n] = \bar{v} - \int_0^{\bar{v}} F(x)^n dx$. Therefore

$$w^U = \pi^U = \bar{v} - \int_0^{\bar{v}} F(x)^n dx - nk \tag{16}$$

The only difference from the mixed-strategy case is that the equilibrium number of bidders, n , is now a deterministic function of the entry cost k , instead of a stochastic function of the same cost.²²

²¹This does not imply that other cases are without interest. It merely reflects our desire to compare as equal cases as possible.

²²As with mixed-strategy entry, if $k > \bar{k} = k_1$ then the entry cost exceeds the expected value of a single bidder, and it is optimal not to have entry at all, implying that $w^U = 0$.

Assuming that potential bidders conform to this asymmetric pure-strategy equilibrium instead of the symmetric mixed-strategy equilibrium complicates the results somewhat. First consider the entry probability. Figure 7 below shows how the entry probability varies with the entry cost (again assuming uniformly distributed values and $N = 2$).²³

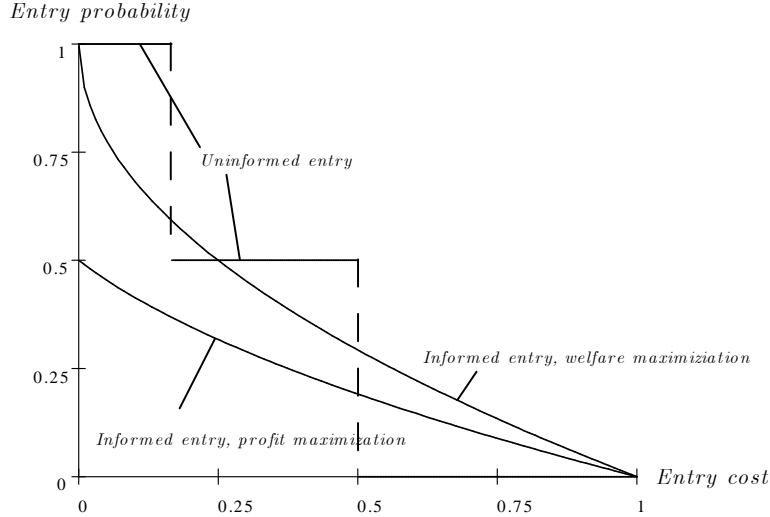


Figure 7: Non-monotonic changes in entry

It is still the case that early information reduces entry for low entry cost and increases entry for high entry costs, but the relation is not necessarily monotonic. In Fig. 7 we see that if the seller maximizes welfare, early information reduces entry for $k < \frac{1}{6}$ and for $\frac{1}{4} < k < \frac{1}{2}$, while entry increases for $\frac{1}{6} < k < \frac{1}{4}$ and for $k > \frac{1}{2}$. (If the seller maximizes profit, the same phenomenon occurs with uniformly distributed values when $N \geq 4$.)

Perhaps more surprising is that the other figures seem to be more robust with respect to the modeling of uninformed entry. As an example, Figure 8 below shows how the value of early information depends on the entry cost when uninformed entry

²³Note that with pure strategy entry, the graphed entry probability equals the average over the potential bidders, not individual probabilities (which of course are either 0 or 1 in a pure-strategy equilibrium).

is in pure strategies:

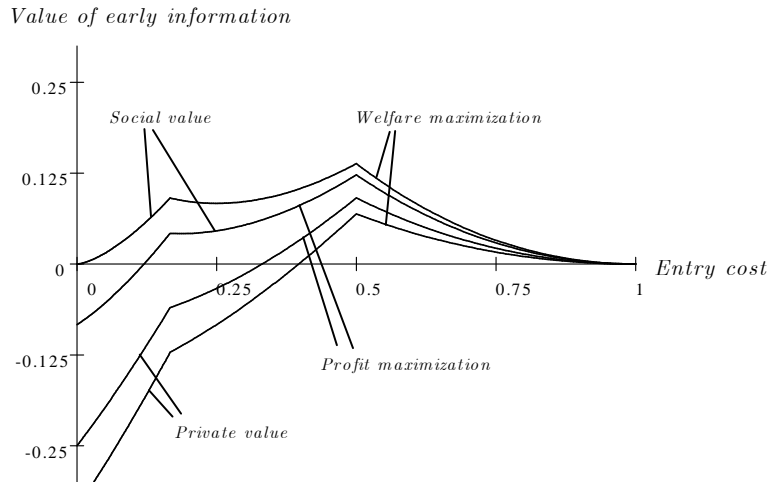


Figure 8: Value of early information

(The figure corresponds to a combination of Figures 5 and 6.) We see that the graphs are less smooth, as entry in the non-release case is less smooth, but the essentials of the figure does not change.

6 Concluding remarks

This paper unifies two approaches to the modeling of auctions with entry. Samuelson (1985) develops a model where the prospective entrants knows their values before they sink the entry investment, while Levin and Smith (1994) works with the opposite assumption: bidders have no private information before they enter. The starting point of the present paper is the fact that sometimes the seller can affect how much information the bidders have before entry – he can choose between a Samuelson world and a Levin-Smith world. At first glance one might suspect that it is in the seller’s interest to provide the best possible information for his buyers, but we have seen that this is not necessarily so. True, if the number of prospective bidders who actually enter is not affected by early information, then early information tend to improve the selection of bidders. Moreover, competition is intensified. However,

increased competition is bad for entry. Reduced entry affects profit negatively, and sometimes this effect is strong enough to dominate the benefits, leaving the seller with lower profit with informed bidders than if they were uninformed.

7 Appendix

7.1 Example details

Suppose v_i is drawn from a uniform distribution on the interval $[0, 1]$ and that $N = 2$.

Then, using (1) and (2) we find that

$$q^* = \begin{cases} 1 & \text{if } k < \underline{k} = \int_0^1 (1-x)x^{2-1} dx = \frac{1}{6} \\ 0 & \text{if } k > \bar{k} = \int_0^1 (1-x) dx = \frac{1}{2} \\ \frac{3}{2} - 3k & \text{if } k \in [\underline{k}, \bar{k}] = [\frac{1}{6}, \frac{1}{2}] \end{cases} \quad (17)$$

Then we use (3) to get

$$\pi^U = w^U = \begin{cases} \frac{2}{3} - 2k & \text{if } k < \frac{1}{6} \\ 0 & \text{if } k > \frac{1}{2} \\ \frac{3}{4} - 3k + 3k^2 & \text{if } k \in [\frac{1}{6}, \frac{1}{2}] \end{cases} \quad (18)$$

(We know that with mixed-strategy entry there will be no information rent, that is, $u^U = 0$.)

Next we move to the cases of informed entry. If the seller maximizes profit, we get (using (5)) $v_\pi = \frac{1}{4} + \frac{1}{4}\sqrt{1+8k} \in [\frac{1}{2}, 1]$. In contrast, if the seller maximizes social surplus, we get (using 9)) $v_w = \sqrt{k} \in [0, 1]$. Then

$$w_w^I = \int_{\sqrt{k}}^1 2v^2 dv - 2k(1 - \sqrt{k}) = \frac{2}{3} + \frac{4}{3}k^{\frac{3}{2}} - 2k, \quad (19)$$

$$u_w^I = 2 \int_{\sqrt{k}}^1 (1-v)v dv = \frac{1}{3} + \frac{2}{3}k^{\frac{3}{2}} - k, \quad (20)$$

$$\pi_w^I = \int_{\sqrt{k}}^1 2v^2 dv - 2 \int_{\sqrt{k}}^1 (1-v)v dv - 2k(1 - \sqrt{k}) = \frac{1}{3} + \frac{2}{3}k^{\frac{3}{2}} - k, \quad (21)$$

and

$$\begin{aligned} w_\pi^I &= \int_{\frac{1}{4} + \frac{1}{4}\sqrt{1+8k}}^1 2v^2 dv - 2k \left(1 - \left(\frac{1}{4} + \frac{1}{4}\sqrt{1+8k} \right) \right), \\ &= \frac{5}{8} - \frac{7}{4}k + \frac{10k-1}{24}\sqrt{1+8k}, \end{aligned} \quad (22)$$

$$u_\pi^I = 2 \int_{\frac{1}{4} + \frac{1}{4}\sqrt{1+8k}}^1 (1-v)v dv = \frac{1-k}{12} \left(3 - \sqrt{1+8k} \right), \quad (23)$$

$$\begin{aligned} \pi_\pi^I &= \int_{\frac{1}{4} + \frac{1}{4}\sqrt{1+8k}}^1 2v^2 dv - 2 \int_{\frac{1}{4} + \frac{1}{4}\sqrt{1+8k}}^1 (1-v)v dv - 2k \left(1 - \left(\frac{1}{4} + \frac{1}{4}\sqrt{1+8k} \right) \right) \\ &= \frac{3}{8} - \frac{3}{2}k + \frac{(8k+1)^{\frac{3}{2}}}{24}. \end{aligned} \quad (24)$$

7.2 Numerical simulations of Conjecture 1

Attention is restricted to the case of a profit-maximizing seller. (It is straightforward to extend the analysis to the simpler case of a welfare-maximizing seller.)

Equilibrium entry satisfies

$$\left(v_\pi - \frac{1 - F(v_\pi)}{f(v)} \right) F(v_\pi)^{N-1} - k = 0, \quad (25)$$

$$\sum_{n=1}^N \binom{N-1}{n-1} (q^*)^{n-1} (1-q^*)^{N-n} E[u_i|n] - k = 0. \quad (26)$$

7.2.1 Uniform distribution

If v_i is drawn from a uniform distribution on $[0, 1]$, the above expressions simplifies to

$$(2v_\pi - 1) v_\pi^{N-1} - k = 0, \quad (27)$$

$$\sum_{n=1}^N \binom{N-1}{n-1} (q^*)^{n-1} (1-q^*)^{N-n} \frac{1}{n(n+1)} - k = 0. \quad (28)$$

Given N , each of these equations implicitly defines the entry probability as a function of the entry cost. Using the functional relationship between cut-off values and entry

probabilities $1 - q_\pi = F(v_\pi) = v_\pi$ to transform the former of these equations into one in which the entry probability q_π for the case of informed entry and a seller who maximizes profit is an implicit function of k .) The functions are plotted in Fig. 1 (in the main text) for 4 different values of N : $N \in \{2, 3, 5, 20\}$.

7.2.2 Exponential distribution

Next suppose v_i is exponentially distributed on $[0, \infty)$ with parameter $\lambda = 1$ (this parameter choice is without loss of generality, since all exponential distributions have the same shape and k is a free parameter). Then $E[u_i|n] = \int_0^\infty e^{-x}(1-e^{-x})^{n-1}dx = \frac{1}{n}$ and equilibrium entry is described by

$$(v_\pi - 1)(1 - e^{-v_\pi})^{N-1} - k = 0 \quad (29)$$

$$\sum_{n=1}^N \binom{N-1}{n-1} (q^*)^{n-1} (1 - q^*)^{N-n} \frac{1}{n} - k = 0 \quad (30)$$

Now $1 - q_\pi = F(v_\pi) = 1 - e^{-v_\pi} \Leftrightarrow v_\pi = -\ln q_\pi$, and the former of these equations can be written

$$(-\ln q_\pi - 1)(1 - q_\pi)^{N-1} - k = 0 \quad (31)$$

The resulting plot demonstrates that each pair of curves cross only once, essentially confirming Conjecture 1 for exponentially distributed values.

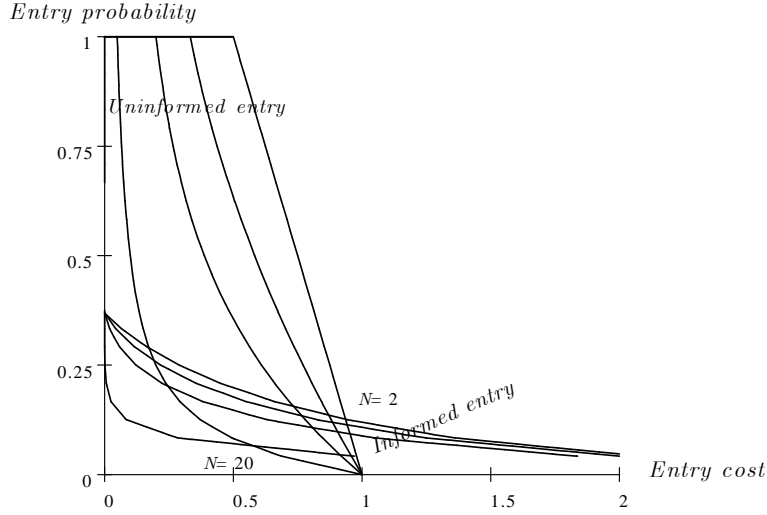


Figure 9: Entry behavior for different N , exponential distribution

7.2.3 Normal distribution

Finally suppose $v_i \sim N(1, 1)$. (The variation can be fixed without loss of generality, while different means yield qualitatively different figures. However, experiments that are not reported suggest that the conjecture holds for the entire family of normal distributions.) Since the support of the normal distribution is $(-\infty, \infty)$, now it might be the case that the highest value is negative even if all potential bidders enter. This complicates derivation of the non-release equilibrium, while little change in the case of informed entry.

To be more specific, with uniformly distributed values, expected utility is given by:

$$\begin{aligned}
 E[u_i|n] &= \int_0^\infty \int_{-\infty}^{v_i} (v_i - \max\{0, x\}) d[F(x)]^{n-1} dF(v_i) \\
 &= F(0)^{n-1} \int_0^\infty x dF(x) + \int_0^\infty [1 - F(x)] F(x)^{n-1} dx
 \end{aligned}$$

Then entry can be described by

$$\left(v_\pi - \frac{1-F(v_\pi)}{f(v_\pi)}\right) F(v_\pi)^{N-1} - k = 0$$

$$\sum_{n=1}^N \binom{N-1}{n-1} (q^*)^{n-1} (1-q^*)^{N-n} \left(F(0)^{n-1} \int_0^\infty x dF(x) + \int_0^\infty [1-F(x)] F(x)^{n-1} dx \right) - k = 0$$

or, substituting $1-F(v^*)$ for q^* , the latter can be written as follows (v^* is the cut-off value that corresponds to an entry probability of q^* , and is introduced because it turns out to be numerically simpler to plot cut-off values against k than to plot entry probabilities against k):

$$\sum_{n=1}^N \binom{N-1}{n-1} (1-F(v_\pi))^{n-1} F(v_\pi)^{N-n} \left(F(0)^{n-1} \int_0^\infty x dF(x) + \int_0^\infty [1-F(x)] F(x)^{n-1} dx \right) - k = 0$$

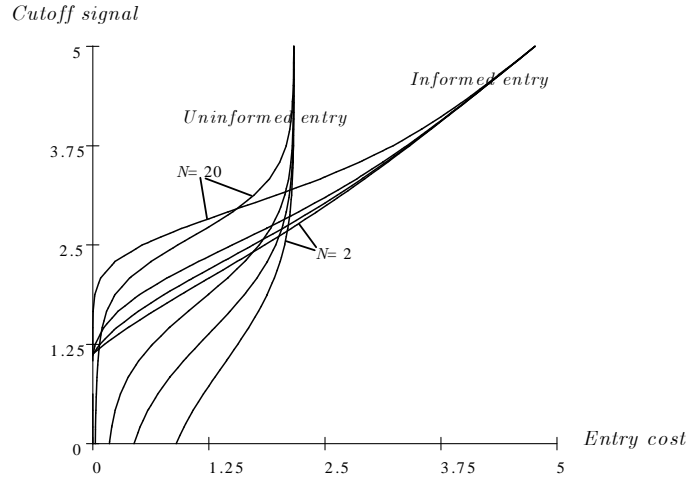


Figure 10: Entry behavior for different N , normal distribution

Also for normally distributed values each pair of curves cross only once, again lending support to Conjecture 1.

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