

Working Paper No. 26/01  
**On how size and composition of customer  
bases affect equilibrium in a duopoly with  
switching costs**

by

**Tommy Staahl Gabrielsen  
Steinar Vagstad**

SNF-project No. 4838:  
«TELENOR - Innelåsing og prising med heterogene konsumenter

The project is financed by Telenor FoU

**THE ECONOMICS OF TELECOMMUNICATIONS**

This report is one of a series of papers and reports on telecommunication economics published by the Foundation for Research in Economics and Business Administration (SNF) as part of its telecommunication economics program. The main focus of the research program is to study the deregulation process of the telecommunication industry, and the economic and organizational consequences of changes in markets, technology and regulation. Being started in 1992, the program is now in its third period ending in 2001/2002. The program is financed by Telenor AS.

FOUNDATION FOR RESEARCH IN ECONOMIC AND BUSINESS ADMINISTRATION  
BERGEN, OCTOBER 2001  
ISSN 0803-4028

© Dette eksemplar er fremstilt etter avtale med KOPINOR, Stenergate 1, 0050 Oslo. Ytterligere eksemplarfremstilling uten avtale og i strid med åndsverkloven er straffbart og kan medføre erstatningsansvar.

# On how size and composition of customer bases affect equilibrium in a duopoly with switching costs\*

Tommy Staahl Gabrielsen and Steinar Vagstad

Department of Economics, University of Bergen,

Fosswinckelsgate 6, N-5007 Bergen, Norway

June 21st 2001

## Abstract

Switching costs may facilitate monopoly pricing in a market with price competition between two suppliers of a homogenous good, provided the switching cost is above some critical level. It is also well known that asymmetric size of customer bases makes monopoly pricing more difficult. Adding consumer heterogeneity to the model we demonstrate that also composition of each firm's customer base affects pricing, and this composition may aggravate or ease the incentives to break out of the monopoly pricing equilibrium.

**JEL classification numbers:** D44, D82, L10, L51

**Keywords:** switching costs, consumer heterogeneity, duopoly, price discrimination

---

\*This research has been sponsored by Telenor, through the Foundation for Research in Economics and Business Administration.

# 1 Introduction

Firms often compete in markets for more or less homogeneous goods, with prices as the main strategic variable. A problem facing such firms is how to escape the Bertrand paradox. In many markets, the most compelling solution to the paradox is the existence of switching costs: the fact that even if consumers don't care about which product they start to buy, there may be costs associated with switching suppliers.<sup>1</sup> Such costs dampen competition in mature markets in a variety of settings, as shown by Paul Klemperer in numerous articles (see his 1995 survey). In particular, if all consumers have positive switching costs, the only possible price equilibrium in pure strategies is monopoly pricing, and such an equilibrium exists if and only if the switching costs exceed some critical level.

The aim of this paper is to shed light on the extent to which the conditions for existence of an equilibrium involving monopoly pricing (i.e., the size of the critical switching cost) are affected by asymmetries between the firms. Already Klemperer (1987) noticed that the critical switching cost may depend on the relative size of the firms. In particular, size asymmetry make monopoly pricing less likely.<sup>2</sup> The critical switching cost may also be affected by heterogeneity of consumer preferences.<sup>3</sup> Moreover, heterogeneity among consumers also give rise to another possible asymmetry between the firms: they may have different compositions of their customer bases. In what follows we will study the effects of each kind of asymmetry and how they blend.

In addition to the already mentioned literature on switching costs, many scholars have studied non-linear pricing in more or less competitive settings. With the exception of Gabrielsen and Vagstad (2000), all these contributions model other

---

<sup>1</sup>Other proposed solutions include product differentiation (physically or informationally) and tacit collusion, as laid out in any modern treatments of Industrial Organization, e.g. Tirole (1988).

<sup>2</sup>In short, a price cut that is large enough to make the rival's start buying from you instead entails a gain (increased sales) but also a loss (lower profits from your "old" customers). Thus, a firm's incentives to undercut decreases in the firm's relative size. Consequently, unequal sizes call for larger switching costs to keep the smaller firm from undercutting.

<sup>3</sup>Gabrielsen and Vagstad (2000) finds that in a symmetric model consumer heterogeneity often aggravate problems of non-existence of a pure-strategy equilibrium.

sources of market power than switching costs. Wilson (1993, part 12.3) consider Cournot competition, while Stole (1995), Armstrong and Vickers (1999) and Rochet and Stole (1999) study situations with differentiated products.

To our knowledge, none has studied the joint effects of size and consumer type composition asymmetries on duopoly pricing. We conduct the analysis within a model allowing any kind of non-linear pricing. Our basic model entails two types of consumers,  $H$  (high-demand) and  $L$  (low-demand), and two firms who have split the market some way or another in a first period that is not modelled. All consumers have a common positive cost of switching supplier, implying that monopoly pricing is the only candidate for Nash equilibrium in pure strategies. Our first task is to characterize monopoly pricing. It turns out that absent any economies or diseconomies of scale, size does not matter for pricing, only the relative numbers of high vs. low-demand consumers within each firm's customer base. However, both size and composition matters for existence of a pure-strategy equilibrium. Our main result is that each type of asymmetry tends to increase the switching cost needed for existence of a pure-strategy equilibrium, but that one source of asymmetry may or may not counteract the effect of the other, primarily depending on whether it is the smaller or larger firm who has the largest share of high-demand consumers in its customer base.

Firms should be concerned about existence of a pure-strategy equilibrium for two reasons. First, the alternative to the proposed equilibrium involving monopoly pricing by both firms is an equilibrium in which each firm draws tariffs from a distribution ranging from competitive pricing to monopoly pricing, with lower profits as an inevitable result. Second, market shares and compositions are not entirely exogenous to the firms, but depend on actions taken in earlier periods. Since we advocate size and composition symmetry, and, in the absence of size symmetry, a certain pattern of composition asymmetry, we also provide guidelines for firms seeking to affect the prospects of ending up in an equilibrium involving monopoly pricing.<sup>4</sup>

---

<sup>4</sup>Clearly, sometimes a firm want just the opposite, e.g. in order to deter entry from a third firm, or in order to discourage investments by its rival. However, also in those situations our results can

The paper proceeds as follows. The basic model is presented in Section 2. Under the assumption that there exists a pure-strategy equilibrium, equilibrium pricing is studied in Section 3. Section 4 gives a general discussion of issues related to the question of existence and presents our results, while some concluding remarks are gathered in Section 5. All proofs are relegated to the appendix, together with a brief description of an example used for illustration in the main text.

## 2 The model

Consider two firms —  $A$  and  $B$  — setting prices in a market with two kinds of consumers —  $H$  ("high" demand) and  $L$  ("low" demand). There are a total of  $l$  low-demand consumers and  $h$  high-demand consumers in the market. The two firms offer functionally identical products, but each consumer has already bought from one of the firms, and if a consumer wants to switch to the other supplier, switching costs are incurred. We assume that all consumers have identical positive switching costs denoted  $s$ .<sup>5</sup> In particular, the costs of switching do not depend on a consumer's demand volume.<sup>6</sup>

Next, we assume that each firm offers a menu of contracts, one intended for each type of consumer (as is well known, in a static model with only two types, it suffices to study menus with only two tariffs). We allow for quite general contracts  $M = (M_L, M_H)$ , including, but not limited to, the possibility of restricting attention

---

be used, just with the opposite prescriptions.

<sup>5</sup>This implies that the only candidate for equilibrium in pure strategies entails monopoly pricing (as specified below).

<sup>6</sup>This is obviously not the only way to model switching costs. Consider switching mobile telephone operator. This would entail some fixed costs, for instance the effort of contacting the operators and make them do what you want, possible penalties for terminating the relationship with your existing operator, and costs of opening a new relationship. Typically there are also volume-dependent switching costs, for instance the costs attached to lack of number portability which is presumably a larger problem for a pizza chain than from a typical private consumer, but may be substantial even for private consumers.

to two-part tariffs<sup>7</sup> or point contracts.<sup>8</sup>

Consumer preferences are described by the following quasi-linear utility function:

$$u(\theta, q, T) = U(\theta, q) - T \quad \text{for } \theta \in \{L, H\} \quad (1)$$

where  $\theta$  is the consumer's "type",  $q$  is demand volume and  $T$  is monetary payment for the good in question. In line with the literature (see e.g., Wilson (1993, Sect. 6.2)) it is assumed that  $U_q > 0$ ,  $U_{qq} < 0$ ,  $U_\theta > 0$  and  $U_{q\theta} > 0$ , where subscripts denote partial derivatives. In words, this means that the marginal utility of the good in question is positive at a decreasing rate, that utility is increasing in the consumer's type (which is just a normalization), and marginal utility that is increasing in the consumer's type. This last assumption corresponds to the single-crossing condition of the mechanism design literature, and is crucial for price discrimination to work.

Firms are allowed to be asymmetric as regards customer bases, while costs are symmetric, for simplicity normalized to zero.<sup>9</sup>

### 3 Equilibrium mechanism

As long as all consumers have positive switching costs,  $s > 0$ , Klemperer (1987) has argued — in a framework of linear pricing — that if there is a pricing equilibrium in pure strategies, this equilibrium must entail monopoly pricing. The argument goes as follows. At any lower common price than the monopoly price, each firm has an incentive to slightly increase its price, in order to exploit its own customers without losing any to its competitor. Note that even small switching costs suffices to make the (possible) equilibrium switch from competitive pricing to monopoly pricing. It

---

<sup>7</sup>That is, contracts of the form  $M_i = (p_i, F_i)$  where tariff  $i$  consists of a fixed fee  $F_i$  and a marginal price  $p_i$ , implying that a consumer consuming  $q$  units pays  $T_i(q) = F_i + p_i q$  under tariff  $i$ .

<sup>8</sup>That is, contracts of the form  $M_i = (q_i, T_i)$  where  $q_i$  is quantity and  $T_i$  is payment.

<sup>9</sup>Similar results can be derived for more general symmetric cost functions, as long as there are not too strong economies of scale. Moreover, it is straightforward to generalize the analysis to situations involving (at least some types of) cost asymmetries, but then some of the statements about the desirability of having symmetric customer bases would have to be rewritten. This is, however, beyond the scope of the present paper.

should be clear that the logic of small deviations applies equally well to situations involving non-linear pricing: even if firm  $A$  uses linear prices, it would pay for firm  $B$  to price non-linearly, for instance using two-part tariffs.

However, the proposed equilibrium may be vulnerable to non-marginal price changes: it is still the case that a sufficiently large price cut will make one firm corner the market, and if the switching costs are too small, cornering the market becomes so attractive that monopoly pricing is not an equilibrium either – implying that there is no equilibrium in pure strategies at all.<sup>10</sup> In this respect the magnitude of the switching cost is important.

In this section we simply assume that there exists an equilibrium in pure strategies and proceed to characterize this equilibrium, while the issue of existence is relegated to Section 4. Hence, consider a monopoly firm which have in its customer base from the first period a number  $l$  of low-demand customers and  $h$  high-demand customers. Moreover, throughout the paper we restrict attention to situations involving internal equilibria, in the sense that in equilibrium, both types of consumers will be served by both firms. Technically, this requires that the group of low-demand consumers must be important enough in terms of demand (e.g. as measured by  $L/H$ ) and numbers (e.g. as measured by  $l_i/h_i$ , where  $l_i$  and  $h_i$  are the numbers of low and high demand consumers in firm  $i$ 's customer base,  $i \in \{A, B\}$ ). The most important consequence of working with internal equilibria is that with such equilibria the solution (i.e., the optimal mechanism) becomes differentiable in the numbers of low and high demand consumers.

Let  $v_\theta(M_i) \equiv \max_q u(\theta, q, T_i(q))$  denote the maximum utility a consumer of type  $\theta$  can obtain when exposed to a mechanism  $M$  involving a quantity-payment scheme  $T_i(q)$ , and let  $q_\theta(M_i) \equiv \arg \max_q u(\theta, q, T_i(q))$ .<sup>11</sup> Then the firm chooses the mecha-

---

<sup>10</sup>There is always an equilibrium in mixed strategies, however, see Klemperer (1987). This equilibrium is rather complicated even in a model with linear pricing and homogeneous consumers, and it is beyond the scope of the present paper to analyze mixed-strategy equilibria of the current model.

<sup>11</sup>In the special case of a point contract  $M_i = (q_i, T_i)$  the maximization vanishes as there is only one possible quantity. Then  $v_\theta(M_i) = u(\theta, q_i, T_i)$ .

nism  $(M_L, M_H)$  to maximize

$$\pi(M_L, M_H) \equiv l(T_L(q_L(M_L))) + h(T_H(q_H(M_H))) \quad (2)$$

subject to the standard incentive and participation constraints:

$$v_L(M_L) \geq 0 \quad (3)$$

$$v_H(M_H) \geq 0 \quad (4)$$

$$v_L(M_L) \geq v_L(M_H) \quad (5)$$

$$v_H(M_H) \geq v_H(M_L) \quad (6)$$

The assumptions made about preferences implies that (4) and (5) are redundant, while (3) and (6) bind for the optimal contract. The solution will be denoted  $(M_L^*, M_H^*)$ . For subsequent reference, we also define  $v_\theta^* \equiv v_\theta(M_\theta^*)$ ,  $T_\theta^* \equiv T_\theta(q_\theta(M_\theta^*))$  and  $\pi^M \equiv \pi(M_L^*, M_H^*)$ .

The optimal contract exhibits the well-known characteristics; no distortion of the high-demand type and downward distortions of the low-demand type's quantity (for a two-part tariff this amounts to a marginal price above marginal cost). Moreover, all rent is extracted from the low-demand customers whereas the high-demand customers earn an information rent:  $v_L^* = 0 < v_H^*$ . Finally, the distortion imposed on the low-demand customers is increasing in the relative number of high-demand customers. The reason is that the more high-demand customers the more important they are, and to extract more rent from the high-demand customers you must distort the low-demand contract to make it unattractive for the high-demand customers. Formally,  $\partial v_H(M_H^*)/\partial(l_i/h_i) > 0$ .

## 4 Existence of pure-strategy equilibria

This section discusses the basic considerations related to the question of existence of a pure-strategy Nash equilibrium of the model outlined in the previous sections. To find the critical switching costs needed to sustain monopoly pricing, we need to derive optimal undercutting strategies for the firms. To attract your rival's customers, you must offer them a contract that compensate them for having to bear their



switching cost  $s$ . Since consumers are of different types, there are two different ways to undercut the rival: one can either try to attract his high-demand customers (to be dubbed strategy *high*) or to go for all the competitor's customers (strategy *all*).<sup>12</sup>

The basic question is how asymmetries in either firm size or customer composition affect existence of the pure-strategy equilibrium, that is, the size of the critical switching costs needed to sustain a pure strategy equilibrium that involves monopoly pricing. First, consider size asymmetry. Proposition 1 generalizes to general nonlinear mechanisms the above-mentioned result first proved by Paul Klemperer (1987):

**Proposition 1** *Pure size asymmetry increases the critical switching cost.*

*Proof:* In the appendix.

The intuition is quite clear: if firms are equal in size and composition, obviously both firms face the same incentive to undercut. Suppose then that the firms have identical composition of customers, but that one firm is larger than the other, where size is measured by a firm's total number of customers. Intuitively, the smaller firm now will have higher incentive to undercut than if firms are symmetric in every respect, simply because the potential gain from undercutting is larger the more customers you get when cutting prices. This intuition applies whichever undercutting strategy the small firm uses. Hence, size asymmetry should make the smaller firm more aggressive which will destabilize the market: higher switching costs is needed to sustain monopoly prices than in a perfectly symmetric setting.

Proposition 1 is illustrated in Figure 1 below, for an example which is described in some more detail in the appendix: To keep firm  $A$  from undercutting using strategy *all*,  $s$  must be at least as large as the function denoted  $s_{all}^A(k)$ , where  $k$  denotes firm  $A$ 's market share (defined by  $k = \frac{l_A}{l} = \frac{h_A}{h} = \frac{l_A+h_A}{l+h}$  for cases of symmetric composition). Similarly, to keep the same firm from using strategy *high*,  $s \geq s_{high}^A(k)$ . These two functions are plotted together with the corresponding functions for firm  $B$ , which by symmetry are given by  $s_{all}^B(k) = s_{all}^A(1-k)$  and  $s_{high}^B(k) = s_{high}^A(1-k)$ . In order to deter any type of undercutting from any of

---

<sup>12</sup>Formally, there is also a third strategy: going for the rival's low-demand customers only. However, with our assumptions, this strategy is always dominated by strategy *all*.

Critical switching cost

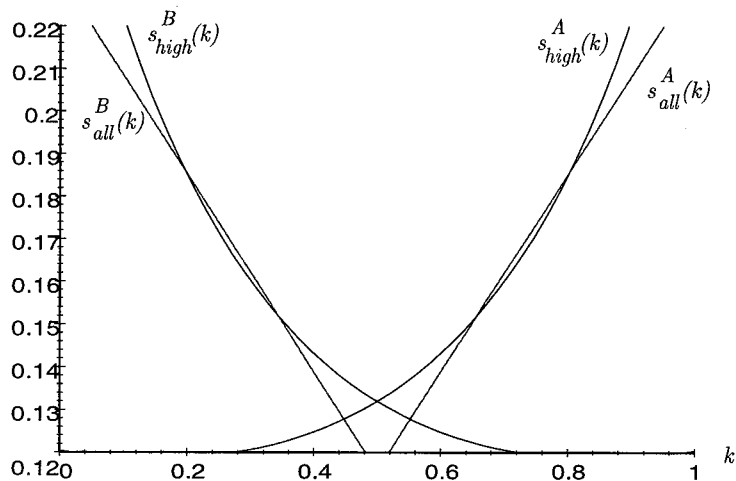


Figure 1: *Critical switching cost for symmetric composition*

the two firms, the switching cost must be larger than or equal to all four functions. Consequently,  $s$  must be larger than or equal to a critical switching cost  $s^*$  defined by

$$s^*(k) = \max \{s_{all}^A(k), s_{all}^B(k), s_{high}^A(k), s_{high}^B(k)\}$$

That is,  $s^*(k)$  is the upper envelope of the four graphs in the figure. It is clear from the figure that asymmetry increases  $s^*(k)$ .

Now consider composition asymmetries. The first thing to notice regarding composition asymmetries is that, given that a pure strategy equilibrium exists, more asymmetry tends to increase industry profit:

**Proposition 2** *Whenever a pure strategy equilibrium exists, industry profit is minimized for symmetric composition.*

*Proof:* In the appendix.

The intuition is that composition asymmetries enable firms to specialize in rent extraction from the group of customers that is most important to each firm. A firm that has relatively many low-demand customers will distort low-demand contracts relatively less, and in this way extract more rent from low-demand customers at the expense of leaving more rent to high-demand customers. Similarly, the firm who has

relatively more high-demand customers will distort low-demand contracts relatively more in order to extract more rent from high-demand customers. Thinking about complete asymmetry - full specialization - makes the argument obvious. This will allow for efficient contracts and full rent extraction of all customers.

>From a situation where firms have symmetric composition of customer bases (but possibly different size), suppose that firm  $A$  swaps low-demand customers for a number of high-demand customers in a way that leaves his profit unchanged. Formally, let a small number  $\varepsilon$  (possibly negative, to capture swaps the other way around) of low-demand customers go from firm  $A$  to firm  $B$ , who gives back a number  $\delta$  of high-demand customers. Moreover, let the relation between  $\varepsilon$  and  $\delta$  be such that firm  $A$ 's profit is left unchanged. Since high-demand consumers are more valuable than low-demand ones,  $\varepsilon > \delta$ . Some important properties of firm  $A$ 's critical switching cost after such a swap is then described by the following Proposition:

**Proposition 3** *After the swap, to prevent firm  $A$  from undercutting with strategy *high*, the switching cost  $s \geq s_{high}^A(k, \varepsilon)$ , where*

$$\frac{\partial s_{high}^A(k, \varepsilon)}{\partial \varepsilon} < 0$$

*To prevent  $A$  from using strategy *all*, the switching cost  $s \geq s_{all}^A(k, \varepsilon)$ , where*

$$\begin{aligned} \frac{\partial s_{all}^A(k, \varepsilon)}{\partial \varepsilon} &= 0 \text{ if } \varepsilon < 0 \\ \frac{\partial s_{all}^A(k, \varepsilon)}{\partial \varepsilon} &< 0 \text{ if } \varepsilon > 0 \end{aligned}$$

*Proof:* In the appendix.

The first part of the proposition, the part dealing with strategy *high*, has a simple interpretation: When  $\varepsilon > 0$ , firm  $A$  receives high-demand consumers from  $B$  in return for giving away low-demand consumers. His equilibrium profit is unaffected by this change. His profit from undercutting his rival decreases — and so does the switching cost needed to prevent this type of undercutting — for two reasons. First, his rival's high-demand consumers are less numerous than before. Second, since firm  $B$ 's high-demand consumers after the swap become under-represented in firm  $B$ 's

customer base (compared to the average), they will be offered a contract leaving them an information rent that is higher than  $v_H^*$ . This makes each of them more difficult to attract than before the swap.

The last part of the proposition — on strategy *all* — is slightly more difficult to understand. Firm *A*'s problem when undercutting this way is similar to the monopolist's problem, with the low-demand consumers' participation constraint tightened by  $s$ . Offering all consumers the monopoly mechanism plus an amount  $s$  to cover the switching cost is the best firm *A* can get away with. This contract is enough to attract firm *B*'s high-demand consumers only if they had an initial contract (being firm *B*'s customers) leaving them a rent not exceeding  $v_H^*$ . As argued above, firm *B*'s high-demand consumers will be difficult to attract whenever  $\varepsilon > 0$ , while they will accept the proposed contract (monopoly contract plus  $s$ ) if  $\varepsilon < 0$ . In the latter case neither the equilibrium profit nor the profit from using strategy *all* depend on  $\varepsilon$ , consequently, the critical switching cost does not either. In contrast, if  $\varepsilon > 0$  the profit when undercutting decreases (due to the added difficulties attracting firm *B*'s high-demand consumers) and so does the critical switching cost.

Two Corollaries follow immediately from Proposition 3:

**Corollary 1** *With full symmetry initially ( $l_A = l_B$  and  $h_A = h_B$ , implying  $k = \frac{1}{2}$ ), a profit neutral swap of customers will increase the critical switching cost if  $s_{high}^A(\frac{1}{2}) > s_{all}^A(\frac{1}{2})$  and not affect the critical switching cost if  $s_{high}^A(\frac{1}{2}) < s_{all}^A(\frac{1}{2})$ .*

**Corollary 2** *With initial size asymmetry ( $k \neq \frac{1}{2}$  but  $\frac{l_A}{h_A} = \frac{l_B}{h_B}$ ), a profit neutral swap of customers may have any effect on the critical switching cost: it may increase (if  $s_{high}^A(k) > s_{all}^A(k)$  and  $\varepsilon < 0$ ), remain unchanged (if  $s_{high}^A(k) < s_{all}^A(k)$  and  $\varepsilon < 0$ ) or decrease (if  $\varepsilon > 0$ ).*

We conclude with a brief discussion of how important the effects we have identified are. Suppose the situation corresponds to the example described in detail in the appendix, and that the swap is of a magnitude of 1% of the total number of low-demand consumers. The associated decrease in critical switching costs depends

on the initial size distribution according to the table below:<sup>13</sup>

$k$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{2}{5}$
$-\frac{\Delta s^*}{s^*}$	0.014	0.0000473	0.0000577	0.0303

Table 1: *Magnitude of effects*

This implies that while there is hardly anything to gain for some values of  $k$ , there are substantial gains for other values: When  $k = \frac{2}{5}$ , a 1% shift of customer composition reduces the critical switching cost by 3%. It turns out that the largest changes in the critical switching cost occur when strategy *high* is the most difficult to stop.

## 5 Concluding remarks

There are different ways to escape the Bertrand paradox threatening the profit of price-setting firms competing in a market for homogeneous products. We have studied one such possibility — the creation of consumer switching costs — in a market with heterogeneous consumers. We have earlier (Gabrielsen and Vagstad, 2000) argued that consumer heterogeneity tend to increase the critical switching cost needed to make a pure-strategy equilibrium involving monopoly pricing exist. This has the immediate implication that the more heterogeneity, the higher efforts to raise barriers for consumers who may want to switch supplier, in order to preserve monopoly pricing.

While our first paper restricted attention to symmetric duopoly, this paper has opened for firm asymmetries. We have shown that pure size asymmetry increases the critical switching cost. Moreover, we have shown that pure composition asymmetry increases or leave unchanged the critical switching cost. Furthermore, we have shown that composition asymmetry increases industry profits whenever the

---

<sup>13</sup>The changes are calculated using the following formula, letting  $s^*$  denote the relevant switching cost (i.e.  $\max\{s_{high}^A(k, \emptyset), s_{all}^A(k, \emptyset)\}$ ):

$$-\frac{\Delta s^*}{s^*} = \frac{\max\{s_{high}^A(k, 0), s_{all}^A(k, 0)\} - \max\{s_{high}^A(k, 0.1), s_{all}^A(k, 0.1)\}}{\max\{s_{high}^A(k, 0), s_{all}^A(k, 0)\}}$$

pure-strategy equilibrium exists. Finally, there are mixed results from blending size and composition asymmetries: the critical switching cost may increase, decrease or remain constant as consumers are swapped in a profit-neutral way, depending mostly on the direction of asymmetries: the critical switching cost is decreased if the smaller firm has a larger share of the high-demand consumers. However, the combined effect also depends on the form of the temptation. In particular, if it is most tempting to try to attract the entire market, then the critical switching cost is not increased if the smaller firm gives up some high-demand consumers in exchange for low-demand ones.

In future work we would also like to extend our analysis in two other ways. First, we wish to study the interplay between the second-period effects on the critical switching cost studied in the present paper, and the competition for (different types of) customers in the first period. Such an extension could build heavily on Paul Klemperer's earlier work (see e.g. his (1995) survey). Second, we would also like to know how things change if we allow for more dynamics, e.g. by allowing for tacit collusion in addition to the switching costs studied here. Beggs and Klemperer (1992) and Padilla (1995) have studied the interplay between switching costs and the scope for reaching a collusive agreement in a repeated version of simpler pricing games, and an extension should take these contributions as starting points.

## 6 Appendices

### 6.1 Proof of Proposition 1

Suppose firm  $A$  has a share  $k$  of both types of consumers. If the pure-strategy equilibrium exists it entails sharing of monopoly profits according to market shares:

$$\pi_A(k) = k\pi^M$$

where  $k$  is firm  $A$ 's market share,  $\pi_A(k)$  is firm  $A$ 's profit, and  $\pi^M$  is the monopoly profit as defined in Section 3. Firm  $A$  may be tempted to cut prices to attract all his rival's customers — strategy *all*, or go for his high-demand customers only, strategy *high*.

First suppose he considers strategy *all*. He must then pay the new consumers their switching costs, which amounts to choosing the mechanism  $(M_L, M_H)$  that maximizes

$$l(T_L(M_L)) + h(T_H(M_H))$$

subject to

$$v_L(M_L) \geq v_L^* + s = s$$

$$v_H(M_H) \geq v_H^* + s$$

$$v_L(M_L) \geq v_L(M_H)$$

$$v_H(M_H) \geq v_H(M_L)$$

This problem is identical to the monopolist's problem described in the previous section except that both types' participation constraints are tightened by an amount  $s$ . The solution then entails reducing the payment in both contracts offered by the same amount  $s$ , leaving the undercutting firm with profit given by

$$\pi_{all}(s) = l(T_L^* - s) + h(T_H^* - s) = \pi^M - (l + h)s$$

To prevent firm *A* from undercutting in this way, the following must hold:

$$\begin{aligned} \pi_{all}(s) &\leq k\pi^M \\ &\Downarrow \\ s &\geq s_{all}^A(k) = \frac{1 - k}{l + h}\pi^M \end{aligned}$$

By symmetry, to prevent firm *B* (having a market share of  $1 - k$ ) from undercutting in this way,

$$s \geq s_{all}^B(k) = s_{all}^A(1 - k) = \frac{k}{l + h}\pi^M$$

Consequently, the larger a firm is, the less tempted to undercut. When the market is split evenly between them, they are both equally tempted, while asymmetries make the smaller firm more aggressive and the larger firm less aggressive. What counts for equilibrium existence, however, is that the most tempted firm becomes more aggressive as we increase the asymmetry:

$$s_{all}(k) \equiv \max \{s_{all}^A(k), s_{all}^B(k)\} = \frac{\pi^M}{l + h} \max \{k, 1 - k\}$$

which is minimized for  $k = \frac{1}{2}$ .

Next suppose that firm  $A$  instead consider to go for his rival's high-demand consumers only, strategy *high*. Then he maximizes

$$k l T_L(M_L) + h T_H(M_H)$$

subject to

$$v_L(M_L) \geq 0$$

$$v_H(M_H) \geq v_H^* + s$$

$$v_L(M_L) \geq v_L(M_H)$$

$$v_H(M_H) \geq v_H(M_L)$$

Let  $M_L^{high}$  and  $M_H^{high}$  denote the solution to the problem. Moreover, let  $\pi^{high}(s, k) = k l T_L(q_L(M_L^{high})) + h T_H(q_H(M_H^{high}))$  denote the corresponding profit. Note first that as  $s$  increases, one of the constraints become tighter, while nothing happens to the other constraints or with the objective function. Moreover, when  $s > 0$  the constraint is binding for the optimal mechanism. As a consequence, the maximum value of the objective function must be decreasing in  $s$ . Formally,  $\partial \pi^{high}(s, k) / \partial s < 0$ .

To prevent firm  $A$  from undercutting this way, the switching cost must be sufficiently high. Formally, the critical switching cost needed to prevent this type of undercutting,  $s_{high}^A$ , is defined by

$$\pi^{high}(s_{high}^A, k) \equiv k \pi^M \tag{7}$$

This equation holds as an identity when we vary  $k$ . Consequently, it implicitly defines  $s_{high}^A$  as a function of  $k$ ;  $s_{high}^A = s_{high}^A(k)$ . Its derivative is given by (employing the implicit function theorem)

$$\begin{aligned} \frac{\partial s_{high}^A(k)}{\partial k} &= \frac{\pi^M - \partial \pi^{high}(s^*, k) / \partial k}{\partial \pi^{high}(s^*, k) / \partial s} \\ &= \frac{\pi^M - l T_L(M_L^{high})}{\partial \pi^{high}(s^*, k) / \partial s} \end{aligned}$$



using the envelope theorem to obtain  $\partial\pi^{high}(s^*, k)/\partial k = lT_L(M_L^{high})$ . Since the profit from the low-demand consumers (irrespective what contract they are offered) must be smaller than the monopoly profit,  $\pi^M - lT_L(M_L^{high}) > 0$ , and from above we have that  $\partial\pi^{high}(s, k)/\partial s < 0$ . Therefore,  $\frac{\partial s_{high}^A(k)}{\partial k} < 0$ . By symmetry, the to prevent firm  $B$  from using this strategy,  $s \geq s_{high}^B(k) = s_{high}^A(1 - k)$  and therefore  $\frac{\partial s_{high}^B(k)}{\partial k} > 0$ . Consequently, the larger a firm is, the less tempted it is to undercut also for this particular strategy. When the market is split evenly between them, they are both equally tempted, while asymmetries make the smaller firm more aggressive and the larger firm less aggressive. What counts for equilibrium existence is again that the most tempted firm becomes more aggressive as we increase the asymmetry:

$$s_{high}(k) \equiv \max \{s_{high}^A(k), s_{high}^B(k)\} = s_{high}^A(\min \{k, 1 - k\})$$

a function that reaches its minimum for  $k = \frac{1}{2}$ .

What remains is to put both strategies together. To prevent any kind of undercutting, the following must hold:

$$s \geq s^*(k) = \max \{s_{all}(k), s_{high}(k)\}$$

As both  $s_{all}(k)$  and  $s_{high}(k)$  increases with asymmetries and is minimized for  $k = \frac{1}{2}$ , the maximum of the two must exhibit the same property. ■

## 6.2 Proof of Proposition 2

If both firms price as if they were monopolists, industry profit would equal monopoly profits:  $\pi^A(M_L^*, M_H^*) = l_A T_L^* + h_A T_H^*$  and  $\pi^B(M_L^*, M_H^*) = l_B T_L^* + h_B T_H^*$ , therefore  $\pi^A(M_L^*, M_H^*) + \pi^B(M_L^*, M_H^*) = (l_A + l_B)T_L^* + (h_A + h_B)T_H^* = \pi^M$ . The firms may choose to deviate from monopoly pricing, however, and if they do so, it is because it increases profits:  $\max_{M_L^A, M_H^A} \pi^A(M_L^A, M_H^A) \geq \pi^A(M_L^*, M_H^*)$  (with strict inequality if mechanisms are different) and  $\max_{M_L^B, M_H^B} \pi^B(M_L^B, M_H^B) \geq \pi^B(M_L^*, M_H^*)$  (same comment applies). Consequently,  $\max_{M_L^A, M_H^A} \pi^A(M_L^A, M_H^A) + \max_{M_L^B, M_H^B} \pi^B(M_L^B, M_H^B) \geq \pi^M$ . That is, industry profit is minimized for symmetric composition. ■

### 6.3 Proof of Proposition 3

To prevent undercutting with strategy *high*,

$$k\pi^M \geq \max_{M_L, M_H} \{hT_H(q_H(M_H)) + (kl - \varepsilon)T_L(q_L(M_L))\} \equiv \pi^{high}(s, k, \varepsilon)$$

where maximization is performed over the set of mechanisms  $\{(M_L, M_H)\}$  that are incentive compatible as well as individually rational for the intended consumers (all high-demand and firm *A*'s low-demand consumers).<sup>14</sup> Note that the number  $\varepsilon$  does not enter the LHS. Changing  $\varepsilon$  has a first order impact on the RHS (in addition there is an effect on the choice of optimal mechanism, but this effect is of second order, due to the envelope theorem). Since the critical switching cost is defined by

$$k\pi^M \equiv \pi^{high}(s_{high}^A, k, \varepsilon)$$

it follows (using the implicit function rule) that

$$\frac{\partial s_{high}^A(k, \varepsilon)}{\partial \varepsilon} = -\frac{\partial \pi^{high}(s_{high}^A, k, \varepsilon)/\partial \varepsilon}{\partial \pi^{high}(s_{high}^A, k, \varepsilon)/\partial s_{high}^A} = -\frac{-T_L(q_L(M_L))}{\partial \pi^{high}(s_{high}^A, k, \varepsilon)/\partial s_{high}^A} < 0$$

since  $T_L(M_L) > 0$  and  $\partial \pi^{high}(s, k, \varepsilon)/\partial s < 0$  for the same reasons that  $\partial \pi^{high}(s, k)/\partial s < 0$  when composition were symmetric (cf. the proof of Proposition 1).

Next consider strategy *all*. Suppose ordinary incentive and participation constraints suffice for the undercutting firm (what is left out is the constraint securing that firm *B*'s high-demand consumers are satisfied with firm *A*'s offer — this will be checked below), who then makes the same profit as if the initial composition were symmetric (from the proof of Proposition 1):

$$\pi^{all}(s) = l(T_L^* - s) + h(T_H^* - s) = \pi^M - (l + h)s$$

To prevent firm *A* from undercutting in this way, the following must hold:

$$\begin{aligned} \pi^{all}(s) &\leq k\pi^M \\ \Downarrow \\ s &\geq s_{all}^A(k) = \frac{1 - k}{l + h}\pi^M \end{aligned}$$

<sup>14</sup>Note the slight abuse of notation, as  $\pi^{high}$  has earlier been defined as a function of only  $s$  and  $k$ .

This constraint is not affected by  $\varepsilon$ . What remains is to check whether the omitted constraint is satisfied or not, that is, whether the proposed contract is good enough for firm  $B$ 's high-demand consumers to make them switch. This depends on how satisfied they are with status quo. If firm  $B$  has relatively few high-demand consumers, less distortions are imposed on the firm's low-demand consumers in order to extract information rent from these high-demand consumers, who will then receive much information rent. This means that for  $\varepsilon > 0$ , an ordinary monopoly contract with paid switching costs is insufficient to make them switch: they need more, and this makes undercutting less tempting, consequently,  $\partial s_{all}^A(k, \varepsilon)/\partial \varepsilon < 0$  for  $\varepsilon > 0$  (again, for the same reasons as  $\partial \pi^{high}(s, k, \varepsilon)/\partial s < 0$ ). By the same lines of reasoning, if  $\varepsilon < 0$  firm  $B$ 's high-demand consumers are easily attracted by the monopoly contract, and in this situation a swap does not change anything and therefore  $\partial s_{all}^A(k, \varepsilon)/\partial \varepsilon = 0$  for  $\varepsilon < 0$ . ■

## 6.4 Example details

Figure 1 is based on a situation featuring quadratic utility of the form  $u(\theta, q, T) = \theta q - \frac{1}{2}q^2 - T$ . Moreover,  $(L, H) = (\frac{2}{3}, 1)$ ,  $l_A + l_B \equiv l = 10$  and  $h_A + h_B \equiv h = 5$ . Attention is restricted to interior equilibria (in the sense that both types of consumers are served in any equilibrium) and two-part tariffs:  $M_i = (p_i, F_i)$ . It is then easily shown that a sufficient and necessary condition for equilibria to be interior is that  $l_i > h_i$ , and that monopoly pricing for firm  $i$  yields  $p_H = 0$  and  $p_L = \frac{1}{3} \frac{h_i}{l_i} = \frac{1}{3} \alpha_i$ . Working out the equilibrium profit and comparing with the maximum profit from each of the undercutting strategies, it can be shown that  $s_{all}^A(k) = \frac{25}{108} (1 - k)$  and  $s_{high}^A(k) = \frac{1}{36} \frac{8k^2 - 5k + 10}{2k + 1}$ , which is all we need to plot the graphs in Figure 1. Basically, this is also what we need to do to compute the numbers in Table 1, but here we refer the interested reader to contact the authors for the full calculations.

## References

- [1] Armstrong, M. and J. Vickers (1999): "Competitive price discrimination." Mimeo, Oxford.
- [2] Beggs, A. and P.D. Klemperer (1992): "Multiperiod competition with switching costs," *Econometrica* 60, 651-66.
- [3] Gabrielsen, T.S. and S.Vagstad (2000): "Consumer heterogeneity and pricing in a duopoly with switching costs," WP 25/2000, Foundation for Research in Economics and Business Administration, Bergen, Norway.
- [4] Klemperer, P. (1987): "Markets with consumer switching costs," *Quarterly Journal of Economics* 102, 375-94.
- [5] Klemperer, P. (1995): "Competition when consumers have switching costs: An overview with applications to Industrial Organization, macroeconomics, and international trade," *Review of Economic Studies* 62, 515-39.
- [6] Padilla, A. J. (1995): "Revisiting Dynamic Duopoly with Consumer Switching Costs," *Journal of Economic Theory* 67, 520-30.
- [7] Rochet, J.-C. and L.A. Stole (1999): "Nonlinear pricing with random participation constraints." Mimeo, Chicago.
- [8] Stole, L.A. (1995): "Nonlinear pricing and oligopoly." *Journal of Economics and Management Strategy*, 4, 529-62.
- [9] Tirole, J. (1988): *The theory of Industrial Organization*, MIT Press.
- [10] Wilson, R. (1993): *Nonlinear pricing*, Oxford University Press.

## PUBLIKASJONER INNEN TELEØKONOMI 1998 –

- Gabrielsen, Tommy Stahl  
Vagstad, Steinar      *On how size and composition of customer bases affect equilibrium in a duopoly with switching cost*  
SNF-Working paper No. 26/2001
- Berge, Laila Kristin      *Prisdiskriminering i oligopol*  
*-en teoretisk gjennomgang og eksempler fra telemarkedet*  
SNF-rapport nr. 13/2001, Bergen.
- Bjørndal, Mette  
Jørnsten, Kurt      *An Analysis of a Combinatorial Auction.*  
SNF-Working paper No. 19/2001, Bergen.
- Vagstad, Steinar      *Price discrimination with uncertain consumers.*  
SNF-Working Paper No. 12/2001, Bergen.
- Gooderham, Paul N.  
Ulset, Svein      *Knowledge transfer between parent and developing country subsidiaries. A Conceptual Framework.*  
SNF-Working Paper No. 08/2001, Bergen.
- Stahl Gabrielsen, Tommy  
Vagstad, Steinar      *Second-period pricing in a duopoly with switching costs: the effect of size and composition of customer bases.*  
SNF-Working Paper No. 09/2001, Bergen.
- Pedersen, Per E.      *Adopsjon av mobil handel (m-handel)*  
*-en forstudie*  
SNF-rapport nr. 07/2001, Bergen.
- Hundsnes, Tore      *The Management of Complexity*  
SNF-Report No. 58/2000, Bergen.
- Knivsflå, Kjell Henry  
Rud, Linda  
Sættem, Frode      *Kapitalnettverk for små og mellomstore bedrifter*  
SNF-rapport nr. 72/2000, Bergen.
- Lommerud, Kjell Erik  
Sørgard, Lars      *Entry in Telecommunication: Customer Loyalty, Price Sensitivity and Access Prices.*  
SNF-Working Paper No. 62/2000, Bergen.
- Munkeby, Trygve Osvoll      *Konkurransen mellom norske internettleverandører.*  
SNF-rapport nr. 48/2000, Bergen.
- Pedersen, Per E.  
Methlie, Leif B.      *Tjenesteintegrering i elektronisk handel.*  
SNF-rapport nr. 21/2000, Bergen.
- Methlie, Leif B.  
Pedersen, Per E.      *MAP-IT: A Model of intermediary Integration Strategies in online Markets.*  
SNF-Working Paper No. 26/2000, Bergen.

- Gabrielsen, Tommy Staahl  
Vagstad, Steinar      *Consumer heterogeneity and pricing in a duopoly with switching costs.*  
SNF-Working Paper No. 25/2000, Bergen.
- Ulset, Svein  
Gooderham, Paul      *Internasjonalisering av telesektoren. Generelle lærdommer og spesielle utfordringer for Telenor.*  
SNF-arbeidsnotat nr. 16/2000, Bergen.
- Ulset, Svein      *Virtual Network Operation. Strategy, Structure and Profitability.*  
SNF-Working Paper No. 15/2000, Bergen.
- Foros, Øystein  
Kind, Hans Jarle  
Sørgard, Lars      *Access Pricing, Quality Degradation and Foreclosure in the Internet.*  
SNF arbeidsnotat nr. 12/2000, Bergen.
- Foros, Øystein  
Sand, Jan Yngve  
Ulset, Svein      *Asymmetrisk regulering innen telekommunikasjon.*  
SNF særtrykk nr. 03/2000, SNF, Bergen.  
*Ekspansive teleselskap. Finansiering, organisering og styring.*  
SNF-rapport nr. 64/1999, Bergen.
- Sannarnes, Jan Gaute      *Ulike reguleringsregimer i telesektoren sett ut fra et dynamisk perspektiv.*  
SNF-rapport nr. 58/1999, Bergen.
- Seime, Gunn Randi      *Konkurransen i det norske mobiltelefonimarkedet.*  
SNF-rapport nr. 49/1999, Bergen.
- Methlie, Leif B.  
Pedersen, Per E.      *Multimedia Banking*  
*Bankenes strategiske situasjon. Ny teknologi – ny konkurransearena – ny struktur.*  
SNF-rapport nr. 41/1999, Bergen.
- Pedersen, Per E.      *Multimedia Banking*  
*Programvareagenter i elektronisk handel. En kartlegging med vekt på agentbaserte tjenester og finanstjenestesektoren.*  
SNF-rapport nr. 40/1999, Bergen.
- Pedersen, Per E.      *Multimedia Banking*  
*En agentbasert tjeneste for produkt- og leverandør-sammenlikning av finanstjenester.*  
SNF-rapport nr. 39, 1999, Bergen.
- Pedersen, Per E.  
Nysveen, Herbjørn  
Jensen, Leif Magnus      *Multimedia Banking*  
*En eksperimentell studie av atferdskonsekvenser ved bruken av agentbaserte tjenester i finanssektoren.*  
SNF-rapport nr. 38/1999, Bergen.

- Fjell, Kenneth  
Foros, Øystein  
Gabrielsen, Tommy S.  
Hagen, Kåre P.  
Sørgard, Lars  
Vagstad, Steinar
- Problemstillinger for videre forskning på prising av tele-tjenester.*  
SNF-rapport nr. 27/1999, Bergen.
- Fjell, Kenneth  
Hagen, Kåre P.
- Oversikt over forskningsprogrammet i teleøkonomi ved SNF: 1996-1998.*  
SNF-rapport nr. 26/1999, Bergen.
- Fjell, Kenneth  
Foros, Øystein  
Hagen, Kåre P.  
Sørgard, Lars
- Telenor – bare lave priser ?  
Drøfting av Telenors rabattstruktur utfra et bedriftsøkonomisk og samfunnsøkonomisk perspektiv.*  
SNF-rapport nr. 23/1999, Bergen.
- Staahl Gabrielsen, Tommy  
Vagstad, Steinar
- Konkurransereform i telesektoren: Hvordan rasjonalisere observert atferd?*  
SNF-rapport nr. 65/1998, Bergen.
- Altenborg, Ellen
- Koordinering og insentiver i samarbeid om produktutvikling mellom forretningsområder i Telenor.*  
SNF-rapport nr. 39/98, Bergen
- Methlie, Leif
- Multimedia Banking  
Strukturendring i bank. Distribusjon – grovanalyse.*  
SNF-arbeidsnotat nr. 31/1998, Bergen.
- Methlie, Leif
- Multimedia Banking  
Strukturendring i bank. Strategisk posisjonering – grovanalyse.*  
SNF-arbeidsnotat nr. 30/1998, Bergen.
- Foros, Øystein  
Ulset, Svein
- Naturlige grenser for teleselskapene.*  
SNF populærvitenskapelig særtrykk nr. 10/1998, Bergen.
- Ulset, Svein  
Spiller, Pablo
- Organizing Global Seamless Networks: Contracts, Alliances and Hierarchies.*  
SNF Reprint No. 26/1998, Bergen.
- Ulset, Svein
- Infrastruktur og konkurranse i telesektoren.*  
SNF særtrykk nr. 27/1998, Bergen.
- Ulset, Svein
- Value-Creating Interconnect  
International Competition in Global Network Services.  
Technology, Organization and Performances.*  
SNF-report No. 28/1998, Bergen.

- Ulset, Svein *Value-Creating Interconnect  
Optimal Organization of the Converging Information and  
Communication Technology (ICT) Industries.  
Theoretical analysis and some illustrative data from the  
Norwegian market.*  
SNF-report No. 27/1998, Bergen.
- Methlie, Leif B.  
Nysveen, Herbjørn *Multimedia Banking  
Kundeatferd ved bruk av Internett og andre kanaler.*  
SNF-rapport nr. 29/1998, Bergen.
- Ulset, Svein *Verdiskapende samtrafikkavtaler.  
Hvordan kan organisering av infrastruktur bidra til utvikling  
av bedre og billigere teletjenester. En analyse av betingelsene  
for konkurranse og samarbeid i telesektoren.*  
SNF-rapport nr. 25/1998, Bergen.
- Spiller, Pablo T. *Value-Creating Interconnect.  
Unbundling and the Promotion of Local Telephone  
Competition: Is Unbundling Necessary in Norway?*  
SNF-Report No. 03/1998, Bergen.
- Bjørnenak, Trond  
Gabrielsen, Tommy Staahl  
Vagstad, Steinar *Verdiskapende samtrafikkavtaler.  
Prising av samtrafikk.*  
SNF- rapport nr. 02/1998, Bergen.
- Andersen, Christian  
Sannarnes, Jan Gaute *Analyse af tilgangsafigifter ved irreversible investeringer under  
usikkerhed.*  
SNF-rapport nr. 07/1998, Bergen.