<u>Working Paper No 03/11</u>

Electricity production in a hydro system with a reservoir constraint

by

Lars Mathiesen
Jostein Skaar
Lars Sørgard

SNF project no 9031
"Improving competition policy"

The project is funded by the Research Council of Norway

INSTITUTE FOR RESEARCH IN ECONOMICS AND BUSINESS ADMINISTRATION
BERGEN, FEBRUARY 2011
ISSN 1503-2140

Electricity production in a hydro system with a reservoir constraint[1]

Lars Mathiesen
Department of Economics, Norwegian School of Economics and Business Administration

Jostein Skaar
The Norwegian Competition Authority

Lars Sørgard
Department of Economics, Norwegian School of Economics and Business Administration

February 07, 2011

Abstract

The purpose of this article is to analyze how market power may affect the allocation of production between seasons (summer and winter) in a hydro power system with reservoir constraints and inflow uncertainty. We find that even without market power the price in the summer season may be lower than the expected price in the winter season. Market power may in some situations lead to higher sales and lower price in summer than the competitive outcome and in other situations to the opposite result. Furthermore, market power may lead to a smaller price difference between summer and winter than in a competitive market.

[1] This is a substantially revised version of an earlier draft with the title 'Water with power: Market power and supply shortage in dry years'. We are indebted to seminar participants at Norwegian School of Economics and Business Administration for helpful comments, and to the Research Council of Norway for funding through the project 'Improving competition policy' at SNF. This paper presents our personal views, and it is not necessarily shared by the Norwegian Competition Authority.

1. Introduction

In some countries electricity production is dominated by hydro power.[2] A system where water can be stored in reservoirs and there is uncertainty about the inflow is quite complex. A large fill in one period may lead to overflow in the next if there is heavy rain. We are concerned with the consequences of market power in this setting. How do we expect a producer with market power to behave? A common argument is that a firm with market power could benefit from supplying much in one period thereby limiting its supply in the next period. This conclusion finds support in the literature.[3] We show that the opposite may be true, and furthermore that market power in a setting with uncertain inflow and a restricted reservoir may lead to smaller price differences between periods than what would be observed in a competitive outcome. This seems to contrast with received wisdom.

> 'In a perfectly competitive hydro system, demand peaks tend to be shaved off.. . Conversely, when hydro generators have market power the temporal separation is further increased.' (Rangel, 2008, p. 1296.)

This study was motivated by observations in the Nordic power market 2002-03. Spot prices in January 2003 were three to four times the average price in a normal year. [4] Producers blamed the price hike on the low inflow to reservoirs in autumn 2002. Data seems to support this claim. [5] Is this the whole story, or could it be that strategic behavior contributed to the price hike? Victor D. Norman, the Minister responsible for competition policy, said the following at the outset of the period with supply shortage:

> 'A situation with low prices during summer and high prices during winter may indicate that there has been an abuse of market power.' (Dagbladet, Nov. 13, 2002)

Our problem relates to the medium term where uncertain inflow combined with restricted storage capacity is of importance.[6] It corresponds to an analysis of transfer of water from a

---

[2] In New Zealand 60% of production is from hydro, in Chile 70%, Brazil 97% and Norway close to 100%.

[3] See for example Crampes and Moreaux (2001), Garcia *et al.* (2001), Johnsen (2001), and Bushnell (2003).

[4] It is of course inappropriate to infer from differences between *ex post* summer and winter prices that market power is abused. Such differences may entirely stem from stochastic events. The use of a theoretical model, however, may indicate under which conditions a firm has an incentive to exploit its market power.

[5] For a discussion of the low inflow 2002/03, see Bye *et al.* (2003). During the winter 2009/10 spot prices repeatedly reached 20-40 times their normal level after a dry and cold autumn/winter, and high spot prices were also observed in the first part of the autumn/winter 2010/11 when we had low temperatures and low inflow.

[6] Kelman *et al.* (2001) argue that uncertainty of inflow is a medium term and not a short term issue.

high-inflow period with low demand to a low-inflow period with high demand, i.e., in the Nordic countries summer and winter, where reservoirs are run down during winter and refilled when snow melts in spring and summer. Summer is the first period of the model. Then available water from melting of snow is known. Rainfall during autumn is uncertain in summer. If there is heavy rain, we assume that the inflow is so large that reservoirs become full and some water may have to be spilled. If there is little rain, however, we assume that the entire inflow will be stored for production in the succeeding winter season.[7]

The basics of this economic problem are well known from other sectors and the literature.[8] Optimal capacity in a system with uncertain demand balances the expected loss from having too much to offer, *i.e.*, having to waste the marginal unit, with the expected loss from carrying too little, *i.e.*, having to forego sale of the marginal unit. Optimal allocation implies that sometimes there will be waste, which in our case is spill of water. In addition to this classical structure – which here translates into a restricted reservoir capacity and uncertain inflow of water – our demand for electricity is price dependent and there is market power. Our model can also be relevant for durable goods markets, where volumes sold in one period will influence the optimal price setting in the following periods. This shares some similarities with the interaction between periods that we have modeled.

A producer may obviously profit if he is allowed to withhold supply and spill water. In that case, comparing operations of a price taker and a price maker provides an easy conclusion.[9] We will, however, adopt the seemingly common assumption in the analysis of hydro power systems that the regulator does not allow a producer to spill water for other reasons than overflow.[10] Unless there is overflow because of insufficient reservoir capacity, he has to supply all available water during the two periods.[11] Thus our problem is essentially one of allocating a given total supply between two periods.

[7] Hydro reservoirs allow the system to smooth extreme variations in inflow over several years. By modeling only one year we disregard this possibility. We do not think this reduces the relevance of our results.
[8] Cf. the classical Newsvendor problem. See any textbook in Operations Research.
[9] The price maker will supply less and obtain a higher price. See figures 3.5 and 3.6 in Kelman *et al*. (2001).
[10] See Crampes and Moreaux (2001), Johnsen (2001), Bushnell (2003), Skaar and Sørgard (2006), Førsund (2007), and Hansen (2009).
[11] Of course, by his operations in period 1 the producer may provoke overflow and spill in period 2. This mechanism is incorporated into our model.

We show that when the probability for spill of water is positive, the expected winter price will be higher than the summer price even under perfect competition. Intuitively, if the summer price equaled expected winter price, a competitive firm would benefit from selling more during summer than to wait and risk a spill of water. The implication is that differences between summer prices and expected winter prices do not warrant a conclusion that there has been an abuse of market power.

Our approach to the analysis uses the competitive outcome as the point of departure and checks whether a monopolist would deviate from this outcome. We derive simple formulas for when he would supply more (less) in period 1 than a competitive firm. It turns out that price elasticities measured at the competitive outcome in the two periods determine in which direction the monopolist would deviate from the competitive outcome. The general flavor is that for a given capacity, the monopolist will supply more than the competitive outcome in the period with the most elastic demand.

The simplest case is when the reservoir capacity is so low that a high inflow leads to spill of water and the opportunity cost of water is zero. Then, only the marginal revenue in the low inflow state is relevant. The monopolist will now sell more than the price taker in period 1 if the price elasticity (at the competitive outcome) in period 1 is than larger in absolute value than the price elasticity in period 2 with low inflow. This is in line with standard theory where a monopolist reallocates sales from a market with low price elasticity to a market with a higher elasticity as this latter market can better sustain a larger volume. If he sells less during summer and more during winter, the price difference between summer and winter becomes smaller than in the competitive outcome.

If demand is equal in the two periods, one might think that the monopolist has no incentive to deviate from the competitive outcome. This is correct when demand is described by a constant price elasticity function, but in general it is not true. When the price elasticity increases (in absolute value) with price, as for example with linear demand, the monopolist will supply less than the competitive outcome in period 1 because the competitive price (and thus the price elasticity) in period 1 is lower than the price (and price elasticity) in period 2 with low inflow.

If the reservoir capacity is sufficient and there is no spill, the price elasticities of demand can still be sufficient information to determine the direction of the distortion. For example, the monopolist will supply less (more) in period 1 if the price elasticity of demand in period 1 is lower (higher) than both price elasticities in period 2. If, however, the price elasticity of period 1 is between the elasticities of period 2, the formula is more complex, and also price levels and the probability of a high inflow matters.

Reducing capacity marginally such that the solution shifts from being unconstrained to being constrained, the optimal response for the monopolist is typically to increase production in period 1 in order to avoid spill of water if there is a high inflow. However, demand for electricity is known to have low price elasticity and the constraint that all available water has to be used for production may result in negative marginal revenue in period 2 with high inflow. In that situation, the monopolist would reduce his supply in period 1 rather than increase it as a response to the reservoir constraint. Reduced supply in period 1 leads to a higher price in period 1. It allows a higher supply in period 2 when there is a low inflow and thus a reduction in the price that at the outset is too high in this state. Now water in the high inflow state is spilled at zero cost, rather than sold at negative marginal revenue.

Our results have implications for the question whether market power aggravates or not a supply shortage in periods with low inflow. According to the existing literature, market power leads to such distortion. We show that this is not always true. It depends on demand conditions during winter and summer, in particular the price elasticities of demand in the two periods. Unfortunately, to our knowledge there are no studies that test empirically the possible difference in price elasticities of demand between periods (summer and winter).

Garcia *et al.* (2001) have also analysed a situation with water inflow and reservoir constraints, although in a very different model. For example, they assume an infinite horizon model with strategic interaction in a duopoly. They offer valuable insight into not only each producer's incentive to store water, but also the incentives for them to collude and the consequences of a price cap. They apply a rectangular demand function, where the price during winter time is exogenously given, and we complement their analysis by assuming a downward sloping demand curve. In our model a reallocation in sales between periods can lead to a change in prices. A shift in production from summer to winter would lead to higher summer prices and lower winter prices, a mechanism not present in Garcia et al. (2001). This explains why we find that market power in some instances can shift production from summer to winter and in other instances lead to a reallocation the other way.

Johnsen (2001) analyses market power, uncertain inflow, and storage in a two-period model with limited transmission capacity between two regions.[12] He assumes that a producer cannot dispose of water, but has to use all for production over the two periods. He disregards reservoir capacity. Compared to the competitive solution a monopolist increases production and stores less in the first period (when available water is certain) in order to become import constrained in the second period and thereby be able to create shortage and higher price. Also by storing less, the monopolist reduces the probability of becoming export constrained in the second period if a high inflow should occur.

Kelman *et al*. (2001), Bushnell (2003), and Evans *et al*. (2010) perform numerical analyses of hydrothermal competition in the electricity markets in Brazil, California and New Zealand respectively, allowing for market power and inflow uncertainty. Evans *et al*. model continuous time, Bushnell studies periods within a time horizon of one month, while Kelman *et al*. studies months within a 3-year time horizon. This time perspective of operations on a seasonal basis resembles ours. While hydro production is almost the same in the monopoly and the competitive solutions both in Kelman *et al* and Evans et al., activity levels of thermal production are strikingly different. Thus total supply is lower and prices are higher. Bushnell observes that a producer who exerts market power, but is not allowed to spill (dispose of) water may experience negative marginal value of water. (See BPA in Table 8.)

This article is organized as follows. In the next section we formulate a theoretical model and report some general results. In Section 3 we provide some numerical examples to illustrate the complex interplay between the reservoir constraint and the functional form of the demand. In Section 4 we offer some concluding remarks.

[12] Hansen (2009) uses a two-period model of one region with uncertain inflow and compares the competitive outcome with both a monopoly and a Cournot outcome. In his model there are no reservoir constraints and thereby no possibility for spill of water.

2. A theoretical model

2.1 The model

Our model has one region, two periods, uncertain inflow of water in the second period and a restricted reservoir capacity (*R*).[13] The uncertainty is represented by two states of the world – a low (*L*) and a high (*H*) inflow of water in (the beginning of) the second period. We assume that capacity *R* is sufficient in the low inflow state, while it may be insufficient in the high inflow state. If so, there will be spill of water. As mentioned above, if a producer can spill water in order to ration supply and drive up the price, the result is obvious: A monopoly will supply less than the competitive outcome and prices will be higher. Cf. Kelman *et al*. (2001). We therefore assume that all available water has to be used for electricity production unless there is overflow. To focus on the medium term problem – which can be interpreted as one year - we also assume that all available water is used by the end of period 2; nothing is stored into the next year. Demand in each period depends on the price in this period (and state). Full use of available water may cause marginal revenues or even prices to become negative, to which we will return below. For the moment, however, we assume that prices and marginal revenues are non-negative.

On the supply side we consider two regimes – monopoly and perfect competition. While the monopoly is one firm – a price maker – the competitive market consists of many price taking firms and it is their aggregate volume (production and reservoir fill) that matters. Expected profit (or revenue as we shall ignore costs other than the opportunity cost of water) of a firm (monopoly) producing from water is profit in period 1 plus expected profit from period 2:

$$(1) \qquad E\pi = p_1 x_1 + \{ q\, p_{2H} x_{2H} + (1-q)\, p_{2L} x_{2L} \}$$
$$= p_1[I_1 - r_1] + \{q p_{2H}[r_1 + I_{2H} - w] + (1-q) p_{2L}[r_1 + I_{2L}]\}.$$

x_1 , $p_1(x_1)$, and I_1 denote production, price, and inflow of period 1, x_{2s} , $p_{2s}(x_{2s})$, while I_{2s} denote production, price, and inflow of period 2 state *s*, *s* = *H*, *L*. r_t denotes storage at the end of period *t* and *q* is the probability of a high inflow (I_{2H}). In state *H*, water may have to be spilled (*w*) such that the reservoir capacity (*R*) is not violated

[13] The model builds on Johnsen (2001) except that his model has restricted transmission between regions, while we focus on restricted storage capacity.

(2) $r_1 + I_{2H} - w \leq R$.

The firm has two non-negative decision variables: r_1 and w. The requirement that all water is used is embedded in the model structure as x_{2s}, $s = H, L$, are not separate decision variables, but determined by supply in period 1 (x_1) and possible spill (w).

Both behavioral types want to maximize profits subject to the reservoir and non-negativity constraints. A price taker considers prices as given beyond his control while a monopoly realizes that it can influence prices. In order to present both behavioral regimes on a compact form, let $z = p$ for the price takers and $z = MR$ (marginal revenue) for the price maker. Finally, let λ denote the shadow price on the reservoir capacity interpreted as marginal cost of capacity. First order conditions for optimal operations are then[14]

(3) i) $v_1 = z_1 + \lambda - (qz_{2H} + (1-q)z_{2L}) \geq 0$, ii) $r_1 \geq 0$, and iii) $v_1 r_1 = 0$,[15]

(4) i) $v_2 = qz_{2H} - \lambda \geq 0$, ii) $w \geq 0$, and iii) $v_2 w = 0$,

(5) i) $v_3 = R + w - (r_1 + I_{2H}) \geq 0$, ii) $\lambda \geq 0$, and iii) $v_3 \lambda = 0$.[16]

Condition (3) is about optimal inventory management. (3i) says that in equilibrium the expected value (price or marginal revenue) from selling the marginal stored unit cannot exceed its marginal cost as determined by forgone sale in period 1 and marginal cost of storage. Furthermore, inventory has to be non-negative (3ii), and if it is positive, the expected value from sale in period 2 equals its marginal cost (3iii). That is, each firm will balance its storage fill (production) such that this obtains.[17] Condition (4) is about spill of water in state

[14] (3)-(5) are equilibrium conditions for two models: The monopoly with one price maker and the competitive model with an aggregation of many price taking firms. See the Appendix. Formulating an equilibrium model directly antedates Samuelson (1952), who suggested the artificial objective of maximizing the sum of producers' and consumers' surpluses in order to obtain a computable model for the competitive equilibrium. The direct approach was again brought forward by Scarf's (1973) development of the fixed-point algorithm and received renewed interest. Mathiesen (1985) and Bushnell (2003), among others, employed this format.

[15] qp_{2H} and $(1-q)p_{2L}$ are Arrow-Debreu prices, where p_{2s} is a price contingent on being in state s, $s = H, L$, and $p_2 \equiv (qp_{2H} + (1-q)p_{2L})$ is the price of electricity in period 2 as evaluated in period 1. Mathiesen (1992) interpreted p_{2s} as spot prices and p_2 as a guaranteed price (irrespective of the outcome of inflow).

[16] A complementary slackness condition $vy = 0$, implies that if $y > 0$, $v = 0$, and if $v > 0$, $y = 0$. In general, $y = 0$ does not imply $v > 0$ and, $v = 0$, does not imply $y > 0$.

[17] This interpretation (in terms of prices) is straight forward for the monopoly – it can influence prices. Each individual price taker cannot, but aggregate volumes adjust such that prices are balanced in equilibrium.

H and hence the reservoir capacity. The marginal value of capacity (λ) is constrained by the expected value of water in state *H*, *i.e.*, the Arrow-Debreu price. If there is spill, (4i) holds with equality, while if there is excess capacity the marginal value of capacity is zero. Condition (5i) is the reservoir constraint. (5ii) requires the shadow price of capacity to be non-negative, and (5iii) says that when this shadow price is positive, capacity binds, and when there is slack capacity, the shadow price is zero. We will return to further interpretations of these conditions below.

Observe that although the model (3)-(5) is solved for both periods simultaneously, it includes the full set of contingent second stage variables (x_{2s}), $s = H, L$, such that optimal choices are being made *ex post* the resolution of uncertainty. Thus the solution is of closed loop type.

Because we focus on the implications of restricted storage capacity, *i.e.*, a full rather than an empty storage, we assume that $r_1 = I_1 - x_1 > 0$, whereby (3iii) implies

(3i') $$z_1 + \lambda = qz_{2H} + (1-q)z_{2L} \equiv E(z_2).$$

2.2 Intuition about the solutions

We want to characterize optimal water allocations for different levels of reservoir capacity treating *R* as a parameter. The model is phrased in terms of storage (r_1), but we find it more convenient to discuss its solutions in terms of production in period 1 ($x_1 = I_1 - r_1$). Production will be in either of three regimes as illustrated in Figure 1.

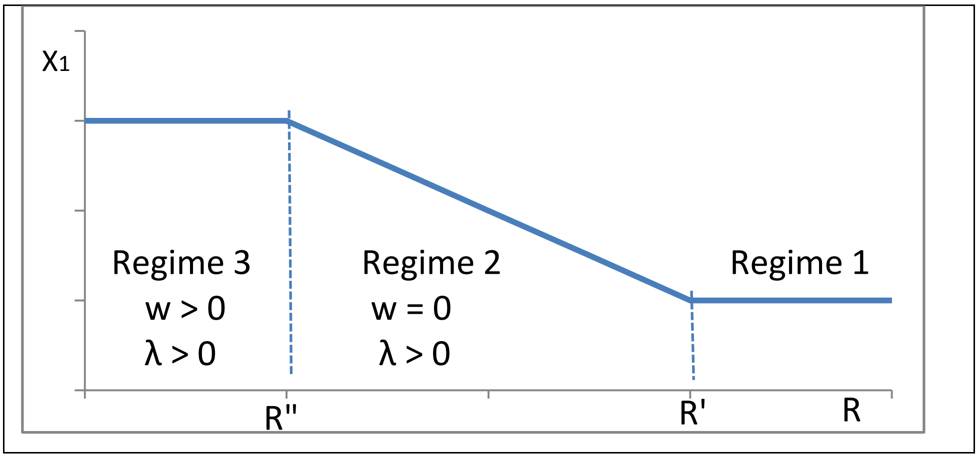


Figure 1. Optimal production x_1 as a function of reservoir capacity *R*

If reservoir capacity is sufficiently large ($R > R'$), the producer can choose the unconstrained production in period 1. That is, x_1 is independent of R ($dx_1/dR = 0$), which is illustrated in the right part of Figure 1. This solution is labeled *Regime 1*.

For a reservoir capacity below R', choosing the unconstrained production in period 1 would lead to spill of water in state H in period 2. Spill implies a loss of revenue compared to the unconstrained solution. For a medium reservoir capacity, with $R'' < R < R'$, it is better to increase production in period 1 and thereby reduce the amount of water stored. This is in Figure 1 labeled *Regime 2*, where the shadow price (λ) is positive because of the lost revenue from reallocating sales, but where there is no spill of water ($w = 0$). Reduced reservoir capacity is matched by a corresponding increase in production in period 1 ($dx_1/dR = -1$).

For a low reservoir capacity, illustrated in Figure 1 with $R < R''$, the production in period 1 would have to be substantially higher than the unconstrained solution in order to avoid spill of water. The costs associated with such distorted sales at a very low price in period 1, are so large that the firm would be better off choosing a production in period 1 that leads to spill of water in state H. This is labeled *Regime 3*, where there is a positive shadow price on the reservoir capacity ($\lambda > 0$) and spill of water ($w > 0$). In this regime a further reduction in reservoir capacity will have no effect on the level of production in period 1 ($dx_1/dR = 0$), but only lead to increased spill of water if there is a high inflow ($dw/dR = -1$).

2.3 Definition of Regimes

Let x_1^{ki} denote the optimal production level in period 1, where $i = M, C$ for monopoly and competition respectively, and $k = 1, 2, 3$ for the regimes. In period 2 the optimal production levels are denoted x_{2s}^{ki}, where $s = L, H$. Optimal prices are defined similarly.

Regime 1 [High reservoir capacity]:

When reservoir capacity is sufficient, *i.e.*, $R > I_1 + I_{2H} - x_1 - w$, (5iii) implies that $\lambda = 0$. From (3i') we have the price condition (or first-order condition for optimal x_1)

(6) $\quad z_1 = E(z_2).$

The price taker allocates water between periods such that the price he obtains in period 1 equals the expected price of period 2,[18] while the monopoly similarly balances marginal revenues.[19] This leads to optimal production levels x_1^{1C} respectively x_1^{1M} in period 1.

When the reservoir capacity becomes scarce (2) may be rewritten as

(2') $x_1 + w = (I_1 + I_{2H}) - R.$

To balance the storage of water one can either increase x_1 beyond x_1^{1i} (reduce storage (r_1)) (*Regime 2*), or aim for spill of water in state H, $w > 0$ (*Regime 3*).

Regime 2 [Medium reservoir capacity]:

Increased x_1 means increased supply in period 1 and reduced supply in period 2. When $p_t(x)$, $t = 1, 2$, is decreasing and strictly monotone, increased x_1 implies lower price in period 1 [$p_1(x_1) < p_1(x_1^{1C})$] and higher average price in period 2 [$E(p_2) > E(p_{2s}^{1C})$]. The same argument applies to the monopoly when $MR_t(x)$, $t = 1, 2$, is decreasing and strictly monotone. When $x_1 > x_1^{1i}$, it therefore follows from (3i) that

(7) $\lambda = E(z_2) - z_1 > 0.$

λ now measures the expected loss of (marginal) revenue from reallocating supply from period 2 to period 1. Thus, when R constrains production, $\lambda > 0$ and thereby

(6') $z_1 < E(z_2).$

Intuitively, it is more profitable to sell a unit at a positive price (marginal revenue) in period 1 than possibly have to spill it in period 2 although the price (marginal revenue) in period 1 is below the expected price (marginal revenue) in period 2.

Regime 3 [Low reservoir capacity]:

[18] This does not say that the individual price taker tries to control prices, but that in a competitive equilibrium prices satisfy this condition because of aggregate adjustments by the firms.

[19] The analysis in this regime is analogous to the analyses in Johnsen (2001) and Hansen (2009).

If the producer spills water ($w > 0$), (4i) and (4iii) imply that $\lambda = qz_{2H}$, and (3i') simplifies to

(6'') $z_1 = (1-q)z_{2L}$.

The price (marginal revenue) obtained in state *H* is irrelevant to the pricing condition because the marginal, stored unit is spilled.

When expected profits are concave in r_1 and w, and inflows I_1, I_{2H} , I_{2L}, and capacity *R* are positive the model has a unique solution for both types. (See Lemma 1 in the Appendix.) Depending on the particular parameter values this solution may be in *Regime 1*, *2*, or *3*.

2.4 Results concerning allocation of production

Our goal is to describe possible consequences of market power. Taking the competitive outcome as a reference we ask whether a monopoly has an incentive to deviate from this solution.[20] Consider the competitive outcome with optimal production and measure price elasticities of demand at these volumes. Let ε_1 , ε_{2H} and, ε_{2L} denote the elasticities in period 1, in period 2 state H and state L, respectively.

While the solution for both behavioral types will be in any of the three regimes illustrated in Figure 1, they may be in the same regime (for a given *R*) or they may be in different regimes. Assume first both solutions are in the same regime.

Proposition 1: Solutions in the same regime.

Assume that $p_t(x) > 0$ *and* $MR_t(x) > 0$, $t = 1, 2$, *for all relevant x. Then*

(i) *In Regime 1 [high reservoir capacity],* $x_1^{1M} > x_1^{1C}$ if

- $\varepsilon_1 < \varepsilon_{2s}$, $s = H, L$ (*sufficient condition*)
- $\dfrac{\frac{1}{\varepsilon_1} - \frac{1}{\varepsilon_{2H}}}{\frac{1}{\varepsilon_{2L}} - \frac{1}{\varepsilon_1}} > \dfrac{p_{2L}}{p_{2H}} \cdot \dfrac{1-q}{q}$ (*necessary and sufficient condition*).

(ii) *In Regime 2 [medium reservoir capacity],* $x_1^{2M} = x_1^{2C}$.

(iii) *In Regime 3 [low reservoir capacity],* $x_1^{3M} > x_1^{3C}$ *if* $\varepsilon_1 < \varepsilon_{2L}$.

[20] Farrell and Shapiro (1990, 2010) used this approach to check whether a merged entity would find it profitable to deviate from the prices set by the merger candidates prior to the merger.

Proof. See the Appendix.

In *Regimes 1* and *3*, supply differs for the two behavioral types when price elasticities differ between periods. The monopolist will supply more than the competitive outcome in the period where demand is most elastic.

A monopoly will set price in accordance with the inverse price elasticity rule. Thus, the Proposition shows that differences in (inverse) price elasticities are decisive for whether and how a monopoly will deviate from the competitive outcome. In *Regime 3* the trade off is between demand in period 1 and demand in state *L* of period 2. With identical elasticities, the monopoly has no incentive to charge other prices than the competitive outcome. If elasticities differ, however, the monopoly would profit from increasing its supply when demand is most elastic.

In *Regime 1, with high reservoir capacity,* three prices are involved and the rule is more complicated. One trivial case is when demand in period 1 is more elastic than demand in both states of period 2. Then the monopoly would produce more than the competitive outcome in period 1. The opposite case is also trivial. If demand in period 1 is less elastic than demand in both states of period 2, the monopoly would supply more than the competitive outcome in period 2. The more complex case is when the price elasticity of demand in period 1 lies between the two elasticities of period 2, $\varepsilon_{2L} < \varepsilon_1 < \varepsilon_{2H}$.[21] Then, a higher monopoly supply than the competitive outcome in period 1 and thereby a lower supply in period 2 is more likely (i) the lower the probability of high inflow (*q*) and (ii) the lower the price with low inflow (p_{2L}).

Proposition 1 applies to situations where both firms' solution are in the same regime. But this may not be the case. For example, the monopoly may spill water (*Regime 3*) while the competitive firm has a higher production (*Regime 2*).

[21] For example, when demand is linear the price elasticity increases in magnitude with price. Being a negative number, it is lower at the high price of a low inflow (supply). Thus, $\varepsilon_{2L} < \varepsilon_{2H}$.

Figure 2 illustrates how the two firms' solution can be in different regimes. Consider the left panel where demand in period 1 is most elastic. When both firms are in the same regime, *i.e.*, $R > R_C^{'}$, $R_C^{''} < R < R_M^{'}$, or $R < R_M^{''}$, the graphs illustrate Proposition 1. Within *Regimes 1* and *3* the monopoly produces more than the competitive outcome in period 1, while their production is equal in *Regime 2*. With unequal demand in the two periods the breakpoints

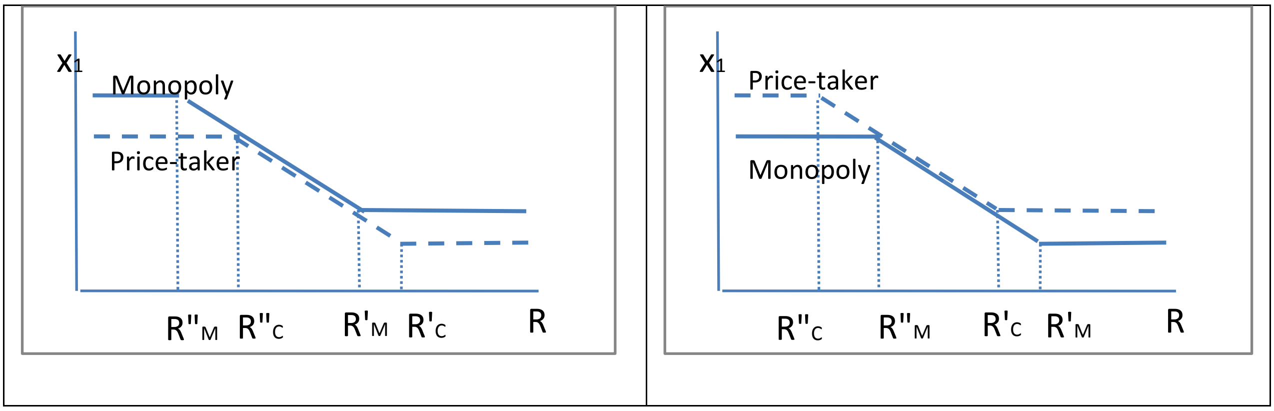


Figure 2: Optimal x_1 as function of reservoir capacity (*R*) with constant price elasticity.
(a) Most elastic demand in period 1. (b) Most elastic demand in period 2.

between the regimes may differ for the two types. Hence, there is an interval ($R_M^{'} < R < R_C^{'}$) where the monopoly solution is in *Regime 1*, while the competitive solution is in *Regime 2* and similarly at the other end, the competitive solution gets into *Regime 3* (at $R_C^{''}$) before the monopoly solution (at $R_M^{''}$). In the right panel demand is most elastic in period 2 and the graphs are switched. Figure 3 below illustrates the possibility that there may be no overlap between regimes.

We can summarize these observations as follows:

Proposition 2:

Assume that $p_t(x) > 0$ *and* $MR_t(x) > 0$, $t = 1, 2$, *for all relevant* x.

Then $x_1^{kM} \geq x_1^{kC}$, $k = 1, 2, 3$ *if* $\varepsilon_1 < \varepsilon_2^L$.

We have assumed positive price and marginal revenue. For some demand functions both may be negative for "large" volumes. For example, with a linear demand function $p = a + bx$, price is negative for $x > X = -a/b$, and the marginal revenue is negative for $x > X/2$.[22] We will maintain the assumption of a positive price, but consider negative marginal revenue.[23] The critical issue is marginal revenue in period 2 state H when $R = R'$, *i.e.*, at the breakpoint between *Regimes 1* and *2*. If $MR_2^H(R') < 0$, (3i') implies $MR_1 > E(MR_2)$, which would signal that reallocation from period 2 to period 1 is constrained, *i.e.*, $r_1 = 0$. However, we have assumed that $r_1 = I_1 - x_1 > 0$ because of our focus is on the spill of water, not an empty reservoir. Thus, increasing x_1 (as in *Regime 2*) is inconsistent with optimality when $MR_2^H(R') < 0$ *and* $r_1 > 0$. In fact, *Regime 2* does not appear when $MR_2^H(R') < 0$.

Proposition 3:
Assume $MR_{2H}(R') < 0$. *Then Regime 2 is no longer present and monopoly production in Regime 3[low reservoir capacity] is lower than in Regime 1 [high reservoir capacity], i.e.,* $x_1^{3M} < x_1^{1M}$.

Proof. See the appendix.

Negative marginal revenue means that (compared to demand) there is too much water available in state H of period 2. If the reservoir does not constrain water allocation, the monopoly dampens this problem by supplying more in period 1 and thereby less in period 2. However, this drives down the price in period 1. If the reservoir constrains allocation, supply in period 2 with high inflow is separated from supply in period 1. Then there is no reason to distort production in period 1 and the monopolist can respond to a reservoir constraint by supplying less in period 1. Figure 4 (below) illustrates the monopoly's optimal production x_1 as a function of R when $MR_{2H}(R') < 0$.

2.5 Results concerning price differences

Finally, consider price differences between periods. It is of interest to see whether a shift from a competitive to a monopoly outcome will lead to larger or smaller price differences. Let us

[22] Bushnell reports that his hydrothermal model of the Western United States computes negative values of water for BPA when the firm exerts market power. (Bushnell, 2003, p 90 and Table 8.)

[23] The results derived below for negative marginal revenues in the monopoly solution apply equally well to the case of a negative price and the competitive equilibrium. In practical application, however, it is easier to envision sales at negative marginal revenue than at a negative price.

define price difference as the difference between average prices in the two periods. Again we assume that $MR_{2H}(R') \geq 0$.

From our analysis, we have the following results:

Proposition 4:

(i) *Regime 1 [high reservoir capacity]*: $p_1 = E(p_2)$ *in the competitive outcome.*

(ii) *Regimes 2 and* 3 [low/medium reservoir capacity]: $p_1 < E(p_2)$ *in the competitive outcome.*

(iii) *In Regime 1, if* $x_1^{1M} \neq x_1^{1C}$*, a shift from the competitive to the monopoly outcome increases the difference in average prices in the two periods.*

(iv) *In Regime 3, if* $x_1^{3M} < x_1^{3C}$*, a shift from the competitive to the monopoly outcome decreases the difference in the average prices in the two periods*

Proof: See the appendix.

If the reservoir does not constrain water allocation, the competitive outcome leads to equal average prices in the two periods. If the reservoir constraint is binding, however, production in period 1 is increased in order to avoid spill of water completely (*Regime 2*) or partly (*Regime 3*) and the price in period 1 is below the average price of period 2.

With a non-binding reservoir (*Regime 1*) average prices in the competitive outcome are equal. Any other water allocation leads to price differences. This is in line with what we expect in markets with market power. In *Regime 3*, there are differences in average prices even in the competitive outcome and market power can lead to either larger or smaller differences. If the monopoly produces less than the competitive outcome in period 1, the price difference is reduced. In particular the monopoly price in period 2 state L will be lower than the competitive price. Interestingly, in this case the difference between the highest and lowest price in the two periods is larger in the competitive outcome than in a situation with market power. Cf. Rangel (2008) who seems to suggest the opposite.

3. Numerical examples

In the literature it has been shown that the choice of demand function affects the magnitude of a price increase in a merger simulation.[24] Although we are only interested in qualitative results, *i.e.*, whether the monopoly will supply more (or less) than the competitive outcome, functional form also affect our results. For illustrations we apply the function

$$p = a + bx^e.$$

Two parameter sets give rise to often used demand functions:

- $a = 0$, $b > 0$ and $e < 0$, which gives demand with *constant price elasticity* (ε=1/e), and
- $a > 0$, $b < 0$ and $0 < e < 2$, which is a *convex* ($e < 1$), a *linear* (e=1), or a *concave* ($e > 1$) function.

(i) Demand with constant price elasticity.

Assume that that demand has constant price elasticity (ε). Inverse demand is $p = bx^e$, where $e = 1/\varepsilon$, and $MR = (e+1)p$. $MR > 0$ when $(e+1) > 0$, *i.e.*, $1/\varepsilon > -1$ or $\varepsilon < -1$, which is required for existence of a profit maximum for the monopoly. For this demand function we can conclude that $MR > 0$ (and $p > 0$) in all periods and states of the world. Thus, $\lambda \geq 0$ and our pricing conditions (6), (6'), and (6") all apply.

When demand is equally elastic in the two periods, optimal allocation of water is identical for the two types. In fact, (6) and (6") reduce to identical conditions for the two types with this function. This follows directly from Proposition 1, and Figure 1 illustrates this case.[25]

Unequal demand is obtained by increasing either of the two parameters b and e in a period. Note that b captures the market size, while e captures the (inverse) price elasticity. Different b's make no difference to the comparison of production volumes from conditions (6) and (6") for the two types. This follows directly as the price elasticity does not change along this demand curve (see Proposition 1). Changing e (the inverse elasticity), however, has the effect

[24] Crooke *et al.* (1999) have shown that the predicted price increase from a merger is much larger when using an iso-elastic rather than a linear demand curve.
[25] Demand is calibrated to P=10, X=10, and ε = -2, giving e = -0.5 and b =31.62. I_1 =16, I_2^H =6, I_2^L =2, and q = ½.

that the optimal allocation of water will differ for the two behavioral types. Figure 2 illustrates two cases. In 2a demand is most elastic in period 1 ($\varepsilon_1 < \varepsilon_2 < -1$) which implies $-1 < e_2 < e_1 < 0$, while in 2b demand is least elastic in period 1 so that $-1 < e_1 < e_2 < 0$.

(ii) Linear demand (a > 0, b < 0, and e = 1)

Let $p = a + bx$. Although $p < 0$ for $x > -a/b$, we consider only supply (x) such that that $p(x) > 0$. We will, however, study supply x for which $MR(x) = a + 2bx < 0$, that is, $x > -a/2b$. As shown above the sign of MR_{2H} matters at the point where reservoir capacity becomes binding. We therefore distinguish between cases where marginal revenue is positive and negative, and start with parameters a and b such that $MR_{2H}(R') > 0$. We consider changes in parameter a, which captures the willingness to pay. An increase in a leads to a parallel upwards shift in the demand curve.

Positive marginal revenue at R'

Consider three parameter sets: i) $a_1 = a_2$, ii) $a_1 > a_2$, and iii) $a_1 < a_2$. Figure 3 displays optimal production (x_1) for the two behavioral types for different reservoir capacities. The patterns of these graphs are similar to those of the examples with constant price elasticity in Figure 2. Going right to left, first there is a segment with constant production, then a segment where production is increased in order to avoid spill, and finally a segment where production again is held constant while water is spilled in state *H*.

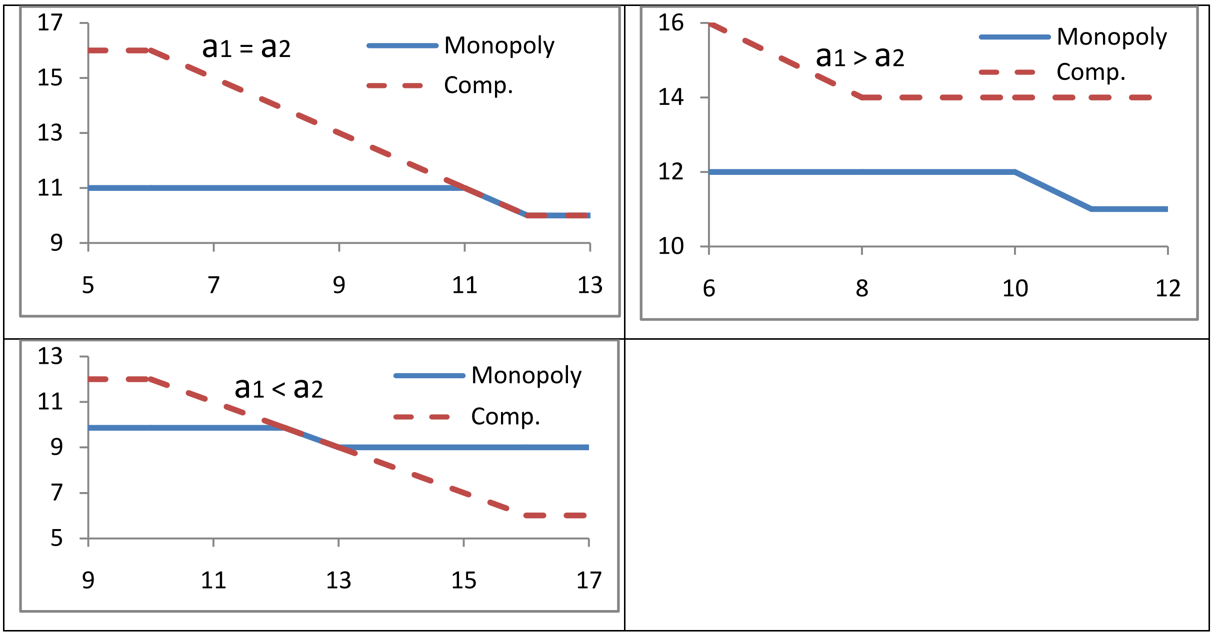
The graphs in the panel with $a_1 = a_2$ coincide in *Regimes 1* and *2*, while they deviate in *Regime 3,* with a lower monopoly production. Observe that Proposition 1 relates to price elasticities measured at the competitive equilibrium. Demand with constant price elasticity has by definition the same elasticity throughout. This is not the case for the linear demand function where the price elasticity increases with price. In particular, the price elasticity of demand is higher in period 2 with low inflow than in period 1 since quantity sold is higher in the latter case. Hence, the different pictures of *Regime 3* in Figures 2 and 3 follow because of non-constant price elasticity along the linear demand in Figure 3.[26]

[26] The elasticities computed at the solution x_1^C are $\varepsilon_1 = -2.3$, $\varepsilon_{2H} = -1.7$, and $\varepsilon_{2L} = -3.3$, whereby $MR_1 = E(MR_2)$.

Consider the panels with $a_1 \neq a_2$.[27] The *Regime 1* segment of the graphs looks like those in Figure 2. When $a_1 > a_2$, elasticities at the competitive outcome x_1^C satisfy $\varepsilon_1 > \varepsilon_2^s$, $s = H, L$, *i.e.*, demand is most elastic in period 2.[28] Then it follows from Proposition 2 that $x_1^C > x_1^M$. Observe that the graphs of the two types are disjoint, *i.e.*, there is no overlap of Regime 2.[29]

The *Regime 3* segments differ from the picture in Figure 2 as $x_1^C > x_1^M$ in both panels. In *Regime 3* it is a_1 versus $(1\text{-}q)a_2$ that matters, not a_1 versus a_2 (see Lemma 2).

Figure 3. Optimal production (x_1) with linear demand.



Negative marginal revenue at R'

A monopoly would never supply such that *MR* is negative if it could restrict supply. We have assumed, however, that the producer is not allowed to spill water for other reasons than overflow (in state *H*). He has to supply all available water (as electricity) during the two

[27] In the base case (with a_1=a_2) linear demand is calibrated at $P_1 = P_2 = 10$, $X_1 = X_2 = 10$, with a price elasticity $\varepsilon = -2$ at this point. Recalibrating demand at prices $P_1 = 10 + \Delta P$ and $P_2 = 10 - \Delta P$, gives solutions $x_1^C = 10 + 2\Delta P$ and $x_1^M = 10 + ½\Delta P$ in *Regime 1*.

[28] Elasticities at the competitive solution $x_1^C = 14$ are $\varepsilon_1 = -1.14$ $\varepsilon_2^L = -6.5$, $\varepsilon_2^H = -2.75$.

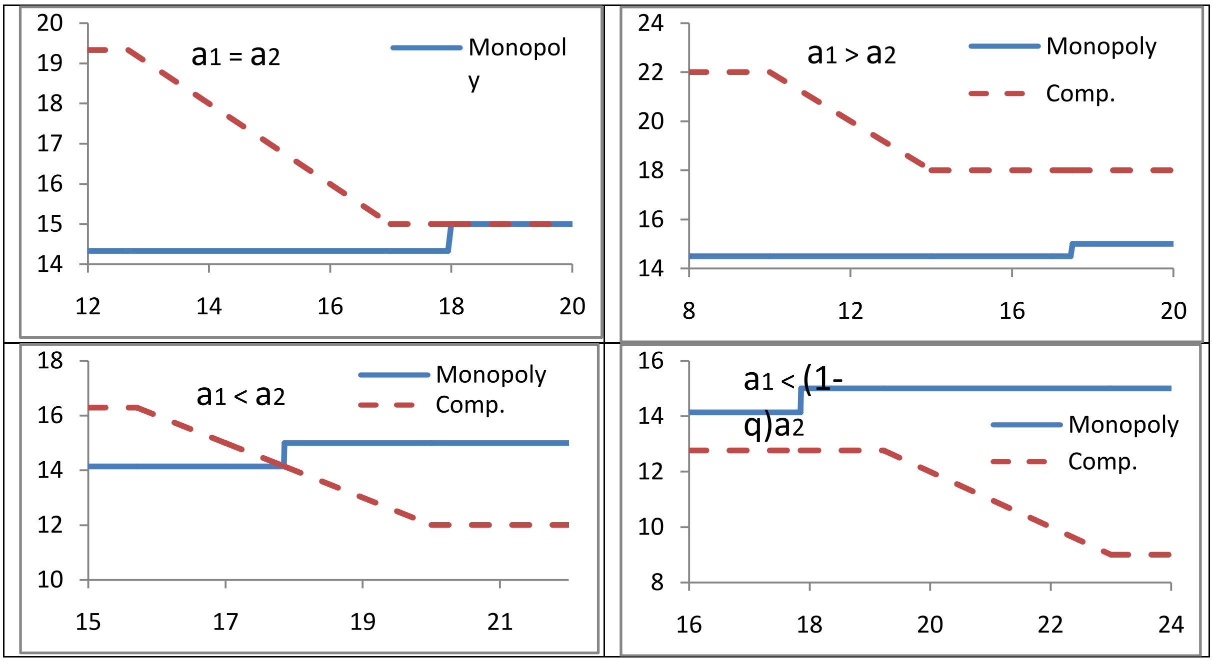
[29] With a fixed volume to be shared over two periods the consumers' surplus (CS) becomes convex in volume and is maximized at a corner solution. Consider $x_1+x_2=1$, $P_t=1-x_t$, t=1,2. CS is maximized at $x_1=1$, $x_2=0$, or at $x_1=0$, $x_2=1$, and not at $x_1 = x_2 = ½$. The objective for the competitive outcome, the sum of consumers' and producers' surpluses, is concave however, and the competitive outcome is not at a corner solution, but it is more sensitive to parameter changes than the monopoly outcome.

periods. Thus the monopoly may have to supply at negative marginal revenue. Our model is constructed such that this may happen in period 2 state *H*. Assume that $MR_{2H}(R') < 0$.

Figure 4 illustrates the optimal production for the two behavioral types for four sets of parameter values: i) $a_1 = a_2$, ii) $a_1 > a_2$, iii) $a_1 < a_2$, and iv) $a_1 < (1-q)a_2$.

The pattern of optimal production in *Regime 1* is as above. Solutions coincide when demand is equal, $x_1^C > x_1^M$ when $a_1 > a_2$, and $x_1^C < x_1^M$ when $a_1 < a_2$. Furthermore, the graph for the price-taker consists of three segments as above. The striking difference is the pattern of monopoly production outside *Regime 1*. As commented upon above, this is related to MR_2^H being negative at the point where *R* becomes binding. We have shown that in this case the middle, upward-sloping segment of a production plan is never optimal. The monopoly bypasses this segment, jumping from constant, unconstrained production to constant constrained production with spill of water, and we observe that monopoly production is reduced going from *Regime 1* to *Regime 3* (see Proposition 3).

Figure 4. Optimal production (x_1) with linear demand when $MR_2^H(R') < 0$.



$x_1^C > x_1^M$ in the first three panels in *Regime 3*. Lemma 2 tells us that this is true when $a_1 > (1-q)a_2$. With $a_1 < (1-q)a_2$, however, $x_1^C < x_1^M$, as shown in the fourth panel.

4. Some concluding remarks

Hydro power systems are complex partly because of reservoirs that allow reallocation of water between periods. Our analysis is not about whether total production is changed, but rather whether and how production is reallocated between periods. We present simple formulas to predict the direction of the distortion caused by market power. Information about the price elasticities of demand can be sufficient to determine whether market power will lead to higher (or lower) production in the first period. This suggests that information about price elasticities of demand, for example differences between summer and winter, is crucial for understanding the consequences of a merger that would have a potential for exploiting market power.

Our analysis illustrates that market power as such will not necessarily lead to a more serious supply shortage in situations with a low inflow of water. To the contrary, a monopoly may find it profitable to produce less in period 1 and thereby more in period 2 state *L*. On the other hand, the lower production in period 1 that provides a basis for higher production in period 2, also increases availability in state *H* and will cause spill of water in this state. Thus, the example illustrates that the benefit from having more water available in case of low inflow is balanced with the drawback of a higher spill of water when there is high inflow.

It is misguided, however, to focus on whether market power leads to a more serious supply shortage if there is low inflow. The competitive outcome is the social optimum balancing the marginal value to society of production in period 1 and the marginal value of storing one unit for supply in period 2. Any deviation from that outcome creates a distortion. By definition then, exertion of market power will most likely lead to a distortion irrespective of the direction of the distortion.

References

Bushnell, J. (2003): "A mixed complementarity model of hydro - thermal electricity competition in the Western U.S.", *Operations Research*, 51(1), 80-93.

Bye, T. N.-H., von der Fehr, C. Riis, and L. Sørgard (2003): Kraft og makt. En analyse av konkurrasneforholdene i kraftmarkedet' (Market and Power), report submitted to the Ministry of Government Administration and Reform, Oslo.

Crampes, C. and M. Moreaux (2001): "Water Resource and Power Generation", *International Journal of Industrial Organization*, 19(6), 975-997.

Crooke, P., L.M. Froeb, S. Tschantz, and G.J. Werden (1999): "Effects of assumed demand form on simulated post merger equilibria", *Review of Industrial Organization*, 15 (3), 205-17.

Evans, L., G. Guthrie, and A. Lu (2010): An intertemporal model of electricity market performance with an application to climate change, New Zealand Institute for Study of Competition and Regulation Inc., Discussion paper.

Farrell, J. and C. Shapiro (1990): "Equilibrium horizontal mergers", *American Economic Review*, 80(1), 107-126.

Farrell, J. and C. Shapiro (2010): "Antitrust evaluation of horizontal mergers: An economic alternative to market definition", *The B. E. Journal of Theoretical Economics*, 10 (1), Article 9.

Førsund, F. (2007): *Hydropower Economics*, Springer, New York.

Garcia, A., J. D. Reitzes and E. Staccheti (2001): "Strategic Pricing when Electricity is Storable", *Journal of Regulatory Economics*, 20(3), 223-247.

Hansen, P. V. (2009): "Inflow uncertainty in Hydropower markets", *Scandinavian Journal of Economics*, 111(1), 189-207.

Johnsen, T. A. (2001): "Hydropower generation and storage, transmission constraints and market power", *Utilities Policy*, 10, 63-73.

Johnsen, T. A., S. K. Verma, and C. Wolfram. (1999): "Zonal pricing and demand-side bidding in the Norwegian Electricity Market", POWER Working Paper PWP-063, University of California Energy Institute.

Kelman, R., L.A.N. Barroso, and M.V.F. Pereira (2001): “Market Power Assessment and Mitigation in Hydrothermal Systems”, *IEEE Trans Power Systems*, 16(3) 354-359.

Mathiesen, L. (1985): “Computational experience in solving equilibrium models by a sequence of linear complementarity problems”, *Operations Research*, 33(6), 1225-1250.

Mathiesen, L. (1992): "Pricing of electricity in the presence of water uncertainty", Norwegian School of Economics and Business Administration, Discussion paper 13/92.

Rangel, L. F. (2008): ‘Competition policy and regulation in hydro-dominated electricity markets’, *Energy Policy*, 36, 1292-1302.

Samuelson, P. (1952): “Spatial price equilibrium and linear programming”, *American Economic Review*, 42, 283-303.

Scarf, H.E. (1973): *The computation of economic equilibria*, Yale University Press, New Haven, Conn.

Scott T. J. and E. G. Read (1997): "Modelling Hydro Reservoir Operation in a Deregulated Electricity Market", *Int. Trans. Opl. Res*, 3(3-4), 243-253.

Skaar, J. and L. Sørgard (2006): “Temporary Bottlenecks, Hydropower, and Acquisitions”, *Scandinavian Journal of Economics*, 108(3), 481-497.

Appendix.

Lemma 1. Assume that expected profits are concave in r_1 and w, and that parameters I_1, I_{2L}, and I_{2H} are positive. For a given $R > 0$, the model (1)-(2) has a unique solution for both behavioral types. Depending on *R*, this solution may be in *Regimes 1* or *3*, or when price (marginal profit) is positive at *R'*, in *Regime 2*.

Proof. It is an established fact that the maximization of a concave function over a non-empty and convex set has a unique solution. We have assumed that both objectives are concave in decision variables. The constraint set is defined by linear constraints. Hence it is convex. When I_1, I_{2L}, I_{2H}, and *R* are all positive, the constraint set is non-empty.

Proof of Proposition 1.

Regime 1: From (8) we write the optimality condition for the competitive outcome as

(C) $p_1 = qp_{2H} + (1-q)p_{2L}$, and the optimality condition for the monopoly as

(M) $MR_1 = p_1\left(1+\frac{1}{\varepsilon_1}\right) = q\left(1+\frac{1}{\varepsilon_{2H}}\right)p_{2H} + (1-q)\left(1+\frac{1}{\varepsilon_{2L}}\right)p_{2L} = E(MR_2).$

The monopolist has an incentive to set $x_1^{1M} > x_1^{1C}$ when $MR_1 > E(MR_2)$, as measured at the competitive outcome. When $\varepsilon_1 < \varepsilon_{2s} < -1$, it follows that $(1+1/\varepsilon_1) > (1+1/\varepsilon_{2s})$, $s = L, H$, whereby $MR_1 > E(MR_2)$, which proves the first bullet-point.

Use (C) to substitute for p_1 in (M) and we get that $x_1^{1M} > x_1^{1C}$ if

(E) $q\left(\frac{1}{\varepsilon_1} - \frac{1}{\varepsilon_{2H}}\right)p_{2H} < (1-q)\left(\frac{1}{\varepsilon_{2L}} - \frac{1}{\varepsilon_1}\right)p_{2L}.$

(E) is easily rearranged as the condition in Proposition 1 and proves the second bullet point.

Regime 2: For any given reservoir capacity *R*, there is a unique production in period 1 that leads to no spill of water and full capacity utilization if there is high inflow. If both producers' solutions are in *Regime 2,* these solutions have to be identical.

Regime 3: From (8'') the first order condition in the competitive outcome is

(C') $p_1 = (1-q)p_2^L$, while the first order condition for the monopolist is

(M') $MR_1 = p_1\left(1+\frac{1}{\varepsilon_1}\right) = (1-q)\left(1+\frac{1}{\varepsilon_2^L}\right)p_2^L = E(MR_2)$.

Again, the monopolist will set $x_1^{1M} > x_1^{1C}$ when $MR_1 > E(MR_2)$. Now, use (C') to substitute for p_1 in (M') to get that $x_1^{1M} > x_1^{1C}$ if $(1+1/\varepsilon_1) > (1+1/\varepsilon_{2L})$ or $\varepsilon_1 < \varepsilon_{2L}$.

Proof of Proposition 3.

Assume that x_1^1 solves (8), i.e., $MR_1 = (1\text{-}q)MR_2^L + qMR_2^H$, and that $MR_2^H(x_1^1) < 0$. Thus, $MR_1(x_1^1) < (1\text{-}q)MR_2^L(x_1^1)$. Observe that MR_1 decreases in x_1 while MR_2^L increases in x_1 (decreases in x_2^L). The x_1-value x_1^3 that solves (8") (in *Regime 3*), i.e., $MR_1 = (1\text{-}q)MR_2^L$, is therefore smaller than x_1^1, *i.e.*, $x_1^3 < x_1^1$.

Lemma 3. Let inverse demand $p(x) = a + bx^e$, and consider *Regime 3*.
If $a_1 = (1\text{-}q)a_2$ and $e_1 < e_2$, $x_1^C < x_1^M$, while if $a_1 > (1\text{-}q)a_2$ and $e_1 = e_2$, $x_1^C > x_1^M$.

Proof.

(8") for the competitive case can be rewritten as

(C') $b_1x_1^{e1} – (1\text{-}q)b_2(x_2^L)^{e2} = (1\text{-}q)a_2 – a_1$

and for the monopoly (8") can be written as

(M') $(1+e_1)b_1x_1^{e1} – (1\text{-}q)(1+e_2)b_2(x_2^L)^{e2} = (1\text{-}q)a_2 – a_1$.

If $a_1 = (1\text{-}q)a_2$ the conditions can be rewritten as $g(x_1) = B$, with $g(x_1) = x_1^{e1}/(x_2^L)^{e2}$ and equal for the two cases, while constant B differs, $B_C = (1\text{-}q)b_2/b_1$ and $B_M = B_C[(1+e_2)/(1+e_1)]$. With $e_1 < e_2$, it follows that $B_C < B_M$ and as g is increasing in x_1, $x_1^C < x_1^M$.

If $a_1 > (1\text{-}q)a_2$, the conditions can be written as $h(x_1) = B$ with $h_C \equiv x_1^{e1} – [(1\text{-}q)b_2/b_1](x_2^L)^{e2}$ and $h_M \equiv x_1^{e1} – [(1\text{-}q)b_2(1+e_2)/b_1(1+e_1)](x_2^L)^{e2}$ and with constants $B_C = [(1\text{-}q)a_2 - a_1]/b_1 > 0$ and

$B_M = B_C/(1+e_1) > 0$. If $e_1 = e_2$, $h_C = h_M$ and increasing, and $B_C > B_M$, whereby $x_1^C > x_1^M$.