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**Optimal Feedback Controls:
Comparative Evaluation of the Cod fisheries in
Denmark, Iceland and Norway**

by

**R. Arnason, L.K. Sandal,
S.I. Steinshamn and N. Vestergaard**

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0. Introduction¹

Most ocean fisheries are subject to quite limited property rights, traditionally referred to as the common property problem (Gordon 1954, Hardin 1968). As a result of the common pool problem, market forces do not provide the appropriate guidance to private enterprise and the harvesting activity tends to be economically wasteful. This suggests the need for some form of strategic interference that is to say fisheries management.

Management of fisheries is normally concerned with implementing an optimal harvesting programme. An optimal harvesting programme is defined as one that maximizes the present value of economic rents from the fishery over time. To identify this programme, a model of the fishing activity is needed. Fisheries models are inherently more complicated than conventional economic models. This is because any reasonable fisheries model comprises both an economic production model and a biological population growth model as well as the connection between the two. Hence, in addition to the usual economic variables, fisheries models must include biological capital, namely the fish stocks. To add to the complexity, the dynamics of fish stocks are generally substantially more complicated than those of conventional physical capital included in economic models of production.

The primary purpose of this paper is to compare the relative efficiency of the fish harvesting policies of Iceland, Norway and Denmark. All three nations harvest a number of fish species. For the purposes of this paper, however, we have chosen to concentrate on the cod fishery as this is the single most important fishery in all three countries.

¹ We are grateful to Sveinn Agnarsson and Frank Jensen for valuable assistance with the computations in this article. Financial support from the Nordic Council of Ministers (Nordisk Ministerråd) is acknowledged.

The three nations conduct their cod fisheries in quite different contexts. First, there is a difference in national control over the respective fisheries. Since the extension of her fisheries jurisdiction to 200 miles in 1976, Iceland has been in virtual sole control of her cod fishery. Norway, on the other hand, shares her cod stock, the Arctic cod, with Russia and must therefore decide on a harvesting policy jointly with Russia. Denmark is only one of several, mainly European Union, countries pursuing the North Sea cod fishery. Since the early 1980s, the European Union has set the overall total allowable catch (TAC) for this fishery of which Denmark merely receives a share. Thus, compared to Iceland and Norway, Denmark probably has the least control over her cod harvesting policy. In view of these differences in autonomy between the three countries, it is clearly of interest whether this shows up in their respective cod harvesting policies.

Second, during the latter part of the period under study in this paper, the fisheries management systems of the three countries have been quite different. Stated very briefly, Iceland has since 1984 operated a more or less complete ITQ-system in her cod fishery. (Arnason 1993). Norway has for about the same period managed her cod fishery on the basis of quasi-permanent individual quotas (Anon. 1996d). In Denmark, however, the fishery has for the past two decades essentially been managed on the basis of a licence limitation program supplemented with very short term (down to two months) non-permanent, non-transferable vessel quotas that, in the case the fishery is closed, may actually turn out to be worthless (Vestergaard 1998). Thus, it is clear that the quality of the harvesting rights held by individual companies in these three cod fisheries has differed greatly in recent years. It is often suggested that differences in the fisheries management regime, especially the quality of individual harvesting rights, may influence harvesting strategies (Arnason 1990a, Johnson

1995, Turrís 1999). Therefore, it is of interest to find out whether in fact empirical indications of this can be detected.

In addition to the comparison between the cod harvesting policies of Denmark, Iceland and Norway, the paper has certain broader theoretical implications. Within the field of fisheries economics essentially two classes of fisheries models have been developed. The first class consists of simple aggregative models treating the fish stocks as one homogenous biomass and characterizing the application of various types of fishing capital as fishing effort. These are the classical fisheries economics models of Gordon (1954), Smith (1968) and Clark and Munro (1982) that have become standard tools in analytical fisheries economics. These models are intended to describe key features common to all fisheries but have little specific empirical content. Consequently, this class of models is primarily used for analytical work. The other class of fisheries models consists of empirical models that are supposed to describe actual fisheries situations in some detail. These models are usually highly complex involving several cohorts of fish and types of fishing vessels. Examples of these models are given by Arnason (1990), Placenti et al. (1992), Olafsson and Wallace (1994) and Rizzo et al. (1998). In what follows we find it convenient to refer to these two classes of models as the analytical and the empirical models, respectively.

The two classes of fisheries models have somewhat different advantages and disadvantages. The advantage of the analytical models is essentially their simplicity and tractability. As a result they are capable of generating informative qualitative solutions to the fisheries problem that are relatively easily explainable with reference to basic economic principles. Analytical models suffer from two major disadvantages, however. First, except for

the simplest of these models², explicit feed-back solutions giving the optimal harvest as a function of the biomass have hitherto not been available. Second, analytical fisheries models do not provide an empirically accurate description of any particular fishery. For this reason, they are not well suited to provide practical management advice to fisheries authorities.

The great advantage of empirical fisheries models is that they are, at least in principle, capable of providing practical management advice for the specific fisheries they are designed to describe. Empirical fisheries models have many serious disadvantages, however. First, they are usually extremely difficult to build. Their construction typically involves the collection of large amounts of data, extensive statistical estimation work and substantial computer programming effort. Second, due to the complexity of these models, they are generally quite unwieldy and cumbersome to operate. Just running these models on a computer is frequently a major undertaking. Third, again due to their complexity, the fisheries policy recommendations generated by empirical fisheries models are often difficult to understand and explain. Therefore, these models tend to be experienced as black boxes churning out what is claimed to be optimal fisheries paths in response to inputs of data. Just as in the case of analytical models, explicit functions describing optimal feedback solutions are generally not available for empirical models. On the other hand, empirical fisheries models may in principle be employed to calculate numerical feedback solutions to the fisheries problem.

The present paper proposes an approach that adds empirical content and specific solution procedures to the analytical fisheries models in order to generate empirically relevant solutions. More precisely, it suggests the statistical estimation of the relationships typically used in analytical fisheries models and then the employment of certain mathematical techniques to generate explicit feed-back solutions to this class of models. In this way, the

² E.g. the linear fisheries model of Clark and Munro 1982.

current approach attempts to bridge a part of the gap between the analytical and empirical fisheries models. It is essentially an aggregative simple description of a fishery, just like analytical models, but with empirically estimated relationships, just like empirical models.

This approach is, of course, not at all original. In fact, prior to the advent of high speed computers that made empirical fisheries models possible, it was not uncommon for in fisheries research to statistically estimate the relationships of analytical fisheries models in order to obtain estimates of the optimal equilibrium solutions (see e.g. Mohring 1973, Spence 1975 and Clark 1990). The current paper, however, improves substantially on this line of research by providing optimal dynamic feed-back solutions to these same models.

We should also stress that our approach does not, in our view, replace either type of the usual analytical and empirical models. In fact, it seems to us that it should be used as a convenient complement to both. For instance in analytical modeling, our approach can serve as an easily obtainable numerical illustration to general analytical results. In empirical modelling, our approach can for instance be used as a benchmark to corroborate or, as the case may be, re-examine some of the outputs of the empirical models.

Of course, the procedure proposed in this paper does not provide detailed solutions to the fisheries problem. In fact, due to the simplicity of the underlying model, it can only provide the approximate attributes of optimal harvesting paths. The approach may nevertheless be quite useful. First, in many fisheries, as well as other natural resource use, it may simply not be feasible, due to lack of data and/or other inputs, to construct a fully-fledged empirical model. Second, in many cases, the management capability is simply inadequate to implement detailed management programmes anyway. Third, as discussed

above, the solution paths generated by our procedure are relatively easily explainable and therefore, perhaps, stand a better chance of being appreciated and adopted. Fourth, the proposed procedure makes it relatively easy to investigate the impact of exogenous changes on the economics of the fishery and optimal harvesting paths. Fifth, the procedure makes it relatively easy to compare, on even footing so to speak, the relative efficiency of the harvesting policies in different fisheries around the world. In fact, this is exactly the use we put our approach to later in this paper where the relative efficiencies of the fisheries policies in Denmark, Iceland and Norway are compared.

Although our approach has been developed for fisheries, there is no reason to restrict its use in this way. The approach can, with only slight modifications, be applied to other use of replenishable natural resources such as water resources, grasslands, forests and the environment in general.

The paper is organized broadly as follows. In section 1 the theoretical model is explained. In section 2 the model is applied to a comparative study of the fisheries policies in Denmark, Iceland and Norway. Finally, section 3 contains a brief discussion of the main results of the paper.

1. Theoretical model

This section sketches the theoretical model that is used to determine an optimal harvesting policy.³ The objective is to discover the time path of harvest that maximizes the following functional:

³ The model can also be generalized to include general stochastic processes (Sandal and Steinshamn, 1997b).

$$\int_0^{\infty} e^{-\delta t} \Pi(h, x) dt$$

(1)

subject to $\dot{x} = f(x, h)$, $x(0) = x_0$, $\lim_{t \rightarrow \infty} x(t) = x^* > 0$.

where x represents the fish stock biomass, h the flow of harvest, Π net revenues and $f(.,.)$ is a function representing biomass growth. Dots on tops of variables are used to denote time derivatives, and δ is the discount rate. x_0 represents the initial biomass and x^* some positive (equilibrium) biomass level to which the optimal programme is supposed to converge.⁴ The functions Π and f can in principle be any functions although it is henceforth assumed that they are sufficiently regular for both the problem and the results to be meaningful.

The current value Hamiltonian corresponding to problem may be written as:⁵

$$H = H(h, x, \lambda) = \Pi(h, x) + \lambda f(h, x),$$

where λ is the costate variable. Assuming an interior solution (i.e. positive biomass and harvest), the necessary or first-order conditions for solving the maximization problem (Kamien and Schwartz, 1991) include:

$$\begin{aligned} H_h &= 0, \\ \dot{\lambda} &= \delta \lambda - H_x. \end{aligned}$$

⁴ Indeed, the last constraint in (1), which can be derived as a transversality condition, may be regarded as the requirement of fishery sustainability.

⁵ It is assumed that the multiplier corresponding to the objective function, $\Pi(h, x)$, is unity.

Upon differentiating the Hamiltonian function with respect to time, these conditions combined with the dynamic constraint in (1) yield⁶

$$\dot{H} = \delta \cdot \lambda \cdot \dot{x} \quad (2)$$

The interior optimum condition, $H_h = 0$, implies that the costate variable, λ , can be rewritten as a function of x and h :

$$\lambda = -\frac{\Pi_h}{f_h} \equiv \Lambda(h, x).$$

As this is a known function (provided the functions Π and f are known), it can be used to eliminate the costate variable, λ , from the problem. More to the point, it is now possible to define the following new function different from the Hamiltonian but always equal to it in value:

$$P(h, x) = \Pi(h, x) + \Lambda(h, x)f(h, x). \quad (3)$$

For fisheries management, and, indeed, the purposes of this paper, it is extremely useful to be able to express the optimal harvest at each point of time as a function of the fish stock biomass at that time. Let us refer to this as the function $h(x)$. In the optimal control literature, this is referred to as feedback control (Seierstad and Sydsæter 1987 p. 161, Kamien

⁶ $\dot{H} = H_h \dot{h} + H_x \dot{x} + H_\lambda \dot{\lambda}$. From the necessary conditions, $H_h = 0$, $H_x = \delta\lambda - \dot{\lambda}$. Finally, by the construction of the Hamiltonian function, $H_\lambda = \dot{x}$

and Schwartz 1991 p. 262). So, we seek the feedback control, $h(x)$, for problem (1). Inserting this unknown function into (3) and differentiating with respect to time yields:

$$\dot{P} = \left(\frac{\partial P}{\partial x} + \frac{\partial P}{\partial h} \frac{\partial h}{\partial x} \right) \cdot \dot{x}.$$

But by construction $\dot{P} \equiv \dot{H}$. Hence, by (2) we obtain the first-order differential equation that can be used to determine the feedback control:

$$\frac{dP}{dx} \equiv \frac{\partial P}{\partial x} + \frac{\partial P}{\partial h} \frac{\partial h}{\partial x} = \delta \cdot \Lambda(h, x). \quad (4)$$

Solving (4) or, if that is more convenient, (3) for the harvest, h , yields the desired feedback control. This, however, is not a trivial task in general.

In the special case where the rate of discount, $\delta = 0$, it is particularly easy to find the optimal feedback control. In this case $\frac{dP}{dx} = 0$ by (4). In other words, P is a constant. This corresponds to the well-known result that with zero discounting the maximized Hamiltonian is constant (Seierstad and Sydsæter, 1987, pp. 110-11). Obviously, if this constant can be determined, the feedback control is given implicitly by (3) and our problem is solved.⁷ Now, the Hamiltonian can be interpreted as the rate of increase of total assets (Dorfman 1969). Profit maximization requires us to make this as large as possible for as long as possible. The

⁷ Of course, without discounting, the integral in (1) may not converge, but with the listed transversality condition in (1) this is not a problem. Although the integral may have an infinite value, there exists one control trajectory that maximizes the integral. This is the trajectory whose value in terms of the objective function ultimately catches up with the value from any other control trajectory (Seierstad and Sydsæter, 1987, pp. 231-3).

largest possible sustainable value of the Hamiltonian is given by the maximum of the sustainable net revenue defined as

$$S(x) = \Pi(h, x)|_{\dot{x}=0} \quad (5)$$

which is a function of x only as $f(h, x) = 0$ can be used to eliminate h . Note that S is simply the net revenue that can be obtained by fixing the stock at any level. When $\delta = 0$, there is no discounting of the future and obviously the constant we are seeking is $P = P_0 = \max[S(x)]$. This constant substituted for the left-hand side of (3) gives the optimal feedback control as an ordinary algebraic equation (not a differential equation). This equation can subsequently be used for comparative dynamics and sensitivity analysis. Note, however, that the feedback control itself, $h(x)$, has normally to be found by numerical means, although in certain special cases it is possible to obtain explicit solutions.

In the more general case, where $\delta > 0$, it is unavoidable to seek the solution on the basis of the differential equation given in (4). This equation can either be solved numerically for the optimal feedback control or perturbation methods can be used in order to find closed form solutions if that is required, see, e.g., Sandal and Steinshamn (1997a).

An example

We now provide a simple example of how our method of finding optimal feedback solutions works. To simplify the presentation we take the case of zero discounting, i.e. $\delta = 0$. Moreover, we adopt relatively simple functional forms under which an explicit expression for the feedback rule is available. Regarding the practical relevance of the case of zero discounting, the reader may consult Mendelsohn (1972) and Sandal and Steinshamn (1997a)

who argue that positive discounting actually has little influence on optimal paths as long as the ratio of the discount rate to the intrinsic growth rate of the biomass is small. The value of the objective function, on the other hand, is, of course, highly sensitive to the discount rate.

Assume the instantaneous profit function:

$$\Pi(h, x) = p(h)h - c(h, x) = (a - bh)h - \frac{kh}{x}$$

and net (i.e. with the harvest subtracted) growth function of the biomass:

$$f(h, x) = g(x) - h = rx(1 - x) - h.$$

Consequently, the expression for the shadow value of biomass along the optimal path is:

$$\Lambda(h, x) = a - 2bh - \frac{k}{x},$$

and the $P(h, x)$ function corresponding to (3) is:

$$P(h, x) = (a - bh)h - \frac{kh}{x} + \left(a - 2bh - \frac{k}{x} \right) [rx(1 - x) - h] \quad (6)$$

The maximum sustainable net revenue revenue function, $S(x)$, corresponding to (5) is:

$$S(x) = br^2(1 - x) \left[x^3 - x^2 + \frac{ax - k}{br} \right].$$

With zero discounting $P(h,x) = P_0 = \max S(x)$. Therefore, in this case, equation (6) (corresponding to the general theoretical equation (3)) then can be solved explicitly for h . The solution is

$$h = g(x) \pm \left[\frac{P_0 - S(x)}{b} \right]^{\frac{1}{2}}$$

which is a function of x only. The subtraction, i.e. “-“ is chosen when biomass has to be increased, i.e. $\dot{x} > 0$ along the optimal path, and vice versa.

By comparison, the traditional approach to solve the same fisheries problem employed by Clark (1990) and others, yields a system of non-linear differential equations

$$\begin{aligned} \dot{h} &= \left(\frac{a}{2b} - h \right) (r - 2x) + \frac{k}{2bx} r, \\ \dot{x} &= rx(1-x) - h, \end{aligned}$$

which is not at all straight-forward to solve for the feedback rule.

2. Application: The performance of the Denmark, Icelandic and Norwegian cod fisheries

In this section we employ the approach developed above to throw some light on the relative efficiency of the cod fisheries of Denmark, Iceland and Norway. In particular, we will use the approach to provide us with estimates of how close to (or distant from) the optimal path the actual utilization of the cod stocks of these three nations has been. For this purpose we will

first obtain statistical estimates of the aggregate profit and biomass growth functions that form the building blocks of the aggregative fisheries model discussed in the previous section. With these in hand, we can employ the optimal feedback methodology of the previous section to calculate the optimal harvesting policy for each of these three countries every year. Finally, comparing the calculated optimal paths with the actual ones for these three fisheries provides us with an estimate of their relative efficiency.

2.1 *The empirical model*

In order to calculate the optimal feedback rule for each country it is necessary to estimate the corresponding biological growth and economic profit functions.

Biological growth functions

Above we have assumed that instantaneous change in stock biomass equals natural growth less harvest:

$$\frac{dx}{dt} \equiv f(x, h) = g(x) - h$$

It is not possible to estimate $g(x)$ directly, because the available data is in discrete time. Consequently, we employ the approximation:

$$g(x) = x_{t+1} - x_t + h,$$

where the subscript t refers to years, x_t refers to biomass at the beginning of each year and h the harvest during the period $[t, t+1]$.

Different forms for the aggregate growth function, $g(x)$, were tried. For Denmark and Norway the following gave the best fit

$$g(x) = rx \left(1 - \frac{x^2}{K^2} \right)$$

whereas for Iceland the logistic function

$$g(x) = rx \left(1 - \frac{x}{K} \right),$$

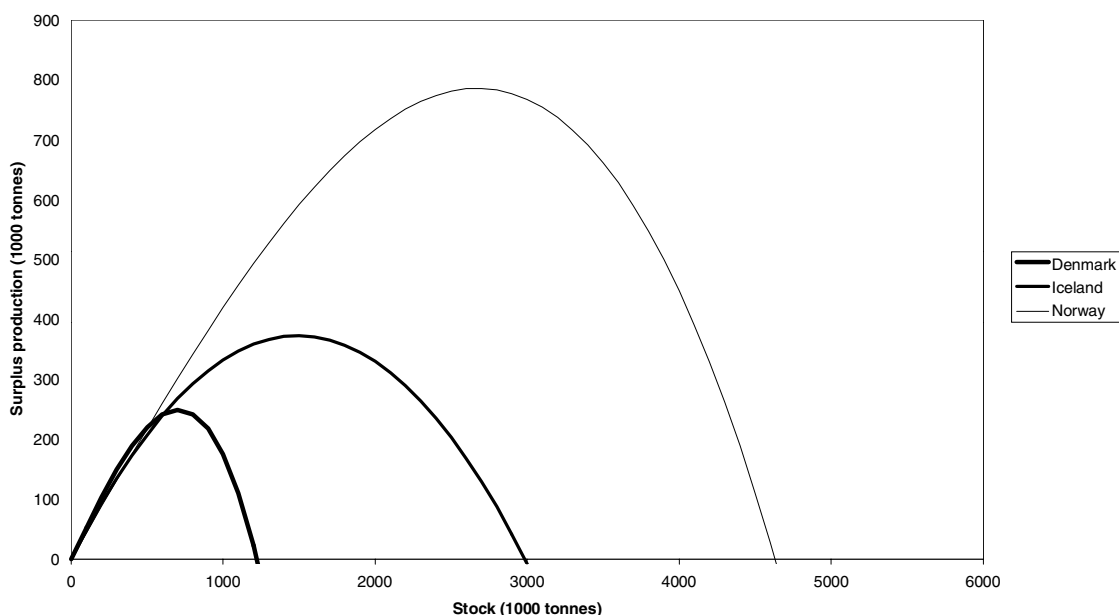
yielded the best fit. In both cases r is the intrinsic growth rate and K is the carrying capacity of the stock (Clark, 1976). The estimations was performed using NLREG⁸ and EViews3. The results of the estimations are shown in Table1.

<i>Table 1. Statistical properties of the biological growth functions. K is 1000 tonnes.</i>				
	Function	Parameters	t-statistic	
Denmark (n = 32)	$rx \left(1 - \frac{x^2}{K^2} \right)$	$r = 0.53$ $K = 1,222$	6.57 11.08	$R^2 = 0.13$ $F = 4.56$
Iceland (n = 42)	$rx \left(1 - \frac{x}{K} \right)$	$r = 0.50$ $K = 2,987$	10.10 12.06	$R^2=0.19$ $F = 127.2$
Norway (n = 50)	$rx \left(1 - \frac{x^2}{K^2} \right)$	$r = 0.44$ $K = 4,631$	10.18 17.5	$R^2 = 0.21$ $F = 12.6$

⁸ NLREG copyright Phillip H. Sherrod, 4410 Gerald Place, Nashville TN, 37205-3806 USA, (phil.sherrod@sandh.com).

For Denmark the whole North Sea stock is used as basis for the estimation. Data for stock and landings are taken from Anon (1997a). For Norway we use data for the Arctic cod stock from Anon (1998). For Iceland the relevant data relating to estimation of a growth function is given in Anon (1998a and 1998b). The differences in scale of the respective fisheries are illustrated in Figure 1a where the growth functions for each country's cod stock are illustrated in the same diagram.

Figure 1a. The biological growth function for Denmark, Iceland and Norway



Economic model

The generic profit function employed in the empirical model is:

$$\pi(h, x) = p(h)h - C(x, h).$$

where $p(h)$ represents the (inverse) demand function for landed cod, and $C(h,x)$ is the cost function associated with the harvest process.

Several forms for the demand functions were estimated for the three countries. The form adopted was:

$$P(h) = a - bh \quad (7)$$

where h represents landings of cod and a and b are coefficients. However, as it turned out, the slope parameter, b , was statistically insignificant for both Denmark and Iceland.

For the harvesting cost function the following functional form was adopted for all three countries:

$$C(h,x) = \alpha \frac{h^2}{x} \quad (8)$$

where α is a parameter. The dependent variable, i.e. costs, is defined as total costs less depreciation and interest payments. This may be regarded as an approximation to total variable costs.

Due to differences in data availability and industry structure between the three countries it is most convenient to discuss the estimation of these functions for each country in turn.

Denmark

With the help of data from Anon (1991) and Anon (1996b) a cod landings demand function of the form in (7) was estimated. The prices used are real prices (deflated by the consumer's price index). It turned out that slope coefficient was statistically insignificant. We therefore proceed on the assumption of constant price of cod landings of DKK 10.4. The estimation results are summarized in Table 2. The reason for this result is that Danish cod plays a smaller role in the total market and hence the price formation is independent of the Danish landings.

Because we only have data for two years in Anon (1996a), we have calibrated a variable cost function (see Hovitt 1995 for an example). We have calibrated on the whole North Sea stock but have used the Danish part of the catches in The North Sea. The calibration gives the following result:

$$C(x, h) = 29.618 h^2/x$$

The Danish variable profit function thus reads:

$$\pi(h, x) = 10.4h - 29.618 h^2/x,$$

when h and x are measured in metric tonnes.

Table 2. Statistical properties of the estimated demand functions.

Estimation procedure OLS				
	Function	Parameters	t-statistic	
Denmark	$p(h) = a$	10.4	11,23	$R^2=0.0$
Iceland ($n=24$)	$p(h) = a$	81.27	34.1	$R^2 = 0.0$
Norway ($n = 28$)	$p(h) = a - bh$	$a = 3.498$ $b = 0.0017$	9.16 2.79	$R^2 = 0.23$ $F = 7.8$

Iceland

The inverse demand function for cod landings, (7), was estimated with the help of monthly data in 1997 and 1998 (Anon 1998b and Anon 1999). The prices used are real prices (deflated by the consumer price index). The slope coefficient turned out to be statistically insignificant. As a result, we must proceed on the assumption of fixed price of cod landings which was estimated to be ISK 81.27 per kg. of landings (1998 prices). The key estimation results are further summarized in Table 2.

The reason for this apparent price inflexibility may be that most of Icelandic landings of cod still occur as a part of vertically integrated fish harvesting and processing firms. In addition to this, many of the independent harvesting operations supply their catches to processing firms according to fixed price contracts. For these reasons, and in spite of the emergence of auction markets for fish landings in recent years, the wetfish price of cod has during the past two decades been largely insensitive to variations in supply.

The cod harvesting cost function (8) was estimated using a combination of cross-section and time series data on variable costs. The data source was Anon(1998c). The time

period was 1990 to 1995 and the total number of observations was 35. The cost data was employed at fixed 1998 prices. A generalized least squares (GLS) estimation technique was employed. The key estimation results are broadly as follows:

$$C(h, x) = 119.0679 \frac{h^2}{x} \quad R^2 = 0.90.$$

(1.90)

The Icelandic variable profit function therefore is:

$$\pi(h, x) = 81.27 \cdot h - 119.0679 \frac{h^2}{x}$$

when h and x are measured in 1000 tonnes.

Norway

The inverse demand function for cod landings, (7), was estimated on the basis of data on real prices and landings obtained from Anon (1997b) and Anon (1997c). The key estimation results are listed in Table 2. The slope coefficient, b , turned out to be significantly different from zero. Consequently, we must reject the null-hypothesis of constant prices. The data are assumed to represent the demand function as the supply function is exogenously given through a binding TAC regulation scheme with little or no alternative harvest outlets.

With regard to the cost function we use data from Anon (1996c) to calibrate a variable cost function. For each vessel group the costs associated with cod are derived as total costs multiplied with the time devoted to cod and then multiplied with the number of vessels. Costs are then the sum over all vessel groups deflated with the 1979 consumer's price index. The calibrated cost function, using only the Norwegian part of total harvest, is:

$$C(h,x) = 6.2 h^2/x$$

Therefore the Norwegian variable profit function is:

$$\pi(h, x) = (3.5 - 0.0017h)h - 6.2 h^2/x$$

when h and x are measured in 1000 metric tonnes.

The profit functions: Summary and comparison

The above variable profit functions are expressed in different currencies and price levels. Moreover, they refer to widely different average biomass levels. Consequently, they are not directly comparable. However, by dividing through each of these functions by the respective cod landings price, profits will be measured in volume terms (biomass units) and hence more comparable between countries. In the case of Norway, where a downward sloping demand function for cod has been estimated the average sample price is employed. This yields the following set of variable profit functions:

Denmark: $h - 2.848h^2 / x,$

Iceland: $h - 1.465h^2 / x,$

Norway: $h - 2.531h^2 / x.$

In terms of costs, it is seen from this that Denmark has the highest costs and Iceland the lowest costs for the same stock and harvest level. Denmark and Norway, however, have quite similar cost structure whereas Iceland's cost parameter is only half of Denmark's.

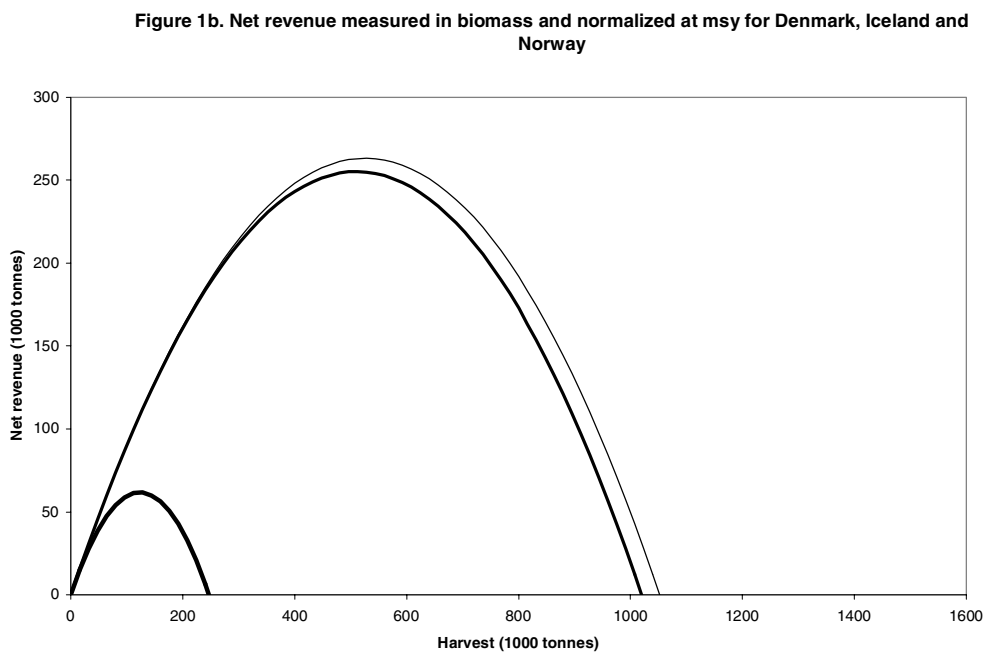
Moreover to normalize for the different cod stock level, we may for instance substitute the msy stock-level for the biomass variable, x , in each country's profit function. This yields the following set of variable profit functions:

$$\text{Denmark: } h - 0.00405 h^2,$$

$$\text{Iceland: } h - 0.00098 h^2,$$

$$\text{Norway: } h - 0.00095 h^2.$$

These profit functions are further illustrated in Figure 1b. When the msy stock level is used as reference point, it turns out that Iceland and Norway are quite similar and that Denmark's costs parameter is more than four times that of the other two due to the lower scale of the Danish fishery. Another explanation for this difference could be that the regulation system in Denmark is more inefficient than the regulation system in Norway and Iceland.



2.3 Comparative efficiency

Having completed the construction of our simple fisheries model we are now in a position to assess the relative efficiency of the cod harvesting policies followed by the three countries in the past. For this purpose we employ two main criteria; (i) the "economic health" of the cod stock and (ii) the "appropriateness" of the annual harvest. The former is measured by the actual stock size relative the optimal steady state level. The latter is measured by the actual annual harvest relative to the optimal one.

To present our results we primarily rely on diagrams. This has the advantage of conveying all data points simultaneously. However in order to facilitate efficiency comparison for the period as a whole we have devised simple numerical measures. The one used to assess the "economic health" of the stock is:

$$\eta \equiv \frac{\sum x_{act}}{\sum x^*} \equiv \sum_{t=1}^T \frac{x_{act}}{x^*} / T \in [0, \infty),$$

where T is length of the period we have observations for, x_{act} is the actual stock each period and x^* is the long-term optimal steady state. Obviously, the most desirable value of η is unity. $\eta < 1$ suggests an economically overexploited stock and $\eta > 1$ suggests underexploitation of the stock.

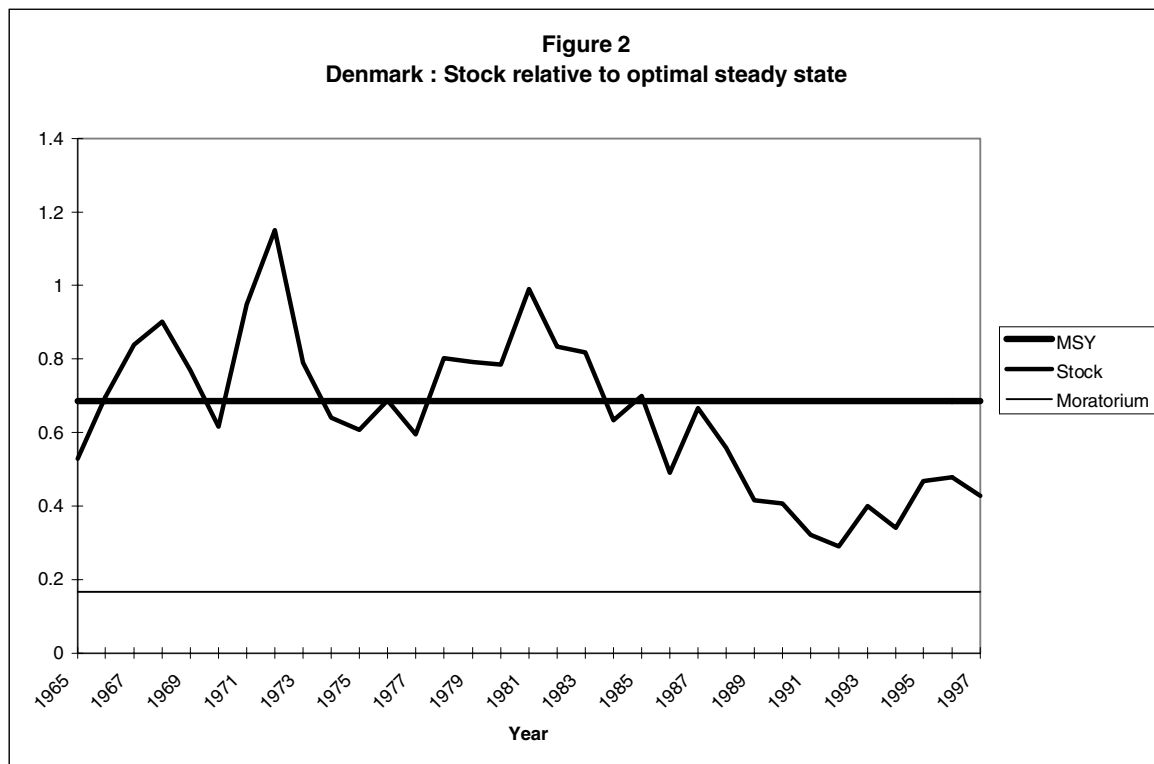
For the appropriateness of the annual harvest we use⁹:

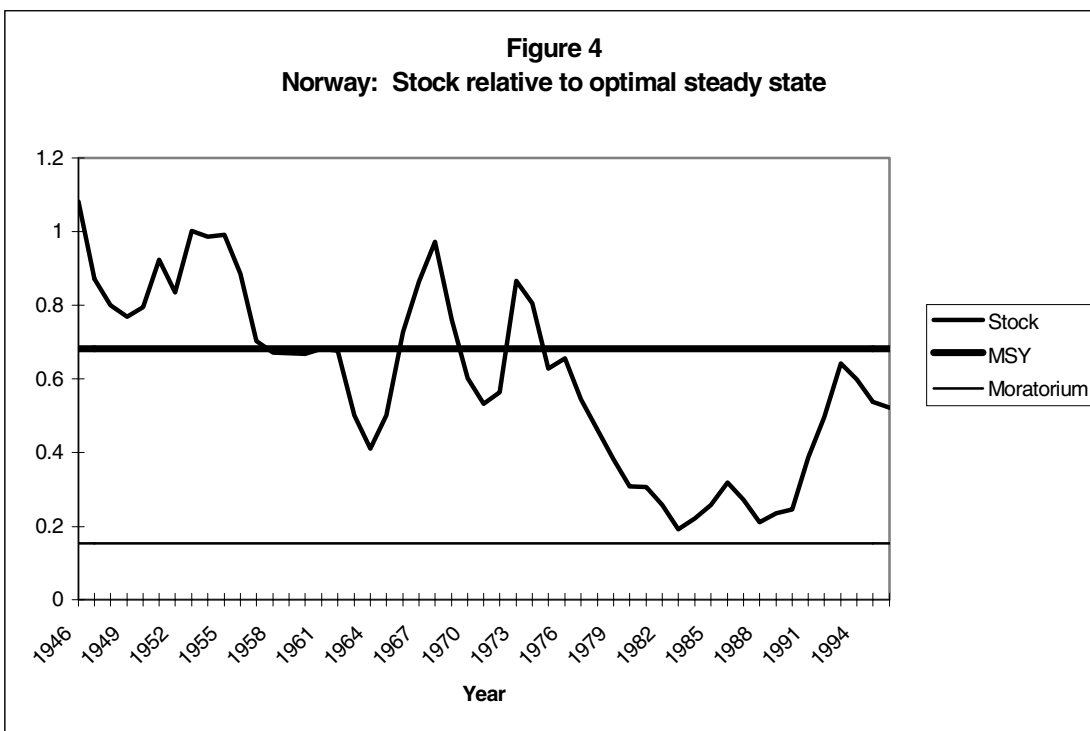
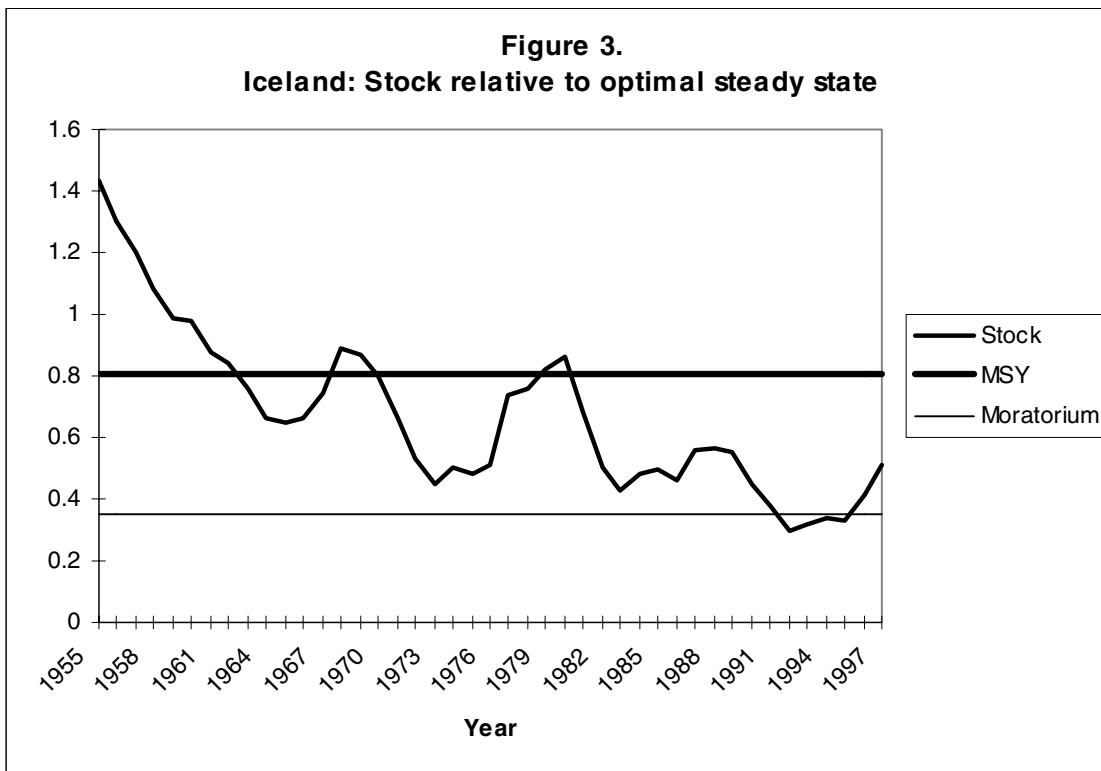
⁹ An alternative measure would be $(h_{act} - h_{opt})/h_{opt}$. This, however, is complementary to η in the sense that they add to one. Which to use is therefore a matter of taste.

$$\varphi \equiv \frac{\sum h_{act}}{\sum h_{opt}} \equiv \sum_{t=1}^T \frac{h_{act}}{h^{\circ}} / T \in [0, \infty),$$

where h_{opt} is the optimal level of harvest, h_{act} the actual one and h° the average optimal harvest. Obviously, $\varphi = 1$ indicates a perfectly efficient harvesting policy. $\varphi > 1$ indicates economic overharvesting and $\varphi < 1$ economic underharvesting.

Consider first the "economic health" of the cod stock in each country. This is illustrated in figures 2 – 4. In these figures, the actual biomass relative to the optimal one as in the η -measure discussed above is presented. Consequently, the optimal stock equilibrium for each country equals one, and the figures trace out the actual development of the cod stock relative to this. In addition, horizontal reference lines corresponding to the msy (maximum sustainable yield) stock level and the fishing moratorium (where it is optimal to cease fishing) stock level are drawn in the diagrams.





As illustrated in Figures 2-4, the cod stock biomass for these three countries is far below the economic optimal level for most of the period for which we have data. Moreover, all three cod stocks exhibit a clear downward trend relative to the optimal level over time. Thus, our model confirms the general view that the North-Atlantic and North-sea cod stocks are overexploited and the overexploitation is getting worse. Only towards the very end of the period, in the 1990s, do we see signs of some improvement in Norway and a smaller one in Iceland.

In Denmark, the cod stock is actually well above the msy level for long periods and never comes close to the moratorium level. However, during the available data period from 1965, the stock only once manages exceed the optimal equilibrium level, in 1973. In Iceland, the cod stock begins in a very good shape and is actually well above our calculated optimal equilibrium level in the 1950s. Since then, however, the stock development has generally been downhill and in the 1990s the stock actually hits our calculated fishing moratorium level. It should be noted, however, that due to the nature of the estimated Icelandic harvesting cost function, this moratorium is at very much higher stock level than for either Denmark or Norway. The development of the Norwegian cod stock is quite similar to the Icelandic one. It begins, in 1946, at quite a high level just above the optimal equilibrium level. Thereafter, it declines especially in the 1970s and languishes at very depressed levels in the 1980s getting very close to the calculated fishing moratorium levels which, due to the special nature of the Norwegian profit function is much lower than Iceland's.

For the period as a whole, the average relative stock for Denmark, i.e. the η -measure, is highest for Denmark ($\eta=0.65$). The corresponding η -measures (using the same data period: 1964-1996) are 0.57 for Iceland and only 0.49 for Norway. Looking at the whole data period

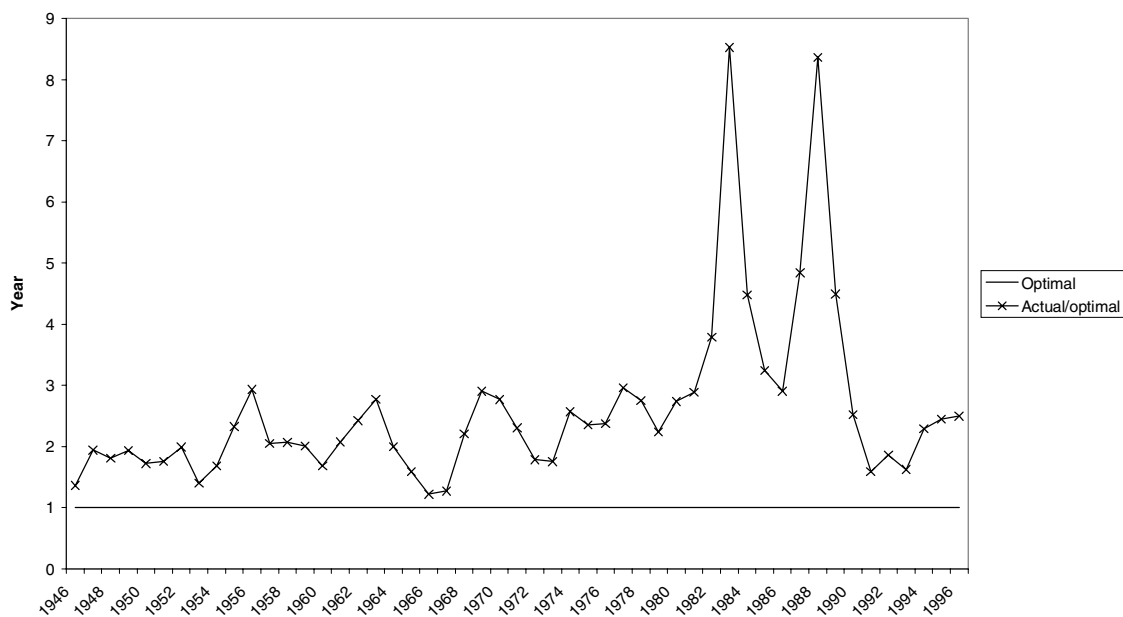
for Iceland (since 1955) and Norway (since 1946), however, the η -measure for these countries improves substantially. This is documented in Table 3

	Common data period 1964-1997	Total data period (Iceland 1955-97, Norway 1946-97)
Denmark	0.65	0.65
Iceland	0.57	0.67
Norway	0.49	0.60

Let us now turn our attention to the "appropriateness" of the cod harvest policies in each of the three countries. This is illustrated in Figures 5-7 below where the optimal actual harvest is represented by unity and the actual harvest is relative to the optimal one every year. From these diagrams it is evident that the harvesting policies¹⁰ for all three cod stocks have been severely excessive. Moreover, the level of overexploitation shows an increasing trend over time for all cod stocks. It is only towards the end of the period, in the 1990s that there are signs of some improvement in the harvesting policies especially of Denmark and Norway.

¹⁰ The term, "hasting policy" may not be appropriate in this context, as all three cod fisheries were open to international fishing for the early part of the period in question.

Figure 7. Norway: Actual harvest relative to optimal harvest



Compared to the other two countries, Denmark has operated the most stable harvesting policy relative to the optimal, but, unfortunately, it has been stable overexploitation. by contrast, Iceland's cod harvesting policy relative to the optimal is the most volatile. It features some years of close to optimal harvesting and even underharvesting especially in the early period, but also has the most severe cases of excessive harvesting relative to the optimal, i.e. when a harvesting moratorium should have been imposed. Again, it should be pointed out that in comparison to Denmark and Norway this is caused by the special nature of the Icelandic harvesting cost function which encourages more stock conservation than the other countries. Norway's cod harvesting policies fall somewhere in between those of Denmark and Iceland.

For the common data period (1964-1996), the average cod exploitation rate relative to the optimal, i.e. the ϕ -measure is highest for Iceland ($\phi=3.01$). This means that during this period, the cod harvest in Iceland has on average been more than three times the optimal one.

Note that the years when a cod harvesting moratorium would have been optimal, 1992-96, play a large role in this high average. In Denmark and Norway the average level of overharvesting has been much lower. For the data period as a whole, Iceland's overall cod harvesting policy improves substantially or to $\phi=2.14$ compared to 2.17 for Norway. These results are further documented in Table 4

Table 4 Efficiency of the cod harvesting policies, ϕ -measures		
	Common data period 1964-1997	Total data period (Iceland 1955-97, Norway 1946-97)
Denmark	2.33	2.33
Iceland	3.01	2.14
Norway	2.35	2.17

It is also informative to examine the actual cod harvest policies of the three countries relative to the optimal feed-back policies in the usual phase-diagram format employed in theoretical fisheries analysis (see e.g. Clark 1976). Figures 8-10 present these type of diagrams in harvest-biomass space for each country respectively. The solid graph in these figures represents the optimal cod harvest feed-back policy as a function of the stock level. The solid thin line represent the growth function, and the optimal long run equilibrium is at the intersection between the optimal path and the growth curve.

Figure 8. Denmark: Growth function, actual and optimal harvest against stock

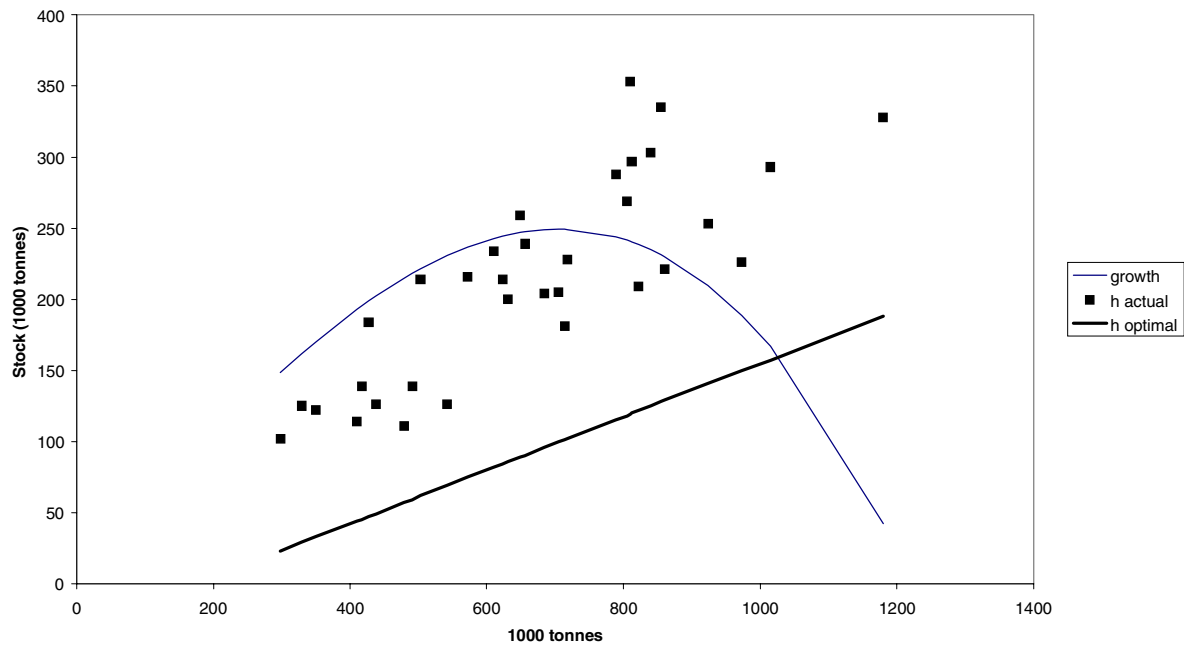


Figure 9. Iceland: Growth function, actual and optimal harvest against stock

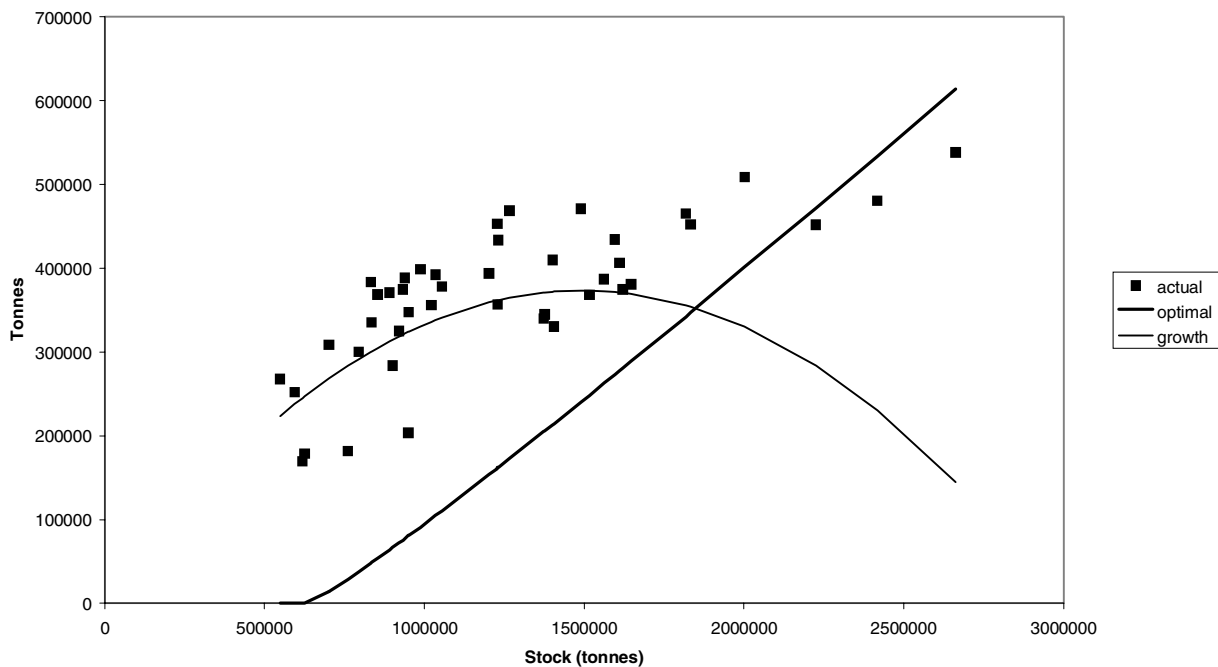
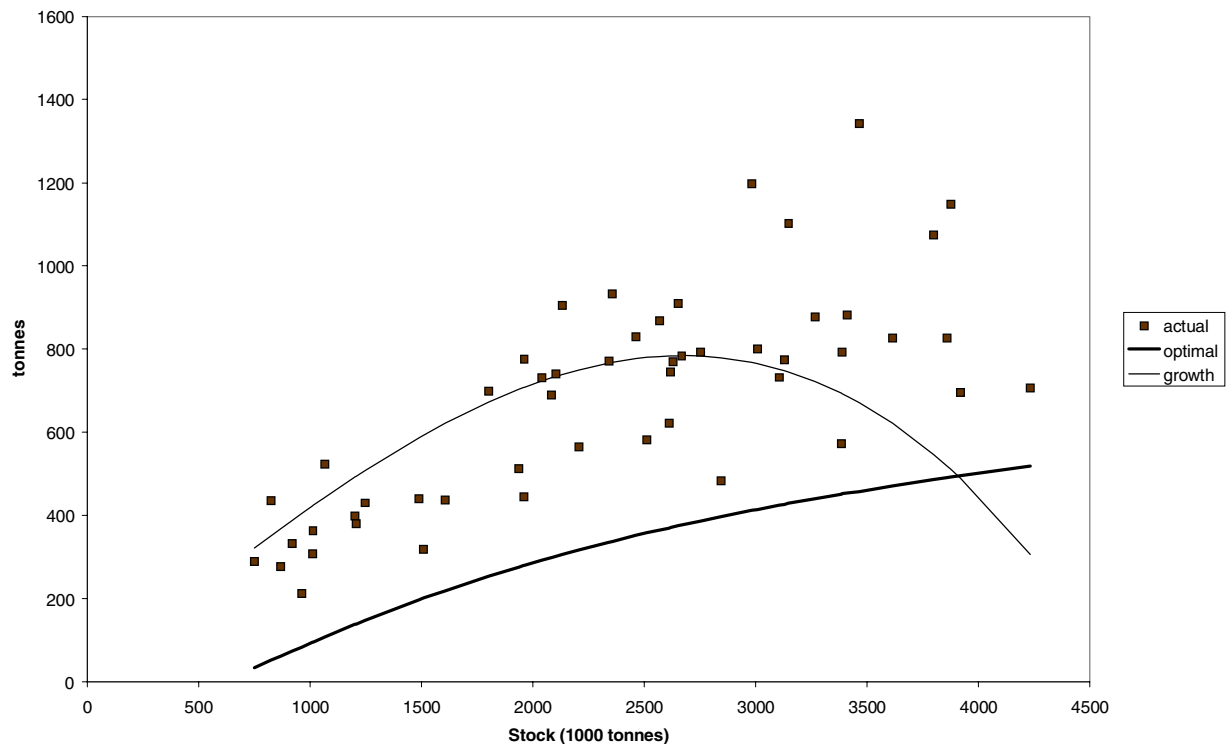


Figure 10: Norway: Growth function, actual and optimal harvest against stock



From Figures 8-10 it is seen that the optimal harvesting paths as a function of the cod stock biomass, i.e. the optimal feed-back curves, are close to straight lines for Denmark and Iceland but quite curved for Norway. This difference is due to the estimated price-harvest relationship for Norway. It is also interesting to note that for Denmark and Norway, the actual cod harvesting policies are not far from being the optimal policy shifted upward by a (substantial) constant. Thus, the actual harvesting policy for these countries, has roughly the same slope as the optimal policy. For Iceland, on the other hand, the actual harvesting policy has a different slope from the optimal one. In the case of Iceland, the actual harvest is more like an attempt at a constant harvesting rate irrespective of the stock size.

3. Discussion

This paper contributes, as we see it, primarily in two different ways. First, the paper demonstrates that it is now quite feasible – in fact relatively easy - to calculate optimal feed-back policies for renewable resource extraction on the basis of simple aggregative models. Secondly, the paper provides quantitative information about the economic efficiency, or rather inefficiency, of the cod fisheries in Denmark, Iceland and Norway in recent decades.

Optimal feed-back policies calculated on the basis of simple aggregative models can be useful in a variety of ways: First, obviously, such calculations can be employed as a quick check on the efficiency of existing policies. Second, such calculations can provide preliminary estimates of optimal paths and the accompanying economic benefits. Depending on the deviation from the current situation and the magnitude of the potential gains, this can then be used to judge whether a more in depth study or, perhaps possibly, efforts to alter the course of events are justified. Third, as in this paper, deviations from calculated optimal paths can be used for international efficiency comparisons and, possibly, to throw light on the efficacy of different organizational frameworks for the activity in question.

The comparative study of the three cod fisheries is perhaps more striking in terms of the similarities it uncovers rather than the differences. For all three countries the efficiency of the cod fisheries, measured as the deviation between the actual and the optimal, is low. For all three countries this efficiency shows a declining trend since the 1960s. This trend is, of course, in broad accordance with the prediction of fisheries economics for common property fisheries and therefore as expected. What is mildly surprising, however, is that in spite of much greater national control over the cod fisheries since the 1970s (at least in Iceland and

Norway) and greatly more extensive fisheries management since then, there are very few signs of a reversal in this trend of declining efficiency. It is only in the most recent years of the 1990's that there are some weak indications that the declining trend may be bottoming out and perhaps reversing in the three countries. This interpretation, however, is based on very few observations, and it remains to be seen whether they represent just a temporary aberration from the same old trend or whether they actually represent a regime shift.

During the last decade or so of our sample the cod fisheries in the three countries have been subject to somewhat different fisheries management systems. The cod fishery in Denmark is managed on the basis of the Danish share of the total allowable catch and short term vessel quotas (up to 3 months) (Vestergaard, 1998). The Norwegian cod fishery has been managed on the basis of individual but nontransferable quotas (Anon., 1996d) and the Icelandic cod fishery has been managed primarily on the basis of transferable quotas (Arnason, 1996). The differential effects of these management systems, if any, do not show up in our efficiency measures. These measures, of course, are restricted to aggregate harvest rates and biomass levels so these results do not exclude different economic returns in the fisheries deriving from the different operational efficiencies of the respective industries. Nevertheless, it is interesting to note that to the end of our data period (1996), the theoretical superiority of individual quota systems (see e.g. Hannesson 1994) does not seem to be reflected in the build up of cod biomass towards the optimal level, neither in Iceland nor Norway. There is some weak evidence, however, toward the end of the period, that the development of the Icelandic and Norwegian cod stocks is becoming relatively more favourable than that of Denmark.

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