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Optimal population and capital dynamics

in fisheries with irreversible investments

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Abstract

A multi-dimensional, non-linear dynamic model in continuous time is presented for the purpose of finding the optimal combination of exploitation and capital investment in optimal renewable resource management. Non-malleability of capital is incorporated in the model through an asymmetric cost-function of investment, and investments can be both positive and negative. Exploitation is controlled through the utilisation rate of available capital. A novel feature in this model is that there are costs associated with the available capital, whether it is utilised or not. And, in contrast to most of the previous literature, the state variables, namely the physical capital and the biological resource, enter the objective function. Due to the nonlinearities in this model, some of the results are in sharp contrast to previous literature.

Key words Irreversible investments, non-malleable capital, renewable resources.
JEL Classification codes: D92, Q20, Q22.

Introduction

Many renewable resources are characterized by overexploitation combined with excessive exploitation capacity. As the existing capacity has a tendency to put additional pressure on the resource, it is more and more agreed upon that models for optimal management should include capital dynamics as well as resource dynamics. By capital is here meant physical capital that can be used for the purpose of exploitation of the resource.

Smith (1968) was the first to consider capital accumulation in resource economics within a model with two capital stocks, biological and physical. The analysis of irreversible investments in physical capital, however, was initiated in the pioneering work by Clark, Clarke, and Munro (1979). They developed a model to analyse the effects of irreversibility of capital investments upon optimal exploitation policies for renewable resource stocks. This is a deterministic linear model with bang-bang policies, and the main conclusion is that whereas the long-run optimal steady state is unaffected by the assumption about irreversibility, the short-term optimal policies may depend significantly upon this assumption. Their typical result derived for an unexploited fishery is an initial pulse investment followed by a period of zero gross investment. McKelvey (1985) studied the same problem within an open-access regime and found the results of Clark, Clarke, and Munro to hold there as well. Charles (1983) and Charles and Munro (1985) perform stochastic analyses of the same problem and find that the effects of uncertainty can go either way with respect to investment. Boyce (1995) was the first to consider non-linearities in the objective function. He presents a model with a general non-linear utility of harvest and cost of investment functions. Then he shows that Clark, Clarke, and Munro's result for an unexploited fishery does not hold in this setting. In his nonlinear model, it is never optimal to stop investment once it has started.

The present work extends Boyce's model by including the resource stock in the objective function and removing the non-negative constraint on investment. In addition, a novel feature in this model is that there are costs associated with the available capital whether it is utilised or not. Typically such costs are insurance, interest on capital, etc. Thus we are able to characterize optimal policies from all possible initial combinations of biological stock and physical capital; not only the unexploited fishery so heavily emphasized in the previous literature. We find that depending on the initial situation there may be both over- and undershooting with respect to the biological stock and undershooting with respect to capital. Undershooting is defined as a period of decline in one of the state variables even when the initial level is below the long-term steady state.

Overshooting is defined correspondingly. In contrast to the previous literature, we find that overshooting with respect to capital is not optimal in phases where the capital is not fully utilised.

Further, in Boyce's model the optimal steady state can only be reached from the first and third quadrant (as divided by the steady state). In the model presented here, the steady state can be approached from all four quadrants.

In the following, the model is first presented and investigated analytically. Towards the end, a numerical example is given.

The Model

The model is a dynamic optimization model in continuous time. It is assumed that the objective is to maximise net present revenue. The fleet is characterized by total physical capital, k, while the renewable resource is characterized by total biomass, x. The instrument used to control the capital is investment, $I \in (-\infty, \infty)$, and the instrument used to control the exploitation of the natural resource is the capital utilisation rate, $\phi \in [0,1]$. The extreme situations with (i) no capacity utilisation or (ii) full capacity utilisation are represented by $\phi = 0$ and $\phi = 1$, respectively. When $0 < \phi < 1$, there is exploitation at reduced capacity utilisation. The rate of harvest function is given by:

$$h(k, x, \phi) = qx\phi k$$

where q is an exogenous coefficient. The net revenue function has the form:

$$\Pi(k, x, \phi, I) = \pi(x, h) - C(I) - K(k),$$

where $\pi(x,h)$ is a concave function representing the net revenue associated directly with the exploitation activity. The term $C(I)$ represents costs (or revenues) associated with investment (or disinvestment) I. It is assumed $I \cdot C(I) \geq 0$, implying that there is a cost associated with postitive investment buying capital, and an income (negative cost) associated with negative investment selling capital. Furthermore, $C' > 0$ and $C'' > 0$ implies that the more we buy, the higher the cost, and the more we sell (put on the market), the lower the income (subscripts denote partial derivatives). In addition, C = 0 when I = 0.

The costs associated with the total level of available capital, *k*, whether it is utilised or not, are denoted $K(k)$. The following assumptions are used[1]:

$$\pi_x \geq 0,\ K' \geq 0,\ C' > 0,\ C'' > 0. \tag{1}$$

The interpretation of $\pi_x > 0$ is that a more abundant resource base reduces harvest costs. The convexity of the investment cost function, *C*, accounts for the non-malleability of investment. There is an asymmetric relationship between the buying price and the selling price of capital due to the assumption $C'' > 0$ on $I \in (-\infty, \infty)$. When $I > 0$ we are buying capital, and the marginal price of capital, C', is higher then the marginal price we receive when $I < 0$ and we are selling an equivalent amount of capital. The marginal price of capital is continuously increasing in investment whether it is positive or negative. The degree of malleability in this model can be controlled through the convexity of the investment cost function. By adjusting *C* we can have anything from almost completely malleable capital to completely non-malleable capital. As investment/disinvestment can take any value on the real axis, optimality in this control variable will be inner optimality.

The variables *x* and *k* are state variables, while *I* and ϕ are controls. The state equations for stock and capital are assumed to have the simple forms:

$$\dot{x} = f(x) - h \tag{2}$$

$$\dot{k} = I - bk, \tag{3}$$

where $f(x)$ is the biological surplus growth function and *b* is the depreciation factor for capital. It is assumed that *f* is a continuously differentiable function with the properties $f(0) = f(\kappa) = 0$, where κ is the natural carrying capacity of the biological stock.

The optimization problem for the managing authority is given as follows:

$$\max_{\phi, I} \int_0^\infty e^{-\delta t} \Pi\big(k(t), x(t), I(t), \phi(t)\big) dt, \qquad \phi(t) \in [0,1],\ I(t) \in R,$$

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[1] Functional dependence is depressed for readability when it does not cause any confusion.

where δ is the discount rate, subject to the dynamic constraints (2) and (3) and subject to $k, x \geq 0$.

The current value Hamiltonian for this problem becomes:

$$H(k,x,I,\phi,\lambda,\mu) = \Pi(k,x,I,\phi) + \lambda[I - bk] + \mu[f(x) - qx\phi k].$$

Some first-order derivatives of the Hamiltonian are:

$$H_\phi = [\pi_h - \mu]qxk,$$
$$H_I = -C' + \lambda,$$
$$H_k = [\pi_h - \mu]qx\phi - K' - b\lambda,$$
$$H_x = \Pi_x + \mu(f' - qk\phi).$$

The dynamic equations for the shadow prices λ and μ are:

$$\dot{\lambda} = \delta\lambda - \frac{\partial H}{\partial k} = (\delta + b)\lambda + K' - [\pi_h - \mu]qx\phi, \tag{4a}$$

$$\dot{\mu} = \delta\mu - \frac{\partial H}{\partial x} = (\delta + qk\phi - f')\mu - \Pi_x. \tag{4b}$$

As investments can take any real value, the rate I that maximizes H must be a critical point and hence:

$$\lambda = C' > 0. \tag{5}$$

As the utilisation rate is constrained by $0 \leq \phi \leq 1$, it gives rise to three natural regions for $k \cdot x > 0$ (table 1).

Table 1

Characterization of the Various egions

Region A	$H_\phi < 0$	$\mu > \pi_h(x,0)$	$\phi = 0$
Region B	$H_\phi = 0$	$\mu = \pi_h(x,h)$	$0 < \phi < 1$
Region C	$H_\phi > 0$	$\mu < \pi_h(x,h)$	$\phi = 1$

In the following, *I* is defined as gross investment whereas the actual change in the capital level, $\dot{k}$, is defined as net investment. Notice that this implies that our approach is more general than e.g. Clark, Clarke, and Munro (1979), as we allow for regions where the capital is not fully utilised. Now we state some general results based on the outline above.

Proposition 1

In regions where capital is not fully utilised (A and B), gross investment will be increasing and the capital stock will never have local maxima.

Proof: In A and B the dynamic equation for the shadow price of capital is given by $\dot{\lambda} = (\delta + b)\lambda + K'$ from equation (4) and Table 1. From equation (5) we get $\dot{\lambda} = C''\dot{I} = (\delta + b)C' + K' > 0$, implying $\dot{I} > 0$ given the assumptions in equation (1). Further, inserting $\dot{k} = 0$ in the expression for $\ddot{k}$, we see that $\ddot{k} = \dot{I} = \dfrac{\dot{\lambda}}{C''} > 0$. Hence, any local extreme points with respect to $k(t)$ are necessarily local minima.

The intuition behind this is that in the case of overcapitalization, disinvestment will take place at a decreasing rate and positive investment at an increasing rate. In other words, it is best to gradually slow down sale of capital and postpone investment. Due to the non-linearity of *C*, this will not be a bang-bang operation.

Proposition 2

In a steady state the capital is fully utilised.

Proof: In steady state $\dot{\lambda} = 0$ by definition. From equations (4a), (1), and (5) this is impossible in A and B; hence, the steady state must be in C.

The intuition behind this is that it will always be wasteful to have unutilised capital in steady state, even if the cost of capital $K = 0$, as there will be a cost associated with the depreciation of the capital. If $K > 0$, there will be an additional cost associated with the idle capital.

Proposition 3

The shadow price, μ, is positive for all times if the condition $\delta + \dfrac{f}{x} > f'$ holds in steady state. Further, in steady state $\delta - f'$ and $\delta + \dfrac{f}{x} - f'$ will have the same sign.

Proof: In steady state we have from equation (4): $(\delta + qk\phi - f')\mu = \Pi_x$. Further, as $\dot{x} = 0$ and $\phi = 1$ according to Proposition 2, we have $h = f = qxk$. This yields $(\delta + \dfrac{f}{x} - f')\mu = \Pi_x$ in steady state. As $\Pi_x > 0$ from equation (1), we have that $\mu > 0$ in steady state when $\delta + \dfrac{f}{x} > f'$. From equation (4) we have $\dot{\mu} < 0$ when $\mu = 0$, hence μ cannot go from negative to positive. As μ is positive in steady state, it must always be positive. The last part of Proposition 3 follows from

$$(\delta + \frac{f}{x} - f')\mu = \Pi_x > 0 \Leftrightarrow (\delta - f')\mu = \pi_x + (\pi_h - \mu)\frac{f}{x} > 0 \text{ as } \pi_h > \mu \text{ and } \frac{f}{x} > 0 \text{ in steady state.}$$

As $\lambda > 0$ from equation (5), we know that both shadow prices are positive. Further, note that the condition $\delta + \dfrac{f}{x} > f'$ is always fulfilled for concave growth functions (e.g., like the widely used logistic growth function). The results in Proposition 3 turn out to be useful later.

In the following it is assumed that the shadow prices are always positive (that is equivalent to f being concave in steady state), which is a very reasonable assumption. A common result from bioeconomic models is that maximum economic yield (MEY) is attained at a higher stock level than the maximum sustainable yield (MSY) (Seijo Defeo, and Salas, 1998). And if $f' < 0$ at steady state, then Proposition 3 is definitely fulfilled.

Proposition 4

The shadow price on the stock, μ, decreases when $\delta < f'$.

Proof: From equation (4) we have $\dot{\mu} = (\delta - f')\mu - \pi_x - (\pi_h - \mu) \cdot qk\phi < (\delta - f')\mu < 0$ as $\mu > 0$ and $(\pi_h - \mu) \cdot qk\phi \geq 0$.

The interpretation of Proposition 4 is that as long as the alternative rate of return, δ, is less than the biological rate of return, f', the scarcity rent, μ, will decrease.

The following definition turns out to be useful. Let *S* be a function of the state variables alone defined as:

$$S(k,x) \equiv \Pi(k,x,bk,1),$$

which can be interpreted as the net revenue when the physical capital is fixed ($I = bk$) and fully utilised ($\phi = 1$). With this definition, we can state the next proposition:

Proposition 5

When the capital is fixed and fully utilised, we have $\dfrac{\partial S}{\partial k} \cdot k = \delta\lambda k + \mu h$, that is the marginal return on capital must equal the alternative return on capital plus the marginal return on the biological stock.

Proof: When $\phi = 1$ and $I = bk$, we have $S(k,x) = \pi(x,qkx) - C(bk) - K(k)$. As $\dot{\lambda} = 0$ according to equation (5) when capital is fixed, we have $\delta\lambda + \mu qx = \pi(k,qkx)qx - b \cdot C'(bk) - K'(k)$. The proposition then follows from $qx = \dfrac{h}{k}$.

The term $\dfrac{\partial S}{\partial k}$ is the rate of return on capital, and this is multiplied by the capital level on the left-hand side. The alternative rate of return is δ, and this is multiplied by the capital

evaluated at its shadow price, λ, plus the harvest (which is the return on the stock) evaluated at its shadow price, μ. Note also that Proposition 5 can be used to characterize the steady state where $h = f(x)$.

The marginal revenue from exploitation at a fixed stock level is given by $\pi_h(x, f(x))$, which is a function in x only. Integrating with respect to x we get $\int \pi_h(x, f(x))dx$, and this can be interpreted as the value of a stock evaluated by its marginal revenue (relative to an arbitrary reference value/point). When this is multiplied by δ, we get the alternative rate of return on the stock. On the other hand, $\pi(x, f(x))$ is the actual rate of return on the stock when it is fixed. This leads to the definition of a new term, B, which is the difference between the alternative rate of return and the actual rate of return on the stock:

$$B(x) \equiv \delta \int_z^x \pi_h(s, f(s))ds - \pi(x, f(x)).$$

This function turns out to be extremely useful. Note, for example, that the classical golden rule (Clark, 1990) used to determine a steady state is simply given by $B'(x) = 0$ as B' can be interpreted as the marginal net revenue from harvesting. This can be generalized to include capital and investment by defining:

$$MC(x) \equiv K'(\tfrac{f}{qx}) + (\delta + b)C'(\tfrac{bf}{qx})$$

and

$$\eta(x) \equiv (\delta + \frac{f}{x} - f')/(qx).$$

The interpretation of MC is that it is the total marginal cost of investment when both state variables are fixed and capital is fully utilised. The term K' is the direct capital cost, $\delta C'$ is the opportunity cost and $b \cdot C'$ is the depreciation cost. The next proposition is useful for determining the biological stock in steady state.

Proposition 6 (Generalized Golden Rule)

The biological stock in a steady state is found by solving the ordinary (algebraic) equation $B'(x) = \eta(x) \cdot MC(x)$.

Proof: This proposition follows from equations (4), (5), and $\dot{x} = \dot{\mu} = \dot{\lambda} = 0$ together with $\varphi = 1$ inserted into Proposition 5.

Proposition 6 is a generalization of the classical Golden Rule for renewable resources (Clark 1990; Sandal and Steinshamn 1997). The terms derived from π are the classical Golden Rule, $(\delta - f')\pi_h = \pi_x$, which is equivalent to $B' = 0$. The term MC is the net marginal capital cost and η is the conversion rate between the two. Capital costs and investment costs enter the equation in the same manner as positive terms. Thus, larger investment costs cause the same kind of changes on the equilibrium stock as larger capital costs. Both costs may have a significant influence on the optimal paths, not least in regions A and B.

It is well known that the resource stock is typically increasing with higher operational costs in models without capital dynamics. It is therefore relevant to ask if this also applies to capital and investment costs. The answer is given in the next proposition.

Proposition 7

Let $\psi(x) \equiv B' - \eta \cdot MC$. *If* $\psi'(x) > 0$ *for any* $x = x*$ *where* $\psi(x*) = 0$, *then the steady state standing stock will increase with higher capital and investment costs.*

Proof: Let MC depend on a cost parameter α, $MC = MC(x; \alpha)$, such that $\dfrac{\partial}{\partial \alpha} MC > 0$. From $B' = \eta \cdot MC$ in steady state it can be deducted that $\dfrac{\partial}{\partial x}[B' - \eta \cdot MC] \cdot \dfrac{\partial x}{\partial \alpha} = \eta \cdot \dfrac{\partial MC}{\partial \alpha}$ and hence $\dfrac{\partial x}{\partial \alpha} > 0$.

The condition that ψ is increasing when it is zero is by far the most common case. For example, in the case of the widely applied logistic growth function, it can be shown that this is always fulfilled. By definition:

$$\Psi' = B'' - \eta' \cdot MC - \eta \cdot MC'$$
$$B'' = f' \cdot (\delta - f') \pi_{hh} + (\delta - 2f') \pi_{xh} - \pi_{xx} - f'' \cdot \pi_h$$
$$\eta' = (qx^2)^{-1} \left[2x(f/x)' - \delta - x \cdot f'' \right]$$
$$MC' = q^{-1} \left[K'' + b(\delta + b)C'' \right] (f/x)'.$$

The signs of η' and MC' are determined by the growth function, f, and in the case of a logistic growth function, both signs are non-positive. Hence, the two last terms are positive. From the definition of $B(x)$ and the fact that $\pi(x,u)$ is concave in (x,u), it is reasonable to assume that $B(x)$ is convex in the vicinity of steady state. It is convex everywhere when $\delta \to 0$. In the special case of the model studied by Clark and Munro (1975) where $\pi(x,h) = (p - c(x))h$, the condition $B''(x) > 0$ is trivially fulfilled. Then $B''(x) = -f'' \cdot (p - c) - c' \cdot (\delta - 2f) + c'' \cdot f$. The first and last terms are positive as $f'' < 0$ and $c'' > 0$. The second term is non-negative because $c'(x) \le 0$ and typically $f' < 0$ in the steady-state optimum.

Corollary 1
Let η', f', and f'' all be non-positive at the steady state. Then the standing stock will increase with higher capital and investment costs.

Proof: In this case we have $MC'(x^*) < 0$, and the condition that $\pi(x,h)$ is concave together with equation (1) yields directly $B''(x^*) \ge 0$; hence, this is sufficient for $\Psi'(x^*) > 0$.

Note that this does not require f to be globally concave. The result that the standing stock increases with higher capital and investment costs is also quite intuitive. It shows, however, that increased convexity in the cost of investment function calls for a more conservative utilisation pattern and also increased capital costs, K.

Numerical Example
In this section, we will present a short numerical example based on fisheries management in order to illustrate some of the features described earlier. The net revenue function is assumed to have the form:

$$\pi(x,h) = p(h)h - c(x,h),$$

where $p(h)$ is the inverse demand function for harvest given as:

$$p(h) = p_0 - \Gamma h,$$

and $c(x,h)$ is the cost function for harvesting

$$c(x,h) = \frac{c_h h}{x}.$$

The investment costs, or income from sale of capital, are represented by the function:

$$C(I) = \rho \begin{cases} -n^2 + (I+n)^2 & ; & I > -n \\ \alpha^2 - n^2 + 2\alpha(\alpha - 2n)\dfrac{I+n-\alpha}{I-n} & ; & I \le -n, \end{cases}$$

where ρ and n are positive constants and $0 < \alpha < n$. Note that C approaches a constant as $I \to -\infty$. This means that there is a saturation point for sale of capital. In other words, for very high levels of disinvestment the marginal sales-price will approach zero. Note that in the limit $\alpha = 0$, the cost of investment, $C(I)$, is a half parabola touching the horizontal line. Small values of α thus represent a strict convexification of this picture.

The cost of capital is given by:

$$K(k) = c_k k,$$

where c_k is a fixed unit cost (for example the rental rate) associated with the total capital, k, whether it is utilised or not.

The state equations for stock and capital are assumed to have the form:

$$\dot{x} = f(x) - h = x(r - sx) - qx\phi k$$

$$\dot{k} = I - bk,$$

where $f(x)$ is the biological growth function, and r and s are biological growth coefficients.

To illustrate the dynamics of the model described above, a numerical example has been constructed. The following specification of the parameters has been used in the construction of the example:

$r = 0.54 \text{ year}^{-1}$

$s = 1.22 \times 10^{-4} \text{ (year} \cdot 10^{6}\text{kg)}^{-1}$

$q = 0.004 \text{ year}^{-1}$

$p_0 = 13.5 \text{ } (10^{6} \text{ NKr})/(10^{6} \text{ kg})$

$c_h = 7500 \text{ } (10^{6} \text{ NKr})$

$c_k = 7 \text{ } (10^{6} \text{ NKr})/(\text{year} \cdot \text{vessel})$

$\Gamma = 0.01 \text{ } (10^{6} \text{ NKr}) \cdot \text{year}/(10^{6} \text{ kg})$

$b = 0.05 \text{ year}^{-1}$

$n = 10 \text{ vessel/year}$

$\rho = 6.67 \text{ } (10^{6} \text{ NKr})\text{year/vessel}^{2}$

$\alpha \rightarrow 0^{+}$

$\delta = 0.05 \text{ year}^{-1}.$

This specification is not based on any thorough empirical analysis, but bears a certain stylized resemblance to the Norwegian cod fishery. When these parameters are used, the long-run, non-exploited stock (i.e., the natural carrying capacity of the stock) is $x_0 = r/s = 4426 \times 10^{6}$ kg. The steady-state situation is characterized by:

$x^* = 3429 \cdot 10^6 \ kg$

$k^* = 30 \ vessels$

$I^* = 1.5 \ vessels / year$

$h^* = 417 \cdot 10^6 \ kg / year$

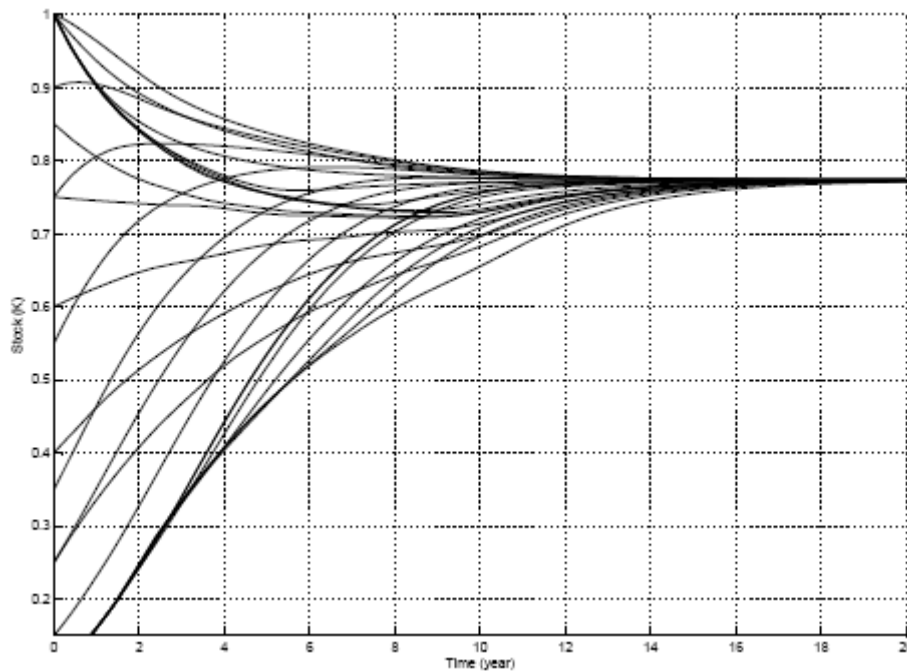$\lambda^* = 154 \ NKr / vessel$

$\mu^* = 1.34 \ NKr / kg.$

Figure 1. Optimal Evolution of Stock over Time with Different Initial Capital Levels Measured Relative to the Carrying Capacity (4,426)

Altogether, 25 different optimal paths with various combinations of initial biological stock and capital are illustrated in figures 1 - 3. The same 25 paths are illustrated in each of the three figures. Figure 1 illustrates the stock development with various starting points for both stock and capital. The stock is measured relative to the long-run carrying capacity (4,426). Note that there are many paths starting with the same stock, but because they represent different initial capital levels they split up either immediately or after a while, but in the long run they all approach the same steady state. Note also that there are several examples of over- and undershooting and this takes place at an early stage. After about 15 years most paths are fairly close to the long-term steady state.

Over- and undershooting can be explained by the combination of nonlinearities in the model, namely the convexity of investment costs and the downward sloping demand function.

Overshooting means that it is suboptimal to harvest more either because it is too costly to invest in more capital or because more harvest will reduce the price too much. Hence, the stock will increase for a while. Undershooting with respect to the stock may seem counterintuitive at first glance, but the explanation is that with high initial capital it will not be optimal to sell too much, as this reduces the price. It may not be optimal to let the capital be idle either, because we have to cover the costs. Therefore, the stock will initially be harvested down slightly before it is allowed to increase to the long-term optimal steady state.
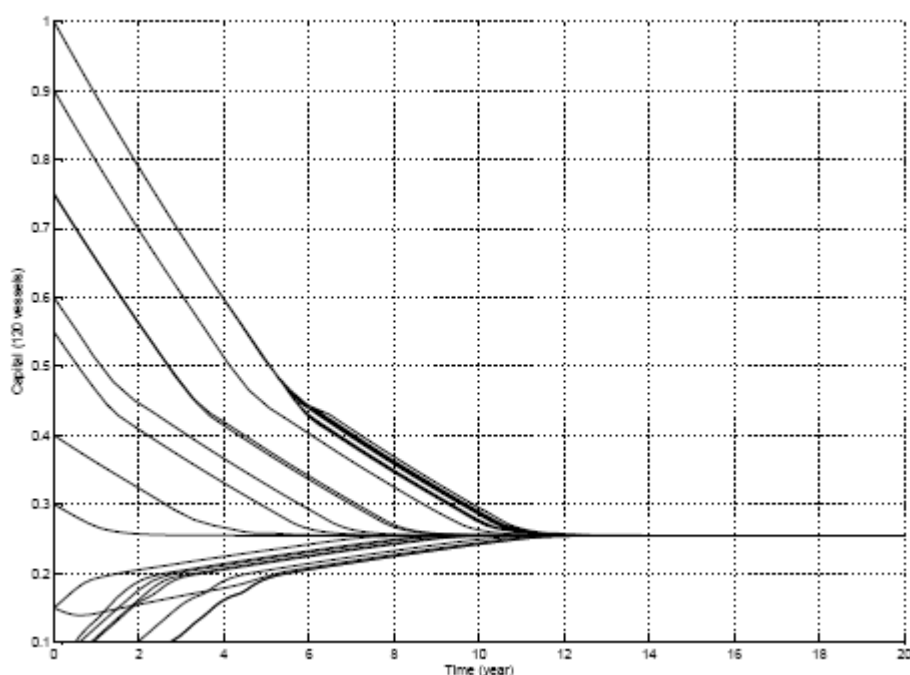

Figure 2. Optimal Evolution of Capital over Time with Different Initial Stock Levels Measured Relative to $\hat{k} = 120$

Figure 2 illustrates the same 25 paths for physical capital over time. In many cases we see that paths starting from the same initial capital level stick together for quite a while and then they depart before they finally approach the steady state. Note that all paths have more or less settled on the steady state in less than 12 years. From figure 2 it is seen that there is undershooting with respect to physical capital. This is in accordance with Proposition 1 and in contrast to previous literature. This follows from the convexity of the cost of investment function, which implies that investment should be delayed as long as possible (see Proposition 1).
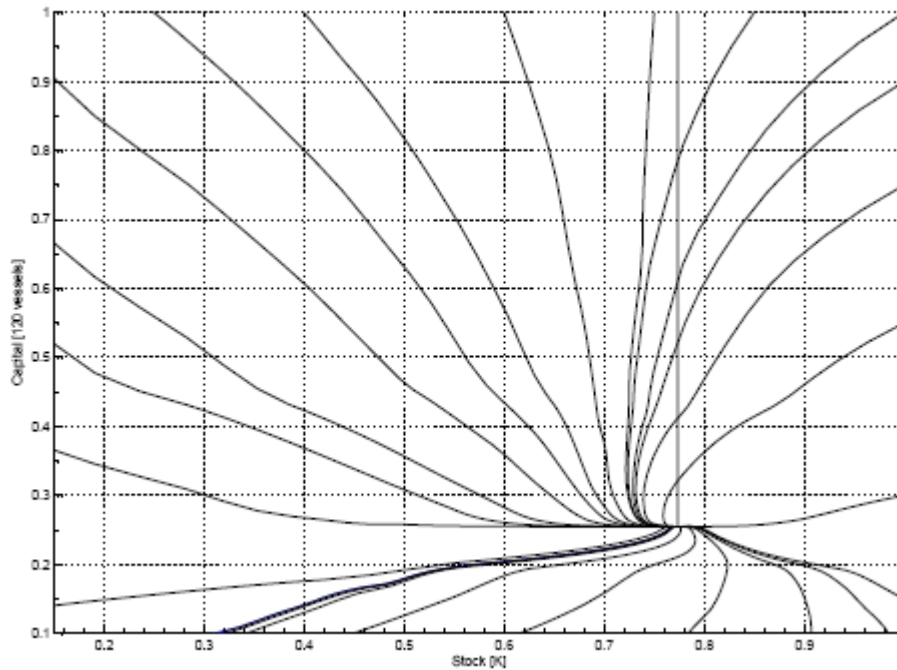
Figure 3. Optimal Paths in State-Space (Stock-Capital)

Figure 3 shows the same 25 paths in state-space, and again it is apparent that many of the paths exhibit over- or undershooting as expected from the previous theoretical discussion. But this also shows that over- and undershooting is not only a theoretical possibility in the vicinity of steady state, but it is quite likely to be an optimal policy in practice and even far from steady state.

Summary and Conclusions

This article introduces a general convex cost/revenue function for investment/disinvestment of capital in the exploitation of renewable resources. This function mimics the second-hand market for capital and is therefore a more realistic representation of irreversible investments than simple non-negativity constraints. Further, the exploitation of the resource is a function of the utilisation rate of the available capital, and there is one cost associated with the capital that is actually used and one cost associated with the total available capital whether it is utilised or not, which is a new feature compared to earlier works. The result is that both state variables enter the objective function in the present application.

It has been shown that both the steady state and the paths leading to steady state are affected by the novel features introduced in this paper. Typically both the convexity of the cost of investment and the cost of capital will call for more conservative utilisation of the resource. It is also shown that, depending on the initial conditions, it is possible to approach the steady state in a

variety of ways, and it is also possible to define regions from which the steady state cannot be approached directly. Therefore, there will be so-called over- or undershooting along the paths. Actually, it is possible to have over- and undershooting in all of the four quadrants from which the steady state can be approached. This contrasts the earlier findings of Boyce (1995).

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