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Spatial management of a fishery under parameter uncertainty

by

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Spatial management of a fishery under parameter uncertainty

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Abstract

Spatial management of a fishery under parameter uncertainty is analyzed. The habitat is divided into two areas, and the effort level in the two areas may be different. The migration of biomass between the areas follows a diffusion process; two different specifications are considered. The model features logistic growth and Schaefer production functions. The intrinsic growth rate is treated as uncertain; the uncertainty is symmetric and spatially homogeneous. It is found that the optimal, spatial distribution of effort with respect to expected harvest is neither homogeneous or heterogeneous everywhere, but homogenous for a given subset of the parameter space and heterogeneous elsewhere.

JEL Classification: Q22.

Keywords: Spatial management, fishery economics, parameter uncertainty.

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Introduction

Recently, there has been a surge in the interest for spatial models among fishery economists. Particularly, an extensive literature on marine reserves has emerged over several years. This development towards spatial notions in fishery economics in particular, and renewable resource economics in general, is welcome (Wilen 2000). Although the model in this paper is simpler than the sophisticated models common among ecologists, it is a step up from the traditional, homogenous models often employed in fisheries economics.

To our knowledge, the literature on spatial management in fisheries which abstracts from the extremity of reserves is limited (reserves are indeed extreme, requiring effort in the reserve to be zero, compared to the range of possible spatially heterogenous effort distributions). Sanchirico and Wilen (1999) develop a broad framework for modeling fisheries in a patchy environment. In fact, our model is a special case of that framework; we add parameter uncertainty and optimal management. Under open-access, Sanchirico and Wilen (1999) find that the equilibrium distribution of fishing activity depends on bioeconomic forces both within patches and between patches, and of course on the underlying ecological structure. Further, the interaction between movement of biomass and effort is described. In a later work, Sanchirico and Wilen (2005) derive an optimal, spatial management scheme in a deterministic and spatially heterogeneous renewable resource model (homogeneity is a special case) with input and output taxes and a focus on equilibria. They compare their findings to the case where taxes cannot be assigned with respect to spatial measures, and find, not surprisingly, that spatial explicit taxes are superior by various measures of superiority. In our model, the underlying resource is spatially homogenous, and we consider uncertainty. We are also focusing on the steady state of the system. Special cases of our and the Sanchirico and Wilen approach (*i.e.*, no uncertainty and homogeneity, respectively) should then be comparable. Herrera (2006) analyzes a multispecies model with persistent spatial heterogeneity, where harvest is non-selective. *Inter alia*, he finds that spatially specific management may be more efficient and provide an increase in the net present value of the fishery.

In the literature concerning marine reserves, economists usually find incentives to establish reserves under 'leading' assumptions, such as sink-source dynamics and spatially heterogenous uncertainty. The evidence from introducing

homogenous uncertainty is ambiguous, apart from the abundant evidence of ecological benefits. This literature is extensively reviewed in Grafton et al. (2005). Our analysis provide a rationale for marine reserves in certain cases, even in the long-term steady state equilibrium, without any 'leading' assumptions. When such results were discussed previously, they were usually conditional on, *e.g.*, a stock previously heavily overfished (Pezzey et al. 2000).

This analysis considers optimal steady states in a simple fishery model under parameter uncertainty. The uncertainty is spatially homogenous and symmetric. Looking at expected harvest, we numerically compute the optimal distribution of effort. The approach is more general than considering marine reserves. Establishing a marine reserve is found to be optimal for several sets of parameters, where a marine reserve is implied when the optimal effort level in one area is zero. An interesting finding is that in parts of the parameter space expected harvest has two local maxima and which one of these represents the global maxima depends on the parameters. A surprising implication is that it may be optimal to focus effort in the *smaller* of the two areas and further to establish a marine reserve in the *bigger* area in particular cases. In other cases there is an other type of solution with a more conventional nature, where effort is focused in the big area and the small area is closed to fishing, *i.e.*, a marine reserve. Ultimately, we are able to discern between homogeneous and heterogeneous optimal distributions of effort in parameter space.

Single Area Model

We consider a single area model in order to examine limitations of the model and demonstrate how the uncertainty is introduced. Growth ($f(x)$) is logistic and harvest ($h(E,x)$) follows Schaefer. The stock dynamic equation is thus

$$\begin{aligned} \dot{x} &= f(x) - h(E,x) \\ &= rx\left(1 - \frac{x}{K}\right) - qEx \end{aligned}$$

where x is stock level, r is intrinsic growth rate, K is carrying capacity, q is catchability, and E is effort. *Cf.* Clark (1990). Both K and q are normalized to unity without loss of generality. The rate of growth is uncertain. To keep it simple, the rate of growth takes on two values and the values may be given as $r_0 \cdot (1+\delta)$ and $r_0 \cdot (1-\delta)$. r_0 is then the mean and $r_0\delta$ the standard deviation.

Expected harvest is then

$$\mathrm{E}[h](E) = \beta h(E, x_h^*) + (1-\beta)h(E, x_l^*)$$

where β is the probability of the high rate of growth and x_h^* is the steady state stock level that will arise from the realization of $r_0 \cdot (1+\delta)$. x_l^* is the steady state stock level that will arise from $r_0 \cdot (1-\delta)$.[1] We have

$$x_h^* = \frac{r_0 \cdot (1+\delta) - E}{r_0 \cdot (1+\delta)}, \quad x_l^* = \frac{r_0 \cdot (1-\delta) - E}{r_0 \cdot (1-\delta)} \tag{1}$$

Note that $x_i^* = 0$ for E big enough, $i = h, l$. Substituting these expressions into the expected harvest and resolving with $\beta = {}^1/_2$ yields

$$\mathrm{E}[h](E) = \frac{E}{2}\left(\frac{r_0 \cdot (1+\delta) - E}{r_0 \cdot (1+\delta)} + \frac{r_0 \cdot (1-\delta) - E}{r_0 \cdot (1-\delta)}\right)$$

Assuming risk neutrality, the optimal effort maximizes expected harvest and is here derived as

$$E_{\mathrm{opt}} = \frac{r_0 \cdot (1+\delta) \cdot r_0 \cdot (1-\delta)}{2r_0}, \quad \text{for } \delta \leq {}^1/_2 \tag{2}$$

Notably, the expression is quadratic in δ.[2]

The discussion above presumes that effort is such that both steady state stock levels are positive. This is not true for effort levels above the 'critical' effort level $E = r_0 \cdot (1-\delta)$ (see expressions in (1)). It turns out that it is optimal to let effort exceed this level when $\delta > {}^1/_2$. Then we have $x_l^* = 0$ and $E_{\mathrm{opt}} = \dfrac{r_0 \cdot (1+\delta)}{2}$ for $\delta > {}^1/_2$, which further implies $x_h^* = {}^1/_2$. Note that the expression for optimal effort in this case is linear in δ, *cf.* earlier, and that x_h^* attains its maximum sustainable yield (MSY) level. It is also worth noting

[1] This formulation violates the dynamic equation, as it implies that the stock level reaches the steady state instantly after the uncertainty is resolved. Thus, the notion of a steady state is more related to a one-shot game, where the fisher (or manager) has one opportunity to chose effort level and must stick to that forever, rather than a dynamic fishery where the uncertainty is resolved several times, fishers learn and adapt and so forth. The simplification is attractive because it promotes tractability. Further, one might sometimes wonder whether particular fisheries, and other common pool resource exploitation as well, actually have a lot in common with the one-shot game (we have the myopic behavior often observed in mind).

[2] In later sections we focus on $\beta = {}^1/_2$ (for reasons of simplicity), which means that the uncertainty of the rate of growth is symmetric. Thus, the symmetric uncertainty case is of primary interest also here. The single area model is readily solvable for any β; the optimal effort level is more generally given as $E_{\mathrm{opt}} = \frac{r_0 \cdot (1+\delta) \cdot r_0 \cdot (1-\delta)}{2(\beta \cdot r_0 \cdot (1+\delta) + (1-\beta) \cdot r_0 \cdot (1-\delta))}$.

that when $x_l^* = 0$, there is only one term contributing in the expected harvest expression. That is, the 'bad' state is not taken (properly) into account. This limits the value of the model at these high levels of effort.

Figure 1 demonstrates expected harvest as a function of effort for two different levels of uncertainty. The mean rate of growth is $r_0 = 1$. E[h] changes fundamentally as effort increases beyond $r_0 \cdot (1 - \delta)$. For $\delta = .5$, expected harvest has two global maxima, one on each side of $r_0 \cdot (1 - \delta)$. For higher levels of uncertainty, the 'optimal' effort levels are always to the right of the 'critical' level of effort. This is demonstrated in Figure 2, where optimal effort is shown as a function of δ. It is evident that we want to disregard situations where $E > r - \delta$. Note how expected harvest behaves close to the 'critical' level; we will recognize a similar behavior of expected harvest in the two area model. A last thing suggested by Figure 1 is that expected harvest decreases as δ increases.

Two Area Model

In this section the model is extended to allow for spatial dispersion of effort. Specifically, there are two areas and separate logistic growth functions that apply to each subarea. Migration of biomass between the areas is density dependent and modeled as a diffusion process. The stock dynamic equations are then

$$\begin{aligned} g_1(x_1, x_2) &= f(x_1, \alpha) + m(\alpha)\left(\frac{x_2}{1-\alpha} - \frac{x_1}{\alpha}\right) - \alpha h(E_1, x_1, \alpha) \\ g_2(x_1, x_2) &= f(x_2, 1-\alpha) - m(\alpha)\left(\frac{x_2}{1-\alpha} - \frac{x_1}{\alpha}\right) - (1-\alpha) h(E_2, x_2, 1-\alpha) \end{aligned}$$

where (x_1, x_2) are the biomass levels in the two areas, $1 - \alpha$ denotes the share of habitat allocated in area 1, α is thus the share allocated to area 2, and $m(\alpha)$ is the rate of migration, which may in general depend on α. $f(x_1, \alpha) = rx_1\left(1 - \frac{x_1}{\alpha}\right)$ is the logistic growth function, where x_1 is the biomass level, α is the carrying capacity, and r is the rate of growth. The carrying capacity in the entire habitat is normalized to 1, and the habitat is further assumed to be homogenous across the entire habitat. $h(E_i, x_i, \alpha_i) = qE_i\frac{x_i}{\alpha_i}$, $i = 1, 2$, is the Schaefer production function, where E_i is the level of effort, α_i is the carrying capacity, and q is the catchability coefficient. (The subscripts denote the area of which the variable belongs.) Note that the way the production function

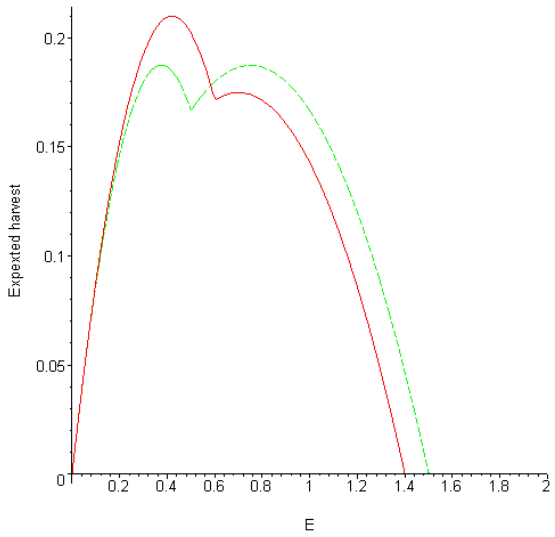


Fig. 1: Expected harvest along effort, E. $r_0 = 1$. $\delta = .4$ (solid line) and $\delta = .5$ (dashed line).

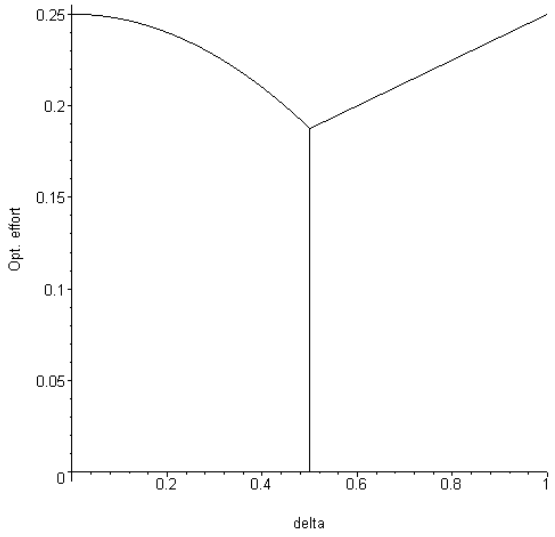


Fig. 2: Optimal effort level along δ, $r_0 = 1$.

is formulated, a proper interpretation is that it is the *harvest per unit area*. Thus, we must control for the area size in the dynamic equations. Further, the effort level is interpreted as *effort per unit area*. The catchability coefficient q is normalized to 1. The difference $\left(\frac{x_2}{1-\alpha} - \frac{x_1}{\alpha}\right)$ governs the diffusion process; the flow of biomass is always directed into the area with the lower density.

We emphasize that in this model, everything about the two areas and substocks are *identical*, apart from the possibility that effort may be different. This is a key point, and is what distinguishes this model from previous research.

It is worthwhile to note that the growth in the two areas is assumed to only depend on the local stock density (as in Kvamsdal and Sandal (2008), and as opposed to Bischi et al. (2007); Flaaten and Mjølhus (2005) treats both alternatives.). This implies that whenever the stock density is distributed heterogeneously, which will occur whenever the effort is distributed heterogeneously, the total growth in the stock is *smaller* than what it would have been if the *same amount* of biomass was distributed homogeneously. This is pointed out in Kvamsdal and Sandal (2008). Moreover, note that the model is equivalent to the homogenous, single area model under homogenously distributed effort; $E_1 = E_2$ (which leads to $\frac{x_1}{\alpha} = \frac{x_2}{1-\alpha}$ and thus a zero migration term).

The expressions describing the steady state stock levels are complex and resist interpretation. We are though, able the express them as functions of effort levels, and will denote them by $x_1^*(E_1, E_2)$ and $x_2^*(E_1, E_2)$. (Although, for simplicity, the arguments are at times omitted.) For the sake of completeness, the full expressions are given in the appendix. Note that the basic model is comparable to the model in Hannesson (1998), excluding the possibility of fishing in both areas; in Hannesson (1998) one of the areas is a marine reserve.

We still consider the rate of growth to be uncertain. As before, the rate of growth can take a high value or a low value with probabilities β and $1-\beta$, respectively. The two values take the form $r_0 \cdot (1+\delta)$ and $r_0 \cdot (1-\delta)$, where $\delta \geq 0$. The mean rate is normalized; $r_0 = 1$. Further, let $H(E_1, E_2)$ denote total harvest. The expected total harvest is then given by

$$\mathrm{E}[H](E_1, E_2) = \beta H(E_1, E_2)|_{r=r_0\cdot(1+\delta)} + (1-\beta)H(E_1, E_2)|_{r=r_0\cdot(1-\delta)}$$

where

$$\begin{aligned} H(E_1, E_2) &= \alpha h(E_1, x_1^*, \alpha) + (1-\alpha) h(E_2, x_2^*, 1-\alpha) \\ &= E_1 x_1^*(E_1, E_2) + E_2 x_2^*(E_1, E_2) \end{aligned}$$

$\mathrm{E}[H](E_1, E_2)$ describes the yield-effort relationship. Under fixed prices, the surface describing expected harvest is proportional to the surface of the revenue function. Figure 3 shows the shape of this surface, when $\alpha = 0.5$, $\beta = 0.5$, $\delta = 0.45$, and $m(\alpha) = 0.5$. Most notably is the 'trench' in the surface. Beyond the 'trench', as observed from the origin, the steady state stock levels are zero in both areas when the rate of growth obtains $r = r_0 \cdot (1 - \delta)$. Thus, the low rate of growth yields no harvest in steady state, *i.e.* such effort levels drives the stock to extinction with probability $1 - \beta$. This 'trench' corresponds to the 'critical' level of effort discussed in the one area model. We will not consider effort levels beyond the 'trench' any further. Also, the analysis of the single area model suggest that there is some upper level of uncertainty at which the model is well-behaved. We do not provide this upper limit in the two area model, but we for most considerations we limit ourself to $\delta \leq 0.5$. This level is naturally inspired by the results from the previous section.

Optimal effort

Under risk neutrality, we are interested in the effort levels that maximize expected harvest. The complexity of the stock level expressions prohibits analytical solutions, and thus we resort to numerical calculations. We focus on the case $\beta = {}^1/_2$. Thus, α, δ, and $m(\alpha)$ forms our parameter space to investigate. Area size (α) may be considered as a decision variable *or* as given exogenously, *e.g.* in a patch model like Sanchirico and Wilen (2002). Both uncertainty (δ) and migration ($m(\alpha)$) are treated as exogenous.

First, let us look at the simple model in which the migration rate is constant and equal to m_0 over all area sizes (Hannesson 1998). It is then possible to draw a few conclusion before diving into the numerical results. When the migration rate is zero, there is no interaction between the two substocks and they can be treated separately. Further, they are identical up to area size, and the optimal effort level per unit area is the same in both areas; $E_1 = E_2$. When the migration rate is infinite, that is, the redistribution of the biomass is instantaneous, the

density will always be identical in both areas, and one might conclude that optimal effort should be equal in both areas as well. However, when redistribution is instantaneous, distribution of effort does not matter, only total aggregated level of effort matters.[3] For simplicity, we may choose the homogeneous distribution of effort ($E_1 = E_2$) to be the solution when $m(\alpha) = m_0 \to \infty$.[4]

The homogeneous distribution of effort seems to play a central role when it comes to optimality. When $E_1 = E_2$, the two area model is equivalent to the one area model, and total optimal effort is described in equation (2) and Figure 2. Indeed, we find that the homogenous distribution is the optimal distribution for a nontrivial subset of the parameter space we are investigating. Figure 4 shows the parameter set where the optimal distribution is homogeneous *for all* area sizes. The plotted points ('boxed') is the smallest level of uncertainty necessary for heterogeneous optimal distributions, given a migration rate m_0.[5] Note that small migration rates are problematic; when solving for $x_1^*(E_1, E_2)$ and $x_2^*(E_1, E_2)$ we end up with a square root which argument is negative for small migration rates in combination with a range of values for the other parameters and variables. This yields complex valued stock levels, and it is unclear to us how to treat and interpret such results. This limits the parameter space we are able to investigate.

The above result is perhaps the most important result in this analysis. However, there are some surprising and interesting observations to be made outside the 'homogeneous area' discussed above. It turns out that there are two different *types* of heterogeneous solutions, and that the parameter space can be characterized further into subsets where the different types are optimal for all

[3] If instantaneous redistribution of biomass seems unlikely, it is worth noting that it is a standard assumption in aggregated biomass models, *e.g.* like those discussed in Clark (1990).

[4] Note that when the density of the stock in the two areas is equal, such as under an infinite migration rate, there exists a total effort level that corresponds to steady state stock levels equal to the MSY stock levels. Thus, when the migration rate is infinite the maximal harvest level is independent of area size. This is exactly the conclusion of Hastings and Botsford (1999). We will return to their findings later.

[5] As m_0 increases, the expected harvest surface 'flattens out', and the numerical procedure we use gets a bit noisy. (*E.g.*, the numerical procedure may report that the optimal effort levels are heterogeneous for seemingly random area sizes, whereas it reports that optimal effort is homogeneous for both smaller and bigger areas. An explanation of this behavior of the optimization procedure and further technical details are available from the corresponding author on request.) Thus, the other set of points in Figure 4 (diamonds) shows where the numerical procedure produces a perfect series of homogeneously distributed optimal effort levels (that is, below the diamond points). The points of interest, however, are the 'boxed' points, where a 'consistent' break from the homogeneous distribution is observed, compared to the random breaks observed between the two set of points.

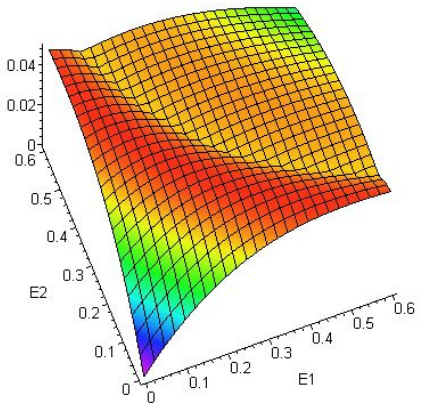


Fig. 3: Expected harvest in (E_1, E_2), where $\alpha = 0.5$, $\beta = 0.5$, $\delta = 0.45$, $r_0 = 1$ and $m_0 = 0.5$.

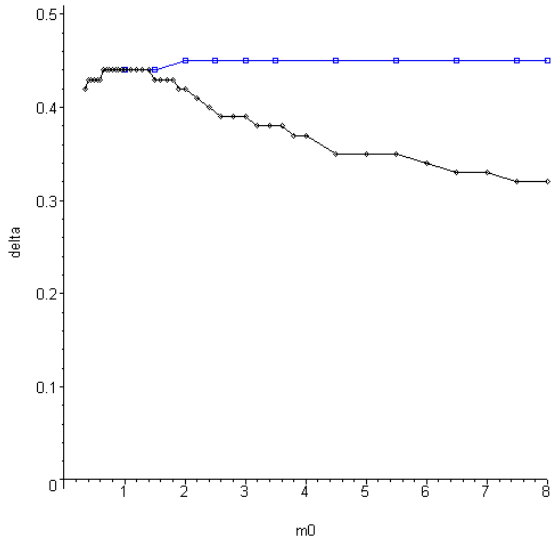


Fig. 4: Characterizing the parameter space: Optimal effort is homogeneously distributed below 'boxed' points.

area sizes. Also, there is a region where one type is optimal for some area sizes and the other type is optimal for other area sizes. We will return to this characterization of the parameter space. First, however, we study the two different types of heterogeneous solutions. The two types are associated with two local maxima of the expected harvest function (Figure 3).

The two types of solutions are demonstrated in Figure 5, where $\delta = .5$ and $m_0 = 1$ for area sizes in the range $[0.05 \dots 0.5]$. The figure shows how the two types of solutions distributes effort between the two areas. Effort in area 1 is shown along the x-axis and effort in area 2 is shown along the y-axis. One of the solutions suggest that the smaller area should be a marine reserve and that all effort should be located in area 2 (note that when the area size is less than half, area 1 is the smaller area of the two). The amount of effort in area 2 is nonzero and depends on the area size; we have $E_2 > E_1$. This solution corresponds to the blue 'diamond' points in Figure 5. From obvious reasons, we will refer to this type of solution as the 'left hand side' (LHS) solution. Effort per unit area increases with area size. The other solution suggest that effort should be higher (or rather, denser) in area 1, however not always zero in the other area. For relatively large area sizes, area 2 is a marine reserve with zero effort per unit area. Notably, we have $E_1 > E_2$. This solution corresponds to the red 'circle' points in Figure 5. We refer to the type of solution as the 'right hand side' (RHS) solution. It is worth noting that the two solutions are identical in the extreme cases of $\alpha = 0$ (not shown in the figure, but both solutions have $E_2 = .375$; effort level in area 1 does not matter as it does not exist when $\alpha = 0$) and $\alpha = 0.5$ (both solutions have $E = .832$ in one area and zero in the other; which area is irrelevant because of the symmetry at $\alpha = 0.5$).

In Figure 5, the RHS solution is the optimal solution for all area sizes. In Figure 6 we have demonstrated a situation where the LHS solution is optimal for all area sizes. Here, $m_0 = 1$ as before, but $\delta = .46$. As we will see, the LHS solution dominates the RHS solution for 'small' levels of uncertainty, whilst it is the other way around for higher levels of uncertainty. We are restricted to the 'heterogeneous' parameter area (see Figure 4), and 'small' levels of uncertainty are to be understood relative to that area. Note that the LHS solution does not change much from the change in uncertainty (from $\delta = .5$ to $\delta = .46$), but that the RHS solution changes quite a bit. In Figure 7 (a) and (b) the level of uncertainty is again $\delta = .5$, but the migration rate is $m_0 = 0.5$ and $m_0 = 2$,

respectively. In (a) the LHS solution is optimal for $\alpha > .225$; in (b) the RHS solution is optimal for all area sizes. Note that effort is smaller for the LHS solution and higher for the RHS solution when the migration rate is higher. Another aspect of these different solutions is that expected harvest is basically the same for both types of solutions for a given parameter set.[6] Undoubtedly, the structure of the LHS solution is 'simpler' than that of the RHS solution, which suggest that LHS may be preferable since expected harvest is more or less the same for both solutions. However, one needs to formally consider the cost related to a more complex management scheme to be able to make such assessments in a satisfactory fashion.

Not only is the expected harvest basically unchanged across the two different solutions, it is also practically constant over different area sizes for a given level of uncertainty. That is, when the optimal distribution of effort changes over area sizes, the change in distribution offsets the potential loss of expected harvest from a different biological structure. This further suggests that a different biological structure do not offer the potential to increase expected harvest. As long as there is no constraint on effort in the model (*e.g.*, budget constraint), this result is perhaps to be expected. Hastings and Botsford (1999) derives the same result analytically in the case of marine reserves with an assumption comparable to an infinite migration rate.[7] It would be interesting to look into which of the two alternative distribution structures described above that add up to most *total* effort. Or rather, which of the alternatives is the least costly one. This requires further work, but preliminary results suggest that solutions of the LHS type require less effort. A thorough investigation of these issues should consider the curvature of the marginal cost of effort.

The result that effort per unit area optimally is higher in the small area ($E_1 > E_2$; the RHS solution) is to a certain extent counterintuitive and unexpected (to us, at least). One may comprehend the deviation to heterogeneous distribution of effort as some kind of hedging towards the uncertainty about the

[6] The relative difference in the expected harvest from the two solutions is in the order of 10^{-3} or smaller.

[7] Hastings and Botsford (1999) assumes that the density of larvae, which is the only 'year class' subject to migration, is always equal in the entire habitat. They further assume that the stock is fished down to zero outside the reserve, and thus that reproduction is restricted to the reserve. In other words, they have a sink-source model. Given the stringency of the assumptions Hastings and Botsford (1999) find necessary to derive their result, we are skeptical about the possibility of analytically proving a similar result in a more general model like ours. However, our numerical calculations support such a conjecture.

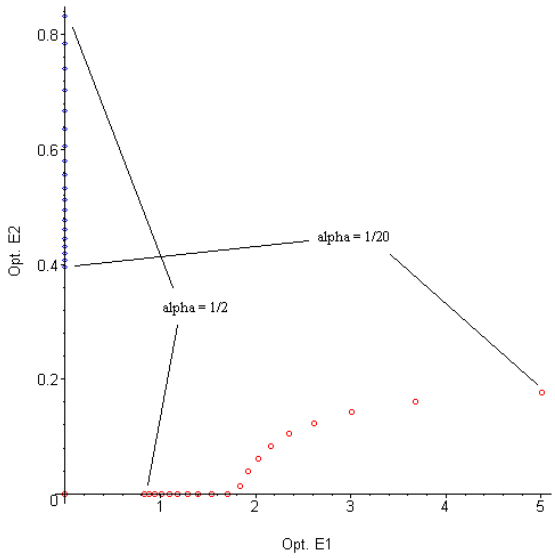


Fig. 5: Two types of heterogeneous distributions of effort. $\delta = .5$ and $m_0 = 1$. Note: The end points corresponds to the area sizes $\alpha = 0.5$ and $\alpha = 0.05$, and each point in between corresponds to a change in area size of .025.

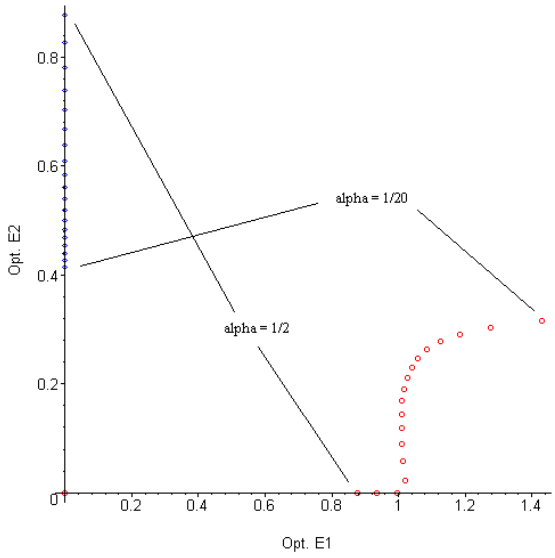
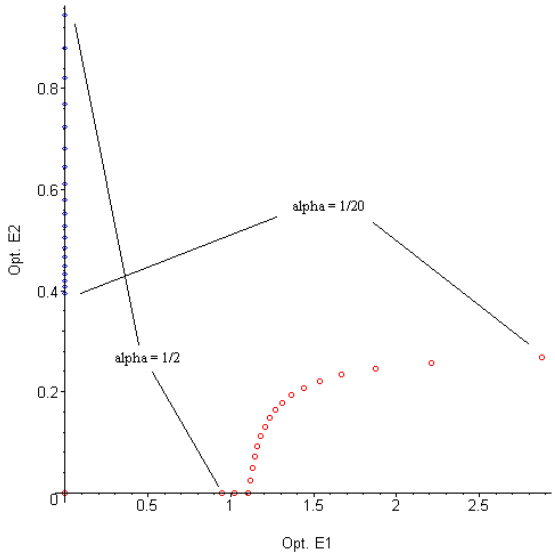
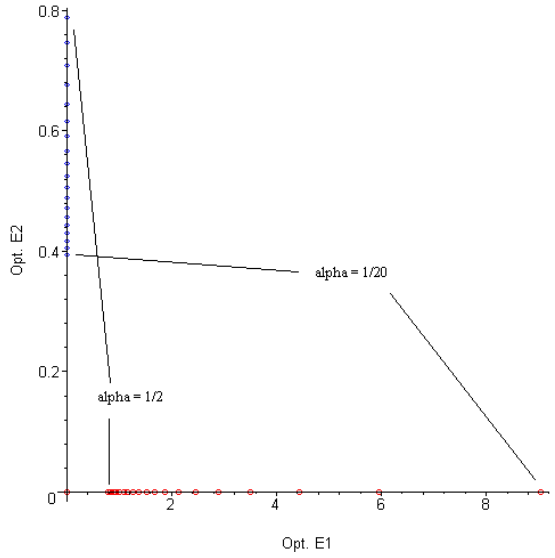


Fig. 6: LHS (blue 'diamond') and RHS (red 'circle') solutions. $\delta = .46$ and $m_0 = 1$.



(a)



(b)

Fig. 7: The two types of solutions (LHS and RHS) under different migration rates: (a) $m_0 = 0.5$, (b) $m_0 = 2$. $\delta = .50$ in both panels.

rate of growth. The idea is to exploit a part of the stock to a lesser degree such that there is a more solid capital stock at hand when the bad state (low rate of growth) occurs. As there is virtually no change to expected harvest across area sizes given the optimal effort levels, it does, however, make sense to sustain a higher stock density in the big area compared to the alternative (of a smaller density in the big area, which intuitively comes across as more vulnerable). The LHS solution, on the other hand, with more effort per unit area in the bigger area ($E_2 > E_1$), is easier to comprehend. The intuition is basically to leave some relatively small area unexploited for a rainy day (not exploited directly, at least; a positive migration rate means that the 'protected' area is indirectly exploited). Note that the LHS solution is equivalent to establishing a marine reserve in area 1. The benefits from marine reserves have been debated, but there seems to be consensus that there are at least biological benefits connected to reserves (see Sanchirico (2000) for a discussion). Again, a deeper economic investigation into these issues should consider the benefits from a simple management structure (compare the LHS and RHS solutions in, for example, Figure 5) and the costs from ignoring the optimal distributions (*e.g.*, from preferring the LHS solution because of its simplicity), but also the value of stability of the biological system. A higher density in the big area means a larger total stock, which intuitively means (to us, at least) a more stabile or robust stock when it comes to uncertainty.

Finally, we address the question of how to characterize the parameter space in terms of the optimality of the different solutions. We have already established the subset of parameters associated with homogeneous distributions in Figure 4. This characterization is further refined in Figure 8. Between the 'box' points (black) and the 'diamond' points (blue), the LHS solution type is optimal *for all* area sizes, and above the 'circle' points (red) the RHS solution is optimal *for all* area sizes. The optimal choice between the to types LHS and RHS depends on the area size in the region between the 'diamond' (blue) and 'circle' (red) points; the LHS solution is preferred for bigger area sizes.

Alternative Migration Specification

It may be more realistic to model the migration rate as dependent on area size. Suppose, for example, $m(\alpha) = m_0 \cdot \alpha(1-\alpha)$. The homogeneous distribution is still optimal in a certain subset of the parameter space. Note that when the

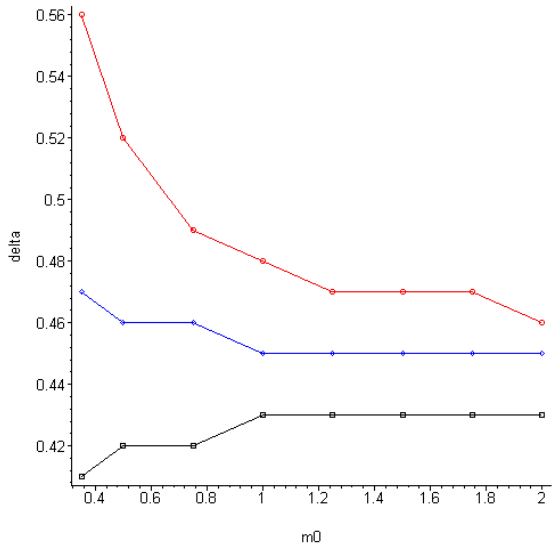


Fig. 8: Characterizing the parameter space: Optimal effort is distributed according to the homogeneous solution below 'box' points (black), according to the LHS solution below 'diamond' points (blue), and according to the RHS solution above the 'circle' points (red).

distribution of effort is homogeneous, the density of fish is identical everywhere and the migration term vanishes, and the resulting expected harvest is identical in the two specifications. Numerical investigations (not provided at present) suggest that the homogeneous distribution is optimal for the alternative migration model in much the same subset of the parameter space as was found in the previous section (Figure 4). The comparison is somewhat troublesome, however, as this specification implies very small migration rates for small area sizes; small migration rates are generally problematic. The two types of solutions (LHS and RHS) are shown in Figure 9. The results resembles the corresponding results from the constant migration rate model (Figure 5). Observer however that the effort level in area 2 does not run off to infinity for small area sizes in the RHS solution, as it does under the constant migration rate specification. This is maybe more a realistic behavior of the solution than seen earlier.

Note that m_0 is set considerably higher in the alternative migration results than previously ($m_0 = 6$ vs. $m_0 = 1$); it is not clear to what values of m_0 should be compared, given the different structure of $m(\alpha)$. (A suggestion is to look at the mean migration rate over area sizes. The mean over all possible area sizes ($\alpha \in [0, 1]$) for the alternative migration specification with $m_0 = 6$ is 1.)

Discussion

The main result in this paper is the characterization of the parameter space into different regions of homogeneous and heterogeneous optimal distributions of effort (Figure 4 and Figure 8). Our characterization is based on solutions to be homogeneous *for all* area sizes. It is possible to make the same characterization and only require that solutions up to a given area size is homogeneous, and thus construct a hyperplane in the parameter space (then consisting of level of uncertainty, rate of migration and area size) which discerns the homogeneous and heterogeneous solutions in more detail. The subset in Figure 4 corresponding to the homogeneous solution may come across as rather large compared to the considered range of the different parameters. There are two objections to such an observation. First of all, given the normalization of different parameters (that is, mean rate of growth, total carrying capacity and catchability coefficient), we are in no position to judge which region of the parameter space is larger' in the sense of being more likely. The likeliness of different regions

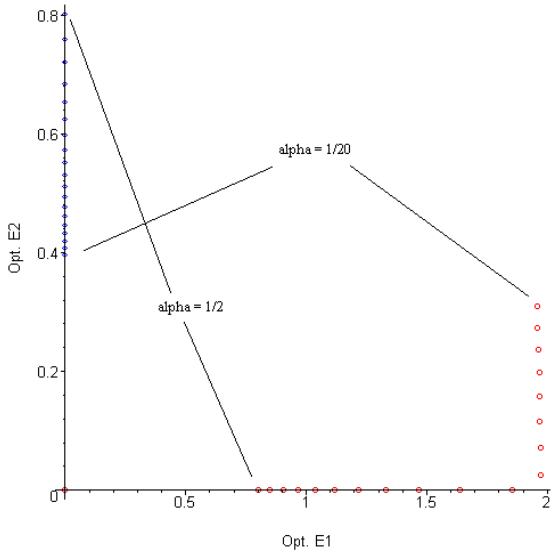


Fig. 9: LHS (blue 'diamond') and RHS (red 'circle') solutions under the alternative migration specification. $\delta = .50$ and $m_0 = 6$.

is obviously of utmost concern when it comes to applications to real fisheries. Such questions are also best addressed through empirical studies. Secondly, we have largely chosen to limit ourselves to the region where $\delta \leq .5$, first of all to avoid computational issues ($\delta = .5$ poses a natural limit in the single area model to which we have sought comparability). In a more comprehensive treatise, the entire parameter space which supports a sound model should be considered. The 'size' of the parameter subset connected to the homogeneous solution can only be determined with these two concerns in mind. Notwithstanding, the size itself of parameter subsets is really of minor concern. We are more concerned with exploring possibilities and limitations of the present model.

When the level of uncertainty reaches a certain level, expected harvest is higher when effort is distributed heterogeneously. Depending on the level of uncertainty, effort distributions implying one area being a marine reserve are optimal. Figure 8 suggests that marine reserves are optimal when the uncertainty is relatively close to the 'homogeneous' region. However, increasing uncertainty further gives rise to different types of heterogeneous effort distributions. Interestingly enough, it is sometimes optimal to put more effort per unit area into the smaller of the two areas ($E_1 > E_2$). This may be interpreted as some kind of hedging strategy and indicates that a larger density of fish in the big area performs better given the bad (low) outcome of the uncertainty compared to how bad the smaller density of fish in the small area performs given the good (high) outcome. However, the distributions where $E_1 > E_2$ have a rather complex structure, and one may argue that the more simple structure of the distributions with $E_2 > E_1$ is valuable, and that it outweighs the error of ignoring the other type of distribution. This argument is particularly compelling given the small difference in expected harvest between the two types of effort distributions. An analysis of these issues is a potential extension of this work.

Finally, we change the shape of the migration term, such that the rate of migration depends on area size. There are convincing arguments for both migration models, and we do not go into that discussion here. However, the findings follow much the same pattern as before; the homogeneous solution is dominant for a considerable subset of the parameter space, and there exist two types of solutions outside this subset which are optimal under different parameter configurations.

Future challenges in this line of research are many. First of all, a dynamic

analysis abstracting from the steady state would be highly valuable. Further, it is natural to think of extensions of the present analysis to uncertainty about other parameters, several parameters, other kinds of uncertainty, and ultimately uncertainty about the modeling itself. Also, a multispecies approach would be interesting. However, what the present analysis lacks is interpretation into an economic framework. Introducing a budget constraint (or rather, marginal costs) on effort, for example, is prone to change the optimal distribution quite a bit, particularly outside the range of the homogeneous solution. Also, several issues of distributing effort heterogeneously, for example congestion, administration and enforcement, or different variable costs, could be addressed.

Appendix: Steady state expressions

Set $g_1(x_1, x_2) = 0$ and solve for x_2 to get[8]

$$x_2^* = \frac{x_1\alpha\left(r - r\alpha - rx_1 - m(\alpha) + E_1(\alpha - 1)\right)}{m(\alpha)(\alpha - 1)}$$

which is the steady state expression of x_2 in terms of x_1, α, and the effort pair (E_1, E_2). Substituting x_2^* into $g_2(x_1, x_2)$ yields a polynomial in x_1 to the fourth degree. The polynomial has two real and two complex roots. The complex roots has no meaningful economic interpretation, and I focus on the two real roots. One of these is always zero, this leaves one interesting solution. Let it be x_1^*.

An underlying condition not yet mentioned is $x_i \geq 0$. The interpretation of this condition is straightforward. The implementation of it, however, tend to complicate things, making both x_i^*, $i = 1, 2$, piecewise continuous functions, instead of continuous functions in all parameters. Temporarily disregarding this technicality, we are able to describe x_1^* in general terms;

$$x_1^* = d_1\psi + \frac{d_2E_1^2 - d_3E_2}{d_4\psi} - d_5E_1 + d_6$$

where

$$\psi = \sqrt[3]{c_1 - c_2E_1E_2 + c_3\sqrt{c_4E_2^3 + c_5E_1^3 - c_6E_1^2E_2^2 - c_7E_1E_2 + c_8}} + c_9E_1^3$$

where all $c_i, d_i \geq 0$ are different combinations of r, α, and $m(\alpha)$. Note that x_1^*,

[8] Note that the catchability coefficient q is normalized to one.

and consequently x_2^*, may be interpreted as a function of the pair (E_1, E_2).

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