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Nodal Pricing in a Coupled Electricity Market

BY
Endre Bjørndal, Mette Bjørndal,
AND **Hong Cai**

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NORWEGIAN SCHOOL OF ECONOMICS

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Endre Bjørndal, Mette Bjørndal, Hong Cai
Department of Business and Management Science
Norwegian School of Economics
Bergen, Norway
Endre.Bjorndal@nhh.no, Mette.Bjorndal@nhh.no, Hong.Cai@nhh.no

***Abstract*—This paper investigates a pricing model for an electricity market with a hybrid congestion management method, i.e. part of the system applies a nodal pricing scheme and the rest applies a zonal pricing scheme. The model clears the zonal and nodal pricing areas simultaneously. The nodal pricing area is affected by the changes in the zonal pricing area since it is directly connected to the zonal pricing area by commercial trading. The model is tested on a 13-node power system. Within the area that is applying nodal pricing, prices and surpluses given by the hybrid pricing model match well with those given by the full nodal pricing model. Part of the network is better utilized compared to the solutions given by the full zonal pricing model. However, the prices given by the hybrid system may send wrong economic signals which triggers unnecessary generation from existing capacities, exacerbates grid congestion, and induces higher re-dispatching costs.**

***Index Terms*—Congestion Management; Nodal Pricing; Zonal Pricing; Electricity Market.**

NOMENCLATURE

A. Sets and Indices

N	Set of nodes
N^{Nodal}	Set of nodes in the nodal pricing area
L	Set of lines
L^{DC}	Set of DC lines
Z	Set of independent price areas
N^{Z}	Subset of nodes included in the price area $z \in Z$

B. Parameters Set and Indices

H_{ij}	Admittance of the line between the nodes i and j
CAP_{ij}	Thermal capacity limit of the line from i to j
CAP_{xz}	Upper limit on the flows from zone x to zone z
$p_i^s(q)$	Supply bid curve at node i
$p_i^d(q)$	Demand bid curve at node i

C. Variables

q_i^s	Generation quantity (MWh/h) at node i
q_i^d	Load quantity (MWh/h) at node i
f_{ij}	Load flow from node i to node j
θ_i	Phase angle at node i

I. INTRODUCTION

In the European spot markets, zonal pricing is the most commonly used method to relieve grid congestion. Zonal pricing applies merit order to dispatch power from one location to another. It is a commercial pricing scheme which only to a limited extent takes physical laws and technical facts into account. A possible consequence of this is that there could be insufficient capacities in the network to transmit the contracted power, which requires the system operator to adjust the generation and consumption in order to change the physical flows in the network and to mitigate congestion [3]. Furthermore, zonal pricing gives a uniform price within each pricing area and thus does not provide sufficient price signals to market participants regarding scarce transmission capacity. In contrast, nodal pricing, which is first discussed by [5], gives the optimal value for each location and produces feasible flows within the network, and is considered to give clearer market signals [2].

Some European countries are considering adopting nodal pricing systems. For instance, Poland has prepared to implement nodal pricing since 2010 and the whole implementation is expected to be finished in 2015 [6]. However, as the Polish power grid is connected to other continental countries, it is inevitable to be affected by (and affect) flows from other areas. It is thus a research question whether nodal pricing in such a case can still work as efficiently as it is supposed to do.

In this paper, we first propose a hybrid pricing model, which could be applied to a joint power market, in which the market is divided into different sub-systems, where some apply nodal pricing and others apply zonal pricing. It is important to note that a nodal pricing sub-system is not isolated from the other parts of the system and still has commercial trading with the connected zonal pricing sub-systems. In such a case, generation or consumption changes in the zonal pricing areas could still have an effect on the nodal pricing area because of the impact of loop flows. A 13-node power system serves to illustrate the hybrid pricing model. We compare the hybrid pricing scheme to the zonal and nodal pricing schemes to investigate how much a single pricing area can gain by applying nodal pricing in the context where its neighborhood areas apply zonal pricing.

The congestion management methods discussed in this paper, i.e., nodal pricing, zonal pricing, and hybrid pricing, are based on centralized optimization subject to the power flow control method chosen by the system operator to relieve grid congestion. The description of the models is provided in Section II. Section III gives a numerical example and compares attained results for different pricing schemes. Some preliminary conclusions are given in Section IV.

II. MODEL

The power market consists of two types of pricing areas, i.e., the nodal pricing and zonal pricing areas. The objective of the system is to maximize the social welfare (1), considering different network constraints ((2)-(5)). Equation (1) is the objective function, expressing the difference between the customers' willingness to pay and the production cost. The difference is defined as social welfare.

$$\max_{q^d, q^s, f, \theta} \sum_{i \in N} \left[\int_0^{q_i^d} p_i^d(q) dq - \int_0^{q_i^s} p_i^s(q) dq \right] \qquad (1)$$

$$q_i^s - q_i^d = \sum_{j:(i,j)\in L} f_{ij} - \sum_{j:(j,i)\in L} f_{ji}, \forall i \in N \qquad (2)$$

$$f_{ij} = H_{ij}(\theta_i - \theta_j), (i,j) \in l \setminus L^{DC}, \forall i, j \in N^{Nodal} \qquad (3)$$

$$-CAP_{ji} \le f_{ij} \le CAP_{ij}, \forall i, j \in N^{Nodal} \qquad (4)$$

$$-CAP_{zx} \le \sum_{\substack{(i,j)\in L \\ i \in N^x \\ j \in N^z}} f_{ij} - \sum_{\substack{(i,j)\in L \\ j \in N^x \\ i \in N^z}} f_{ji} \le CAP_{xz} \qquad (5)$$

In the nodal pricing area, the DC approximation [7] is used to approximate the power flow. The DC approximation gives much faster solution than the full alternating current (AC) solution, and the results given by the DC approximation match fairly well with the full AC solution [4]. The network flows in the nodal pricing areas are constrained by (2) to (4). Equation 2 is the energy balance equation, ensuring the difference of supply q_i^s and demand q_i^d at node *i* is equal to the difference of the power which is transported from (f_{ij}) and to (f_{ji}) node *i*. Equation (3) is the loop flow law, which determines the power flow f_{ij} on a transmission line by the admittances H_{ij} of the line and the difference of load angles (q_i - q_j) of its two connected points. Equation (3) also introduces a set for high voltage direct current (HVDC) transmission lines, L^{DC} . This set does not follow the loop flow law because flows on HVDC lines can be treated as controllable. Power flows on transmission lines are restricted by the thermal capacity limits CAP_{ij} (4). Flows within the nodal pricing area are physically feasible and thus are called physical flows. Physical flows could go from a high price node to a low price node, because of the loop flow constraints.

Within each zonal pricing area, there are no restrictions on the physical flows, i.e. loop rule and thermal capacity limits. Therefore, power will always go from a low price node to a high price node until prices for all nodes are the same, i.e., there are no opportunities to buy power from a lower price node. These flows are not necessarily feasible because they only take the economic but not physical restrictions into account. We refer to such flows as commercial flows. The networks in the zonal pricing areas are constrained by the energy balance equations (2) and aggregate capacity limits CAP_{xz} are used to restrict inter-zonal trading between two connected pricing areas x and z (5). This creates price differences among zones.

As the zonal pricing model does not include the loop flow law (3), the model does not give solutions for the phase angle variables q_i . Hence, flows on the lines connecting the zonal pricing areas and the nodal pricing areas cannot be modeled taking into account the physical law (3). That is, traded flows between the different pricing areas have to be treated as commercial flows. Therefore nodes in a nodal pricing area connected to a zonal pricing area are constrained by both the physical power exchange within the nodal pricing area and the commercial exchange within the zonal pricing area. Trading between the zonal and nodal pricing markets is also restricted by aggregate capacity limits (5), which is the same as in a full zonal pricing market (i.e., the whole network applies zonal pricing).

The dual variables of (2), which are the marginal costs/benefits of increasing injections in the nodes by one unit, are the nodal prices. Prices within each zonal pricing area are uniform as there are no restrictions on the intra-zonal trading. However, prices within the nodal pricing areas could be different, as the model takes both the physical laws and thermal capacity limits into account.

III. NUMERICAL EXAMPLE

A. Data

[1] uses a strongly simplified and rather aggregated model of the Nordic power market with different load scenarios to investigate the possibility of improving the capacity utilization of the transmission grid by varying the zone definitions. We choose this power system as an example for our analysis. Fig. 1 exhibits the topology and the zone definition of this power market.

There are in total 13 nodes in this system. Nodes 1 to 5 are within Norway (NO) and Nodes 6 to 10 are within Sweden (SE). Node 11 represents Finland (FI) and Node 12 and Node 13 represent Denmark (DK). This power market is decomposed into 4 zones according to their jurisdictions. There are in total 21 lines in the model and most of them are AC interconnections, except for Lines 1-13, 10-13, and 9-11, corresponding to HVDC cables. All the lines are assumed to have identical admittances.

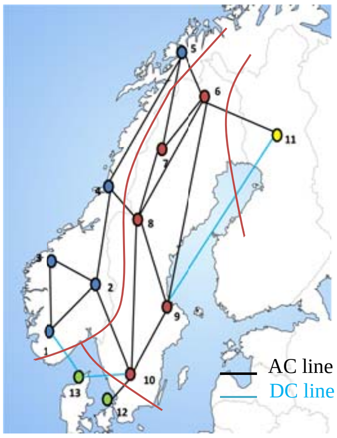

Fig. 1: Topology

This power system and its corresponding data are used as a starting point for examining the hybrid pricing method. We assume in the hybrid model that zone NO applies nodal pricing and that the rest use area prices.

B. *Aggregate capacity limits*

Aggregate capacity limits are used to restrict commercial trading between different pricing areas. In practice, setting adequate aggregate capacity limits is a challenging task because low limits would fail to fully use the network capacity while high ones could cause lots of congestion within a pricing area. In our analysis, we use the flows given by the full nodal pricing solution, i.e., where the whole network applies nodal pricing, as a basis to set the aggregate capacity limits. The limits are equal to the absolute value of accumulated flows between two pricing areas given by the nodal pricing solution.[1] The main reason for setting aggregate capacity limits in such a way is that the nodal pricing solution could be regarded as the optimal benchmark as it takes both the physical and economic constraints into account. These limits could be considered to optimize the utilization of the network given perfect information. Furthermore, this setting makes all the three pricing mechanisms (i.e., nodal pricing, zonal pricing, and hybrid pricing) comparable, because the traded volumes between two pricing areas are the same. When there is a price difference between two nodes connecting two different pricing areas, trading will continue until the price difference is eliminated or the aggregate capacity limit is reached. Note however that the actual flows resulting from the zonal and hybrid market clearings may still be infeasible.

We also assume that the aggregate capacity limits between two price areas are the same in both directions. For instance, the aggregate capacity limits from Norway to Sweden are equal to those from Sweden to Norway.

C. *Some results from a high load scenario*

Since congestion is likely to happen when demand is high, we choose a high demand hour for the following analyses. The total consumption volume given by the full nodal pricing solution is approximately 86% of the consumption prognosis at "10 years" winter temperature [1]. Data on the model and supply and demand information[2] are presented in the appendix.

1) *Prices*

Fig. 2 gives the prices at each node in different congestion management schemes. Prices within the zonal pricing market (Nodes 6 to 13) given by the hybrid pricing solution are identical to those given by the zonal pricing solution. This shows that if the aggregate capacity limits remain the same and the same proportion of the aggregate capacity limits is used, the prices within the zonal pricing market will not be affected by the congestion management scheme in the nodal pricing market.

The comparison between the prices in the nodal part of the hybrid system (i.e., Nodes 1 to 5) and the nodal prices for the whole system generates some interesting observations. In general, the two series of prices, presented in Fig. 2, match fairly well, with a notable exception for Node 5. At Node 5 the price given by the hybrid system is 132.5 NOK, while the full nodal price is only 91.6 NOK.

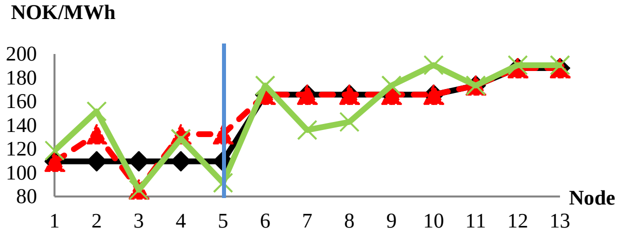

Fig. 2: Prices in different congestion management schemes

The reason for the high price at Node 5 in the hybrid system is that the three nodes that are directly connected with Sweden (i.e., Nodes 2, 4 and 5) face high demands from Sweden. In the hybrid system, the prices at these three points are set to be identical because flows going from these nodes to Sweden are modeled as direct flows without considering physical restrictions (i.e., the loop flow law).

As long as the thermal capacity of the lines connecting these three nodes to the zonal pricing area has not been fully used, i.e., there is no congestion in these lines, the prices at the three nodes should be equal. Otherwise, Sweden could always choose to buy power from the node with the lowest price, since the zonal pricing model does not take the laws of physics entirely into account. Therefore, Node 5 in the hybrid system gets a price as high as those at Nodes 2 and 4.

2) *Fully loaded and overloaded lines*

Physical flows[3] given by the zonal pricing scheme might not be feasible because it does not take scarce transmission capacity and the laws of physics into account. In the hybrid pricing model, the physical constraints are modeled for only parts of the system, so that there can still be infeasible flows in the zonal pricing area. Furthermore, areas applying nodal pricing are connected to other AC network areas applying zonal pricing, and could be affected by the loop flows in such areas. Investigating the capacity utilization of a transmission line, which is defined as the ratio of the physical flow to thermal capacity, helps to explain the reason why the price at Node 5 in the hybrid system is higher than the one in the nodal pricing system.

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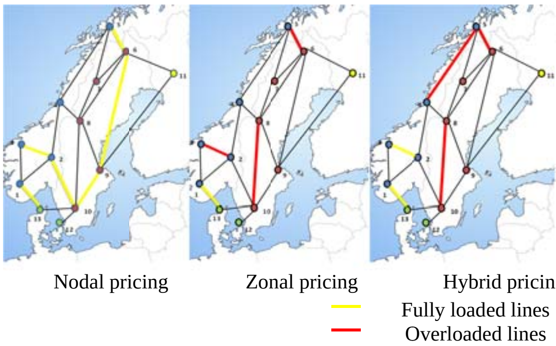
[1] For instance, the transfer capacity from Norway to Sweden is calculated as $CAP_{NO,SE} = \sum_{\substack{(i,j)\in L \\ i\in N^{NO} \\ j\in N^{SE}}} f_{ij}^{*} - \sum_{\substack{(i,j)\in L \\ j\in N^{NO} \\ i\in N^{SE}}} f_{ji}^{*}$, where f_{ij}^{*} and f_{ji}^{*} are solutions given by nodal pricing model.

[2] Formats of Supply and demand curves are displayed in Fig. A1. The corresponding data for parameters can be founded in Table AI and Table AII.

[3] To calculate the physical power flows of the zonal and hybrid pricing solution, we fix the values of nodal load q_i^d, generation q_i^s and flows over the DC lines $f_{ij}\ (\text{where}(i,j) \in L^{DC})$ using the solutions given by the models. We use these values as inputs for a detailed network model to re-compute the final line flows. This network model takes loop flow into consideration ((2) to (3)), minimizes the losses caused by dispatching, but does not consider thermal capacity constraints (4). Thus we obtain the power flows that will result from injections and withdrawals in the nodes given by the zonal and hybrid pricing solutions.

In the full nodal system, Nodes 2, 4 and 5 also face high demand from Sweden. Nodes 2 and 4 are indeed given high prices because of this. In comparison, the price at Node 5 is much lower, because Line 5-6 is fully-loaded. Fig. 3 displays the overloaded and fully loaded lines regarding these three different congestion management schemes. Congestion in Line 5-6 makes it impossible to transmit more power generated at this node to other areas, so the extra generation has significantly less value. In other words, low generation cost is not the only reason for the low price at Node 5. More importantly, the low price is due to the congestion which limits the power to be supplied to other areas. Without the congestion, production at Node 5 will be higher and at a higher marginal cost, implying a higher nodal price.


Nodal pricing Zonal pricing Hybrid pricing

🟨 Fully loaded lines
🟥 Overloaded lines

Fig. 3: Congestion for different pricing schemes

Consequently, the high price at Node 5 in the hybrid system gives wrong economic signals, which may cause short term and long term problems. First, more power will be generated by the existing generation capacity. This extra generation is unnecessary, because it cannot be transmitted to other areas due to the capacity constraint. Second, more generation will exacerbate congestion in those lines connecting Node 5 and other nodes. Note that Line 5-6 is fully loaded in the full nodal price solution. In the hybrid system, however, it becomes overloaded. Line 4-5 also becomes overloaded, despite it being within limits in the full nodal and zonal price solutions. Finally, the situation may worsen if the high price triggers more investments in generation capacity. Extra generation capacity in this area is unnecessary, and it will only intensify grid congestion. The extra congestion must be solved by re-dispatching, which leads to increased cost because the system uses more costly power in re-dispatching.

As discussed before, the nodal pricing area can be affected by the changes in the zonal pricing area. Therefore, there can also be infeasible flows in the nodal pricing area. As in Fig. 3, compared to the zonal pricing scheme, congestion in Line 2-3 is alleviated in the hybrid pricing scheme. However, congestion happens in Lines 5-6 and 4-5, even if the flow on Line 4-5 is feasible in the zonal pricing scheme. This can be explained by the previous discussion regarding the high generation at Node 5. Increased generation at Node 5 causes both Line 5-6 and Line 4-5 to be overloaded.

Table summarizes the traded volumes between different pricing areas for all three pricing schemes. Traded volumes between the nodal pricing area (Nodes 1-5) and other pricing areas are the same for all three mechanisms. However, the zonal pricing and hybrid pricing schemes fail to optimally utilize the existing network.

We notice that in the full nodal pricing model, the price at Node 7 in zone SE is relatively low, which creates counter flows going from Node 7 to Nodes 5 and 6. The counter flows alleviate the congestion in Line 5-6 and Line 4-5. However, the full zonal pricing or hybrid pricing models do not give clear price signals at Node 7 to reflect its cost competitiveness. Furthermore, prices in Norway are much lower than those in other pricing areas, so there will not be counter flows in the zonal and hybrid system to relieve congestion.

Table I: Traded volumes between pricing areas (Unit: MWh)

		Zonal pricing	**Nodal pricing**	**Hybrid pricing**
1 to 5 (NO)[a]	6 to 10 (SE)	2804	2804	2804
1 to 5 (NO)	12 to 13(DK)	1000	1000	1000
6 to 10 (SE)	11 (FI)	219	219	219
12 to 13(DK)	6 to 10 (SE)		31[b]	

a. NO is the area applying nodal pricing while SE, DK, FI are the pricing areas applying zonal pricing.
b. Among Node 10, 12 and 13, Node 13 has the lowest price. However, this fact is not known in either the zonal or the hybrid pricing schemes. Therefore, there will not be flow going from DK to SE.

Nodal pricing in a hybrid pricing context could help to relieve grid congestion to a certain extent. However, we find that it could also intensify the grid congestion. For instance, in Lines 4-5, 5-6 and 8-10, the utilization rates all increase compared to those given by the zonal pricing scheme. This example also shows that congestion not only becomes worse in the area applying nodal pricing (Line 4-5, from 98% to 107%), and on the cross border links (Line 5-6, from 130% to 140%), but also in the area applying zonal pricing (Line 8-10, from 108% to 110%). Increased congestion in these lines could increase cost associated with re-dispatching.

Table II: Utilization rate of overloaded lines for different pricing schemes

	Zonal pricing	**Nodal pricing**	**Hybrid pricing**
Line 2-4	114%	100%	100%
Line 4-5	98%	71%	107%
Line 5-6	130%	100%	140%
Line 8-10	108%	100%	110%

In conclusion, the wrong price signal given at Node 5 and the corresponding increased congestion is the result of two factors. First, the flows over the cross-border lines between the nodal pricing and zonal pricing areas cannot be modeled taking into account the full power flow laws. Second, one of the lines connecting Node 5 and the zonal pricing area (i.e., Line 5-6) is the bottleneck of the whole system. The two factors together lead to the wrong price signal at Node 5. These results highlight the importance of the interface between the nodal pricing and zonal pricing areas in the design of the hybrid pricing system.

3) Surplus

Table III summarizes the social surpluses and grid revenue in different pricing solutions. The total surpluses are not directly comparable because the flows in the zonal and hybrid solutions in general are infeasible and re-dispatching costs are not addressed. However, the different surpluses reflect that the zonal pricing area is affected by the pricing scheme in the nodal pricing area. Within the zonal part of the hybrid system, i.e., Nodes 6 to 13, the consumer and producer surpluses are identical to the zonal price solution, but the grid revenue decreases. As the zonal pricing area has the more expensive power sources in this case, it is always willing to import power from the nodal price area. Given the same traded volumes, the average price to import power from the nodal price area increases greatly from 109.7 in the zonal pricing scheme to 132.5 in the hybrid pricing scheme. This reduces the grid revenue obtained by the zonal pricing area from 120 to 88.

Table III: Surpluses differences (Unit: 1000 NOK)

	Nodes 1 to 5 (Nodal pricing area, i.e., NO)			
	Producers	***Consumers***	***Grid***[a]	***Sum***
Zonal pricing	1501	19301	118	20920
Hybrid pricing	1588	19064	282	20934
Nodal pricing	1638	18931	393	20963
	Nodes 6 to 13 (Zonal pricing areas, i.e., SE,DK and FI)			
Zonal pricing	4237	38912	120	43268
Hybrid pricing	4237	38912	88	43236
Nodal pricing	4220	38708	257	43185

a. Also referred to merchandizing surplus (MS) (see [7]). The mathematical formulation for MS of an area is $MS = \sum_i p_i q_i = \sum_i \sum_j (p_j - p_i) f_{ij}$. Revenues from cross-border commercial trading are equally shared by the two system operators.

Meanwhile, the grid revenue for the nodal pricing area (i.e., Nodes 1 to 5) is greatly improved from 118 to 282. The total social welfare in the hybrid pricing scheme increases slightly by 14 compared to the zonal pricing scheme. The increase in grid revenue comes at the expense of a reduction in consumer surplus. The decrease in consumer surplus is associated with a decrease in consumption in Norway, as displayed in Table 4. This means that the nodal pricing part of the hybrid model reallocates the producer surpluses, consumer surpluses and grid revenue compared to the zonal pricing model. The surpluses of the hybrid solution are becoming closer to those given by the full nodal system.

Table IV: Production and consumption

	Zonal pricing		**Nodal pricing**		**Hybrid pricing**	
	Production	***Consumption***	***Production***	***Consumption***	***Production***	***Consumption***
NO	24225	20421	24026	20223	24098	20294
SE	21583	24168	21448	24064	21583	24168
FI	11958	12177	11958	12177	11958	12177
DK	5212	6212	5234	6203	5212	6212

IV. CONCLUSION

This paper presents a model with hybrid congestion management methods for a hypothetical joint market and tests it in a 13-node power system. Results show that the hybrid pricing model works well in such a context, using the full nodal pricing solution as a benchmark. However, when cross-border lines happen to be the bottlenecks of the whole system, the hybrid pricing model may give wrong price signals for the nodes connecting such lines and trigger more congestion. The results highlight the importance of the interface between the nodal pricing and zonal pricing areas in the design of the hybrid pricing system.

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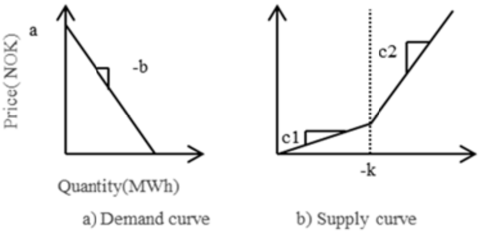
APPENDIX

Table AI: Line capacity

Line	**Lower limit**	**Upper limit**	**Line**	**Lower limit**	**Upper limit**
1-2	2000.0	2000.0	6-7	16500.0	16500.0
1-3	16500.0	16500.0	6-8	16500.0	16500.0
1-13	1000.0	1000.0	6-9	2000.0	2000.0
2-3	2800.0	2800.0	6-11	1500.0	900.0
2-4	800.0	800.0	7-8	16500.0	16500.0
2-10	2000.0	2000.0	8-9	2000.0	2000.0
4-5	400.0	400.0	8-10	2000.0	2000.0
4-8	600.0	600.0	9-10	2000.0	2000.0
5-6	400.0	400.0	9-11	550.0	550.0
5-7	400.0	400.0	10-12	1300.0	1700.0
			10-13	670.0	640.0



a) Demand curve

b) Supply curve

Fig. A1: Supply and demand Curves

Table AII: Parameters for bidding curves at nodes

Node	**Demand**		**Supply**		
	a	***b***	***c1***	***c2***	***K***
1	2000	0.88	0.025	0.15	3600
2	2000	0.2	0.016	0.09	5500
3	2000	0.5	0.011	0.1	9000
4	2000	0.5	0.023	0.25	4400
5	2000	1.5	0.05	0.25	2000
6	2000	1.7	0.04	0.2	2500
7	2000	1.7	0.04	0.2	2500
8	2000	0.5	0.02	0.1	5000
9	2000	0.2	0.018	0.2	5500
10	2000	0.2	0.025	0.15	3600
11	2000	0.15	0.011	0.035	10,000
12	2000	0.7	0.047	0.22	1910
13	2000	0.5	0.047	0.22	2545