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Enforcing the N-1 Criterion in Power Transmission Networks:

An Analysis of a Theoretical Model

MSc thesis in Economics and Business Administration
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Abstract

We construct a numerical model of a power transmission network and analyse the effect of implementing N-1 security constraints. We find that implementing N-1 can have widespread effect on prices and quantities through changes in congestion patterns, particularly by increasing the role low-capacity lines have in causing congestion. This can have considerable implications for optimal investment strategies. Enforcing the N-1 criterion improves the security, but at increased cost of dispatch. While the overall costs increase, it can make individual actors on the market better off, especially the collector of grid fees. Finally we compare N-1 security constraints and other types of security constraints, namely cut constraints and de-rating of lines.

Acknowledgements

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Contents

1 Introduction

> The more nearly perfect a market is, the stronger is the tendency for the same price to be paid for the same thing at the same time in all parts of the market: but of course if the market is large, allowance must be made for the expense of delivering the goods to different purchasers. (Marshall, A., 1920, Book V, Chapter I).

Markets for power are no different from other markets in the respect that prices are determined by buyers' and sellers' willingness to trade. It is the "expense of delivering the goods" that distinguishes it from other commodity markets. The physical laws that determine the flow of electrical power through power transmission networks result in flow constraints that are very different from those that apply to traditional transportation networks.

The purpose of this thesis is to examine how enforcing the N-1 criterion in an electric power transmission network affects optimal dispatch through changes in prices and quantities of power traded. The N-1 criterion dictates that power flow be constrained so that a transmission network does not fail in a case of a failure of one of the network's components. Under this rule the failure of a single power line, for example, would not cause power outages. Enforcing the N-1 criterion improves security, but at a cost. It requires adding stricter constraints to power flow. If the objective were to maximize social welfare, these added constraints would negatively affect the social welfare through changes in prices and quantities.

Chapter 2 contains an introduction to power flow modelling and presents a simplified DC-power flow approximation. DC-approximation is common practice in economic analysis of power transmission networks and yields a fairly accurate approximation of power flows in high-voltage transmission networks. In Chapter 3 we discuss optimal dispatch of power under nodal pricing and solve a stylized steady-state numerical model for optimal dispatch by maximizing social welfare. Further, we examine the relationship between nodal prices and congestion. Chapter 4 contains the main contribution of the thesis. In it we will examine how enforcing N-1 criterion affects optimal dispatch, limiting our scope to the contingencies of power line failure only. Implementing the N-1 criterion can have extensive effects on optimal dispatch through major changes in the congestion pattern of networks. This is highly relevant when considering optimal investment strategies, as we will discuss in detail. We further compare optimal dispatch under the N-1 criterion with optimal dispatch obtained when other types of security constraints are employed.

The constraints associated with the N-1 criterion are dependent on the topology of transmission networks, and for some networks it would even be impossible to implement N-1. Bjørndal, M., Jörnstein, K. and Rud, L. (2012) discuss a practical example for the Norwegian transmission network, where in the winter of 2009-2010 enforcing a security constraint was economically infeasible, while technically possible, for a certain power line. Implementing N-1 constraints can also be technically impossible, in particular for radial networks. Despite the influence of network topologies on the effects enforcing the N-1 criterion, we will try to deduce some generalities about the effects of enforcing it.

2 Power Flow Modelling

In the context of this dissertation, a power transmission network refers to inter-regional high-voltage lines that transmit power from power plants to local distribution substations – and in some cases to energy-intensive end-users, such as aluminium smelters. The physical laws that determine the flow of electrical power through power lines result in flow constraints that are very different from those that apply to traditional transportation networks. Furthermore, electric power travels at nearly the speed of light and cannot be stored viably: it has to be consumed at the moment it is produced. These characteristics distinguish the markets for electric power from other markets and need to be taken into consideration if optimal dispatch is to be facilitated.

The dominant form of power transmission is three-phase alternating-current (AC). Accurate models of power flow in AC-networks include a set of non-linear power balance equations for real and reactive power flow over every line, where the relationship between real and reactive power involves complex numbers. Current and voltage depend on time, even in a steady state situation. Due to non-convexity and multiple local optima, obtaining the power flow solution to such models requires iterative algorithms and the process can be computationally demanding for models of large networks, especially when contingencies are to be considered.

Due to these complications, simpler approximations are often used to model power flow. In Chapter 2.1 we will introduce a "linear lossless DC-approximation". While the DC-approximation involves major simplifications it yields results accurate enough to be widely used by energy economists and, for some purposes, power engineers.

Overbye, T.J., Cheng, X. and Sun, Y. (2004, January) compare the results obtained from accurate AC-models and DC-approximations for two networks, one large and one small, and conclude that the DC-approximation is fairly accurate in revealing congestion patterns: the major factor in price differences across nodes.

Purchala, K., Meeus, L., Van Dommelen, D. and Belmand, R. (2005, June) analyse the factors that affect the accuracy of DC-approximation of AC-networks and find that it can give an accurate approximation of active power flow given that certain assumptions holds, such as the power lines being high-voltage, which they tend to be in inter-regional transmission networks.

2.1 Linear DC Approximation

Here we describe three simplifications to power flow modelling and present a linear lossless DC-approximation of power flow, which we will employ when modelling power flow in later chapters.

We use power, the product of current and voltage, to describe the rate of electric energy transferred by power lines, measured in watts (*W*). In AC-power lines, current and voltage depend on time. More specifically, they are sinusoidal. Power in AC-lines involves both real and reactive power, where reactive power is the result of current moving out of phase with voltage. DC-approximation models ignore reactive power, assuming that phase angles are zero. As a result, only real or active power, capable of doing work, is included in such models.

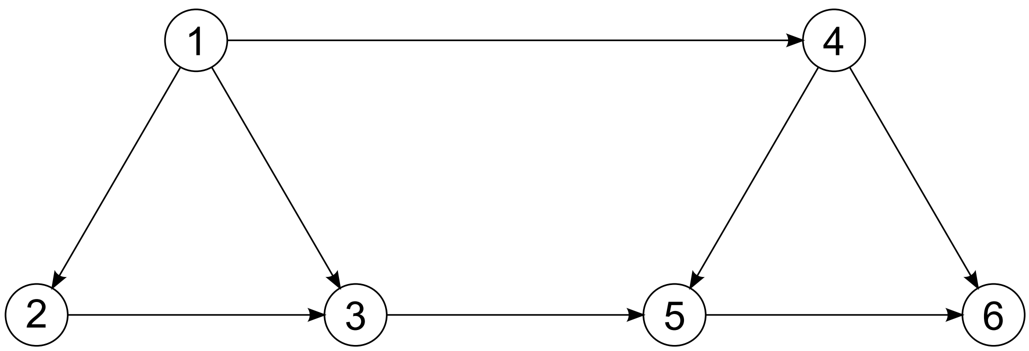
When electric power flows along a line, some of it is lost as heat due to the resistance of power lines. The influence of resistance is lower for high-voltage lines. As we will be modelling high-voltage transmission networks, we assume that losses will be minimal and that the accuracy of the models can withstand losses being ignored (Purchala, K., et al, 2005, June). It is worth noting that a number of methods do exist to incorporate losses into linearized DC-approximation models, as is for example discussed in Stott, B., Jardim, J. and Alsac, O. (2009). We postulate that the stylized models employed in this dissertation suffice to deduce some generalities about the implications of implementing N-1, even with losses ignored. However, we will implicitly include losses through maximum flow capacities of power lines. Line capacities are a result of heating and constitute a thermal limit power lines can withstand. There is a direct relationship between heating and losses, but in our models the flow capacities are to be interpreted as the maximum flow of active power a line can bear; a constant for each line.

Thirdly, we assume we have a flat voltage profile with all voltage magnitudes equal to one. Power flow models with this simplification give fairly accurate results of power flow as long as voltages show little variations over nodes in the network being modelled (Purchala, K., et al, 2005, June).

Of course, when approximations similar to the aforementioned are used to calculate power dispatch for existent networks, reactive power, losses and voltage variations must still be dealt with in actual dispatch. The components of the network must, for example, be able to withstand flow of not only active power, but reactive power as well, although reactive power can partly be managed with fixed equipment. However, in many cases it is possible to calculate a base case of power flow with simplified DC-model that will be very close to viable dispatch obtained from more sophisticated models.

Adopting these approximations, we now introduce the equations governing power flow using as a case in point the six-node network depicted in Figure 1.

Figure 1: The Topology of the six-Node Network



We refer to points of production and consumption as nodes. They are symbolized by the circles in Figure 1. Power can be both produced and consumed within a single node. The net injection of each node is equal to the difference between production and consumption. [1] We denote the net injection in Node i with q_i, which takes a positive value for nodes of net production and a negative value for nodes of net consumption.

The fundamental physical laws that govern power flow are known as Kirchhoff's current and voltage laws; and the requirement of energy balance. Kirchhoff's current law dictates that the current flow into a node of an electrical network is equal to the current flow out of it. As we assume there are no losses in the network and that all voltage magnitudes are equal, this implies that the net injection of a node is equal to the power flow out of the node less power flow into the node. We refer to the equations that describe this law as the node rule equations.

As an example we can examine the node rule equation for Node 2. Line 12 is the power line that connects nodes 1 and 2 and Line 23 is the line that connects nodes 2 and 3. We denote the power flow along these lines as x_{12} and x_{23} respectively.[2] The direction of the arrows in Figure 1 is purely arbitrary. A positive value for x_{12} indicates that power flows from Node 1 to

[1] You could say that net production nodes "export" power while net consumption nodes "import" it.

[2] We likewise refer to flow in all lines with x_{ij}, where j is the node the arrow points at in Figure 1, and i is the node where the arrow originates.

Node 2 along Line 12, while a negative value indicates that power flows in the opposite direction. Now, the node rule equation for Node 2 is $q_2 = -x_{12} + x_{23}$. The node rule equations for all nodes in the network are equivalent:

$$
\begin{aligned}
q_1 &= x_{12} + x_{13} + x_{14} \\
q_2 &= -x_{12} + x_{23} \\
q_3 &= -x_{13} - x_{23} + x_{35} \\
q_4 &= -x_{14} + x_{45} + x_{46} \\
q_5 &= -x_{35} - x_{45} + x_{56} \\
q_6 &= -x_{46} - x_{56}
\end{aligned}
\tag{2.1}
$$

where any single equations may be omitted from the model due to redundancy, as the sum of any five equations always equals the sixth.

Kirchhoff's voltage law dictates that the sum of potential differences across each cycle in a network is equal to zero. For our purposes this means that the sum of power flow across each cycle is zero. As an example, we can form a cycle consisting of the three lines: 12, 23 and 13. The sum of power flow across every line in this cycle must be equal to zero:

$$x_{12} + x_{23} - x_{13} = 0$$

where the sign of x_{ij} denotes the directionality of the cycle. Not all of the cycles that can be possibly formed in a network are independent. Dolan, A. and Aldous, J. (1993) describe an algorithm to construct equations for the smallest number of independent cycles in a network. The number of independent cycles in a network is equal to the number of lines less the number of nodes, plus one: in our case three. For the topology of our six-node network we can for example form these three independent equations, which we refer to as the loop rule equations:

$$x_{12} + x_{23} - x_{13} = 0$$
$$x_{13} + x_{35} - x_{45} - x_{14} = 0 \quad (2.2)$$
$$x_{45} + x_{56} - x_{46} = 0$$

The requirement of energy balances can be derived from (2.1). It expresses that the sum of all net injections is equal to zero. This implies that no power disappears.[3] For the given topology this equation is:

$$q_1 + q_2 + q_3 + q_4 + q_5 + q_6 = 0 \quad (2.3)$$

Given the net injections of all six nodes, using the node and loop rule equations – and the requirement of energy balance – we can now solve for the power flow across every line in the network.

Equations (2.1.), (2.2) and (2.3) further allow us to calculate the so called power transfer distribution factors (PTDFs) for the network, also referred to as load factors. PTDFs express flow changes in power lines if one unit of power is injected into one node to be withdrawn at another. Table 1 shows the PTDFs for the topology represented in Figure 1, with Node 6 arbitrarily chosen as the reference node.[4]

Table 1: PTDFs for the Six-Node Network

	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6
Line 12	0.13	-0.50	-0.13	0.03	-0.03	0
Line 13	0.27	0	-0.27	0.07	-0.07	0
Line 14	0.60	0.50	0.40	-0.10	0.10	0
Line 23	0.13	0.50	-0.13	0.03	-0.03	0
Line 35	0.40	0.50	0.60	0.10	-0.10	0
Line 45	0.07	0	-0.07	0.27	-0.27	0
Line 46	0.53	0.50	0.47	0.63	0.37	0
Line 56	0.47	0.50	0.53	0.37	0.63	0

[3] If we would include losses, some electric power would be transformed into heat.
[4] Column 1 in Table 1 – where Node 6 is the reference node – can be obtained by setting $q_1 = 1$ and $q_6 = -1$ and solving equations (2.1), (2.2) and (2.3) for power flow. Columns 2 through 5 can be obtained in the same manner, with the net injection of the reference node always set at minus one. All values in the column for the reference node are equal to zero.

To clarify, say that we inject one MW of power into Node 1 and withdraw it at Node 2. This will results in a change in power flow across every line in the network. If we examine Line 12 in particular, the values in row 12, and column 1 and 2 tell us that the power flow across it will change by $0.13 - (-0.50) = 0.63$. That is, the power flow across Line 12 will increase by roughly 0.63 MW in the direction from Node 1 to Node 2 (or, equivalently, decrease in the opposite direction). Likewise, if we inject one MW into Node 5 and withdraw it at Node 6, the flow across every line in the network will change. The flow across Line 45, for example, will change by $-0.27 - 0 = -0.27$, or decrease by roughly 0.27 MW in the direction from Node 4 to Node 5.

3 Optimal Dispatch with Nodal Prices for a six-Node Model

In this chapter we develop a stylized numerical model of a transmission network, with a single supplier and a single consumer at each node, using the six-node topology depicted in Figure 1. We define a flow capacity for each power line which sets a limit to the potential power flow over it. This can lead to congestion in the network. We will examine how optimal dispatch can be achieved by allowing for different prices at each node and examine the relationship between nodal price differences and congestion. The methodology of obtaining economically efficient dispatch by using nodal prices was developed by Schweppe, F.C., Caramanis, M.C., Tabors, R.D. and Bohn, R.E. (1988). While it possible to achieve optimal dispatch using nodal prices, in reality, it can be difficult to obtain the necessary information to implement it. This is for example discussed in Bjørndal, M., Jörnstein, K. and Rud, L. (2010).

An alternative to nodal prices, for example employed in Norway and elsewhere in N-Europe, are zonal prices. The method involves defining areas consisting of several nodes and enforcing uniform prices within them. Generally, this method does not lead to optimal dispatch, but it is an improvement over enforcing uniform prices over whole power networks. Bjørndal, E. et al (2012) compare dispatch under nodal and zonal pricing for the Nordic power market.

The PTDFs in Table 1 clearly show that transporting power between two nodes affects the power flow in the whole six-node network. Power flow is clearly not analogous to transportation of goods, where traffic can be increased along a path without affecting other paths in the network. If Line 13, for example, is congested, with power flowing from Node 3 to Node 1, a producer at Node 5 cannot increase production and sell power to a consumer at Node 6, all else equal, as this would lead to an increase in the flow across the congested line, even though it lies far from the shortest path between node 5 and six.

Wu, F., Varaiya, P., Spiller, P.T. and Oren, S.S. (1996) rebut a number of claims about power markets, commonly made by falsely drawing analogies between transportation of goods and electric power. They conclude:

> The first error is to say that just as the price of a good sold at two locations will differ by the cost of transporting that good between those two locations, so will the nodal price difference for power equal the cost of transporting it from one node to the other. The second, and more serious, error is to say that competition will drive the difference in nodal prices to the cost of transporting power, just as competition drives the difference in locational prices of goods to the cost of transporting that good. (p. 22).

When solving for optimal dispatch, our objective will be to maximize social welfare, which consists of producer and consumer surplus, as well as grid fees which are collected when congestion occurs. As an example of another objective, commonly used in power dispatch calculations, we can name the minimization of total costs.

To illustrate the basic concept of consumer and producer surplus, say we have one producer and one consumer of power. The producer's willingness to produce power can be described by a simple reverse supply function:

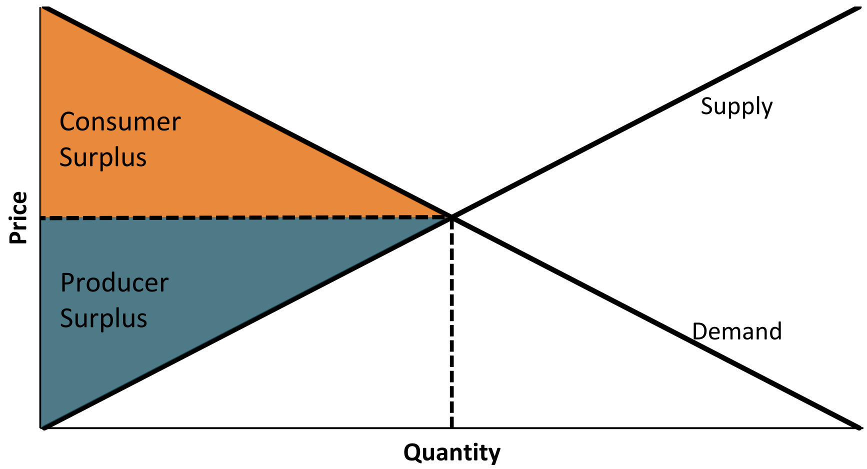
$$p = cq^S$$

where p is the price of power; q^S is the quantity supplied; and c is a constant. The function corresponds to a marginal cost function in a perfect competition situation. The consumer's willingness to purchase power can likewise be described by a reverse demand function:

$$p = a - bq^D$$

where q^d is the quantity consumed; and a and b are constants. The quantity and price are determined by the intersection of the two functions. Now, the producer surplus is equal to sales revenues less the cost of production and the consumer surplus is equal to the difference between the willingness to pay and the actual price. This is graphically represented in Figure 2.

Figure 2: Social Welfare in a Market with a single Producer and single Consumer



In order to examine social welfare in a power market consisting of a whole transmission network we construct a numerical six node-model, using the topology portrayed in Figure 1. Supply and demand functions are associated with each node:

$$p_i = a_i - b_i q_i^D \tag{3.1}$$

$$p_i = c_i q_i^S \tag{3.2}$$

where p_i is the price of power in Node i; q_i^D is the quantity consumed and q_i^S the quantity produced; and a_i, b_i and c_i are positive parameters. The values of the parameters for each node are presented in Table 2.[5]

Table 2: Supply and Demand Parameters for the six-Node network

	a_i	b_i	c_i
Node 1	20	0.05	0.2
Node 2	20	0.05	0.1
Node 3	30	0.10	0.7
Node 4	20	0.05	0.2
Node 5	30	0.10	0.7
Node 6	30	0.10	0.1

[5] We use the same topology and paramters as Bjørndal, M., Jörnstein, K. and Rud, L. (2010). As a consequence our model yields the same optimal results when solving for nodal prices when the N-1 criterion is not enforced.

The net injection of every node is equal to the difference between power supplied and consumed:

$$q_i = q_i^S - q_i^D \tag{3.3}$$

where q_i is net injection in Node *i*. If more power is supplied than consumed in a node, the net injection will be positive.

The model allows for different prices of power in different nodes and imbalances between production and consumption in each node. As a result, the aggregate amount paid by all consumers will not be equal to the aggregate amount paid to all producers. The grid revenue consists of the difference. Aggregate social welfare, our objective function, equals:

$$\sum_{i=1}^{6}\left(\frac{1}{2}\left(p_i q_i^S\right) + \frac{1}{2}(a_i - p_i)q_i^D + p_i(q_i^D - q_i^S)\right) \tag{3.4}$$

where the first term represents producer surplus; the second term represents consumer surplus; and the third term represents grid revenues. Grid revenues function similarly to taxes in basic economic analyses. Collecting them creates a wedge, resulting in lower equilibrium quantities, consumers paying more and producers receiving less. This leads to higher prices in consumption nodes and lower prices in production nodes. However, the wedge created by collecting grid fees is different from tax wedges in one important way; it facilitates optimality rather than obstructing it. It is the limited capacity of power lines that prevents market equilibrium between aggregate supply and aggregate demand, not the collection of grid fees.

Due to thermal constraints, the power flow across each particular line cannot exceed a certain limit, the maximum capacity:

$$-x_{ij}^{MAX} \leq x_{ij} \leq \ x_{ij}^{MAX} \tag{3.5}$$

where x_{ij} is power flow over the line connecting Node *i* and Node *j* and x_{ij}^{MAX} is its capacity. A negative value of x_{ij} denotes that power flows from Node *j* to Node *i*. It is worth noting that in actual networks the capacities may not always be equal in both directions, as in equation (3.5). The capacity parameters we have chosen for our model are presented in Table 3.

Table 3: Maximum Capacity of Lines for the six-Node Network

	x^{MAX} (MW)
Line 12	60
Line 13	60
Line 14	60
Line 23	60
Line 35	10
Line 45	30
Line 46	8
Line 56	60

The power flow is constrained by the node and loop rule equations, as well as the requirement of energy balance: equations (2.1.)-(2.3) as presented in the previous chapter.

We implement the model in the algebraic modelling language GAMS (n.d.) and maximize social welfare, equation (3.4); subject to equations (2.1.)-(2.3), (3.1)-(3.3) and (3.5); using the parameters presented in Table 2 and Table 3. When solving for optimal dispatch we employ the solver MINOS using non-linear programming.

The optimal dispatch results are presented in Table 4 and Table 5. Flows on congested lines are presented in bold in Table 5. Optimal power flow and net injections are graphically represented in Figure 3, with the dashed lines representing congested lines.

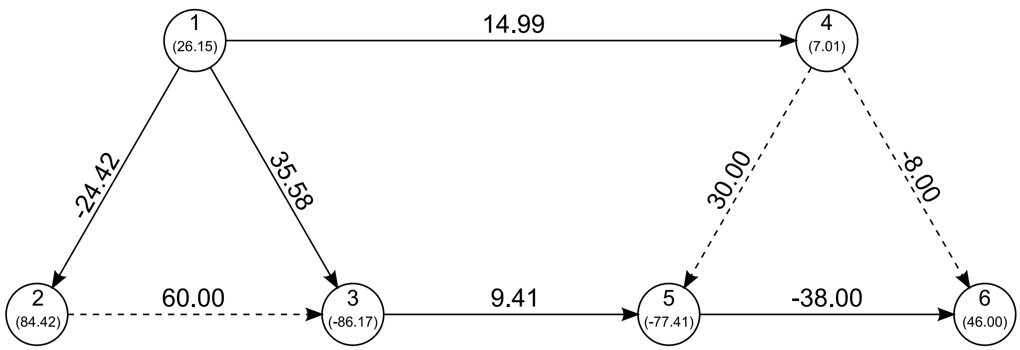
Table 4: Optimal Dispatch for the six-Node Network

Node	Price	Supply	Demand	Net injection	Producer surplus	Consumer surplus	Grid revenue	Social welfare
1	17.05	85.23	59.08	26.15	726.42	87.26	-445.78	367.90
2	16.15	161.47	77.05	84.42	1303.69	148.43	-1363.17	88.94
3	18.71	26.73	112.90	-86.17	250.06	637.26	1612.20	2499.52
4	16.28	81.40	74.39	7.01	662.62	138.36	-114.08	686.91
5	19.48	27.82	105.24	-77.41	270.95	553.74	1507.73	2332.42
6	17.30	173.00	127.00	46.00	1496.45	806.45	-795.80	1507.10
Sum		555.66	555.66	0	4710.18	2371.50	401.11	7482.79

Table 5: Optimal Flow in the six-Node Network

Line	12	13	14	23	35	45	46	56
Power flow	-24.42	35.58	14.99	**60.00**	9.41	**30.00**	**-8.00**	-38.00
Maximum capacity	60	60	60	**60**	10	**30**	**8**	60
Shadow price	0	0	0	**3.46**	0	**6.14**	**-1.16**	0

Figure 3: Flows and Net Injections in Optimal Dispatch of the six-Node Network



In Table 5 we see that shadow prices are associated with three lines. They represent the marginal value of relaxing the capacity constraint. For example, if the capacity of Line 23 was increased by one MW, social welfare would increase by approximately 3.46. Although the shadow price is reported as negative for Line 46, social welfare would increase if the capacity of the line was relaxed. The negative sign only indicates the direction of the flow in the optimal solution. As would be expected, we only have non-zero shadow prices for lines which are congested.

In Figure 3 we see an example of how a loop rule equation limits the flow of an uncongested line. If we examine the right-hand side triangle, formed by nodes 4, 5 and 6, we see that lines 45 and 46 are congested. If we would increase the flow over Line 56, in the direction from Node 6 to Node 5, the flow would also have to increase over one or both of the congested lines. The results in the flow of Line 56 effectively being limited to 38 MW by the loop rule equation $x_{56} = x_{46} - x_{45}$. Thus the net injection of Node 6 is effectively bound at 46 MW, the sum of maximum flows across lines 46 and 56.

Oren, S.S. (2013) presents a fundamental relationship between the shadow prices of congested lines and price differences across two nodes. The relationship can be described by the following equation:

$$p_j - p_i = \sum_{all\ lines\ hk} \left(SP_{hk} \left(PTDF_{hk,i} - PTDF_{hk,j} \right) \right) \tag{3.6}$$

where SP_{hk} is the shadow price of Line *hk*; and $PTDF_{hk,i}$ is the power transfer distribution factor for Line *hk* when injecting one unit of power into Node *i*.[6] For clarification we can examine the right-hand side of (3.5) for two specific nodes: Node 1 as *i* and Node 6 as *j*. We choose Node 6 – the reference node – as one of the nodes to simplify, as $PTDF_{hk,6} = 0$ for all lines. We have:

$$p_6 - p_1 = \sum_{all\ lines\ hk} \left(SP_{hk} \left(PTDF_{hk,1} - PTDF_{hk,6} \right) \right)$$
$$= 3.46 \left(\frac{2}{15} \right) + 6.14 \left(\frac{1}{15} \right) - 1.16 \left(\frac{8}{15} \right) = 0.25$$

where the first, second and third terms represent the PTDFs and shadow prices for the congested lines 23, 45 and 46 respectively. The outcome, 0.25, is equal to the price difference between Node 6 and Node 1, as can be seen in Table 4.

It is intuitive that there exists a relationship between prices on the one hand and PTDFs and shadow prices on the other. If there was no congestion, there would be no price differences between nodes to begin with and PTDFs express with how much trading between the nodes would pressure the congested lines. We see that congestion in two of the three congested lines contributes towards lowering the price in Node 1 compared with the price in Node 6: lines 23 and 45. This corresponds with the fact that injecting power to Node 1 and withdrawing it at Node 6 would increase the flow across these lines. The congestion on Line 46 has an opposite, but smaller, effect as the aforementioned injection changes would relieve congestion on it. When comparing prices between two nodes, prices are higher in the node that would increase flow through congested lines, weighted by shadow prices, if it were to receive power from the other.

[6] PTDFs for all nodes are listed in Table 1 in Chapter 2, with Node 6 as the reference node.

In order to examine how the price in each node deviates from the average price, we create a modified PTDF table, Table 6, where each column reflects how the flow over each congested line would change if one MW were injected into the node in question to be withdrawn evenly at all other nodes. Or, to paraphrase, the columns display how reducing consumption in a node by one MW to redistribute the power evenly across all nodes would affect power flows for all lines. We only included congested lines in Table 6.

For our six-node network the modified PTDFs equal:

$$PTDF_{hk,i}^{*} = \frac{\sum_{j=1}^{6}\left(PTDF_{hk,i} - PTDF_{hk,j}\right)}{6}$$

where $PTDF_{k,i}^{*}$ describes the change in flow across Line *hk* if one unit of power was injected into Node *i* to be withdrawn evenly at all nodes in the network. The modified PTDFs allow us to examine the relationship between shadow prices of congested lines and the deviation of nodal price from average price. We modify equation (3.6):

$$p_i - \bar{p} = \sum_{all\ lines\ hk} \left(SP_{hk} \cdot \left(-PTDF_{hk,i}^{*}\right)\right) \tag{3.7}$$

Table 6: Modified PTDFs for Congested Lines and their Shadow Prices

	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Shadow price
Line 23	0.05	0.42	-0.22	-0.05	-0.12	-0.08	3.46
Line 45	0.07	0.00	-0.07	0.27	-0.27	0.00	6.14
Line 46	0.12	0.08	0.05	0.22	-0.05	-0.42	-1.16
$p_i - \bar{p}$	-0.45	-1.35	1.22	-1.21	1.98	-0.19	

Table 6 allows us to see at a glance how congestion patterns and shadow prices are related to prices. Node 3, for example, has a price 1.22 above the average price. This is no surprise as, on average, injecting power into Node 3 increases flow through all congested lines.[7]

[7] Keeping in mind, once again, that signs denote direction of flow. If the sign of the shadow price for line *hk* and the sign of $PTDF_{hk,i}^{*}$ is the same, withdrawing additional power at Node *i* to redistribute it evenly decreases the flow through Line *hk*.

In general, when models of power markets are solved for optimal dispatch, prices will be highest at nodes that tend to receive their power through congested lines.

4 Enforcing the N-1 Criterion

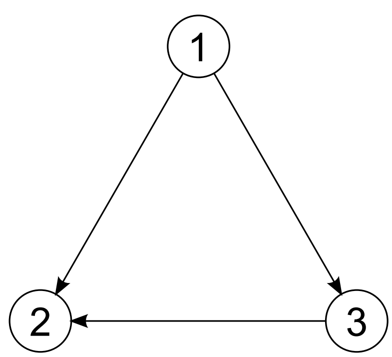
In this chapter we will examine the implications of enforcing the N-1 criterion, so that in the case of a contingency – in case a line fails – the instantly redirected power flow will be within the bounds of the thermal capacity constraints.[8] This significantly improves the security of power delivery, but at a price of lower social welfare in optimal dispatch.[9] It depends on the value of security and the probability of line failure whether increasing the security by implementing N-1 is worthwhile. While costs of blackouts and the value of security is beyond the scope of this thesis, it is important to keep in mind that it is of vital importance when deciding whether to enforce the N-1 criterion in a power network or not.

In order to better understand certain aspects of congestion patterns that can occur when the N-1 criterion is enforced, we start by looking at a smaller three-node example before we expand the six-node model developed in the previous chapter.

4.1 A Simple three-Node Model

The topology of the three-node model is presented in Figure 4.

Figure 4: The Topology of the three-Node Network



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[8] We will not consider other types of contingencies, such as breakdown of power plants.
[9] Or, if we were minimizing costs, we would increase security at the price of a higher cost.

As before, three sets of equations – equivalent to (2.1)-(2.3) – describe power flow: Kirchhoff's loop and node rule equations; and the requirement of energy balance. For the three-node network they are:

$$q_1 = x_{12} + x_{13}$$

$$q_2 = -x_{12} + x_{23}$$

$$q_3 = -x_{13} - x_{23}$$

$$x_{12} + x_{23} - x_{13} = 0$$

$$q_1 + q_2 + q_3 = 0$$

As in the six-node model, equations (3.1)-(3.5) describe supply, demand, net injections, line capacities and social welfare. The parameters we have selected are presented in Table 7.

Table 7: Parameters for the three-Node Network

	a_i	b_i	c_i		x^{MAX} (MW)
Node 1	20	0.05	0.15	Line 12	20
Node 2	20	0.05	0.35	Line 13	20
Node 3	20	0.05	0.45	Line 23	100

In the contingency of Line 12 failing, Node 1 will be connected to Line 13 only as can be seen in Figure 4. Line 13 has a maximum capacity of 20 MW. Thus, the net injection in Node 1 must lie within the range [-20, 20] in the pre-contingency situation. Otherwise, in case of a failure in Line 12, it would be impossible to instantaneously redirect the power flow and stay within the capacity of Line 13. Note that the post-contingency flow across Line 13 is not part of a loop. The flow across it is bound by the maximum capacity only. The node rule equations in this contingency are:

$$q_1 = x_{13}^{12}$$

$$q_2 = x_{23}^{12}$$

$$q_3 = -x_{13}^{12} - x_{23}^{12}$$

where x_{ij}^{hk} is the flow across Line *ij* in the post-contingency situation of Line *hk* failing. Finally, post-contingency flows must respect the capacity of lines:

$$-x_{ij}^{MAX} \leq x_{ij}^{12} \leq x_{ij}^{MAX}$$

Now, after having constructed equivalent equations for the contingencies of failure in the other two lines, we maximize social welfare in GAMS – with and without implementing N-1 – in order to obtain optimal dispatch. The results for optimal dispatch without N-1 are presented in Table 8 and Table 9. The results for optimal dispatch including N-1 constraints are presented in Table 10 and Table 11. Table 11 includes flows in the post-contingency situations. Flows over congested lines are presented in bold. The flows and injections when N-1 is implemented are graphically represented in Figure 5, including flows and injections for all contingency situations.

Table 8: Optimal Dispatch in the three-Node Network

Node	Price	Supply	Demand	Net injection	Producer surplus	Consumer surplus	Grid revenue	Social welfare
1	16.48	109.87	70.41	39.46	905.27	123.92	-650.28	378.91
2	16.67	47.64	66.55	-18.92	397.09	110.74	315.42	823.25
3	16.87	42.16	62.70	-20.54	355.53	98.29	346.41	800.23
Sum		199.66	199.66	0	1657.90	332.95	11.55	2002.40

Table 9: Optimal Flow in the three-Node Network

Line	12	13	23
Power flow	-19.46	**20.00**	0.54
Shadow price	0	**0.58**	0

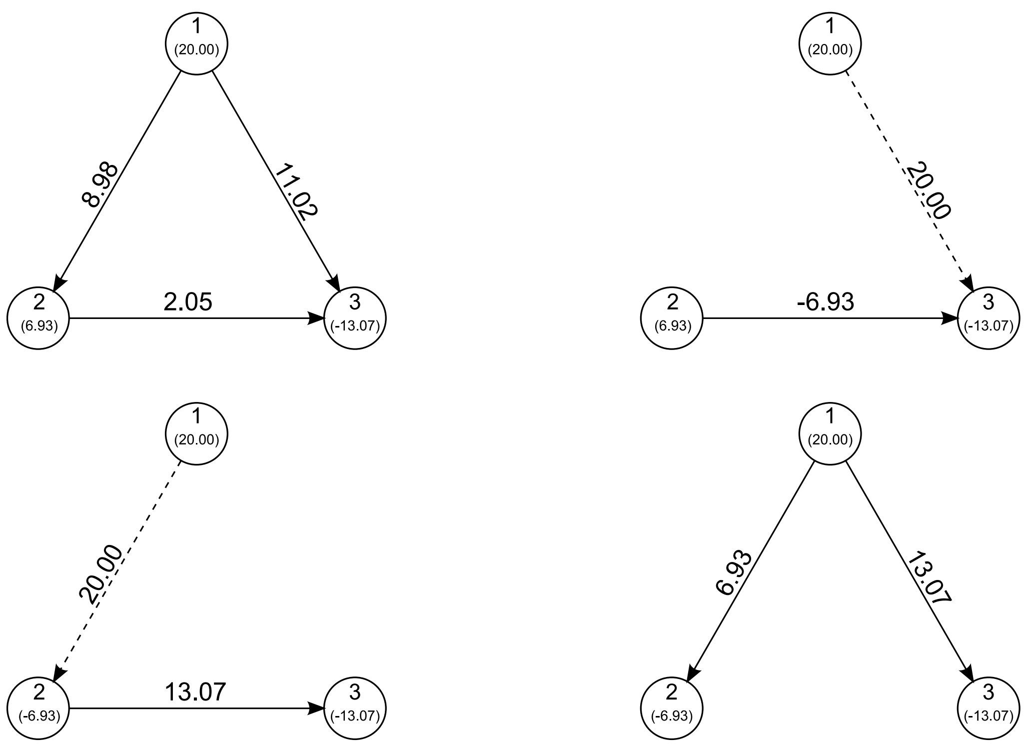
Table 10: Optimal Dispatch in the three-Node Network with N-1 Enforced

Node	Price	Supply	Demand	Net injection	Producer surplus	Consumer surplus	Grid revenue	Social welfare
1	15.75	105.00	85.00	20.00	826.88	180.63	-315.00	692.50
2	17.20	49.13	56.06	-6.93	422.47	78.58	119.16	620.21
3	17.20	42.99	56.06	-13.07	369.67	78.58	224.78	673.02
Sum		197.13	197.13	0.00	1619.01	337.78	28.94	1985.73

Table 11: Optimal Flow in the three-Node Network with N-1 Enforced

Line	12	13	23
Power flow	8.98	11.02	2.05
Maximum Capacity	**20**	**20**	100
Flow if Line 12 Fails		**20.00**	-6.93
Flow if Line 13 Fails	**20.00**		13.07
Flow if Line 23 Fails	6.93	13.07	

Figure 5: Optimal Dispatch in the three-Node Network with N-1 Enforced



A comparison of optimal dispatch with and without enforcing N-1 shows that the sign of net injection doesn't change for any node between the solutions: Node 12 is a net production node in both solutions and the other two nodes are net consumption nodes. However, when the N-1 criterion is enforced, the flow over every line – and the net injection of every node – will be closer to zero. In other words, enforcing the N-1 criterion reduces trading amongst nodes. Implementing N-1 entails adding constraints on power flow so it is unsurprising that it can negatively influence trading. Social welfare has also decreased, as would always the case for an objective function when adding binding constraints to a maximization problem. However, it is worth noting that surplus has not decreased for all market participants. Both grid revenue and total consumer surplus has increased, although individual consumer surplus has decreased in two nodes. Total producer surplus has decreased, but increased for one individual producer.

As stated above, for the three-node network, implementing N-1 leads to a decrease in the power flow of all lines. It follows that not a single line operates at its maximum capacity. The capacities of lines bind the dispatch through post-contingency flows only. Table 11 reports that congestion takes place in two contingency situations: In case of a Line 12-failure, Line 13 will be congested, and in case of a Line-13 failure, Line 12 will be congested. GAMS reports a shadow price of 1.45 for Line 13 in the Line 12-contingency; and a shadow price of zero for Line 12 in the Line 13-contingency. We would generally not expect a shadow price of zero for a congested line, and indeed the shadow price of zero on Line 12 is due to a special circumstance which will be discussed later.

The relationship between shadow prices and nodal price differences, described by equation (3.6), holds when N-1 is implemented. However, the equation has to be adjusted to account for congestion occurring in post-contingency situations. Due to that, the PTDFs for pre-contingency network will not be of use when examining the relationship. We will have to employ the PTDFs for the post-contingency networks. We modify equation (3.6) to account for that:

$$p_j - p_i = \sum_{all\ hk,\ \ all\ qr} \left(SP_{hk}^{qr} \left(PTDF_{hk,i}^{qr} - PTDF_{hk,j}^{qr} \right) \right) \tag{4.1}$$

where SP_{hk}^{qr} is the shadow price of Line hk in the post-contingency situation of Line qr failing; and $PTDF_{hk,i}^{qr} - PTDF_{hk,j}^{qr}$ represents the power transfer distribution factor of Line hk when power is injected into Line i to be withdrawn at Line j in the post-contingency situation of a failure of Line qr.

In a contingency situation, the three-node network becomes radial. As a result all PTDFs will equal zero; or positive or negative one. If we calculate, for example, the price difference between Node 3 and Node 1, using equation (4.1), we have:

$$p_3 - p_1 = \sum_{all\ hk,\ \ all\ qr} \left(SP_{hk}^{qr} \left(PTDF_{hk,1}^{qr} - PTDF_{hk,3}^{qr} \right) \right) = 1.45 \cdot 1 = 1.45$$

As we mentioned earlier, GAMS reports a shadow price of zero for Line 12 in the Line 13-contingency, despite the line being congested in that contingency situation. Even though equation (4.1) holds using a shadow price of zero for Line 12, things are a little more complicated. We cannot interpret the shadow prices reported for lines 12 and 13 as the marginal value of increasing their capacities. This is due to a special circumstance that can occur when N-1 is enforced in a network wherein a node is connected to lines that have exactly the same capacity. As we stated in the beginning of the chapter, the potential contingency of a failure of Line 12 limits the net injection of Node 1 to the range of [-20, 20], as total flow to or from Node 1 would have to be redirected to Line 13. But the potential failure of Line 13 also limits the injection of Node 1 to the same range, as in that situation, flow would have to be redirected to Line 12, which has the same capacity of 20 MW. This is easy to see by examining the upper right and lower left diagrams in Figure 5.

If we would increase the capacity of either of those lines, it would have no effect on optimal dispatch. Thus, it would have no marginal effect on social welfare. This is because the capacities of lines 12 and 13 jointly bind the net injection of Node 1. The net injection of Node 1 cannot exceed the capacity of the line that has the lower capacity. However, this also means reducing the capacity of either line would affect optimal dispatch. When reducing the capacity of Line 13, the shadow price effectively takes the value reported by GAMS: the value of 1.45. The shadow price of 1.45 is also valid for Line 12 when decreasing its capacity.[10]

[10] If we increase the capacity of Line 13 by a close to infinitesimal value, GAMS in fact reports a shadow price of zero for Line 13 for the Line 12-contingency; and a shadow price of 1.45 for Line 12 in the Line 13-contingency. Which one is reported as zero to begin with is arbitrary, and a matter of the solver's optimization algorithms.

Perhaps the most accurate way to frame this is to say that the congestion itself – jointly caused by Line 12 and Line 23 – has the shadow price of 1.45. Relieving or increasing that congestion has the marginal value of 1.45 per MW. It is sufficient to reduce maximum flow over either line to increase congestion but necessary to increase maximum flow over both lines to relieve it. In the context of calculating price differences using PTDFs and shadow prices, as in equation (4.1), it doesn't matter whether we use the shadow price of 1.45 for line 12 or 13, as long as we use the shadow price of zero for the other line. We could also divide it between them, for example using a shadow price of $1.45/2$ for both lines.

Perhaps it is unlikely in an actual network that a situation would arise where two lines would have exactly the same capacity. Nevertheless, the capacities of two lines connected to the same node could be very close together. In that case, the line with the lower capacity would correctly be ascribed a non-zero shadow prices in N-1 dispatch, but not the other.[11] In that case the non-zero shadow price would be extremely sensitive to increase in capacity as it would drop to zero as soon as the capacity would catch up with the capacity of the other.

In general, when implementing N-1, the net injection of a node connected to two lines can never exceed the capacity of the line with lesser capacity of the two lines. If a node is connected to *m* lines, its net injection can never exceed the aggregate capacities of the *m-1* weakest lines. Of course, this is not always binding, and perhaps rarely so for a node connected to multiple lines. This constraint is in fact closely related to the node rule, which dictates that the net injection of a node must be equal to the flow from it less the flow to it. It follows that the positive or negative net injection of a node can never exceed the aggregate capacity of all lines connected to it. When enforcing the N-1 criterion, this requirement becomes stricter, as it will have to hold in the contingency of a failure of the strongest line.

4.2 The six-Node Model

Now, after having taken the detour of examining the implementation of the N-1 criterion for a simple three-node model, we direct our focus again onto six-node model developed in Chapter 3.

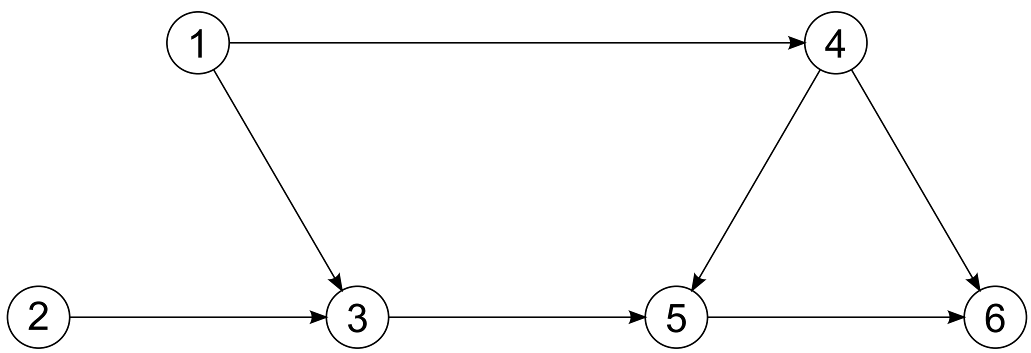
In order to examine N-1 dispatch, we construct a set of constraints for each potential contingency. Below we use as an example the contingency of a failure of Line 12. When

[11] That is, if we assume that we observe congestion to begin with.

implementing the model in GAMS, we will include equivalent constraints for all potential contingencies.

If Line 12 fails, Node 2 will be connected to Line 23 only, as can be seen in the topology presented in Figure 6. Line 23 has a maximum capacity of 60 MW and, thus, the net injection in Node 2 must lie within the range of [-60, 60] in the pre-contingency situation. If Line 23 fails, Node 2 will be connected to Line 12 only. Line 12 has the same capacity as Line 23 (60 MW). Thus, the situation of Node 2 is analogous to the situation of Node 1 in the three-node model discussed above; it is connected to two lines with the same capacity.

Figure 6: The Topology of the six-Node Network if Line 12 fails



In the post-contingency of a failure of Line 12, flow across Line 23 is not part of a loop. The line is now radially connected to the network and the flow across it is bound by the maximum capacity only. For the post-contingency network we have two independent loops:

$$x_{13}^{12} + x_{35}^{12} - x_{45}^{12} - x_{14}^{12} = 0$$

$$x_{45}^{12} + x_{56}^{12} - x_{46}^{12} = 0$$

where x_{ij}^{hk} is the flow across Line *ij* in the post-contingency situation of Line *hk* failing.

The node rule equations in the post-contingency situation are:

$$q_1 = x_{13}^{12} + x_{14}^{12}$$

$$q_2 = x_{23}^{12}$$

$$q_3 = -x_{13}^{12} - x_{23}^{12} + x_{35}^{12}$$

$$q_4 = -x_{14}^{12} + x_{45}^{12} + x_{46}^{12}$$

$$q_5 = -x_{35}^{12} - x_{45}^{12} + x_{56}^{12}$$

where we have chosen to omit the node rule equation for Node 6, as only five are independent. The post-contingency flow must respect the maximum capacity of power flow across every line:

$$-x_{ij}^{MAX} \le x_{ij}^{12} \le \ x_{ij}^{MAX}$$

After having constructed analogous sets of constraints for the possible failure of the other seven lines in the network, we maximize social welfare using GAMS. The results are presented in Table 12 and Table 13; and in Figure 7. Table 13 includes post-contingency flows. The dashed lines represent lines which are congestion in at least one contingency situation.

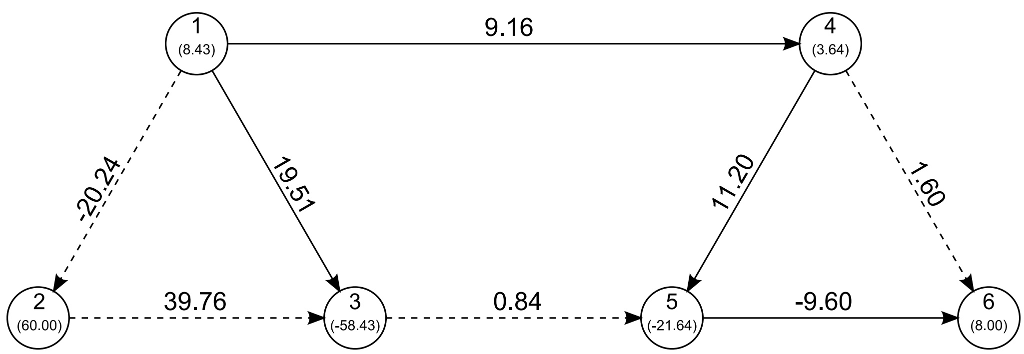
Table 12: Optimal Dispatch in the six-Node Network with N-1 Enforced

Node	Price	Supply	Demand	Net injection	Producer surplus	Consumer surplus	Grid revenue	Social welfare
1	16.34	81.69	73.26	8.43	667.26	134.17	-137.70	663.72
2	15.33	153.33	93.33	60.00	1175.56	217.78	-920.00	473.33
3	21.14	30.20	88.63	-58.43	319.14	392.72	1235.03	1946.89
4	16.15	80.73	77.09	3.64	651.71	148.56	-58.82	741.45
5	24.36	34.80	56.44	-21.64	423.73	159.26	527.14	1110.13
6	15.40	154.00	146.00	8.00	1185.80	1065.80	-123.20	2128.40
Sum		534.74	534.74	0.00	4423.20	2118.28	522.46	7063.93

Table 13: Optimal Flow in the six-Node Network with N-1 Enforced

Line	12	13	14	23	35	45	46	56
Power flow	-20.24	19.51	9.16	39.76	0.84	11.20	1.60	-9.60
Maximum Capacity	**60**	60	60	**60**	**10**	30	**8**	60
Flow if line 12 fails		4.79	3.64	**60.00**	6.36	7.52	-0.24	-7.76
Flow if line 13 fails	-9.09		17.52	50.91	-7.52	16.78	4.39	-12.39
Flow if line 14 fails	-17.19	25.62		42.81	**10.00**	5.10	-1.45	-6.55
Flow if line 23 fails	**-60.00**	48.43	20.00		**-10.00**	18.43	5.21	-13.21
Flow if line 35 fails	-20.52	18.95	10.00	39.48		11.76	1.88	-9.88
Flow if line 45 fails	-18.64	22.71	4.36	41.36	5.64		**8.00**	-16.00
Flow if line 46 fails	-20.10	19.81	8.72	39.90	1.28	12.36		-8.00
Flow if line 56 fails	-19.37	21.26	6.54	40.63	3.46	18.18	**-8.00**	

Figure 7: Optimal Flows and Net Injection with N-1 Enforced



When comparing the results with the previous results – optimal dispatch without implementing N-1 constraints – we see that the sign of net injection hasn't changed in any node. But there is less flow through all lines and the net injection of every node is closer to being zero. As in the three-node model, enforcing the N-1 criterion has reduced trading amongst nodes. Social welfare has also decreased, as expected, but grid revenue has increased. In some nodes individual producer surplus has increased and in some nodes individual consumer surplus has increased.

No lines operate at maximum capacity in the optimal dispatch solution. The maximum capacities bind the dispatch through post-contingency flows only: lines 12, 23, 35 and 45 would each operate at maximum capacity in one or more post-contingency situations, as can be seen in Table 13. We observe a change in the pattern of congestion. Lines 23 and 46 were also congested when we did not include security constraints, but we observe congestion in two lines that were not congested before: lines 12 and 35. However, the limited capacity of Line 45 is no longer a binding constraint.

In order to examine the relationship between congested lines and nodal prices, described by equation (4.1), we will have to employ the PTDFs for the post-contingency networks. We construct a PTDF-table for the five post-contingency congestions: Table 14. The table further includes shadow prices. We only include rows for congested lines in order to keep the size of the table manageable. For radially connected lines PTDFs can equal negative or positive one; or zero.

Table 14: Post-contingency PTDFs and Shadow Prices for the six-Node Network

		Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Shadow price
Line 12 failure:	Line 23	0	1	0	0	0	0	2.23/2
Line 14 failure:	Line 35	1	1	1	0	0	0	5.83
Line 23 failure:	Line 12	0	-1	0	0	0	0	-2.23/2
	Line 35	0.36	0.36	0.64	0.09	-0.09	0	-8.64
Line 45 failure:	Line 46	0.57	0.50	0.43	0.79	0.21	0	17.12
Line 56 failure:	Line 46	1	1	1	1	1	0	-13.41
	$p_6 - p_i$	-0.94	0.07	-5.74	-0.75	-8.96	0	

We observe the same interconnectedness of the capacities of Line 12 and Line 23 as we did with Line 12 and Line 13 in the three-node model. Analogously to the situation that occurred in the three-node model, the shadow price of 2.23 represents the shadow price of the post-contingency congestion, jointly caused by both lines. This was discussed in detail in Chapter 4.1. Thus, in Table 14 we report the shadow prices of both lines as (positive and negative) half of 2.23. In the context of calculating price differences using equation (4.1), we could just as well report one as 2.23 and the other as zero.[12]

In contrast to optimal dispatch when the N-1 criterion is not enforced, there can be several shadow prices associated with each line. Line 35, for example, is congested in the post-contingency situation of a failure in Line 14 as well as a failure of Line 23. Increasing the capacity of Line 35 thus affects optimal dispatch – with the effect of increasing social welfare – through two separate mechanisms.

[12] Where that would be a negative 2.23 for Line 12.

In cases where we have more than one shadow price associated with a line, they have to be interpreted differently from regular shadow prices. Each can obviously not represent the total marginal value of increasing the capacity of that line. Instead, each represents a partial marginal value of increasing the capacity of it. However, if we add, for example, the absolute partial shadow prices associated with Line 35 together, we get an approximation of the marginal value of increasing the capacity of the line. This is only an approximation as the contingencies are interconnected. If it were possible to increase the capacity of Line 35 in only one contingency, it would affect the shadow price of the congestion in the other contingency.[13] For Line 35, we obtain the following approximation of the overall marginal value – with respect to social welfare – of increasing its capacity: $|\,5.83\,| + |-8.64\,| = 14.47$. Numerical analysis has verified that this approximation is very accurate for all lines with partial shadow prices in models we have included in this dissertation.

In Table 15 we report modified PTDFs, where each value reflects how the flow of the line in question would change if one MW were injected into the node in question and withdrawn evenly at all other nodes. As in Table 6, this allows us to examine the deviation from average price using a modified version of equation (3.7):

$$p_i - \bar{p} = \sum_{all\ hk,\ \ all\ qr} \left(SP_{hk}^{qr} \cdot \left(-PTDF^{*}{}_{hk,i}^{qr}\right)\right)$$

Table 15: Modified Post-Contingency PTDFs and Shadow Prices for the six-Node Network

		Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Shadow price
Line 12 failure:	Line 23	-0.17	0.83	-0.17	-0.17	-0.17	-0.17	2.23/2
Line 14 failure:	Line 35	0.50	0.50	0.50	-0.50	-0.50	-0.50	5.83
Line 23 failure:	Line 12	0.17	-0.83	0.17	0.17	0.17	0.17	-2.23/2
	Line 35	0.14	0.14	0.41	-0.14	-0.32	-0.23	-8.64
Line 45 failure:	Line 46	0.15	0.08	0.01	0.37	-0.20	-0.42	17.12
Line 56 failure:	Line 46	0.17	0.17	0.17	0.17	0.17	-0.83	-13.41
	$p_i - \bar{p}$	-1.78	-2.79	3.02	-1.97	6.24	-2.72	

[13] While this might not be possible for actual lines, it is indeed possible in mathematical models.

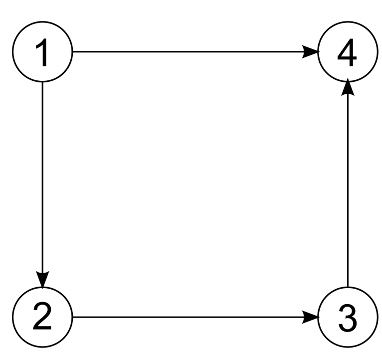
4.3 Implications of N-1 Security Constraints for Investment

Shadow prices play a fundamental role in analyses of optimal investment in power networks as they can be interpreted as the marginal value of increasing the capacity of a line.

Of course actual investment does not take place in marginal steps and often consists of building new lines, rather than strengthening old ones. But power flow models can easily be used to examine the benefits of realistic investment options. All things considered, the information provided by congestion and shadow prices provides vital information on weak links of transmission networks: links that it would be beneficial to strengthen. Enforcing the N-1 criterion in a transmission network transforms and intensifies patterns of congestion. It is self-evident that this has widespread implications for optimal investment when security is an issue.

An important case is the larger role low-capacity lines can play in congestion patterns when N-1 is implemented. For example, say we have a four-node network, with all nodes connected to two lines, as in Figure 8. We assume that Line 12 has a high flow capacity and that Line 14 has a low flow capacity. Now, without N-1 constraints, the flow over Line 12 can be high, as long as it is countered by a high flow in the other direction somewhere in the loop; on Line 23 and/or Line 34. However, if we enforce N-1, the net injection of Node 1 will be limited to the capacity of Line 14. It is unlikely that enforcing the N-1 security constraint in this situation would be economically viable. But it might be worthwhile to invest in the network to be able to implement N-1 and increase security. In that case, an analysis of N-1 flows would reveal Line 14 as the weak link, while its shadow price might very well be zero in optimal dispatch without security constraints, depending on parameters. It would be possible to enforce N-1 at a much lower cost if the weak link on Line 14 would be eliminated by directly strengthening it, or by constructing a new line with a high capacity between nodes 1 and 3.

Figure 8: A Diagram of the four-Node Network



We can deduce a rule of thumb for networks well adapted to implementing N-1: If a line connected to a node tends to carry high flows, other lines connected to the same node must have the capacity to carry that flow in case it fails. In other words, the capacities of power lines must be balanced.

It is important to distinguish between socially optimal investments and optimal investments for individuals. While an analysis of dispatch can reveal optimal investment options, it is not necessarily so that incentives exist for individual actors to invest optimally, as is for example discussed in Bjørndal, M. (2000). Investments in power networks that increase social welfare, or lower cost of dispatch, may not make every actor on the market better off, especially not the receiver of grid revenues. That is, optimal investments in networks tend not to be Pareto-efficient.

In order to be able to analyse the benefits of strengthening the lines in the six-node model, we examine the sensitivity of the shadow prices to changes in capacities. We employ absolute values of shadow prices, which are more relevant to investment analysis, as they represent the marginal value of increasing capacities. For dispatch without security constraints, this is depicted in Figure 9, Figure 10 and Figure 11; for the congested lines 23, 45 and 46 respectively. The graphs further depict social welfare, enabling us to see how large-step increases in capacities affect social welfare.

Figure 9: The Sensitivity of the Shadow Price of Line 23 (no Security Constraints)

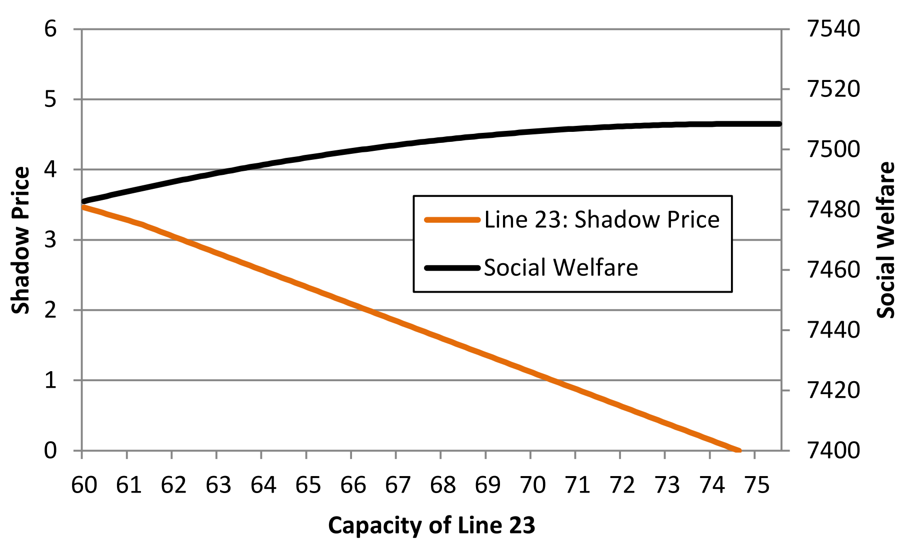


Figure 10: The Sensitivity of the Shadow Price of Line 45 (no Security Constraints)

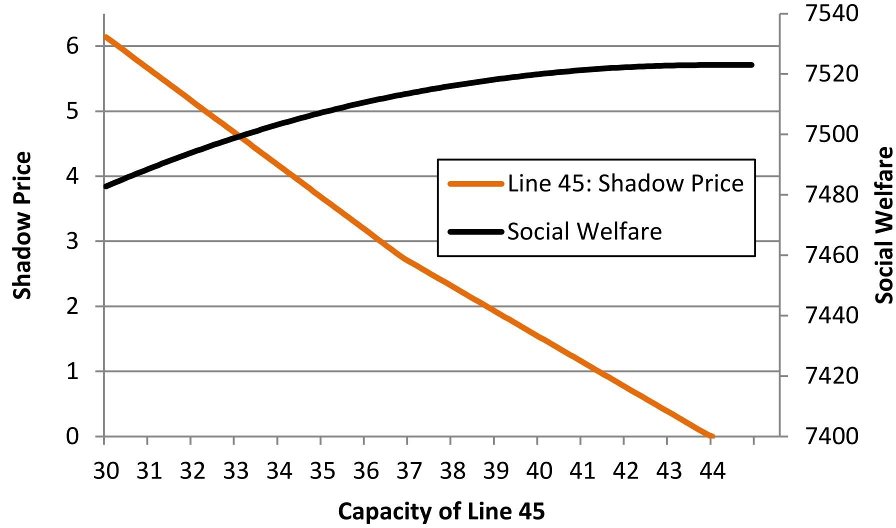
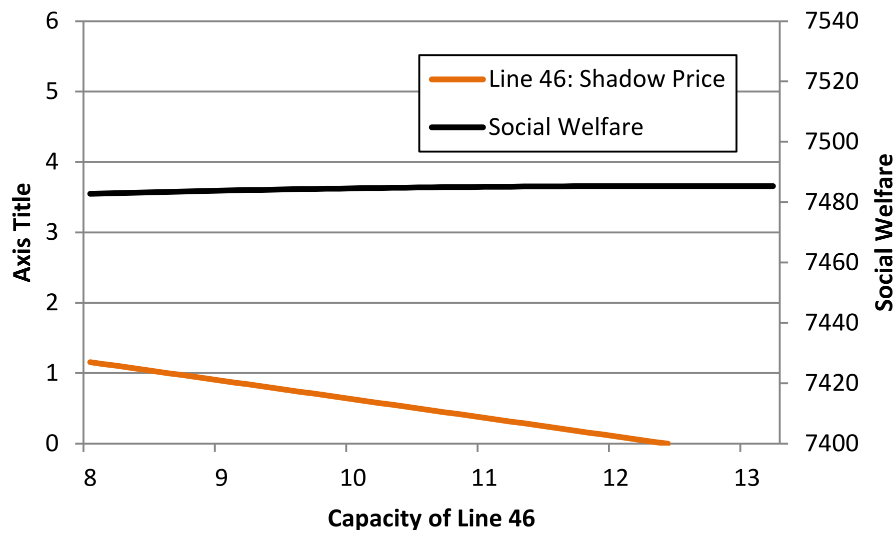


Figure 11: The Sensitivity of the Shadow Price of Line 46 (no Security Constraints)



For each line, we observe a decreasing marginal benefit of increasing capacity, with the shadow prices eventually reaching zero. In one case, for Line 45, we observe a change in the slope of the shadow price. This happens at a point between a capacity of 36 and 37 MW and signals a change in the congestion pattern of the network. Line 45 is still congested at the capacity of 37 MW as is indicated by the non-zero shadow price, indicating that the change in congestion pattern happens elsewhere in the grid. In Chapter 3 we discussed how the capacities of lines 45 and 46 in the six-node model effectively limited the net injection of Node 6 through the loop rule equations for the right-hand side triangle. At the point where the slope changes in Figure 10, this stops being a binding constraint. That is, the flow over Line 56 will no longer be bound by the loop rule equation $x_{56} = x_{46} - x_{45}$. The net injection of Node 6 will have reached a point where its optimal net injection will be determined by relative shadow prices rather than the absolute limit imposed by network design. The increase in the capacity of Line 45 will thus effectively have relieved the congestion of another line.

We likewise examine the sensitivity of shadow prices when N-1 is enforced. While four lines are congested in one or more contingency situations, two have shadow prices that are extremely over-sensitive to increases in their capacities: lines 12 and 23. If we increase the capacity of either of these lines by an infinitesimal amount, the shadow price immediately drops to zero. A graph of that process would be very uninformative. This happens as both lines jointly cause congestion, as was discussed in detail in Chapter 4.1.

Lines 35 and 46 have two partial shadow prices each, as they are congested in two different contingency situations each. For each line, if we add together their partial shadow prices, we get an approximation of the marginal value of increasing their capacity, as is discussed in Chapter 4.2. The partial shadow prices of lines 35 and 46 are depicted in Figure 12 and Figure 13 respectively; as well as the sum of their partial shadow prices.

For both lines 35 and 46 we observe a decreasing marginal benefit of increasing capacity, but opposed to before, shadow prices do not decrease continuously. In both cases they suddenly drop to almost zero. As before, this is due to changes in congestion patterns. Keeping track of which mechanisms affect social welfare when we enforce the N-1 criterion can be demanding, as it can involve eight different sets of post-contingency equations. Nevertheless we will try to give as simple of an overview as possible of why these drops in shadow prices occur.

Figure 12: The Sensitivity of the Shadow Price(s) of Line 35 (with N-1 Constraints)

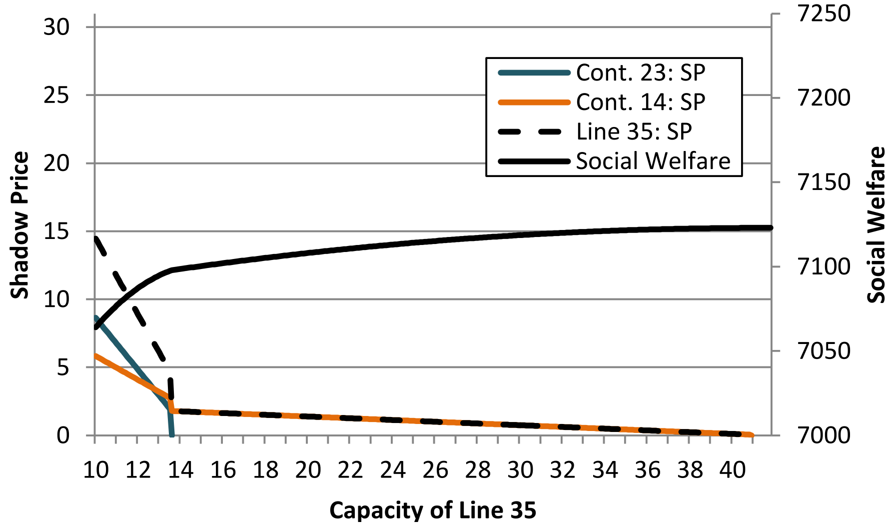
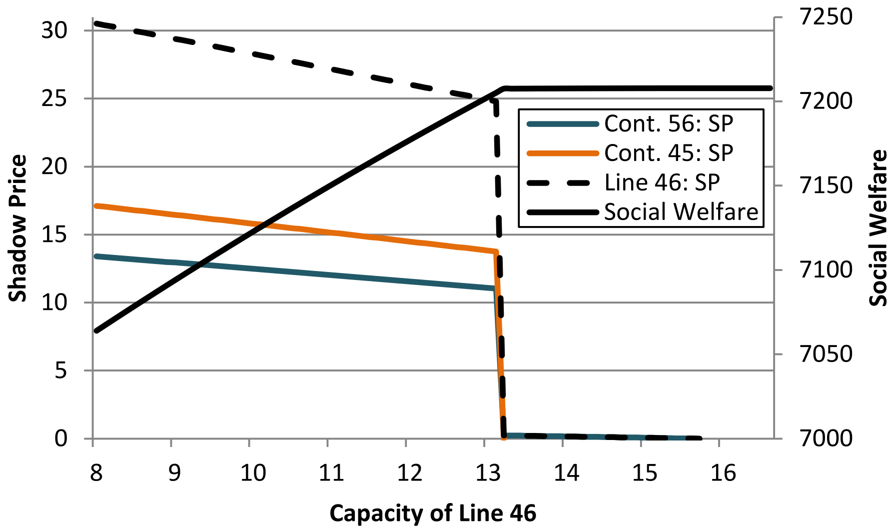


Figure 13: The Sensitivity of the Shadow Price(s) of Line 46 (with N-1 Constraints)



We will start by examining the effects of increasing the capacity of Line 35. Power between the left and right side of the six-node network can only flow through lines 14 and 35. When we implement N-1, the aggregate flow between the two sides will be limited by the capacity of Line 35. This is a consequence of the Line 14-contingency, which causes all flow between the left and right sides of the network to go through Line 35. Now, as we increase the capacity of Line 35, more power will flow from the left side to right in optimal dispatch. To begin with, this also allows for increased flows within the left-hand side triangle between nodes 1 and 3; and within the right-hand side triangle between nodes 4 and 5. This is happens as the increased capacity of Line 35 makes certain loop flow equations less constricting. It becomes possible to increase flows over certain power lines by responding with an increase in the flow on Line 35. At a certain point, however, other lines that form a part of these post-contingency loop rule equations will become congested. After that, it will be impossible to further increase trading between nodes 1 and 3; and between nodes 4 and 5 by utilizing the increased flexibility in the loops. At this point we observe the sudden drop in the shadow price of Line 35. As we want to examine why the shadow prices suddenly drop, we have to depict capacity changes on both sides of the drop in shadow prices. Figure 14 depicts the how flows change in the Line 23-contingency if we increase the capacity of Line 35 from 12 to 13 MW. For each line we display the flows corresponding to a capacity of 13 MW on Line-35, while in parentheses we display by how much the flows changed when we increased the capacity from 12 to 13 MW. Figure 15 is arranged in the same manner, but depicts a change in the capacity of Line 35 from 14 to 15 MW.

After the drop of the shadow prices, increasing the capacity of Line 35 will still increase trading between the left-hand side and the right-hand side of the network, but only by increasing the net injection of nodes 1 and 3 evenly to withdraw power in nodes 4 and 5 evenly. We have to satisfy the loop rule equation for the loop formed by nodes 1-3-5-4-1.

Figure 14: N-1 Flows in the Line 23-contingency when the Capacity of Line 35 is increased from 12 to 13 MW

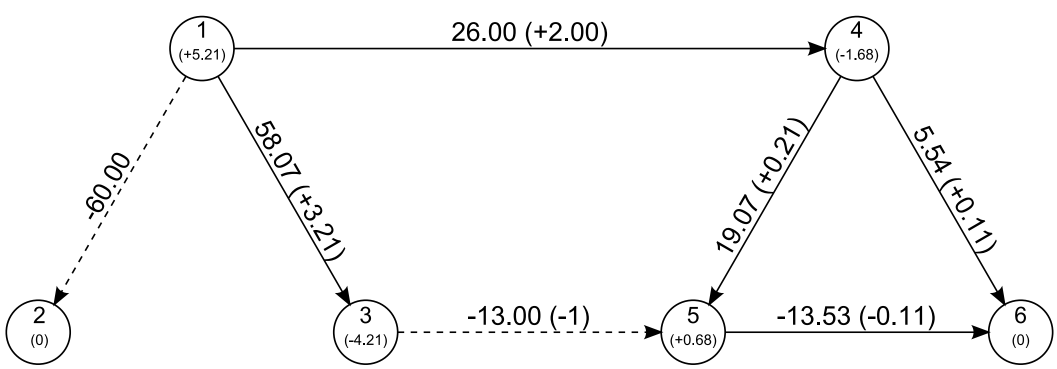
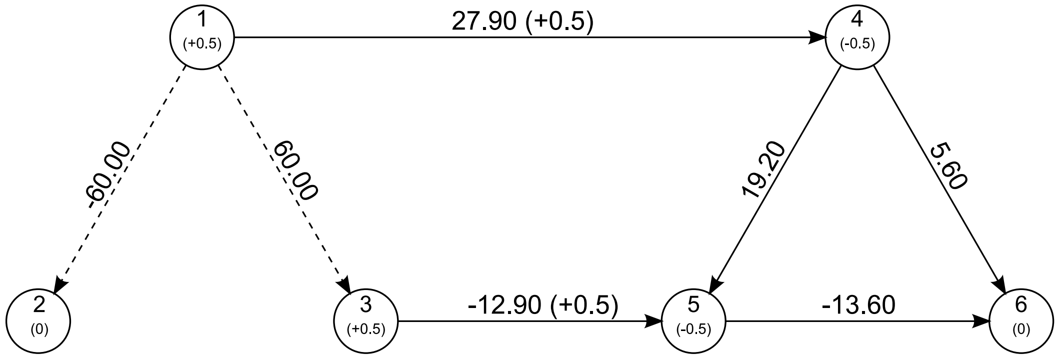


Figure 15: N-1 Flows in the Line 23-contingency when the Capacity of Line 35 is increased from 14 to 15 MW



Now we turn our attention to the shadow price of Line 46. If Line 56 fails, Line 46 it must carry the whole net injection of Node 6. Thus, the net injection of Line 6 is bound by the capacity of Line 46. If we increase the capacity of Line 46 the producer at Node 6 will increase his net injection, as he has relatively low marginal costs, and the flow on Line 46 will increase in the direction from Node 6 to Node 4. To begin with, this also allows for nodes 1 and 5 to increase their production to be withdrawn in nodes 3 and 5, as certain loop flow equations in other contingencies become less binding. However, at a certain point, Line 45 will become congested in the Line 56-contingency. After that, increasing the capacity of Line 46 will have less influence on social welfare and most of the increased injection in Node 6 will be withdrawn at Node 4.

Figure 16: N-1 Flows in the Line 56-contingency when the Capacity of Line 46 is increased from 12 to 13 MW

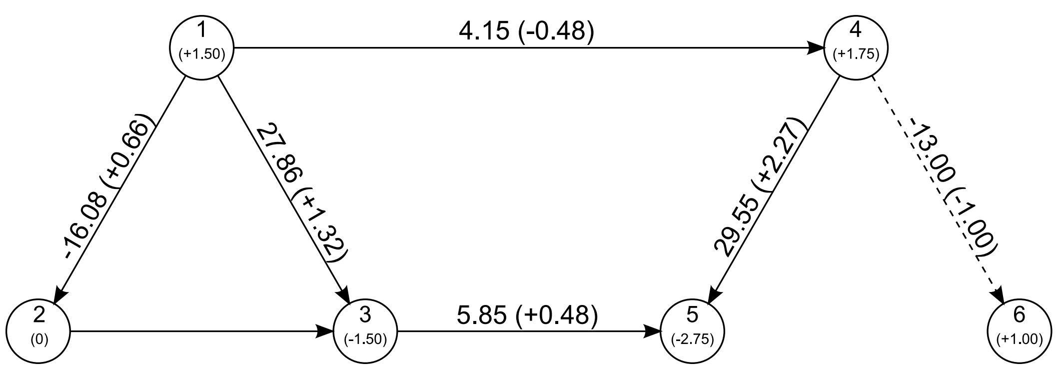
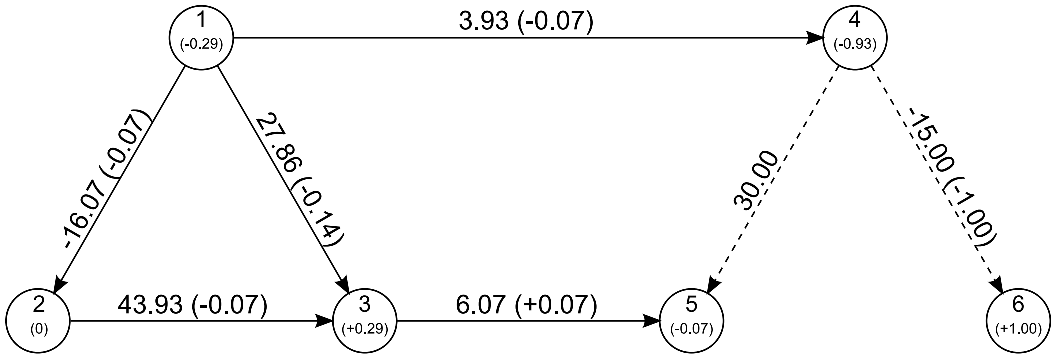


Figure 17: N-1 Flows in the Line 56-contingency when the Capacity of Line 46 is increased from 14 to 15 MW



Erroneously using a model without N-1 constraints to identify profitable line reinforcements in a network can result in flawed results. In the context of the six-node model we might for example conclude that strengthening Line 45 would be most beneficial. But if N-1 were enforced, it wouldn't even affect optimal dispatch.

We have analysed the benefits of investment in a fictional model in a steady-state situation. While analysing optimal investments in actual power transmission networks would involve much richer, dynamic models, there is no doubt that optimal investment can depend heavily on whether security will be taken into account or not. As the cost of blackouts is very real, security of delivery should be taken into account in analyses of actual investment options.

4.4 A Comparison with other Types of Security Constraints

In this chapter we will compare optimal dispatch under the N-1 security rule with results obtained when other methods of implementing security constraints are employed. We start by examining how contingencies affect flows in the six-node model when no security constraints are implemented. This is presented in Table 16. When no security constraints are implemented, flows in all contingencies but one exceed capacities on some lines.

Table 16: Post-Contingency Flows for the six-Node Network with no Security Constraints implemented

Line	12	13	14	23	35	45	46	56
Power flow	-24.42	35.58	14.99	**60.00**	9.41	**30.00**	**-8.00**	-38.00
Maximum Capacity	**60**	**60**	60	**60**	10	**30**	**8**	60
Flow if line 12 fails		17.82	8.33	**84.42**	**16.07**	25.56	**-10.22**	-35.78
Flow if line 13 fails	-4.09		30.24	**80.33**	-5.83	**40.17**	-2.92	-43.08
Flow if line 14 fails	-19.42	45.58		**65.00**	**24.41**	20.01	**-13.00**	-33.00
Flow if line 23 fails	**-84.42**	**79.22**	31.36		-6.95	**40.91**	-2.55	-43.46
Flow if line 35 fails	-27.56	29.30	24.41	56.86		**36.28**	-4.86	-41.14
Flow if line 45 fails	-20.14	44.15	2.14	**64.29**	**22.27**		**9.14**	-55.14
Flow if line 46 fails	-25.15	34.13	17.18	59.27	7.23	24.18		-46.00
Flow if line 56 fails	-20.97	42.49	4.63	**63.46**	**19.78**	**57.64**	**-46.00**	

Neuhoff, K et al (2011) use the method of de-rating lines to 80% of their normal capacity in order to approximate N-1 security constraints. When this method is employed to calculate the dispatch of the six-node model it approximates the effects of implementing N-1 in the sense that it lowers social welfare while increasing the grid revenue; it reduces flows on all lines; and it increases the average deviation of nodal prices from average price. Optimal dispatch with line capacities de-rated to 80% is presented in Table 17 and Table 18. De-rating lines affects injections with relative uniformity compared with N-1, as is displayed in Table 19. Congestion occurs in the same lines as when no security constraints were implemented, albeit for some lines, not in as many contingency situations. The main improvement in terms of security is that the network would withstand a failure of Line 35, unlike before. Also, it causes flows on congested lines to exceed their capacities by less. This method thus seems to be relatively ineffective in increasing security in the six-node model. In fact, we would have to de-rate the lines to less than to 20% of normal capacities to ensure that flow would never exceed capacities in any contingency, wiping out much of the benefits of trading. This reduces the social welfare by more than 719 compared with dispatch with no security constraints, while enforcing the N-1 criterion reduces it by roughly 419.

Table 17: Optimal Dispatch of de-Rated Lines

Node	Price	Supply	Demand	Net injection	Producer surplus	Consumer surplus	Grid revenue	Social welfare
1	17.17	85.83	56.68	29.16	736.70	80.30	-500.48	316.52
2	15.48	154.84	90.33	64.51	1198.69	203.99	-998.76	403.92
3	19.88	28.40	101.22	-72.83	282.23	512.31	1447.62	2242.16
4	16.14	80.69	77.25	3.44	651.04	149.20	-55.44	744.79
5	20.91	29.87	90.94	-61.07	312.20	413.46	1276.74	2002.40
6	16.84	168.40	131.60	36.80	1417.93	865.93	-619.71	1664.14
Sum		548.02	548.02	0.00	4598.79	2225.18	549.96	7373.94

Table 18: Post-Contingency Flows of de-Rated Lines

Line	12	13	14	23	35	45	46	56
Power flow	-16.51	31.50	14.16	48.00	6.67	24.00	-6.40	-30.40
Maximum Capacity	**60**	**60**	60	**60**	**10**	**30**	**8**	60
Flow if line 12 fails		19.49	9.66	**64.51**	**11.17**	21.00	-7.90	-28.90
Flow if line 13 fails	1.49		27.66	**66.00**	-6.83	**33.00**	-1.90	-34.90
Flow if line 14 fails	-11.78	40.94		52.72	**20.83**	14.56	**-11.12**	-25.68
Flow if line 23 fails	**-64.51**	**66.40**	27.26		-6.42	32.73	-2.04	-34.76
Flow if line 35 fails	-18.73	27.05	20.83	45.78		28.45	-4.18	-32.62
Flow if line 45 fails	-13.08	38.35	3.88	51.43	**16.96**		7.31	-44.11
Flow if line 46 fails	-17.09	30.33	15.91	47.42	4.92	19.35		-36.80
Flow if line 56 fails	-13.74	37.02	5.87	50.76	**14.96**	46.11	**-36.80**	

Table 19: %-Reduction in Injections when N-1 and de-Rating is implemented

	1	2	3	4	5	6
Injection without security constraints	26.15	84.42	-86.17	7.01	-77.41	46
Lines de-rated to 80%	8.43	60	-58.43	3.64	-21.64	8
%-reduction	32%	71%	68%	52%	28%	17%
N-1 implemented	29.16	64.51	-72.83	3.44	-61.07	36.8
%-reduction	112%	76%	85%	49%	79%	80%

The effect of implementing the N-1 criterion has a more lopsided effect, with for example the net injection of Node 6 falling dramatically. By definition, the post-contingency flows never exceed line capacities when N-1 is implemented. For our six-node network, it seems we would have to de-rate the lines more than to 80% of the normal capacity in order to achieve comparable reductions in net injections on average as we do when we implement the N-1 constraints. The method of de-rating lines can probably be effectively used to approximate the costs of enforcing the N-1 criterion in a power transmission network. But our results imply that it does a poor job of improving security effectively and capturing the effect N-1 has on limiting flows in specific parts of networks.

Bjørndal, E. et al (2012), in their comparative study of power dispatch under nodal and zonal pricing regimes in the Nordic power market, use cut constraints to model the security restrictions imposed by Statnett in Norway. They define a cut as a set of transmission lines over which aggregate flow must not exceed the selected cut capacity. We will examine the effect of imposing such a constraint on lines 14 and 35 in our six-node model. If one of these lines fails, the power would be redirected to the other. Therefor we constrain the aggregate flow of these two lines to the range of [-10, 10], corresponding with the capacity of Line 35, which has lower maximum capacity. We have:

$$10 \leq x_{12} + x_{35} \leq \ 10$$

Enforcing this constraint will guarantee that the flow between the left-hand side triangle and the right hand-side triangle will not be disrupted in case either line 12 or 35 fails. Optimal dispatch including the cut constraint is represented in Table 20 and Table 21. It is worth noting that cut constraints can be implemented over cuts that do not separate the network into two parts. Bjørndal, E. et al (2012) mention an example where a line receives a certain percentage of the flow of another line in case it fails. In that case the cut constraint is formulated to constrain the flow of the receiving line so that it can accept that particular percentage of flow from the other.

Table 20: Optimal Dispatch with a Cut Constraint across lines 12 and 35

Node	Price	Supply	Demand	Net injection	Producer surplus	Consumer surplus	Grid revenue	Social welfare
1	16.68	83.40	66.39	17.01	695.60	110.19	-283.80	521.99
2	16.17	161.73	76.54	85.19	1307.85	146.45	-1377.86	76.44
3	18.18	25.97	118.18	-92.21	236.13	698.35	1676.51	2610.99
4	16.58	82.92	68.32	14.60	687.56	116.70	-242.08	562.18
5	20.07	28.68	99.27	-70.60	287.80	492.75	1417.08	2197.63
6	17.30	173.00	127.00	46.00	1496.45	806.45	-795.80	1507.10
Sum		555.70	555.70	0.00	4711.39	2370.89	394.05	7476.33

Table 21: Post-Contingency Flows with a Cut Constraint across lines 12 and 35

Line	12	13	14	23	35	45	46	56
Power flow	-25.194	34.806	7.403	**60**	2.597	**30**	**-8**	-38
Maximum Capacity	**60**	**60**	60	**60**	**10**	**30**	**8**	60
Flow if line 12 fails		16.48	0.53	**85.19**	9.47	25.42	**-10.29**	-35.71
Flow if line 13 fails	-5.31		22.32	**79.89**	**-12.32**	**39.94**	-3.03	-42.97
Flow if line 14 fails	-22.73	39.74		**62.47**	**10.00**	25.07	**-10.47**	-35.53
Flow if line 23 fails	**-85.19**	**78.44**	23.77		**-13.77**	**40.91**	-2.55	-43.46
Flow if line 35 fails	-26.06	33.07	10.00	59.13		**31.73**	-7.13	-38.87
Flow if line 45 fails	-20.91	43.38	-5.45	**64.29**	**15.45**		**9.14**	-55.14
Flow if line 46 fails	-25.92	33.35	9.59	59.27	0.42	24.18		-46.00
Flow if line 56 fails	-21.74	41.72	-2.96	**63.46**	**12.96**	57.64	**-46.00**	

The congestion patterns in the post-contingency flows are curious. Enforcing the cut constraint over lines 14 and 35 secures that the flow of both lines stay within capacities if either one fails, where flow over Line 35 will be at the limit if Line 14 fails. But, it does not secure that flow over Line 35 stays within capacity if some other lines fail. This is due to flow that takes place over loops that include both triangles. This is not necessarily a problem. We see in Table 21 that the aggregate flow from the left-hand side triangle to the right-hand side triangle – through lines 14 and 35 – is always equal to 10 MW. It would be possible to respond to the contingency of a failure in line 13, 23, 45 or 56 by manually disconnecting either line 14 or 35. Breaking the loop will cause total flow over the cut to stay within limits. However, flows over other lines would still be above limits, but the cut constraint was not designed to deal with problems in other parts of the network to begin with.[14]

Hedman, K. W., O'Neill, R. P., Fisher, E. B. and Oren, S. S. (2009) demonstrate that considerable cost cutting can be achieved by allowing for transmission switching when implementing N-1. While transmission switching lies beyond the scope of this thesis, the above example demonstrate the increased flexibility such methods allow for.

[14] Usually there exists some operating reserve in power networks available with short notice to meet disruption to supply. Perhaps these could be employed to deal with problems in other parts of the network.

5 Conclusions and Final Remarks

In this dissertation we have developed a simple steady-state model and analysed how enforcing the N-1 criterion in power transmission networks affects optimal dispatch. We have employed major simplifications in our models. In reality power markets operate in a dynamic environment with demand and supply fluctuating throughout the day and year. In addition, to accurately describe power flow, and the costs associated with it, we would have to employ more complicated power flow models; as well as more accurate supply and demand models. However, we have been able to deduce some generalities about the implications of enforcing the N-1 criterion in the power markets, which would be valid for real transmission networks:

I. *Enforcing the N-1 criterion tends to increase congestion and reduce trading. This can lead to lower social welfare and higher cost of production in optimal dispatch. The value of the increased security must be higher than these costs in order for it to be beneficial to enforce the N-1 criterion.*

II. *While enforcing the N-1 criterion leads to lower social welfare (with the value of security not considered), it can lead to higher surplus for individual market actors, especially the receiver of grid revenues. This leads to a potential conflict of interests.*

III. *Enforcing the N-1 criterion changes the patterns of congestion. This has considerable consequences for optimal investment. In particular, it can lead to low-capacity lines playing a much larger role in congestion. Special care must be taken to incorporate effects on security when investment options are analysed.*

There is ample room for further research, for example analysing the N-1 criterion using more accurate dynamic models; including probabilities of lines failures and costs of blackouts; and analysing the effects on optimal investments deeply.

The few examples analysed in this dissertation seem to suggest that enforcing the N-1 criterion can have a severe effect on limiting trade between nodes, and thus decrease social welfare and increase the cost of production. But, then again, the costs of power outages can be severe as well. For actual power transmission networks, less severe security constraints might be more viable. However, the analysis conducted by Hedman, K et al (2009) strongly suggests that the flexibility of power networks could be increased considerably by using transmission switching as a control variable. It would be of particular interest to analyse the effects of implementing N-1, perhaps while allowing for transmission switching, in an existing power market, using real demand and supply data, such as by expanding the OptFlow model developed by Bjørndal, E. et al. (2012).

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