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Cost Uncertainty in Petroleum Investments

A Real Options Model

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Abstract

There are a number of uncertainties that are important to consider when planning a large petroleum investment. Oil price uncertainty has, in particular, been incorporated in many examples of real options analyses of petroleum investments. However, in the context of real options, there are few academics or professionals who discuss cost uncertainty in the petroleum industry. This real options analysis on cost uncertainty in petroleum projects demonstrates that there are significant effects of introducing stochastic costs. We find that volatility and correlation between income and cost components have important effects on both option value and the optimal investment time, especially for projects that are initially less profitable. Moreover, the choice of convenience yield can magnify these effects. We conclude that although the analysis of these effects is tedious and computationally demanding, petroleum companies should consider incorporating a real options framework that includes cost uncertainty in their evaluations of future prospects.

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List of Acronyms

ACF	Autocorrelation function
AR	Autoregressive
ARIMA	Autoregressive integrated moving average
ARMA	Autoregressive moving average
BBL	Barrel
CAPM	Capital asset pricing model
EMM	Equivalent martingale measure
EOU	Exponential Ornstein-Uhlenbeck
FOB	Free on board
GARCH	General autoregressive conditional heteroskedastic
GBM	Geometric Brownian Motion
HWWI	Hamburg Institute of International Economics
LME	London Metal Exchange
LSMC	Least Square Monte Carlo
MA	Moving average
MC	Monte Carlo
MMA	Money market account
MT	Metric tons
MUSD	Million US dollars
NCS	Norwegian continental shelf
NPV	Net present value
OU	Ornstein-Uhlenbeck
PACF	Partial autocorrelation function
PDE	Partial differential equation
RNV	Risk-neutral valuation
SDE	Stochastic differential equation
USD	United States dollar
WTI	West Texas Intermediate

Preface

This thesis is written as a concluding part of the Master of Science in Economics and Business Administration at the Norwegian School of Economics and Business Administration, within the major Financial Economics.

Real options and risk analysis have long been topics close to our hearts. Courses taken in derivative pricing and risk management amplified this interest. In the literature, real options frameworks are often applied on petroleum projects, and we find the challenges of this industry intriguing. Since there are few publications on cost uncertainty, we found it interesting, with good help from our supervisor Jørgen Haug, to analyze the effect of cost uncertainty in petroleum projects.

The process of writing this thesis has been both challenging and enlightening. We have gained valuable knowledge in programming, as well as extending our understanding of time series analysis and financial frameworks. Moreover, discerning the implications of our analysis for the petroleum industry has broadened our interest in real options. The lessons learned from writing this paper, with all its challenges, are invaluable.

We would like to thank our supervisor Jørgen Haug for valuable feedback and meaningful dialogues throughout the period. We would also like to thank Jonas Andersson for helpful discussions on time series analysis. All errors and opinions contained herein are solely our own.

Bergen, June 2010

Line L. Mjell and Kristian Støre

1 Introduction

This thesis examines real options prevalent in petroleum projects. Specifically, we examine how modeling cost uncertainty will affect the valuation and decision analysis made by petroleum companies. We have created a model that incorporates different flexibilities that are typical for petroleum investments, including the option to postpone an investment decision, the option to switch between production rates, the option to temporarily shut down, and the option to abandon. There are three main underlying risk factors in the model: oil prices, steel prices, and a variable cost index. We analyze historical data for oil and steel prices in order to estimate processes to describe the dynamics of the spot prices.

1.1 Background

There are numerous examples in the literature of real options analyses applied to the valuation of investment projects. Because of the high volatility of oil prices, the mature markets, and the large initial investments, projects in the petroleum industry have been used as examples by many academics. However, most of the literature only considers income uncertainty.

Evidence suggests that costs of petroleum investments are cyclical, and show more signs of volatility than the effects of inflation would imply (NOU, 1999; Statoil, 2010). There are few applications of this in the literature. We are interested in the extent to which modeling uncertain costs would change the optimal decisions and the estimated value of the investments in the industry.

1.2 Purpose

The aim of our thesis is to model a petroleum investment in order to explore whether the inclusion of stochastic costs will affect the valuation of projects and optimal investment decisions made by companies in the petroleum industry. We analyze historical data for the underlying risk factors in order to find realistic parameters to include in the model.

Every petroleum field has specific features and there are no general traits that apply to all investment opportunities. We model a field with characteristics similar to those found in the

North Sea. It is important to note that within this region the fields also exhibit different characteristics; our parameters are chosen to represent a plausible project.

The model is based on real options methodology and uses risk-neutral valuation techniques. This approach more accurately values flexibility with several risk factors than the traditional net present value approach. We use the Least Square Monte Carlo method proposed by Longstaff and Schwartz (2001) in order to price the complex American option features.

1.3 Disposition

We start the thesis with an introduction to petroleum investment projects in Chapter 2. Relevant uncertainties are outlined, focusing on uncertainty in costs. This chapter will define and narrow the scope of our analysis. Chapter 3 describes the financial frameworks for investment analysis and the theoretical concepts applied for valuation under uncertainty. In Chapter 4 we analyze the dynamics of oil and steel prices in order to estimate appropriate processes as well as the correlation between them. The subsequent chapter describes the model and the flexibilities included in it. In Chapter 6 we examine the results from the model and the associated sensitivity analysis. The last chapter presents our findings and concludes the thesis with suggestions for future research.

2 Petroleum Investments and Uncertainty

Extraction, production and refining of petroleum are complex exercises. Each location has specific geological and physical specifications. Performing capital investment decisions for petroleum investments is thus a difficult exercise in which many uncertainties need to be considered. Many of the projects include flexibilities which further complicate both valuation and optimal decision analysis. Projects with such inherent flexibilities are referred to as *real options*.

This chapter will discuss the main features of petroleum investments and their inherent flexibilities. Key uncertainties in these projects will be outlined, with focus on cost uncertainty.

2.1 Introduction to Real Options in Petroleum

Hull (2009) discusses five major types of real options embedded in investment projects: abandonment options, expansion options, contraction options, options to defer, and options to extend. Operational options to shut down or suspend operations for a short term are often mentioned for natural resource investments; Lund (2000) discusses these operational options in the context of a petroleum project. In addition to these flexibilities come project specific options.

There are many examples of real options analysis applied to natural resources and petroleum investments in the literature. Tourinho (1979) originated the application of real option pricing to value reserves of natural resources. Among the early applications of real options analysis, Brennan and Schwartz (1985) used a self-financing portfolio approach to evaluate natural resource investment under oil price uncertainty. McDonald and Siegel (1985) analyzed the option value of postponing an irreversible investment. Majd and Pindyck (1987) used option pricing methods to derive optimal decision rules for sequential investment outlays. Ekern (1988) proposed an option pricing approach for evaluating petroleum projects that include development and operations options of satellite fields. Paddock, Siegel and Smith (1988) developed an option pricing methodology for valuation of claims on an offshore petroleum lease.

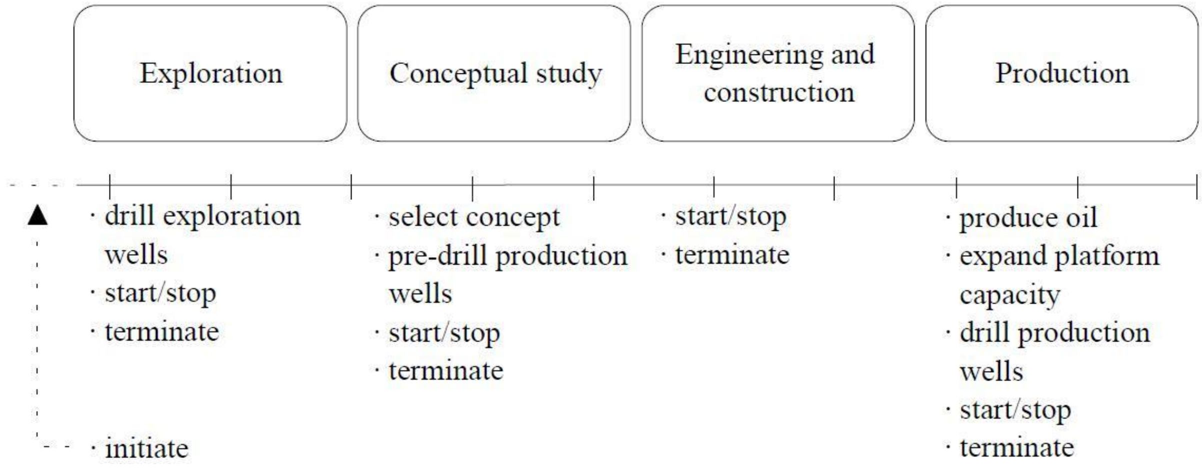
Most of the real options literature on petroleum investments focuses on modeling uncertain revenues, specifically oil price uncertainty. Some models include a stochastic[1] convenience yield[2] (Gibson and Schwartz, 1990), stochastic interest rates (Schwartz, 1997), stochastic volatility (Lin, 2007), or other features of multiple risk factors of income uncertainty. However, there are few examples of cost uncertainty in the real options literature. Among the handful of examples, Pindyck (1993) incorporates both technical and input cost uncertainty in his model and applies it to an example of developing a nuclear reactor. Schwartz and Zozaya-Gorostiza (2003) builds a model which they use to value IT development projects with both cost and income uncertainty.

2.2 Petroleum Investment Projects

Lund (2000) divides a petroleum project into four phases: exploration, conceptual study, engineering and construction, and production, as depicted in Figure 2.1. In the exploration phase an estimate of the size of the reservoir is calculated and the opportunities for drilling are explored. The potential production capacity is decided in the conceptual study phase, as well as any operational flexibilities. The engineering and construction phase carries out the decisions made in the conceptual study. If the decision is made to invest in extraction capabilities, the project enters the production phase after construction is complete. Laughton et al. (2005) argue that there is a fluid transition between the phases in petroleum projects. New information will become available throughout the life of the project, which will require continuous appraisal.

[1] A stochastic process is a process whose future value is uncertain because of a random term (Hull, 2009).
[2] Brennan (1991) defines convenience yield as “the flow of services which accrues to the owner of a physical inventory but not to the owner of a contract for future delivery” (Brennan, 1991, pp.33), cf. section 2.1.8.

Figure 2.1 The Four Phases of Petroleum Projects



Source: Lund (2000)

It is not unusual for 10-15 years to elapse from the time oil is discovered until production can start (Norwegian Petroleum Directorate, 2009). In Norway, a company first applies for a license to start test drilling or exploration. If the test drilling is successful and reveals petroleum reserves, there is a new round of concession applications. Generally the production license lasts 30 years, with the possibility of applying for an extension (Norwegian Petroleum Directorate, 2010).

We will focus on the construction and production phases of a petroleum investment project for a field in which the presence of extractable oil reserves has already been confirmed. The major revenues and costs occur in these phases of the project, which implies that cost uncertainty will have the greatest effect on value and optimal decisions during these periods. By including flexibilities that allow the company to make decisions on the basis of the changing underlying risk factor prices, we also take into account some of the new information that is revealed throughout the life of the project.

2.3 Uncertainty in Petroleum Investments

Different types of risks and uncertainties are associated with the exploration and production of an oil field. Laugthon, Sagi and Samis (2000) and Jonsbråten (1998) classify uncertainties in two categories: “exogenous” and “endogenous”. Exogenous uncertainties will be revealed independent of project decisions, while endogenous uncertainties will be revealed throughout the project, as a function of project decisions.

Jones (1988), Smith and McCardle (1999), Jonsbråten (1998), and Bøhren and Ekern (1987) are among the many that discuss the different features of uncertainty in petroleum investment projects. We sum up the different uncertainties in seven categories:

1. Geological uncertainty
2. Political uncertainty
3. Oil price uncertainty
4. Cost uncertainty
5. Fiscal regimes
6. Technological uncertainty
7. Market risk, exchange rates, interest rates, inflation

Geological uncertainty concerns the size and type of the reservoir. It will affect the amount of reserves that can be extracted and the cost of doing so. This is a type of risk factor that will mainly affect the optimal decisions and the estimated value of a field in the early phases of a project. New information will however also be obtained about the geology during the exploration and production phases, which will further adjust the estimates. Geological uncertainty is mainly endogenous.

Political uncertainty includes uncertainty regarding the stability of the regulations and the political situation in a country, which can affect both the operations and oil prices. The political environment may impact on the value of a petroleum project. This exogenous uncertainty is country and region-specific, which makes it difficult to model.

Fiscal regimes include the taxes and regulations that regulate the activity of a petroleum project. Because this exogenous uncertainty will affect the cash flows of a project, fiscal regimes will influence the value of an investment to a great extent. The tax systems are country specific and often complex; a petroleum company will need to take this into consideration when new projects are considered.[3]

Technological uncertainty is both exogenous and endogenous. While the company influences the technological advances specific to its own operations, the general level of technology is exogenously given. Technology advancements may improve the rate of extraction. It may also affect the operational and investment costs. Fields that appear unprofitable when they are first explored may become profitable as the technology improves.

[3] See Hannesson (1998) for further discussions on fiscal regimes in petroleum projects.

Market risk, exchange rates, inflation and interest rates are other risk factors that will affect project value. These are all examples of exogenous uncertainties.

In this thesis we focus on the effects of risk and uncertainty in the cost drivers of petroleum investments, as there has been limited research performed in this area. We also include oil price uncertainty, but leave other types of uncertainties constant.

2.3.1 Income Factors and Uncertainty

The main source of uncertainty in the revenue of petroleum extraction projects is the oil price. As mentioned above, there are many applications of oil price uncertainty in the real options literature. Not only do oil prices fluctuate with the market conditions in the overall economy, oil prices are also greatly affected by other features of supply and demand. The political situation in oil producing countries, notably OPEC[4] countries, is a major determinant of the formation of oil prices. The oil price shocks of 1973 and 1979 are examples of this. Moreover, features of storage and transportation affect oil price formation (Hannesson, 1998).

2.3.2 Cost Factors and Uncertainty

The costs of a petroleum investment in the North Sea can be divided into eight categories: management and project administration, platform, modifications, underwater installations, marine operations, concluding work, drilling and complementing, and miscellaneous (NOU, 1999). The costs can also be split based on the timing of the expenses: initial investment, development costs, operating costs, and dismantling costs (Hannesson, 1998; Beck and Wiig, 1977).

Beck and Wiig (1977) state that the main cost drivers are rig capital costs and drilling and equipment time involved. Production well costs represent approximately 30 percent of total development costs on the Norwegian continental shelf (NOU, 1999). Rig rates are the most important cost element in production wells. The majority of these costs are based on day rate contracts. Drilling of the production wells starts relatively late in the project completion, which increases uncertainty regarding the cost estimate. Emhjellen and Osmundsen (2009)

[4] The Organization of the Petroleum Exporting Countries is a cartel of twelve countries consisting of Algeria, Angola, Ecuador, Iran, Iraq, Kuwait, Libya, Nigeria, Qatar, Saudi Arabia, the United Arab Emirates, and Venezuela.

assert that two major cost factors in petroleum investments are materials and costs of planning, building and drilling, which are represented by steel and labor costs in their study.

Uncertainty in the cost components of petroleum investments affects the valuation and estimation of these projects. The main types of uncertainty in costs are based on geological characteristics (for example the depth and type of field), technological factors, and factor prices (Adelman and Shani, 1989; NOU, 1999; Pindyck, 1993; Devine and Lesso, 1972). We focus on the uncertainty based on factor prices, the market prices of the factors of production..

Two types of uncertain cost components are included in our model. The first factor is steel, which is a major component of rigs and platforms, and will thus affect investment costs. The second factor that is included is a variable cost index that affects the production cost. Since it is difficult to discern and model one or two major variable cost components in production, we will model a hypothetical index of variable costs that includes engineering hours, labor costs, material prices, and other variable production costs.

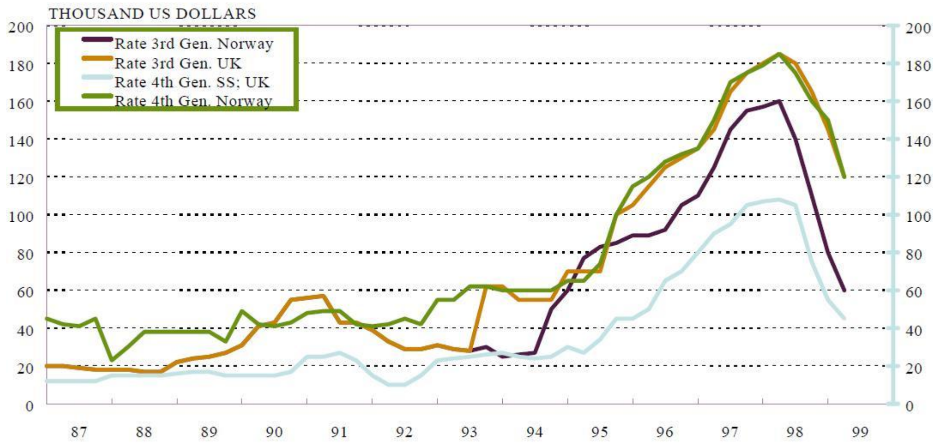
2.3.3 Cost Cyclicality

Players in the petroleum industry find that their costs have pro-cyclical patterns (NOU, 1999; Statoil, 2010). Projects appear more valuable for companies in times with high oil prices and high revenues. There are increases in demand of competent personnel and suppliers as a result of increased extraction of petroleum from existing oil fields, as well as development of additional new fields.

Already in 1983 Shell stated in the Petroleum Handbook that the "international" construction costs (e.g. platform construction costs, pipe-laying barge costs) *"show high inflation rates at time of rapid industry growth and competition for oilfield construction services, while the local costs come under particular strain in periods of accelerated local economic activity in oilfield areas (e.g. in the cities of Aberdeen and Stavanger)"* (Shell, 1983, pp. 191).

NOU (1999) finds consistent results indicating that costs are higher when the level of activity is high. Cyclicality is particularly noticeable through increases in rig rates for drilling and increases in hourly rates or wages for suppliers. Figure 2.2 displays the cyclicality in rig rates for the UK and Norway in given market segments.

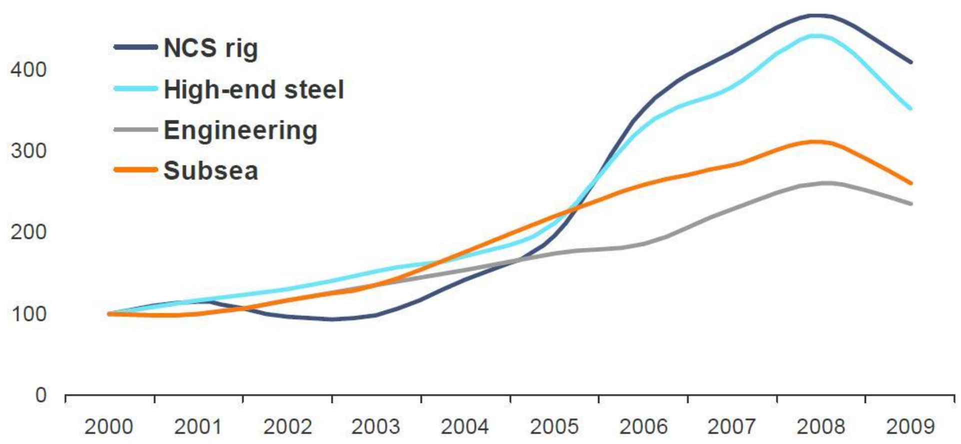
Figure 2.2 North Sea Semi-Submersible Drilling Units



Average dayrates in given market segments over time. Source: Fearnley Offshore AS / NOU (1999)

Figure 2.3 depicts how some of the cost indices are more cyclical than others (Statoil, 2010). Rig and steel prices increased significantly more than engineering and subsea cost during the expansion cycle 2006-2008.

Figure 2.3 Supplier Market Cost Indices



Source: Statoil (2010). NCS denotes the Norwegian Continental Shelf.

NOU (1999) reports evidence that a strong growth in demand in the years prior to 1999 caused unusually high rig rates, even for rigs that were considered old (older than 15 years). Higher factor prices, for example in engineering hours, workshop services, vessel rates, material prices, and drilling rates, increased costs in petroleum investments. The cost rates were especially pro-cyclical within drilling activities, but pro-cyclicality was also found within project planning and construction activities because of non-planned activities that had to be performed by agents that were already working at full capacity. In addition, the shortage of competent engineers in times of high market activity creates problems and delays. These delays, may in turn, cause further cost increases.

Cyclicality in costs will influence uncertainty in investment decisions. Petroleum companies may already take this cyclicality into account in their valuations and decision making processes, but there has been little formal application of it in investment and real options analyses. We examine how this cost uncertainty affects the investment decisions and valuation estimates of projects in the petroleum industry. Moreover, the evidence of cyclicality in costs produces an opportunity to analyze the effect of correlation between the cost factors and oil prices.

2.4 Price Processes for Uncertainty

In order to model the underlying risk factors in revenues and costs of petroleum investments, we discuss the most appropriate processes for these input factors. We focus the analysis on oil and steel, which are commodities with prices that follow complex and dynamic patterns. There have been many publications that have attempted to find models that fit historical data for commodities. The discussion on commodity price models focuses mainly on whether commodity prices are mean reverting, i.e. follow an exponential Ornstein-Uhlenbeck process (EOU), or follow a random walk with drift like a Geometric Brownian Motion (GBM). Both of these models are stochastic processes. A stochastic process is a process whose future value is uncertain because of a random term (Hull, 2009). The term *stochastic* is herein used to describe processes that exhibit volatility. We will present both the GBM and the EOU in this section in order to find appropriate models for steel and oil prices.

2.4.1 Literature on Commodity Prices

Commodity and energy prices have some common attributes with financial products, but they also possess distinctive features. Pilipovic (1998) points out that energy prices are influenced by factors that financial products are less affected by, such as geography, weather, and political turbulence. Commodity prices also show characteristics of seasonality, fat tails, asymmetric distributions, and time-variant volatilities (Lin, 2007). Cortazar and Schwartz (2003) describe the role played by the convenience yield and demonstrate that the number of factors used to describe uncertainty differentiate commodities from financial products.

Commodity prices will vary according to the access to downstream customers, the ability to store the product, the correlation with market price risk, and the negotiation strength of the players in the industry (Geman, 2005a).

GBM has traditionally been the usual choice of spot price process to describe commodities. For example, Brennan and Schwartz proposed in 1985 a model with GBM and a constant convenience yield. Ekern (1988) and Pindyck (1981) also model commodity prices using GBM. In the 1990s, a number of authors proposed mean-reversion as an alternative model since commodity prices seemed to be reverting back to a long-term mean. This was based on the seminal paper by Vasicek, who in 1977 proposed a mean-reverting Ornstein-Uhlenbeck process to describe short-term interest rate dynamics, to account for the fact that interest rates in general do not increase on average (Geman, 2005b). Moreover, Lund (1993) argued that a GBM cannot be an equilibrium price process for an exhaustible resource under assumptions that suppliers choose the time to extract, and deposits have different costs.

Mean reversion in commodities may be consistent with the notion that the resource is sold in a competitive market, so that the price reverts slowly to the long-run marginal cost (Pindyck, 1999). Pindyck found that there are traits of a mean reverting process in oil prices. However they are mean reverting very slowly. He also found that the trend that oil prices revert to is changing over time. Smith and McCardle (1999) refer to managers within the petroleum industry who argue that when prices are high compared to a long-run average (or equilibrium price level), new production capacity comes on line, or older production facilities continue longer than expected. This will drive oil prices down. Conversely, if prices are lower than the equilibrium price level, there will be fewer new investments and older facilities will close early, thus driving up the price. These dynamics imply “mean reverting prices”. Smith and McCardle (1999) also find mean reverting oil prices in their study of historical prices 1990-1994. Baker et al. (1998) find that futures prices often strongly support the mean reversion phenomenon. Gibson and Schwartz (1990) and Miltersen and Schwartz (1998) model commodity prices that display mean reverting features stemming from mean reverting convenience yields. Other researchers who have modeled mean reverting prices are Geman (2005a), Cortazar and Schwartz (2003), and Schwartz and Smith (2000).

In 2005, Geman suggested that the signs of mean-reversion were gone in gas and oil prices, and that the theory of GBM was again introduced as the most suitable model (Geman, 2005b). Lin (2007) finds structural changes in the oil price series, and concludes that GBM with several stochastic volatility features has performed better than EOU since the 1990s. "*The mean reversion, if it exists, might be much slower than ever"* (Lin, 2007, pp.39). Other authors that model commodity prices with GBM in recent years are Cortazar and Casassus (2000) and Armstrong et al. (2004).

Some authors find that none of the suggested models fit the data, or that a mix of the two is more appropriate. In 1988, Pindyck found that both GBM and a mean-reverting process were consistent with the observed data, such that an analyst may pick either one (cited in Bjerksund and Ekern, 1990). Baker et al. (1998) argue for a mean-reversion in the short run, while a random walk with drift (GBM) fits better in the long run.

We conclude that there is a split view in the literature on how to model commodity prices. Consequently we will describe and test both GBM and EOU in this thesis.

2.4.2 Geometric Brownian Motion

A process is called Geometric Brownian Motion with drift if it satisfies the stochastic differential equation (SDE):

$$dS_t = \mu S_t dt + \sigma S_t dz \tag{2.1}$$

Where *dz* is a Wiener Process[5], μ is the drift, and σ is the volatility of the spot price. In such a model, percentage changes in *S* are normally distributed with stationary increments and absolute changes in the spot price are lognormally distributed (Dixit and Pindyck, 1994).

It can be shown that the exact analytical solution to the SDE (2.1) is:

$$S_t = S_0 e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma dW_t} \tag{2.2}$$

If a commodity follows the process of *S*(*t*) in equation (2.1), then it can be proved using Itô's lemma that *X* = *ln S* will follow:

$$dX_t = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dz \tag{2.3}$$

[5] A Wiener process is a "*stochastic process where the change in a variable during each short period of time of length Δt has a normal distribution with a mean equal to zero and a variance equal to Δt"* (Hull, 2009, pp.792).

Discretization of equation (2.3) will give the following equation:

$$\Delta X_t = (\mu - \tfrac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\varepsilon_t \tag{2.4}$$

We notice that equation (2.4) is an integrated process of first degree, which in time series terminology is called an ARIMA(0,1,0) model. Time series will be discussed further in Chapter 4.

2.4.3 Exponential Ornstein-Uhlenbeck

The main alternative to GBM is that commodity prices are mean reverting. It can be argued that commodity prices may mean-revert to a level which may be viewed as the marginal cost of production (Geman, 2005a). An Ornstein-Uhlenbeck process is mean reverting and follows the stochastic differential equation:

$$dX_t = \kappa\left(\alpha - X_t\right)dt + \sigma dW_t \tag{2.5}$$

As we model commodity prices, we will use an *exponential* Ornstein-Uhlenbeck (EOU) process in this thesis, which will ensure positive values. The dynamics in an exponential Ornstein-Uhlenbeck process are represented by the following equation:

$$dS_t = \kappa\left(\mu - \ln\left(S_t\right)\right)S_t dt + \sigma S_t dW_t \tag{2.6}$$

In this model κ is the force of mean-reversion, while μ is the long-term average. A small κ suggests slow mean reversion. For prices following the dynamics described in (2.6), the *logarithm* of these prices follow an Ornstein-Uhlenbeck process, as in equation (2.5). Thus *ln S = X*.

It can be proven that the analytical solution to an Ornstein-Uhlenbeck process is:

$$\ln(S_t) = e^{-\kappa}\ln(S_{t-1}) + \alpha(1 - e^{-\kappa}) + \xi_t \tag{2.7}$$

where

$$\xi_t \sim N\left(0, \eta^2\right) \tag{2.8}$$

and

$$\eta^2 = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa}) \tag{2.9}$$

The EOU is thus an autoregressive process of first degree, an AR(1) in time series terminology (Dixit and Pindyck, 1994).

2.4.4 Multi-Factor Models

A number of authors include multiple uncertainties in order to provide a more realistic stochastic behavior of the commodity price.

Schwartz and Smith (2000) develop a two-factor model that allows mean reversion in short-term prices and uncertainty in the equilibrium level to which prices revert. Gibson and Schwartz (1990) develop a model with the spot price of oil and the instantaneous convenience yield as factors. Lin (2007) finds that models with two stochastic volatility components fit the market data better than those with only one stochastic volatility component.

Schwartz (1997) presents a three-factor model with a stochastic mean reverting convenience yield and stochastic interest rates. The three-factor model proposed by Cortazar and Schwartz (2003) is related to Schwartz' (1997) model but all three factors are calibrated using only commodity prices. Cortazar and Schwartz (1994) develop a three-factor model for copper prices, using a no-arbitrage approach.

Multi-factor models are out of scope of this thesis. We include and test only one-factor processes for the underlying uncertainty factors.

2.4.5 Convenience Yield

Brennan (1991) defines convenience yield as *"the flow of services which accrues to the owner of a physical inventory but not to the owner of a contract for future delivery"* (Brennan, 1991, pp.33). The convenience yield of commodities could include storage costs, insurance, and a benefit received from the flexibility of having an available inventory. The convenience yield referred to in our analysis represents the net effect of all these costs and benefits.

The theoretical relationship between futures and spot price represented in equation (2.10) can be used to deduce the convenience yield (Hull, 2009).

$$F_t\left(S,T\right) = S_t e^{(r-\delta)(T-t)} \tag{2.10}$$

Here S_t is the spot price, F_t the futures price, r the risk-free interest rate, δ the convenience yield, and (T-t) the difference in maturity between spot and futures.

Equation (2.10) can be used to determine the annualized forward convenience yield by using two futures prices which are adjacent to each other in terms of maturity:

$$\delta_{t,T} = r_{t,T} - \frac{1}{(T-t)} \ln\left[\frac{F_0(S,T)}{F_0(S,t)}\right] \tag{2.11}$$

Here $\delta_{t,T}$ is the T-t periods ahead annualized convenience yield and $r_{t,T}$ denotes the (T-t) periods ahead annualized riskless forward interest rate.

3 Financial Frameworks for Project Valuation

The choice of framework for capital investment decisions is the source of much debate; academics have explored a plethora of different approaches in order to capture the complexities inherent in projects. Although the traditional *net present value* (NPV) approach for project valuation has a strong foothold among practitioners, managers and academics alike are aware of its many pitfalls and shortcomings. The method is also referred to as *discounted cash flow* by many authors. In modern finance, the *risk-neutral valuation* (RNV) [6] techniques for valuing financial options have been suggested by academics as an improved tool for pricing real assets or projects. These techniques seemingly lend themselves to analyzing the option-like features prevalent in many projects, such as the option to wait, the option to abandon a project, or the option to temporarily shut down. The value of this inherent flexibility in projects, first coined *real options* by Myers (1977), is poorly captured by the NPV approach (Trigeorgis, 1993).[7]

This chapter will introduce and contrast the NPV and RNV frameworks for valuing projects. The implementation of cost uncertainty in these frameworks will also be discussed.

3.1 Net Present Value and Risk-Neutral Valuation

An important feature of a valuation framework is its ability to incorporate the risk elements of a project. The systematic risk[8] of the cash flows dictates whether a risk premium should be included in the discount rates used. It is however important to note, in a setting with flexibility, that the unsystematic risk could also influence the value of a project. To adjust for the systematic risk, NPV relies on risk-adjusted discount rates, while the RNV approach risk-adjusts the expected cash flows. By risk-adjusting the expected cash flows, the RNV method allows for the use of alternative discount rates (see section 3.1.2). How risk is incorporated makes the two approaches distinctively different. Although the NPV approach is the most widely used by practitioners (Laughton et al. (2008); Schwartz and Trigeorgis (2001)), academics and managers have long realized the need for tools which more correctly take into account the unique risk profile of a project. The risk-adjusted discount rate for real options is

[6] See Hull (2009) for a thorough introduction to RNV.
[7] For a comprehensive overview of the developments in the real options field, as well as arguments contrasting the NPV and the RNV approaches, see Schwartz and Trigeorgis (2001).
[8] Systematic risk is *"risk that cannot be diversified away"*, such as general market risk (Hull, 2009 pp.790).

particularly difficult to calculate, and the RNV approach offers a potentially superior way to consider a project's risk.

3.1.1 Net Present Value

The NPV approach is widely used in capital investment decisions and effectively illustrates the difficulties surrounding the valuation of uncertain cash flows. The NPV of a project can be defined as "*the present value of its expected future incremental cash flows*" (Hull, 2009, pp.745). This can be expressed as:

$$NPV = \sum_{t=1}^{T} \frac{E\left(CF_t\right)}{\left(1+\mu\right)^t} \tag{3.1}$$

where T is the life of the project and μ is an "appropriate" risk-adjusted discount rate. $E(CF_t)$ is the expected future cash flow at time t. When making capital investment decisions, a company can use the NPV approach to estimate the value of a project. If the NPV is positive (negative), investing in the project will in theory increase (decrease) the company's value. The "risk-adjusted" discount rate is chosen to reflect the riskiness of the project. The discount rate used is often a company average which is usually referred to as a "weighted average cost of capital" (WACC), or an industry standard (Brealey et al., 2008).

Using the NPV approach for evaluating real options has a major drawback in the use of a single discount rate for all cash flows. This risk-adjusted discount rate is difficult to calculate for most projects, and it becomes especially challenging when the project has embedded option elements. Using an incorrect discount rate might, for example, lead a company to discard a project that has a negative NPV, when in reality it has a positive value because of the future growth opportunities it provides. The aforementioned weaknesses of the NPV approach have long been a topic of research in academia. Among the early works, Hayes and Garvin (1984) showed that using the NPV approach could often undervalue projects, and thus lead to underinvestment. Myers (1984) further acknowledged the shortcomings of the NPV approach in correctly valuing real options.

Another complicating factor in estimating the appropriate discount rate for a project is the fact that the incremental cash flows may originate from many different sources, with potentially numerous underlying risk factors. For example, the aggregated revenues most likely have a different risk profile than the aggregated costs. This makes the estimation of a single discount rate for the project more difficult.

Consider a project with known fixed costs and uncertain revenues. For simplicity we assume that the project is all equity financed and that revenues have a constant beta according to the CAPM[9]. Estimating the project beta that would correctly discount all cash flows is surprisingly difficult, and becomes exceedingly challenging as the complexity of the cash flows increase. Even if the beta for the revenues is constant, the overall beta for the project is state and time dependent. Following the reasoning of Sick and Gamba (2005), the ratio P/W is a variable describing the leverage effect from fixed costs, where P is the price of the underlying asset value (impacting the revenue) and W is the value of the real option. Combining the leverage with the hedge ratio ($\Delta P = \frac{\partial W}{\partial P}$) we can express the elasticity of the option price with respect to the underlying price as:

$$\eta(P) = \frac{P}{W}\frac{\partial W}{\partial P} \tag{3.2}$$

As the beta of the underlying asset is a measure of the risk per dollar invested, the beta of the project β_W can be expressed as the product of the underlying asset's beta β_P and the above mentioned elasticity:

$$\beta_W = \eta(P)\beta_P = \frac{P}{W}\frac{\partial W}{\partial P}\beta_P \tag{3.3}$$

If the right hand side stays constant, the beta of the project is constant, and one could rely on a constant discount rate for the cash flows of the project. As argued by Sick and Gamba (2005), this is not likely. They highlight this with the example of a finite-lived development option, similar to a call option, showing that both the elasticity and P/W would change with P. This issue would occur regardless of flexibility, and makes the use of a constant discount rate difficult at best.

[9] A common approach to calculating the discount rate for a project is by using the "capital asset pricing model" (CAPM). According to the CAPM model, well-diversified investors only demand excess returns above the risk-free rate if an asset has systematic risk that is undiversifiable. The CAPM model assumes that the only relevant systematic risk for investors is the market risk. The CAPM discount rate, or expected return, is defined as: $E(r_i) = r_f + \beta_i(E(r_m) - r_f)$. Here beta is a measure of the market risk of the asset, r_f is the risk-free rate, and $E(r_m)$ is the expected return of the market (Bodie et al., 2009).

Emhjellen and Osmundsen (2009) suggest splitting the cost and revenue cash flows to avoid some of the difficulties in estimating a risk-adjusted discount rate for the entire project. This recommendation relies on the value additivity principle (Mossin, 1969; Schall, 1972), which states that the cash flows of an asset can be evaluated separately, and that the sum of present values of the separate cash flows is equal to the present value of the entire asset. With the example of revenues and costs this can be stated as:

$$NPV = \sum_{t=1}^{T} \frac{E(R_t - C_t)}{(1+\mu)^t} = \sum_{t=1}^{T} \frac{E(R_t)}{(1+\mu_R)^t} - \sum_{t=1}^{T} \frac{E(R_t)}{(1+\mu_C)^t} \tag{3.4}$$

For oil projects specifically, Emhjellen and Osmundsen (2009) argue that the costs (exemplified by steel and labor in their article) have no systematic risk and may thus be discounted using a risk-free rate. If this were to hold true, the costs could be discounted using a risk-free rate, and the income would be discounted by the discount rate appropriate for oil revenues. This approach might avoid some problems surrounding the estimation of a project discount factor, but it does not effectively take into account how to estimate discount rates for projects with embedded options. Moreover, one must assume a model, such as the CAPM, in order to estimate the risk-adjusted discount rate.

3.1.2 Risk-Neutral Valuation

The general idea of risk-neutral valuation is to price an asset as if investors are risk-neutral. In such a world the risk-free rate is used for discounting the cash flows. The asset value obtained in the risk-neutral world will be the same as the value in other worlds with different risk preferences, including the real world (Hull, 2009). Using RNV the exercise of estimating an appropriate risk-adjusted discount rate is avoided entirely. The actual adjustment for risk happens in the expected cash flows instead of the discount rate.

The somewhat conceptual idea of a risk-neutral world is perhaps best illustrated by the binomial model developed by Cox, Ross and Rubenstein (1979) and Rendleman and Bartter (1979) for pricing financial derivatives. Consider a stock whose dynamics can be described in a one-period binomial tree. At each step the price of the stock will either go up by a factor of *u*, or down by a factor of *d*. The risk-free rate is denoted *r*, and the probability of an upward movement is *p*.

Following the RNV methodology, the riskiness of the stock should be reflected in the expected cash flows instead of the discount rate. One way to achieve this is to change the probabilities used. The new risk-adjusted "probabilities" would only exist in a fictional, risk-neutral universe. One can show through the use of no-arbitrage arguments that the risk-adjusted probability of an upward movement must be:

$$q = \frac{r-d}{u-d} \tag{3.5}$$

Note that neither the real probability of an upward movement, nor the expected return, is needed to find this probability measure. The expected cash flow using the risk-neutral probabilities may be discounted using the risk-free rate. The present value of the stock may then be expressed as:

$$S_0 = \frac{uS_0\left(q\right) + dS_0\left(1-q\right)}{r} \tag{3.6}$$

Here the denominator represents an expected stock price in the risk-neutral world. Analogously, this tree may be expanded to several time periods. Cox, Ross, and Rubenstein (1979) showed how this approach could be used to price European stock options. The expected cash flows from the option in the risk-neutral world could be discounted using the risk-free rate.

In continuous time, the risk-adjusted probabilities are reflected in the dynamics of the underlying assets. Specifically, the drift of the asset is adjusted such that the resulting cash flows can be discounted by a risk-free rate. In the famous Black-Scholes model (Black and Scholes, 1973) (BS) the risk adjustment of the drift term of a stock results in changing the drift from the expected growth rate of the stock to the risk-free rate. The binomial model of Cox, Ross, and Rubenstein (1979) can be expanded to show that, when the binomial tree has infinitely many jumps until maturity of the option, it produces the same results as the BS model.

The RNV framework will be used in this thesis because of its advantage over the NPV approach; RNV better prices flexibility and avoids the exercise of calculating a risk-adjusted project discount rate.

3.2 Equivalent Martingale Measure

To perform the risk adjustment of the underlying stochastic processes in the RNV framework, one can rely on the results of the equivalent martingale measure (EMM). Using the EMM allows for discounting cash flows by means of the risk-free rate.

Consider two traded assets *f* and *g* with only one source of uncertainty, where:

$$\phi = \frac{f}{g} \tag{3.7}$$

and f and g follow the processes:

$$dg_t = \mu_g g_t dt + \sigma_g g_t dW_t \tag{3.8}$$

$$df_t = \mu_f f_t dt + \sigma_f f_t dW_t \tag{3.9}$$

Here *g* is referred to as the *deflator*, or the *numeraire security*. If there are cash flows to the assets (e.g. convenience yield) the relationship in equation (3.7) holds for the gains processes[10] of *f* and *g*. The EMM result shows that for some choice of market price of risk ϕ is a martingale; i.e. ϕ follows a zero-drift stochastic process. To achieve the desired market price of risk, the well known Girsanov's theorem (Neftci, 2000) may be used to adjust the Wiener process of the original process of ϕ. Deciding which asset to use as the numeraire is effectively the same as choosing the market price of risk. This is sometimes referred to as defining the probability measure. As shown in Hull (2009, chapter 27), the choice of market price of risk that makes ϕ a martingale is the volatility σ_g of *g*. Under this probability measure, which could be referred to as the *g-measure*, the process of *f* becomes:

$$df_t = \left(r + \sigma_f \sigma_g\right) f_t dt + \sigma_f f_t dW_t \tag{3.10}$$

The purpose of adjusting the probability measure such that ϕ becomes a martingale is that it allows the use for alternative discount rates. In particular, we want to use a probability measure such that we may discount using the risk-free rate. In order to discount using the

[10] The gains process for an asset is a process which reflects the total capital and cash flow gains accruing to the holder of the asset. Examples of cash flows to the holder of the assets are dividends, storage costs, and convenience yield. The latter is not necessarily a cash flow, but rather a net benefit resulting from the flexibility afforded by holding the asset.

risk-free rate we must use the probability measure which adjusts the cash flows to be "risk-neutral". The market price of risk after adjusting for the new probability measure would consequently have to be zero and this corresponds to using a "money market account" (MMA), our proxy for a riskless investment, as the numeraire. In the remainder of our thesis we will use the MMA with a constant interest rate as the numeraire, and refer to processes under this "risk-adjusted" or "risk-neutral" probability measure as being under the EMM. Many authors refer to processes under the EMM, with an MMA as the numeraire, as being under the Q-measure.

As explained in Chapter 2, the two main stochastic processes we focus on are a GBM and a mean reverting EOU. Under the EMM the GBM may be written as (cf. Appendix 1):

$$dS_t = (\mu - \lambda\sigma) S_t dt + \sigma S_t dW_t \tag{3.11}$$

where λ is the market price of risk for the asset. For underlying processes which are not traded assets, but where historical data for the evolution of the process is observed, λ may be estimated. However, one is confined to assuming a model, such as the CAPM[11], for this estimation. If the asset is a traded asset and markets are complete, it is not necessary to estimate the market price of risk, as the cash flows may be replicated in the market. For many real options the project itself is not a traded asset and the cash flows may not be replicated. To avoid the issue of estimating a market price of risk, one has to be able to replicate the cash flows from the project. This assumption is in the literature phrased as an assumption that *spanning* holds, or that a *twin asset* exists. If this assumption holds, the GBM under the EMM becomes (cf. Appendix 1):

$$dS_t = (r - \delta_t) S_t dt + \sigma S_t dW_t \tag{3.12}$$

where δ_t represents the convenience yield.

For the EOU, the process of the *logarithm of the price* under the EMM is (See Schwartz, 1997; Bjerksund and Ekern, 1995):

$$dX_t = \kappa\big((\alpha - \lambda) - X_t\big) dt + \sigma dW_t \tag{3.13}$$

[11] Using the CAPM to find λ, the expression $\lambda = \frac{\rho}{\sigma_m}(\mu_m - r)$ may be used for estimation (Hull 2009, pp. 748), where ρ is the correlation between the market returns and the returns on the asset, σ_m is the volatility of the market returns, and μ_m is the expected return in the market.

If spanning holds, the process of the price itself reduces to:

$$dS_t = (r - \delta_t) S_t dt + \sigma S_t dW_t \tag{3.14}$$

We see from equations (3.11) and (3.13) that when spanning does not hold the drift terms for both the GBM and the EOU are determined by the estimated parameters for the original processes, and an estimate of the market price of risk λ. When spanning does hold it is clear from equations (3.12) and (3.14) that the drift term for both the GBM and EOU processes are only dependent on the riskless rate of return and the convenience yield.

3.3 Option Pricing with Risk-Neutral Valuation

By relying on the EMM and risk-adjusting the drift terms of the underlying stochastic processes, the issues surrounding the estimation of a project discount rate may be avoided. However, calculating the value of flexibilities in real options is still complicated. Closed form solutions are the most computationally effective valuation techniques, and are therefore to be preferred over other solutions if attainable. However, most real options have American features. Closed form solutions do not exist for most[12] American options and one must use numerical methods to attain an appropriate valuation.

Schwartz and Trigeorgis (2001, pp.10) argue that there are generally two different types of numerical techniques for option valuation:

"(1) those that approximate the underlying stochastic process directly, and are generally more intuitive; and (2) those approximating the resulting partial differential equations [PDE]."

Among the first group, notable examples are binomial or trinomial lattices and Monte Carlo simulation. The second group includes numerical solutions to PDEs. We will briefly introduce solving PDEs numerically before outlining the Monte Carlo Simulation technique.

[12] One of the few examples of an American style option with a closed form solution is the American call option, which with no cash flows over the life of the option will never be exercised early, and thus has the same value as a European call option with the same underlying, strike price, and maturity (Hull, 2009).

3.3.1 Numerical Solutions for PDEs

Numerically solving PDEs is a widely used numerical technique among academics for valuing options. Solving PDEs using finite difference methods was first introduced by Brennan and Schwartz (1977). However, as suggested by Schwartz and Trigeorgis (2001), one may not always be able to write down the set of PDEs necessary to value a real option. Sick and Gamba (2005) go further to claim that practitioners face real options situations that are too diverse to justify building a PDE for each problem they encounter. Moreover, the finite difference methods are limited to three underlying risk factors because higher dimension PDEs may not be attained (Rodrigues and Armada, 2005).

3.3.2 Monte Carlo Simulation

Simulation techniques, such as the Monte Carlo simulation technique first introduced by Boyle (1977), model the distribution of the underlying asset(s) in order to evaluate options. For example, one could simulate the evolution of a stock price and use the resulting cash flows at maturity to price a European call option on the stock. Using the RNV framework, the risk adjustment would be reflected in the drift of the stock. The 'expected' cash flow to the call option, as estimated by the simulations, can then be discounted using a risk-free rate.

Monte Carlo simulation takes advantage of the law of large numbers[13] to obtain an accurate estimate of the option value. By simulating the underlying process a large number of times, Monte Carlo simulation is a method that enables a good representation of multiple interacting stochastic processes with fewer resources than what is required when creating binomial trees.

Since Monte Carlo simulation is forward looking it was at first not seen as appropriate for valuing American style options, which require backward-looking recursive techniques. More recently, several researchers have made suggestions for how backward-looking techniques may be used with Monte Carlo simulation to value American style options; the first solution was published in 1993 (Tilley, 1993). We will make use of a more recent contribution by Longstaff and Schwartz (2001): the Least Square Monte Carlo (LSMC) approach.

[13] The law of large numbers implies that when the underlying process is simulated enough times the average value obtained through simulation will be close to the actual expectation. Moreover, the higher moments observed through simulation, such as the variance, should also converge to the actual variance implied by the processes as the number of simulations increase. (Brandimarte, 2006)

3.4 Least Square Monte Carlo Simulation

With the use of the LSMC technique (Longstaff and Schwartz, 2001), one can estimate the optimal exercise strategy for an American style option. Since the underlying asset is already risk-adjusted, the resulting cash flows from the simulation can be discounted with the risk-free rate, and the option may be priced.

The power of the technique lies in its ability to estimate a continuation value for the option. Going backwards in time, starting at the maturity of the option, one must recursively solve for the optimal decisions in each state. To do this, one must be able to compare the value of exercising the option in that state, and the value of holding on to the option. Ordinary least square regression is used on the underlying risk factors as well as the value of the option in the subsequent period, in order to determine the expected continuation values.

"*No-arbitrage theory implies that the value of continuation is given by taking the expectation of the remaining discounted cash flows with respect to the risk-neutral pricing measure Q.*" (Longstaff and Schwartz, 2001, pp.121)

This exercise is repeated backwards in time until time zero, effectively finding the value of the option, given that the holder of the option follows the "optimal decisions" path calculated from the LSMC technique. Details of this technique will be explained further in Chapter 5, as the real options model is described.

4 Data Analysis

As the main purpose of this thesis is to examine how risk factors influence petroleum projects, we perform a time series analysis in order to find the underlying dynamics in some of the main risk factors that affect these investments. We analyze historical time series in order to test if the dynamics in oil and steel spot prices correspond to either of the two main theoretical models outlined in the previous chapters: Geometric Brownian Motion (GBM) and exponential Ornstein-Uhlenbeck (EOU).

4.1 Data

The time series analysis is based on several sets of data: spot and futures steel prices, spot and forward Brent oil prices, and U.S. treasury securities short-term yields. Prices for immediate delivery and current month delivery are used as proxies for spot rates of steel and Brent oil. The data series are based on daily observations. All values are nominal.

4.1.1 Steel Prices

In our analysis, we use a steel price index developed by HWWI, Hamburg Institute of International Economics: HWWI Iron Ore Steel Scrap price index in USD/ton. This index is composed of 70 percent iron ore prices, and 30 percent steel. Iron Ore is the raw material needed to make pig iron, which in turn is one of the main raw materials in steel. The data set includes daily prices from April 1, 1996 to April 1, 2010.

The second data series used is steel futures prices from London Metal Exchange (LME). LME Steel Billet Mediterranean cash spot in USD per metric ton (USD/MT) and LME Steel Billet Mediterranean 3 month futures USD/MT are used in order to find an estimate of the convenience yield of steel. The data set includes daily prices from July 24, 2008 to May 5, 2010.

The steel price data are summarized in Table 4.1.

Table 4.1 Steel Price Data

	HWWI Steel prices	LME Steel prices	
	Index	LME Cash	LME 3 month
Period	1996-2010	2008-2010	2008-2010
Number of observations	3654	465	465
Mean	192	423	425
Median	118	374	385
Standard deviation	117	156	143
Max	552	1120	1045
Min	92	260	255

The prices are quoted in USD/ton.

Different data sets are used for estimation of the steel price process and the estimation of the convenience yield of steel. Trading of steel futures on mercantile exchanges did not start until 2008, and short time series like the LME Cash generate thus insignificant answers regarding the dynamics of the steel price. Consequently, we use the HWWI Iron Ore Steel Scrap price index as a proxy for the steel price. This data set cannot be used to estimate the convenience yield however, since there are no available corresponding futures prices for this index. We therefore choose the LME spot and futures prices for the convenience yield calculations.

We are aware that there are weaknesses in these datasets and their ability to estimate a model for the traded steel prices, which may lead to model errors. There may also be additional errors because we estimate the convenience yield from a different time series than the one used for the process estimation. Moreover, production capacity may influence the price of finished steel, and the price index of steel and iron ore may not incorporate these restrictions fully. Quantity restrictions in the production capacity in steel production may be a major determinant for the steel price, a factor that may not be included in the iron ore price. However, we find these to be the best available data sets for our purposes.

4.1.2 Oil Prices

For the analysis of oil price dynamics, we use Brent crude oil prices, which is the type of crude oil delivered from the North Sea. In contrast to West Texas Intermediate (WTI) oil, which is perhaps the most quoted oil price, there is no traditional “spot” market for Brent oil. Buyers inform the sellers in advance how much oil they want to buy for a particular month, and the oil is then delivered according to contract conditions (Lin, 2007). We thus use an approximate spot price of oil, Crude Oil Brent Current Month Free-on-board[14] (FOB) in USD per barrel (USD/BBL) from ICIS Pricing. The data set includes daily prices from January 4, 1982 to April 26, 2010. In order to estimate the convenience yield, we use Crude Oil Brent 1 Month Forward FOB USD/BBL from ICIS Pricing.

Table 4.2 summarizes the oil price data.

Table 4.2 Oil Price Data

	Oil Brent Prices	
	Spot prices	**1 Month Forward**
Period	1982-2010	1985-2010
Number of observations	7386	6540
Mean	32	32
Median	25	21
Standard deviation	22	24
Max	146	147
Min	9	9

All prices are in USD/BBL.

With a time series as volatile as the oil price, daily observations for 28 years may not be enough to capture the long term price process of the commodity. This may cause noise or errors in the estimation. Even though time series data spanning a longer time frame may exist for WTI oil prices, we decide to use Brent oil prices because we are using investments in the North Sea as base cases.

4.1.3 Interest Rates

We use market yields on U.S Treasury securities at one month and three months constant maturity as interest rate prices for the estimation of convenience yields. The interest rate data is summarized in Table 4.3.

[14] Under a Free-On-Board (FOB) contract, the commodity is provided by the seller at a lifting installation and the buyer is responsible for shipping and freight insurance (Geman, 2005a).

Table 4.3 Interest Rate Data

	US Treasury Securities	
	1 Month	**3 Months**
Period	2001-2010	2001-2010
Number of observations	2286	2286
Mean	2.11	2.19
Median	1.71	1.72
Standard deviation	1.64	1.65
Max	5.27	5.19
Min	0.00	0.00

All values are in percentages.

US Treasury interest rates are used in order to estimate the convenience yield of steel and oil prices since the commodity prices used are quoted in USD. These can be used as a proxy for risk-free rates as there is generally little risk associated with these types of securities. Interest rates of different maturities are chosen to match the observed oil and steel futures and the observations are daily.

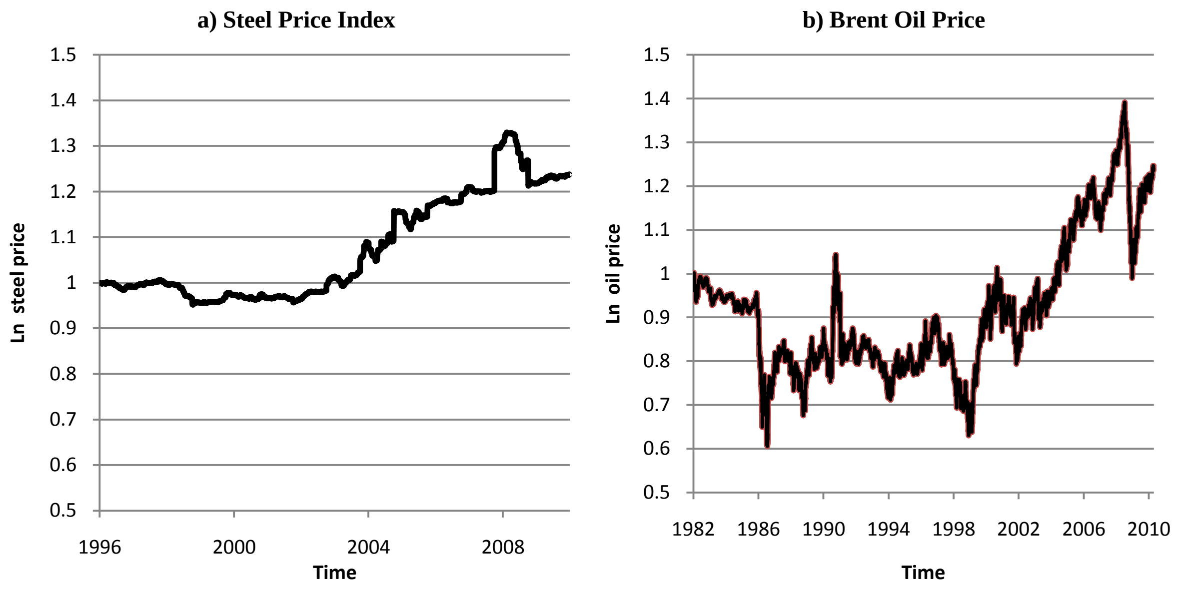
4.2 Price Process Analysis

We want to estimate processes that steel and oil spot prices follow over time. As explained in Chapter 2, the two main processes that are fitted to commodities are the mean reverting EOU process and the random walk with drift, i.e. GBM. The Box-Jenkins methodology is used to test these models on the data; the three steps are identification, estimation, and diagnostics.

4.2.1 Identification

Figure 4.1 displays the evolution of steel and oil prices over time. There are no evident structural breaks or missing values in the data. The boom and the subsequent financial crisis in 2007-2008 may be considered as extreme values for the commodity prices compared to the rest of the data series. However, the time series are short, and boom and busts are part of business cycle variations that will occur from time to time. The values in this period are assumed not to be extreme for these prices, and the entire data sets are thus included for our computations.

Figure 4.1 Historic Steel and Oil Prices



The values are the logarithm of the prices, normalized to start at 1. Note that the time scales differ between the graphs.

Both data series appear non-stationary in mean and variance. It is difficult to conclude whether the price indices follow a GBM (ARIMA(0,1,0)) or a long term mean reversion model like the EOU (AR(1)) from these data series. There might be an upward trend in both the steel prices and oil prices from 2002 until today, but this may also be just a peak in a longer term mean reversion.

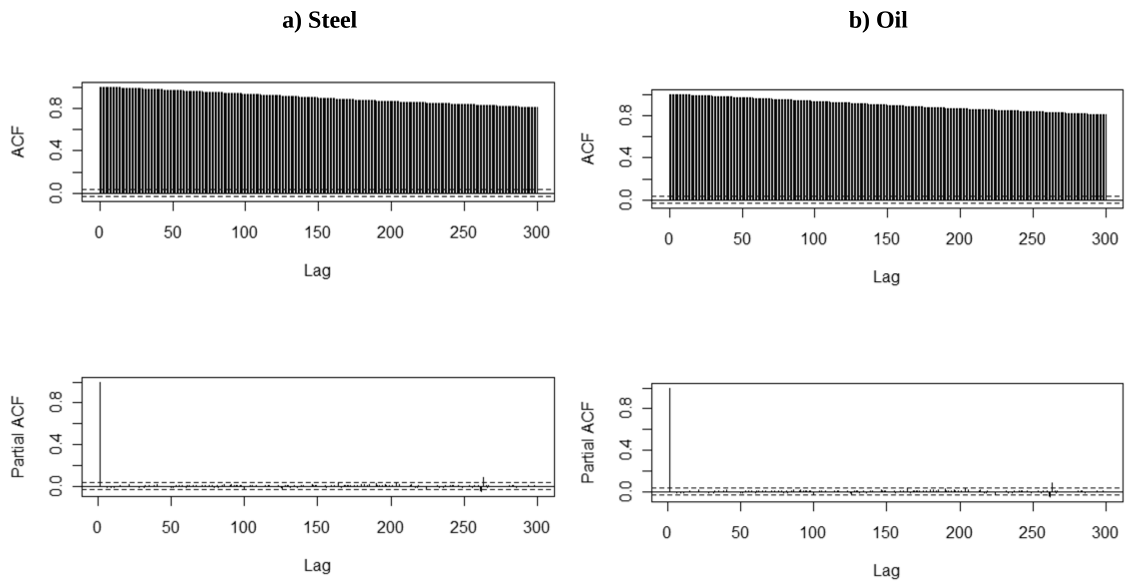
Autoregressive processes (AR) are generally characterized by a geometrically decaying autocorrelation function (ACF). The partial autocorrelation function (PACF) indicates the autoregressive order of the process. An AR(1) process will thus display a spike in the first lag of the PACF, with the rest of the lags insignificantly different from zero (Enders, 2004). If the underlying process has one or more unit roots, the ACF will decay slowly or not at all. Strict moving average (MA) processes display a geometrically decaying PACF and an ACF that indicate the order of the process. ARMA-models will display inconclusive autocorrelation graphs because of the interplay between the two process types. A time series is integrated of order d, denoted $I(d)$, if the dth difference of the time series is white noise[15] (Cowpertwait and Metcalfe, 2009). An integrated process ARIMA(0,1,0) will appear as an ARMA(0,0) process on the integrated time series, and thus exhibit no spikes in the PACF of the integrated series.

[15] A time series is white noise if the variables in the time series are independent and identically normally distributed with a mean of zero (Cowpertwait and Metcalfe, 2009).

Both data series have geometrically decaying autocorrelation functions (ACF) on the logarithmic prices, as depicted in Figure 4.2. The partial autocorrelation functions (PACF) both display a significant peak in the first lag. This may suggest that an autoregressive model of order one, AR(1), would fit. However, the ACFs are geometrically decaying very slowly, and both data series appear non stationary. This suggests that there are one or more unit roots. If unit roots exist, the time series will need to be differentiated (Enders, 2004).

Moreover, we notice seasonality in steel prices. There are approximately 260 days of trading per year and we notice a spike in the partial autocorrelation function at this lag. Seasonality in the data set may interfere with our model estimates. However, it is out of the scope of this thesis to deseasonalize the data before estimating the process of steel prices.

Figure 4.2 Autocorrelation Function and Partial Autocorrelation Function of Steel and Oil Prices



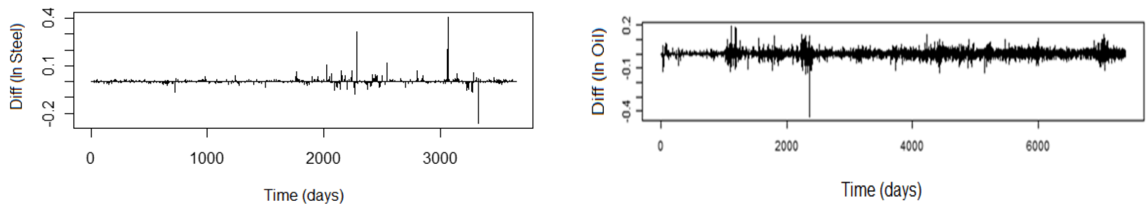
The autocorrelation and partial autocorrelation functions are estimated on logarithmic prices.

The Dickey-Fuller test (Enders, 2004) indicates that we cannot exclude the notion that there may be a unit root in the prices of both steel and oil. The 95 percent confidence interval for the estimated coefficient includes 1 for both data sets.

We take the first difference of the logarithmic series in order to construct stationary time series. Figure 4.3 depicts the differenced time series of steel and oil. It is now even more apparent that the oil price displays more volatility than the steel price index.

Figure 4.3 First Difference of the Logarithm of the Steel and Oil Prices

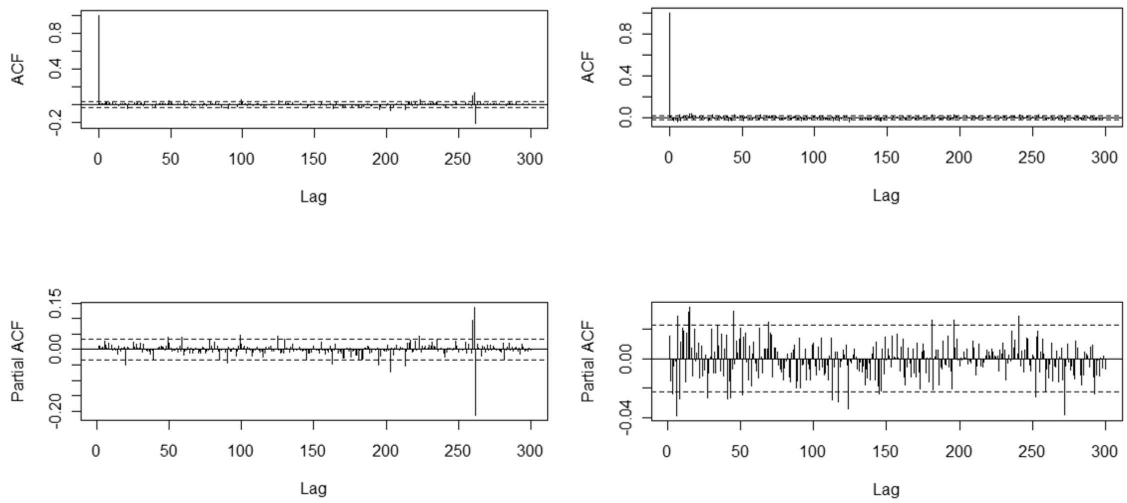
a) Steel **b) Oil**



The differenced time series can now be examined in order to estimate a suitable model. For the steel prices, apart from the apparent seasonality, there are few spikes or signs of an underlying model in the integrated data (Figure 4.4). The ACF and PACF converge to zero from the first lag. There are thus indications that the logarithmic returns display random walk, and that the logarithmic steel price index thus follows an ARIMA(0,1,0) with trend, which is analogous to a GBM.

Figure 4.4 ACF and PACF of the Difference of the Logarithmic Steel Price Index

a) Steel **b) Oil**



The oil price displays several small spikes in the partial autocorrelation function. This makes it difficult to predict which model is the best fit. We will thus test different models in the ARIMA-family in the next step of the Box-Jenkins methodology.

4.2.2 Estimation

The hypothesis for the steel price is that the logarithmic steel price follows a Geometric Brownian Motion. We want to estimate whether this fits the data, as well as looking at other ARIMA-models to see if they would be a better fit. For the oil price, it is difficult to predict which model would be the best fit. We will thus estimate different ARIMA-models in order to find the one that fits the best at the lowest order possible.

The two most common model selection criteria are the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) (Enders, 2004). Both information criteria value parsimony versus increased explanatory power. The lower their value, the better fit the model provides. BIC has superior properties for large samples (Enders, 2004) and will thus be our preferred criterion of choice.

The estimated parameters of different tested ARIMA-models are displayed in Table 4.4. Fitting different ARIMA models to the data set confirms to some extent our hypothesis for the logarithmic steel price. Although the results are not significant, the best fitting model for steel prices appears to be an ARIMA(0,1,0) according to the information criteria.

The results for oil prices are less conclusive. ARIMA(0,1,0) appears to be the best fitting model, according to the BIC, although neither of the tested models display significant results. We thus perform diagnostic test on both ARIMA(0,1,0) and ARIMA(1,1,0) on the logarithmic steel prices, as these appear to be the most appropriate models for the time series.

Table 4.4 ARIMA Models for Historic Steel and Oil Prices

Model	Constant/intercept	AR(1)	MA(1)	AIC	BIC
Steel					
ARIMA(0,1,0)	0.0003 (0.0002)			-22250	-22244
ARIMA(1,1,0)	0.0003 (0.0002)	0.0120 (0.0165)		-22249	-22236
ARIMA(0,1,1)	0.0003 (0.0002)		0.0118 (0.0163)	-22249	-22236
ARIMA(0,2,0)	0.0000 (0.0004)			-19758	-19746
Oil					
ARIMA(0,1,0)	0.0001 (0.0003)			-34857	-34850
ARIMA(1,1,0)	0.0001 (0.0003)	0.0153 (0.0116)		-34857	-34843
ARIMA(0,1,1)	0.0001 (0.0003)		0.0157 (0.0118)	-34857	-34843
ARIMA(0,2,0)	0.0000 (0.0004)			-29847	-29835

The models are fitted on the logarithm of the daily commodity prices. Coefficient estimates are followed by standard errors in parenthesis. Models with higher ARMA(p,q)-orders have been tested, but do not display better fits than the models presented above.

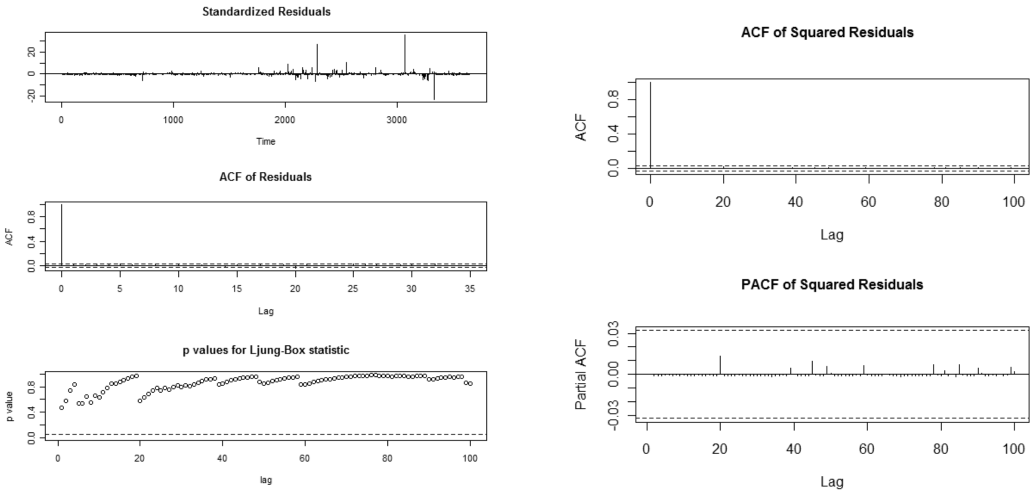
4.2.3 Diagnostics

Diagnostic tests are performed in order to check if the residuals of the fitted models are white noise. A well fitted model will have residuals that are white noise, i.e. that they have a mean of zero, a constant variance, and are uncorrelated over time (Enders, 2004).

The residuals of the ARIMA(0,1,0) model for the logarithmic steel price appear to be white noise, as presented in Figure 4.5. The figure shows that the residuals have a mean of zero, and that they are not significantly autocorrelated. The plotted p-values are significant at a 5 percent level, which indicates that we fail to reject the hypothesis that the residuals are white noise. The squared residuals are not autocorrelated at a 5 percent level, which supports the conclusion that the residuals appear to be white noise and the model fits with the data.

Figure 4.5 Residuals and Squared Residuals of ARIMA(0,1,0) on Logarithmic Steel Prices[16]

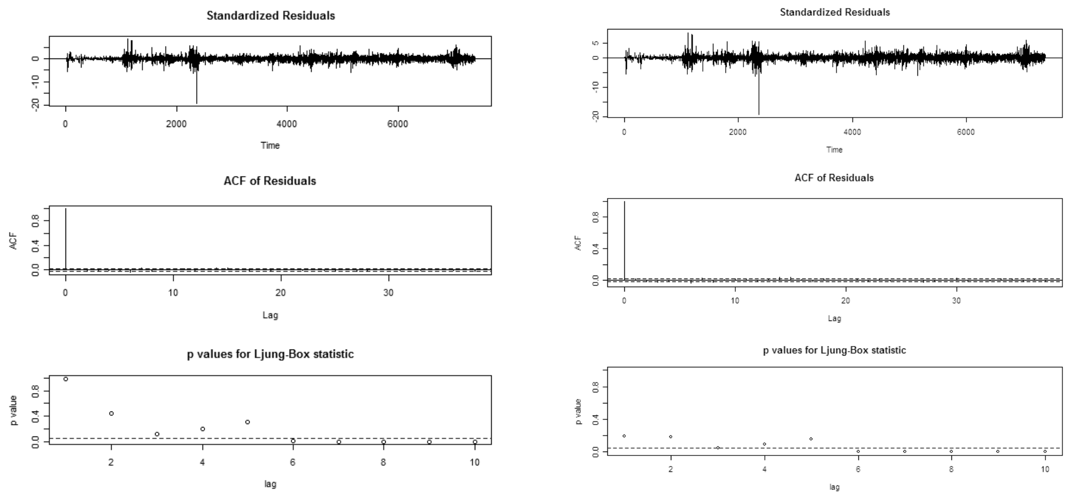
a) Residuals **b) Squared residuals**



For the oil price process estimation, we test the residuals of both ARIMA(0,1,0) and ARIMA(1,1,0) models. Both models display residuals that are not autocorrelated, as depicted in Figure 4.6. However, the p-values reject the hypothesis of white noise for most lags for both the ARIMA(1,1,0) and the ARIMA(0,1,0) model. None of the models appear to fit the oil price time series well.

Figure 4.6 Residuals of ARIMA(1,1,0) and ARIMA(0,1,0) on Logarithmic Oil Prices

a) ARIMA(1,1,0) **b) ARIMA(0,1,0)**

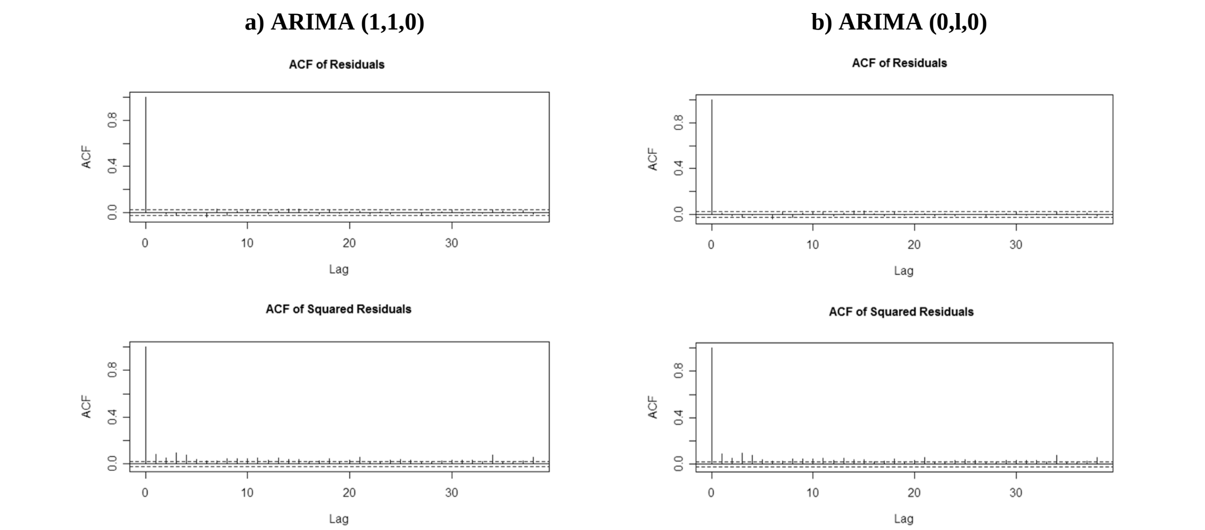


We test if the squared residuals of ARIMA(1,1,0) and ARIMA(0,1,0) on logarithmic oil prices are correlated; the results are depicted in Figure 4.7. There are volatility clusters in the

[16] Diagnostic tests are also performed on an ARIMA(1,1,0). However, the results reject that the corresponding residuals are white noise, which indicates that it is not a better fitting model.

squared residuals for both models, which indicates that there is conditional heteroskedasticity, or volatility clusters. This may indicate that a fitted General Autoregressive Conditional Heteroskedastic model (GARCH-model) on the residuals would be a better fit.

Figure 4.7 Autocorrelation Functions of the Residuals and the Residuals Squared for the ARIMA(1,1,0) and ARIMA(0,1,0) Models on Logarithmic Oil Prices



Note that there is a spike in lag zero in the ACF when using the program R, which is not a sign of autocorrelation in the residuals.

Lin (2007) finds that the AR(1)-GARCH(1,1)[17] is the most favorable model on the integrated logarithmic Brent spot price. Fitting such a model to our Brent spot prices, there are indications that this model could be a better fit. However, more tests would need to be performed in order to conclude whether the results from this model are significant, which is out of the scope for this thesis.

In conclusion, our results indicate that GBM may be the best fitting model between the two candidates, GBM and EOU, for both steel and oil prices.

4.3 Convenience Yield

The convenience yield, as defined in Section 2.1.8, is "*the flow of services which accrues to the owner of a physical inventory but not to the owner of a contract for future delivery*" (Brennan, 1991, pp.33).

[17] Where the GARCH model includes a stochastic volatility.

An annualized convenience yield, based on our daily observations, can be found by equation (4.1). We will use this relationship to find the instantaneous convenience yield, and will use spot (or a proxy for spot) as the $F_0\,(S,t)$.

$$\delta_{t,T} = r_{t,T} - \frac{1}{(T-t)} \ln\left[\frac{F_0\left(S,T\right)}{F_0\left(S,t\right)}\right] \tag{4.1}$$

The time series that are used to estimate the convenience yield are short, which implies that the estimates are imprecise and may not be good estimates of the convenience yield for the commodities in question. These are, however, the best estimates we can compute given the information available, and they will thus be used in the real options valuation.

Figure 4.8 displays the estimated convenience yields for steel and oil prices. Both estimated convenience yields appear mean reverting around an average slightly below zero percent. The average estimated convenience yield of steel is -4.18 percent, while the average convenience yield of oil is -3.30 percent. The extreme negative values during the financial turmoil in 2008 appear to be the main reason these averages are negative.

Figure 4.8 Estimated Convenience Yield for Historic Steel and Oil Prices

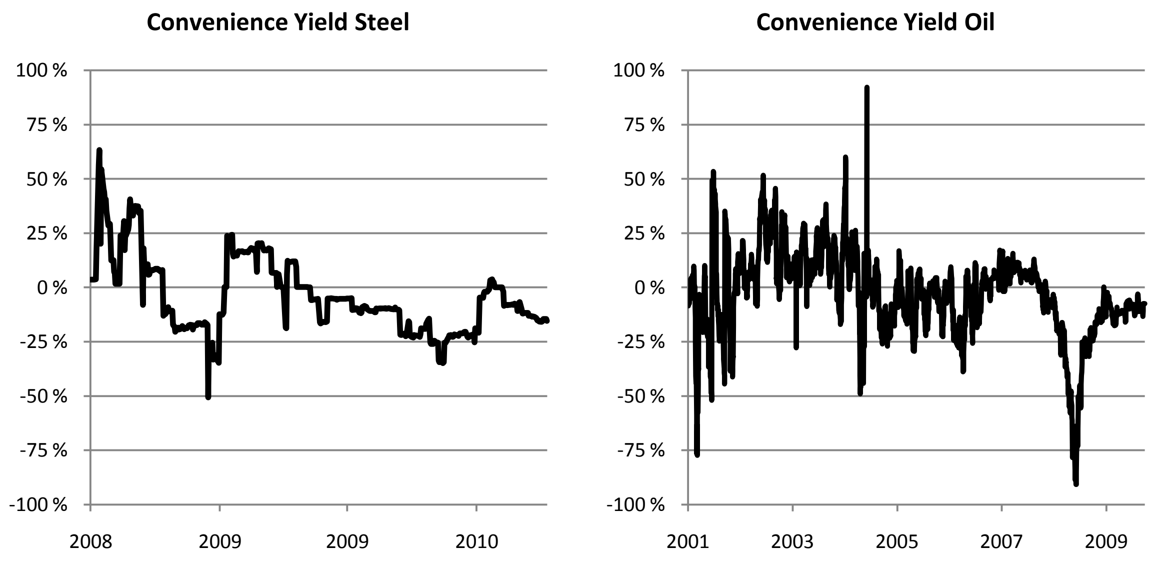


Table 4.5 shows descriptive statistics on the estimated convenience yields.

Table 4.5 Convenience Yield Data

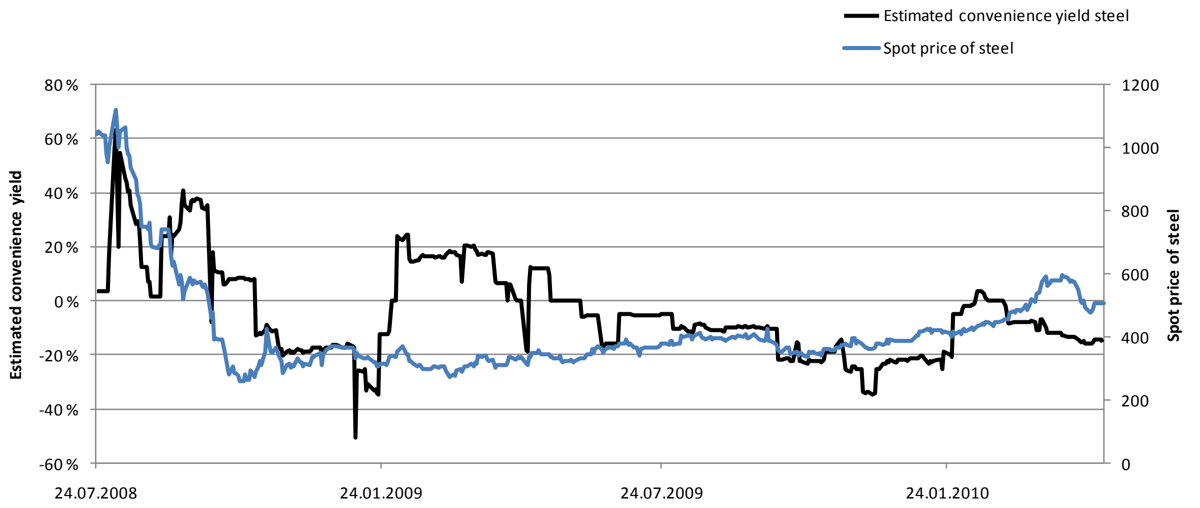
	Estimated Convenience Yield	
	Steel	**Oil**
Period	2008-2010	2001-2010
Number of observations	464	2280
Mean	-4.18 %	-3.30 %
Median	-8.51 %	-1.46 %
Standard deviation	17.51 %	20.23 %
Max	63.40 %	92.18 %
Min	-50.66 %	-90.77 %

These data are based on the estimated convenience yields.

The net convenience yield may be considered the difference between the value of having the commodity in stock less the storage cost. It is therefore reasonable to think that the convenience yield will be high when the commodity price is high, as the benefits of holding the commodity become apparent (Lin, 2007).

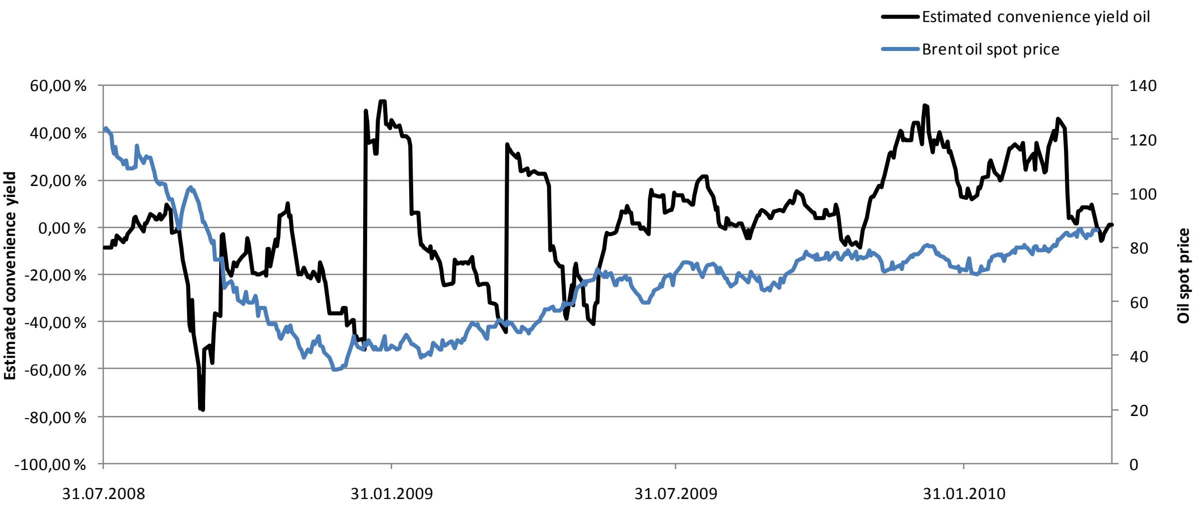
When comparing the estimated convenience yield of steel with the historic steel price, as depicted in Figure 4.9, we notice that there is some co-movement. However, the estimated convenience yield displays more volatile patterns than the steel price itself.

Figure 4.9 Estimated Convenience Yield and Steel Price



As depicted in Figure 4.10, the estimated convenience yield of oil displays even more volatility than the spot price over the entire period. There is co-movement; the convenience yield increases as the oil price increases. However, the convenience yield displays large jumps that are not present in the oil price. The most apparent examples are the jumps in convenience yield in first quarter of 2009. Although the time series used for estimation of convenience yield is short, these differences might indicate that a stochastic convenience yield may be appropriate, especially for oil prices.

Figure 4.10 Estimated Convenience Yield and Brent Spot Price

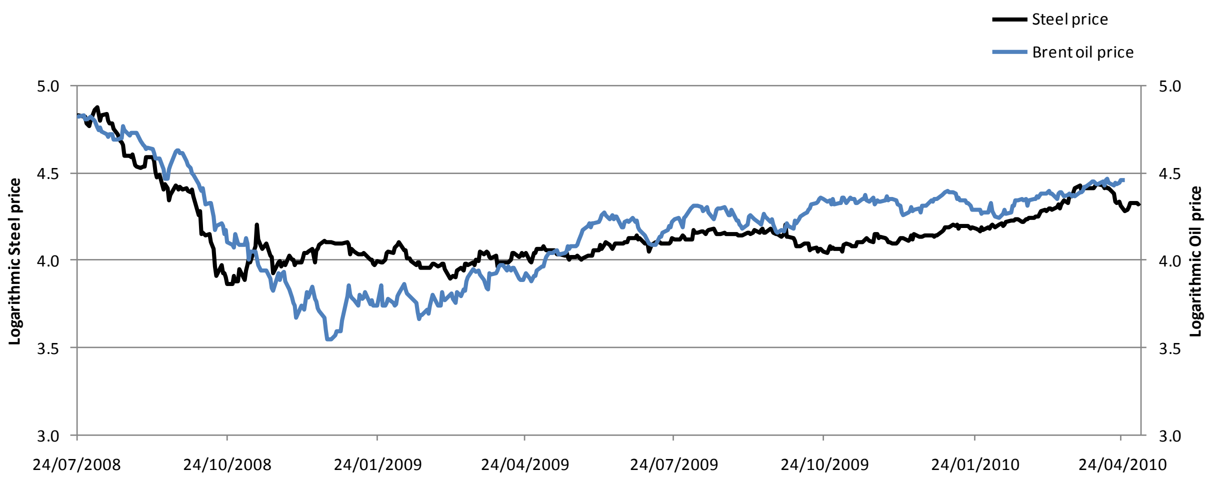


4.4 Correlation

In order to describe the stochastic features, we estimate the correlation between the steel price and the oil price data series. As displayed in Figure 4.11, the time series appear to be co-varying, especially for certain intervals.

The estimated correlation between the logarithmic data series is 0.9002. However, as they display a similar trend this is not a good estimate of the correlation. The correlation between the logarithmic returns of the price series is the variable that is interesting in this case. This correlation is estimated to be -0.0076, which is close to zero. This lack of correlation might be because the time series are different in that oil is considerably more traded than steel, which gives a significant difference in volatility in the logarithmic returns. It might also be that one of the time series lags the other.

Figure 4.11 Logarithmic Steel and Oil Prices



The logarithmic steel and oil prices are normalized to start at the same start value.

4.5 Concluding Remarks on the Data Analysis

Our aim was to test the steel price and oil price to find a realistic process to include in our real options model. Although the results are not statistically significant, we choose to use an ARIMA(0,1,0) to model both the steel price and the oil price. This is equivalent to a Geometric Brownian Motion model in continuous time. We emphasize that the results are weak – and especially inconclusive for the oil price – and our conclusions are mainly drawn in order to use illustrative processes in real option modeling. The results must therefore be viewed with some skepticism.

Models that include stochastic volatility and extra volatility factors to capture certain attributes (e.g skewness and kurtosis) might fit the data better. See Lin (2007) for a more advanced modeling of the Brent oil price process. Lin (2007) includes two volatility driven processes for the oil price, where one represents short-term shocks and the other a slow mean-reverting process. However, such complex models are out of scope of this thesis, and we focus on basic modeling in order to simulate this in the real options model.

Table 4.6 summarizes the structural annual parameters for an estimated Geometric Brownian Motion on the steel and oil prices. The parameters are used in the real options model in Chapter 6.

Table 4.6 Structural Parameters GBM for Steel and Oil Prices

	Oil	**Steel**
μ	9.74%	9.88%
σ	36.83 %	18.55 %
δ	-3.30 %	-4.18 %

The parameters are estimated from daily observations and converted to annual figures.

Baker et al. (1998) comment that unit root tests like Dickey-Fuller make it difficult to reject random walk because it is hard to find large enough samples. Pindyck and Rubinfeld (1991) applied the test to a large enough sample (114 years) for crude oil, copper and lumber and found that they could reject random walk for crude oil and copper at a 5% level, but not lumber. Our samples are much smaller than this, which may be why we obtain inconclusive results.

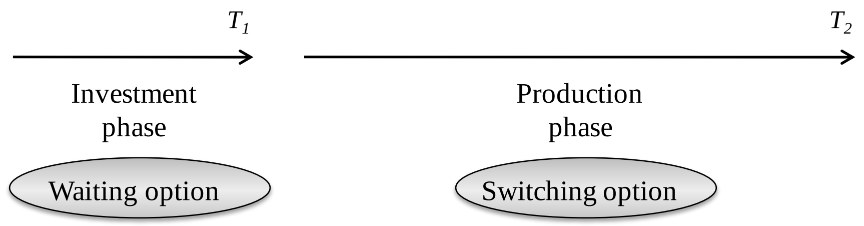
Our results coincide with the conclusions of a number of authors in recent literature who find that the oil price follows a GBM process, for example Geman (2005a), or that find that a GBM may be just as good as EOU, such as Pindyck (1988, cited in Bjerksund and Ekern,1990).

5 Real Options Model

To evaluate different flexibilities prevalent in petroleum projects we have constructed a model that includes two option types with distinctively different features. The two types of real options we look at are a so-called switching option, which allows the managers to switch between operating modes, and an option to invest. The option to invest may be seen as an option to postpone an investment decision and we will refer to it as a waiting option. While a switching option is an operational flexibility, the option to wait allows flexibility in an investment decision. This allows us to examine flexibilities that are inherent in two separate phases of a petroleum project.

For the sake of notational convenience, the duration of the waiting option will be denoted T_1, while the equivalent measure for the switching option is denoted T_2.

Figure 5.1 Timeline of Project Phases and Options



The flexibility in our switching option model allows managers to switch between operating modes, or levels. More specifically, for an oil field in production, this could include an option to adjust the extraction rate, temporarily cease production, or even abandon the field entirely. In our switching option model we look at projects with all of these flexibilities combined, but also allow for combinations of fewer flexibilities in order to discern, to some extent, how they individually add value to the project. The two sources of uncertainty that we model in the switching option are oil prices and variable costs. We are particularly interested in the effects of going from deterministic variable costs (with constant drift, but without volatility) to stochastic variable costs, and how correlation between the oil price and the variable costs affect the value of the project.

For the waiting option, the managers have the flexibility to defer an investment decision. While for some projects an investment decision may have to be made at a specific point in time, the waiting option has a window of time in which the company may choose to invest. The value obtained from this option, if the company chooses to invest, is the value of all future cash flows from the project less the investment costs. The investment costs are, in our model, closely related to steel prices. The model is used to examine the effects of stochastic investment costs on the optimal decision time and the value of the option. Moreover, the effects of correlation between the value of all future cash flows from the field and the investment cost will be analyzed.

To price the switching option and the waiting option we rely on Monte Carlo simulation. As discussed in earlier chapters we simulate the underlying risk processes under the equivalent martingale measure in order to use a risk-free rate for discounting. The Least Square Monte Carlo method is used to calculate expected values, or continuation values. This is necessary in order to find the optimal decisions for all the simulated paths, and thus ultimately derive the value of the option. For the remainder of the chapter we will go through the framework of our model and the technicalities of pricing the switching option and the waiting option. At the end of the chapter, some issues surrounding simulation and the general build-up of the model are discussed.

5.1 Option to Switch Between Operating Modes

Our model for valuing a switching option in a petroleum project is extended from models by Brennan and Schwartz (1985) and Cortazar, Gravet and Urzua (2008). Brennan and Schwartz (1985) looked at an American-style switching option for a copper mine with the options to initiate production, temporarily cease production, and abandon. Their analysis was based on a one-factor model for uncertain copper prices and the valuation was done using finite difference methods. Cortazar, Gravet and Urzua (2008) developed this model further by introducing a multi-factor model for the uncertain commodity prices. They show how a Least Square Monte Carlo approach may be used to value the real option in such a high-dimension problem. However, neither Brennan and Schwartz (1985) nor Cortazar, Gravet and Urzua (2008) incorporate uncertain cost factors in their models.

In our base case for the switching option, we include the flexibilities to choose between producing at a high level, producing at a low level, temporarily ceasing production, or abandoning the project. An abandoned project may not be reopened and will not have any value or cash flows after abandonment. The three different modes of production where managers still have flexibility and value, are thus:

$$j = [producing\ high, producing\ low, closed] \tag{5.1}$$

The terms *closed* and *temporarily closed* will be used interchangeably. The term *open* will be used to denote situations in which the platform is in production, producing at either a *high* or a *low* rate.

There are costs associated with switching from one mode of production to another. From all modes, the production may be abandoned by paying a cost *A* to scrap the platform, but then the production can never start again. However, if the platform is temporarily closed, it may be reopened again at a cost. Likewise, the platform can go from any of the open states to closed at a cost. There are also switching costs for switching between the production states of high and low. During the period in which the platform is closed, there is a periodic maintenance cost of *M*. These costs are summarized in table 5.1.

Table 5.1 Cost Input Switching Option

Input name	Explanation
Chl	Cost of reducing from high to low production
Clh	Cost of increasing from low to high production
Ch0	Cost of reducing from high production to temporarily closed
Cl0	Cost of reducing from low production to temporarily closed
C0l	Cost of increasing from temporarily closed to low production
C0h	Cost of increasing from temporarily closed to high production
M	Annual cost of maintenance for a temporarily closed platform
A	Cost to scrap the platform (abandonment)

There are two underlying risk factors that are included in our switching option model: the price of oil and variable costs. The spot price level of these factors will be denoted S_1 and S_2 respectively. The development of both spot prices will be simulated with *N* number of replications, and ω will refer to a particular simulated path. When the platform is open, the cash flow is represented by equation (5.2). The cash flow depends on the current reserve level *Q*, the production rate q_j for the given production mode *j*, and the level of the underlying risk factors. When the platform is temporarily closed, the only cash flow is the maintenance cost. If the platform is abandoned, the cash flow is equal to the scrap costs, but thereafter there is no cash flow from a platform that is considered abandoned. The cash flow from production is represented by:

$$CF_t(S_1(\omega), S_2(\omega), q, Q) = q_j \Delta t[Revenue(S_1(\omega)) - VarCost(S_2(\omega))] \tag{5.2}$$

We consider a switching option with value $V_t\,(S_1, S_2, j, Q)$ and cash flow $CF_t\,(S_1, S_2, j, Q)$ at time t. The LSMC (Longstaff and Schwartz, 2001) framework may be applied to find the initial value V_t of a switching option producing for T_2 years. Following the LSMC method the value may be found by starting at the final time period of the project, and recursively working backwards to find the optimal decisions for all simulated paths. We will use discrete time steps of Δt in our model. For the switching option we assume that the project is terminated at time T_2, and there is no longer a continuation value or cash flows from production after this. There will however be a scrapping cost at time T_2 if the platform has not already been abandoned. The continuation value $\hat{F}^{j}_{T_2-\Delta t}$ for all states at time T_2-Δt (one step before the terminal date) therefore becomes the discounted scrap cost. Equation (5.3) and (5.4) represent the value at time T_2 and the continuation value at time T_2-Δt respectively:

$$V^{j}_{T_2}\left(S_1(\omega), S_2(\omega), Q\right) = -A \qquad \forall\,\omega,\ \forall\, j,\ \forall Q \tag{5.3}$$

$$\hat{F}^{j}_{T_2-\Delta t}\left(S_1(\omega), S_2(\omega), j, Q\right) = -Ae^{(-r\Delta t)} \qquad \forall\,\omega,\ \forall\, j,\ \forall Q \tag{5.4}$$

The value at time T_2-Δt may now be calculated for all possible states, by maximizing across the different options available for all the respective modes and possible states. To illustrate this, consider a platform which operates at high production at T_2-Δt, and the reserves available are enough to extract at any of the possible production levels in the last period. The platform may continue in the same mode, producing high, or it may reduce to producing low, temporarily close, or even abandon. Optimizing the value at T_2-Δt, one needs to maximize the expected value given these four options. We denote $E\left[V^{High}_{T_2-\Delta t}\right]$ the expected value of the option at time T_2-Δt if the platform was producing in *high* mode in the previous period. The maximizing problem thus becomes:

$$E\left[V^{High}_{T_2-\Delta t}\right] = \max\left(\hat{F}^{High}_{T_2-\Delta t} + CF^{High}\ ;\ \hat{F}^{Low}_{T_2-\Delta t} + CF^{Low} - Chl\ ;\ \hat{F}^{Closed}_{T_2-\Delta t,} - M - Ch0\ ;\ -A\right) \tag{5.5}$$

The procedure for determining the values at T_2-Δt may quite trivially be transferred to cases in which the previous production mode was different from *high:*

$$E\left[V^{Low}_{T_2-\Delta t}\right] = \max\left(\hat{F}^{High}_{T_2-\Delta t} + CF^{High} - Clh\ ;\ \hat{F}^{Low}_{T_2-\Delta t} + CF^{Low}\ ;\ \hat{F}^{Closed}_{T_2-\Delta t} - M - Cl0\ ;\ -A\right) \tag{5.6}$$

$$E\left[V_{T_2-\Delta t}^{Closed}\right]=\max\left(\hat{F}_{T_2-\Delta t}^{High}+CF^{High}-C0h\ ;\ \hat{F}_{T_2-\Delta t}^{Low}+CF^{Low}-C0l\ ;\ \hat{F}_{T_2-\Delta t}^{Closed}-M\ ;\ -A\right) \quad (5.7)$$

Since the continuation value at time T_2-Δt is solely the discounted scrap cost, there is no uncertainty regarding this value. The expected value at T_2-Δt is thus equal to the realized value for all production modes:

$$E\left[V_{T_2-\Delta t}^{j}\right]=V_{T_2-\Delta t}^{j} \qquad \forall j \quad (5.8)$$

Moving one more time step backwards to T_2-$2\Delta t$, the calculation of the continuation values is not as trivial anymore, and we will rely on least square regression as proposed by Longstaff and Schwartz (2001). The complexity of the switching option in general, and the continuation values in particular, increase significantly because of the number of dimensions that need to be considered. Continuation values must be estimated along four dimensions:

- **Time**: The time dimension ranges from t to the maturity of the option, including all intermediary points where decisions can be made.
- **Underlying risk factors**: All possible outcomes of the risk factors S_1 and S_2 must be considered. The simulation represents an estimate of the distribution of the underlying risk factors across time.
- **State of production**: In our base case there are four different states of production. The state of production at a given decision point is the level chosen at the previous decision point.
- **Level of reserves:** The level of reserves may vary from the initial level of reserves Q_{max} to 0 (given that there is enough time to potentially deplete the reservoir).

To find the continuation values at time T_2-$2\Delta t$, we regress the discounted values from time T_2-Δt on the underlying risk factors at time T_2-$2\Delta t$. This regression, as mentioned above, is done for all possible production states and all possible reserve levels. The regression is performed on a combination of regression basis functions for the underlying risk factors $\boldsymbol{L}(\boldsymbol{S})$, where $\boldsymbol{L}$ is the basis of functional forms of the combination of underlying risk variables $\boldsymbol{S}$:

$$\left[V_{T_2-\Delta t}^{j}\left(\boldsymbol{S},Q\right)\times e^{(-r\Delta t)}\right]=\boldsymbol{L}_{T_2-2\Delta t}\left(\boldsymbol{S}\right)\left[a_{j,Q,T_2-2\Delta t}\right]+\varepsilon \quad (5.9)$$

The choice of basis functions will be discussed in section 5.1.2. We then use the optimal coefficients $\hat{a}$ from the regression to estimate the expected continuation values:

$$\left[\hat{\boldsymbol{F}}^{j}_{Q,T_2-2\Delta}\right] = \boldsymbol{L}_{T_1-2\Delta t}\left(\boldsymbol{S}\right)\left[\hat{a}^{j}_{Q,T_2-2\Delta t}\right] \tag{5.10}$$

For a set B of basis functions the continuation values dependent on the vector of prices $\boldsymbol{S}$ at time T_2-$2\Delta t$ is then determined by:

$$\hat{F}^{j}_{Q,T_2-2\Delta}\left(\boldsymbol{S}\right) = \sum_{i=1}^{B} \hat{a}^{j,i}_{Q,T_2-2\Delta t} L^{i}_{T_2-2\Delta t}\left(\boldsymbol{S}\right) \tag{5.11}$$

This continuation value will be used in equations analogous to (5.5), (5.6), and (5.7) to find the optimal decisions at time T_2-$2\Delta t$. However, after choosing the optimal decision for all possible states, the values used for $\hat{V}^{j}_{T_2-2\Delta t}$ should be the realized and not the expected values. This is because the maximum functions used in equations (5.5), (5.6), and (5.7) are convex functions and therefore the expected values estimated by the regression would, following the reasoning of Jensen's inequality (cf. Jensen, 1906 or Musiela and Rutkowski, 2005), on average be higher than the realized values. To avoid a potential upward bias in the option values, we use the realized values instead of the expected values. Proceeding by using the realized values as the values for T_2-$2\Delta t$, the above procedure is repeated recursively until reaching time $t+\Delta t$.

The continuation value at the start time t of the switching option is found by calculating the average of all the discounted values at time $t+\Delta t$. We assume that there is a construction period with no cash flows spanning from time t and until time Δt. Moreover, the initial state after the construction period is assumed to be high production. There are no decisions at time t or cash flows during the first period. With N number of simulated paths the value at time t thus becomes:

$$V_t\left(S_{1,t=t}, S_{2,t=t}, Q_{max}\right) = \frac{1}{\mathrm{N}} \sum_{i=1}^{\mathrm{N}} e^{-r\Delta t}\left(V^{High,i}_{t+\Delta t}\right) \tag{5.12}$$

5.1.1 Reserve Levels

The level of oil reserves is a complicating factor that adds to the dimension along which the continuation values have to be calculated. The initial amount of oil reserves is Q_{max}, but as time progresses the possible states of reserves increases. The total number of possible reserve levels tends towards:

$$R = \frac{Q_{\max}}{q_{Low}\Delta t} \tag{5.13}$$

The lowest production level is represented by q_{Low}, and Δt is the time between each decision point.

This relationship only holds true at a point in time where it is a possible state that the field is depleted. At an intermediary point in time the possible states of the reserves will be limited by the maximum level that could have been extracted. For example, one time step after production has started, the possible states of reserves are Q_{max}, $(Q_{max} - q_{High}\Delta t)$, and $(Q_{max} - q_{Low}\Delta t)$. In other words, with the maximum reserves as a starting point, the production could have been either *0*, *high*, or *low* over the first time period. Given the possible states of the reserves and the four different states of production, the total possible states of the field thus tend towards $4R$.

5.1.2 Regression and Basis Functions

In order to perform the LSMC regression, we apply a constant and polynomials of both state variables up to the fifth degree as basis functions. We also add polynomials up to the order of five of the relationship between the oil price and the variable costs $\left(\frac{s_1}{s_2}\right)$. Increasing the number of basis functions beyond this does not change the option values obtained significantly, and for the sake of computational efficiency we want to use as few basis functions as possible without losing accuracy. As argued by Longstaff and Schwartz (2001), one should continue to increase the number of basis functions until the estimated value does not change. With too few basis functions the estimated values should have a negative bias. Longstaff and Schwartz (2001) extensively discuss the possible basis functions to use in the regression. Based on their conclusions we have chosen quite simple basis functions for our analysis. Although not a robust result, their tests on a higher dimension problem suggest that using simple basis functions should not alter our results significantly. Moreover, if more risk factors were to be introduced, Longstaff and Schwartz argue that the number of basis functions will be quite manageable, and will need to increase less than exponentially when more state variables are introduced.

5.2 Option to Wait

The other basic flexibility evaluated in our model is an option to invest in a petroleum project. A company has the right, but not the obligation, to start investment in the project at any given time between time t until time T_1. An option to invest in a project can be considered as a finite-lived American option since the decision can be made once within this time period, where T_1 is the end of the right to start investment in the field. This could also be referred to as an *option to wait*, or postpone an investment decision.

In order to maximize the value of the flexibility in this option, the company needs to analyze when they should invest, given the price paths of underlying risk factors. The company will invest when the value of the investment exceeds the continuation value.

The *investment value* is the value of investing in a platform that will be able to produce for T_2 years after the investment is initiated. Ideally we would calculate the investment option value using our switching option model, using the asset prices at the time of investment as the start prices of the simulation. As explained in section 5.1, we use the simulated asset prices S_1 and S_2 to determine the optimal decision at each time step of the switching option, and thus the value of the investment. This would effectively be an *option to invest in a switching option* (a compound option). However, this approach is too computationally demanding for the resources we have available. We will therefore use highly simplified and stylized examples when determining the value of investing in the field for a given level of the underlying risk factors. Specifically, the investment cost of developing the field will be a linear function of simulated steel prices S_3, and the value of all future cash flows from the developed field (referred to here as the *investment value*) will be a linear function of the simulated oil prices S_1.

Similarly to the method used to price the switching option, we start at the expiration date T_1 and work our way backwards in time to find the optimal choice of investment. At expiration the value of the option is given by:

$$V_{T_1} = \max\left(InvestmentValue_{T_1}(S_1) - InvestmentCost_{T_1}(S_3)\ ;\ 0 \right) \tag{5.14}$$

The value of exercising at a given point in time, the investment value less the investment costs, is defined as the *intrinsic value*:

$$IntrinsicValue_t = InvestmentValue_t(S_1) - InvestmentCost_t(S_3) \tag{5.15}$$

Going one time step back in time to T_1-Δt, we find the continuation values by regressing the discounted values $e^{-(r\Delta t)}V_{T_1}$ on the underlying factor prices S_1, and S_3 at time $T_1 - \Delta t$. By using the regression coefficients and the vector of simulated spot prices at time $T_1 - \Delta t$, we can compute an estimate of the continuation values $\hat{F}_{T_1-\Delta t}$. Continuation values are calculated using the procedure in equations (5.9), (5.10), and (5.11) where the chosen basis functions for the waiting option are the same as for the switching option. Unlike for the switching option however, not all simulated paths are included in the regression. As noted by Longstaff and Schwartz (2001), by removing all the price paths that would not give a positive value from immediate exercise at T_1-Δt the regression is improved significantly. Therefore the regression to find continuation values is only performed for paths that are "in the money":

$$InvestmentValue_{T_1-\Delta t}(S_1(\omega)) > InvestmentCost_{T_1-\Delta t}(S_3(\omega)) \tag{5.16}$$

At time T_1-Δt the company chooses the highest value of the expected continuation value and the immediate exercise of the option. Illustrating this for the specific paths ω of simulated spot prices:

$$E\left[V^{\omega}_{T_1-\Delta t}\right] = max(IntrinsicValue_{T_1-\Delta t}(S_1(\omega), S_3(\omega))\ ;\ \hat{F}^{\omega}_{T_1-\Delta t}) \qquad \text{for } \omega = 1, \ldots, \text{N} \tag{5.17}$$

Similarly to the procedure used for the switching option, we use the realized values as the option values at time T_1-Δt, instead of the expected value. If the optimal choice is exercising the option, then it is apparent that the expected value is the same as the realized value. Going back in time, these steps are repeated at each decision point, until we reach time $t+\Delta t$. The option value at time t is calculated as the maximum of immediate exercise or the average value for all the price paths at time $t+\Delta t$. At time t the prices are at the current spot price, so the intrinsic value is known. The value at time t is calculated as:

$$V_t = max\left(IntrinsicValue_t \ ; \ \frac{1}{\mathrm{N}}\sum_{i=1}^{\mathrm{N}}\left(V_{t+\Delta t}^{i}\right)e^{-r\Delta t}\right) \tag{5.18}$$

5.3 Simulation

In accordance with the theoretical framework we use, all underlying risk factors must be simulated under the equivalent martingale measure. This technique allows us to use a discount rate equal to the "riskless" rate of return, which is assumed to be constant in our model. To simulate all the underlying risk factors most efficiently, and allow for correlation, the model performs the simulation in four intuitive steps:

1. All the needed standard normal variables are randomly drawn using a random number generator.
2. If any of the underlying risk factors are correlated, a Cholesky transformation[18] of the correlation matrix is used to correlate the standard normal variables.
3. The paths are simulated using the standard normal variables from step 1 and 2, taking into account the stochastic process of all the underlying risk factors.
4. Our model uses an antithetic variates technique to reduce the variance in our value estimates. The most obvious way to reduce the variance is by increasing the number of simulations, effectively drawing more random numbers. However, this may be very resource demanding. Consider an option value V that is calculated by drawing a set ε of random numbers. Using antithetic variates the value of the option would instead be found by initially calculating two option values. First we draw a set ε_1 of random numbers which is half as many as in ε. We then calculate an option value V_1 from ε_1, and another option value V_2 from ε_2, where $\varepsilon_2 = (-\varepsilon_1)$. The final option value is calculated by finding the average of the two values, $V_A = \frac{V_1+V_2}{2}$. While both techniques simulate the same amount of paths by drawing the same amount of random numbers, the variance for the value V_A, found by using antithetic variates, is less than for the value V (see for example Hull, 2009 pp.433 or Brandimarte, 2006 pp.244-251).

[18] See Brandimarte (2006)

In our analysis of a switching option, the underlying risk factors are the oil price and an aggregated variable cost index. For the waiting option, the price of oil and the price of steel are the risk factors. Depending on the stochastic process used, the model has two different methods of simulating. If the process has a known solution, as is the case for both GBM and EOU, the simulation is performed using this solution. However, the model may also simulate using a discretization technique: the Euler scheme[19]. By using a known solution for the stochastic differential equation, the potential problem of too large time intervals is avoided, which may be a problem when using the Euler scheme. Since our analysis will use GBM processes with known solutions, the issues of discretization are not considered further.

The model is programmed using the Matlab programming language. Matlab is an efficient tool for working with matrices and vectors. There are several factors in the model that make effective vectorization a necessity to utilize computer processor capacity, such as the need to work with a high number of simulated price paths, and many different decision dates.

[19] See Brandimarte (2006)

6 Model Analysis

Our analysis of the switching and waiting options examines how different flexibilities and parameter values affect the value of the real options and the optimal decisions of petroleum companies. Our main focus is to look at the effects of introducing stochastic costs and correlations between income and cost risk factors. The switching option will be analyzed first followed by similar analyses for the waiting option.

6.1 General Assumptions

Our assumptions are mainly based on Norwegian conditions. All values are nominal. We assume a constant nominal risk-free interest rate of 4 percent. Since our valuation analysis includes long term projects and we do not include stochastic interest rates, 4 percent is chosen to represent a reasonable long-run average of risk-free interest rates. For simplicity we have not included taxes in this model. All values are in US dollars (USD).

As explained in Chapter 2, petroleum fields have different features of costs, amount of reserves, and geology. The project-specific parameters chosen for the switching and waiting option are thus assumed to be reasonable examples, but cannot be thought of as general parameters applicable to all field types.

We assume that the underlying risk factors follow a Geometric Brownian Motion price process with parameters as estimated in Chapter 4. Complete markets and spanning assets are assumed, which means that prices are simulated using the risk-adjusted process in equation (3.12). The base case parameters used for the risk-neutral simulation of price processes are summarized in Table 6.1. In the base case we assume a constant net convenience yield δ for oil and steel spot prices, equal to the average estimated convenience yields from Chapter 4. The simulated prices are all spot prices. In all cases the oil price starts at USD 83, which is the price per April 26, 2010.

Since we do not have good time series available for the variable costs, and they encompass a plethora of underlying risk factors, we must make some simplifying assumptions. The variable costs that we simulate will represent the total production costs, and is thus an index consisting of several underlying cost factors. We will refer to these aggregated variable costs simply as variable costs henceforth. For the purpose of illustration we will assume that the variable costs also follow a Geometric Brownian Motion. Furthermore, we assume that there is no net convenience yield associated with the variable costs.

Table 6.1 Process Parameter Assumptions

	Oil GBM	Steel GBM	Variable cost index
S0	83	350	60
δ	-3.30 %	-4.18 %	-
σ	36.83 %	18.55 %	30.00 %

The number of replications used in the simulations depends on the analysis performed and is set to the maximum number given the available computer capacity. The replications vary from 300 000 to 1 000 000. This is done in order to get an estimate as accurate as possible given the available resources. For simplicity we assume that the company can make decisions according to the different flexibilities once a year. This means that, for the waiting option, the decision makers must evaluate once per year whether or not to invest. Similarly, for the switching option, the company will have the opportunity once per year to switch operating mode.

We focus the analysis on examining the effects of introducing stochastic costs. To illustrate these effects we compare the effects in terms of option values and optimal decisions in a field with "*deterministic*" costs with the introduction of *stochastic* costs. The deterministic costs have a risk-adjusted drift equal to the risk-free rate and display no volatility. This means that the deterministic costs are equal to the *expected* value of the stochastic costs at all times.

6.2 Switching Option

In the switching option we consider a project that starts after the test drilling has revealed a known amount of reserves Q_{max} in a field, and after the decision to invest has been made. The investment cost is not included in the switching option, as it is considered in the waiting option. We simplify the analysis by suggesting that it will take only a year from the decision to invest until the company can start the extraction. As in the case for most Norwegian projects, we assume that the license to extract and develop a field is granted for a period of 30 years (Norwegian Petroleum Directorate, 2010). We denote this period T_2. The base case assumptions for the switching option are summarized in Table 6.2.

Table 6.2 Switching Option Parameters Assumptions

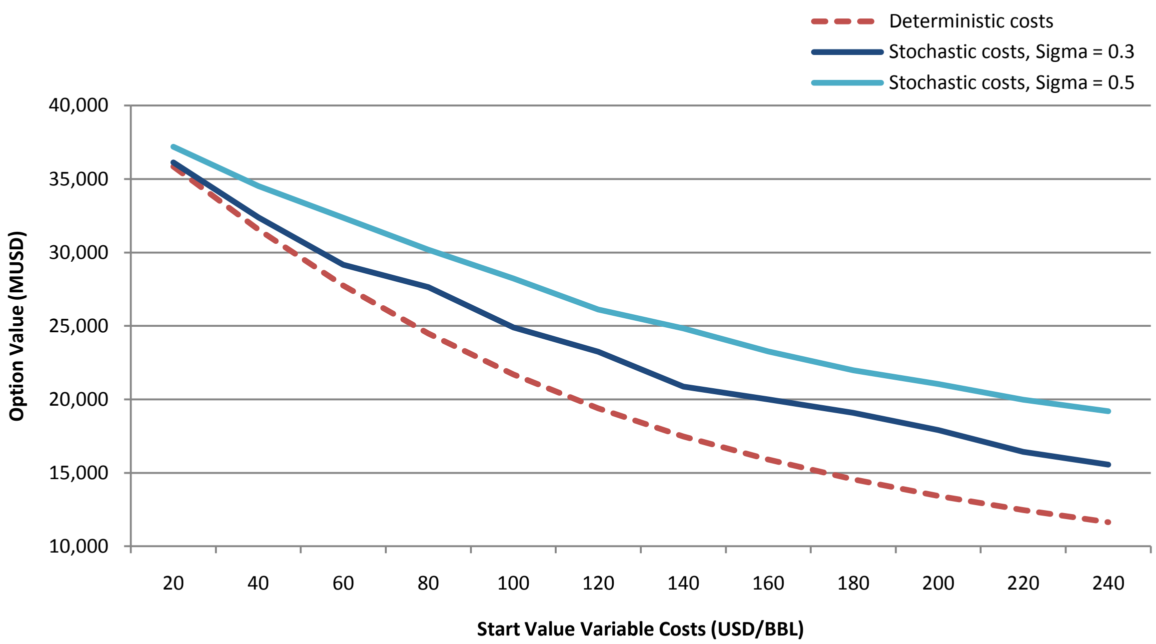
Input name	Base case value	Explanation
T_2	30 years	The time of production
Q_{max}	300 million barrels	The maximum amount of extractable reserves in a field
q_{High}	10 million barrels per year	The high production rate per year
q_{Low}	5 million barrels per year	The low production rate per year
Chl	50 million USD	Cost of reducing from high to low production
Ch0	100 million USD	Cost of reducing from high to zero production
Clh	50 million USD	Cost of increasing from low to high production
Cl0	80 million USD	Cost of reducing from low to zero production
C0h	80 million USD	Cost of increasing from zero to high production
C0l	50 million USD	Cost of increasing from zero to low production
M	300 million USD	Annual cost of maintenance for a temporarily closed platform
A	100 million USD	Cost to scrap the platform (abandonment)

6.2.1 The Value Effect of Volatility

We run the model on a base case with a stochastic oil price determining the revenue, for both stochastic and deterministic variable costs. The deterministic variable costs have no volatility and will increase with a drift of 4 percent, equal to the risk-neutral drift of the stochastic variable costs. We compare deterministic and stochastic variable costs for different values of the initial variable cost index price. The stochastic costs are assumed to have a volatility of 30 percent in the base case.

Testing for different start prices of the variable cost index, we find that the option value is higher for stochastic costs than for deterministic costs, as shown in Figure 6.1. Because volatility generally increases option value, this is an expected effect. The option features of the project will capture the extra upside that the volatility represents over the 30 years of production, while the downside risk is reduced by the option to temporarily close or abandon. The effect is magnified for higher starting values of the variable cost index, which means that the value of volatility is higher when the initial production margins are low for the project.

Figure 6.1 Switching Option: Option Value for Deterministic and Stochastic Costs



The standard deviation of the results is approximately USD 200 million, or 0.6 -1.7 % of the option value. Correlation between variable costs and the oil price is assumed to be zero. The risk-neutral drift of oil prices is assumed to be 7.3 percent, in line with findings in Chapter 4. All values are in MUSD.

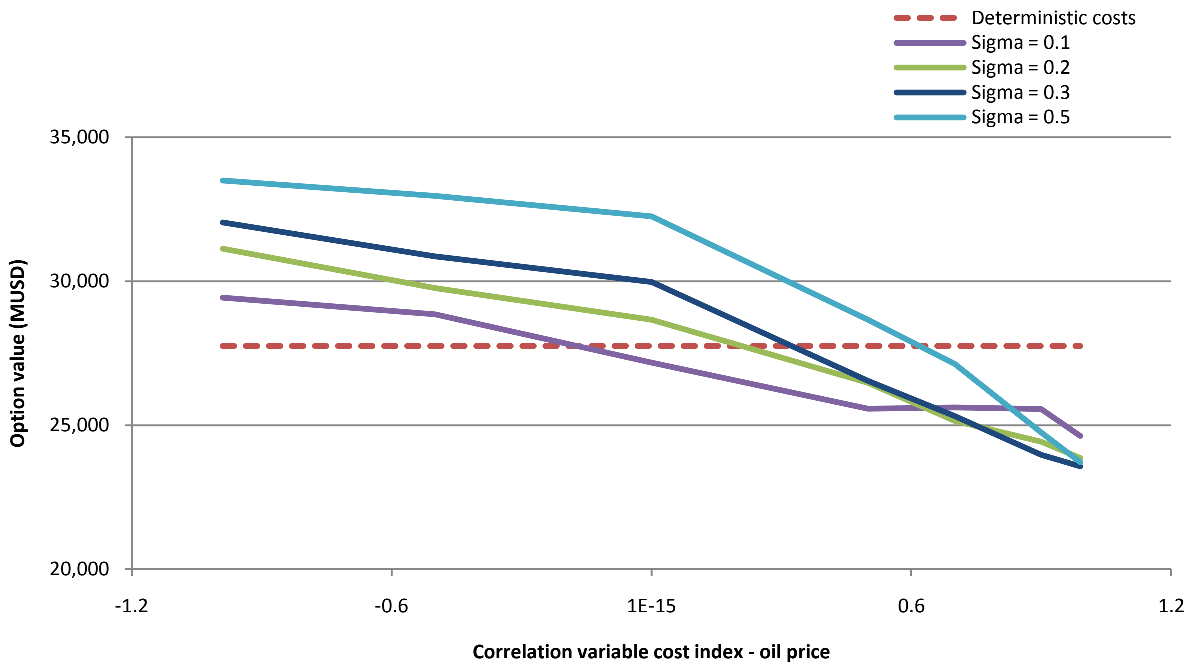
We find that modeling costs as stochastic instead of deterministic changes the estimated option value by up to 65 percent, with a volatility in costs of 50 percent. The implications for a petroleum company in terms of projecting the value of a field may thus be considerable.

6.2.2 The Value Effect of Correlation

We consider different correlation coefficients between the oil price and the variable cost index and how these influence the value of the project. Furthermore, the combined effects of correlation and volatility are examined. The results are depicted in Figure 6.2.

There is a negative relationship between the option value and the correlation coefficient. When correlation is negative, there is a higher chance that when the oil price increases, the variable costs will decrease, which is particularly beneficial for a holder of an option to invest. If the opposite happens, that the oil price decreases and the variable costs increase, the holder of the option does not need to exercise and can wait and see if the prices become more beneficial.

Figure 6.2 Switching Option: The Effect of Correlation and Volatility on the Option Value.



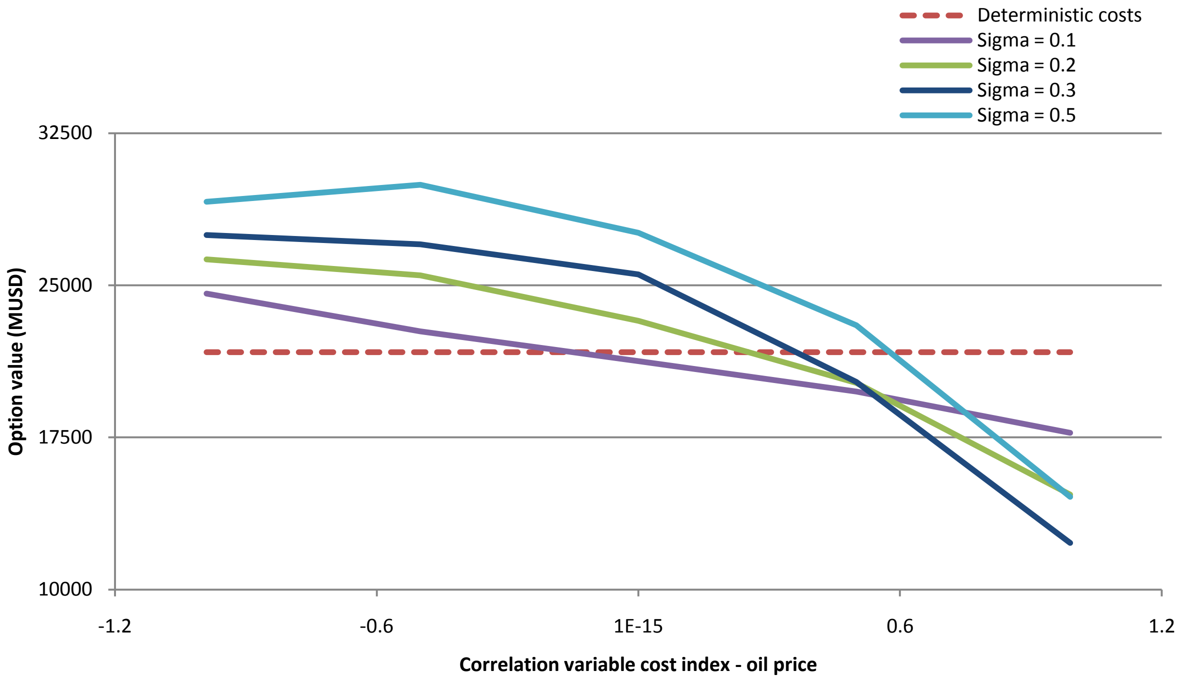
The variable cost index is assumed to start at USD 60. The risk-neutral drift of oil prices is assumed to be 7.3 percent, in line with findings in Chapter 4. The standard deviation of the results are approximately 0.7 percent, or around USD 200 million. Values are in MUSD.

Evaluating the interplay between correlation and volatility, we find that the option value generally increases with higher volatility levels in the variable cost index. This result is, however, only true for low or negative correlation coefficients. For a high and positive correlation coefficient the interplay between different effects make it difficult to conclude on the effect of volatility. A negative correlation has a positive effect on the option value, and the increased volatility should magnify this effect. Conversely the value reducing effect of a positive correlation should interplay with the value augmenting effect of an increased volatility. The direction of the net effect is inconclusive in this case.

A negative correlation between variable costs and oil results in a 20 percent higher value when volatility is 50 percent, compared to the case for deterministic costs. Conversely, the option value is up to 15 percent lower if costs are stochastic and correlation is positive.

Comparing the base case in Figure 6.2 to a less profitable case, in which the variable cost index starts at USD 100, reveals that the aforementioned effects of correlation are magnified when projects start out less profitable (Figure 6.3). Both in percentage and absolute changes, the less profitable case displays larger effects for the different levels of correlation. Moreover, the inconclusive effects of the interplay between a high correlation and the volatility appear for lower values of the correlation coefficient in the less profitable case.

Figure 6.3 Switching Option: The Effect of Correlation and Volatility on Option Value for a Less Profitable Case.



The variable cost index is assumed to start at USD 100. The risk-neutral drift of oil prices is assumed to be 7.3 percent, in line with findings in Chapter 4. The standard deviation of the results are approximately 0.7 percent, or around USD 200 million. Values are in MUSD.

When correlation is negative (positive) the option value may be 35 percent higher (lower) for stochastic costs (for volatilities of the variable costs of 10-50 percent) compared to deterministic cost.

In general the effects of correlation and volatility are significant and represent large differences in values, especially for less profitable cases. Consequently, the choice of these parameters may significantly influence the valuation made by petroleum companies.

6.2.3 The Value of Flexibility

The presented switching option model incorporates several types of flexibilities. The value of the different flexibilities can be examined by including and excluding specific flexibilities in the model.

Table 6.3 illustrates how the value of the switching option changes when certain flexibilities are excluded. The base case includes all the modeled flexibilities: the option of producing at a high or a low production rate, the option to temporarily close and the option to abandon. Based on these base case assumptions, we show that all the flexibilities have positive values. We analyze the value of flexibility for two cases: the base case in which variable costs start at USD 60, and a less profitable case in which the variable costs start at USD 100. For both cases we analyze the effect of introducing stochastic costs.

Table 6.3 Switching Option: The Option Value for Different Types of Flexibilities

	Base case (VC0 = 60)						Less profitable case (VC0 = 100)					
	Stochastic costs			Deterministic costs			Stochastic costs			Deterministic costs		
Flexibility	**Value**		**Δ from FF**	**Value**		**Δ from FF**	**Value**		**Δ from FF**	**Value**		**Δ from FF**
Full flexibility (FF)	28 123	(0.68 %)	-	27 221	(0.72 %)	-	22 517	(0.83 %)	-	20 882	(0.93 %)	-
Excluding the option to:												
- Abandon	28 024	(0.69 %)	**-0.35 %**	26 153	(0.75 %)	**-3.92 %**	22 301	(0.84 %)	**-0.96 %**	19 308	(1.01 %)	**-7.54 %**
- Temp close	27 949	(0.69 %)	**-0.62 %**	27 221	(0.72 %)	**0.00 %**	22 060	(0.85 %)	**-2.03 %**	20 484	(0.94 %)	**-1.91 %**
- Low production	27 815	(0.69 %)	**-1.10 %**	26 489	(0.74 %)	**-2.69 %**	22 092	(0.85 %)	**-1.89 %**	20 360	(0.95 %)	**-2.50 %**
- High production	13 777	(0.70 %)	**-51.01 %**	12 993	(0.76 %)	**-52.27 %**	10 733	(0.87 %)	**-52.33 %**	9 221	(1.05 %)	**-55.84 %**
- Abandon and temp close	26 555	(0.73 %)	**-5.58 %**	26 153	(0.75 %)	**-3.92 %**	18 697	(1.04 %)	**-16.96 %**	18 119	(1.08 %)	**-13.23 %**
- Abandon, temp close and low	23 627	(0.84 %)	**-15.99 %**	23 618	(0.84 %)	**-13.24 %**	12 033	(1.70 %)	**-46.56 %**	12 018	(1.65 %)	**-42.45 %**
- Abandon, temp close and high	11 763	(0.84 %)	**-58.17 %**	11 759	(0.84 %)	**-56.80 %**	5 967	(1.71 %)	**-73.50 %**	5 959	(1.67 %)	**-71.46 %**
- Temp close and low	27 304	(0.70 %)	**-2.91 %**	26 063	(0.76 %)	**-4.25 %**	21 016	(0.89 %)	**-6.67 %**	18 015	(1.07 %)	**-13.73 %**
- Temp close and high	13 656	(0.70 %)	**-51.44 %**	12 993	(0.76 %)	**-52.27 %**	10 523	(0.89 %)	**-53.27 %**	8 959	(1.08 %)	**-57.10 %**
- Abandon and low	27 545	(0.70 %)	**-2.06 %**	25 014	(0.79 %)	**-8.11 %**	21 956	(0.86 %)	**-2.49 %**	18 701	(1.05 %)	**-10.44 %**
- Abandon and high	13 231	(0.73 %)	**-52.95 %**	11 759	(0.84 %)	**-56.80 %**	9 874	(0.97 %)	**-56.15 %**	7 143	(1.38 %)	**-73.76 %**

All values are in MUSD. Numbers in parentheses are standard deviations. The values are calculated with the same simulated asset paths. The correlation between the variable cost index and the oil price is assumed to be 0.3. The variable cost index is assumed to have a volatility of 30 percent. The risk-neutral drift of oil prices is assumed to be 7.3 percent, in line with findings in Chapter 4.

The value effect when removing several flexibilities is, in general, higher than what the value of the separate flexibilities would suggest. The value of several flexibilities put together is higher than the sum of values for the individual flexibilities. Removing one type of flexibility reduces the value by 2 percent on average. In the base case, a project that does not include the options to abandon and temporarily close will reduce the value by approximately 4-6 percent. However, in the less profitable case, the value is reduced by 13-17 percent if these options are excluded. If the only possibility is to produce at a high rate constantly (i.e. the options of abandon, temporarily closing and low production are excluded), the project value is reduced by 13-16 percent in the base case. For the less profitable case, the value is reduced by 42-47 percent. Excluding the flexibility of producing at a high level reduces the value of the option by more than 50 percent.

The value of flexibility appears to be slightly lower for deterministic compared to stochastic costs. This is consistent with our general results that the option value is higher for cases with stochastic costs. However, the results suggest that the value of the option to abandon is more valuable when the costs are deterministic than when the costs are stochastic. The reason may be that with no volatility, it is more valuable to completely shut down operations in unprofitable circumstances than to temporarily close because there are lower probabilities that the project may become valuable again.

We conclude that the value of all flexibilities in our switching option is positive and they are especially valuable in less profitable projects. The flexibilities seem to conjointly create value, such that the value of flexibilities put together is higher than the value of the individual flexibilities. Flexibilities appear more valuable if costs are modeled as stochastic, but we find indications that abandonment is more valuable in cases with deterministic costs.

6.3 Waiting Option

The waiting option provides the flexibility to postpone an investment decision. This flexibility is similar to an American call option, where the underlying asset is the value of the developed platform, and the strike price is the investment cost. In our model we will also allow the investment cost to be stochastic. We are especially interested in examining the effect of a stochastic investment cost, and how this affects the value of the project and the average time before investment. Moreover, we assess how correlation between the value of the developed field and the investment cost affect the value of the option to wait and the optimal timing. The main underlying risk factor for the investment cost is assumed to be the steel price.

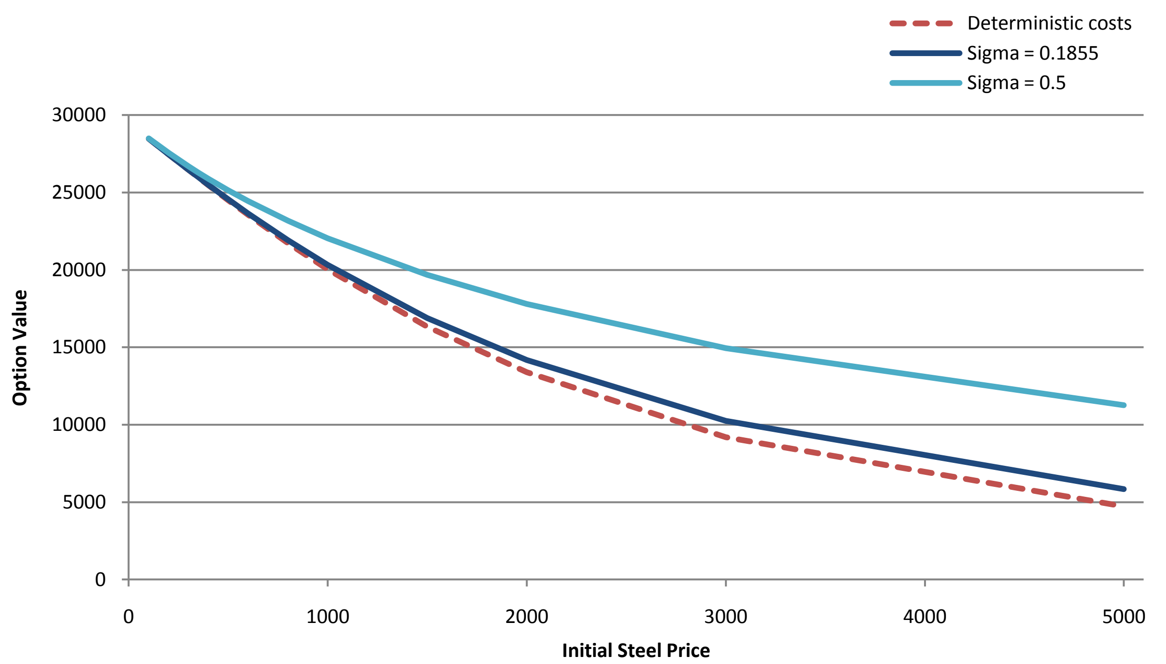
We assume that the company has the right, but not the obligation, to start investment in a platform at any point in time between time t and T_1. For simplicity we assume that this period is 5 years, and that decisions can be made once a year. The investment value and investment costs are calculated as linearly dependent on oil and steel prices respectively. The initial oil price is assumed to be USD 83 per barrel, which corresponds to an investment value of USD 25 billion. A steel index value of 350 corresponds to an investment cost of USD 3 billion.

6.3.1 The Value Effect of Volatility

We examine how stochastic investment costs will affect the value of the option, compared to a case with deterministic costs. The investment costs are based on a stochastic steel price index as an underlying risk factor.

The effects on option value of different initial steel prices, and thus different initial investment costs, are depicted in Figure 6.4.

Figure 6.4 Waiting Option: Option Value as a Function of the Initial Steel Price



All values are in MUSD. The correlation between the steel price and the oil price is assumed to be zero. The risk-neutral drift of oil prices and steel prices are assumed to be 7.3 percent and 7.18 percent respectively, in line with the average findings of convenience yields in Chapter 4. The standard deviation is 20-30 MUSD, or 0.1-0.5 percent of the option value.

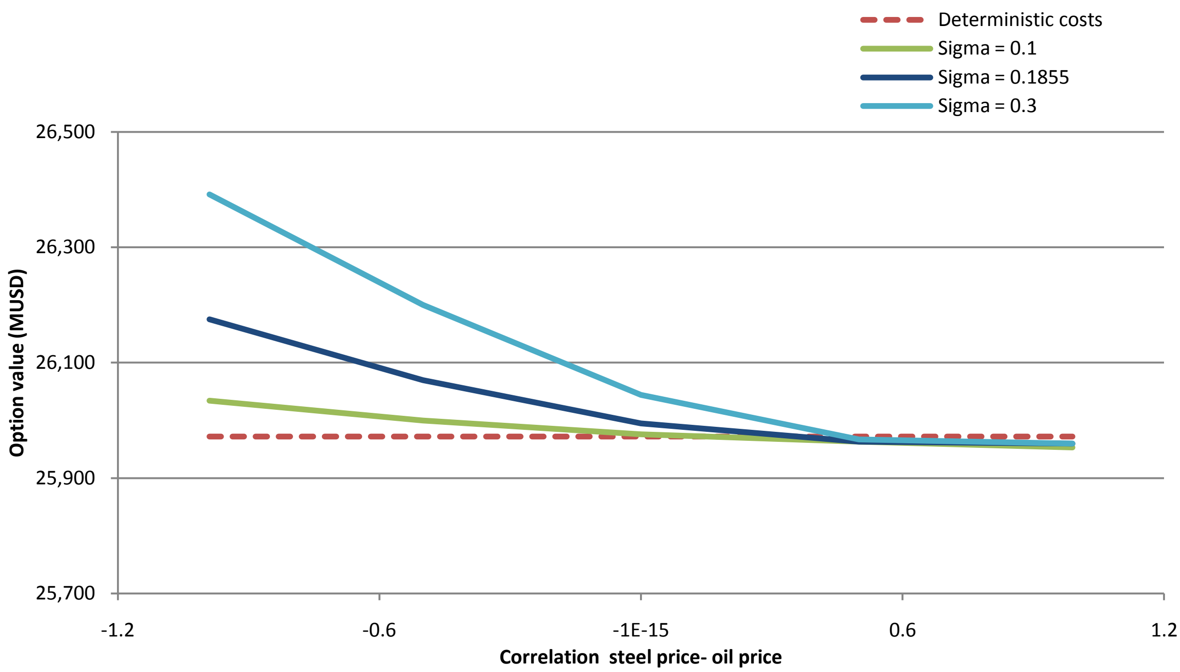
The option value decreases when the initial steel price increases, and our analysis shows that the value is generally higher for cases in which the volatility of the steel price is high. Moreover, this effect is magnified for high initial steel prices. This is consistent with the fact that volatility increases the probability of very high and very low steel prices. Because of the option feature, the upside of low steel prices is captured in the option value, while the downside of high steel prices does not negate this effect. For higher initial steel prices this effect is even more prevalent. High volatility increases the chances that the project may turn valuable over the waiting period, and the option value will thus increase. Deterministic investment costs display no volatility, and give lower option values for all cases of initial steel prices.

Similarly to the results for the switching option, we find that volatility in the cost factors significantly affects the value of the waiting option. The large differences between cases with stochastic and deterministic costs imply that there may be differences in value projections between companies that make different assumptions. This indicates that the choice of assumptions have significant effect on the option value.

6.3.2 The Value Effect of Correlation

In order to examine the effect of correlation on the option value, we simulate the waiting option with different correlation coefficients between steel and oil prices. As depicted in Figure 6.5 these effects are significant, and the effect increases with increased volatility. For low and negative correlation between the steel price and the oil price, the cases with stochastic costs are more valuable than the case with deterministic investment cost. This relation is consistent with the fact that a negative correlation implies that steel prices tend to decrease (increase) when oil prices increase (decrease). This increased "upside" of high oil prices and low steel prices is captured more in the value of the option than the downside of the opposite effect. This is due to the option features of the project. We notice that when there is a positive correlation between the oil and steel prices, the option value is approximately the same for all cases.

Figure 6.5 Waiting Option: The Effect of Correlation and Volatility on the Option Value



All values are in MUSD. The risk-neutral drift of oil prices and steel prices are assumed to be 7.3 percent and 7.18 percent respectively. The initial steel price is assumed to be USD 350. The standard deviation is 20-30 MUSD, or 0.11-0.13 percent of the option value.

6.3.3 The Timing Effect of Cost Uncertainty

This section discusses the optimal time of investment and how volatility, correlation, convenience yield, and the start price of steel affect the optimal investment time. Furthermore, we discuss the fraction of options that are not exercised.

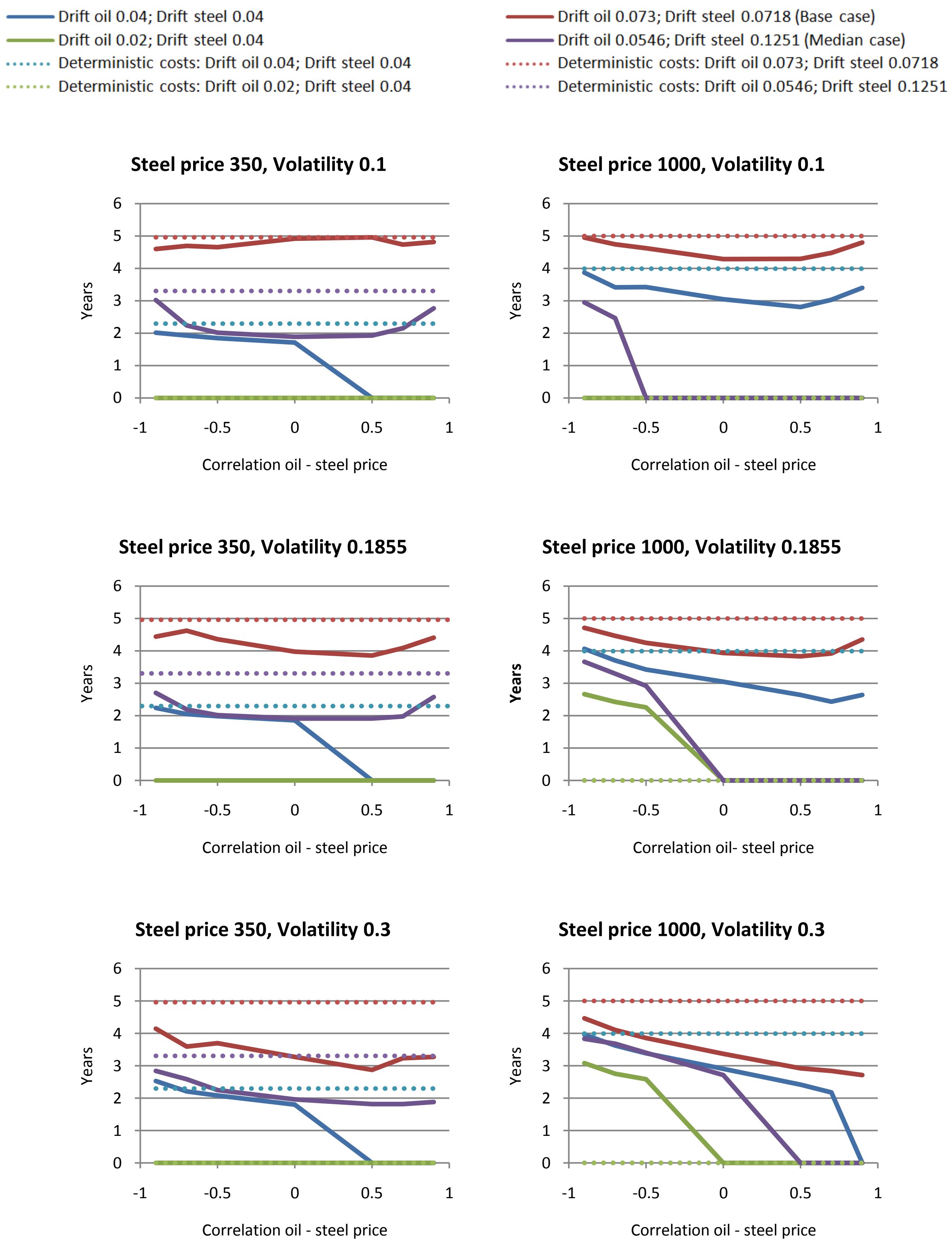
Optimal Timing

Figure 6.6 depicts the optimal exercise time for different combinations of variables. Optimal exercise time is calculated as the average time (in years) at which investment occurs across the simulated paths.

As explained in Chapter 3, the risk-neutral drift is composed of the risk-free interest rate and the convenience yield if spanning is assumed. We look at the optimal timing for different combinations of risk-neutral drift for the oil and steel price. Moreover, we compare a profitable case with a less profitable case. The profitable case supposes a steel price that starts at 350, which implies that immediate exercise has a value of USD 22 billion. In the less profitable case the steel price starts at 1000, which implies a value of immediate exercise of USD 16.4 billion. In addition, we examine the effects of the volatility of the steel price as well as the correlation between steel and oil prices.

When the drift of oil is higher than the drift of steel, the isolated effect on the value of waiting is positive. This is the case if the convenience yield of oil is below that of steel. If additionally the drift of oil prices is higher than the risk-neutral rate (i.e. negative convenience yield), it will be optimal to wait as long as possible in most cases, in order to benefit from the higher return of oil over time. In the opposite case, where the drift of oil is less than both the risk-free rate and the drift of steel, it is most often optimal with immediate investment. For cases in between these two we find two opposite effects. On one hand a drift of steel that is higher than the drift of oil makes it less beneficial to wait as the steel prices are expected to grow more than oil prices. On the other hand a drift of oil that is higher than the risk-free rate increases the value of waiting. The magnitude of the two effects will determine the optimal time of investment, as is depicted in Figure 6.6 for the Median case (violet line).

Figure 6.6 Waiting Option: Optimal Timing of Investment



The graphs display the average number of years before the investment takes place for different parameters of the underlying risk factors.

In addition, volatility and correlation may affect the optimal investment time. Generally, a combination of high volatility and positive correlation makes early investment optimal, while a high volatility and negative correlation has the opposite effect. As explained throughout this chapter, volatility will generally increase the option value because the holder of the option can benefit from the upside. A negative correlation will add more value as the steel price is more likely to fall if the oil price increases. Should the steel price increase and the oil price fall the option does not need to be exercised. In total, waiting might increase the value if the volatility is high and the correlation between steel and oil prices is negative. A positive correlation however, could negate the positive effects of high volatility. If the oil price increases, so will the steel price in most cases, and there is little to gain by waiting. The optimal decision may be to invest earlier in those cases. Immediate investment is especially valuable for less profitable cases with high volatility and positive correlation.

The average level of time of optimal investment for cases in which the steel price is deterministic is represented by the dotted lines in Figure 6.6. As deterministic costs are not affected by volatility or correlation, they are constant lines in the graphs. We find that the average waiting time is generally longer for models with deterministic costs. However, in the case where the drift of oil is 2 percent and the drift of steel is 4 percent (the green line) it is always optimal to choose immediate investment when costs are deterministic, while in some instances it may be optimal to wait in cases with stochastic costs. This is because the drift rates indicate that waiting will deteriorate the value. With no stochastic feature in deterministic costs, there is very little chance that the situation will improve by waiting.

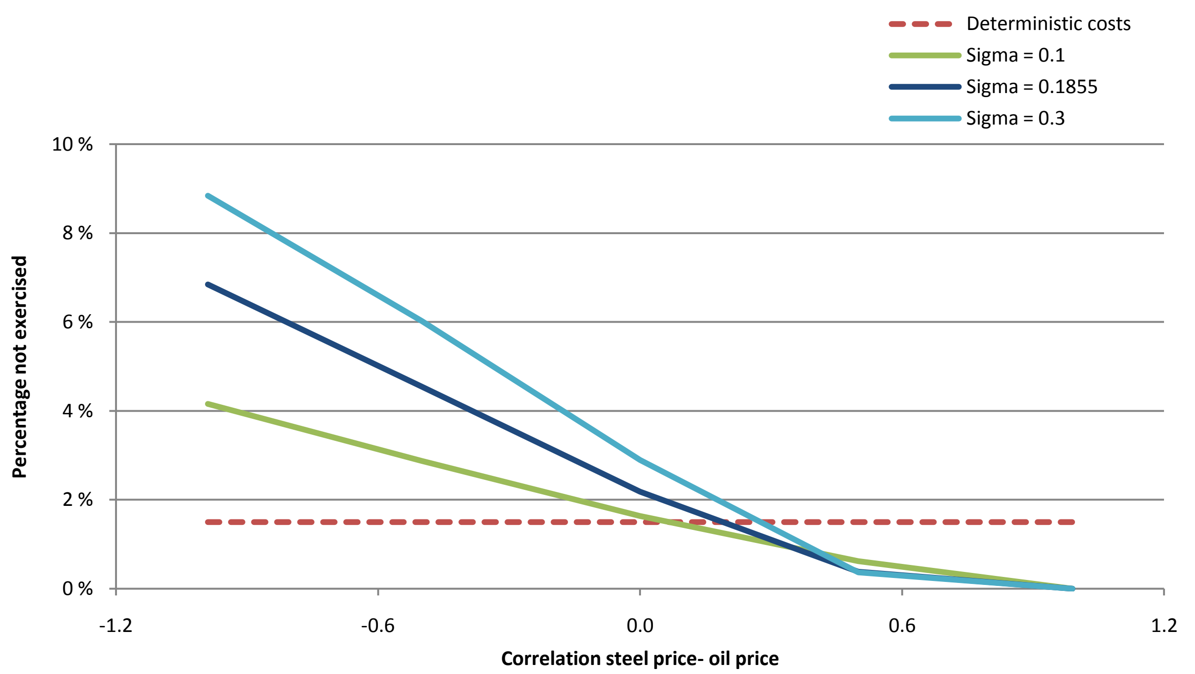
We conclude that the investment decisions differ significantly between models with stochastic and deterministic costs. In addition we find that the parameters chosen for volatility, drift, and correlations are important for the optimal decision time. The implications for petroleum companies may be significant.

Fraction of Simulated Paths Not Exercised

The fraction of simulated paths that are not exercised is also an indication of the optimal decisions in a waiting option. The fraction is calculated as the percentage of paths that lead to no investment during the 5 years of the option to invest.

The number of cases that are not exercised increases with a negative correlation between the steel and oil price, as depicted in Figure 6.8. While 1.5 percent of cases with deterministic investment costs are not exercised, up to 9 percent of cases are not exercised in the event of a volatility of 30 percent and negative correlation. Conversely, the number of cases without investment converges to zero when the correlation coefficient tends towards 1 for cases with stochastic investment costs. This effect of negative correlation is a result of the aforementioned option feature that increases the value with higher volatility. A negative correlation increases the chances of an increase (decrease) in steel prices when the oil price decreases (increases). The latter effect increases option value, and although the former effect does not reduce the option value, the number of cases in which it is never optimal to exercise increases. This is the reason why cases with stochastic costs and high volatility have a higher fraction of cases where it is not optimal to exercise the option.

Figure 6.7 Waiting Option: Fraction of Cases that Are Not Exercised



The risk-neutral drift of oil and steel prices are assumed to be 7.3 percent and 7.18 percent respectively. The initial steel price is assumed to be USD 350.

The results indicate that there is a significant effect from the volatility in costs on whether or not it is optimal to invest.

6.4 Summary

In general, the introduction of stochastic costs increases project value for both the switching option and the waiting option. Moreover, the positive effects on project value are larger for less profitable cases. The effects of correlation and the level of volatility are also important. When introducing correlation between income and costs factors, we observe that a negative correlation coefficient increases project value. The positive value effect of volatile stochastic costs is magnified when correlation is negative. However, for positive correlation values, the interplay between volatile costs and correlation between costs and income factors produce inconclusive effects on project value.

The different flexibilities in the switching option all add value to the project, and the combination of flexibilities adds value that is not apparent when looking at the value of the flexibilities individually. The value of flexibilities appear larger for stochastic than deterministic costs, with the exception of the value of abandonment that is slightly higher for deterministic costs.

For a waiting option, we find that the optimal investment time is dependent on several factors, including the convenience yield (and thus drift), as well as the volatility and correlation of the underlying processes. In most cases, a company would wait longer before investing in a project with deterministic costs than in one with stochastic cost features. In cases with stochastic costs, a larger (smaller) fraction of cases will never be exercised if the correlation between the cost process and the income process is negative (positive), compared to a case with deterministic costs.

In conclusion we find the effects of stochastic costs to be important in many aspects of the valuation and for the optimal decisions in a petroleum project. Modeling the costs as deterministic or stochastic may significantly influence the investment and production decisions.

6.5 Limitations of the Model

Our model and the analyses we have performed are based on a number of simplifying assumptions. There are a number of analyses and features that we have had neither the time nor capacity to perform or test. We will therefore include a section on the possible limitations of our work, with suggestions for model improvements.

The construction of the model may not be realistic in all its components. Some of the parameter assumptions that are chosen for our analyses may not be plausible, or they may change over time. This is especially true for the variable costs of a platform, as well as the maintenance and switching costs. In the switching option, we assume constant rates of extraction. In reality, the extraction rate may not only depend on the choices of production, but also on the "life cycle" of the field, because the pressure in the reservoir falls with time (Hannesson, 1998). An extraction rate that changes with time may influence the choices of production and corresponding costs in the switching option. Moreover, we suggest performing an analysis for the compound option *waiting option on the switching option*, which could further enhance the level of sophistication.

The assumption that spanning holds is used throughout our analysis. For this assumption to be realistic it must be possible to replicate the cash flows of the real option in the market. The mature markets for oil futures suggest that it to some extent is plausible to replicate the revenue streams of a petroleum project. Recently initiated futures trading in steel also supports an argument that spanning could hold for the cost-side of the waiting option. It is, however, important to note that the costs consist of more factors than steel, complicating the assumption of spanning. The argument that spanning holds is difficult to present for the variable costs in the switching option. However, the lack of good time series for these costs prevents us from making stronger assumptions. Relaxing the assumption of spanning necessitates the estimation of market prices of risk for the underlying risk factors.

Our model assumes single-factor processes for the underlying risks. Because multifactor models may be better fits for the historical data (for example by modeling stochastic volatility and convenience yields), including such processes in the real options valuation could model the underlying uncertainty in a more sophisticated and plausible manner. Such models may result in more precise estimates of the effects of cost uncertainty.

There are several other types of uncertainty that will affect the decisions and the valuation of oil investments, as explained in Chapter 2. Geological uncertainty will influence the expected size of the field, the technology used to extract the oil, and the risks that can be encountered once the production has started. The levels of production available for a platform could change alongside geological uncertainty, and with the level of the remaining reservoir. Not including this type of risk may lead to unrealistic estimates of oil reserves and the cost of extracting them. Moreover, we have, for simplicity, assumed that there are no taxes in our model. This could induce a higher estimated project value than what is the reality. However, in Norway, tax regulations are developed with the intention that they will not affect optimal decisions. Other uncertainties that are not included are political and technological uncertainties. This may cause erroneous valuation estimates. Moreover, volatility in interest rates and inflation rates may add further sources of uncertainty.

Furthermore, we have used the LSMC method described by Longstaff and Schwartz (2001) in order to find the continuation values for the options. In the original framework, Longstaff and Schwartz include only in-the-money paths in order to minimize the noise in the regression. Our switching option has several flexibilities available at each decision point, with corresponding cash flows and continuation values, which makes it difficult to exclude any paths. We have thus chosen to include all paths in the regression analysis for the switching option. This may have caused noise in the regression.

Finally, the decision makers in petroleum companies will in practice base their decisions on both financial and qualitative analyses. Extreme cases will thus be dealt with in a more rational way than our model is able to. For example, should the steel price increase dramatically, the companies will search for substitutes that can replace steel in order to reduce their costs. One could for example introduce a maximum price of certain costs in the model. Considering strategic and game theory aspects in the valuation is at the forefront of real options analysis (see Schwartz and Trigeorgis, 2001, Chapter 1), and is not included in our analysis.

7 Conclusion

This thesis set out to analyze what effect stochastic costs have on the valuation and optimal decisions in petroleum projects with inherent flexibilities. Two different kinds of projects are chosen, from different phases of a petroleum project: a waiting option and a switching option. As risky costs have mostly been ignored in the real options literature, and by the industry, our goal is to shed light on whether or not this topic should be given more attention. Moreover, the effects on project value and optimal timing for different projects and input parameters could be of importance for industry players.

Our conclusion is that the petroleum industry may find it valuable to model the risk in cost components of petroleum projects, as it affects both the value and the optimal decisions of the project. As petroleum projects involve large investment costs and the possibility of considerable revenues these effects may influence substantial values. The significant effects emphasize how important the assumptions of a project are, which again highlights how fundamental it may be for a petroleum company to consider all types of underlying risk. If petroleum companies base their analyses and projections on erroneous assumptions, this might lead to suboptimal decisions.

For both the waiting and switching options the value effects of stochastic costs are significant. This effect is due to the option features, which capture the up-side effect of volatility, while the down-side is reduced. The increase in value is much higher both in percentage terms and absolute terms for switching options that start out less profitable, and for waiting options where immediate investment is less valuable. For such projects the flexibilities afforded are more likely to be exercised, and accordingly the changes in volatility are more likely to add value and influence which choices the companies would make. For instance, for a project where it is extremely profitable to continue production, it is unlikely that a company will end up in a situation in which it is optimal to temporarily shut down. For a less profitable project, however, a large jump in the underlying cost factor is more likely to make production unprofitable, and a temporary shutdown may be optimal. Modeling stochastic costs may consequently be especially important in circumstances that involve less profitable conditions.

There is in general a negative relationship between the correlation between income and cost components, and the option value. Moreover, the effects of volatility are not unambiguously positive when correlation is introduced. For negative correlations the positive effects of volatility in costs are magnified, both for the switching and the waiting option. However, for high positive correlations, the effects on project value when introducing volatility in costs are inconclusive.

The values of the different flexibilities in the switching option are strictly positive and the combined value of the different flexibilities is larger than the sum of the individual flexibilities. Similarly to the effects on overall project value when introducing cost uncertainty, we find that the value of flexibility is higher for projects that are less profitable than for highly profitable projects. Modeling stochastic instead of deterministic costs will slightly increase the value for most of the different flexibilities. However, our findings suggest that the value of abandonment is higher in the case of deterministic costs compared to stochastic costs.

Introducing stochastic costs in the waiting option also has significant impact on the average time of investment. The convenience yield, which adjusts the risk-neutral drift of the underlying processes, is in addition to correlation and volatility, the factor with the greatest effect on the optimal investment time. In general, a company would invest later if the costs are deterministic than if they are stochastic. The exception is the case in which the drift of revenues is below the drift of costs as well as the risk-free rate, where immediate investment is always optimal (assuming that the project is initially profitable). Because of the significant effect of the drift term we therefore find it important to evaluate the drift in conjunction with volatility and correlation when introducing stochastic costs.

The risk-neutral valuation framework has been suggested by many academics as the new valuation regime, replacing the traditional net present value approach. Risk-neutral valuation avoids the issues of estimating risk-adjusted discount rates, and is better suited for pricing flexibilities. Monte Carlo simulation is able to handle high-dimensional problems, and because of techniques such as the Least Square Monte Carlo approach, simulation may also be used for pricing options with American features. Despite the tedious computations and analysis required to perform a project evaluation in a real options framework, we have shown that modeling the stochastic features of such a project may be highly valuable in the decision process.

Both our analysis and previous work in the literature on commodity prices show that the underlying dynamics are difficult to estimate. Our data show that oil and steel prices display highly volatile patterns, but it is challenging to conclude whether they are mean reverting or follow a random walk with drift. As our analysis shows that the introduction of stochastic costs may significantly influence value and optimal decisions, we find that these volatile features are important to model.

We suggest that future work analyze how our results would be affected by the choice of price processes for the underlying risk factors, given the inconclusive results of our time series. Moreover, while the convenience yield is assumed constant in our model, our analysis suggests that it is extremely volatile. Since the convenience yield has a significant effect on the optimal time of investment we suggest further research on how our results would change if a stochastic convenience yield is introduced. Finally, incorporating more sophisticated approaches for dealing with extreme cases would ameliorate the degree of realism in the model.

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Data sources

Downloaded from Datastream.

Steel prices

HWWI Iron Ore, Steel Scrap USD - PRICE INDEX [downloaded 24.04.10]

London Metal Exchange LME Steel Billet Med. Cash U$/MT [downloaded 06.05.10]

London Metal Exchange LME Steel Billet Med. 3 Mth U$/MT [downloaded 06.05.10]

Oil prices

Crude Oil-Brent Cur. Month FOB U$/BBL [downloaded 27.04.10]

Crude Oil-Brent 1Mth Fwd FOB U$/BBL [downloaded 06.05.10]

Interest rates

Market yield on U.S. Treasury securities at 1-month constant maturity, quoted on investment basis [downloaded 06.05.10]

Market yield on U.S. Treasury securities at 3-month constant maturity, quoted on investment basis [downloaded 06.05.10]

Appendix 1: Risk adjustment of GBM

This appendix illustrates how the risk adjustment is done for a GBM with the MMA as the numeraire. The underlying follows the SDE of a GBM:

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

If there are any cash flows to the asset, here denoted D_t, the gains process must be adjusted instead. Defining the Gains and the Gains process:

$$G_t = S_t + D_t \qquad \rightarrow \qquad dG_t = dS_t + dD_t$$

The Gains process is deflated by our numeraire, the MMA (A_t), with interest rate r

$$MMA \quad : \quad A_t = e^{-\int_0^t r(u)du} \qquad\qquad Deflated\ Gains: \quad G_t^* = \frac{G_t}{A_t}$$

Applying Itô's Lemma:

$$G_t^* = \left(\mu + \delta_t - r_t\right) G_t^* dt + \sigma G_t^* dW_t$$

Applying Girsanov's theorem and assuring that the drift of the process is equal to zero:

$$G_t^{*,Q} = \left(\mu + \delta_t - r_t\right) G_t^{*,Q} dt + \sigma G_t^{*,Q} \left(dW_t^Q - \lambda\right)$$

$$G_t^{*,Q} = \left(\mu + \delta_t - r_t - \lambda\sigma\right) G_t^{*,Q} dt + \sigma G_t^{*,Q} dW_t^Q$$

$$\lambda = \frac{\mu + \delta_t - r}{\sigma}$$

The same adjustment is applied in the stochastic process for the asset

$$dS_t = (\mu - \lambda\sigma) S_t dt + \sigma S_t dW_t$$

If spanning holds, or a twin asset exists, the process under the EMM must be

$$dS_t = \left(r - \delta_t\right) S_t dt + \sigma S_t dW_t$$