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Volatility on Oslo Stock Exchange

Structural Breakpoints in Volatility Using the ICSS Algorithm

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Contents

1. Abstract

This thesis uses the Iterative Cumulative Sum of Squares (ICSS) algorithm (Inclán and Tiao, 1994) to search for structural breakpoints in volatility in the Norwegian stock market. I analyze data both with the original method, and with a revised version by Bacmann and Dubois (2001).

I also analyze the stock markets in the other Scandinavian countries, a European index, the US stock market, the oil market and exchange rates. I then state tentative explanations for the breakpoints.

I present a problem with the algorithm; the number of, and dating of breakpoints in a period is dependent on the choice of time period. I also show that the algorithm has problems discovering breaks occurring in the beginning or end of the time series. Through simulations, I show that the algorithm almost always fails to discover the correct number of breakpoints when there are several breaks quite close in time.

2. Method

There exists a lot of different literature on the behavior of volatility of asset returns. The GARCH model and variations of it are used to analyze the volatility in many different time series. Lamoureux and Lastrape (1990) questioned whether persistence found in financial time series was overstated because of the existence of deterministic structural shifts.

They analyzed 30 exchange traded stocks, and found that when regime shifts were incorporated directly into an ARCH/GARCH model, the persistence of variance indicated by the GARCH model decreases dramatically. In lack of a method to detect the breakpoints in variance, they divided the study period into equally spaced, non overlapping intervals, and tested the impact of sudden changes of variance parameters to the estimated models.

2.1 The ICSS Algorithm

The Iterative Cumulative Sums of Squares (ICSS) algorithm was first presented by Inclán and Tiao (1994). This method was developed to study "the detection of multiple changes of variance in a sequence of independent observations". The ICSS algorithm uses cumulative sums of squares to find breakpoints in a series, by systematically searching different pieces of the series. The method deals with a problem with Lamoureux and Lastrape's method, allowing the researcher to endogenously find the breakpoints, before the dummies are introduced to the model.

Aggarwal et al (1999) applied the algorithm to nine emerging markets, six major markets and four large indices from 1985 to 1995, and found several breakpoints in those series. Bacmann and Dubois (2001) revised the findings from Aggarwal et al, concluding that several of the breakpoints they found are due to GARCH effects in the financial series. Bacmann and Dubois introduced a 2-pass method were the GARCH effects are removed from the financial series before the ICSS algorithm is run.

This expanded model has rarely been used. Malik (2003) applied the original method to five currencies from 1990 to 2000, and Ewing and Malik (2005) used the original method in an analysis of 'spill-over effects' in regime shifts in volatility from large-cap stocks to small-cap stocks. Nouira et al (2004) use the simple method on the Euro/USD exchange rate from

1999-2003, while Aragó and Fernandez-Izquierdo (2003) apply it to the Spanish stock index from 1993-1999.

Viviana Fernandez (2006) compares the ICSS algorithm to the alternative method wavelet analysis in a study of stock indices for Emerging Asia, Europe, Latin America and North America, and interest rates for Chile from 1997-2002.

The ICSS algorithm assumes stationary variance over an initial period, until a breakpoint occurs. The variance is then stationary until another breakpoint occurs. This process is repeated, giving a time series of observations with an unknown number of breakpoints.

$$\sigma_t^2 = \begin{cases} \tau_0^2 & 1 < t < t_1 \\ \tau_1^2 & t_1 < t < t_2 \\ & \ldots \\ \tau_M^2 & t_M < t < t_n \end{cases} \quad (1)$$

To estimate the number of and placement of the breakpoints, the cumulative sums of squares is used; $C_k = \sum_{t=1}^{k} \varepsilon_t^2$, k=1,2,….,T, where ε_t is a series of uncorrelated random variables with mean 0 and variances σ_t^2 as in (1). The centered cumulative sums of squares is defined as

$$D_k = \frac{C_k}{C_T} - \frac{k}{T}, \ \ k = 1, 2...., T, \quad D_0 = D_T = 0 \ \ (2)$$

and will oscillate around zero, as long as there are no breakpoints in volatility. If there is a breakpoint, D_k will departure from zero; to a negative value if the variance changes to a lower level and to a positive value if the variance changes to a higher level.

The algorithm searches for variance change points by $\max_k |D_k|$. If this value exceeds a predetermined boundary, we conclude that there exists a change point around this point. Inclán and Tiao (1994) estimated the predetermined boundary (an extract for 95% significance level is presented here, for the full table refer to Inclán and Tiao's paper)

Table 1: Empirical and asymptotic quintiles of $\max_k \sqrt{\frac{T}{2}} |D_k|$

T	**100**		**200**		**300**		**400**		**500**		∞
p	qp	SE	qp	SE	qp	SE	qp	SE	qp	SE	D_{1-p}^*
.95	1.27	.009	1.30	.004	1.31	.008	1.31	.010	1.33	.009	1.358

Since we are interested in searching for several different breakpoints we have to use an iterative scheme of the D_k applied to different parts of the time series.

With $\varepsilon[t_1:t_2]$ as the notation for $\varepsilon_{t_1}, \varepsilon_{t_1+1}, \ldots, \varepsilon_{t_2}$, $D_k(\varepsilon[t_1:t_2])$ to indicate the range over which the cumulative sums are obtained, and D^* is D^*_{1-p} from table 1, the procedure is as follows (see Appendix 1 for procedure in R):

Step 0:

$t_1 = 1$

Step 1:

Calculate $D_k(\varepsilon[t_1 : T])$

Let k^* bet the point were $\max_k |D_k(\varepsilon[t_1 : T]|)$ is obtained.

$$M = \max_{t_1 \le k \le T} \sqrt{\frac{T - t_1 + 1}{2}} |D_k(\varepsilon[t_1 : T])| \quad (3)$$

If $M(t_1 : T) > D^*$ consider that there is a breakpoint at time k*.

If $M(t_1 : T) < D^*$ the algorithm stops.

Step 2a:

Let $t_2 = k^*$

Calculate $D_k(\varepsilon[t_1 : t_2])$. If $\max(D_k(\varepsilon[t_1 : t_2]))$, then we have a new point of change. Again let k^* bet the point were $\max_k |D_k(\varepsilon[t_1 : t_2]|)$.

Repeat this step until $M(t_1 : t_2) < D^*$. Then the *first* point of change is $k_{first} = t_2$

Step 2b:

Let $t_1 = k^*(\varepsilon[t_1 : T]) + 1$ (the change point found in step 1) and evaluate $\max_k |D_k(\varepsilon[t_1 : T]|)$.

Evaluate $D_k(\varepsilon[t_1 : T])$ and let t_1 be the point at which $\max(D_k(\varepsilon[t_1 : T]))$ is obtained, and repeat this step until $M(t_1 : T) < D^*$.

Let $k_{last} = t_1 - 1$.

Step 2c:

If $k_{first} = k_{last}$ there is only one breakpoint.

If $k_{first} < k_{last}$ repeat Step 1 and Step2 with $t_1 = k_{first} + 1$ and $T = k_{last}$. Call $\hat{N}_T$ the number of breakpoints found.

Step 3:

Sort the breakpoints in increasing order.

Let *cp* be the vector of the breakpoints, with $cp_0 = 0, cp_{N_{t+1}} = T$.

Check all the breakpoints by calculating

$$D_k\left(\varepsilon\left[cp_{j-1} + 1 : cp_{j+1}\right]\right), j = 1, 2....N_T \qquad (4)$$

If $M\left(p_{j-1} + 1 : cp_{j+1}\right) > D^*$ keep the point. Else; eliminate it.

Repeat Step 3 until the number of breakpoints does not change, and points found in each pass are "close" to those of the previous pass.

2.2 GARCH(p,q)

One of the main assumptions for linear regression is that the variance of the errors is constant over time – homoscedasticity. In many financial series only the unconditional variance is constant while the conditional variance is time-varying. In many cases the volatility occurs in bursts, where large changes in asset prices are followed by other large changes, and small changes are followed by small changes.

One model that copes with this problem is the Autoregressive Conditionally Heteroscedastic (ARCH) model, first presented by Engle (1982). This model introduces conditional variance to a random variable, u_t. The conditional variance of u_t is given by $\sigma_t^2 = \text{var}\left(u_t \mid u_{t-1}, u_{t-2}, ...\right)$ In the ARCH(p) model, the volatility is given by

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + ...\alpha_p u_{t-p}^2 \ . \qquad (5)$$

The conditional mean equation can take almost any form.

The Generalized ARCH (GARCH) model was developed independently by Bollerslev (1986) and Taylor (1986). The GARCH model is an expansion of the ARCH model, and can also be dependent on its own previous values, not only the previous values of the mean equations error values. The extended version can be expressed as

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \alpha_2 u_{t-2}^2 + \beta_1 \sigma_{t-1}^2 + ... + + \alpha_p u_{t-p}^2 + \beta_q \sigma_{t-q}^2 . \quad (6)$$

With the extension of ARCH, the model becomes more parsimonious and it avoids overfitting. A very commonly used version of this model is the GARCH(1,1) model, and it is that basic GARCH model I will use throughout this thesis.

2.3 The ICSS Algorithm Applied

Aggarwal et al (1999) presented a combined model with GARCH(1,1) and the ICSS algorithm. First breakpoints are detected by the ICSS algorithm, before dummy varibables are introduced into the variance equation of the GARCH model to account for the sudden changes in variance.

In series with autocorrelation an AR(1) is estimated, and the residuals used in the algorithm. Aggarwal et al applied this model to the stock markets of ten developing countries, six major markets and four indices from May 1985 to April 1995, both in local currency and USD. They found that many of the shocks in volatility were driven by local factors, rather than shifts in exchange rate regimes.

In the GARCH(1,1) model, without dummy variables, Aggarwal et al found both the alpha and the beta to be significant in 16 of the 20 series. When they introduced the dummy variables, both parameters were significant in only two of the series.

Bacmann and Dubois (2001) reviewed their finding and presented several problems with the ICSS algorithm. They found that:

- Existence of conditional heteroscedasticity in the time series makes the specification of the ICSS algorithm disastrous.

- When unconditional kurtosis exists and step size is larger than 30, the ICSS is well-specified, i.e. when monthly or less frequent data is used.
- Structural breakpoints found at a daily frequency must be confirmed with a test of monthly data.
- The ICSS algorithm are almost sure to find significant jumps, defining significant jumps as a doubling or tripling of the unconditional variance.

Bacmann and Dubois presents two different methods to search for breakpoints; (1) to use monthly stock returns or (2) the two-pass method, where the ICSS algorithm is applied to the "estimates of the standardized and normalized residuals of the conditional heteroscedastic model estimated in a first pass". They found the ICSS algorithm to be well-specified for a GARCH(1,1) model.

With this model they found the number of breakpoints to be lower (zero or one breakpoint in their study of ten emerging markets over ten years) than what Aggarwal et al found.

3. Data

I start in chapter 3.1 with a presentation of the data I study in this paper, before the descriptive statistics are presented in chapter 3.2.

3.1 Data selection

The main focus of this thesis is the Norwegian Stock Market (Oslo Stock Exchange), and I start by analyzing this market both in terms of USD and NOK, using the Morgan Stanley Country Indexes (MSCI). I also analyze the share Norsk Hydro (in USD).

The rest of the time series are all analyzed in USD. If local currency is used, the analysis would be of the market in itself, while for reasons of comparison I am interested in studying the markets as financial indicators, and I will therefore use a single currency for all.

I go on by analyzing the Scandinavian markets Denmark (Copenhagen Stock Exchange), Finland (Helsinki Stock Exchange) and Sweden (Stockholm Stock Exchange), using MSCI data. The same is done for Europe and the US. For the oil market I use the Brent price, 1 month delivery, and I also analyze the NOK/USD exchange rate.

All series start 1.January 1980 and end 30.November 2004, except the index for Finland that starts in January 1982, and the oil price that starts in 1984 for daily data, and 1982 for weekly and monthly data.

Other literature using the ICSS algorithm uses weekly return to find the structural breakpoints. In this thesis, I will also study daily and monthly results. For the weekly results, the values on Wednesdays are used (the previous day if the Wednesday was a holiday), while the monthly analysis is done with the first day of the month where a market value is noted.

In the analysis, most focus is put on the results for the 2-pass methods and for the results from the monthly series, as Bacmann and Dubois are suggesting.

All analyses are done with a 95% significance level.

3.2 Descriptive Statistics

Table x in the Appendix presents the descriptive statistics for the series I have studied. The statistics is :

Annualized Geometric Returns is given by $r = \tau * \sqrt[T]{\frac{V_2}{V_1} * \frac{V_3}{V_2} \dots \frac{V_T}{V_{T-1}}} - 1$, where τ is the number of periods in a year (i.e. $\tau = 52$ for weekly return).[1]

Standard Deviation is defined by $sd(X) = \sqrt{E(X^2) - (EX)^2}$ and is the expected deviation from the mean for each observation.

Skewness measures the extent to which a distribution is not symmetric about its mean value, and is defined by $\frac{E\left[(X-\mu_X)^3\right]}{E\left[(X-\mu)^2\right]^{3/2}}$.

Kurtosis is given by $\frac{E\left[(X-\mu_X)^4\right]}{E\left[(X-\mu)^2\right]^2}$ and measures how fat the tails of the distribution are.

The *Ljung-Box* statistic checks for autocorrelation in the series. It is defined by $Q^* = T(T+2)\sum_{k=1}^{m} \frac{\hat{\tau}_k^2}{T-\kappa} \sim \chi_m^2$.

The *Jarque-Bera* statistic tests for normality by combining the test for skewness and the test for kurtosis. The test is defined by $W = T\left[\frac{b_1^2}{6} + \frac{(b_2-3)^2}{24}\right]$.

I also test for autocorrelation and GARCH effects in the residuals from an ARMA(1) model.

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[1] Annualized Geometric Return can be calculated using logarithmic return: $r = \tau * \sum_1^T \frac{\ln \frac{V_t}{V_{t-1}}}{T}$, where τ is the number of shorter periods in a year. (i.e. $\tau = 52$ for weekly return)

4. Results

I start chapter 4.1 by running the ICSS algorithm on the different time series. Here I find strong indication of a negative break in volatility in the early 1990s in Norway, and I study this closer in chapter 4.2. In chapter 4.3, I go on by running simulations to control the algorithms strength at finding breakpoints when different time periods are used.

4.1 Time Series Analysis

I present the number of structural breaks with both the simple ICSS algorithm as presented by Inclán and Tiao (1994), and the two-pass method presented by Bacmann and Dubois (2001).

I apply the simple method to daily, weekly and monthly data, while I apply the two-pass method to daily and weekly data.

For each time series, a table shows the number of breakpoints in each analysis and the GARCH effects in the initial data. Then tables with the volatility periods, and the periodic volatility is presented for some of the tests.

4.1.1 Norway 1980-2005

For the Norwegian market, I first analyze the data in the local currency, Norwegian Krone (NOK), before I analyze it in USD. Then I study Norsk Hydro.

Norway NOK

The return on the Norwegian market has been 8.10% over the period, with a volatility of 21.78% (daily return). Compared to the other markets I study, the return has been low, with a moderate volatility.

Table 2: Norway NOK, Descriptive

	The ICSS Algorithm		GARCH(1,1)		
Period	**Simple Method**	**Two-pass method**	**α**	**β_0**	**β_1**
Daily	39	10	1.2e-05* (20.18)	0.13* (34.48)	0.80* (129.12)
Weekly	10	1	5.9e-05* (3.72)	0.08* (5.30)	0.86* (33.57)
Monthly	3		3.0e-03 (1.25)	0.06 (1.20)	0.37 (0.79)

The Ljung-Box test shows strong evidence of autocorrelation in the daily series, and an AR(1) is estimated to deal with it.

The number of breakpoints when the simple method is applied to daily data is 39, an average of 1.5 points per year. This seems to be too much to give meaning to the term ‘structural breakpoints’. Bacmann and Dubois’ (2001) description of the results as “disastrous” seems to be appropriate, as the breakpoints appear not only during large international or national events, but regularly.

When the 2-pass method is applied to daily data, and when the simple method is applied to weekly data, I get 10 breakpoints. These are mostly explainable breakpoints, but the 2-pass method applied to weekly data indicates that the correct number of breakpoints is lower.

The breakpoints found with the simple method applied on weekly and monthly return series, and the two-pass method applied on the weekly series is listed in table 3.

Table 3: Volatility periods, Norway NOK

NORWAY NOK	
MONTHLY SIMPLE METHOD	
Period	**τ**
1.2.1980-1.11.2005	25.41
1.2.1980-1.9.1992	28.33
1.9.1992-3.8.1998	19.08
3.8.1998-1.12.1998	63.03
1.12.1998-1.11.2005	21.98

NORWAY NOK	
WEEKLY 2-PASS METHOD	
Period	**τ**
16.1.1980-30.11.2005	22.45
16.1.1980-10.3.1993	24.46
10.3.1993-30.11.2005	20.23

NORWAY NOK	
WEEKLY SIMPLE METHOD	
Period	**τ**
9.2.1980-30.11.2005	22.44
9.2.1980-24.8.1983	24.31
24.8.1983-29.7.1987	19.15
29.7.1987-18.5.1988	44.03
18.5.1988-10.3.1993	23.89
10.3.1993-22.7.1998	17.42
22.7.1998-3.2.1999	41.85
3.2.1999-22.5.2002	17.69
22.5.2002-31.7.2002	43.31
31.7.2002-4.6.2003	25.12
4.6.2003-5.10.2005	14.91
5.10.2005-30.11.2005	35.59

NORWAY NOK	
DAILY 2-PASS METHOD	
Period	**τ**
3.1.1980-30.11.2005	21.78
3.1.1980-12.4.1983	23.27
12.4.1983-6.3.1987	18.28
6.3.1987-11.5.1988	40.60
11.5.1988-16.8.1991	19.93
16.8.1991-11.12.1992	31.63
11.12.1992-2.5.1995	17.29
2.5.1995-14.10.1997	13.68
14.10.1997-12.5.1999	28.55
12.5.1999-28.5.2002	17.90
28.5.2002-19.6.2003	24.53
19.6.2003-30.11.2005	16.18

The simple method applied to weekly data marks the October crash in 1987, with a 10 month period of high volatility. The volatility stays at a higher level than before also for the five years after the crash. Then there was a period of very low volatility in the mid-1990s, before the volatility sky-rocketed in a 6-month period in 1998-1999. This was a period when the market fell sharply. A new period of high volatility came in 2002, when the market again fell sharply.

The simple method applied to the monthly data also marks the period in 1998 with high volatility, and it seems to have stayed somewhat higher in the last seven years too, than during the 1990s.

The main difference between the simple method applied to monthly data and the 2-pass method applied to weekly data is that the former marks a 3-month high-volatility period in the autumn of 1998, that the Weekly Simple method dates late July to early February 1998-1999. This was during the severe fall in the market from an index of 1449 in July 1998 to a low of 800 in mid-October. The volatility during this period was almost the double of the average volatility during the period.

What is clear from the four different algorithm runs is that there was a break in late 1992 or early 1993 (September1992 – March 1993), half way into the series. Since then, the volatility on OSE has been significantly lower than during the 1980s.

This break is probably a sign of the market becoming more mature, and that the Norwegian volatility decreases down towards the European volatility.

Norway USD

It is well-known among investors that investing in a stock market using a foreign currency is more risky than investing in a market using the home currency. This is very clear from the Norwegian data, where the average return has been lower and the volatility higher for investors in USD than for investors in NOK. Average return has been 6.91% (compared to 8.10%) while volatility has been 23.05% (compared to 21.78%).

Table 4: Norway USD, Descriptive

	The ICSS Algorithm		GARCH(1,1)		
Period	**Simple Method**	**Two-pass method**	**α**	**β_0**	**β_1**
Daily	40	6	1.7e-05* (14.62)	0.13* (39.33)	0.79* (99.76)
Weekly	12	1	6.4e-05* (3.40)	0.09* (4.72)	0.85* (28.36)
Monthly	3		3.3e-03 (1.33)	0.06 (1.23)	0.36 (0.77)

The differences from the NOK index to the USD index are quite small. The monthly test dates the break in the early 1990s to August 1992, while the 2-pass method applied to weekly data dates it to December 1992. The 2-pass method applied to daily data dates this break to February 1993. This is in the same range as the results for the series in NOK.

The simple method applied to monthly data also marks the break in the fall of 1998, but here I see that the volatility was considerably lower for US investors than for Norwegian investors; opposite of the period overall.

Table 5: Volatility periods, Norway USD

NORWAY USD	
DAILY 2-PASS METHOD	
Period	**τ**
3.1.1980-30.11.2005	23.05
3.1.1980-16.10.1987	22.75
16.10.1987-21.10.1987	245.53
21.10.1987-9.2.1993	26.98
9.2.1993-10.2.1997	15.73
10.2.1997-31.7.1998	19.60
31.7.1998-13.1.1999	45.18
13.1.1999-30.11.2005	19.97

NORWAY USD	
MONTHLY SIMPLE METHOD	
Period	**τ**
1.2.1980-1.11.2005	26.41
1.2.1980-1.9.1992	29.31
1.9.1992-3.8.1998	18.81
3.8.1998-3.5.1999	52.40
3.5.1999-1.11.2005	22.38

NORWAY USD	
WEEKLY SIMPLE METHOD	
Period	**τ**
9.1.1980-30.11.2005	23.53
9.1.1980-24.8.1983	26.36
24.8.1983-26.8.1987	21.16
26.8.1987-2.12.1987	58.09
2.12.1987-25.7.1990	21.56
25.7.1990-4.3.1992	30.09
4.3.1992-20.7.1994	23.07
20.7.1994-29.4.1998	15.75
29.4.1998-17.2.1999	37.71
17.2.1999-19.6.2002	18.21
19.6.2002-31.7.2002	62.48
31.7.2002-5.3.2003	27.86
5.2.2003-29.6.2005	15.65
29.6.2005-30.11.2005	27.48

NORWAY USD	
WEEKLY 2-PASS METHOD	
Period	**τ**
16.1.1980-30.11.2005	23.54
16.1.1980-2.12.1992	25.81
2.12.1992-30.11.2005	21.12

Norsk Hydro

The largest company on OSE the last 25 years (except the last five years since Statoil was listed) has been Norsk Hydro, an oil, fertilizer and aluminum producer. Both historic return (10.05%) and volatility (30.99%) are higher for Hydro than for the stock index as a whole.

What is interesting from the results from Norsk Hydro is the fact that there are fewer breakpoints for that single stock than for the stock market overall, even though the volatility for Norsk Hydro is so high.

Table 6: Norsk Hydro, Descriptive

	The ICSS Algorithm		GARCH(1,1)		
Period	**Simple Method**	**Two-pass method**	**α**	**β_0**	**β_1**
Daily	27	4	3.0e-05* (17.39)	0.13* (38.98)	0.79* (140.87)
Weekly	3	1	7.2e-05* (3.57)	0.07* (5.69)	0.89* (44.58)
Monthly	1		6.3e-03 (1.81)	0.11 (1.93)	0.08 (0.17)

As for the Norwegian index, the volatility has been significantly lower in the second part of the period. The 2-pass method applied to daily and weekly data and the simple method applied to the weekly data indicates that there was a break in late 1992 or early 1993. The test on monthly data dates this shift to 1988, indicating that this shift occurred, to some degree because of the market crash in 1987. Also the 2-pass method applied to daily data and the simple method applied to weekly data indicates a break shortly after the market crash. The volatility seems therefore to have shifted downwards in 1988 after the crash, and then shift down again in 1992 or 1993.

The breakpoint marked in 1997 was at the start of a sharp decrease in stock price for Norsk Hydro. Also the oil price started to fall, while the new break in 2000 indicates the start of a positive development for the stock.

Table 7: Volatility Periods, Norsk Hydro

NORSK HYDRO	
DAILY 2-PASS METHOD	
Period	**τ**
12.12.1980-30.11.2005	30.98
12.12.1980-11.1.1988	38.96
11.1.1988-11.12.1992	31.71
11.12.1992-10.2.1997	20.21
10.2.1997-17.4.2000	31.30
17.4.2000-30.11.2005	25.60

NORSK HYDRO	
WEEKLY SIMPLE METHOD	
Period	**τ**
17.12.1980-30.11.2005	30.31
17.12.1980-9.8.1989	36.70
9.8.1989-17.2.1993	30.07
17.2.1993-12.5.1999	25.68
12.5.1999-30.11.2005	24.95

NORSK HYDRO	
WEEKLY 2-PASS METHOD	
Period	τ
24.12.1980-30.11.2005	30.32
24.12.1980-2.12.1992	35.00
2.12.1992-30.11.2005	25.40

NORSK HYDRO	
MONTHLY SIMPLE METHOD	
Period	τ
2.2.1981-1.11.2005	30.67
2.2.1981-1.4.1988	38.39
1.4.1988-1.11.2005	26.99

4.1.2 Scandinavia

The tests for Denmark, Finland and Sweden are done to see if the results are regional or national. Denmark is the country with the lowest volatility (18.40%), and also the country that has given the least in return over the 25 years (11.37% yearly average). Finland has been the market with highest return (15.05%), but also the most risky one (30.54%). Sweden is in the middle, with an average return of 13.94% and a standard deviation of 23.67%. All these three markets have paid better than the Norwegian market, and Denmark has even been less volatile than the Norwegian market.

Denmark

The low volatility might be one reason that no changes are found in the Danish series; the market there has been more mature than the Norwegian for a longer time, and the volatility is only 2-3 percentage points higher than for the European and US markets.

Table 8: Denmark, Descriptive

	The ICSS Algorithm		GARCH(1,1)		
Period	Simple Method	Two-pass method	α	β_0	β_1
Daily	41	3	2.4e-06* (9.78)	0.06* (16.13)	0.92* (205.59)
Weekly	3	0	3.0e-02* (2.39)	0.06* (3.90)	0.90* (33.16)
Monthly	0		2.4e-03 (0.55)	0.04 (0.41)	0.23 (0.17)

But the breakpoints found for the weekly data with the simple method strengthens the view that the market has seen shorter periods of change the last 10 years; with a higher volatility during the dot.com boom and crash and then a lower volatility the last few years. The volatility the last three years has been especially low, only 14.55%. I found no period of extremely high volatility in Denmark, the maximum is 22.31% in the four years of the dot.com boom.

Table 9: Volatility periods, Denmark

DENMARK	
DAILY 2-PASS METHOD	
Period	τ
3.1.1980-30.11.2005	18.40
3.1.1980-19.1.1993	18.87
19.1.1993-30.6.1997	13.89
30.6.1997-29.10.2002	22.24
29.10.2002-30.11.2005	14.55

DENMARK	
WEEKLY SIMPLE METHOD	
Period	τ
9.1.1980-30.11.2005	18.82
9.1.1980-18.8.1993	19.76
18.8.1993-11.3.1998	13.94
11.3.1998-16.10.2002	22.31
16.10.2002-30.11.2005	15.04

Finland

Finland is the most volatile of the Scandinavian markets, and the one that has given the highest return over the last 25 years.

Table 10: Finland, Descriptive

	The ICSS Algorithm		GARCH(1,1)		
Period	Simple Method	Two-pass method	α	β_0	β_1
Daily	48	21	1.2e-06* (14.86)	0.01* (26.29)	0.98* (1398.41)
Weekly	8	1	2.5e-05* (4.30)	0.05* (7.71)	0.94* (113.867)
Monthly	3		2.3e-04 (1.37)	0.09* (2.58)	0.88* (19.60)

The number of breakpoints found is also quite high compared to the other series I run the algorithm on. Most interesting here is the fact that Finland has had the opposite development compared to Norway and Denmark; the volatility has increased heavily in the last part of the period. This positive shift is here dated to only a couple of weeks after the negative shift occurred in Norway. The simple method applied to monthly data shows that it is especially through the last 7 years, since July 1998 that the volatility has increased with more than 50% during the dot.com boom and with 35% the last few years. Opposite from Norway and Denmark, Finland experienced its highest volatility when the market grew, not when it fell.

Table 11: Volatility Periods, Finland

FINLAND	
WEEKLY SIMPLE METHOD	
Period	**τ**
13.1.1982-30.11.2005	31.33
13.1.1982-30.4.1986	23.57
30.4.1986-29.8.1990	19.80
29.8.1990-19.8.1992	23.12
19.8.1992-25.8.1993	44.04
25.8.1993-1.4.1998	25.66
1.4.1998-1.12.1999	33.86
1.12.1999-16.10.2002	57.39
16.10.2002-12.5.2004	31.96
12.5.2004-30.11.2005	18.42

FINLAND	
WEEKLY 2-PASS METHOD	
Period	**τ**
20.1.1982-30.11.2005	31.34
20.1.1982-19.8.1992	21.57
19.8.1992-30.11.2005	37.33

FINLAND	
MONTHLY SIMPLE METHOD	
Period	**τ**
1.3.1982-1.11.2005	31.26
1.3.1982-1.8.1990	20.30
1.8.1990-3.8.1998	28.49
3.8.1998-1.11.2001	52.59
1.11.2001-1.11.2005	35.07

Sweden

The largest market in Scandinavia is the Swedish market, and it can therefore be expected to be more closely correlated to the European index (which it is a large part of) and less correlated to the US market, since the latter will have less influence on the Swedish market than on the Danish, Finnish and Norwegian markets.

Table 12: Sweden, Descriptive

	The ICSS Algorithm		GARCH(1,1)		
Period	**Simple Method**	**Two-pass method**	**α**	**β_0**	**β_1**
Daily	39	5	3.9e-06* (11.27)	0.08* (28.38)	0.90* (217.32)
Weekly	6	0	2.7e-05* (4.20)	0.09* (7.69)	0.89* (85.24)
Monthly	1		92e-04 (1.53)	0.11* (2.14)	0.71* (5.19)

There are no breakpoints found for the two-pass method applied to weekly data, but a breakpoint is found in the monthly series. This break is dated to 1998, indicating that there has been a structural shift in volatility during the period of dot.com boom, crash and recovery.

Five and six breakpoints are found in the 2-pass method applied to daily data and the simple method applied to weekly data. These breaks are quite similar to the points found for several of the other stock series. The 2-pass method applied to daily data dates the first break to 1987, just after the market crash, where the Swedish market shows the somewhat surprising result of going into a period of lower volatility. This is on the contrary to the result in Norway, where the volatility was quite high after this crash. Also Sweden has a period of low volatility during the mid-90s, but the 2-pass method dates the shift as late as 1994, a year after the shift occurred in Norway and Denmark. Contrary to the result from the test on monthly data, the 2-pass method applied to daily data indicates that the volatility shifted downwards again after the dot.com boom and crash.

The simple method applied to weekly data also dates the downward shift in volatility in the mid-1990s to 1994, but 6 months later than in the daily data. This test also shows four breakpoints between 1998 and 2005; increased volatility during the dot.com boom which further increased during the market crash, before falling in the two periods of market growth from 2002 to 2005.

Table 13: Volatility Periods, Sweden

SWEDEN	
DAILY 2-PASS METHOD	
Period	**τ**
3.1.1980-30.11.2005	23.67
3.1.1980-13.11.1987	21.08
13.11.1987-13.10.1989	15.93
13.10.1989-24.2.1994	24.11
24.2.1994-16.10.1997	17.16
16.10.1997-15.10.2002	33.84
15.10.2002-30.11.2005	20.22

SWEDEN	
WEEKLY SIMPLE METHOD	
Period	**τ**
9.1.1980-30.11.2005	24.98
9.1.1980-16.10.1985	19.95
16.10.1985-24.8.1994	24.77
24.8.1994-29.7.1998	17.59
29.7.1998-15.11.2000	31.58
15.11.2000-6.11.2002	42.72
6.11.2002-19.5.2004	25.35
19.5.2004-30.11.2005	13.87

SWEDEN	
MONTHLY SIMPLE METHOD	
Period	**τ**
1.2.1980-1.11.2005	24.93
1.2.1980-3.8.1998	22.44
3.8.1998-1.11.2005	30.19

4.1.3 Europe and the US

Europe

To see if the breakpoints for Norway also can be found for the whole continent, I analyze an index for the European market. As expected, the volatility of the European index is lower than for Norway (14.48), and the return is also higher (9.61). This indicates that the Norwegian stocks are moderately risky without paying a return that is on the level of the volatility.

Table 14: Europe, Descriptive

	The ICSS Algorithm		GARCH(1,1)		
Period	**Simple Method**	**Two-pass method**	**α**	**β_0**	**β_1**
Daily	43	5	1.9e-06* (15.09)	0.11* (28.39)	0.87* (145.08)
Weekly	4	3	2.5e-05* (5.28)	0.15* (8.20)	0.79* (30.81)
Monthly	3		1.4e-03* (2.00)	0.11 (1.77)	0.26 (0.76)

The number of breakpoints found in the European index is quite high compared to the other series, with three breaks found for both the two-pass method applied to weekly data and the simple method applied to monthly data.

Table 15: Volatility Periods, Europe

EUROPE	
DAILY 2-PASS METHOD	
Period	**τ**
3.1.1980-30.11.2005	14.48
3.1.1980-22.9.1987	9.30
22.9.1987-13.10.1989	16.67
13.10.1989-21.8.1991	14.49
21.8.1991-24.6.1997	9.94
24.6.1997-17.3.2003	22.10
17.3.2003-30.11.2005	12.68

EUROPE	
WEEKLY SIMPLE METHOD	
Period	**τ**
16.1.1980-30.11.2005	15.09
16.1.1980-7.10.1987	9.77
7.10.1987-13.1.1988	41.67
13.1.1988-22.7.1998	12.28
22.7.1998-19.3.2003	23.89
19.3.2003-30.11.2005	13.53

EUROPE	
WEEKLY 2-PASS METHOD	
Period	**τ**
16.1.1980-30.11.2005	15.09
16.1.1980-7.10.1987	9.77
7.10.1987-28.10.1987	60.07
28.10.1987-20.8.1997	13.27
20.8.1997-30.11.2005	19.73

EUROPE	
MONTHLY SIMPLE METHOD	
Period	**τ**
1.2.1980-1.11.2005	15.97
1.2.1980-1.10.1987	11.74
1.10.1987-1.12.1987	51.91
1.12.1987-2.6.1997	13.44
2.6.1997-1.11.2005	18.53

The results found for the 2-pass method applied to weekly data and the simple method applied to monthly data are close to similar. There was a break in volatility during the stock market crisis in the autumn of 1987, where the volatility skyrocketed. Then there was a ten year period of low volatility, until the summer 1997, when it increased to its highest level (except the 87' crash) of about 18%. The exact dating of this increased volatility might be closer to October, when the stock markets around the world crashed on October 27th because of fear of economic crisis in Asia.

As for Sweden, but opposite of Norway, the European index has had a higher volatility over the last 13 years, than during the 1980s. From 1980 to 1987 the European volatility was less than 10%, while it has been 16.84% since 1987.

The 2-pass daily method shows that this is mainly due to two higher-volatility periods; one after the market crash in 1987 and one from 1997 to 2003, and that Europe, as Norway, had low volatility during the mid-90s.

The simple method applied to weekly data and the 2-pass method applied to daily data indicates that the volatility has fallen and is on a low level again.

The US

The US market has had volatility and return at the same level as the European index, with average return of 9.33% and standard deviation of 14.48%.

Table 16: The US, Descriptive

	The ICSS Algorithm		GARCH(1,1)		
Period	**Simple Method**	**Two-pass method**	**α**	**β_0**	**β_1**
Daily	23	5	9.9e-07* (8.52)	0.06* (49.44)	0.92* (362.60)
Weekly	11	0	1.1e-05* (3.32)	0.09* (6.75)	0.89* (50.71)
Monthly	0		3.2e-05 (0.88)	0.10* (2.59)	0.89* (22.77)

The most interesting here are the breakpoints found by the 2-pass method applied to daily data, where 5 breakpoints are found. These are quite similar to the breakpoints found for Europe; a break during the 1987 market crash, followed by a four-year period of volatility higher than before the crash, a period in the mid-90s with very low volatility, and then higher volatility from 1996 to 2002. The volatility the last 3 years has been lower than for the whole period, but not as low as what is the case for Denmark and Europe.

Table 17: Volatility Periods, the US

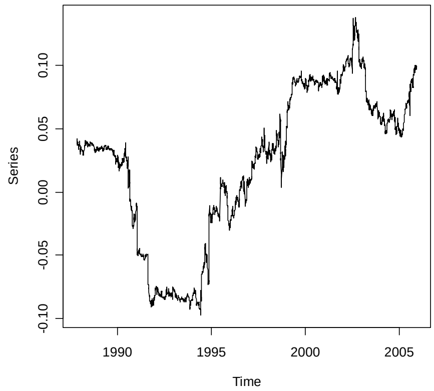
USA	
SIMPLE METHOD WEEKLY	
Period	**τ**
9.1.1980-30.11.2005	15.84
9.1.1980-7.7.1982	14.95
7.7.1982-20.4.1983	21.72
20.4.1983-21.5.1986	11.96
21.5.1986-7.10.1987	17.09
7.10.1987-4.11.1987	62.01
4.11.1987-10.8.1988	20.08
10.8.1988-1.8.1990	11.40
1.8.1990-15.4.1992	17.00
15.4.1992-26.3.1997	10.23
26.3.1997-4.10.2000	17.48
4.10.2000-19.3.2003	23.51
19.3.2003-30.11.2005	11.55

USA	
DAILY 2-PASS METHOD	
Period	**τ**
3.1.1980-30.11.2005	16.60
3.1.1980-5.8.1987	14.46
5.8.1987-21.10.1987	58.03
21.10.1987-30.12.1991	17.19
30.12.1991-2.12.1996	9.55
2.12.1996-15.10.2002	21.09
15.10.2002-30.11.2005	14.23

‘

4.1.4 Oil and Exchange Rate

Fig 1: Oil Correlation



Oslo Stock Exchange (OSE) is dependent on the oil price, and Figure 1 shows that the correlation has increased the last years, since Statoil went public. It is therefore logical to also search for breakpoints in the oil price, to possibly explain the breakpoints in the Norwegian series. The results give no significant breakpoints for any of the series when the two-pass method is used, a strong indication that the volatility does not have any breaks.

Table 18: Oil, Descriptive

	The ICSS Algorithm		GARCH(1,1)		
Period	**Simple Method**	**Two-pass method**	**α**	**β_0**	**β_1**
Daily	49	0	1.9e-06* (7.20)	0.10* (23.47)	0.91* (234.11)
Weekly	7	0	4.9e-05* (3.80)	0.13* (8.77)	0.86* (62.69)
Monthly	0		1.0e-03* (3.05)	0.34* (4.44)	0.63* (13.95)

Table 19: Volatility Periods, Oil

OIL	
WEEKLY SIMPLE METHOD	
Period	**τ**
11.1.1984-30.11.2005	36.70
11.1.1984-8.1.1986	14.12
8.1.1986-6.8.1986	90.70
6.8.1986-11.7.1990	34.71
11.7.1990-10.4.1991	87.70
10.4.1991-10.1.1996	23.09
10.1.1996-16.9.1998	32.85
16.9.1998-14.3.2001	42.26
14.3.2001-30.11.2005	31.94

The simple method applied to the monthly series indicates 7 breakpoints. Three of them occurred before 1986, and show that the first years of the 1980s were turbulent in the oil market. The volatility was extremely high around the first Gulf-war before it fell to a low level during the 1990s. This is the same as for several of the stock indexes, and it can seem like the 1990's was a quite calm period in the international markets, both for financial instruments and for commodities.

Surprisingly, the volatility over the last five years have been lower than for the overall period, even though the oil market has been somewhat turbulent because of 9/11 and the wars in Afghanistan and Iraq.

NOK/USD

To see if there has been volatility breaks in the exchange rate market that has infected the stock market, I also run the algorithm on the Norwegian Krone to US Dollar exchange rate.

Breakpoints are only found when the simple method is applied to the daily and weekly series, the two tests that return the least trustworthy results.

Table 20: Exchange Rates, Descriptive

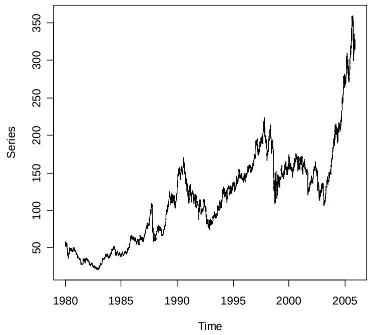
	The ICSS Algorithm		GARCH(1,1)		
Period	**Simple Method**	**Two-pass method**	**α**	**β_0**	**β_1**
Daily	36	0	6.1e-07* (8.15)	0.06* (18.17)	0.92* (237.25)
Weekly	5	0	8.9e-06* (3.16)	0.09* (4.99)	0.86* (32.66)
Monthly	0		3.7e-04 (1.21)	0.12 (1.58)	0.47 (1.21)

Of the five breakpoints found in the weekly series, only a break in May 2002 seems to occur at approximately the same time as for the Norwegian series, with a period of higher volatility the last years.

Table 21: Volatility Periods, Exchange Rates

NOK TO USD EXCHANGE RATE	
WEEKLY SIMPLE METHOD	
Period	**τ**
17.12.1980-30.11.2005	10.02
17.12.1980-20.2.1985	8.22
20.2.1985-21.5.1986	14.08
21.5.1986-19.12.1990	7.64
19.12.1990-3.2.1993	14.63
3.2.1993-15.5.2002	9.36
15.5.2002-30.11.2005	11.06

Fig 2: Exchange



The changes in volatility are quite small, only varying from a low of 7.64% to a high of 14.63%.

4.2 Volatility Change

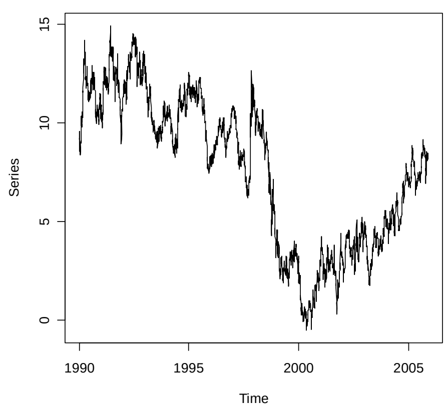
The ICSS algorithm indicates that the volatility in Norway has been significantly lower over the last 12-13 years, than during the 1980s. To further study this, I calculate volatility for overlapping ten year periods, both in USD and NOK. Four of these periods are listed in the table, while the graph shows the development of volatility for daily overlapping ten year periods, starting with 1980-1989 for the USD series. The x-axis states the last year of the period.

Table 22: Ten-Year Return and Volatility, Norway

	USD			NOK		
	Mean	**St.dv**	μ/σ	**Mean**	**St.dv**	μ/σ
Weekly	6.91	23.05	0.30	8.10	21.78	0.37
1980-1989	8.53	25.18	0.34	11.43	23.68	0.48
1985-1994	12.59	25.04	0.50	9.64	24.16	0.40
1990-1999	3.19	22.47	0.14	5.05	21.48	0.24
1995-2004	6.99	20.65	0.34	6.03	19.46	0.31

The breakpoint found in the middle of the series is clearly marked in Figure 5 (time=1997). The period from 19.10.1987 to 7.10.1997 had a volatility of 23.35, while the period starting the next day has a volatility of 21.97, a fall of about 6%. The middle of these periods is around October 14th 1992, only 5 months before the date given for the shift by the ICSS algorithm.

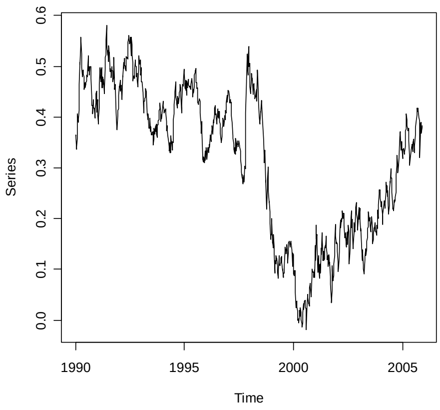
Fig 3: Average Return



After this shift in 1992/1993 the graph shows that the volatility has stayed lower than during the 1980s and early 1990s.

Figure 3 shows the 10-year return on the Norwegian market. The graph indicates that also the ten-year mean experienced a large drop just shortly after the volatility fall, and has since then been somewhat lower than in the 80s.

Fig 4: Mean/Volatility



It is clear that some of the changes here are due to the market crash in 1987. The market fell sharply on October 19th, and then increased sharply again the next day.

Fig 5: Volatility

Figure 4 shows that the fall in volatility is not only due to a lower return. The Norwegian stock market has given less return over the last 12-13 years, than during the 1980s, but it seems like the relative return is increasing again.

4.3 The ICSS and Time Periods

4.3.1 Indication

The breakpoints found in my research, differ to some degrees from those found in previous research by Aggarwal, Inclán and Tiao (1999) and Bacmann and Dubois (2001), but they used different time periods in their analysis. This raises the question of whether the breakpoints found are dependent on the time period.

To test for this, the ICSS algorithm is applied to the weekly return series of the Norwegian Index in USD in different period lengths, to see if the breakpoints found in each search are similar to those found when the algorithm is applied to the whole series.

I split it in two equal periods, four overlapping ten-year periods, and five five-year periods. The simple method is used, which originally returned 12 breakpoints (for the ten and five-year period tests, the algorithm is run on data till 2004, so the number of breaks in the original test were 11.)

Table 23: Breakpoints, 2-periods and 10-year periods

	2 Periods		**10-year Periods**			
80-05	**80-92**	**93-05**	**80-89**	**85-94**	**90-99**	**95-04**
24.8.1983	24.8.1983		24.8.1983			
26.8.1987	26.8.1987		26.8.1987			
2.12.1987	2.12.1987		2.12.1987			
25.7.1990	25.7.1990					
5.3.1992						
					6.10.1993	
20.7.1994		20.7.1994				
		17.1.1996				17.1.1996
		25.12.1996				25.12.1996
29.4.1998					29.4.1998	
		29.7.1998				29.7.1998
17.2.1999		17.2.1999			13.1.1999	17.2.1999
19.6.2002		19.6.2002				19.6.2002
31.7.2002		31.7.2002				31.7.2002
5.3.2003		5.3.2003				5.3.2003
29.6.2005		29.6.2005				
	4	**9**	**3**	**0**	**3**	**7**
12	**13**		**11**			

When I split the series in two, I get three 'new' breakpoints, but 'lose' two and end up with a total of 13. The breakpoint in 1992 is probably not found in the 1980-1992 test because it is too close to the end of the period, so the shift in volatility is not clear. A break in 1998 is moved to three months later, while two new ones are found in 1996.

For the test on ten year periods, I end up with a total of 11 breakpoints, the same as for the original test, but only 8 of them are on the same date as in the original series. Some are lost, and a few others are discovered, and it seems quite clear that breakpoints occurring in the end of the shorter periods are not discovered. This is an indication of the problem of using this method to discover breakpoints in recent time; they might be there, but are not discovered because there has not been enough time to prove its existence, and they might *not* be there, but are discovered because of a short period of time with very high or low volatility.

Table 24: Breakpoints, 5-year Periods

	5-year Periods				
80-05	**80-84**	**85-89**	**90-94**	**95-99**	**00-04**
24.8.1983					
26.8.1987		26.8.1987			
2.12.1987					
25.7.1990					
5.3.1992					
			2.12.1992		
20.7.1994					
				25.12.1996	
29.4.1998					
				29.7.1998	
17.2.1999					
19.6.2002					19.6.2002
31.7.2002					31.7.2002
5.3.2003					5.3.2003
11	**0**	**1**	**1**	**2**	**3**
	7				

The problem of losing breakpoints when short time periods are used becomes especially clear when the algorithm is run on the five 5-year periods. Only seven breakpoints are found, and it seems like two years might not always be enough to determine if there has been a break in the volatility (neither the 1983 break nor the second 1987 break are found). None of the breakpoints that occur in the first or last year of the short periods and originally were found, are found in these tests.

4.3.2 Simulation

Table 25: Volatility Breaks, Simulation

SIMULATION VOLATILITY		
12 BREAKS	VOLATILITY	6 BREAKS
1-190	26.36	1-190
191-399	21.16	191-551
400-413	58.09	
414-551	21.56	
552-635	30.09	552-635
636-759	23.07	636-759
760-956	15.75	760-998
957-998	37.71	
999-1172	18.21	999-1209
1173-1178	62.48	
1179-1209	27.86	
1210-1330	15.65	1210-1352
1331-1352	27.48	

To further study this problem, I create a normally distributed time series with breaks in the volatility of the same size as the breaks found in the simple method applied to weekly data of the Norwegian Index in USD.

I run four different simulations for different comparisons. When I use 12 breakpoints, the 12 showed in the first column of table 25 are analyzed, while I use the 6 in the last column for the analysis of 6 breakpoints. The volatility for the periods are in the centre column.

Sim 1: 12 breaks in 1352 observations, series split in 2

The first test is on 1352 observations (26 years of 52 weeks) with the volatility shifts from the initial test. The results are summed up in Figure 6. The x-axis shows the result for each simulation of "[number of breaks] - 12", so a negative number shows that fewer than 12 breaks were found. The results are given so the sum is 1. The normal distribution function with a standard deviation of 2.66 (average of the two series) is inserted.

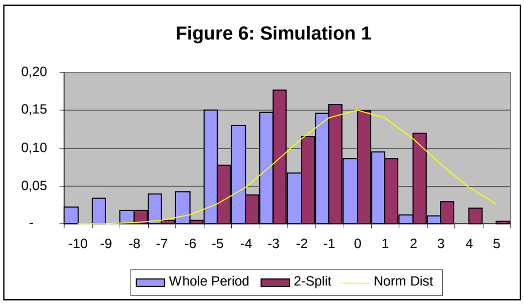


Table 26: Simulation 1

	Whole	Split
Return	8.96	10.85
St.Dev	2.84	2.49
<12	71 %	59 %
=12	9 %	15 %
>12	20 %	26 %

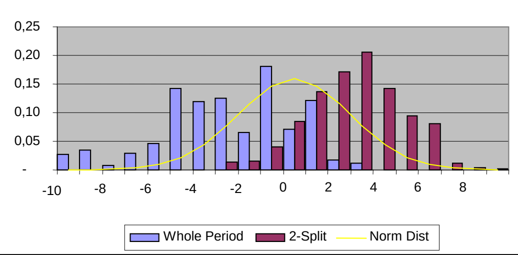
First of all, the simulation shows that the algorithm has problems finding all the breakpoints in the series. The average for the test on the whole period is 8.96, while it is 10.85 when the period is split in two. The exact number of breaks is only found in 9% and 15% of the simulations, and the algorithm seems to have a strong tendency of failing to discover breakpoints. In 71% of the simulations on the whole data set and 59% of those with a divided data set, the test discovers less than 12 breakpoints. The test on the whole period has a tendency to underestimate the number of breakpoints (extreme negative observations), while the test on the divided series has extreme positive observations, which makes it seem more evenly distributed.

In 78% of the simulations, a higher number of breakpoints are found when the divided series is analyzed, than when the whole series is analyzed.

Sim 2: 12 breaks in 1352 observations, series split in 4

In the second test the data is divided into four series of equal length, to study the effect of a larger number of periods. The results are disastrous for the algorithm.

Figure 7: Simulation 2



On average, the algorithm finds 14.62 breakpoints in the data set when it is divided into four equal parts of 338 observations (6.5 years of 52 weeks). In 85% of the simulations the number of breaks are larger than 12, leading the algorithm to return several non-existing breaks.

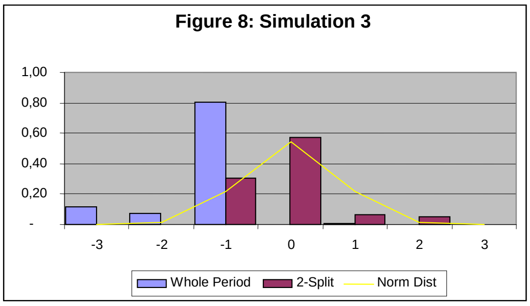
Table 27: Simulation 2

	Whole	Split
Return	9.20	14.62
St.Dev	2.91	2.11
<12	78 %	7 %
=12	7 %	8 %
>12	15 %	85 %

These first two simulations clearly show the need for using a large time period when the ICSS algorithm is run.

The last two simulations are done to look at the behaviour of the algorithm when the number of observations to breakpoints is higher. In the first two tests a new break on average occurred at every 113th observation. In the third simulation, the smallest and closest breakpoints from the original test are removed, so only half of the breakpoints are left. In the last simulation, I used 12 breakpoints on 6760 observations, just multiplying the number of observations and the time of the volatility break with 5. A break will then occur on average every 563rd observation.

Sim 3: 6 breaks in 1352 observations, series split in 2



When the number of breaks is halved, the accuracy of the algorithm gets somewhat better, even though it seems to have a tendency to underestimate the number of breakpoints. This is especially true when the algorithm is run on the whole data set of 1352 observations, where the average number of breakpoints found are only 4.72, and 99% of the simulation returns a too low number of breaks.

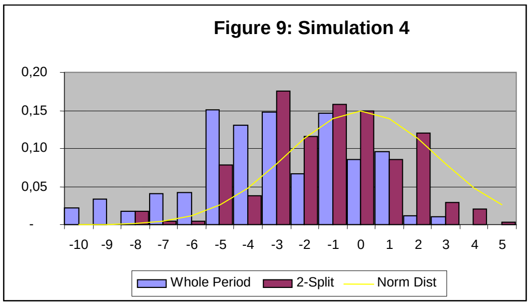
Table 28: Simulation 3

	Whole	Split
Return	4.72	5.87
St.Dev	0.70	0.76
<6	99 %	31 %
=6	0 %	57 %
>6	1 %	12 %

The results are a lot better when the series is divided in two parts, with 57% of the simulations returning the correct number of breakpoints.

The ICSS algorithm is clearly more trustworthy when the number of breakpoints is moderatly low, but still it has a tendency to find too few points.

Sim 4: 12 breaks in 6760 observations, series split in 2



The results here are almost exactly the same as for the first simulation, even though the number of observations is five times higher.

The ICSS algorithm does therefore seem to be dependent on the relative placement of breakpoints in the time series, and is therefore not necessarily more accurate when the number of observations increases.

Table 29: Simulation 4

	Whole	Split
Return	9.10	10.85
St.Dev	2.83	2.56
<12	79 %	54 %
=12	8 %	12 %
>12	14 %	34 %

All these simulations show that the results from the ICSS algorithm are questionable when the number of breakpoints found is high. This is also in line with the findings of Bacmann and Dubois (2001), which states that the algorithm are almost sure to find all the large shifts in volatility, but are less accurate when it comes to smaller shifts.

5. Conclusion

5.1 Comparison and Implications

In this thesis I have studied the volatility on Oslo Stock Exchange from 1980 to November 2005, focusing on structural breakpoints in volatility. I started searching for these breakpoints without an initial view of the location of these time points, and later looked for explanations for them.

The ICSS algorithm strongly marked a clear shift in the volatility in Oslo Stock Exchange in March 1993. In the period from 1980-1993, the volatility was 24.45%, before it shifted down to 20.26% in the years after that. Some of this is due to the fall in average return, but a more mature market is also a probable explanation. The volatility now seems to increase again, possibly due to the increased dependence on the energy markets.

It is somewhat surprising that this is the only break that is found in the weekly data when the two-pass method is used, and therefore the only break that can, with strong certainty, be said to have occurred. The only other clearly probable break was in the late summer/early autumn of 1998 with a strong increase in volatility.

The break found in 1993 is confirmed also by the test of the Norwegian index in USD, and seems to have occurred in Hydro a couple of years earlier. The same shift down in volatility also happened in Denmark in 1993, while the volatility in Finland and Sweden (the latter in 1998) shifted upwards.

The changes in volatility in Norway have also happened in other markets. Denmark, Europe and the US have the same negative volatility shift in the early 1990s, even though only Norway and Europe have this break pointed out in highly reliable tests. This break is not regional, as it is found neither for Finland nor Sweden. Also changes in volatility the last few years are found in other series.

Changes in volatility in oil price and exchange rate are found neither in the two-pass tests nor in monthly data. There is therefore no clear indication of contagion in regime shifts from these markets to the Norwegian stock market.

The shift in volatility in the Norwegian market has implications for financial modeling. Inserting the volatility for the whole period in capital allocation models, will lead to a lower allocation of capital to Oslo Stock Exchange than to other securities/exchanges, because it will seem like the exchange pays less for the risk than what is true.

The question is whether these results are permanent or only a temporary change in volatility that will switch back up again. It is natural to assume that the shift in volatility in Norway is a shift down toward Europe and the US, as a result of a growing market and internationalization of the financial markets. But the Norwegian market is growing more and more dependent on the energy market, and the volatility seems to have increased somewhat over the last few years again.

5.2 Strenghts of methods

In this thesis, I have shown that there are some problems with the ICSS algorithm. Specifically, if many breakpoints exist in a series, the algorithm has problems discovering them all, and, only in quite few of the simulations, are the algorithm able to 'discover' all the breaks.

Simulating with different length of time periods, I have shown that if a long period is used, the method is likely not to return non-existing breakpoints, but will fail to discover all. The shorter the period, the more likely the algorithm is to return too many breakpoints.

One alternative approach to detect breakpoints is the wavelet analysis. The wavelet method has been extensively used to analyze economic and financial data the last few years. Fernandez (2006) compared the ICSS algorithm to wavelet analysis, concluding that the latter is the more robust of the two. A problem with Fernandez' test is the use of very short time periods in the comparison (stock return series over six years and interest rate series over 2 years). The ICSS algorithm seems to easily miss out on breaks happening shortly after the beginning of the series or shortly before the end of the series, making the results for the algorithm in that test untrustworthy.

5.3 Further research

To clearly identify the breakpoints in the time series I have analyzed, they should also be studied using other methods, i.e. wavelet analysis. From the ICSS algorithm, there is still a question of the number of breakpoints in the Norwegian data (i.e. was the high volatility period in the autumn of 1998 a structural shift?) that can be answered using other methods.

My simulations are run using volatility from the Norwegian Index in USD, and further research should study the problem of choice of time period length using other volatility change sizes and change points.

The question of how long after a break occurred the algorithm is able to discover it, should also be answered. It is probable that large shifts in volatility are more easily discovered, while smaller changes take longer time to find. But small changes might also be wrongly assumed to be breakpoints, and further research should try to answer at what time a breakpoint can be confidentially stated to have happened.

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Appendix A: Descriptive Data

		Mean	St.dv	Skew.	Kurt.	Ljung-B	Jarque-B	ARMA(1,1) Autocorr	ARMA(1,1) GARCH
Norway NOK	Monthly	7.99	25.41	-0.72	5.54	2.39	158.65*	0.10	3.43
	Weekly	8.16	22.44	-0.36	6.27	3.78	699.06*	0.90	57.75*
	Daily	8.10	21.78	-0.84	22.51	57.28*	109170.7*	0.01	854.59*
Norway	Monthly	6.92	26.41	-0.55	5.06	0.28	99.06*	0.45	3.47
	Weekly	6.95	23.53	-0.36	5.94	2.75	565.27*	0.40	39.73*
	Daily	6.91	23.05	-0.93	21.64	50.11*	99745.08*	0.01	676.86*
Hydro	Monthly	10.14	30.67	-0.09	3.72	0.0002	15.47*	0.33	7.44*
	Weekly	10.05	30.31	-0.04	5.58	2.74	377.96*	0.02	16.74*
	Daily	10.05	30.99	-0.35	14.47	9.31*	36134.77*	0.01	859.19*
Denmark	Monthly	11.41	19.45	0.25	3.34	2.83	5.38	0.10	0.04
	Weekly	11.37	18.82	-0.004	4.74	0.08	194.50*	0.04	12.66*
	Daily	11.37	18.40	-0.17	7.17	28.75*	5083.70*	0.001	243.12*
Finland	Monthly	15.13	31.22	0.20	4.71	5.26*	39.00*	0.0002	1.68
	Weekly	15.04	31.33	-0.16	7.35	0.03	1048.09	0.04	17.29*
	Daily	15.05	30.54	-0.02	14.72	10.18*	35776.15*	0.01	70.30*
Sweden	Monthly	13.97	24.93	0.18	3.46	0.50	8.30*	0.09	16.77*
	Weekly	13.93	24.98	-0.19	5.72	1.30	489.16*	0.0002	55.57*
	Daily	13.94	23.67	-0.22	9.86	38.02*	13538.39*	0.004	400.92*
Europe	Monthly	9.56	15.97	-0.89	7.68	2.03	500.52*	0	2.80
	Weekly	9.67	15.09	-0.62	10.53	0.09*	3618.92*	0.38	223.60*
	Daily	9.61	14.48	-0.54	11.52	26.06*	21426.5*	0.004	718.89*
USA	Monthly	9.24	14.61	-0.30	6.24	0.41	215.00*	0.01	2.41
	Weekly	9.41	15.84	-0.22	6.64	1.77	845.93*	0.12	108.29*
	Daily	9.33	16.60	-1.63	42.80	1.92	455260.3*	0.07	90.42*
NOK-USD	Monthly	0.87	10.37	0.38	3.64	0.08	9.82*	0.01	1.87
	Weekly	1.04	10.02	0.19	4.55	0.26	135.45*	0.02	4.70*
	Daily	1.04	10.36	-0.29	10.12	8.26*	13811.36*	0.01	20.90*
Oil	Monthly	3.25	37.77	0.07	4.82	0.06	37.67*	0.33	5.32*
	Weekly	2.72	36.70	-0.32	8.85	2.23	1669.98*	0.0004	60.69*
	Daily	2.82	37.86	-1.04	29.76	1.60	171965*	0.01	33.22*

Appendix B: ICSS algorithm in R

Preperations:

x = data series (after AR(1) or GARCH(1,1) if necessary)

date = the dates matching x

T = number of observations in x and date.

In the case were the data are filtered through a GARCH(1,1), the volatility for each period must be calculated after the algorithm is run, as this code will return the volatility for the GARCH(1,1) residuals.

```
y=0
y[1]=0
t=te=y[2]=T
a=2
i=l=yy=D=0
a=j=2
m=10

tb=1

while(j>1) {
j=0

while(m>D) {
j=j+1
q=x[tb:te]
k=1:(te-tb+1)
DK=cumsum(q^2)/cumsum(q^2)[te-tb+1]-k/(te-tb+1)
M=sqrt((te-tb+1)/2)*DK
m=max(abs(M))
te2=te
te=which.max(abs(M))+tb-1
N=te2-tb+1
if(N>500) {D=1.358} else {
if(N>400) {D=1.33} else {
if(N>200) {D=1.31} else {
if(N>100) {D=1.30} else {
D=1.27}
}}}}

a=a+1
y[a]=te2
tb=te2+1
te=t
m=10


if(j>1) {
m=10
j=0
while(m>D) {
j=j+1
q=x[tb:te]
k=1:(te-tb+1)
DK=cumsum(q^2)/cumsum(q^2)[te-tb+1]-k/(te-tb+1)
M=sqrt((te-tb+1)/2)*DK
m=max(abs(M))
tb2=tb-1
tb=which.max(abs(M))+tb
N=te2-tb+1
if(N>500) {D=1.358} else {
```

```
if(N>400) {D=1.33} else {
if(N>200) {D=1.31} else {
if(N>100) {D=1.30} else {
D=1.27}
}}}
}
if(j>1) {
a=a+1
y[a]=tb2} else{}
te=tb2-1
t=te
tb=y[a-1]+1
m=10
} else {"stop"}
}
y=sort(y)
l=10

while(l>0) {
j=1
i=i+1
while(j<a-1) {
j=j+1
tb=y[j-1]+1
te=y[j+1]-1
q=x[tb:te]
k=1:(te-tb+1)
DK=cumsum(q^2)/cumsum(q^2)[te-tb+1]-k/(te-tb+1)
M=sqrt((te-tb+1)/2)*DK
m=max(abs(M))
N=te-tb+1
if(N>500) {D=1.358} else {
if(N>400) {D=1.33} else {
if(N>200) {D=1.31} else {
if(N>100) {D=1.30} else {
D=1.27}
}}}
if(m>D) {
yy[j]=which.max(abs(M))+tb-1
} else {yy[j]=0}
}
yy[a]=T
j=1
k=0

while(j<a) {
j=j+1
l=abs(y[j]-yy[j])+l
}
yy=sort(yy)
j=1
while(j<a) {
if(yy[j]==yy[j+1]) {
yy[(j+1):(a-1)]=yy[(j+2):a]
yy=yy[1:(a-1)]
a=a-1
} else {j=j+1}
}
if(i==20) {
l=0} else {}
if(l==0) {} else {y=yy}
}
l=10
s=0
while(l>0) {
```

```
s=s+1
l=0
j=1
while(j<a) {
if(y[j]==y[j+1]) {
y[(j+1):(a-1)]=y[(j+2):a]
a=a-1
} else {j=j+1}
}
j=1

yy=1:a
yy[1]=0
yy[a]=T

while(j<a-1) {
j=j+1
tb=y[j-1]+1
te=y[j+1]
q=x[tb:te]
k=1:(te-tb+1)
DK=cumsum(q^2)/cumsum(q^2)[te-tb+1]-k/(te-tb+1)
M=sqrt((te-tb+1)/2)*DK
m=max(abs(M))
which.max(abs(M))+tb-1
N=te-tb+1
if(N>500) {D=1.358} else {
if(N>400) {D=1.33} else {
if(N>200) {D=1.31} else {
if(N>100) {D=1.30} else {
D=1.27}
}
}
}
if(m>D) {
yy[j]=(which.max(abs(M))+tb-1)
} else {yy[j]=0}

}

yy=sort(yy)
j=1
l=0
while(j<a) {
j=j+1
l=abs(y[j]-yy[j])+l
}

if(l==0) {
} else {
y=yy
}
j=1
while(y[2]==0) {
y[2:(a-1)]=y[3:a]
y=y[1:(a-1)]
a=a-1
}
if(s==50) {l=0} else {}
}
y[1]=1
y=sort(y)
j=0
yv=0
yv[1]=100*sd(x)*sqrt(260)
```

```
while(j<(a-1)) {
j=j+1
tb=y[j]
te=y[j+1]
yv[j+1]=100*sd(x[tb:te])*sqrt(260)
}
if(Month[3]-Month[1]>1) {d=12} else {
if(Day[2]-Day[1]<5) {d=260} else {d=52}}
yd=date[y]
q=x
z=data.frame(x=yd,y=yv)
a=nrow(z)
y[1]=1
y=sort(y)
j=0
yv=0
yv[1]=100*sd(q)*sqrt(d)
while(j<(a-1)) {
j=j+1
tb=y[j]
te=y[j+1]
yv[j+1]=100*sd(q[tb:te])*sqrt(d)
}
YYY=data.frame(yd,yv)

YYY
```