<u>Working Paper No. 63/01</u>

Equilibrium and Price Stabilization

by

S.D. Flåm
I. Gaasland
E. Vårdal

SNF-project No. 2625:
Markeds- og produksjonsreguleringer i jordbruket

The project is financed by: The Research Council of Norway

FOUNDATION FOR RESEARCH IN ECONOMIC AND BUSINESS ADMINISTRATION

BERGEN, DECEMBER 2001

ISSN 0803-4028

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December 20, 2001

Abstract. The main objects here are markets with stochastic demand and supply. Agriculture provide prime instances. A key concern is how a buffer agency may learn to stabilize prices. We model such learning and identify conditions under which the process may generate necessary input for comparative welfare analysis.

Key words: Rational expectations, competitive equilibrium, price stabilization, buffer agency, stochastic approximation.

1. Introduction

Some commodities, notably those produced by agriculture, are subject to highly variable supply (or demand). The ups and downs of exogenous factors (such as weather) cause abundance to be stochastic so that prices fluctuate accordingly. Producers who face inelastic demand may then prefer that price stabilization be undertaken by some buffer agency. Essentially, such an institution would provide a particular form of insurance. It is not clear though that stable prices will provide greater welfare. Indeed, the seminal study of Newbery and Stiglitz (1981) brings out that such prices may, in general, cause well-founded concerns with efficiency.

This note also deals with price stabilization. The said study provided a negative impulse, mainly qualitative in nature. In contrast, what we explore here is a supplementing, positive perspective, namely: Suppose the welfare economics of a price-regulated market should be compared with that of a laissez faire regime. Then, in terms of implementation and modelling, what challenges are likely to emerge? Specifically, under either regime, we wonder: *can rational agents - while imperfect in their competence, experience or information - finally reach equilibrium by decentralized modes of behavior? If so, how?* Also, but now in terms of tractable numerics, we ask: *Under what conditions is equilibrium stable and computable*?

Clearly, welfare under different settings cannot be compared unless equilibrium, in each case, be stable, computable, reachable - and at best also unique. So, the above questions lead us to explore and identify here conditions under which there are positive, constructive answers. In fact, this note can be read as a recipe about how - and when - all necessary input to welfare analysis can be furnished by the economy itself or by simulation.

Our main motivation came in *three* forms: *First*, the problem of computing either equilibrium is far from trivial. In fact, we face here issues that relate to existence and construction of so-called *rational expectations.* Must these non-tangible items really be grasped by each and every concerned agent? *Second*, the modelling and computation/simulation had better be firmly based on information flows that are easily accessible at the level of each agent. *Third*, we find that explicit modelling of the buffer agency may be called for. It seems reasonable then to require that its operation be geared towards feasibility and cost minimization.

The subsequent analysis can be interpreted in terms of agriculture, producing commodities in random amounts. Major decisions underlying the production must then be made before uncertainty about growth is resolved. This setting is precisely the one which dominates the large literature on cobwebs. The standard cobweb story, which goes in terms of price expectations, is however, here turned around: Producers are promised a price - and the agency must learn how to stipulate that eventually stable price so as to ensure both market clearing and financial viability in the mean.

Arguments are organized as follows. Section 2 spells out how unregulated equilibrium, organized by spot markets, may be realized in decentralized, fairly non-informed manner. Section 3 deals with the alternative setting in which a market-balancing buffer agency eventually may learn to find a sustainable price level. In section 4 we provide a numerical example from the market for grain in Norway. We here discuss the welfare for producers and consumers in stabilized compared with competitive markets. As main vehicle we use stochastic approximation theory. An appendix provides a brief summary of some useful results from that theory.

2. Competitive Spot Markets

Consider henceforth a finite, fixed set I of producers, all supplying the same homogeneous, perfectly divisible goods to common markets. Everybody is a price-taker and faces uncertainty. More precisely, individual profit is affected by the state ω of the world. That state belongs to a finite, complete list Ω of mutually exclusive outcomes. An exogenous, maybe unknown probability distribution governs the likelihood of diverse outcomes, and it defines an expectation operator E. Agent i has state-dependent utility $u_i[\pi_i, \omega]$ of his uncertain profit

$$\pi_i(x_i, \omega) := p(\omega) \cdot q_i(x_i, \omega) - c_i(x_i, \omega). \tag{1}$$

In principle, if well informed, trained and competent, he would seek to

$$\text{maximize } Eu_i[\pi_i(x_i, \omega), \omega] \quad \text{with respect to own input} \quad x_i \in X_i.$$

In (1) $p(\omega)$ denotes the (contingent) price vector that clears markets in state ω. If G is the finite set of produced goods, then clearly $p(\omega) \in \mathbb{R}_+^G$. The quantity vector $q_i(x_i, \omega) \in \mathbb{R}_+^G$ records the output/supply provided by agent i when ex ante using input bundle x_i and ex post being exposed to state ω. The function $c_i(x_i, \omega)$ accounts for his cost.

We emphasize that x_i must be decided before the uncertainty is resolved. Also, the input bundle x_i is constrained; it must belong to a prescribed, nonempty, compact, convex subset X_i of some finite-dimensional space. Note that costs, technologies and risk exposure may differ across agents.

To close the model we must specify how prices are formed. On this account we simply let a "curve" $P(Q,\omega) \in \mathbb{R}^G$ tell the price vector at which consumers are willing to purchase altogether the commodity bundle Q in state ω. As customary, $P(\cdot,\omega)$ is the inverse of the state-contingent demand "curve" $D(\cdot,\omega)$.

Note that this reduced production economy is organized around spot markets in each of which consumers are represented simply via their aggregate demand schedule. An input profile $x = (x_i)$ is therefore said to constitute an *equilibrium* if $x_i \in \arg\max Eu_i[\pi_i(\cdot,\omega),\omega]$ for all i, and *rational price expectations* prevail:

$$p(\omega) = P(\sum_{i \in I} q_i(\omega), \omega) \quad \text{for all } \omega \in \Omega. \tag{2}$$

Existence of such an equilibrium follows in standard manner under customary conditions. Here we take existence for granted. Assume for each i, that $Eu_i[\pi_i(x_i,\omega),\omega]$ be concave and smooth with respect to x_i. (We regard this a fairly innocuous assumption.) Let

$$M_i(x,\omega) := \frac{\partial}{\partial x_i} u_i[\pi_i(x_i,\omega),\omega]$$

denote the marginal utility of agent i. Then equilibrium is characterized by satisfaction of (2) as well as the sufficient optimality conditions:

$$x_i = Proj_i\{x_i + sEM_i(x,\omega)\} \quad \text{for all } i \text{ and all } s > 0. \tag{3}$$

Here $Proj_i$ is short notation for the *orthogonal projection* onto the closest approximation in X_i. The question was whether equilibrium can be reached via an adaptive procedure? Since (3) invites a dynamic version, we advocate the following iterative, decentralized adaptations:

Reaching equilibrium with spot markets:
Start with individual choices $x_i^0, i \in I$, determined by (historical) factors not discussed here. Sample $\omega^0 \in \Omega$ according to the fixed, exogenously given distribution. Set $p(\omega^0) := P(\sum_{i \in I} q_i(x_i^0, \omega^0), \omega^0)$.
Update inputs x_i^t by the rule

$$x_i^{t+1} = Proj_i\left\{x_i^t + s_t M_i(x^t,\omega^t)\right\} \quad \text{for all } i. \tag{4}$$

Increase time by 1.
Sample a new state ω^{t+1} from the said distribution and set the spot price as

$$p(\omega^{t+1}) := P(\sum_{i \in I} q_i(x_i^{t+1}, \omega^{t+1}), \omega^{t+1}).$$

Continue to update inputs until each bundle x_i^t converges towards a steady state x_i. □

Proposition (Attaining equilibrium with spot markets) *Suppose the limit set $\mathcal{L}$ of the differential equation $\dot{x} = Proj_{Tx}EM(x,\omega)$ is finite (8). Then process (4) converges with probability one to equilibrium.* □

3. PRICE STABILIZATION

Suppose in this section that some (maybe public) buffer agency seeks to stabilize price to producers. Instruments for that enterprise is provided by active use of inventory and/or trade. Good years yield net inputs to inventory or some export; lean years reduce inventory or justify some import.

Whatever measures taken by the said agency, a natural constraint is that net changes of inventory - seen as a time average - be nil. Formally, this means that, in the long run, expected supply equals expected demand:

$$ES(p, \omega) = ED(p, \omega). \tag{5}$$

We write here $\pi_i = \pi_i(x_i, p, \omega)$ to stress that profit depends on the regulated, fixed price p. Correspondingly, a vector $p \in \mathbb{R}^G_+$ is now said to be a *stabilized equilibrium price* if the resulting optimal choices $x_i \in \arg\max Eu_i [\pi_i(\cdot, p, \omega), \omega]$ yields an aggregate supply $S(p, \omega) := \sum_{i \in I} q_i(x_i, \omega)$ that satisfies (5).

To find of such an equilibrium seemingly demands considerable information - and also some competence. Indeed, as formulated, it presumes knowledge of both functions $S = \sum_{i \in I} q_i$ and D as well as the underlying probability distribution. One can hardly suppose that so much information be quickly or readily available. So, we shall here take an extreme opposite position, namely: suppose the agency knows neither. Indeed, suppose it must contend with sequential observation of realized supply and demand. But then, how can it eventually learn to stabilize the price vector? For that purpose classical microeconomics immediately suggests a procedure: Why not use a Walrasian scheme? So, let now

$$M_i(x, p, \omega) := \frac{\partial}{\partial x_i} u_i [p \cdot q_i(x_i, \omega) - c_i(x_i, \omega)]$$

denote the marginal utility of agent i when promised price p, and consider the following process:

Price stabilization with no direct buffer cost:
Start at a reasonably informed guess p_0. Suppose the producers have already committed to inputs $x_i^0, i \in I$. Sample the state ω^0.
Update prices and inputs as follows:

$$\left.\begin{array}{rl} p^{t+1} := & Proj_+ \left\{ p^t + s_t \left[\sum_{i \in I} q_i(x_i^t, \omega^t) - D(p^t, \omega) \right] \right\} \\ x_i^{t+1} := & Proj_i \left\{ x_i^t + s_t M_i(x^t, p^t, \omega^t) \right\} \quad \text{for all } i \in I. \end{array}\right\} \tag{6}$$

Increase time by 1.
Sample a new state ω^{t+1} independently from the given distribution.
Continue to update prices and inputs until convergence. □

In (6) $Proj_+$ denotes projection onto the non-negative price orthant $\mathbb{R}^G_+$. Let $Proj_{T_+ p}$ denote projection onto the corresponding tangent cone. Note that (6) presumes no knowledge about supply or demand functions or the underlying probability distribution. Instead, and much more modestly, it merely requires the possibility to observe the aggregate supply $\sum_{i\in I} q_i(x_i^t, \omega^t)$ and demand as these items evolve over time.

Proposition. (Convergence under no handling cost) *Suppose the limit set $\mathcal{L}$ of the coupled set of differential equations*

$$\begin{aligned} \dot{p} &= Proj_{T_+ p}\left[E\sum_{i\in I} q_i(x_i,\omega) - D(p,\omega)\right] \\ \dot{x}_i &= Proj_{T_i x_i} EM_i(x,p,\omega) \text{ for all } i. \end{aligned}$$

is finite (8). Then, provided prices remain bounded, process (6) converges with probability one to a stabilized equilibrium. □

4. EXAMPLE: THE NORWEGIAN MARKET FOR GRAIN

We now give examples of equilibria in the case of a competitive and a stabilized market respectively. The examples are based on data from the Norwegian market for grain. We assume that the farmers are faced with a choice between renting out the land or growing grain. Due to uncertain crops growing grain is risky, while renting out land is riskless. We assume that the farmers are risk averse, so with equal expected profitability in the two alternatives they will rent out the whole land.

In the case of price stabilization (fixed prices) farmers face uncertainty in the form of a variable crop. In the spot market case both production and price are uncertain. In the following we go through the specific assumptions in our case study. Farmers' output equals:

$$q_i(x_i,\omega) := y_i(\omega)x_i = \left(y_i^1(\omega)x_i^1, x_i^2\right)$$

where x_i^j is farmer i's use of input (acreage) in the production of commodity *j*, *j=1,2.* 1 represent grain while 2 is land-hire. $y_i^1(\omega)$ is the crop yields (per hectare) which depend on the states of the world. In addition we assume constant marginal costs.

(1) can then be written as:

$$\pi_i(x_i,\omega) := p(\omega)y_i(\omega)x_i - c_i x_i.$$

Demand is assumed to be of the constant elasticity type, so (2) specializes to

$$p(\omega) := K\left(\sum_{i\epsilon I} q_i(\omega)\right)^{-\varepsilon}.$$

K is a constant and ε is the price elasticity of demand. We assume an exponential utility function;

$$u_i(\pi_i) := -ke^{-A\pi_i}$$

which give a constant absolute risk aversion. k is a constant and A is the measure of absolute risk aversion.

The main assumption of the analysis is written into table 1. We look at the land use in Norwegian agriculture. Total acreage suited for grain production is .5 million hectare, of which .35 million hectare are used for grain production. The remainder is utilized to different forms of riskless activities. Currently, the Norwegian grain price is stabilized to around NOK 2. The price elasticity of demand is assumed to be -.6. The marginal cost is assumed to be NOK 41.10 per hectare, while rent NOK 30 per hectare. Estimate of crop yields is based on information over actual crop yields the last ten years, and is given in the data appendix.

Table 1: Assumptions

Variables in base year (2000)[1]		
Total acreage, mill. hectare	$\bar{x}$	.5
For grain, mill. hectare	x^1	.345
For rent, mill. hectare	x^2	.155
Price of grain, NOK per kilo	p^{BASE}	2.
Consumption (average) mill. kilo		1295.
Supply[1]		
Absolute risk aversion	A	.001
Marginal cost grain, 1,000 NOK per hectare	mc_i	41.10
Rent, 1,000 NOK per hectare		30.
Demand		
Price elasticity of grain	ε	.6

Table 2 gives the results of the experiments. One might expect that the competitive case is less attractive as both price and crop are uncertain. However, farmers are concerned with uncertainty in profits, not prices, and profits become more stable when the price is flexible. This is one reason why farmers decide to plant more in the competitive compared to the price stabilization case. When farmers plant more, prices go down, and they reach a limit where it is no longer profitable to expand production. Consequently, production increases only slightly.

From table 2 we see that consumers gain if markets are competitive. There are at least two reasons for this. First, consumers tend to gain from variable prices. Second, consumers gain from increased production. Observe that farmers' expected profit goes down in the competitive case. On the other hand variance of profit also

[1] Source: NILF (1998)

declines. However, this is not enough to avoid farmers' utility to be lower in the competitive compared to the stabilized case. According to this analysis, producers prefer the stabilized case. For consumers it is the other way around.

Table 2: Welfare comparison

	Price stabilization	*Competitive spot market*
Land use (mill. hectare):		
Grain	.345	.358
Hire	.155	.142
Expected price of grain (NOK per kilo)	2.	2.01
Expected production (mill. kilo)	1,297	1,344.4
Change in welfare:		
Consumer surplus		36.9
Expected profit		-78.3
Variance of profit		-48,977.5
Farmers' utility		-.007

5. Appendix: Stochastic Approximation

For convenience and presentation this section collects some technicalities which will serve us time and again. It also outlines some proofs. Readers concerned only with economic issues may skip this part.

We repeatedly consider some stochastic process z^t which evolves within an appropriate, finite-dimensional (Euclidean) vector space $\mathbb{E}$. The key issue is always whether z^t will converge to a stationary point. z^t moves in discrete time $t = 0, 1, ...$ and is at any stage affected by the upcoming realization ω^t of "the state of the world." The latter belongs to a complete list Ω, comprising all relevant, mutually exclusive outcomes. For simplicity we take Ω to be finite. Also for simplicity, we posit that $\omega^0, \omega^1, ...$ be independent and all distributed according to a (possibly unknown) probability measure μ over Ω. That measure generates an expectation operator denoted E.

The state z^t is defined recursively (and evolves) as follows:

$$z^{t+1} := Proj_Z \left\{ z^t + s_t M(z^t, \omega^t) \right\} \tag{7}$$

Here $Proj_Z(\cdot)$ denotes the *orthogonal projection* in the ambient space $\mathbb{E}$ onto the closest approximation in Z. The latter item is a nonempty, compact, convex subset of $\mathbb{E}$. Thus z^t always belongs to $Z \subseteq \mathbb{E}$. The function $M : Z \times \Omega \to \mathbb{E}$ is a priori given. So is also the sequence $\{s_t\}$ of *step-sizes* $s_t > 0$. It is chosen once and for all subject to

$$\sum_{t=0}^{\infty} s_t = +\infty, \quad \sum_{t=0}^{\infty} s_t^2 < +\infty$$

A point $z \in Z$ is declared *stationary* iff

$$z = Proj_Z \{z + sEM(z, \omega)\} \quad \text{for all } s > 0.$$

We deem it highly desirable that $\{z^t\}$ converges to such a point. To inquire about such behavior we introduce the auxiliary function $h(z) := Proj_{Tz} EM(z, \omega)$. Here $Proj_{Tz}$ denotes the orthogonal projection onto the *tangent cone* $Tz := Cl_+ \mathbb{R}(Z - z)$ of Z at z. We assume that for any initial point $z^0 \in Z$ the differential equation

$$\frac{dz(\tau)}{d\tau} =: \dot{z} = h(z)$$

has a unique, infinitely extendable solution trajectory $z(\tau) \in Z, \tau \geq 0$. Moreover, and most crucial, *we suppose that*

the limit set $\mathcal{L}$ *of all accumulation points of such trajectories is finite.* (8)

Now, finally we have:

Theorem (Asymptotic stability and convergence) *Under the above assumptions process* $\{z^t\}$ *as defined by (7) converges with probability one to a stationary point.* □

6. DATA APPENDIX

Crop yield; kilo per hectare:

					ω					
	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
y	4396	4069	2815	3927	2840	3626	4026	3876	4084	3875

Source: NILF (2000)

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