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Market Power in a Power Market with Transmission Constraints

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Abstract-- **In this paper we present a model for analysing the strategic behaviour of a generator and its short run implications on an electricity network with transmission constraints. The problem is formulated as a *Stackelberg leader-follower* game. The upper level problem is generator's *profit maximisation* subject to the solution of the lower level problem of *optimal power flow* (OPF) solved by system operator. Strategic bidding is modelled as an iterative procedure where the supply functions of the competitive fringe are fixed while the strategic player's bids are changed in a successive order until the bid giving maximum profit is found. This application rests on the assumption of supply function Nash equilibrium when the supplier believes that changes in his bids will not influence other actors to alter their bid functions. Numerical examples are presented on a simple triangular network.**

***Index Terms*—Electric power market, Supply function equilibria, Bilevel games, Strategic energy bidding, Irrelevant constraints**

I. Nomenclature

Sets:

N - set of nodes (buses)

K_i - set of producers at node i, $i \in N$

L - set of links in the network

L_l - set of directed links within a loop l

Variables:

p_i - locational price at node i, $i \in N$, $p = (p_i, i \in N)$

s_{ik} - supply function slope at node i for producer k, $i \in N$, $k \in K_i$, $s = (s_{ik}, i \in N, k \in K_i)$

q_i^d - quantity consumed in node i (demand), $i \in N$, $q^d = (q_i^d, i \in N)$

q_{ik}^s - quantity produced in node i by producer k (supply), $i \in N$, $k \in K_i$, $q^s = (q_{ik}^s, i \in N, k \in K_i)$

x_{ij} - flow over link $(i, j) \in L$, $x = (x_{ij}, (i, j) \in L)$

Parameters:

n - number of nodes

m - number of links $\{i, j\}$ in the network, $i, j \in N$

a_i - inverse demand function intercept at node i, $i \in N$

b_i - inverse demand function slope at node i, $i \in N$

Q_{ik} - production capacity at node i for producer k, $i \in N$, $k \in K_i$

M_{ij} - capacity of link $(i, j) \in L$

II. Introduction

Liberalisation of the electricity industry has been underway for the past two decades or more. The process of moving from strictly regulated monopolies to functional specialisation and competitive markets was justified by changes in technology and the desire for improved efficiency through better pricing. Types of liberalisation models vary from country to country and there is no overall consensus on which one is most appropriate. The more the market is open to competition the more important becomes the issue of its design that will be robust and efficient. The task of creating an optimal electricity market design is complicated by the technological characteristics of the transmission of electric power. We all know that the specifics of a power flow restrict us in the ability to effectively monitor it, and the storage of energy is too expensive. These two features contribute to the incompleteness of electricity markets together with constraints on transmission and weather conditions. Power flows in an electricity network obey certain physical laws that give rise to the phenomenon of *loop flow*, when the flow cannot be routed and will take all available paths between origin and destination. In transmission networks the capacity of transmission lines determines the degree of competition between different producers in different locations, even though this is not always the case. Congested lines may isolate parts of the network from competition thus possibly increasing market power for agents with a favourable location to the congested line. Producers in liberalised markets with transmission constraints may be interested in inducing congestion to rip the benefits of being a monopolist in a certain area. Another possibility for agents in the market to exercise horizontal market power is their size or a high geographic concentration. If competition in the market is weak the market

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may fail to bring prices down to marginal cost.

The existing literature has many examples of modelling competition in electricity markets to study opportunities for strategic bidding and, as a result, exercise of market power. The amount of market power that can be exercised is dependent on a number of factors, such as; physical characteristics of the network, transmission pricing methods, power mitigating arrangements, and auction design. A summary paper in a tabular format [1] describes eight models measuring market power applied to different geographic markets. A brief literature survey of strategic bidding in competitive electricity markets was presented in [2]. In [3] authors attempt to identify, classify and characterise electricity generation market models that include a representation of the physical system and are suited for analysing imperfectly competitive markets.

Mathematical modelling allows a generalisation of market and network structure and participants' interactions. It produces sensitivity data that can be used for further analyses. A number of studies use bilevel games as a model basis for the electricity market, see, among others, [4], [5], [6], [7], [8], [9], [10]. Similar studies including [4], [7], [5], [11], [9] and [12] model supply function bidding on meshed networks with transmission constraints. A much larger number of papers apply supply function bidding for analysis but fail to include consideration of transmission constraints and loop flow in the network.

Reference [13] employ a bilevel game to model electricity markets with looped networks when generators/consumers bid their linear supply/demand functions by modifying a single parameter in the function. The findings of the paper employing an SFE model contradict some of the results of previous simpler models and intuitive logic, and the authors conclude that probably each given situation in the electricity market requires its own specific model.

In [14] authors study a bilevel model of restructured electricity markets, where each strategic player faces a problem formulated as a mathematical program with equilibrium constraints (MPEC). The whole game is an example of an equilibrium problem with equilibrium constraints (EPEC). Authors demonstrate situations where pure Nash equilibria can be found and further study the weaker concepts of local Nash and Nash stationary equilibria that can be viewed as solutions to complementarity problems. Some numerical examples of methods finding Nash stationary points are presented for some randomly generated EPECs.

Reference [15] study the effect of the market pricing mechanism (pay-as-bid or marginal pricing) on the profit of strategically bidding supplier (manipulation of the intercept). Strategic behaviour is formulated as a bilevel problem using a supply function equilibrium (SFE) model and a solution is found by employing the MPEC approach introduced in [9]. The results of numerical simulations show that market clearing prices were the same under both pricing mechanisms provided there were no transmission constraints. In the presence of transmission constraints supplier's profit was dependent on the pricing rule as well as his position on the network.

In [16] authors model the deregulated electricity market as a bilevel mathematical program and demonstrate that this formulation helps reveal the impact of simplifying the electricity network and omitting the inclusion of physical constraints.

In this paper we present a model for analysing the strategic behaviour of a generator and its short run implications on an electricity network with transmission constraints. The problem is formulated as a *Stackelberg leader-follower* game where the leader is the individual bidding strategically (in our case a supplier), while the follower is the system operator that chooses its strategy while having full knowledge about the leader's decision (but not his true cost/supply curve), a fact that the leader also takes into account while making his decisions. The rest of the suppliers act as a competitive fringe that take the strategy of the bidding suppler (leader) as given, bid their marginal cost, act as price takers and achieve a restricted competitive equilibrium.

Since our model is a bilevel program we will be interested to study the effect of irrelevant constraints first discussed by [17]. Unlike standard mathematical problems the inclusion of an irrelevant constraint into a bilevel program makes the solution deviate from optimal solution. The consideration of this property is important when modelling real world situations as a seemingly unimportant constraint that actually would affect the optimal solution might be left out.

The rest of the paper is organised as following. In Section III we describe problem structure and introduce the example of a simple three-node network. Section IV presents the mathematical model formulation. Section V analyses the numerical results on the simple network. In Section VI we conclude and outline ideas for further research.

III. Problem Structure and a Simple Network Example

We study generator bidding strategically in a power network that consists of a number of nodes and edges connecting them. There is both production and consumption in every node, and edges represent transmission lines. Both generators and consumers provide bids that describe their demand and supply at various prices on an hourly basis to the system operator. Generators' bids are represented by linear nondecreasing marginal price functions, and consumers' bids are represented as downward sloping curves. The system operator then finds optimal nodal prices, and demand and supply quantities based on the solution to the OPF model, in order to maximise total welfare (sum of generator and consumer surpluses measured by their bids). Generators can however choose bid functions that do not represent their true costs by manipulating the slope of the supply functions seeking to maximise their individual profits.

We provide a simple example of the electric power network with three interconnected nodes as in Fig. 1. This triangular connection demonstrates the phenomenon of *loop flow*. In this network each node hosts a consumer (demand) and a set of producers (supply).

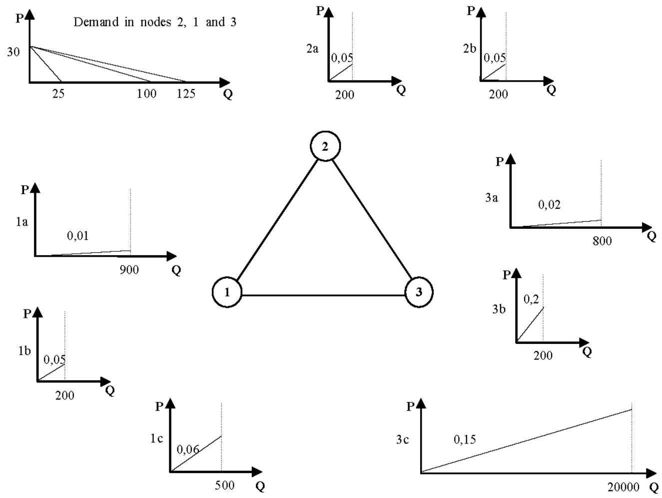

Fig. 1. Simple three-node network.

Specific supply and demand data with capacities is summarised in Table 1.

TABLE I
ASSUMPTIONS ON DEMAND AND SUPPLY FUNCTIONS

Node	Producer	Function type		Supply capacity
1		Inverse demand	$3000 - 0{,}3q$	
	a	Marginal cost	$0{,}01q$	9000
	b	-	$0{,}05q$	2000
	c	-	$0{,}06q$	5000
2		Inverse demand	$3000 - 1{,}2q$	
	a	Marginal cost	$0{,}05q$	2000
	b	-	$0{,}05q$	2000
3		Inverse demand	$3000 - 0{,}24q$	
	a	Marginal cost	$0{,}02q$	8000
	b	-	$0{,}20q$	2000
	c	-	$0{,}15q$	20000

IV. MODEL

The model presented is a linearised direct current (DC) flow model that assumes full information and linear supply and demand functions. We assume that all lines have the same electrical characteristics and that small total losses are neglected. Transmission capacity constraints are the supposed thermal capacity levels. So far we only consider a single period (one hour) static model.

From a mathematical perspective our model is formulated as a mathematical program with equilibrium constraints (MPEC). Strategic bidding is modelled as an iterative procedure where the supply functions of the competitive fringe are fixed while the strategic player's bids are changed in a successive order until the bid giving maximum profit is found. This application rests on the assumption of supply function Nash equilibrium when the supplier seeks to maximise his own profit while believing that changes in his bids will not influence other actors to alter their bid functions.

Model formulation is as following:

For a given (*i*,*k*): $$\max_{s} \Pi(p(s), q^d(s), q^s(s)) \quad (1)$$

st $(p(s), q^d(s), q^s(s))$ are determined by

$$\max_{q^d, q^s, x} B(q^d, s) - C(q^s, s) \quad (2)$$

$$st \quad \sum_{k \in K_i} q_{ik}^s - q_i^d = \sum_{(i,j) \in L} x_{ij}, \quad i \in N \setminus \{n\} \quad (3)$$

$$\sum_{(i,j) \in L_l} x_{ij} = 0, \quad l = 1, \ldots, m - n + 1 \quad (4)$$

$$\sum_{i \in N} \left(\sum_{k \in K_i} q_{ik}^s - q_i^d \right) = 0 \quad (5)$$

$$p_i = a_i - b_i q_i^d, \quad i \in N \quad (6)$$

$$s_{ik} q_{ik}^s \le p_i, \quad i \in N, k \in K_i \quad (7)$$

$$0 \le x_{ij} \le M_{ij} \quad (8)$$

$$0 \le q_{ik}^s \le Q_{ik}, \quad i \in N, k \in K_i \quad (9)$$

The model above is a two-level optimisation problem, where the upper level is individual's (supplier or consumer) *profit maximisation* subject to the solution of the lower level problem of *optimal power flow* (OPF) solved by system operator, where optimal generation and load dispatches as well as system spot prices are determined while total social welfare is maximised assuming the bid supply functions are 'true'.

Individual's profit function (1) is given by

$$\Pi_{ik}(p(s), q^d(s), q^s(s)) = p_i q_{ik}^s - 0{,}5 s_{ik} (q_{ik}^s)^2. \quad (10)$$

The objective of OPF (2) is a social welfare function where total consumer benefit is given by

$$B(q^d, s) = \sum_{i \in N} 0{,}5 q_i^d (a_i + p_i),$$

and total supplier cost is given by

$$C(q^s, s) = \sum_{i \in N} \sum_{k \in K_i} 0{,}5 s_{ik} (q_{ik}^s)^2 .$$

Constraints (3) represent Kirchhoff's junction rule that insures that net ingoing flows into a node is equal to the sum of all outgoing flows. Constraints (4) stand for Kirchhoff's loop rule saying that the sum of voltages in a closed circuit is zero. Constraint (5) corresponds to the law of conservation of energy guaranteeing that total production is equal to total consumption. Constraints in (6) are the inverse demand functions for consumers. Constraints (7) guarantee that the price at each node is determined by the least cost efficient producer or by demand.

Constraints (8) and (9) represent the transmission link and production capacities' constraints respectively.

V. NUMERICAL RESULTS

We first look at the competitive equilibrium solution when all parties act as price takers. So far there is no transmission constraint present in the network, the system price is uniform at 115.4 units per MWh. The competitive equilibrium price exceeds the marginal costs of generators *a* and *b* at nodes 1 and 2. This difference is the scarcity rent due to the limited capacity of these generators, which is not in itself an evidence of the exercise of market power. All results are summarised in Table II below.

A couple of interesting changes are observed compared to the unconstrained solution when we look at a situation where a transmission capacity restriction of 2000 units is presented on line 1-3. Prices have fallen for the suppliers at node 1 and 2 and quantities have decreased for the most efficient producers at node 1. At the same time price at node 3 has increased and the less efficient producers increase their outputs. Price changes reflect the congestion costs introduced by the constraint. Suppliers in node 1 are forced to sell locally and due to the inelasticity of the demand the prices are brought down, same reasoning explains the rise in the prices at node 3 where local producers can increase their output due to the scarcity of supply from other generators induced by the constraint. Now that we have moved away from the unconstrained solution total social welfare has decreased by 0.08%. This is only a slight decrease but if we look at consumers and producers separately we see that in aggregate producers' surplus is decreased and consumers' surplus increased. And then separately, producers at node 3 experiences a positive change in surplus of over 30% compared to the competitive solution, while producers at node 1 experience a negative change of over 40%. In their turn consumers at node 1 benefit slightly, while consumers at node 3 face a negative surplus change. These income redistribution effects give rise to a complicated discussion on whether to invest in new transmission capacity, as even though the total social surplus increases as a result some parties may find themselves being worse off. These results are summarised in the first two columns of Table III.

Finally we look at the situation where one of the suppliers will bid strategically. We choose supplier *a* at node 1 to be the strategic player. Equilibrium is found through a complete enumeration procedure where the slope of the supply function of the strategically bidding supplier is allowed to vary with an increment of 0.0025 from 0 to 1, while his profit is maximised. The system settles at a uniform price of 144.9 units per MWh. The efficient generator 1*a* manages to increase the system price by withholding its production at the same time as other generators do not have enough capacity to offset this decrease. Total supply of energy is decreased with ca 1%, and this slight change draws the price from 115.4 to 144.9 due to the inelastic demand.

Next, we test the importance of inclusion of irrelevant constraints into our bilevel model. As pointed in [17], when an additional inactive constraint is included in the bilevel program, the original solution may no longer be optimal. We take the same situation as above, where producer in 1*a* bids strategically, and add a capacity constraint of 210 units on link 1-2 that seemingly should not disturb our optimal solution from the previous example. However, the solution has changed as we can see from results in Table II, the new constraint becomes binding although for the flow going from node 2 to 1. The price is no longer uniform across the network, with the highest being in node 1 of 205.05 units per MWh, as the total supply in node 1 decreased from 10855.95 to 10242.11 units. Our strategic bidder has under current conditions picked a different slope of his supply function (0.0425 compared to 0.0225 in previous example) and is now earning more (20.3% increase) by producing less (25.1% decrease). Total social welfare has decreased by 0.84%.

As discussed previously by [16], in bilevel programs based on problems arising in electricity networks loop flow constraints cannot be omitted, which might happen if one for example relies on the analogy with transportation networks, where the flow can be routed. The solution of a bilevel problem is then influenced by this misunderstanding in the same way as if one of the necessary conditions of the lower level problem was not taken into account. We introduce two additional constraints to our model: on link 1-2 constraint of 400, and on link 2-3 a constraint of 1000.

TABLE II
NUMERICAL RESULTS

		Unconstrained solution	**Constrained solution (C_{13} = 2000)**	**Strategic bidder 1*a* (C_{13} = 2000)**	**Irrelevant constraint (C_{13} = 2000, C_{12} = 210)**	**No account of loop flow, strategic bidder 1*a*, (C_{12} = 400, C_{13} = 2000, C_{23} = 1000)**
Price	p_1	115.4	87.17	144.92	205.05	223.64
	p_2	115.4	109.70	144.92	99.32	95.51
	p_3	115.4	132.22	144.92	152.18	183.16
Welfare	Total	36218077	36187612	36082570	35866064	35623964
	Producer	1547219	1297070	2117949	2444754	2731534
	Consumer	34670858	34755411	33964620	33421310	32892430
Production	Node 1	9615.38	9709.42	9516.95	9316.51	9254.55
	Node 2	2403.85	2408.59	2379.24	2417.23	2420.41
	Node 3	12019.2	11949.1	11896.2	11865.9	11736.8
Consumption	Node 1	12923.08	11913.71	10855.95	10242.11	9454.54
	Node 2	4000.00	4000.00	4000.00	3972.84	3820.4
	Node 3	7115.38	8153.39	8936.43	9384.70	10136.84
Profit	Bidder 1*a*	633462	379961	725944	872897	764091
Flow	1-2	570.51	204.29	-93.92	-210	-400
	2-3	2166.67	1795.72	1526.84	1345.6	600
	1-3	2737.18	2000.00	1432.92	1135.6	1000

Numerical results show how the solution changes when the loop flow constraint is dropped. Total social welfare has increased in the last case compared to the situation with a strategic bidder 1*a* and constraint of 2000 on 1-2 line by 1,2%. Producers and consumers at node 2 have not been affected as a result of dropped loop flow constraint, their prices and quantities remain the same, and so are the flows from node 2 to nodes 1 and 3 which utilise all the capacity available. At the same time the flow on route 1-3 has increased more than three times as a result of the change in the model, prices at both nodes have settled at 146.61 units per MWh. While producers at node 1 have increased their production, overall production at node 3 has dropped. Our strategic bidder 1*a* produces 75% more in the new case than in the original one while his profits have gone down by 2.8%.

In Fig. 2 we show how the profit function of strategically bidding producer develops in a constrained problem and after the inclusion of the irrelevant constraint. We see that the profit function in constrained solution (C_{13} = 2000) has a local maxima, which shifts upwards and becomes the new solution once we have introduced the irrelevant constraint C_{12} = 210.

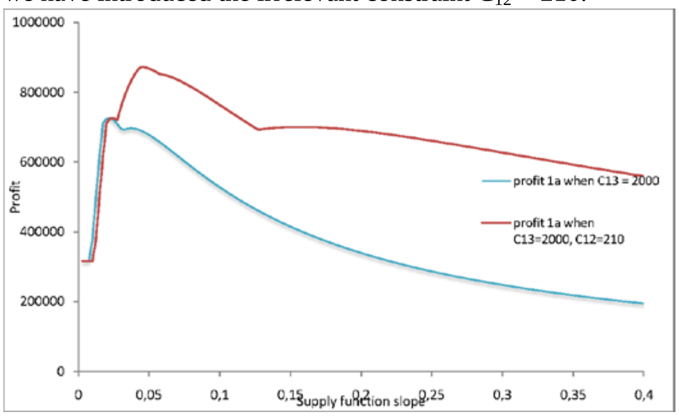

Fig. 2. Profit of supplier in 1*a*.

TABLE III
INCOME REDISTRIBUTION EFFECTS

Surplus	Unconstrained solution	Constrained solution	Strategic bidder 1*a*
Grid revenue	0	135131	0
Consumers	34670858	34755411	33964620
Node 1	13868343	14140930	13585848
Node 2	3467985	3480775	3396462
Node 3	17335428	17133706	16982310
Producers	1547219	1297070	2117949
Node 1	875178	519280	1090775
Node 2	261538	238781	379661
Node 3	410503	539009	647513
Total	36218077	36187612	36082570

VI. CONCLUSIONS AND FUTURE RESEARCH

Our results show that a strategically bidding generator may reduce its production leading to an optimal dispatch in the network without binding transmission constraints, making the prices equal in the network. A generator may be able to increase its profits in the presence of transmission constraint(s), even though this might not lead to separating parts of the market. Another finding is that seemingly irrelevant constraints may change the equilibrium solution in a system with imperfect competition. On the other hand, introducing a tighter capacity constraint does not always result in a decrease in total surplus.

Our future research is to study the case of a strategically bidding generator owning capacity in several different nodes. Another case is to study the effect of size of a generator on the amount of market power it can exercise. A natural progression would be to model and analyse coordinated bidding of several generators with an EPEC model.

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VIII. Biographies

Mette Bjørndal received a doctorate in Economics from Norwegian School of Economics, Norway, in 2000. She is currently Professor and Vice Rector at Norwegian School of Economics, Norway.

Victoria Gribkovskaia graduated from Molde University College, Norway, with BSc and MSc in Logistics. She is currently pursuing a PhD degree in Management Science at Norwegian School of Economics, Norway. Her major research interests are deregulated electricity markets, strategic bidding in electricity markets and mathematical modelling.

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