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**Can Exclusive Territories Limit
Strategic Location Downstream?**

by

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Abstract

Research on spatial price discrimination demonstrates that strategic (off center) location choices by downstream firms can increase downstream profit and reduce both the profit of an upstream monopoly and social welfare. This paper examines exclusive territories as a vertical control mechanism and shows that such territories can force downstream firms to return to the center of the market. Yet, exclusive territories cannot completely eliminate the influence of strategic downstream location - the profit maximizing exclusive territories are either too small or too large to be socially efficient.

(JEL L19, L22) [Keywords: strategic location, exclusive territories]

1. Introduction

Recent literature on spatial price discrimination has shown that strategic (off center) location choices by downstream firms can increase downstream profit and reduce both the profit of an upstream monopoly and social welfare. Gupta, Kats and Pal (1994) show that for a successive monopoly with delivered pricing, off center location doubles downstream profit, reduces upstream profit by one half and doubles the cost of serving the same market. This results from the downstream firm's move from the center to the corner of the market and the upstream firm's accommodation by lowering price (see figure 1). If the downstream market is characterized by Cournot competition, the strategic behavior and its effects are reduced, but not eliminated.¹ As a consequence of the strategic behavior downstream, an incentive for vertical integration emerges.

Despite a vast literature on vertical controls, relatively little research has been done in the context of spatial models. We introduce exclusive territories as a vertical control mechanism designed to counteract the strategic use of location downstream. As Blair and Kaserman (1983) emphasize, vertical controls can often substitute for full integration. Our model indicates that exclusive territories that maximize upstream profit can, indeed, force downstream firms to return to the center of the market.² Yet, establishing such territories cannot completely eliminate the influence of strategic downstream location. The exclusive territories that maximize upstream profit will either be too small or too large relative to the welfare maximizing size.

The paper is organized as follows: Section 2 sets forth the model in which we introduce exclusive territories and derives their welfare maximizing size. Section 3 solves the game. Section 4 discusses the results, highlighting the role of fixed cost downstream. Section 5 explores the consequences of endogenous downstream fixed cost and presents a unique

equilibrium. Section 6 concludes the paper. A variety of technical proofs are relegated to an appendix available upon request.

2. Structure of the game and optimal exclusive territories

An upstream monopoly supplies an input to identical downstream firms with fixed proportion technology and constant marginal cost. Without loss of generality, we assume that one unit of input is needed for each unit of output and that marginal cost is zero. Including other factors of production will not change the point we wish to make. Each downstream firm has a fixed cost F and transportation cost which increases linearly in distance at rate t .³

The final market for the product is of unit length with consumers uniformly distributed along the line segment. Each consumer has a perfectly inelastic demand for one unit at reservation price r .

We imagine that the upstream monopoly determines the exclusive territory size, m , for each downstream firm. Further, we assume that the number of downstream firms, $\frac{1}{m}$, is an element of \mathbb{N}^+ eliminating the possibility of any residual market.⁴

We solve a four stage game. In stage one, the upstream firm determines the size of the exclusive territory, m , and hence how many firms will be serving the market. In stage two, each downstream firm chooses location, α , within its territory. We assume that each downstream firm locates in the range $0 \leq \alpha \leq \frac{m}{2}$. The prohibition of competition across territories and symmetry insures no loss of generality. We assume that costs of relocation are such that location cannot be altered for the period of the game. In stage three, the upstream firm determines the price, P_u , to the downstream firms. In stage four, each downstream firm determines the delivered price

to the consumers, P_d .⁵ Our assumption of delivered pricing follows from the earlier literature and from the contention by Thisse and Vives (1988) that firms will adopt such pricing whenever possible. The game is solved for one firm and generalized. The sequence of the game is appropriate. The downstream firm needs to know its territory before it can make a rational location decision. Prices are set later as they can be more easily altered than location.

Before solving the game, we derive the socially optimal exclusive territory size by assuming that strategic behavior is absent and each downstream firm cost minimizes by locating at the center of its territory.

The aggregate costs (fixed plus transport) are $\frac{1}{m} \left(\frac{m^2 t}{4} + F \right)$. Minimizing with respect to m yields the socially optimal size as a function of F and t :

$$(1) \quad m^* = 2\sqrt{\frac{F}{t}}$$

As can be seen by equation (1), the greater the fixed cost, the larger the optimal territory. On the other hand, as transportation cost increases the optimal size decreases. Territories of size m^* become the benchmark against which those from our model will be compared.

3. The relationship between territory size and fixed cost

We now solve the game through backward induction, starting with the downstream delivered price schedule. Upstream price, location and territory size are taken as given by the downstream firm although they are endogenous variables at earlier stages in the game. However, reservation price, transportation cost and fixed cost remain exogenous at all stages of the game.

Proposition 1. The delivered price schedule downstream is: $P_d(P_u, \alpha, m; r, t, F) = r$ if $r \geq P_u + |x - \alpha|t$ where x is the location of the consumer. If $r < P_u + |x - \alpha|t$, consumers will not be served.

As consumer demand is perfectly inelastic, the optimal price will be r provided it covers the costs of serving the consumer (Hurter and Lederer, 1986).

Next, we determine the upstream price schedule by maximizing upstream profit. There are four continuous segments of the upstream profit function, yielding four segments to the price schedule, as shown in proposition 2.

Proposition 2. The upstream price schedule is:

- (i) $\hat{P}_u(\alpha, m; r, t, F) = r - (m - \alpha)t$ if $\max\left[0, 2m - \frac{r}{t}\right] \leq \alpha \leq \frac{m}{2}$
- (ii) $\hat{P}_u(\alpha, m; r, t, F) = \frac{r + \alpha t}{2}$ if $0 < \alpha < \min\left[2m - \frac{r}{t}, \frac{r}{3t}, \frac{m}{2}\right]$
- (iii) $\hat{P}_u(\alpha, m; r, t, F) = r - \alpha t$ if $\frac{r}{3t} \leq \alpha \leq \min\left[\frac{r}{2t}, \frac{m}{2}\right]$
- (iv) $\hat{P}_u(\alpha, m; r, t, F) = \frac{r}{2}$ if $\frac{r}{2t} < \alpha \leq \frac{m}{2}$

The upstream price depends on downstream location as expressed by the non-overlapping boundary conditions listed after it. The boundary conditions are expressed in terms of the endogenous variable of the preceding stages.

Depending on upstream price, all or part of the exclusive territory will be served. As P_u rises, increasing portions of the territory eventually go unserved. The territory is fully served only if $P_u \leq r - (m - \alpha)t$. Next, the territory is cut from one side if $r - (m - \alpha)t < P_u < r - \alpha t$ (the downstream firm

is located off center), and is fully cut from one side if $P_u = r - \alpha t$.

Finally, the territory is cut from two sides if $r - \alpha t < P_u \leq r$.⁶

As the upstream price rises, the firm moves through the four segments of the profit function. To derive the complete price schedule in proposition 2, we first derive the optimal price within each segment. Then we determine the boundary conditions for when each segment of the schedule maximizes profit over all segments. We do this by evaluating the shape of the profit function.

Proof of Proposition 2 (i)

The upstream firm will increase price as long as the entire territory remains served. This segment of the profit function is:

$$(2) \quad \pi_u[P_u \leq r - (m - \alpha)t] = P_u m$$

The optimal price is the upper bound, $\hat{P}_u(\alpha, m, r, t, F) = r - (m - \alpha)t$. This price maximizes upstream profit over all segments if profit declines as P_u is further increased.

The profit in the next segment is $\pi_u[r - (m - \alpha)t < P_u < r - \alpha t] = P_u \left[\alpha + \frac{r - P_u}{t} \right]$. The first derivative will be non-positive at the continuous transition point between the segments, $P_u = r - (m - \alpha)t$, when:

$$(3) \quad \alpha \geq 2m - \frac{r}{t}$$

which yields the boundary condition. The boundary condition in (3) is increasing in m and t , but decreasing in r . This concludes the proof of proposition 2 (i).⁷

Next we solve for the downstream location schedule.

Proposition 3. *The downstream firm will locate within its territory as follows:*

$$\begin{aligned}
 \text{(i)} \quad \hat{\alpha}(m,r,t,F) &= \max\left[0, 2m - \frac{r}{t}\right] & \text{if} \quad 0 \leq m \leq \min\left[\frac{4r}{7t}, 1\right] \\
 \text{(ii)} \quad \hat{\alpha}(m,r,t,F) &= \frac{r}{7t} & \text{if} \quad \frac{4r}{7t} < m < \min\left[\sqrt{\frac{4}{7}}\left(\frac{r}{t}\right), 1\right] \\
 \text{(iii)} \quad \hat{\alpha}(m,r,t,F) &= \frac{m}{2} & \text{if} \quad \sqrt{\frac{4}{7}}\left(\frac{r}{t}\right) \leq m \leq \min\left[\frac{r}{t}, 1\right] \\
 \text{(iv)} \quad \frac{r}{2t} < \hat{\alpha}(m,r,t,F) &\leq \frac{m}{2} & \text{if} \quad \frac{r}{t} < m \leq 1
 \end{aligned}$$

The choice of location is related to upstream territory size as expressed by the boundary conditions.

Proof of Proposition 3 (i)

The downstream profit function under (i) is given by:

$$(4) \quad \pi_d \left\{ \max\left[0, 2m - \frac{r}{t}\right] \leq \alpha \leq \frac{m}{2} \right\} = (r - P_u)\alpha - \frac{\alpha^2 t}{2} + \frac{(r - P_u)^2}{2t} - F = \frac{m^2 t}{2} - \alpha^2 t - F$$

The optimal location within this segment is the lower bound,

$$\hat{\alpha}(m,r,t,F) = \max\left[0, 2m - \frac{r}{t}\right]. \text{ For this solution to maximize profit over all}$$

segments, it is sufficient that profit declines as the downstream firm

moves to the left of $\alpha = 2m - \frac{r}{t}$ and the territory becomes cut from one side.

The profit in the next segment is $\pi_d \left\{ 0 < \alpha < \min \left[2m - \frac{r}{t}, \frac{r}{3t}, \frac{m}{2} \right] \right\} = \frac{r^2 + 2\alpha tr - 7\alpha^2 t^2}{8t}$.

The first derivative is non-negative for $\alpha = 2m - \frac{r}{t}$ when:

$$(5) \quad m \leq \frac{4r}{7t}$$

which yields the boundary condition for segment (i) of the location schedule. This concludes the proof of proposition 3 (i).⁸

Each downstream location choice in proposition 3 generates a separate segment of the final stage upstream profit function.

Proposition 4. *The upstream monopoly will set exclusive territory size:*

$$(i) \quad \hat{n}(r, t, F) = \sqrt{\frac{2F}{t}} \quad \text{if} \quad 0 \leq F \leq \min \left[\frac{r^2}{14t}, \frac{t}{2} \right]$$

$$(ii) \quad \hat{n}(r, t, F) = \emptyset$$

$$(iii) \quad \hat{n}(r, t, F) = \max \left[\sqrt{\frac{4}{7} \frac{r}{t}}, 2\sqrt{\frac{F}{t}} \right] \quad \text{if} \quad \frac{r^2}{14t} \leq F \leq \min \left[\frac{r^2}{4t}, \frac{t}{4} \right]$$

$$(iv) \quad \hat{n}(r, t, F) = \emptyset$$

Each of the four segments in proposition 4 corresponds to the respective price schedule segments of proposition 2 and location choices of

proposition 3. The transition in (iii) between $\sqrt{\frac{4}{7} \left(\frac{r}{t} \right)}$ and $2\sqrt{\frac{F}{t}}$ is

continuous and occurs at $F = \frac{r^2}{7t}$. Further, the optimal solutions for

segments (ii) and (iv) are dominated by the other segments and will never be chosen.⁹

Proof of Proposition 4 (i)

Upstream profit is:

$$(6) \quad \Pi_u = \frac{1}{m}(P_u s)$$

where s is share of territory served by each downstream firm. When the territory is fully served, $s=m$. Recall that the downstream firm locates at $\max\left[0, 2m - \frac{r}{t}\right]$. This yields two profit functions associated with these two possible locations:

$$(7a) \quad \Pi_u\left(\alpha = 0, 0 \leq m \leq \min\left[\frac{r}{2t}, 1\right]\right) = r - mt$$

$$(7b) \quad \Pi_u\left(\alpha = 2m - \frac{r}{t}, \frac{r}{2t} < m \leq \min\left[\frac{4r}{7t}, 1\right]\right) = mt$$

For the first function, the optimal territory will be the smallest which allows the downstream firm to cover costs. We find this by solving (8) for m .

$$(8) \quad \pi_d\left(\alpha = 0, 0 \leq m \leq \left[\frac{r}{2t}, 1\right]\right) = \frac{m^2 t}{2} - F \geq 0$$

$$m \geq \sqrt{\frac{2F}{t}}$$

Thus, the optimal territory size is $\hat{m} = \sqrt{\frac{2F}{t}}$. For the second function, the optimal territory size is the upper bound, $\hat{m} = \frac{4r}{7t}$, which will never be chosen. Next, we determine the boundary condition.

As the profit function over all segments has more than one maximum, we derive the boundary condition from the upper envelope of the optimal profit segments with respect to F . The optimal profit for segment (i) is derived by inserting the optimal territory sizes back into the profit functions in (7) yielding two new optimal profit functions. The range for each optimal profit function is limited by the boundary conditions on m in (7) and the non-negative downstream profit constraint. This yields the following range on F for the first optimal profit function:

$$(9) \quad \text{For } \hat{m} = \sqrt{\frac{2F}{t}} :$$

$$0 \leq \sqrt{\frac{2F}{t}} \leq \min\left[\frac{r}{2t}, 1\right]$$

$$0 \leq F \leq \min\left[\frac{r^2}{8t}, \frac{t}{2}\right]$$

and for the second:

$$(10) \quad \text{For } \hat{m} = \frac{4r}{7t} :$$

$$\pi_d\left(m = \frac{4r}{7t}\right) = \frac{r^2}{7t} - F$$

$$\frac{r^2}{7t} - F \geq 0, \text{ or } F \leq \frac{r^2}{7t}$$

The optimal profit for segments (ii) through (iv) are derived analogously. Expressing them in terms of optimal territory size and range, we have:

$$(11a) \quad (ia) \quad \Pi_u \left(m = \sqrt{\frac{2F}{t}}, 0 \leq F \leq \min \left[\frac{r^2}{8t}, \frac{t}{2} \right] \right) = r - \sqrt{2Ft}$$

$$(11b) \quad (iia) \quad \Pi_u \left(m = \sqrt{\frac{4}{7}} \left(\frac{r}{t} \right), 0 \leq F \leq \frac{r^2}{7t} \wedge r \leq \sqrt{\frac{7}{4}t} \right) = \left(1 - \frac{1}{\sqrt{7}} \right) r$$

$$(11c) \quad (iiib) \quad \Pi_u \left(m = 2\sqrt{\frac{F}{t}}, \frac{r^2}{7t} \leq F \leq \min \left[\frac{r^2}{4t}, \frac{t}{4} \right] \right) = r - \sqrt{Ft}$$

Segments (ib), (ii), and (iv) are dominated by the remaining segments and will never be chosen.

We trace the upper envelope as F increases from segment (ia) into (iia) to (iiib). Solving the following inequality for F , yields the boundary condition for proposition 4 (i):

$$(12) \quad r - \sqrt{2Ft} \geq \left(1 - \frac{1}{\sqrt{7}} \right) r$$

$$F \leq \frac{r^2}{14t}$$

This completes the proof of proposition 4 (i).

The schedule of optimal territories for the upstream firm is shown by the upper envelope of the profit segments (i) through (iv) in figure 1. As can be seen, part of segment (i) and all of segments (ii) and (iv) are dominated by (iii). This completes the four stage backward induction and the development of the propositions. In the next section we will emphasize the two types of equilibria that exist.

4. Exclusive territories and vertical control

In this section, we discuss the implications of the exclusive territories which maximize upstream profit in a series of corollaries.

Corollary 1. *The upstream monopoly chooses territory size and upstream price such that each territory, and therefore the entire market, will always be fully served.*

This result has some intuitive appeal and differs from that for successive monopolies by Gupta, Kats and Pal (1994) in which portions of the downstream market sometimes go unserved. The upstream monopoly in our case either decreases or increases territory size depending on the downstream fixed cost. As it alters the size, it changes the number of downstream firms to ensure that the entire market is served.

For successive monopolies, portions of the market go unserved if the gain to the upstream monopoly from raising price is greater than the loss from cutting the market. When the market size of the downstream firm is given, this makes sense. However, in our case, the upstream monopoly adjusts the territory size to fit the portion of the downstream market which is being served. This territory size is then replicated for adjacent territories, until the entire market is served.

Corollary 2. *Exclusive territories are chosen such that the downstream firm locates either at the corner or in the center.*

A range of territory sizes does exist for which the downstream firms would choose to locate somewhere between the corner and the center, but these are not chosen by the upstream firm. If the territory size is made small enough, the downstream firm locates at the corner. Alternatively, if the territory size is made large enough, the downstream firm locates in the

center. For both cases, the upstream price is higher than for any size territory that generates a downstream location between the corner and the center.

The upstream monopoly accepts the strategic corner location downstream provided the fixed cost is sufficiently small. Specifically, it is necessary that $F < \frac{r^2}{14t}$ for the upstream monopoly to choose a small territory size as can be seen from proposition 4. If $F > \frac{r^2}{14t}$, the upstream monopoly can maintain a higher price by significantly increasing territory size thereby eliminating strategic behavior downstream. If fixed cost is $F = \frac{r^2}{14t}$, the upstream monopoly will be indifferent between choosing a small territory in segment (i), or doubling territory size according to segment (iii).

From a social perspective, the larger territory is superior as the sum of the aggregate fixed cost and transportation cost is less. This can be shown by comparing total costs, Ω , for the two territory sizes.

$$(14) \quad \Omega_i = \frac{F}{m_i} + \frac{m_i t}{2} \quad \text{in segment (i)}$$

$$\Omega_{iii} = \frac{F}{m_{iii}} + \frac{m_{iii} t}{4} \quad \text{in segment (iii)}$$

where $m_i = \sqrt{\frac{2F}{t}}$, $m_{iii} = \sqrt{\frac{4}{7} \left(\frac{r}{t} \right)}$ and $F = \frac{r^2}{14t}$. Taking the difference, we get:

$$(15) \quad \Omega_i - \Omega_{iii} = \frac{\sqrt{7}r}{28} > 0$$

Corollary 3. *The exclusive territory size chosen by an upstream monopoly will generally be either too small or too large to be socially optimal.*

For relatively small values of fixed cost downstream, $0 \leq F \leq \frac{r^2}{14t}$, the

upstream monopolist will set territory size to $\hat{m} = \sqrt{\frac{2F}{t}}$ as seen in proposition 4 (i). This territory size just allows the downstream firm to cover costs. However, the downstream firm locates strategically at the corner. When compared to what is socially optimal, this territory size is too small.

$$(16) \quad m^* = 2\sqrt{\frac{F}{t}} > \sqrt{\frac{2F}{t}} = \hat{m}$$

For fixed costs of $\frac{r^2}{14t} \leq F \leq \frac{r^2}{7t}$ it will be optimal for the upstream firm

to significantly increase the exclusive territory size to $\hat{m} = \sqrt{\frac{4}{7}} \left(\frac{r}{t} \right)$ while reducing the number of downstream firms accordingly. This results in a location in the center and allows the upstream monopoly to maintain its profit level as seen in figure 1, instead of making incremental increases in territory size combined with price concessions. Thus, for relatively large fixed costs downstream, the upstream firm chooses a large territory size and the downstream firm locates in the center. However, in this case the territories will be too large relative to the socially optimal size as seen in (17).

$$(17) \quad m^* = 2\sqrt{\frac{F}{t}} < \sqrt{\frac{4}{7}} \left(\frac{r}{t} \right) = \hat{m}$$

Only for very large fixed costs of $\frac{r^2}{7t} \leq F \leq \frac{r^2}{4t}$ will the socially optimal territory size result. As fixed cost becomes greater than $\frac{r^2}{7t}$, the upstream firm will again need to increase territory size to allow each downstream firm to just cover costs. Given that the downstream firm already locates in the center, we now have a condition where the socially optimal territory size results and aggregate costs are minimized.

In the next section, we discuss the implications of endogenous downstream fixed cost.

5. Endogenous downstream fixed cost and a unique equilibrium

In this section, we allow fixed cost downstream to be endogenous. This extension seems realistic as in the longer run a firm could obtain some control over its fixed cost through its choice of technology. In essence, it faces a tradeoff between fixed and variable costs when making investments (Heywood and Pal, 1996a). This analysis shows that downstream firms can make strategic use of fixed cost to enhance profit and it yields a unique equilibrium.

Proposition 5. If fixed cost downstream is endogenous, a unique equilibrium will result for which the downstream firm earns a positive profit, but does not locate strategically.

To determine the optimal fixed cost downstream, we extend the backward induction by an additional stage. Thus, the very first move is that of the downstream firm choosing fixed cost, F . The solution to this stage can be obtained based on information already available.

As the downstream profit is always zero for $\hat{m} = \sqrt{\frac{2F}{t}}$ and $\hat{m} = 2\sqrt{\frac{F}{t}}$,

downstream fixed cost will never be chosen such that $0 \leq F \leq \frac{r^2}{14t}$ or

$\frac{r^2}{7t} \leq F \leq \frac{r^2}{4t}$. One possible range remains; that of $\frac{r^2}{14t} \leq F \leq \frac{r^2}{7t}$ for which

$\hat{m} = \sqrt{\frac{4}{7}}\left(\frac{r}{t}\right)$. Downstream profit for this range is given by (18).

$$(18) \quad \pi_d = \frac{m^2 t}{4} - F$$

As territory size is fixed, any increase in fixed cost will reduce profit at unit rate, and the profit maximizing choice of fixed cost

downstream is thus $F^* = \frac{r^2}{14t}$. This yields a downstream profit of:

$$(19) \quad \pi_d(F^*) = \frac{r^2}{7t} - \frac{r^2}{14t} = \frac{r^2}{14t}$$

Given the full set of propositions, the following unique equilibrium emerges:

$$(20) \quad F^* = \frac{r^2}{14t}, \quad m^* = \sqrt{\frac{4}{7}}\left(\frac{r}{t}\right), \quad \alpha^* = \sqrt{\frac{1}{7}}\left(\frac{r}{t}\right), \quad P_u^* = \left(1 - \sqrt{\frac{1}{7}}\right)r, \quad P_d^* = r$$

The downstream firm chooses F^* to maximize profit by just forcing the upstream monopoly to set the territory size sufficiently large that the downstream firm locates in the center. For any fixed cost within the range

$\frac{r^2}{14t} \leq F < \frac{r^2}{7t}$, the downstream firm would still earn a positive profit, whereas for any other fixed cost economic profit downstream will be zero.

6. Conclusion

Previous research has demonstrated the presence of downstream strategic location in spatial price discrimination models. This paper studies the impact of exclusive territories on such strategic behavior in the presence of downstream linear transportation cost and fixed cost.

We find that for a small fixed cost, the upstream monopoly accepts the presence of strategic behavior, but adopts a territory size which just allows the downstream firm to cover costs. For a large fixed cost, territory size is increased and the downstream firm locates in the center. In this case, profit can be earned downstream. The entire market will be served in either case. However, the territory size will generally be either too small or too large to be socially optimal, implying that there always remains an incentive for vertical integration. Exclusive territories are not a substitute.

If fixed cost is endogenous, the result is a unique equilibrium with no strategic downstream location. The downstream firm retains a positive profit. However, the territory size remains too large to be socially optimal and an incentive for vertical integration still exists.

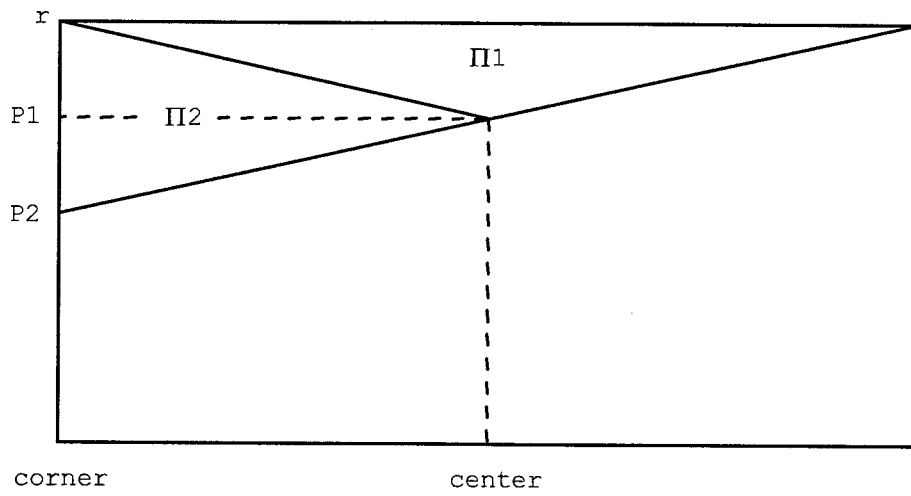


Figure 1. Strategic behavior downstream. The firm locates at the corner, increasing profit by $\Pi2$ as the upstream firm reduces price from $P1$ to $P2$.

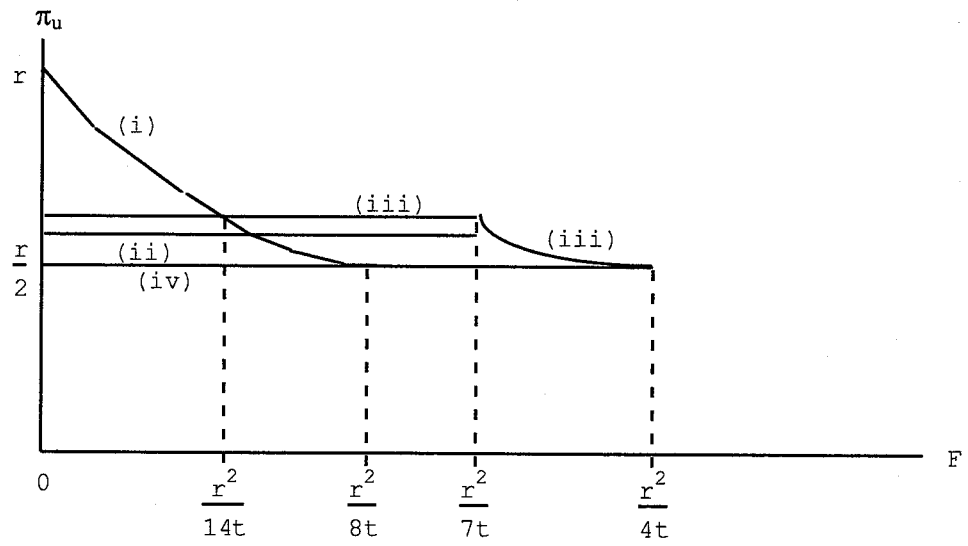


Figure 2. Upstream profit versus downstream fixed cost.

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Endnotes

¹ Location can be a strategic variable in other contexts. For example, Gupta, Heywood and Pal (1997) find that for horizontal mergers, the expectation of acquiring a competitor will induce a firm to locate closer to the target to reduce profit and thereby lowering the acquisition price. Heywood and Pal (1996b) show that an output tax will make a monopolist locate inefficiently to avoid profit losses.

² Absent the upstream monopoly the downstream firms would adopt the center of any market (Hurter and Lederer, 1986).

³ Absent a fixed cost, it would be optimal for the upstream firm to have an infinite number of downstream firms, essentially eliminating transportation cost, and charge the reservation price while serving the entire market.

⁴ As m becomes smaller, this should have a decreasing impact on the conclusions of the model. Maximum residual is $m - \varepsilon$, where ε is some small, positive number. Hence, as $m \rightarrow 0$, so will the residual.

⁵ We exclude two-part pricing by the upstream firm. If the firm could charge both franchise fee and per unit price, it might generate optimal behavior. There are, however, practical reasons to suspect that such franchise fees may not be feasible (see Tirole, 1988, p. 176-82).

⁶ For certain combinations of t and m , one or more of these strategies may be unattainable to the upstream firm. For example, it may not be possible to serve all of a large territory if transportation cost is also large.

⁷ The proofs for proposition 2 (ii) through (iv) are found analogously. Proofs for (ii) through (iv) for this and subsequent propositions are available from the authors.

⁸ The proofs of proposition 3 (ii) through (iv) follow the same procedure as the subsequent proof of proposition 4 (i) due to the presence of more than one maximum over these segments.

⁹ For $F > \frac{r^2}{4t}$, downstream profit would be negative thereby forcing the downstream firms to close and no market would be served.

Can Exclusive Territories Limit Strategic Location Downstream?

Reviewers' Appendix

Proof of Proposition 2 (ii)-(iv)

(ii) Upstream profit from a single territory when cut from one side:

$$\pi_u[r - (m - \alpha)t < P_u < r - \alpha] = P_u \left(\alpha + \frac{r - P_u}{t} \right)$$

Maximizing profit:

$$\frac{\partial \pi_u}{\partial P_u} = \alpha + \frac{r - 2P_u}{t} = 0, \text{ implying } \hat{P}_u = \frac{r + \alpha}{2}. \text{ Note that: } \frac{\partial^2 \pi_u}{\partial P_u^2} = \frac{-2}{t} < 0$$

Boundary condition; lower bound:

$$\left. \frac{\partial \pi_u[r - (m - \alpha)t < P_u < r - \alpha]}{\partial P_u} \right|_{P_u = r - (m - \alpha)t} \geq 0$$

$$\alpha + \frac{r - 2[r - (m - \alpha)t]}{t} \geq 0$$

$$\alpha \leq 2m - \frac{r}{t}$$

Boundary condition; upper bound:

$$\pi_u[r - \alpha \leq P_u \leq r] = 2P_u \frac{r - P_u}{t}$$

$$\left. \frac{\partial \pi_u[r - \alpha \leq P_u \leq r]}{\partial P_u} \right|_{P_u = r - \alpha} \leq 0$$

$$\alpha + \frac{r - 2(r - \alpha)}{t} \leq 0$$

$$\alpha \leq \frac{r}{3t}$$

(iii) When territory is fully cut from one side, $\hat{P}_u = r - \alpha$.

Boundary condition; lower bound:

$$\left. \frac{\partial \pi_u [r - (m - \alpha)t < P_u < r - \alpha t]}{\partial P_u} \right|_{P_u = r - \alpha t} \geq 0$$

$$\alpha + \frac{r - 2(r - \alpha t)}{t} \geq 0$$

$$\alpha \geq \frac{r}{3t}$$

Boundary condition; upper bound:

$$\left. \frac{\partial \pi_u [r - \alpha t \leq P_u \leq r]}{\partial P_u} \right|_{P_u = r - \alpha t} \leq 0$$

$$2 \frac{r - 2(r - \alpha t)}{t} \leq 0$$

$$\alpha \leq \frac{r}{2t}$$

(iv) Upstream profit from a single territory when cut from two sides:

$$\pi_u [r - \alpha t \leq P_u \leq r] = 2P_u \frac{r - P_u}{t}$$

$$\frac{\partial \pi_u [r - \alpha t \leq P_u \leq r]}{\partial P_u} = 2 \frac{r - 2P_u}{t} = 0$$

$$\hat{P}_u = \frac{r}{2}$$

Note that: $\frac{\partial^2 \pi_u}{\partial P_u^2} = \frac{-4}{t} < 0$

Boundary condition:

$$\left. \frac{\partial \pi_u [r - \alpha t \leq P_u \leq r]}{\partial P_u} \right|_{P_u = r - \alpha t} \geq 0$$

$$2 \frac{r - 2(r - \alpha t)}{t} \geq 0$$

$$\alpha \geq \frac{r}{2t}$$

Proof of Proposition 3 (ii)-(iv)

(ii) Downstream profit when territory is cut from one side:

$$\pi_d = \left\{ 0 \leq \alpha < \min \left[2m - \frac{r}{t}, \frac{r}{3t}, \frac{r}{2t} \right] \right\} = (r - P_u)\alpha - \frac{\alpha^2 t}{2} + \frac{(r - P_u)^2}{2t} - F$$

$$\text{where } P_u = \frac{r + \alpha t}{2}$$

Maximizing profit:

$$\frac{\partial \pi_d}{\partial \alpha} = \frac{r - 7\alpha t}{4} = 0$$

$$\hat{\alpha} = \frac{r}{7t}$$

$$\text{Note that: } \frac{\partial^2 \pi_d}{\partial \alpha^2} = \frac{-4}{t} < 0$$

Boundary condition: (For the remaining proofs, profit over all segments has more than one maximum, and we determine the boundary conditions by tracing the upper envelope of the optimal profit function. We obtain the optimal profit function by inserting the optimal solution back into the relevant profit function, but first we determine the range for the optimal profit segment.) The range of segment (ii) is:

$$\text{For } \hat{\alpha} = \frac{r}{7t}$$

$$0 \leq \hat{\alpha} < \min \left[2m - \frac{r}{t}, \frac{r}{3t}, \frac{r}{2t} \right]$$

This implies:

$$\frac{r}{7t} \leq 2m - \frac{r}{t}, \quad \frac{r}{7t} \leq \frac{r}{3t}, \quad \frac{r}{7t} \leq \frac{r}{2t}$$

$$\text{Also: } \frac{r}{7t} \leq \frac{m}{2}$$

Solving for territory size yields:

$$m \geq \frac{4r}{7t}, \quad \text{true } \forall m, \quad \text{true } \forall m$$

$$\text{Also: } m \geq \frac{2r}{7t}$$

Thus, the range is:

$$m \geq \frac{4r}{7t}$$

(iii) When territory is cut fully from one side:

$$\pi_d \left\{ \frac{r}{3t} \leq \alpha \leq \min \left[\frac{r}{2t}, \frac{m}{2} \right] \right\} = \frac{(r - P_u)^2}{t} - F = \alpha^2 t - F \quad \text{where } P_u = r - \alpha t.$$

$$\frac{\partial \pi_d}{\partial \alpha} = 2\alpha t > 0$$

$$\hat{\alpha} = \min \left[\frac{r}{2t}, \frac{m}{2} \right] \quad \text{Note that: } \frac{\partial^2 \pi_d}{\partial \alpha^2} = 2t > 0$$

Range:

$$\frac{r}{3t} \leq \frac{m}{2}, \text{ or } m \geq \frac{2r}{3t}$$

(iv) When territory is cut from both sides:

$$\pi_d \left(\frac{r}{2t} < \alpha \leq \frac{m}{2} \right) = \frac{(r - P_u)^2}{t} - F = \frac{r^2}{4t} - F \quad \text{where } P_u = \frac{r}{2}.$$

Range:

$$\frac{r}{2t} < \hat{\alpha} \leq \frac{m}{2}$$

$$\frac{m}{2} > \frac{r}{2t}, \text{ or } m > \frac{r}{t}$$

Optimal profit in terms of optimal locations and ranges:

$$(ia) \quad \pi_d \left(\alpha = 0, 0 \leq m \leq \min \left[\frac{r}{2t}, 1 \right] \right) = \frac{m^2 t}{2} - F$$

$$(ib) \quad \pi_d \left(\alpha = 2m - \frac{r}{t}, \frac{r}{2t} \leq m \leq \min \left[\frac{2r}{3t}, 1 \right] \right) = \frac{8mrt - 2r^2 - 7m^2 t^2}{2t} - F$$

$$(ii) \quad \pi_d \left(\alpha = \frac{r}{7t}, \frac{4r}{7t} < m \leq 1 \right) = \frac{r^2}{7t} - F$$

$$(iii) \quad \pi_d \left(\alpha = \frac{m}{2}, \frac{2r}{3t} \leq m \leq \min \left[\frac{r}{t}, 1 \right] \right) = \frac{m^2 t}{4} - F$$

$$(iv) \quad \pi_d \left(\frac{r}{2t} < \alpha \leq \frac{m}{2}, \frac{r}{t} < m \leq 1 \right) = \frac{r^2}{4t} - F$$

Determining shape and critical values of each segment:

(ia) Shape:

$$\frac{\partial \pi_d \left(\alpha = 0, \frac{2r}{3t} \leq m \leq \frac{r}{t} \right)}{\partial m} = mt > 0 \quad \text{Increasing}$$

$$\frac{\partial^2 \pi_d \left(\alpha = 0, \frac{2r}{3t} \leq m \leq \frac{r}{t} \right)}{\partial m^2} = t > 0 \quad \text{Convex}$$

Critical values:

$$\pi_d(\alpha = 0, m = 0) = 0$$

$$\pi_d \left(\alpha = 0, m = \frac{r}{2t} \right) = \frac{r^2}{8t} - F$$

$$\pi_d \left(\alpha = 2m - \frac{r}{t}, m = \frac{r}{2t} \right) = \frac{r^2}{8t} - F$$

(ib) Shape:

$$\frac{\partial \pi_d \left(\alpha = 2m - \frac{r}{t}, \frac{r}{2t} \leq m \leq \frac{2r}{3t} \right)}{\partial m} = 0$$

$$\frac{1}{2t} (8rt - 14mt^2) = 0$$

$$m = \frac{4r}{7t}$$

$$\frac{\partial^2 \pi_d \left(\alpha = 2m - \frac{r}{t}, \frac{r}{2t} \leq m \leq \frac{2r}{3t} \right)}{\partial m} = -14t < 0 \quad \text{Concave (maximum at } m = \frac{4r}{7t} \text{)}$$

Critical values:

$$\pi_d \left(\alpha = 2m - \frac{r}{t}, m = \frac{r}{2t} \right) = \frac{r^2}{8t} - F$$

$$\pi_d \left(\alpha = 2m - \frac{r}{t}, m = \frac{4r}{7t} \right) = \frac{r^2}{7t} - F$$

$$\pi_d \left(\alpha = 2m - \frac{r}{t}, m = \frac{2r}{3t} \right) = \frac{r^2}{9t} - F$$

Thus we have continuous transition between (ia) and (ib). Also, the maximum profit of segment (ib) is equal to the profit of segment (ii).

Thus, (ib) is dominated by (ii) for the range $\frac{4r}{7t} < m \leq \frac{2r}{3t}$ and the boundary

condition in proposition 3 (i) is $0 \leq m \leq \frac{4r}{7t}$.

(ii) Fixed profit.

(iii) Shape:

$$\frac{\partial \pi_d}{\partial m} \left(\alpha = \frac{m}{2}, \frac{2r}{3t} \leq m \leq \frac{r}{t} \right) = \frac{mt}{2} > 0 \quad \text{Increasing}$$

$$\frac{\partial^2 \pi_d}{\partial m^2} \left(\alpha = \frac{m}{2}, \frac{2r}{3t} \leq m \leq \frac{r}{t} \right) = \frac{t}{2} > 0 \quad \text{Convex}$$

Critical values:

$$\pi_d \left(\alpha = \frac{m}{2}, m = \frac{2r}{3t} \right) = \frac{r^2}{9t} - F$$

$$\pi_d \left(\alpha = \frac{m}{2}, m = \frac{r}{t} \right) = \frac{r^2}{4t} - F$$

Thus, segment (iii) is continuous with (ib) and (iv).

Determining the transition point between (ii) and (iii):

$$\pi_d \left(\alpha = \frac{r}{7t}, m \geq \frac{4r}{7t} \right) \leq \pi_d \left(\alpha = \frac{m}{2}, \frac{2r}{3t} \leq m \leq \frac{r}{t} \right)$$

$$\frac{r^2}{7t} - F \leq \frac{m^2 t}{4} - F$$

$$m \geq \sqrt{\frac{4}{7} \frac{r}{t}}$$

The boundary condition in proposition 3 (ii) is: $\frac{4r}{7t} < m < \sqrt{\frac{4}{7} \frac{r}{t}}$

The boundary condition in proposition 3 (iii) is: $\sqrt{\frac{4}{7} \frac{r}{t}} \leq m \leq \frac{r}{t}$

As profit for segment (iv) is fixed, it is sufficient to note that the transition between (iii) and (iv) is continuous and occurs at $m = \frac{r}{t}$. The boundary condition for (iv) is: $m > \frac{r}{t}$

Proof of Proposition 4 (ii)-(iv)

(ii) Total upstream profit when each territory is cut from one side:

$$\Pi_u \left(\frac{4r}{7t} < m < \sqrt{\frac{4r}{7t}} \right) = \frac{1}{m} P_u s = \frac{1}{m} \left(\frac{16r^2}{49t} \right)$$

where $P_u = \frac{4r}{7t}$ and served territory $s = \alpha + \frac{r - P_u}{t} = \frac{4r}{7t}$.

Maximizing profit:

$$\frac{\partial \Pi_u}{\partial m} = \frac{-1}{m^2} \left(\frac{16r^2}{49t} \right) < 0$$

Optimal territory size is $\hat{m} = \frac{4r}{7t}$ conditional upon:

$$\pi_d \left(\frac{4r}{7t} < m < \sqrt{\frac{4r}{7t}} \right) \geq 0$$

$$\frac{r^2}{7t} - F \geq 0, \text{ or } F \leq \frac{r^2}{7t}$$

and $\frac{4r}{7t} \leq 1, \text{ or } r \leq \frac{7t}{4}$

(iii) Total upstream profit when each territory is fully cut from one side:

$$\Pi_u \left(\sqrt{\frac{4r}{7t}} \leq m \leq \min \left[\frac{r}{t}, 1 \right] \right) = \frac{P_u s}{m} = r - \frac{mt}{2} \quad \text{where } P_u = r - \frac{mt}{2} \text{ and } s=m.$$

$$\frac{\partial \Pi_u}{\partial m} = -\frac{t}{2} < 0$$

Optimal territory size is the smallest possible conditional upon:

$$\pi_d \left(\sqrt{\frac{4}{7}} \frac{r}{t} \leq m \leq \frac{r}{t} \right) \geq 0$$

$$\frac{m^2 t}{4} - F \geq 0, \text{ or } m \geq 2\sqrt{\frac{F}{t}}$$

Thus, the optimal territory size is $\hat{m} = 2\sqrt{\frac{F}{t}}$. The range is:

$$\sqrt{\frac{4}{7}} \frac{r}{t} \leq \hat{m} \leq \min \left[\frac{r}{t}, 1 \right]$$

$$\frac{r^2}{7t} \leq F \leq \min \left[\frac{r^2}{4t}, \frac{t}{2} \right]$$

(iv) When territory is cut from two sides:

$$\Pi_u \left(\frac{r}{t} < m \leq 1 \right) = \frac{P_u s}{m} = \frac{1}{m} \left(\frac{r^2}{2t} \right)$$

$$\text{where } P_u = \frac{r}{2} \text{ and } s = 2 \frac{r - P_u}{t} = \frac{r}{t}.$$

Maximizing profit:

$$\frac{\partial \Pi_u}{\partial m} \left(\frac{r}{t} < m \leq 1 \right) = -\frac{1}{m^2} \left(\frac{r^2}{2t} \right) < 0$$

The optimal territory size is the lower bound conditional upon:

$$\pi_d \left(m > \frac{r}{t} \right) \geq 0$$

$$\frac{r^2}{4t} - F \geq 0, \text{ or } F \leq \frac{r^2}{4t}$$

Optimal profit function in terms of optimal territory sizes and ranges:

$$(ia) \quad \Pi_u \left(m = \sqrt{\frac{2F}{t}}, 0 \leq F \leq \min \left[\frac{r^2}{8t}, \frac{t}{2} \right] \right) = r - \sqrt{2Ft}$$

$$(ib) \quad \Pi_u \left(m = \frac{4r}{7t}, 0 \leq F \leq \frac{r^2}{7t} \wedge r \leq \frac{7t}{4} \right) = \frac{4r}{7}$$

$$(ii) \quad \Pi_u \left(m = \frac{4r}{7t}, 0 \leq F \leq \frac{r^2}{7t} \wedge r \leq \frac{7t}{4} \right) = \frac{4r}{7}$$

$$(iiia) \quad \Pi_u \left(m = \sqrt{\frac{4}{7}} \left(\frac{r}{t} \right), 0 \leq F \leq \frac{r^2}{7t} \wedge r \leq \sqrt{\frac{7}{4}} t \right) = \left(1 - \frac{1}{\sqrt{7}} \right) r$$

$$(iiib) \quad \Pi_u \left(m = 2\sqrt{\frac{F}{t}}, \frac{r^2}{7t} \leq F \leq \min \left[\frac{r^2}{4t}, \frac{t}{4} \right] \right) = r - \sqrt{Ft}$$

$$(iv) \quad \Pi_u \left(m = \frac{r}{t}, 0 \leq F \leq \frac{r^2}{4t} \wedge r \leq t \right) = \frac{r}{2}$$

The boundary conditions for each optimal territory size is found by identifying the transition points of the upper envelope of the above profit functions. First, we determine the shape and critical values of the optimal profit function within each segment.

(ia) Shape:

$$\frac{\partial \Pi_u}{\partial F} \left(m = \sqrt{\frac{2F}{t}}, 0 \leq F \leq \frac{r^2}{8t} \right) = \frac{\partial (r - \sqrt{2Ft})}{\partial F} = -\sqrt{\frac{2t}{F}} < 0 \quad \text{Decreasing}$$

$$\frac{\partial^2 \Pi_u}{\partial F^2} \left(m = \sqrt{\frac{2F}{t}}, 0 \leq F \leq \frac{r^2}{8t} \right) = \left(\frac{1}{2} \right) \frac{\sqrt{2t}}{\sqrt{F}^3} > 0 \quad \text{Convex}$$

Critical values:

$$\Pi_u \left(m = \sqrt{\frac{2F}{t}}, F = 0 \right) = r$$

$$\Pi_u \left(m = \sqrt{\frac{2F}{t}}, F = \frac{r^2}{8t} \right) = \frac{r}{2}$$

Transition point between segments (ia) and (iiia):

$$r - \sqrt{2Ft} \geq \left(1 - \frac{1}{\sqrt{7}} \right) r$$

$$F \leq \frac{r^2}{14t}$$

The boundary condition in proposition 4 (i) is $0 \leq F \leq \frac{r^2}{14t}$.

Profits in segments (ib) and (ii) are both dominated by segment (iia) and are never chosen.

(iiib) Shape:

$$\frac{\partial \Pi_u}{\partial F} \left(m = 2\sqrt{\frac{F}{t}}, \frac{r^2}{7t} \leq F \leq \frac{r^2}{4t} \right) = \frac{\partial (r - \sqrt{Ft})}{\partial F} = -\sqrt{\frac{t}{F}} < 0 \quad \text{Decreasing}$$

$$\frac{\partial^2 \Pi_u}{\partial F^2} \left(m = 2\sqrt{\frac{F}{t}}, \frac{r^2}{7t} \leq F \leq \frac{r^2}{4t} \right) = \left(\frac{1}{2} \right) \frac{\sqrt{t}}{\sqrt{F}^3} > 0 \quad \text{Convex}$$

Critical values:

$$\Pi_u \left(m = 2\sqrt{\frac{F}{t}}, F = \frac{r^2}{7t} \right) = \left(1 - \frac{1}{\sqrt{7}} \right) r$$

$$\Pi_u \left(m = 2\sqrt{\frac{F}{t}}, F = \frac{r^2}{4t} \right) = \frac{r}{2}$$

Segment (iv) is dominated by segment (iii).