NORGES HANDELSHØYSKOLE

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Analytical VaR for Nord Pool Electricity Derivatives

An adjusted RiskMetrics approach

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NORGES HANDELSHØYSKOLE

Summary

VaR has emerged as the industry standard for risk reporting, applicable for virtually all corporations which are exposed towards market prices, but is especially relevant for banks and other speculative parties which are regulated under the Basel II accord. The Nord Pool electricity derivative market is extensively applied by hedgers as well as speculators, thus the necessity of estimating VaR for portfolios including such contracts.

The model presented in this thesis is based upon the RiskMetrics approach, but is ultimately somewhat adjusted due to the special characteristics of the electricity markets. Moreover, because the calculations and amount of data required for this thesis are extensively, it has resulted in the development of an application written in C#.Net, complemented by SQL commands for easier and faster calculations. A great deal of the workload of this thesis has been in the development of this application.

The validity of the model has been examined by back testing 12 real-world portfolios over year 2009 as the sample period. The null hypothesis of the back test is that the expected exception ratio is equal to the actual exception ratio. The results of the back test has been failure to reject the null hypothesis for any of the 12 real-world portfolios, thus this thesis cannot present any statistical evidence that the model is faulty.

Acknowledgements

First and foremost I would like to express gratitude to my advisor Professor Gunnar Stensland for insightful discussions and guidance during the work of this thesis.
Also I offer my regards to many of the Viz Risk Management employees for constructive comments and technical assistance.

Lastly, I would like to thank "Company" giving me access to their portfolios. Without this, it would not have been possible to back test the model presented in this thesis.

"Company" is Scandinavian electricity portfolio management corporation, which for reasons of discretions wishes to be anonymous, thus it will only be referenced to as "Company".

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1 Introduction

The Nord Pool exchange has over the years become the largest electricity derivative exchange in world, whereas the participants being hedgers and speculators. Because Value at Risk (VaR) has emerged as an industry standard of market risk measurement, its application toward Nord Pool electricity derivatives should not be limited. However, because electricity derivatives in general are considered more complex *(continuous delivery, non-storability*[1]*, seasonality etc)* than regular commodity derivatives, multi factor simulation models are often the preferred choice.

This thesis investigates whether an analytical VaR model can be utilized for Nord Pool electricity derivatives, applying the RiskMetrics approach developed by J.P. Morgan in the early nineties with some adjustments due to the characteristics of electricity market.

The scope of this thesis is limited to Nord Pool System exchange traded electricity derivatives only. Although there are more available products from brokers, which could have been applied as additional input for a *"richer"* model.

The first section discusses risk and risk management, emphasizing on how closely related analytical VaR is to the earliest models of modern finance, types of risk and the necessitate for corporate risk management. Section 3 presesents VaR methodology in a general perspective, benefits and criticism. The following section presents concepts concerning Nord Pool electricity derivatives. The main part of this thesis is Section 6 which concerns the methods applied, firstly presenting RiskMetrics approach for standard commodity derivatives. Second, adjustment to the RiskMetrics approach and theory of arbitrage free pricing and splitting the electricity derivatives exposed cash flows. Third, theory and application of the back test which will be used for evaluating the model. The last part concerns practical implication of the necessary calculations needed for this thesis. Section 7 presents chosen parameters, a simple example of the model and discussions of the results of the back test for 12 real-world portfolios.

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[1] This creates a major obstacle for extenting the notion of convenience yield (Eydeland and Geman 1998)

2 Risk

Although there is no exact definition for risk, the word can be traced back to ancient Greece from the word *(riza)* meaning root. The word did later appear in Latin and Italian vocabulary *(risicare)*, translated from Italian; “to dare”.

In modern time Knight (1921: 231) defined “risk” as measurable uncertainty, i.e. the physical probabilities are known, while “uncertainty” being defined as subjective probabilities. Knight did however not take into account the outcomes associated with the physical probabilities in his definition of “risk”. Markowitz (1952) suggested that in a financial context the variance of return could serve as a very close proxy for “risk”, where the return of each security is a random variable. On the other hand, this approach distanced itself from physical probabilities such as Knight had proposed, because in order to find the variance(s) Markowitz (1952: 89) suggested;

“…should combine statistical technique and the judgement of practical men”.

The reason for this suggestion is that the physical *(and true)* probabilities in the financial markets are *not* known, since prices and subsequent returns are ultimately decided by man. Thus, applying statistical techniques on historical datasets can only serve as the best estimate of the physical (and true) probabilities.

In Markowitz’ model, the portfolio variance stems only from the variances and covariance of the returns of the securities, implying that all financial risk originates from price changes. Jorion (2007) expands the definition of financial risk by taking into account other sources which would increase the variance of the portfolios change in value. Table I exhibits the Jorion’s classifications of financial risks, note that the main category Market Risk, is essentially the same as Markowitz’ suggestion.

Table I : Categories of Financial Risk

Market Risk	The risk of losses due to price movements of such as security prices, interest rates and currency exchange rates. Value at Risk is a risk measurement of market risk.
Liquidity Risk	Can be split into two subcategories, first being asset-liquidity risk which is how much the trades affect the market prices. If the security is not heavily traded such as OTC securities, then one or very few trades may significantly affect prices. Second, which is funding-liquidity risk is the risk of not meeting future obligations. A usual tool for this is a close relative of VaR; CFaR (Cash Flow at Risk)
Credit Risk	Losses which are associated with counterparty not being able to fulfil they're contractual obligations. This type of risk also includes sovereign risk which occurs when governments facilitate such circumstances that it is not possible for the counterparty fulfil they're obligations.
Operational Risk	Risk from internal processes, individuals or systems which result in losses.

2.1 Risk Management

Under Markowitz definition of risk, risk management is in its simplest form the identification and control of the variance according to the owner's preferences. Whereas the first step can be viewed as passive risk management, while incorporating both steps can be considered as active risk management. Because variance is just *a* number, it yields very limited information. More extensive information could be obtained by assessing the entire distribution of the stochastic returns or P&L[2].

In a corporate finance perspective, there are several aspects of incorporating a risk management program, whereas the ultimate question is if risk management adds value. Now considering only passive risk management, it is arguable that the owners understanding of the corporation's risk will somewhat add value. However, when risk is reported by the management to the owners, it does not include correlations with other business entities or securities. This leads to the fact that such risk reporting is of no use for the investor who holds a diversified portfolio. Moreover, it is arguable that the owners could measure their risk themselves, something which is typically done by large investors, mutual funds and so on. However, in some businesses the measurement of risk

[2] P&L : Profit and Loss: return in a monetary unit

can be extremely complex, requiring information that the owners may not have available. Also, the process of measurement can require a substantial amount of resources, consequently the rationale for keeping it on a corporate level rather than an individual. Legal concerns can also be one of the reasons for passive risk management, such as VaR reporting under the Basel legislations (see section 3.1.2)

Because this thesis focuses on VaR, which is a passive form for risk management, it will not go into discussions regarding active risk management.

3 Value at Risk

From Jorion (2007: 17) Value at Risk is defined as;

> *VaR summarized the worst loss over a target horizon that will not be exceeded with a given level of confidence.*

In the context of passive risk management, (1- α)VaR is the α percentile of the estimated future P&L distribution. VaR has developed into an industry standard and is often considered synonymous with the word “risk” (Dowd 1999, Jorion 2007)

3.1 Benefits of VaR

The predominantly advantage of using VaR as risk reporting is its simplicity. It is relatively easy to understand and is summarized and interpreted by a single number.

3.1.1 Internal Control Mechanism

During the 1990s, several banks and other large corporations went bankrupt due to extensive exposure in financial and commodity derivatives. Barings Bank, Metalgesellschaft and Proctor & Gamble U.S. were some of these entities, in some cases had the bankruptcy been caused by a single trader. Obviously, the internal control mechanism of these corporations had failed, and the higher levels of management as well as the shareholders had been given a misleading image of the company’s risk. VaR can typically be used as a control tool for traders, presenting each trader with a VaR limit. However this does not say anything about the aggregate VaR, because it will be affected by correlations between all positions. Making it is possible that all traders are within their limit, while the overall VaR is extensive. On the other hand, all traders may breach their limits; however the overall can be very low or even non-existing *(typically if all traders were doing arbitrage strategies*[3] *where some traders were buying while others selling).*

3.1.2 Social Welfare

Bankruptcies or bailouts of large corporations are in general costly for the society. Most recent, during the financial turmoil of 2008-2009 several financial and insurance institutions have gone bankrupt or been bailed out by the government. The associated bailout cost for the U.S. is the substantial estimate of $ 89 Billion ('U.S. bailout cost seen lower at $89 billion: report', 12/4/2010)

Other associated costs are loss of potential future taxation, welfare expenditure for the employees who have lost their jobs and insurance claim for deposits (if banks). These costs will ultimately have to be paid by the taxpayers, thus it is in the societies interest to avoid bankruptcies or bailouts. Thereby the government will have an incentive to pass legislations which reduces the probability for such cases.

Per today, commercial banks are regulated under the Basel II accord[4], which amongst others emphasize on an internal model approach, meaning in this context that banks are free to choose their VaR model. Briefly explained concerning market risk, the banks will have to keep a higher level of *capital* than the *total risk charge*, which is constructed by credit, market and operational risk charge. Where *capital* is divided into two subcategories; *Tier 1* and *2 capital*, being adjusted book value of equity and other "inferior" capital respectively. Market risk charge can be either a standardized approach or the 99% 10-Trading day VaR times a penalty multiplier. The penalty multiplier is set by the numbers of VaR breaches over the last 252 trading days, whereas an exceptionally high number of breaches indicate a faulty model. The reason behind the Basel II legislations is that the *capital* held should serve as safeguard against huge losses, reducing the probability for bankruptcy and following its associated costs.

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[3] See Hull (2007: 14-15)

[4] For a more extensive description of the Basel II legislations see www.bis.org/publ/bcbsca.htm

3.2 VaR Methods

Because VaR is taken from an estimated future P&L distribution, there are numerous ways of getting the VaR. The most popular being analytical VaR, which is probably due to the fact that RiskMetrics major influence on VaR development.

Table II : VaR Methods

Analytical	Assumes that securities or risk factors which the securities are exposed to follow some statistical distribution (usually Gaussian). Thus the portfolio P&L's α percentile can be found by a closed form solution. If assuming a Gaussian distribution such as RiskMetrics, VaR is simply the variance times the confidence multiplier, thus being virtually the same as Markowitz' definition of risk.
Historical Simulation	The α percentile of the historical P&L or return distribution given the report day's positions. Whereas the sample usually being a rolling window. This method is however not very eligible for forward contracts since historical prices may not be available.
Monte Carlo Simulation	Simulates future prices, by drawing random numbers where the correlations between securities are taken into account. For each simulation, each security is completely re-evaluated by the simulated price (and change in time), thus yielding a far more accurate distribution than analytical methods regarding non-linear securities.

3.3 Criticism

Although VaR has reaped an increasing popularity for more than a decade, it has met some scepticism among practitioners as well as academics; whereas some has signalled that it is a right out dangerous measurement of risk. One of the sharpest criticisers Taleb (1997) points out;

"…VaR is like a Maginot line. In other words, there is a tautological link between the harm of the events and their unpredictability, since harm comes from surprise"

Because VaR does not yield any information regarding losses exceeding the level of confidence, extreme negative outcomes may be underestimated. As mentioned in Section 2.1, assessing the entire estimated P&L distribution should provide superior information.

An extension of VaR is Conditional VaR (CVaR), which is the expectation of losses exceeding the α percentile of the distribution. However, CVaR is restricted to Historical and Monte Carlo Simulation. As for analytical VaR applying the Gaussian distribution, assessing CVaR and the future estimated P&L distribution, does not yield any additional information because the shape of the distributions is always Gaussian. It follows that CVaR is then just a function for the CDF[5] .

Moreover, VaR only emphasise on the loss part of the estimated P&L distribution, telling nothing of potential upsides that comes with a given VaR. Remember that the actual (and true) future P&L distribution is *not* known, statistical methods on historical data are applied as the best estimate. If one were to forget this fact, by treating VaR as an exact science, the measurement and interpretation of risk can be significantly misleading.

4 The Nord Pool Electricity Market

4.1 General

Nord Pool is a multi national electricity, carbon emission and gas exchange. Whereas being the largest electricity derivative exchange in the world ('Nasdaq OMX buys Nordic power bourse Nord Pool', 17/3/2010). The Nord Pool participants (more than 385[6]), consists of generators, retailers, power consuming manufacturing corporations, market makers, brokers and financial participants such as hedge funds and banks.

The financial market of Nord Pool is applied by its members to either hedge their future exposure against the spot price (generators, retailers …) or pure speculation (hedge funds, banks …) . Moreover, Nord Pool also facilitates OTC trades between its members by running all settlements by the Nord Pool Clearing House. Note that this and the following sections are somewhat simplified, but should give a general overview of the Nord Pool electricity market.

[5] Cummulative Densitiy Function

[6] By 31/4/2010; www.nordpool.com

4.2 Brief History

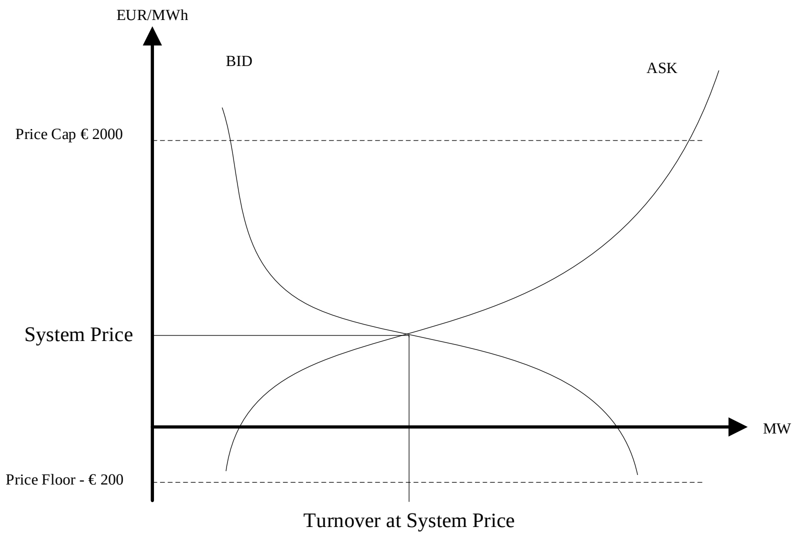
In the early 1990s Norway and Sweden were among the first countries which formed a deregulated electricity market. In 1996, the two countries formed the common electricity exchange; Nord Pool ASA. Two years later, Denmark and Finland joined the exchange; during this period the financial contracts traded at Nord Pool grew in scale and scope. In 1999, standardized financial options became applicable for trading. From 2000 until today, Nord Pool has introduced a range of new commodities such as contract for difference (CfD), carbon emissions, gas and green certificates. Fall 2008 Nasdaq OMX acquired the clearing and consulting parts of Nord Pool and on March 17 this year Nasdaq OMX also acquired the remaining parts of Nord Pool ASA, which consisted of amongst other all power derivative and carbon emission trading. However, these deals did not include the physical market (ELSPOT and ELBAS) which was carved out into a separate company (Nord Pool Spot AS) in 2002.

4.3 Products

The Nord Pool power exchange consists predominantly of two types of trading; physical and financial.

The spot market (ELSPOT) is the day-ahead market for physical delivery because the non-storability of electricity. For the next day ahead Nord Pool participants can place bids for buying or selling electricity down to an hourly level. For each hour, the spot price is settled by the price equilibrium. The spot price for the entire day is simply the average of the hourly spot prices. The Nord Pool spot price is also known as the System Price. Figure 1 exhibits how the spot price is found in equilibrium by the aggregate supply and demand curves (Ask and Bid). Note that the spot price is subject to a minimum and maximum price, which is currently (- € 200) and € 2000 respectively.

Figure 1 : System Price Equilibrium



Source : Adapted from www.nordpoolspot.com

The physical market also includes an intra-day market (ELBAS), which trades can be made until one hour before the delivery; this market is mainly used for balancing the power grid.

The second type of trading is financial derivatives, primarily consisting of forwards and futures as well as some options[7]. The Nord Pool forward/future market offer contracts with delivery from day after tomorrow up six years ahead. The short end consisting of futures on days and weeks, followed by forward contracts on months, quarters and years in the long end. The main difference between the futures and forwards is the settlement agreement.

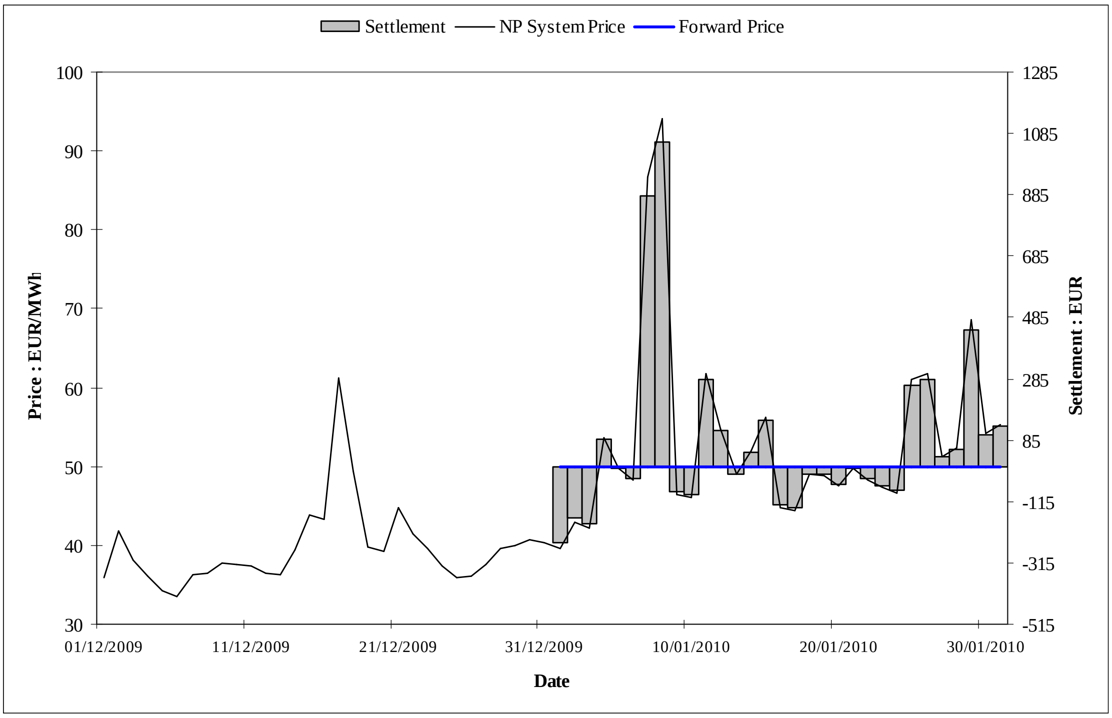
Regarding Futures, the change in value, i.e. the change in closing prices from one trading day until the next *(before last trading day)* is settled in full, meaning for the total quantity of the delivery period. During the delivery period, the difference between spot and last trading day's closing price is settled multiplied by the number of hours on the given day. While remaining hours in delivery period will be mark-to-market by the last closing price.

[7] See Eydeland and Wolyniec (2006): 34-46 for more detailed descripitons of electricity forwards, futures and options

Forwards do not settle anything before the delivery period, but will instead mark-to-market the difference between the forward price and the purchase price *(B)*. In the delivery period the settlement will be the difference between the spot price and the purchase price on a daily basis[8].

Figure 2 exhibits the development of the spot price in the period 1/12/2009 to 31/1/2010 *(Left Y-axis)* and the cash settlements for ***one*** January 2010 forward contract *(Right Y-axis)*.

Figure 2 : Example Nord Pool System (Base) M1-10 Forward (purchase price = 50 EUR/MWh)



Although Futures and Forwards differ from each other in terms of settlement, the exposure with respect to relative price changes is virtually the same if assuming deterministic interest rate[9], making the difference an issue of liquidity rather than market risk. For this reason and practical purposes this thesis will treat futures as forwards.

[8] For a more detailed explenation about Nord Pool Futures/Forwards settlment, see Nord Pool (2008)

[9] See Hull (2003: 126-127)

Moreover, System contracts can have two types of load; the first which is Base, covers the entire delivery period, while Peak load offers delivery each weekday from 08:00 to 20:00. Also, in addition to System contracts, Nord Pool offers CfD's for different areas; which is basically a two legged contract; first leg consisting of a long position in a given area, while the second leg is a short position in System *(Base)*.

4.4 Nord Pool and VaR

VaR is primarily used by commercial banks and investment firms, some of the Nord Pool speculative parties fall directly within this category. However, there may be reason to believe that the speculative parties are accountable for a great deal of the financial trading at Nord Pool. This is due to the fact that in 2008 the exchange traded financial contracts was about four times the total generation in the Nord Pool area (Nord Pool 2008: 23).

Nevertheless, it is also applicable among participants such as generators and retailers; reason being that these firms often have their own trading desks, thus VaR could be used as an internal control mechanism. Moreover, because VaR has developed into an industry standard, it is most common as the reporting of risk for virtually all major corporations which are exposed to some factors which are sensitive to market price changes[10].

Because, the Nord Pool forward contracts significantly differ from standardized commodity contracts (such as crude oil etc), the most well-established methodologies require some adjustment in order to be compatible.

Bjerksund et. al. (2000) proposes a three-factor model, arguing that the model is better suited for risk management purposes opposed to their single factor model for contingent claims. This model is applied for Monte Carlo simulations, whereas under each simulation a forward curve is generated by the function of the (correlated) three random factors. Under each simulated curve, all the contracts in the portfolio are re-evaluated by the arbitrage free pricing formula (see section 6.3). Although such a method is considered superior to any analytical method concerning non-linear instruments, it has

[10] By market prices including: currencies, interest rates, raw material and so on.

the drawback of being time consuming when the portfolio is large and the instruments are complex (for example customized load profile contracts).

In the empirical work of Koekebakker and Ollmar (2005), which analyzed the forward curve dynamics at Nord Pool from 1995 to 2001, found that a two factor model explained 75%, while needing more than 10 factors in order to explain 95%. However, the analysis performed was under the assumption that the dynamics where constant over the time-series, but as the authors themselves pointed out; the volatility seemed non-stationary and exhibited a seasonal pattern, thus the possibility of a missing time-dependent component. Although these findings could be applied in a VaR model, one would still run into the same time-consuming issues as for Bjerksund et. al. (2000)'s model.

On the basis of the time-consuming issues regarding Monte Carlo simulation, this thesis proposes the use of an analytical VaR model, which would be considerably faster than any multifactor models. The analytical model, utilizes the RiskMetrics methodology such as EWMA and Cash Flow Mapping methods, but is adjusted somewhat in order to be in line with the Nord Pool financial market.

5 Data

5.1 Historical Closing Prices

Historical closing prices for Nord Pool System (Base and Peak) futures/forwards for the period 1/9/2008 – 30/12/2009 were downloaded from the Nord Pool FTP server. The purpose of these closing prices is to construct smooth forward curves (see Section 6.3.1). Where the forward curves are applied for: firstly, pricing of Time Buckets in order to calculate VCV estimates. Second, calculate daily forward prices from the forward curve in order to map exposure for portfolios into Time Buckets, which is in crucial when closing price does not exists for a given contract, for example when it is in delivery.

The reason for applying closing prices, opposed to for example bid/ask is because in the existence of closing prices, MtM value is based on these while in the opposite case MtM value is calculated from the forward curve.

5.2 Portfolios

In order to back test the analytical VaR model presented in this thesis, a total of 12 portfolios have been subjected for back tests over year 2009. All these portfolios are managed by “Company”. However, in order to keep anonymity, the actual names of the portfolios will not be disclosed. Also, keeping further discretion the positions over the sample period in these portfolios will not be disclosed in detail apart from the content being exchange traded System contracts at Nord Pool. Moreover, trading has been ongoing over the sample period. Also, to exclude the additional complication of currency risk, it is assumed that all portfolios reports in Euros.

6 Method

6.1 Value at Risk Methodology

Applying the RiskMetrics Technical Document (1996) framework for analytical VaR *(also known as the Variance-Covariance method)*, following the assumption that return or relative changes in portfolio value is a stochastic variable in discrete time, being essentially the same as Markowitz’s portfolio model. Also assuming this stochastic variable is Gaussian, the α percentile of the P&L distribution (VaR) can then be defined as*;*

$$\begin{aligned}
&\tilde{W}_{P,t+h} = W_{P,t}(1+\tilde{R}) \quad , \quad \tilde{R} \sim N(\mu_{P,h}, \sigma^2_{P,h}) \\
&VaR_{t+h}\left((1-\alpha)\right) = W_{P,t}\left(-\mu_{P,h} + z_\alpha \sigma_{P,h}\right) \\
&where \\
&W_{P,t} = \text{Market Value of initial portfolio at time } t \\
&\mu_{P,h} = \text{The mean return of the portfolio for timeframe } h \\
&\sigma_{P,h} = \text{The standard deviation of the return of the portfolio for timeframe } h \\
&z_\alpha = \int_0^{(1-\alpha)} \varphi^{-1}(s)ds \quad , \quad \varphi^{-1} = N^{-1}(0,1)
\end{aligned} \tag{1a}$$

Moreover, when assuming the return of the individual securities or risk factors of the securities to be Gaussian distributed ensures that the portfolio return will be Gaussian as well. Because the timeframe *(h)* is considerably short *(usually one to 10 days)* it is most common to assume that the mean return is zero, thus profit or loss will be equally likely, hence VaR can be written as;

$$VaR_{t+h}\left((1-\alpha)\right)=W_{P,t}z_{\alpha}\sigma_{P,h} \tag{1b}$$

Under this assumption the only estimation concern is the standard deviation or variance of the return of the portfolio.

By definition, a portfolio will contain more than one security, which leads to the fact that the variance of the portfolio is not only subject to the variance of each of its securities, but also the covariance *(or correlations)* between the securities or risk factors.

For a portfolio containing three or more securities, the variance can be more easily expressed in matrix form;

$$\begin{aligned}
&VaR_{t+h}\left((1-\alpha)\right)=z_{\alpha}\left(\mathbf{w}^{\mathbf{T}}\boldsymbol{\Sigma}\mathbf{w}\right)^{\frac{1}{2}}\\
&\Rightarrow VaR_{t+h}\left((1-\alpha)\right)=z_{\alpha}\left(\begin{bmatrix} w_i & \cdots & w_N\end{bmatrix}\begin{bmatrix}\sigma_i^2 & \cdots & \sigma_{N1}\\ \vdots & \ddots & \vdots\\ \sigma_{1N} & \cdots & \sigma_N^2\end{bmatrix}\begin{bmatrix}w_1\\ \vdots\\ w_N\end{bmatrix}\right)^{\frac{1}{2}}\\
&\textit{where}\\
&w_i=\text{Position } i\\
&\sum_{i=1}^{N}w_i=W_P
\end{aligned} \tag{1c}$$

6.2 The RiskMetrics Approach

6.2.1 Returns: The Basis for VCV Estimation

The basis for all Variance or Covariance (VCV) estimate is returns. RiskMetrics (1996: 46) argues that log returns *(r)* are more preferable than simple returns *(R)* because they have more attractive statistical properties than simple returns or prices. Firstly, log returns can be time-aggregated by an additive process; also empirical evidence suggests that log returns tend to have a closer fit towards the Gaussian distribution than simple returns.

When aggregating returns over time, log returns are additive opposed to simple returns which is calculated by a geometric process;

$$R_t(N) = (1+R_t)\times(1+R_{t-1})\times\ldots\times(1+R_{t-N-1})$$
while
$$r_t(N) = \sum_{i=t-N}^{t} r_i$$

However applying VCV estimates from log returns for a portfolio raises some issues of consistency. Assuming Gaussian distributed log returns for the individual securities or risk factors does not ensure that the portfolio's log return to be the weighted average return of the securities as it is for simple returns.

For simplicity consider a two security portfolio *(P)*, each securities log return is Gaussian distributed, thus the securities prices will be log normal. However the log return of the portfolio is not Gaussian distributed because:

$$\begin{aligned}
&S_{1,t+h} = S_{1,t}e^{\tilde{r}_{1,h}} \\
&S_{2,t+h} = S_{2,t}e^{\tilde{r}_{2,h}} \\
&\ln\left(\frac{P_{t+h}}{P_t}\right) = \ln\left(\frac{P_t w_1 e^{\tilde{r}_{1,h}} + P_t w_2 e^{\tilde{r}_{2,h}}}{P_t}\right) = \ln\left(w_2 e^{\tilde{r}_{1,h}} + w_2 e^{\tilde{r}_{2,h}}\right) \nsim N(\mu_{P,h}, \sigma^2_{P,h}) \qquad (2a) \\
&P = S_1 + S_2 \\
&\tilde{r}_{i,h} \sim N(\mu_{i,h}, \sigma^2_{i,h})
\end{aligned}$$

When the security prices are log normally distributed, the sum of the prices are *not* lognormal, thus the log return of the sum of prices can not by definition be Gaussian. As for RiskMetrics, even though VCV estimates stems from log returns it is assumed that future value of the securities are characterized by returns in discrete time. RiskMetrics (1996: 8) argument for this is that continuous and discrete returns does not differ much from each other when the return is relatively small, thus security prices and portfolio value will be Gaussian by approximate.

$$\begin{aligned}
&\tilde{R}_h \approx e^{\tilde{r}_h} - 1 \\
&\Rightarrow \tilde{S}_{t+h} \approx S_t(1+\tilde{r}_h)
\end{aligned} \qquad (2b)$$

6.2.2 EWMA

RiskMetrics (1994) suggests Exponentially Weighted Moving Average (EWMA) method for estimating the variances and covariances, thus implying that the VCV estimates are heteroskedastic (time-varying) and autocorrelated (history dependent)[11].

Also, because the time horizon is very short it is assumed that the mean return is zero. Variance or Covariance can under this assumption be defined as;

$$\begin{aligned}
&\sigma_t^2 = E\left[r_t^2\right] - E\left[r_t\right]^2 = E\left[r_t^2\right] \\
&\sigma_{12,t}^2 = E\left[r_{1,t}r_{2,t}\right] - E\left[r_{1,t}\right]E\left[r_{2,t}\right] = E\left[r_{1,t}r_{2,t}\right] \\
&\textit{thus} \\
&\hat{\sigma}_t^2 = E\left[r_t^2\right] \\
&\hat{\sigma}_{12,t}^2 = E\left[r_{1,t}r_{2,t}\right]
\end{aligned} \tag{3a}$$

It follows from Equation 3a that the squared or joint returns for time *t*, is somewhat correlated with returns with time less than *t*. EWMA variance, covariance and correlations estimates are defined by;

$$\begin{aligned}
&\hat{\sigma}_{1,t}^2 = (1-\lambda)r_{1,t-1}^2 + \lambda\hat{\sigma}_{2,t-1}^2 \\
&\hat{\sigma}_{12,t} = (1-\lambda)r_{1,t-1}r_{2,t-1} + \lambda\hat{\sigma}_{12,t-1} \\
&\hat{\sigma}_{12,t} = \frac{\hat{\sigma}_{12,t}}{\hat{\sigma}_{1,t}\hat{\sigma}_{2,t}}
\end{aligned} \tag{3b}$$

Alternatively, the EWMA estimates is expressed in matrix form, thus the VCV Matrix can be calculated by;

$$\begin{aligned}
&\mathbf{\Sigma_t^{EWMA}} = (1-\lambda)\mathbf{r_{t-1}^T r_{t-1}} + \lambda\mathbf{\Sigma_{t-1}^{EWMA}} \\
&\textit{thus} \\
&\mathbf{\Sigma_t^{EWMA}} = (1-\lambda)\begin{bmatrix} r_{1,t-1} \\ \vdots \\ r_{N,t-1} \end{bmatrix}\begin{bmatrix} r_{1,t-1} & \dots & r_{N,t-1} \end{bmatrix} + \lambda\begin{bmatrix} \hat{\sigma}_{1,t-1}^2 & \cdots & \hat{\sigma}_{1N,t-1} \\ \vdots & \ddots & \vdots \\ \hat{\sigma}_{N1,t-1} & \cdots & \hat{\sigma}_{N,t-1}^2 \end{bmatrix}
\end{aligned} \tag{4a}$$

[11] See Woolridge (2003: 333-334)

Following the relationship between correlations and covariances, the correlation matrix **(P)** can easily be found by;

$$\mathbf{P}=\begin{bmatrix}\hat{\sigma}_1^{-1} & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \hat{\sigma}_N^{-1}\end{bmatrix}\begin{bmatrix}\hat{\sigma}_{1,t-1}^2 & \cdots & \hat{\sigma}_{1N,t-1}\\ \vdots & \ddots & \vdots\\ \hat{\sigma}_{N1,t-1} & \cdots & \hat{\sigma}_{N,t-1}^2\end{bmatrix}\begin{bmatrix}\hat{\sigma}_1^{-1} & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \hat{\sigma}_N^{-1}\end{bmatrix} \tag{4b}$$

The EWMA estimate for time *t*, is the sum of the previous trading day's *(t-1)* squared or joint return plus the previous day's VCV estimate. The decay factor λ determines the weights assigned to each past observation. From Equation (3c), EWMA variance can also be expressed as;

$$\begin{aligned}\hat{\sigma}_{1,t}^2 &= (1-\lambda)\sum_{i=1}^{T}\lambda^{i-1}r_{1,t-i}^2\\ &\Rightarrow (1-\lambda)\left[r_{1,t-1}^2+\lambda r_{1,t-2}^2+\lambda^2 r_{1,t-3}^2\ldots+\lambda^{T-1}r_{1,t-T}^2\right]\end{aligned} \tag{5}$$

Table III, exhibits some of the weights for past observation when using a decay factor of 0.94.

Table III : EWMA Weights λ = 0.94

Time	*t-1*	*t-2*	*t-3*	*t-4*	*t-T*
Assigned Weight	$(1-\lambda)$ 0.06000	$(1-\lambda)\lambda$ 0.05640	$(1-\lambda)\lambda^2$ 0.05302	$(1-\lambda)\lambda^3$ 0.04984	$(1-\lambda)\lambda^{T-1}$ …

One could theoretically assign an infinite number of past observations to the EWMA estimator, thus ensuring a weight sum of one. This is of course not possible; normally one would have *T* observations available. The efficient number of observations can be defined by tolerance level *(γ)*, i.e. the weight sum of observations beyond a given threshold of observations *(l)* will not count as efficient observations. RiskMetrics (1996) shows that the efficient number of observations for a given tolerance level is given by;

$$
\begin{aligned}
& l = \frac{\ln(\gamma)}{\ln(\lambda)} \\
& \textit{because} \\
& (1-\lambda)\sum_{t=l}^{\infty} \lambda^{t} = \gamma \\
& \lambda^{l} \underbrace{(1-\lambda)(1+\lambda+\lambda^{2}+\ldots)}_{=1} = \gamma
\end{aligned} \tag{6}
$$

For example, with λ = 0.94, the efficient number of observations are 74 days at a 1% tolerance level, thus the 74 past squared or joint returns explain 99% of the EWMA estimate.

RiskMetrics argues that EWMA is preferable because firstly returns are more affected by recent returns *(or shocks)* than past returns, thus they should be weighted accordingly. This feature will not be captured if using a rolling sample, which weighs each past observation in the sample equally. Second, EWMA is a special case of IGARCH[12], i.e. a GARCH(1,1) without the constant term and where the parameters sums up to one. Under these restrictions, the model will only have the parameter (λ). With only one parameter, optimizing a maximum likelihood function to find the optimal parameter becomes considerably simpler than for GARCH or IGARCH models. RiskMetrics used RMSE[13] as the maximum likelihood function, which is essentially the same as any squared residual function. In RiskMetrics findings of the optimal decay factor consisted of time series for more than 480 different securities, whereas each security was given an optimal decay factor. Following, the optimal *overall* optimal decay factor was the *average* optimal decay factor weighted by individual RMSE by total RMSE. Note that RiskMetrics took only into account variances or volatilities and *not* covariances in finding the optimal

---

[12] (I)GARCH – (Integrated) Generalized Autoregressive Conditional Hetroscedisic, for more on these models see Engle (1982)

[13] Root Mean Squared Error, see RiskMetrics(1996: 98)

$$RMSE_v = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(r_{t+1}^2 - \hat{\sigma}_{t+1|t}^2(\lambda)\right)^2}$$

decay factor. The results from this optimization were an optimal λ of 0.94 for 1-day volatility and 0.97 for monthly volatilities.

6.2.3 Mapping

General Concept

Various securities can have several cash flows, where each cash flow (CF) will occur at a specific time. This is a somewhat different concept than for simpler securities such as stocks, because in a liquid market one could at any time turn in the stocks in exchange for cash. However, securities such as bonds or forward contracts will have CF's occurring at specific future time points or periods. Having each CF at a given time for a given security as unique would ultimately require VCV estimates for all of these CF's. This is of course quite unfeasible, considering the number of VCV estimates as well as computational time. The general idea is therefore to map each cash flow for all the securities in the portfolio to a set of risk factors, thus aggregating the exposure on risk factors. In general each cash flow is mark-to-market *(MtM)*, due to the fact that it's a measurement of market risk. Also, CF's can be discounted at the risk free rate to reflect Present Value (PV).

In general, the mapping concept is just another way of viewing the value of the portfolio or MtM value. Thus, weighting the (present value) of the exposure of the risk factors rather than on individual securities. Also, one could view a single security as portfolio with weights in multiple risk factors.

RiskMetrics (1996: 118) describes three following conditions to hold when mapping the CF's

1. **Market value is preserved**. The total market value of the two RiskMetrics cash flows must be equal to the market value of the original cash flow.
2. **Market risk is preserved[14].** The market risk of the portfolio consisting of RiskMetrics cash flows must also be equal to the original cash flows.
3. **Sign is preserved.** The RiskMetrics cash flows have the same sign as the original cash flow.

Moreover, Jorion (2007) states that mapping is the only solution when the characteristics of the security changes over time. This is especially true for securities which cash flows occur at a specific date(s), such as bonds or forward/future contracts.

For example consider a long position in a 3-month Crude Oil Forward today, in two months time the purchased Forward will no longer have a 3-month maturity, but a 1-month maturity, thus it is not really comparable and the contract cannot be assigned a specific VCV estimate.

RiskMetrics (1996: 169) points out that the solution to this problem is to construct a term structure i.e. construct contracts with constant maturity independent of trading day. VCV coefficient estimates can be obtained by for example linear interpolation to define constant maturity.

The *problem* which arises from any mapping procedure is that it is possible that some of the risk information will be lost.

[14] This only concerns bonds or interest related securies. See RiskMetrics (1996: 119-120)

Figure 3 : Example of VaR Mapping

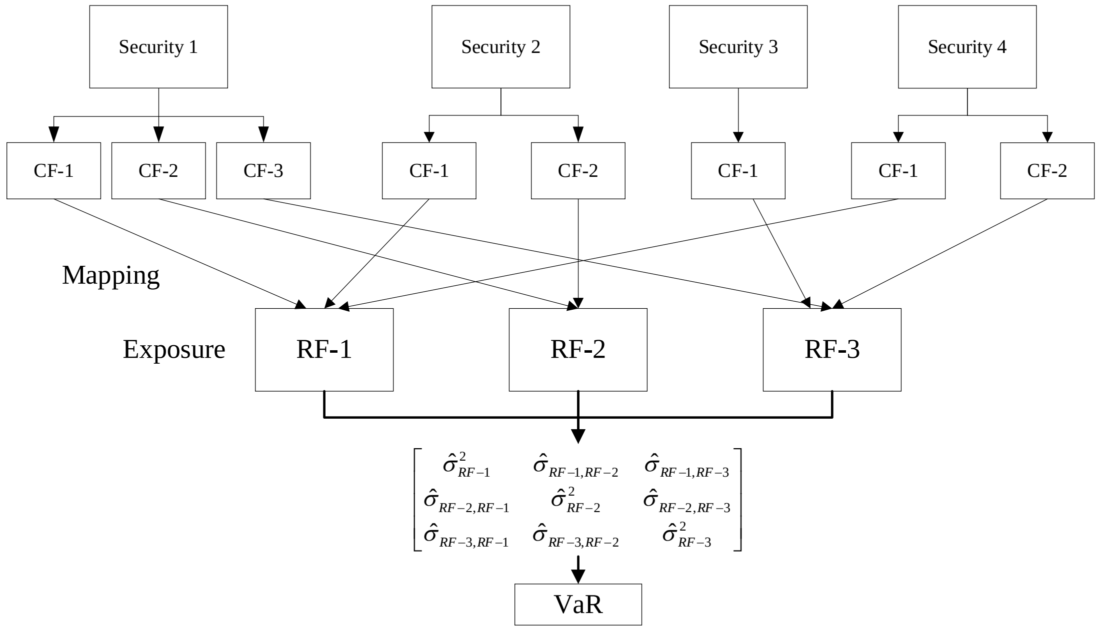


Figure 3 exhibits an example of how four securities with a total of eight unique cash flows can be assigned to three risk factors. With a VCV Matrix for the three risk factors VaR can be estimated as in Equation 1, with weights equal the exposure in the risk factors.

Commodity Forward Contracts

Forward contracts offer either physical or financial settlement upon delivery. Although physical delivery can not directly be translated into a CF, it does offer an implied benefit to buy or sell at given price *(B)*, thus creating a benefit which is equivalent to a CF. Moreover, if one were to enter into *M* forward contracts at current market price at time *t* with bulk delivery at time *T,* and the continuous compounded deterministic interest rate *r* the present MtM value of the position is zero.

$$\begin{aligned} &MtM_t = e^{-rs}\left(F(t,T) - B\right)M_t = 0 \\ &when \\ &F(t,T) = B \end{aligned} \qquad (7)$$

However, this does not indicate the market risk of the position is zero, because the MtM value will change accordingly with the market price. Assuming zero interest rate[15], the change in MtM value with respect to the forward price can from Equation 8a be expressed as;

$$\begin{aligned}&\Delta MtM_t = \frac{\partial MtM_t}{\partial F(t,s)}\Delta F(t,T)\\ &\Rightarrow M_t \Delta F(t,T)\\ &when\\ &r = 0\end{aligned} \tag{8a}$$

Meaning the MtM value of an *M* position in the contract *F(t,T)* changes by *M* monetary units by the absolute change in price in terms of present value when assuming zero interest rate. Because the VCV estimates are based on the returns or relative price change this expression can be slightly rewritten;

$$\begin{aligned}&\Delta MtM_t = \left(F(t,s)M_t\right)\frac{\Delta F(t,T)}{F(t,T)}\\ &where\\ &\frac{\Delta F(t,T)}{F(t,T)} = \tilde{R}\end{aligned} \tag{8b}$$

Note that when convenience yield is non-existing; the forward price can be directly linked to the spot price in absence of risk free arbitrage, such that the forward price is only exposed to the spot and interest rate changes. However, because storable commodities are subject to convenience yield, this will not be possible. Here, exposure Ex(·) will be defined as underlying values of the position which is exposed against the relative market price changes, thus;

$$Ex\left(F(t,T)M_t\right) = F(t,T)M_t \tag{8c}$$

Furthermore, the exposure of the given contract can be mapped to different risk factors as shown in Figure 3. Forward contracts on storable commodities offer bulk delivery at a

---

[15] Section 6.3 explains the reasons for assuming zero interest rate

given time. However this point may fall between two of the fixed maturity contracts *(Risk Factors)*, thus the contract must be mapped to these factors.

The RiskMetrics Approach for commodity forwards consists of the following steps:

1. Calculate VCV estimates from the constant maturity contracts by linear interpolation.
2. Discount the contracts(s) CF to PV if zero-interest is not assumed.
3. Map the PV of the contracts into the same term structure as the fixed maturity contracts.
4. Calculate VaR; include interest rate CF volatilities and correlations if zero-interest is not assumed.

Example:

Consider the one month constant maturity to be 30 days; today the 1-month contract expires in 24 days while the 2-month contract expires in 55 days. By linear interpolation the contract maturing in 30 days would then equal weight sum of the two contracts thus;

$$F(t,30) = \frac{24}{30}F(t,24) + \frac{6}{30}F(t,55)$$

Consequently, the log can be found by the log difference from the previous trading day's *(t-1)* constant maturity price;

$$r_{30,t} = \ln\left(F(t,30)\right) - \ln\left(F(t-1,30)\right)$$

Under this framework one could easily estimate the VCV of the term structure i.e. 1M, 2M and 3M constant maturity contracts. A problem which arises from this method is that the constant maturity contract can not be assessed as a risk free arbitrage valuation due to the fact that the convenience yield for the constant maturity is unknown.

As for mapping, consider an M position in a T day maturity contract *(where* $30 < T < 90$*)* with VCV estimates of the term structure V_i , $i = (30, 60, 90, \ldots)$, the exposure of the contract can be expressed as:

$$
\begin{aligned}
&Ex\left(F(t,T)M_t\right) = Ex(V_{60}) + Ex(V_{90}) \\
&\Rightarrow \left(\frac{T - V_{60}}{V_{90} - V_{60}}\right) F(t,T)M_t + \left(\frac{V_{90} - T}{V_{90} - V_{60}}\right) F(t,T)M_t = F(t,T)M_t
\end{aligned}
$$

Thus the VaR of this contract can then be expressed as;

$$
VaR_{t+h}((1-\alpha)) = z_\alpha \left(\begin{bmatrix} Ex(V_{30}) & Ex(V_{90}) \end{bmatrix} \begin{bmatrix} \sigma^2_{V_{30}} & \sigma_{V_{90}V_{30}} \\ \sigma_{V_{30}V_{90}} & \sigma^2_{V_{90}} \end{bmatrix} \begin{bmatrix} Ex(V_{30}) \\ Ex(V_{90}) \end{bmatrix} \right)^{\frac{1}{2}}
$$

Options

Because options are non-linear instruments, their returns will not be identical to their respective underlying when applying the option valuation models of Black and Scholes (1973) or Black (1976), referred to as B&S and B-76 from here. Both these models demonstrate under a set of assumptions and absence of risk free arbitrage, that the options cash flow could be replicated by a self-financing portfolio of a position in the underlying and in a risk free interest bearing security.

The underlying of the Nord Pool electricity options are forward contracts and not spot prices. The standard valuation method used for forward contracts is the B-76 model[16], which for a European Call can be expressed by;

$$
\begin{aligned}
&C(F(t,T),K,t,\tau,r,\sigma) = e^{-r(\tau-t)}\left(F(t,T)\cdot N(d_1) - K\cdot N(d_2)\right) \\
&where \\
&d_1 = \frac{\ln(F(t,T)/K) + \frac{1}{2}\sigma^2(\tau - t)}{\sigma\sqrt{\tau - t}} \qquad (9) \\
&d_2 = d_1 - \sigma\sqrt{\tau - t} \\
&t < \tau < T
\end{aligned}
$$

[16] See Section 6.3.2 for the assumptions made of applying this model to contingent claims of electricity forward contracts

The change in the options value can be found by taking a Taylor series expansion of the B-76 model[17].

$$\Delta C = \frac{\partial C}{\partial F(t,T)} \Delta F(t,T) + \frac{1}{2} \frac{\partial^2 C}{\partial F(t,T)^2} \Delta F(t,T)^2 + \frac{\partial C}{\partial \tau} \Delta \tau + \frac{\partial C}{\partial r} \Delta r + \frac{\partial C}{\partial \sigma} \Delta \sigma + \dots \quad (10a)$$

However, for short time frames it can be assumed that the only significant subject to change is the underlying. Thus the change in the options value can be roughly approximated by its first order partial derivative with respect to the underlying;

$$\begin{aligned} &\Delta C \approx \delta_t \Delta F(t,T) \\ &where \\ &\delta_t = \frac{\partial C}{\partial F(t,T)} = e^{r(\tau - t)} N(d_1) \end{aligned} \quad (10b)$$

It turns out that the first order derivative is equal to the options delta *(δ_t)*, which is also equivalent to position in the underlying when replicating the cash flow of the contingent claim. It is therefore arguable the options risk with respect to the underlying is equal to the risk of the underlying times the delta. Because the B&S and B-76 argument of a self financing replicating portfolio is in continuous time, the delta will as well change continuously. Thus a delta approximation of the change in the options value will therefore be somewhat inaccurate for larger changes in the price of the underlying.

Figure 4 exhibits the P&L *(Y-Axis)* by changes in the forward price *(X-Axis)* for one at-the-money call option with purchase price equal the market price. The price of the forward, volatility, time to maturity and continuous compounded interest rate are 45, 20%, 1Y and 5% respectively. In this case, the P&L from delta opposed to full valuation does not differ much for price changes in the underlying of ±2. However, as expected, for larger price changes of the underlying, the delta approximation is far less accurate.

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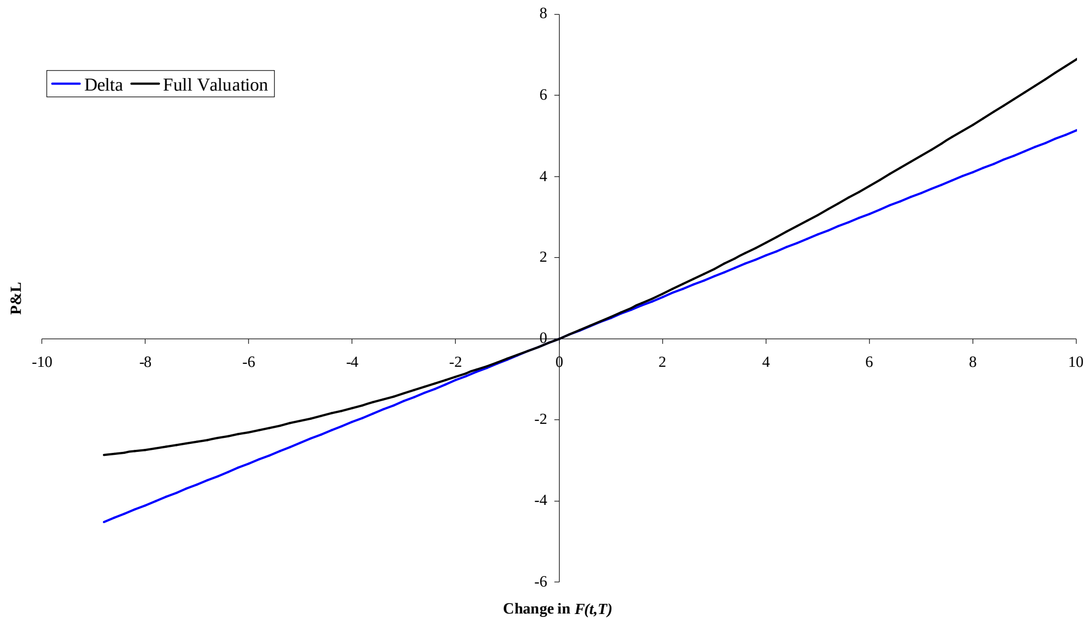
[17] See Jorion (2007: 299)

Figure 4 : Delta vs Full Valuation P&L : one call



Jorion (2007: 258) points out that it is fairly easy to construct a delta-neutral portfolio which VaR number is close to nil *(Such as the Short Straddle Strategy*[18]*)*. However for large unfavourable price movements such a portfolio would generate substantial losses. RiskMetrics (1996) suggests a delta-gamma-theta approximation for options, however implementing this into options on electricity derivatives is rather complex and is therefore forfeited.

From Equation 8d and 10a, the exposure of *M* options can by delta-normal valuation be expressed as;

$$\begin{aligned} \Delta C &= \delta_t \Delta F(t,T) M_t \\ \Delta C &= \left(\delta_t F(t,T) M_t\right) \frac{\Delta F(t,T)}{F(t,T)} \\ Ex(C \cdot M_t) &= \delta_t F(t,T) M_t \end{aligned} \tag{10c}$$

[18] See Hull (2008 : 231-232), note that in order to be completly delta-neutral the put to call ratio cannot be one.

6.3 Applying the RiskMetrics Framework to the Nord Pool electricity derivative market

The Nord Pool forward market, consists of contracts of risk free arbitrage average future prices for given time intervals *(could also be referred to as swaps)*, such as weeks, months, quarters and years. In the delivery period; the difference between the spot price *(NP System Base/Peak)* is settled financially for each day in the delivery period. Bjerksund et. al. (2000) shows that the price of the contracts can be expressed as;

$$F(t,T_1,T_2) = \int_{T_1}^{T_2} w(s;r)\, f(t,s)ds \quad , \quad t \le T_1 \le T_2$$

$$\text{where} \qquad \qquad (11a)$$

$$w(s;r) = \frac{e^{-r(s-t)}}{\int_{T_1}^{T_2} e^{-r(s-t)} ds}$$

Where the weight function; *w(s;r)* is an adjustment to exclude risk free arbitrage if the interest rates is not equal to zero. However, in this thesis, it is explicitly assumed that the interest rate is zero. This is because firstly, the yield curve movements has a quite insignificant effect on the Nord Pool contracts, considering that the typically price movement is ± 2-3% on a daily basis. Benth et. al. (2007) also proposes this assumption arguing among others that seasonality has a greater impact than interest rate levels. Moreover, interest rates have been extremely low for the last years, both in Europe and the Nordic countries. Also, incorporating interest rates complicates both the VCV estimations *(because the interest rate component will have to be excluded)* and when mapping the positions which are to be assigned to different Time Buckets. Furthermore, if the zero interest rate assumption is left out, one will have to estimate VaR from the interest rate risk as well, thus incorporating covariance between the forward and yield curve.

Bjerksund et. al. (2000) also shows that the when the interest rate is zero the price of the contract can be expressed by;

$$F(t,T_1,T_2)=\frac{1}{T_2-T_1}\int_{T_1}^{T_2} f(t,s)ds \quad , \quad t\le T_1\le T_2$$

$$\textit{because} \qquad (11b)$$

$$w(s;0)=\frac{e^{-0(s-t)}}{\int_{T_1}^{T_2} e^{-0(s-t)}ds}=\frac{1}{T_2-T_1}$$

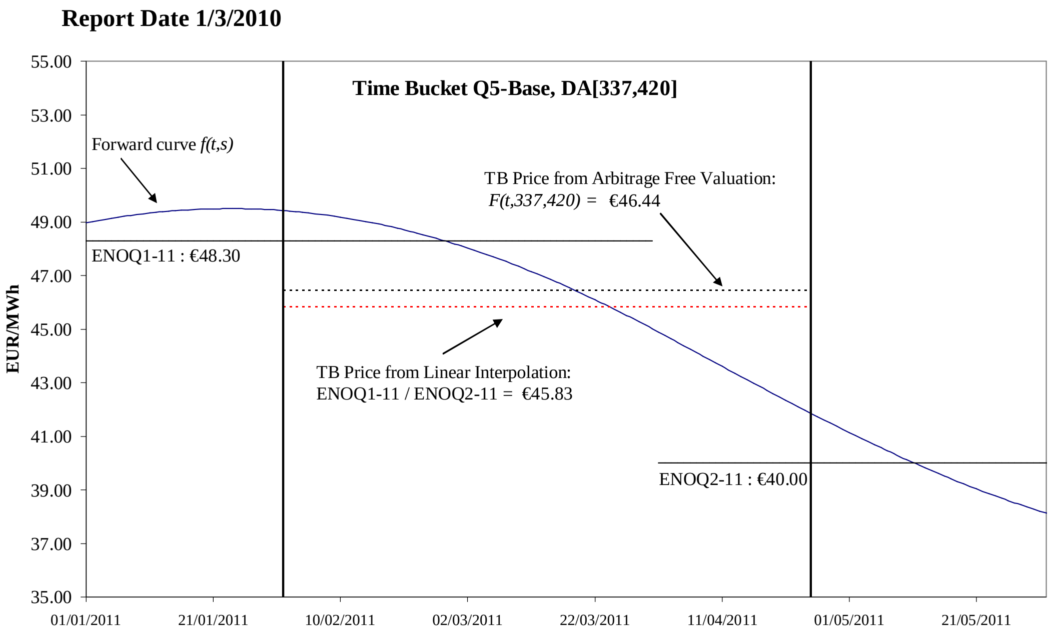
6.3.1 VCV Estimates

General

The electricity market differs from standard commodity forwards in the way that electricity contracts offer a continuous flow of delivery over a time interval while a standard commodity forwards offers only a bulk delivery at a given time. Thus it makes more sense to construct fixed maturity contracts which span over a period *(Time Bucket)* rather than points as well as a maturity structure which resemblance the actual traded contracts. Moreover, because the Nord Pool contracts are written on the arbitrage free average price of the continuous forward prices in a time interval, linear interpolation is considered to be inadequate. There are several reasons for this assessment. Firstly that several contracts can span over a given Time Bucket, such as three calendar months, making linear interpolation more complicated. Second, the existence of perfectly overlapping contracts such as the three first calendar month as well as the first quarter, which results in a sorting dilemma *(although it is common to exclude the longest contract due to the fact that including a greater number of contracts will increase market information)*. Moreover, seasonality i.e. high prices during the winter and low during the summer can make linear interpolation to under or overestimate the price of the Time Bucket.

This results in the necessity of extrapolating dynamic forward prices from *t* to the end of the delivery period for the contract with longest maturity, such that the prices for the time buckets in absence of risk free arbitrage can be found.

Figure 5 : Pricing of Time Buckets



Pricing of Time Buckets

The price of a Time Bucket for a given date can be found by arbitrage free pricing from the forward curve. In the electricity market there are traded contracts for given time periods such as weeks, months, quarters and years. The forward curve is a curve which spans from the given date until (at least) the end of the tradable longest contract. In general the forward curve can be expressed as a mathematical function which among others depends on time to maturity and is subject to the arbitrage free valuation of tradable contracts to equal the observed prices. This function can be found by numerous different optimization algorithms.

In this thesis, the forward curves are generated by Elviz Curve Server[19] 10.1 (ECS). ECS generates forward curves based on the maximum “smoothness” principle along with a sinusoidal Prior function which models seasonality where there are missing market information, typically long end contracts such as quarters or years can not by themselves

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[19] For additional information on Elviz Curve Server; www.viz.no

model the markets seasonality. Firstly, now ignoring the Prior function, ECS forward curve is a polynomial Spline function. Consider the total of *N* tradable forward contracts, where the contracts cover the entire timeline $[\ t\ ,\ T_{N+1}\ ]$. Moreover each observed contract, with maturity $(\ T_i\ ,T_{i+1}\ )$ has its own set of the coefficients *a, b* and *c*;

$$\begin{aligned} &f(t,s) = a_i + b_i s + c s_i^4 \\ &where \\ &i \in F(T_i, T_{i+1}) \end{aligned} \qquad (12a)$$

This is showed by Stensland (2008) and is the same as Adams and Deventer (1994) applied for yield curves. Adams and Deventer (1994) showed that a forward curve for the interval $(\ t,\ T_{N+1}\ )$ based on the maximum "smoothness" principle is the set of coefficients which minimizes the integral of squared second order derivative with respect to time *(s)* of the Spline function. Adams and Deventer (1994: 54) also notes that *"This expression is a common mathematical definition of smoothness used in engineering application"*.

$$\begin{aligned} &Min \quad \int_t^{T_{N+1}} \left[\frac{\partial^2}{\partial s^2} f(t,s) \right]^2 ds \\ &\Rightarrow \sum_{i=0}^{N} \int_{T_i}^{T_{i+1}} \frac{1}{144} c_i^2 s^4 ds \\ &where \\ &f(t,s) = \begin{bmatrix} a_0 + b_0 s + c_0 s^4 & s \in [T_0, T_1] \\ a_1 + b_1 s + c_0 s^4 & s \in [T_1, T_2] \\ \cdots & \cdots \\ a_N + b_N s + c_0 s^4 & s \in [T_N, T_{N+1}] \end{bmatrix} \end{aligned} \qquad (12b)$$

However, Forsgren (1998) proved that Adams and Deventer's Spline function was not the optimal solution to the minimization problem. Instead, the optimal solution (smoothest) is given by *natural* Splines, thus including the quadratic and cubic terms[20]. On the other hand, incorporating these terms would considerably complicate the optimization problem.

---

[20] Note that this would require Constraint *III* (See page 38) to be extendend to the second and third order derivative as well. While in the Adams and Deventer case, the second and third derivative conditions are fulfilled by the first order condition by design.

Thus, it is believed that these terms would substantially change the solution given by Adams and Deventer (1994).

Furthermore, Stensland (2008) shows that the Target function is subject to the following constrains *(I-IV)*[21]. Also, Adams and Deventer (1994) and Benth et. al. (2007) presents similar constraints to the Target function.

$$
\begin{aligned}
& I. \\
& \frac{1}{s_{i+1}-s_i}\int_{s_i}^{s_{i+1}} w(0;s)\, f(t,s)ds = F(t,T_i,T_{i+1}) \\
& \Rightarrow \frac{1}{s_{i+1}-s_i}\left(a_i\left(s_{i+1}-s_i\right)+\tfrac{1}{2}b_i\left(s_{i+1}^2-s_i^2\right)+\tfrac{1}{5}c_i\left(s_{i+1}^5-s_i^5\right)\right) = F(t,T_i,T_{i+1}) \quad i=0,1,2,\ \dots,N \\
& II. \\
& \left(a_i-a_{i-1}\right)+\left(b_i-b_{i-1}\right)s_i+\left(c_i-c_{i-1}\right)s_i^4 = 0 \quad i=1,2,\ \dots,N \\
& III. \\
& \lim_{h\to 0}\frac{f(t,s+h)-f(t,s)}{h} = \lim_{h\to 0}\frac{f(t,s-h)-f(t,s)}{-h} \quad , \quad s=s_i \\
& \Rightarrow \left(b_i-b_{i-1}\right)+4\left(c_i-c_{i-1}\right)s_i^3 = 0 \quad i=1,2,\ \dots,N \\
& IV. \\
& \frac{\partial^2 f(t,s)}{\partial s^2} = 0 \quad , \quad i=N \\
& \Rightarrow 12c_N s^2 = 0 \\
& \Rightarrow c_N = 0
\end{aligned}
$$

Where

I. The arbitrage free valuation from the forward curve of observed contracts has to equal its observed price.

II. The curve's connecting points between contracts have to be equal, i.e. the forward curve has to be continuous.

---

[21] These constraints are somewhat simplefied oposed to ECS algorithm, but should demonstate the general concept.

III. The *right* and *left* first order derivative with respect to time has to be equal in the preceding and following intervals shared connecting points. Thus ensuring smoothness of the curve in the connecting points.

IV. The second order derivative with respect to time has to equal zero, thus the c_N coefficient has to be zero.

In order to solve this minimization problem in ECS, it is required as input a set of N contracts, which spans over time s from t to T_{N+1} i.e. from trading day until the last day of delivery of the longest contract. Note that all the contracts have to cover the entire timeline perfectly *(although a solution is feasible with some adjustments to the constraints)* and perfectly overlapping contracts *(such as Jan, Feb, Mar and Q1)* is not possible, because it would hinder a unique solution. ECS has a build in sorting algorithm which drops the longest contracts in such cases *(i.e. Jan, Feb and Mar would be chosen over Q1).* Moreover ECS also adjusts for Daylight Savings Time, the last week in March will be treated as 167 hours, while the last week in October as 169 hours.

Now taking the Prior function into account, Stensland (2008) explains that the algorithm consists of three steps;

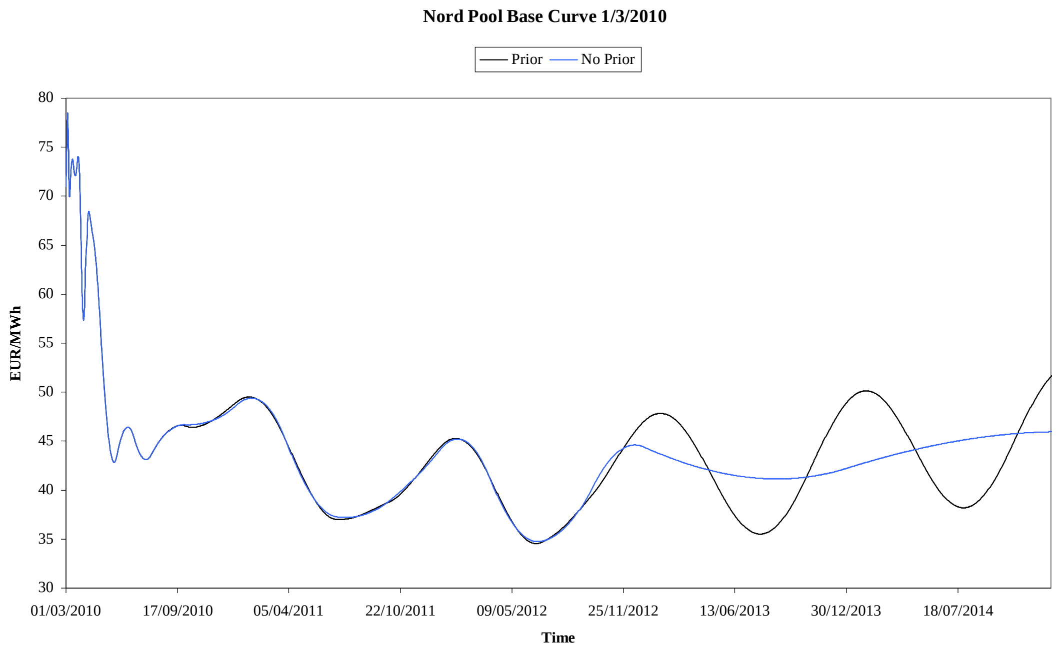
1. Subtract the prior function from all observed contracts.

2. Construct the Spline function described above.

3. The Forward curve will now consist of two parts: Spline function + Prior function

Figure 6 exhibits the effect of the prior function, as mentioned earlier and also commented by Benth et. al. (2007) that the short end of the curve is literally unaffected by adding the Prior function, reason being that observed contracts will override the Prior function. Here, the two curves are virtually identical until end 2012, thereafter it only exists annual contracts. Observe that the season profile from the prior function is relatively similar to the season profile for 2012.

Figure 6 : Prior vs No Prior



From the forward curve, represented by the Spline function it is possible to extract an arbitrage free average forward price for any given maturities (represented by time intervals), where the end-maturity is less than the last delivery date for the longest contract. If the desired maturity *(V_1,V_2)* falls within *one* Spline, the average forward price can be expressed as;

$$F(t,V_1,V_2) = \frac{1}{V_2 - V_1} \int_{V_1}^{V_2} a_i + b_i s + c_i s^4 ds \quad , \quad T_i \leq V_1 < V_2 \leq T_{i+1} \qquad (12c)$$

Similarly, if the desired maturity falls between several Splines, the price can be expressed as;

$$F(t,V_1,V_2) = \frac{1}{V_2 - V_1} \sum_i^H \int_{Max(V_1,T_i)}^{Min(V_2,T_{i+1})} a_i + b_i s + c_i s^4 ds \qquad (12d)$$

$$T_i \leq V_1 < T_{i+1} .... < T_H < V_2 \leq T_{H+1}$$

Peak and Off-Peak

The Nord Pool forward market consists of two types of contracts; Base and Peak Load, consequently Peak-Base or Peak-Off-Peak covariance is required. Recall that Base is just the sum of Peak and Off-Peak hours. Fundamental (co)variance calculus illustrates the relationship between Base *(B)*, Peak *(P)* and Off-Peak *(OP)* returns;

$$
\begin{aligned}
&COV(r_B, r_P) = COV(w_{OP} r_{OP} + w_P r_P, w_P r_P) = w_{OP} w_P COV(r_{OP}, r_P) + w_P^2 COV(r_P, r_P) \\
&\Rightarrow w_{OP} w_P COV(r_{OP}, r_P) + w_P^2 VAR(r_P) \\
&VAR(r_B) = w_{OP}^2 VAR(r_{OP}, r_P) + w_P^2 VAR(r_P) + 2 w_{OP} w_P COV(r_{OP}, r_P) \qquad (13a) \\
&where \\
&w_{OP} + w_P = 1
\end{aligned}
$$

It follows that these relationships are valid if and only if the ratio between Peak and Off-Peak hours are constant for all Time Buckets independent of trading days. Constructing Time Buckets on a weekly granularity ensures a (60/108 hours) constant ratio[22]. Following, when traded Base contracts are split between buckets they will typically *not* have this constant ratio in first and last bucket which the contract is exposed to, thus the Base variance and Base/Peak covariance estimates of these buckets are biased. It is therefore believed that applying Peak/Off-Peak Time Buckets is a better procedure than Base/Peak. Because, the VCV coefficients and mapping ensures that the volatility of these Time Buckets are correct, because they are weighed by the actual Peak and Off-Peak hours, and not the constant ratio.

Moreover, if the portfolio also consists of contracts with customized load profiles, it is impossible to map according to Base/Peak Time Buckets, thus further justification for applying Peak/Off-Peak.

ECS will generate *two* individual forward curves (Base and Peak) based on the described optimization problem due to the fact that Base and Peak hours are semi overlapping over the entire curve[23]. This results in the need of calculating Off-Peak prices

[22] By ignoring Day Light Saving Time.

[23] Although a solution is possible which results in a single curve, however this is not possible without a Price Profile which complicates the optimization problem considerable. For more information regarding Price Profiles see ECS Manual.

for Time Buckets and days *(for mapping)*. Consider a contract maturity *(V_1,V_2)*, under the zero interest rate assumption and in absence of risk free arbitrage, Equation 13b illustrates the arbitrage free relationship between Base *(B)*, Peak *(P)* and Off-Peak *(OP)* prices*;*

$$\begin{aligned}
&F_{OP}(t,V_1,V_2)=\frac{F_B(t,V_1,V_2)Q_B-Q_PF_P(t,V_1,V_2)}{Q_{OP}}\\
&\textit{where}\\
&Q_B=Q_P+Q_{OP}\\
&F_B(t,V_1,V_2)Q_B=F_P(t,V_1,V_2)Q_P+F_{OP}(t,V_1,V_2)Q_{OP}\\
&Q_B=\text{Base hours in the interval }(V_1,V_2)\\
&Q_P=\text{Peak hours in the interval }(V_1,V_2)\\
&Q_{OP}=\text{Off-Peak hours in the interval }(V_1,V_2)
\end{aligned} \tag{13b}$$

Log Returns

RiskMetrics (1996) suggests the basis for log returns of constant maturity contracts is the log difference between a contract with same maturity *(in days)* for a given day and previous day. For simplicity consider a bulk forward contract with the arbitrary constant maturity *V*;

$$r_V=\ln\left(F(t,V)\right)-\ln\left(F(t-1,V)\right)\quad,\quad t<V \tag{14a}$$

Note that there are two timelines here, *t* which is in trading days, while maturity *V* is in calendar days. If *t* where a Monday, then *(t-1)* would be previous Friday, thus the difference *t-(t-1)* would equal three calendar days. The problem which arises from Equation 14a is that comparison of constant maturity contracts are not directly comparable because if one were to enter into for example a 30 day *(V)* contract yesterday (*t-1)*, and hold to *t*, the contract would then be a 29 day contract (if *(t-(t-1))* = 1 day) and *not* a 30 day contract.

To address this issue one will instead compare the constant maturity *(V)* contract for trading day *t* by the previous trading day's *(t-1)* with maturity *(V + (t-(t-1)))*. Which then yields the return of entering into a contract on previous trading day, which on the given trading day will have the maturity *(V)*. In other words, defining the constant maturity for a given trading day in calendar time and comparing it with the same calendar time on previous trading day;

$$r(t,V) = \ln\left(F(t,V)\right) - \ln\left(F(t-1, V+(t-(t-1)))\right) \quad , \quad t < V \tag{14b}$$

The length of the Time Buckets are set to increase with time to maturity, because there are more contracts in the short end *(weeks, months)* than in the long end *(annual contracts)* in order to be similar to the actual maturity structure. A Time Buckets interval is defined from a given Day-Ahead to a given Day-Ahead (DA) on the forward curve. The length of the Time Buckets is weekly, because it ensures a constant Peak to Off-Peak ratio and avoids the complications of intra-week price variations.

Table IV : Example of Time Buckets (u_i)

Time Bucket	From DA	To DA	Continuous time *s (Start)*	*s (End)*	Discrete Time V_{bi} *(Start)*	V_{ei} *(End)*	Bucket Length (in days)
u_0	1	7	V_0	V_1	V_{b0}	V_{e0}	7
u_1	8	14	V_1	V_2	V_{b1}	V_{e1}	7
u_2	15	42	V_2	V_3	V_{b2}	V_{e2}	28
...	...	...	…	…	…	…	...
u_{19}	841	1008	V_{19}	V_{20}	V_{b19}	V_{e19}	168

Under the zero interest assumption the log returns are calculated by comparing today's integral *(* V_i , V_{i+1} *)* of the forward curve on date *t* by last trading day *(t-1)* forward curve over the integral *(*V_i *+(t-(t-1)),* V_{i+1} *+(t-(t-1)))*.

$$\begin{aligned} r(t,u_i) &= r(t,V_i,V_{i+1}) \\ &= \ln\left(\frac{1}{V_{i+1}-V_i}\int_{V_i}^{V_{i+1}} f(t,s)ds\right) - \ln\left(\frac{1}{V_{i+1}-V_i}\int_{V_i+(t-(t-1))}^{V_{i+1}+(t-(t-1))} f(t-1,s)ds\right) \\ t &< V_i < V_{i+1} \end{aligned} \tag{14c}$$

6.3.2 Mapping

Forward Contracts

Because it is assumed that each Time Bucket has distinctive VCV coefficients, a contract which spans over two or more Time Buckets for a given valuation date will have to be mapped between these. In absences of risk free arbitrage, a contract can be viewed as a portfolio of two or more Time Buckets. Formally when a contract has constant delivery throughout the maturity *(T_1 ,T_2)* and which also spans over two Time Buckets u_1 and u_2. When ignoring the existence of Peak hours and, the Exposure for *M* such contracts are;

$$
\begin{aligned}
&F(t,T_1,T_2) = \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} f(t,s)ds \\
&F(t,T_1,T_2) = \frac{1}{T_2 - T_1}\left(\int_{T_1}^{V_1} f(t,s)ds + \int_{V_1}^{T_2} f(t,s)ds\right) \\
&Ex\left(F(t,T_1,T_2)M_t\right) = Ex(u_1) + Ex(u_2) \qquad (15a) \\
&= M_t\int_{T_1}^{V_1} f(t,s)ds + M_t\int_{V_1}^{V_2} f(t,s)ds \\
&t \le V_1 \le T_1 < V_2 \le T_2
\end{aligned}
$$

Contracts can also offer a specific quantity of delivery upon specific future time intervals (such as peak contracts which offer 12 hours of delivery on weekdays and nil during weekends) which is generalized by a specific load contracts; $L(T_1, T_2)$. By discretionizing Equation 15a and adding the variables $q(i)$ and $F(t,i,i+1)$ which indicates the delivered quantity and the daily forward price on the i.th day in delivery. V_{bi} and V_{ei} is the first and last forward date in time bucket *i* respectively. The Exposure can as following be written as;

$$
\begin{aligned}
&Ex\left(L(t,T_1,T_2)M_t\right) = Ex(u_1) + Ex(u_2) \\
&\Rightarrow \sum_{i=T_1}^{V_{e1}} F(t,i,i+1)q(i) + \sum_{i=V_{b2}}^{T_2} F(t,i,i+1)q(i) \\
&where \\
&V_{e1} = \text{Last Day Ahead for Time Bucket 1} \qquad (15b) \\
&V_{b2} = \text{First Day Ahead for Time Bucket 2} \\
&Q = \sum_{i=T_1}^{T_2} q(i) \\
&t \le V_{b1} \le T_1 \le V_{e1} < V_{b2} \le T_2 \le V_{e2}
\end{aligned}
$$

$Ex(u_z)$ denotes the exposure, which is the sum of all market prices times quantity for a given Time Bucket *(z)*. Note that here the Time Buckets for a given interval are split between Off-peak and Peak hours, which together form the exposure for base contracts.

Now, consider a Base contract with equal amount of delivery throughout its entire delivery period, it is also subject to *two* Time Buckets thus the exposure for *M* such contracts equals;

$$
\begin{aligned}
& Ex(F(t,T_1,T_2)M_t) = Ex(u_1^P) + Ex(u_1^{OP}) + Ex(u_2^P) + Ex(u_2^{OP}) \\
& \Rightarrow M_t \sum_{i=T_1}^{V_{e1}} F_P(t,i,i+1)q_P(i) + M_t \sum_{i=T_1}^{V_{e1}} F_{OP}(t,i,i+1)q_{OP}(i) \\
& + M_t \sum_{i=V_{b1}}^{T_2} F_P(t,i,i+1)q_P(i) + M_t \sum_{i=V_{b1}}^{T_2} F_{OP}(t,i,i+1)q_{OP}(i) \\
& t \le V_{b1} \le T_1 \le V_{e1} < V_{b2} \le T_2 \le V_{e2}
\end{aligned}
\tag{15c}
$$

Options

Options on Nord Pool forward contracts can be priced by the B-76 model, however due to the fact that most commodity forwards does not exhibit constant volatility as Bjerksund et. al. (2000: 2) points out for most commodities *"Typically the implicit volatility is a decreasing and convex function of time to maturity"*. Thus, the volatility must be approximated in some manner in order to be compatible with the B-76 model. Bjerksund et. al. (2000) proposes the following single factor model for forward contracts with the following price process under the equivalent martingale measure and delivery at time *s*, where the volatility is a function of time to maturity *(s)*.

$$
\begin{aligned}
& \frac{df(t,s)}{f(t,s)} = \sigma(t,s)dW(t) \\
& = \left(\frac{a}{s-t+b} + c \right) dW(t)
\end{aligned}
\tag{16a}
$$

Where *a*, *b* and *c* are positive constants. In this model it is assumed that dynamics of all the forward prices for the same security is only subject to *one* Wiener Process, thus the correlations between forward prices with maturities are implicit in the volatility function. This leads to the approximation of the price dynamics of an electricity contract, which offers a continuous flow of delivery between the maturities (T_1, T_2), under the zero interest assumption;

$$\frac{dF(t,T_1,T_2)}{F(t,T_1,T_2)} \approx \frac{1}{T_2-T_1}\int_{T_1}^{T_2}\frac{df(t,s)}{f(t,s)}ds = \left[\frac{a}{T_2-T_1}\ln\left(\frac{T_2-t+b}{T_1-t+b}\right)+c\right]dW(t) \tag{16b}$$

Following, considering a European option, with maturity *(τ)*, less than the beginning of the delivery of the forward contract (i.e. $\tau < T_1$). Because it is assumed that the price dynamics from Equation 16b is only subject to *one* Wiener Process it is applicable in the Black 76 model. Bjerksund et. al. (2000) shows that for such an option that the B-76 annualized fixed volatility can be estimated by a plug-in volatility;

$$v_E = \sqrt{\frac{1}{\tau-t}\int_{z=t}^{\tau}\left(\int_{s=T_1}^{T_2}\frac{\sigma(z,s)ds}{T_2-T_1}\right)^2 dz} \tag{16c}$$

Thus,

$$\begin{aligned} &C(F(t,T_1,T_2),K,t,\tau,r,v_E) = e^{-r(\tau-t)}\left(F(t,T_1,T_2)\cdot N(d_1)-K\cdot N(d_2)\right) \\ &\textit{where} \\ &d_1 = \frac{\ln(F(t,T_1,T_2)/K)+\frac{1}{2}{v_E}^2(\tau-t)}{v_E\sqrt{\tau-t}} \\ &d_2 = d_1 - v_E\sqrt{\tau-t} \\ &t<\tau<T_1 \end{aligned} \tag{16d}$$

Note that Nord Pool only offers European Call and Put options; hence from Equation 16d, the value of put options can be found by for example put-call parity[24].

Following RiskMetrics approach in a discrete Peak and Off-Peak setting, the exposure for ***one*** call option, given that the contract spans over only *one* Time Bucket, is simply the exposure for the underlying (Equation 15c) multiplied by the delta for time *t*;

[24] See Hull (2008: 331)

$$Ex(C(F(t,T_1,T_2),K,t,\tau,r,\sigma)) = Ex(u_1^{OP}) + Ex(u_1^P)$$
$$\Rightarrow \delta_t \sum_{i=T_1}^{T_2} F_P(t,i,i+1)q_P(i) + \delta_t \sum_{i=T_1}^{T_2} F_{OP}(t,i,i+1)q_{OP}(i) \qquad (17)$$
$$t \le T_1 < T_2$$

Thus, exposure can now be defined as Delta-Adjusted Exposure, because for the forward contracts Delta is by definition 1.

6.3.3 Calculating VaR

Having defined exposure, the weight in a given Time Bucket equals the aggregate exposure for the Time Bucket.

$$w_i^P = Ex(u_i^P) \quad and \quad w_i^{OP} = Ex(u_i^{OP}) \quad , \quad \forall \ i \qquad (18a)$$

From Equation 2 it follows that the portfolios *VaR((1-α))* equals:

$$\hat{\sigma}_{P,h}^2 = \begin{bmatrix} w_0^P & \cdots & w_N^P & w_0^{OP} & \cdots & w_N^{OP} \end{bmatrix} \begin{bmatrix} \hat{\sigma}_{u_0^P}^2 & \cdots & \hat{\sigma}_{u_0^P u_N^P} & \hat{\sigma}_{u_0^P u_0^{OP}} & \cdots & \hat{\sigma}_{u_0^P u_N^{OP}} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \hat{\sigma}_{u_N^P u_0^P} & \cdots & \hat{\sigma}_{u_N^P}^2 & \hat{\sigma}_{u_N^P u_0^{OP}} & \cdots & \hat{\sigma}_{u_N^P u_N^{OP}} \\ \hat{\sigma}_{u_0^{OP} u_0^P} & \cdots & \hat{\sigma}_{u_0^{OP} u_N^P} & \hat{\sigma}_{u_0^{OP}}^2 & \cdots & \hat{\sigma}_{u_N^{OP} u_0^{OP}} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \hat{\sigma}_{u_N^{OP} u_0^P} & \cdots & \hat{\sigma}_{u_N^{OP} u_N^P} & \hat{\sigma}_{u_N^{OP} u_0^{OP}} & \cdots & \hat{\sigma}_{u_N^{OP}}^2 \end{bmatrix} \begin{bmatrix} w_0^P \\ \vdots \\ w_N^P \\ w_0^{OP} \\ \vdots \\ w_N^{OP} \end{bmatrix} \qquad (18b)$$
$$VaR_{t+h}((1-\alpha)) = z_\alpha \hat{\sigma}_{P,h}$$

6.4 Back Testing

6.4.1 General

A common way of testing whether the model is adequate is Back Testing. Jorion (2007: 139) states that Back Testing is a formal statistical framework that consists of verifying that actual losses are in line with projected losses *(VaR)*. Furthermore, this method of validating the model is a way to check whether the assumptions, parameters and methods needs to be further calibrated.

With a *VaR(1-α)* estimate, losses should only exceed the *VaR(1-α)* estimate $\alpha \times N$ of N observations. Observations of Losses exceeding the VaR estimates are defined as exceptions *(e)*. However, the number of actual exceptions are likely to somewhat differ from expected exceptions *($\alpha \times N$)* for N which is not considerably large. This could occur because for example *bad* or *good* luck. In the opposite case where N is large, one could perform back tests by straightforward statistical inference; nevertheless such cases are seldom, because it is unlikely that a portfolio has been using the exact same model, assumptions and methods over a longer time.

6.4.2 Theoretical P&L

The necessitate for Theoretical P&L rather than Actual P&L is due to the fact that $VaR_{t+h}(1-\alpha)$ is an estimate of the maximum loss which should occur for a given level of confidence *(α)*, time horizon *h* and also given the positions held at time *t.* Actual P&L for time *t+h* originates from MtM value from the positions held at time *t+h* less the MtM value of positions held at *t*. Thus Actual P&L will be affected by changes in positions from time *t* to *t+h*. Actual P&L will also consist of trading P&L, fees and interest income or expenses. Although, these additional components could be relatively small when *h* is short. Theoretical P&L *(π^*_{t+h})* is then MtM value at *t+h* given the positions held at *t* less the MtM value at *t*.

Concerning forward contracts, which MtM value is determined of contract prices *(B)* as well as generalized forward prices *F(t,T)*, the actual nor the theoretical P&L are affected by the contract price. Consider the VaR portfolio to consist of only one forward contract *F(t,T)* at contract price *B* with position *M* assuming zero interest rate.

$$\begin{aligned} MtM_t &= \left(F(t,T) - B\right) M_t \\ \pi^*_{t+h} &= \left(F(t+h,T) - B\right) M_t - \left(F(t,T) - B\right) M_t \\ \pi^*_{t+h} &= \left(F(t+h,T) - F(t,T)\right) M_t \end{aligned} \tag{19}$$

6.4.3 Kupiec Test

Kupiec (1995) proposes a likelihood ratio (LR) test for back testing the exceptions in a given sample.

Firstly, defining the actual exception and expected exception ratio;

$$p = \frac{e}{N} \quad , \quad p^* = \alpha \tag{20}$$

Under the null hypothesis that actual exception ratio is equal to the expected exception ratio, Kupiec (1995: 79) shows that the LR test statistics is chi-squared distributed with *one* degrees of freedom;

$$H_0 : e = p^* \quad , \quad H_A : e \neq p^*$$
$$LR(e, p^*) \sim \chi_1^2$$
$$LR(e, p^*) = -2\ln\left[(1-p^*)^{N-e}(p^*)^e\right] + 2\ln\left[(1-\tfrac{e}{N})^{N-e}(\tfrac{e}{N})^e\right] \quad (9)$$
With Critical Value (Confidence = k)
$$CV((1-k)) = \chi_{1,k}^2$$

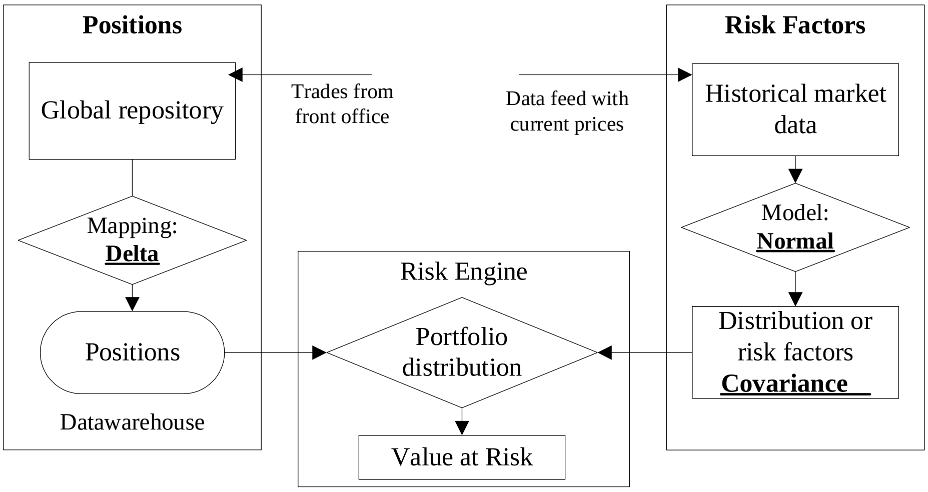
Kupiec (1995) also points out that for small samples; this test has poor power characteristics, by having relatively large acceptance regions for the null hypothesis when the sample size is small.

6.5 Practical Implications

Although forward curves for the Nord Pool Base and Peak curves can be calculated and exported on an hourly granularity in ECS, there are no known applications that could firstly calculate VCV estimates from continuous forward curves based on the basis of the methodology presented in this thesis. Nor are there any known applications which can map Peak and Off-Peak exposure into Time Buckets and calculate the VaR estimate. The solution for the calculations necessary in this thesis has therefore been to develop an application for the calculations needed.

Jorion (2007) exhibits a flowchart on the highest level of how an Analytical VaR System could be constructed. The system Jorion (2007) describes is split into two main procedures which together form the input for the VaR calculation. The VaR calculation is performed by the Risk Engine. The first procedure, showed on the left hand side of Figure 7 which maps all positions for a given trading day into positions of Exposure. On the right hand side, VCV matrices are calculated by a given model and historical prices.

Figure 7 : Delta-normal VaR System



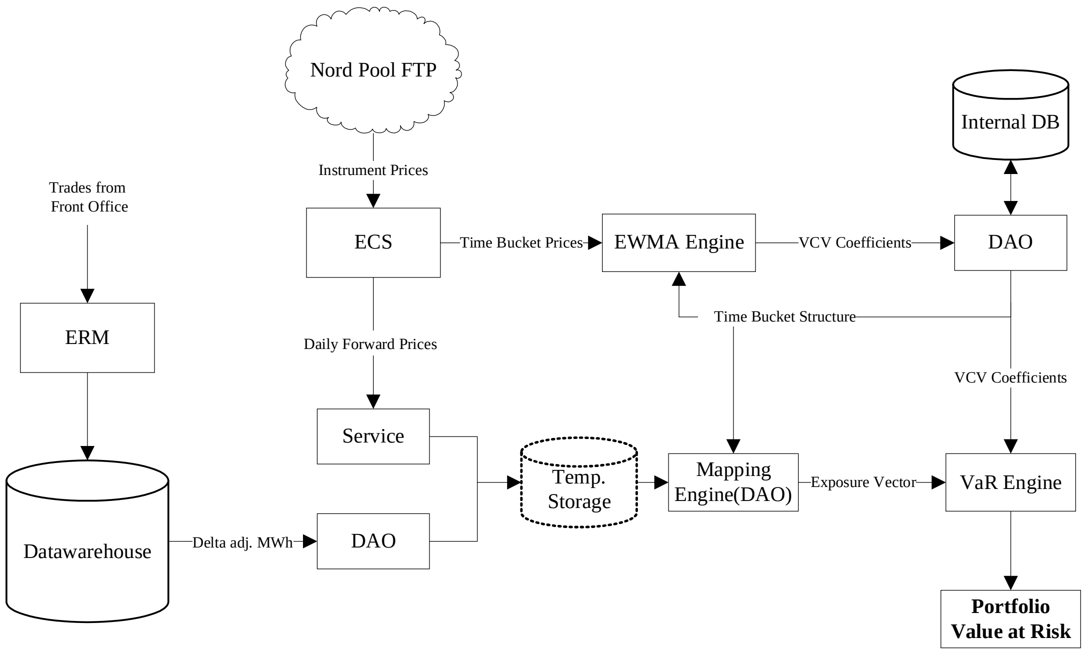
Source: Jorion (2007: Figure 10-9)

The application named *AVAR* for further references, which is written in C#.NET, following the Model-View-Controller software-architecture[25], making it easy to run VaR reports for a range of dates by Batch Processing which is needed for Back Testing. *AVAR's* objective is firstly to calculate VCV estimates for each Time Bucket within the time series, and then storing them in a SQL Database. Second, calculate VaR for each portfolio for each day in the time series, by extracting delta-adjusted future volumes on a daily granularity from the Datawarehouse. The data in the Datawarehouse is from dumps done by Elviz Risk Manager (ERM). The daily volume will then be split into Peak and Off-Peak hours. From ECS, daily Peak and Off-Peak prices will be extracted as showed in Equation 13, applying the same curves settings as for the VCV estimates, in order to keep consistency. From there, *AVAR* will utilize SQL commands in order map the exposure into Time-Buckets. *AVAR's* calculation engine will, by extracting the coherent VCV estimates from the internal database calculate VaR for the entire time series, as well as theoretical P&L by the next trading days forward curve. Thus from there it is fairly easy to extract the exceptions over the time series and running the Kupiec Test.

[25] For more about the M-V-C architecture see Larman (2002)

Figure 8 exhibits the flowchart of *AVAR*, on a higher level of detail than Jorion's example, exhibiting the dataflow from prices to curves to coefficients and exposure. Note that *AVAR* is utilizes DAO (Data Access Objects) and Services in order extract future delta adjusted volumes and forward prices.

Figure 8: *AVAR* Flowchart



Appendix 10.1 will provide the core calculation code applied for the EWMA and Value at Risk Engine.

7 Results

7.1 Choice of Time Bucket Structure

In this thesis, the Time Bucket structure chosen is exhibited in Figure 9. As discussed in Section 6.3.1 the structure is constructed to resemblance the actual traded contracts on the Nord Pool exchange. The reason for the first Time Bucket to start one day ahead of the report date is because VaR is from the next trading day's estimated P&L distribution, thus the given report dates exposure is irrelevant for the next day's P&L. However, this is not entirely true, considering weekends and other holydays where the next trading day is not one day ahead. Nevertheless, it is considered sufficient because the VCV estimates cannot be consistently constructed from a dynamic Time Bucket Structure.

Figure 9: Time Bucket Structure

Bucket Number	Bucket Name	From Days Ahead	To Days Ahead	Bucket Length in Days
1	1W	1	7	7
2	2W	8	14	7
3	3W	15	21	7
4	4W	22	28	7
5	2M	29	56	28
6	3M	57	84	28
7	4M	85	112	28
8	5M	113	140	28
9	6M	141	168	28
10	Q3	169	252	84
11	Q4	253	336	84
12	Q5	337	420	84
13	Q6	421	504	84
14	Q7	505	588	84
15	Q8	589	672	84
16	Y2.5	673	840	168
17	Y3+	841	2016	1176

7.2 VCV Estimates

Having set a tolerance level at 1% and the decay factor (λ) of 0.94, the first EWMA estimates start from 2/1/2009, which is the first trading day of 2009. The sample period for the back tests, year 2009 (249 trading days) resulted in 148,155 Peak and Off-Peak VCV estimates[26]. Obviously, because of the sheer number of coefficients, they cannot be

[26] Excluding the symetrical coefficients

presented in any fashionable way in paper. However, presenting a volatility surface and Peak/Off-Peak Time Bucket correlations over the sample period is feasible. Here it is chosen to present the volatility surface in Base[27]. This because short end Peak and Off-Peak Time Buckets may give an impression of unnatural high volatility, however these Time Buckets also exhibit a strong negative correlation. Following, viewing Base volatiles should provide a better understanding of volatility term structure. One can easily see from the surface chart that the volatility is typically decreasing with time to maturity, although there are some spikes which occur for reasons unknown. Furthermore, the surface seems to exhibits a seasonal pattern, i.e. increasing volatility curves during fall/winter and decreasing during spring/summer. This supports the proposition of Koekebakker and Steen (2001) that the volatility exhibits a seasonal pattern. Thus it seems like the EWMA volatility estimates captures the seasonal pattern. However one cannot exclude the possibility that it is somewhat lagging.

[27] From Equation 13a, the volatilty of Base Time Buckets can be derived from Peak/Off-Peak volatility and Peak/Off-Peak covariance when Peak to Off-Peak hours is a constant ratio.

Figure 10 : Volatility Surface 2009 (Base)

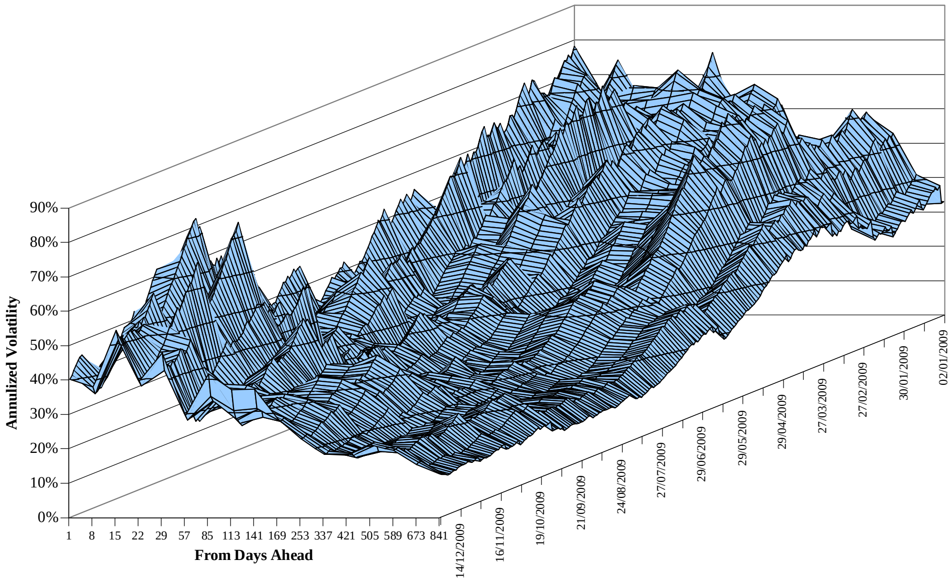
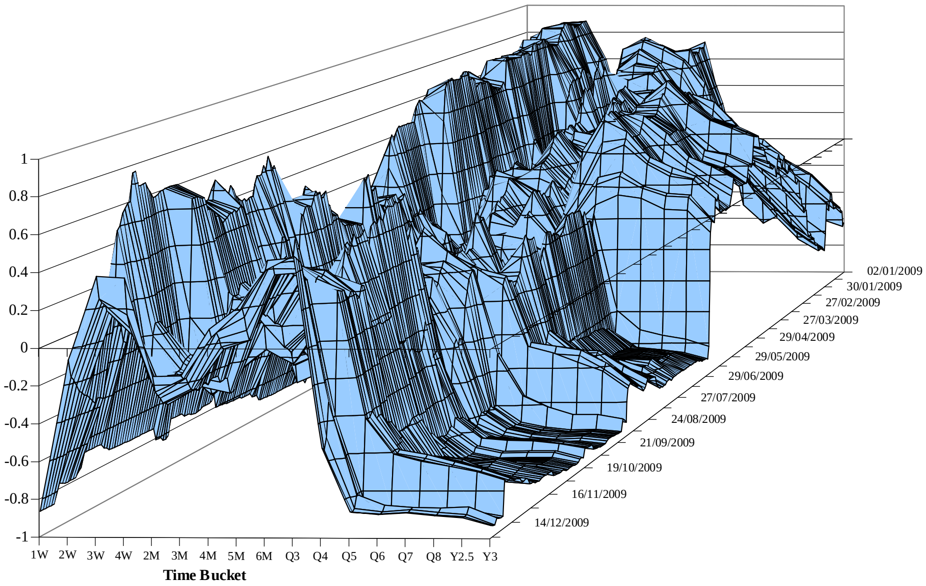


Figure 11 exhibits the correlation estimates between Peak and Off-Peak Time Buckets over the sample period. In general the correlations seem fairly stable over the sample period. The correlations from the very short end Time Buckets being strongly negative, moving to very positive until the end of the first two months. The following Time Buckets seems to be somewhat unstable and uncorrelated. Furthermore, the correlations increases from the first year and afterward steady moving towards a strong negative correlation again, with exception of spring time, whereas the long end seems to be a bit uncorrelated.

Figure 11 : Correlation Surface Peak/Off-Peak Time Buckets



7.3 Mapping and VaR Example

This section will present a simple example of how to map a single forward contract into four Time Buckets and following its VaR calculation. The motivation for this is to show how VaR is calculated on a very small scale. Regarding the real-world portfolios which hold a range of contracts, the calculations are far more extensive than presented here. Nevertheless, the essence is very much the same, apart from some of the calculation methods which are optimized in *AVAR*, by avoiding duplicate calculations.

Relevant information

Date *(t)*	01/10/2009
Horizon *(h)*	One trading day

Assessed Contract	**ENOMDEC-09**
Total Hours	744
Total Exposure	€24,403.2

Closing prices	
ENOMDEC-09 *(Dec 09 Base)*	€32.80
ENOMPLDEC-09 Jan 10 *(Dec 09 Peak)*	€38.15

Time Buckets

Bucket Name	From Days Ahead	To Days Ahead
3M-Peak (u_{3M}^{P})	57	84
4M-Peak (u_{4M}^{P})	85	112
3M-Off-Peak (u_{3M}^{OP})	57	84
4M-Off-Peak (u_{4M}^{OP})	85	112

Firstly, smooth continuous forward curves are constructed for Base and Peak as discussed in Section 6.3.1. Second, daily Base and Peak prices for the delivery period are calculated from Equation 12c-d, and following Off-Peak prices from Equation 13b. These prices are exhibited in Table V, along with the daily Peak/Off-Peak volume in MWh which is simply 12/12 during weekdays and 0/24 during weekends.

Table V : Relevant Forward Prices / Quantity

	DA (i)	Forward Date	Day	Base Price $F_B(t,i,i+1)$	Peak Price $F_P(t,i,i+1)$	Off-Peak Price $F_{OP}(t,i,i+1)$	Peak Vol. $q_P(i)$	Off-Peak Vol. $q_{OP}(i)$		
Time Bucket: 3M-OffPeak and 3M-Off-Peak	61	01/12/2009	Tue	32.7439	37.9359	27.5518	12	12	$Ex(u^P_{3M}) = \Sigma F_P(t,i,i+1)\, q_P(i) = 8{,}224.3$	$Ex(u^{OP}_{3M}) = \Sigma F_{OP}(t,i,i+1)\, q_{OP}(i) = 10{,}630.7$
	62	02/12/2009	Wed	32.7317	37.9419	27.5216	12	12		
	63	03/12/2009	Thu	32.7209	37.9486	27.4932	12	12		
	64	04/12/2009	Fri	32.7114	37.9560	27.4668	12	12		
	65	05/12/2009	Sat	32.7033		32.7033		24		
	66	06/12/2009	Sun	32.6965		32.6965		24		
	67	07/12/2009	Mon	32.6913	37.9825	27.4001	12	12		
	68	08/12/2009	Tue	32.6876	37.9928	27.3823	12	12		
	69	09/12/2009	Wed	32.6854	38.0040	27.3668	12	12		
	70	10/12/2009	Thu	32.6848	38.0160	27.3536	12	12		
	71	11/12/2009	Fri	32.6859	38.0289	27.3430	12	12		
	72	12/12/2009	Sat	32.6888		32.6888		24		
	73	13/12/2009	Sun	32.6934		32.6934		24		
	74	14/12/2009	Mon	32.6999	38.0732	27.3265	12	12		
	75	15/12/2009	Tue	32.7082	38.0899	27.3265	12	12		
	76	16/12/2009	Wed	32.7185	38.1077	27.3294	12	12		
	77	17/12/2009	Thu	32.7309	38.1266	27.3352	12	12		
	78	18/12/2009	Fri	32.7453	38.1466	27.3440	12	12		
	79	19/12/2009	Sat	32.7619		32.7619		24		
	80	20/12/2009	Sun	32.7807		32.7807		24		
	81	21/12/2009	Mon	32.8017	38.2137	27.3898	12	12		
	82	22/12/2009	Tue	32.8251	38.2386	27.4117	12	12		
	83	23/12/2009	Wed	32.8510	38.2648	27.4371	12	12		
	84	24/12/2009	Thu	32.8793	38.2924	27.4662	12	12		
Time Bucket: 4M-OffPeak and 4M-Off-Peak	85	25/12/2009	Fri	32.9102	38.3214	27.4989	12	12	$Ex(u^P_{3M}) = \Sigma F_P(t,i,i+1) \cdot q_P(i) = 2{,}306.5$	$Ex(u^{OP}_{3M}) = \Sigma F_{OP}(t,i,i+1) \cdot q_{OP}(i) = 3{,}241.6$
	86	26/12/2009	Sat	32.9437		32.9437		24		
	87	27/12/2009	Sun	32.9799		32.9799		24		
	88	28/12/2009	Mon	33.0189	38.4175	27.6204	12	12		
	89	29/12/2009	Tue	33.0608	38.4527	27.6689	12	12		
	90	30/12/2009	Wed	33.1057	38.4896	27.7218	12	12		
	91	31/12/2009	Thu	33.1535	38.5281	27.7789	12	12		

The exposure of the four Time Buckets follows from Equation 15c, along with the relevant estimated VCV matrix forms the foundation for the 95%VaR calculations presented in Equation 18b. The actual numbers and a summary are presented in Figure 12.

Figure 12 : 95%VaR Example (ENOMDEC-09)

$$\hat{\sigma}_{P,h}^2 = \begin{bmatrix} w_{3M}^P & w_{4M}^P & w_{3M}^{OP} & w_{4M}^{OP} \end{bmatrix} \begin{bmatrix} \hat{\sigma}_{u_{3M}^P}^2 & \hat{\sigma}_{u_{4M}^P u_{3M}^P} & \hat{\sigma}_{u_{3M}^{OP} u_{3M}^P} & \hat{\sigma}_{u_{4M}^{OP} u_{3M}^P} \\ \hat{\sigma}_{u_{3M}^P u_{4M}^P} & \hat{\sigma}_{u_{4M}^P}^2 & \hat{\sigma}_{u_{3M}^{OP} u_{4M}^P} & \hat{\sigma}_{u_{4M}^{OP} u_{4M}^P} \\ \hat{\sigma}_{u_{3M}^P u_{3M}^{OP}} & \hat{\sigma}_{u_{4M}^P u_{3M}^{OP}} & \hat{\sigma}_{u_{3M}^{OP}}^2 & \hat{\sigma}_{u_{4M}^{OP} u_{3M}^{OP}} \\ \hat{\sigma}_{u_{3M}^P u_{4M}^{OP}} & \hat{\sigma}_{u_{4M}^P u_{4M}^{OP}} & \hat{\sigma}_{u_{3M}^{OP} u_{4M}^{OP}} & \hat{\sigma}_{u_{4M}^{OP}}^2 \end{bmatrix} \begin{bmatrix} w_{3M}^P \\ w_{4M}^P \\ w_{2M}^{OP} \\ w_{4M}^{OP} \end{bmatrix}$$

$$\hat{\sigma}_{P,h}^2 = \begin{bmatrix} 8{,}224.3 & 2{,}306.5 & 10{,}630.7 & 3{,}241.6 \end{bmatrix} \begin{bmatrix} 0.001584 & 0.000662 & -0.000738 & -0.000472 \\ 0.000662 & 0.000928 & -0.000168 & -0.000490 \\ -0.000738 & -0.000168 & 0.001225 & 0.000508 \\ -0.000472 & -0.000490 & 0.000508 & 0.000839 \end{bmatrix} \begin{bmatrix} 8{,}224.3 \\ 2{,}306.5 \\ 10{,}630.7 \\ 3{,}241.6 \end{bmatrix}$$

$$VaR_{t+h}(95\%) = z_\alpha \hat{\sigma}_P = 1.648 \cdot 386.91 = 636.41$$

Date *(t)*	01/10/2009
Horizon *(h)*	One trading day
Total Hours	744

Contract	Closing Price	Total Exposure	St. Dev	Annualized St.Dev(Return)[28]	95%VaR	Relative 95%VaR
ENOMDEC-09	32.8	24403.2	386.91	25.17%	636.41	-2.61%

7.4 Back Test Results

Having applied the methodology discussed in this thesis, 95%VaR and theoretical P&L has been calculated for 12 real-world Nord Pool portfolios over the sample period of year 2009. The reason for choosing 95%VaR opposed to 99% as the Basel legislations suggest is that the sample is rather small (249 trading days), thus increases the possibility for having zero actual exceptions if assuming the model is not faulty. In such case the Kupiec Test will not yield any test statistics.

Kupiec Test with 95% *(k)* confidence has been performed on all the portfolios. In order of keeping the holder of the portfolios anonymous, they will be referred as Portfolio A-L. Keeping further discretion and yielding a better picture of VaR against returns, the results will be presented in 95%VaR actual exceptions, relative VaR and theoretical return[29].

---

[28] By 252 trading days pr annum

[29] Relative VaR = - (VaR / Absolute Total Exposure)

Theoretical Return = Theoretical P&L / Absolute Total Exposure

The results have been most encouraging due to the fact that the null hypothesis has not been rejected for any of the portfolios. Thus this thesis cannot yield any statistical evidence that the VaR model presented here is faulty based on the Kupiec Test.

Table VI exhibits the actual exceptions and Kupiec Test statistics for the for portfolios. The actual exception ratio corresponds remarkably well with the expected ratio (5%) for virtually all portfolios, apart from Portfolio A and J. The actual exception ratio is for Portfolio A somewhat higher than expected (6.82%) while Portfolio J is lower (2.81%). However, these deviations are not significantly sufficient to reject the null hypothesis at a 95% *(k)* confidence.

Table VI : Back Test Results Portfolio A-L

	N	*e*	$\hat{p}$	*p**	*LR(e,p*)*	*CV(k=95%)*	**Conclusion**
Portfolio A	249	17	0.0682	0.05	1.5788	3.841459	Cannot Reject H_0
Portfolio B	249	12	0.0481	0.05	0.0173	3.841459	Cannot Reject H_0
Portfolio C	249	11	0.0441	0.05	0.1847	3.841459	Cannot Reject H_0
Portfolio D	249	11	0.0441	0.05	0.1847	3.841459	Cannot Reject H_0
Portfolio E	249	12	0.0481	0.05	0.0173	3.841459	Cannot Reject H_0
Portfolio F	249	12	0.0481	0.05	0.0173	3.841459	Cannot Reject H_0
Portfolio G	249	15	0.0602	0.05	0.5175	3.841459	Cannot Reject H_0
Portfolio H	249	13	0.0522	0.05	0.0252	3.841459	Cannot Reject H_0
Portfolio I	249	14	0.0562	0.05	0.1956	3.841459	Cannot Reject H_0
Portfolio J	249	7	0.0281	0.05	2.9632	3.841459	Cannot Reject H_0
Portfolio K	249	11	0.0442	0.05	0.1847	3.841459	Cannot Reject H_0
Portfolio L	249	11	0.0442	0.05	0.1847	3.841459	Cannot Reject H_0

Further details of portfolios can be viewed in Figure 13 and Figure 14, which illustrates the relative returns (red dots) and the relative 95%VaR (black line) over the sample period. The relative 95%VaR of the majority of the portfolios is typically around -(2-5)%. The exception is Portfolio A, J and K, which for shorter time periods exhibits substantial relative 95%VaR. The explanation is that in these time periods, the portfolios shift their exposure extensively against products in the very short end of the curve. Following, the theoretical returns in these periods are also extensive (especially concerning Portfolio J and K), however the drastically increase in relative 95%VaR results in few breaches in these periods.

Figure 13 : Relative 95%VaR (Black Line) and Theoretical Return (Red Dots) Portfolios A-J

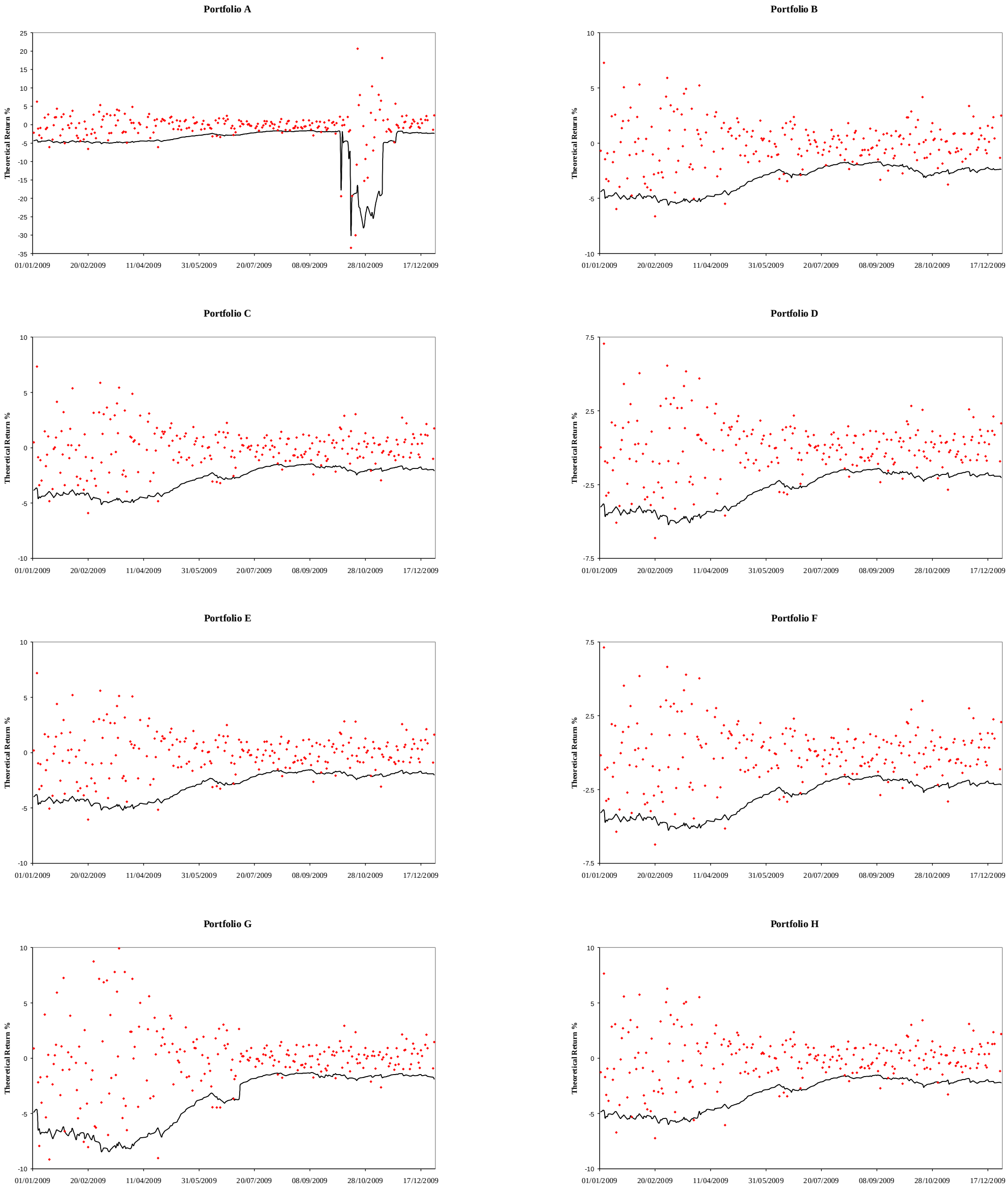
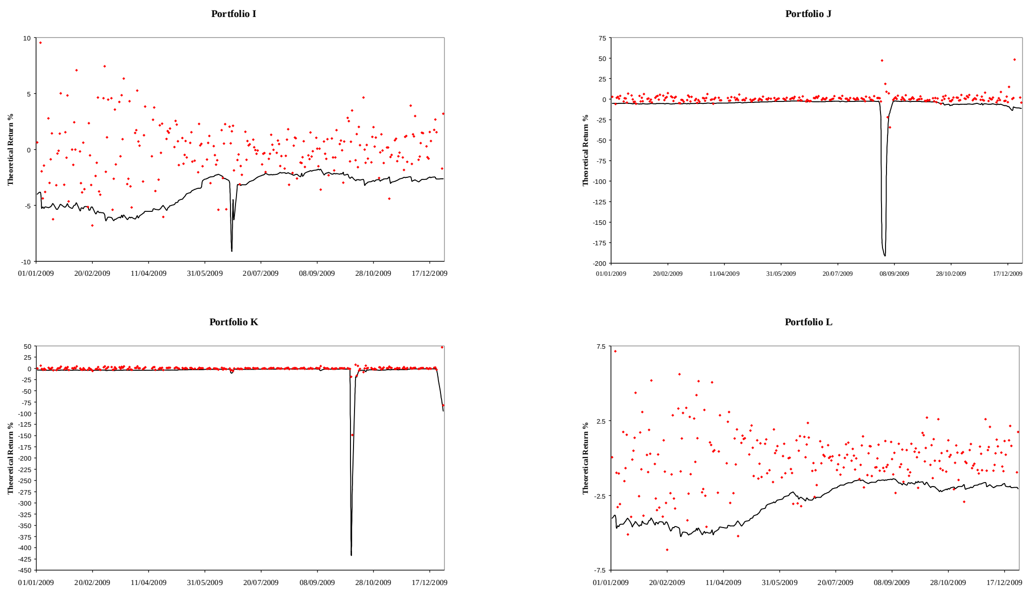


Figure 14 : Relative 95%VaR (Black Line) and Theoretical Return (Red Dots) Portfolios I-L



8 Conclusions

This thesis has investigated the application of an analytical VaR model for Nord Pool exchange traded electricity derivatives. Although building on the RiskMetrics approach the model has been given some adjustments due to the special characteristics of the electricity market.

Firstly opposed to linear interpolation for constructing constant maturity contracts in order to calculate log returns, this thesis advocate arbitrage free pricing by smoothed forward curves to price constant maturity intervals (Time Buckets). Also, the Time Buckets are split between Peak and Off-Peak prices. However, the VCV coefficients from the log returns are estimated in the same manner as RiskMetrics, utilizing the time-varying and autocorrelated estimator EWMA with the suggested decay factor.

Second, the exposure from the Nord Pool electricity derivatives are not performed by linear two-way splitting as RiskMetrics suggests for commodity derivates. Instead they are mapped into Time Buckets by daily Peak and Off-Peak prices and delta adjusted volumes over the delivery period.

The model has been evaluated by back-testing 12 real-world portfolios, where the actual numbers of exceptions are tested against the expected exceptions. The test statistics of the Kupiec test has not yielded rejection of the null hypothesis that the actual exception ratio is equal to the expected exception ratio for any of the portfolios. Thus, these tests cannot present any statistical evidence that the model is inadequate.

Finally, emphasizing on the fact that analytical VaR models are closely related to Markowitz’ portfolio optimization model; VaR is virtually the same as the estimated variance of portfolio returns. However, the failure of rejecting the null hypothesis does *not* conclude that the model is the perfect predictor of future variance. Instead, based upon the results from back tests, one cannot reject to possibility that the model is able keep projected losses in line with expected losses, thus in this perspective the model does not require any further calibration.

Although, scope of this thesis is limited towards exchange traded Nord Pool electricity derivatives, the exact same methodology could in theory be applied for OTC contracts as well as other electricity markets. Note if MtM were calculated on an hourly granularity, this model would not support contracts with an intraday load profile other than Peak or Off-Peak. Reason being that in such case the MtM value could differ (violation of RiskMetrics CF mapping principles).

In suggestion of further work, firstly concerning non-linear derivates it is possible that the delta valuation is too *inaccurate* due to the larger price changes which are common for electricity derivatives. Thus, the model could be improved by delta-gamma valuation. Also, it would have been most interesting to perform a similar back test as presented in this thesis, but including several markets as well as more “complex” contracts.

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10 Appendixes

10.1 Code Snippets

Matrix Calculus

```
using System;

namespace AVAR.Model.Domain.Calculus
{
    class MatrixCalc
    {
        public static double[,] MatrixMult(double[,] matrixA, double[,] matrixB)
        {
            if (matrixA.GetLength(1) != matrixB.GetLength(0))
            {
                throw new ArgumentException("Matrix A and B have invalid dimensions to perform matrix multiplication");
            }

            double[,] resultMatrix = new double[matrixA.GetLength(0), matrixB.GetLength(1)];
            for (int i = 0; i < resultMatrix.GetLength(0); i++)
            {
                for (int j = 0; j < resultMatrix.GetLength(1); j++)
                {
                    for (int z = 0; z < matrixA.GetLength(1); z++)
                    {
                        resultMatrix[i, j] +=  matrixA[i, z] * matrixB[z, j];
                    }
                }
            }

            return resultMatrix;

        }

        public static double[,] Transpose(double[,] matrix)
        {
            double[,] resultMatrix = new double[matrix.GetLength(1), matrix.GetLength(0)];
            for (int i = 0; i < resultMatrix.GetLength(0); i++)
            {
                for (int j = 0; j < resultMatrix.GetLength(1); j++)
                {
                    resultMatrix[i, j] = matrix[j, i];
                }
            }

            return resultMatrix;

        }

        public static double[,] ScalarMult(double[,] matrix, double scalar)
        {
            double[,] resultMatrix = new double[matrix.GetLength(0), matrix.GetLength(1)];
            {
                for (int i = 0; i < resultMatrix.GetLength(0); i++)
                {
                    for (int j = 0; j < resultMatrix.GetLength(1); j++)
                    {
                        resultMatrix[i, j] = matrix[i, j] * scalar;
                    }
                }
            }
            return resultMatrix;
        }

```

```
        public static double[,] MatrixAdd(double[,] matrixA, double[,] matrixB)
        {
            if(matrixA.GetLength(0)!= matrixB.GetLength(0)|matrixA.GetLength(1) != matrixB.GetLength(1))
            {
                throw new ArgumentException("Matrix A and B must have equal dimensions");
            }

            double[,] resultMatrix = new double[matrixA.GetLength(0), matrixA.GetLength(1)];
            {
                for (int i = 0; i < resultMatrix.GetLength(0); i++)
                {
                    for (int j = 0; j < resultMatrix.GetLength(1); j++)
                    {
                        resultMatrix[i, j] = matrixA[i, j] + matrixB[i, j];
                    }
                }
            }
            return resultMatrix;
        }
    }
}
```

EWMA Engine

```
using System;

namespace AVAR.Model.Domain.VCV
{
    class EwmaEngine

        /// <summary>
        /// Runs the EWMA VCV Analysis, will return output in a DataTable of Correlation and Volatility Coefficients
        /// which can be BulkInserted into a Database
        /// </summary>
        /// <param name="corrVolAnalysis">The Analysis, stores parmaterers such as Decay Factor (Lambda)
        /// and Tollerance level</param>
        /// <param name="logRetMatrixOutPut">The Log Return Matrix Object</param>
        public void RunEwma(CorrVolAnalysis corrVolAnalysis,LogRetMatrixOutPut logRetMatrixOutPut)
        {
            if(corrVolAnalysis.Method!=CorrVolMethod.EWMA)
                throw new ArgumentException("CorrVolAnalysis Must BE EWMA");

            if(corrVolAnalysis.TolleranceLevel==null)
                throw new ArgumentException("CorrVolAnalysis does Not have Tollerance Level");

            decimal tolleranceLevel =(decimal) corrVolAnalysis.TolleranceLevel;
            decimal lambda = corrVolAnalysis.MethodVariable;
            decimal oneMinusLambda = 1m - lambda;
            int analysisID= corrVolAnalysis.AnalysisID;

            int matLength = logRetMatrixOutPut.LogRetMat.GetLength(1);
            DateTime[] reportDateVector = logRetMatrixOutPut.ReportDateVector;
            double[,] logRetMatrix = logRetMatrixOutPut.LogRetMat;
            int[,] marketBucketArray = logRetMatrixOutPut.MarketBucketArray;

            //k is the first observation which will be stored
            int k = (int)Math.Floor((Math.Log((double)tolleranceLevel) / Math.Log((double)lambda)));

            double[,] prevEwmaMatrix = new double[matLength, matLength];

            double[,] inverseVolDiagonalMatrix= new double[matLength, matLength];

            for (int dateIndex = 0; dateIndex < reportDateVector.GetLength(0); dateIndex++)
            {
                //Copies the The Log Return Vector for the previous report date
                double[,] logRetVector = CopyLogRetVectorRow(logRetMatrix,dateIndex );

                //Calcualtes the EWMA VCV Matrix
               double[,] ewmaMatrix = MatrixCalc.MatrixAdd(MatrixCalc.ScalarMult(MatrixCalc.MatrixMult(logRetVector,
                                    MatrixCalc.Transpose(logRetVector)),(double) oneMinusLambda),
                                    MatrixCalc.ScalarMult(prevEwmaMatrix,(double) lambda));

                //Extracts the inverse volatility trace in order to store the coefficients as correlations and volatilities rather than VCV
                for (int i = 0; i < matLength; i++)
                {
                    for (int j = 0; j < matLength; j++)
                    {
                        if (i == j)
                            inverseVolDiagonalMatrix [i, j] = 1/Math.Pow(ewmaMatrix[i, j],0.5);
                        else
                            inverseVolDiagonalMatrix [i, j] = 0;
                    }
                }

                double[,] ewmaCorrelationMatrix = MatrixCalc.MatrixMult(inverseVolDiagonalMatrix,
                                                 MatrixCalc.MatrixMult(ewmaMatrix,
                                                            inverseVolDiagonalMatrix));

                //Stores the Coefficients if the Date Index is greater than the Mininium numbers of observations requiered by
               //tollerance level
                if (dateIndex > k)
                {
```

```
                DateTime reportDate=reportDateVector[dateIndex];

                StoreCoefficients(matLength,reportDate, inverseVolDiagonalMatrix, analysisID, marketBucketArray,
                ewmaCorrelationMatrix);
            }

            prevEwmaMatrix = ewmaMatrix;
        }
    }
}
```

VaR Engine

```
using System;

namespace AVAR.Model.Domain.VaR
{
    class ValueAtRiskEngine
    {
        public static double CalculateSigma(VcvMatrix vcvMatrix,ValueExposureVector valueExposureVector)
        {
            double[,] matrix = vcvMatrix.Matrix;
            double[,] exposureVector = valueExposureVector.Vector;

            if(exposureVector.GetLength(0)!=vcvMatrix.Matrix.GetLength(0)
            |exposureVector.GetLength(0)!=vcvMatrix.Matrix.GetLength(1))
                throw new ArgumentException("VCV Matrix and Exposure Vectors Dimensions are Mismatch");

            if(exposureVector.GetLength(1)!=1)
                throw new ArgumentException("Invalid dimensions for ExposureVector");


            ////Returns the Variance of the portfolio by the following matrix operation
            ////   _______________________________   |                  |    |Exp.    |
            ////  |  Exposure Vector Transposed   | x |       | VCV Matrix  | x  |Vector |
            ////                                      |                  |    |        |

            double[,] variance = MatrixCalc.MatrixMult(MatrixCalc.Transpose(exposureVector),
                                    MatrixCalc.MatrixMult(matrix, exposureVector));

            //Calculates the sigma
            double sigma = Math.Pow(variance[0, 0], 0.5);

            return sigma;

        }

    }
```