NHH NORWEGIAN SCHOOL OF ECONOMICS AND BUSINESS ADMINISTRATION

# Empirical Studies of Nord Pool's Financial Market

## Unbiasedness, Risk Premiums & Hedging

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## Abstract

The theme of this thesis is empirical studies of Nord Pool's financial market. Future contract with maturity of one, four and twelve weeks and its underlying spot prices were studied between 1995 to 2009. The volatility of power is exceedingly volatile compared to other assets. Although some of this volatility may be due to seasonal price movements, there is a substantial basis risk in this market. In this thesis, I will discuss two subjects, the unbiasedness hypothesis and hedging. First part addresses the issue whether prices of future contracts can be used as reliable predictors for the future spot prices. The main finding of this section is that Nord Pool's financial market can be described as a market that has gradually improved itself in terms of market efficiency. There are also some indications for a time-varying risk premium in the future contract. The second part deals with risk management of spot and future contracts. Various hedging models such as OLS, VAR and multivariate GARCH-models were put into use to calculate hedging ratios and hedging efficiency. In this section, it is argued that all hedging models outperformed a naïve hedge and the time-varying hedge ratios gave better results than the constant hedge ratios by using an in-the-sample analysis. However, the time-varying hedging-strategy lost some of its properties when employing an out-of-sample analysis. The hedging efficiency varied between 20 per cent for the shorter contracts and as much as 50 per cent for the longer contracts.

## Acknowledgements

The work on this thesis has been challenging. Several people have offered invaluable help. First and foremost, I want to thank my advisor, Bård Støve. As a patient advisor, he has been available at all times, and the quality of his help has been priceless.

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# Contents

# 1. Introduction

This master thesis will address spot-futures relationships and hedging. I chose the Nordic electricity as a theme since I find this market quite fascinating due to its complex nature. The goal of the thesis is to unravel some of the unique characteristics of the power market. Here I have suggested two topics that I would like to explore in detail.

Many financial theories, involving market efficiency and rational expectations of investors, imply that the futures prices should be a good predictor for future spot prices. Said in other words, future contracts serves as an unbiased predictor of the future spot prices. This is also known as the unbiasedness hypothesis. The underlying assumption that is to be tested is whether power futures is informational efficient, and that arbitrage opportunities for producers does not exist.

The second topic addresses hedging. An important property with futures market is that risk can be measured and traded. This could be of great interest for energy intensive industries, since futures contracts enable market participants to hedge and protect their assets while incorporating market flexibility into their transactions. A challenge for the risk manager who operates in this market is to find the optimal hedge ratio in order to reduce risk. Therefore, I have looked further into the hedging effectiveness for the market by using different estimation procedures.

In my thesis, I have used two software applications, R and S-plus (with the Finmetrics module) when analyzing the data. In addition, I have used PcGive to give a graphical presentation of my results.

## 2. About Nord Pool

Nord Pool commenced trading in 1993. In 1996 Sweden was integrated into the exchange, followed by Finland (31st of August 1998) and western Denmark (1st of January 1999). Since then it has established itself as the only liquid spot and financial market for electricity in the Nordic countries.

As the only power exchange in the Nordic, it provides a market place for physical power. As much as 70 per cent of the Nordic energy consumption is traded through this market place. As of today Nord Pool is the world's largest exchange for physical power, with a turnover of almost 2000 TWh for 2008.

Nord Pool consists of more than 420 members in total (as of 2008), representing the whole value chain for electricity. This includes exchange members, clearing clients, members and representatives in 20 countries. The membership includes energy producers, energy intensive industries, large consumers, distributors, funds, investment companies, banks, brokers, utility companies and financial institutions.

What makes the Nordic power market so unique is the composition of energy sources. In this market, hydropower consists of roughly 50 per cent of the production. This is unique in the sense that hydropower is cheap to produce, has low marginal costs and the ability to store energy indirectly in the reservoir. In European perspective, this makes the market unique since the Norwegian reservoir capacity alone represents almost half of Europe's reservoir capacity (statkraft.no).

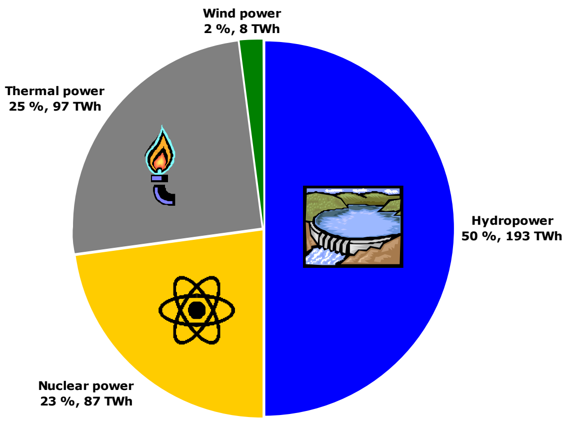


**Exhibit 2.1:** Production mix in the Nord Pool area (Pettersen 2009)

# 3. Spot, Futures and Forwards contracts

In this section, I will outline the basics of the spot and the financial market of Nord Pool. In addition, I will discuss the fundamentals of the markets. In the exhibit below, a graphical summary of the market is presented.

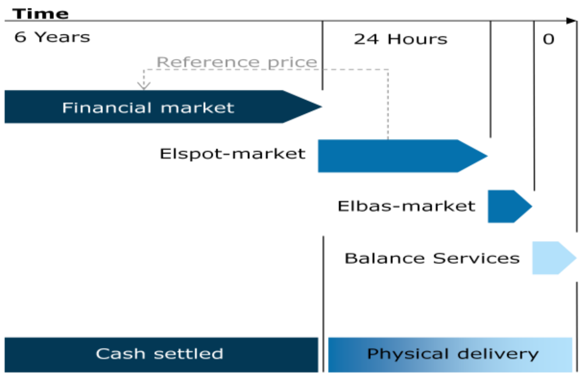


**Exhibit 3.1:** Overview of the spot and financial market (Pettersen 2009)

## 3.1 The Spot Market

The primary role of the spot market is to establish equilibrium between supply and demand with a certain scope of forward planning. It is an auction based day-ahead market. A classical spot market would not be possible, since the producers need a notification in advance to verify that the production schedule is feasible within transmission constraints. Every morning Nord Pool participants post their orders to the auction for the coming day. The consumers and producers have a deadline at 12:00 am to submit their orders for the next day. Each order specifies the volume in MWh that a participant is willing to buy or sell at specific price levels (EUR/MWh) for each individual hour. Then at 1:30 p.m., Nord Pool Spot publishes hourly spot price for the coming day in order to balance supply and demand. In addition to the Elspot market, Nord Pool also has a market place for physical balance, the Elbas market and Balance Services.

### 3.2 Forward and Futures Contracts

A *forward contract* is a financial instrument (derivative) which represents an agreement between two parties prospective delivery of an asset during a given time period at an agreed price. At the initial time of agreement, the agreed price (forward price) is fixed such that there is no payment between the two parties. Nord Pool uses a slightly modified definition for the term forward contract. In the context of other standard textbooks, such as Hull (2009), the term forward is used for over-the-counter trades, which implies that there is no market-to market settlement. Nord Pool use the term forward for contracts with no daily market-to-market settlement in the trading period before maturity (Burger, Graeber & Schindlmayr 2007). For periods from 1-4 years ahead, Nord Pool offers forward contracts.

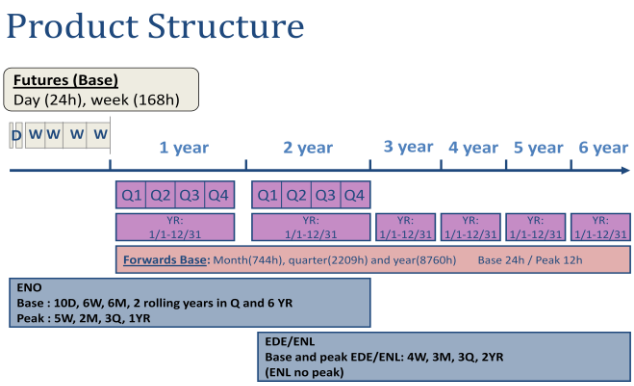


**Exhibit 3.2:** Product structure of the financial contracts

The contracts are settled by comparing the average system price for the specific period with the forward price in the contract. The difference in price is multiplied with the volume in the contract, and this amount of money is transferred between the parties. If it is higher than the system price, the investor will be compensated. On the contrary, if the price is lower than the system price, he or she will have to compensate the opposite party of the forward contract. A forward contract is therefore not only a mutual insurance, but also a mutual obligation.

*Futures contracts* are very similar to forward contracts, but these contracts typically have some features that make them more useful for hedging and less useful for merchandising than forward contracts. These include the ability to extinguish positions through offset, rather than actual delivery of the commodity, and standardization of contract terms. Futures contracts are typically traded on organized exchanges in a wide variety of physical commodities and financial instruments. A futures contract is mainly a financial instrument for dealing with economic risk, and is typically settled in cash. In order to attract sufficient trading liquidity, futures contracts are standardized in terms of the underlying commodity (quality, volume, location, etc.) as well as the delivery period. The potential problem of counterpart risk is limited by the mark-to-market mechanism. Basically, market-to-market works as follows: A clearing house acts as an intermediary for all trades. In order to trade, each trader has to deposit security in an account with the clearinghouse. When today's futures price is quoted in the market, the contract with yesterday's futures price is replaced by a contract with today's futures price, and the gain or loss for each position following from this price change is settled against each trader's accounts. In case of insufficient funds, the trader will have to make an additional deposit or the positions will be closed. As opposed to forward contracts, where each contract is settled at maturity, the futures are settled daily. This procedure is known as market-to-market. At Nord Pool's financial market, cash-settlement is made throughout the delivery period, starting at the due date of each contact. Settlement is conducted between Nord Pool's clearing service and individual members.

From September 1995 to July 2003, all contracts were registered as futures contracts. After July 2003, the contracts that implied delivery with 4 and 12 weeks were categorized as Forwards. This is because Nord Pool introduced monthly Forwards. However, this can be regarded as a trivial issue, since the forward contracts are priced the same as s contracts (See chapter 4.1).

# 4. Pricing Forward and Futures Contracts

Futures prices are often interpreted as the markets anticipation of future spot prices (Avsar and Goss 2001). The simplest form of a spot-future relation stated as the current price of a futures contract is equal to the present value of the future spot price:

$$F_{t,T} = S_t e^{r_f T} \tag{4.1}$$

Where $S_t$ and $F_{t,T}$ are the spot and futures price respectively observed at time t for a contract with maturity at T. The equation states that the futures prices are an unbiased predictor of the future spot price in a frictionless world. In economic analysis, the hypothesis often appears under the guise of rational expectations (Bilson 1981) – a somewhat more advanced pricing model that includes the cost of storage (W). The storage cost, including rent of storage space, insurance, physical deterioration or wastage, can be written with the following equation:

$$F_{t,T} = S_t e^{r_t(T-t)} + W \tag{4.2}$$

The right hand side of the equation quantify the cost of buying, funding and storing the commodity. Specifically, $S_t$ is the spot price at t; r is the risk-free interest rate. In the case of water reservoirs, the cost of storage equals zero. The producers face a neglectable production cost (Bye, Hva bestemmer kraftprisene? 2006).

Compared to regular commodities, electricity is a non-storable commodity. The spot-futures relationship is discontinuous and the basis will reflect expectations about the future spot price changes, possibly adjusted for a risk premium (Gjølberg and Johnsen 2003). For producers, who can effectively store electricity in water reservoirs, the storage cost function depends on the water level in the reservoirs and the probability of overflow (Gjølberg and Johnsen 2003).

## 4.1 Forward price = Futures price

As mentioned, forward contracts only have settlement at maturity, while futures have daily settlements until maturity. This covers gains or losses from day-to-day changes in the market price of each contract. The final settlement, which begins at maturity, covers the difference between the last closing price of the futures contract and the system price in the delivery period (markedskraft.no). It is well known from standard textbooks that given some assumptions, the forward price and the futures price are equal for similar contracts when delivery occurs at a single point in time. The contracts are identical in all respects, except for the fact that a holder of a futures contract will realize his gains or losses every day. This means that the future holder can reinvest the proceeds, something a holder of a forward cannot. The essential insight is that the forward can be replicated by trading with a corresponding futures contract. Margrabe (1976) demonstrates that if the interest rates were not random, forward and futures prices would be the same. The appendix of Chapter 5 in Hull (2009) provides a formal proof. Because of the market-to-market and randomly varying interest rates, forward and futures prices differ. However, as a word of caution, it is not advisable to make a large-scale futures or forward trade without investigating this relationship further. For simplicity, I have adopted the assumption of a constant riskless interest rate in this thesis.

### 4.2 Normal backwardation and Contango

Backwardation is a condition in the market where futures prices are lower in the distant delivery months than in the near delivery months. The shape of the futures curve is important to commodity hedgers and speculators. If a hedger wants to avoid the risk of price fluctuations, the hedger needs to take a short hedge. This means that the hedger needs to take short position to deliver the underlying in the future at a guaranteed price. In order for the speculators to take over the risk exposure, the hedger has to offer the speculators an expected profit (risk premium). Speculators will enter the corresponding long side of the contract only if he or she expects that the future spot price is higher than the current futures price. For the speculator this would yield an expected profit of $E(S_T)-F_0$. The speculator's expected profit is the hedger's expected loss, but the hedger is willing to take the expected loss on the contract in order to eliminate the risk of uncertain prices changes. Hence, if there are more short hedgers than long hedgers, a risk premium will occur.

The theory of backwardation suggest that the futures price will be bid down to a level below the expected spot price and will rise over the life of the contract until the maturity date, at which point $F_T=S_T$ (Bodie, Kane and Marcus 2009). If there is some correlation, the price of the futures contract and the spot should converge when approaching expiration.

The polar hypothesis to backwardation holds that the natural hedgers are the purchasers of a commodity. Contango is a terminology used to describe a condition, in which distant delivery prices for futures exceed spot prices, often due to the costs of storing and ensuring the underlying commodity. In the case of contango if there are more long hedgers than short hedgers there can be risk premium in the market. Because long hedgers will agree to pay high futures prices to eliminate risk, and because speculators must be paid a premium to enter into the short position, the contango theory holds that $F_{t,T}$ must exceed $E(S_T)$.

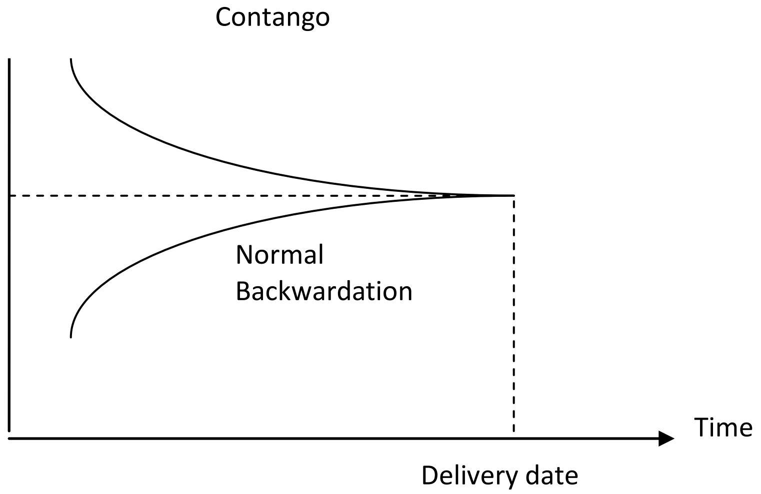


**Exhibit 4.1:** Normal backwardation and contango

Normal backwardation and contango assume that the speculators in the market are rational. They can of course predict wrongly from time to time, but on the average, they will make a reasonable prediction of the price. The second assumption is that the speculators and hedgers have the same expectations as the spot price at maturity. The reader should note that normal backwardation and contango sometimes refers to whether the futures price is below or above the current spot price, and not the expected (Hull 2008).

Whilst hedgers are in the market to reduce their risk, speculators take on extra risk in order to profit from their futures transactions. This implies that in order for speculators to get attracted to the market, there has to be a risk premium. John Maynard Keynes and John Hicks argued that if speculators tend to be long while hedgers are at the short end, the expected future spot price should be above the futures price (Hull 2008).

### 4.3 Convenience Yield

Convenience yield may be defined as; the advantage or premium, that firms derive from holding an underlying product or physical good, rather than the contract or derivative product. Studies by Brennan (1958) and Telser (1958) have documented that there is a relationship between convenience yield and inventories. It is driven by relative scarcity and the inventory serves as a state variable summarizing the effect of past supply and demand. Users of a consumption asset may obtain a benefit from physically holding the asset (as inventory) prior to maturity, which is not obtained from holding the futures contract. These benefits include the ability to profit from temporary shortages, and keeping a running production.

One of the main reasons why convenience yield appears is due to availability of stocks and inventories of the commodity. A negative basis at very low inventory levels indicates a high convenience yield. When inventories are low, we expect that scarcity now is greater than in the future. Unlike the previous case, the investor cannot buy inventory to make up for demand today. In a sense, the investor wants to borrow inventory from the future but is unable. Therefore, we expect future prices to be lower than today and hence that $F_{t,T} < S_t$. Consequently, the convenience yield decreases with the inventory. This makes it possible to observe normal backwardation.

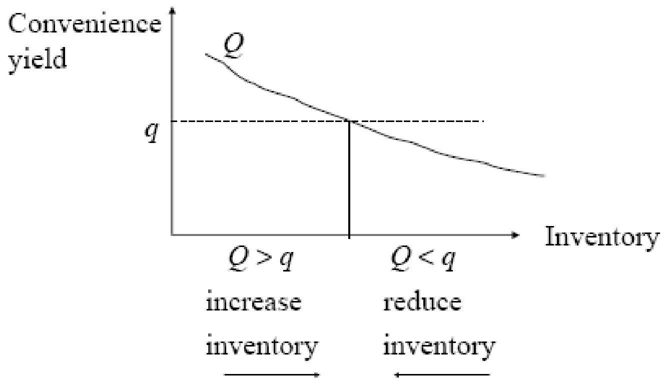


**Exhibit 4.2:** Relationship between inventory and convenience yield

## 4.4 Price Determining Factors

For the sake of completion and consideration for the reader, I have included a discussion of what governs the price development in the power market. Price determination in the electricity market is complex and any factors affect the price at any time, such as operating costs, reservoir levels and capacity constraints for both the production and the transfer of power between regions and the energy and environmental policy (Bye, Hva bestemmer kraftprisene? 2006). I will discuss what has been retrieved from the homepages of Nord Pool and Statkraft as well as earlier studies. One of the challenges for the risk manager is to keep track of these variables and make reliable forecast of these in order to make a complete risk assessment.

### 4.4.1 Factors that Affects Demand

*Temperature*. In the Nordic countries heating of households is largely based on electricity. Hence, the temperature has a direct impact on demand. Lower temperatures give a higher demand, which in turn causes the price to rise (statkraft.no).

*Business cycles*. The Nordic electricity market is affected by fluctuations in other commodity and currency markets, particularly in Europe, but also to some extent worldwide. General economic fluctuations also affect the power consumption and thus trade. This has been noted in the recent financial crisis.

*Electricity consumption over time*. In Norway alone, electricity consumption is rising by about 1-1,5 TWh per year. One TWh is enough electricity to supply for 50 000 households (kraftkartet.no). The last 10 years the consumption in Norway rose five times more than the increase in production capacity. Increased consumption leads to higher demand on the power exchange, which in turn leads to higher prices.

### 4.4.2 Factors that Affect Supply

*Water reservoir level*. Fifty per cent of the Nordic power market is supplied by hydropower. Increased water reservoir levels, increase the supply and force the prices down. The relationship between the spot price and reservoir levels becomes less significant the further we go into the future. Uncertain reservoir capacity and other market conditions mean that future prices can differ from current expectations (Ericson, Halvorsen og Hansen 2008).

*Inflow*. Much of the snow and the precipitation does not go directly into the reservoirs, but in the areas around the reservoir and eventually finds its way to the reservoir. An important task for the geologist and engineers is therefore to take samples of the snow and groundwater to estimate the future reservoir level.

*Nuclear Power*. There are several major nuclear power plants in the Nordic countries. Whether we get more or less nuclear power in the Nordic countries is primarily a political issue. In Sweden, there are plans to shut down some nuclear power plants and upgrade some other plants. In Finland, it is planned to develop a major new nuclear power plant that will provide a significantly improved supply in the Nordic electricity market.

*New production capacity*. Eventually, demand for electricity will increase by economic activity. With the given capacity prices will rise, and it will continue to rise until it reaches the cost of developing new capacity (Bye, Hva bestemmer kraftprisene? 2006). Forecasts show that it costs around 25-30 cents / KWh to build new power plants (statkraft.no). For this reason, we can expect that this will be a maximum average level for the price of the electricity market over time, provided that it is possible to develop new production to meet demand.

*Exchange with countries outside the Nordic market*. The Nordic market is also connected with other electricity markets such as Russia, Germany and Poland. Thus, supply and demand on the continent also affect prices in the Nordic countries. Price differences between day and night are much larger in Germany than in Scandinavia. The power market in Europe provides buying cheap electricity at nighttime in Germany and transport northward as much as the network capacity allows to, and to purchase electricity at daytime relatively cheap in the Nordic region and transporting it south as much as the network can handle.

*Bottlenecks*. The capacity is utilized such that power from high price areas is transferred to low price areas. Thereby, the price in high price areas is reduced, whereas the price in low price areas is raised. Due to limitations in the grid system electricity, there will normally be price differences for price areas in the Nord Pool market.

*Coal, gas and oil prices.* Unlike wind or hydroelectric power, for which the "raw material" is free, coal, gas and oil plants in Europe need to buy the raw material in order to produce electricity. When these prices rise, it becomes more expensive to produce electricity. Raw material prices are considered to be quite important since it is the dominating primary source for thermal power (Bye and Rosendahl, SSB 2005). High prices reduce production, which in turn decreases the supply in Europe and generates higher prices.

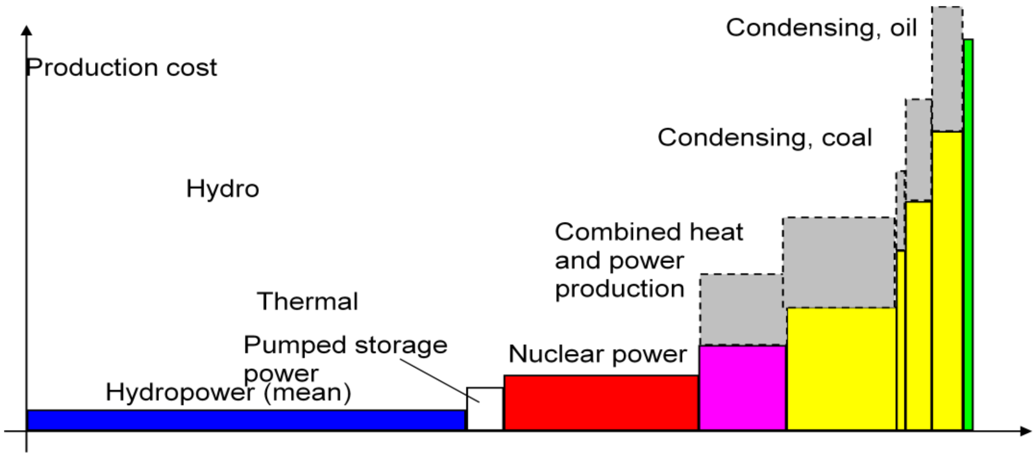


**Exhibit 4.3:** Production cost of the different energy sources (Pettersen 2009)

*CO2-prices*. From January 1st 2005 $CO_2$ emission quotas were introduced. Power plants that emit $CO_2$ must buy $CO_2$-allowances to cover their emissions. These quotas are traded in a separate market, and if the prices of $CO_2$ quotas are high, it may be unprofitable to produce electricity in coal plants. High price of $CO_2$ allowances could provide a lower supply in the electricity market, which in turn leads to higher prices. In the exhibit above, the $CO_2$-prices are market in the grey area.

*U.S. dollars*. Lower USD result in lower coal prices, as coal prices are traded in U.S. dollars. Low USD give better conditions, for example, for the German coal power, which may result in increased exports from the German power market to the Nordic countries.

# 5. Econometric Techniques

In this chapter, I will briefly go through the basics of the linear regression model based on OLS and its underlying assumption. Afterwards I will describe a vector autoregressive model and Granger causality. Lastly, I will discuss how to model volatility in time-series by using ARCH/GARCH-modeling. Since the data sample is represented as a time series, this section will focus mainly on time series properties.

## 5.1 The Basic of the OLS Regression Model

In econometrics, ordinary least square (OLS) is a technique for estimating the unknown parameters in a linear regression model. In a generalized form, one can express a regression models as:

$$y_t = \alpha_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + \cdots + \beta_k x_{tk} + \varepsilon_t \quad (5.1)$$

This equation tells us that one unit increase in $x_{t1}$ changes the expected value of $y_t$ by $\beta_1$ and equally for the other variables. Since there typically is some randomness in economic data, we need to include an error term ($\varepsilon_t$) into the equation. The parameters are found by minimizing the sum of squared distances between the observed responses and the estimated regression line. The ordinary least squares computational technique provides simple expressions for the estimated parameters and the associated statistical values such as the standard error of the parameter in equation 5.1. For a more in debt discussion around the OLS technique, the reader is referred to Wooldridge (2006).

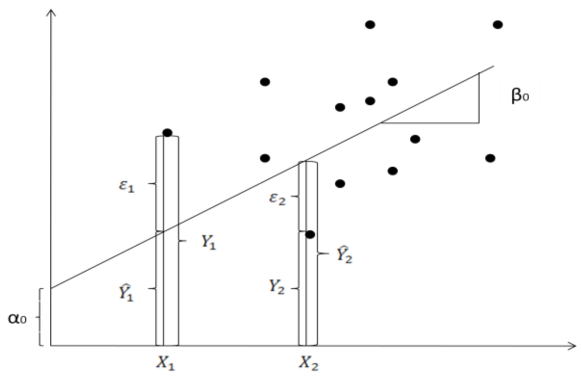


**Exhibit 5.1:** A graphical presentation of a simple OLS-regression

In order to find the linear relationship that gives the best fit to the dataset, one minimizes the sum of the squared residuals of the residuals from *i* to *n* observations for each variable (*i* to *k*).

$$Min \sum_{i=1}^{n} \varepsilon_t^2 = Min \sum_{i=1}^{n} \left(\hat{Y}_t - Y_t\right)^2 = Min \sum_{i=1}^{n} \left(y_i - \hat{\alpha}_0 + \hat{\beta}_1 x_{i1} + \cdots + \hat{\beta}_k x_{ik}\right) \quad (5.2)$$

When trying to evaluate how good a regression model is, we need to determine what fraction of the sample variation in the dependent variable is explained by the regression line. R-square (goodness-of-fit) gives an important insight whether the model captures the variation in the model. However, one should be careful in using *R*-square as the main criterion for evaluating a regression equation.

To decide whether there a proper variable is included into the regression model, p-values can be used to evaluate the significance level. The p-value is defined as the smallest significance level at which the null hypothesis can be rejected (Wooldridge 2006). In a context of a regression analysis, we test whether the coefficient in the model is significantly different from zero.

In order to evaluate whether the regression model has provided us with reliable results, we have to check whether the underlying properties are fulfilled. In the next chapter, will discuss these assumptions further and possible remedies if they are violated.

### 5.1.1 Model is Linear in Parameters

This assumption simply states that the functional linear relationship between the dependent and independent variable when holding other factors fixed. The linearity of the regression equation implies that one-unit change in the independent variable has the same effect on the dependent variable, regardless of the initial value of dependent variable (Woolridge 2006). Sometimes one of the independent variables can be a non-linear function of another variable, e.g. polynomial variables or variables with logs. The model remains linear as long as it is linear in the parameter. The most difficult issue to address, is whether our model allows us to draw conclusions about how the independent variable affects the dependent variable with other things being the same (ceteris paribus)

### 5.1.2 No Perfect Collinearity

In the sense that the calculations of the parameters in the regression models are calculated correctly, multicollinearity is not a problem, only perfect collinearity is. The problem arises if some or all of the explanatory variables are highly correlated with one another. If we want to try to measure the separate effects of the different regressors, we also want to choose a set of dependent variables that are relatively uncorrelated with each other. One problem is that the individual p-values can be misleading. In an extreme case, it is possible to find that no coefficients are significantly different from zero, while R-square is quite large. Intuitively, this means that the explanatory variables together provide a great deal of explanatory power, but multicollinearity makes it impossible for the regression to decide which variable is providing explanation to the model. A simple approach to this problem is to drop out some of the highly correlated variables from the regression. Generally, multicollinearity will lead us to reject the hypothesis that an independent variable influences the dependent variable even when it should not. Note that this only affects the variance of the estimates in the variables; it is not a direct violation of the OLS assumptions.

### 5.1.3 Zero Conditional Mean

When discussing causal relationship, the first thing we should look for is whether the zero conditional mean is fulfilled. For instance, if we have not included enough significant variables that correlate with the dependent variable, we will probably not fulfill the condition for zero conditional mean. Whenever we omit an important independent variable in the regression model, the other variables will try to absorb some of the variation that is left out. Consequently, the coefficients will change. This causes trouble if we want to look at the separate regressions of the different coefficients of the independent variables. Since those coefficients tried to compensate for leaving something out, they try to pick up the remaining variation with the dependent variable. The result is that there is an omitted variable bias in the coefficients of the included dependent variables. If we know we are leaving out an important variable from our regression model, we should be very reluctant to give meaning to the separate coefficients. We should avoid saying a one unit increase in X implies a β-unit change in the predicted Y, because there may be a hidden association somewhere.

### 5.1.4 Homoscedasticity

The homoscedastic assumption states the residuals have a constant variance for all values of the independent variable. If the residuals are heteroscedastic, we can risk that the OLS estimators are inefficient and provide incorrect standard errors. This is particularly important for this data sample since we know that asset prices often has evidence for systematic conditional heteroscedasticity. (Chinn & Coibion 2009). For consequential modeling of time-series this has to be accounted for in order for regressions to be meaningful. Heteroscedasticity can be tested with several tests. Here I will use the White test, which is based on the null hypothesis of homoscedasticity. *White test* for heteroscedasticity can be computed by storing estimates and residuals from the regression, and then running a regression of squared residuals on the fitted values and the squared fitted values. After this, one can use an F-test, to see if all coefficients are jointly equal zero. If we can reject the hypothesis that the explanatory variables being jointly equal to zero, we then assume there is no heteroscedasticity. White test also includes the squared explanatory variables and the cross product. I will come back to a possible remedy for heteroscedasticity in chapter 5.2

### 5.1.5 Autocorrelation

A typical property of time series data is that past values of a variable affect future value of the same variable. In regressions with time series, the residuals associated with observations, typically affects the residuals into future periods. If one run a model without lagged variables, and detect autocorrelation, the estimators will have incorrect standard errors. The Ljung–Box test is a statistical test of whether any of a group of autocorrelations of a time series is different from zero. Instead of testing serial correlation at each distinct lag, it tests the "overall" serial correlation based on a number of lags. In this test, the null hypothesis states that the residuals are random. The alternative hypothesis states that the residuals are not random. The test statistic is:

$$Q(n) = n(n+2)\sum_{j=1}^{h}\frac{\hat{\rho}_j^2}{n-j} \qquad (5.3)$$

where n is the size of the sample and $\hat{\rho}_j$ is the autocorrelation at lag *j.* All tests in this thesis include ten lags. Large values of the Ljung-Box statistics indicate serial correlations. If the residuals are autocorrelated, it can have consequences for the computed result since the computed standard error of the estimated parameter may underestimate the true standard errors. A simple method to correct autocorrelation is to use Newey-West standard error, also known as heteroscedastic and autocorrelated consistent standard error (HAC in short). This method estimates the OLS parameters and adjusts the standard error for autocorrelation and heteroscedasticity. For more details, the reader is referred to Newey and West (1987).

## 5.2 Stationarity

Financial time series often show properties of non-stationarity. To explain this term, we can start the other way around and define stationary time series. A data sample is said to be stationary if its mean, variance and covariances are constant for each given lag. If they are not, the series is non-stationary. When a time series is non-stationary, shocks to the series will not expire away over time and return to its trend. It will only change when a new shock occurs.

Market efficiency requires that price changes are uncorrelated, and implies a unit root in the level of the price or log of the price series (Serletis, Unit Root Behavior in Energy Futures Prices 1992). One way of depicting whether a time series is stochastic or deterministic is to run the following regression:

$$\Delta y_t = \alpha + \beta t + \gamma_1 y_{t-1} + \gamma_2 \Delta y_{t-2} + \cdots + \gamma_k \Delta y_{t-k} + \varepsilon_t \qquad (5.4)$$

This regression is also known as an Augmented Dickey Fuller-test. The values on parameters can be obtained by running a standard OLS-regression on the data. The test statistics on the Augmented Dickey Fuller-test is based on the following calculation:

$$ADF_i = \frac{\gamma_1}{SE(\gamma_1)} \qquad (5.5)$$

The null hypothesis for this test is that the time-series are non-stationary (stochastic trend) against the alternative hypothesis that the time-series are stationary. If the test-statistics is less than the ADF critical value, then the null hypothesis is rejected and we can assume the unit root does is not present. If the process has a unit root, at it is a non-stationary time series.

A critical aspect of the ADF-test is to determine the optimal number of lags. Including too few lags will not remove all of the autocorrelation, thus giving biased result. While using too many will increase the coefficient standard errors (Brooks 2002). One approach is to drop the number of lags in the regression until the last lag is statistically significant. An alternative approach is to minimize an information criterion such as the Schwarz Bayesian Information Criterion (BIC), which I will return to in chapter 5.8.

A regression on non-stationary time series can lead to spurious results. The regression can then have significant coefficient estimates and a high coefficient of determination (R-Square), even when the two variables in the regression are unrelated. In addition, it can be proved that the standard assumptions for asymptotic analysis will not be valid. A standard remedy for the presence of stationary is to difference the time series. If a series is non-stationary, but becomes stationary after differencing once, it is said to have one unit root. The time series is then I(1). In general, if a time series has to be differenced *n* times to become stationary, the time series has *n* unit roots and is I(n).

## 5.3 Cointegration

An important question when determining the properties of two non-stationary time series is whether there exist a long-term relationship between the time series. If two or more series are non-stationary, but a linear combination of them is stationary, then the series are said to be cointegrated.

Cointegrating relationships between time series can be tested through Johansen's cointegration-test (Johansen, Statistical analysis of cointegration vectors 1988), which is based on a vector auto-regression error correction model (VECM). Alternatively, one can use other tests such as the Engle and Granger's two-step procedure (Engle og Granger 1987). When comparing these two methods, Johansen's test has a number of desirable properties, including the fact that all variables are treated as endogenous variables. To test for cointegration, Johansen proposed to specify the following VECM:

$$\Delta X_t = \alpha_1 + \sum_{i=1}^{p-1} \Gamma_i \Delta X_{t-i} + \Pi X_{t-1} + \varepsilon_t \tag{5.6}$$

where $\alpha_1$ is a constant, $X_t$ is the vector of spot and futures prices and $\Gamma_i$ and $\Pi$ are 2 x 2 coefficient matrices measuring the short- and long-run adjustment of the system. The error term $\varepsilon_t$ is a 2 x 1 vector of residuals that are assumed to be uncorrelated, have a mean of zero and a finite variance (White noise). The Johansen procedure tests for cointegration by examining the rank of Π. For this, the $\lambda_{max}$ and $\lambda_{trace}$ statistics are estimated. The max test, with the corresponding $\lambda_{max}$ statistics, is a test for rank(Π) = r against the null hypothesis that rank(Π) = r + 1. The trace test then examines the null hypothesis that the number of cointegrating vectors is less than or equal to r, with the alternative hypothesis that the number of cointegrating vectors is greater than r.

Johansen's cointegration-test subsequently uses the information of rank(Π) to determine whether a cointegrating relationship is present. If the rank of Π is 0, there are no cointegrating relationships. If the rank of Π is 1, a single cointegrating relationship is present.

If the log likelihood of unconstrained model that includes the cointegrating equations is significantly different from the log likelihood of the constrained model that does not include the cointegrating equations, we reject the null hypothesis of no cointegration.

### 5.4 VAR Models and Granger Causality

The vector autoregression-model (VAR) is a natural extension of the univariate autoregressive model to dynamic multivariate time series. It describes the development of a set of variables over a sample, as a linear relationship of only their lagged values. The VAR model has proven to be especially useful for describing the dynamic behaviour of economic and financial time series and for forecasting. It often provides superior forecasts to those from univariate time series models and elaborate theory-based simultaneous equations models (Koop 2006). Forecasts from VAR models are quite flexible because they can be made conditional on the potential future paths of specified variables in the model. In addition to data description and forecasting, the VAR model is also used for structural inference. Consider the following bivariate VAR-model:

$$\mathrm{Y_t} = \alpha_1 + \rho_{11}\mathrm{Y_{t-1}} + \ldots + \rho_{1p}\mathrm{Y_{t-p}} + \beta_{11}\mathrm{X_{t-1}} + \cdots + \beta_{1q}\mathrm{X_{t-q}} + \varepsilon_{1t}$$

$$X_t = \alpha_2 + \rho_{21}Y_{t-1} + \ldots + \rho_{2p}Y_{t-p} + \beta_{21}X_{t-1} + \cdots + \beta_{2q}X_{t-q} + \varepsilon_{2t} \qquad (5.7)$$

where Y and X are two time series, ρ and β is the coefficient on the lagged variables for Y and X respectively, and ε is the residuals.

In time series models such as a bivariate VAR-model, causality between the variables can be tested with Granger causality. If past X contains useful information (in addition to the information in past Y) to predict future Y, we say X "Granger causes" Y. The null hypothesis of the test is that there is no Granger causality. This can be tested by an F-test on whether the coefficients $\beta_{11}$to $\beta_{1q}$are jointly significant. Despite its name, Granger causality does not necessarily imply true causality and should be interpreted with caution (Woolridge 2006). If both $X_t$ and $Y_t$ are driven by a third variable with different lags there could be indication for Granger causality, even in the absence of a true causal relationship.

## 5.5 ARCH and GARCH

As already mentioned, a feature of asset volatility is that it tends to change over time. In the finance literature, this is referred to as volatility clustering. To capture the effects of time-varying distribution of the errors in a model, Engle developed the ARCH models. In the simplest case, modelling the second moments of a univariate model, we assume that the conditional variance at time t depends on the squared errors from the preceding *p* periods. The error at time t depends on the information given in the market in the previous period. A general version of the ARCH(p) model is given as:

$$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \cdots + \alpha_p\varepsilon_{t-p}^2 \quad (5.8)$$

where $\alpha_0$ is a constant, $\varepsilon_{t-\text{p}}^2$ is the squared error from period t-1, $\sigma_t^2$ is the conditional variance at time t and $\alpha_1$ (i = 1, 2, 3, …, p) are coefficients.

Since the influential work of Engle (1982), there have been introduced several extensions to the ARCH-model. Bollerslev introduced a generalized version of the ARCH in 1986 by including the lagged volatility estimates into the ARCH-model

$$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \cdots + \alpha_p\varepsilon_{t-p}^2 + \beta_1\sigma_{t-1}^2 + \beta_2\sigma_{t-2}^2 + \cdots + \beta_q\sigma_{t-q}^2 \quad (5.9)$$

The GARCH(p,q) has proven to work well for various financial series (Wang, 2003). The GARCH(p,q) is typically a more describing model than an ARCH model, as it is more parsimonious. The estimated variance is a function of the previous variance and the squared errors. This approach captures volatility clustering observed in the data. Conditioning on the normally distributed errors with zero mean and variance of $\sigma_t^2$, the GARCH(p,q) could be estimated by the maximum likelihood method, i.e. finding the most likely values of the parameters given the actual data. When estimating the parameters in the model, it is important to note that there is no guarantee to obtain a global maximum of the likelihood function by using a standard optimization technique (Andersen, et al. 2009). For more information on the maximum likelihood technique, see Brooks (2002) or Wooldridge (2006).

Note that the model assumes that positive and negative error terms have a symmetric effect on the volatility. In other words, good and bad news have the same effect on the volatility in this model. This is not included, since a preliminary analysis quickly revealed that there is no significant effect of leverage in the time series.

## 5.6 Multivariate GARCH

In 1988 Bollerslev, Engle and Wooldridge proposed a generalized version of the univariate GARCH to a multivariate dimension, in order to model the conditional variance and covariance of several time series simultaneously. This multivariate GARCH model can be applied to the calculation of dynamic hedge ratios based on the conditional variance and covariance of the spot and futures prices.

### 5.6.1 DVEC

A problem with MGARCH models is that the number of parameters can quickly increase, as more variables are included into the model. This creates difficulties in the estimation of the models, and therefore an important goal in constructing a MGARCH models is to make it reasonably parsimonious, while maintaining flexibility. For empirical implementation, an excessively large number of parameters can be removed from the MGARCH-model. Numerous MGARCH models have been proposed, each imposing a different set of restrictions on the dynamic process that governs the covariance matrix. A diagonal vectorized model was proposed by Bollerslev, Engle and Wooldridge (1988)

$$H_t = C + \sum_{i=1}^{q} A_i \otimes \varepsilon_{t-i}\varepsilon'_{t-i} + \sum_{j=1}^{p} B_j \otimes H_{t-j}$$

where, $H_t$ is a vector of the variance and covariance matrix, C represent a vector of constants and A and B are parameters. The symbol $\otimes$ denotes the Hadamard product (element by element matrix multiplication). The variance-covariance matrix has appositive number on its leading diagonal and is symmetrical around this diagonal. Bera and Higgins (1993) point out that the positive definiteness of $H_t$ matrix is difficult to impose during estimation and easy to check, and that no interaction is captured between the different conditional variances and covariances.

### 5.6.2 BEKK

An alternative approach to estimate a time-varying variance-covariance matrix that guarantees a positive definite constraint is the BEKK-model (named after Baba, Engle, Kraft and Kroner, 1991).

$$H_t = CC' + \sum_{i=1}^{q}\sum_{k=1}^{K} A'_{ki}\varepsilon_{t-i}\varepsilon'_{t-i}A_{ki} + \sum_{j=1}^{p}\sum_{k=1}^{K} B'_{kj}H_{t-j}B_{kj} \quad (5.10)$$

where $A_{kj}$ , $B_{kj}$ are diagonal matrixes for parameters, and C is an upper triangular matrix. $H_t$ is the vector for the conditional variances of the errors ($\varepsilon_{t-i}$) and lagged variances $H_{t-j}$. The decomposition of the constant term into a product of two triangular matrices is to ensure positive definiteness of $H_t$,

In the BEKK representation, the conditional variances are functions of their own lagged values and their own lagged error terms, while the conditional covariance is a function of lagged covariances and lagged cross products of the error terms. For any parameterization to be sensible, the formulation guarantees that conditional variances are positive and allows the conditional covariance to change sign over time. Each element of the conditional covariance matrix is assumed to evolve independently, meaning that shocks to the variances of each time series have no impact on the future covariance between them (Harris, Stoja og Tucker 2007).

## 5.7 Model Selection Criteria

In this thesis, I have presented several autoregressive models. Most of these models will be sensitive to the number of lags that are included in the model. There is no formal method to choose the most appropriate model. However, the number of lags should be large enough to remove autocorrelation, but without removing too many degrees of freedom. One way to select the number of appropriate lags is to use an information criterion. The most common information criteria are Akaike (AIC) and Schwartz (BIC). These criteria can be viewed as measures that combine fit and complexity. Fit is measured by -2*ln(likelihood). As the fit of the model improves, the information criteria approaches -∞. Given two models fitted on the same data, the model with the smaller value of the information criterion is considered to be better.

$$AIC = \ln(\tilde{\sigma}^2) + \frac{2k}{T} \tag{5.11}$$

$$BIC = \ln(\tilde{\sigma}^2) + \frac{k}{T} lnT \tag{5.12}$$

where σ is error variance, k is the number of parameters to be estimated and T is the size of the sample. It is perhaps best to use the BIC since, it is theoretical and asymptotically correct (Mills 1999). For this purpose, the BIC will be considered the best-fitted information criteria. However, Monte Carlo studies have shown that in small samples, the AIC can work better than BIC (Enders 2004).

# 6. Models for Spot-Futures Relations

When testing a financial theory, it is necessary to formulate a theoretical model that can be tested in an econometric context. In this section, I will lay out a framework for how one can measure the unbiasedness of futures contracts and risk premiums in the context of econometric modeling.

## 6.1 Unbiasedness Hypothesis

Market efficiency and the unbiasedness hypothesis implies that the futures price at time t for delivery at time T differs from the spot price realized at time T only by a random error, $\varepsilon_T$. To test of the performance of the future price as forecast of the spot price on a later period would be the same as to test for unbiasedness. The simplest form for unbiasedness can be written as:

$$E(S_T) = F_{t,T} \tag{6.1}$$

which implies:

$$S_T = F_{t,T} + \varepsilon_T \tag{6.2}$$

In econometric modeling, the unbiasedness hypothesis can be tested as (Bilson 1981):

$$S_T = \alpha_0 + \beta_1 F_{t,T} + \varepsilon_T \tag{6.3}$$

Where the null hypothesis is given as $\alpha_0 = 0$, $\beta_1 = 1$. A modified version of 6.3, is given by subtracting the current spot-price:

$$S_T - S_t = \alpha_0 + \beta_1 (F_{t,T} - S_t) + \varepsilon_T \tag{6.4}$$

This regression model is the same used in other articles such as Chinn and Coibion (2009). The role of futures prices as market expectations will be the basis to forecasting error. Here we want to test whether α is significantly different from 0 and $\beta_1$ is significantly different from 1. If the unbiased hypothesis is rejected, either it can be explained by lack of rationality among the market players, or a risk premium in the futures prices. Lack of rationality, on the other hand, might for example stem from the calculation of the settlement price. The procedure for calculating the spot price is complex and hard to grasp. If the unbiasedness hypothesis is rejected, it is hard to conclude which of the two assumptions are violated. If we assume that there is a risk premium and the market participants act rational, we can split the futures price into an expected spot rate and a risk premium (P).

$$F_{t,T} = E(S_T) + P_T \tag{6.5}$$

If the market price does not follow this equilibrium, a speculator can gain an arbitrage. If the price of the futures contract is larger than the product on the right hand side of equation 6.5, one could go short in the underlying asset to gain an arbitrage. In other words, one can buy a futures contract and sell the underlying asset. In the other case, if the product on the left hand side would be smaller than the futures price, one can buy the underlying asset and get an arbitrage.

## 6.2 Dynamic model for Risk Premiums

The existence of a risk premium in the power market provides an alternative hypothesis to the proposition that the futures prices are an unbiased predictor of the future spot price. It would be interesting to see whether the risk premium changed with the reservoir levels. In order to explore this further we can use the framework proposed by Lucia & Torró (2005 & 2008) that calculates the fitted values of risk premium and basis by using a VAR-model. By following Fama & French (1987), we can test whether the basis contains information about the expected change in the spot price and the risk premium. In order to see whether the risk premium and the basis changes with the reservoir level, we can include the reservoir levels (R) as an exogenous variable.

$$P_{t,T_j} = \alpha_\mathrm{p} + \sum_{i=1}^{k} \varphi_{pi} P_{t-i,T_j} + \sum_{i=1}^{k} \theta_{pi} B_{t-i,T_j} + \gamma_\mathrm{p}\mathrm{R}_\mathrm{t} + \varepsilon_{p,t}$$

$$\mathrm{B}_{t,T_j} = \alpha_b + \sum_{i=1}^{k} \varphi_{bi} P_{t-i,T_j} + \sum_{i=1}^{k} \theta_{bi} \mathrm{B}_{t-i,T_j} + \gamma_\mathrm{b}\mathrm{R}_\mathrm{t} + \varepsilon_{b,t} \tag{6.6}$$

where α is the intercept term and $\varphi$ and θ are parameters and ε are the error terms. P and B represent risk premiums and basis respectively for each contract j. The number of lags in the VAR-model is chosen by using BIC.

# 7. Models for Optimal Hedge Ratio

Given the volatile characteristics of the spot and futures contract, risk management becomes an important tool to deal with the risk associated by trading in the underlying asset. A portfolio consisting of spot and futures contracts can be used to diminish some volatility. However, it is crucial that an optimal portfolio of spot and futures contracts is determined. Since there are a relative low number of producers who control a large fraction of production and total reservoir capacity (Gjølberg and Johnsen 2003) it is most likely an excess of long hedging demand at Nord Pool with consumers paying a risk premium.

To achieve a perfect hedge, it is required that the basis of the portfolio at expiration is zero. As mentioned earlier, the basis could be different from zero at expiration of the hedge. Finding the optimal futures contract to use as a hedging vehicle is thus a matter of finding the futures contract that gives the lowest basis risk when put in a portfolio with the spot. When the best futures contract has been identified, the challenge is then to find the number of futures per unit of exposure in the spot market that minimizes total return variance. This relationship is called the optimal hedge ratio. In this chapter, I will present various models to calculate the optimal hedge ratio.

In traditional hedging it is assumed that the best hedge is achieved by taking a futures position equal in magnitude but of opposite position in the spot market. This is called a naïve hedge or a 1:1 hedge. This is the right hedge if the spot and futures have the same mean and variance and are perfectly correlated. As we will see later, a naïve hedge most often does not give the best risk reduction. In this thesis, I will use this as a benchmark to compare it to more sophisticated models.

After Markowitz introduced modern portfolio theory in 1952, it did not take long before the theory was applied to hedging as well. Several studies were made on the subject, but the breakthrough came when Ederington in 1979 presented his paper on hedging. In this section, I will first present Ederington's framework for calculating the optimal hedge ratio and hedging efficiency. Afterward I will introduce a framework for time-varying hedge ratios and present models that deal with that specific issue, and compare them to see which one achieves the best variance reduction.

There has not been produced many studies on hedging in the power market. Nevertheless, two published papers have been found on this subject. The first paper by Byström from 2003 compared different short term hedging approaches in the Nordic electricity by using daily data in the period

December 1997 to October 1999. He found that hedging appeared to give risk reducing benefits. A similar result was produced by Yang, Zhang, Liu & Luo (2009) in their analysis stretching from 1996 to 2003. They argued that using a dynamic hedge could reduce market risk to a certain extent, but stressed that there is a high basis risk.

## 7.1 Ederington's Framework

In this section, I will elaborate on the concept of basis and basis risk. In the context of hedging, the basis is defined as the difference between the spot price of the asset that is being hedged and the futures price of the contract. Often a portfolio manager wants to hedge against possible movements in a stock, or to mimic the effects of a portfolio without actually buying that portfolio. Futures contracts can be used to mimic the behavior of an underlying security. For example, if an investor is long in an asset then he or she can temporarily remove the risk of price movements in that stock by shorting a futures contract in the asset. This will remove the risk of price movements in the stock entirely, and effectively converts the portfolio into a risk-free asset. When the futures contract matches the underlying asset exactly, then risk is eliminated. However, if there are differences between the underlying and the forward, then *basis risk* is still suffered by the hedger. The basis will vary as the contract moves closer to maturity. If the underlying security we are trying to hedge is the same as the security delivered under the forward contract, then the basis will approach zero as the contract nears maturity. Thus, the hedger can eliminate risk entirely from the hedge.

This presentation follows the framework by Ederington (1979) with the exception of transaction and brokerage costs. Let ΔR represent the return on a portfolio, which includes both spot market holdings, $X_S$. The hedger has a return and variance function by holding futures contracts.

$$\Delta R_t = (S_t - S_{t-1})X_S + (F_{t,T} - F_{t-1,T})X_F \tag{7.1}$$

$$\mathrm{Var}(\Delta R_t) = X_S^2\sigma_S^2 + X_F^2\sigma_{\Delta F}^2 + 2X_S X_F \sigma_{\Delta S,\Delta F} \tag{7.2}$$

In these equations $X_S$ and $X_F$ represent the spot and futures market holdings. We can find the minimum by introducing a new variable that represent the proportions between the holding, $h=X_F/X_S$. Since in a hedge $X_S$, and $X_F$ have opposite signs, h is usually negative.

$$\Delta R_t = X_s(\Delta S_t + h\Delta F_t) \quad (7.3)$$

which can be rewritten to:

$$\frac{\Delta R_t}{X_s} = \Delta S_t + h\Delta F_t \quad (7.4)$$

The variance of equation (7.4) can be expressed as

$$Var\left(\frac{\Delta R_t}{X_s}\right) = Var(H) = \sigma_{\Delta S}^2 + h^2\sigma_{\Delta F}^2 + 2h\sigma_{\Delta S,\Delta F} \quad (7.5)$$

The spot market holdings, $X_S$, are viewed as fixed proportions, and the challenge is therefore to find the futures market holdings that minimize total variance. This can be found by solving 7.5 with respect to h:

$$\frac{\partial \mathrm{Var(H)}}{\partial \mathrm{h}} = 2\mathrm{h}\sigma_{\Delta \mathrm{F}}^2 + 2\sigma_{\Delta \mathrm{S},\Delta \mathrm{F}} = 0 \quad (7.6)$$

which gives the risk minimizing hedge ratio:

$$h^* = -\frac{\sigma_{\Delta S,\Delta F}}{\sigma_{\Delta F}^2} \quad (7.7)$$

According to Ederington (1979), the hedging efficiency can be measured as the proportion of the variance that is eliminated by the hedged portfolio relative to the unhedged portfolio. The variance of the unhedged return per unit of the commodity is given by$Var(U) = \sigma_{\Delta S}^2$, whereas the variance of the risk minimising portfolio of spot and futures per unit of the commodity is given by $\sigma_h$. Hedging effectiveness, HE, is then given by:

$$\mathrm{HE} = 1 - \frac{\mathrm{Var(H)}}{\mathrm{Var(U)}} \quad (7.8)$$

## 7.2 Hedging with OLS

The minimum-variance hedge ratio method takes a purely statistical approach to the problem of constructing the hedge ratio by regressing changes in the spot prices on the changes in of futures price with OLS. This is a simple regression with an intercept term, $\alpha_0$, a slope coefficient, $\beta_1$, and an error-term, $\varepsilon_t$, which are assumed to be normally distributed with mean zero. The regression thus assumes that changes in the spot price are proportional to changes in the futures price, but with a random element included.

$$\Delta S_t = \alpha_0 + \beta_1 \Delta F_{t,T} + \varepsilon_t \qquad (7.9)$$

The model is intuitively reasonable, since we require h* to correspond to the ratio of changes in ΔS to changes in ΔF. The coefficients are usually based on historical data on ΔS and ΔF, under the implicit assumption that the future will in some sense be like the past (Hull 2008). To derive the optimal hedge ratio by using the regression model, we rewrite equation (7.4) and substitute (7.9) as follows:

$$\frac{\Delta R_t}{X_s} = \Delta S_t + h\Delta F_{t,T} = \alpha_0 + \beta_1 \Delta F_{t,T} + h\Delta F_{t,T} + \varepsilon_t = \alpha_0 + (\beta_1 + h)\Delta F_{t,T} + \varepsilon_t \qquad (7.10)$$

The variance of the risk-minimizing portfolio of spot and futures per unit of the commodity can be expressed as:

$$\mathrm{Var}(\frac{\Delta R_t}{X_s}) = \mathrm{Var(H)} = \beta_1^2 \sigma_{\Delta F}^2 + h^2 \sigma_{\Delta F}^2 + 2h\beta_1 \sigma_{\Delta F}^2 + \sigma_\varepsilon^2 \qquad (7.10)$$

The value of h that minimizes variance of the portfolio is found by taking the derivative of the equation above with respect to h and setting it equal to zero:

$$\frac{Var(H)}{\partial h} = 2\beta_1 \sigma_{\Delta F}^2 + 2h\sigma_{\Delta F}^2 = 0 \qquad (7.11)$$

By solving for h, the optimal hedge ratio is obtained:

$$h^* = \frac{2\beta_1 \sigma_{\Delta F}^2}{2\sigma_{\Delta F}^2} = -\beta_1 \qquad (7.12)$$

In the regression model, this can be measured as the R-square from the regression of ΔS on ΔF, which is the amount of risk we manage to hedge. The naïve hedge in this context corresponds to equation 7.11 when $\alpha_0$= 0 and $\beta_1$= 1.

$$HE = 1 - \frac{Var(H)}{Var(U)} = 1 - \frac{\beta_1^2\sigma_{\Delta F}^2 + h^2\sigma_{\Delta F}^2 + 2h\beta_1\sigma_{\Delta F}^2 + \sigma_\varepsilon^2}{\sigma_{\Delta S}^2} \tag{7.13}$$

Substitute (7.13) into (7.16)

$$HE = 1 - \frac{Var(H)}{Var(U)} = 1 - \frac{\beta_1^2\sigma_{\Delta F}^2 + (-\beta_1)^2\sigma_{\Delta F}^2 + 2(-\beta_1)\beta_1\sigma_{\Delta F}^2 + \sigma_\varepsilon^2}{\sigma_{\Delta S}^2} = 1 - \frac{\sigma_\varepsilon^2}{\sigma_{\Delta S}^2} = \rho^2 = R^2 \tag{7.14}$$

From the definition of the optimal hedge ratio, $h^*$ can be estimated from a regression of the returns on the underlying asset to be hedged against the returns on the forward contract. The slope of this regression gives the minimum-variance hedge ratio. The R-square of this gives the systematic link between the asset and the hedge instrument. Thus, (1 – R-square) gives a measure of the basis risk.

### 7.3 Hedging with VAR

If autocorrelation is present when estimating optimal hedge ratios the following bivariate Vector Autoregressive (VAR) model, might work better:

$$\Delta S_t = \rho_s + \sum_{i=1}^{k}\varphi_{si}\Delta S_{t-i} + \sum_{i=1}^{k}\theta_{si}\Delta F_{t-i,T} + \varepsilon_{st}$$

$$\Delta F_{t,T} = \rho_f + \sum_{i=1}^{k}\varphi_{fi}\Delta S_{t-i} + \sum_{i=1}^{k}\theta_{fi}\Delta F_{t-i,T} + \varepsilon_{ft} \tag{7.15}$$

where $\rho_s$ and $\rho_f$ are the intercepts and $\varphi$ and θ are parameters, and ε are the error terms. In the VAR-model, all of the variables are endogenous. This is necessary because the two equations in the model are simultaneously estimated. This fact makes the VAR model less restrictive than the regression models presented earlier.

The VAR-model specified in (7.15) is in first difference and can therefore only capture the short-term properties of the spot and futures prices. To account for a possible long-run solution, we have to check for cointegration of the variables and include a correction term in the model.

The VAR model seeks to remedy the problem of autocorrelation by allowing the value of a variable to depend on its own lags and the lags of the other variables. This implies that the model might be able to capture more features of the data and thus offers a richer structure. When using a VAR model, the optimal hedge ratio can be derived as:

$$h^* = -\frac{Cov(\varepsilon_{st}, \varepsilon_{ft})}{Var(\varepsilon_{ft})} \tag{7.16}$$

Where the optimal hedge ratio (h*) is covariance of the error terms $\varepsilon_{st}$ and $\varepsilon_{ft}$ from the VAR-model divided by the variance of the error term of the futures price.

### 7.4 Hedging with MGARCH

In the previous sections, we found the optimal hedge ratio with a constant hedge ratio. However, this assumes that the joint distribution of the spot and futures price does not change in time. In a volatile market, one might consider to implement a time-varying hedge ratio. Since futures are dynamic and often have a time-varying risk premium, an investor should be concerned about adjusting the hedge position from one time to the other.

When using GARCH models to specify the conditional second moments of the time series, the optimal hedge ratio will consequently be time-varying and given by the following formula:

$$H_t = CC' + \sum_{i=1}^{q} \sum_{k=1}^{K} A'_{ki} \varepsilon_{t-i} \varepsilon'_{t-i} A_{ki} + \sum_{j=1}^{p} \sum_{k=1}^{K} B'_{kj} H_{t-j} B_{kj} \tag{7.17}$$

The hedging position we will take each period is then given by the conditional covariance divided by the conditional variance of the futures contract at time t.

$$h_t^* | \Omega_{t-1} = -\frac{Cov(\Delta S_t, \Delta F_t | \Omega_{t-1})}{Var(\Delta F_t | \Omega_{t-1})} \tag{7.18}$$

At each time, we can calculate the return on the portfolio consisting of spot and futures.

$$r_{h_t} = \Delta S_t + (h_t^* | \Omega_{t-1}) \Delta F_t \tag{7.19}$$

And finally we can find the hedging efficeny by measuring how much the variance is reduced by comparing it to an unhedged portfolio:

$$HE = 1 - \frac{Var(r_{h_t})}{Var(\Delta S)} \tag{7.20}$$

## 7.5 Hedging with VAR-MGARCH

This model is much alike the regular VAR-model that was presented in section 7.3, but the model includes GARCH-error terms. By estimating a VAR and a GARCH-model simultaneously, we might obtain a richer model. The following model is based on a DVEC representation:

$$\Delta S_t = \rho_s + \sum_{i=1}^{k} \varphi_{si} \Delta S_{t-i} + \sum_{i=1}^{k} \theta_{si} \Delta F_{t-i,T} + \varepsilon_{st}$$

$$\Delta F_{t,T} = \rho_f + \sum_{i=1}^{k} \varphi_{fi} \Delta S_{t-i} + \sum_{i=1}^{k} \theta_{fi} \Delta F_{t-i,T} + \varepsilon_{ft}$$

$$H_t = C + \sum_{i=1}^{q} A_i \otimes \varepsilon_{t-i}\varepsilon'_{t-i} + \sum_{j=1}^{p} B_j \otimes H_{t-j}$$

$$\varepsilon_{it}|\Omega_{t-1} \sim \mathrm{distr}(0, \mathrm{H_t}) \tag{7.21}$$

where $H_t$ is a matrix for the conditional variances $h_{ss}$, $h_{ff}$ and the covariance $h_{sf}$. Since this model is solved with the maximum likelihood estimator, this will imply that the error terms in the model are normally distributed. The time-varying hedge ratio is calculated by using (6.21). The number of lags in the VAR-MGARCH-model is selected by minimizing the BIC.

# 8. Data and descriptive statistics

Most of the data are collected from the Nord Pool's FTP-server. As agreed upon with Nord Pool, none of the data is reproduced in tables. Here I have collected daily prices from 1995 to 2009. By using a high frequency, such as daily data, we use all available information in the data, but on the other side, one will often struggle with serial correlation in the residuals. However, this is not a valid justification. Serial correlation is often a question of how many lags of residuals we need to include in the model.

## 8.1 Spot and Futures Prices

The Spot prices have been registered since 1992 at Nord Pool. The earliest observations on Futures and Forward contracts were registered at 25th of September 1995. In order to be able to construct a comparable dataset for both spot and futures and forward prices, the sample stretches from 1/10/96 to 12/31/09. The system price is common to all four Nordic countries and does not consider the capacity constraints. The spot price in this thesis is a daily arithmetic average of the hourly spot prices.

Futures contract that are included in the study are futures contracts with 1 week, 4 and 12 weeks maturity. Since most futures and forward contracts are traded as quarters, this was a natural selection. In addition, shorter contracts, such as 1 month and 1 week, are interesting since it is reasonable to expect that contracts with shorter maturity have incorporated more fundamental information and therefore better predictors of future spot prices.

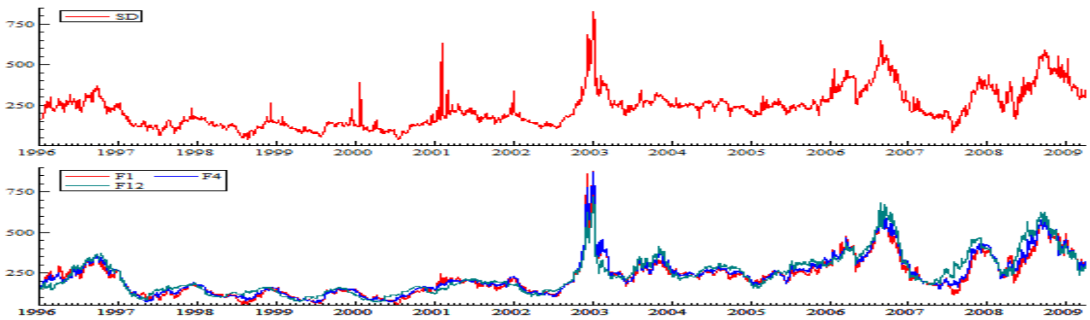


**Exhibit 8.1:** Daily spot and futures prices

The upper graph shows the daily average spot prices, while the lower graph shows daily prices for futures contracts with 1, 4 and 12 weeks until maturity, marked as F1, F4 and F12 respectively. As the graph shows us, these two time series are closely related to each other, indicating a high correlation. Compared to the spot prices, we can see that the futures contracts tend to follow much smoother patterns. This can be explained with a higher intraday volatility in the spot. The time series seems to be quite stable until 2003 when compared to the variation after 2003. The period after 2003 also seems to be a period with higher volatility. This shift could be taken as an argument for structural changes in the sample. It could be interesting to divide the data set into sub periods when investigating the different models.

| | Mean | Std.dev. | Kurtosis | Skewness | Jarque-Bera | Ljung-Box | ADF |
|---|---|---|---|---|---|---|---|
| SD | 232.7 | 111.6 | 1.4 | 1.0 | 855.5 *** | 36829.3 *** | -5.8 ** |
| F1 | 231.6 | 113.4 | 2.3 | 1.1 | 1498.9 *** | 38003.0 *** | -4.5 ** |
| F4 | 238.7 | 116.5 | 1.6 | 1.0 | 983.1 *** | 38992.5 *** | -3.7 * |
| F12 | 245.7 | 119.6 | 0.8 | 1.0 | 634.7 *** | 39712.5 *** | -3.5 * |
| Log.diff | | | | | | | |
| Weekly spot return | 0.2% | 13.8% | 10.4 | 0.1 | 15657.6 *** | 3307.3 *** | -23.3 ** |
| Monthly spot return | 1.5% | 24.4% | 2.2 | 0.1 | 700.6 *** | 16191.3 *** | -13.1 *** |
| Quarterly spot return | 2.2% | 38.4% | 0.9 | 0.5 | 267.3 *** | 29900.2 *** | -7.1 *** |
| Weekly F1 return | 0.1% | 9.5% | 5.2 | 0.3 | 3920.4 *** | 4776.0 *** | -20.3 *** |
| Monthly F4 return | 0.5% | 18.5% | 1.4 | 0.1 | 270.7 *** | 25811.2 *** | -7.5 *** |
| Quarterly F12 return | -1.8% | 35.2% | 0.2 | 0.4 | 106.3 *** | 36791.3 *** | -3.8 ** |

*Significance level: *p<0.05, **p<0.01, ***p<0.001*

**Exhibit 8.2:** Descriptive statistics for level and logarithmic difference

In the table I have outlined a standard table with descriptive statistics of the data. In addition to the level data, I have also calculated weekly, monthly and quarterly logarithmic differences on the daily data. The Logarithmic price changes is given as $r_t = \ln x_t - \ln x_{t-1}$. The graph below gives a representation of the logarithmic price changes the spot and futures contracts.

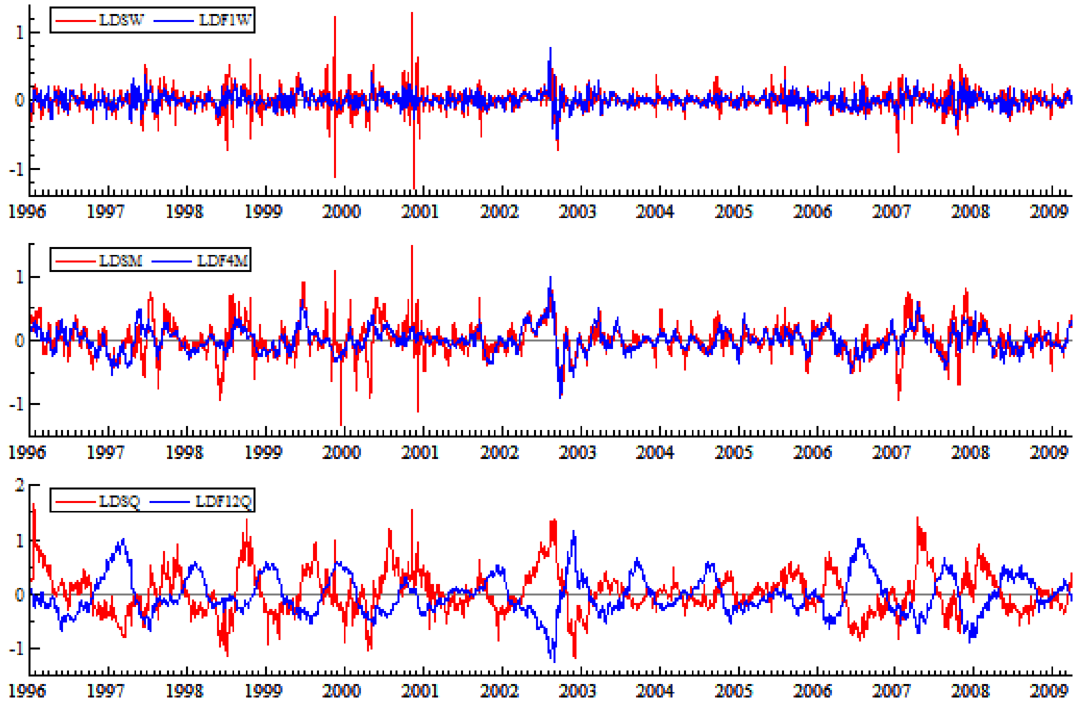


**Exhibit 8.3:** Logarithmic price changes

Perhaps the most remarkable feature of this time series is the volatility. On a daily basis, the quarterly spot return has a daily volatility of 38.4 per cent. On an annual basis, this corresponds to 76 per cent. Few, if any, other commodities have coefficients of variation above 40 per cent, see Gjølberg and Johnsen (2003) or Weron (2000). Although some of this volatility may be due to seasonal price movements, one must conclude that there is a substantial price risk in this market. We can also observe that the F12 time series is less volatile than the other futures contracts (F1 and F4). From the fact that it takes longer to implement changes in information, while the shorter contracts can more easily adjust to the new information.

When it comes to the distribution of the data material, we can note that the sample seemingly does not follow a normal distribution, but something that resembles a leptokurtic distribution. A graph over the empirical distributions with a reference to the normal distribution is provided in the appendix. The Jarque-Bera statistic tests the null hypothesis of zero skewness and excess kurtosis, and has a critical value of 5.99 at the 5 per cent significance level. Almost all studies of forward and futures prices reject the hypothesis about a normally distributed data set. Instead, Leptokurtic distribution is described as a better description of the financial data. Due to a higher frequency of extreme observation, it has fatter tails and higher peaks. Since there is a non-normality feature in the dataset, one has to be more careful interpreting the results. In addition, most studies of forward and futures prices show a first order autocorrelation. However, it is not strong enough to establish a trading strategy (Kolb 1997).

There also seems to be a problem with autocorrelation when using a Ljung-Box-test in the level data. The null hypothesis in Ljung-Box-statistics states that there are no autocorrelation in the data. The test indicates that there is autocorrelation after taking the logarithmic difference of the level. In many ways, this is an expected result, since there are overlapping periods in the calculated returns.

Another important property for time-series is stationarity. The intuition behind this is that when the time series is exposed to a shock, the series will not return to its trend. We can characterize the time series as being persistent, and it will only change when exposed to a new economic shock. In this thesis, the time-series is tested by using an ADF-test. The number of lags in the test was decided by using Schwarz Bayesian Information Criteria (BIC). When it comes to the time-series represented with logarithmic difference, there is strong evidence for stationarity. However, there is no clear evidence that supports the hypothesis that the time series is non-stationary when the data is represented in levels. When looking at the significance levels in the ADF-test, only the daily spot price have a p-value level lower than 5 per cent. We can also note that t-values for the ADF-test will probably be reduced when seasonal components are introduced into the test. One might suspect that the mean-reverting process in the time-series is explained due to strong seasonality in the sample.

Unit root can be found by using a different frequency, adjusting for seasonality or simply looking at a different sample. The existing literature shows that the time series of daily electricity prices in Nord Pool have low mean reversion with long memory, and therefore a unit root hypothesis is acceptable (Torró 2007). Nord Pool daily spot time series is studied by Escribano et al. (2002), Koopman et al. (2007) and Goto and Karolyi (2004). Escribano et al. (2002) and Koopman et al. (2007) analyzed the daily system price in the period 1993-1999. They concluded that the system price had a slow mean reversion. Goto and Karolyi (2004) analyzed the system price from 1993 to 1999. By using ADF-tests with different assumptions, they rejected the unit root hypothesis in most cases.

The last preliminary test performed on the data is a cointegration-test. Since there were some doubts about the stationary properties of the time-series, a cointegration-test is also conducted based on maximum likelihood estimation of the vector error correction model (VECM) with an unrestricted trend.

**Johansen's cointegration test**

| | Level | | | Log.diff | |
|---|---|---|---|---|---|
| **1 Week** | **Eigenvalue** | **Trace Stat** | **1 Week** | **Eigenvalue** | **Trace Stat** |
| H(0)++** | 0.13 | 487.60 | H(0)++** | 0.12 | 534.89 |
| H(1)++** | 0.01 | 17.44 | H(1)++** | 0.03 | 104.90 |
| Lags: 8 | | | Lags: 9 | | |
| **4 Weeks** | | | **4 Weeks** | | |
| H(0)++** | 0.05 | 186.78 | H(0)++** | 0.06 | 319.51 |
| H(1)++** | 0.01 | 20.47 | H(1)++** | 0.03 | 122.49 |
| Lags: 3 | | | Lags: 0 | | |
| **12 Weeks** | | | **12 Weeks** | | |
| H(0)++** | 0.01 | 69.09 | H(0)++** | 0.02 | 119.16 |
| H(1)++** | 0.01 | 17.89 | H(1)++** | 0.01 | 47.44 |
| Lags: 1 | | | Lags: 1 | | |

*Significance level: *p<0.05, **p<0.01*

**Exhibit 8.4:** Johansen's cointegration-test

As we can see from the exhibit, several cointegrating tests have been performed. H(0) test the null-hypothesis of zero cointegrating vectors and H(1) test the null-hypothesis of one cointegrating vector. A common trait for all of the tests is that a cointegrating relationship is rejected for all time series on a 1 per cent significance level. There is little evidence for a cointegrating relationship between the lagged futures prices and spot prices. This is a plausible result, since there were unclear, but weak indications for non-stationary time series. Since there is no long-term relationship between the variables, it is not necessary to construct a VECM-hedging-model.

## 8.2 Reservoir Levels

The data for reservoir levels are collected from the web pages of Norwegian Water Resources and Energy Directorate (nve.no), Svensk Energi, the Finnish Environment Institute SYKE and Nord Pool. The Finnish reservoir level is only included just after the integration in the Nord Pool market. Denmark is not included in these observations, since the power production does not include any hydropower. In the graph below the reservoir are measured as the percentage of maximum reservoir filling for the Nord Pool area.

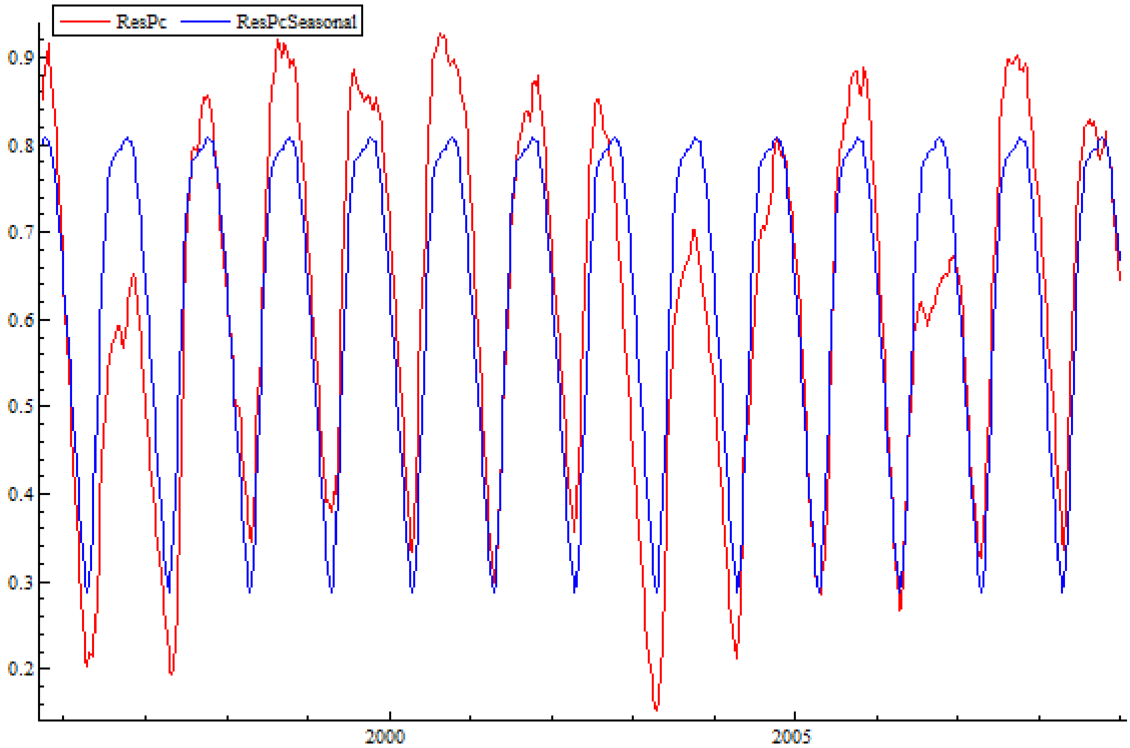


**Exhibit 8.5**: Reservoir level

During fall/early winter, reservoirs are typically holding 80-90 per cent of maximum capacity, recording a maximum filling of 95.1 per cent in November 1995. Before the snow starts melting, reservoirs are normally down to 40 per cent. During some years, 1996, 1997 and 2003 fillings reached below 20 per cent for 1996 and 1997 and 15 per cent for 2003. This is important to stress, since the reservoir level has an impact on the spot price, futures price and the basis.

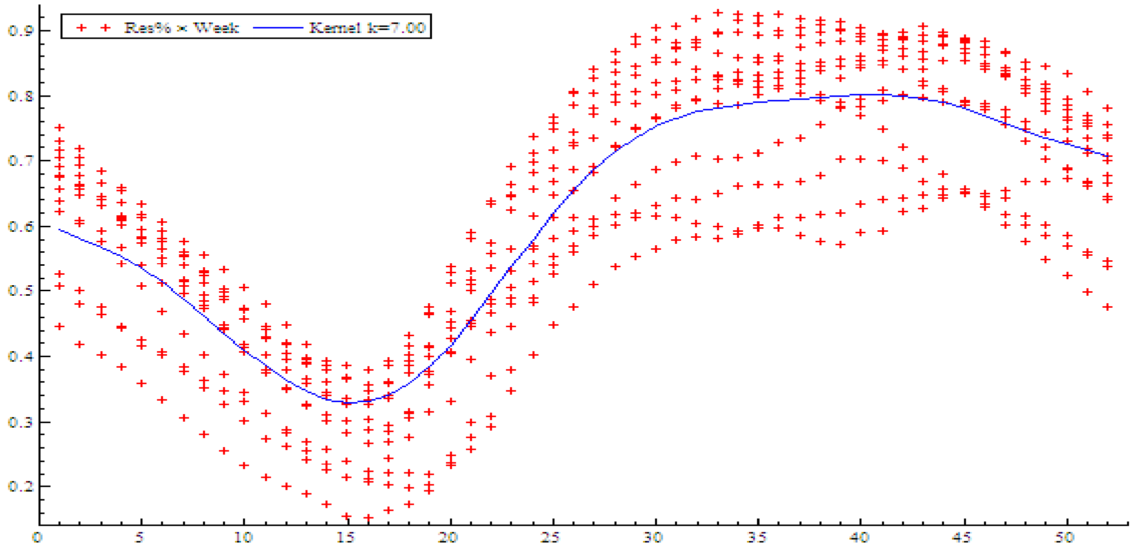


**Exhibit 8.6:** Seasonality of the reservoir level

In the exhibit above, the week numbers are displayed along the horizontal axis while the reservoir level is represented on the vertical axis. The exhibit seeks to illustrate the seasonal pattern in the dataset. The blue line is a smoothened curve based on Kernel. According to these historic data, we can normally expect the reservoirs to be a bit lower than 30 per cent around week 15 to 17. Nord Pool has divided the seasons into summer and winter. The summer season stretches from week 14 to 44. The rest of the year is considered as winter season.

# 9. Econometric Results for Spot-futures Relations

## 9.1 Unbiasedness Hypothesis

The unbiasedness hypothesis has been tested as described in section 6.1. In the exhibit below one can see the estimated parameters and residual diagnostics of the regression model.

| | 1 Week | 4 Week | 12 Week |
|---|---|---|---|
| $\alpha_0$ | 0.0062 | -0.0132 | -0.0346 * |
| Std.error | 0.0039 | 0.0098 | 0.0158 |
| $\beta_1$ | 0.8334 *** | 0.7948 *** | 0.9288 |
| Std.error | 0.0485 | 0.0624 | 0.0561 |
| R-square | 0.3549 | 0.2645 | 0.3257 |
| *Residual diagnostics* | | | |
| *White test* | 22.19 | 12.33 | 9.463 |
| *Jarque-Bera* | 16995.41 *** | 821.04 *** | 177.42 *** |
| *Ljung-Box* | 3993.148 *** | 19950.7995 *** | 58388.0535 *** |

*Null hypothesis: α=0 & β=1. Significance level: *p<0.05, **p<0.01, ***p<0.001*

**Exhibit 9.1:** Unbiasedness hypothesis on log.diff.

As we can see from the table, the constants are significantly different from zero for all contracts and β is significantly different from one for contracts that matures in 1 and 4 weeks. This implies that the unbiasedness hypothesis fails to hold in this market and that the futures prices do not reflect all relevant information about the future spot price. One can also note that the R-square is generally lower for the contracts with longest maturity, which indicates that there are higher basis risk for longer contracts.

The positive constant reflects the fact that typically, the electricity futures point upwards, even though the futures are not trending downward over time. This configuration of futures prices that are higher than spot prices is referred to as backwardation, and involves the interplay of cost-of-carry and convenience yield in driving a wedge between the futures rate and the expected future spot rate, e.g. see French (2005). In this case, we can see that the longer contracts have a larger β than the shorter. This might imply that the risk premium is higher for the contracts with longer maturity than the shorter contracts.

Futures prices may be biased forecasts of subsequent spot prices if there is systematic risk premium. However, if forecast errors are systematic over time or if predictions can be improved by utilizing available information, this is hard to reconcile with rational expectations and efficiency. It could be taken as a motivation for market participants to develop other prediction model of the spot price, e.g. see Torró (2007).

As we already know, there are a relatively low number of producers who control a large portion of production capacity in the market. This suggests that there is an excess of long hedging demand at Nord Pool with consumers paying a risk premium. In addition to such a risk premium, the concentration of supply might generate a "reservoir rent". In many ways, this model seems sensible and one might assume that futures prices are not an unbiased predictor for the future spot price.

In the next exhibit, the same test has been used for different sub-periods. The data sample was divided into 4 periods of equal length. The first period stretched from January 10$^{th}$ 1996 to July 19$^{th}$ 1999. The second period starts July 20$^{th}$ 1999 and end January 16$^{th}$ 2003. Third period lasted from January 17$^{th}$ 2003 to July 7$^{th}$ 2006. The fourth and final period began in July 13$^{th}$ 2006 and ended in December 30$^{th}$ 2009.

The exhibit on the next page is a summary of the econometric result of the unbiasedness hypothesis for each sub-period. For the first sub-period, there is weak evidence for market efficiency for the 1 and 4 week futures contracts. The market seemingly moved into a state of efficiency in the second sub-period for all contracts. The futures contracts were seemingly good predictors of the future spot prices and there are no strong indications for risk premium in the futures prices. In the third sub-period the market lost some of its market efficiency properties. It seems like the futures lost some its ability to predict the future spot prices. At the same time, there are some indications for market risk premium. The last sub-period of the sample, we can again see some indications for risk premium. However, the futures contracts have some predictable abilities.

As a general comment, we can say that Nord Pool has gone through several stages since the market opened. The recent period suggest that there is a risk premium, but futures prices have the ability to some degree to reflect the future spot prices.

**Period 1**

| | 1 Week | 4 Week | 12 Week |
|---|---|---|---|
| $\alpha_0$ | 0.0101 | -0.018 | -0.0643 |
| Std.error | 0.0081 | 0.0184 | 0.03 |
| $\beta_1$ | 0.7403 *** | 0.6383 *** | 0.8814 |
| Std.error | 0.0959 | 0.1033 | 0.0956 |
| R-square | 0.2786 | 0.2357 | 0.3621 |
| *Residual diagnostics* | | | |
| *White test* | 18.88 *** | 10.89 ** | 0.1454 |
| *Jarque-Bera* | 190.9101 *** | 49.4045 *** | 9.8568 *** |
| *Ljung-Box* | 1226.8445 *** | 5626.5948 *** | 14671.3318 *** |

**Period 2**

| | 1 Week | 4 Week | 12 Week |
|---|---|---|---|
| $\alpha_0$ | 0.0083 | 0.0285 | 0.0753 ** |
| Std.error | 0.0085 | 0.0172 | 0.0274 |
| $\beta_1$ | 0.8705 | 1.0045 | 1.0667 |
| Std.error | 0.0738 | 0.0904 | 0.0704 |
| R-square | 0.4086 | 0.4234 | 0.4286 |
| *Residual diagnostics* | | | |
| *White test* | 4.803 | 1.383 | 10.73 ** |
| *Jarque-Bera* | 7275.0707 *** | 487.5334 | 296.4619 |
| *Ljung-Box* | 877.8283 *** | 4404.7647 | 11420.554 |

**Period 3**

| | 1 Week | 4 Week | 12 Week |
|---|---|---|---|
| $\alpha_0$ | -0.0035 | -0.0166 | -0.0308 |
| Std.error | 0.0054 | 0.0148 | 0.022 |
| $\beta_1$ | 0.7126 *** | 0.3228 *** | 0.5829 *** |
| Std.error | 0.0781 | 0.1462 | 0.1297 |
| R-square | 0.2676 | 0.0349 | 0.132 |
| *Residual diagnostics* | | | |
| *White test* | 15.68 *** | 11.04 ** | 15.25 |
| *Jarque-Bera* | 3004.7863 | 599.9661 | 271.651 |
| *Ljung-Box* | 889.7178 | 4528.1999 | 10573.487 |

**Period 4**

| | 1 Week | 4 Week | 12 Week |
|---|---|---|---|
| $\alpha_0$ | 0.0108 | -0.0384 *** | -0.1258 *** |
| | 0.006 | 0.0173 | 0.0296 |
| $\beta_1$ | 1.013 | 1.0337 | 1.139 |
| Std.error | 0.0696 | 0.1132 | 0.1125 |
| R-square | 0.4344 | 0.3155 | 0.3561 |
| *Residual diagnostics* | | | |
| *White test* | 4.943 | 1.099 | 9.375 |
| *Jarque-Bera* | 1618.2397 | 57.1379 | 30.1759 |
| *Ljung-Box* | 987.8469 | 4716.4277 | 13484.6645 |

*Null hypothesis: $\alpha=0$ & $\beta=1$. Significance level: *p<0.05, **p<0.01, ***p<0.001*

**Exhibit 9.2:** Unbiasedness hypothesis tested under different period

## 9.2 Dynamic Model for Risk Premiums

As discussed, the poor forecasting power of futures prices can be caused by a risk premium (constant or dynamic). Since there were uncertain evidence for risk premium in the previous regression analysis, it could be interesting to see whether a dynamic approach would give clearer results. In this section I will test the VAR-model for dynamic risk premium as described in section 6.2.

A continuously varying premium impairs the effectiveness of the futures market. Note that the risk premium can also be regarded as the forecasting error. The premium is calculated as $P_t = F_{t,T} - S_T$ and the basis is calculated as $B_t = F_{t,T} - S_t$. The calculated risk premiums displayed in the exhibit below can be understood as an ex post or realized forecasting errors under the assumption of rational expectations.

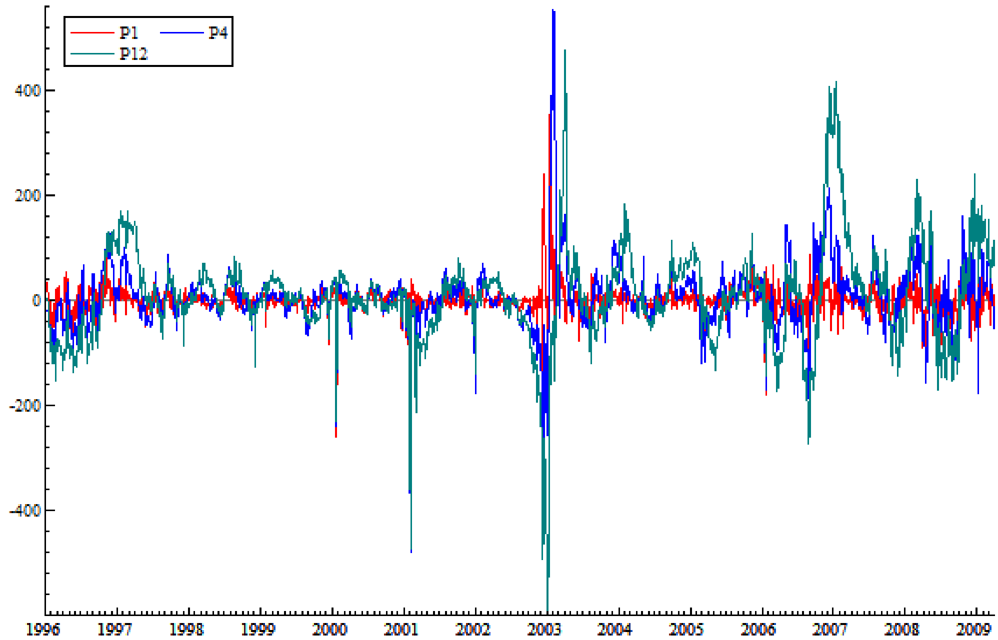


**Exhibit 9.3:** Risk premiums in futures when using level

As we can see from the graph the risk premium seems to been varying considerably over the sample. In winter of 2003 the risk premium reached a remarkable peak. The low reservoir level in that period in conjunction with the economic situation gave rise to an astonishing risk premium for the longer contracts in this sample. It can also be noted that the risk premium seemingly to be more volatile in the period after 2003 compared to the period before.

| | 1 Week | | | | 4 Weeks | | | | 12 Weeks | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ΔB | | ΔP | | ΔB | | ΔP | | ΔB | | ΔP | |
| α | -0.027 | *** | -0.021 | *** | -0.023 | *** | -0.018 | *** | -0.005 | | -0.016 | *** |
| (std.err) | 0.004 | | 0.004 | | 0.004 | | 0.004 | | 0.004 | | 0.005 | |
| Res | 0.043 | *** | 0.033 | *** | 0.045 | *** | 0.037 | *** | 0.015 | * | 0.034 | *** |
| (std.err) | 0.006 | | 0.006 | | 0.007 | | 0.007 | | 0.007 | | 0.008 | |
| $\Delta B_{t-1}$ | 0.520 | *** | -0.514 | *** | 0.605 | *** | -0.421 | *** | 0.766 | *** | -0.274 | *** |
| (std.err) | 0.027 | | 0.026 | | 0.029 | | 0.030 | | 0.029 | | 0.030 | |
| $\Delta P_{t-1}$ | 0.150 | *** | 1.147 | *** | 0.199 | *** | 1.198 | *** | 0.081 | ** | 1.108 | *** |
| (std.err) | 0.027 | | 0.027 | | 0.028 | | 0.029 | | 0.029 | | 0.030 | |
| $\Delta B_{t-2}$ | 0.177 | *** | 0.283 | *** | 0.253 | *** | 0.315 | *** | 0.179 | *** | 0.217 | *** |
| (std.err) | 0.036 | | 0.035 | | 0.029 | | 0.030 | | 0.030 | | 0.031 | |
| $\Delta P_{t-2}$ | -0.165 | *** | -0.267 | *** | -0.188 | *** | -0.225 | *** | -0.092 | * | -0.126 | *** |
| (std.err) | 0.037 | | 0.036 | | 0.028 | | 0.029 | | 0.029 | | 0.030 | |
| $\Delta B_{t-3}$ | -0.014 | | -0.017 | | | | | | | | | |
| (std.err) | 0.034 | | 0.033 | | | | | | | | | |
| $\Delta P_{t-3}$ | 0.021 | | 0.024 | | | | | | | | | |
| (std.err) | 0.034 | | 0.033 | | | | | | | | | |
| $\Delta B_{t-4}$ | -0.020 | | -0.037 | | | | | | | | | |
| (std.err) | 0.034 | | 0.033 | | | | | | | | | |
| $\Delta P_{t-4}$ | 0.058 | | 0.021 | | | | | | | | | |
| (std.err) | 0.034 | | 0.033 | | | | | | | | | |
| $\Delta B_{t-5}$ | -0.023 | | 0.481 | *** | | | | | | | | |
| (std.err) | 0.034 | | 0.033 | | | | | | | | | |
| $\Delta P_{t-5}$ | 0.039 | | -0.353 | *** | | | | | | | | |
| (std.err) | 0.034 | | 0.033 | | | | | | | | | |
| $\Delta B_{t-6}$ | -0.087 | * | -0.572 | *** | | | | | | | | |
| (std.err) | 0.036 | | 0.036 | | | | | | | | | |
| $\Delta P_{t-6}$ | 0.066 | | 0.502 | *** | | | | | | | | |
| (std.err) | 0.035 | | 0.035 | | | | | | | | | |
| $\Delta B_{t-7}$ | 0.066 | * | 0.177 | *** | | | | | | | | |
| (std.err) | 0.029 | | 0.028 | | | | | | | | | |
| $\Delta P_{t-7}$ | -0.038 | | -0.162 | *** | | | | | | | | |
| (std.err) | 0.026 | | 0.026 | | | | | | | | | |
| BIC | -20612 | | | | -19950 | | | | -19864 | | | |
| R-squared | 0.5 | | 0.6 | | 0.8 | | 0.9 | | 0.9 | | 0.9 | |
| *Jarque-Bera* | 146327.0 | *** | 172235.9 | *** | 178417.7 | *** | 82559.1 | *** | 117881.6 | *** | 89177.5 | *** |
| *Ljung-Box* | 4.7 | | 10.7 | | 10.9 | | 30.7 | *** | 25.2 | ** | 22.0 | * |

*Significance level: *p<0.05, **p<0.01, ***p<0.001*

**Exhibit 9.4:** Dynamic risk premium model

As the table tells us, the basis for all contracts is considerably larger than the cost of capital. This means that the return on selling futures on average is considerable higher than the price of tying up the necessary capital. The basis also reflects storage costs and convenience yield, but these should only be present in the electricity market when the risk of a stock-out or overflow is considerable. This is usually not the case during most of the year.

The advantage by using a VAR-model to calculate the risk-premium by using fitting values is to smoothen out some of excessive variation in the data. This way, we can make a generalization out of the data. In the graph below, I have organized the average risk premium from the fitted values on the vertical axis and weeks on the horizontal axis. The graph gives an indication for time-varying risk premium following a cyclical pattern.

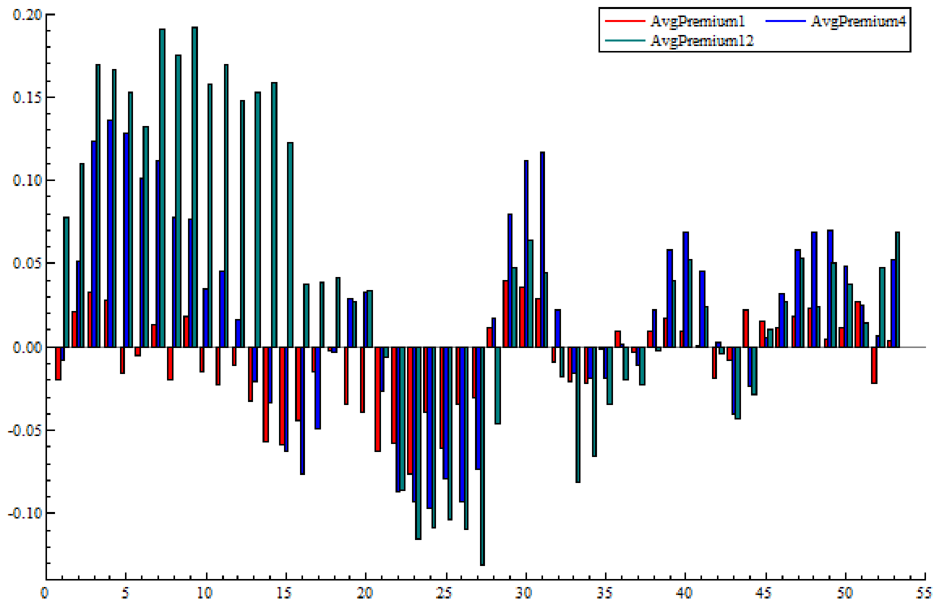


**Exhibit 9.5**: Time-varying risk premium

In the summer months the risk premium tend to be positive and negative for the winter season. When testing for the statistical properties for the seasonal risk premium, it is found that the average risk premium in week 10,15,17,23 and 24 is statistical different from zero. For the futures contracts with 4 weeks to maturity, the average premium in week 16-26 is shown to be significant different from zero. For the contract that matures in 12 weeks, the average risk premium is statistical different from zero. In summary, it seems that the hedger has been most eager to hedge their position when the reservoir reaches its seasonal trough.

This time-varying risk premium can be considered as the cause for market inefficiency in the futures market. In the stock market the risk premium tend to vary between, i.e. 5-8 per cent on an annual basis (see Gorton and Rouwenhorst (2006). An alternative approach to measure the realized risk premium is to test the unbiasedness hypothesis with a seasonal dummy, see Gjølberg and Brattested (2010). By using this approach they found that forecasting error varied between 7-9 per cent on a monthly basis over the period 1995-2008. Given its magnitute they argeued that it is hard to explain the forecast error as a risk premium only.

In the classical view of hedging pressure as a determinant of risk premiums, when it is positive (futures prices below expected spot prices), the futures market is said to be in *normal backwardation* (short hedging pressure). On the other hand, if the *forward bias* is negative (futures prices above expected spot prices), the futures market is said to be in *contango* (long hedging pressure).

# 10. Econometric Results for Hedging

In this section I will introduce the econometric result I obtained by running the different hedging models, both in-the-sample as well as out-of-sample. These include both constant and time-varying hedge ratios. In order to determine the best hedging model, all models will be summarized in the end of this chapter

## 10.1 Naïve Hedge

The simplest hedging approach is to take an equal position in the spot markets as in the futures market. This approach requires no sophisticated modeling, but can give unreliable results. The main reason to use this approach is to use it as a benchmark for the more sophisticated models.

| **Naïve** | 1 Week | 4 Weeks | 12 Weeks |
|---|---|---|---|
| Whole period | 25.4 | 38.3 | -194.6 |

**Exhibit 10.1:** Naïve hedge

As we can see from the exhibit above a naïve hedge would give a risk reduction in the variance of the spot as much as 25.4 and 28.3 per cent for contracts that mature in 1 and 4 weeks, and a 194.6 per cent increase for 12 weeks. The closer the contracts are to maturity, more information is incorporated into the futures contracts. In many aspects, the naïve hedge approach may be a better approach for the shorter contracts, but one should be careful to use a naïve approach for contracts since it does no real assessment of variance reduction.

## 10.2 Hedge with OLS

In this section, I will present the result from the constant hedge ratio by employing a classical OLS-regression. In the graph below, one can see the regression on logarithmic return of the spot on the futures. When assessing the hedging efficiency of the futures contract with constant hedge ratios, it is assumed that hedger operates constantly in the spot where the hedger rolls over the futures contract from one period to the next.

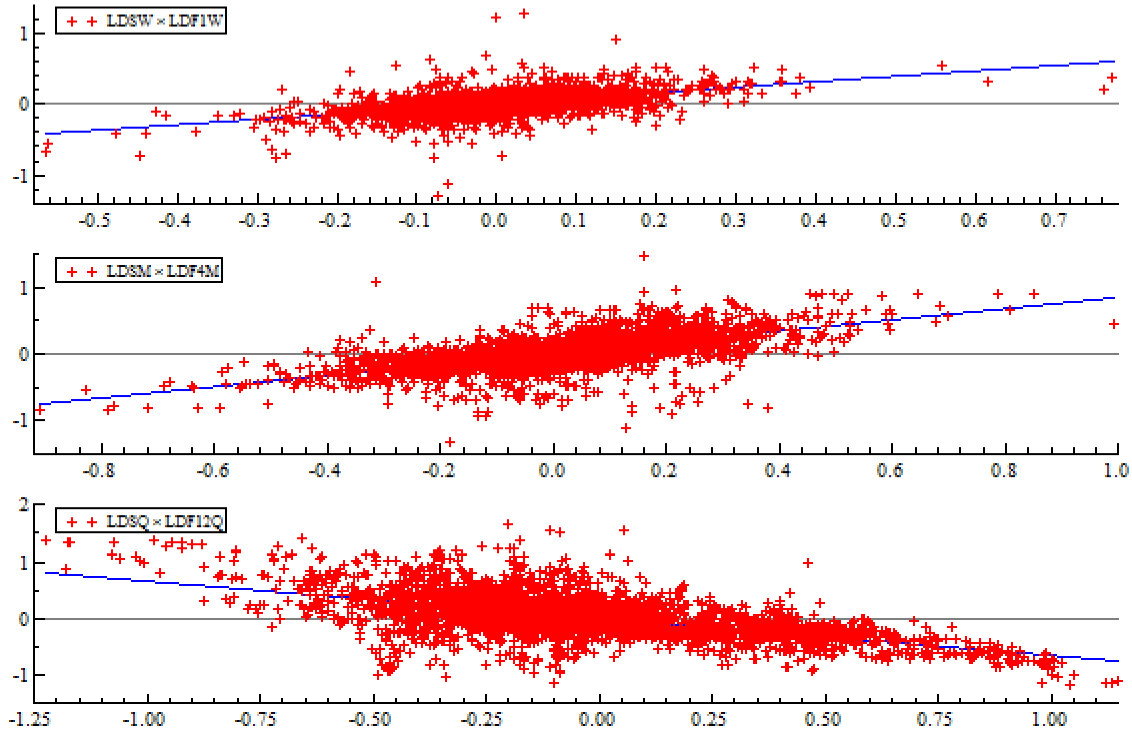


**Exhibit 10.2:** OLS-regression

As we can see from the graphs above, there seems to be an interesting linear relationship between logarithmic difference of the spot and futures prices. For the shorter contracts that mature in 1 and 4 weeks, the positive slope indicates that one could obtain a risk-reducing portfolio by going short in the futures and long in the spot. For contracts that mature in 12 weeks, one can go long in futures in order to obtain risk reduction. When comparing the R-square (hedging efficiency) with the naïve hedge, we can see that OLS-hedge is a more reliable strategy since it give risk reduction for all contracts. One can clearly see an improvement for the portfolio consisting of spot and futures contracts that matures in 12 weeks. In addition, one can note that R-squares lower than 50 per cent indicates that there is more basis risk than diversifiable risk.

| | 1 Week | 4 Week | 12 Week |
|---|---|---|---|
| $\alpha_0$ | 0.002 | 0.010 | 0.010 |
| | 0.003 | 0.008 | 0.014 |
| $\beta_1$ | 0.768 *** | 0.835 *** | -0.656 *** |
| | 0.043 | 0.044 | 0.039 |
| R-square | 0.279 | 0.399 | 0.363 |
| *Residual diagnostics* | | | |
| *White test* | 13.0 *** | 29.0 *** | 173.9 *** |
| *Jarque-Bera* | 40302.8 *** | 5685.2 *** | 661.4 *** |
| *Ljung-Box* | 2933.9 *** | 13783.4 *** | 44763.1 *** |

*Std.error in parentheses. Significance level: *p<0.05, **p<0.01, ***p<0.001*

**Exhibit 10.3:** Econometric result for constant hedging ratio

As we can see from the exhibit, the constant hedge ratio for the period varies from 5 to 42 per cent. This model seems to give reliable result since we obtain coefficients that are significantly different from zero for the variables on a 1 per cent significance level.

However, there are some worrying elements with this regression. It seemingly violates the heteroscedacity assumption, autocorrelation and normality assumption. The remedy for this problem is to use Newey-West HAC-standard errors. This is crucial for the interpretation of the regression-statistics since there are clear signs for heteroscedacity and autocorrelation in the time-series.

## 10.3 Hedge with VAR

One can also use a VAR-model to calculate a constant hedge-ratio as described in chapter 5.4, by constructing a bivariate model where the current spot and futures prices is a function of their own lagged values. The first thing we should check for is how many lags we should include into the model. As augmented for earlier, an appropriate way of determining the optimal number of lags in a VAR-model, is to use the model that minimizes Schwarz's Bayesian information criterion (BIC).

| | 1 Week | | 4 Weeks | | 12 Weeks | |
|---|---|---|---|---|---|---|
| | ΔS | $\Delta F_{1\,week}$ | ΔS | $\Delta F_{1\,week}$ | ΔS | $\Delta F_{1\,week}$ |
| α | -0.002 | 0.000 | -0.001 | 0.000 | 0.000 | 0.000 |
| (std.err) | 0.001 | 0.001 | 0.002 | 0.001 | 0.002 | 0.001 |
| $\Delta S_{t-1}$ | 0.588 *** | -0.027 * | 0.834 *** | -0.017 | 0.844 *** | 0.021 ** |
| (std.err) | 0.017 | 0.011 | 0.017 | 0.008 | 0.017 | 0.008 |
| $\Delta F_{t-1}$ | 0.616 *** | 1.011 *** | 0.455 *** | 1.015 *** | -0.371 *** | 1.037 *** |
| (std.err) | 0.028 | 0.017 | 0.037 | 0.017 | 0.037 | 0.017 |
| $\Delta S_{t-2}$ | 0.027 *** | 0.037 ** | 0.012 | 0.010 | 0.103 *** | -0.011 |
| (std.err) | 0.020 | 0.012 | 0.017 | 0.008 | 0.017 | 0.008 |
| $\Delta F_{t-2}$ | -0.301 *** | -0.069 ** | -0.318 *** | -0.042 * | 0.328 *** | -0.039 * |
| (std.err) | 0.040 | 0.025 | 0.037 | 0.018 | 0.037 | 0.017 |
| $\Delta S_{t-3}$ | -0.026 | -0.009 | | | | |
| (std.err) | 0.020 | 0.012 | | | | |
| $\Delta F_{t-3}$ | 0.045 | -0.036 | | | | |
| (std.err) | 0.040 | 0.025 | | | | |
| $\Delta S_{t-4}$ | -0.028 | -0.033 ** | | | | |
| (std.err) | 0.020 | 0.012 | | | | |
| $\Delta F_{t-4}$ | 0.024 | -0.006 | | | | |
| (std.err) | 0.040 | 0.025 | | | | |
| $\Delta S_{t-5}$ | -0.523 *** | 0.006 | | | | |
| (std.err) | 0.019 | 0.012 | | | | |
| $\Delta F_{t-5}$ | 0.142 | -0.453 *** | | | | |
| (std.err) | 0.038 | 0.024 | | | | |
| $\Delta S_{t-6}$ | 0.268 *** | -0.005 | | | | |
| (std.err) | 0.019 | 0.012 | | | | |
| $\Delta F_{t-6}$ | 0.283 *** | 0.497 *** | | | | |
| (std.err) | 0.038 | 0.023 | | | | |
| $\Delta S_{t-7}$ | 0.025 | 0.035 ** | | | | |
| (std.err) | 0.020 | 0.012 | | | | |
| $\Delta F_{t-7}$ | -0.128 *** | -0.067 ** | | | | |
| (std.err) | 0.040 | 0.025 | | | | |
| $\Delta S_{t-8}$ | -0.028 | -0.013 | | | | |
| (std.err) | 0.020 | 0.012 | | | | |
| $\Delta F_{t-8}$ | -0.006 | -0.075 ** | | | | |
| (std.err) | 0.040 | 0.025 | | | | |
| Hedging Efficiency | 16.12% | | 21.59% | | 17.05 | |
| BIC | -19310 | | -17461 | | -17844 | |
| R-squared | 0.71 | 0.77 | 0.84 | 0.94 | 0.93 | 0.98 |
| *Jarque-Bera* | 94594.5 *** | 6236.9 *** | 75454.6 *** | 6239.1 *** | 29668.7 *** | 14951.1 *** |
| *Ljung-Box* | 110.6 *** | 125.1 *** | 77.5 *** | 42.1 *** | 33.8 *** | 110.6 *** |

*Null hypothesis: α=0 & β=1. Std.error in parentheses. Significance level: *p<0.05, **p<0.01, ***p<0.001*

**Exhibit 10.4:** Constant hedge ratio with VAR

In this model the residual were used to calculate the hedging ratio and the hedging efficiency. The result is found the exhibit above. In general, we can note that the VAR-estimates normally give the same results as the VAR-hedge, but the OLS-portfolio has a slight better ability to reduce variance of the portfolio. Since there is some evidence for significant autoregressive variables, it can be interpreted as a predictor for a good hedge ratio for the next period.

There are some concerns with the estimates from the VAR-model. Firstly the residuals appear not to follow a normal-distribution. If the residuals are not normally distributed, then the model might be misspecified or an important variable might be missing. Secondly they also seem to violate the assumption of no autocorrelation. As already mentioned, autocorrelation does not generate a biased coefficient but they give misleading standard errors. Compared to the OLS-model, we can see that the VAR-model has much lower Ljung-Box-statistics.

Another feature of the VAR-model is that we can also check for Granger Causality between the variables. It is a test for the lead-lag relationship between the futures and spot markets and their difference in the rate of absorbing new information. For instance, we can check whether the futures price has an leading role on the price formation and whether the price discovery market is efficient. This discussion comes as an anecdote to the discussion around unbiasedness-hypothesis. The null hypothesis in the first row states that spot do not Granger-cause the futures prices, while the null hypothesis in the second row states that futures prices do not Granger cause spot prices.

| | 1 Week | | 4 Week | | 12 Week | |
|---|---|---|---|---|---|---|
| | F-test | P-value | F-test | P-Value | F-test | P-Value |
| Future | 93.47 | - | 130.10 | - | 77.05 | - |
| Spot | 4.21 | 0.00 | 2.86 | 0.06 | 9.41 | 0.00 |

**Exhibit 10.5:** Granger Causality

As we can see from this table, we reject the null hypothesis that changes in the futures prices do not Granger causes the changes in the spot price. At the same time we do not reject the null hypothesis that changes in spot prices is Granger causing the changes in futures price. This is interesting, since it suggest that futures prices might have some predictability on the spot. This is result is in line with the founding's by Yang, et al. (2009). This can indicate that there exists unidirectional Granger causality from the futures price to the spot price, and it suggests that the futures price can lead the spot price.

### 10.4 Hedge with MGARCH

The first time-varying hedging-model is the MGARCH-model represented with BEKK(2,1). This representation was chosen since it gave the lowest BIC-value when compared to DVEC. Hedge ratios have been estimated by using multivariate GARCH-models. All multivariate GARCH-models were estimated by employing the BHH algorithm in FinMetrics. In this study, several versions of the MGARCH-models were tested out with the respect to the number of lagged variables. I found the simplest model would give the smallest BIC-value. In order to avoid too many parameters three separate bivariate models were estimated.

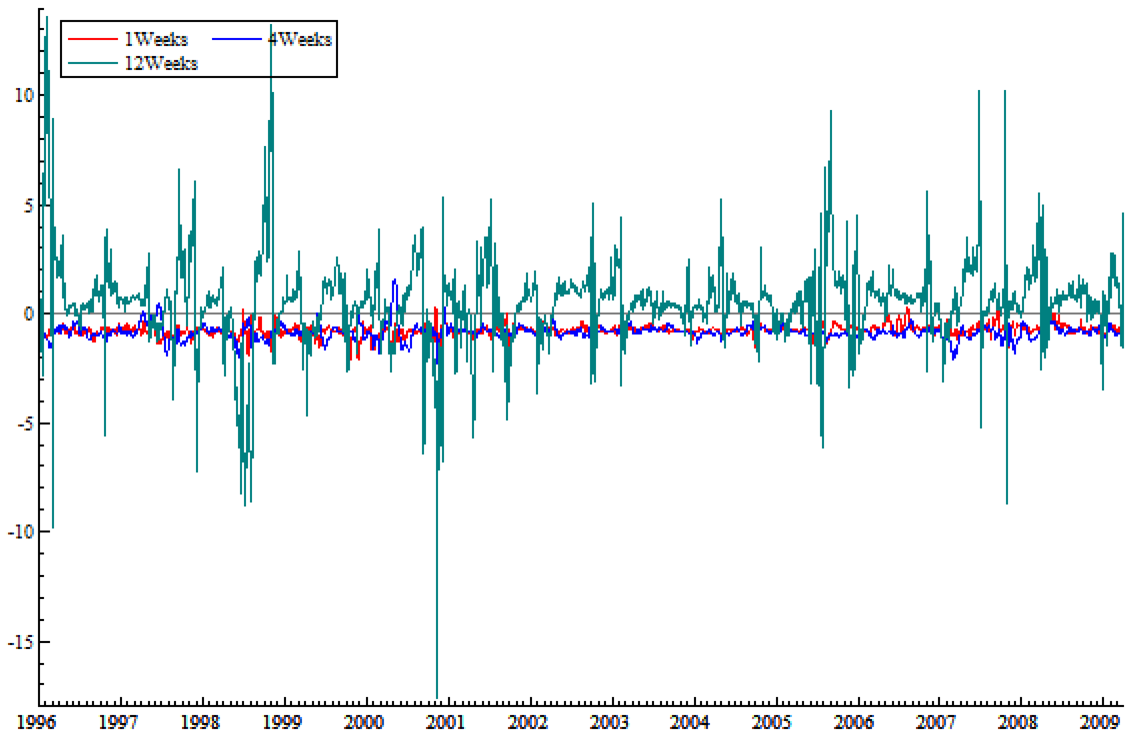


**Exhibit 10.6:** Time-varying hedge-ratio

As we can see from the graph, the hedge ratio tends to vary around a level below the horizontal axis for the shorter contracts that matures in 1 and 4 weeks. With some few exceptions, most of the time-varying hedge ratio indicates that the best risk reducing position is to go long in the spot and short in the futures contracts. For contracts that mature in 12 weeks, the hedging position will vary over time depending on the season and other market conditions.

| | 1 Week | | | 4 Weeks | | | 12 Weeks | | |
|---|---|---|---|---|---|---|---|---|---|
| | Value | Std.Error | | Value | Std.Error | | Value | Std.Error | |
| $c_{ss,t}$ | 0.041 | 0.004 | *** | 0.073 | 0.009 | *** | 0.060 | 0.002 | *** |
| $c_{sf,t}$ | 0.015 | 0.003 | *** | 0.035 | 0.008 | *** | (0.000) | 0.002 | |
| $c_{ff,t}$ | 0.024 | 0.003 | *** | 0.043 | 0.005 | *** | 0.031 | 0.001 | *** |
| $\alpha_{ss,t-1}$ | 0.316 | 0.029 | *** | 0.316 | 0.036 | *** | 1.022 | 0.051 | *** |
| $\alpha_{sf,t-1}$ | (0.000) | 0.018 | | 0.000 | 0.021 | | 0.007 | 0.004 | |
| $\alpha_{fs,t-1}$ | 0.000 | 0.028 | | 0.000 | 0.036 | | (0.043) | 0.008 | *** |
| $\alpha_{ff,t-1}$ | 0.316 | 0.027 | *** | 0.316 | 0.037 | *** | 1.069 | 0.054 | *** |
| $\alpha_{ss,t-2}$ | 0.900 | 0.096 | *** | 0.900 | 0.134 | | 0.249 | 0.029 | *** |
| $\alpha_{sf,t-2}$ | 0.000 | 0.038 | | (0.000) | 0.061 | | (0.008) | 0.005 | |
| $\alpha_{fs,t-2}$ | (0.000) | 0.084 | | (0.000) | 0.113 | | 0.066 | 0.010 | *** |
| $\alpha_{ff,t-2}$ | 0.900 | 0.116 | | 0.900 | 0.154 | | 0.108 | 0.035 | ** |
| $\beta_{ss,t-1}$ | 0.018 | 4.170 | | 0.021 | 4.854 | | 0.191 | 0.023 | *** |
| $\beta_{sf,t-1}$ | (0.001) | 1.500 | | 0.000 | 2.158 | | 0.006 | 0.007 | |
| $\beta_{fs,t-1}$ | 0.005 | 2.976 | | 0.003 | 3.816 | | (0.045) | 0.012 | *** |
| $\beta_{f,t-1}$ | 0.025 | 3.741 | | 0.024 | 4.983 | | 0.268 | 0.019 | *** |
| | HE | 33.7% | | HE | 56.1% | | HE | 86.8% | |
| | BIC(16) | -14645 | | BIC(16) | -7273 | | BIC(16) | -6496 | |
| | Jarque-Bera | | | Jarque-Bera | | | Jarque-Bera | | |
| | LDSW | 8299.2 | *** | LDSM | 181.8 | *** | LDSQ | 41.9 | *** |
| | LDF1W | 148.4 | *** | LDF4M | 972.6 | *** | LDF12Q | 55492.2 | *** |
| | Ljung-Box | | | Ljung-Box | | | Ljung-Box | | |
| | LDSW | 110.4 | *** | LDSM | 857.2 | *** | LDSQ | 7.0 | |
| | LDF1W | 481.2 | *** | LDF4M | 387.3 | *** | LDF12Q | 3.9 | |

*Significance level: *p<0.05, **p<0.01, ***p<0.001*

**Exhibit 10.7:** Hedging with MGARCH

The MGARCH-model gives a remarkable high hedging efficiency when comparing it to OLS, VAR and the Naïve-model. It gives us an indication that a time-varying hedge ratio is a better procedure to construct a risk reducing portfolio.

The exhibit below shows us that the residuals follow a non-normal distribution. When it comes to the other assumptions concerning the residuals, it seems like that the MGARCH-model is able to capture the autocorrelation. The residual diagnostics indicates that the MGARCH-models do not fulfill the requirement for a normal distribution. This indicates that the MGARCH-model might not correctly specified or insufficient to describe the volatility of the time series. Unfortunately, when there are outliers from the normally distribution in the data, classical methods can have a poor performance. The recent years risk measurements such as hedge ratio has been criticized since they seldom are able to encompass the fat tails into the calculations (e.g. see Taleb 2005 & 2007). Despite these fact, the models gives better results than the OLS- and VAR-model.

Over a year, we can see that there are some seasonal changes in the hedging-position. The shorter contracts that matures in 1 and 4 weeks, will typically imply that one should be long in the spot and short in the futures contract over the whole year, while contracts that matures in one quarter, will have some seasonal changes. The exhibit below shows the weekly average of hedge ratios. The graph gives a weak indication between the hedge ratios and traded volume of financial contracts.

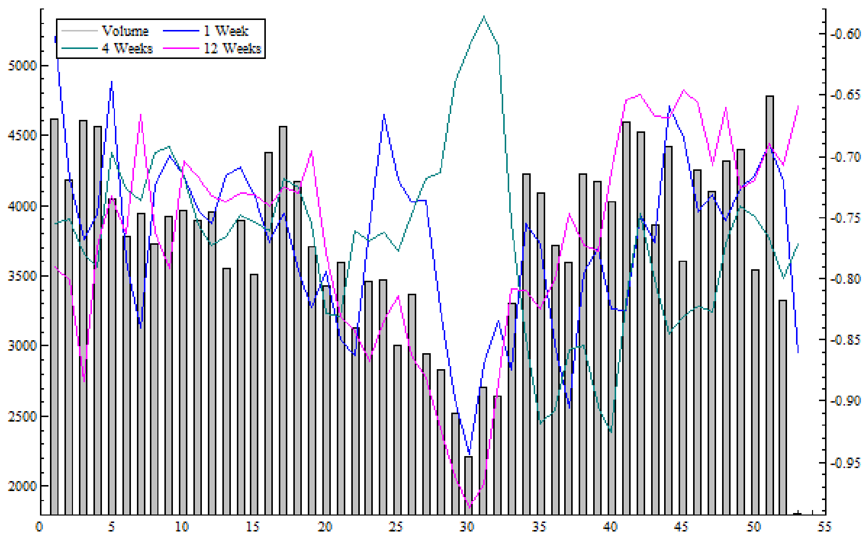


**Exhibit 10.8**: Average seasonal hedge ratios

## 10.5 Hedging with VAR-MGARCH

The last and final model is the VAR-MGARCH, which considers the heteroscedastic volatility effect of the time series and calculate time-varying hedge ratio. This is a VAR-model with GARCH error-terms. This model is a mix between the regular VAR-model and the MGARCH-model. This model is then used to calculate the dynamic hedge ratios based on the conditional variance and covariance of the spot and futures prices. Here, I have used a DVEC-based MGARCH-model since it gave the lowest BIC-values compared to BEKK. In table below, we can see the estimated parameters from the VAR-MGARCH-model

| | 1 Week | | | 4 Weeks | | | 12 Weeks | | |
|---|---|---|---|---|---|---|---|---|---|
| | Value | Std.Error | | Value | Std.Error | | Value | Std.Error | |
| $\rho_s$ | 0.000 | 0.001 | | -0.001 | 0.001 | | 0.004 | 0.003 | |
| $\rho_f$ | 0.000 | 0.001 | | 0.000 | 0.001 | | -0.002 | 0.002 | |
| $\theta_{s1}$ | 0.609 | 0.010 | *** | 0.854 | 0.007 | *** | 0.920 | 0.010 | *** |
| $\theta_{f1}$ | -0.050 | 0.006 | *** | -0.009 | 0.003 | | 0.031 | 0.006 | *** |
| $\varphi_{s1}$ | 0.381 | 0.011 | *** | 0.145 | 0.008 | *** | -0.061 | 0.012 | *** |
| $\varphi_{f1}$ | 0.923 | 0.009 | *** | 0.990 | 0.004 | *** | 0.995 | 0.007 | *** |
| $c_{ss,t}$ | 0.000 | NA | | 0.000 | NA | | 0.003 | NA | |
| $c_{sf,t}$ | 0.000 | NA | | 0.000 | NA | | 0.000 | NA | |
| $c_{ff,t}$ | 0.000 | NA | | 0.000 | NA | | 0.003 | NA | |
| $\alpha_{ss,t-1}$ | 0.328 | NA | | 0.265 | NA | | 0.157 | NA | |
| $\alpha_{sf,t-1}$ | 0.019 | 0.000 | *** | 0.062 | 0.000 | *** | 0.087 | 0.001 | *** |
| $\alpha_{ff,t-1}$ | 0.027 | 0.000 | *** | -0.023 | 0.000 | *** | 0.088 | 0.000 | *** |
| $\alpha_{ss,t-2}$ | -0.024 | 0.000 | *** | -0.130 | 0.000 | *** | -0.061 | 0.001 | *** |
| $\alpha_{sf,t-2}$ | 0.047 | 0.045 | | -0.029 | 0.047 | | 0.004 | 0.057 | |
| $\alpha_{ff,t-2}$ | 0.097 | 0.025 | *** | 0.113 | 0.029 | *** | 0.020 | 0.101 | |
| $\beta_{ss,t-1}$ | 0.715 | 0.016 | *** | 0.873 | 0.011 | *** | 0.830 | 0.046 | *** |
| $\beta_{sf,t-1}$ | 0.834 | 0.047 | *** | 0.903 | 0.043 | *** | 0.765 | 0.055 | *** |
| $\beta_{f,t-1}$ | 0.860 | 0.028 | *** | 0.912 | 0.029 | *** | 0.452 | 0.088 | *** |
| | | | | | | | | | |
| | HE | 15.24% | | HE | 8.04% | | HE | 24.03% | |
| | BIC | -21920 | | BIC | -20674 | | BIC | -16509.01 | |
| | | | | | | | | | |
| | Jarque-Bera | | | Jarque-Bera | | | Jarque-Bera | | |
| | LDSW | 339747 | *** | LDSM | 15342 | *** | LDSW | 34192 | *** |
| | LDF1W | 2573 | *** | LDF4M | 3197 | *** | LDF1W | 1335600 | *** |
| | | | | | | | | | |
| | Ljung-Box | | | Ljung-Box | | | Ljung-Box | | |
| | LDSW | 4.809 | | LDSM | 25.06 | * | LDSW | 14.461 | |
| | LDF1W | 136.941 | *** | LDF4M | 66.28 | *** | LDF1W | 1.717 | |

**Exhibit 10.9:** Hedging with VAR-MGARCH

As we can see from the exhibit, the VAR-MGARCH-model tends to give reliable results with a high hedging efficiency. However, hedging strategy suggested by VAR-MGARCH model may require frequent shift in hedging positions and would result in increased transaction costs.

According to the residual diagnostics, the residual appear to have some non-normal feature. But the Ljung-Box-statistics suggest that autocorrelation is no longer a problem. In many ways it gives us an indication on that the model is fairly robust.

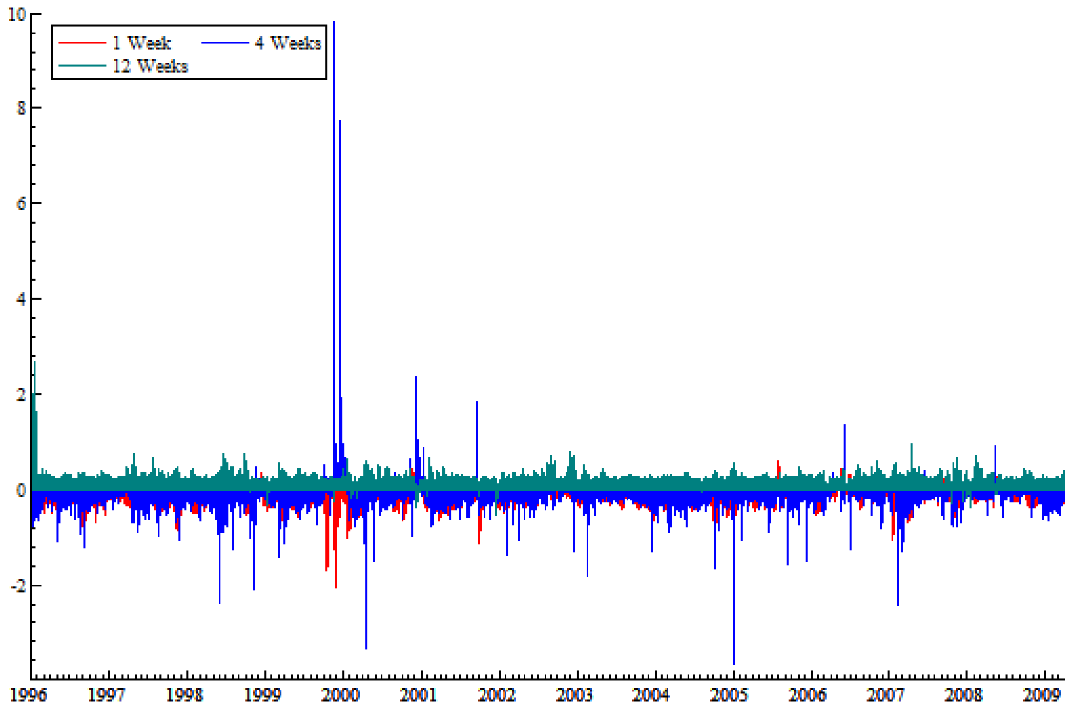


**Exhibit** 10.10: Hedge ratios with VAR-MGARCH

## 10.6 Summary for in-the-sample

In this chapter, I will quickly summarize the result from the different hedging models with the respect to residual diagnostics and hedging efficiency. In the next chapter I will discuss the same models, but in the context of an out-of-sample. The models are also tested done in four different sub-periods of equal length.

When evaluating the hedging efficiency by using an in-the-sample analysis, we should be concerned about three things. Firstly, it should give a good measurements of hedging efficiency since we want diminish most of the variability as possible. Secondly, the result should give a consistent and reliable result for all periods. Thirdly, the hedging efficiency should not go at the expense of high risk premiums.

A common problem for most of the models was the assumption concerning the residuals. When using the whole sample, we could easily see the models had problems measuring the heteroscedacity, autocorrelation and normality assumption. Comparing this to earlier studies, it has shown to be a common problem. Even though the residuals are not normally distributed, it is still possible to make compelling inferences of the coefficient, due to the Central Limit Theorem, which states that a sum of independent or weakly dependent random variables, with finite mean and variance, has a distribution approximately to the normally distribution as the sample size grows (Wooldridge 2006). For large samples, deviation from a normal distribution has little effect on the inferences.

Earlier studies have shown mixed results. Byström (2003) showed that by using daily data for the period 1996 to 1999 one would obtain a distribution of the residuals that resembles a normal distribution fairly well, but with some presence of autocorrelation. Yang, Zhang, Liu & Luo (2009) found expressed some concerns regarding to the violation of heteroscedasticity in their hedging models.

We could also note that the spot price had a much lower durability in its volatility estimates compared to the futures contract when using MGARCH and VAR-MGARCH. There were a slight tendency that longer futures contract had higher durability than the shorter. At the same time, it is found that the volatility process for the spot and futures follow a mean reverting process. If there is an expected shock in the market, the fluctuations will not die out in the short run.

| Risk reduction | | | In-the-sample | | | |
|---|---|---|---|---|---|---|
| Whole period | | Naïve | OLS | VAR | MGARCH | VAR-MGARCH |
| | 1 Week | 25.35 | 27.90 | 16.12 | 33.68 | 15.66 |
| | 4 Weeks | 38.31 | 39.87 | 21.59 | 56.12 | 12.13 |
| | 12 Weeks | -194.60 | 36.25 | 17.05 | 86.83 | 16.07 |
| Period 1 | | | | | | |
| | 1 Week | 39.26 | 39.40 | 23.96 | 52.44 | 29.11 |
| | 4 Weeks | 29.38 | 31.50 | 29.52 | 51.37 | 20.03 |
| | 12 Weeks | -153.02 | 21.05 | 19.67 | 52.09 | 14.53 |
| Period 2 | | | | | | |
| | 1 Week | 21.81 | 23.18 | 12.23 | 27.13 | 5.09 |
| | 4 Weeks | 26.97 | 28.93 | 5.49 | 66.22 | -0.14 |
| | 12 Weeks | -153.91 | 29.12 | 12.09 | 86.94 | 1.01 |
| Period 3 | | | | | | |
| | 1 Week | 31.15 | 36.09 | 17.53 | 38.68 | 25.20 |
| | 4 Weeks | 55.78 | 60.01 | 33.91 | 74.30 | 24.23 |
| | 12 Weeks | -289.88 | 48.11 | 24.96 | 69.79 | 32.33 |
| Period 4 | | | | | | |
| | 1 Week | 10.58 | 21.10 | 12.85 | 30.97 | 12.92 |
| | 4 Weeks | 50.73 | 50.83 | 17.85 | 69.34 | 14.20 |
| | 12 Weeks | -242.00 | 55.23 | 12.49 | 50.24 | 5.15 |

**Exhibit 10.11:** Summary for hedging efficiency with in-the-sample

As we can see from this exhibit, MGARCH model is performing best in 14 out of 15 cases. When looking at the whole period, the MGARCH-model is performing the best. Nevertheless, this is the case in the short run. For contracts that mature in the coming three months, we might be better off by implementing a dynamic hedge strategy. Most of the futures contracts that are traded on Nord Pool mature in one quarter.  For some consumers huge values are at risk. For intensive energy consumers, it might be wise to investigate further into dynamic hedges in order to manage their risk. However, as discussed in chapter 9, the hedger should be concerned with paying high premiums.

## 10.7 Summary for out-of- sample

As mentioned earlier, one should be careful to interpret the results by only relying on an in-the-sample analysis since there are limitations to what we can predict with this method. One could therefore use an out-of-sample hedging in addition in order to support the conclusion.

Doing an out-of-sample analysis is basically the same econometric exercise as in-the-sample but one is excluding some remaining observations from the sample. The model is then used to predict the remaining sample. All the models were used to predict the optimal hedge ratio. Which models that would give the best results, will be indicated with the highest variance reduction measurement. In this thesis, I withhold roughly 20 per cent of the remaining observations on doing the out-of-sample analysis.

| **Risk reduction** | | | **Out-of-sample** | | | |
|---|---|---|---|---|---|---|
| **Whole period** | | **Naïve** | **OLS** | **VAR** | **MGARCH** | **VAR-MGARCH** |
| | 1 Week | 11.43 | 19.41 | 15.63 | 18.87 | 4.76 |
| | 4 Weeks | 51.99 | 50.54 | 24.78 | 50.52 | 5.55 |
| | 12 Weeks | -204.83 | 46.66 | 22.20 | 49.02 | 11.45 |
| **Period 1** | | | | | | |
| | 1 Week | 25.94 | 26.01 | 16.57 | 23.28 | 13.66 |
| | 4 Weeks | 23.31 | 27.27 | 26.83 | 28.88 | 21.49 |
| | 12 Weeks | -113.76 | 6.23 | 7.87 | 8.82 | 7.75 |
| **Period 2** | | | | | | |
| | 1 Week | 29.72 | 32.93 | 29.62 | 33.46 | 13.60 |
| | 4 Weeks | 69.94 | 71.88 | -6.02 | 72.00 | 3.11 |
| | 12 Weeks | -153.71 | 40.08 | -3.39 | 77.95 | 24.91 |
| **Period 3** | | | | | | |
| | 1 Week | -151.42 | 28.72 | -24.86 | -101.91 | 28.77 |
| | 4 Weeks | 65.53 | 55.99 | 21.24 | 64.53 | 46.40 |
| | 12 Weeks | -68.98 | 19.14 | 7.50 | 21.44 | 5.73 |
| **Period 4** | | | | | | |
| | 1 Week | -26.96 | -5.20 | 2.43 | -5.23 | -0.61 |
| | 4 Weeks | 40.91 | 41.39 | 16.99 | 41.39 | 8.09 |
| | 12 Weeks | -135.84 | 44.13 | 9.76 | 22.18 | 0.52 |

**Exhibit 10.12:** Summary for hedging efficiency with out-of -sample

When looking at the result from the naïve hedge, we can see that it can yield a high return and hedging efficiency. However, the result tends not to be very reliable across the sub-periods. It might seem like the method is generating erratic and random results.

As we can see from the exhibit, the hedging efficiency tends to vary over the different sub-periods. It gives us an indication that the risk manager would benefit to pick out relevant data when assessing hedging ratios from the different models.

When using a constant hedge ratio, OLS method tends to give reliable results. It does not tend to give random results like the naïve model. In 3 out of 15 cases, it gives the best result. The next model is the VAR-model. It tends to give lower hedging efficiencies than the OLS-model. In the second and third period, the VAR-model gives a negative result while the OLS and MGARCH-model gives a positive variance reduction. When looking at the time-varying hedging strategies it has lost some of its nice properties when using out-of sample analysis, the MGARCH-model does not appear to be the superior model for all periods. When looking at the exhibit the MGARCH-model seems to be the best in 6 out of 16 cases.

This result is in line with what is found in Byström (2003). Since the dynamics hedge ratio are less stable and having pronounced fluctuations, the hedger has to adjust their futures positions more often. In his article Byström argued that one should favor unconditional hedges since there are cost associated with updating the portfolio when using a dynamic model. These cost includes a marginal additional cost of trading at Nord Pool and the cost of spending time and developing sophisitcated dynamic hedges. The actual transaction costs one probably ends up with a significant additional daily cost.

In conclusion, when comparing the OLS and MGARCH, it seems like it should be a choice of either OLS or MGARCH. In most cases the result tends to be much alike and only differ between a few percentages. A trader following time-varying hedging-strategy will not be penalized too heavily for the rare times when simpler models outperform the time-varying strategy.

A final remark to the table is that the hedging efficiency measurement tends to be unstable across the different methods for the longer contracts. Price changes are in focus when analyzing hedging and finding optimal hedge ratios. It is therefore crucial to decide the appropriate length of the interval to measure the price changes. The result indicates that it is harder to give reliable and consistent hedge for the longer contracts that accounts for all the risk factors mentioned in chapter 3.7.

## 11. Further Research

In this section, I will discuss some of the limitations and possible further research areas for other who wants to study a comparable market. I have suggested four possible topics that could be looked further into.

The first topic that would be worth taking a further look into, is whether it is possible to include a exogenous variables into the hedging models. One could include risk factors for accounting for all variables described in chapter 4.4 Beside what has been written in this thesis , I have tried to construct models that incorporated information from variables such as reservoir, temperature, consumption, other assets prices, etc., without any luck.

As we have seen in this thesis, the futures contracts can be successfully used to manage risk in the Nord Pool area. It could be interesting to investigate options as well. Since we detected some risk premium in the futures contracts, it would be interesting to see whether options prices included risk premiums. These premiums can occur when there are large jumps in realized volatility (Erake 2008). This issue can also be complicated by looking further into how one estimates the volatility. Weron (2000) argues that the estimated volatility is not consistent with the different frequency one use. When measuring the volatility in the spot prices in the California Power Exchange, he found that the measured monthly volatility was less than the predicted volatility from the daily frequency. Thus, Black-Scholes-type formulas should in general overestimate premiums of long-term options written on electricity (Weron 2000). To my knowledge, risk premiums in options in Nord Pool has not been investigated.

Two important stylized facts of financial time series are volatility clustering and excessive kurtosis. The ARCH and GARCH models attempt to capture these features, however many empirical studies observe that these models are unable to capture all the excess kurtosis. One possible explanation is that the time-series are affected by occasional unpredictable events, which make the conditional distribution heavy-tailed. One major issue is to find an appropriate underlying distribution. The tests performed here, indicated that neither the Gaussian n or Student's t-distribution appear to provide full fit.

According to the hedging efficiency measurements, the OLS-model seemed to give stable results compare to the time-varying hedge ratios. Lamoureux and Lastrape (1990) questioned whether persistence found in financial time series was overstated because of the existence of deterministic structural shifts. Models, such as the Iterative Cumulative Sums of Squares (ICSS) algorithm could be used to look further into the persistence of volatility (see Inclán and Tiao (1994).

# 12. Concluding Remarks

The purpose of this exercise was to construct and test models for the unbiasedness hypothesis and hedging models by employed different sets of econometric techniques to unravel some of the characteristics of the Nordic power market.

The preliminary analysis of the sample showed that the data material is characterized by high volatility and excessive skewness. The hypothesis of normal distribution was strongly rejected by the Jarque-Bera test. The spot and futures prices could be described as stationary and there were little indication for a cointegrating relationship between the spot and futures when looking at the whole sample.

The first topic of the thesis was to test whether the futures prices are reliable predictors for the future spot prices. By using a standard OLS, it could be proven that the market was not informational effective for all periods. There were some indication for a gradual improvements in the market efficiency the unbiasedness. However, when testing for unbiasedness, two tests are done simultaneously; test for risk premium and irrational behavior. If either one of these elements are present in the futures prices, the unbiasedness hypothesis fails to hold. The data material indicated that the futures contracts would be a biased predictor for the future spot prices for the shorter contracts with maturity of 1 and 4 weeks.

As mentioned the unbiasedness hypothesis is a joint test for rational behavior and risk premiums. As argued, a risk premium in the market would not seem too unreasonable since the market has a remarkable high volatility. Ensuring some stability in the cost a hedger would be required to pay a premium to pass over the risk to a speculator. In order to explore whether there were a systematic risk premium a VAR-model was used to estimate a dynamic risk premium model. It is found that the risk premium tended to vary over time and react to the reservoir levels. However, there were weak indications for significant seasonal changes in the risk premium.

The last topic of this thesis is hedging. A market with such volatility hedging with futures contracts is an appropriate tool for risk management. Despite the fact that futures prices have some lacking ability to predict future spot prices and often contain a time-varying risk premium, they can be used as a tool to manage risk by holding a portfolio of futures and spot contracts. Various models were put to use to explore a reasonable proportion of futures and spot contacts. It was shown that a simple portfolio consisting of both the spot and futures would give a considerable risk reduction. When doing the residual analysis of each model, it was detected some violations of the models underlying assumptions, such as autocorrelation, white noise and homoscedasticity. When doing this, it was found that even the simplest hedging approach would give beneficial variance reduction. However, it was found that the market is characterized with more basis risk than diversifiable risk. Most of the methods would generate similar results. When using an in-the-sample approach the MGARCH-model gave the superior results, indicating a time-varying strategy would yield the best risk-reducing portfolio. When using an out-of-sample approach the MGARCH-model lost some of its superiority in variance reduction. This property indicates that it is hard to identify a single superior model. None of the models could prove to be the best alternative in all of the sub periods. When taking into account the cost and time spent on developing and estimating a more sophisticated dynamic models as well as operational risk associated with the use of the more complex models to the actual transaction costs one probably ends up with a significant additional cost compared to the constant hedge ratio. Nord Pool's financial markets provide a reasonably high level of hedging effectiveness when taking into account the high basis risk and it the result indicates that it provides a useful risk management tool for hedgers

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# 14. Appendix

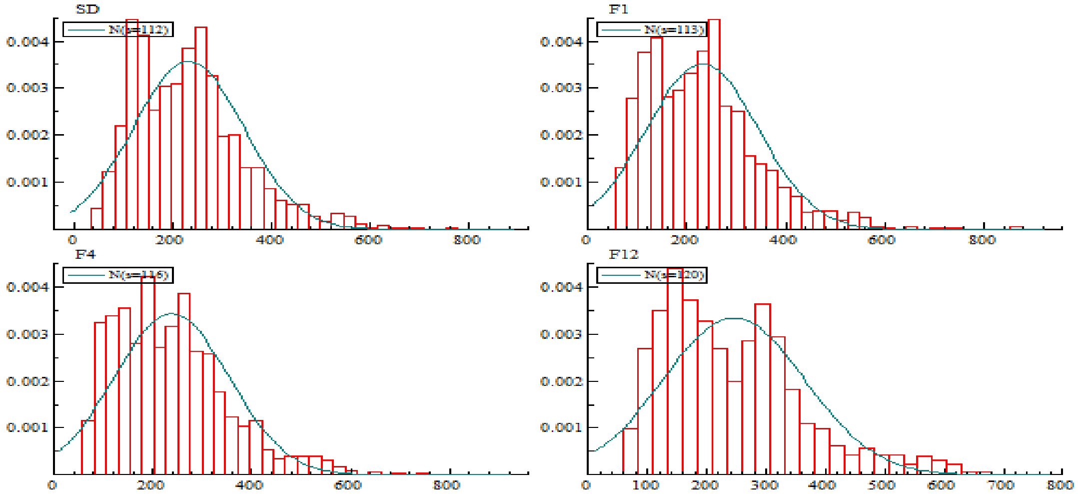


**Exhibit 14.1:** Distribution of spot prices (SD), 1 Week future contracts (F1), 4 weeks contracts (F4) and 12 Week future contracts (F12)

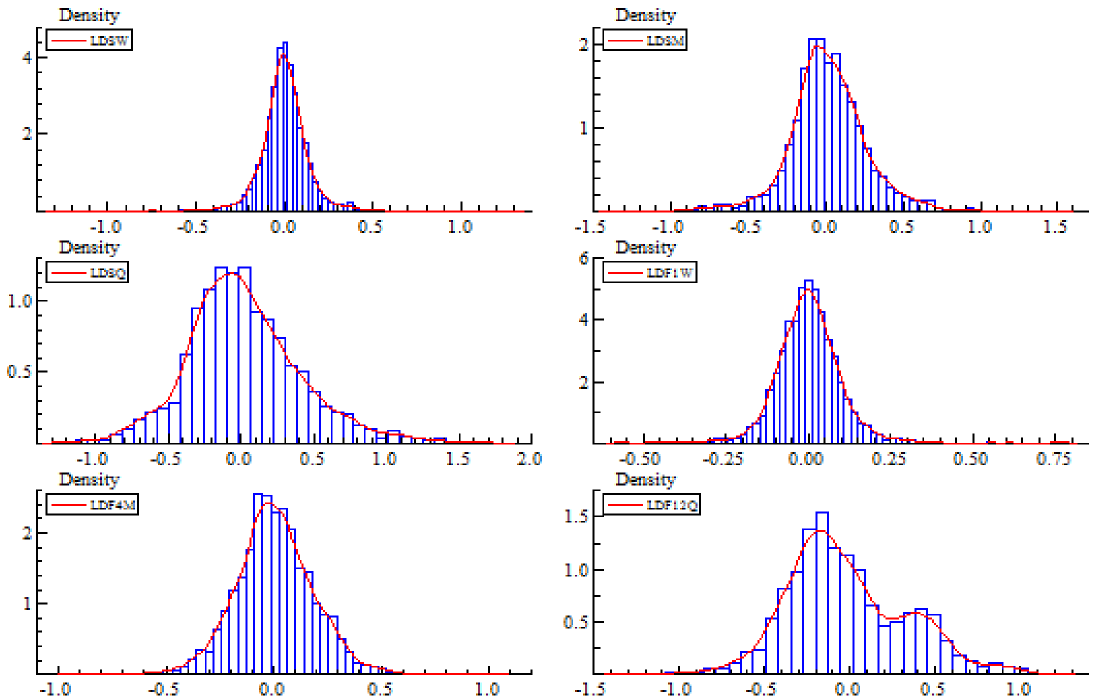


**Exhibit 14.2:** Distribution of returns.