Norges Handelshøyskole

Spring 2008

An Econometric Analysis of the Office Real Estate Market in Oslo and Bergen

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Handed in the 19th June, 2008

This master thesis has been written following my study curriculum, which embeds a technical background in construction and a master degree in finance and administration. Being involved in both, it turned out interesting to tailor a project drawing from these two domains, and especially real estate development.

Abstract: *This master thesis exposes the application of two existing econometrical and dynamic models originally written for London to the Norwegian cities of Oslo, the capital, and Bergen, the second largest city of the country. Both models presented simultaneously two unique equilibrium-based systems of equations using similar variables to estimate and forecast the London office real estate market. In a first time, the thesis reverts to quantify the efficiency of these models that fit very well an internationally-sized city to other candidates with an obvious smaller dimension but also a dissimilar development, compared to London and to each other. Secondly, some additional expressions of both models are presented and estimated. Finally, this work devotes a section to analyse and compare estimates together before drawing conclusions.*

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Introduction

The composition of this thesis has been motivated by the will to write something exploitable for professional use, especially for the sector of the real estate development in Norway.

The primary idea, which was to study the housing price in Norway, appeared rapidly difficult because this latter grew somewhat irrationally during the last years in almost every European country, certainly in Norway due to a tight correlation with the oil price, but also more generally for macroeconomic reasons like regime switches between a high and a low interest rate regime for instance. Predicting the housing price in Norway involved using a broader macroeconomic scope rather than a more standard framework such as "cost of land/ cost of materials/overhead cost" for instance.

On the other hand even in Norway, the market of office construction seemed more inclined to have features linked to some exogenous and traditional determinants. The application of econometrical methods over the example of Norwegian cities appeared thus susceptible to fit and give significant results.

The office real estate market in the main cities of every developed country experienced a common symptom through the 1980s and 1890s decades, a myopic construction of office spaces clustered in financial centres, generally called central business districts. It was mostly based on unfunded expectations from developers lacking data about the market and led to a large oversupply of office surface. This caused eventually the apparition of a new phenomenon: the cyclical behaviour of the office market. This widespread problem entailed in the late 1980s researchers to elaborate models based on rational information which are able to trace and forecast office space supply and demand.

Since Rosen in 1984, who is recognized being the first to build a multi-equation model, many research papers and works were published and now a panel of empirical models are available on this specific subject. As far as the literature review went, the most recent model was published in 2006 by Öven and Pekdemir. They expose a quantitative and qualitative multi-variable model able to predict the office rent level in the city of Istanbul. This model presents however several downsides: it requires a cumbersome collection work of many particular data which are generally not available for Oslo and Bergen, and it explains only the rent level. Other more complete models such as those utilized in this paper study the rent level but add also other components, such as the office space supply and demands.

During the review of the literature, two models appeared to be very interesting. They both start by setting up the basic framework exposed by Rosen to which they add identities and change the formulations of the equations to increase the ability of the model to fit the reality. The models were devised at the same moment and both present the remarkable feature to study the office market of London. They are supposedly using close data as they both cite the same data research corporation, DTZ Debenham Thorpe at London. Nevertheless, one considers only the market of the City of London whereas the other encompasses a larger radius of roughly 40 miles around the same area of central London. It turned out interesting to use both models in order to check for any common trend in the outcomes but also to comfort the conclusions by supplying a larger panel of results.

The two models use sensibly the same variables and compute similar identities and equations, but arrange them in different order. This resemblance is puzzling and a special heed has been to expose the differences between the workings of the different models.

Finally, the perspective of the paper is to provide the most complete representation of the market behaviour in two largest cities of Norway, Oslo and Bergen. Through the utilization of dynamic models, the paper attaches importance to identify what factors drives each one of the markets, and what are the markets' patterns. This could eventually be employed further in forecasting.

Preamble

Chronologically, both articles were written in 1997, even if one was officially published only in 1999. The first to come out was **"The Cyclic Behavior of the Greater London Office Market"**, written by *Wheaton, Torto and Evans* in 1997. Wheaton and Torto are two American professors while Evans was research director at the data research company mentioned earlier, DTZ Debenham Thorpe. The two former had several prior publications written either on their own or in collaboration with other researchers, and most of their work is focused on the analysis and forecast of various real estate markets across the planet. Their names are currently associated with Torto Wheaton Research, which is an independent company specialized in observation, analysis and prediction of the real estate of USA and Canada, among others.

The second research team, *Hendershott, Lizieri and Matysiak*, published **"The Workings of the London Office Market"** in 1999. Like Wheaton and Torto, these three professors and especially Hendershott wrote previously several studies related to real estate market in USA, Great Britain and Australia.

Each article pictures a complete econometrical and equilibrium-based but dynamic model of the office market of London, built on more partial ones published over time. Both models are structured in a similar manner, as said earlier. However, they embed the variables into their equations and identities differently. This had been caused by the different approaches of the authors. According to the elaboration path described above, each work is largely based upon the previous work of the most preeminent members of each team, namely Wheaton and Torto on one side, and Hendershott on the other.

Although the *Wheaton*, *Torto* and *Evans* paper (WTE) was published earlier, this thesis conveniently presents the *Hendershott*, *Lizieri* and *Matysiak* work (HLM) always in first position. In addition, an extra formulation is presented and estimated for each equation, in order to compare the original models with some more recent formulation. All models produce influential outcomes that are compared with each other at every step of the procedure.

As a matter of fact, the data utilized in this thesis covers the period 1987-2007, i.e. 21 years against 20 for HLM and 21 for WTE. This is however a somewhat limited time series and all estimations based on such a short period would generally be biased by the so-called "small sample bias". However, the models produce very suitable results that permit drawing a representation of the office markets in each city.

The first section presents briefly the functioning of the office real estate market to get an introductive insight to the empirical part.
In the second section, the two models are thoroughly exposed and then summarized side by side in a table. This section explains also the functioning of the models and their reaction to exogenous variable variations.
The third section is devoted to the description of the data utilized commonly by the two models. It is though judicious to notice that the models do not input exactly identical data and a final table summarizes the variables for the two models.
The fourth section describes graphically the models to be regressed and presents their respective estimates. Additional formulations are proposed and estimated statistically. These results are compared and commented within the cities.
Finally, the two last parts in a first time compare the outputs obtained by the models, before defining what possible improvements to make and drawing general conclusions.

A brief and indicative review of the antecedents of each article is presented on the next page.

Date	*Name*	*Date*	*Name*
1999	**Hendershott, Lizieri and Matysiak.** ***The Workings of the London Office market***	1997	**Wheaton, Torto and Evans.** ***The Cyclical Behavior of the Greater London Office Market***
1997	***WTE. The Cyclical Behavior of the Greater London Office Market***	**1994**	**Barras.** ***Property and the economic cycle: Building cycles revisited***
Provided an alternative model, which has inspired the authors, and presumably impact the way they designed at least one equation.		Attempted to identify the causes of cyclical behaviour of office real estate markets. Considers employment as one of the explanatory variables responsible whereas before construction pace had only been identified.	
1996	**Hendershott.** ***Rental Adjustment and Valuation of Real Estate in Overbuilt Markets: Evidence of the Sydney Office Market***	**1988**	***Gardiner and Hennebery. The development of a simple regional model of office rent prediction***
Reutilized the Valuation model exposed in Hendershott and Kane, and defined the rental adjustment equation as it is in the HLM model. However, it is beforehand presented in previous papers of the author in 1995 and again in 1996.		Presented a rent adjustment model which embeds supply and demand measures. Included the notion of delays in the effectiveness of the variables.	
1995	***Hendershott and Kane*. *US Office Markets Values During the Last Decade: How Distorted Have Appraisals Been?***	**1988**	***Voith and Crone*. *National Vacancy Rate and the Persistence of Shocks in U.S, Office Markets***
Improved the model from Corcoran with more complete equations including measures of interest rates, inflation, and taxes too. Exposed the Valuation model reutilized from there on in Hendershott following publications.		Exposed a model of vacancy-adjustment around a long-run equilibrium level estimated by the model. This adjustments model concept inspired the authors for equations of other variables in the TWE model.	
1987	**Corcoran.** ***Explaining the Commercial Real Estate Market***	**1987**	***Wheaton*. *The Cyclical Behavior of the National Office Market***
Integrated the concept of equilibrium-based model, linking the rental income with the return on market securities, and also including also some tax appraisals.		Added space identities for Demand and Supply to the Rosen model, instead of directly using equations. However, it approximated rents with vacancies, and especially did not link the model to capital markets yet.	
1984	**Rosen.** ***Toward a Model of the Office Building Sector***		
Set up the basic framework of equations "supply-demand-rent adjustment" and used several variables still employed in the both HLM and WTE models. Unfortunately, statistical results were inconclusive regarding its ability to work.			

A. Functioning of the Office Market

This section presents rapidly the reasons of the cyclical behaviour of the office markets. However, even if the patterns are shared by every city worldwide, there are notable differences in the development, i.e. the amplitude and the length of the cycles.

Starting from structural, say "in equilibrium", conditions on a given market, there are market features associated, such as the natural rent level and the natural vacancy rate which goes in line with each other. In this situation, a potential tenant of an office space may pay the naturally fair price to rent. As soon as he enters the rental market, the vacancy diminishes and eventually the price goes very high because of scarcity of space (Boom market). At that state, property investors can make abnormal high returns on investment and are willing to invest. This triggers orders of new office space construction which increases the vacancy on the market again. Thus, the rent level adjusts negatively depending on the new supply (Overbuilt market). Eventually, the rent level goes under equilibrium and the vacancy rate goes up inversely. At that time, new tenants are willing to rent because of low rents due to large vacancies, which leads finally back to the initial equilibrium situation. This cyclical phenomenon entails more or less large deviations around the equilibrium levels of both the vacancy and the rent, which subsequently affect the variation of the rent level.

This functioning engendered the persistence of cycles in the office market. To illustrate this, consider that some investors decide to invest because of favourable conditions, it is likely that the market demand will not be identical, or even will have reversed, when the construction is achieved after 2 or 3 years. This would lead to extra vacancy and persistent low rent until the market finally absorbs the extra supply. In addition, the increasing length of lease contracts leads to a stickier demand, which reflects a persistent rigidity in the rent level. One extra reason would be that the developers normally do not bear the risk of failure, but rather the lenders, like for a long-term nonrecourse lending, or secured lending. When a borrower repeatedly misses payment of principal and interest, the bank becomes legally the owner of the collateral, i.e. the real estate asset in this case. The developers will just balk away and wait for a new opportunity.

Anyway, these reasons are not a complete picture of all markets since there are differences in size, development, renting behaviour, lending habits, city regulations, etc. But the phenomenon of overbuilt office real estate has been experienced worldwide, and it has been evidenced that the rent remains at a high or low level several periods.

Yet, property developers, which are considered as rational, should learn from their past errors and adjust rents more quickly to restore structural conditions of the market. Across literature, this is seen as the fact that property investors may miss "necessary information to set the market-clearing rent"[1] and thus, base their expectations on "gross guesses about movements to the equilibrium"[1].

Through historical monitoring of vacancy rate and rent level, two major characteristics of the real estate office markets have been evidenced. The first is that the peaks on each side of the structural "equilibrium" level are inclined to gain in amplitude, and the second is that the equilibrium had a slight upward trend over time. It evidences that developers do not rely on their past mistakes but rather on their current presumptions, and confirms the tendency to an "overbuilding" development.

The purpose of the models presented below, as well as any empirical research on this topic, is to understand the factors responsible of this behaviour and capture accurately the adjustment process in order to produce more reliable predictions.

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[1] Wheaton and Torto, 1988. Vacancy Rates and the Future of Office Rents.

B. Exposition and workings of the models

1. Hendershott, Lizieri and Matysiak (1999)

The model is composed of 7 equations: 3 of them lead to a statistical regression whereas the 4 remaining are space identities. They link two exogenous variables (employment and real interest rate) with six endogenous variables (real rent, supply of office space, occupied space, absorption, vacancy rate, construction completions).

The core identities of the models are the supply S and demand OS equations. One has in addition the vacancy rate identity v since it links the two former together.

- The supply equation of office space indicates that the total space of the current year is the total space of the past year constrained by the depreciation rate, plus the office construction starts and completions effectively done during the current year, encompassed under the variable *Completion*:

$$S_t = (1 - \delta)S_{t-1} + Comp_t \qquad (1)$$

where δ is the depreciation rate, also called discard rate to recall the idea of demolition of obsolete office space.

- The demand equation returns the Occupied office Space. It consists in the past year demand plus the difference between the current and the same past year, which is called *Absorption*:

$$OS_t = OS_{t-1} + AB_t \qquad (2)$$

- The vacancy rate expresses the difference in percentage between the total space (supply S_t) and the occupied space (demand OS_t) every year:

$$v_t = 100\frac{S_t - OS_t}{S_t} \qquad (3)$$

➢ The model considers thereby several equations that need to be estimated through a statistical regression. There are 3 equations, to which one last identity is added to calculate the so-called equilibrium rent R*:

$$R^* = (r + \delta + oper)RC \tag{4}$$

The variables of the equation are as following: the real interest rate r is the risk-free return to the capital plus a measure of risk premium, δ is the depreciation rate enunciated above, $oper$ is the operating-expense ratio, which is representative of the overhead cost and is treated as a percentage of the nominal rent level, and finally RC is the replacement cost of the real estate asset. The Equilibrium rent intervenes into the computation of both the *Rent adjustment* and the *Completion*. From this, one can deduce that one of the exogenous variables, the real interest rate, affects negatively both supply and demand. This makes sense since it increases the Equilibrium rent, i.e. the return required by lenders. The smaller it is, the more numerous are new constructions and inversely.

➢ The *Rent adjustment* equation is dedicated to replicate the variations of the rent level over time.

It is designed in the paper as $\%\Delta \mathrm{R} = \mathrm{f}(v^* - v,\ \mathrm{R}^* - \mathrm{R})$ (5)

According to the formula, the percentage change of the real rent level $\%\Delta R$ varies according to what it is called "*gaps*" in the paper, i.e. the difference between the equilibrium vacancy and rent levels (denoted by a *) and the actual values. The equilibrium rent is here supposed to be varying over time, contrary to the replacement cost which is constant[2]. The story behind that they supposed that the variations in the interest rate capitalized mainly in the price of land, which basically leads to offset changes in interest rates and in the replacement

[2] At that moment the article mentions also the Valuation model exposed previously in Hendershott and Kane (1992). It says that V, the value of the asset (building) may equal its replacement cost RC. This is more detailed in part C, section Replacement cost.

cost. On the other hand, WTE stated the replacement cost being variable, and the equilibrium fixed in the long-run. This latter view seems more realistic, and is more developed later.

A critical issue is to determine the time lag of adjustment. The authors of the paper used the real rent R at the beginning of the current period, which is actually the rent of the previous year R_{t-1} . This entails that the gaps deviations might affect the rent the next year, thereby almost immediately. The authors have thereby stated the equation to be estimated as following:

$$\frac{\Delta R_t}{R_{t-1}} = \alpha - \lambda v_{t-1} + \beta(R_t^* - R_{t-1}) + \varepsilon_t \qquad (5')$$

This equation is drawn from research papers published by Hendershott and Kane in 1995[3] that Hendershott used again in other papers he published later on his own (Hendershott 1995, 1996a, 1996b). These articles mention the ratio V/RC, between V the value of the building and its replacement cost. Whenever the ratio equals 1, the market is in equilibrium, which also means that $R_t^* = R_{t-1}$ and $v_t^* = v_{t-1}$. The problem is to assess what the true value of the replacement cost is. In fact, as a rule of thumb the replacement cost of a building is 2/3 for the land and 1/3 for the construction. Unfortunately, neither official, nor reliable data of it or on the cost of land were available in any cities.

For this reason, the natural vacancy rate v^* cannot be directly calculated, but is indicatively estimated through the regression, the constant α being equalled to $\alpha = \lambda v^*$ to retrieve backwards $v^* = \lambda/\alpha$.

Even if Hendershott, MacGregor and Tse (2002)[4] states it being an Error Correction Model(ECM), this expression resembles to a Partial Adjustment model as the parameter β

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[3] Hendershott P., Kane E., 1995. U.S. Office Market Values During the Past Decade: How Distorted Have Appraisals Been? *Real Estate Economics* 23:2, 101-116.

[4] Hendershott P., MacGregor B. and Tse R., 200. Estimation of the Rental Adjustment Process. *Real Estate Economics*; 30, 2; 165-183.

expresses indeed the rapidity of adjustment of the last period rent level to the natural one, or in other words the proportion of the rent level which adjusts towards the equilibrium level each year. An explanation about both mechanisms can be found in the appendixes.

- The second equation to be estimated is the *Completion* equation. As a recall from Equation (1), the completions indicate the difference between the (office) supply of the current year and the one of the previous year scaled by the depreciation rate.

Therefore, *Completion* supposedly includes the surface of every effective construction start and achievement of the current year. Similarly to the rent adjustment mechanism exposed above, property development is triggered whenever the vacancy is really small and the rents are at a high level. This situation has been overtaken recently in the office markets in Oslo and Bergen where there is nowadays a scarcity of office surface to rent, the rental price being steady. The situation is similar in Bergen, but with generally a lag compared to Oslo and of course a much lower amplitude. More generally, this problem is often common to every city of the world.

Another characteristic of the office market is related to the environmental features of the location: whatever the expansion can be, it should eventually reach a problem such as a natural barrier or distance from the city centre. This is currently the case in Oslo where the actual financial centres are overbuilt and the actual level of office real estate production seems to stagnate. Nonetheless, one notices the apparition of new locations such as Bjørvik, and even Tjuvholmen, as the rent started to grow again last year.

Thus the authors stated the construction completion being naturally linked to the same variables, at the difference that the real rent gap is reversed here, R needing to be very higher than R^*.

The equation is stated as $Comp = g(v^* - v,\ R - R^*)$. (6)

Nevertheless even the rent should adjust almost immediately, the construction starts should normally respond to the rent gap with a lag which has to be assessed. On the other hand, completions should not be influenced by these variables and thus simply progress at their normal construction pace. The authors unclearly proposed this equation and encountered some problems during estimation which affected its formulation. This will be developed in the part D.

- The last equation is the *Absorption* equation. It is used to estimate the net variation in occupied space every year. As a recall from Equation (2), the absorption is just the difference between the previous and the current demand.

The equation has thus been stated as $\frac{Absorp}{D_{t-1}} = h(\%EMPL, R)$ (7)

The authors have simply related *Absorption* positively to the employment percentage variation and negatively to the real rent level. The evidence here is pretty straightforward, it is obvious that a larger amount of workers necessitates a larger office surface, but higher rents encourage employers to reduce the office space per worker.

But its calculation requires indeed the utilization of a more complicated mechanism of Error Correction (ECM). Two equations need to be calculated, one for the desired office demand and another one for the dynamic absorption adjustment:

$$OS_t^* = \gamma_0 + \gamma_1 E_t - \gamma_2 R_t + \varepsilon_t \qquad (7\text{'a})$$

$$\frac{AB_t}{OS_{t-1}} = \mu_1 \frac{AB_{t-1}}{OS_{t-2}} - \mu_2 \frac{\Delta R_{t-1}}{R_{t-2}} - \mu_3 \varepsilon_{t-1} \qquad (7\text{'b})$$

The desired demand is designed as the maximum occupied space, i.e. the demand itself. The desired space is actually the demand *ex ante*, or the potential demand disregarding imperfections in the rental market such as adjustment costs or tenants moving out. Like in the relation (7'a), a high employment increases the equilibrium demand level whereas a high rent reduces demand to space. The lagged residual from this equation is thereafter inputted as a correction term of the equilibrium demand level into the dynamic adjustment equation (7'b). The estimation of the equation (7'b) gives the approximation of the error correction term μ_3, which expresses the speed of adjustment of the model to the equilibrium. It connotes the sticky feature of the absorption which cannot adapt to its equilibrium level rapidly.

The absorption is also linked to its lagged value and to the lagged variation of the rent level. Both convey information on the past values of variables that impact directly the absorption, the vacancy rate and the rent level.

2. Wheaton, Torto and Evans (1997)

The WTE model counts 6 equations in total, so be it one less than the HLM one, but there are still three of them that have to be estimated through a regression. However, it links not only two but three exogenous variables (nominal interest rate, employment and replacement cost of construction) but still with six endogenous variables (supply of office space, occupied space, absorption, vacancy rate, new construction orders).

The two models share the basic framework composed by the space identities *Supply*, *Demand* and *Vacancy* as following:

- *Supply* $S_t = S_{t-1} + Const_t(1-\delta)$ (I)
- *Demand* $OS_t = OS_{t-1} + AB_t$ (II)
- *Vacancy* $v_t = 100\frac{S_t - OS_t}{S_t}$ (III)

The *Supply* formula shows two differences with the HLM model.

While the latter used $Comp_t$ being the completed development, WTE utilizes $Const_t$ the construction orders of the current year. This is a more reasonable denomination as it has been mentioned in the review of the HLM model. It is in effect possible to track down the total surface of construction orders by collecting construction contracts. On the other hand, measuring achievements accurately seems almost impossible in reality, for instance because of the system of construction reservations when finishing a building or some complications occurring during the construction phase, which will postpone the delivery date reported in the contracts.

Second difference in WTE, the depreciation is now applied to constructions orders rather to the total office stock of the past year. Thus, the authors seem to consider irrationally that the existing stock stays available on the market infinitely. They do not account for the dilapidation of the building and rely on the data gathered on the office space.

However, this makes the *Construction* curve of WTE parallel and situated under the *Completion* curve of HLM. It might be understood as *Construction* only amounting the starts and not both starts and completions like in the HLM model. Nevertheless, the one-year lag assumption seems quite unrealistic in both cases, because it implies that all construction projects started in the year are finished at the end of the year.

On the other hand, the *Demand* and the *Vacancy* identities are identical for both models.

- The first equation to be regressed is *Absorption*. Again, it accounts for the net variation in occupied space every year. It is stated as:

$$AB_t = \tau_1[\alpha_0 + E_t(\alpha_1 - \alpha_2 R_{t-1})] - \tau_1 OS_{t-1} + \varepsilon_t \qquad \text{(IV)}$$

Like for HLM, the equation links *Absorption* to employment and rents but the formulation of the two equations are different because WTE utilizes a Partial Adjustment mechanism.

First of all, the parameter τ_1 was also available in the HLM as μ_3, even if it is different as HLM uses an ECM. Like for HLM, it indicates the proportion of the stock which actually adjusts to the long-run equilibrium demand every year. The equilibrium is however identical than in the HLM adjustment equation, since it represents the potential office demand, or demand *ex ante*, if there were no imperfections This equilibrium noted OS_t^* is actually the fragment of the equation (IV) between square brackets:

$$OS_t^* = \alpha_0 + E_t(\alpha_1 + \alpha_2 R_{t-1}).$$

τ_1 expresses the rate of adjustment of the Absorption level to its equilibrium.

Secondly, the equilibrium equation tells that the demand is the product of E_t the employment and the office space demanded per worker $(\alpha_1 + \alpha_2 R_{t-1})$. α_1 defines the baseline surface per worker and α_2 the proportion (in addition or subtraction) depending upon the level of rent. Both models employ very similar "equilibrium equations".

Finally, the equation (IV) is obtained by the difference from equation (II):

$$AB_t = OS_t - OS_{t-1} = \tau_1(OS_t^* - OS_{t-1})$$

This implies that the Absorption equation can be also reformulated using directly occupied space:

$$OS_t = \tau_1[\alpha_0 + E_t(\alpha_1 - \alpha_2 R_{t-1})] + (1 - \tau_1)OS_{t-1} + \varepsilon_t \qquad \text{(IV')}$$

➢ The second equation concerns the Rental adjustment. The explanatory variables are the rent, the vacancy rate and the absorption growth. Like previously, the equation integrates an "equilibrium" towards the endogenous variable will tend over time without reaching it:

$$R_t = \mu_3[\mu_0 - \mu_1 v_{t-1} + \mu_2(AB_{t-1}/OS_{t-2})] + (1 - \mu_3)R_{t-1} + \varepsilon_t \qquad \text{(V)}$$

This long-run equilibrium is the target rent $R^*$and again is the equation between square brackets, $R^* = \mu_0 - \mu_1 v + \mu_2(AB/OS)$, where v and AB/OS are the historical averages of the vacancy rate and the absorption variation. As a note, the equation was originally using $\frac{AB_{t-1}}{OS_{t-1}}$ rather than the variation rate reported in (V), but it almost let unchanged the results for Oslo and Bergen since the changes in occupied space are small each year. This has been done conveniently to be used in section D.

Each article presents a different equation: while HLM states that the rent adjusts around a varying equilibrium, WTE designed it being more steady as a baseline level of rent μ_0. Then, the tenants who pull out from the rent market (augmentation of vacancy v_{t-1}) are automatically replaced by new ones (augmentation of the absorption growth AB_{t-1}/OS_{t-2}). One notices that vacancy rate affects negatively the rent, like in the HLM model.

The equation (V) is found following the same procedure than in (IV), using the variation in rent between two consequent years: $R_t - R_{t-1} = \mu_3(R^* - R_{t-1})$.

The two models differ drastically about the equilibrium rent because it is supposed constant over time contrary to the HLM rent adjustment equation (5'). Having a fixed equilibrium demand appears more realistic because the long-run target should be stable, especially in order to make forecasts, even in the short or mid-term.

- The last equation is *Construction*. It accounts for the construction orders given out in every calendar year. As exposed earlier, this assumption seems more reasonable than the one in the HLM model. However, the formulation of the equation is very different here as the authors consider "the construction of office spaces as a form of investment in new capital". Thus they stated their equation based on more general investment theory as following:

$$Cst_t = \beta_0 + \beta_1 R_t - \beta_2 v_t - \beta_3 I_t - \beta_4 RC_t + \varepsilon_t \qquad \text{(VI)}$$

Construction relies on four parameters: a measurement of the return on investment with the rent R_t, which itself depends upon the vacancy associated v_t, a measure of financial capitalization rate I_t and finally a replacement cost RC_t, which is a benchmark of incurred costs.

The property investors should start constructions according to the result of this equation. Similarly to the HLM model, the augmentation of the interest rates is susceptible to lower the amount of construction orders passed in the year. Obviously, this is full of sense since higher the rate is, more difficult are the requirement from lenders to cope with.

There are some similarities with the HLM model in the explanatory variables utilized in this equation, but the WTE counts two extra variables for as many differences. The interest rate employed differs, as WTE uses directly nominal while HLM considers a measure in real terms. As to the replacement cost, WTE uses an existing series as a proxy whereas HLM approximates it ex ante using the extra identity (4). Doing so, the HLM estimation of *Completion* employs the variables *Gap*s instead of the gross variables directly and this embodies one other major dissimilarity between the models: the replacement cost series utilized in the WTE model varies over time, whereas the equilibrium rent included in the HLM rent gap depends upon a constant replacement cost. Besides, this last assumption appears pretty unrealistic, since it is compared to the selling value of the asset[5], which is itself certainly varying and the replacement cost should do too.

On the next page is presented a summary table of the model frameworks.

[5] See the section about the Replacement cost in Part C, page 33.

Designation	Eq	Hendershott, Lizieri and Matysiak	Eq	Wheaton, Torto and Evans
Supply	1	$S_t = (1-\delta)S_{t-1} + Comp_t$	I	$S_t = S_{t-1} + Const_t(1-\delta)$
Demand	2	$OS_t = OS_{t-1} + AB_t$	II	$OS_t = OS_{t-1} + AB_t$
Vacancy	3	$v_t = 100\frac{S_t - OS_t}{S_t}$	III	$v_t = 100\frac{S_t - OS_t}{S_t}$
Replacement cost	4	$R^* = (r + \delta + oper)RC$		
Rental adjustment	5’	$\%\Delta R = \alpha - \lambda v_{t-1} + \beta(R^* - R_{t-1})$	V	$R_t = \mu_3[\mu_0 - \mu_1 v_{t-1} + \mu_2(AB_{t-1}/OS_{t-2})] + (1-\mu_3)R_{t-1}$
Completion/ Construction	6	$Comp = g(v^* - v, R - R^*)$	VI	$Const_t = \beta_0 + \beta_1 R_t - \beta_2 v_t - \beta_3 I_t - \beta_4 RC_t$
Absorption	7’	$S_t^* = \gamma_0 + \gamma_1 E_t - \gamma_2 R_t$ $\frac{AB_t}{OS_{t-1}} = \mu_1 \frac{AB_{t-1}}{OS_{t-2}} - \mu_3 \frac{\Delta R_{t-1}}{R_{t-2}} - \mu_2 \varepsilon_{t-1}$	IV	$AB_t = \tau_1[\alpha_0 + E_t(\alpha_1 + \alpha_2 R_{t-1})] - \tau_1 OS_{t-1}$

C. Description of the data employed

The gathering of the data has been a crucial issue for this thesis. The project of an empirical model is meaningless without ensuring that the data employed is valid and truly utilizable. Thus, a special heed has been given at each step concerning the data, namely gathering and adaptability through transformation. Furthermore, when data was not always fully available about a required item, then an approximation has been realised with care and concern about continuity and consistency of the data. Each step is thoroughly detailed and accompanied with comments.

1. Gathering of data

During this exercise, two main sources were particularly supportive: the real estate development and financial company *Aberdeen Property Investors* in Oslo and the national statistical office of Norway *Statistisk Sentral Byrå*. The two taken together brought almost 100% of the time series. Below are listed what each of them respectively supplied.

Aberdeen Property Investors

- Annual nominal office rent R_t series for Oslo (1985 – 2007) and Bergen (1988 – 2007)
- Annual vacancy rate v_t for Oslo (1985 – 2007) and Bergen (1988 – 2007)
- Yearly total office spaces S_t in Oslo (1985 – 2007) and Bergen (1988 – 2007)
- Yearly full-time equivalent services employment E_t in Oslo and Bergen (1995 – 2007)

Statistisk Sentral Byrå (SSB)

- Yearly full-time equivalent services employment E_t in Norway (1982 – 2007)
- Annual Consumer Price Index in Norway (1985 – 2007) (supplied also by Aberdeen)
- Yearly population in several cities of Norway (1982 – 2007)
- Construction costs index RC_t in Norway (1987 – 2007)

Norges Bank (Bank of Norway)

- Yearly nominal yields on Norwegian 5-year government bonds (1986 – 2007)

2. Transformation and reconstruction of data

(a) Rental data

Aberdeen provided time series of the nominal market rent in Oslo and Bergen. The unity employed is the Norwegian crown per square meter per annum (NOK/m²/year). The latter city is pretty small (ca 250,000 inhabitants) and its office market is assumed being the entire city, while the figure for Oslo (ca 600,000 inhabitants) is indeed as an weighted average of rents from the different office sectors of the city (Central Business District, Oslo Vest..).

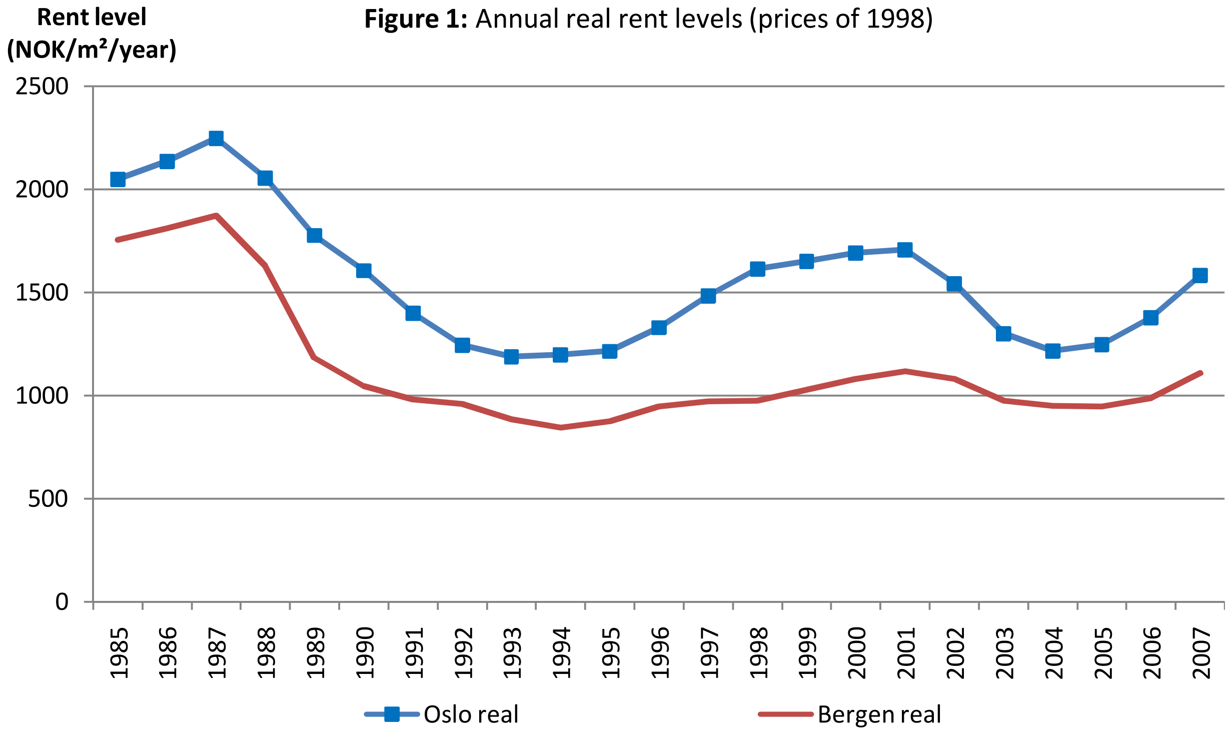
Though, the series did not cover entirely the period of study (1987 – 2007) in the case of Bergen although it did for Oslo. Three extra years needed to be added in front of the existing series, notably to calculate variations between years back to 1986. Fortunately, the curves for the two cities are very similar and the Bergen figures have been conveniently reconstructed using a smoothed comparison with the variation in Oslo. This is of course an estimation, and may induce some marginal error- source effects.

The series have thereafter been adjusted for inflation using the series of Consumer Price Index, which has been used as a GDP deflator. This allows obtaining series of real constant rent levels for both cities. This procedure has not changed the general path of the curves and the result is plotted in Figure 1. The usage of the GDP deflator has been tried to deflate the rental series but led to less significant results in regressions.

However, data used here lacks a last transformation compared to the data employed by the team HLM. They were able to get a time series about rental incentives, i.e. present values of rent-free periods allocated by landlords when vacancy rates were high[6]. This gave them what they called real effective rate series, which supposedly react faster to vacancy variations. After discussion with market participants, it appears that this feature of the London market does not fit for Norway. As a note, the WTE paper does not account for it either, using directly a rent index series.

One notes that the rent cycles have larger amplitude in Oslo than in Bergen.

Figure 1: Annual real rent levels (prices of 1998)



(b) Office stock data

Aberdeen provided also data on total office spaces for the two cities. The unity is the thousand square meters. The data obtained for Oslo is reckoned as an official measure[7], while the one for Bergen is an estimate made by Aberdeen itself. It has been impossible to get office space data on Bergen from another source. Nevertheless, data provided by the

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[6] Confer Hendershott (1995), p.130. The two-step procedure utilized is presented.

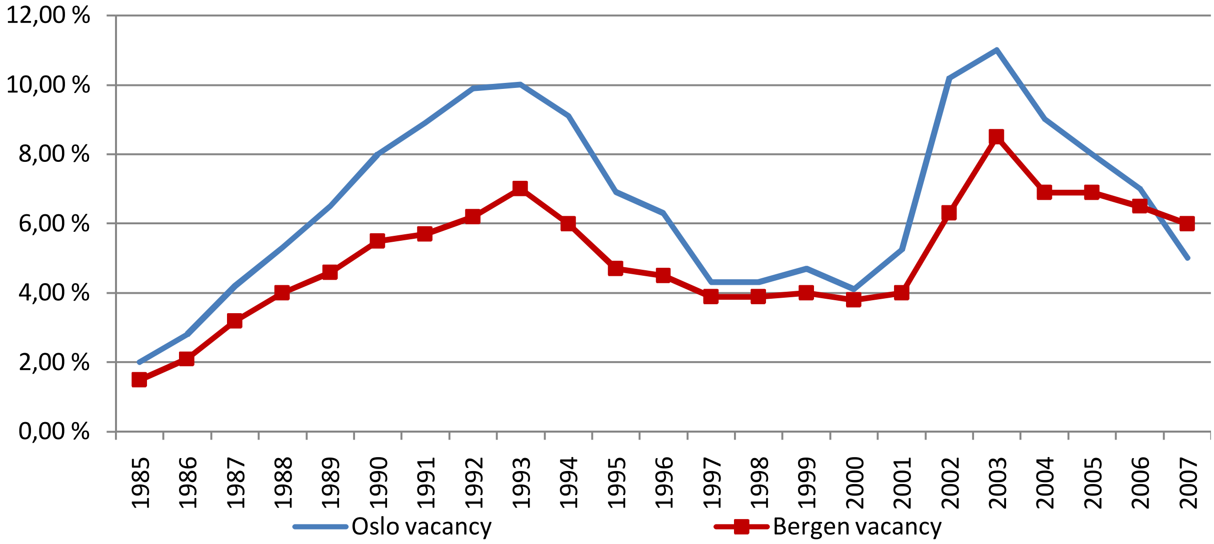
[7] The series is extracted from a study called "Oslostudiet" realised by the company Eiendomsspar, but it is commonly used as an official measure by market participants.

company is considered as reliable, as being acknowledged that Aberdeen has business activities in the city of Bergen.

As for the rental data, the series for Bergen has to be reconstructed over 3 years prior to 1988. The same procedure has been adopted, i.e. using instead the growth of the Oslo series. Both series have a very straight path, which have not been represented graphically, but as a matter of fact the supply of Oslo is nearly four times larger than in Bergen in average.

♦ The vacancy rates were historically values monitored over time in Oslo and Bergen. Again, there is a lack of the 3 first years for Bergen which have been approximated using the variation experienced in Oslo over these years. As it can seen from Figure 2, vacancy rates were low when rents, starts and completions were high, which again supports the picture of the market's functioning.

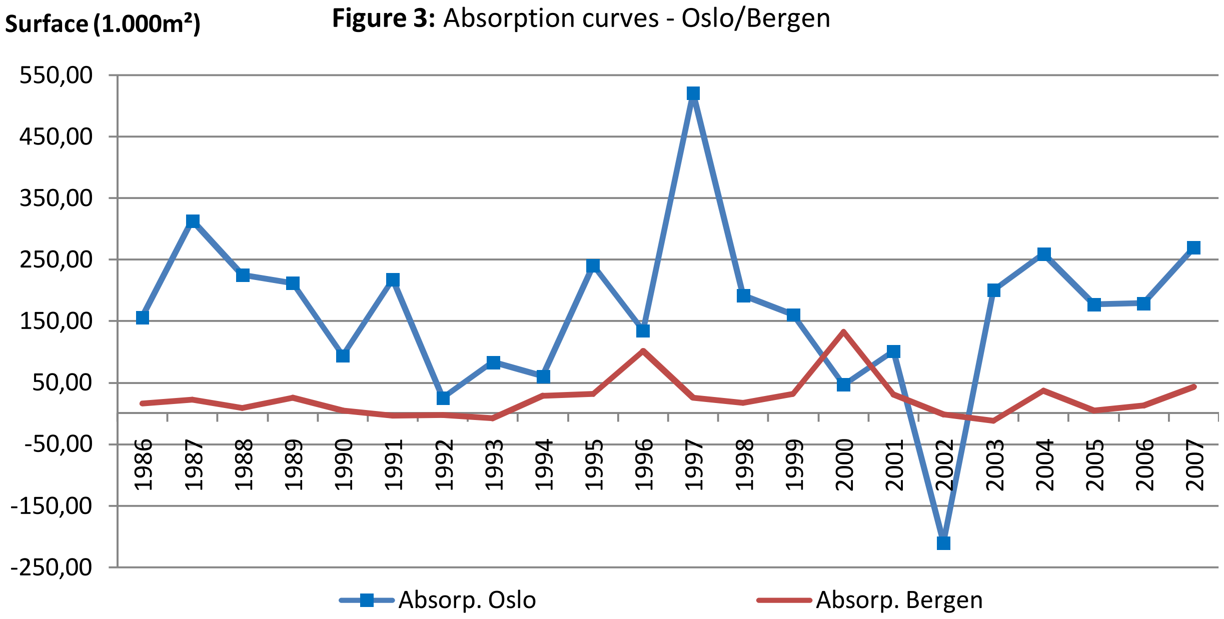
Figure 2: Historical vacancy rates in Oslo and Bergen



Both curves have a very similar path. Nevertheless, the average vacancy rate is higher in Oslo than in Bergen (7.19% against 5.45%) and the Oslo curve is generally situated above the Bergen one, as the rent levels always do. Comparing both Figures 1 and 2 highlights the reciprocal negative trajectories of the two variables.

♦ The total space series is considered as the *office Supply* for both models. From that, it has been simple to extract the *office Demand* series (occupied space) using the vacancy rates in either Equation (3) or (III). The calculation of *Absorption* entails from the identical formulas (4) or (IV), the two models sharing stock data. It turns out that the variation in Oslo is much wider than in Bergen, as it could be guessed from the sizes of total stock series and from that Oslo has historically a much volatile office market than smaller cities in Norway. Besides, there are extreme values in both curves between 1996 and 2002 as one can see large spikes around the respective means.

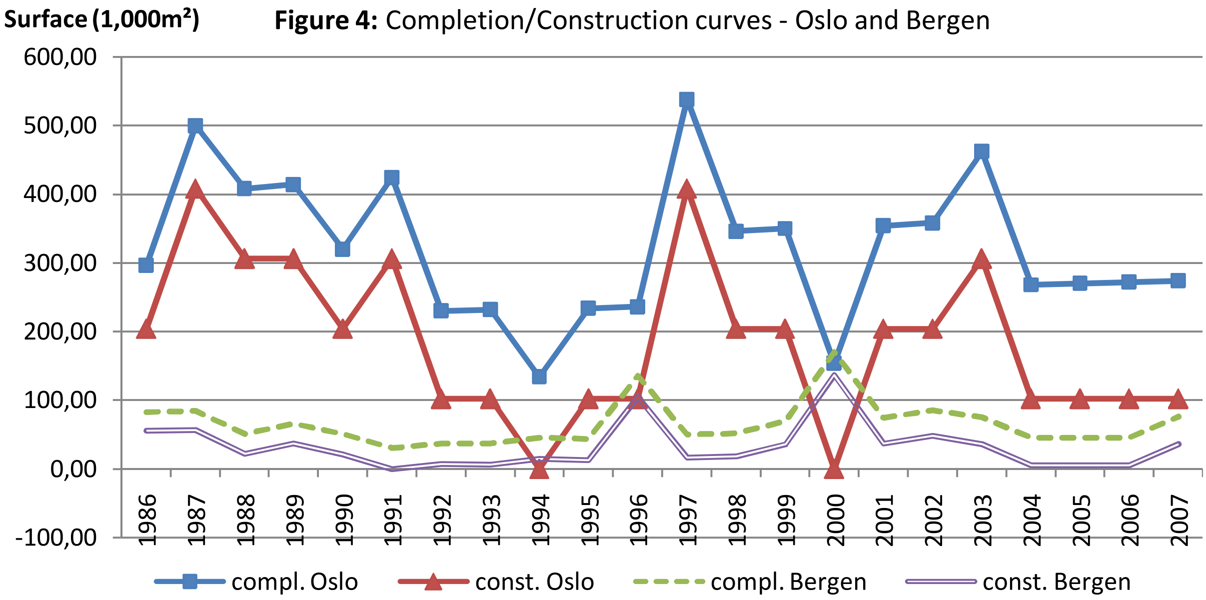
Figure 3: Absorption curves - Oslo/Bergen



♦ As mentioned in the part A, the models utilize different formulas for *Completion* (HLM) and *Construction* (WTE). Each one has been computed using respectively Equations (1) and (I) plus setting a constant depreciation rate δ of 2% as suggested by Aberdeen. The authors of the HLM paper managed to get a series of completions and then to extract yearly depreciations which averages to 1.83%. They however equalize the depreciation to 2% constant to calculate their replacement cost.

Figures 3 and 4 show respectively *Absorption* and *Completion*/*Construction* series. The curves are parallel among cities. It results from the way the series were computed and as indicated earlier, the difference would represent the amount of construction completions achieved every year.

Figure 4: Completion/Construction curves - Oslo and Bergen



It appears obvious that the variations in Oslo and Bergen have different scales, like in Figure 2. One notes as well the presence of spikes around the middle of the sample too. This is somewhat natural phenomenon explained by boom periods in the market where rents were increasing. As seen in Figure 1 real rents averaged at a high level in 1997-2001, which triggered numerous construction orders like it already happened from 1985 to 1988. This evidences the market functioning exposed earlier in section A.

These extreme values appear also in the article graphs of the two models. The next part details how the authors addressed their treatment to do the regressions.

(c) Employment

Aberdeen supplied time series about services employment for several cities of Norway. These series were in full-time equivalent (FTE), i.e. man-hours per year. They were particularly interesting since the figures were divided between cities. SSB also publishes data of that kind, but only for the whole country.

The data from Aberdeen presented nevertheless the flaw to start only from 1995, which involved reconstructing backwards to 1982, i.e. 12 years since the employment growth lagged 5 years is employed in one regression. This required using the only available series on the missing part, i.e. the SSB series about Norway. The Aberdeen series were though utilized as a starting point for each city.

Thereby, Aberdeen FTE figures concerned the four largest and most dynamic cities of Norway: Oslo, Bergen, Trondheim and Stavanger. The total of figures averaged 600 thousands of man-hours per year over 1995-2007.

It allows calculating the proportional weight of every city compared to this average over the same period[8]:

City	Oslo	Bergen	Trondheim	Stavanger	Total
Average	295.52	126.05	82.487	96.423	600.494
Proportion	49.21 %	20.99 %	13.74 %	16.06 %	100.00 %

The FTE numbers above report the employment for all kinds of services in the four cities. The reconstruction of the series was done in two steps.

First, it appeared inconsistent that only these four cities taken together account from the total office space of Norway. Effectively, Oslo cannot be responsible for 49% of whole Norway's cities office stock, even if it is the capital city and the country's population is very scattered. In order to smooth the figures, they were weighted with some estimators of population in Norway over 1982-2007. The estimator considered the population of a larger panel of the first 8 cities of Norway (including 6 out of 8 above 100,000). The data comes from SSB as well and the complete table is reported in the appendixes. Therefore in 2007, the total figure amounted to ca 1.5 million inhabitants against only 1 million for the four

[8] Details about the procedure about employment approximation are reported in appendix.

original cities. Norway counts only a few cities largely populated. For example Bergen is the second largest with only a quarter of a million.

Below are the results of this calculation presented.

City	Oslo	Bergen
Proportion FTE	49.21%	20.99 %
Proportion pop.	37.54%	17.34%
Average FTE/pop.	43.38%	19.17%

Averaging with population reduces the proportion in the cities to a more reasonable measure: 43% for Oslo and 19% to Bergen.

The second step of the transformation consisted in weighting the FTE series from SSB using the averages just obtained.

However, it remained to assess a series of services employment figures consistent with those from Aberdeen. Thus, several items from the Norway's FTE employment table of SSB were selected thoroughly and added together as "Services Norway" in order to be compared with the series from Aberdeen over the period 1995-2007.

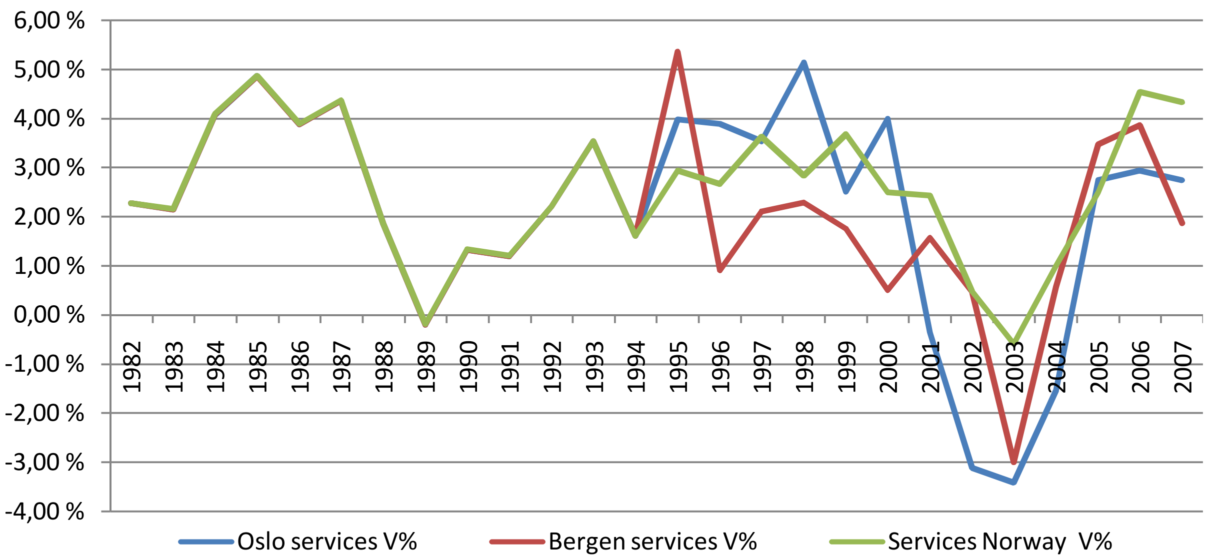
Below are the results of the comparison presented:

Year	1995	1996	1997	1998	1999	2000	2001
Services Norway	**595.6**	**611.5**	**633.7**	**651.7**	**675.7**	**692.6**	**709.5**
FTE Total (Aberdeen)	536.9	552.3	569.4	591.4	602.9	616.1	620.9
Proportion Total	*90.15 %*	*90.32 %*	*89.86 %*	*90.75 %*	*89.22 %*	*88.96 %*	*87.51 %*
FTE Oslo (Aberdeen)	260.9	271.2	280. 8	295.2	302.6	314.7	313.6
Proportion Oslo	*43.82 %*	*44.35 %*	*44.31 %*	*45.30 %*	*44.79 %*	*45.44 %*	*44.20 %*
FTE Bergen (Aberdeen)	116.8	117.9	120.4	123.2	125.3	126.0	128.0
Proportion Bergen	*19.62%*	*19.28%*	*19.00%*	*18.90%*	*18.55%*	*18.19%*	*18.04%*
Year	2002	2003	2004	2005	2006	2007	***Average***
Services Norway	**712.9**	**708.7**	**715.8**	**733.7**	**767**	**800.3**	**693.0**
FTE Total (Aberdeen)	615.5	597.9	598.2	614.9	637.7	652.2	600.5
Proportion Total	*86.34 %*	*84.37 %*	*83.56 %*	*83.81 %*	*83.14 %*	*81.50 %*	*86.88 %*
FTE Oslo (Aberdeen)	303.8	293.5	288.9	296.9	305.6	314.0	295.5
Proportion Oslo	*42.62 %*	*41.41 %*	*40.36 %*	*40.46 %*	*39.85 %*	*39.24 %*	*42.73 %*
FTE Bergen (Aberdeen)	128.6	124.8	125.5	129.9	134.9	137.4	126.1
Proportion Bergen	*18.04 %*	*17.60 %*	*17.53 %*	*17.70 %*	*17.59 %*	*17.17 %*	*18.25 %*

This table leads to several conclusions:

- First, the proportions remain very stable over time which indicates that the "Services Norway" figures are consistent with the FTE numbers from Aberdeen.
- Secondly, the average proportions for both cities are very close to the estimators previously calculated (43.38% against 42.73% for Oslo and 19.17% versus 18.25% for Bergen) which shows consistence with the estimator FTE/population.
- Thirdly, it reveals indicatively that the four main cities represented supposedly 87% of the total employment in services in Norway, which evidences the clustering of activities. Also, Oslo represents Bergen, Trondheim and Stavanger standing alone.
- Finally, one notes that every proportion is evenly decreasing since 2000. This indicates the development of service activities in other locations than in the main four poles, phenomenon that can be observed globally. Though, the decrease is very slow certainly due to the modest size of cities in Norway.

Figure 5: Service employment variation - Norway, Oslo & Bergen



The series "Services Norway" has thus been utilized as a representation of the employment behaviour over 1982-2007, with Oslo and Bergen explaining respectively 43.38% and

19.17% of the Norway total services employment. The Figure 5 plots the three variation curves on the same referential axis. As it can be seen, the reconstruction method forces the Oslo and Bergen curves to vary like Services Norway before 1995. Some variations must have occurred like between 1995 and 2007 (where Bergen and Oslo variations are sometimes larger or smaller than the one for Norway), but it seems that graphically Norway might be a relevant representation of the variation over time. This assumption allows reconstructing employment figures series E required in the Equation (IV) of WTE.

(d) Interest rate

Both models are using interest rates, but each one differently. However, it is always present as an estimator of the capitalization rate. It is a component of the user cost of capital which will impact the rent level thereafter. It has been considered by the authors that a new real estate asset is above all an investment.

➢ The calculation of its profitability should thus include a measure of an expected return plus an eventual risk premium, which were reckoned as following:

- A real risk-free rate of return – knowing that the average length of lease contracts in Norway is 5 years in accordance with market participants, a series of 5-year government bond yields from Norges Bank has been employed. From this nominal series, a measure of annual expected inflation has been removed to obtain a real risk-free interest rate series over the period. The inflation measure consists in averaging the inflation (extracted from the Consumer Price Index) of the current and the two previous years.
- A risk premium – a constant value of 2% has been entered into the calculation of Equation (4). Having a higher premium obliges property lenders requiring a higher return, which would have a negative effect on construction starts.

➢ On the other hand, the Equation (VI) of the WTE model uses only a nominal measure of risk-free rate of return, only because it worked better over real and real net of tax rates. They also seem to assume that a risk premium, if there is, must be included in the replacement cost series.

(e) Replacement cost

The replacement cost series accounts for the amount of the initial investment, i.e. a new building project. In the HLM model, it is thus employed in the calculation of the long-run equilibrium rent. As introduced earlier, construction is triggered when real rent is really above the equilibrium rent, which consequently depends upon the replacement cost. The utilization and the obtainment of the Replacement cost differ drastically between the models.

➢ HLM model is based on the idea that whenever on the market the value of the real estate asset *V* exceeds its replacement cost *RC*, new construction orders are automatically given out (new investments). This is also known as a Tobin's q Argument: when the market value of the asset *V* is more than its replacement cost *RC*, companies are willing to invest more in the capital, since they obtain more for what they pay for. This concept gives a good illustration of the model functioning, but it could not be directly measured by the authors as no data is available for the ratio *V/RC*. This ratio is thus only abstract here, and moreover the Tobin's model also appeared very weak among literature. The authors nevertheless related it to the real rent and the vacancy gaps as follows:

$$Boom\ market\text{: } \frac{V}{RC} > 1\ ;\ v < v^*\ ;\ R > R^*$$

$$Overbuilt\ market\text{: } \frac{V}{RC} < 1\ ;\ v > v^*\ ;\ R < R^*$$

New construction orders (Boom market) occurs when the vacancy is very low and the rent well over the equilibrium rent and inversely. This is an assumption endowed of theoretical sense but not clear from a reality point of view. McDonald (2002) doubted about the faculty of the rents to grow whenever $v < v^*$ because of the sticky feature of the market.

Anyway, the HLM model defines the Replacement cost RC thanks to the Equation (4). By reversing it, they evaluate it as the real rent of a specific year divided by the sum of several factors concerning this year: r the real interest rate, δ the depreciation rate and $oper$ the operating-expense ratio.

$$RC = \frac{R^*}{r + \delta + oper}$$

It reverts to find what year to select, knowing that the rent might have equalled the natural rent this year. It is arbitrary chosen over the period when the rent appeared relatively stable. For instance, Hendershott (1995) opted for the final year of a "steady" period whereas they chose the one in the middle in the HLM paper. For the same reason, the observation of the rent data (see Figure 1 above) elected 1998 and 2001 being this specific year for respectively Oslo and Bergen.

After a discussion with Aberdeen, δ was fixed at 2% constant and $oper$ at 7% of nominal rent. 7% nominal averaged 7.5% real when deflating it with the Consumer Price Index like previously. The equilibrium series was thereafter reconstituted by reversing again the Equation (4) thanks to RC just found and constant over time.

- Oppositely, the WTE model used an existing and available series on commercial space construction costs index supposed approximate the Replacement cost.

Since a series about RC is not available, the authors claimed that it should equal a weighted sum of cost of land and cost of construction. But it was difficult to get and then rely on data about prices of land. They also argued the value of real estate assets is highly related to cost of construction since the price of land is inclined to rise linked to the value of the edifice built on it. Finally, they stated that "the most appropriate measure of replacement cost is for capital alone", i.e. the cost of construction. It is also important to point out that the unity of this series must be consistent with the unities of employment and real rent when testing statistically the Equation (V). To do so, the use of the natural logarithm is convenient since it removes the dimensional aspect.

Below is presented a summary table of the variables employed.

Denomination	Unity	Equations	Derivatives
Real rent *R*	NOK/m²/year(y)	4, 7, IV, V, VI	Rent gap, Replacement cost (HLM)
Vacancy rate v	% of office space	3, 5, III, V,VI	Demand, Absorption
Office supply	1000 m²	1, 3, I, III, IV, V	Demand, Absorption, Completion, Construction
Employment *E*	1000 man-hour/y	IV	Employment growth
Replacement cost $RC_{(HLM)}$	NOK/m²/y	5, 6	Rent gap (and equilibrium rent R*)
Nominal interest rate *I*	Percentage	VI	
Real interest rate *r*	Percentage	4	
Replacement cost $RC_{(WTE)}$	Index	VI	
Rent gap *(R*-R)* (also Ln)	NOK/m² /y	5, 6	
Demand *D* or *OS*	1000 m²	2, 3, 7, II, III, V	
Absorption *AB*	1000 m²	2, 5, II, IV, V	
Completion *Comp*	1000 m²	1,6	
Construction *Const*	1000 m²	I, III, VI	
Employment growth *eg*	% of employment	7	

The whole work about statistical estimations of this paper has been done under the statistical program Stata/IC 10.0. Among others, it provided several functions very useful, which are mentioned later along the part D.

D. Descriptions and estimates of the models

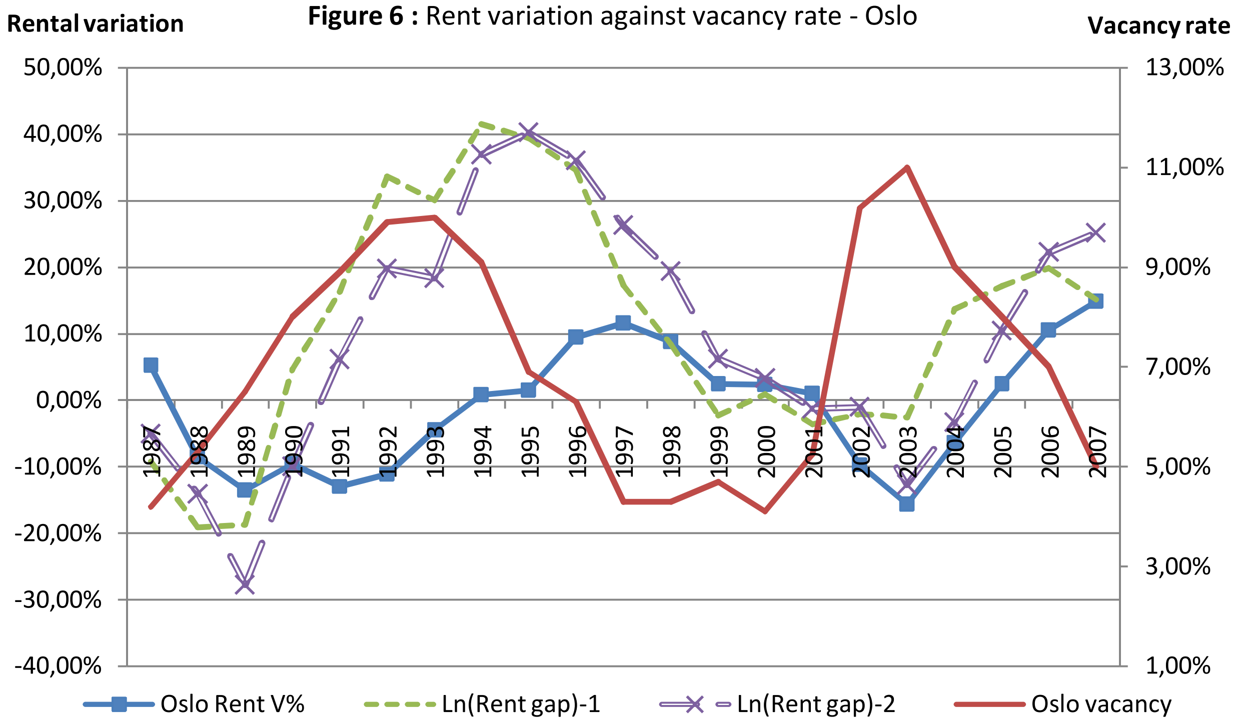
1. *Rental adjustment Equation*

(a) Hendershott, Lizieri and Matysiak

The authors had stated the equation as it follows:

$$\%\Delta R_t = \alpha - \lambda v_{t-1} + \beta(R_t^* - R_{t-1}) \qquad (5')$$

Figure 6 : Rent variation against vacancy rate - Oslo



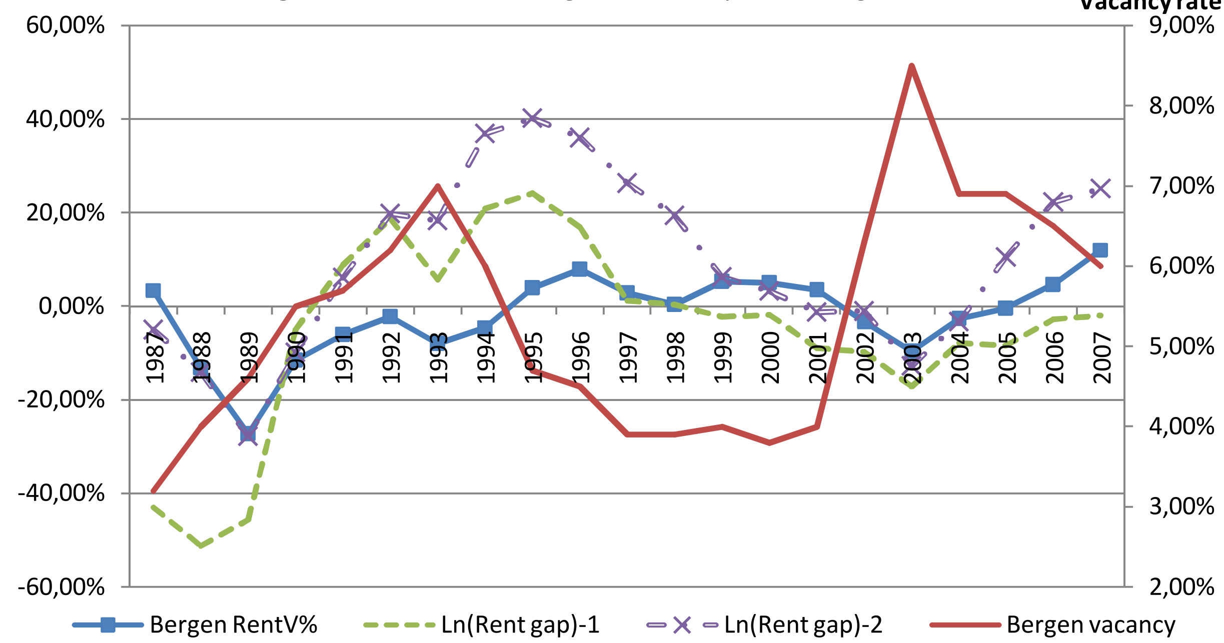
The Oslo graph shows the two patterns expressed in the rent adjustment equation: an almost simultaneous and inverse correlation between the two curves. For Bergen, the negative correlation is also obvious, although it turns positive from 1989 to 1992.

Both real rent gaps lagged one and two years have an obvious correlation for Oslo and Bergen graphically speaking. Then it surely explains a part of the rent variation and its parameter should be significant in both cases. The employment of the natural logarithm has been preferred over the figure in NOK/m²/year for sake of consistence in the dimension of the other variables utilized, namely the percentage.

However, to run validly an Ordinary Least Squares estimation (OLS) with time series, it requires the stationary of the variables. If this is not the case, spurious estimates will stem from the OLS regression. This concept and the test employed are presented more in detail in the appendixes. It reverts to test for unit roots to and if found, it must be evidenced further the existence of a valid cointegrating relationship between 2 variables or more to proceed the OLS. A stationary process is noted I(0), or integrated of order zero, while one variable having a unit root is non-stationary, and is noted I(1). Such a series increasingly wanders from the long-run equilibrium over time. A typical solution to this problem is to replace the I(1) process, say y_t, by its first difference $y_t - y_{t-1} = \Delta y$. When the variables are integrated by more than 1 order, say K, the solution will be to take the Kth difference of the variable.

Another possibility more convenient since it does not restrict to interpret the first difference is thus to find a cointegrating relationship between a pair and even more than two variables. It indicates that these variables have a forcing effect on each other to move towards a common long-run equilibrium. If such a relationship exits, the whole process becomes stationary, and can be used.

Figure 7 : Rent variation against vacancy rate - Bergen



The appendixes reports the tests for Unit Root and Cointegration for every series and equation estimated in this paper, such as the associated findings are only briefly presented as following: I(1) series are evocated and the result of the cointegration test is generally reported in the tables. For instance in Oslo, the test for the rent gap lagged one year did not reject the unit root. Knowing that the gap has a presumably unit root, and even if the vacancy is a weak I(0), this equation might be invalid. Before some adaptations, the results for Oslo and Bergen are presented below in the Table 1.

Table 1 - $\Delta ln(R_t)$[9] $= \alpha - \lambda v_{t-1} + \beta[ln(R_t^*) - ln(R_{t-1})]$ ***- Eq. (5")***

City	*Variables*	*Parameter*	*t-Value*
Oslo	Constant	.1521	2.64
	Vacancy v_{t-1}	-3.1077	-3.38
	Rent gap $lnR_t^* - lnR_{t-1}$	.4396	3.54
	R^2	44.26%	
	Durbin-Watson test	.507	
	Estimated v^*	4.89%	
	Number of observations	21	
	Cointegration: Augmented Dickey-Fuller test	-5.600	
Bergen	Constant	.02340	0.33
	Vacancy v_{t-1}	-1.29	-0.92
	Rent gap $lnR_t^* - lnR_{t-1}$	.2799	2.62
	R^2	28.23%	
	Durbin-Watson test	.677	
	Estimated v^*	1.81%	
	Number of observations	21	
	Cointegration: Augmented Dickey-Fuller test	-5.197	

First, one notes that the logarithm was not applied to the vacancy rate, since it is required to calculate the natural vacancy rate, logarithm produces wrong outcomes in that regard.

Second, one notices that the signs are correctly oriented, negative for the vacancy and positive for the rent gap. For Oslo, results are satisfactory considering that every parameter

[9] The variation rate can be replaced for small changes of one variable X_t over time, knowing that the difference in natural logarithm is approximating the variation rate.

$$\Delta ln(X_t) = ln(X_t) - ln(X_{t-1}) = ln\left(\frac{X_t}{X_{t-1}}\right) \approx \frac{X_t - X_{t-1}}{X_{t-1}}$$

is significant, but the R-squared value is small. As to Bergen, the results are quite poor: only the rent gap has a significant parameter and there is a small R-squared.

Third, about the natural vacancy rate, London and Sydney data gave reasonable appraisals, so did in Oslo but not in Bergen, which is certainly due to the non-significance of the vacancy rate.

However, a larger drawback appeared in both cities, there is a large and positive autocorrelation (the Durbin-Watson statistic[10] is much lower than 2, the value indicating no autocorrelation in the residuals). The results above appeared to be biased by serial correlation included in the residuals and it has been thus important to remove it every time it occurred. This has been done using the eponym procedure implied by Prais and Winsten in 1954, which was an improvement of the original Cochrane-Orcutt procedure in 1949[11]. Fortunately, this procedure which must normally be iterated several times can be conveniently done thanks to a package under Stata. In addition to this procedure, the removal of serial correlation involves the computation of serial correlation-robust (SC-robust) standard errors. The purpose of this technique is presented in detail in the appendixes, but it is simply important to know that the application of both methods normally permits to obtain non-biased estimates from OLS.

Before, there is a special comment to add regarding the cointegration. The result of the Test reported in the table 1 should be compared to the critical values calculated by MacKinnon. A relevant extract can be found in the appendixes, as well as an explanation about using it.

In the case of these two cities, it evidenced the existence of cointegrating relationship between the variables of the equation thanks to a very significant trend. Then, in spite of negative presumptions, it turns out that the formulation of HLM can be validly interpreted in Oslo and Bergen.

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[10] A more detailed explanation about both tests and procedures is attached in the appendixes.

However, the explanatory power of the equation was pretty poor and should be possible to improve. Then Table 2 below presents the results of the HLM equation corrected for autocorrelation, but also testing for different time lags for all explanatory variables are reported in table 2 below.

Table 2 - $\Delta ln(R_t) = \alpha - \lambda v_t + \beta[ln(R_t^*) - ln(R_{t-2})]$ ***- Eq. (5"b)***

City	*Variables*	*Parameter*	*t-Value*
SC-Robust			
Oslo	Constant	.1429	3.28
	Vacancy v_t	-2.5818	-6.82
	Rent gap $lnR_t^* - lnR_{t-2}$	.3068	4.26
	R^2	73.59%	
	Durbin-Watson test	1.98	
	Estimated v^*	5.53%	
	Coefficient of correlation ρ	.5079	
	Cointegration: Augmented Dickey-Fuller test	-4.888	
	Number of observations	21	
SC-Robust			
Bergen	Constant	.1116	1.44
	Vacancy v_t	-2.553	-2.25
	Rent gap $lnR_t^* - lnR_{t-2}$	.3853	2.70
	R^2	53.10%	
	Durbin-Watson test	1.94	
	Estimated v^*	4.37%	
	Coefficient of correlation ρ	.6182	
	Cointegration: Augmented Dickey-Fuller test	-4.353	
	Number of observations	21	

The removal of serial correlation has been done successfully for Oslo and Bergen. Some finest adjustments of lag for the rent gaps and the vacancy rates gave better results in both cases. The formulation of the HLM model fits best here with no lag in vacancy and two years lag in the rent. The rent gap parameter, which conveys the speed of adjustment, lies always between 0 and 1, as expected using a Partial Adjustment mechanism.

Indeed, the results tell an interesting story: the rent adjusts automatically to the level of vacancy since CBDs are quite small and so information circulates quickly. However, the market needs respectively a bit less than 3 years in Bergen and a bit more than 3 years in

Oslo to adjust the rent to the equilibrium level. This adjustment is then quicker in Bergen than in Oslo since the closest to one, the fastest[11]. All that makes sense considering the size of the cities, Bergen market should adjust to the market rapidly since there is a limited office space (the rent reaches 38% of the equilibrium rent in one year), and so does Oslo but at a slower pace (30%).

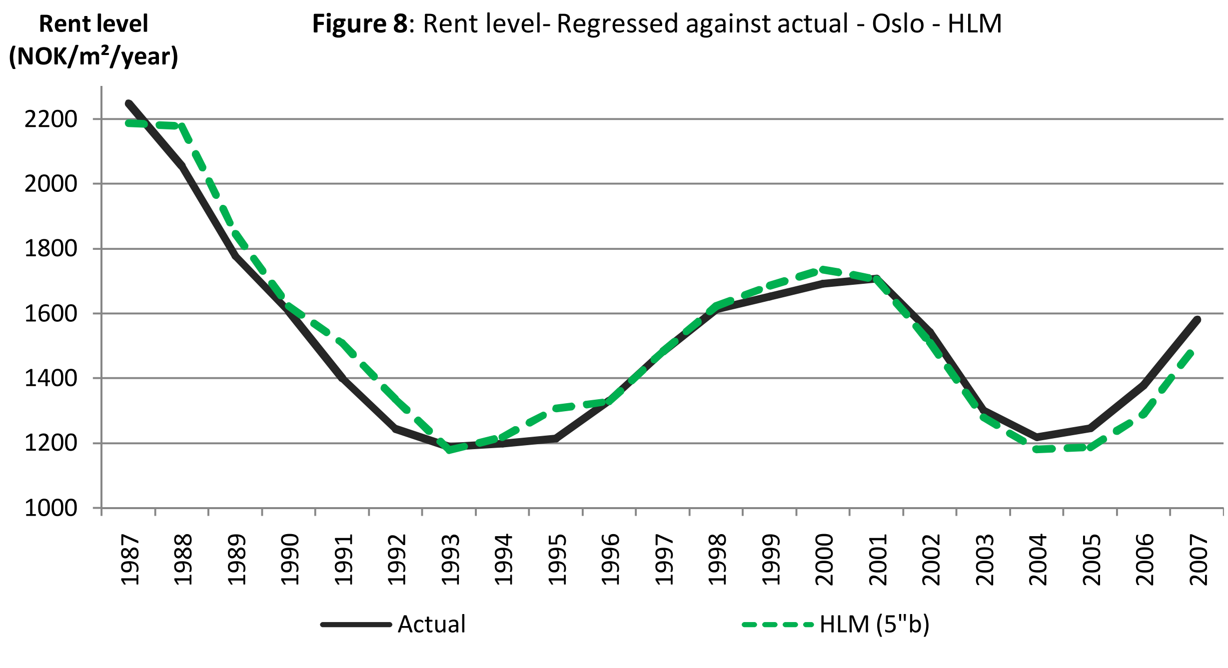
All variables are significant and are correctly sign-oriented. Both models explain much more of the variation than in Table 1 and fit remarkably well.

The double correction for autocorrelation also improved the natural vacancy rate which appears very plausible in both cities. The one in Oslo is slightly greater than in Bergen, which can easily be understood with the different sizes of the office surface.

Fortunately, one was completely unable to circumvent with certitude the problem of unit root process, but the Augmented Dickey-Fuller ensured that one cointegration relationship exists in both cities.

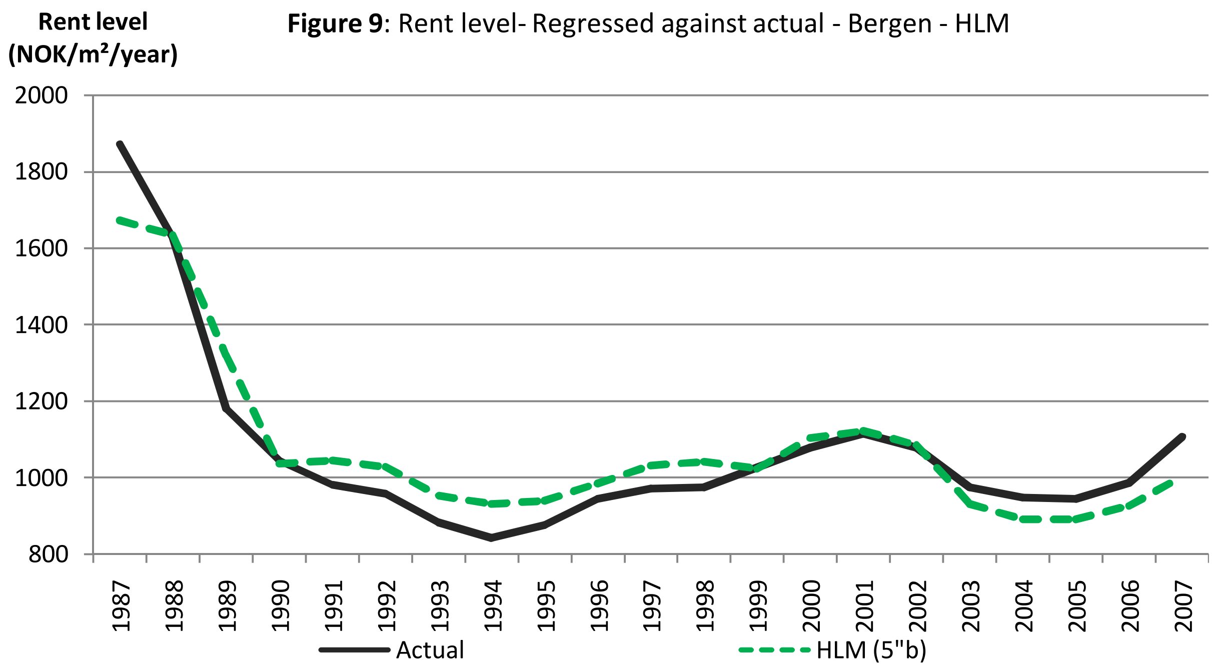
Below, the Figures 8 and 9 plot both curves (5"b) against the actual rent in both cities.

Figure 8: Rent level- Regressed against actual - Oslo - HLM



[11] The rent gap parameter was always comprised between 0.3 and 0.4, for every lags of vacancy and rent gap in both cities. Even if the rent adjusts fastest when using the lagged vacancy, some autocorrelation persisted.

Figure 9: Rent level- Regressed against actual - Bergen - HLM



In both cities, the curves are very close from each other, and the estimated one follows quite well the original movements. There is however a small deviation at the first year which is certainly due to the Prais-Winsten procedure.

In spite of that, it appears that the original log-formulation captures very well the trajectory of the rent in both cities and presents no autocorrelation.

(b) Wheaton, Torto & Evans

The rental formula of WTE consisted in a Partial Adjustment as it follows:

$$R_t = \mu_3[\mu_0 - \mu_1 v_{t-1} + \mu_2(AB_{t-1}/OS_{t-1})] + (1-\mu_3)R_{t-1} \qquad \text{(V)}$$

where μ_3 is the speed of adjustment of the past period rent level towards the equilibrium rent which is the segment between square brackets. The rent changes relatively to the lagged vacancy rate, the variation in absorption and the rent level, and Figures 10 and 11 plot the rent against current vacancy rate and absorption growth.

Figure 10: Real rent vs vacancy and absorption rates - Oslo

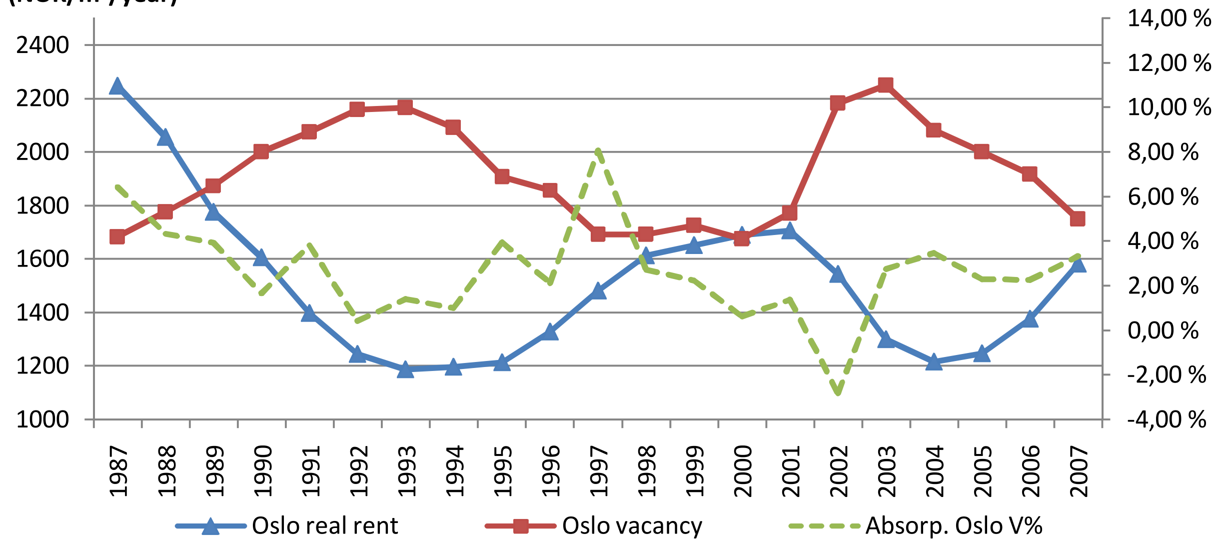
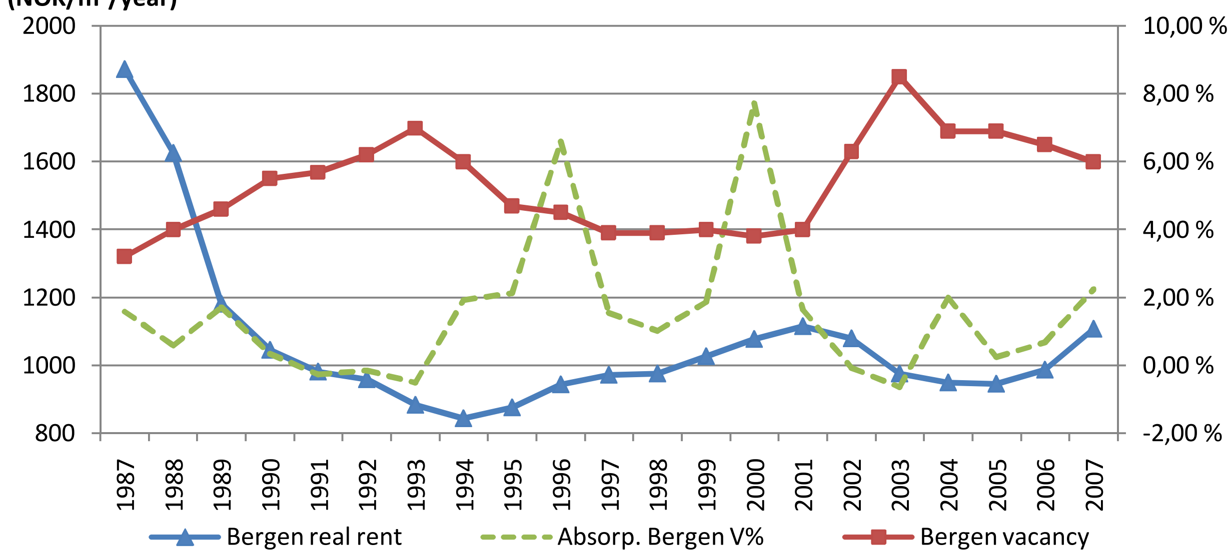


Figure 11: Real rent vs vacancy and absorption rates - Bergen



In both cities, the current vacancy rate presents like previously a negative correlation with the real rent, and the absorption seems to be positively related to rent from a graphic point of view.

As found in the HLM model, the vacancy seems to react more or less instantly to the vacancy. To test for that, both vacancies lagged zero and one year are tested.

On the other hand, it is difficult to attribute directly some explanatory power to the presence of spikes in absorption since the rent has very smooth ups and downs. However, the large spikes are always oriented similarly to the rent, except in 2000 in Oslo. Again in order to remove the dimensional effect, both original version using natural logarithms and another with difference in natural logarithm are presented respectively in the Table 3 and 4 below. The second version is interesting since it expresses the equation in variations like in HLM.

Table 3 – $ln(R_t) = \mu_3[\mu_0 - \mu_1 v_t + \mu_2(AB_{t-1}/OS_{t-2})] + (1-\mu_3)ln(R_{t-1})$-***Eq.(V'L0)***

$ln(R_t) = \mu_3[\mu_0 - \mu_1 v_{t-1} + \mu_2(AB_{t-1}/OS_{t-2})] + (1-\mu_3)ln(R_{t-1})$- ***Eq.(V'L1)***

City	*Variables*	*Simple logarithms (V'L0)*		*Simple logarithms (V'L1)*	
		Parameter	*t-Value*	*Parameter*	*t-Value*
SC-robust					
Oslo	Constant	2.5278	5.60	4.3877	6.42
	Absorption AB_{t-1}/OS_{t-2}	-.059	-0.23	-.3018	-0.86
	Vacancy v_t	-3.5641	-7.06		
	Vacancy v_{t-1}			-4.1231	-6.45
	Rent $ln(R_{t-1})$	.6871	11.27	.4403	4.74
	Speed of adjustment μ_3	31.29%		55.97%	
	R^2	98.39%		99.70%	
	Durbin-Watson test	1.66		1.49	
	Number of observations	21		21	
	Correlation of correlation ρ	.1011		.6079	
SC-robust					
Bergen	Constant	2.2343	2.20	2.5195	2.15
	Absorption AB_{t-1}/OS_{t-2}	-.0471	-0.10	.2232	0.54
	Vacancy v_t	-3.5558	-3.58		
	Vacancy v_{t-1}			-2.8046	-1.68
	Rent $ln(R_{t-1})$	.7052	4.68	.6572	3.92
	Speed of adjustment μ_3	29.48%		34.28%	
	R^2	99.05%		98.88%	
	Durbin-Watson test	1.50		1.43	
	Number of observations	21		21	
	Correlation of correlation ρ	.4651		.5978	

First of all, the equation (V'L) proposed by WTE leads to a moderate autocorrelation in each case. The authors reported as well a Durbin-Watson statistic of 1.38 in their paper. One will see whether serial correlation can be removed successfully later.

Secondly, it is satisfactory regarding that all parameters are always correctly oriented, except from the absorption. Moreover, it appears that this latter does not fit, and does not improve the model for either Oslo or Bergen, while it had an effect in the WTE paper. In fact, vacancy rate depends negatively on Absorption, because it is simply the difference in occupied space from one year to another. Absorption can be rewritten in function of the vacancy rate v: $AB_t = OS_t - OS_{t-1} = S_t(1 - v_t) - S_{t-1}(1 - v_{t-1}) = S_t(1 - v_t)(1 - L)$ where L is the back lag operator, which returns the lagged value of the variable. This shows that Absorption is negatively linked to the contemporaneous vacancy rates. The non-significance is probably due to the large and opposite spikes.

As to the fitting of the model, it is almost perfect in each case, even without the absorption. The WTE article also boasts a R-squared value of 89%, with, as said above, a larger autocorrelation. These figures seem even a bit odd, as they are that high. The serial autocorrelation has probably its share of the explanation.

At last, Table 3 provides some indications of the speed of adjustment that can be compared to those obtained with HLM. They both deliver a quicker speed of adjustment when relying on the past vacancy, and one notes that relying on the immediate vacancy rate entails in both models a speed from 30 to 40%. The WTE model indicates indicatively that the rent adjusts quicker to the past information in only 2 years with WTE.

As to the Unit Root linked problem, all variables used here are weakly dependent and can be validly used for estimation. One will see whether the log difference version of the equation confirms the previous findings. Their results are presented on the next page in Table 4.

Table 4 –$\Delta ln(R_t) = \mu_3[\mu_0 - \mu_1 \Delta ln(v_t) + \mu_2(AB_{t-1}/OS_{t-2})] + (1-\mu_3)\Delta ln(R_{t-1})$-***Eq.(V"DL0)***

$\Delta ln(R_t) = \mu_3[\mu_0 - \mu_1 \Delta ln(v_{t-1}) + \mu_2(AB_{t-1}/OS_{t-2})] + (1-\mu_3)\Delta ln(R_{t-1})$-***Eq.(V"DL1)***

City	*Variables*	*Difference in logarithms (V"DL0)*			*Difference in logarithms (V"DL1)*		
		Parameter		*t-Value*	*Parameter*		*t-Value*
SC-robust							
Oslo	Constant	.012		0.18	.0625		1.68
	Absorption AB_{t-1}/OS_{t-2}	.0798		0.04	-1.4815		-1.47
	Vacancy $\Delta ln(v_t)$	-.7227		-4.87			
	Vacancy $\Delta ln(v_{t-1})$				-.5043		-3.93
	Rent $\Delta ln(R_{t-1})$	.7256		8.64	.5618		4.30
	Speed of adjustment μ_3		27.44%			43.82%	
	R^2		83.34%			77.86%	
	Durbin-Watson test		2.01			1.65	
	Number of observations		21			21	
	Correlation of correlation ρ		-.2815			.1064	
Bergen	Constant	.00171		0.03	.0266		0.82
	Absorption AB_{t-1}/OS_{t-2}	.1954		0.14	-.8972		-0.96
	Vacancy $\Delta ln(v_t)$	-.4605		-2.04			
	Vacancy $\Delta ln(v_{t-1})$				-.4695		-3.01
	Rent $\Delta ln(R_{t-1})$	.5911		2.95	.5037		2.14
	Speed of adjustment μ_3		40.89%			49.63%	
	R^2		46.62%			52.88%	
	Durbin-Watson test		1.92			2.04	
	Number of observations		21			21	

The variation rate formula[12] presents very satisfactory results and evidences the explanatory power of the vacancy and the lagged rent in all cases. Unfortunately, the absorption is again non-significant. As a note, it has not been log-transformed since it represented already the variation in occupied space.

For Oslo, the results of equations (V"DL) are quite close than those obtained using the original formulas (V'L), for both vacancy lags. There is however a moderate autocorrelation when taking the lagged vacancy rate, but the model fits still very much in Oslo. On the other side, Bergen lost explanatory power and obtains results a bit lower than in Oslo. In both cities, all parameters are correctly signed, excepted for the absorption variation. Bergen did not require any correction for autocorrelation.

[12] By variation rate, it means the configuration utilizing the difference in logarithms.

The table 4 gives results about speed of adjustment which supports what was found in WTE (V'L) and HLM. Here, Bergen adapts its rent to the long-run value a bit faster than Oslo whatever is the vacancy lag if one disregards the original WTE model with one year lag since it suffers from autocorrelation:

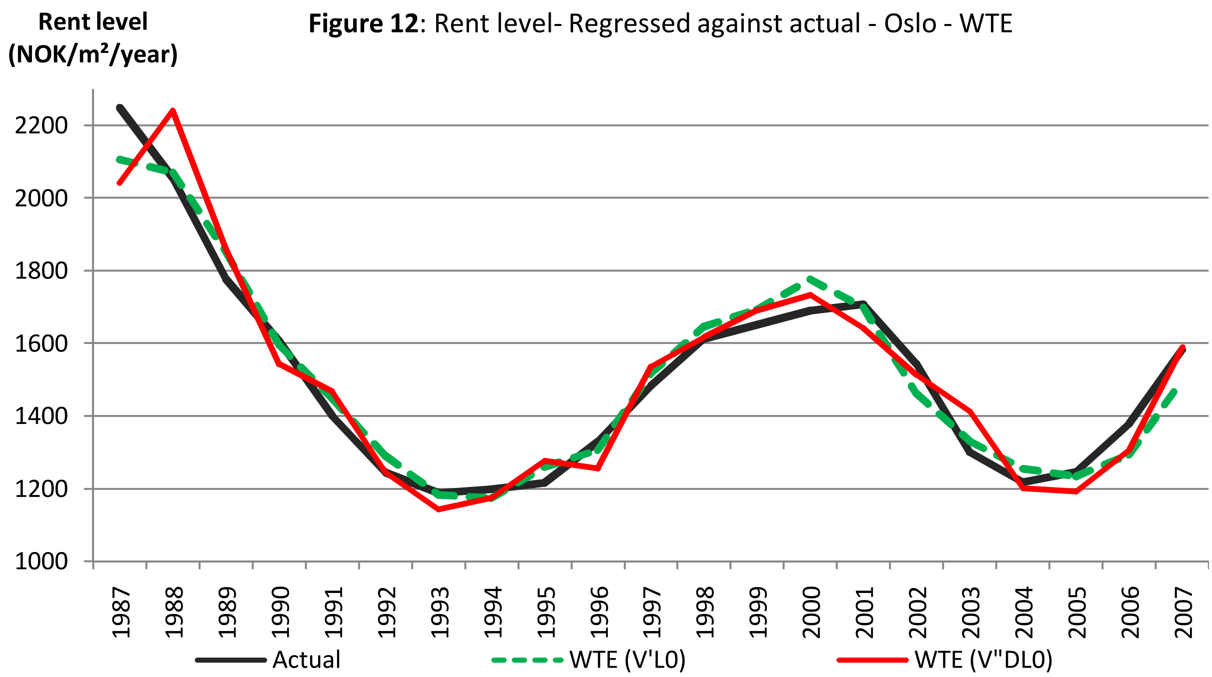
City	Vacancy lag	HLM (5")	WTE (V'L)	WTE (V"DL)
Oslo	Current vacancy v_t	30.68%	31.29%	27.44%
	Past vacancy v_{t-1}	-	55.97%	43.82%
Bergen	Current vacancy v_t	38.53%	29.48%	40.89%
	Past vacancy v_{t-1}	-	34.28%	49.63%

As shown above, the results are very uniform, especially between HLM and WTE (V"DL) since they both compute rental variation. This confirms the rent adjustment speed in each city. Bergen adjusts its rent to the current vacancy between 2 and 3 years, while Oslo needs a bit more than 3 years optimally. When considering the vacancy of the past year, both cities reach structural rent conditions within 2 years.

This is in line with the theory of cycle persistence mentioned in part A, which was induced also by the asynchrony between construction start and achievement. The authors of HMT reported a speed of adjustment of 37% and the WTE model 29%, which again evidences a delay of adjustment of around 3 years. While, in a perfect market with fluid information (between landlords and tenants) and substitute products (in terms of space and location), the rent should adjust automatically to restore structural conditions, this process lasts in reality around 3 years in Oslo and Bergen, as it was in London too.

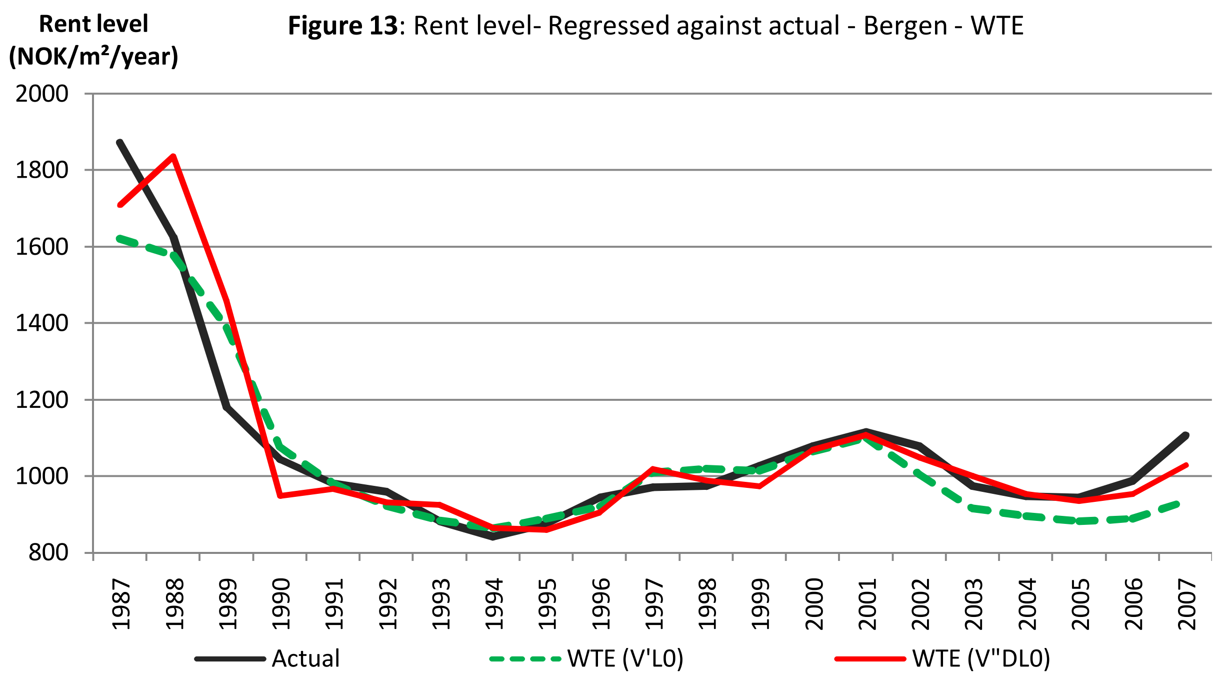
The Figures 12 and 13 presents the graphical results of the WTE equations for Oslo and Bergen. Since all curves are very close two by two, only the ones without vacancy lags are reported in order not to load the graphics too much.

Figure 12: Rent level- Regressed against actual - Oslo - WTE



The WTE model gives very satisfactory outcomes among the two cities. The degree of fitting is really high and both versions of the WTE model follow the actual path very well. These results are similar to those found using the HLM model, which give validity to both of them, as well as their common findings. Namely, it takes around 3 years for the rent to adjust to the equilibrium rent in both cities, and Bergen achieved it a bit faster than Oslo.

Figure 13: Rent level- Regressed against actual - Bergen - WTE



(c) Error Correction Mechanism (Hendershott, MacGregor and Tse)

In addition to the models presented above, one utilizing an Error Correction Model (ECM) are presented and estimated. Reporting one ECM appeared interesting because this mechanism has been utilized in the real estate literature since the early 2000s. It is theoretically able to capture efficiently some of the main features of the real estate, i.e. time persistence due to construction delay and cyclical behaviour.

Hendershott, MacGregor and Tse (2002) have proposed another rental equation for London using an ECM. They mentioned a bit misleadingly the resemblance of the HLM equation with an ECM, which is more similar to a Partial Adjustment in this case and have used data utilized in HLM in the following model. They have first linked the demand to the rent and the employment level, as they respectively affect negatively and positively the demanded space.

$$OS = \lambda_0 R^{\lambda_1} E^{\lambda_2}$$

Then they resorted to the known demand identity $OS(R, E) = (1 - v)S$ in order to re-express the first equation having R on the left-hand side and depending upon the vacancy:

$$R = \gamma_0 E^{\gamma_1} [(1 - v)S\,]^{\gamma_2}$$

Finally, they applied logarithms and reformulated equation using the one-year lagged version of the short-run part as an error correction "long-run" part:

$$\left.\begin{aligned} \Delta\ln(R_t) &= \alpha_0 + \alpha_1\Delta\ln(E_t) + \alpha_2\Delta\ln(1 - v_t) + \alpha_3\Delta\ln(S_t) + \alpha_4 u_{t-1} \\ u_t &= \ln(R_t) - \hat{\beta}_0 - \hat{\gamma}_1\ln(E_t) - \hat{\gamma}_2\ln(1 - v_t) - \hat{\gamma}_3\ln(S_t) \end{aligned}\right\} \begin{aligned} &\text{(Short Run)} \quad (8b) \\ &\text{(Long Run)} \quad (8a) \end{aligned}$$

The equation (8a) is the "targeted" long-run equilibrium rent and the parameter α_3 is the error correction term, which forces the current rent level towards it[13]. The equations (8a)

[13] See section about ECM in the appendixes for a more developed explanation.

and (8b) are commonly estimated separately, but in the case of small sample it is also preferred to estimate one single equation[14].

$$\Delta\ln(R_t) = \alpha_0 + \alpha_1\Delta\ln(E_t) + \alpha_2\Delta\ln(1 - v_t) + \alpha_3\Delta\ln(S_t) \qquad \text{(Long Run)} \qquad (8)$$
$$+ \alpha_4[\ln(R_{t-1}) - \mu_1\ln(E_{t-1}) - \mu_2\ln(1 - v_{t-1}) - \mu_3\ln(S_{t-1})]$$

In the case of one single equation, the error parameter is indeed the one of the lagged rent. To compare all parameters of both long-run estimations, it reverts to divide the parameters of the fragment in the square brackets by α_4. The figures reported below account for this.

As a note, the authors have tested several combinations in addition to the one above, another one when clustering the components of the demand together and another without the vacancy terms. They are not reported here because the results were less satisfactory. Only estimates already corrected for autocorrelation are reported in the table 5 below.

Table 5 – ECM rental equation – Hendershott, MacGregor and Tse (HMT)

ECM(HMT)		*Variables*	Oslo		Bergen	
			Parameter	*t-Value*	*Parameter*	*t-Value*
			SC-Robust			
Simultaneously (8)	Long-run	Constant	6.8542	3.53	-1.5903	-1.13
		Employment $\Delta\ln(E_t)$	.3553	0.59	2.6029	2.41
		Vacancy $\Delta\ln(1 - v_t)$	1.5538	3.03	1.2024	0.60
		Supply $\Delta\ln(S_t)$	-.2061	-0.50	1.5755	1.42
		Rent $\ln(R_{t-1})$	-.5742	-6.05	-.2207	-1.56
		Employment $\ln(E_{t-1})$	.9328	1.34	-.5218	-0.15
		Vacancy $\ln(1 - v_{t-1})$	5.9885	4.89	10.5426	1.16
		Supply $\ln(S_{t-1})$	-1.0536	-1.86	2.2519	0.77
		R^2	82.98%		72.40%	
		Durbin-Watson test	1.76		1.83	
		Number of observations	21		21	
		Coefficient of correlation ρ	.9434		-	
		Cointegration: ADF test	-4.614		-4.675	

[14] See Davidson and MacKinnon, 1993. Estimation and inference in econometrics. p 723-725.
They argued that in presence of small samples, it is preferable to use the single equation. They supportively said that the parameters of the "long-run" part estimated separately are likely to be biased. However, this problem appears less severe when the R-squared statistic is close to 1.

ECM (HMT)		*Variables*	Oslo *Parameter*	Oslo *t-Value*	Bergen *Parameter*	Bergen *t-Value*
				SC-Robust		SC-Robust
Separately (8a+b)	Short-Run	Constant	-.0095	-0.14	-.0649	-1.24
		Employment $\Delta\ln(E_t)$	1.1758	3.19	.9425	0.97
		Supply $\Delta\ln(S_t)$	-.2134	-0.39	.8225	2.58
		Vacancy $\Delta\ln(1-v_t)$	.3219	0.66	1.529	2.92
		Error Correction Term α_4	-.5416	-4.01	-.3464	-4.34
		R^2	60.32%		38.84%	
		Durbin-Watson test	1.32		1.90	
		Number of observations	21		21	
		Coefficient of correlation ρ	.9419		.6845	
				SC-Robust		SC-Robust
	Long-run	Constant	13.6406	4.46	9.1666	2.00
		Employment $\ln(E_t)$	1.4793	2.42	-.9575	-1.01
		Vacancy $\ln(1-v_t)$	1.7618	1.30	5.5113	2.91
		Supply $\ln(S_t)$	-1.6244	-3.03	.3642	0.67
		R^2	98.98%		98.33%	
		Durbin-Watson test	0.91		1.16	
		Number of observations	21		21	
		Coefficient of correlation ρ	.888		.7919	
		Cointegration: ADF test	-4.242		-5.174	

The first comment is that the "separate" configuration gives lower results in terms of explanatory power than the "simultaneous" one, which boasts a high R-squared in both cities. The negative sign of the correction term in all cases indicates a propensity of the rent to overshoot the long-run level, as it did in the HMT article too.

Secondly, each estimation suffers from positive serial correlation. This is in fact normal, since the equations involve lagged dependent variables. In this case, computing SC-robust standard error terms is valid and provides estimates that account for serial correlation and provides sound results that can be used for interpretations, but however without discarding the correlation included in the residuals. As said earlier, it can be done with quasi-differencing using for instance an AR(1) correction method[15], such as Prais-Winsten.

[15] It is also important to ensure the AR(p) order that errors follow. Utilizing Prais-Winsten with an order greater than 1 leads to incorrect estimators and implies manual differencing for instance. The test Breusch-Godfrey evidenced errors being AR(1) here.

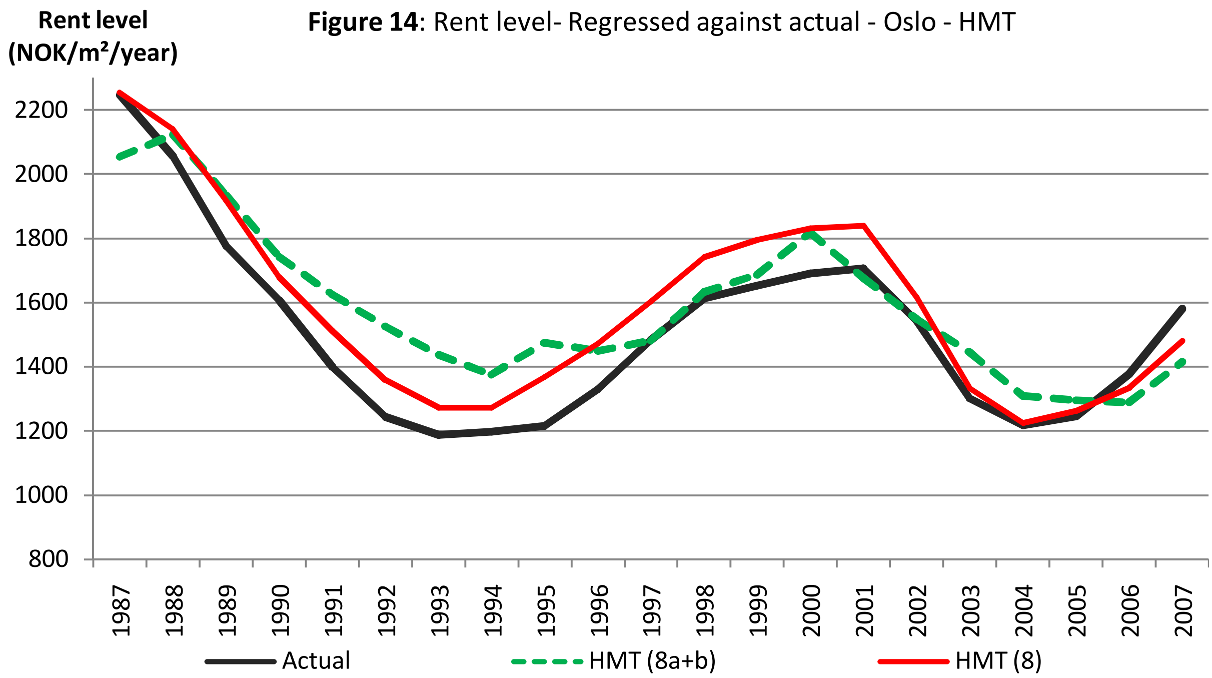
However with substantial serial correlation, the corrections applied here does not manage to clean up everything. The HMT paper also exhibits traces of serial correlation in their results.

One notes that only few variables are significant in both versions of the model, and they are not identical in all estimations. For instance, in the separate configuration, employment seems to have an effect in Oslo and not in Bergen and inversely considering the vacancy. This becomes even more puzzling with the single equation because the situation becomes completely the opposite than for the separate configuration, regarding the significance of the variables. Bergen seems not to fit very well, as only the log difference of employment variable is truly significant in the single equation, whereas four of them are in Oslo.

There are also contradictions regarding the signs of the findings of the HMT paper. The authors have also encountered problems with different signs when estimating directly equation (8) and (8a+8b).

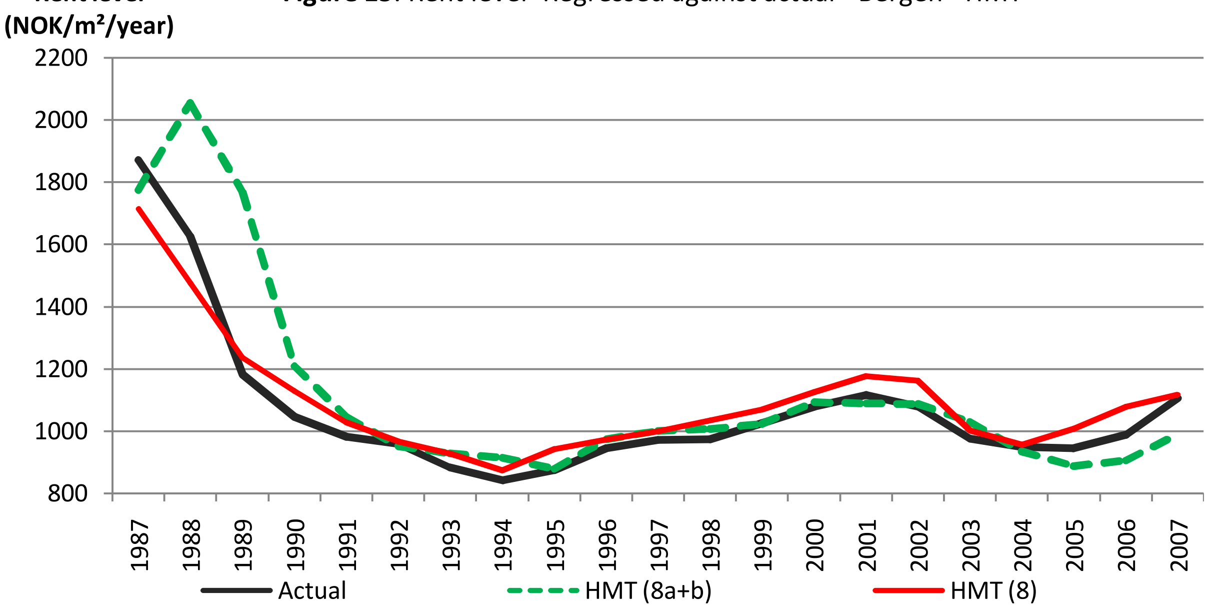
Regarding dissimilarities between Bergen and Oslo, it confirms that the markets behave differently. The Figures 14 and 15 shows the actual rent against the estimates from HMT in both cities.

Figure 14: Rent level- Regressed against actual - Oslo - HMT



Both configurations in Oslo manage to imitate quite accurately the paths of the actual rent, especially the single one. The separated configuration appears however much less performing. It entails certainly from the residual autocorrelation and the non-significance of certain parameters, certainly the vacancy rate. The single curve seems very "timely accurate", i.e. the ups and downs of actual and regressed curves are synchronized, and have also a much more fluid trajectory.

Figure 15: Rent level- Regressed against actual - Bergen - HMT



The results for Bergen are also very satisfactory as both curves manage behaving close to the actual one, and capture the trajectory. The estimation (8a+b) has however a large deviation at the beginning of the sample.

As one can see, there is a more generally one-year lag between the estimated curve (8a+b) and the actual one. As Davidson and MacKinnon (1993) thought, it seems better to use the simultaneous configuration when having a reduced number of observations. The ECM (8) from HMT, which has been tested in once, provides estimations with high explanatory power and seems more able to reproduce accurately the rental path than the separated (8a+b), even when a few variables are significant.

About the error correction term α_4 presented in the tables, its negative sign indicates that the rent is overshooting in both cities and α_4 forces it to the long-run equilibrium. Although it resembles to the rent adjustment parameter found in both HLM and WTE models, it cannot be compared directly with them as it has a different function in an ECM.

After estimation of all models on the cities candidates, it turns out that mainly a measure of both the vacancy and the structural equilibrium rent suffice to accurately reproduce the rental behaviour. The theoretical mechanism exposed in part A seems to hold also in Oslo and Bergen, and providing the right inputs one can be able to foresee the occurrence and the amplitude of the rental cycles. Having in addition some other exogenous variables such as employment level makes outcomes even closer to the reality, but does not appear crucial.

2. Completion Equation

(a) Hendershott, Lizieri and Matysiak

The formulation originally proposed by HLM is $Comp = g(v^* - v,\ R - R^*)$. It tells that the completions should react to the vacancy and negative rent gaps movements. The regression of this equation caused some problems to the authors, which occurred in Oslo and Bergen as well.

- The first one concerns the Vacancy gap, which was presumably calculated using the natural vacancy rate found thanks to the original rent adjustment formula. Several measurements with different lags have been tried out in Oslo but the vacancy parameter is never significant. On the other hand in Bergen, the vacancy variable is significant improves largely the explanatory power in this original formulation.

- The second problem is linked to the large spikes in the Completion curve (see Figure 3, page 27). It involves that the rent gap parameters are either poor compared to the HLM results (for Oslo) or even non-significant (for Bergen), and leads to a low R-squared in both cases.

The authors adopted the following solutions for each problem:

- They did not display any vacancy variable in their regression results, since it was not significant[16]. Nonetheless graphically in Oslo and Bergen, vacancy gaps seem to be correlated with the completion curve (see below).

- Using only a rent gap variable, they were unable to reproduce the extreme values. Therefore, they integrated a dummy variable to handle years where there were extreme values that they reckoned as being exceptionally flourishing in terms of construction for

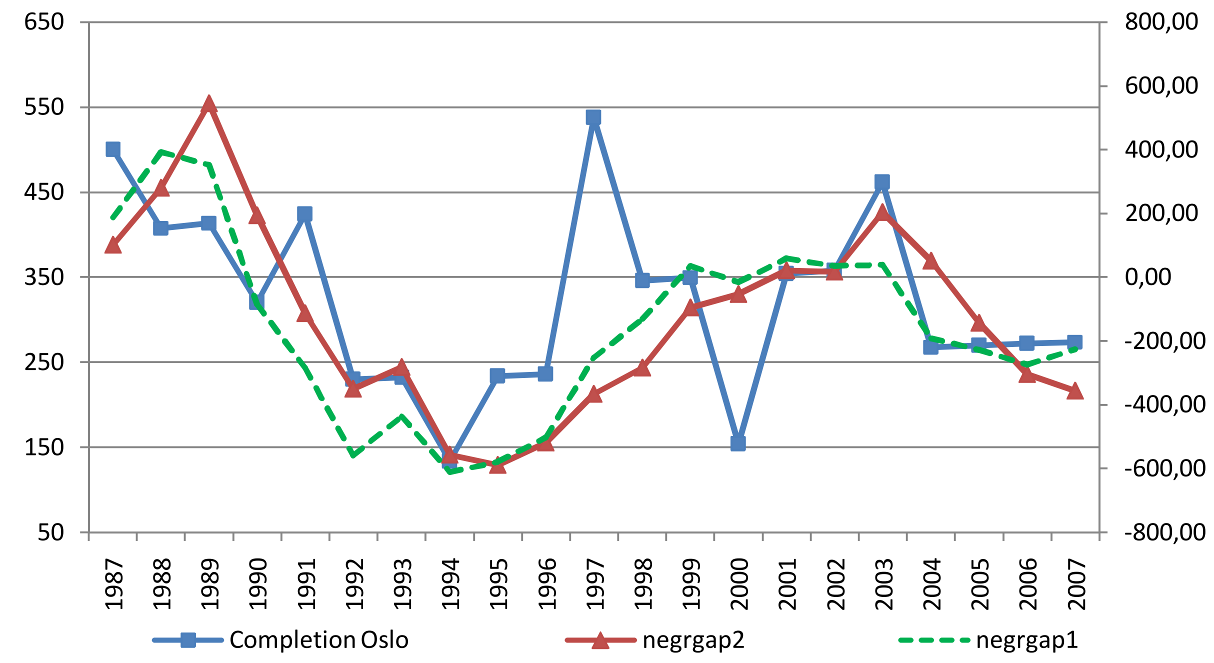
[16] See Ball, Lizieri and MacGregor (1998), p. 240

some reasons. The authors did not give many details about it and their final conclusions at the end of the paper only exposed the estimated vacancy rate and real rent. Doing so, they managed to obtain both rent gap and dummy variables being very significant when testing them together.

Their procedure may suggest that they lack real justification about their formula. For instance, the development of construction is reasonably linked to the rent gap, but again it is unclear why this latter should have an effect on completions started in the past.

To clear up this gradually, the figures 16 and 17 plot the completion versus reversed rent gaps for 1 and 2-year lag in both cities in order to deem what reaction delay fits *Completion* best. Thereby, *negrgap1* and *negrgap2*[17] are the negative rent gap lagged one and two years respectively. This means that for instance *negrgap2* equals $R_{t-2} - R^{*}{}_{t}$, recalling that the rent gap is used to determine what span of time is necessary for past values to reach the current equilibrium.

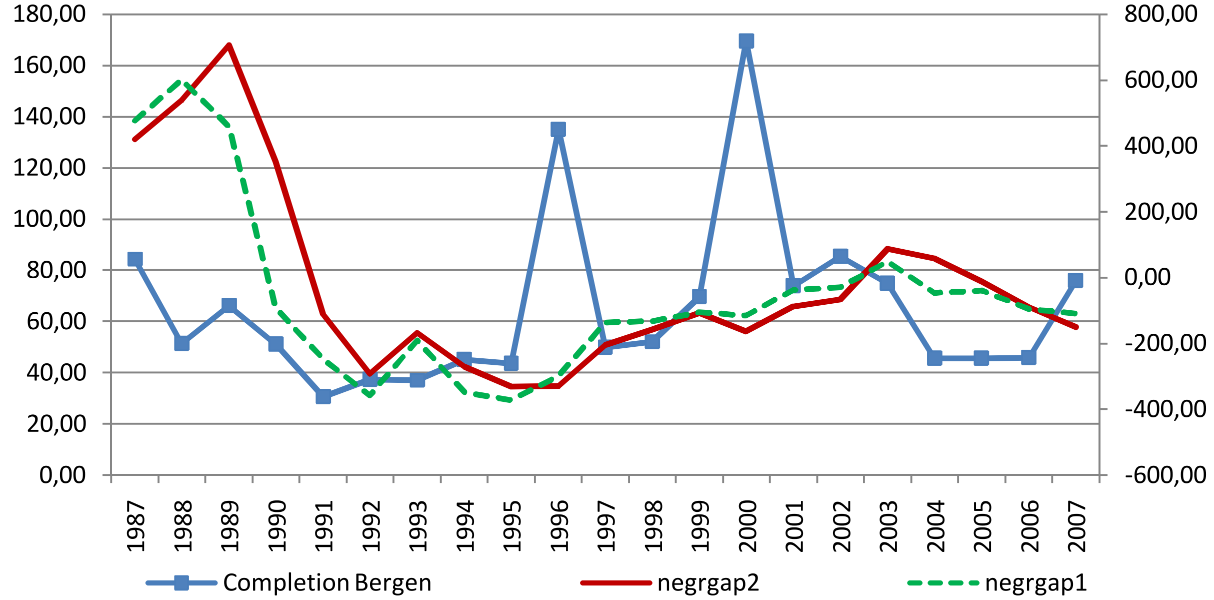
Figure 16: Construction completions against reversed real rent gaps - Olso



[17] As a remark, the reversed rent gap lagged 3 years could not be plotted or estimated here, by lack of data.

For Oslo, both curves seem to fit the path of the completions with the same accuracy. In the paper, the authors had added up these negative rent gaps (which reverts supposedly to a rent gap lagged 1.5 years). They support it saying that the larger the rent gap, the greater the completion. Nevertheless, the regression calculations will identify the most fitting lagged variable in our cities.

Figure 17: Construction completions against reversed real rent gaps - Bergen

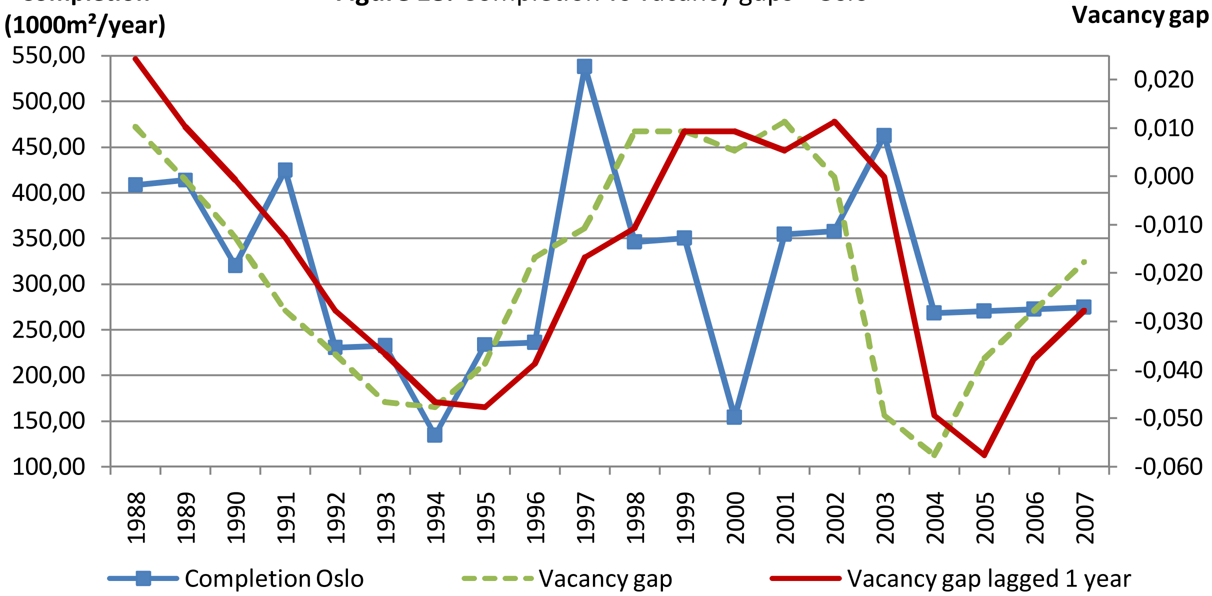


As to Bergen the rent gap curves are very similar but contrary to Oslo, there are some points of dissimilarity between them and the *Completion* curve. Except for the start until 1994, the paths are very different, almost negatively correlated. Again, the presence of the spikes really hampers any potential resemblance.

The figures 18 and 19 below presents the *Completions* against the vacancy gaps obtained using the natural vacancy rates obtained in HLM estimation (5"b), i.e. 5.53% for Oslo and 4.37% for Bergen. As a reminder, the vacancy gap is the difference between the natural vacancy rate and the actual one of a given year. The result works like a threshold to "reorder" construction, whenever $v < v^*$ as expressed previously. The results obtained in the original equation (5") were however not entirely satisfactory for two reasons. First, they were obtained thanks to an equation presenting large autocorrelation and should be biased. Second, they were obviously too low as almost none of the historical vacancy rates is below 4.89% and 1.81% respectively, and thus could not be utilized as a reasonable measure.

The general look of the vacancy gap will be always the same, since its level being the only component varying (the natural vacancy rate is constant). This means in other words that the natural vacancy rate is simply an axe of symmetry. Thus, utilizing only the lagged vacancy rate under a statistical regression will change uniquely the sign of the estimate. All other t-values, other coefficients and the R-squared value remain equal, except for the constant for which the t-value drops drastically and gets a different coefficient. Thus, this weakens the potential existence of a vacancy gap evaluated as in HLM.

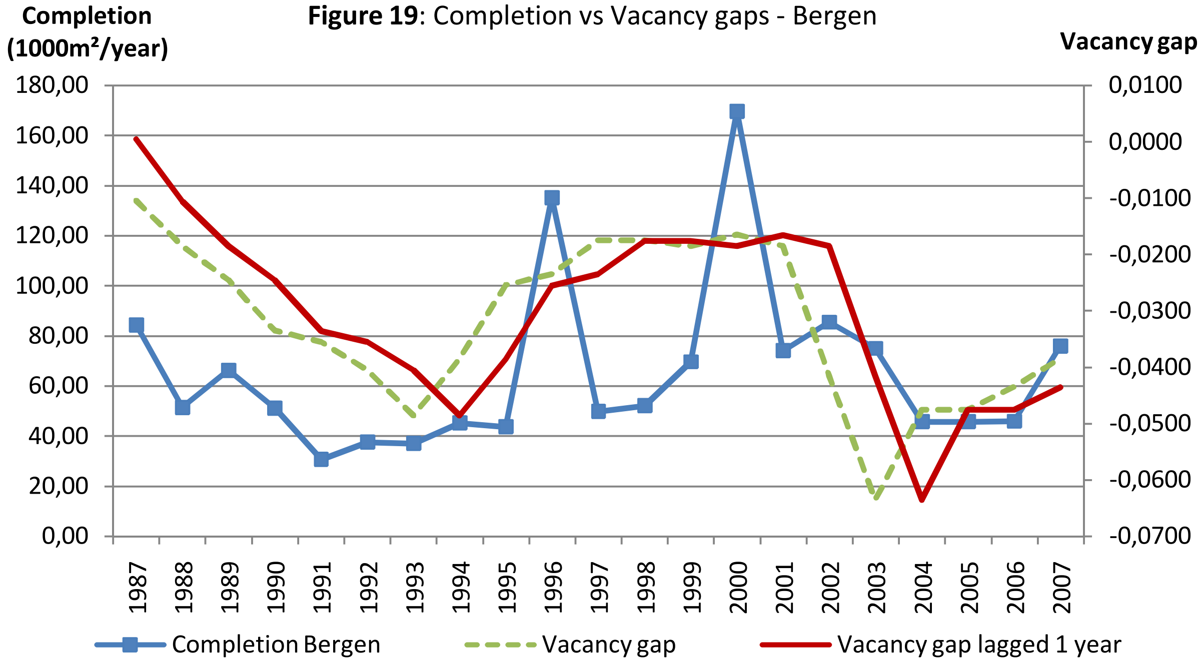
Figure 18: Completion vs vacancy gaps - Oslo



The vacancy gap lagged one year seems to follow positively the trajectory of the completions except for the spike in 2000. Graphically, it seems that the gap with "instantaneous" vacancy fits better, namely the current vacancy. It gives the hunch that the completion reacts to the vacancy within one year in Oslo.

As to Bergen, the conclusions on the graphic are very similar. The vacancy gap is positively correlated, even with the two spikes as both are relatively high at this moment. Like in Oslo, the Bergen *Completion* seems to lag vacancy rates with less than one year.

Figure 19: Completion vs Vacancy gaps - Bergen



One explained that in the formulation of the equation the vacancy gap is unnecessary since it overstates the power of the constant. Thus, the estimation below would consider the vacancy rate alone.

Applying the logarithm to the rental gap returns to having a term with the dimension of a log difference. Now, recalling from equation (1) that *Completion* is the difference between the office supplies of 2 consequent years:

$S_t = S_{t-1}(1-\delta) + Comp_t \Leftrightarrow Comp_t = S_t - S_{t-1}(1-\delta)$.

Completion has thus the same path than the variation but a different dimension. The logarithms will correct for this, and one obtains the estimation:

$$ln(Comp_t) = \beta_0 + \beta_1[ln(R_{t-X}) - ln(R_t^*)] - \beta_2 ln(v_{t-X}) + \varepsilon_t \,,\; X = 1 \; or \; 2 \qquad (6')$$

As to the cointegration, it needs not being tested as all variables are stationary as it is reported in the tables in appendixes. As usually, the Prais-Winsten procedure has been used in order to remove autocorrelation when necessary.

Table 6: $[ln(Comp_t) = \beta_0 + \beta_1[ln(R_{t-X}) - ln(R_t^*)] - \beta_2 ln(v_{t-X}) + \varepsilon_t\ ,\ X = 1\ or\ 2\ ]$ – **(6')**

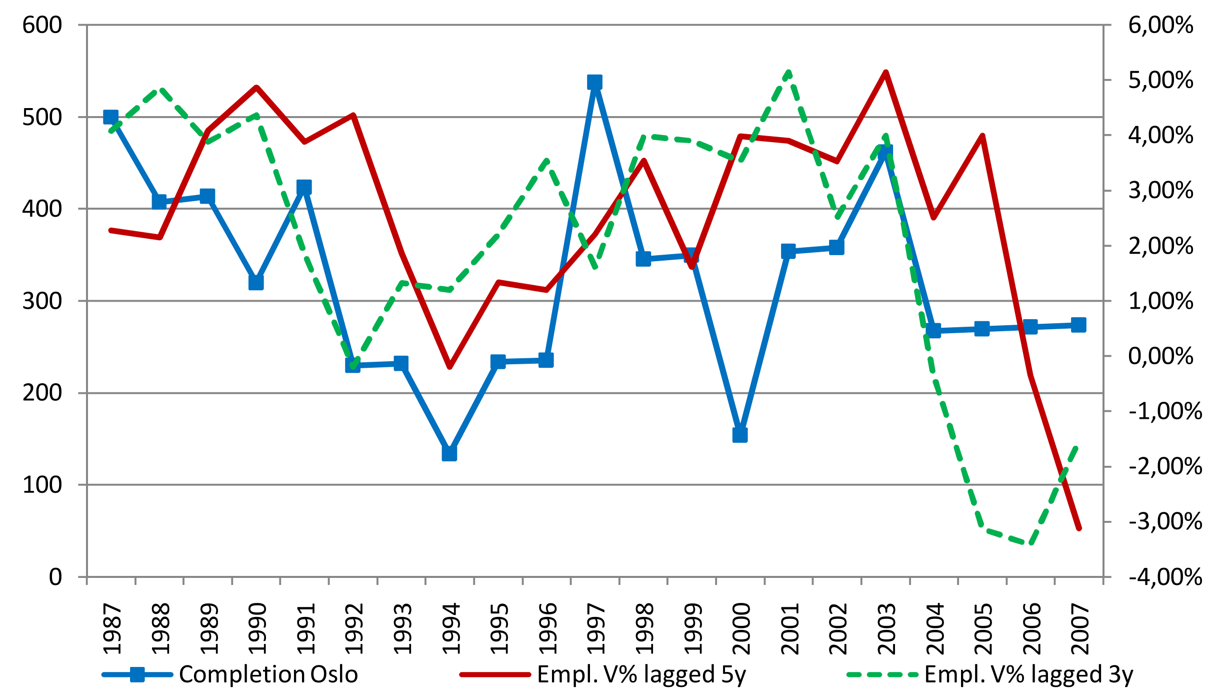
City	*Variables*	*Parameter*		*t-Value*
Oslo	Constant	5.1185		9.17
	Vacancy $ln(v_{t-1})$	-.24774		-1.26
	Rent gap $ln(R_{t-2}) - ln(R_t^*)$	.8262		2.04
	R^2		33.25%	
	Durbin-Watson test		1.97	
	Number of observations		21	
SR-Robust				
Bergen	Constant	2.2739		2.90
	Vacancy $ln(v_{t-1})$	-.5923		-2.22
	Rent gap $ln(R_{t-2}) - ln(R_t^*)$	-.4065		-0.79
	R^2		28.23%	
	Durbin-Watson test		1.92	
	Number of observations		21	
	Coefficient of correlation ρ		.1666	

The regressions give some information. First, the lags giving the best results among others are one year for the vacancy and two for the rent. The differences were sometimes very small but these lags gave the most efficient results in every city. Second, the results differ between cities. For instance, the negative rent gap variable fits for Oslo but not great for Bergen, where it bears an incorrect sign. This is the contrary regarding the vacancy variable. This confirms that both markets had a different development. Here, Bergen considers a small vacancy rate to trigger construction while the rent level dominates the decision of investors in Oslo.

On the other hand, even if there is virtually no autocorrelation the model gives very limited results in both cities. The R-squared is limited to 33% in Oslo and drops to 28% in Bergen. These poor results as well as the high significance of the constants when employing the "vacancy gap" imply that the equation such as HLM conceived it lacks variables. The results of (6') are plotted on Figure 22 at the end of this section.

The insertion of other exogenous variables should improve these results. MacDonald in his article "A Survey of Econometric Models of Office Markets" (2002) reviewed several model of office space, including HLM and WTE, and argued that inserting dummy variables was "an ad hoc procedure without a convincing theoretical explanation". He assumed that the starts and completions should be triggered following office employment growth, with nonetheless a delay that he reckoned being 5 years in London after examination of the original data. He also claimed this was another version of myopia, but it was nevertheless a way to improve the model. Anyway, employment growth might be correlated positively to the completions. The Figure 20 confronts *Completion* with employment growth.

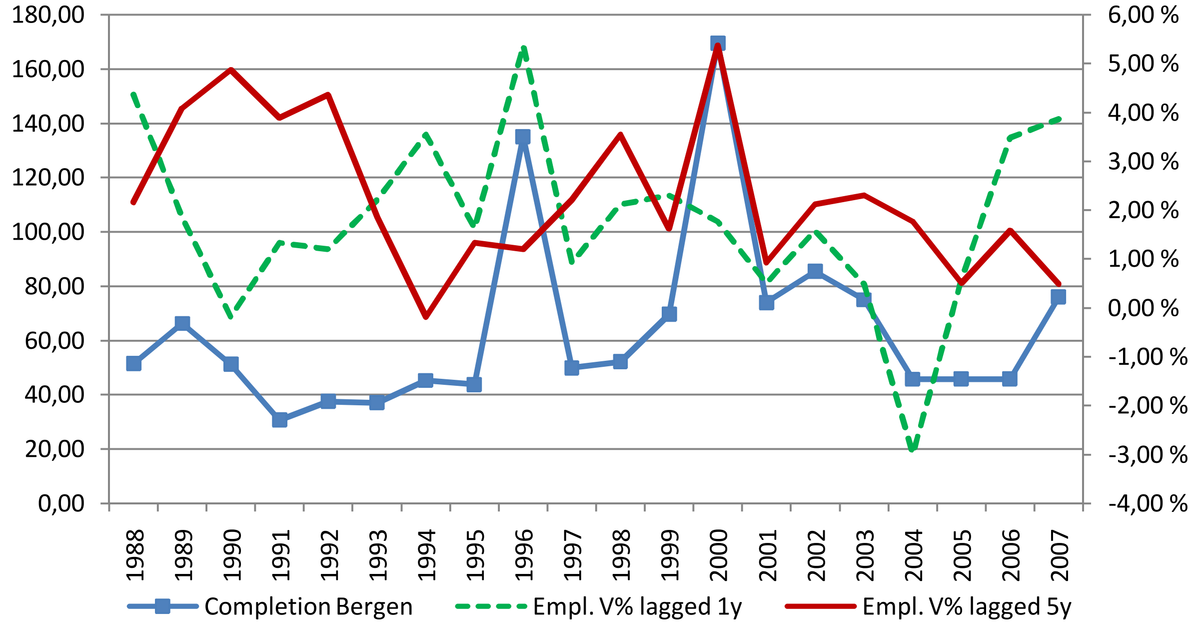
Figure 20: Construction completions vs employment growth - Oslo



Graphically, it seems that there may be a correlation between completions and employment variations delayed by 3 and 5 years. Some of the patterns of *Completion* are common with the employment growth, for instance between 1991 and 1995, or 2000 and 2004. Nonetheless, it does not supply concrete explanation for the zone around the two spikes.

The Figure 21 below plots the Completion against employment growth in Bergen. It certainly gives an intuition about where the spikes come from. The assumption of McDonald seems to be verified in Bergen, at least graphically. The general paths of employment growths follow the Completion curve.

Figure 21: Construction completions against employment growth - Bergen



Testing under regression various lags of employment growth has been done in order to improve the results at best, and it confirmed the predominance of the 1 and 5-year lags in Bergen, but did not identify any in Oslo. Thereby, while in Bergen the insertion of employment variables produced quantifiable effects, in Oslo the rent-related variables remain the only one having an explanatory power, and the best results possible using the HLM model in Oslo are those in table 6 above. This shows that in spite of a graphical correlation, employment does not affect the degree of completions in every city.

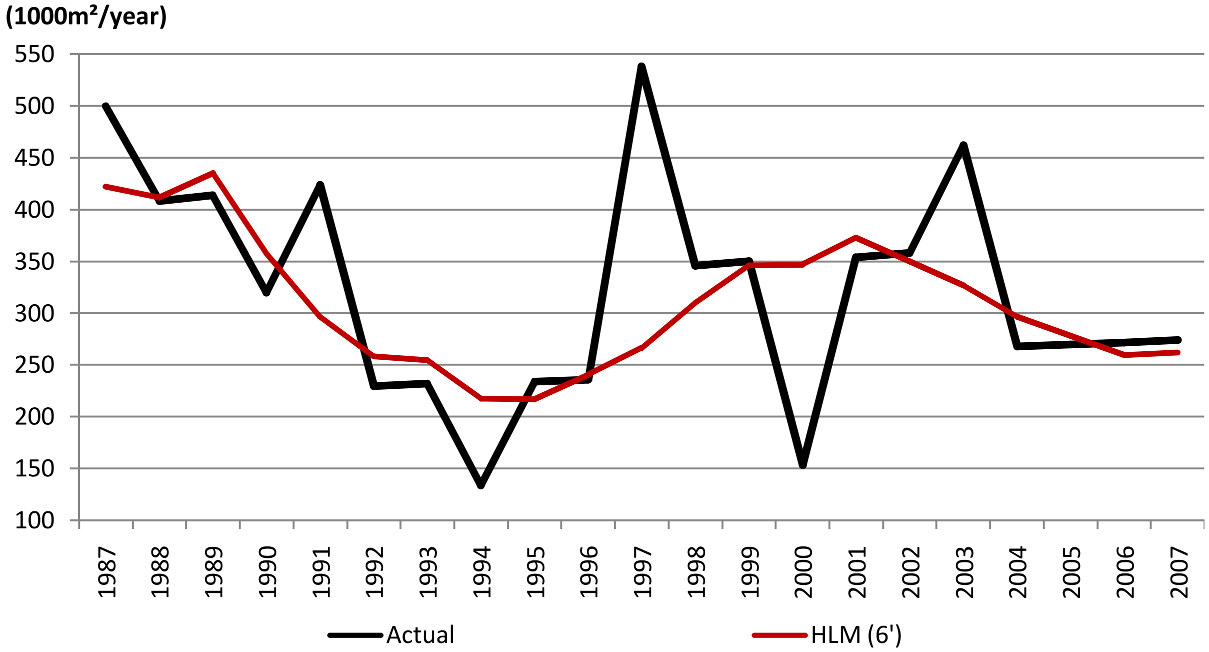
On the next page are the best results for Bergen in Table 7.

Table 7: $[ln(Comp_t) = \beta_0 + \beta_1[ln(R_{t-2}) - ln(R_t^*)] - \beta_2 ln(v_{t-1}) + \beta_3 eg_{t-1} + \beta_3 eg_{t-5} + \varepsilon_t]$ – (6'E)

City	*Variables*	*Employment level (6'E)*	
		Parameter	*t-Value*
SR-Robust			
	Constant	3.2119	3.57
	Vacancy $ln(v_{t-1})$	-.1331	-0.45
	Employment growth eg_{t-1}	12.034	2.26
	Employment growth eg_{t-5}	10.3222	1.67
Bergen	R^2	65.44%	
	Durbin-Watson test	1.825	
	Number of observations	21	
	Coefficient of correlation ρ	.5805	
	Cointegration: ADF Test	-4.033	

The insertion of employment growth boosted the R-squared from 28 to 65%, which proves again a different development pattern among cities. As a matter of fact, the vacancy kept its negative sign but lost its explanatory power in favour of employment.

Figure 22: Completion - Actual against Estimated (WTE) - Oslo

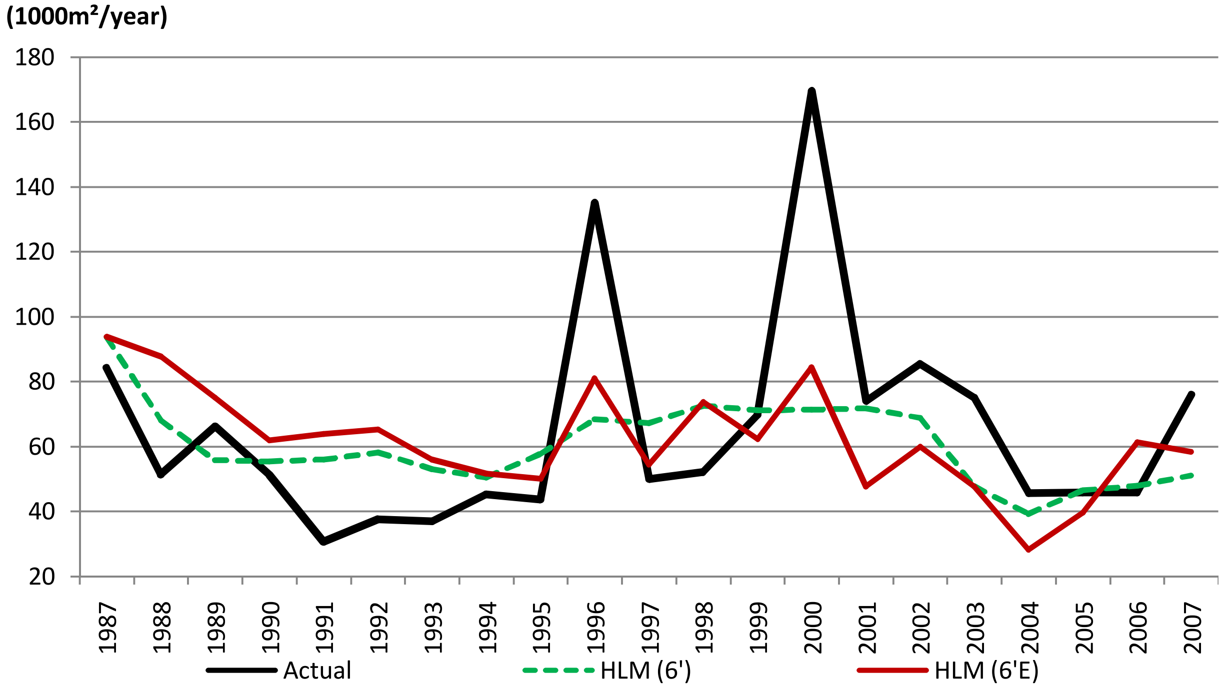


In Oslo, only the rental gap appeared statistically significant which suggests that among the explanatory variables availables the rent level alone suffices to trigger constructions, i.e. the investors consider mainly the prospects of high returns as a decision tool. The rent gap parameter reported in Table 6 page 63 supports this: it is 86%, which evidences the very rapid adjustment of the development to the rental gap.

As a consequence, the plotting in Oslo shows that the HLM formulation only manages to reproduce the average of the Completion curve, and completely fail to capture any deviations. This enhances that the rent is one major component about *Completion* in Oslo. The spikes are thus the consequence of other reasons, which could be overstated future expectations by the investors. One notices that ups and troughs follow one after the other every 3 years without exception, that is more or less the building delay.

On the contrary in Bergen, the rent variable is powerless and seems not to influence the starts and completions. It has been dropped in the estimation without having consequences on the path. This seems a bit odd, since property investors always consider the return on a project. However, the vacancy rate and especially the employment level have an effect on the completion, as the Figure 23 below presents some improvements with the original equation (6').

Figure 23: Completion - Actual against Estimated (HLM) - Bergen



Even if the estimation is far from perfect, there are very notable changes with the equation (6'E) compared to (6'). Both versions show as it could be guessed graphically, employment growth clearly initiated the apparition of the two big spikes, and seems to follows more accurately the movements of the actual curve.

Moreover, it increased the R-squared without having autocorrelation. MacDonald's proposition identified another decision factor for construction orders in Bergen. This can be understood as Bergen has experienced in 1996 an accelerated increase in employment and thus in office surface development for roughly a decade. Thus, property suppliers had to adapt quickly to the demand and considered more the vacancy and the employment demand relative to the actual profitability of the constructions.

The HLM equation for Completion certainly lacks insertion of extra explanatory variables, as the R-squared is limited in Oslo. The authors had purposely employed convenient dummy variables to sidestep these problems linked to the incapacity to capture the large spikes during construction booms. The insertion of a measure like employment managed to improve the initial formula at least in Bergen.

(b) Wheaton, Torto and Evans

The authors specified their *Construction* equation as a linear function of several variables:

$$Cst_t = \beta_0 + \beta_1 R_t - \beta_2 v_t - \beta_3 I_t - \beta_4 RC_t + \varepsilon_t \quad \text{(VI)}$$

According to WTE, construction orders are triggered in function of the actual values of rent level, vacancy rate, nominal interest rate[18] and replacement cost. The Figures 24 and 25 plots the Construction against the Real rent level in Oslo and Bergen.

Figure 24: Construction vs real rent - Oslo

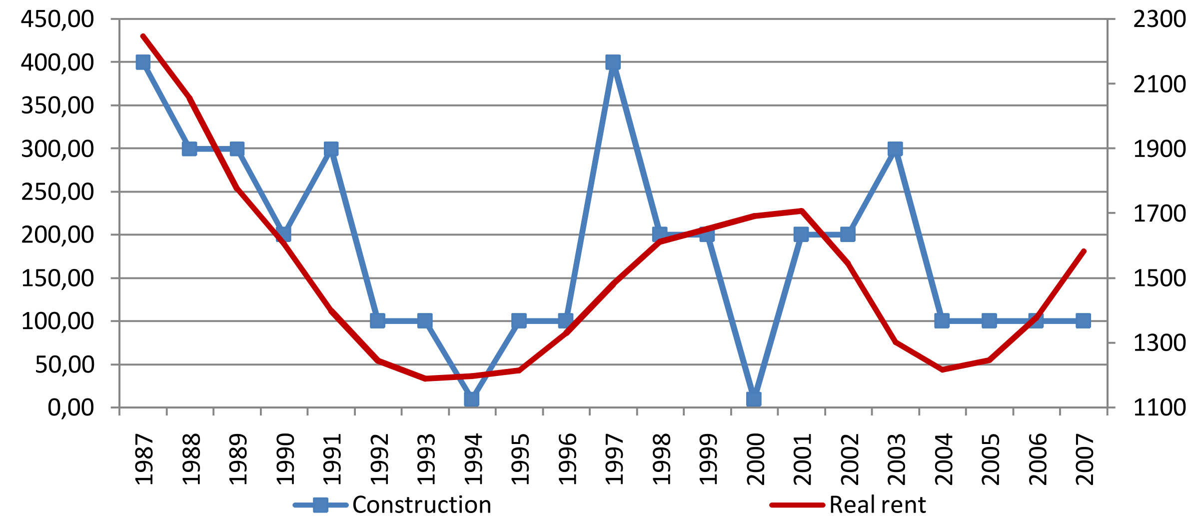
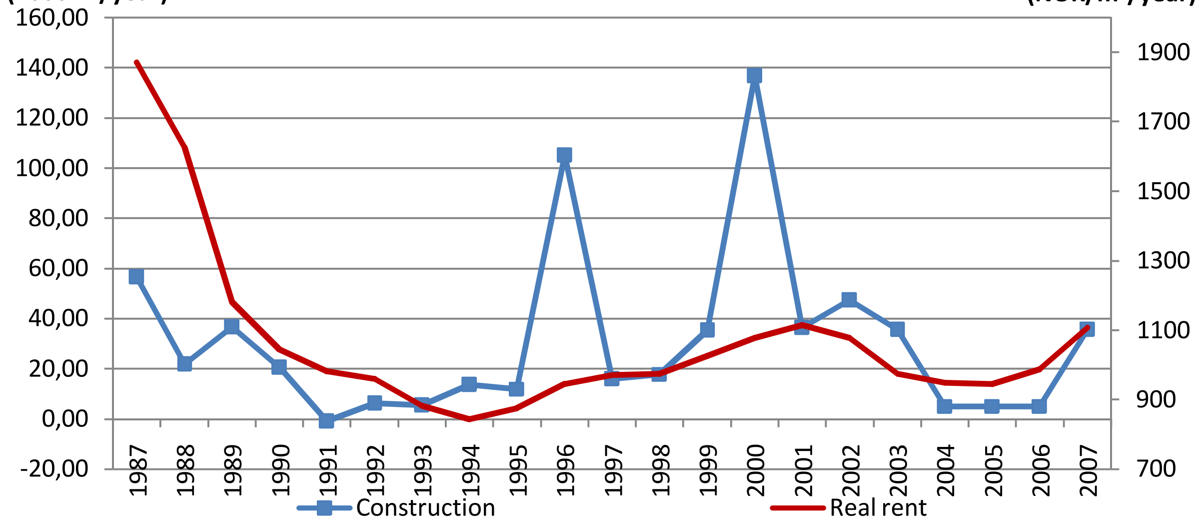


Figure 25: Construction vs real rent - Bergen



[18] Real has been tested here and in the WTE paper, and led to poorer results (confer article, p. 91, note 13)

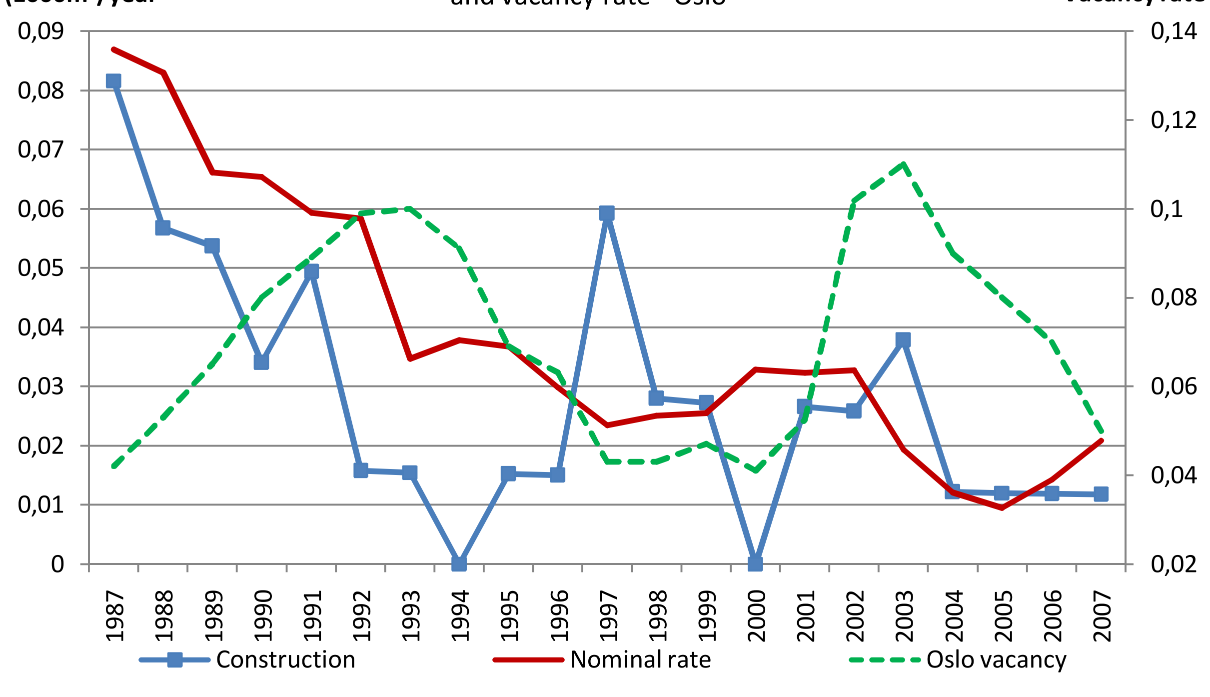
In both cities, the rent appears positively correlated to the Construction, as was the rental gap in the previous HLM equation. Again, it provides nonetheless no explanation for the spikes, between 1995 and 2001.

The equation (VI) of WTE will be transformed using the logarithms before being estimated, to remove the dimensional aspect of the equation. The equation is then as following:

$$ln(Const_t) = \beta_0 + \beta_1 ln(R_t) - \beta_2 ln(v_t) - \beta_3 ln(I_t) - \beta_4 ln(RC_t) + \varepsilon_t \qquad \text{(VI')}$$

To have an additional insight of (VI'), Figures 26 and 27 plot the Construction against the nominal interest rate and the vacancy rate. Both should have a negative path compared to *Construction*. Higher the interest rate, lower the incentives to borrow for the first and identically to HLM for the second variable.

Figure 26: Construction vs nominal interest and vacancy rate - Oslo

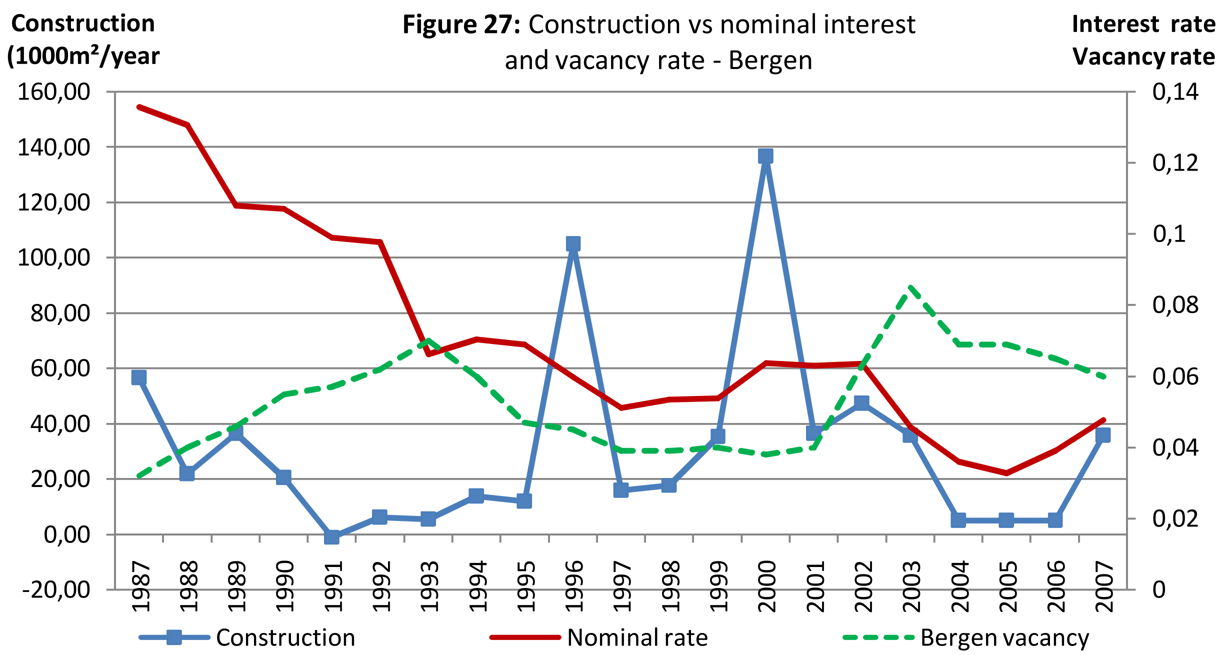


In both cities, the opposition between the construction and the nominal interest rate seems more or less true, graphically speaking. In Oslo, it helps understanding why the spikes went up or down. On the other hand in the beginning of the sample, it makes perplex because even if the interest went down, the completion decreased as well. This was maybe due to the very large inflation during these years, which made the real rent indeed increasing from 1988 to 1992.

In Bergen, the development has another relation to the interest rate. The negative correlation is more obvious at the beginning but the end of the period is puzzling: why the constructions orders followed the interest rate so accurately while it should be the opposite? One can notice that the first spike in 1996 probably signed the beginning of 10 years of important construction in Bergen, and this is rational when looking at the real rent. On the other hand, spikes in 1996 and 2000, as well as the slump in 2004 do not fit since they follow the same direction than the interest rate movement. Knowing from HLM that employment has a role to play, it does certainly not explain the whole amplitude of the spikes. One notices that they all occurred one year after new elections for the city hall. Moreover, history tells that a new mayor was elected at every time, which suggests that the spikes and this puzzling behaviour might also have been driven by political decisions, but the statistical estimation will probably give some indications.

As to the vacancy rates compared to the construction, the trajectories behave oppositely except in the region of the spikes in Oslo, which should hamper the results.

Figure 27: Construction vs nominal interest and vacancy rate - Bergen



As to the replacement cost, it should also theoretically affect the completion negatively according to the Tobin's q argument. When RC increases, the ratio V/RC becomes less than one, which is a situation where the market is overbuilt and the vacancy is large (confer page 33). This phenomenon is somewhat limited in Bergen, since the market is smaller but also because of the decade of rapid expansion started around 1996 which probably forced construction disregarding market conditions.

In the Equation (VI'), $ln(RC_t)$, $ln(v_t)$ and $ln(I_t)$ are I(1)[19], i.e. they are presumably wandering from their long-run equilibrium as long as one goes ahead on time. All other parameters of the equation are I(0). Like it was done earlier in previous equations, it reverts to ensure that there is a valid cointegrating relationship between the variables.

The Table 8 below presents the results of WTE equations (VI') in both cities:

Table 8: $ln(Const_t) = \beta_0 + \beta_1 ln(R_t) - \beta_2 ln(v_t) - \beta_3 ln(I_t) - \beta_4 ln(RC_t) + \varepsilon_t$ ***- (VI')***

City	*Variables*	*Simple logarithms (VI')*	
		Parameter	*t-Value*
Oslo	Constant	-23.2957	-1.54
	Rent $ln(R_t)$	-0.3324	2.37
	Vacancy $ln(v_t)$	1.9336	1.77
	Interest rate $ln(I_t)$	-2.2285	-1.37
	Replacement cost $ln(RC_t)$	-3.5839	-1.28
	R^2	30.35%	
	Durbin-Watson test	2.187	
	Number of observations	21	
	Cointegration: ADF Test	-4.803	
Bergen	Constant	-15.1396	-1.38
	Rent $ln(R_t)$	-.662	-0.44
	Vacancy $ln(v_t)$	-2.2595	-2.44
	Interest rate $ln(I_t)$	2.1696	1.69
	Replacement cost $ln(RC_t)$	4.8437	1.94
	R^2	42.13%	
	Durbin-Watson test	2.14	
	Number of observations	21	
	Cointegration: ADF Test	-4.942	

[19] See the appendixes for the Unit Root and Cointegrating test results.

The results from the WTE equation bring several answers at the same time. First of all, they are very similar to those from HLM in terms of explanatory power. Bergen benefits again from a more accurate estimation that Oslo, but yet is much lower than what found WTE in their article: they discarded the two first years of the sample where large rents did not produce an increase in construction orders and obtained an R-squared of 88%, with however a Durbin-Watson statistic of 2.56, i.e. presenting doubtlessly some negative autocorrelation. This evidences the difficulty to capture accurately the "construction" expansion in both cities using the variables utilized here.

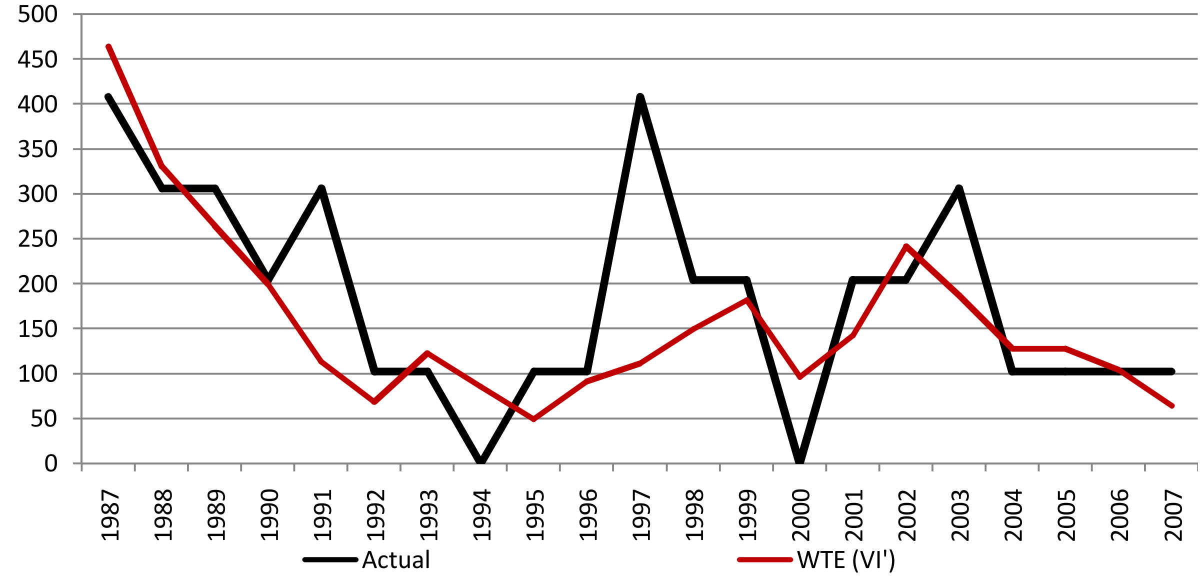
The variables have a multi-cointegration relationship, as the Augmented Dickey-Fuller statistic gives witness to it, and the estimation is free of serial correlation in both cities.

One notices that the rent and the vacancy rate are again the most significant variable in respectively Oslo and Bergen, and inversely the other is not significant. This is in line with the results found using the HLM model which also identified these variables being the main decision factors in each city, at least during the sample period.

One drawback of the equation (VI') to Oslo and Bergen is related to the signs of the parameters. None of the cities has every sign oriented correctly in line with the theory. Disregarding the constant, the variables wrongly signed are the vacancy in Oslo, and the rent, the replacement cost and the interest rent in Bergen. While the former is certainly due to the spike of 2000, the three latter tends to support that the period of high development stemmed from compelled construction orders.

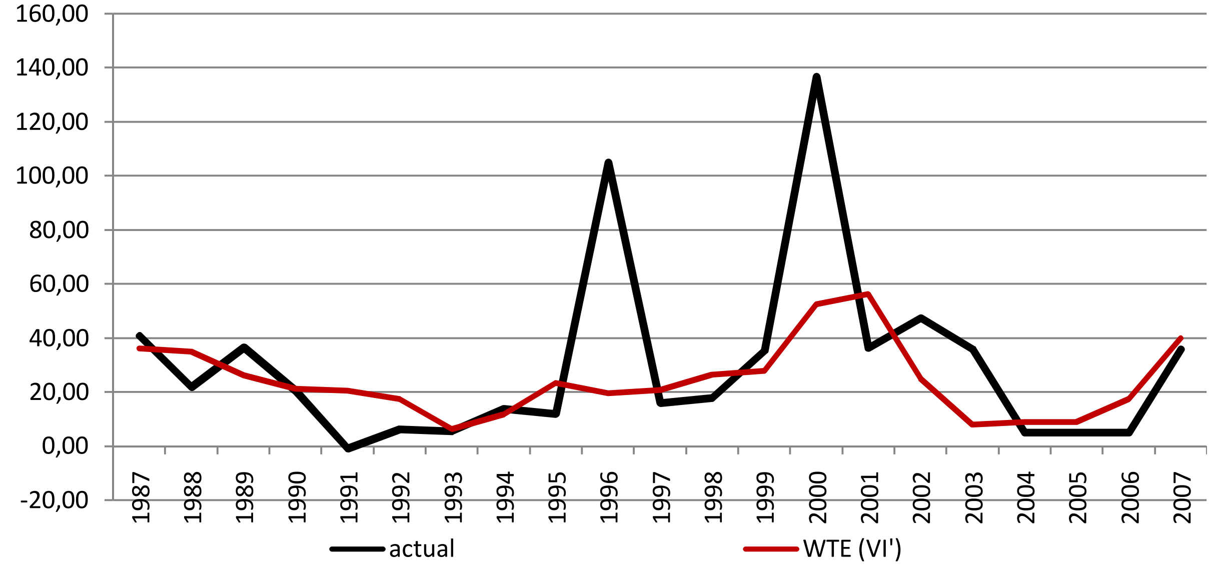
The results of (VI') are plotted on the figures 28 and 29 below and give some elements of answer about the spikes in *Construction*, and thus probably also in *Completion*.

Figure 28: Construction - Actual against Estimates (WTE) - Oslo



In both cities, the curve of the original WTE model (VI') appears capable to follow the general path of the *Construction* curve. Moreover, even if it is still unable to mark the first peak, one notices an improvement compared to the HLM figure. This entails that the way WTE linked interest rate and replacement to the development is more efficient than in equation (6'). It also means that *Construction* must depend upon something else. Knowing that employment had not effect, this might come from external sources, as it is suspected in Bergen.

Figure 29: Construction - Actual against Estimated (HLM) - Bergen



In Bergen, the results are also better than in Oslo as it was already the case with the HLM model. One notes that in both cities the estimated curve (V') manages to capture at least partly the second "large" spike in 2000 and this, despite its opposite orientation. After some manipulations of the plotting and even if all variables are of course entangled, it appears that the rent and the replacement level has mainly an effect on the general path and level of the estimated curve, whereas the vacancy and the interest rates produces mostly deviations. This makes sense when comparing with HLM, the vacancy rate in Oslo was almost non-significant and the curve captured none of the deviations. Furthermore, the opposite signs of the diverse variables in the two cities permits capturing the different deviations and spikes. It evidences certainly the somewhat irrational behaviour that happened in Bergen during a period.

WTE created a model which delivers a good estimation of the general level of construction in the two candidate cities. The results are in accordance with those found in the HLM model, and their comparison permits drawing a more complete representation of the office development in Oslo and Bergen. It particularly helps identifying their differences. On the other hand, the model could not capture the full amplitude of extreme variations around the mean either.

Comparing both models, one reckons that employment and vacancy had a major effect on the office real estate development in Bergen, whereas the rent is one of the major, at least a necessary factor to trigger development in Oslo. On a smaller scale, it appeared that the interest rate and a benchmark measure of replacement cost generate some effects as well.

The model helped identify potential reasons for development but like in the HLM model, it certainly lacks insertion of other variables, or perhaps it identified features of some irrational cycles in the markets.

(c) Error Correction Mechanism (Englund, Gunnelin, Hendershott and Söderberg)

In the subsequent literature to HLM and WTE, other research teams have set up various new models, complete or partial, often starting from the inconsistencies experienced in prior papers. One of them is the one by Hendershott, MacGregor and Tse (HMT) from which was drawn the ECM utilized previously for the rental estimation.

Another team composed by Englund, Gunnelin, Hendershott and Söderberg (EGHS) used the HMT model as a starting point for establishing an innovative and complete model using data about the Stockholm office market.

As one can see, Hendershott is always present among every team, and he is certainly one of the most prolific author about office real estate econometrics. After have read through its work, it appears that some of his papers are very repetitive, and that he is often updating his precedent work. However he and his teams proposed complete and precursory models about this topic where it is true only a few exists, despite there are numerous partial ones, especially about the rent.

In the EGHS model, the long-run part of the equation about the supply utilizes an innovative aspect of HMT compared to HLM, i.e. the estimation of the equilibrium rent rather than its calculation prior to the estimation using the rental gap.

The EGHS model utilizes a two-step procedure to evaluate the equilibrium rent series. The first consists in elaborating a long-run equilibrium series.

Recalling what was presented in the ECM in the section about the rent earlier, the authors reutilize the long-run equation (8a) page 49. Specifying the long-run demand for office space as a log-linear function of real rent and employment level:

$$ln[D(R,E)] = \lambda_0 - \lambda_R ln(R) + \lambda_E ln(E) \qquad (D_{LR})$$

In this equation, the rent reduces the office demand while the employment increases it. D_{LR} stands for Demand Long-Run.

Now, considering the known identity about occupied space in the long-run equilibrium, one has:

$OS^{*} = D(R^{*}, E) = (1 - v^{*})S_t$

Now taking the logarithm, and reformulating these two equations in function of the rent in the long-run:

$ln(R^{*}) = \gamma_S[ln(1 - v^{*}) - \lambda_0] + \gamma_E ln(E) + \gamma_S ln(S)$ (9a)

According to EGHS, this equation has been already estimated among European cities by several research papers, and the parameters have typically high significance. It nonetheless involves the existence of the same cointegrating relationship on the long term than in HMT, which has been evidenced. The results of the estimation are reported below in Table 9.

Table 9: $ln(R^{*}) = \gamma_S[ln(1 - v^{*}) - \lambda_0] + \gamma_E ln(E) + \gamma_S ln(S)$ ***- (9a)***

City	*Variables*	*Long-Run demand (D_{LR})*		*Equilibrium rent (9a)*	
		Parameter	*t-Value*	*Parameter*	*t-Value*
SR-Robust					
Oslo	Constant λ_0	4.1894	4.49		
	Rent $ln(R)$	-.1324	-2.04		
	Vacancy $[ln(1 - v^{*}) - \lambda_0]$			-1.7968	-1.37
	Employment $ln(E)$	.9998	7.45	2.8786	4.41
	Supply $ln(S)$			-1.8446	-2.09
	R^2	99.88%		99.56%	
	Durbin-Watson test	1.74		0.96	
	Number of observations	21		21	
	Coefficient of correlation ρ	.8676		.8644	
SR-Robust					
Bergen	Constant λ_0	4.4026	5.30		
	Rent $ln(R)$	.0167	0.36		
	Vacancy $ln(1 - v^{*})$			-1.7975	-1.36
	Employment $ln(E)$	.6099	3.96	-.7099	-0.52
	Supply $ln(S)$			.3181	0.36
	R^2	99.78%		99.75%	
	Durbin-Watson test	1.78		0.88	
	Number of observations	21		21	
	Coefficient of correlation ρ	.9135		.6888	

The results are very impressive in terms of explanatory power, but there is a lot of autocorrelation and Bergen does not have all significant coefficients. Another issue to consider here is the cointegration. It has been proven that it exists thanks to equation (8a) but the insertion of the constant makes suspicious about cointegration between all the variables, even if some cointegrating relationships appear partially. Employment, supply were evidenced as I(1) and also there are some doubts about the vacancy rate. EGHS had troubles with this as well, and conveniently discarded some years at the beginning of their data before testing for cointegration.

Then, the model defines the ECM of the supply in a first time when including example of the rent gap from HLM but using the structural equilibrium rent of HMT:

$$\Delta ln(S_t) = \alpha_0 - \alpha_1 \Delta ln(v_t) + \alpha_2 \Delta ln(R_t) + \alpha_3 \Delta ln(E_t) - \psi_v v_{t-X} + \psi_\varepsilon \varepsilon_{R_{t-X}} + u_t \qquad (9b)$$

The changes in office supply, or *Completion*[20], react positively to employment and real rent but negatively to the vacancy, since it indicates an oversupply.

The authors thereafter asserted that the changes from one year to another ("short-run") would not have effect on the development due to the delay that construction requires, and then tested the equation only with the two last terms ("long-run").

The first one v_{t-X} is the vacancy rate. It was originally again a vacancy gap, utilizing a natural constant vacancy rate. But as mentioned earlier for the HLM completion equation, it is generally not available beforehand and moreover it affects only the sign of the coefficient.

The second one $\varepsilon_{R_{t-X}}$is the rent gap, i.e. the error term of the equation (9a). This is the same notion than in the original HLM model, which makes think that the model is perhaps mistakenly called an ECM as it does not use the same variable in both long and short-run.

[20] Completion should however be accounted for depreciation, but it makes only marginal differences.

Their notation without defined lag indicates an unspecified time lag which will be determined by the data itself under statistical estimation.

In order to check whether the one-year lagged variables are inefficient or not, two versions of equation (9b) are tested, one with the short-run part and one without. Their results are reported in Table 10 below:

Table 10: $\Delta ln(S_t) = \alpha_0 - \psi_v v_{t-X} + \psi_\varepsilon \varepsilon_{R_{t-X}} + u_t$- ***(9b')***

$\Delta ln(S_t) = \alpha_0 + \alpha_1 \Delta ln(v_t) + \alpha_2 \Delta ln(R_t) + \alpha_3 \Delta ln(E_t) - \psi_v v_{t-X} + \psi_\varepsilon \varepsilon_{R_{t-X}} + u_t$- ***(9b")***

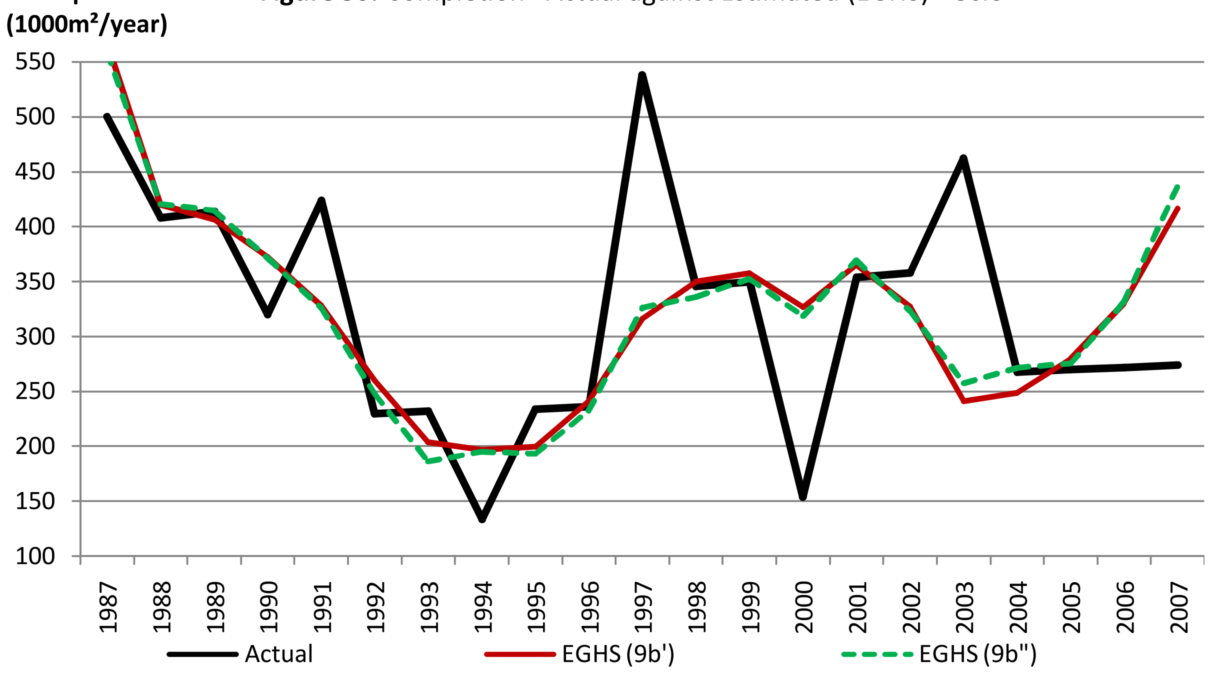
City	*Variables*	*Long-Run only (9b')*		*Long-Run & Short-Run (9b")*	
		Parameter	*t-Value*	*Parameter*	*t-Value*
SR-Robust					
Oslo	Constant	.0465	5.37	.0498	2.57
	Vacancy variation $\Delta ln(v_t)$			-.006	-0.22
	Rent variation $\Delta ln(R_t)$			.0061	0.11
	Employment variation $\Delta ln(E_t)$			-.0871	-0.44
	Vacancy v_{t-2}	-.2094	-1.52	-.2255	-0.80
	Rent gap ε_{R_t}	.0595	3.36	.0631	2.46
	R^2	66.00%		66.16%	
	Durbin-Watson test	1.86		1.83	
	Number of observations	21		21	
	Coefficient of correlation ρ	-		.0284	
SR-Robust					
Bergen	Constant	.0482	3.65	.0725	3.29
	Vacancy variation $\Delta ln(v_t)$			.0017	0.10
	Rent variation $\Delta ln(R_t)$			.1433	1.53
	Employment variation $\Delta ln(E_t)$			-.7363	-2.57
	Vacancy v_{t-1}	-.5713	-2.92	-.7473	-2.41
	Rent gap $\varepsilon_{R_{t-5}}$	-.023	-2.10	.022	0.88
	R^2	44.39%		67.94%	
	Durbin-Watson test	2.26		2.04	
	Number of observations	16		16	
	Coefficient of correlation ρ	-.2973		-.3869	

The results are satisfactory and at the same time very different among cities.

First, in Oslo as it was expected by the authors, contemporaneous changes of rent, employment and vacancy rate do not affect the variation in office space. This gives witness that the office market in Oslo reaches maturity compared to Bergen, where 2 out of 3 variables turned out having a notable effect.

Secondly, the coefficients, their significance and their sign confirm what was found in the previous two models. Namely, the rent was responsible for the decision to build in Oslo, whereas this was the employment and the vacancy in Bergen. In addition to that, the lags determined by the data identify that the markets reacts quickly to these main decision factor. As an example in Bergen, employment and vacancy of the past year affects the completion, as it did too in the WTE model. On the other hand, only the rent lagged 5 years supposedly has an effect on development, which even in the construction sector appears unrealistic. The sign being negative, it certainly evidences the previous cycle and then the inefficiency of the rent on the development as one see on the figure 31 next page. The estimated curve (9b') reproduces a slightly smoothed representation of the path of the "vacancy gap"[21]. The figures 30 and 31 plot the ECM estimation against the actual Completion in both cities.

Figure 30: Completion - Actual against Estimated (EGHS) - Oslo



As one can see, the two estimated curves are very close, which proves the inefficiency of contemporaneous changes over development next year, in cities with mature office market. One realizes when comparing with the plotting of the other models that there is mostly the

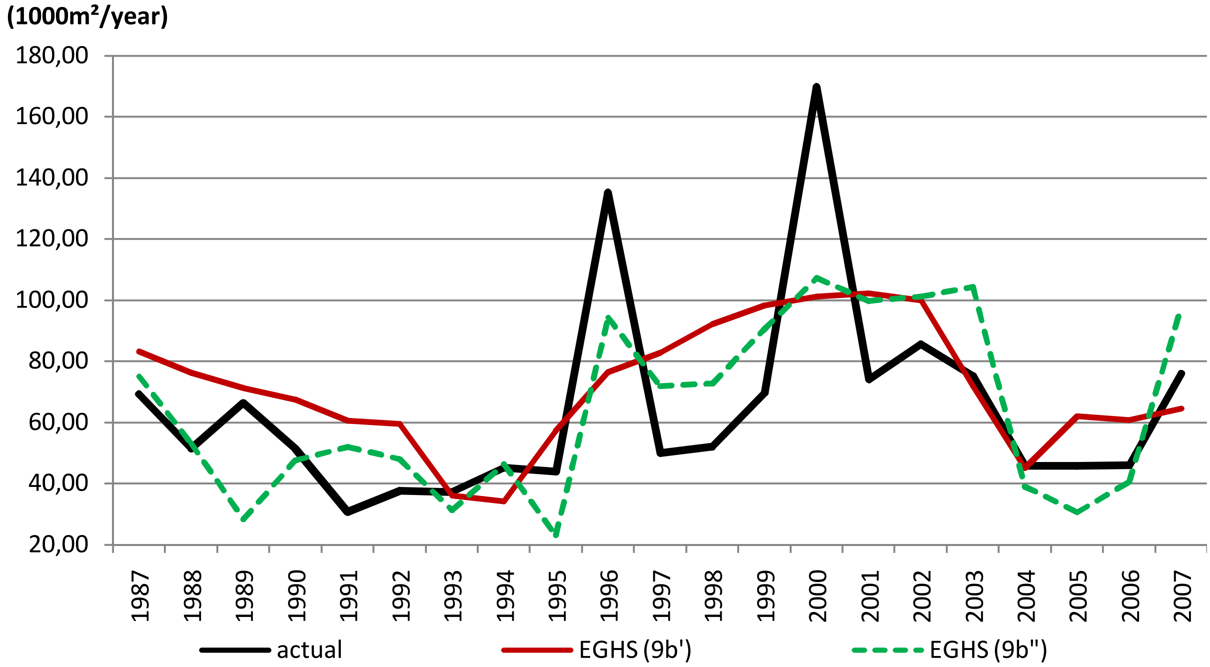
[21] See Figure 19 page 62.

variation in rent which initiated the spike in 2000. On the contrary, it appears that it has much less to do with the spikes of 1997 and 2003. According to the WTE plotting, those were maybe related to the interest rate but as it was proposed earlier, they might be due to a spread "overanticipation" of the market future by property developers, which relied on exaggerated rent expectations a few years before 1997 to account for the building delay.

Following that, the drastic negative spike of 2000 stemmed from the fact that they found themselves with a huge amount of surface compared to the demand, combined with a downward-sloping rent level. As one can see, even when the estimated curve actually "captures" partly one spike, the generally large gap in amplitude indicates the difference between what suppliers "blindly" proposed and the potential demand according to the market conditions. To the extremely severe misrepresentation of 1997, suppliers responded in 2000 with a reaction with the same scale. This following period suffered from the largest vacancy rates of the sample, and was certainly very harmful for the developers.

On the contrary in Bergen, the situation appears much more comprehensible according to market conditions.

Figure 31: Completion - Actual against Estimated (EGHS) - Bergen



As one can see, the vacancy plays the main role explaining the general trajectory of the completion curve. On the other hand, the insertion of immediate previous changes modifies massively the path of the estimated curve. It fits the spikes much more completely than in the HLM model, and matches accurately the ups and downs. The Bergen market experienced a development in accordance with what the model is able to predict. In effect, Bergen is still in development and does not have deviations as volatile than in Oslo, or another larger city. Bergen has not reached a maturation state, where there are periods of scarcity which leads to cycles. The participants adjust rent in function of the demand in an ordered fashion, and the vacancy rate experiences only small deviations due to adaptation periods between completion and leasing of office space.

As to the spikes, the HLM and especially the EGHS results illustrate the effectiveness of the contemporaneous changes in employment in Bergen. It must be an extra reason to their amplitude, which might come from external institutions but the model does not account for that.

The interpretation of the EGHS model helps understanding the development of the two cities, in accordance with the two other models. Moreover in Oslo, the set of causes leading to the apparition of cycles in the market has been highlighted; one extra condition to that is of course the building period necessary for the constructors to propose new office supply to the market.

3. Absorption Equation

(a) Hendershott, Lizieri and Matysiak

The model stated the Absorption equation as the following Error Correction Mechanism:

$$\left.\begin{aligned} OS_t^* &= \gamma_0 + \gamma_1 E_t - \gamma_2 R_t + \varepsilon_t \\ \frac{AB_t}{OS_{t-1}} &= \mu_1 \frac{AB_{t-1}}{OS_{t-2}} - \mu_2 \frac{\Delta R_{t-1}}{R_{t-2}} - \mu_3 \varepsilon_{t-1} \end{aligned}\right\} \qquad \begin{aligned} &\text{(Long Run)} \quad (7\text{'a}) \\ &\text{(Short Run)} \quad (7\text{'b}) \end{aligned}$$

The estimation has been however transformed with use of logarithms for the Long Run part and log differences for the Short Run. As done for the Construction, replacing Absorption by the difference in occupied space from Equation (2), one obtains:

$$\left.\begin{aligned} ln(OS_t^*) &= \gamma_0 + \gamma_1 ln(E_t) - \gamma_2 ln(R_t) + \varepsilon_t \\ \Delta ln(OS_t) &= \mu_1 \Delta ln(OS_{t-1}) - \mu_2 \Delta ln(R_t) - \mu_3 \varepsilon_{t-1} + u_t \end{aligned}\right\} \qquad \begin{aligned} &\text{(Long Run)} \quad (7\text{''a}) \\ &\text{(Short Run)} \quad (7\text{''b}) \end{aligned}$$

Both Figures 32 and 33 shows the Absorption against the rent variation lagged one year for Oslo and Bergen respectively

Figure 32: Absorption against Rent variation - Oslo

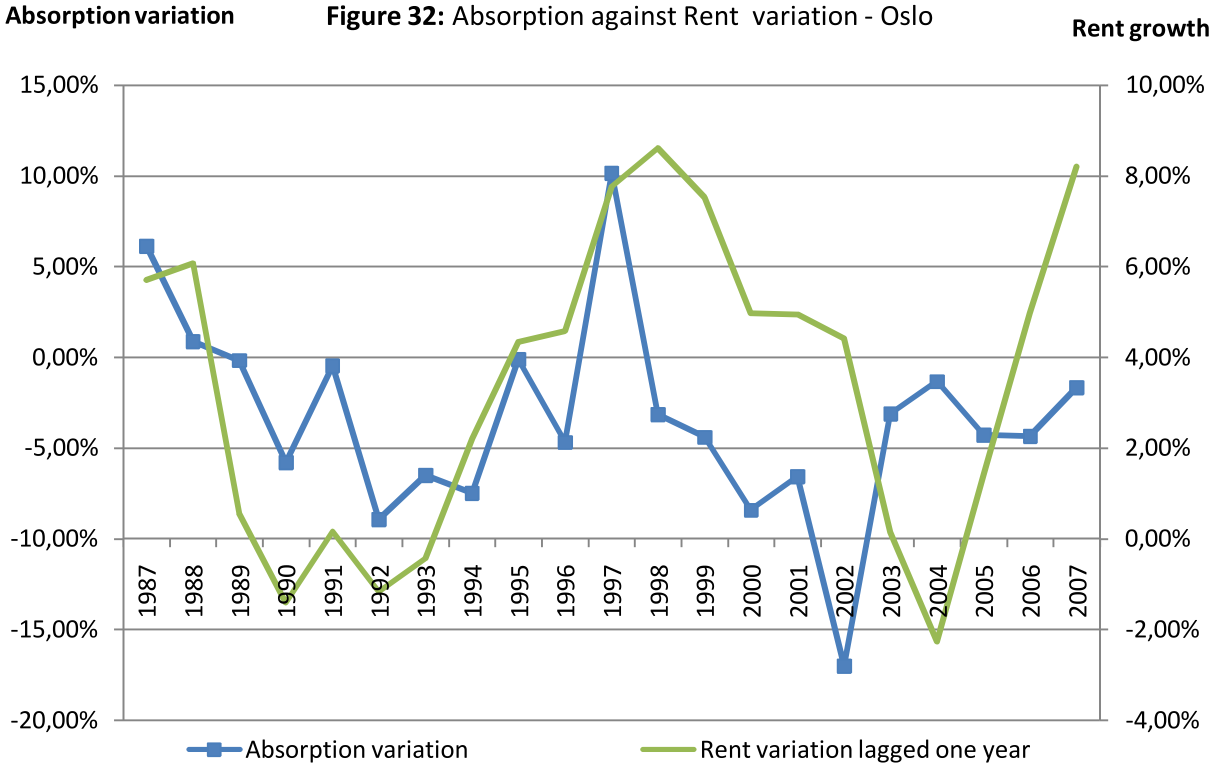
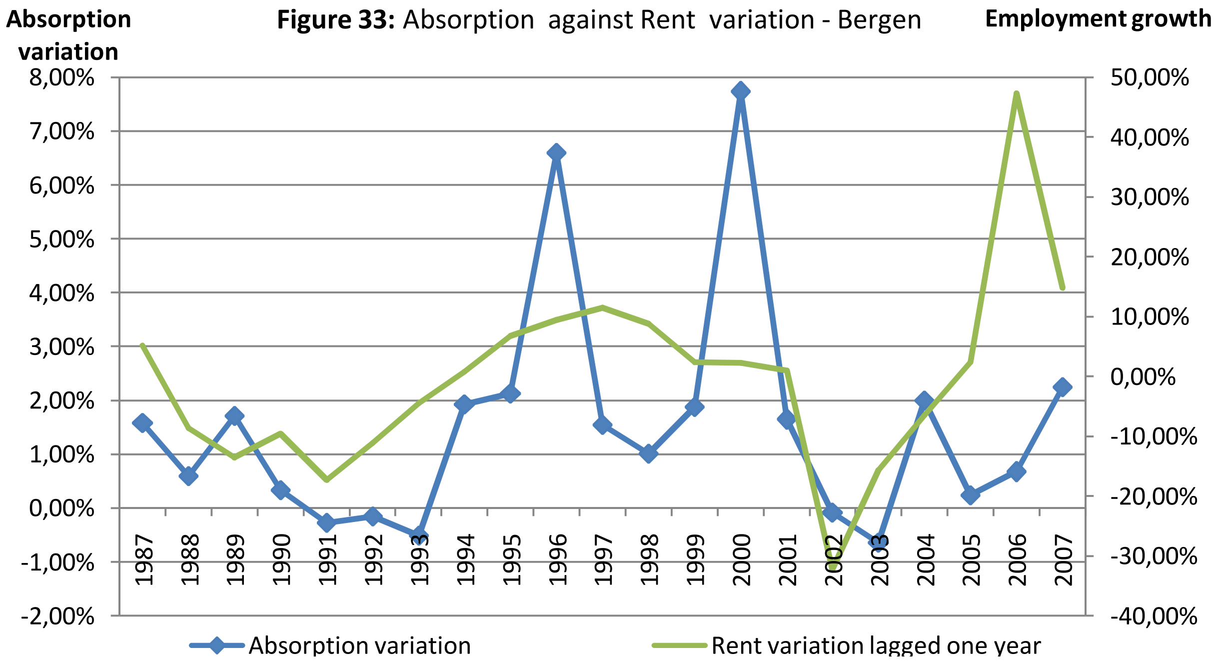


Figure 33: Absorption against Rent variation - Bergen



According to the authors, the rental growth should be inversely correlated to the Absorption, which is again not clear graphically speaking. Anyway, after a sharp examination, the curve seems to have a weak negative correlation at least for Oslo. In Bergen, this is even less clear.

The authors specified in the paper that the Long-Run equation they tested for London profited from a cointegrating relationship. As one can notice, all the variables utilized in this equation are I(1) and one special heed has been given to evidence the relation if it is. Unfortunately, the test did not evidence a relation linking all three variables. On the other hand, there are at least two cointegration relationships which link series two by two. Again, all related results can be found in the appendixes.

To test a variant of the previous equation, the version tested simultaneously is also measured. Here is its formulation:

$$\Delta ln(OS_t) = \mu_1 \Delta ln(OS_{t-1}) - \mu_2 \Delta ln(R_t) \qquad \text{(Long Run)} \qquad (7")$$

$$-\mu_3[ln(OS_{t-1}) - \gamma_1 ln(E_{t-1}) + \gamma_2 ln(R_{t-1})] + u_t$$

The figures presented the results of Equations (7") and (7"a+7"b) in the Table 11 on the next page. They have as usually been corrected for residual autocorrelation.

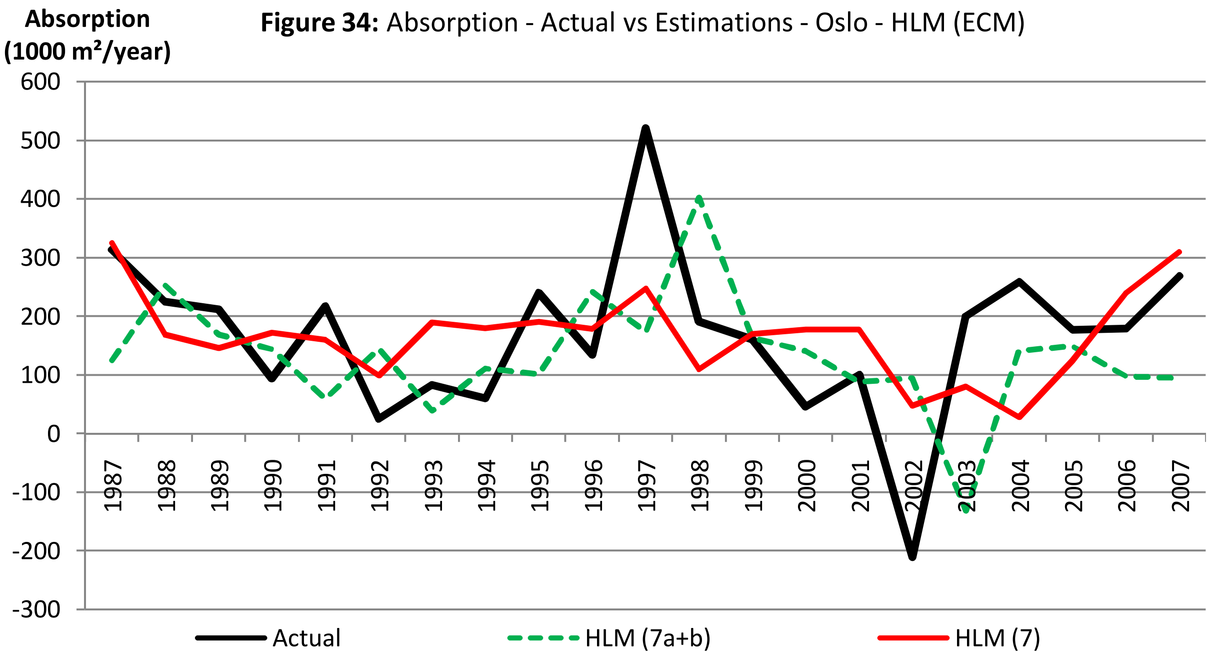
Table 11: Absorption - Error Correction Model (HLM) – Oslo & Bergen

ECM (HLM)		*Variables*	Oslo *Parameter*	Oslo *t-Value*	Bergen *Parameter*	Bergen *t-Value*
				SR-Robust		
Separately (7"a+7"b)	Short-Run	Occupied space $\Delta\ln(OS_{t-1})$	.7517	9.68	.4583	2.31
		Rent $\Delta\ln(R_t)$	.017	0.42	.0544	1.04
		Error Correction Term μ_3	-.1499	-1.36	-.1583	-1.68
		R^2	77.78%		34.36%	
		Durbin-Watson test	1.82		2.00	
		Number of observations	20		20	
		Coefficient of correlation ρ	-.5232		-	
				SR-Robust		*SR-Robust*
	Long-run	Constant	4.1894	4.49	4.4026	5.30
		Employment $\ln(E_t)$	.9998	7.45	.6099	3.96
		Rent $\ln(R_t)$	-.1324	-2.04	.0167	0.36
		R^2	99.88%		99.78%	
		Durbin-Watson test	1.74		1.78	
		Number of observations	21		21	
		Coefficient of correlation ρ	.8677		.9135	
				SR-Robust		
Simultaneously (7")	Long-run	Occupied space $\Delta\ln(OS_{t-1})$	-.3853	-1.80	-.0112	-0.06
		Rent $\Delta\ln(R_t)$	.149	2.01	.0671	1.30
		Occupied space $\ln(OS_{t-1})$	.0657	0.82	-.3004	-2.96
		Employment $\ln(E_{t-1})$	-.1596	-1.25	.385	2.91
		Rent $\ln(R_{t-1})$	.0481	2.97	.0593	2.55
		R^2	69.09%		65.74%	
		Durbin-Watson test	1.96		2.03	
		Number of observations	21		21	
		Coefficient of correlation ρ	.1426		-	

First, considering the long-run part (7"a) of the separated configuration, the fitting is again almost perfect. This time, the correction for autocorrelation managed to clean it almost completely. One notes that every variable being significant is also correctly signed. The rent turns out being non-significant in the separate configuration while it is when testing one single equation. As one can see, the single equation seems to offer the best alternative among the models, since in both cities, only the lagged absorption is significant. The fitting power drops drastically to 34% in Bergen for more than the double in Oslo. But, this power seems to come only from the lagged absorption variation.

The assumption of Davidson and MacKinnon about testing preferably one single equation with a limited number of observations appears again being verified. The R-squared statistic is quite stable around 65% without autocorrelation in this configuration and one retrieves some features observed in the precedent part about Completion, namely the opposition of sign and significance when looking at the employment. Bergen considered employment to trigger development, whereas this was non-significant in Oslo. This is still the case in the single configuration. Moreover, the rent appears to be wrongly signed in Bergen in (7). This is puzzling but it shows that tenants rented office space even if the rent increased. The figures 34 and 35 plot both configurations against the actual absorption in Oslo and Bergen respectively.

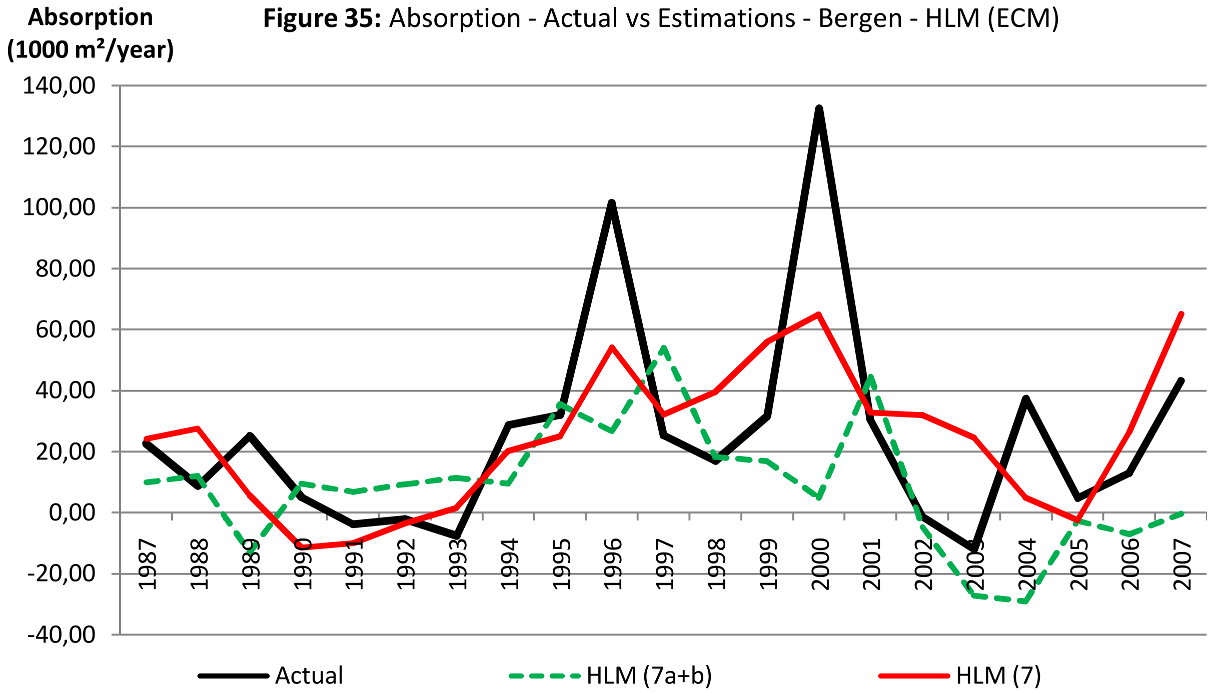
Figure 34: Absorption - Actual vs Estimations - Oslo - HLM (ECM)



The plotting of the ECM (7a+b) in Oslo confirms that it relies excessively on the lagged absorption variation as the Table 11 suggested. It consists only in a smooth and lagged replication of the actual curve, which is not satisfactory. The non-significance of the rent variation and the error correction term μ_3 did not enable the model to work as it should do.

On the other hand, the model treated as one unique equation gives some more reasonable estimation even if it is far from perfect. The curve follows correctly the general path of the actual one, and initiated substantial movements corresponding to the large spikes. Anyway, the actual curve is very volatile and is difficult to fit very great. The result from the single equation is thus quite satisfactory and thus the rent has a substantial power on the office rental.

Figure 35: Absorption - Actual vs Estimations - Bergen - HLM (ECM)



The curves in Bergen appears much more accurate, first because of the actual absorption experiences less deviations than in Oslo. The curve for (7a+b) suffers as well from the same problem than in Oslo, i.e. it is more or less a lagged copy of the absorption, and then does not appear usable.

On the contrary, the estimation done simultaneously is highly satisfactory, and manages capturing quite precisely both spikes and the general allure of the absorption. Looking at the Table 10, it appears that only the lagged variables composing the long-run part of the equation are indeed significant. The short-run part seems quite inefficient, or in all cases limited when looking at the plotting above. It implies that considering absorption, the contemporaneous changes did not matter. Comparing with Oslo, only the rent was significant and the estimated curve did not capture the deviations as well as in Bergen. This is surely related to the increasing demand during one decade in Bergen, and as soon as the supply entered the market, it was instantly rented out even in this period of high rent level.

In Oslo, the large drop in Completion in 2000 was effectively accompanied by a large drop in Absorption, but only 2 years later certainly due to the financial crash known as "internet bobble burst". This gives witness to a large oversupply and the delay it took the market to absorb it. Meanwhile, the rent level had to drop from 2000 by almost 10% in 2 years and almost 30% in 4 years to regain in absorption from 2004 on.

(b) Wheaton, Torto and Evans

The authors designed what was the first equation of their model as following:

$$AB_t = \tau_1[\alpha_0 + E_t(\alpha_1 - \alpha_2 R_{t-1})] - \tau_1 OS_{t-1} \quad \text{(IV)}$$

As presented earlier in part B, the formula links absorption with employment, rent and lagged occupied space, which are the variables used in HLM too. They have adapted their formula differently and designed it as a Partial Adjustment, like for the Construction equation. It utilizes variables with different unities, and therefore different dimension. This equation involves several variables being I(1), and thus requires a multi-cointegration relationship to produce non-spurious, valid results. Unfortunately, testing did not evidence the existence of such a relation in this case. To solve this situation, one begins with reformulating the equation as mentioned in the part B. Equation (IV) can be rewritten as:

$$OS_t = \tau_1[\alpha_0 + E_t(\alpha_1 - \alpha_2 R_{t-1})] + (1 - \tau_1)OS_{t-1} + \varepsilon_t \quad \text{(IV')}$$

It permits avoiding the problem linked to negative values included in Absorption. Using the logarithm, one gets:

$$ln(OS_t) = \tau_1[\alpha_0 + \alpha_1 ln(E_t) - \alpha_2 ln(E_t R_{t-1})] + (1 - \tau_1)ln(OS_{t-1}) + \varepsilon_t$$

Utilizing the calculation rule $ln(XY) = ln(X) + ln(Y)$, one obtains:

$$ln(OS_t) = \tau_1[\alpha_0 + (\alpha_1 - \alpha_2)ln(E_t) - \alpha_2 ln(R_{t-1})] + (1 - \tau_1)ln(OS_{t-1}) + \varepsilon_t \quad \text{(IV'L)}$$

Then, when taking the log difference one has:

$$\Delta ln(OS_t) = \tau_1[\alpha_0 + (\alpha_1 - \alpha_2)\Delta ln(E_t) - \alpha_2 \Delta ln(R_{t-1})] - (1 - \tau_1)\Delta ln(OS_{t-1}) + \varepsilon_t \quad \text{(IV"DL)}$$

The equation in levels (IV'L) actually suffers from the same problem than the one in HLM, namely it reproduces only a lagged copy of the absorption and thus cannot be utilized or interpreted in a correct manner.

The results of the log-difference configuration are presented in Table 12 on the next page.

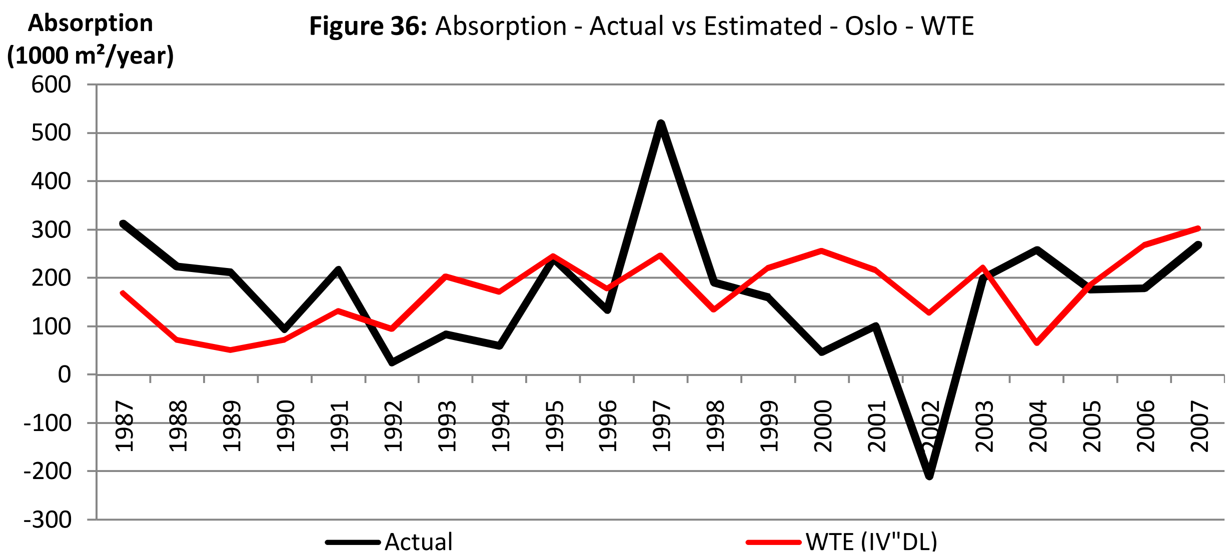
Table 12 – Log difference – WTE (IV"DL)

City	*Variables*	*Difference in logarithms (IV"DL)* *Parameter*	*t-Value*
SC-robust			
Oslo	Constant	.0327	2.80
	Employment growth $\Delta ln(E_t)$	.317	0.87
	Rent variation $\Delta ln(R_{t-1})$	.0445	0.54
	Demand variation $\Delta ln(OS_{t-1})$	-.4255	-1.97
	R^2	25.06%	
	Durbin-Watson test	1.84	
	Number of observations	21	
	Correlation of correlation ρ	.4106	
SC-robust			
Bergen	Constant	.0252	2.18
	Employment growth $\Delta ln(E_t)$	-.3958	-1.08
	Rent variation $\Delta ln(R_{t-1})$	.1099	1.53
	Demand variation $\Delta ln(OS_{t-1})$	-.048	-0.32
	R^2	21.04%	
	Durbin-Watson test	1.89	
	Number of observations	21	
	Correlation of correlation ρ	.2185	

According to the results, the formulation from WTE does not seem to fit very well in any city. The explanatory power is limited to 25%. Only the constant is significant, and the other variables have different significance, sign and magnitude.

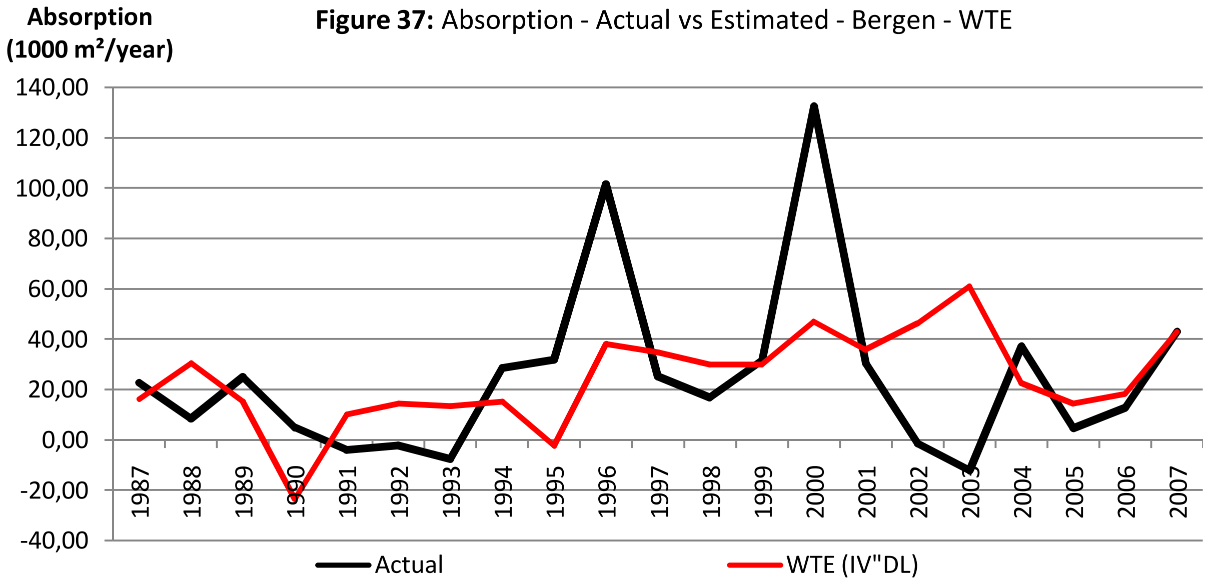
It tells also that the speed of adjustment of the variation of the market to one shock in demand is supposedly 142% and 100% in Oslo and Bergen. This is doubtless a fallacious figure. It is then quite difficult to interpret the outcomes reported in the table above.

Figure 36: Absorption - Actual vs Estimated - Oslo - WTE



The Figures 36 and 37 presents the estimated absorption in Oslo and Bergen. As it can be guessed, they should have difficulty to reproduce most features quite accurately. In Oslo, the curve behaves in the average of the actual absorption, but does not impact the amplitude of the spikes at all.

Figure 37: Absorption - Actual vs Estimated - Bergen - WTE



In Bergen, the result appears again better since there are fewer variations and like in Oslo the curve marks the spikes, but on several points of the period, estimated and actual curves actually go in the opposite way.

More generally, it turns out that the WTE equation about Absorption fit better than HLM in neither of the cities, even if it uses the same variables. It has been tried again using other time lags, but the explanatory power did not take off like it did in HLM.

It evidences the validity of the Error Correction Mechanism over the Partial Adjustment in the case of office demand and that its variation reacts thus to the past levels rather than the past variations. The equation (IV'L) of WTE presented just above did respond very well in terms of significance, exactly as it happens with the single equation of the ECM. But comparing directly levels led to the kind of lagged duplication experienced with the ECM tested in two separated parts, when the explanatory went mainly from the past absorption.

(c) Long-Run Vacancy Change

So far, one has already had the opportunity to test one ECM, thanks to HLM. It turns out that the outcomes were quite satisfactory and captured efficiently the largest variations along the sample period.

Another possibility to capture the absorption is to estimate the vacancy rate, since having the supply it is easy to obtain the demand (or occupied space), and further the absorption.

Barras in his article published in 2005 called "A Building Cycle Model for an Imperfect World" pictured a complete dynamic model of office real estate, based on data about the City of London. This model utilizes variables in common with all equations presented above, and of course arranges his model around supply, demand, completion equations like the others did.

As to the demand, he argued that the natural vacancy rate as calculated by HLM for example is a structural vacancy rate, which is artificially created by "landlords as a buffer to accommodate turnover, allow for demand uncertainty and provide option to delays lettings in expectation of better market future conditions". Below is presented how it made his formulation.

First, he defined the vacancy, not as the variation between supply and occupied space but only as the difference between them:

$$\boldsymbol{V_t} = S_t - OS_t \qquad (10)^{22}$$

Then he created a new variables called take-up, decomposed into two components: the net absorption AB_t which represents the part of new tenants entering the rental market as a consequence of economic growth, utilized before and being $AB_t = OS_t - OS_{t-1}$, and in addition the turnover T_t which are the part of existing tenants moving from one office to

[22] The notation **V** is in bold to differentiate it with V, the value of the building evocated earlier.

another, certainly because the new ones match better their current requirements, which returns to $T_t = \tau OS_{t-1}$ with τ being this proportion called the turnover rate and considered as constant.

The take-up becomes then:

$$U_t = AB_t + T_t \qquad (11)$$

Now from (10), taking the difference between vacancies of two consecutive years, one gets:

$$\boldsymbol{V_t} - \boldsymbol{V_{t-1}} = (S_t - S_{t-1}) - (OS_t - OS_{t-1}) \qquad (12)$$

Transforming the first term between parentheses with use of the known supply identity (1) page 11, one obtains:

$$S_t = Comp_t + (1-\delta)S_{t-1} \Rightarrow S_t - S_{t-1} = Comp_t + (1-\delta)S_{t-1} - S_{t-1} = Comp_t - \delta S_{t-1}$$

Transforming the second term utilizing the equation (11) gives:

$$AB_t = U_t - T_t \Rightarrow OS_t - OS_{t-1} = AB_t = U_t - T_t = U_t - \tau OS_{t-1}$$

Replacing into (12):

$$\boldsymbol{V_t} - \boldsymbol{V_{t-1}} = (Comp_t - \delta S_{t-1}) - (U_t - \tau OS_{t-1})$$

Then replacing S_{t-1} in function of the vacancy rate v_t and OS_{t-1}, as $S_t = OS_t(1 + v_{t-1})$ and moreover dividing by the lagged occupied space, one obtains finally:

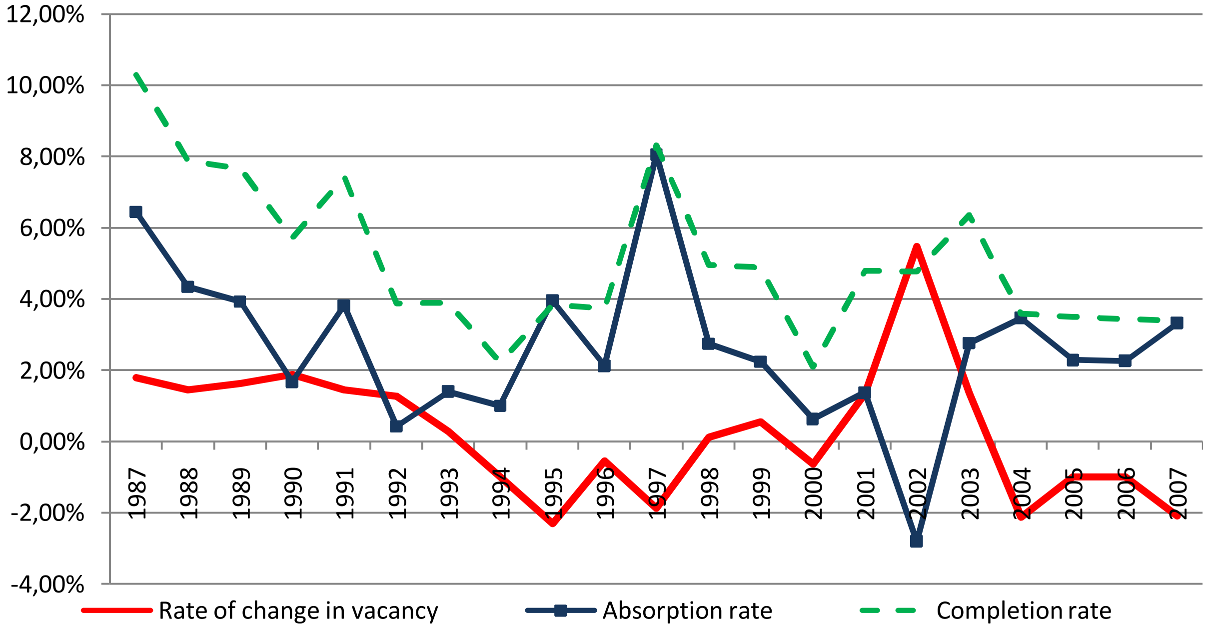
$$\Delta \boldsymbol{v}_t = c_t - u_t - \delta v_{t-1} + (\tau - \delta) \qquad (13)$$

The several components of the formula are:

- The rate of change in vacancy: $\Delta \boldsymbol{v}_t = {\boldsymbol{V_t} - \boldsymbol{V_{t-1}}}/{OS_{t-1}}$
- The rate of completion: $c_t = {Comp_t}/{OS_{t-1}}$
- The rate of take-up: $u_t = {U_t}/{OS_{t-1}}$

Unfortunately, contrary to London, data on take-up is not available directly in Norway and should be appraised in a different manner. To do so, one starts with plotting the rate of change in vacancy against both rate of completion and absorption, since the take-up is unknown.

Figure 38: Rate of change in vacancy vs completion and absorption rates -Oslo

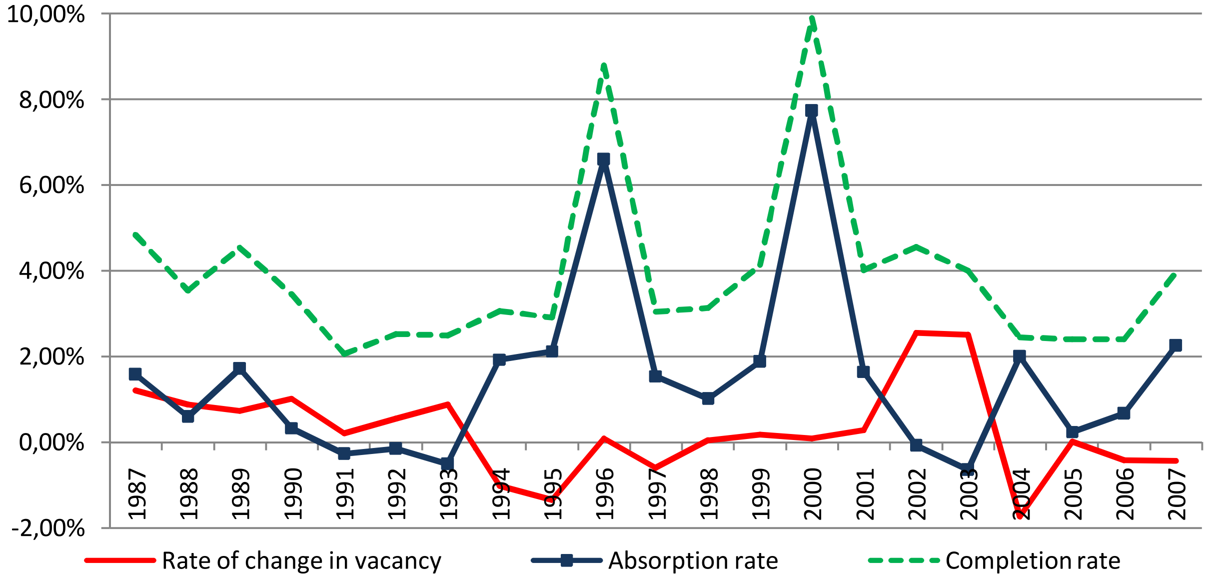


On both graphs, one sees that absorption behaves negatively to the change in vacancy, and the completion is positively correlated. The first is fairly logical as tenants entering the market reduce the vacancy, and the curves seem to get apart with the same amplitude.

Completion on the other hand is positively correlated with the vacancy, since increasing supply automatically increases vacancy. All three curves are closely related, *Completion* is the upper bound of the movements while the two others curves behaves around 0%.

One notices that absorption is not always dependent of completion, which evidences the overestimation of property investors once in a while. The example highlighted in the *Completion* part can be retrieved from 2000 to 2003 in Oslo, where both completion and absorption go apart and the negative spike of completion does not match the one of absorption. On the other hand in Bergen, both spikes are synchronal, which proves the high demand of office space during this period.

Figure 39: Rate of change in vacancy vs completion and absorption rates -Bergen



The graph above shows values in variation, which is maybe not relevant unless the same conclusion emerges from the comparison of values in thousands of squared metres. The table below compared the level values in Oslo and Bergen between 1997 and 2004:

	Years	1996	**1997**	1998	1999	2000	2001	**2002**	2003	2004
Oslo	Completion	236	**538**	346	350	154	354	**358**	462	268
	Absorption	134	**521**	191	161	46	101	**-210**	201	259
		102	**17**	155	189	108	253	**568**	261	9
Bergen	Completion	**135**	50	52	70	**170**	74	85	75	46
	Absorption	**101**	25	17	32	**132**	30	-1	-12	37
	Difference	**34**	25	35	38	**38**	44	86	87	9

According to this table and especially the four years of large spikes which are been boldfaced, one retrieves the patterns detected earlier. Since 1997 which a very good year in Oslo, the supply persisted being high whereas the absorption decreased over the years. Then, after have reduced drastically the supply in 2000 followed assorted with a light regain in absorption in 2001, investors believed that absorption will raise in 2002 but unfortunately the market demand plunged all over the world certainly as the consequence of the market crash about the internet bubble burst the same year.

Even in years where the absorption is at its "normal" level, the difference between completion and absorption is sometimes large and moreover does not decrease over years. Knowing that the rent is basically the only source of revenues for an office property, such a situation should not be sustainable for investors and logically banks and financial institutions should stop lending at loss. Thus, this difference is certainly the consequence of what Barras defined in his article: on the first hand, a structural vacancy rate that property owners retain on purpose, and on the other hand, a constant turnover in the market of existing tenants to a new location.

Thus U_t, the take-up level, is the difference between the Supply constrained by the structural vacancy rate and the demand, plus the absorption as normally calculated earlier:

$$U_t = S_t(1 - v^*) - OS_t + AB_t$$

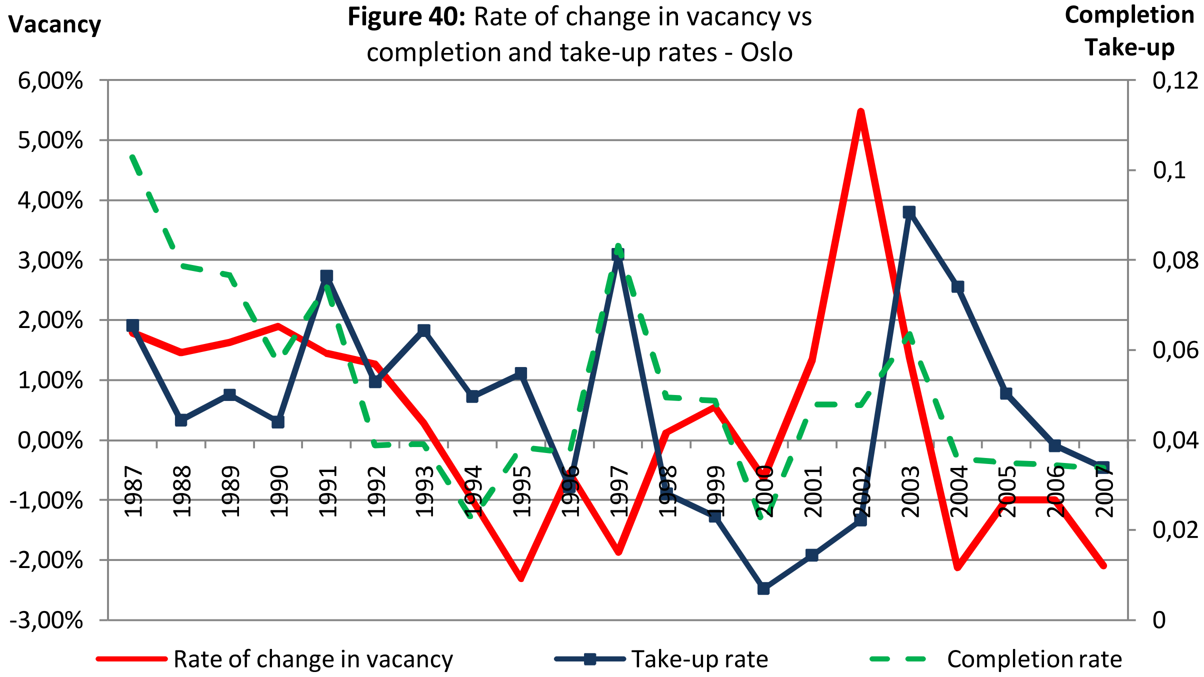
All variables are known, except the natural vacancy rate here. One way to approximate it is to employ the one found in the rental equation from HLM, namely 5.53% in Oslo and 4.37% in Bergen. One way to verify whether these values are some correct estimates is to compared the value of the turnover rate obtained by Barras under which is a bit over 6%, knowing that Barras identifies the natural vacancy as the difference between the actual occupied space OS_t and the desired occupied space S_t^*. In this way, the natural vacancy accounts for both the buffer retained by property investors and the turnover.

Considering that the natural vacancy rate is around 5%, the turnover rate should be rather small. After their application, one obtains respectively 2% for Oslo and 1.28% for Bergen[23].

The estimation of a take-up series can be done since the figures from HLM seem to give a good appraisal. Figures 40 and 41 present the same plotting than previously, but instead with take-up replacing absorption.

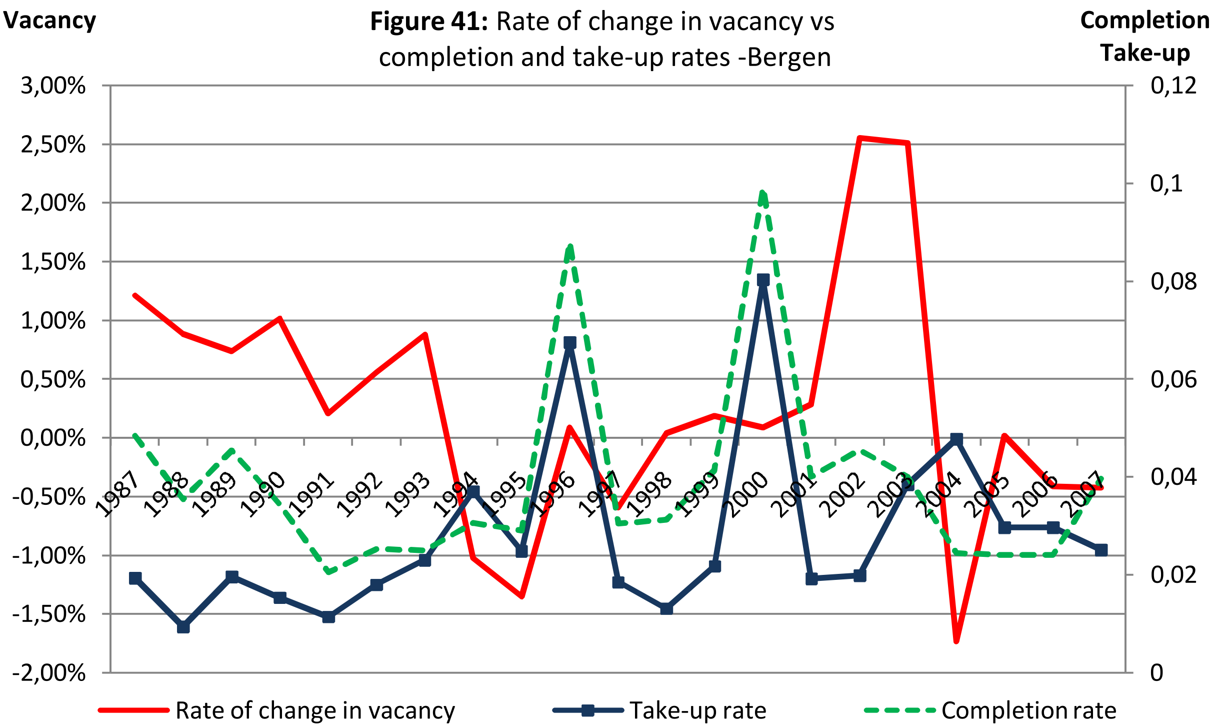
[23] The calculation of the take-up is attached in appendix.

Figure 40: Rate of change in vacancy vs completion and take-up rates - Oslo



The negative correlation seems also obvious, whereas one notices that during several years, the turnover made the take-up curve go beyond completion. This makes sense since there is still some available supply on the market. This happens years when completion is low, and then owners rented out a part of the buffer they retain to accommodate the demand. since it happened only when rents were low here.

Figure 41: Rate of change in vacancy vs completion and take-up rates -Bergen



Then, the equation (13) to be estimated is:

$$\Delta \boldsymbol{v}_t = (\tau - \delta) + \beta_1 c_t - \beta_2 u_t - \delta v_{t-1} + \varepsilon_t \quad (13')$$

The results of the equation are presented below in table 13:

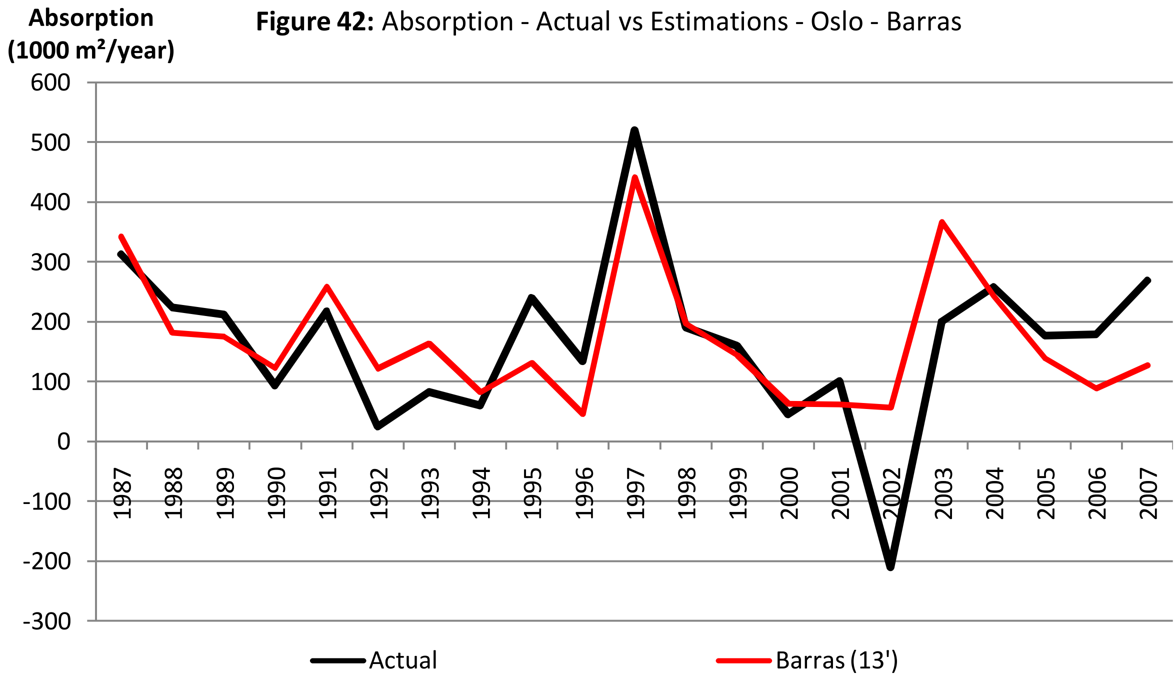
Table 13 – Rate of changes in vacancy - Barras (13')

City	*Variables*	*Barras (13')*	
		Parameter	*t-Value*
SC-robust			
Oslo	Constant $(\tau - \delta)$	-.0915	-2.54
	Completion rate c_t	1.7537	3.07
	Absorption rate u_t	-1.7054	-2.96
	Lagged vacancy rate v_{t-1}	1.235	2.30
	R^2	45.53%	
	Durbin-Watson test	1.71	
	Number of observations	21	
	Correlation of correlation ρ	.4687	
SC-robust			
Bergen	Constant $(\tau - \delta)$	-.0313	-1.64
	Completion rate c_t	1.1111	2.96
	Absorption rate u_t	-1.1267	-2.96
	Lagged vacancy rate v_{t-1}	.3586	1.21
	R^2	53.97%	
	Durbin-Watson test	1.76	
	Number of observations	21	
	Correlation of correlation ρ	.3209	

The results are really satisfactory even if the lagged vacancy rate is not correctly signed in neither of the cities. As well as the constant which gives -7.15% and -1.13% when estimating turnover rates respectively in Oslo and Bergen. This does not make sense since the part of the occupied space cannot be negative, but this is due to the positive sign of the vacancy rate.

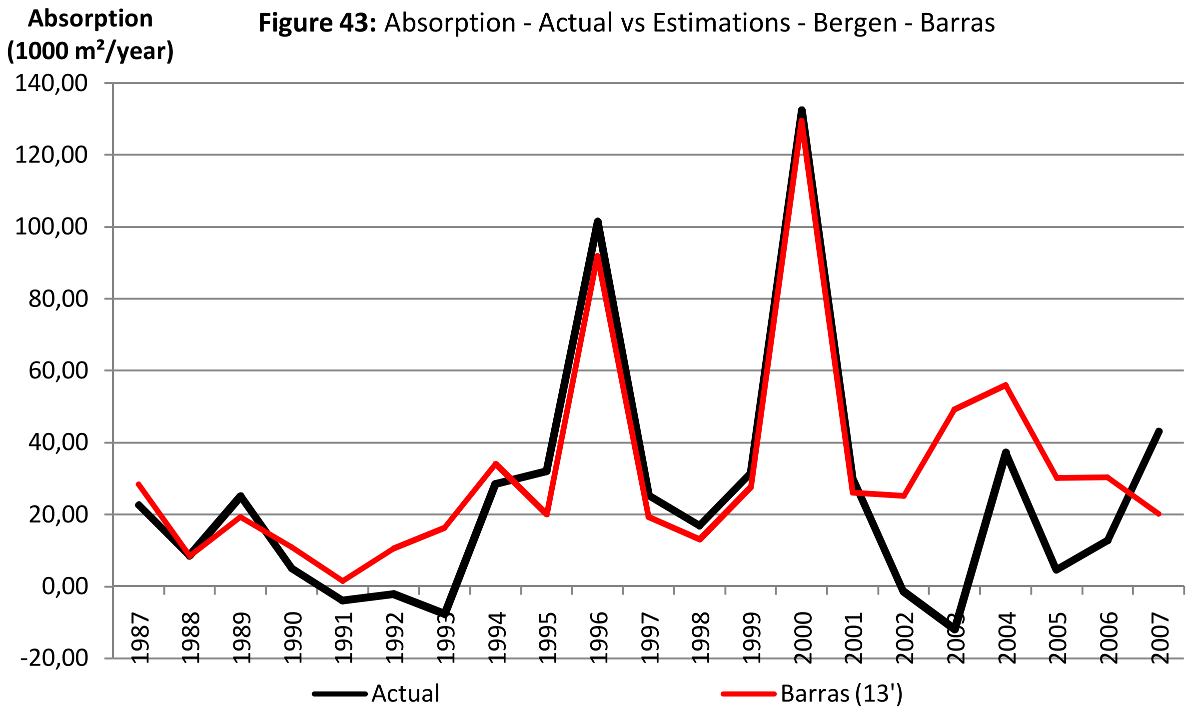
Almost every parameter is significant, and the explanatory power is good. Barras reported an adjusted R-squared of 58%, which reverts probably to a R-squared slightly greater than 60%. He had also a larger sample of 35 years. There are also some traces of autocorrelation, but the model still fits very great. The results are plotted on the figures 42 and 43 on the next page.

Figure 42: Absorption - Actual vs Estimations - Oslo - Barras



The model of Barras gives outstanding accuracy, especially with the largest spikes. The ability to capture them stems entirely from the joint utilization of the take-up and the completion, since both are measures of the changes in supply and demand. This estimation is very impressive, but requires the monitoring of supply, take-up and vacancies over time. Moreover, it does not utilize exogenous variables such as employment.

Figure 43: Absorption - Actual vs Estimations - Bergen - Barras



On the other hand, one very good thing about the Barras model is that it completes the "picture" when including the turnover, since it drives the level of occupied as well. It made a large improvement by capturing entirely the spikes, and it also indicates that it should be simpler to estimate the vacancy than the absorption. But, one question remains about the non-utilization of the rent as a driver of the space occupation, since the vacancy depends directly upon it. The model of Barras is indeed composed of 6 equations and all other utilizes the rent as variable. According to him, the rent variation is also a directly related to the lagged vacancy rate and thus he seems to assume that the vacancy rate characterizes the rent.

The testing of these three models established that the historical rent and the employment are essential factors driving the office demand, like it could obviously be guessed. But the vacancy can easily be estimated through completion and the variable "take –up" encompassing the absorption and the turnover, but this implies certainly issues about endogeneity of the variables. On the other hand, one considers often that the apparition of cycles in the office real estate market is an endogenously created problem, even if it is solidly related to the rent.

E. Comparison of estimates

It turned out that some of the estimations from the models above managed to fit very well in the two cities, and it reverts to identify which one may be the most appropriate in each case. For each equation, rental, completion and absorption, a table reports both the R-squared value and the Sum of Squared errors. Its square root divided by the number of observations is also reported for a clearer interpretation.

Sum of squared errors $SSE = \sum_{t=1987}^{T=2007}(y_t - \hat{y}_t)^2$

R-squared $R^2 = 1 - \frac{SSE}{SST} = 1 - {SSE}/{\sum_{t=1987}^{T=2007}(y_t - \bar{y}_t)^2}$

a) Rental equation (page 36 to 54)

City	*Test*	*Models*						
		HLM (5"b)	*WTE (V'L0)*	*WTE (V"L1)*	*WTE (V"DL0)*	*WTE (V"DL1)*	*HMT (8)*	*HMT (8a+b)*
Oslo	*SSE*	*75692.6*	***68486.7***	*139399.1*	*136559.8*	*101843.9*	*223945.8*	*477825.6*
	R^2	*73.59%*	*99.09%*	***99.75%***	*83.34%*	*77.98%*	*60.32%*	*82.98%*
	$\sqrt{SSE}/T$	*13.10*	***12.46***	*17.78*	*17.60*	*15.20*	*22.53*	*32.92*
Bergen	*SSE*	*117203.1*	*170096.2*	*140756.3*	*176368.2*	*140359.0*	*100109.38*	*603992.5*
	R^2	*53.10%*	*99.17%*	***99.31%***	*40.89%*	*52.88%*	*72.40%*	*38.84%*
	$\sqrt{SSE}/T$	*16.30*	*19.64*	*17.86*	*20.00*	*17.84*	***15.07***	*37.01*

In Oslo, the minimal error is 12.46 NOK/m²/year for the WTE model in levels with no vacancy lag. This error is really small, as well as the other configurations of the model. In effect the maximum error of the 2 first models is then 22.53 NOK/m²/year when disregarding the last column. In Bergen, the HMT model is the most accurate with an error of 15.07 NOK/m²/year, and all other models give very close results with a maximum of 20 NOK/m²/year if one disregards again the HMT model estimated separately.

Comparing the coefficient of determination, or R-squared, makes no hesitation about the most fitting equation, as the statistic is almost 100%. The WTE model in level is almost perfectly accurate in all cases but this formulation in levels is hampered by persistent serial correlation, and then the version in log difference would rather be preferred.

As to the ECM single equation, it offers of very good results, even if it suffers also from traces of autocorrelation. On the other hand, the HLM model produces the second best results free of correlation when considering only vacancy and rent gap to appraise the rent variations.

b) Completion equation (page 55 to 76)

City	*Test*	*Models*				
		HLM (6')	*HLM (6'E)*	*WTE (VI')*	*EGHS (9b')*	*EGHS (9b")*
Oslo	*SSE*	***164556.76***	-	*177854.01*	*176256.93*	*168975.10*
	R^2	*33.25%*	-	*30.25%*	*66.00%*	***66.16%***
	$\sqrt{SSE}/T$	***19.32***	-	*20.08*	*19.99*	*19.57*
Bergen	*SSE*	*19042.93*	*18116.34*	*17402.11*	*16035.62*	***11964.92***
	R^2	*28.23%*	*53.10%*	*42.13%*	*44.39%*	***67.94%***
	$\sqrt{SSE}/T$	*6.57*	*6.41*	*6.28*	*6.03*	***5.21***

In Oslo and Bergen, the results are very close the ones to the others and one notices that they are much more accurate in Bergen. This is the consequence of the inefficiency of the employment and the vacancy in Oslo which enabled the equation to capture only the average of the completion curve. The best result is a deviation of 19,320m²/year for the HLM model. As to the explanatory power, EGHS produces R-squared values double than the two original models, since it uses more explanatory variables. On the other hand, it unfortunately did not improve the model accuracy.

In Bergen, the results are nearly 3 times more accurate than in Oslo, but completions in Oslo are also 5 times larger than in Bergen. It explains certainly the difference, but it is clear here that the validity of employment and vacancy improved the results in Bergen. The smallest deviation and the best fitting model is the ECM from EGHS which uses the short-run part of the model and provides an averaged error of 5,210m²/year and virtually no autocorrelation.

However, the model lies on an equilibrium rent to which the estimation appears biased by serial autocorrelation, and one has to cautious with the results even if it does not appear in the final estimation. It should be possible to refine the equilibrium rent equation to discard the serial correlation.

c) Absorption equation (page 77 to 98)

City	*Test*	*Models*			
		HLM (7"a+b)	*HLM (7")*	*WTE (IV"DL)*	*Barras (13')*
Oslo	*SSE*	*545370.25*	*299572.33*	*411746.45*	***179323.79***
	R^2	*77.78%*	***69.09%***	*25.06%*	*45.53%*
	$\sqrt{SSE}/T$	*35.17*	*26.06*	*30.56*	***20.17***
Bergen	*SSE*	*32784.57*	*13451.56*	*23271.40*	***7525.00***
	R^2	*34.36%*	***65.74%***	*21.04%*	*53.97%*
	$\sqrt{SSE}/T$	*8.62*	*5.52*	*7.26*	***4.13***

One first comment on the results here is that again the accuracy seems greater in Oslo, and again the difference comes partly from the scale between Oslo and Bergen but also because results fitted better in Bergen. The best fitting results are those provided by the single ECM from HLM, which gives also little deviations without autocorrelation.

The Barras approach as seen above gave astonishing results since it captured with accuracy the large deviations of the absorption series. Barras defined more precisely the components of the absorption with adding the turnover. However, data about it is surely unavailable in most of the cities, even if it can be approximated. In addition, it does not use the rent as a driver of the vacancy which is subject to a bit of scepticism. Finally, it displays a little autocorrelation in its equation while the HLM does not. The Barras approach is certainly of good quality, but the HLM one would be preferred for the reasons enunciated above.

F. Conclusions

Models

With some simple adaptations, the model from Hendershott, Lizieri and Matysiak provided among the best results in both Oslo and Bergen. It has certainly the most accurate results free of serial correlation among all models experimented in Oslo and Bergen. Its rental equation however relies on a measure of replacement cost which is constant and obviously out of the real picture, since the replacement cost depends upon the interest rate, the price of materials and the price of land among others.

The model proposed by Wheaton, Torto and Evans also gave satisfactory well, but its absorption equation work in neither of the cities. It has also suffered from moderate autocorrelation as it was originally presented. The transformation using the variations instead of direct level did not handle all serial correlation, but the model generally supplied results confirming those from HLM.

The extra formulations have on their side great outcomes:

- The application of an Error Correction Mechanism worked efficiently with the completion and the rent. However, their formulation involved evidencing a cointegrating relationship between variables, which is not guaranteed in the office real market at least between variables altogether. Moreover, it appears that the elaboration of a long-run equilibrium is very tricky, as autocorrelation is hard to discard in the formulation of HMT.
- The long-run vacancy change of Barras proposed an interesting alternative to capture absorption, but suffers from a formulation free of exogenous components.

Markets

The utilization of several models permitted a more detailed picture of the market behaviour in the two largest cities of Norway, each experiencing a different development.

Oslo's market seems to have reached a certain maturity since the rent and vacancy cycles have larger amplitude than in Bergen, which is certainly related to the available space of the main researched locations. The market available space has a constant growth as the completion figure is more or less identical every year, but the demand has not responded to the supply every year, which led to disturbances. Since the market is very slow to adjust because of the building delay, it requires an accurate prediction of the future demand by the constructors. Some exaggerated expectations like it happened after the construction boom in 1997 combined with the internet bobble burst in 2002 has put out the synchronic regulation of the market by the vacancy, which in turn affected negatively the rent. Because of the tightness due to the crash, the demand did not adjust to the supply and put property developers in distress. The market finally recovered in 2004, which means that it took 2 more years because of the unexpected crash.

In Bergen, the situation is very different. First, one discerns a period of roughly a decade where construction of office real estate was on an upward trend and some measures verified that construction was certainly compelled by more than the unique employment. In effect, the rent was high and the nominal interest rate was increasing. One can also see that while the absorption was declining in Oslo, it was on the contrary increasing steadily until the crash in 2002. Consequently, one notices also that the" damages" to the demand were much less severe than in Oslo. The market in Bergen is then still in development, the supply caters the demand and the rent adapts smoothly to the vacancy.

Improvements

The data are certainly possible to improve in a certain extent:

- First, it must be some small-sample bias linked to the limited period of time. For instance, it would be great to monitor series semi-annually instead of annually. Doing so, more updated information enables developers to sharpen their expectations.
- Secondly, the measures themselves can be more precise. For example, instead of considering the rent to measure the demand, it should also integrate the cost incumbent to the tenant as well, like guarantees.
- Third, of course the accuracy of data employed should be as close as possible to the city or the land studied. For example, a portion of the employment series was approximated on the entire land, which even if it is close is not the most appropriate. More generally, it reverts to study in detail the close past behaviour of the market studied and current indicators, such as interest rate.

General summary

The comparison of the outcomes given by the different models had identified several variables having substantial effects within cities, but also within equations. They are responsible for the apparition of cycles in the Oslo market which functions as exposed in part A. However, ones can envisage modifying further existing linear models to find the best fitting formulation in every city studied. Most of the models considered were transformed in order to produce non-biased estimates, and then has proven that they can be trustworthy utilized further into forecast. Relying on estimated features such as those here constitutes an efficient decision help tool, which confirms or supports the presumptions of the developers. In such a fashion, the apparition of hurtful periods due to overestimation can be limited and very likely shortened.

Acknowledgements

I want to thank particularly Lars Flåøyen from Aberdeen Property Investors in Oslo for his contribution concerning the data about Oslo and Bergen and his comments. I also want to thank warmly my thesis supervisor Fred Schroyen for his assistance and his advices, anonymous personnel from SSB for their help, as well as Øyvind Anti Nilsen, Svein Olav Krakstad, Alief Rezza, Johannes M. Uhde and Heddy Anne Torp Lund and Erik Lund for their observations.

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Appendix

1. Calculation of the Equilibrium Rent under HLM model

The natural rent level, or equilibrium rent, is estimated using the formula:

$$R^* = (r + \delta + oper)RC \qquad (4)$$

This is a two-step procedure, where the first is to appraise the replacement cost RC. It reverts to choose a year where rent level is stable, as it is considered being in equilibrium. In both Oslo and Bergen, this year has been chosen being 1998 and 2001 respectively. The rent level of this year is then used as an approximation of the equilibrium in the reversed equation (4) to retrieve the replacement cost: $RC = \mathrm{R}^*_{1998}/(r_{1998} + \mathrm{r}_{\mathrm{pm}} + \delta + oper)$.

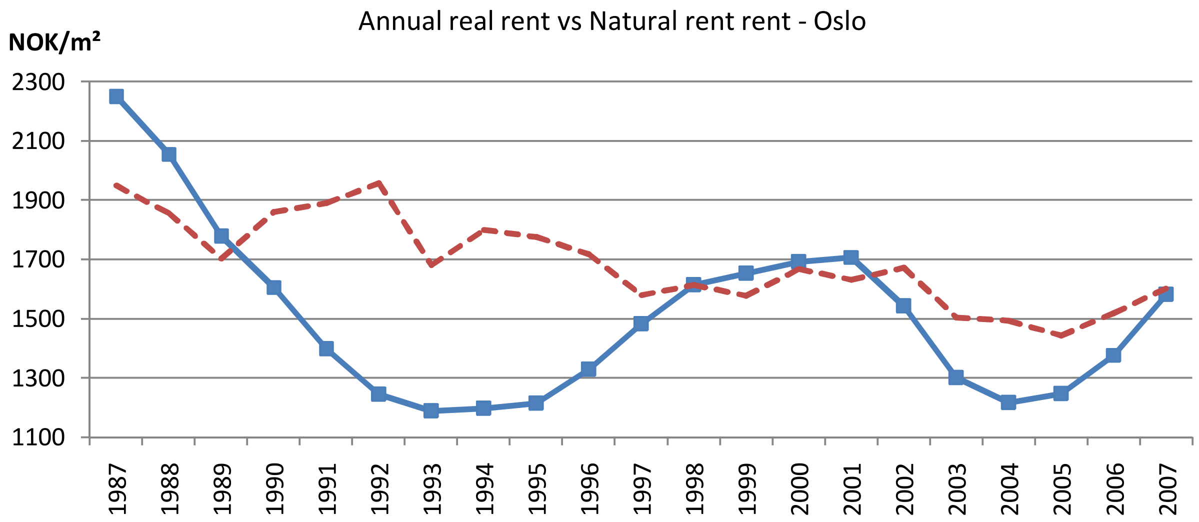
Risk premium, depreciation and operating ratio assumed constant, the replacement cost is:

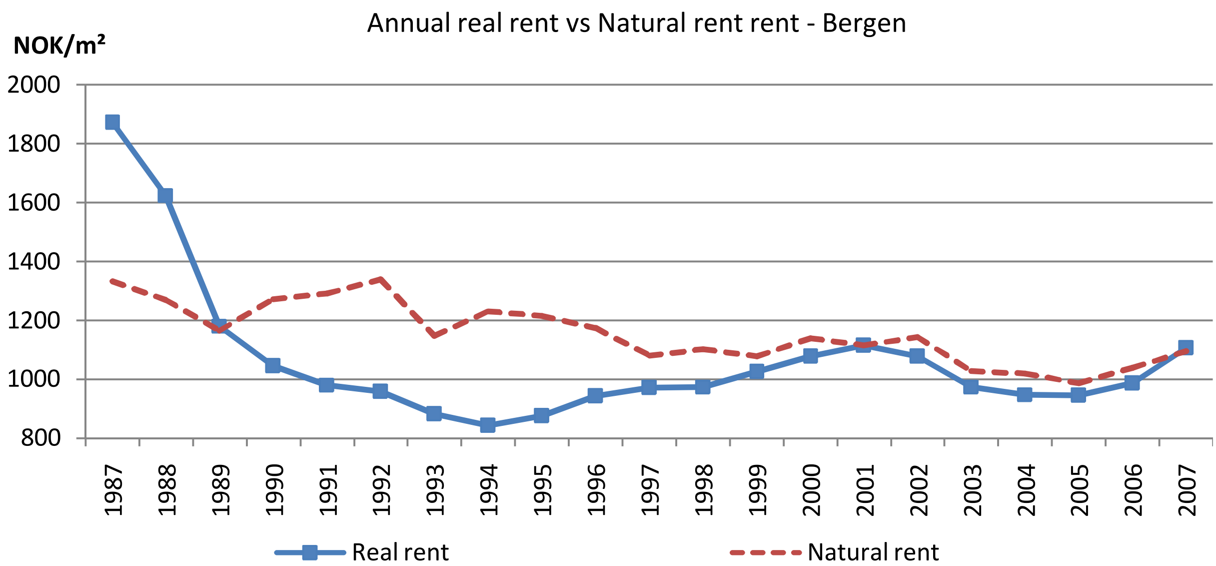
Oslo $$RC = \frac{1613\ NOK/m^2}{3.3265\%+2\%+2\%+7.5\%} \approx 10880\ NOK/m^2$$

Bergen $$RC = \frac{975\ NOK/m^2}{3.3265\%+2\%+2\%+7.5\%} \approx 6576\ NOK/m^2$$

Reversing again equation (4) and plugging RC allows obtaining time series of equilibrium rent R^*:

$Year$	r	r_{pm}	$oper$	δ	RC (Oslo)	R^* (Oslo)	RC (Bergen)	R^* (Bergen)
1987	0.064174	0.02000	0.075	0.02	10880.02	1949.42	6576.065	1178.26
1988	0.055654	0.02000	0.075	0.02	10880.02	1856.72	6576.065	1122.23
1989	0.041577	0.02000	0.075	0.02	10880.02	1703.56	6576.065	1029.66
1990	0.056056	0.02000	0.075	0.02	10880.02	1861.09	6576.065	1124.87
1991	0.058698	0.02000	0.075	0.02	10880.02	1889.84	6576.065	1142.25
1992	0.064871	0.02000	0.075	0.02	10880.02	1956.99	6576.065	1182.84
1993	0.039428	0.02000	0.075	0.02	10880.02	1680.18	6576.065	1015.53
1994	0.050394	0.02000	0.075	0.02	10880.02	1799.49	6576.065	1087.64
1995	0.04825	0.02000	0.075	0.02	10880.02	1776.16	6576.065	1073.54
1996	0.042782	0.02000	0.075	0.02	10880.02	1716.67	6576.065	1037.59
1997	0.030221	0.02000	0.075	0.02	10880.02	1580	6576.065	954.98
1998	0.033265	0.02000	0.075	0.02	10880.02	1613.12	6576.065	975
1999	0.029991	0.02000	0.075	0.02	10880.02	1577.5	6576.065	953.47
2000	0.038208	0.02000	0.075	0.02	10880.02	1666.91	6576.065	1007.51
2001	0.034896	0.02000	0.075	0.02	10880.02	1630.87	6576.065	985.72
2002	0.038769	0.02000	0.075	0.02	10880.02	1673.01	6576.065	1011.20
2003	0.023222	0.02000	0.075	0.02	10880.02	1503.85	6576.065	908.96
2004	0.022155	0.02000	0.075	0.02	10880.02	1492.25	6576.065	901.94
2005	0.017752	0.02000	0.075	0.02	10880.02	1444.35	6576.065	872.99
2006	0.024697	0.02000	0.075	0.02	10880.02	1519.91	6576.065	918.66
2007	0.032326	0.02000	0.075	0.02	10880.02	1602.91	6576.065	968.82





2. Employment data approximation

The employment data about FTE-numbers is not reported here for discretion reasons, but the next steps are. Averaging the FTE figures gave then the following results:

City	Oslo	Bergen	Trondheim	Stavanger	Total
Average	295.52	126.05	82.487	96.423	600.494
Proportion	49.21 %	20.99 %	13.74 %	16.06 %	100.00 %

From there, a measure of the population proportion was run from data drawn from SSB. The sample considered the 8 cities reported in the table next page. The figures reported concerns the whole population of the cities. Limiting age between the age range 16 to 67 years old has before been envisaged, but figures were not directly available. Even if it was possible to approximate, it could not have been done for every city considered. In all cases, this measure of the population appears appropriate in this situation, i.e. only to smooth the FTE numbers.

Year	*Oslo*	*Trondh.*	*Bergen*	*Stavang.*	*Tromsø*	*Kristians.*	*Bærum*	*Sarp./Fred.*	***Total***
19	450 386	134 690	207 419	91 021	47 148	61 476	80 816	39 742	*1 112 698*
198	448 775	134 665	207 292	91 964	47 316	61 824	80 908	39 721	*1 112 465*
198	447 257	134 143	207 332	92 883	47 406	61 704	81 295	39 303	*1 111 323*
198	447 351	143 075	207 416	94 193	47 753	62 197	82 918	39 194	*1 124 097*
198	449 395	134 362	207 922	95 084	48 091	62 646	84 724	38 716	*1 120 940*
198	451 345	134 527	208 886	95 463	48 838	63 314	86 541	149 855	*1 238 769*
198	453 730	135 524	209 831	96 439	49 459	63 491	87 773	151 264	*1 247 511*
198	456 124	136 601	211 095	96 948	50 228	64 395	88 594	152 989	*1 256 974*
199	459 364	137 346	211 826	97 570	50 548	64 888	89 221	154 109	*1 264 872*
199	461 644	138 058	213 344	98 180	51 328	65 690	90 579	156 269	*1 275 092*
199	467 441	139 630	216 066	99 808	52 504	66 347	91 692	158 039	*1 291 527*
199	473 454	140 656	218 144	101 403	53 456	67 100	92 748	159 848	*1 306 809*
199	477 481	142 188	219 884	102 637	54 614	67 863	94 098	161 961	*1 320 726*
199	483 401	142 927	221 717	103 590	55676	68 609	95 548	164 157	*1 335 625*
199	488 659	143 829	223 238	104 373	56 646	69 269	97 034	166 303	*1 349 351*
199	494 793	144 670	224 308	105 626	57 384	70 069	98 298	168 367	*1 363 515*
199	499 693	145 778	225 439	106 856	57 485	70 640	99 590	170 230	*1 375 711*
199	502 867	147 187	227 276	108 019	58 121	71 498	100 773	172 271	*1 388 012*
200	507 467	148 959	229 496	108 818	59 145	72 395	101 494	173 889	*1 401 663*
200	508 726	150 166	230 948	108 848	60 086	73 087	101 340	174 427	*1 407 628*
200	512 589	151 408	233 291	109 710	60 524	73 977	101 497	175 474	*1 418 470*
200	517 401	152 699	235 423	111 007	61 182	74 590	102 529	177 119	*1 431 950*
200	521 886	154 351	237 430	112 405	61 897	75 280	103 313	178 593	*1 445 155*
200	529 426	156 161	239 209	113 991	62 558	76 066	104 690	180 756	*1 462 857*
200	538 411	158 613	242 158	115 157	63 596	76 917	105 928	182 845	*1 483 625*
200	547 617	161 730	244 620	117 315	64 492	77 840	106 932	184 772	*1 505 318*
Aver	*484 488*	*143 998*	*221 577*	*103 050*	*54 903*	*68 584*	*94 264*	*142 701*	*1 313 565*
Prop	***36.88 %***	***10.96 %***	***16.87 %***	***7.85 %***	***4.18 %***	***5.22 %***	***7.18 %***	***10.86 %***	***100.00 %***

In addition, the consistence of the technique is demonstrated later in the paper section about employment data.

The proportion of population reported in the paper is slightly greater than the one of the bottom line of the table next page because they encompass only the period 1982-1995, since only this period is missing in the FTE data about cities.

Finally, the figure "Services Norway" has been made when adding several items of the table from the SSB website, "ÅRLIG NASJONALREGNSKAP 1970 – 2007 - Tabell 18. Sysselsatte normalårsverk etter hovednæring. Lønnstakere og selvstendige. 1000" - "Yearly National Accounting 1970-2007 – Table 17. Full Time Equivalent Employment by sector. Employees and independent workers. 1000". Below are presented the items selected in Norwegian and English, as well as an example from 1995.

Norwegian	English	1995	
Post og telekommunikasjon	Post and telecommunications		44.4
Finansiell tjenesteyting	Financial sector		49.9
Boligtjenester (husholdninger)	Real estate development		1.2
Forretningsmessig tjenesteyting	Services agreement sector		120.5
Offentlig administrasjon og forsvar	Civil administration and Defence	158.3	
minus Forsvar	Minus Defence	-45.6	
Offentlig administrasjon	Civil administration		112.7
Total (Man-hours/year)			595.6

3. General observations about OLS statistical regression

The models employed here are some linear processes and are estimated under Ordinary Least Square, or OLS. It means that the regression computes what is the optimal linear function which minimizes the squared errors between all explanatory variables and the dependent one. Some conditions are required to run the regression, such as linearity in parameters for instance.

One important assumption here which caused problems is to have weakly dependent residuals. It means that they must be free of serial correlation, which hampers the regression with the apparition of an artificial trend for example. It has thus devoted a particular heed to ensure that serial correlation has been removed from the outcomes of the estimations.

Another important point to enhance is that the reduced number of observations here with 21, or even in the enquired articles, implies that the results are likely to be biased by small sample bias. However, all models seem to behave quite well and the biases are marginal.

4. Durbin-Watson autocorrelation test

This test gives indication about the presence of autocorrelation in the residuals. Autocorrelation biases the results and need being removed in order to interpret validly the outcomes. The Stata command after estimation is dwstat with beforehand tsset *variable* to indicate the time unity of the data.

The test statistic is formally designed as $DW = \sum_{t=2}^{n}(\varepsilon_t - \varepsilon_{t-1})^2 \Big/ \sum_{t=1}^{n} {\varepsilon_t}^2$ where n is the number of observations and ε_t is the error term of the estimation

Using an algebraic transformation, it becomes $DW \approx 2(1-\rho)$, which means that having a Durbin Watson statistic of around 2 evidences the absence of autocorrelation. There are some Durbin-Watson tables reporting Up and Down bounds about the statistic which is used to obtain a precise diagnosis about the presence of residual autocorrelation or not.

5. The Cochrane-Orcutt and Prais-Winsten procedures

These procedures permit removing serial correlation contained in the residual of a statistical estimation when and only when the error term follows an autoregressive process, noted AR(1). In the case of an order superior to 1, they are not utilizable.

The first one has been created by two English statisticians, Cochrane and Orcutt (CO) in 1949, whereas the Prais-Winsten (PW) procedure was devised in 1954.

This algorithm estimates the coefficient of correlation through iterated regressions, and then subtracts this value to every variable composing the original equation. This step normally cleans up for serial correlation, except for excessive one. This persistence of serial correlation is often synonym of a weak model. One problem with the Cochrane-Orcutt procedure is that it removes one observation out of the sample, which is unwanted when having a small sample like in this paper.

Then, one would prefer using the Prais-Winsten procedure, which does the same correction than the Cochrane-Orcutt but without removing the first observation. The Prais-Winsten procedure was then preferred over the original, and utilized thanks to the STATA command Prais.

In addition to that, one option on STATA enables to compute the serial-correlation robust standard deviation of the variables when running a regression. This is efficient since it identified the true significance of the parameters, and gives them free of serial correlation. It has to be distinguished the serial correlation integrated in the errors (that CO and PW correct for) and its effect onto the parameters standard deviation. According to Wooldridge (2006), the joint utilization of both methods would ensure that any serial correlation is accounted for in statistical inference. The STATA command is *Prais* [...], *robust*.

6. Partial Adjustment and Error Correction Mechanisms

Both mechanisms were widely utilized among economical models, and especially those attempting modeling real estate-linked variables. They are quite useful since common adjustments in this sector needs normal time of construction before reaching the targeted level associated with the changes in decision variables.

a. Partial adjustment model

It comprises two parts, one traducing the static amount that one seeks to reach, which is comparable to a long-term equilibrium and another describing the dynamic adjustment process over one period lag:

Equilibrium $y_t^* = \alpha_0 + \alpha_1 x_t + u_t$

Dynamic $y_t - y_{t-1} = \lambda(y_t^* - y_{t-1})$

where λ is the speed of adjustment of y_t towards the desired amount y_t^*, and u_t the residual. When plugging the static equilibrium part into the adjustment process, one obtains:

$$y_t = \lambda(\alpha_0 + \alpha_1 x_t + u_t - y_{t-1}) + y_{t-1} \Leftrightarrow y_t = \lambda\alpha_0 + (1-\lambda)y_{t-1} + \lambda\alpha_1 x_t + \lambda u_t$$

It reverts then to estimate $y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 x_t + v_t$in order to retrieve $\lambda = 1 - \beta_1$, and finally the α parameters. Closer λ is to 1, faster is the adjustment.

Some equations of the models are using a partial adjustment mechanism, like both rental equations (4) and (VI), but also the absorption of TWE (IV). However, the desired rent level of the HLM equation is special, since it is calculated and not estimated.

b. Error Correction Model (ECM)

It resembles to the Partial Adjustment, as one way to start is also having two equations, one static and one dynamic. The static equation can be like previously a (long-run) desired amount linking the dependent variable series with others. Considering an equilibrium, one gets: $y_t^* = \alpha_0 + \alpha_1 x_t + u_t$ (like in the Partial adjustment case)

The residual u_t of the static equation is $u_t = y_t^* - \alpha_0 - \alpha_1 x_t$. It is also extracted from the estimation to be reinserted later in the dynamic equation.

The dynamic (short-run) part includes always first differenced variables, which implies that all are I(0), and then stationary. On the other hand, a series integrated of order 1, I(1), does necessitate any transformation to be stationary and could not be utilized in an ECM.

However, the calculation of the residual of the "equilibrium" equation allows considering the hypothesis that y_t^* and x_t are cointegrated with parameter α_1 (for simplicity, disregard the constant α_0). Cointegration means that for economic reasons, two I(1) series are both forced towards a long-run relationship, in this case with the parameter α_1, and are then stationary together. Cointegration has nonetheless to be positively tested to the dynamic equation to hold. One test by MacKinnon is mentioned below.

The dynamic ECM is thereby composed of two parts, short-run and long-run ones. First, the short-run part includes the first difference variables (including the dependent one) with several lags if necessary, but for simplicity no lag is included: $\Delta y_t = \beta_0 + \beta_1 \Delta x_t + v_t$
This equation measures the immediate short-run changes of explanatory variable over the dependant one, as usual. Now, integrating the one-year lagged long-run section obtained previously, and supposedly cointegrated and stationary:

$$\Delta y_t = \beta_0 + \beta_1 \Delta x_t + \delta(y_{t-1}^{*} - \alpha_0 - \alpha_1 x_{t-1}) + v_t$$

The ECM can be estimated in one single equation, it is commonly done in two steps with using instead the residual u_t, which gives δ the error correction term. The working of the long-run is the following: if $y_t^{*} > \alpha_0 - \alpha_1 x_t$ $(or\, u_t > 0)$, the dependent variable y has overshot the equilibrium and δ will be negative in order to push back y towards the equilibrium in the next period, and inversely whether $y_t^{*} < \alpha_0 - \alpha_1 x_t$.
One example of ECM is utilized in the HLM model, in the Absorption equation (7).

7. Cointegration and Unit Root testing

The Unit Root notion comes from terminology of times series processes and basically indicates that the process studied is not stationary. Being, it cannot be employed in an OLS. One typical solution is to take the first difference of the process (or the variation), but the interpretation would not have necessarily to do with a relationship in levels. Another more convenient solution is to find a cointegration relationship between variables entailing that they altogether form a stationary process. This has been first identified by Granger and Newbold in 1986, followed the next year by Engle and Granger. Still nowadays, it is an active area of research.
Testing for Unit Root has been done here utilizing the Augmented Dickey –Fuller, or ADF, test. It has been devised following the original paper written in 1979 by these two American statisticians. The ADF test enables testing for different lags, but also for a potential trend such it is likely to be in the variables linked to the office space market. The variables pass the test when the result is lower than the asymptotic critical values calculated by Dickey and Fuller. The basic test is actually testing whether the residuals of the regression between the first difference of a variable and its lagged value are perfectly correlated.

The STATA command for that is *dfuller* which has been preceded however by *tsset* (time setting). Three tests have been run here, for several lags until 3 years:

1: No constant, no trend

2: Constant but no trend

3: Constant and trend

The results are presented in the tables below. When the process is I(0), it should pass any test for the level (i.e. as it is) depending on its path. An I(1) process would fail to reject the test in the first columns but passes the test "1" when being differenced.

Oslo

Variables	Level				Difference				Result
	Lags	1	2	3	Lags	1	2	3	
$ln(R_t)$	1	FTR	R 1%	R 5%	1	R 5%	FTR	FTR	I(0)
$\Delta ln(R_t)$	1	R 5%	FTR	FTR	0	R 1%	R 10%	FTR	I(0)
$ln(R_{t-1})$	1	FTR	R 1%	R 5%	1	R 10%	R 10%	FTR	I(0)
$\Delta ln(R_{t-1})$	1	R 1%	R 10%	FTR		–	–	–	I(0)
$ln(R_t^*) - ln(R_{t-1})$	0	FTR	FTR	FTR	1	R 5%	R 10%	FTR	I(1)
$ln(R_t^*) - ln(R_{t-2})$	1	R 10%	FTR	FTR		–	–	–	I(0)
$ln(R_{t-2}) - ln(R_t^*)$	1	R 10%	FTR	FTR		–	–	–	I(0)
v_t	1	FTR	R 10%	FTR	0	R 5%	FTR	FTR	I(0)
$ln(v_t)$	1	FTR	FTR	FTR	0	R 5%	FTR	FTR	I(1)
$ln(1 - v_t)$	1	FTR	R 10%	FTR	0	R 5%	FTR	FTR	I(0)
$\Delta ln(1 - v_t)$	0	R 5%	FTR	FTR	0	R 1%	R 1%	R 1%	I(0)
v_{t-1}	0-1-1	FTR	R 10%	FTR	0	R 1%	FTR	FTR	I(0)
$\Delta ln(v_{t-1})$	0	R 1%	FTR	FTR		–	–	–	I(0)
AB_{t-1}/OS_{t-2}	0	R 5%	R 5%	R 5%		–	–	–	I(0)
$ln(E_t)$	1	FTR	FTR	FTR	0	R 10%	FTR	FTR	I(1)
$\Delta ln(E_t)$	0	R 10%	FTR	FTR	0	R 1%	R 1%	R 5%	I(1)
$ln(E_{t-1})$	1	FTR	FTR	FTR	0	R 10%	FTR	FTR	I(1)
$ln(S_t)$	3-3-1	FTR	FTR	FTR	3-0-2	R 10%	R 5%	R 10%	I(1)
$\Delta ln(S_t)$	3-0-2	R 10%	R 5%	R 10%	0	R 1%	R 1%	R 1%	I(0)
$ln(S_{t-1})$	3-3-0	FTR	FTR	FTR	3-2-2	FTR	R 5%	R 10%	I(1)
eg_{t-1}	0	R 10%	FTR	FTR	3	R 5%	FTR	FTR	I(0)
eg_{t-1}	0-3-0	FTR	R 10%	FTR		–	–	–	I(0)
$ln(Comp_t)$	3-0-2	FTR	R 5%	R 5%	1-2-2	R 1%	R 1%	R 1%	I(0)
$ln(Const_t)$	1-2-2	FTR	R 1%	R 5%	2	R 1%	R 1%	R 5%	I(0)
$ln(I_t)$	0	FTR	FTR	FTR	0-2-2	R 1%	R 5%	R 10%	I(1)
$ln(RC_t)$	1	FTR	FTR	FTR	0	R 1%	R 1%	R 1%	I(1)

Bergen

Variables	Level				Difference				Result
	Lags	1	2	3	Lags	1	2	3	
$ln(R_t)$	1-1-0	FTR	R 1%	R 10%	0-0-1	R 5%	FTR	R 10%	I(0)
$\Delta ln(R_t)$	0-0-1	R 5%	FTR	R 10%	2	R 1%	R 1%	R 1%	I(0)
$ln(R_{t-1})$	2-1-1	FTR	R 1%	R 1%	1-0-1	R 5%	FTR	R 5%	I(0)
$\Delta ln(R_{t-1})$	1-0-1	R 5%	FTR	R 5%		–	–	–	I(0)
$ln(R_t^*) - ln(R_{t-1})$	0	R 10%	FTR	FTR	0-0-1	R 5%	FTR	R 5%	I(0)
$ln(R_t^*) - ln(R_{t-2})$	1	R 5%	R 10%	FTR		–	–	–	I(0)
$ln(R_{t-2}) - ln(R_t^*)$	1	R 5%	R 10%	FTR		–	–	–	I(0)

Bergen (continued)									
Variables	Level				Difference				Result
	Lags	1	2	3	Lags	1	2	3	
v_t	0	FTR	FTR	FTR	0	R 1%	R 5%	R 5%	I(1)
$ln(v_t)$	0	R 10%	FTR	FTR	0	R 1%	R 10%	FTR	I(0)
$\Delta ln(1-v_t)$	0	R 1%	R 5%	R 10%	0	R 1%	R 5%	R 10%	I(0)
$ln(1-v_t)$	0	FTR	FTR	FTR	0	R 1%	R 5%	R 10%	I(1)
v_{t-1}	0-3-3	FTR	R 5%	FTR	0	R 1%	R 5%	FTR	I(0)
$\Delta ln(v_{t-1})$	0	R 1%	R 10%	FTR		–	–	–	I(0)
AB_{t-1}/OS_{t-2}	3-0-0	FTR	R 5%	R 10%		–	–	–	I(0)
$ln(E_t)$	0	FTR	FTR	FTR	0	R 5%	R 5%	R 10%	I(1)
$\Delta ln(E_t)$	0	R 5%	R 5%	R 10%	0	R 1%	R 1%	R 1%	I(0)
$ln(E_{t-1})$	0	FTR	FTR	FTR	0	R 5%	R 5%	FTR	I(1)
$ln(S_t)$	0	FTR	FTR	FTR	0	R 1%	R 1%	R 5%	I(1)
$\Delta ln(S_t)$	0	R 5%	R 1%	R 5%	0-2-0	R 1%	R 1%	R 1%	I(0)
$ln(S_{t-1})$	0	FTR	FTR	FTR	0	R 5%	R 1%	R 5%	I(1)
eg_{t-1}	0	R 5%	R 5%	FTR	0	R 5%	R 1%	FTR	I(0)
eg_{t-5}	3-0-0	R 10%	R 5%	FTR		–	–	–	I(0)
$ln(Comp_t)$	0	FTR	R 5%	R 10%	0	R 1%	R 1%	R 1%	I(0)
$ln(Const_t)$	0	FTR	R 10%	FTR	0	R 1%	R 1%	R 1%	I(0)
$ln(I_t)$	0	R 10%	FTR	FTR	0	R 1%	R 10%	FTR	I(1)
$ln(RC_t)$	0	FTR	FTR	FTR	0-2-2	R 1%	R 5%	R 10%	I(1)

"FTR" indicates "Fail to reject" and "R" reject with a certain degree of confidence. When the test "rejects" the null hypothesis of the ADF test, it indicates that there is proof against unit root, i.e. the series is stationary. The column "lags" indicates the number of lags necessary and significant to obtain the results in the columns on the right. One single numbers indicates a constant lag, which is very often 0.

The results evidence unit roots mainly in the employment and supply levels, in the nominal rate and the replacement cost. But also and this is more surprising the vacancy turns out having an unit root, even if it passes the test in logarithms rejects the test 1 at 10%.

This leads to testing for cointegration in several equations of the work above. Davidson and MacKinnon is their book "Estimations and Inferences in Econometrics" (1993) reports a table of asymptotic critical values for cointegration test calculated by MacKinnon. This table can be found at the page 722 and below is presented a relevant extract.

These values have to be compared with the values reported in the tables. The concept is identical to the unit root and it differs only by the regression estimated. While looking for unit root reverts to test for perfect correlation in the residuals of one variable regressed by its lagged value, the test for cointegration tests the residuals of the equation linking variables altogether. Whenever the test does not evidence the relationship, the result of the ADF test is over the last critical values at 10%.

The number of variables indicates the total of explanatory variables and the dependent one. τ_c is the critical value corresponding to the ADF test only with a constant, and τ_{ct} with both constant and trend. In effect, it is very rare that a cointegration relationship has no constant.

Below are reported in detail the cointegration tests mentioned along part D.

The tests below give witness to cointegration into the equations estimated above. However if almost all are proven without uncertainty, certain are somewhat weakened by the lag which can be very high (4 or 5), or by the non-significance of the trend and constant parameters. The first can be understood since it takes years to build, it might be fairly logical that it takes several years for the variables to adjust together. One notices often the presence of a small upward trend into the relationship, as it was suggested in the introductive part A.

Oslo

Eq.	Page	Variables: Dependent	Variables: Explanatory	τ_c	τ_{ct}	Constant (t-value)	Trend (t-value)	Lags
5''	38	$\Delta ln(R_t)$	$ln(R_t^*) - ln(R_{t-1}), v_{t-1}$	–	-5.600	-0.177 (-4.42)	0.0135 (4.74)	4
5''b	40	$\Delta ln(R_t)$	$ln(R_t^*) - ln(R_{t-2}), v_t$	–	-4.888	-.0563 (-3.75)	0.0052 (4.08)	0
8	50	$ln(R_{t-1})$	$ln(1 - v_t), ln(S_{t-1}), ln(E_{t-1})$	–	-4.614	-.1316 (-2.40)	0.0076 (1.94)	5
8a	51	$ln(R_t)$	$ln(1 - v_{t-1}), ln(S_t), ln(E_t)$	-4.242	–	1.113 (3.43)	–	5
VI'	69	$ln(Const_t)$	$ln(v_t), ln(R_t), ln(I_t), ln(RC_t)$	-4.803	–	–	–	0
7''a	82	$ln(OS_t)$	$ln(R_t)$	–	-4.123	-.1348 (-3.84)	0.0122 (4.20)	2
7''a	82	$ln(E_t)$	$ln(R_t)$	–	-3.803	-.0579 (-3.12)	0.0056 (3.52)	1

Bergen

Eq.	Page	Variables: Dependent	Variables: Explanatory	τ_c	τ_{ct}	Constant (t-value)	Trend (t-value)	Lags
5''b	40	$\Delta ln(R_t)$	$ln(R_t^*) - ln(R_{t-2}), v_t$	–	-5.465	-.0763 (-4.80)	.0068 (5.16)	0
8	50	$ln(R_{t-1})$	$ln(1 - v_t), ln(S_{t-1}), ln(E_{t-1})$	-4.675	–	–	–	1
8a	51	$ln(R_t)$	$ln(1 - v_{t-1}), ln(S_t), ln(E_t)$		-5.515	-.1001 (-2.55)	.0067 (2.11)	1
VI'	69	$ln(Const_t)$	$ln(v_t), ln(R_t), ln(I_t), ln(RC_t)$	-4.899	–	–	–	0
7''a	82	$ln(OS_t)$	$ln(R_t)$	–	-3.662	-.1599 (-3.16)	.0135 (3.40)	4
7''a	82	$ln(E_t)$	$ln(R_t)$	–	-3.223	-.034 (-1.37)	.0021 (1.14)	4

Asymptotic critical values											
Variables		2		3		4		5		6	
Test statistic		τ_c	τ_{ct}	τ_c	τ_{ct}	τ_c	τ_{ct}	τ_c	τ_{ct}	τ_c	τ_{ct}
Degree of confidence	1%	-3.9	-4.32	-4.29	-4.66	-4.64	-4.97	-4.96	-5.25	-5.25	-5.52
	2.50%	-3.59	-4.03	-4	-4.37	-4.35	-4.68	-4.66	-4.96	-4.96	-5.23
	5%	-3.34	-3.78	-3.74	-4.12	-4.1	-4.43	-4.42	-4.72	-4.71	-4.98
	10%	-3.04	-3.5	-3.45	-3.84	-3.81	-4.15	-4.13	-4.43	-4.42	-4.7

8. Turnover and take-up series

The turnover series were calculated according to the equation:

$$U_t = S_t(1 - v^*) - OS_t + AB_t$$

with v^* being different among cities, 5.53% in Oslo and 4.37% in Bergen.

However, in the years where the buffer $S_t v^*$ retained by the owners is actually more than the difference between supply and occupied space, it is automatically set at 5,000m²/year since it is likely to have some turnover every year even if all the supply is allocated.

Obtaining the turnover series in this fashion, it reverts to average the ratio T_t / OS_{t-1} on the period to obtain the turnover rate τ. Oslo provides 2% and Bergen 1.28%, the total take-up rate is then 5.53+2=7.53% in Oslo and 4.37+1.28=5.65% in Bergen against slightly above 6% in London, according to Barras and knowing he had a take-up series at hand.

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