The Norwegian School of Economics
and Business Administration
Bergen, Fall 2010

Capital structure decisions

An empirical study of structured credit models

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Master thesis in Financial Economics

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NORGES HANDELSHØYSKOLE

1. Preface

This thesis is a part of the Master program at The Norwegian School of Economics and Business Administration (NHH). Its purpose being to use to use the knowledge gained and the methods learned at the Master level. My goal is to make this thesis understandable for academics and finance professionals with the same prerequisites as myself. I believe a key course to follow the content easily is Risk Management at NHH, or equivalent. I also hope that this thesis will be of help for them who read it when it comes to take capital structure and financing decisions.

I also want to thank Kristian Miltersen, now a professor at Handelshøyskolen i København, for helping and guiding me with the framework and the models used here. His advices have been useful for accomplishing the empirical testing.

A thank you to Robert Goldstein, Nengjiu Ju and Hayne Leland too, for being helpful and contribute with explanations to their numerical examples in their article.

And finally, but most of all, I would like to thank Jøril Mæland! She became my supervisor for this thesis at the most critical point, and has provided useful feedback and guidance for the work presented here. Her quick response and availability is deeply appreciated.

2. Abstract

There are several issues with adapting dynamic structured credit models to be applicable for values observable in reality. To use the theories, and with that the models developed, to find optimal capital structures is challenging in many ways. No conclusion can be drawn from this thesis whether it is the theories that are imperfect, or companies not optimally financed. Some of the companies refinanced in a manner which could be in line with the dynamic optimal, while others lacked such indications. To find the optimal leverage ratio, by testing for the amount of debt optimal to issue in a restructuring, failed with respect to what is realistic. Possible weaknesses with existing models could be that they do not allow for the high volatility in EBIT which could be observed empirically. Work is still to be made, to use structured credit models as a decision tool for capital structure decisions. But the theories behind them could be of help to the people making the financing decisions.

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3. Introduction

I have chosen to examine whether companies are making choices to hold an optimal level of debt to maximize their stakeholders' wealth, by empirically test them with a dynamic structured credit model. Further would I like to find the amount of debt the model suggests, if I would use a structured credit model as a tool in a financing decision. Another interesting observation would be if the debt level suggested by the model would be realistic, and in line with debt and interest ratios observed empirically.

To really understand what affects the value of debt, equity and other claims to a company's cash flow, fundamentals of prevailing capital structure views, and what separates them from former theories is important. I would therefore look into some capital structure history, as the model and tests here could be mistaken for the traditional capital structure view.

4. Some capital-structure history.

There have been several ways of considering choices of capital-structures through time. Before Modigliani and Miller (Miller and Modigliani, 1958) described their theory in 1958, the prevailing view was the existence of an optimal mix of debt and equity that maximized the total value of a firm, the Traditionalists (Brealey, Myers and Allen, 2008). This view was also under the assumption of perfect capital markets. This being the case, the Return on Assets (ROA) of a company would have a minimum and the value a maximum at a certain rate of debt. Their argument was that by changing the way a company was financed, the return on the same assets would be different, and at a certain mix optimal. The basis of the assumptions was the belief that debt remained more or less risk-less by low ratios, and so did the equity. By higher debt-ratios the risk of both debt and equity increased. I want to emphasize the traditional view prevailing before Modigliani and Miller (MM) as well, since this thesis will try to find exactly what the traditionalists claimed, videlicet the existence of an optimal capital structure. But in contrast to the traditional view, my approach is based on MM's findings.

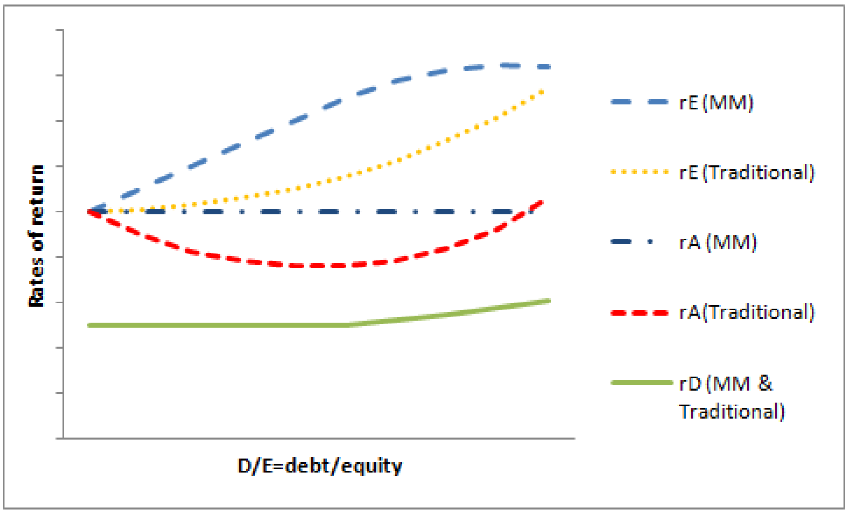

Figure 1 – (Brealey, Myers and Allen, 2008)Shows the view of Miller and Modigliani in contrast to the traditional opinion with return under different leverage ratios. The red line shows the return on assets under the traditional view. The traditionalists believed borrowing increased the return on equity at a lower ratio in the beginning, compared to MM, but increased rapidly from a certain leverage ratio. This resulted in the smiling return on assets line. This had a minimum point where the value of the company would have a maximum (lowest discount rate on the company's total cash flow). In contrast MM believed the return on equity increased proportionally with the leverage ratio in the beginning, but at a certain point (because of equities limited liability) the increased return on debt exactly offset the increase in equity return. Hence, the return on assets was constant with MM's theory, and so was the total value of the company (debt plus equity).

The traditionalistic view is in contrast to MM's findings. MM said the total value of a company was unaffected by how it was financed. They assumed perfect capital markets, zero taxes and zero bankruptcy-costs. I will further explain how the same applies even though taxes, bankruptcy-costs and restructuring-costs are present. I will stress that it is the total-value which is unaffected. Taxes and bankruptcy-costs do affect value of equity and debt.

The novelty of MM's findings was that they claimed the total value was independent of the capital structure. The value of a firm is the present value of all its future cash-flows, discounted at a risk-adjusted rate. This value has to be the same no matter how the assets are financed. The assets are the same, and so is the potential output.

Consider an all equity firm in a zero tax world. The value of that company would be the total present value of all future cash-flows. By adding debt to the company, some of the cash-flows would be streamed to the creditors rather than equity-holders. It would be the same cash-flow, generated by the same assets, but now the creditors have a claim to a part of it. If slicing an identical cash-flow into different streams would add to the total value, would mean managers optimizing capital-structures was alchemists. In other words must the value come from somewhere.

The “somewhere” is consistent with MM’s findings. As long as there is an MM-world, the value of equity will be exactly offset by the amount of debt added, and handed out as dividend. The extra value that could be observed virtually comes from the tax-shield effect of debt. In Norway the effective tax-rate on equity is 48.16%, and 28% on debt (OECD). For equity its 28% tax on the pre-tax earnings to the company, and when paid out as dividend there is 28% tax too, or stocks are sold with profit, reflecting the earnings. Since the tax is a profit tax, interests are tax deductable. By taking on debt, and pay it out to equity-holders, investors get a shield towards taxes, by the amount of the tax-spread between interests and capital taxation. The value added to equity is from the part of future cash-flow claims previously held by the government. By taking on debt and hand it out to equity-holders, some of the value are taken from the government and given to equity-holders. If EBIT=Interests, the total taxes paid from a company’s operations would be 28%, the tax on interests. This is consistent with MM’s findings. Values are neither created nor lost by mixing the financing between equity and debt. The values are only transferred between different claimants.

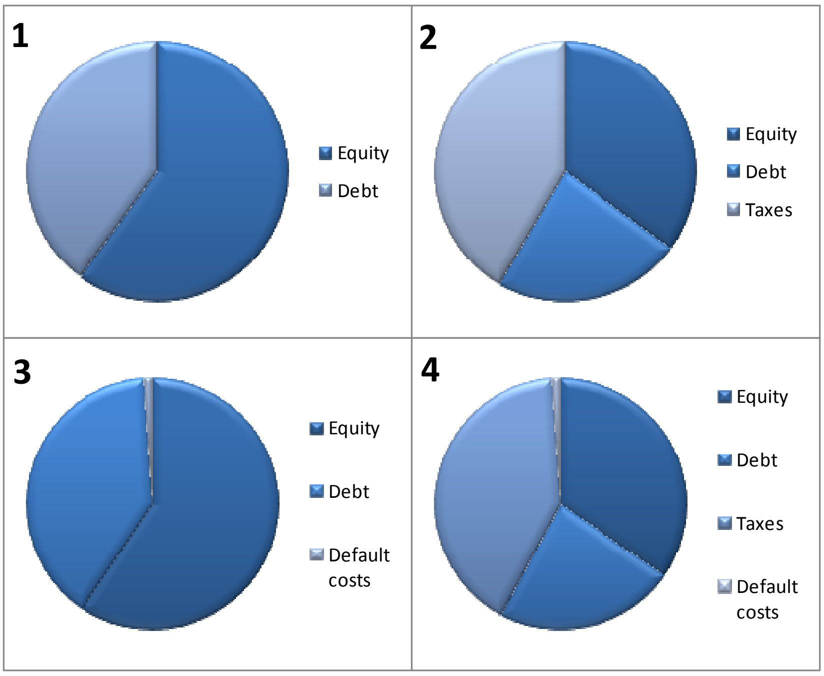

Figure 2: Shows the distributions of the same firm, under the four different market assumptions. 1 – The zero-tax zero bankruptcy costs. 2 – With taxes, but zero bankruptcy-costs. 3 – With default costs, but zero tax. 4 – With both taxes and default-costs. The total present value of the firm is the same but the distribution of the company's value among its claimants is different. This is for illustration purpose only, and not from any real example later in this thesis. In 1 the capital-structure is irrelevant. In 2 the optimal would be to have debt only, or as much debt as possible. In 3 all equity, or very low debt ratios would be preferred. In 4 there is a trade-off between taxes and increased interests, as distress-costs increase.

This takes us further to the existence of bankruptcy-costs. If there weren't any costs connected to bankruptcy, the optimal level of debt would be as much as the companies' cash-flow allowed. In Norway the direct costs of bankruptcy is 1% of the estate and 7% in the US (World Bank, Doing Business). The average recovery-rate on claims to the estate is 89% and 76.7% respectively. In other words, there is risk in debt and a loss of value connected to a bankruptcy for the pre-default claimants. By adding debt the risk of default increases. When a company levers up, it also puts on liabilities it has to service. The interest-rate to be paid on debt is therefore an increasing function of the debt-rate and the loss in the event of default. A real-estate company would cause less harm to creditors than a firm with strictly intangible assets. The risk of equity therefore starts to increase by adding debt. By a certain debt-ratio the debt also starts to take on some risk. This risk is a transfer from the equity, which by this ratio have a declining rate of increase in risk and return. By this debt-level the risk of bankruptcy starts affecting the debt since equity-holders have limited liability. Hence, this risk doesn't add strictly to the equity. In this framework the ROA is constant at all debt ratios, in contrast to the Traditionalists, and the total value of the firm is, given all other factors, constant for all debt-levels.

The required fundamentals to understand the trade-off theory should now be explained. The optimal ratio of debt is that which maximizes the existing equity-holders value. This is a tradeoff between the increased cost of debt and the gain obtained by the tax-shield. A certain leverage-ratio would maximize the existing equity-holders wealth. A company with strictly debt and equity financing would have five claimants. The government through taxes, equity and debt holders, bankruptcy claimants and the claimants of restructuring costs involved in changing the capital-structure. The sum of all these claims adds up to the total value of the company. At every restructuring point, it takes place a transfer of claims. The cash-flow is the same, and so is the present total value. Only the composition of the claims is changed.

5. Capital Structure Models

The pioneers in capital structure model theories were Kane, Marcus and McDonald, who introduced their theories already in 1982 and 1985 (Goldstein, Leland and Ju, 2001). These theories were further developed by Fisher, Heinkel and Zechner in 1989 (Goldstein, Leland and Ju, 2001). Their theories and models focused strictly on corporate finance theories which could explain managements' capital structure choices, but lacked the focus on credit risk and costs related to bankruptcy and restructuring. These links were picked up in Leland's article (Leland, 1994) from 1994, and connected.

A thorough problem with the early models was the lack of a consistent use of parameter variables, which was missing. The values of interest, hence those which the model was trying to solve or figure out, were derived from the properties of an unlevered firm value. With this approach the models tested for optimal debt ratios, credit spreads et cetera by first un-levering the characteristics of a company, and then optimized matters relating debt side by side with the unlevered firm. In other words did they allow the existence of two conditions that in reality could not exist simultaneously.

It may also be questioned if the state of the properties of an unlevered firm actually can exist. Even corporations without debt have the option to issue debt in the future, and historical prices could account for this potential. Goldstein et al (2001) uses Microsoft as an example of an un-levered firm, which had guided that it would not put on any debt in the future. The company is also of the size that a buy-out or the purchase of a controlling majority for the purpose of instructing management seems unlikely. They argue that such companies may be priced as unlevered companies. As recent as May 11th 2009 Microsoft announced that they had filed for a substantial debt-issue (Techflash, 2009) (6 billion USD). This strengthens the argument from Ammann and Genser's paper from 2004 (Ammana and Genser, 2004). They conclude that dynamic capital structure models of unlevered corporations cannot be used for model-testing.

This brings us to the model this thesis will be based on. By using EBIT as the state variable, the issues of leverage and financing vanish. EBIT (**E**arnings **B**efore **I**nterests and **T**ax) is a variable independent of a company's financing decisions, and also exist simultaneously no matter how a company has mixed its debt and equity. In other words it consists of all the cash-flows that are transferred to the company's different stakeholders.

6. The Structured Credit Models

As mentioned, Leland made a link between the established corporate capital models and the structured credit models in his paper "Debt Value, Bond Covenants, and Optimal Capital Structure" from 1994. This was not a total new way of thinking, but Leland structured the theories in a new uniform framework.

As a basis, the models have EBIT as its state variable. Assume that over time depreciation and amortization are equivalent with required capital expenditures to stay in business, which is a reasonable assumption. Over time EBIT is therefore the income unit financing the company's claimants.

Further, the total value of the company is the present value of all future EBITs. To put this in a framework, Leland assumed that EBIT was following a standard Brownian motion (he does not specify whether it is geometric or arithmetic). With this the total value of the company, which is derived from the cash-flow, would also follow a Brownian motion. In the traditional framework this characteristic was applied to the unlevered value of equity.

The optimal mix of equity and debt is the mix that maximizes the value of equity, and with that the value of debt and equity (claims to the cash flow is transferred from the government). Here Leland had a static approach. By a static approach we mean that a company can issue debt once, and then hold the issued nominal amount thereafter. The static approach also returns an optimal solution on closed form. But assuming that you cannot put on debt in the future also motivates for higher debt-levels today, to maximize the benefits of the tax-shield. The model therefore yields a higher debt-level than what is observed in reality.

To compensate for this, the model must account for potential future debt-issues. By accounting for future debt, the level it needs to take on today can be reduced. Management does not need to overshoot the debt amount, but rather keep the debt-ratio in a certain range, and when it reaches a certain ratio, refinance with a greater amount of debt.

This is the model that will be presented in this thesis. The model was presented by Goldstein, Ju and Leland in 2001, and is a dynamic version of Leland's model from 94. This model does not have a closed form solution, so to find the optimal ratio a computer tool is required to find the numerical solution.

The target with the dynamical approach is to find a debt level which triggers restructuring of the debt. As long as the company is in the range between a high and a low debt ratio, management does not make any restructuring choices. Once a low ratio is reached (Debt to Total value), the company restructures by issuing new debt, and restores the initial level. This will assure the company always is in an optimal debt-range, regarding maximizing its shareholders value. It trades off some risk of being too highly levered by holding the option to issue debt in the future, as well as holding the benefits of the tax-shield.

This takes us over to the next part, the description of the model used.

7. The Framework of the Model

I will start by describing the mathematics of the model used. This is a short version (or more a summary) of Goldstein et al's findings from their article in 2001. Then I will explain how I have modeled the equations used, so they were applicable for the collected data. Some changes have been made, making only observable parameters needed as input.

All values are present value of future cash claims. As described above, a company has five different claimants, the government, creditors, equity holders, restructuring costs to legal and financial advisers and bankruptcy costs.

I will start with introducing the static model which only allow for one debt issue, and from that derive the dynamic model, as done by Goldstein et al. This will give a more intuitive understanding of how the model works, and the more complicated matters could be explained when they arise.

We start with a simple model of firm dynamics, which is supported by the rational expectations general equilibrium framework. Imagine the payout flow of a pure exchange economy:

$$\frac{d\delta}{\delta} = \mu_P dt + \sigma dz$$

where μ_P and σ are constants. It is well known that any asset can be valued by discounting its expected future cash flow, generated under the risk neutral measure. The value of the claim will be:

$$V(t) = E_t^Q\left(\int_t^\infty ds\delta_s e^{-rs}\right) = \frac{\delta_t}{r - \mu}$$

where $\mu = (\mu_p - \theta\sigma)$ is the risk neutral drift. Since r and μ are constants, both V and δ share the same dynamics. $\frac{dV}{V} = \mu dt + \sigma dz^Q$ which gives $\frac{dV + \delta dt}{V} = r dt + \sigma dz^Q$.

The risk neutral expected return will be the risk free rate, as for an equivalent martingale measure, so every claimant receives a fair expected return for the exposed risk. Further, Goldstein et al assume a simple tax structure, which also I have done. Assumed is a structure which includes tax on effective dividends (τ_d), tax on interests (τ_i) and tax on corporate profits (τ_c), with full loss offset provision.

With this framework there is no such thing as an all equity firm. Even with zero debt, the government has a claim to the taxed revenue. The cash flow to the company is streamed to the different claimants, and each stream is then valued. The sum of all claims adds up to the total value of the firm. For a company with a management that does not take on any debt, the value is: $E = \left(1 - \tau_{eff}\right)V_0$ for equity and $G = \tau_{eff}V_0$ for the government and the effective tax rate is given by $\left(1 - \tau_{eff}\right) = (1 - \tau_c)(1 - \tau_d)$ for equity investors.

If we want to maximize the equity-holders wealth by issuing debt once (the static approach), the interesting is the amount of debt to issue. Assume the company can service debt up to the level where the coupon is C, and will choose bankruptcy if the debt level reaches V_B. Then we define the cost of a potential bankruptcy to be αV_B if the company reaches V_B. We now have four claims to the company, which each has to satisfy the following partial differential equation (PDE), P being the payout flow,

$$\mu V F_V + \frac{\sigma^2}{2} V^2 F_{VV} + F_t + P = rF$$

The debt is assumed perpetual, and by that the claim to the principal is independent of time. The rationale for the perpetual assumption is that they want to keep the nominal debt issued, which at the time of the issue should be optimal. Perpetual debt instruments exist, but a more common rationale would be that at maturity the amount is refinanced. We may therefore write:

$$0 = \mu V F_V + \frac{\sigma^2}{2} V^2 F_{VV} - rF$$

The general solution is $F_{GS} = A_1 V^{-y} + A_2 V^{-x}$ where $x = \frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right]$

and

$y = \frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2}\right) - \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right]$. A_1 and A_2 are constants, determined by boundary conditions. The general solution does not account for inter-temporal solutions. These must be derived from the particular solutions. We define $p_B(V)$ as the present value of a claim paying 1 contingent of firm value if V_B is reached. This claim does not result in any immediate payout, hence the claim must be on the form $p_B(V) = A_1V^{-y} + A_2V^{-x}$. Accounting for the boundary conditions: $\lim_{V\to\infty} p_B(V) = 0$, $\lim_{V\to V_B} p_B(V) = 1$ we find:

$$p_B(V) = (\frac{V}{V_B})^{-x}$$

When the firm is solvent, the government, debt holders and equity holders share the payout from the company. If all claims were held by one owner, he would be entitled to the entire payout, as long as the value stays above V_B. This value is the solvent value of the company, and I will refer to it as V_{solv}. Values adequately over V_B approaches the total firm value V, meaning $V_{solv} = V - V_Bp_B(V)$. Each single claim can be found in the same way. The interest claim will be

$$V_{int} = \frac{C}{r}[1 - p_B(V)]$$ [1]

The claims to the solvent company will be (remember the debt is perpetual):

Equity: $E_{solv}(V) = \left(1 - \tau_{eff}\right)(V_{solv} - V_{int})$

Government: $G_{solv}(V) = \tau_{eff}(V_{solv} - V_{int}) + \tau_iV_{int}$

Debt: $D_{solv}(V) = (1 - \tau_i)V_{int}$

The sum of these adds up to V_{solv}.

We assume the creditors are protected by certain covenants, e.g. sale of assets and change of control covenants. Due to the protective covenants, none of the EBIT generating assets are allowed to be sold. If the payout falls below promised interest, the equity holders are entitled to pay in more cash. Still, at a certain point they will rather choose to default, than throw new

[1] Since the debt is perpetual, a claim to the principal isn't included.

money after a firm that does not contribute with the required return. The equity claim will disappear, and the company becomes divided between the government[2], debt holders and bankruptcy costs.

The present value of the default claim can be written: $V_{def}(V) = V_B p_B(V)$

The sum $V_{def} + V_{solv}$ equals the total claim to EBIT, V. Hence, values are neither created nor lost as described introductorily. The value is only redistributed among its claimants, exactly as shown in the pies (figure 2). We define α as the default cost ratio. The claims to the bankrupt estate are:

Debt: $D_{def}(V) = (1-\alpha)(1-\tau_{eff})V_{def}(V)$

Government: $G_{def} = (1-\alpha)\tau_{eff}V_{def}(V)$

Bankruptcy Costs: $BC_{def}(V) = \alpha V_{def}(V)$

We must also account for the cost of restructuring, which is connected to issue debt. The amount is deducted from the proceeds distributed to equity. This amount is a portion (given by a ratio) of the initial value of the debt issued, give by: $RC(V_0) = q\left[D_{solv}(V_0) + D_{def}(V_0)\right]$.

To maximize the wealth of the equity holders, management must choose the coupon level C, and the level of where bankruptcy is preferred over continued operations, V_B. V_B is found by invoking the smooth-pasting condition (Dixit, 1991 and Dumas, 1991).

$$\frac{\partial E}{\partial V}_{V=V_B} = 0$$

Solving, we get:

$$V_B^* = \lambda \frac{C^*}{r}$$

where:

[2] Assuming that the EBIT generating machinery are sold off to a competing company, which continues to operate. The government will then have a claim to the cash-flow generated by the new company.

$$\lambda \equiv (\frac{x}{x+1})$$

As mentioned, management must find the optimal coupon, C^*. If we assume the payout ratio is independent of C (which empirical studies oppose[3], but theory keeps holding to), the optimal coupon can be obtained in closed form. Let's define q as the restructuring cost ratio. To maximize the wealth of the shareholders, management must find the solution of:

$$\max_{C}\{(1-q)D[V_0, C, V_B(C)] + E[V_0, C, V_B(C)]\}$$

This level ensures that the equity receives a fair value for their claim to the cash-flow (claim of the interests) sold, less the portion of cost for restructuring. By differentiating the max function above, with respect to C, and set the equation to zero, we find the optimal coupon to be:

$$C^* = V_0\left(\frac{r}{\lambda}\right)\left[\left(\frac{1}{1+x}\right)\left(\frac{A}{A+B}\right)\right]^{\frac{1}{x}}$$

where:

$$A = (1-q)(1-\tau_i) - (1-\tau_{eff})$$

$$B = \lambda(1-\tau_{eff})[1-(1-q)(1-\alpha)]$$

To be a tax advantage with debt, A must be greater than zero. If q=0, it will require $\tau_{eff} > \tau_i$. If q>0, the effective tax rate must be sufficient to cover both taxes paid by the debt holders and the cost of restructuring, to be a tax-benefit.

We now have the necessary framework to find the optimal level of debt, independent of how the company is currently capitalized. By putting the results from above together, we find the value of the equity claim just before the debt-issue to be:

$$E(V_{0-}) = \{(1-q)D[V_0, C^*, V_B(C^*)] + E[V_0, C^*, V_B(C^*)]\}$$

$$= V_0\left[\left(1-\tau_{eff}\right) + AQ\right]$$

[3] Goldstein et al mention this in the article, but still argues for independence to be rational

where

$$Q \equiv \left[\left(\frac{A}{A+B}\right)\left(\frac{x}{1+x}\right)\right]^{\frac{1}{x}}$$

and $E(V_{0-})$ is the equation to maximize. There is a transfer of value to present equity-owners from the government, of which some of the values go to restructuring-costs, RC.

The percentage increase of the equity-value is

$$\%\, increase = 100 * \frac{AQ}{(1-\tau_{ett})}$$

which captures the tax-advantage of debt in this static model.

I will now go on with the framework for the dynamical model, which also is the one I have been using in the empirical tests. The basics are more or less the same as the static. What's new is to open the model for the option of issuing further debt in the future.

Management may decide to operate for years before taking on debt, or chooses to raise an optimal level of debt. In a start up phase for instance, debt might not be the preferred capital. But as the cash flows get more steady or necessary security to get reasonable terms on debt are acquired, debt instruments become more attractive. If a company commits to optimize its debt-level at time T, the value of the future tax-shield must still be encountered in the present value. As the company waits for T, an acquisition of the company from a competitor or random investor may occur. We assume that an acquisition will be at the optimally levered value of the company, which is reasonable. Most acquisitions come with a premium to the level where the company traded prior to the acquisition-announcement. By all means, this makes T random, even if the company has announced when it will lever. The present value of equity must therefore account for some of the potential capital structure optimizing in the future, and with that the tax benefit from it, even without having occurred.

If we consider an acquisition event for absent, the value of future optimizing must still be accounted for. When T is adequately larger than t, the tax advantage will not be realized for a long time. The value of equity will only be slightly higher than in a company that theoretically refuses to take on any debt at this point, ref Amman and Genser. When T approaches t, almost the whole tax advantage must be accounted for. Moments before the restructuring takes place the unlevered equity value will equal the levered one minus RC.

In the dynamic model the company will have a threshold, V_B, where it optimally chooses to default, as in the static case. In addition there will be a new threshold, V_U, where management chooses to call outstanding debt, and issue a greater amount (restructure). The ratio the larger amount of debt increases with, stems from a scaling feature inherent in modeling the EBIT claims as log-normal or proportional.

Implications of the scaling feature is already reviewed, with V_B being proportional to C^* and C^* to V_0. Goldstein et al illustrates this with two firms, A and B, which by all means are identical except their value. This gives $V_0^B = \gamma V_0^A$. Because of the scaling feature, C and V_B will also differ by the same factor γ. The same argument as with the two firms applies to a company that initially issues debt, grow, and then finds it optimal to issue new debt. At this point, the time when its optimal to restructure, the firm will have grown with the factor γ. After the new issue, the firm will appear as the same as at time zero, except all factors are γ larger. We see $V_0^0 = V_u^0 = \gamma V_0^0$. All other claims scale up by the same factor, and period 1 starts.

Assume debt is callable at par. For this to apply we must prove that when V_U is reached, the debt has the same value as when it was issued. The bankruptcy level increases with the same factor $\gamma = V_u / V_0$, the value the firm has increased with. The risk and probability of default at the beginning of each period will be the same as at the end of each period. The reason is that at the end of each period, the company will issue new debt (increased by γ), so the risk level is the same. In other words, the old debt has the same relative coupon flow, and faces the same risk as at issue, hence the value must be the same.

The dynamic model is exploiting this scaling feature. First we find the value of all period zero claims. Then management must decide at which debt level it is optimal to restructure and issue further debt, and also increase $V_B, C \; and \; V_u$ with the factor γ, which EBIT increases with for each period. With this strategy being optimal, we find the present value of all future period claims by discounting them.

We define $p_U(V)$ as the present value of a claim paying 1 if firm value reaches V_U before V_B. This is the same principle as for $V_B$explained earlier. The claim will not have any dividends, hence it can be written:

$$p_U(V) = A_1 V^{-y} + A_2 V^{-x}$$

and satisfy the boundary conditions: $p_U(V_B) = 0$ and $p_U(V_U) = 1$. This gives: $p_U(V) = -\frac{V_B^{-x}}{\Sigma} V^{-y} + \frac{V_B^{-y}}{\Sigma} V^{-x}$, where $\Sigma \equiv V_B^{-y} V_U^{-x} - V_U^{-y} V_B^{-x}$. We also define $p_B(V)$ for a claim reaching V_B before V_U.

We are aware that: $V_{solv}^0(V) + V_{def}^0(V) + V_{res}^0(V) = V$, so EBIT is independent of the chosen capital structure. With full tax-offset, period zero claims after restructuring are:

$$d^0(V) = (1-\tau_i)V_{int}^0(V) + (1-\alpha)\left(1-\tau_{eff}\right)V_{def}^0(V)$$

$$e^0(V) = \left(1-\tau_{eff}\right)(V_{solv}^0(V) - V_{int}^0(V))$$

$$g^0(V) = \tau_{eff}\left(V_{solv}^0(V) - V_{int}^0(V)\right) + \tau_i V_{int}^0(V) + (1-\alpha)\tau_{eff} V_{def}^0(V)$$

$$bc^0(V) = \alpha V_{def}^0(V)$$

The sum of these claims adds up to $V_{solv}^0(V) + V_{def}^0(V) = V - p_U(V)V_U$. Meaning the value of future period claims is not included. We know $V_0^1 = V_U^0 \equiv \gamma V_0^0$. V_U and V_B scale by the same factor γ too. Hence, it follows that: $p_B^1(V_0^1) = p_B^0(V_0^0), p_U^1(V_0^1) = p_U^0(V_0^0)$.

The firm will look identical at every restructuring, since all parameters are scaled by the same factor. We now define e(V) as the equity claim to the intertemporal cash-flow for all future periods. Similarly for d(V), g(V) and bc(V). To get the total equity claim, E(V), we use an intuitive expansion of the claims to each period. $\frac{e(V_0)}{e^0(V_0)} = \frac{d(V_0)}{d^0(V_0)} = \frac{g(V_0)}{g^0(V_0)} = \frac{V_0}{V_0 - V_U^0(V_0)p_U(V_0)} = \frac{1}{1-\gamma p_U(V_0)}$. By multiplying with $e^0(V_0)$ we get $e(V_0) = \frac{e^0(V_0)}{1-\gamma p_U(V_0)}$.

At each restructuring there will be a transfer of cash flow. The present claim to the debt is therefore $D^0(V_0) = d^0(V_0) + p_U(V_0)D^0(V_0) = \frac{d^0(V_0)}{1-p_U(V_0)}$. This value is divided between equity an restructuring cost at the proportion (1-q) and q. Using the scaling argument, the total claim of the restructuring is: $RC(V_0 -) = qD^0 \frac{1}{1-\gamma p_U(V_0)}$.

Total claim to equity right before restructuring is the sum of the intertemporal claims of debt and equity less the restructuring costs.

This chapter was a brief summary of Goldstein et al's article, and must therefore not be red in another context, or perceived as being my theories (some sentences might be similar and all equations are the same). It is presented like this since I believe the summary is critical knowledge to understand the thesis and the tests done.

The empirical tests below, has this framework as their basis. But many of the variables and parameters described here cannot be collected or verified. For instance cannot the total value of a company be verified, as the government claim is not traded or quoted. I have therefore made adjustments to the equations, to the extent possible, to avoid such parameters in the empirical test.

8. The framework used for the empirical testing.

It is hard to define when companies observed last restructured, and by that define time zero. I will, as a consequence, focus on whether the companies observed are within the defined debt-levels, restructuring and default boundaries. If they are, the companies must be assumed inter-temporarily optimally levered, and in a phase between two restructurings or an event of default. If a company is observed with a lower debt ratio than the boundary suggests, the wealth of its equity owners should theoretically have a potential to increase. If a debt ratio higher than the upper bound would be observed, it should theoretically have defaulted. None of the companies which were possible to test had debt ratios suggesting default, hence the issue has not been further examined.

I will explain the input parameters used for the empirical testing. It's done with the purpose of identifying the upper and lower band of the optimal debt level, the V_B and V_U as a part of the EBIT. Since the total value of a company is a function of its cash-flow it is actually EBIT which is tested. See appendix for the derivation of the used model in Wolfram Mathematica. By putting in V_B and V_U, I run the test for the optimal coupon, relative to EBIT, and corresponding levels of changes in EBIT, which substitutes $V_B$and V_U. The upper and lower band is EBIT's growth before a restructuring should take place (or decrease before default). For the avoidance of doubt, I do not actually value the whole firm with all the claims, but make the calculations with the relevant factors of what I analyze. EBIT is therefore the reference value for all the variables, and equal to 1.

The input is from Thompson Financial. I will specify the sample period for each company, since data were not available over the same period for all companies. The test is for an intertemporal period of the company, which the data samples are from. By this I check whether the companies have an optimal average level of debt over the period, compared to the value ex ante. The same applies to the restructuring level and the default level. In other words, will I try to find the optimal amount of debt to issue, with the average intertemporal values for each company as input, and the corresponding restructuring levels. As output I hope to get an upper and lower relative bound of EBIT, and the coupon, C, implying the debt level.

The framework used is based on the dynamics of Goldstein et al's article. As input variables I use historical accounting numbers from the companies, gathered from Thompson. Be aware that accounting principles have changed over the sample period, and that the US and Norwegian principles differ. The consequences of this will be discussed further below. The inputs are the following equations, which is solved in Wolfram Mathematica.

```
E1[ξ_] := e1 * ξ^(x1) + e2 * ξ^(x2) + (1 - τe) * ξ / (r - μ) - (1 - τe) * c / r
D1[ξ_] := d1 * ξ^(x1) + d2 * ξ^(x2) + (1 - τi) * c / r
{E1[B] == 0, E1'[B] == 0}
```

The function E1 is the present value of all future equity claims to EBIT, and D1 is the value of the debt claim to EBIT. As you see from the input, the value of equity is zero if EBIT reaches B, the lower boundary.

```
{D1[B] == (1 - α) * A * B, D1[F] == (1 + λ) * P}
```

If reaching the lower boundary, the default boundary, the value of the debt claim is the value of the company, less the restructuring costs. At the upper bound, the restructuring boundary, the value of the debt claim is the same as the principal.

```
{E1[1] == A - (1 - q) * P, D1[1] == P}
```

At the start of each period the equity value is the total value of the company, less the debt after the restructuring costs are taken out.

```
{E1[F] == A * F - (1 + λ) * P, E1'[F] == A}
```

E1 is the value of the cash stream to equity. D1 is the value of the cash stream to debt, which at issue is P, the principal. ξ is the EBIT from the company, *te* the effective tax rate for private investors and *ti* is the tax on interest. *σ* is the standard deviation to EBIT, *r* the post tax interest rate, q the cost of restructuring, α the cost of bankruptcy, λ the tenor of the debt which is set to

0 for all calculations giving infinite debt. The rationale of assuming infinite debt, is that at maturity the company will refinance outstanding debt unless they are bankrupt. *B* is the lower bound of EBIT where the equity should default. *F* is the upper bound, suggesting a restructuring and increase the debt, and *c* the coupon. All parameters are calculated with EBIT as reference, which is 1. A is the total value of debt and equity, stated as the relative size of EBIT. It can be referred to as" Value to EBIT," a multiple similar to PE (Price/Earnings) which I will name it from here. μ is the implied growth rate (the drift described in previous chapter) of the company's EBIT, which is tested towards A, the "PE" used in the model.

9. What does the data sample include, and how is it processed

The data sample is gathered from Thompson Financial, and is each company's annual figures. The collected inputs are each company's market capitalization of equity, total interest expenses, EBIT and total debt. The data samples are from 1984 and including 2008, where they are available. To make the data available to the framework described they had to be processed. The standard deviation of EBIT is calculated by first finding each year's relative change in EBIT

$(EBIT_{-1} - EBIT_0)/EBIT_0$ and then use Excel's *"stdev"* function. The coupon used for input is calculated by taking each year's interest expenses and divided by EBIT. Finally, the leverage ratio is found by dividing total debt to total debt plus the market capitalization of equity (Debt/(Debt + Equity)).

The tax rates was found at the World Bank's web page "*Doing business*," which is, according to them, *"a Project that provides objective measures of business regulations and their enforcement across 183 economies and selected cities at the subnational and regional level."* Bankruptcy costs were collected from OECD's web page. The interest rate used as input (the risk free) is Norway's and the US' average 10 year government bond rate over the sample period, which is adjusted to be post tax according to each countries taxation on interests. The rates are also downloaded from Thompson Financial, and for Norway the "*10 year government benchmark yield*" is used. Be aware that the US and Norway have changed their tax policy over the sample period, but this policy change is not reflected in the adjustments. It is the prevailing policy which is used.

Most companies either did not exist in 1984 or did not have the accounts publically available. Therefore the time for the input for each company is not comparable to each other. There will not be a comparison between companies based on the results here. Still, since the testing is for an intertemporal period, the result will give an indication whether the companies are staying with leverage levels within the theoretical optimal boundaries. But the results for each company will not be comparable to the next one.

Further, the model assumes a lognormal calculation of the changes in EBIT, and the standard deviation. In reality EBIT may be negative for companies, even over a period of many years. This is also the case for many of the companies in the sample. Hence, calculating the variance and standard deviation by log normality is not possible. The numbers are therefore from simple return, which will bias the results from the test. Still, the deviation should not be so different that the results would not give any meaningful results of where the companies are regarding choice of capital structure over the period.

All input and variables are divided by EBIT each corresponding year. EBIT is therefore 1 in each case, and the other variables are their value relative to EBIT. The input in the model is then the average of the variables over the period. This gives the intertemporal testing, which rather allow saying whether the companies are within the boundaries in the period or not, as an average than if there were a year they were not.

A valuation of the whole company is not necessary to get the results we are looking for. By using this approach, we can focus on the measurable variables, and stick to the calculations we may observe.

To see the processed input for each company, see appendix.

10. The results from the testing

Please see appendix for a complete table of results, and companies tested. The inputs for the calculations are available on request.

Many of the companies in the original sample were missing data points and removed. The US tested companies all have 24 observations. For the Norwegian companies the average is 16.6 observations, with three of them less than 10. One observation represent one year of accounting data. Those with less than 10 observations are specified where mentioned, and in the table showing the input for the test.

For companies fully testable, there are three corresponding graphs (See appendix for all of them). The model, programmed in Wolfram Mathematica the computer tool, did not allow for sufficient iterative calculations to make all critical variables dynamic, the way I modeled it. Hence, in the testing either My or C was fixed. I have therefore found the corresponding My to the company's PE ratio, as described, and then I have tested for the optimal coupon with the corresponding My and finally checked if that affected My. Optimally this process should have been completed repeatedly, converging to the optimal solution. I was in dialogue with others to solve the problem, but the scope of the number of calculations which had to be made for each iteration seem to have set a limit to the number of variables possible. A more powerful computer, flexible programming or another mathematical program may provide better testing.

The result we want to find is the level of the coupon relative to EBIT. Then I will present how the company has accommodated their capital structure compared to the optimal level I have as output.

The US Companies

For the tested US companies, all companies which returned a result gave an optimal coupon over 1. According to the testing the optimal level of c should be higher than the cash generated by the company's assets, EBIT, in the period. An optimal level of debt giving a higher interest cost than what the assets in the company can support is not sustainable over time. For a shorter period a coupon higher than EBIT is not a problem, as long as it is lower than the debt service capacity of the company. Depreciation and amortization are non cash costs, which can service debt in the short term. In the long run depreciation and amortization costs must be assumed real costs, equal to required capital expenditures to stay in business. The debt will in other words consume the company in the long run if the coupon is higher than EBIT. Without infusion of new funds the company will have no other choice than to default. To support a higher coupon than the companies' cash flow supports, there has to be growth. I will therefore test for the growth rate of the companies to their respective PE average, and use the growth rate as input for testing the optimal coupon at restructuring.

One of the companies which allowed testing of all variables was Chevron, a US fully integrated Oil Company.

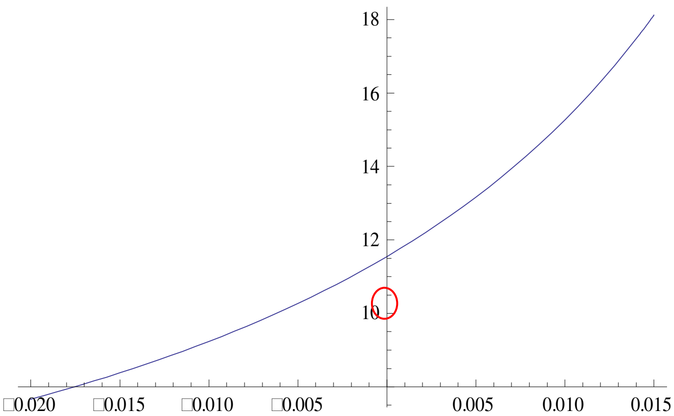

Figure 3: The X-axis is the growth ratio, while the Y-axis is the PE ratio. To find the relevant growth rate, we use the observed PE ratio, and read off the corresponding growth rate. For Chevron the PE ratio is 10.18 which give a growth rate of -0.010 or -1.0%.

The first test for all companies was to find μ (the growth rate) for the relevant PE ratio (Value of the company, here debt plus equity divided by EBIT). Graphed is the output for Chevron that has a PE of 10.18, implying a growth rate of -0.01. By all means, a negative growth rate for a major oil company seems reasonable, as replacement of oil reserves is the key challenge in the business. Most companies in the business have accepted the peak oil theory[4], and by that will the organic growth be negative.

By putting the growth rate in, and test for the optimal coupon gave the following result.

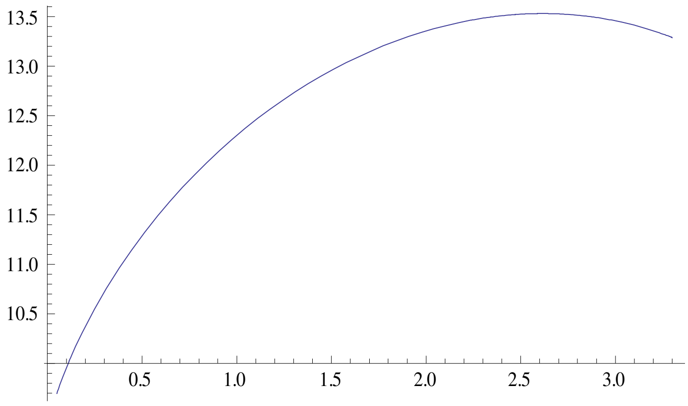

Figure 4: Here the X-axis shows the coupon and the corresponding Chevron-value at the Y-axis. As we see the optimal coupon is 2.6 times EBIT, meaning that the optimal debt level will be a coupon 2.6 times the cash flow Chevron has to service its debt with.

The X-axis represents the coupon, and the Y-axis gives the PE or corresponding value of the company for different debt levels. According to the results, the optimal coupon of Chevron should be 2.6, meaning that the optimal coupon is 2.6 higher than Chevron generates in cash. A first assumption would be that the coupon should not be higher than 1, but lower if the risk in the company is high. Chevron has an EBIT standard deviation of 0.713 which among the tested companies is average, or in the lower range. In theory, e.g. as the numerical examples presented by LJG the σ is from 0.23 to 0.27 which is substantially lower. The risk in the tested companies is therefore higher than the levels used in most theoretical and numerical examples. I will discuss this further below, but consequences of this should be in mind while reading the following results. A debt level with a coupon more than twice the cash generated to service the debt indicate that the equity owners are taking out all future profit right now, and leave the risk to the debt-holders. For a negative growth company this might be the optimal solution for the equity as well, but it's not likely that creditors will accept such levels of debt.

[4] Theory presented by M. King Hubbert, that at a certain point the major oil resources in the world will be empty, and that there will be a peak in the oil production, which will result in a decline in rate at a certain point in time.

The company returning the lowest coupon among those available for testing, or were possible to test is a US company called Legg Mason. Legg Mason is a US listed global asset management company with a σ of 0.38.

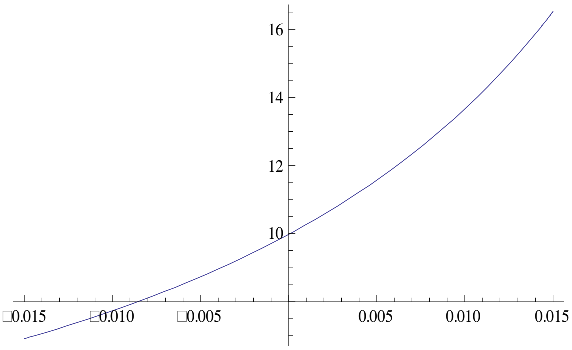

Figure 5 - The X-axis is the growth ratio, while the Y-axis is the PE ratio. To find the relevant growth rate, we use the tested PE ratio, and read off the corresponding growth rate. For Legg Mason the PE ratio is 10.09 which give a growth rate of -0.012 or -1.2%.

The PE ratio implies a growth rate of -0.012. With the credit crunch in 2008 freshly in mind, a negative growth rate for an asset management firm is not too unreasonable. By using this growth rate as input for the testing, the optimal coupon is 1.05, or 5% larger than the EBIT they are generating. With negative growth and a coupon larger than the EBIT, the company will not have the best prospects to fulfill its obligations.

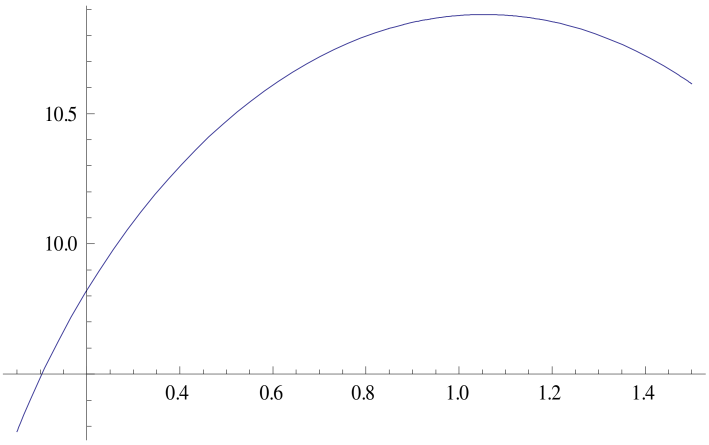

Figure 6 - The X-axis shows the coupon and the Y-axis the corresponding Legg Mason-value. As we see the optimal coupon is 1.05 times EBIT, hence 5% larger than the cash flow to service it.

The Norwegian Companies

For the Norwegian companies, none of them returned an optimal point of c. There were also many strange results compared to the US tested corporations. Please have in mind the difference in tax on interest and capital, as this is one of the differences between the countries. Also the cost of bankruptcy is different, 7% in the US versus 1% in Norway. I will touch more upon these issues in the conclusion chapter. The growth rate is also tested for the Norwegian companies, in the same way as for the US corporations. Please see Appendix for the output.

Still most companies returned almost optimal coupon levels. Those were in line with the US results, that the coupon at each restructuring point optimally was larger than EBIT. As an illustration for one of those close to an optimal point is Ekornes, a Norwegian furniture manufacturer. With a growth rate of -1% and a PE a bit over 9.2 this was the result:

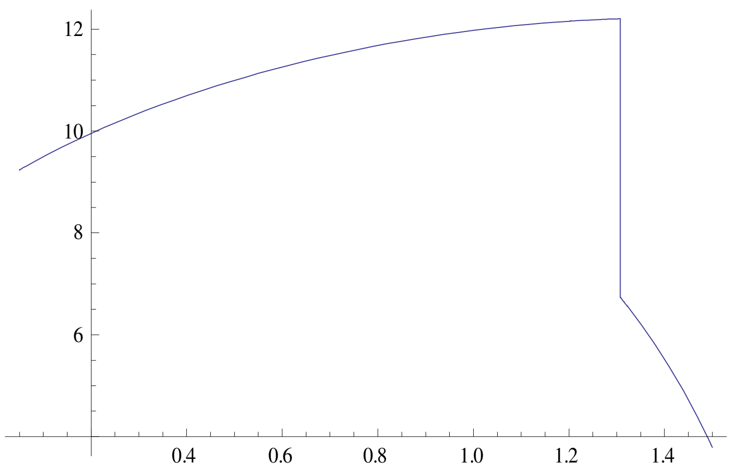

Figure 7 – The X-axis is the coupon and the Y-axis shows the corresponding PE value for Ekornes. As we may see, the curve breaks down right before an optimal max point seems reached. Since the testing is numerical, there will not necessarily be an optimal point for a company at all. But a weakness in the model or with the used computer tool might also be the case.

The curve breaks down right before the optimal level seems reached.

There does not need to exist an optimal solution for Ekornes or the other Norwegian companies, as the solutions are numerical. Since the result from the Norwegian tested companies is that none of them returned an optimal solution, the inputs for the Norwegian companies are limiting the feasible sample space. For the avoidance of doubt, the model might be correct, but with the restrictions the Norwegian input add, a solution does not need to exist.

This input is, among others, the tax rate on interest and the effective tax on equity for instance. Another difference is the cost of bankruptcy, which is 6 percent points lower in Norway than in the US. The lack of an optimal solution is therefore a combination of all these factors. The model will have intervals, where no solution is possible, as the input limits the sample space. As an example of such “illegal” intervals, could be the optimal solution would be close to or an imaginary number[5], and by that outside the results allowed here. A solution does not need to exist at all either, as the input might restrict the realistic interval to have a solution.

To illustrate how strange some of the other results were, I also show Statoil (figure 9). Statoil is a Norwegian fully integrated oil and gas company, with the government as majority owner. Bear in mind that the effective tax rate for oil companies are higher than regular, 78.16%, hence the benefit of holding debt increases even more(effective tax on equity less tax on interest). Though Statoil operates worldwide, the absolute majority of its operations are at the Norwegian continental shelf, at least over the tested period, so the Norwegian tax rate should be representative. The tax on interest is the same. With a growth rate of -5.3% to a PE of 4.1 the optimal coupon level was more exponential than optimal. The result does not look like a graph for optimizing at all. As commented on Chevron, the optimal solution for the equity seems to be taking out of all profit, and leave the company to the creditors.

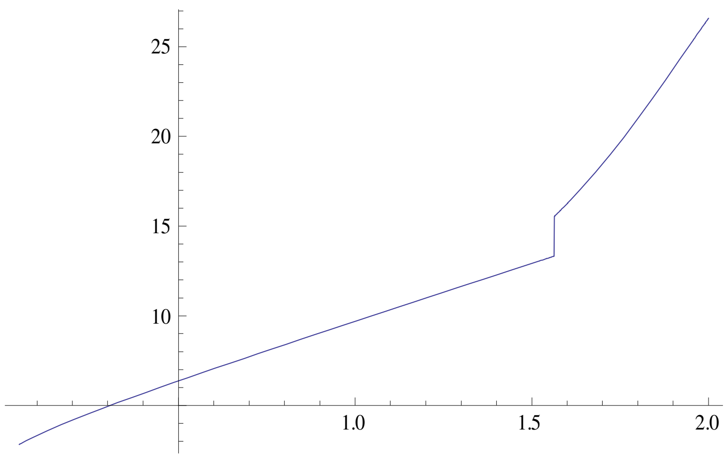

Figure 9 – The graph is the output for Statoil, with the X-axis being the coupon and the Y-axis the corresponding value of the company. Compared to TGS all coupon levels are positive here, but the output suggests that the optimal solution is an exponential increase in debt.

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[5] The square root of a negative number

11. Are the companies within the leverage bounds?

I will here present the findings for the test of the upper and lower bounds. Note that the levels are tested with EBIT as reference, and not the value of the companies. Still, those values are directly linked, and a function of each other. The restructuring level, F, is measured with the average EBIT the companies had through the sample period. The table below shows the input for each company, of those which were testable. Of a list of over 100 companies, only 15 companies were applicable for the model used, and three of the companies have less than 10 observations (starred in the left column). Of these companies again, only 6 of the US were testable in order to find the optimal coupon at a restructuring point, with reference to previous chapter. The number of observations is stated in the right column.

<10	Name	r-post tax	Stdev EBIT	Alpha	IssueCost	Teff	Ti	Average C	StdevC	"PE"	Stdev "PE"	Observations
	USA							Average	Stdev	Average	Stdev	
	CHEVRON	0,04	0,71	0,07	0,01	0,50	0,35	0,15	0,11	9,03	5,78	24
	EXXON MOBIL	0,04	0,38	0,07	0,01	0,50	0,35	0,08	0,05	9,75	4,87	24
	SCHLUMBERGER	0,04	0,95	0,07	0,01	0,50	0,35	0,14	0,10	20,70	9,06	24
	AT&T	0,04	0,37	0,07	0,01	0,50	0,35	0,21	0,06	9,06	3,13	24
	VERIZON COMMUNICATIONS	0,04	0,41	0,07	0,01	0,50	0,35	0,23	0,06	7,70	2,72	24
	SOUTHWEST AIRLINES	0,04	0,51	0,07	0,01	0,50	0,35	0,23	0,15	12,52	5,79	24
	LEGG MASON	0,04	0,38	0,07	0,01	0,50	0,35	0,30	0,14	7,35	3,66	24
	Norway							Average	Stdev	Average	Stdev	
	NORSK HYDRO	0,05	0,77	0,01	0,01	0,48	0,28	0,29	0,26	6,31	4,85	24
	ORKLA	0,05	0,52	0,01	0,01	0,48	0,28	0,35	0,18	5,67	2,38	23
*	YARA INTERNATIONAL	0,05	0,32	0,01	0,01	0,48	0,28	0,10	0,03	7,97	3,64	7
	VEIDEKKE	0,05	0,65	0,01	0,01	0,48	0,28	0,38	0,24	6,00	3,40	21
	EKORNES	0,05	0,20	0,01	0,01	0,48	0,28	0,05	0,06	8,50	2,42	15
	RIEBER & SON	0,05	0,31	0,01	0,01	0,48	0,28	0,23	0,14	7,00	4,28	24
	STATOILHYDRO	0,05	0,53	0,01	0,01	0,78	0,28	0,19	0,21	2,85	0,30	24
*	PA RESOURCES (OSL)	0,05	1,08	0,01	0,01	0,48	0,28	0,17	0,14	4,36	2,20	7
	TGS-NOPEC GEOPHS.	0,05	0,70	0,01	0,01	0,48	0,28	0,05	0,04	7,32	1,67	12
*	DOF	0,05	0,59	0,01	0,01	0,48	0,28	0,56	0,29	4,23	2,18	9

Table 1 – Table shows the data used as input for the empirical testing. All figures are measured over the sample period, with number of observations in the right column. I have also included the standard deviation for C and PE, to illustrate how much the measure varies over the period. The column to the left indicates with a * if there is less than 10 observations.

Anyway, what we are really interested in is if the companies have made any choices with respect to optimally lever the company over the sample period. To see this we want to observe how the level of debt changes when EBIT reaches the restructuring boundary, F, and see if there are some trends. If a company really makes managerial choices with respect to optimal leverage we should expect a debt-ratio graph with slightly decreasing level, which goes back to the initial level every time EBIT reaches F. Hence, what we may get are the levels of the upper and lower bound, given that the average EBIT is the initial EBIT. The results are presented in the tables below.

<10	Name	Parameters - for observed period					
	USA	μ	B	F	Debt-Low	A	P
	CHEVRON	-0,0100	0,0223	1,2797	0,0728	10,1755	0,9482
	EXXON MOBIL	-0,0050	0,0257	1,2740	0,0682	10,7605	0,9354
	SCHLUMBERGER						
	AT&T	-0,0050	0,0688	1,2501	0,1555	11,2726	2,1909
	VERIZON COMMUNICATIONS	-0,0130	0,0742	1,2408	0,1727	9,7459	2,0882
	SOUTHWEST AIRLINES	0,0050	0,0409	1,2750	0,1078	14,4523	1,9868
	LEGG MASON	-0,0120	0,1033	1,2294	0,2197	10,0856	2,7238
	Norway						
	NORSK HYDRO	-0,0330	0,0508	1,2347	0,1491	7,7347	1,4241
	ORKLA	-0,0300	0,1033	1,2084	0,2424	8,0325	2,3529
*	YARA INTERNATIONAL	-0,0120	0,0430	1,2256	0,1096	9,2575	1,2439
	VEIDEKKE	-0,0320	0,0833	1,2191	0,2106	8,0214	2,0591
	EKORNES	-0,0100	0,0299	1,2034	0,0659	9,2365	0,7320
	RIEBER & SON	-0,0140	0,1017	1,2058	0,2221	9,5229	2,5498
	STATOILHYDRO	-0,0530	0,0610	1,1492	0,2651	4,1049	1,2508
*	PA RESOURCES (OSL)	-0,0730	0,0256	1,2467	0,0932	4,9203	0,5720
	TGS-NOPEC GEOPHS.						
*	DOF	-0,0490	0,1578	1,1905	0,3292	6,8776	2,6955

Table 2 – This is the output figures from Wolfram Mathematica, after the input from table 1 has been used as input. The relevant output from this table is the value of My, which will be used as growth rate when we test for which coupon to me optimal at each restructuring. An interesting observation is that all the companies gave a negative My.

In table 2 is the output after plotting in the input in table 1. All values are relative to EBIT, except μ, which is the growth rate to the corresponding average PE ratio over the sample period. It is My we are interested in, since it will be used as input to find the optimal coupon for each company in the following test (see output in table 3). An interesting observation is that all companies, except Southwest Airlines, had a My below zero. Meaning the implied growth rate to the corresponding PE average over the period suggests the companies will have negative growth. My initial thoughts to this result were that the credit crunch in 2008 had to be the reason. But a conclusion is not possible to make, without further test, for instance over a different sample period. None of these values are of optimal levels of any kind. Since time zero is impossible to find for the empirical study, a change in debt level at each time EBIT crosses the upper bound might be the only observable. Even that may be hard to quantify, since a decision may take time, and it is not necessarily a capital-structure choice if the debt level is changed.

Name	Parameters - with optimzed C, My not adjusted					
USA	C	B	F	Debt-Low	A	P
CHEVRON	2,6000	0,2635	1,1887	0,4320	13,5311	6,9490
EXXON MOBIL	1,2000	0,3208	1,1768	0,4829	12,6992	7,2171
SCHLUMBERGER						
AT&T	1,1500	0,3223	1,1765	0,4840	12,6425	7,1983
VERIZON COMMUNICATIONS	1,1000	0,3057	1,1762	0,4735	10,7677	5,9960
SOUTHWEST AIRLINES	2,2000	0,2869	1,1880	0,4486	18,0839	9,6382
LEGG MASON	1,0500	0,3180	1,1735	0,4847	10,8813	6,1887

Table 3 – The output for the companies which were testable with the used computer, frame-work and Wolfram Mathematica. None of the companies had an optimal debt level with a coupon equal to or less than their EBIT. Since most companies in reality have a coupon which the can serve with their cash flow, the results are not in line with decisions taken by the management of the firms.

Table 3 shows the output for the companies returning an optimal coupon, when tested. These companies are all US. Not even all the US companies returned a value, as seen.

To give an indication of how the result may be tested, I have made an illustration with the output from Chevron. I would like to stress that this is an illustration only, and with the existing data sample it cannot be used for really testing the optimality.

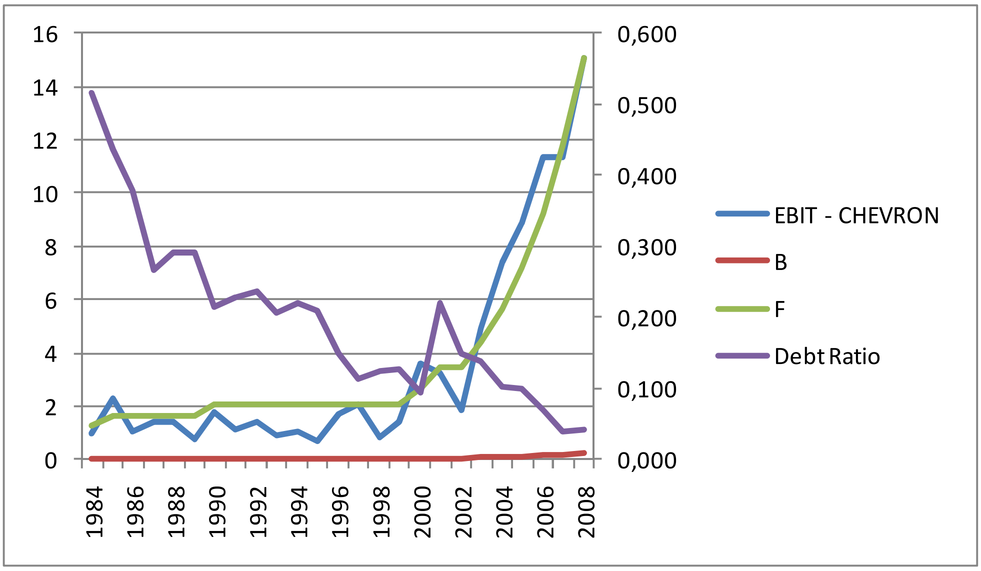


Figure 10 – The X-axis is year of the observations, starting in 1984. The left Y-axis is the level for EBIT (year 1, here 1984 being equal to 1), B (the default boundary) and F (the restructuring boundary). The right Y-axis refers to the Debt Ratio, being Debt/(Debt+Equity). F and B are here increased by γ (the scaling factor), being equal to F. Every time EBIT is equal to or larger than F, the level of F and B is increased proportionally to γ. This graph does not indicate any connection between the level of debt and the suggested optimal time to restructure and issue a greater amount of debt.

I have used the results from the empirical testing over the sample period to see if there are any trends. Since all variables are relative to EBIT, EBIT is 1 in 1984. All values are therefore relative to EBIT in 1984, also the development in EBIT. Every time EBIT reaches F, both F and B is restructured correspondingly with the relative factor, F. For the avoidance of doubt, at each point EBIT is equal to or larger than F an increase in the upper and lower bound is made. If EBIT ever goes below B, an event of default should have happened according to the theory. We should also be able to see if the company has increased the debt when the F level is reached.

For Chevron such a change in debt may be seen in year 2000 (figure 10), where EBIT breaks through the F bound, and the relative debt level increases. But as a trend it is obvious that the relative debt level is being reduced. Over the period we should have seen that the level of debt had moved slightly downwards between each restructuring, and then gone back to the initial level each time EBIT reached F, and an increased amount of debt was issued (figure 11).Figure 11 illustrates one possible scenario of how the corresponding debt level could have looked like, had the company had an active capital structure policy. All the figures are the same as in figure 10, except the debt level, which is se to the initial level each time F is reached.

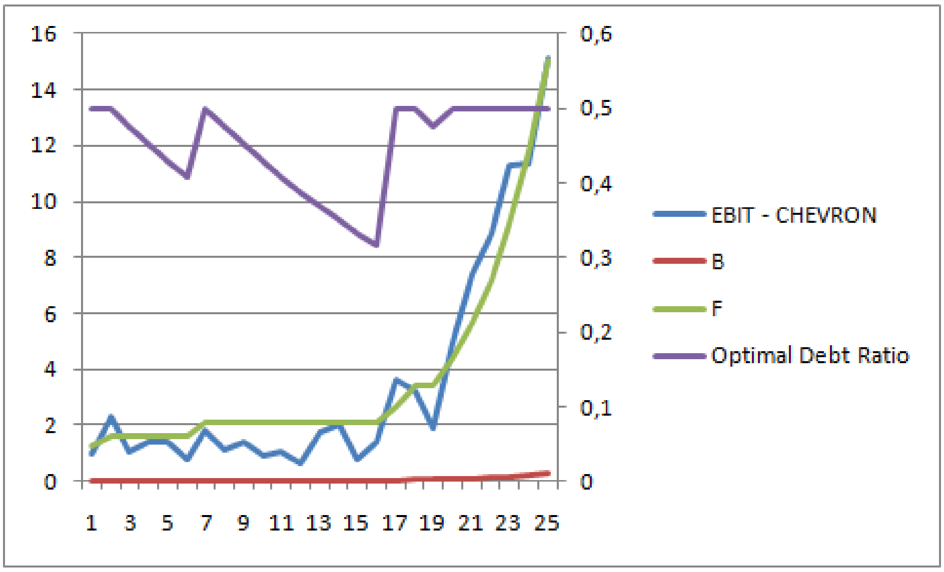


Figure 11 – All variables are here identical to figure 10, except for the debt ratio. What I have tried to illustrate is how the debt ratio could have looked like, had Chevron actively used the model to make their capital structure decisions. In this example, the debt level goes down by a ratio of 5% each year, except the years where the EBIT breaks the F boundary. Then a refinancing takes place, and the initial level of debt is restored (here 50% debt).

As this not being the case, I have also looked at Verizon, an US telecom company (figure 12). In Verizon the theoretical trend of how a company should optimize are more distinctive. There are several times the EBIT is breaking through the F bound with a corresponding increase in the debt level. If Verizon actually are optimizing its level of debt is still not possible to say. But over the sample period it certainly look more like what to expect from the theory. It seems like the relative level of debt goes back to a certain ratio, of close to 0.4 in relation to EBIT breaking through the F bound. This should be in line with what to expect, with an active capital structure policy among the management.

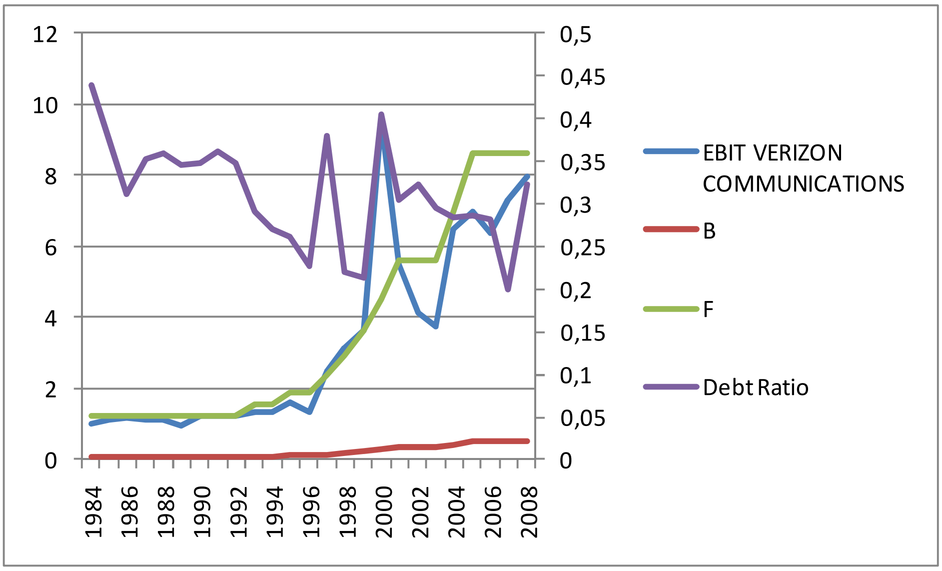


Figure 12 – The X-axis is year of the observations, starting in 1984. The left Y-axis is the level for EBIT (year 1, here 1984 being equal to 1), B (the default boundary) and F (the restructuring boundary). The right Y-axis refers to the Debt Ratio, being Debt/(Debt+Equity). F and B are here increased by γ, being equal to F. Every time EBIT is equal to or larger than F, the level of F and B is increased proportionally to γ. In contrast to the Chevron (figure 9) some kind of relation might be observed. We see that the level of debt goes back to approximately 0.4 on several occasions, and this happens close to observations where Verizon's EBIT crosses the F level. Still, no conclusion may be taken from this, as the test is made over the same period as the sample have been collected.

12. So what may be taken away from the empirical test?

What we have seen is that the companies possible to test for an optimal coupon all resulted in coupons higher than the cash generated to service the debt. Still, the number of companies tested is not large enough to draw a general conclusion.

What we can assume is that the test made here did not allow the parameter variable, EBIT and the relative standard deviation to EBIT, to differing too much from those presented in theoretical examples, as Leland's paper from 1994 or Goldstein et al's from 2001. The observed variables differed a lot from those presented in the articles mentioned. I will present the variables I believe could be a key element for the tests to fail.

Since the solutions to these tests are numerical, and not in closed form solution, there does not need to exist a solution. There might be optimal solutions for those that did not return any result too, but they might be so extreme that they are outside of the boundaries made to keep the results virtually possible, or even outside real numbers (R).

One of the variables is the standard deviation of EBIT. As mentioned, this has been tested on a simple return basis, since EBIT was negative for many of the companies on several occasions. This alone will bias the variance to some extent, but should be in line with what is reasonable. To only test companies which have a positive EBIT in the sample period could therefore improve the quality, if the purpose is to examine the validity of the theories. At least it would eliminate some of the doubt of the results which is a consequence of mixing simple and logarithmic features. Still, the standard deviations here are so much larger than those presented in theory, that the use of simple return cannot explain the difference in variance alone.

In Goldstein et al's paper the standard deviations used in the numerical examples are in the range 0.23 to 0.27. The average standard deviation in this test was 0.55 for the testable companies. The companies that could not be tested for any optimal level of coupon had higher standard deviations than this. Especially did the various standard deviations seem to be the key among the US companies whether they were testable or not. Those with a high stdev did not give any results, while those with levels in the lower end were testable. None of the Norwegian companies was possible to test for optimal coupon, so there has be additional differences for them. Please see the appendix for the input of all companies tested. The average standard deviation was 2.8 among the complete US sample over the complete sample period (1984-2008) and 7.6 for the Norwegians'. In other words does it seem like the variance in EBIT in reality is a lot higher than what's common to assume in theory.

For the Norwegian companies there has to be further differences, since none of them gave a reasonable result. Among the fixed input applying to all the Norwegian companies that differs from the US ones are the default costs and the spread in taxes between tax on interests and tax on equity.

In Norway the default cost is 1% of the estate, while its 7% in the US. This means intuitively that debt is less risky in Norway than in the US. If the cost of default is less, then it encourages higher ratios of debt. The model will take this into account, and should therefore return higher debt ratios when the cost of default is lower. I will not conclude with this is the reason for the Norwegian companies not giving any results, but mention it as one of the differences between the US and Norwegian companies.

The difference in taxation is also a fixed difference. In Norway the difference between fixed income taxation and equity taxation is 48.16% less 28% is 20.16%. In the US the difference is 50% less 35% is 15%. The benefit of holding debt is in other words higher in Norway than in the US. Both the cost of default and the benefit of holding tax encourage higher debt ratios in Norway relative to the US. If this is the reason for the testability I cannot say, but its differences in fixed input which differs in the two countries.

Another reason for so many of the companies failing to be tested may also be the way the equations are modeled, or the computer tool used. All tests have been deviated with Wolfram Mathematica, a computer tool which should be more powerful than Microsoft's Excel for instance, especially when it comes to testing for numerical solutions.

So is it possible to conclude whether companies are holding optimal levels of debt, which amount they should issue when restructuring, and are the results here realistic and in line with observed levels? There cannot be made any conclusion of the optimal debt levels. The amount of debt to issue resulted all in coupons higher than the cash flow to service the coupon, so the results are in that respect not realistic either. The optimal coupons according to the test were all higher than the coupons which could be observed in reality. To make an empirical test of whether companies are optimally levered with respect to cost of default and corresponding tax shields, will need further model adjustments most likely, and probably allow for more extreme variables than the theory take into account today. Using this approach should therefore not be the key element in a capital structure decision. But the theories and dynamics of how external parameters as tax and risk affect the value of the company could be of help to the decision makers.

13. Reflections

During the study of this paper I was confronted with several issues, which I have taken notes of and would like to present. Among them were thoughts I got while testing, and direct issues regarding capital structure choices.

First of all, when testing for capital structures in a quantitative way, the data used has to be consistent. It does not necessarily matter how all inputs are calculated, as long as the same parameter is calculated in the same way throughout the sample period. This has not been the case in this study. To take the Norwegian data sample first. In Norway the accounting principles have changed over the period, from Norwegian GAAP in the 80's to IFRS after the millennium. The way the input is calculated is of this reason changed over the time. I can't say if EBIT is calculated in the same way now, as in the 80's or 90's, or if Thompson Financials have taken this development into account.

For comparing the results between countries the issue becomes even more piquant. IFRS is not practiced consistently all over the world, where it is the prevailing standard, and the US has another accounting standard, the US GAAP. Comparing the results to, say, whether companies in one country are better in accommodating a capital structure which benefits the owners' best to another country becomes difficult.

There are also theories which the structured credit models do not take into account. The issues regarding asymmetric information is hard to quantify, and hence not taken into account. Another issue is equity's ability to force concessions, like in a chapter 11 process in the US. This brings us also to how the different legal systems between countries also will affect the optimal debt level. Some countries are of the opinion that creditors should have strong protection, while other countries protect the equity holders instead. I will not make any statement of what's best or worst, but enjoy bringing up issues which may be of interest in a more comprehensive analysis.

In this thesis all companies existed at the time the data was collected. To test whether companies choose to default when having earnings below the default boundary, defaulted companies must be tested. This test is therefore only a test if companies are making managerial restructuring choices and increase the debt, not to test whether the default barrier is followed.

Another issue which struck me was the knowledge of how exposed the government is to company earnings, even without holding any equity in them. This is a political issue, so I will not argue whether governments holding equity is good or bad. But for an all equity company, the government holds a claim of 48.16% of the value in Norway. By acquiring equity as well, the community is further exposed to the risk of each company. On the other side does not the government claim allow any voting rights unless they hold equity too.

Appendix

14. The model used for testing

Below are the equations from Wolfram Mathematica, which has been used for the empirical testing. It starts with defining the different inputs, and then they are all put together, so they become dependent upon each other.

In[1]:= `E1[ξ_] := e1 * ξ^(x1) + e2 * ξ^(x2) + (1 - τe) * ξ / (r - μ) - (1 - τe) * c / r`

In[2]:= `D1[ξ_] := d1 * ξ^(x1) + d2 * ξ^(x2) + (1 - τi) * c / r`

In[3]:= `{E1[B] == 0, E1'[B] == 0}`

Out[3]= $\left\{B^{x1}\,e1 + B^{x2}\,e2 - \frac{c\,(1-\tau e)}{r} + \frac{B\,(1-\tau e)}{r-\mu} == 0,\ B^{-1+x1}\,e1\,x1 + B^{-1+x2}\,e2\,x2 + \frac{1-\tau e}{r-\mu} == 0\right\}$

In[4]:= `FullSimplify[%]`

Out[4]= $\left\{B^{x1}\,e1 + B^{x2}\,e2 + \frac{c\,(-1+\tau e)}{r} + \frac{B - B\,\tau e}{r-\mu} == 0,\ B^{-1+x1}\,e1\,x1 + B^{-1+x2}\,e2\,x2 + \frac{1-\tau e}{r-\mu} == 0\right\}$

In[5]:= `Solve[%, {e1, e2}]`

Out[5]= $\left\{\left\{e1 \to -\frac{B^{-x1}\,(-B\,r + B\,r\,x2 - c\,r\,x2 + c\,x2\,\mu)\,(-1+\tau e)}{r\,(x1-x2)\,(r-\mu)},\ e2 \to \frac{B^{-x2}\,(-B\,r + B\,r\,x1 - c\,r\,x1 + c\,x1\,\mu)\,(-1+\tau e)}{r\,(x1-x2)\,(r-\mu)}\right\}\right\}$

In[6]:= `FullSimplify[Flatten[%]]`

Out[6]= $\left\{e1 \to -\frac{B^{-x1}\,(B\,r\,(-1+x2) + c\,x2\,(-r+\mu))\,(-1+\tau e)}{r\,(x1-x2)\,(r-\mu)},\ e2 \to -\frac{B^{-x2}\,(B\,r\,(-1+x1) + c\,x1\,(-r+\mu))\,(-1+\tau e)}{r\,(-x1+x2)\,(r-\mu)}\right\}$

In[7]:= `{D1[B] == (1 - α) * A * B, D1[F] == (1 + λ) * P}`

Out[7]= $\left\{B^{x1}\,d1 + B^{x2}\,d2 + \frac{c\,(1-\tau i)}{r} == A\,B\,(1-\alpha),\ d1\,F^{x1} + d2\,F^{x2} + \frac{c\,(1-\tau i)}{r} == P\,(1+\lambda)\right\}$

In[8]:= `Solve[%, {d1, d2}]`

Out[8]= $\left\{\left\{d1 \to \frac{-B^{x2}\,c + c\,F^{x2} - A\,B\,F^{x2}\,r + B^{x2}\,P\,r + A\,B\,F^{x2}\,r\,\alpha + B^{x2}\,P\,r\,\lambda + B^{x2}\,c\,\tau i - c\,F^{x2}\,\tau i}{\left(B^{x2}\,F^{x1} - B^{x1}\,F^{x2}\right)\,r},\ d2 \to -\frac{-F^{x1}\left(-A\,B\,(1-\alpha) + \frac{c\,(1-\tau i)}{r}\right) + B^{x1}\left(-P\,(1+\lambda) + \frac{c\,(1-\tau i)}{r}\right)}{-B^{x2}\,F^{x1} + B^{x1}\,F^{x2}}\right\}\right\}$

In[9]:= `FullSimplify[Flatten[%]]`

Out[9]= $\left\{d1 \to \frac{B^{x2}\,(P\,r\,(1+\lambda) + c\,(-1+\tau i)) + F^{x2}\,(c + A\,B\,r\,(-1+\alpha) - c\,\tau i)}{\left(B^{x2}\,F^{x1} - B^{x1}\,F^{x2}\right)\,r},\ d2 \to \frac{F^{x1}\,(-A\,B\,r\,(-1+\alpha) + c\,(-1+\tau i)) - B^{x1}\,(P\,r\,(1+\lambda) + c\,(-1+\tau i))}{\left(B^{x2}\,F^{x1} - B^{x1}\,F^{x2}\right)\,r}\right\}$

In[10]:= `{E1[1] == A - (1 - q) * P, D1[1] == P}`

Out[10]= $\left\{e1 + e2 - \frac{c\,(1-\tau e)}{r} + \frac{1-\tau e}{r-\mu} == A - P\,(1-q),\ d1 + d2 + \frac{c\,(1-\tau i)}{r} == P\right\}$

In[11]:= `FullSimplify[%]`

Out[11]= $\left\{e1 + e2 + \frac{1-\tau e}{r-\mu} + \frac{c\,(-1+\tau e)}{r} == A + P\,(-1+q),\ d1 + d2 + \frac{c - c\,\tau i}{r} == P\right\}$

In[12]:= `% /. %6`

Out[12]=
$$\left\{\frac{1-\tau e}{r-\mu}+\frac{c\,(-1+\tau e)}{r}-\frac{B^{-x2}\,(B\,r\,(-1+x1)+c\,x1\,(-r+\mu))\,(-1+\tau e)}{r\,(-x1+x2)\,(r-\mu)}-\frac{B^{-x1}\,(B\,r\,(-1+x2)+c\,x2\,(-r+\mu))\,(-1+\tau e)}{r\,(x1-x2)\,(r-\mu)}==A+P\,(-1+q),\ d1+d2+\frac{c-c\,\tau i}{r}==P\right\}$$

In[13]:= `% /. %9`

Out[13]=
$$\left\{\frac{1-\tau e}{r-\mu}+\frac{c\,(-1+\tau e)}{r}-\frac{B^{-x2}\,(B\,r\,(-1+x1)+c\,x1\,(-r+\mu))\,(-1+\tau e)}{r\,(-x1+x2)\,(r-\mu)}-\frac{B^{-x1}\,(B\,r\,(-1+x2)+c\,x2\,(-r+\mu))\,(-1+\tau e)}{r\,(x1-x2)\,(r-\mu)}==A+P\,(-1+q),\right.$$
$$\left.\frac{F^{x1}\,(-A\,B\,r\,(-1+\alpha)+c\,(-1+\tau i))-B^{x1}\,(P\,r\,(1+\lambda)+c\,(-1+\tau i))}{\left(B^{x2}\,F^{x1}-B^{x1}\,F^{x2}\right)\,r}+\frac{c-c\,\tau i}{r}+\frac{B^{x2}\,(P\,r\,(1+\lambda)+c\,(-1+\tau i))+F^{x2}\,(c+A\,B\,r\,(-1+\alpha)-c\,\tau i)}{\left(B^{x2}\,F^{x1}-B^{x1}\,F^{x2}\right)\,r}==P\right\}$$

In[14]:= `FullSimplify[%]`

Out[14]=
$$\left\{\frac{1-\tau e}{r-\mu}+\frac{c\,(-1+\tau e)}{r}==A+P\,(-1+q)+\frac{B^{-x2}\,(B\,r\,(-1+x1)+c\,x1\,(-r+\mu))\,(-1+\tau e)}{r\,(-x1+x2)\,(r-\mu)}+\frac{B^{-x1}\,(B\,r\,(-1+x2)+c\,x2\,(-r+\mu))\,(-1+\tau e)}{r\,(x1-x2)\,(r-\mu)},\right.$$
$$\left.\frac{1}{\left(B^{x2}\,F^{x1}-B^{x1}\,F^{x2}\right)\,r}\left(-A\,B\left(F^{x1}-F^{x2}\right)r\,(-1+\alpha)+B^{x2}\left(P\,r\,(1+\lambda)-c\left(-1+F^{x1}\right)(-1+\tau i)\right)+B^{x1}\left(-P\,r\,(1+\lambda)+c\left(-1+F^{x2}\right)(-1+\tau i)\right)+c\left(F^{x1}-F^{x2}\right)(-1+\tau i)\right)==P\right\}$$

In[15]:= `Solve[%, {A, P}]`

Out[15]=

$$\Bigg\{\Bigg\{A \to -\frac{c}{r}+\frac{1}{r-\mu}-\frac{B^{1-x1}}{(x1-x2)\,(r-\mu)}+\frac{B^{1-x1}\,x2}{(x1-x2)\,(r-\mu)}-\frac{B^{-x1}\,c\,x2}{(x1-x2)\,(r-\mu)}-\frac{B^{1-x2}}{(-x1+x2)\,(r-\mu)}+$$

$$\frac{B^{1-x2}\,x1}{(-x1+x2)\,(r-\mu)}-\frac{B^{-x2}\,c\,x1}{(-x1+x2)\,(r-\mu)}+\frac{B^{-x1}\,c\,x2\,\mu}{r\,(x1-x2)\,(r-\mu)}+\frac{B^{-x2}\,c\,x1\,\mu}{r\,(-x1+x2)\,(r-\mu)}+\frac{c\,\tau e}{r}-\frac{\tau e}{r-\mu}+$$

$$\frac{B^{1-x1}\,\tau e}{(x1-x2)\,(r-\mu)}-\frac{B^{1-x1}\,x2\,\tau e}{(x1-x2)\,(r-\mu)}+\frac{B^{-x1}\,c\,x2\,\tau e}{(x1-x2)\,(r-\mu)}+\frac{B^{1-x2}\,\tau e}{(-x1+x2)\,(r-\mu)}-\frac{B^{1-x2}\,x1\,\tau e}{(-x1+x2)\,(r-\mu)}+$$

$$\frac{B^{-x2}\,c\,x1\,\tau e}{(-x1+x2)\,(r-\mu)}-\frac{B^{-x1}\,c\,x2\,\mu\,\tau e}{r\,(x1-x2)\,(r-\mu)}-\frac{B^{-x2}\,c\,x1\,\mu\,\tau e}{r\,(-x1+x2)\,(r-\mu)}+\Bigg((-1+q)\,\Bigg(\frac{1}{B^{x2}\,F^{x1}-B^{x1}\,F^{x2}}$$

$$B\,\left(F^{x1}-F^{x2}\right)\,(-1+\alpha)\,\left(\frac{1-\tau e}{r-\mu}+\frac{c\,(-1+\tau e)}{r}-\frac{B^{-x2}\,(B\,r\,(-1+x1)+c\,x1\,(-r+\mu))\,(-1+\tau e)}{r\,(-x1+x2)\,(r-\mu)}-\right.$$

$$\left.\frac{B^{-x1}\,(B\,r\,(-1+x2)+c\,x2\,(-r+\mu))\,(-1+\tau e)}{r\,(x1-x2)\,(r-\mu)}\right)+$$

$$\frac{B^{x2}\,c\,\left(-1+F^{x1}\right)\,(-1+\tau i)}{\left(B^{x2}\,F^{x1}-B^{x1}\,F^{x2}\right)\,r}-\frac{c\,\left(F^{x1}-F^{x2}\right)\,(-1+\tau i)}{\left(B^{x2}\,F^{x1}-B^{x1}\,F^{x2}\right)\,r}-\frac{B^{x1}\,c\,\left(-1+F^{x2}\right)\,(-1+\tau i)}{\left(B^{x2}\,F^{x1}-B^{x1}\,F^{x2}\right)\,r}\Bigg)\Bigg)\Big/$$

$$\left(1+\frac{B\,\left(F^{x1}-F^{x2}\right)\,(1-q)\,(-1+\alpha)}{B^{x2}\,F^{x1}-B^{x1}\,F^{x2}}+\frac{B^{x1}\,(1+\lambda)}{B^{x2}\,F^{x1}-B^{x1}\,F^{x2}}-\frac{B^{x2}\,(1+\lambda)}{B^{x2}\,F^{x1}-B^{x1}\,F^{x2}}\right),$$

$$P \to -\Bigg(\frac{1}{B^{x2}\,F^{x1}-B^{x1}\,F^{x2}}B\,\left(F^{x1}-F^{x2}\right)\,(-1+\alpha)$$

$$\left(\frac{1-\tau e}{r-\mu}+\frac{c\,(-1+\tau e)}{r}-\frac{B^{-x2}\,(B\,r\,(-1+x1)+c\,x1\,(-r+\mu))\,(-1+\tau e)}{r\,(-x1+x2)\,(r-\mu)}-\right.$$

$$\left.\frac{B^{-x1}\,(B\,r\,(-1+x2)+c\,x2\,(-r+\mu))\,(-1+\tau e)}{r\,(x1-x2)\,(r-\mu)}\right)+$$

$$\frac{B^{x2}\,c\,\left(-1+F^{x1}\right)\,(-1+\tau i)}{\left(B^{x2}\,F^{x1}-B^{x1}\,F^{x2}\right)\,r}-\frac{c\,\left(F^{x1}-F^{x2}\right)\,(-1+\tau i)}{\left(B^{x2}\,F^{x1}-B^{x1}\,F^{x2}\right)\,r}-\frac{B^{x1}\,c\,\left(-1+F^{x2}\right)\,(-1+\tau i)}{\left(B^{x2}\,F^{x1}-B^{x1}\,F^{x2}\right)\,r}\Bigg)\Big/$$

$$\left(1+\frac{B\,\left(F^{x1}-F^{x2}\right)\,(1-q)\,(-1+\alpha)}{B^{x2}\,F^{x1}-B^{x1}\,F^{x2}}+\frac{B^{x1}\,(1+\lambda)}{B^{x2}\,F^{x1}-B^{x1}\,F^{x2}}-\frac{B^{x2}\,(1+\lambda)}{B^{x2}\,F^{x1}-B^{x1}\,F^{x2}}\right)\Bigg\}\Bigg\}$$

In[16]:= `FullSimplify[Flatten[%]]`

Out[16]=

$$
\begin{aligned}
\{A \to & \Big(B^{-x1-x2} \Big(-\frac{1}{r-\mu}\left(B^{1+x1}\,(-1+x1)+B^{x2}\left(B-B\,x2+B^{x1}\,(-x1+x2)\right)\right)\left(B^{x1}\left(-1+F^{x2}-\lambda\right)+B^{x2}\left(1-F^{x1}+\lambda\right)\right) \\
& (-1+\tau e)+\frac{1}{r}c\,\Big(B^{2\,x2}\,x2\left(-1+F^{x1}-\lambda\right)(-1+\tau e)+B^{2\,x1}\,x1\left(-1+F^{x2}-\lambda\right)(-1+\tau e)+ \\
& B^{x1+2\,x2}\,(x1-x2)\left(\lambda-\lambda\,\tau e+\left(-1+F^{x1}\right)(\tau e-\tau i)+\left(-1+F^{x1}\right)q\,(-1+\tau i)\right)- \\
& B^{2\,x1+x2}\,(x1-x2)\left(\lambda-\lambda\,\tau e+\left(-1+F^{x2}\right)(\tau e-\tau i)+\left(-1+F^{x2}\right)q\,(-1+\tau i)\right)+ \\
& B^{x1+x2}\Big(x1\left((1+\lambda)(-1+\tau e)+F^{x2}(-1+q)(-1+\tau i)+F^{x1}(q-\tau e+\tau i-q\,\tau i)\right)+ \\
& x2\left((1+\lambda)(-1+\tau e)+F^{x1}(-1+q)(-1+\tau i)+F^{x2}(q-\tau e+\tau i-q\,\tau i)\right)\Big)\Big)\Big)\Big) \Big/ \\
& \left((x1-x2)\left(-B\left(F^{x1}-F^{x2}\right)(-1+q)(-1+\alpha)+B^{x2}\left(-1+F^{x1}-\lambda\right)+B^{x1}\left(1-F^{x2}+\lambda\right)\right)\right), \\
P \to & -\Bigg(\frac{1}{x1-x2}B^{1-x1-x2}\left(F^{x1}-F^{x2}\right)(-1+\alpha)\Bigg(\frac{c\left(-B^{x1}\,x1+B^{x2}\left(B^{x1}\,(x1-x2)+x2\right)\right)}{r}+ \\
& \frac{B^{1+x1}\,(-1+x1)+B^{x2}\left(B-B\,x2+B^{x1}\,(-x1+x2)\right)}{r-\mu}\Bigg)(-1+\tau e)+ \\
& \frac{B^{x2}\,c\left(-1+F^{x1}\right)(-1+\tau i)}{r}-\frac{B^{x1}\,c\left(-1+F^{x2}\right)(-1+\tau i)}{r}+\frac{c\left(-F^{x1}+F^{x2}\right)(-1+\tau i)}{r}\Bigg) \Big/ \\
& \left(-B\left(F^{x1}-F^{x2}\right)(-1+q)(-1+\alpha)+B^{x2}\left(-1+F^{x1}-\lambda\right)+B^{x1}\left(1-F^{x2}+\lambda\right)\right)\}
\end{aligned}
$$

In[17]:= `{E1[F] == A * F - (1 + λ) * P, E1'[F] == A}`

Out[17]=

$$
\left\{e1\,F^{x1}+e2\,F^{x2}-\frac{c\,(1-\tau e)}{r}+\frac{F\,(1-\tau e)}{r-\mu}==A\,F-P\,(1+\lambda),\ e1\,F^{-1+x1}\,x1+e2\,F^{-1+x2}\,x2+\frac{1-\tau e}{r-\mu}==A\right\}
$$

In[18]:= `FullSimplify[%]`

Out[18]=

$$
\left\{e1\,F^{x1}+e2\,F^{x2}+P\,(1+\lambda)+\frac{c\,(-1+\tau e)}{r}+\frac{F-F\,\tau e}{r-\mu}==A\,F,\ e1\,F^{-1+x1}\,x1+e2\,F^{-1+x2}\,x2+\frac{1-\tau e}{r-\mu}==A\right\}
$$

In[19]:= `% /. %6`

Out[19]=

$$
\begin{aligned}
\Big\{ & P\,(1+\lambda)+\frac{c\,(-1+\tau e)}{r}-\frac{B^{-x2}\,F^{x2}\,(B\,r\,(-1+x1)+c\,x1\,(-r+\mu))\,(-1+\tau e)}{r\,(-x1+x2)\,(r-\mu)}- \\
& \frac{B^{-x1}\,F^{x1}\,(B\,r\,(-1+x2)+c\,x2\,(-r+\mu))\,(-1+\tau e)}{r\,(x1-x2)\,(r-\mu)}+\frac{F-F\,\tau e}{r-\mu}==A\,F, \\
& \frac{1-\tau e}{r-\mu}-\frac{B^{-x2}\,F^{-1+x2}\,x2\,(B\,r\,(-1+x1)+c\,x1\,(-r+\mu))\,(-1+\tau e)}{r\,(-x1+x2)\,(r-\mu)}- \\
& \frac{B^{-x1}\,F^{-1+x1}\,x1\,(B\,r\,(-1+x2)+c\,x2\,(-r+\mu))\,(-1+\tau e)}{r\,(x1-x2)\,(r-\mu)}==A\Big\}
\end{aligned}
$$

In[20]:= `FullSimplify[%]`

Out[20]=

$$
\begin{aligned}
\Big\{ & P\,(1+\lambda)+\frac{c\,(-1+\tau e)}{r}+\frac{F-F\,\tau e}{r-\mu}==A\,F+\frac{1}{r\,(x1-x2)\,(r-\mu)} \\
& B^{-x1-x2}\left(B^{x1}\,F^{x2}\,(B\,(r-r\,x1)+c\,x1\,(r-\mu))+B^{x2}\,F^{x1}\,(B\,r\,(-1+x2)+c\,x2\,(-r+\mu))\right)(-1+\tau e), \\
& -\frac{1}{r-\mu}\Big(-1+\frac{1}{F\,r\,(x1-x2)}B^{-x1-x2}\left(-B^{x1}\,F^{x2}\,x2\,(B\,r\,(-1+x1)+c\,x1\,(-r+\mu))+\right. \\
& \left.B^{x2}\,F^{x1}\,x1\,(B\,r\,(-1+x2)+c\,x2\,(-r+\mu))\right)(-1+\tau e)+\tau e\Big)==A\Big\}
\end{aligned}
$$

In[21]:= `% /. %16`

Out[21]=

$$\Bigg\{\frac{c\,(-1+\tau e)}{r}+\frac{F-F\,\tau e}{r-\mu}-$$

$$\Bigg((1+\lambda)\Bigg(\frac{1}{x1-x2}B^{1-x1-x2}\left(F^{x1}-F^{x2}\right)(-1+\alpha)\left(\frac{c\left(-B^{x1}\,x1+B^{x2}\left(B^{x1}\,(x1-x2)+x2\right)\right)}{r}+\right.$$

$$\left.\frac{B^{1+x1}\,(-1+x1)+B^{x2}\left(B-B\,x2+B^{x1}\,(-x1+x2)\right)}{r-\mu}\right)(-1+\tau e)+$$

$$\frac{B^{x2}\,c\left(-1+F^{x1}\right)(-1+\tau i)}{r}-\frac{B^{x1}\,c\left(-1+F^{x2}\right)(-1+\tau i)}{r}+\frac{c\left(-F^{x1}+F^{x2}\right)(-1+\tau i)}{r}\Bigg)\Bigg)\Big/$$

$$\left(-B\left(F^{x1}-F^{x2}\right)(-1+q)(-1+\alpha)+B^{x2}\left(-1+F^{x1}-\lambda\right)+B^{x1}\left(1-F^{x2}+\lambda\right)\right)==\frac{1}{r\,(x1-x2)\,(r-\mu)}$$

$$B^{-x1-x2}\left(B^{x1}\,F^{x2}\,(B\,(r-r\,x1)+c\,x1\,(r-\mu))+B^{x2}\,F^{x1}\,(B\,r\,(-1+x2)+c\,x2\,(-r+\mu))\right)(-1+\tau e)+$$

$$\Bigg(B^{-x1-x2}\,F$$

$$\left(-\frac{1}{r-\mu}\left(B^{1+x1}\,(-1+x1)+B^{x2}\left(B-B\,x2+B^{x1}\,(-x1+x2)\right)\right)\left(B^{x1}\left(-1+F^{x2}-\lambda\right)+B^{x2}\left(1-F^{x1}+\lambda\right)\right)\right.$$

$$(-1+\tau e)+\frac{1}{r}c\left(B^{2\,x2}\,x2\left(-1+F^{x1}-\lambda\right)(-1+\tau e)+B^{2\,x1}\,x1\left(-1+F^{x2}-\lambda\right)(-1+\tau e)+\right.$$

$$B^{x1+2\,x2}\,(x1-x2)\left(\lambda-\lambda\,\tau e+\left(-1+F^{x1}\right)(\tau e-\tau i)+\left(-1+F^{x1}\right)q\,(-1+\tau i)\right)-$$

$$B^{2\,x1+x2}\,(x1-x2)\left(\lambda-\lambda\,\tau e+\left(-1+F^{x2}\right)(\tau e-\tau i)+\left(-1+F^{x2}\right)q\,(-1+\tau i)\right)+$$

$$B^{x1+x2}\,\Big(x1\left((1+\lambda)(-1+\tau e)+F^{x2}\,(-1+q)(-1+\tau i)+F^{x1}\,(q-\tau e+\tau i-q\,\tau i)\right)+$$

$$\left.\left.x2\left((1+\lambda)(-1+\tau e)+F^{x1}\,(-1+q)(-1+\tau i)+F^{x2}\,(q-\tau e+\tau i-q\,\tau i)\right)\Big)\right)\right)\Bigg)\Big/$$

$$\left((x1-x2)\left(-B\left(F^{x1}-F^{x2}\right)(-1+q)(-1+\alpha)+B^{x2}\left(-1+F^{x1}-\lambda\right)+B^{x1}\left(1-F^{x2}+\lambda\right)\right)\right),$$

$$-\frac{1}{r-\mu}\Bigg(-1+\frac{1}{F\,r\,(x1-x2)}B^{-x1-x2}\left(-B^{x1}\,F^{x2}\,x2\,(B\,r\,(-1+x1)+c\,x1\,(-r+\mu))+\right.$$

$$\left.B^{x2}\,F^{x1}\,x1\,(B\,r\,(-1+x2)+c\,x2\,(-r+\mu))\right)(-1+\tau e)+\tau e\Bigg)==$$

$$\Bigg(B^{-x1-x2}\left(-\frac{1}{r-\mu}\left(B^{1+x1}\,(-1+x1)+B^{x2}\left(B-B\,x2+B^{x1}\,(-x1+x2)\right)\right)\left(B^{x1}\left(-1+F^{x2}-\lambda\right)+B^{x2}\left(1-F^{x1}+\lambda\right)\right)\right.$$

$$(-1+\tau e)+\frac{1}{r}c\left(B^{2\,x2}\,x2\left(-1+F^{x1}-\lambda\right)(-1+\tau e)+B^{2\,x1}\,x1\left(-1+F^{x2}-\lambda\right)(-1+\tau e)+\right.$$

$$B^{x1+2\,x2}\,(x1-x2)\left(\lambda-\lambda\,\tau e+\left(-1+F^{x1}\right)(\tau e-\tau i)+\left(-1+F^{x1}\right)q\,(-1+\tau i)\right)-$$

$$B^{2\,x1+x2}\,(x1-x2)\left(\lambda-\lambda\,\tau e+\left(-1+F^{x2}\right)(\tau e-\tau i)+\left(-1+F^{x2}\right)q\,(-1+\tau i)\right)+$$

$$B^{x1+x2}\,\Big(x1\left((1+\lambda)(-1+\tau e)+F^{x2}\,(-1+q)(-1+\tau i)+F^{x1}\,(q-\tau e+\tau i-q\,\tau i)\right)+$$

$$\left.\left.x2\left((1+\lambda)(-1+\tau e)+F^{x1}\,(-1+q)(-1+\tau i)+F^{x2}\,(q-\tau e+\tau i-q\,\tau i)\right)\Big)\right)\right)\Bigg)\Big/$$

$$\left((x1-x2)\left(-B\left(F^{x1}-F^{x2}\right)(-1+q)(-1+\alpha)+B^{x2}\left(-1+F^{x1}-\lambda\right)+B^{x1}\left(1-F^{x2}+\lambda\right)\right)\right)\Bigg\}$$

```
In[22]:= FullSimplify[%]
```

Out[22]=

$$
\begin{aligned}
&\Big\{\frac{c\,(-1+\tau e)}{r}+\frac{F-F\,\tau e}{r-\mu}==\\
&\Bigg((1+\lambda)\Bigg(\frac{1}{x1-x2}B^{1-x1-x2}\left(F^{x1}-F^{x2}\right)(-1+\alpha)\Bigg(\frac{c\left(-B^{x1}\,x1+B^{x2}\left(B^{x1}\,(x1-x2)+x2\right)\right)}{r}+\\
&\qquad\frac{B^{1+x1}\,(-1+x1)+B^{x2}\left(B-B\,x2+B^{x1}\,(-x1+x2)\right)}{r-\mu}\Bigg)(-1+\tau e)+\\
&\qquad\frac{B^{x2}\,c\left(-1+F^{x1}\right)(-1+\tau i)}{r}-\frac{B^{x1}\,c\left(-1+F^{x2}\right)(-1+\tau i)}{r}+\frac{c\left(-F^{x1}+F^{x2}\right)(-1+\tau i)}{r}\Bigg)\Bigg)\Big/\\
&\left(-B\left(F^{x1}-F^{x2}\right)(-1+q)(-1+\alpha)+B^{x2}\left(-1+F^{x1}-\lambda\right)+B^{x1}\left(1-F^{x2}+\lambda\right)\right)+\frac{1}{x1-x2}B^{-x1-x2}\\
&\Bigg(\frac{1}{r\,(r-\mu)}\left(B^{x1}\,F^{x2}\,(B\,(r-r\,x1)+c\,x1\,(r-\mu))+B^{x2}\,F^{x1}\,(B\,r\,(-1+x2)+c\,x2\,(-r+\mu))\right)(-1+\tau e)+\\
&\quad\Bigg(F\Bigg(-\frac{1}{r-\mu}\left(B^{1+x1}\,(-1+x1)+B^{x2}\left(B-B\,x2+B^{x1}\,(-x1+x2)\right)\right)\left(B^{x1}\left(-1+F^{x2}-\lambda\right)+B^{x2}\left(1-F^{x1}+\lambda\right)\right)\\
&\qquad(-1+\tau e)+\frac{1}{r}c\,\Big(B^{2\,x2}\,x2\left(-1+F^{x1}-\lambda\right)(-1+\tau e)+B^{2\,x1}\,x1\left(-1+F^{x2}-\lambda\right)(-1+\tau e)+\\
&\qquad B^{x1+2\,x2}\,(x1-x2)\left(\lambda-\lambda\,\tau e+\left(-1+F^{x1}\right)(\tau e-\tau i)+\left(-1+F^{x1}\right)q\,(-1+\tau i)\right)-\\
&\qquad B^{2\,x1+x2}\,(x1-x2)\left(\lambda-\lambda\,\tau e+\left(-1+F^{x2}\right)(\tau e-\tau i)+\left(-1+F^{x2}\right)q\,(-1+\tau i)\right)+\\
&\qquad B^{x1+x2}\,\Big(x1\left((1+\lambda)(-1+\tau e)+F^{x2}\,(-1+q)(-1+\tau i)+F^{x1}\,(q-\tau e+\tau i-q\,\tau i)\right)+\\
&\qquad\quad x2\left((1+\lambda)(-1+\tau e)+F^{x1}\,(-1+q)(-1+\tau i)+F^{x2}\,(q-\tau e+\tau i-q\,\tau i)\right)\Big)\Big)\Bigg)\Bigg)\Big/\\
&\quad\left(-B\left(F^{x1}-F^{x2}\right)(-1+q)(-1+\alpha)+B^{x2}\left(-1+F^{x1}-\lambda\right)+B^{x1}\left(1-F^{x2}+\lambda\right)\right)\Bigg),\\
&-\frac{1}{r-\mu}\Bigg(-1+\frac{1}{F\,r\,(x1-x2)}B^{-x1-x2}\left(-B^{x1}\,F^{x2}\,x2\,(B\,r\,(-1+x1)+c\,x1\,(-r+\mu))+\right.\\
&\qquad\left.B^{x2}\,F^{x1}\,x1\,(B\,r\,(-1+x2)+c\,x2\,(-r+\mu))\right)(-1+\tau e)+\tau e\Bigg)==\\
&\Bigg(B^{-x1-x2}\Bigg(-\frac{1}{r-\mu}\left(B^{1+x1}\,(-1+x1)+B^{x2}\left(B-B\,x2+B^{x1}\,(-x1+x2)\right)\right)\left(B^{x1}\left(-1+F^{x2}-\lambda\right)+B^{x2}\left(1-F^{x1}+\lambda\right)\right)\\
&\qquad(-1+\tau e)+\frac{1}{r}c\,\Big(B^{2\,x2}\,x2\left(-1+F^{x1}-\lambda\right)(-1+\tau e)+B^{2\,x1}\,x1\left(-1+F^{x2}-\lambda\right)(-1+\tau e)+\\
&\qquad B^{x1+2\,x2}\,(x1-x2)\left(\lambda-\lambda\,\tau e+\left(-1+F^{x1}\right)(\tau e-\tau i)+\left(-1+F^{x1}\right)q\,(-1+\tau i)\right)-\\
&\qquad B^{2\,x1+x2}\,(x1-x2)\left(\lambda-\lambda\,\tau e+\left(-1+F^{x2}\right)(\tau e-\tau i)+\left(-1+F^{x2}\right)q\,(-1+\tau i)\right)+\\
&\qquad B^{x1+x2}\,\Big(x1\left((1+\lambda)(-1+\tau e)+F^{x2}\,(-1+q)(-1+\tau i)+F^{x1}\,(q-\tau e+\tau i-q\,\tau i)\right)+\\
&\qquad\quad x2\left((1+\lambda)(-1+\tau e)+F^{x1}\,(-1+q)(-1+\tau i)+F^{x2}\,(q-\tau e+\tau i-q\,\tau i)\right)\Big)\Big)\Bigg)\Bigg)\Big/\\
&\left((x1-x2)\left(-B\left(F^{x1}-F^{x2}\right)(-1+q)(-1+\alpha)+B^{x2}\left(-1+F^{x1}-\lambda\right)+B^{x1}\left(1-F^{x2}+\lambda\right)\right)\right)\Big\}
\end{aligned}
$$

15. The input to the tests

Below is the complete list of tested companies, and their sample values respectively. An interesting observation here is to look at the level of the standard deviation for the companies over the sample period. They are in average very high, and for all the US companies the average is 2.8 and 7.6 for the Norwegians'. This means that the average standard deviation for the companies in the samples is 280% for the US companies and 760% for the Norwegian companies. In other words, EBIT is a lot more volatile than the commonly used in academia and articles trying to explain capital structure decisions mathematically.

<10	Name	r-post tax	Stdev EBIT	My	Alpha	IssueCost	Lambda	Teff	Ti	Average C	StdevC	"PE"	Stdev "PE"	Obser EBIT
	USA									Average	Stdev	Average	Stdev	
	CHEVRON	0,043689	0,7130	0,01	0,07	0,01	0	0,498	0,35	0,15	0,11	9,03	5,78	24
	EXXON MOBIL	0,043689	0,3805	0,01	0,07	0,01	0	0,498	0,35	0,08	0,05	9,75	4,87	24
	CABOT OIL & GAS 'A'	0,043689	2,2023	0,01	0,07	0,01	0	0,498	0,35	0,38	0,28	13,60	6,99	18
	CHESAPEAKE ENERGY	0,043689	2,1022	0,01	0,07	0,01	0	0,498	0,35	0,42	0,32	9,65	10,75	15
	TESORO	0,043689	4,9222	0,01	0,07	0,01	0	0,498	0,35	0,45	0,44	5,50	6,20	24
	XTO EN.	0,043689	3,4292	0,01	0,07	0,01	0	0,498	0,35	0,32	0,28	10,49	12,19	16
	ANADARKO PETROLEUM	0,043689	5,8294	0,01	0,07	0,01	0	0,498	0,35	0,39	0,25	15,31	10,80	22
	SUNCOR ENERGY INCO.(NYS)	0,043689	1,6896	0,01	0,07	0,01	0	0,498	0,35	0,30	0,55	9,56	4,39	24
	BAKER HUGHES	0,043689	2,3702	0,01	0,07	0,01	0	0,498	0,35	0,26	0,19	15,08	10,29	24
	HALLIBURTON	0,043689	1,6325	0,01	0,07	0,01	0	0,498	0,35	0,22	0,18	19,59	13,25	24
	SCHLUMBERGER	0,043689	0,9549	0,01	0,07	0,01	0	0,498	0,35	0,14	0,10	20,70	9,06	24
	WEATHERFORD INTL.	0,043689	1,7958	0,01	0,07	0,01	0	0,498	0,35	0,39	0,37	21,47	22,42	22
	SMITH INTL.	0,043689	4,8911	0,01	0,07	0,01	0	0,498	0,35	0,64	1,27	20,42	28,83	24
	STANDARD ENERGY	0,043689	1,5084	0,01	0,07	0,01	0	0,498	0,35	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	15
	GLOBAL INDS.	0,043689	2,1942	0,01	0,07	0,01	0	0,498	0,35	0,67	1,62	48,91	104,22	15
	PARKER DRILLING	0,043689	1,9377	0,01	0,07	0,01	0	0,498	0,35	1,20	1,37	21,98	24,16	24
	DIAMOND OFFS.DRL.	0,043689	5,5119	0,01	0,07	0,01	0	0,498	0,35	0,30	0,60	25,07	27,80	13
	NOBLE	0,043689	3,1618	0,01	0,07	0,01	0	0,498	0,35	0,72	1,65	21,47	20,05	17
	PIONEER DRILLING	0,043689	10,7028	0,01	0,07	0,01	0	0,498	0,35	0,86	1,69	53,15	98,40	10
	ENSCO INTL.	0,043689	1,6563	0,01	0,07	0,01	0	0,498	0,35	0,22	0,30	21,00	23,98	17
	NABORS INDS.	0,043689	1,4490	0,01	0,07	0,01	0	0,498	0,35	0,18	0,15	16,33	10,32	22
	TRANSOCEAN	0,043689	1,8785	0,01	0,07	0,01	0	0,498	0,35	0,28	0,28	22,45	17,52	15
	PRIDE INTL.	0,043689	2,3221	0,01	0,07	0,01	0	0,498	0,35	0,37	0,34	11,26	5,34	17
	KIRBY	0,043689	5,2606	0,01	0,07	0,01	0	0,498	0,35	0,27	0,16	10,44	3,45	24
	TIDEWATER	0,043689	1,3412	0,01	0,07	0,01	0	0,498	0,35	0,55	1,60	18,89	14,76	24
	TEEKAY	0,043689	2,7232	0,01	0,07	0,01	0	0,498	0,35	0,65	0,64	7,41	3,77	12
	MICROSOFT	0,043689	0,2553	0,01	0,07	0,01	0	0,498	0,35	0,00091	0,00	21,50	6,56	19
	ORACLE	0,043689	2,1200	0,01	0,07	0,01	0	0,498	0,35	0,12	0,44	21,90	20,61	24
	AT&T	0,043689	0,3676	0,01	0,07	0,01	0	0,498	0,35	0,21	0,06	9,06	3,13	24
	VERIZON COMMUNICATIONS	0,043689	0,4122	0,01	0,07	0,01	0	0,498	0,35	0,23	0,06	7,70	2,72	24
	MOTOROLA	0,043689	19,5647	0,01	0,07	0,01	0	0,498	0,35	0,19	0,13	15,10	8,12	24
	APPLE	0,043689	1,9271	0,01	0,07	0,01	0	0,498	0,35	0,05	0,05	24,76	22,60	20
	CISCO SYSTEMS	0,043689	2,4678	0,01	0,07	0,01	0	0,498	0,35	0,0042	0,01	27,74	24,66	20
	INTERNATIONAL BUS.MCHS.	0,043689	1,2914	0,01	0,07	0,01	0	0,498	0,35	0,11	0,20	11,43	7,01	24
	HEWLETT-PACKARD	0,043689	1,1800	0,01	0,07	0,01	0	0,498	0,35	0,08	0,05	15,72	10,61	21
	ALCOA	0,043689	0,9674	0,01	0,07	0,01	0	0,498	0,35	0,33	0,57	11,85	8,83	24
	CONT.AIRL.B	0,043689	5,2395	0,01	0,07	0,01	0	0,498	0,35	1,27	1,72	3,42	2,53	21
	SOUTHWEST AIRLINES	0,043689	0,5140	0,01	0,07	0,01	0	0,498	0,35	0,23	0,15	12,52	5,79	24
	DELTA AIR LINES	0,043689	2,4108	0,01	0,07	0,01	0	0,498	0,35	0,31	0,32	#DIV/0!	#DIV/0!	24
	LEGG MASON	0,043689	0,3809	0,01	0,07	0,01	0	0,498	0,35	0,30	0,14	7,35	3,66	24
	HONEYWELL INTL.	0,043689	1,4576	0,01	0,07	0,01	0	0,498	0,35	0,29	0,41	17,04	37,53	24
	POTASH CORPORATION (NYS)	0,043689	2,4715	0,01	0,07	0,01	0	0,498	0,35	0,3454996	0,777358	9,47	2,90	21

Appendix Table 1 – The figures used for input to the model. This is the processed values from the sample period for the US companies.

<10	Name	r-post tax	Stdev EBIT	My	Alpha	IssueCost	Lambda	Teff	Ti	Average C	StdevC	"PE"	Stdev "PE"	Obser EBIT
	Norway									Average	Stdev	Average	Stdev	
*	AKER SOLUTIONS	0,047877	4,5234	µ	0,01	0,01	0	0,4816	0,28	1,25	2,29	11,28	3,24	7
	NORSK HYDRO	0,047877	0,7710	µ	0,01	0,01	0	0,4816	0,28	0,29	0,26	6,31	4,85	24
	ORKLA	0,047877	0,5186	µ	0,01	0,01	0	0,4816	0,28	0,35	0,18	5,67	2,38	23
*	YARA INTERNATIONAL	0,047877	0,3206	µ	0,01	0,01	0	0,4816	0,28	0,10	0,03	7,97	3,64	7
*	AKER	0,047877	1,5328	µ	0,01	0,01	0	0,4816	0,28	0,56	0,54	4,31	2,56	7
	ROYAL CRBN.CRUISES (OSL)	0,047877	0,2320	µ	0,01	0,01	0	0,4816	0,28	0,41	0,08	64,92	19,83	12
	SCHIBSTED	0,047877	1,2384	µ	0,01	0,01	0	0,4816	0,28	0,14	0,13	13,07	5,84	19
	TOMRA SYSTEMS	0,047877	1,5575	µ	0,01	0,01	0	0,4816	0,28	0,11	0,10	19,26	10,03	21
	INTL.GOLD EXP.IGE	0,047877	2,5895	µ	0,01	0,01	0	0,4816	0,28	0,05	0,07	7,60	4,96	11
	NORSKE SKOGINDUSTRIER	0,047877	2,3706	µ	0,01	0,01	0	0,4816	0,28	0,50	0,25	5,84	3,98	24
	KONGSBERG GRUPPEN	0,047877	1,8247	µ	0,01	0,01	0	0,4816	0,28	0,35	0,38	13,02	14,77	15
	VEIDEKKE	0,047877	0,6490	µ	0,01	0,01	0	0,4816	0,28	0,38	0,24	6,00	3,40	21
	EKORNES	0,047877	0,1988	µ	0,01	0,01	0	0,4816	0,28	0,05	0,06	8,50	2,42	15
*	NORWEGIAN AIR SHUTTLE	0,047877	5,3635	µ	0,01	0,01	0	0,4816	0,28	0,18	0,17	35,61	34,42	7
	RIEBER & SON	0,047877	0,3139	µ	0,01	0,01	0	0,4816	0,28	0,23	0,14	7,00	4,28	24
	STATOILHYDRO	0,047877	0,5320	µ	0,01	0,01	0	0,4816	0,28	0,19	0,21	2,85	0,30	24
*	PA RESOURCES (OSL)	0,047877	1,0815	µ	0,01	0,01	0	0,4816	0,28	0,17	0,14	4,36	2,20	7
	DNO INTERNATIONAL	0,047877	13,2372	µ	0,01	0,01	0	0,4816	0,28	0,50	0,70	11,83	12,74	20
	CANARGO ENERGY (OSL)	0,047877	1,9599	µ	0,01	0,01	0	0,4816	0,28	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	14
	HAFSLUND 'A'	0,047877	192,3633	µ	0,01	0,01	0	0,4816	0,28	0,45	0,36	4,68	2,39	24
	FRED OLSEN ENERGY	0,047877	5,1450	µ	0,01	0,01	0	0,4816	0,28	0,51	0,61	17,77	12,58	11
	ACERGY	0,047877	5,3824	µ	0,01	0,01	0	0,4816	0,28	0,27	0,30	89,42	51,13	12
	PROSAFE	0,047877	1,4200	µ	0,01	0,01	0	0,4816	0,28	0,24	0,15	9,69	4,42	12
	PETROLEUM GEO SERVICES	0,047877	1,7108	µ	0,01	0,01	0	0,4816	0,28	0,43	0,47	10,39	5,51	18
*	SUBSEA 7	0,047877	5,2864	µ	0,01	0,01	0	0,4816	0,28	1,04	2,15	9,15	0,34	8
	TGS-NOPEC GEOPHS.	0,047877	0,6994	µ	0,01	0,01	0	0,4816	0,28	0,05	0,04	7,32	1,67	12
	PETROLIA DRILLING	0,047877	5,8793	µ	0,01	0,01	0	0,4816	0,28	1,62	1,58	43,37	37,96	11
*	GLOBAL GEO SERVICES	0,047877	2,9196	µ	0,01	0,01	0	0,4816	0,28	2,61	4,24	172,11	235,63	9
	NORSE ENERGY CORP.	0,047877	8,3028	µ	0,01	0,01	0	0,4816	0,28	1,82	1,37	24,19	22,20	10
	STOLT-NIELSEN	0,047877	1,6950	µ	0,01	0,01	0	0,4816	0,28	0,40	0,24	51,06	28,14	13
	WILHS.WILHELMSEN 'A'	0,047877	1,1403	µ	0,01	0,01	0	0,4816	0,28	0,66	0,55	4,19	3,61	24
*	FRONTLINE	0,047877	2,0640	µ	0,01	0,01	0	0,4816	0,28	0,29	0,24	28,24	22,02	8
	GREEN REEFERS	0,047877	11,9328	µ	0,01	0,01	0	0,4816	0,28	1,12	1,39	17,06	20,54	20
*	DOF	0,047877	0,5861	µ	0,01	0,01	0	0,4816	0,28	0,56	0,29	4,23	2,18	9
	SOLSTAD OFFSHORE	0,047877	0,7409	µ	0,01	0,01	0	0,4816	0,28	0,40	0,61	7,87	10,58	12
	BELSHIPS	0,047877	1,9154	µ	0,01	0,01	0	0,4816	0,28	0,69	0,75	7,41	6,50	20
	MARINE HARVEST	0,047877	4,1311	µ	0,01	0,01	0	0,4816	0,28	0,56	0,39	12,14	13,31	12
	TELENOR	0,047877	1,4888	µ	0,01	0,01	0	0,4816	0,28	0,19	0,12	7,18	1,08	10
	TANDBERG	0,047877	3,6838	µ	0,01	0,01	0	0,4816	0,28	0,80	1,85	20,94	20,95	21
	ELTEK	0,047877	3,6607	µ	0,01	0,01	0	0,4816	0,28	0,26	0,30	18,15	17,65	12
*	OPERA SOFTWARE	0,047877	16,8029	µ	0,01	0,01	0	0,4816	0,28	0,00	0,00	74,14	87,43	8
	EDB BUSINESS PARTNER	0,047877	7,7973	µ	0,01	0,01	0	0,4816	0,28	0,20	0,16	9,88	3,36	12
*	TELECOMPUTING	0,047877	3,5026	µ	0,01	0,01	0	0,4816	0,28	0,15	0,13	25,36	6,68	9
	ATEA	0,047877	4,1252	µ	0,01	0,01	0	0,4816	0,28	0,15	0,09	8,24	5,57	18

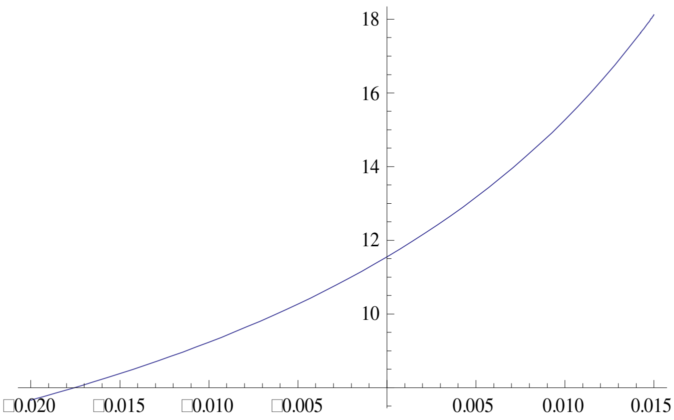
Appendix Table 2 - The figures used for input to the model. This is the processed values from the sample period for the Norwegian companies.

16. The output from the tests

Below is all the output from the empirical testing. I have commented each figure, and added the corresponding output for each company possible to test.

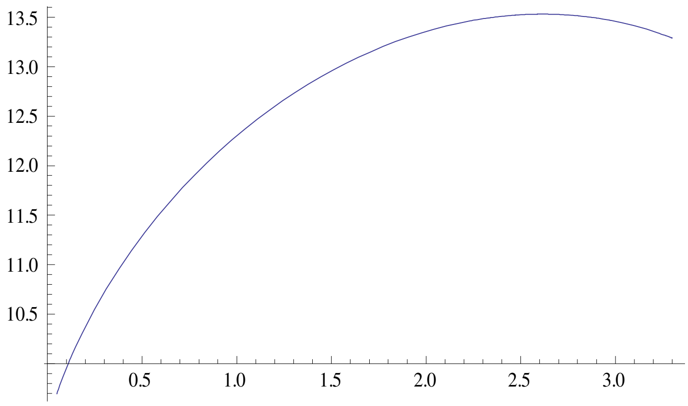
The US companies

CHEVRON



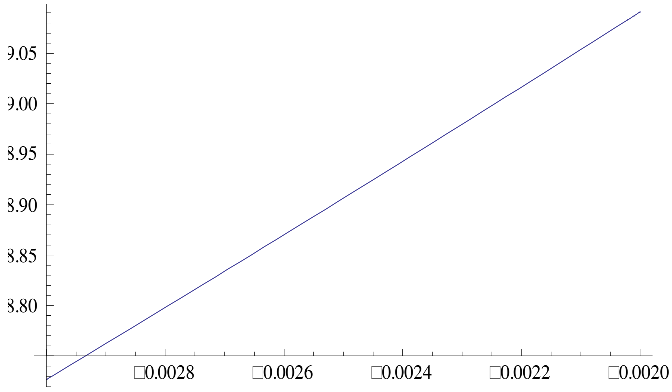
Appendix figure 1 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

μ	B	F	Debt-Low	A	P
−0,0100	0,0223	1,2797	0,0728	10,1755	0,9482



Appendix Figure 2 – The X-axis is the coupon relative to EBIT and the Y-axis the corresponding PE ratio.

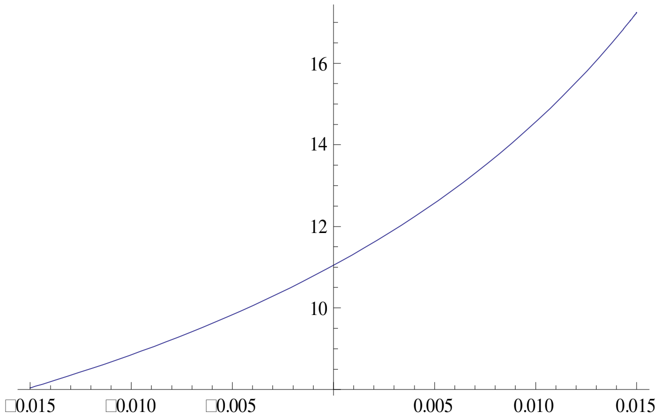
C	B	F	Debt-Low	A	P
2,6000	0,2635	1,1887	0,4320	13,5311	6,9490



Appendix Figure 3 - The X-axis is the growth rate (My), and the Y-axis is the PE ratio

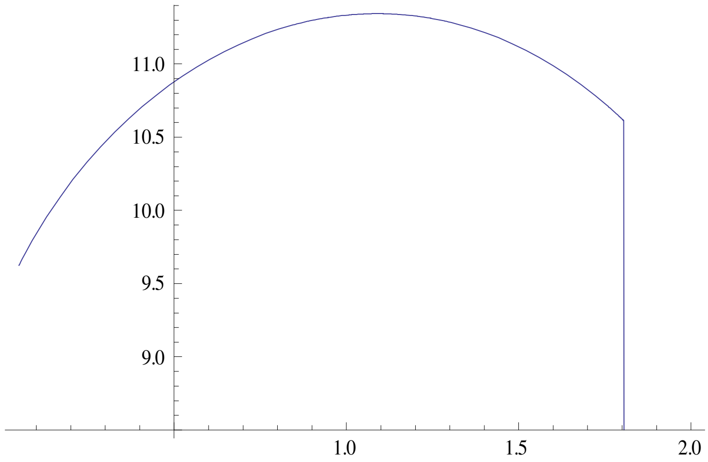
μ	B	F	Debt-Low	A	P
−0,0020	0,2234	1,2003	0,3868	16,8253	7,8125

EXXON MOBIL



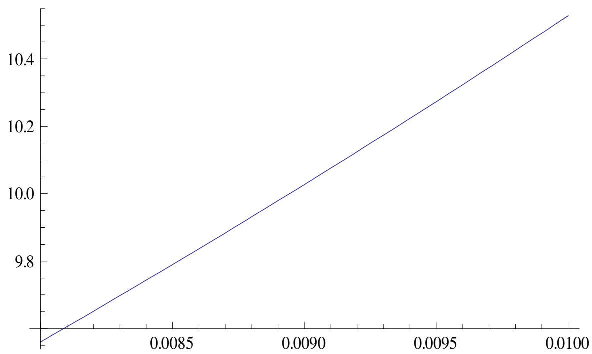
Appendix figure 4 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

μ	B	F	Debt-Low	A	P
−0,0050	0,0257	1,2740	0,0682	10,7605	0,9354



Appendix Figure 5 – The X-axis is the coupon relative to EBIT and the Y-axis the corresponding PE ratio

C	B	F	Debt-Low	A	P
1,2000	0,3208	1,1768	0,4829	12,6992	7,2171

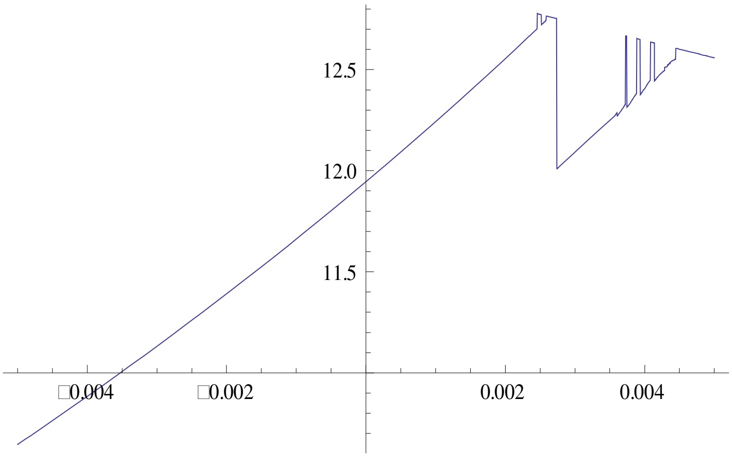


Appendix figure 6 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

μ	B	F	Debt-Low	A	P
0,0084	0,2460	1,2042	0,3950	18,4136	8,7578

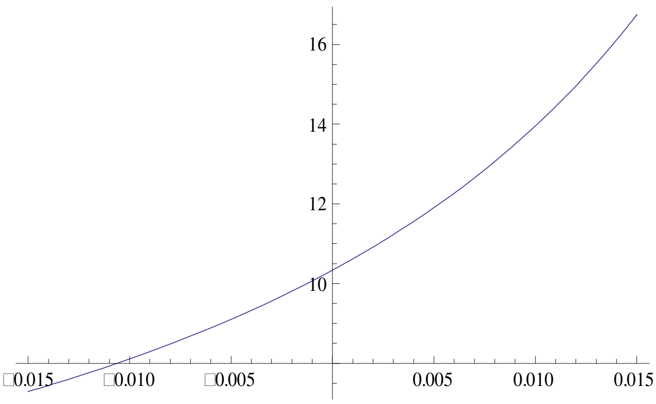
SCHLUMBERGER

$$\text{Plot}[A - (1 - q) * P /. \text{EBITGJLc}[0.043689256, 0.9549, \mu, 0.07, 0.01, 0, 0.498, 0.35, 0.14], \{\mu, -0.005, 0.005\}]$$



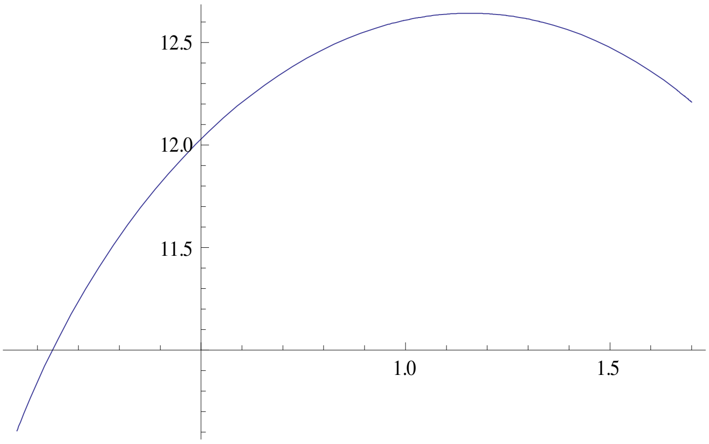
Appendix figure 7 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio. There were no solution outside a interval of My being -0.005 and 0.005 for Schlumberger, while PE of Schlumberger was 20.7. A corresponding growth rate to the PE could therefore not be found.

AT&T



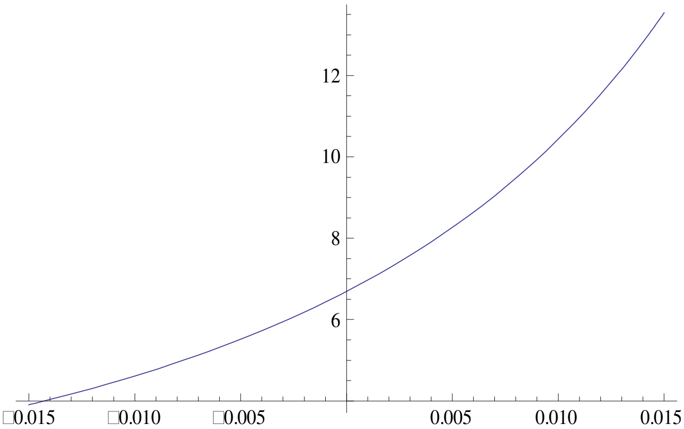
Appendix figure 8 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

μ	B	F	Debt-Low	A	P
-0,0050	0,0688	1,2501	0,1555	11,2726	2,1909



Appendix Figure 9 – The X-axis is the coupon relative to EBIT and the Y-axis the corresponding PE ratio

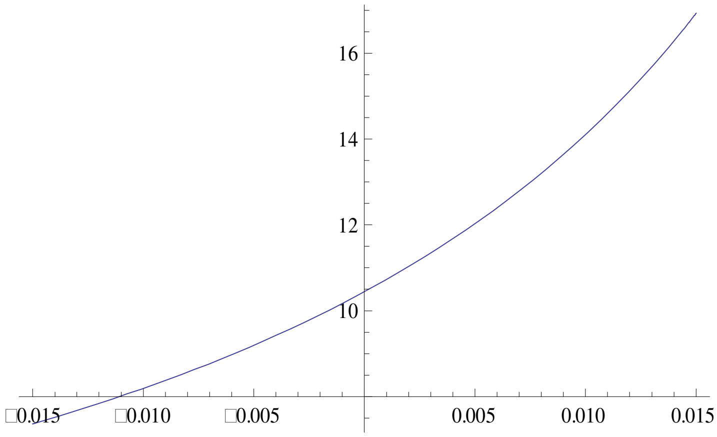
C	B	F	Debt-Low	A	P
1,1500	0,3223	1,1765	0,4840	12,6425	7,1983



Appendix figure 10 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

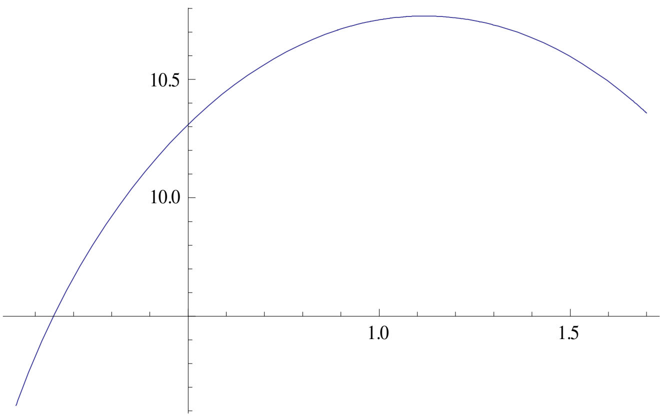
μ	B	F	Debt-Low	A	P
0,0070	0,2562	1,2009	0,4062	17,4936	8,5328

VERIZON COMMUNICATION



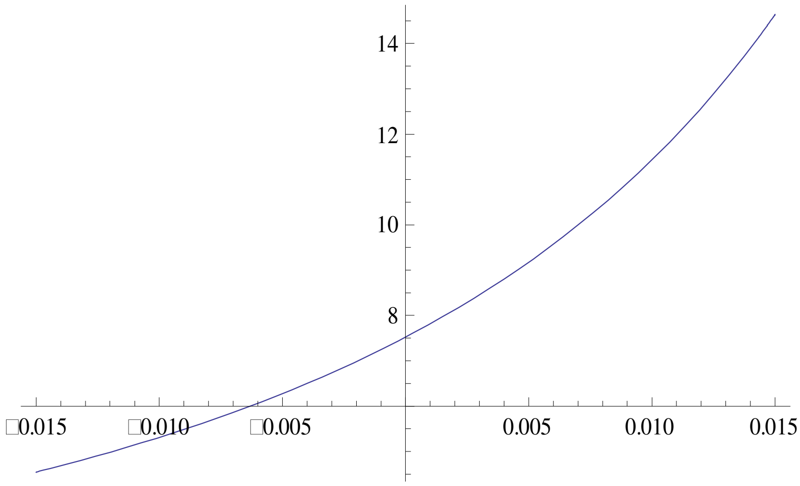
Appendix figure 11 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

μ	B	F	Debt-Low	A	P
−0,0130	0,0742	1,2408	0,1727	9,7459	2,0882



Appendix Figure 12 – The X-axis is the coupon relative to EBIT and the Y-axis the corresponding PE ratio

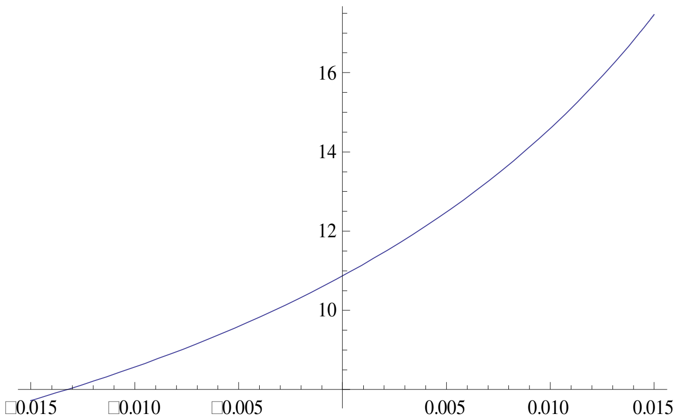
C	B	F	Debt-Low	A	P
1,1000	0,3057	1,1762	0,4735	10,7677	5,9960



Appendix figure 13 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

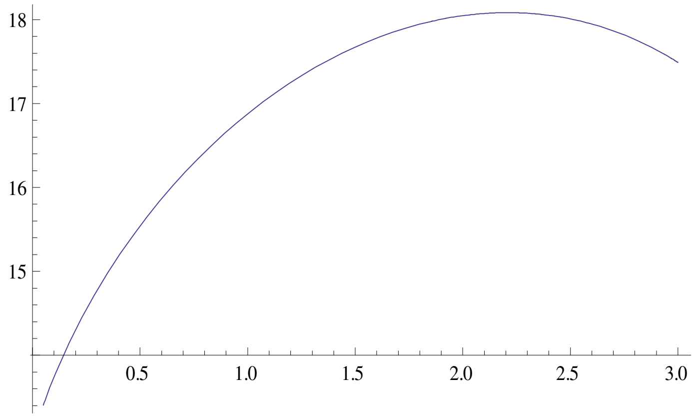
μ	B	F	Debt-Low	A	P
0,0010	0,2436	1,1985	0,3997	14,8677	7,1214

SOUTHWEST AIRLINES



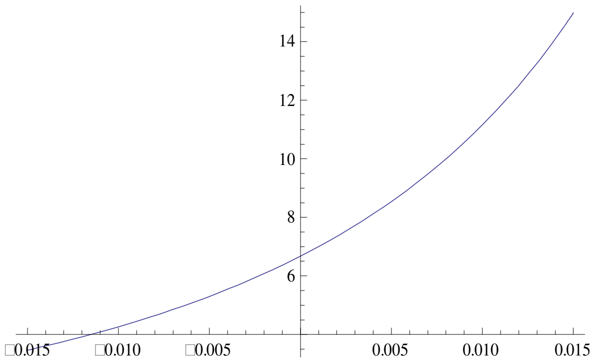
Appendix figure 14 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

μ	B	F	Debt-Low	A	P
0,0050	0,0409	1,2750	0,1078	14,4523	1,9868



Appendix Figure 15 – The X-axis is the coupon relative to EBIT and the Y-axis the corresponding PE ratio

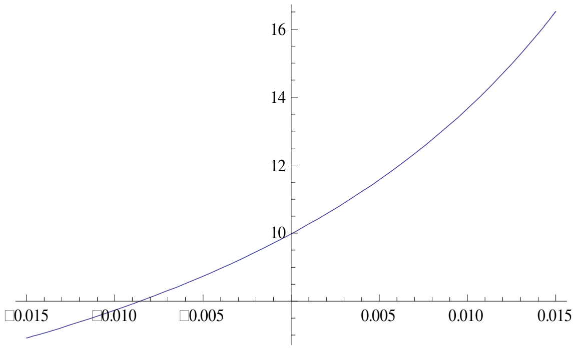
C	B	F	Debt-Low	A	P
2,2000	0,2869	1,1880	0,4486	18,0839	9,6382



Appendix figure 16 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

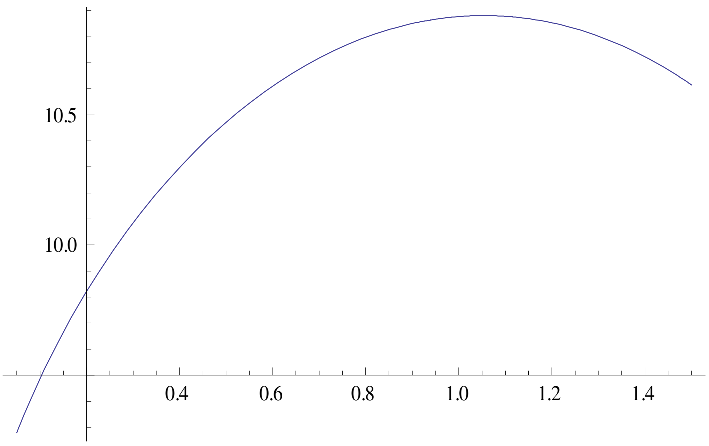
μ	B	F	Debt-Low	A	P
0,0120	0,2360	1,2039	0,3913	23,4417	11,0428

LEGG MASON



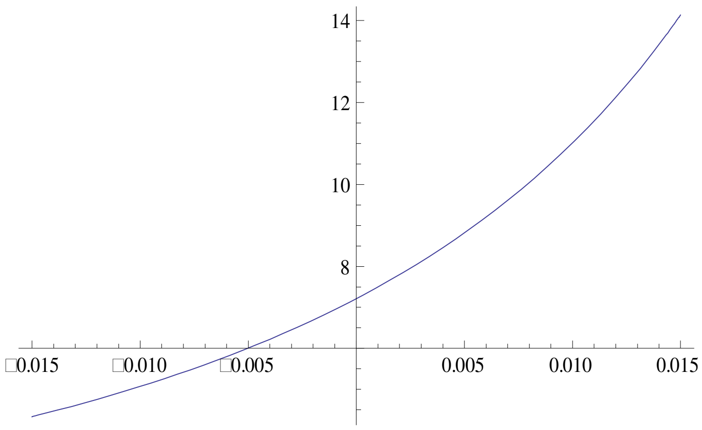
Appendix figure 17 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

μ	B	F	Debt-Low	A	P
−0,0120	0,1033	1,2294	0,2197	10,0856	2,7238



Appendix Figure 18 – The X-axis is the coupon relative to EBIT and the Y-axis the corresponding PE ratio

C	B	F	Debt-Low	A	P
1,0500	0,3180	1,1735	0,4847	10,8813	6,1887

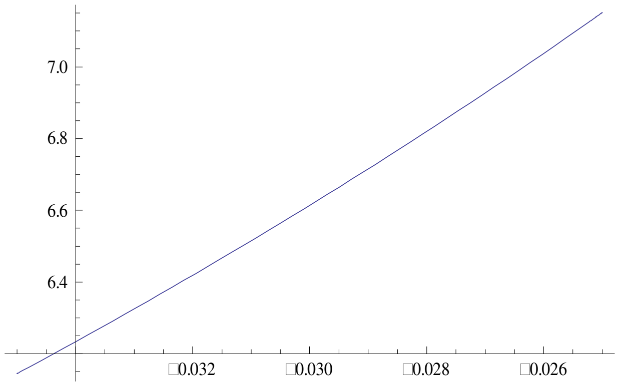


Appendix figure 19 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

μ	B	F	Debt-Low	A	P
0,0000	0,2633	1,1934	0,4201	14,3229	7,1801

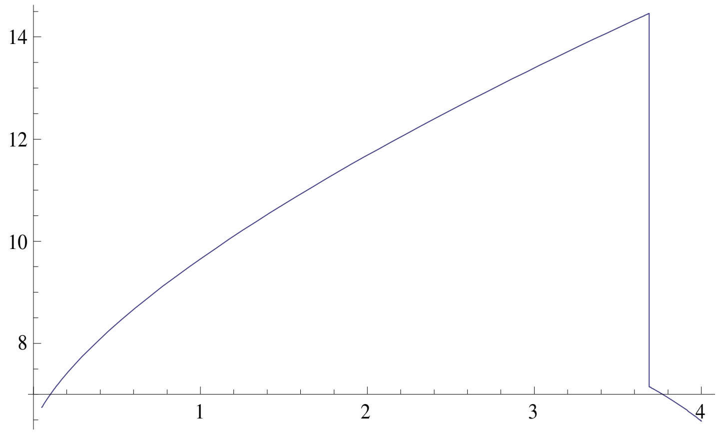
The Norwegian companies

NORSK HYDRO



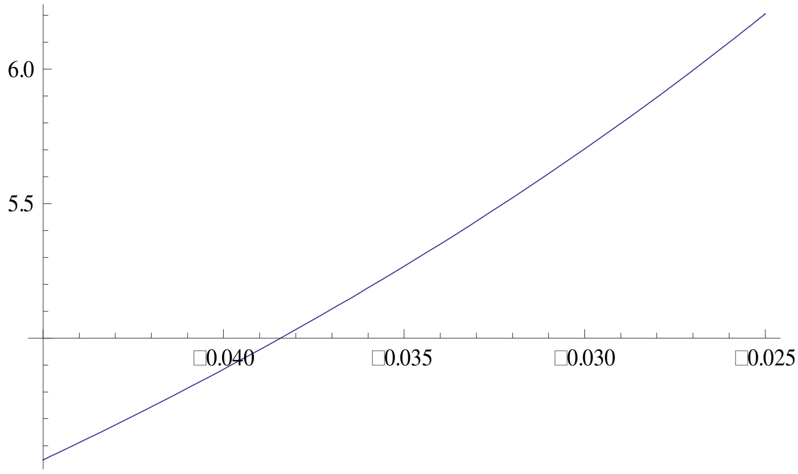
Appendix figure 20 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

μ	B	F	Debt-Low	A	P
−0,0330	0,0508	1,2347	0,1491	7,7347	1,4241



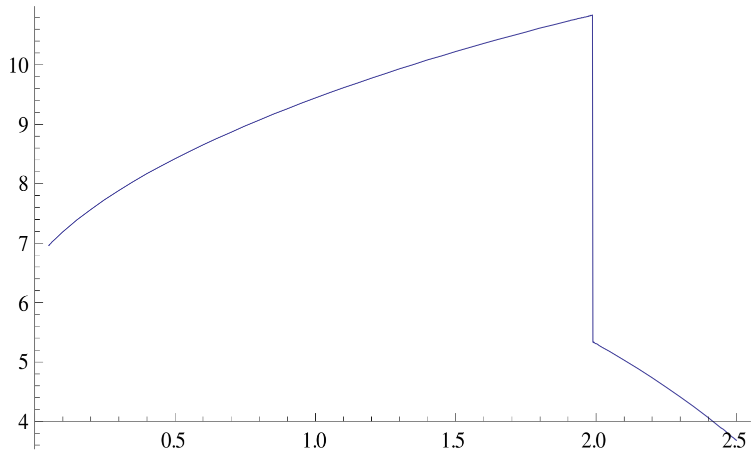
Appendix Figure 21 – The X-axis is the coupon relative to EBIT and the Y-axis the corresponding PE ratio. There is no optimal solution with a C*.

ORKLA



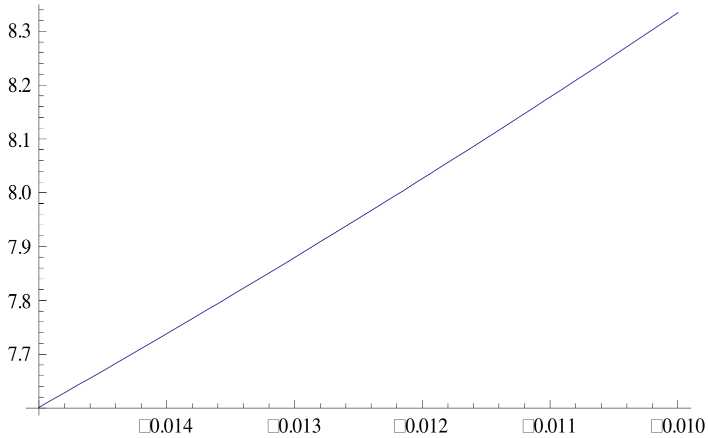
Appendix figure 22 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

μ	B	F	Debt-Low	A	P
−0,0300	0,1033	1,2084	0,2424	8,0325	2,3529



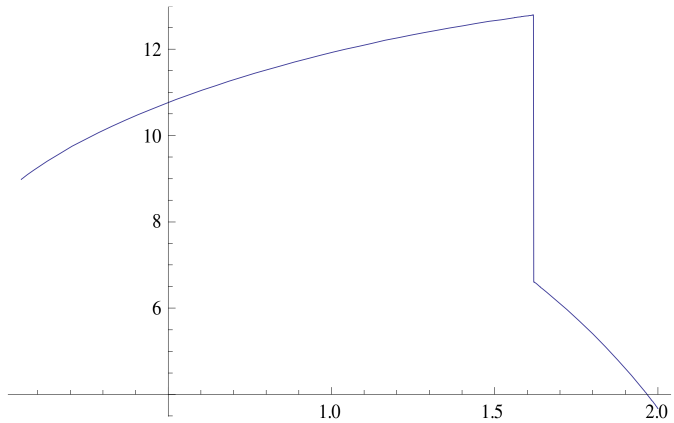
Appendix Figure 23 – The X-axis is the coupon relative to EBIT and the Y-axis the corresponding PE ratio. There is no optimal solution with a C*.

YARA INTERNATIONAL



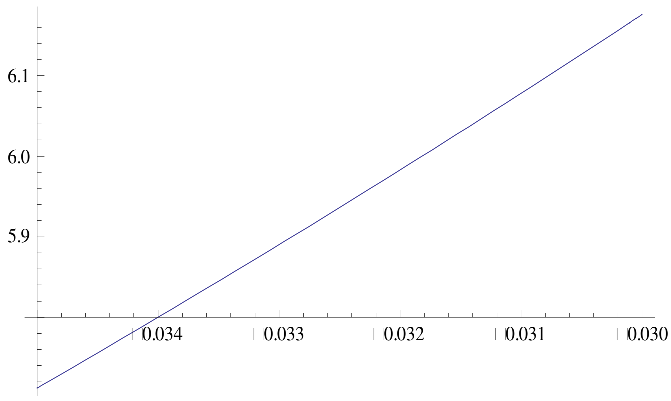
Appendix figure 24 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

μ	B	F	Debt-Low	A	P
−0,0120	0,0430	1,2256	0,1096	9,2575	1,2439



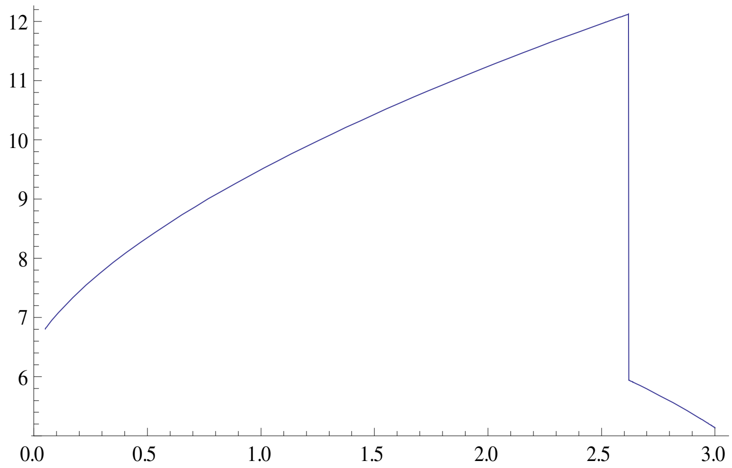
Appendix Figure 25 – The X-axis is the coupon relative to EBIT and the Y-axis the corresponding PE ratio. There is no optimal solution with a C*.

VEIDEKKE



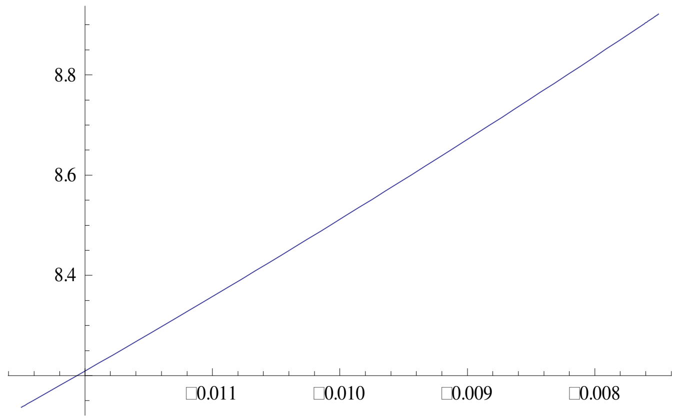
Appendix figure 26 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

μ	B	F	Debt-Low	A	P
−0,0320	0,0833	1,2191	0,2106	8,0214	2,0591



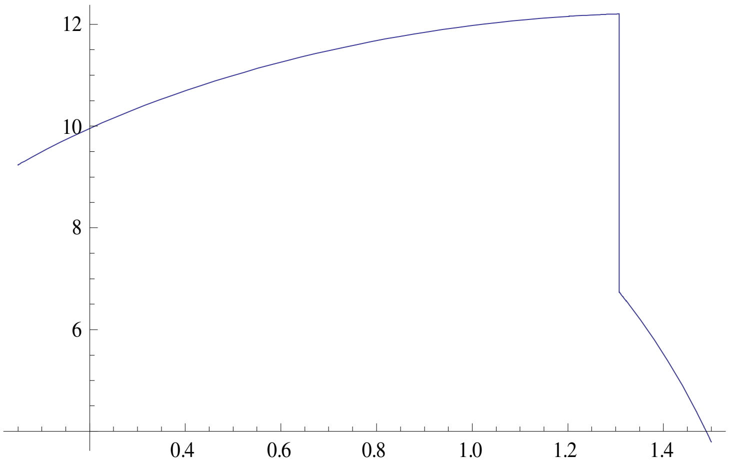
Appendix Figure 27 – The X-axis is the coupon relative to EBIT and the Y-axis the corresponding PE ratio. There is no optimal solution with a C*.

EKORNES



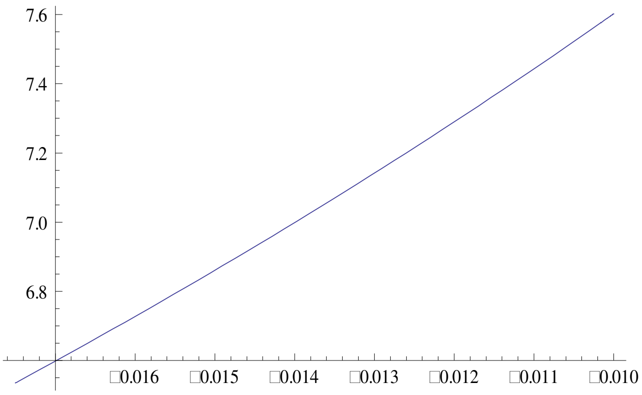
Appendix figure 28 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

μ	B	F	Debt-Low	A	P
−0,0100	0,0299	1,2034	0,0659	9,2365	0,7320



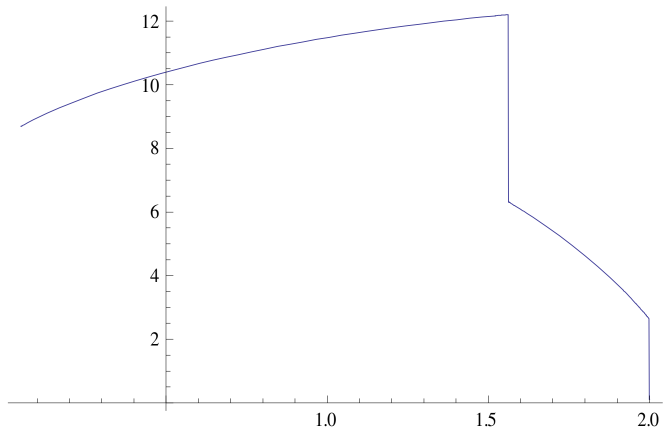
Appendix Figure 29 – The X-axis is the coupon relative to EBIT and the Y-axis the corresponding PE ratio. There is no optimal solution with a C*.

RIEBER & SON



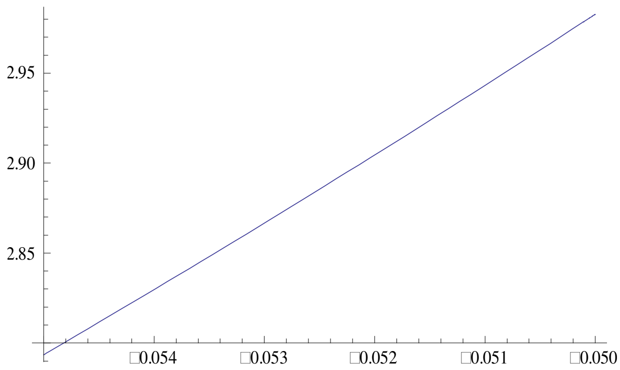
Appendix figure 30 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

μ	B	F	Debt-Low	A	P
−0,0140	0,1017	1,2058	0,2221	9,5229	2,5498



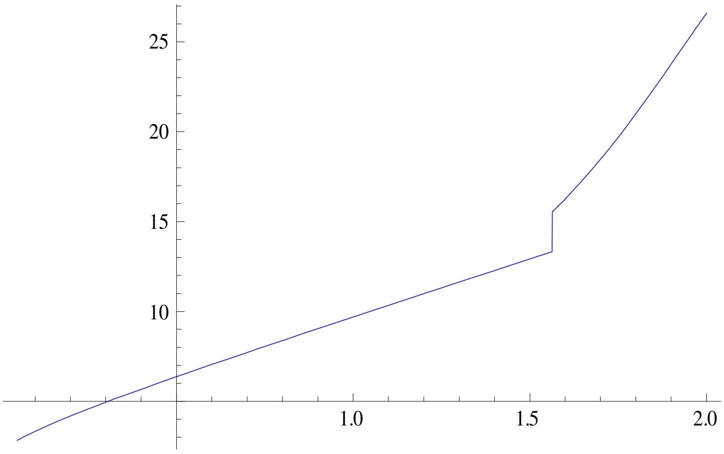
Appendix Figure 31 – The X-axis is the coupon relative to EBIT and the Y-axis the corresponding PE ratio. There is no optimal solution with a C*.

STATOILHYDRO



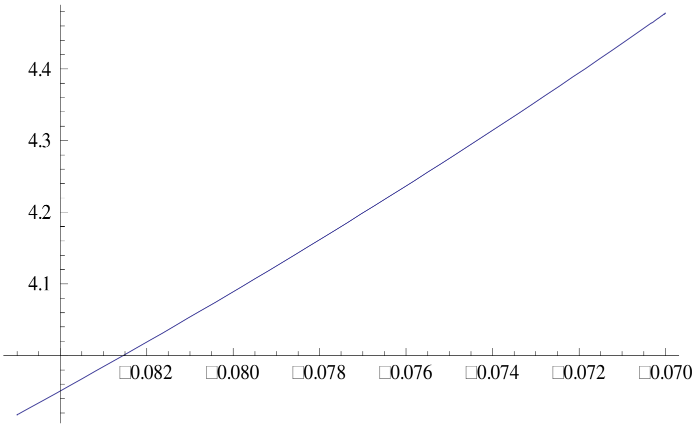
Appendix figure 32 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

μ	B	F	Debt-Low	A	P
−0,0530	0,0610	1,1492	0,2651	4,1049	1,2508



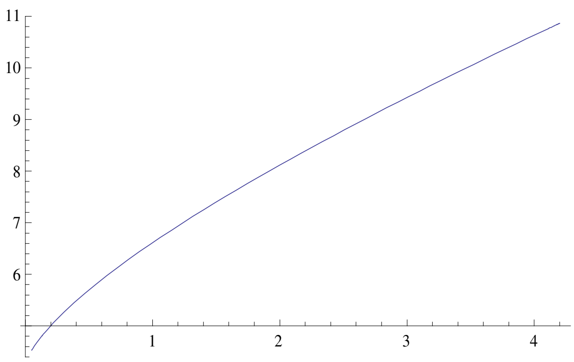
Appendix Figure 33 – The X-axis is the coupon relative to EBIT and the Y-axis the corresponding PE ratio. There is no optimal solution with a C*. Please see the result chapter for further comments and discussions.

PA RESOURCES (OSL)



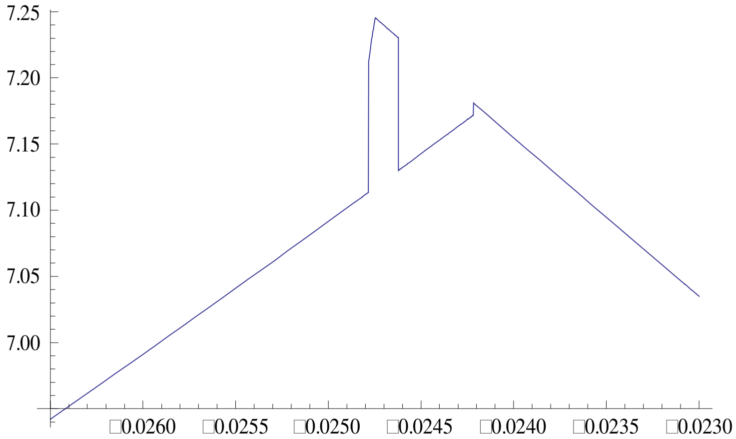
Appendix figure 34 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

μ	B	F	Debt-Low	A	P
−0,0730	0,0256	1,2467	0,0932	4,9203	0,5720



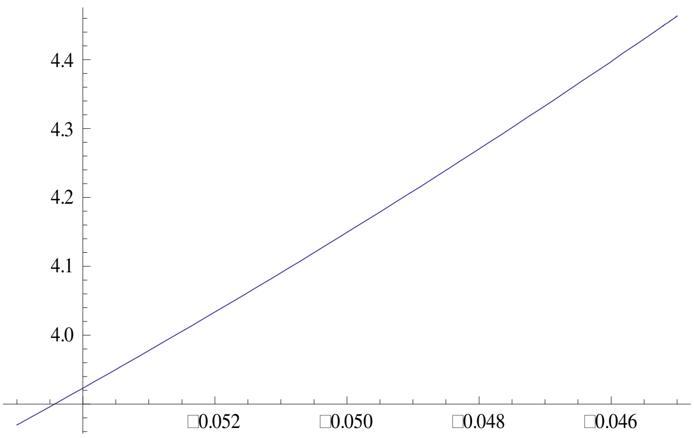
Appendix Figure 35 – The X-axis is the coupon relative to EBIT and the Y-axis the corresponding PE ratio. There is no optimal solution with a C*

TGS-NOPEC GEOPHS



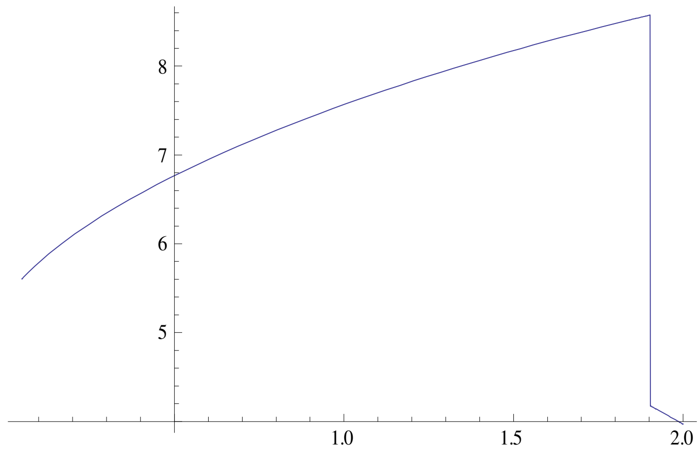
Appendix figure 36 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio. Please see the result chapter for further comments. With a PE of 7.35 a reasonable My was not feasible to resolve.

DOF



Appendix figure 37 – The X-axis is the growth rate (My), and the Y-axis is the PE ratio

μ	B	F	Debt-Low	A	P
−0,0490	0,1578	1,1905	0,3292	6,8776	2,6955



Appendix Figure 38 – The X-axis is the coupon relative to EBIT and the Y-axis the corresponding PE ratio. There is no optimal solution with a C*.

17. Reference list

- F. Modigliani and M. H. Miller, The Cost of Capital, Corporation Finance and the Theory of Investment, American Economic Review 48 (June 1958), pp. 261-297
- Brealey, Richard A., Myers, Stewart C. and Allen, Franklin, Corporate Finance, Eighth edition (2008), McGraw – Hill International Edition
- Goldstein, Leland and Ju, An EBIT-Based Model of Dynamic Capital Structure, Journal of Business, Vol. 74, No. 4 (Oct. 2001), pp. 483-512.
- Hayne E. Leland, Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, The Journal of Finance, Vol. 49, No. 4 (Sep. 1994), pp 1213-1252.
- Manuel Ammann and Michael Genser, A Testable EBIT-Based Credit Risk Model, Swiss Institute of Banking and Finance, University of St. Gallen, January 13, 2004.
- Deffeyes, Kenneth S. (2005). *Beyond Oil: The View from Hubbert's Peak*. Hill and Wang
- The World Bank's web page: http://www.doingbusiness.org/
- OECD's Tax Data Base http://www.oecd.org/document/60/0,3343,en_2649_34533_1942460_1_1_1_1,00.html
- News about Microsoft raising debt - http://www.techflash.com/Microsoft_debt_offering_raises_questions_about_plans_for_funds_44725857.html
- Dixit, A. 1991. A simplified treatment of the theory of optimal regulation of Brownian motion, Journal of Economic Dynamics and Control 15: 657-673
- Dumas, B. 1991. Super contact and related optimality conditions, Journal of Economic Dynamics and Control 15: 675-685