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information for prospective bidders?**

by

Steinar Vagstad

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Should auctioneers supply early information for prospective bidders?

Steinar Vagstad*

Department of Economics, University of Bergen,
Fosswinckelsgate 6, N-5007 Bergen, Norway

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Abstract

Consider an auction in which potential bidders must sink an entry investment before learning their values (Levin and Smith, 1994). Suppose the auction designer can make the bidders learn their value before entry (as in Samuelson, 1985). Such early information will induce screening of high-value bidders, and it will give rise to information rents and thereby a difference between the socially optimal auction and the auctioneer's preferred mechanism. Therefore, the auction designer has too weak incentives to produce early information. Early information may increase or reduce equilibrium entry. If entry is sufficiently reduced, early information will harm the auction designer. (100 words).

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1 Introduction

Much of the received auction theory is concerned about how to sell an object to one out of a number of potential buyers, each of whom privately informed about his or her valuation of the object — independent private value (IPV) auctions. While early contributions dealt with situations with a given information structure and a given number of bidders, more recent contributions have allowed the number of bidders to be endogenous — so-called "auctions with entry."¹ Participation in an auction often entails costs that do not depend on how much the buyer actually bids, or on whether he ends up with the object. A prospective bidder will participate in – or enter – such an auction if and only if the expected gains from participating cover the entry costs. Consequently, the number of bidders is not fixed, but depends on how the particular auction is designed (see, e.g., Samuelson, 1985, McAfee and McMillan, 1987b, and Levin and Smith, 1994).²

Also the present paper deals with IPV auctions with entry, and also allows for endogenous information structure. In particular, situations in which the auction designer may affect how much information the bidders have at different points are explicitly modeled. Intuitively, the outcome of an auction with entry will depend on the extent to which prospective bidders have private information before they decide whether to sink the entry cost and participate in the auction. Early information will make prospective bidders self-select, and due to this self-selection, the competition at the bidding stage will also be affected. The present paper studies the auctioneer's incentives to affect the information structure by producing early information. For

¹Traditional auction theory is excellently surveyed by Milgrom (1987) and McAfee and McMillan (1987a). Relevant contributions to the theory of auctions with entry include Samuelson (1985), McAfee and McMillan (1987b), Tan (1992), Engelbrecht-Wiggans (1993), Levin and Smith (1994) and Kjerstad and Vagstad (2000).

²The entry costs may take different forms. Contenders for procurement contracts often have to sink relation-specific (or auction-specific) investments before they submit their bids (bid preparation costs; costs of establishing the necessary organization to carry out the project on time). People interested in buying a second-hand car at a car auction have to travel from their homes, and spend time at the auction site before they can submit bids for the car they wish to buy. In either case the costs do not depend on whether the bidder ends up winning the auction or not.

instance, a car auctioneer may examine the cars and publish the result in the ads for the auction. Has he the right incentives to do so? A government that wants a private firm to build a new bridge may spend resources to survey the available (e.g. technical) solutions before potential bidders establish the organizations necessary to be taken seriously as contenders for the contract. Has the government the right incentives to conduct such a survey?

Attention is restricted to information that enables the bidders to learn their values before entry, without enabling the auction designer to learn anything at all about the bidders' values.³ In the car auction example, the information that can be produced may be about, say, the make, year and color of the cars to be sold. The auctioneer knows only the distribution of tastes for these attributes in the population. Each bidder, in contrast, knows his willingness-to-pay for a car of a given make, year and color, but (unless the auction designer produces the information) does not know which (i.e. the attributes of) cars will be for sale at a given auction.⁴

Similarly, in the procurement example the information may be about, say, which technology (e.g. steel vs. concrete construction) will be cost efficient for the actual project. The procurer knows the distribution of abilities to build different types of bridges among the population of construction firms. Each bidder, in contrast, knows his ability to build bridges using each given technology, but (unless the technical information is produced) does not know which (i.e. the attributes of) technology will be efficient for this particular bridge.⁵

The analysis is based on two papers that study two polar, exogenously imposed, information structures. Levin and Smith (1994) study situations in which the poten-

³Often, information will inform not only the prospective bidders, but also the auction designer. This will surely add another reason to produce such information, but it is outside the scope of the present paper to assess how this effect blends with the effects examined here.

⁴Clearly, the list of attributes can be made very long. The important feature is that the considered attributes can be costlessly observed once a prospective bidder shows up at the auction site, and also, at some cost, communicated to each prospective bidder prior to his entry decision.

⁵This structure suggests that the auction in consideration is not a pure IPV (independent private values) auctions, but has elements of CV (common values) as well: The technological information is not firm-specific, but project-specific. However, the project-specific information is resolved before bidding takes place, and the auction can therefore be analyzed using the methods of IPV auctions.

tial bidders do not learn their private information until after entry. With sufficiently many potential bidders, the only symmetric equilibrium is one in which each potential bidder enters with a common probability, q^* . Levying entry fees or introducing reservation prices will reduce the equilibrium probability of entry, and this, they show, will be harmful to welfare as well as to expected revenue. The other extreme is studied by Samuelson (1985). In his model the bidders have perfect information before entry. Consequently, in equilibrium the potential bidders are screened such that only potential bidders with values above some common cut-off level actually enter and submit bids. In this case the bidders will earn information rents.

Early information has the following effects. First, among those who enter, high-value bidders will be overrepresented, due to the screening. For a given number of bidders, this is good news for the auction designer, as it implies harder competition due to reduced bidder heterogeneity, and a higher average quality of the bidders. The prospective bidders, on the other hand, have more mixed feelings. A higher average quality is good for each bidder's utility, while increased competition is bad. It turns out that the combined effects of the two can be either positive or negative, and this ambiguity also drives some other mixed results: early information may increase or decrease entry, welfare and revenues, while early information is always good for the bidders.

Since advertising is one type of information production that fits our description (cf. the car auction example above), it is worthwhile to compare my findings with those of the economic theory of advertising.⁶ One issue in that literature is whether the level of advertisement is appropriate from a social welfare point of view. The results are ambiguous; there might be underprovision of advertising because of non-appropriability of social surplus, and overprovision because of 'business stealing.' In our model there is no business stealing, hence we should expect to get – and we do

⁶See, e.g., Tirole (1988) for a discussion. Tirole divides the literature on advertising into two broad categories. The 'partial' view sees advertising as providing information to customers and thus enabling them to make rational choices. The 'adverse' view, in contrast, claims that advertising is meant to persuade and fool consumers. Clearly, the former view is the basis of our analysis.

get – underprovision.

The remainder of this paper is organized as follows. The model is introduced in the next section, the main results are laid out in Section 3, an example with uniformly distributed values and only two prospective bidders is presented in Section 4, while some concluding remarks are gathered in Section 5.

2 The model

Consider a seller who wishes to sell an object to one out of a number $N \geq 2$ of potential bidders. Before a potential bidder can bid, however, he must sink an entry investment of size $k \geq 0$. Before the investment decision is made, each potential bidder i learns an estimate (denoted \tilde{v}_i) of his value (denoted v_i). The estimates are drawn independently from a common distribution $F(\cdot)$ with support $[0, \bar{v}]$. The seller puts value 0 on the object.

The relation between the estimates and the true values is as follows: With probability p the true value of bidder i equals his estimate (that is, $v_i = \tilde{v}_i$), and with probability $1 - p$ the estimate is just noise and the true value is independently drawn from the same distribution F .⁷

The seller controls the information the potential bidders have before entry, i.e., she controls p . Unless explicitly stated, attention is restricted to situations in which $p \in \{0, 1\}$, meaning that the estimate is either perfect (as in Samuelson, 1985) or has no information content (as in Levin and Smith, 1994). Once chosen, p becomes common knowledge.

⁷This information structure was introduced by Erbenová and Vagstad (1999). As long as attention is restricted to the polar cases studied by Samuelson (1985) and Levin and Smith (1994) this notation is slightly excessive. The virtues of this notation are as follows. First, it simplifies notation when comparing different cases: comparing entry amounts to compare cutoff levels of the estimate. Second, this notation highlights a reason to focus on the symmetric mixed-strategy equilibrium when information is symmetric before entry: the symmetric mixed-strategy equilibrium is the limit of the (symmetric) pure-strategy equilibrium when p approaches zero. Finally, I plan an extension to cases of "some" early information, and such an extension can be performed without having to introduce new notation.

The timing of events in the simplest version of our model is as follows:

1. The seller chooses the information structure $p \in \{0, 1\}$.
2. The seller designs an auction mechanism (to be specified below).
3. The potential bidders decide simultaneously whether to enter or not.
4. Those who have entered submit bids.
5. A winner is selected, performs, and is rewarded, according to the auction mechanism in 2.

We make the following assumptions:

- A1. The seller and all potential bidders are risk neutral.
- A2. The auction mechanism and the number of potential bidders (N) are common knowledge, and the number n of actual bidders is revealed prior to bidding.
- A3. Discrimination of bidders or coordination among bidders are infeasible.
- A4. The environment is such that a unique symmetric Nash equilibrium bidding function exists, and that this bidding function is increasing.

By symmetry, equilibrium entry can be described by a common cut-off value denoted v^p such that bidder i enters if and only if $\tilde{v}_i \geq v^p$.⁸ In what follows I will characterize the equilibria in each of the polar cases.

2.1 Entry before bidders learn their values ($p = 0$).

Suppose a plain auction without entry fees or reservation prices is used. Then, for entry costs in a certain range — $k \in (\underline{k}, \bar{k})$ — there is, roughly speaking, "room for" more than one bidder but not for all bidders. This is the case analyzed by Levin and Smith (1994). The only symmetric equilibrium in such a situation have each potential bidder enter with a common probability $q^* \equiv 1 - F(v^0)$. Levin and Smith show that if potential bidders conform to such a symmetric equilibrium, then for $k \in (\underline{k}, \bar{k})$ the optimal mechanism is indeed a plain auction without reservation prices or entry fees. The number of actual bidders follows a binomial distribution. The expected gross utility, denoted $E[u_i]$ of any potential bidder i is, if he enters,

⁸Without loss of generality, if $p = 0$ the estimates are used as a randomizing device when a such is needed.

equal to the entry cost:

$$E[u_i] = \sum_{n=1}^N \binom{N-1}{n-1} [1 - F(v^0)]^{n-1} F(v^0)^{N-n} E[u_i|n] = k, \quad (1)$$

where

$$E[u_i|n] = \int_0^{\bar{v}} \int_0^{v_i} (v_i - x) d[F(x)]^{n-1} dF(v_i) = \int_0^{\bar{v}} [1 - F(x)] F(x)^{n-1} dx \quad (2)$$

is the expected gross utility conditioned on there being n entrants.

Using (2), now we can also characterize the range of entry costs giving rise to the mixed-strategy entry equilibrium: $\bar{k} = E[u_i|1] = \int_0^{\bar{v}} [1 - F(x)] dx$ and $\underline{k} = E[u_i|N] = \int_0^{\bar{v}} [1 - F(x)] F(x)^{N-1} dx$.

With mixed strategy entry, potential bidders are indifferent between entering the auction and staying out, implying that there is no information rent on average. Therefore, the seller's surplus π^0 equals the social surplus w^0 , which can be written as the expected maximum value minus the aggregate entry costs. The expected maximum value equals $\int_0^{\bar{v}} x d[F(x)^n] = \bar{v} - \int_0^{\bar{v}} F(x)^n dx$. Therefore

$$w^0 = \pi^0 = \sum_{n=1}^N \binom{N}{n} [1 - F(v^0)]^n F(v^0)^{N-n} \left[\bar{v} - \int_0^{\bar{v}} F(x)^n dx \right] - (1 - F(v^0)) Nk \quad (3)$$

What remains is to characterize equilibria when $k \notin (\underline{k}, \bar{k})$. First, if $k > \bar{k}$ then the entry cost exceeds the expected value of a single bidder, and it is optimal not to induce entry at all, implying that $w^0 = 0$. Second, if $k < \underline{k}$ then with a plain auction all N potential bidders will enter. Now it is no longer the case that the plain auction without entry fees or reservation prices is optimal, however, because each bidder will earn an information rent (equal to $\underline{k} - k$). Then the optimal mechanism consists of an entry fee of $\underline{k} - k$ followed by a plain auction. Then, in equilibrium, all N potential bidders enter, and there is no information rent. Therefore there is no difference between social surplus w^0 (which includes both the principal's payoff as well as the aggregate payoff of the bidders) and the auctioneer's surplus π^0 (which

equals the expected highest bid). They are both given by

$$w^0 = \pi^0 = \bar{v} - \int_0^{\bar{v}} F(x)^N dx - Nk \quad (4)$$

2.2 Entry after bidders learn their values ($p = 1$)

When the prospective bidders have information before entry they will in general earn information rent. This rent drives a wedge between social surplus and the payoff to the seller, and it becomes desirable to restrict the seller's choice of mechanism. We start with an unregulated seller, and treat cases in which the seller is regulated to maximize social surplus below.

2.2.1 The seller maximizes profit

In a symmetric equilibrium bidder i will enter if and only if $\tilde{v}_i = v_i \geq v_\pi^1$, where v_π^1 is a cutoff estimate common for all prospective bidders (the subscript denotes which objective the auction is designed to maximize – profits – while the superscript denotes the information structure – as before). Again we start by studying interior equilibria, that is, situations in which $v_\pi^1 \in (0, \bar{v})$. From Samuelson (1985) we know that in this case the optimal mechanism, as seen from the auctioneer's point of view, is a plain auction with reservation price $b = [1 - F(v_\pi^1)] / f(v_\pi^1)$.

Consider a potential bidder who has learnt that his value equals the cutoff value v_π^1 . If he enters and none of his competitors do, he will earn $v_\pi^1 - b$. If he enters and at least one of his competitors also does, he cannot profit from entry (since with probability one his competitors have value exceeding v_π^1). Therefore, when the auctioneer is maximizing profit, equilibrium entry must satisfy

$$\left[v_\pi^1 - \frac{1 - F(v_\pi^1)}{f(v_\pi^1)} \right] F(v_\pi^1)^{N-1} = k. \quad (5)$$

Noting that the left-hand side of (5) is strictly increasing in v_π^1 , it must be the case that the equilibrium cutoff value v_π^1 is unique and strictly increasing in k . Inspection reveals that as k approaches 0, v_π^1 approaches $\underline{v} \equiv \{v | v = [1 - F(v)] / f(v)\} > 0$. Moreover, as k approaches \bar{v} , also v_π^1 approaches \bar{v} . From this we can conclude that entry will be interior (and described by equation (5)) for all $k \in (0, \bar{v})$.

I will now derive expressions for social surplus w_π^1 , private surplus π_π^1 and aggregate information rent, denoted u_π^1 . Following the steps of Samuelson (1985), we now exploit the fact that we need not condition on the actual number of bidders n , as n follows from the realization of values (v_1, \dots, v_N) . Therefore, social surplus can be written

$$w_\pi^1 = \int_{v_\pi^1}^{\bar{v}} v d [F(v)^N] - Nk [1 - F(v_\pi^1)] \quad (6)$$

Moreover, as individual expected utility can be written $E[u_i] = \int_{v_\pi^1}^{\bar{v}} [1 - F(v)] F(v)^{N-1} dv$, aggregate expected utility — aggregate information rent or u_π^1 — can be written

$$u_\pi^1 = NE[u_i] = N \int_{v_\pi^1}^{\bar{v}} [1 - F(v)] F(v)^{N-1} dv \quad (7)$$

Finally, as social surplus equals the sum of private surplus π_π^1 and aggregate information rent u_π^1 , private surplus can be written

$$\pi_\pi^1 = \int_{v_\pi^1}^{\bar{v}} v d [F(v)^N] - N \int_{v_\pi^1}^{\bar{v}} [1 - F(v)] F(v)^{N-1} dv - Nk [1 - F(v_\pi^1)] \quad (8)$$

2.2.2 The seller maximizes social surplus

When the auctioneer maximizes profit, we have seen that he imposes a strictly positive reservation price (cf. the preceding subsection), inducing too little entry from a social welfare point of view.⁹ Samuelson (1985) has shown that social welfare is maximized by a plain auction with a reservation price equal to the auctioneer's own value: $b = 0$. Since this is a simple mechanism, it should pose only minor problems to enforce a regulation saying that the reservation price should be zero. I will now study the ex post (i.e., after entry) effects of such a policy. (The before entry effects are discussed below.)

If $b = 0$ then, by the same logic as above, equilibrium entry — v_w^1 — must satisfy

$$v_w^1 F(v_w^1)^{N-1} = k \quad (9)$$

⁹It can be shown that if individual values are drawn from a uniform distribution on $[0, 1]$, then if $N = 2$ the equilibrium cutoff $v_\pi^1 = \frac{1}{4} + \frac{1}{4}\sqrt{1 + 8k} \geq \frac{1}{2}$ for all k . (See the appendix for details.) The auctioneer's preferred mechanism will induce entry in a fraction $1 - (v_\pi^1)^2 \leq \frac{3}{4}$ of cases, despite the fact that when the entry cost becomes very small, entry is socially profitable almost always.

Inspection reveals that as k approaches 0, v_w^1 approaches $\underline{v} = 0$. Moreover, as k approaches \bar{v} , also v_w^1 approaches \bar{v} . From this we can conclude that entry will be interior (and described by equation (9)) for all $k \in (0, \bar{v})$. Performing the same steps of calculus as in the preceding section, we get the following expressions for private and social surpluses as well as information rent in this case (the equations are equal to eqs. (6)-(8) except that the equilibrium cutoff differ – v_w^1 instead of v_π^1):

$$w_w^1 = \int_{v_w^1}^{\bar{v}} v d [F(v)^N] - Nk [1 - F(v_w^1)], \quad (10)$$

$$u_w^1 = N \int_{v_w^1}^{\bar{v}} [1 - F(v)] F(v)^{N-1} dv \quad (11)$$

$$\pi_w^1 = \int_{v_w^1}^{\bar{v}} v d [F(v)^N] - N \int_{v_w^1}^{\bar{v}} [1 - F(v)] F(v)^{N-1} dv - Nk [1 - F(v_w^1)] \quad (12)$$

3 Results

We are now ready to compare the outcomes in the three different cases characterized above. First we consider equilibrium entry.

Proposition 1 *Equilibrium entry may go up (for high levels of k) or down (for low values of k) as a consequence of early information.*

Proof: With no information before entry, the equilibrium probability of entry (that is, $1 - F(v^0)$) equals 1 iff $k < \underline{k}$ and 0 if $k > \bar{k}$. In both cases with information before entry we have that the equilibrium entry probability $1 - F(v^1) \in (0, 1)$ for any $k \in (0, \bar{v})$. This implies that early information increases entry for $k \in [\bar{k}, \bar{v})$ and reduces entry for $k \in (0, \underline{k})$. Q.E.D.

(For well-behaved distribution functions $F(\cdot)$ there will typically exist a number $\bar{k} \in (\underline{k}, \bar{k})$ such that early information increases entry for $k > \bar{k}$ and reduces entry if $k < \bar{k}$. The two cases of informed entry would then yield different values of \bar{k} .)

Proposition 2 *When bidders have early information, then regulating the auctioneer lead to increased entry.*

Proof: The result follows from the fact that the only difference between regulation and not is the reserve price. Given an arbitrary reserve price b , the equilibrium cutoff value v^1 is given by

$$[v^1 - b] F(v^1)^{N-1} \equiv k \quad (13)$$

This identity implicitly defines v^1 as a function of b , and straightforward differentiation (using the implicit-function rule) yields

$$\frac{dv^1}{db} = \frac{F(v^1)}{F(v^1) + [v^1 - b] N dF(v^1)} > 0 \quad (14)$$

Since regulation reduces the reserve price, the cutoff value is reduced and entry is increased. Q.E.D.

The intuition for this result is straightforward: The reason to use reserve prices is to reduce the bidders' expected rent, and this will make them more reluctant to enter.

The next result is obvious and therefore stated without proof:

Proposition 3 *Early information creates information rent on the hands of the bidders: $0 = u^0 < u_\pi^1 < u_w^1$.*

This rent drives a wedge between the social benefits of early information and the benefits that accrue to the principal.

Proposition 4 *For any $k \in [0, \hat{k})$, $\pi_w^1 < \pi_\pi^1 < w_\pi^1 < w_w^1$.*

Proof: First note that for any $k \in [0, \hat{k})$, $w_\pi^1 = \pi_\pi^1 + u_\pi^1 > \pi_\pi^1$. Moreover, regulation increases welfare and reduces profits, i.e., $\pi_w^1 < \pi_\pi^1$ and $w_\pi^1 < w_w^1$. Q.E.D.

Proposition 5 *When $k = 0$, $w_w^1 = w^0$.*

Proof: When $k = 0$, all prospective bidders enter whether the bidders have early information or not, as long as the seller is regulated (in the former case). After entry, the bidder with the highest value is always chosen. Consequently, efficiency is not affected by early information, while distribution certainly is. Q.E.D.

The following result indicates a strong link between equilibrium entry behavior and early information's effect on the seller's payoff:

Proposition 6 *If in the regulation regime entry increases as a consequence of early information (sufficient condition), the seller benefits from early information whether he is regulated or not. If entry decreases (necessary condition), the seller may lose.*

Proof: We start by noting that the auctioneer likes informed entrants – ceteris paribus – as they have their values drawn from a more favorable distribution than uninformed entrants (in a first order stochastic dominance sense). This implies that if the entry probability is unaffected by early information, the auctioneer will be better off with informed entrants. The same holds of course if early information increases entry. Also note that since the mechanism under informed entry screens high-valuation bidders, the auctioneer may benefit even if entry is somewhat reduced, hence the sufficiency but not necessity of increased entry.

Since increased entry is sufficient but not necessary for profits to increase, reduced entry must be necessary but not sufficient for profits to decrease. What remains is to point at a case in which profits actually go down as a result of early information. This is e.g. the case for $k = 0$, cf. the proof of Proposition 7. (By continuity it will also be the case in a neighborhood of $k = 0$.) Q.E.D.

Proposition 7 *If $F(\square)$ is twice differentiable (sufficient condition), then there exists a non-empty interval of entry costs (k_1, k_2) such that for any $k \in (k_1, k_2)$, early information is socially valuable whether the seller is regulated or not, but privately valuable iff the seller is not regulated.*

Proof: Proposition 4 implies that (using $\pi^0 = w^0$)

$$\left(\pi_w^1 - \pi^0\right) < \left(\pi_\pi^1 - \pi^0\right) < \left(w_\pi^1 - w^0\right) < \left(w_w^1 - w^0\right) \quad (15)$$

Consequently, early information is socially valuable whenever it is privately valuable, and all we need to do is to find values of k such that

$$\left(\pi_w^1 - \pi^0\right) < 0 < \left(\pi_\pi^1 - \pi^0\right) \quad (16)$$

A sufficient condition for the existence of such values of k is that i) both $(\pi_w^1 - \pi^0)$ and $(\pi_\pi^1 - \pi^0)$ are continuous function of k ; and ii) both $(\pi_w^1 - \pi^0)$ and $(\pi_\pi^1 - \pi^0)$ can

take on positive as well as negative values. Concerning i), since all three equilibrium cutoffs are continuous functions of k when $F(\cdot)$ is twice differentiable, so are all the expressions for profit, welfare and utility. Concerning ii), we know that for $k \in [\underline{k}, \bar{k}]$, both $(\pi_w^1 - \pi^0)$ and $(\pi_\pi^1 - \pi^0)$ are positive. Similarly, for $k = 0$, $(\pi_w^1 - \pi^0) < (\pi_\pi^1 - \pi^0) < (w_\pi^1 - w^0) < (w_w^1 - w^0) = 0$ (from Proposition 5), implying that both are negative. Q.E.D.

Corollary 8 *For any $k \in (k_1, k_2)$, regulation reduces welfare.*

4 Example

I will now illustrate some of my findings in an example in which v_i is drawn from a uniform distribution on the interval $[0, 1]$ and $N = 2$.¹⁰

First we plot the different cutoff values against k to get a picture of entry behavior in the three regimes:

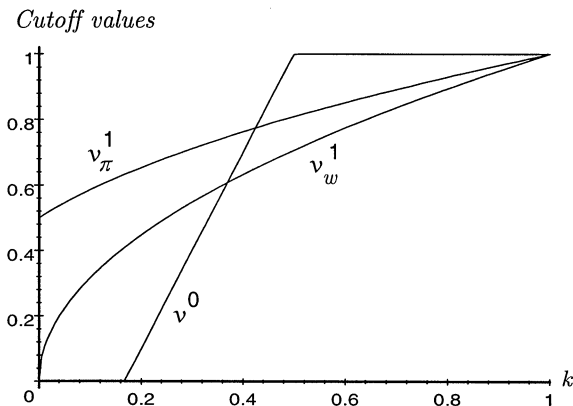


Figure 1: Entry behavior

Here we see clearly that i) early information increases the cutoff value (i.e. reduces entry) for low entry costs and decreases the cutoff (i.e. increases entry) for high entry cost (Proposition 1), and ii) when the potential bidders receive early information, then regulation of the seller encourages entry (Proposition 2).

¹⁰Increasingly complicated closed-form solutions to the three auction games can also be found for $N = 3$ and $N = 4$, while no such solutions can be found for higher numbers of potential entrants. Numerical solutions are easily found also for higher numbers. However, since the essentials do not change as N increases, only the technically simpler case is reported.

Next we plot expected profit in the three cases:

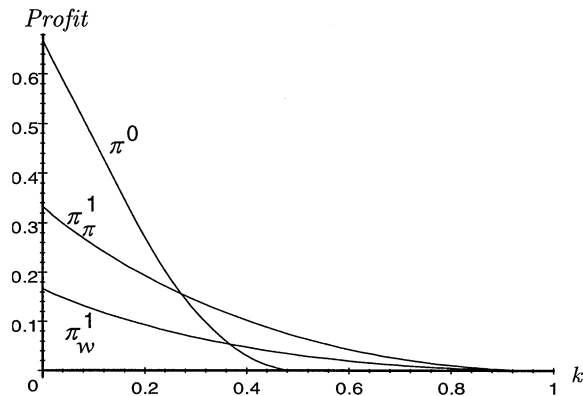


Figure 2: Profit

This picture essentially shows that early information is bad for the seller when entry costs are low, but good for the seller if the entry cost is high (cf. Proposition 6). It also shows the rather obvious result that early information is more beneficial to an unregulated seller than a regulated one.

Next we take a closer look at social surplus:

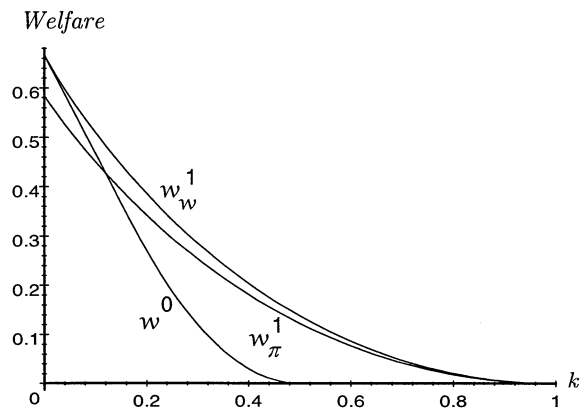


Figure 3: Social surplus

For obvious reasons, social welfare is decreasing in the entry cost. What is perhaps more interesting is to get pictures of the *changes* in information rent, and private and social surpluses as early information is provided. The next figure shows these

numbers for the case of an unregulated seller:

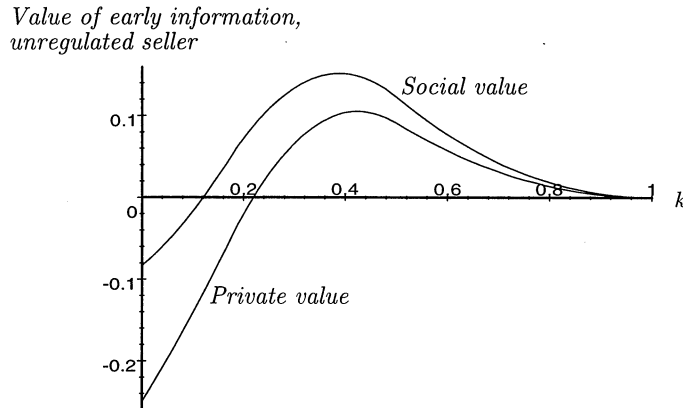


Figure 4: Value of early information (unregulated seller)

In contrast, if the seller is regulated, we get the following picture:

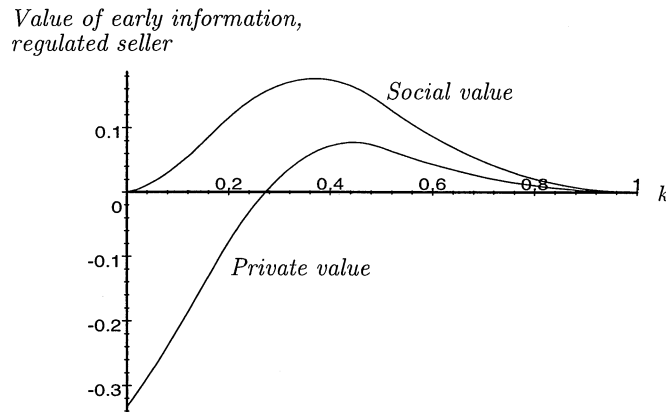


Figure 5: Value of early information (regulated seller)

From these last two figures we can identify a region of entry costs such that (i) early information is of social value, and (ii) early information is of private value iff the seller is not regulated, cf. Proposition 5. (This can be shown to be the case when $k \in (.22, .27)$.) Hence there is a tradeoff between the ex post concerns about efficiency and the ex ante concerns about the seller's incentives to provide early information.

5 Conclusions

This paper unifies two approaches to the modeling of auctions with entry. Samuelson (1985) develops a model where the prospective entrants knows their values before

they sink the entry investment, while Levin and Smith (1994) works with the opposite assumption: bidders have no private information before they enter. The starting point of the present paper is the fact that sometimes the seller can affect how much information the bidders have before entry – he can choose between a Samuelson world and a Levin-Smith world. At first glance one might suspect that it is in the seller’s interest to provide the best possible information for his buyers, but we have seen that this is not necessarily so. True, if the number of prospective bidders who actually enter is not affected by early information, then early information tend to improve the selection of bidders. Moreover, competition is intensified. However, increased competition is bad for entry. This affects profit negatively, and sometimes this effect is strong enough to dominate the benefits, leaving the seller with lower profit with informed bidders than if they were uninformed.

Second, if bidders have no information before entry, the seller extracts all information rent ex ante, and there is therefore no conflict between social welfare and profits. In contrast, when bidders receive early information, not all rent is dissipated. This drives a wedge between social surplus and profit. We have seen that regulating the seller is good for social welfare once early information has been provided, but it is bad for the seller’s incentives to provide early information. We have identified situations in which early information is of social value, but it is of private value only if the seller is not regulated. Hence, regulation is a two-edged sword in this environment.

6 Appendix: Example details

Suppose v_i is drawn from a uniform distribution on the interval $[0, 1]$ and that $N = 2$. Then, using (1) and (2) we find that

$$v^0 = \begin{cases} 0 & \text{if } k < \underline{k} = \int_0^1 (1-x)x^{2-1} dx = \frac{1}{6} \\ 1 & \text{if } k > \bar{k} = \int_0^1 (1-x) dx = \frac{1}{2} \\ -\frac{1}{2} + 3k & \text{if } k \in [\underline{k}, \bar{k}] = \left[\frac{1}{6}, \frac{1}{2}\right] \end{cases} \quad (17)$$

Then we use (3) to get

$$\pi^0 = w^0 = \begin{cases} \frac{2}{3} - 2k & \text{if } k < \frac{1}{6} \\ 0 & \text{if } k > \frac{1}{2} \\ \frac{3}{4} - 3k + 3k^2 & \text{if } k \in \left[\frac{1}{6}, \frac{1}{2}\right] \end{cases} \quad (18)$$

(We know that with mixed-strategy entry there will be no information rent, that is, $u^0 = 0$.)

Next we move to the cases of informed entry. If the auctioneer maximizes profit, we get (using (5)) $v_\pi^1 = \frac{1}{4} + \frac{1}{4}\sqrt{1+8k} \in [\frac{1}{2}, 1]$. In contrast, if the auctioneer maximizes social surplus, we get (using (9)) $v_w^1 = \sqrt{k} \in [0, 1]$. Then

$$w_w^1 = \int_{\sqrt{k}}^1 2v^2 dv - 2k(1 - \sqrt{k}) = \frac{2}{3} + \frac{4}{3}k^{\frac{3}{2}} - 2k \quad (19)$$

$$u_w^1 = 2 \int_{\sqrt{k}}^1 (1-v)v dv = \frac{1}{3} + \frac{2}{3}k^{\frac{3}{2}} - k \quad (20)$$

$$\pi_w^1 = \int_{\sqrt{k}}^1 2v^2 dv - 2 \int_{\sqrt{k}}^1 (1-v)v dv - 2k(1 - \sqrt{k}) = \frac{1}{3} + \frac{2}{3}k^{\frac{3}{2}} - k \quad (21)$$

and

$$\begin{aligned} w_\pi^1 &= \int_{\frac{1}{4} + \frac{1}{4}\sqrt{1+8k}}^1 2v^2 dv - 2k \left(1 - \left(\frac{1}{4} + \frac{1}{4}\sqrt{1+8k} \right) \right) \\ &= \frac{5}{8} - \frac{7}{4}k + \frac{10k-1}{24}\sqrt{1+8k} \end{aligned} \quad (22)$$

$$u_\pi^1 = 2 \int_{\frac{1}{4} + \frac{1}{4}\sqrt{1+8k}}^1 (1-v)v dv = \frac{1-k}{12} (3 - \sqrt{1+8k}) \quad (23)$$

$$\begin{aligned} \pi_\pi^1 &= \int_{\frac{1}{4} + \frac{1}{4}\sqrt{1+8k}}^1 2v^2 dv - 2 \int_{\frac{1}{4} + \frac{1}{4}\sqrt{1+8k}}^1 (1-v)v dv \\ &\quad - 2k \left(1 - \left(\frac{1}{4} + \frac{1}{4}\sqrt{1+8k} \right) \right) \\ &= \frac{3}{8} - \frac{3}{2}k + \frac{(8k+1)^{\frac{3}{2}}}{24} \end{aligned} \quad (24)$$

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