Norwegian School of Economics

Bergen, Spring Semester, 2014

VPP evaluation from a small player's perspective

Dong Energy as an example

Ming Su

Supervisor: Petter Bjerksund

Master thesis in

Energy, Natural resources and the Environment

NORWEGIAN SCHOOL OF ECONOMICS

Abstract

Virtual power plant (VPP),which can be interpreted as a strip of hourly European options, gives the buyers (normally the competitors of the big player in the electricity market) the rights but not obligation to buy an agreed amount of electricity from the big player at a pre-determined price during one specific period. VPP has been used as a tool for regulators to encourage competition in the electricity market since the very beginning of 21th century. During the past years, VPP has mainly been discussed from regulators' perspectives. In this thesis, we are going to explore the value of VPP from a small player's standpoint, who wants to participate into VPP auctions (Dong Energy's VPP product as an example). Before joining the auction, pricing VPP is very crucial for small player to better understand, value and utilize this product. We would like to demonstrate that Black 76 can be used as an effective way for VPP pricing. Pricing is a complex process on an hourly basis. Main factors have been discussed and explained during the pricing process. We conclude that the overall value of VPP is beyond the bottom line, which is the minimum value from Black 76 pricing. It should also include the added value, flexibility in the second nomination. The flexibility value is addressed by illustrating the situation under which the total return from re-nomination is much higher than the first exercise strategy. We also present the risks Dong Energy will face to issue such VPP product in order to reflect the benefits for owning VPP as a small player in a competitive electricity market. Delta analysis and dynamic hedging can also be replicated for small players to manage their potential risk in the future.

Contents

1. Introduction

Virtual Power Plant (VPP) was introduced at the beginning of 21 century. Since then it has been widely accepted and utilized as a supplementary tool for government supervisor to regulate monopoly in deregulated electricity market. VPP is by definition as an option to buy a given amount of output from the producer (normally the dominant company) at a predetermined price per unit (typically equal to marginal cost), which the buyer then sells in the final product market to compete with the producer [Maurer]. The competitors of the dominant company acquire VPP through auctions. For auction, there are different auction design strategies. The most often used is a simultaneous ascending-clock auction with a discrete round structure.

There are also different voices on the effectiveness of VPP as a regulation tool for preventing monopoly and promoting competition in the academic and industry field. Lawrence concludes that VPP auctions have been used as effective devices for facilitating new entry into electricity markets and for developing wholesale power markets [Ausubel]. While Christian Schultz shows that the incumbent can maintain the monopoly outcome by being prudent and withhold production from the market so that the total production equals the monopoly production [Schultz].The effectiveness of VPP as a regulation tool is still under discussion. But electricity spot prices in the emerging power markets are volatile. Uncontrolled exposure to market price risks can lead to devastating consequences for market participants in the electricity industry. So it has been widely accepted that VPP, as a series of European call options, can be used as a supplementary financial tool for small players in the electricity market to hedge risks and avoid unnecessary loss. The financial characteristics of VPP further address the importance of owning a VPP product which also brings the question we are going to explore in this thesis. That is the evaluation and utilization of VPP.

The purpose of this thesis is to help small players in the electricity market to better evaluate and utilize their VPP products. We would like to act as a market player, such as a median size electricity company, to evaluate VPP. There are few researches on the pricing of VPP. KYOS used an intrinsic value and extrinsic value approach to evaluate the total value of VPP. It concludes that the VPP becomes more valuable when the Energy Price is low [KYOS Energy].The methodology used by KYOS however is very complicated which contains various advanced techniques. In our analysis, we choose a comparable simple but straightforward approach. We divide the total value of VPP into two parts. The first part is the minimum value (bottle line) of VPP, which can be calculated through some financial methodologies. The second part is the added value, which can be the flexibility value added from the nomination rules and can also be depend on various views from different industries (for example the benefits for a wind electricity company for joining the VPP is obviously larger than the benefits for a hydro power plant company).

The main methodology we use here is Black 76. We use Black 76 to predict our option values (a strip of European options), which is inspired from Bjerksund, P [2008]. As we know the two core elements of using Black 76 to evaluate the European options is to find the corresponding forward price and volatility. There are different models to predict hourly forward price. For example Fleten and Lemming [2003] recommended bottom-up models to combine all the determinants into one equation. KYOS Energy Consulting BV [2010] used Monte Carlo simulations to construct their forward curve. Corchero,G [2011] suggested stochastic programming approach. But their models require large amount of knowledge in computer science area. We choose a more applicable and understandable way by observing the patterns of seasonally, daily and hourly electricity prices, to apply those patterns into the prediction of hourly forward prices. For volatility Bjerksund suggested continuous time approach for hourly exercised options. The volatility is related to the variance of the log-return of the underlying price during the life-time of the option. We derived the hourly forward price by observing the weekly forward price in the current financial market and use the formula which reflects the daily and hourly pattern of electricity price. And we find their corresponding volatility by using continuous time approach. Finally we calculated the option price for each individual hour. By adding these values together we get the overall price for a specific VPP product. In our analysis, we give the example of the second quarter VPP product in 2014 from Dong Energy.

Through our analysis from Black 76, we get the bottom line or minimum price for VPP, which is the smallest price we would like to pay to own a VPP product. This evaluation has been made based on the assumptions under Black 76. But the overall value of VPP is more than the theoretical value. It also depends on the nomination procedure. In our example from Dong Energy, the added value from second nomination cannot be ignored. Actually it makes a big difference when the small players are not good at forecasting day-ahead spot electricity price or when they make mistakes in their forecasting (some small players may lack of information, resource or skills). Re-nomination will remedy the “out of the money” decision and may turn the transaction into “in the money”. The benefits of re-nomination will also be amplified when extreme situation like spikes occurs in reality. The VPP owner can buy the electricity in the pre-decided price and sell it at the market high price to gain profits from the huge gap. The overall explanation addresses that the minimum value of owning a VPP can be derived from Black 76. But the total value should be analyzed by considering the characteristics of the electricity company, their ability of forecasting spot prices and the probability of extreme situation.

After the financial evaluation of VPP by using Black 76, we come to the decision making stage, whether to exercise our VPP rights, assuming that we have won a VPP contract. We give specific analysis to each different stage during the exercise period. The success of first nomination depends on the forecasting of day-ahead spot prices. The methodologies we used here is multiple linear regression models for day-ahead spot price forecast. We use the idea from Fleten [2003] to find the main factors that influence tomorrow’s spot price and put them into one equation, which is the multiple linear regression models we use in our analysis, to derive the relations of these factors and their influence on tomorrow’s electricity prices. We found two elements that are most related to tomorrow’s spot price and derived 24 different functions for each hour. Our regression model is a simplified model. There are some factors we did not include because of the difficulty to measure.Different methodologies for forecasting will influence the accuracy for exercise. So choosing the right model and including the relevant factors is very essential for making the right decision.Our purpose here is not to find the idealist forecasting model. We want to demonstrate an idea that maybe forecasting model will help VPP owners to make better decision in the first round. But it does not mean that the decision will be the optimal one. The re-nomination should also be considered when the market information has been published. By comparing these two options, we choose the one that will bring most profits.

In the second nomination stage, we use optimization model to find the optimal hours to maximize our profits. There are different ways to find the optimal decision such as dynamic programming. We use AMPL software to help us find the optimal value under the realized electricity prices. It is very important to find the optimal way to re-nominate the hours. The proper decision in second nomination will help VPP owners to adjust their mistakes in the first forecasting and minimize their risks. Our conclusion is that the forecasting technique in the first round will increase the chances to get optimal exercise strategy; but the second nomination helps owners to mitigate their risks and adjust their plan in time.

At last we discuss the risk management of Dong Energy in order to reflect the benefits of owning a VPP for small players. The risk for Dong to short call options is unlimited. On the contrary, the benefit for the small players to long call options is huge. Through delta analysis Dong Energy can detect how much the value of daily VPP option will change as the underlying electricity price varies. As a result, we find that dynamic hedging is necessary for offset the huge risks come from short call options.

The whole thesis is composed by two main parts. The first part, starts from chapter 2 to chapter 4, primarily talks about the background information of VPP, the introduction of Dong Energy's VPP product and its mechanism, and the evaluation of VPP by using related financial models. The second part of this thesis begins from chapter 5 .It is a decision making process in reality, mainly talking about the two VPP nominations and our respective strategy in these two stages. In chapter 6 we start from another perspective to talk about the risks of Dong Energy to reflect the benefits of owning VPP products. We also introduce delta and dynamics hedging for Dong Energy to manage the risk. chapter 7 is the conclusion part.

2. Background

In this chapter first we will have a brief introduction of VPP. Then we will talk about the history and development of VPP in different European countries. Finally we would like to discuss the objectives of VPP for regulators.

2.1 VPP definition

Virtual Power plants (VPP) are sales of electricity capacity by one or more dominant firms in electricity market through virtual divestitures instead of physical divestitures. The main firm remains management and controls of the physical power plants rather than selling them. At the same time the main firm offers the contracts which give the bidders (normally the competitors) the rights but not obligation to exercise certain amount of electricity capacity during a fixed period at an agreed price. Virtually all VPP auctions have followed the simultaneous ascending clock auction design with discrete rounds [Ausubel].

The VPP products are mainly option contracts with four basic elements [Maurer]:

- Option price: the price (EUR/MW) that gives the buyer the right to exercise a certain amount (MW) of energy during a specific time period;
- Strike price: the price (EUR/MWh) which are supposed to be paid by the VPP owners when exercise the option;
- Duration: the time period during which the contract is valid;
- Energy Capacity: the amount of energy (MW) the buyer has the right to buy at the agreed strike price.

2.2 VPP history and development

VPP has been used as the tool for government regulators to promote competition and liberalization in electricity market. The goal of VPP is to increase competition in the wholesale energy market by reducing the incumbents' market share and facilitating new entry. Several regions throughout the world have used such auctions. VPP auctions were first introduced in France in 2001. The same concept has also been used in Belgium, the

Netherlands, Denmark, Spain, Portugal, Germany and US. The Canadian province of Alberta conducted an auction in 2000 that was not formalized as a VPP divestiture but has similar characteristics [Maurer]. Examples from France, Belgium Spain and the Netherlands will be described in the following.

2.2.1 France

The first VPP auction was EDF (Electricité de France) auction which was began in 2001. EDF was the dominant electricity company in France and it was required to release the auction by regulatory “quid pro quo” for agreeing the acquisition of a joint controlling stake in Energie Baden-Württemberg AG (EnBW), the fourth largest electric utility in Germany. EDF offers access to 5,400 MW of generation capacity in France. This capacity can be acquired by generators, suppliers and traders already operating in France and also by those who wish to enter the market, through quarterly auctions conducted on the internet. EDF is the world longest-running VPP auctions company. It has been successfully running 42 auctions until to today [EDF].

There are two groups of VPP contracts offered in EDF’s auctions: base load products and peak-load products. Six base load products and five peak load products have been given in the auctions, ranging from 3 months to 48 months. Before each auction, the seller will decide the prices for different products in a given group. Each VPP product is an energy option contract. Buyers have to pay an option premium to the sellers no matter the option will be exercised or not in the future. And the option premium will be decided during the auction process. Whenever the electricity spot price exceeds the strike price, the option will be exercised by the owner. The strike prices normally equal to the variable cost of producing respective energy. And the VPP auctions follow the simultaneous ascending clock auction design with discrete rounds.

In the meanwhile of EDF issued its VPP products, the VPP auction has becoming popular with regulators throughout Europe. The basic mechanism has been replicated: Electrabel in Belgium, Nuon in the Netherlands, Elsam in Denmark, Endesa and Iberdrola in combined auctions in Spain, Ren and Edp in combined auctions in Portugal, and E.On and Rwe in separate voluntary auctions in Germany.

2.2.2 Belgium

Following EDF in France, Electrabel in Belgium absorbed the same mechanism. Electrabel is the largest power producer in the Belgium. On 4th July 2003, Electrabel struck a deal with Belgian competition authorities to auction off around 1,200 MW of virtual capacity at its two nuclear plants in Tihange and Doel to new entrants in the market, in return for becoming the default supplier for several municipal distribution companies. Electrabel has issued a Preliminary Information Memorandum (PIM), which reiterated the decision of the Belgian competition authorities that the auctions will be conducted in a manner similar to those organized by French power giant EDF. Electrabel will sell VPP on a quarterly basis until December 2008. The first auction for 250 MW relating to deliveries started on 1st January 2004 [ICIS].

2.2.3 Netherlands

In approving Nuon’s purchase of assets from Reliant Europe, the Dutch competition authority declared that 900 MW of generating capacity from Nuon had to be put at a length of five years through the use of a virtual power plant (VPP) auction. The 900 MW was divided into 90 identical blocks of 10 MW, with an imposed capacity of 23 blocks (230 MW) for any one bidder. There was also a bidder qualification process to ensure financial and technical capability. The auction followed a hybrid design. It started with an ascending clock auction phase, whereby bidders had to stipulate the number of blocks they would buy at a specified price. The price raised an increment round by round. Once excess demand had reached a “trigger level”, a final sealed-bid uniform price round was held. At this stage, bids were ranked block-by-block in descending order in terms of price, and the top 90 bids were the winners [Maurer].

2.2.4 Spain

The Spanish power system had an installed capacity of 91,000 MW. The capacity mix includes 18% hydropower, 30% natural gas, 13% coal, 8% nuclear, and 32% renewable sources. Most of the expansion in renewable sources was comprised of wind generation, where Spain has made remarkable progress, albeit at a very high cost to be paid by customers.

In 2005, the government published the White Paper of the Spanish electricity Market, which set forth directives and proposals aimed at reducing market concentration and increasing the competition and efficiency of the electricity market. After a process of mergers and acquisitions, the Spanish market was dominated by only two companies (Iberdrola and Endesa) which together represented around 80 percent of all electricity generated in the country. In order to increase competition, the two dominant market players were required to hold Virtual power plant (VPP) auctions so that a wider range of companies and investors had access to the existing generation capacity. The total amount of energy to be auctioned is 14.9 TWh, which will be divided into five lots. The first will be held in June 2007 and every three months thereafter until June 2008. There have been seven auctions so far in which two products were available: one for peak hours and the other that could be exercised on a 24 hours-7 days basis. Both products are option contracts for energy, and were offered with durations of three, six, and twelve months. The Endesa-Iberdrola auctions were initially held quarterly, like the EDF auctions, but they later became semi-annual. These auctions also follow the simultaneous ascending clock auction design with discrete rounds [Maurer].

2.3 Objectives of VPP

There are different objectives for regulators to launch VPP auction. Although the exact design of the auctions can be varied from countries to countries and time to time, but most of them have been playing effective roles in building a dynamic electricity market.

The first objective here is to facilitate entry into the electricity market by assuring the availability to new entrants of electricity supplies on the high-power grid. The access to generation capacity will enable foreign suppliers to become active on the market for supply to eligible customers to a significant extent. For example in 2008, Iberdrola acquired 1,500 MW of capacity in VPP auctions in Germany, France and Portugal in order to become active in those markets.

The second objective of VPP auction is to promote the development and add liquidity to the wholesale electricity market. In 2001, the wholesale electricity market was close to non-existent in France. After eight years of VPP auctions, France is now generally considered to have about the third most active wholesale electricity market in Europe.

The third objective of VPP auction is to reduce market power in the spot electricity market. Today's VPP auctions involve a relatively small fraction of electricity capacity in the given market, which will help to mitigate market power in the spot market [Ausubel].

VPP auctions act as an active tool for regulators to promote competition and build an active wholesale electricity market and reduce monopoly phenomenon. It also helps participants to find market opportunities in other opening markets and expand their business. It can also be used as a financial tool to manage potential risks, which is the core part we are going to discuss in this thesis.

3. VPP in Nordic electricity market

Nordic electricity market is one of the first deregulated electricity markets in the world. It has been playing significant role in the process of liberalization of world electricity market. In previous chapter, we have talked about the VPP history in other European countries. Our discussion on VPP products in this thesis will mainly focus on Nordic electricity market. First we would like to talk about the VPP product provided by Dong Energy, which is the largest electricity supplier in Denmark and one of the biggest players in Nordic electricity market. Dong Energy is also the only company who has issued VPP under the requirement of Danish regulator in Nordic electricity markets. Following that we will give a brief introduction about Nord Pool based on the consideration that Nord Pool performs as the counterparty of Dong Energy's VPP products and is responsible as nomination aggregator who is also responsible for the actual delivery of electricity. In the last part we will talk about observation of Denmark west price area (DK1) from historical data.

3.1 Dong energy

DONG Energy is one of the leading energy groups in Northern Europe. Their business is based on procuring, producing, distributing and trading energy and related products in Northern Europe. On 27th March 1972, the Danish state founded Dansk Naturgas A/S. The name was changed to Dansk Olie og Naturgas A/S in 1973 and then to DONG in 2002. It was created as a vehicle to develop Danish energy activities. The company has expanded significantly through organic growth and acquisitions both in Denmark and across Europe .DONG Energy was formally established in 2006 by the merger of six Danish energy companies: DONG, Elsam, Energi E2, Nesa, Københavns Energi and Frederiksberg Forsyning. The merge brought Dong more resource, experience and activities. It also led Dong to be the largest electricity supplier in Denmark. Its customers range from Denmark to Sweden, Germany, Holland and United Kingdom [Dong]. And it has been playing an important role in the European electricity exchange market especially in the Nordic electricity market.

Activities conducted by Dong Energy include:

- Oil and natural gas exploration and production. Dong's activities are focused in the waters around Denmark, Norway, the United Kingdom (West of Shetland area), the Faroe Islands and Greenland.
- Electricity generation at power stations and renewable energy facilities. Dong's most electricity and heat today is generated at central coal-fired, gas-fired and biomass-fired CHP plants in Denmark and at new gas-fired power stations in Norway, the Netherlands and the United Kingdom. Dong Energy has more than 20 years' experience in offshore wind farm development which makes DONG Energy the current market leader in offshore wind power. DONG Energy's offshore wind farms are predominantly based in North West Europe, and they have built more offshore wind farms than any other company in the world.
- Natural gas and electricity distribution. DONG Energy distributes natural gas to West and South Zealand and South Jutland and sells natural gas across Denmark, South Sweden and the Netherlands. Since the start of the 20th century, Dong Energy's electricity grid has grown to include all of North Zealand, the city of Roskilde, and Greater Copenhagen. Following the deregulation of the electricity market, they have also been selling electricity and related services to customers outside these grid districts.
- Sales and energy advice. Dong Energy has also been selling electricity and related services to customers outside these grid districts and has developed a strong profile in the fields of energy advice, customer service and reliable energy supplies [Dong].

3.2 Dong's VPP product

In 2003 Elsam was obligated to supply virtual power plant (VPP) capacity as the condition for the acquisition of Danish Nesa, which traded, distributed and transmitted electricity. After the merge, Dang has to fulfil the obligation of selling VPP to its competitors, who has the right but not an obligation to purchase power at a fixed price. And the power will be delivered to the Denmark west price area.

3.2.1 VPP option price and energy set price

The VPP capacity is offered at four auctions each year. The whole price of VPP is composed by two parts: the option price and the energy set price. The VPP option price[1] is the price of owning the VPP capacity during the supply period, which is determined at the auction and priced in EUR per MW per month. The VPP option price is paid regardless of whether to exercise option and produce the electricity. And there is no minimum option price. So the VPP option price depends on the evaluation of bidders, the price they would like to pay for owning the VPP products. This requires VPP bidder especially the small player to have a good understanding and evaluation on the VPP products they are interested in. We are going to further explore this evaluation for small players to make better decisions during their bidding process.

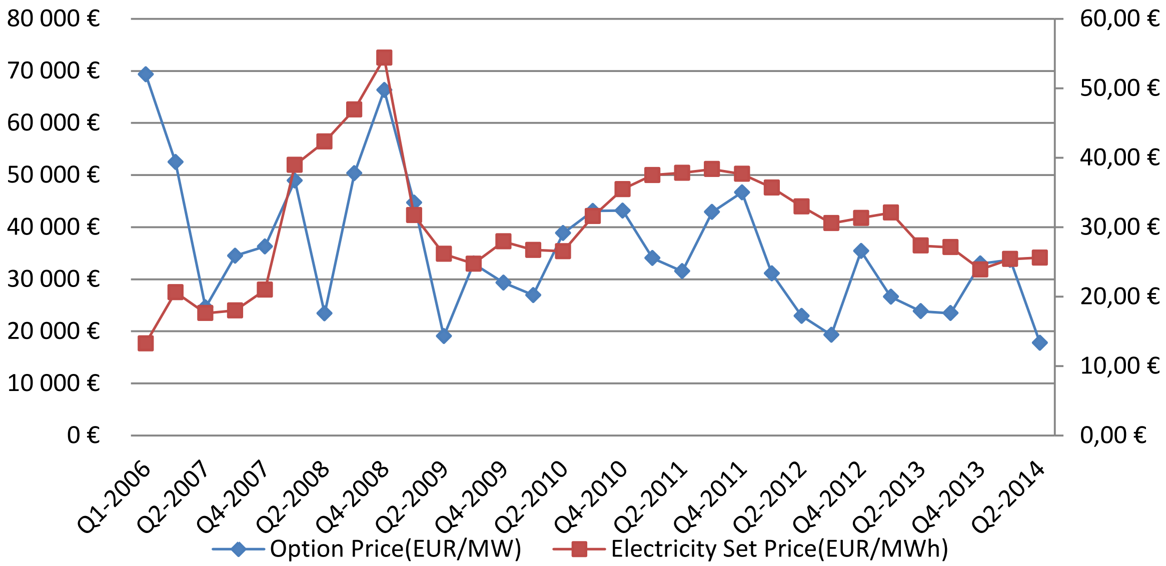
The energy set price[2] of VPP is the price VPP owners should pay for the real production and delivery of electricity, which is also determined ahead of the auction and will be fixed through the whole supply period. The calculation of the energy set price EUR/MWh is based on DONG Energy's most cost effective power plant in price area DK1 and also influenced by fuel prices and exchange rates. Each buyer of the auction cannot acquire more than 50% of the capacity offered.

Appendix 1 gives an overview of all the successfully VPP auctions released by Dong Energy from 2005 to 2014. There are generally three kinds of VPP: quarterly VPP, yearly VPP and 3-years VPP. The most frequently traded VPP is quarterly VPP. The following graph compare the option price and electricity set price of quarterly options from 2006 to 2014.

Graph 1 Realized quarterly VPP from Dong Energy

[1] VPP option price stands for the price premiums owners have to pay for owning the option in electricity financial market.

[2] VPP set price is the strike price in one option. We will continue use this name instead of strike price in this thesis.



Source: Data collected from Dong energy's website

The above graph shows some patterns of quarterly options. For example the option prices at quarter 4 normally higher than prices at other quarters in the same year. And the electricity set prices follow the same pattern. This can be explained by the seasonality in electricity market. During the winter demand for electricity is higher than other seasons, which brings up the electricity price. From 2006 to 2009, both option prices and electricity set prices varied to a big degree. Especially the price peak came at quarter 4 in 2008, which is the highest price point in history. There could be many reasons for this phenomenon. One of the biggest was the global economic crisis which put more uncertainty and volatility on future electricity prices. From 2009 to 2014, the price line became smoother for both option price and electricity set price. This can be explained by economy recovery, cost decreasing, less volatile of exchange rate, and etc. But other reasons we cannot ignore are the gradually maturity of VPP product market and the increasing experience of VPP bidders. Bidders are becoming better at learning from their past experience.

3.2.2 Nomination

The decision of exercise option to product electricity is made on an hourly basis and is called nomination. VPP owners have two nominations. In the first nomination, VPP owner should inform the independent nomination aggregator (Nord Pool in Nordic electricity market) the day-ahead electricity delivery hours before 11 am. The nomination aggregator will inform Dong the aggregated hourly delivery power for the coming day. In the first nomination, VPP owners have full flexibility from hour to hour. In another word, VPP owner can exercise any hour with any amount that will not excessed its bidding capacity. After the first nomination, the VPP owner can also have the chance to re-nominate the hourly based electricity delivery before 2 pm, which is the second nomination. The re-nomination can be made after the hourly based spot price in Nord Pool for the coming day is already published. Based on the new information, VPP owners can adjust their delivery plan. They can change both their nominated hours and amount of electricity. But the re-nomination is restricted by the condition that deviation from hour to hour cannot be more than 40% of the maximum capacity of the individual VPP product.

The whole nomination process adds value for owning VPP products. The first nomination requires VPP owners to have the skill and ability to accurately forecast the spot electricity price for the coming days. We are going to demonstrate how this forecasting will influence the decision making for exercising VPP products. The second nomination gives VPP owners the flexibility and further chances to adjust their plan. We will also talk about how to optimize the second chances and compare it with the first performance to find the optimal decision plan in the coming chapters.

3.3 Nord Pool Spot

Nord Pool, the Nordic Power Exchange, was the world's first multinational exchange for trading electric power and is generally regarded as the most mature and stable power market in the world [R.W]. Nord Pool use to organize two markets; a physical market, Elspot, and a financial market, Eltermin and Eloption. Now the financial market is run by NASDAQ OMX Commodities. Nord Pool also provides clearing services.

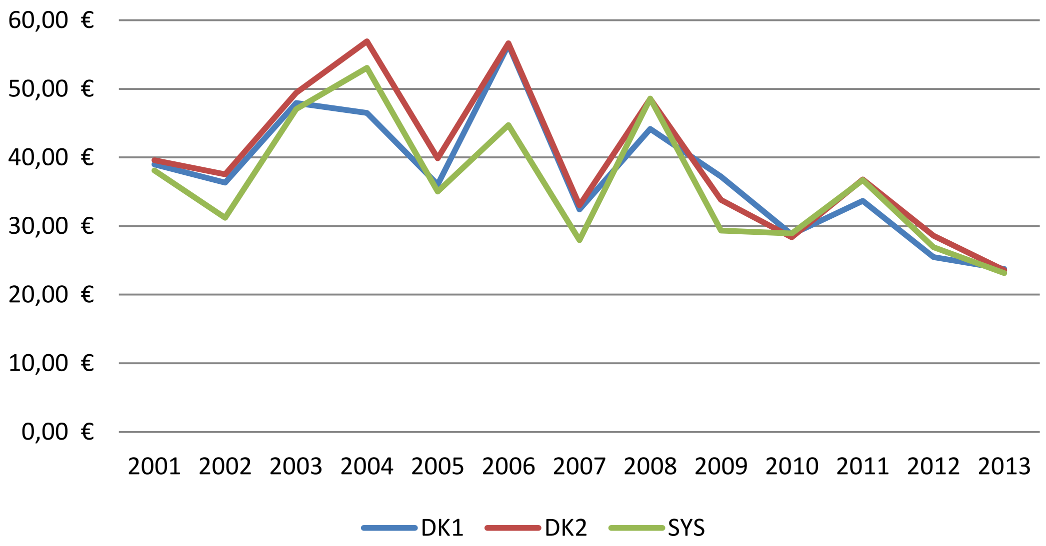
Nord Pool Spot runs the leading power market in Europe and offers both day-ahead and intraday markets to its customers. Nord pool spot is the world’s largest market for buying and selling power. In 2013, the group had a total turnover of 493 TWh. And 84% of all power in the Nordic and Baltic region was traded on Nord Pool Spot. There are 370 companies from 20 countries trade on the market, which includes the auction volume in the

UK market N2EX. Nord Pool Spot is the nomination aggregator in Dong Energy's VPP process. It is independent from Dong energy and informs Dong energy the aggregated hourly delivery of electricity. Nord Pool Spot also act as technical auction to enable and provide technical infrastructure for the VPP auctions [Nord Pool].

3.4 Denmark west price area (DK1)

Nord Pool Spot market (Elspot) is divided into several bidding areas. The available transmission capacity may vary and congest the flow of power between the bidding areas, and thereby different area prices are established [Nord Pool]. In Denmark there are two price areas: Denmark west price area (DK1) and Denmark east price area (DK2). The VPP set price of Dong is calculated based on the most cost effective power plant in the DK1 price area.

Graph 2 Yearly Elspot Price of DK and SYS [3](EUR/MWh)



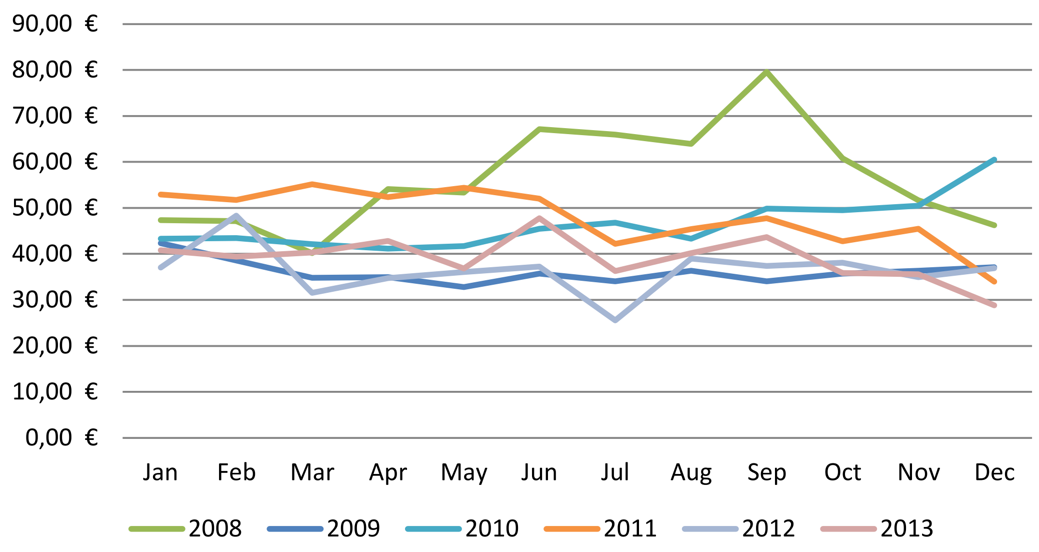
Source: Data collected from Nord Pool website

We can see from above graph that the yearly Elspot price in DK1 at most of the cases was lower than price in DK2 from 2001 to 2013. Compare to the system Elspot price, price in

[3] SYS is the system price which is the formed when demand equals supply.

DK1 followed the pattern that if one year was higher the following year would be lower, vice visa. But the difference between DK1 and SYS is smaller than the difference between DK2 and SYS. That reflects the yearly Elspot price at DK1 is comparable stable and less volatile than DK2. The following is another graph which compare monthly Elspot price in DK1 from 2008 to 2013.

Graph 3 DK1 Monthly Elspot Price (EUR/MWh)

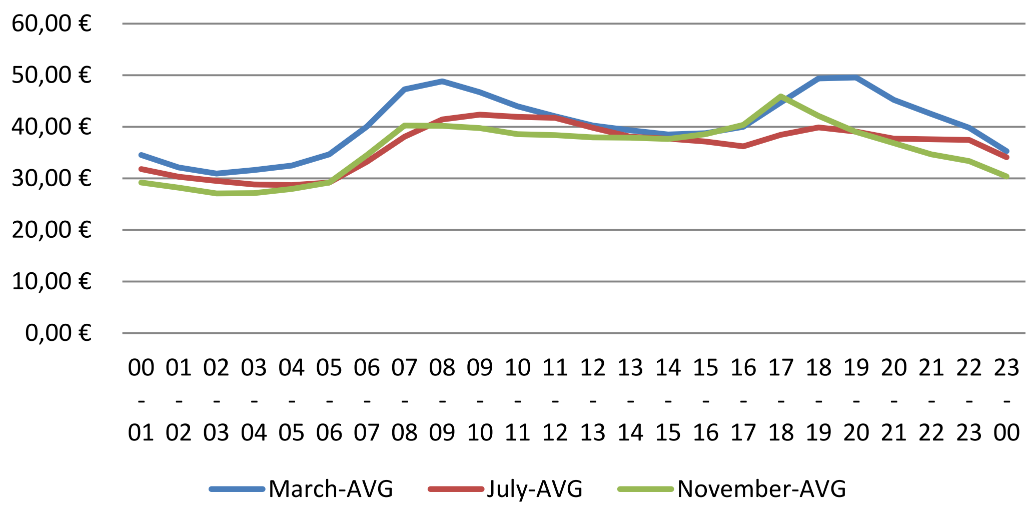


Source: Data collected from Nord Pool website

We can see from the above graph that except in 2008, there was not big variation of DK1 monthly spot price in the past five years. The monthly Elspot price also follows a pattern which reflects the seasonality in electricity market. For example the electricity prices at July normally lower than the prices of other months in the same year. But the monthly Elspot price in 2008 deviated largely from other years. The electricity price in 2008 jumped very high from May to November compare with other years. Especially in September of 2008 the monthly Elspot price arrived at 80 EUR/MWh, the highest point in the past history. That corresponded to quarterly VPP option price of Q4-2008 we demonstrated in graph 1, which reached the highest price point in the quarterly VPP history due to the influence of global economic crisis. The big economic situation and environment will influence the market participants' expectation of future electricity prices. It is more precious to own VPP products and set down the electricity price to avoid unnecessary risks when the global economy is unstable. So VPP values more at that stage in the eyes of bidders.

After the observation of yearly Elspot pattern and monthly Elspot pattern of DK1, we will continue to have a look at the hourly pattern. From the above graph, we observed seasonality difference in months. We choose the most typical moth: March, July and November in 2013 to discuss the average hourly pattern in these months.

Graph 4 Average hourly DK1 pattern at different moths in 2013 EUR/MWh



We can see clearly from above the above graph. The hourly electricity spot prices follow a certain pattern even in different month. There are two peaks from 7:00 to 10:00 and 18:00 to 21:00. The lower price hours normally happened from 2:00 to 4:00 in the early morning. By observing price pattern, it is easy for VPP owners to find the peak hours and off peak hours in a single day. This makes owning VPP more valuable because the special characteristics of electricity price, the variation of which follow a certain pattern and can be predictable. We are going to analysis more about the electricity pattern and its application in our evaluation process for predicting hourly forward price in the coming chapter.

4. VPP evaluation and bidding strategy

In section 4, we are coming to one of the most essential part of our thesis. First we will talk in detail on the method and financial tools we are going to use for evaluation, the coming problem we will confront and the method we use to solve the problems. Finally we will discuss the overall evaluation of VPP.

4.1 VPP evaluation

The VPP evaluation is an important and complex process. Based on the evaluation, the small player in the Nordic electricity market, who are competitors of Dong Energy and have the qualification to join the bidding process of VPP, will set its own bidding price. As we mentioned before, the price of VPP is composed by two parts: the option price and the electricity set price. Since the second part is decided by the regulator based on the most cost effective power plan of Dong in the DK1 pricing area, the fuel price, the exchange rate and so on; we will take the electricity set price, in another work the strike price, as given in our analysis. Our main focus in this thesis will be on how to evaluate the option price.

As we know, VPP has two nominations. The first nomination gives the owner of VPP the right but not obligation to choose whether to exercise its right to receive certain amount electricity from Dong at an agreed set price on an hourly base. Each day the owner of VPP has to notice the nomination aggregator the specific hours of the coming day that decided to receive electricity from Dong. And the whole process will sustain according to the duration of VPP, the shortest of which is quarterly VPP. The second nomination will be held after the publication of market spot price information. According to the new information, the VPP owners can adjust its previous plan under certain limitation on the bidding capacity. It is obvious that the second nomination gives VPP owners certain more rights and flexibility. The added value of the second nomination cannot be ignored. Since the value added by second nomination is impossible to be quantitative and measured, we will first evaluate the VPP based just on the first nomination. According to this evaluation, we can find the lower bound for the total value of owning VPP. It will help small players to arrive at the minimum price they would like to offer to own this product. After that, we will have a discussion on the overall value of VPP by including the second nomination.

There are three VPP products offered by Dong Energy: quarterly VPP, yearly VPP and 3-years VPP. The most frequently traded is quarterly VPP. Considering the frequency of trading, in our analysis we will use the quarterly VPP as evaluation example. We also assume that Nordic electricity market is a competitive market with no frictions and no riskless arbitrage opportunities. The free interest rate will be constant in the near future. These conditions give that a forward price today will be equal the expected spot price.

We can interpret our quarterly VPP as a strip of hourly based European options. The whole value of our quarterly VPP is the aggregated value of each individual hourly-based European option, defined by

$$V_Q = \sum_{D=1}^{84} \sum_{H=1}^{24} V[\max\{\widetilde{f_{D-1,D(H)}} - K, 0\}]$$

For simplify, we assume there are 28 trading days in every month. So there will be 84 trading days in our quarterly VPP. $\widetilde{f_{D-1,D(H)}}$ is the future forward price with exercise at Day D-1 and delivery at day D hour H.

D stands for the trading days of our VPP contract. D-1 stands for the exercise day of our VPP contract. In our case the exercise day will always be fixed at 11am the day before the delivery day. The delivery day has been divided into 24 delivery hours. There are 24 hourly based options for each day. And the 24 hourly based options in one single day share the same exercise time (decisions have already been made at 11am the day before). For example, if we have a Q1-2014 VPP which will be valid from January 2014 to March 2014. $\widetilde{f_{0,1(1)}}$ is the future forward price with exercising at day 0 (11am on 31th December 2013) and with delivery at hour 1, 1st of January 2014. K is the strike price of the option which is also called VPP set price and is already decided before the auction.

$\mathrm{V}[\max\{\widetilde{f_{D-1,D(H)}} - \mathrm{K}, 0\}]$ is the option value for each individual hourly based option. The price of individual hourly option can be calculated through financial tools, which is going to be discussed in the coming sections.

4.2 Black 76

Black 76 model sometimes known as the Black model is a generalization of the Black–Scholes option pricing model. It was first presented in a paper written by Fischer Black in 1976. At that time, a challenge in pricing options on commodities is non-randomness in the evolution of many commodity prices. For example, natural gas tends to be more expensive during winter months than summer months. Because of such non-randomness, many spot commodity prices cannot be modeled with a geometric Brownian motion, and the Black-Scholes (1973) or Merton (1973) models for options on stocks do not apply. Black's (1976) option pricing formula modeled a forward price as an underlying place in place of a spot price to solve this problem [Black].

The model is widely used for modeling European options on physical commodities, forwards or futures. It is also used for pricing interest rate caps and floors. It assumes that the trading in the market is continuous, no frictions, no riskless arbitrage opportunities and the interest rate can be determined.

The values of a European call option described by Black 76 as:

$$\mathrm{C} = \mathrm{V}\left[\max\left\{\widetilde{f_{t,T}} - \mathrm{K}, 0\right\}\right] = e^{-rt}\{f_{0,T}N(d_1) - KN(d_2)\}$$

Where

$$d_1 = \frac{In\left(\frac{f_{0,T}}{K}\right) + \frac{1}{2}\sigma_f^2 t}{\sigma_f\sqrt{t}}$$

$$d_2 = d_1 - \sigma_f\sqrt{t}$$

$f_{0,T}$ is the current forward price of delivery at time T

$\sigma_f{}^2 t = Var_0[Ln(\tilde{f}_{t,T}/f_{0,T})]$

K is the strike price; r is the constant interest rate.

N (d1) and N (d2) are probability factors: N is the cumulative standard normal distribution function

N (d2) is the risk-adjusted probability that the option will be exercised (the option will only be exercised when the spot price larger or equal than the strike price).

According to Black 76, the value of our VPP contract can be defined as:

$$V_Q = \sum_{D=1}^{84}\sum_{H=1}^{24} e^{-rt_{0,D-1}}\{f_{0,D(H)}N(d_1) - KN(d_2)\}$$

Where

$$d_1 = \frac{In\left(\frac{f_{0,D(H)}}{K}\right) + \frac{1}{2}\sigma_{f(D-1)}^2 t_{0,D-1}}{\sigma_{f(D-1)}\sqrt{t_{0,D-1}}}$$

$$d_2 = d_1 - \sigma_{f(D-1)}\sqrt{t_{0,D-1}}$$

Volatility $\sigma_{f(D-1)}$ is defined by:

$$\sigma_{f(D-1)}{}^2 t_{0,D-1} = Var_0[In(\widetilde{f_{D-1,DH}}/f_{0,DH})]$$

D here stands for valid trading days within the contract period.

H stands for 24 hours in one calendar day.

K is the electricity set price for VPP; r is the constant interest rate in the market.

We also assume that all the hours in the same calendar day will be exercised at the same time the day before delivery day (11 am).We use $t_{0,D-1}$ to symbolize the time from today to the exercise day.

4.3 Hourly forward price

In previous discuss, we introduced Black 76 to evaluate our quarterly VPP contract, which is the total value of a strip of hourly-based European contracts. The two main parameters in the evaluation process are hourly forward price and its corresponding volatility. In reality, the hourly forward price cannot be observed directly from financial market. So how to derive the hourly forward price based on historical and current market information will be discussed in this sector.

There are various ways to discover hourly forward price. For example Fleten and Lemming [2003] recommended bottom-up models to combine all the determinants into one equation. KYOS Energy Consulting BV used Monte Carlo simulations to construct their forward curve. Corchero,G suggested stochastic programming approach. But their models require large amount of knowledge in computer science area. The complexity of their model also requires massive amount of resource, capital and time. Here we discover a comparable simple, straightforward and more applicable method to derive hourly forward price. Our purpose here is not to compare and test which method is the best for hourly forward price prediction. By observing the monthly, daily and hourly pattern of electricity price; we include the history information from the competitive and mature market Nord Pool. At the same time, the observed current financial information (weekly forward price) has also been included in the analysis.

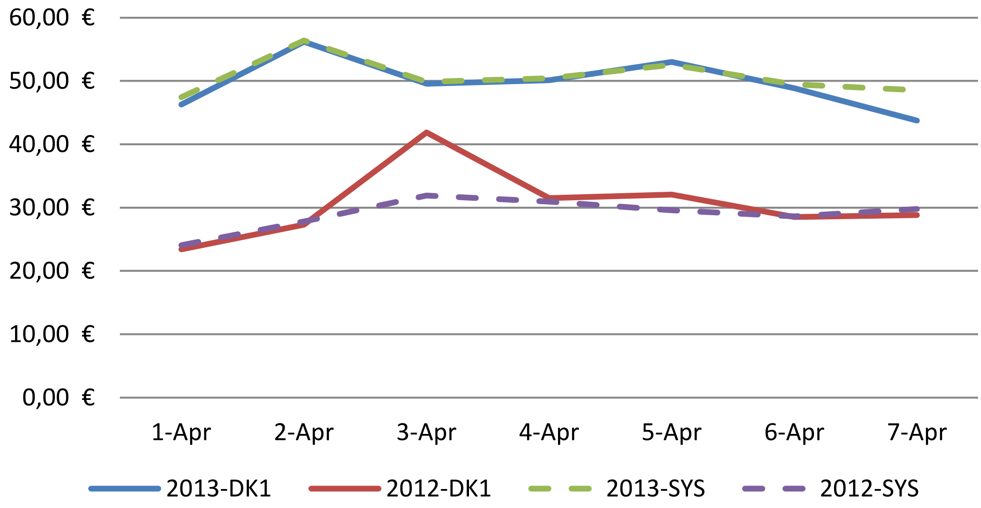
First we are going to introduce our method for predicting hourly forward Price. For example, we are going to bid for the Q2-2014 VPP, which will give us the rights but not obligation to receive X MW/h in 24 sequential hours per day during the second quarter of 2014. The nearest information we can observe from real market (NASDAQ OMX) is the weekly forward price. The weekly forward price in current financial market reflects the market information and participants' expectation on future electricity spot price. But the problem is how we can get hourly forward price from the observed weekly forward price.

As we know the electricity price show seasonality pattern during one year. So the monthly prices vary from season to season. But for a quarterly product, seasonality is not the big problem to consider. Months in the same season show similar monthly patterns. It is important to observe the daily price pattern and hourly price pattern for a quarter VPP. The daily price can be different during one week. For example there are more factory usage in electricity in week days and more household usage in electricity in weekend. It is also well known the hourly price varies during a single day because the difference between peak hours and off-peak hours.

In our method, we observed daily and hourly pattern from the Nord Pool market based both on current and historical information to derive the hourly forward price. In the following we give an example that assuming we know the market forward price of Week 14, the first week of Q2-2014, we are going to analysis the hourly forward price during this week. The explanation of approach is demonstrated below.

First, we find the daily price pattern of the first week of Q2 based on historical data. As you can see from the following graph:

Graph 5 Realized daily price pattern EUR/MWh

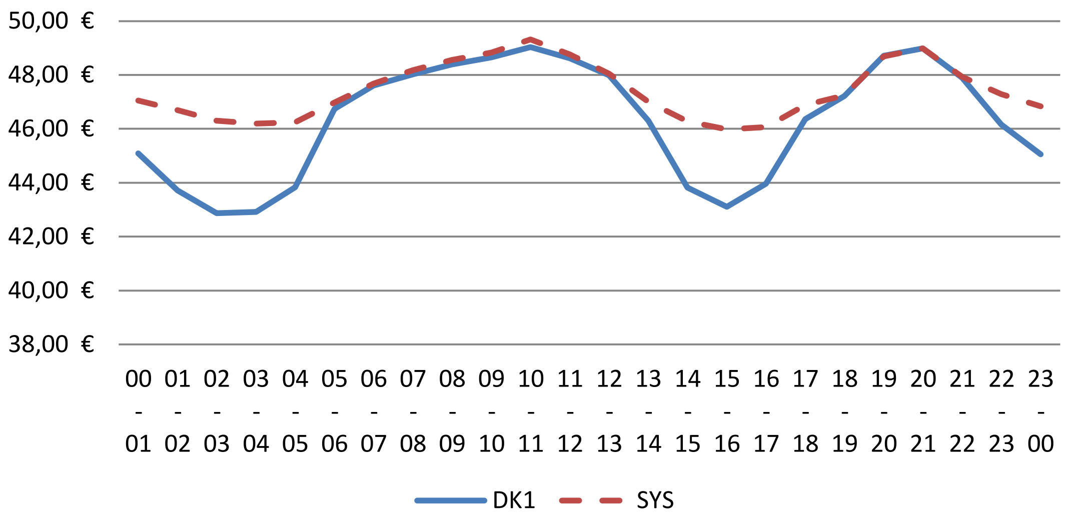


Source: Data collected from Nord Pool website

The above graph describes the pattern of daily price in the first week of April. Data are collected from the past two years. The sold line is the daily price pattern in DK1 area. The dash line is the daily price pattern of system price. We can see clearly DK1 price follow system price closely especially in year 2012. The price in the middle of the week is normally higher than the price at the beginning.

Now, we are coming to the hourly price pattern during a single day. It has been known that there are peak hours and off-peak hours in a day time horizon. In the following we give an example of the hourly price pattern realized on 1st April, 2013.

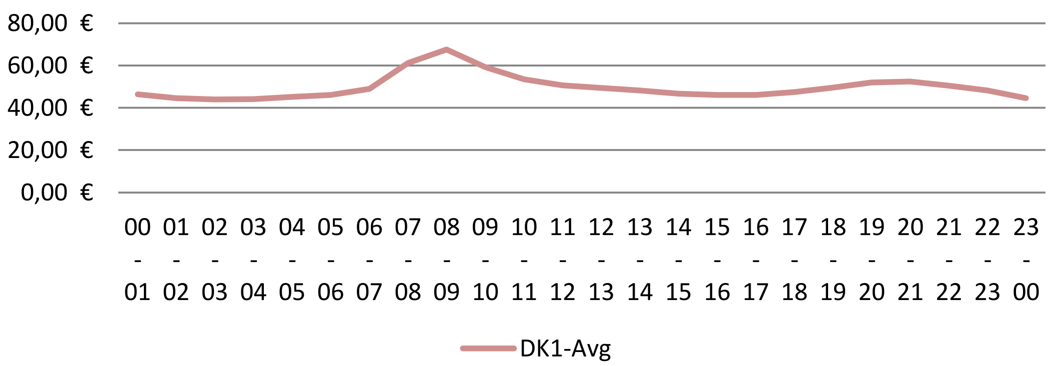
Graph 6 Hourly Price Pattern 01-Apr-2013 EUR/MWh



Source: Data collected from Nord Pool website

DK1 hourly price showed the same moving trend as the system price, but it was lower than system price especially in the off-peak period. There were two peaks in the day. One was from 8:00- 11:00 and another was from 19:00-21:00. There were also two bottoms in the day: 2:00-4:00 in the morning and 15:00-17:00 in the afternoon.

Graph 7 DK1 Average Hourly Price (W1-Q2-2013) EUR/MWh



Source: Data collected from Nord Pool website

Further we calculated the average prices for each hour based on the hourly profile from 1st Apr, 2013 to 7th Apr, 2013 in DK1 area (Graph 7). We can use this profile as the average prices for each hour in the first week of quarter 2. It is easy to notice that everyday follow the regular peak hour (8:00-11:00 and 19:00-21:00) and off-peak hour (2:00-4:00 and 15:00-17:00) pattern. So the corresponding hourly forward price will be influenced both by the daily pattern within the week and by the hourly pattern during one day.

Until now we have observed daily price pattern and hourly price pattern in the electricity market from historical information. And we also compared the daily DK1 area price with the daily system price during a specific week. In our further analysis we will use the historical market data from April 1 to April 7, 2013 as reference to predict the hourly forward price of our first week of Q2-2014. Also based on the current information which gives ENOW14-14, the forward price of the first week of quarter 2, closed at 29.23 EUR/MWh on 17th Feb, 2014. The reason we choose 2013 as the reference year is because we believe in the competitive market the latest realized market information has the closet connection with the current market. The price realized in 2013 has already included all the past market information. The new coming information in 2014 has also been reflected by the current weekly forward price which we observed in the financial market.

Based on our previous observation and analysis, we derive a formula for predicting hourly forward price .Our formula for the hourly forward price of W1-Q2-2014 can be described as:

$$f_{0,DH} = \frac{P_{r-D}}{AVG_{W_i}(S_{r-D})} * \frac{P_{r-H,D}}{AVG_D\left(P_{r-H,D}\right)} * f_{0,Wi}$$

Where

$\mathrm{D} \in W_i \in Q2$

$\mathrm{D} \in [1,81], \mathrm{H} \in [1,24], \mathrm{i} \in [1,12]$

W_i is the i-th week of Q2 and Q2 is the second quarter of a year

P_{r-D} is the realized DK1 daily price for the delivery day D of Q2, $\mathrm{D} \in [1,90]$

S_{r-D} is the realized daily system price for the delivery day D of Q2.

$AVG_{W_i}(S_{r-D})$ is the realized average daily system price for week i of Q2

$P_{r-H,D}$ is the realized DK1 hourly price for the specific hour H at the delivery day D of Q2;.

$AVG_D\left(P_{r-H,D}\right)$ is the realized average hourly price for delivery day D of Q2

$f_{0,Wi}$ is the current forward price for week i of Q2

In our formula $\frac{P_{r-D}}{AVG_{W_i}(S_{r-D})}$ is the factor which reflects the daily pattern in a specific week. $\frac{P_{r-H,D}}{AVG_D(P_{r-H,D})}$ is the factor that demonstrates the hourly pattern in an individual day during the specific week. In our formula these two factors include all the past market information. $f_{0,Wi}$ on the other hand provide the current market information and expectation for the future market spot price.

Given the information in the market that ENOW14-14 closed at 29.23 EUR/MWh on 17th Feb, 2014; we get $f_{0,W1} = 29.23\, EUR/MWh$. Also based on the realized information in 2013, we can calculate the hourly forward price for the first week of VPP Q2-2014. The results are presented in the following table.

As we can see from the table below, the predicted hourly forward prices reflect both hourly pattern as well as daily pattern during the week period.

Table 1 Predicted Hourly Forward Price

Hourly Forward price for W1-Q2-2014 (EUR/MWh)							
	1-Apr-14	2-Apr-14	3-Apr-14	4-Apr-14	5-Apr-14	6-Apr-14	7-Apr-14
00 - 01	26.01	27.43	27.03	25.90	26.01	28.76	26.03
01 - 02	25.22	26.94	26.53	25.57	25.14	28.51	21.92
02 - 03	24.73	26.79	26.27	25.57	25.02	28.46	20.87
03 - 04	24.76	26.82	26.76	25.60	25.25	28.44	20.57
04 - 05	25.29	27.05	27.27	26.68	27.10	28.60	20.56
05 - 06	26.97	24.49	28.65	27.99	28.77	28.76	20.20
06 - 07	27.47	35.19	28.74	27.70	29.62	29.36	19.79
07 - 08	27.70	55.06	34.10	34.99	40.31	28.86	26.38
08 - 09	27.92	63.20	35.74	39.85	51.32	28.76	26.00
09 - 10	28.07	45.04	31.28	36.39	42.63	29.38	26.67
10 - 11	28.29	33.57	30.05	33.30	34.36	29.51	26.67
11 - 12	28.05	30.15	29.33	29.92	31.71	28.82	26.54
12 - 13	27.69	29.50	28.82	28.23	30.77	28.48	25.99
13 - 14	26.71	28.80	28.40	27.77	29.68	27.48	25.71
14 - 15	25.28	28.44	27.53	27.43	28.61	25.76	25.25
15 - 16	24.87	28.05	27.34	27.22	28.18	25.10	25.04
16 - 17	25.36	28.02	27.32	27.15	27.85	25.17	25.30
17 - 18	26.75	28.51	27.66	27.63	28.92	25.97	25.94
18 - 19	27.24	30.79	27.74	28.23	29.32	29.54	27.51
19 - 20	28.10	33.99	29.45	29.36	29.48	30.52	29.00
20 - 21	28.26	34.06	29.96	29.62	29.73	30.24	29.98
21 - 22	27.63	31.44	28.06	29.36	29.12	28.86	29.18
22 - 23	26.63	28.55	26.70	28.04	28.36	27.70	28.35
23 - 00	26.00	25.97	25.57	24.37	26.54	25.16	26.49

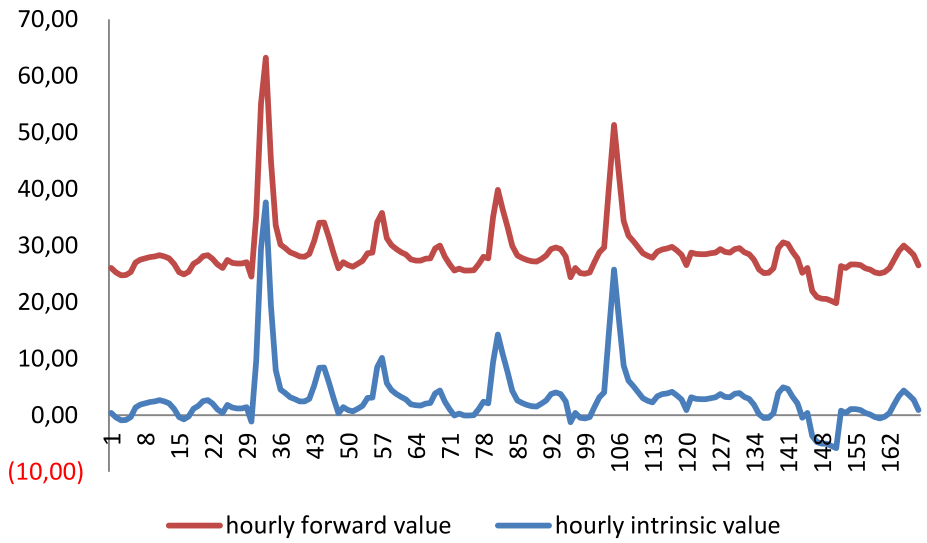
4.4 VPP intrinsic value

In previous sector, we have derived the hourly forward price for the first week of Q2-2014. If the electricity market is healthy and competitive enough, and all the information has already been reflected in the electricity prices; the predicted forward price of today would be the spot price in that specific hour in the future. The intrinsic value is the difference between the underlying price and the strike price.

$$\text{Intrinsic Value} = \text{Current spot price} - \text{strike price}$$
$$= \text{Predicted hourly forward value}^4 - \text{electricity set price}$$

The VPP set price, in another word the strike price, is 25.61 EUR/MWh. We can easily calculate the intrinsic value of the hourly option. Here we make a graph of our predicted hourly forward value and its corresponding intrinsic value for the first week of Q2-2014.

Graph 8 Predicted hourly forward value and intrinsic value EUR/MWh (W1-Q2-2014)



Source: Data comes from table 1 after calculation

[4] Hourly forward value is also hourly forward price. In order to in line with hourly intrinsic value here, we use hourly forward value.

As we can see, the hourly intrinsic value follows the same pattern of the hourly forward value. Most of the hourly intrinsic values are above zero, which means we should exercise the option when the intrinsic values are positive. There are also some situations where the intrinsic values are lower than zero. VPP owners should choose not to exercise the options when the intrinsic value is negative. By looking at VPP intrinsic values, VPP owners will have a clear picture when the option will be “at the money” and when it will be “out of the money”. So VPP owners can choose to exercise option or not. Assuming that in the financial market, it is possible to sell/buy electricity forward in the predicted price. So by doing this, VPP owner can also lock their future cash flow by engaging into a forward transaction and make sure all the future transactions will be in the money. But all these decisions are made under the assumption that the predicted forward price will be the realized spot price in the future. Since the electricity market is not flawless. Other factors for example the weather changes can influence the incoming day’s prices. The decision making just based on financial theories will not be accurate.

4.5 Volatility

Volatility is another big factor which influences our value prediction. Our VPP is a strip of hourly options. Volatility varies from different exercising days. It has been recommended continuous time approach [Bjerksund,P] to evaluate volatility for hourly options. The volatility is related to the variance of the log-return of the underlying price during the life-time of the option. The mathematic expression of volatility can be expressed as:

$$\begin{aligned}
v^2 \cdot (T-t) &= \mathrm{var}_t\left[\ln\left(\frac{f(T,T)}{f(t,T)}\right)\right] \\
&\approx \mathrm{var}_t\left[\int_{s=t}^{T} \frac{df(s,T)}{f(s,T)}\right] \\
&= \mathrm{var}_t\left[\int_{s=t}^{T} \left(\frac{a}{T-s+b}+c\right) dW_s\right] \\
&= \int_{s=t}^{T} \left(\frac{a}{T-s+b}+c\right)^2 ds \\
&= \int_{s=t}^{T} \left(\frac{a^2}{(b+T-s)^2}+\frac{2ac}{b+T-s}+c^2\right) ds \\
&= \left[\frac{a^2}{b+T-s} - 2ac\ln(b+T-s) + c^2 s\right]_{s=t}^{T} \\
&= \frac{a^2}{b} - \frac{a^2}{b+T-t} - 2ac\ln\left(\frac{b}{b+T-t}\right) + c^2(T-t)
\end{aligned}$$

By calibrating the above parameters to the short, medium, and long overnight volatilities

$$\sigma_S = \lim_{t_j \to t_i} \sigma_{i,j} = \frac{a}{b} + c$$

$$\sigma_M = \frac{a}{\frac{1}{2}+b} + c$$

$$\sigma_L = \lim_{t_j \to \infty} \sigma_{i,j} = c$$

The three constants a, b, and c are then determined by

$$c = \sigma_L$$

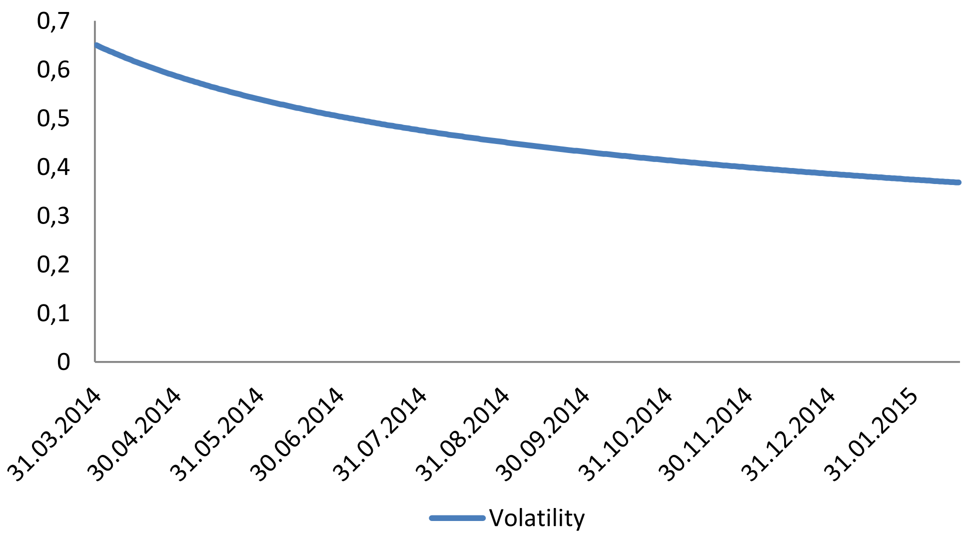
$$b = \tfrac{1}{2} \cdot \frac{\sigma_M - \sigma_L}{\sigma_S - \sigma_M}$$

$$a = \tfrac{1}{2} \cdot \frac{\sigma_M - \sigma_L}{\sigma_S - \sigma_M} (\sigma_S - \sigma_L)$$

We get $\sigma_S = 80\%, \sigma_M = 20\%, \sigma_L = 10\%$; It translates into the constants a=0.13125, b=0.1875, c=0.1.

The following graph shows volatility per annual for hourly settlement options with different time to exercise. The observed date is Feb 17th, 2014

Graph 9 Volatility with different exercising day



As we can see volatility decreases as the time to exercise increases. With time to exercise one year the volatility is close to 36%. But after one year, the decreasing speed slow down. The volatility can approach 10% when the time to exercise is long enough.

4.6 Evaluation of VPP using black 76 (one week example)

After we have found our hourly forward price and volatility use the methods we discussed before, we now come to evaluate VPP by using Black 76. In order to simplify our work, here we just give example on the evaluation of the first week of our VPP contract.

First we rewrite our black 76 formula:

$$V_{w1} = \sum_{D=1}^{7}\sum_{H=1}^{24} e^{-rt_{0,D-1}}\{f_{0,D(H)}N(d_1) - KN(d_2)\}$$

Where

$$d_1 = \frac{\ln\left(\frac{f_{0,D(H)}}{K}\right) + \frac{1}{2}v^2 t_{0,D-1}}{v\sqrt{t_{0,D-1}}}$$

$$d_2 = d_1 - v\sqrt{t_{0,D-1}}$$

$f_{0,D(H)}$ is the hourly forward price at day D hour H

K is the VPP set price 25.61 EUR/MWh

N() is the normal probability function

$t_{0,D-1}$ is time to exercise day before the delivery day D

r is the free interest rate, here we make it equal zero

$$v^2 t_{0,D-1} = \frac{a^2}{b} - \frac{a^2}{b + t_{0,D-1}} - 2aln\left(\frac{b}{b + t_{0,D-1}}\right) + c^2 t_{0,D-1}$$

a,b,c are parameters we have explained before:

$a = 0.13125, b = 0.1875, c = 0.1$

Table 2 Hourly option value for W1-Q2-2014(EUR/MWh)

	1-Apr-14	2-Apr-14	3-Apr-14	4-Apr-14	5-Apr-14	6-Apr-14	7-Apr-14
00-01	2.49	3.38	3.14	2.48	2.56	4.38	2.60
01-02	2.06	3.07	2.83	2.30	2.09	4.20	0.82
02-03	1.82	2.97	2.68	2.30	2.02	4.16	0.55
03-04	1.83	2.99	2.97	2.31	2.14	4.15	0.49
04-05	2.10	3.14	3.30	2.94	3.22	4.26	0.49
05-06	3.07	1.72	4.25	3.80	4.37	4.38	0.42
06-07	3.39	9.82	4.32	3.60	5.01	4.82	0.35
07-08	3.55	29.45	8.81	9.64	14.77	4.45	2.81
08-09	3.70	37.59	10.34	14.31	25.71	4.38	2.58
09-10	3.80	19.44	6.33	10.96	17.05	4.84	2.98
10-11	3.96	8.32	5.32	8.10	9.07	4.94	2.98
11-12	3.79	5.38	4.76	5.23	6.71	4.42	2.90
12-13	3.54	4.87	4.38	3.97	5.93	4.17	2.58
13-14	2.91	4.35	4.07	3.65	5.06	3.48	2.42
14-15	2.09	4.08	3.47	3.42	4.25	2.43	2.17
15-16	1.88	3.81	3.34	3.28	3.95	2.08	2.07
16-17	2.13	3.79	3.33	3.24	3.72	2.12	2.20
17-18	2.93	4.13	3.56	3.55	4.48	2.55	2.55
18-19	3.24	5.90	3.61	3.97	4.78	4.96	3.52
19-20	3.83	8.70	4.85	4.80	4.90	5.74	4.57
20-21	3.94	8.77	5.25	5.00	5.10	5.51	5.32
21-22	3.50	6.45	3.83	4.80	4.63	4.45	4.70
22-23	2.86	4.16	2.93	3.83	4.07	3.63	4.10
23-00	2.48	2.48	2.28	1.70	2.87	2.11	2.87
Average	**2.95**	**7.87**	**4.33**	**4.72**	**6.19**	**4.03**	**2.46**

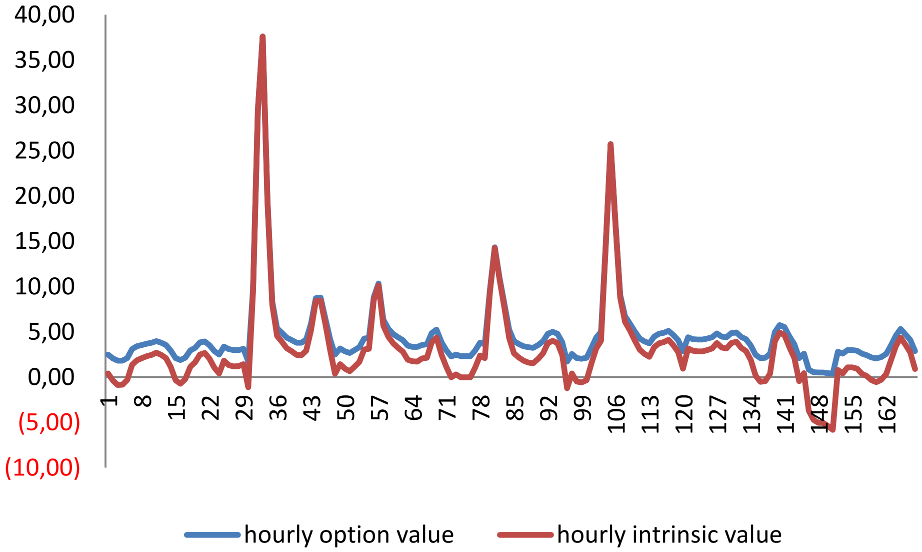
4.7 VPP time value

We have predicted the option value by using our price pattern model and black 76.The option premium (here we use option value) is always greater than intrinsic value. The difference between these two values is the extra money for the risk the option seller/ writer is going to take.

$$\text{Time value} = \text{Option value} - \text{Intrisic value}$$

By using the information we discussed above, we calculated the hourly intrinsic value for options. And we compare the difference between our hourly option value and intrinsic value. As we can see from the following graph:

Graph 10 Hourly option value[5] and intrinsic value EUR/MWh



Source: Data collected from table 1&2 after calculation

The difference between the blue line and the red line is the time value of our hourly VPP. We can see that the red line follows the pattern of the blue line. And the red line is always under the blue line. So the time value of our VPP is no less than zero. Especially the longer the hourly VPP, the larger the gap will be between the blue line and the red line. That

5 Hourly option values here are hourly option premiums which have been predicted in table 2.

demonstrates that the time value of VPP increasing as the time period to be exercised. There may be some exceptions in our observation, but from general it applies to the option theory of time value.

4.8 Quartely VPP evaluation (Q2-2014)

In previous sectors at Chapter 4, we have introduced the Black 76 model to evaluate our VPP. We have also built the hourly forward price forecasting model which reflect the hourly, daily and seasonally pattern of electricity price. In addition we discussed the volatility and found out the formula to use in our evaluation. And continue to that we calculated the value of weekly VPP by using the example of the first week at the second quarter of 2014, W1-Q2-2014. Furthermore, we discuss the intrinsic value and time value of an option. Before we come to the overall evaluation, we would like to give an illustration of the minimum price the small player would like to pay by just looking at the evaluation result from Black 76.

Here we are going to use the same model and method we discussed before to evaluate the Q2-2014 VPP product. All the assumptions we made before will not be addressed here. One more assumption we have to make is the forward prices for week 14 to week 25 in 2014 are available in the financial market on Feb 17th, 2014.

Table 3 Observed weekly price for Quarter 2-2014 EUR/MWh

ENOW14-14	**ENOW15-14**	**ENOW16-14**	**ENOW17-14**	**ENOW18-14**	**ENOW19-14**
29.23	28.10	25.90	26.28	25.50	27.70
ENOW20-14	**ENOW21-14**	**ENOW22-14**	**ENOW23-14**	**ENOW24-14**	**ENOW25-14**
25.25	25.38	23.98	25.00	27.50	24.75

Source: Data was collected from NASDAQOMX

By assuming that there are 4 weeks each month, 84 days in a quarter, and using the formula we discussed in Black 76.

$$V_Q = \sum_{D=1}^{84} \sum_{H=1}^{24} e^{-rt_{0,D-1}} \{f_{0,D(H)} N(d_1) - KN(d_2)\}$$

First, we calculated the option value in each hour during this period. Then we sum the daily option values of each day in quarter 2-2014.

The following table gives detail information of daily option value of quarter 2-2014.

Table 4 Daily option value of VPP Q2-2014 EUR/MWh

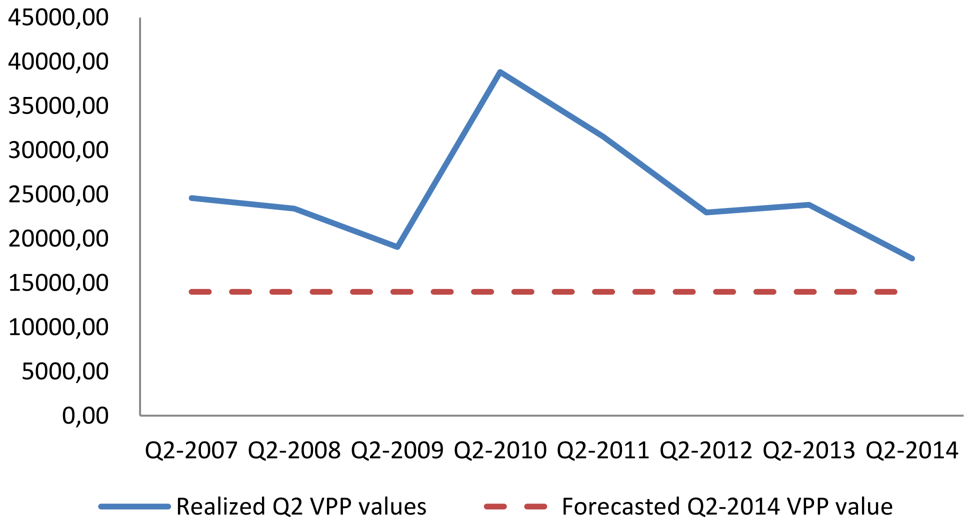
Day1-Q2-2014	Day2-Q2-2014	Day3-Q2-2014	Day4-Q2-2014	Day5-Q2-2014
2.95	7.87	4.33	4.71	6.19
Day6-Q2-2014	Day7-Q2-2014	Day8-Q2-2014	Day9-Q2-2014	Day10-Q2-2014
4.03	2.46	6.33	4.07	3.64
Day11-Q2-2014	Day12-Q2-2014	Day13-Q2-2014	Day14-Q2-2014	Day15-Q2-2014
4.43	4.04	3.02	0.79	5.20
Day16-Q2-2014	Day17-Q2-2014	Day18-Q2-2014	Day19-Q2-2014	Day20-Q2-2014
5.44	3.11	0.61	0.84	1.06
Day21-Q2-2014	Day22-Q2-2014	Day23-Q2-2014	Day24-Q2-2014	Day25-Q2-2014
1.05	2.95	7.87	4.33	4.71
Day26-Q2-2014	Day27-Q2-2014	Day28-Q2-2014	Day29-Q2-2014	Day30-Q2-2014
6.19	4.03	2.46	2.09	3.88
Day31-Q2-2014	Day32-Q2-2014	Day33-Q2-2014	Day34-Q2-2014	Day35-Q2-2014
3.21	1.68	2.14	5.64	4.14
Day36-Q2-2014	Day37-Q2-2014	Day38-Q2-2014	Day38-Q2-2014	Day40-Q2-2014
1.30	2.61	2.12	1.02	1.32
Day41-Q2-2014	Day42-Q2-2014	Day43-Q2-2014	Day44-Q2-2014	Day45-Q2-2014
3.97	2.81	4.28	3.93	3.21
Day46-Q2-2014	Day47-Q2-2014	Day48-Q2-2014	Day49-Q2-2014	Day50-Q2-2014
1.56	1.40	2.34	6.82	2.08
Day51-Q2-2014	Day52-Q2-2014	Day53-Q2-2014	Day54-Q2-2014	Day55-Q2-2014
8.31	9.0	1.83	0.67	8.82
Day56-Q2-2014	Day57-Q2-2014	Day58-Q2-2014	Day59-Q2-2014	Day60-Q2-2014
5.47	1.32	0.93	2.41	3.29
Day61-Q2-2014	Day62-Q2-2014	Day63-Q2-2014	Day64-Q2-2014	Day65-Q2-2014
4.22	16.66	267.64	2.11	2.12
Day66-Q2-2014	Day67-Q2-2014	Day68-Q2-2014	Day69-Q2-2014	Day70-Q2-2014
4.21	5.34	4.93	2.27	1.61
Day71-Q2-2014	Day72-Q2-2014	Day73-Q2-2014	Day74-Q2-2014	Day75-Q2-2014
2.53	1.76	5.80	5.39	4.44
Day76-Q2-2014	Day77-Q2-2014	Day78-Q2-2014	Day79-Q2-2014	Day80-Q2-2014
6.02	2.02	0.43	1.48	7.09
Day81-Q2-2014	Day82-Q2-2014	Day83-Q2-2014	Day84-Q2-2014	**AVG Daily Price**
7.22	3.98	5.22	5.26	**6.95**

$$V_{VPP-Q2-2014} = 24 * \sum_{i=1}^{84} V_D = 14{,}011 \text{ EUR/MW}$$

We calculated the VPP price of Q2-2014 by using the methods we described previously. The predicted value based on the first nomination from Black 76 will be 14,011 EUR/.MW. The forecasted daily value varies from each other and some hours even reflect the spikes (for example day 63). But those variations are the best reflection of the uncertainty and potential risks in electricity market. And our forecasted option price already captured these factors.

Let's have a look of the previous realized Q2 VPP option premiums and compare it with our prediction.

Graph 11 Realized Q2 VPP value and predicted Q2-2014 value EUR/MW



Source: Data from appendix 1& table 4

We can see from above graph. The realized VPP Q2 values from 2007 to 2014 are above our predicted Q2 value for 2014. But the realized Q2-2014 VPP has the smallest distance from our predicted value. The reason for the variation partly is because our evaluation is just based on the first nomination. The second nomination did not be considered in the pricing process. The other reason could be different methods for prediction may lead to different results which will further influence the evaluation accuracy. But our prediction is quite close to the realized value in Q2-2014 just by adding some added value from the flexibility in the second nomination.

The first predicted result for VPP will be the minimum price we would recommend small players to bid for an option. That means we would like to pay for at least 14,011EUR for owning 1 MW of electricity capacity on an hourly basis in quarter 2, 2014. We have to address again the total value of owning a VPP is more than just theoretical value we calculated before. The second nomination should also be taken into consideration in order to better evaluate the VPP products. That is what we are going to explain in the coming sector.

4.9 VPP overall evaluation

In previous sections, we discussed the evaluation of VPP as a strip of European options. And we give illustration of our method for analyzing the weekly options. Furthermore we calculated the Q2-2014 VPP by using black 76 models. The predicted price is the minimum amount we would like to pay for owning the Q2-2014 VPP. As a small player in European electricity market, the evaluation of VPP based on market information by using financial tools will help us to form a basic idea on the benefits and cost of joining VPP auction. It further instructs us to give proper bidding price and guarantee the success of winning VPP contract. But as we mentioned before, the evaluation of our VPP was made under the assumption that there will be just one nomination of VPP. The added value from flexibility of re-nomination was not taken into consideration. Auction bidders should also added the flexibility value into the total value of VPP products. And another information such as the length of VPP contract, which will add more time value into option premium for a longer contract. Other factors such as technology constraints between different areas, competition degree in electricity market, government's policy, global economy situation and etc., should also be considered when deciding the auction strategy. In addition to those elements, the historical bidding price and confidential information of VPP published at Dong Energy's website can be good reference to the final decision as well.

The final decision of bidding strategy is a complex process which also requires other knowledge in addition to all the possible information related to the market. And different bidders may put different weight on the potential benefits of re-nomination. In our previous discussion we evaluate VPP from financial perspective and some possible assumptions about the electricity market have been made during the analyze process. In reality the market is changeable and cannot be perfect. So the business intuition about market from previous many years of experience will help participants to make wise decisions.

In all, VPP overall evaluation is far more from easy. Through Black 76, we get the bottom line of the value of VPP product, the minimum value. The right choice on method to derive hourly forward price and volatility will influence the final result to some degree. To address again, our purpose here is not to find the optimal hourly forward price curve. We would like to demonstrate that by using Black 76 VPP owners can make more reasonable decision on the bidding price they are going to offer in the future. Our evaluation of VPP based on the first nomination helps smaller players to understand its basic values, which is the fundamental price for joining the VPP auctions. And by understanding it, VPP owners can better utilize their VPP products and combine other financial tools to manage their portfolio and hedge their potential risks in electricity market. The re-nomination value can be estimated by quantifying the price VPP owners would like to pay for flexibility. As we know the flexibility means different for electricity companies. For example, the value added from flexibility for wind electricity companies will be larger for hydro power electricity companies due to the resource’s availability and predictability. Individual companies should evaluate re-nomination value based on the characteristics of their corresponding industries and their preference for risks. For companies who are less tolerate for risks, the second nomination will bring more value than for companies who like risks. The second part evaluation will be based on financial behaviors of different type of electricity companies.

5. VPP nomination

5.1 VPP market value

Assuming that we succeed in the second quarter auction from Dong Energy and won the Q2-2014 VPP and it will give us the right but not obligation to purchasing 20MW electricity on an hourly basis from Dong energy during the period of April to June 2014. According to the nomination rules, there will be two nominations in the daily exercising process.

Our first nomination should be decided no later than 11am before the delivery day, after which the hourly day-ahead DK1 area price will be published. In the first nomination VPP owners can have full flexibility from hour to hour. In another way, there will be no hourly electricity capacity constrained in the first nomination.

When it comes to Feb 31, 2014 we have to make the decision for purchasing tomorrow's hourly capacity before 11 am. Today's market value of tomorrow's cash flow can be expressed as:

$$V_D = \sum_{i=1}^{24} e^{rt_{D,i}} Q_{D,i} Max\{S_{D,i} - K, 0\}$$

Where

$Q_{D,i}$ is the capacity of electricity for each hour in day D

r is the constant interest rate

$t_{D,i}$ is the time from the exercising day to delivery day D hour i

$S_{D,i}$ is the spot electricity price for hour i at delivery day D

K is the electricity set price written in VPP contract

Our decision for the first nomination is based on the difference between $S_{D,i}$ and K:

If $S_{D,i} \geq K$,we will exercise full capacity for that specific hour.

If $S_{D,i} < K$,we will choose not to exercise that specific hour. Because for not exercising the option instead that we can buy electricity in a cheaper price from the spot market.

In our previous analysis, we assume that the predicted forward price will become the future spot price. But just as we said the market is volatile and changeable. There comes new information will influence the day-ahead spot prices. The imperfect characteristics of commodity markets require us to not depend on the hourly forward price we predicted before. We should absorb the latest market information and analyze other factors which could possibly influence the coming day's market price. So here we will start a new approach and build another model to help us make decisions and choose the right hours to exercise.

5.2 Factors influence day-ahead spot price

In our first nomination before 11 am on the exercise day, the spot price for DK1 area at tomorrow has not been published yet. We have to make the first round nomination decision based on our prediction for the day-ahead hourly price. As we know the electricity prices are strongly related to many fundamental physical drivers such as loads, hydrological conditions, fuel prices, unit operating characteristics, emission allowances, transmission capability, and etc. In order to better understand these factors that affect the incoming electricity spot prices. First we will have a more specific discuss on the main factors which influence electricity prices in the following.

Some of the most important factors we have to take consideration are seasonality (electricity monthly pattern), daily pattern and hourly pattern. We already discussed previously demand itself varies depending on the season of the year, day of the week and hour of the day. Season of the year is one of the factors that the electricity consumption depends on. During winter, for instance, there is a high demand of electricity, which in turn can cause an increase in the electricity price. In contrast, during summer the demand is considerably low and so does the market price. Hence, this can be considered as one of the factors that affect the system price in any electricity market. Similarly, there is a clear variation of demand for electricity between each day of the week and even between each hour of the day. In week days demand is mostly high at hours between 8:00 -11:00 in the morning and also at about 18:00 – 20:00 in the evening; however, weekends have high demand hours at around 18:00-20 in the evening.

Another important factor is the weather forecast. For example in Norway, almost all of the electricity generation comes from hydropower; where the amount of generated power is greatly dependent on the content of the reservoir. Therefore, the reservoir level and the precipitation level in the current year have a significant effect on the electricity market price. Since Norway is the exporter of electricity in Nordic electricity countries. The electricity price in Norway will also influence the electricity price in Denmark. An increase of the reservoir level (assuming other variables kept constant) normally leads electricity prices to decrease. The level of reservoir can be considered as one input variable that could determine the market clearing price. For Denmark, since lots of electricity is produced by the wind. A precise prediction on the degree of windy in different seasons can influence the load of electricity production.

The third factors we consider here is reserved power. In a power system, there are times where there might be an immediate and unexpected energy demand. In such a case, unless enough reserve power is available, there will be a supply-demand imbalance that might lead the market price to rise. Hence, there always is a need to acquire a reserve power. Consequently, the more reserve available in a system, the more stable the market price will be. It is very important for the participants to be aware of the reserved power level in the industry and also to be alert of incoming emergency due to extreme situation (which is also caused by the weather situation).

One more factors we will take into consideration will be transmission capacity, which also has played a role in determining area prices and spikes. With better transmission capacity, it facilitates the electricity flow from low price area to high price area which will narrow down the area price difference in the whole market. Sometimes transmission failure can lead to spikes in electricity market which normally ended with the transmission capacity go back to the normal level.

5.3 Multiple linear regression models

We have analyzed the factors that will influence day-ahead spot prices in previous discussion. Some factors are easy to measure but others are not quantitative. In order to simplify our cases, we only choose the factors that are easy to be measured. Even electricity prices can be determined by various factors, it also follows some pattern. After carefully study of the price pattern in the Nord Pool market, we found that the electricity market price for hour “H” of the following day is highly correlated with the electricity price of previous day hour “H”, price of same day at hour “H” in previous week,”H-1” hour price in the same day. Since the data of last factor is not available, simply we just consider the first two factors in our analysis. Here we choose multiple linear regression models to predict the electricity prices for tomorrow by using time series data.

The multiple linear regression models are given as:

$$\mathrm{Y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots \beta_k x_k + e$$

Where the parameters $\beta_j, j = 0,1 \dots k$ are called regression coefficients and e is an error. The parameter β_j represents the expected change in response Y per unit change in x_j when all the remaining regressors $x_j (i \neq j)$ are held constant.

We can use the least square method to estimate the regression coefficients. In our simplified model price forecast based on x_1 (electricity price of previous day hour “H”) and x_2 (price of same day at hour “H” in previous week).

$$\mathrm{Y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + e$$

The least square equations are:

$$\mathrm{n}\beta'_0 + \beta'_1 \sum_{i=1}^{n} x_{i1} + \beta'_2 \sum_{i=1}^{n} x_{i2} = \sum_{i=1}^{n} Y_i$$

$$\beta'_0 \sum_{i=1}^{n} x_{i1} + \beta'_1 \sum_{i=1}^{n} {x_{i1}}^2 + \beta'_2 \sum_{i=1}^{n} x_{i1} x_{i2} = \sum_{i=1}^{n} x_{i1} Y_i$$

$$\beta'_0 \sum_{i=1}^{n} x_{i2} + \beta'_1 \sum_{i=1}^{n} x_{i1} x_{i2} + \beta'_2 \sum_{i=1}^{n} x_{i2}^2 = \sum_{i=1}^{n} x_{i2} Y_i$$

The multiple linear regression model we developed using a series of data from previous days and previous weeks to predict the next day market hourly price has the form:

$$P_H = \beta_0 + \beta_1 P_{PDH} + \beta_2 P_{PWH}$$

Where

P_H is tomorrow's price for hour "H"

P_{PDH} is previous day price of the same hour

P_{PWH} is the previous week day price of the same hour

We collect price data from Feb 22th, 2014 to Mar 30th, 2014 and use least square method to forecast the coefficient. The sample size is 30 in this particular model. The following shows the developed equations:

$$30\beta_0 + \beta_1 \sum_{i=1}^{30} P_{PDHi} + \beta_2 \sum_{i=1}^{30} P_{PWH\mathrm{i}} = \sum_{i=1}^{30} P_{Hi}$$

$$\beta_0 \sum_{i=1}^{30} P_{PWHi} + \beta_1 \sum_{i=1}^{30} P_{PDHi}^2 + \beta_2 \sum_{i=1}^{30} P_{PDHi} P_{PWHi} = \sum_{i=1}^{30} P_{PDHi} P_{Hi}$$

$$\beta_0 \sum_{i=1}^{n} P_{PWHi} + \beta_1 \sum_{i=1}^{30} P_{PDHi} P_{PWHi} + \beta_2 \sum_{i=1}^{30} P_{PWHi}^2 = \sum_{i=1}^{30} P_{PWHi} P_{Hi}$$

We use Matlab to calculate the equation (see Appendix 2). The predicted hourly equations can be seen as the follow:

Hour 1: $P_1 = 11.6841 + 0.5393 P_{PD1} - 0.0724 P_{PW1}$

Hour 2: $P_2 = 8.0362 + 0.5806 P_{PD} + 0.0238 P_{PW}$

Hour 3: $P_3 = 8.3146 + 0.5840 P_{PD} - 0.0024 P_{PW}$

Hour 4: $P_4 = 10.7288 + 0.3782P_{PD} + 0.0649P_{PW}$

Hour 5: $P_5 = 10.2315 + 0.4201P_{PD} + 0.0676P_{PW}$

Hour 6: $P_6 = 12.0120 + 0.3034P_{PD} + 0.151P_{PW}$

Hour 7: $P_7 = 7.5491 + 0.5002P_{PD} + 0.1947P_{PW}$

Hour 8: $P_8 = 10.9688 + 0.2739P_{PD} + 0.3277P_{PW}$

Hour 9: $P_9 = 12.1528 + 0.3073P_{PD} + 0.2954P_{PW}$

Hour 10: $P_{10} = 13.9577 + 0.3312P_{PD} + 0.1924P_{PW}$

Hour 11: $P_{11} = 13.5348 + 0.337P_{PD} + 0.1745P_{PW}$

Hour 12: $P_{12} = 12.2278 + 0.3999P_{PD} + 0.1451P_{PW}$

Hour 13: $P_{13} = 11.0606 + 0.416P_{PD} + 0.1517P_{PW}$

Hour 14: $P_{14} = 10.7659 + 0.4322P_{PD} + 0.1325P_{PW}$

Hour 15: $P_{15} = 9.8723 + 0.2808P_{PD} + 0.3096P_{PW}$

Hour 16: $P_{16} = 10.9323 + 0.2592P_{PD} + 0.2863P_{PW}$

Hour 17: $P_{17} = 10.6060 + 0.3874P_{PD} + 0.1795P_{PW}$

Hour 18: $P_{18} = 8.5284 + 0.4473P_{PD} + 0.228P_{PW}$

Hour 19: $P_{19} = 15.5713 + 0.4307P_{PD} + 0.0935P_{PW}$

Hour 20: $P_{20} = 14.7417 + 0.3031P_{PD} + 0.2312P_{PW}$

Hour 21: $P_{21} = 13.3509 + 0.2028P_{PD} + 0.3257P_{PW}$

Hour 22: $P_{22} = 12.9804 + 0.16P_{PD} + 0.3552P_{PW}$

Hour 23: $P_{23} = 13.1649 + 0.4984P_{PD} - 0.0084P_{PW}$

Hour 24: $P_{24} = 14.2966 + 0.5545P_{PD} - 0.1756P_{PW}$

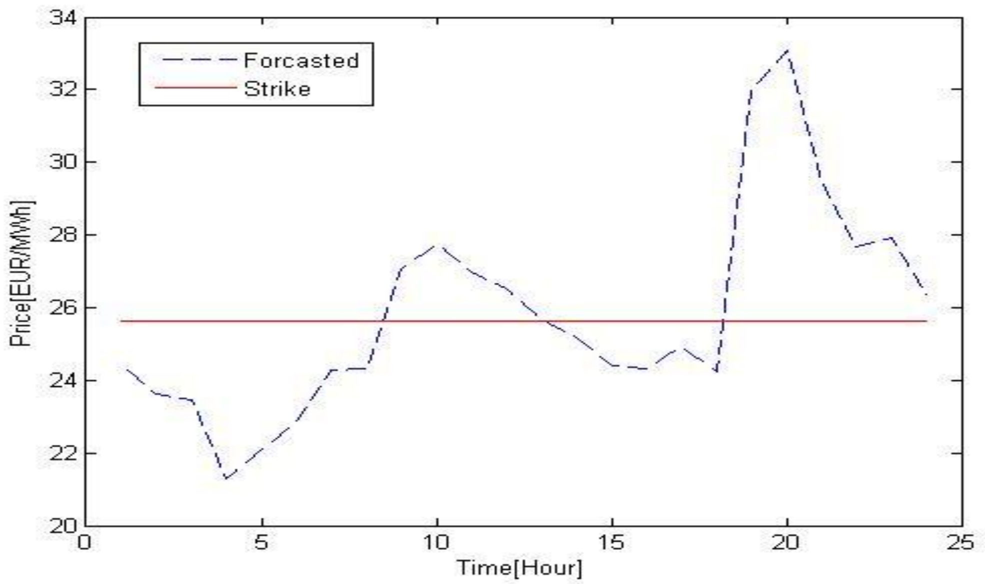
5.4 VPP first nomination

By carefully observe the price pattern and using multiple linear regression models, we get 24 individual formulas for each hour in a calendar day. On the day before our first exercise day, we use our previous formula to predict the electricity price on April 1st, 2014. And we compare the predicted price with our VPP set price, which is K=25.61 EUR/MWh, to decide which hours are going to be taken.

Our principle is to take full load 20MW for hours which have the predicted price larger than VPP set price. Otherwise we will choose to not exercise that hour.

The following graph from Matlab show us the differences between predicted price and our VPP set price:

Graph 12 Forecasted market prices



Source: Forecasted market price, Matlab.

The graph above gives us a clear picture of our prediction result. The results will be demonstrated together with our exercise plan for the first day of April, 2014 in the following table.

Table 5 Action strategy of the first nomination

April 1st,2014	Predicted price(EUR/MWh)	VPP set price(EUR/MWh)	Action
00-01	24.45	25.61	NO
01-02	23.62	25.61	NO
02-03	23.44	25.61	NO
03-04	21.28	25.61	NO
04-05	22.09	25.61	NO
05-06	22.84	25.61	NO
06-07	24.28	25.61	NO
07-08	24.33	25.61	NO
08-09	27.07	25.61	**Exercise**
09-10	27.78	25.61	**Exercise**
10-11	26.96	25.61	**Exercise**
11-12	26.52	25.61	**Exercise**
12-13	25.66	25.61	**Exercise**
13-14	25.19	25.61	NO
14-15	24.41	25.61	NO
15-16	24.30	25.61	NO
16-17	24.92	25.61	NO
17-18	24.24	25.61	NO
18-19	31.98	25.61	**Exercise**
19-20	33.09	25.61	**Exercise**
20-21	29.48	25.61	**Exercise**
21-22	27.69	25.61	**Exercise**
22-23	27.94	25.61	**Exercise**
23-00	26.35	25.61	**Exercise**

5.5 Model further discussion

As we can see from our prediction in table 5, it follows the daily peak-off hour pattern, which includes the peak hours 9:00 am-1:00 pm and 7:00pm to 9:00 pm. It also reflects the seasonality since spring comes the weather is turning warm people don't use too much electricity after 12pm.The result of our prediction captures the characteristics of seasonality and daily pattern. These characteristics are the most important factors should be considered for predicting electricity market prices. By following our multiple linear regression models, the first day we will take full load 20 MW by paying 25.61 EUR/MWh to Dong Energy from hour 9 to 13 and 19 to 24. The prediction accuracy of our model should still be discussed in reality.

The prediction of tomorrow's spot market price is very important for the decision of our first nomination. Because the model we choose will influence our decision making directly. Our model here is to find the main factors which influence tomorrow's spot price for each individual hour. This model is built after careful observation of electricity price pattern. But in order to simplify our analysis we ignore other factors and incoming market information which are hard to be measured but will influence electricity prices in the coming day. For example the weather forecast is an important factor. We have discussed before the weather can have big influence on the electricity price during one day. The problem in reality is the accuracy of weather forecast can be varied from country to country and day to day even hour to hour. So this is a potential risk for electricity bidders if the weather forecast cannot be totally dependent on. But at most of the cases we get the right information for the weather tomorrow, which will be considered for our final decision. Besides weather forecast, another element we often mention is the unpredictable spikes due to some technical or transmission bottlenecks. The more often it happens, the larger influence it will have on the prediction.

Another point we have to address here is that for more accurate analysis we can expand our data pool and update the data information in time. Of course different bidder may interpreter market information according to the degree of relevance. So our spot price prediction model is just the general and simple version for all. Our purpose here is offering a basic and general analysis for predicting the coming day's spot prices. The accuracy of the prediction can be improved by including further factors that may influence the price. It can also be adjusted by using more advanced prediction model, which is not the main purpose of our discussion. Our purpose here is to address the added value from the second nomination based on the possibility that the exercise strategy from the first prediction is not the ideal plan. The second nomination gives the VPP owners the chances to adjust their plans and find the optimal strategy which will bring the largest value.

5.6 VPP second nomination

Re-nomination will be taken place before 2 am after the hourly day-ahead DK1 area price has been announced. The re-nomination will give the VPP owners the opportunity to adjust their plans to re-nominate the day-ahead hourly purchasing capacity from Dong energy. But during the re-nomination the VPP owner will be subject to the constrain that the hour-to-hour deviation may not be more than 40 % of the maximum capacity of the VPP product owned by the VPP owner

Here we are given two options:

- Option 1, we keep out original nomination strategy and take full load 20 MW/h in each exercised hour.
- Option 2, we can change our plan to re-choose the exercising hours but we cannot take full capacity for each hour.

Our optimal strategy is to maximize our profits. First we will calculate the value for each option based on the realized price in the coming day (see information from table 6). And then choose the one which can bring the largest value.

Based on the result from table 5 and table 6 we can calculate the value for the first option.

$$V_1 = 20 * (28.93 - 25.61 + 28.30 - 25.61 + 27.93 - 25.61 + 27.42 - 25.61 + 26.38 - 25.61 + 26.89 - 25.61 + 25.70 - 25.61 + 26.05 - 25.61 + 27.31 - 25.61 + 26.84 - 25.61 + 20.32 - 25.61) = 207.2\ EUR$$

So by choosing option 1, it will bring us 207.2 EUR return for the first day of April.

Assuming that the realized DK1 price on April 1st, 2014 is observed as the prices in the following table:

Table 6 Realized DK1 price vs VPP set price

April 1st,2014	Realized DK1 Price(EUR/MWh)	VPP set price(EUR/WMh)
00-01	25,00	25.61
01-02	23,91	25.61
02-03	19,07	25.61
03-04	21,44	25.61
04-05	21,03	25.61
05-06	22,49	25.61
06-07	28,65	25.61
07-08	28,52	25.61
08-09	28,93	25.61
09-10	28,30	25.61
10-11	27,93	25.61
11-12	27,42	25.61
12-13	26,38	25.61
13-14	25,74	25.61
14-15	25,40	25.61
15-16	25,21	25.61
16-17	22,78	25.61
17-18	26,07	25.61
18-19	26,89	25.61
19-20	25,70	25.61
20-21	26,05	25.61
21-22	27,31	25.61
22-23	26,84	25.61
23-00	20,32	25.61

Source: data collected from Nord Pool

For option 2, the value can be expressed as:

$$V_2 = Max \sum_{i=1}^{24} Q_i * (S_i - K)$$

Where

$$0 \leq Q_i \leq 20$$

$$-8 \leq Q_{i+1} - Q_i \leq 8 \ (i = 1 \ldots 23)$$

Our VPP product is the quarterly product with 20 MW capacities on an hourly basis. The re-nomination is restricted by the condition that deviation from hour to hour cannot be more than 40% of the maximum capacity of the individual VPP product. The above equations have expressed the requirement in mathematic languages.

We use AMPL (see appendix 3) to solve this optimization problem and find the optimized value and its corresponding capacity Q for each hour. The result by running AMPL is showed as follow:

Table 7 Re-nomination result

April 1st, 2014	Q (MWh)
00-01	0
01-02	0
02-03	0
03-04	0
04-05	0
05-06	4
06-07	12
07-08	20
08-09	20
09-10	20
10-11	20
11-12	20
12-13	20
13-14	20
14-15	12
15-16	0
16-17	0
17-18	8
18-19	16
19-20	20
20-21	20
21-22	16
22-23	8
23-00	0

Source: result from AMPL calculation

The total value of option 2 after calculation:

$$V_2 = 4 * (22.49 - 25.61) + 12 * (28.65 - 25.61) + 20 * (28.52 - 25.61) + 20 * (28.93 - 25.61) + 20 * (28.30 - 25.61) + 20 * (27.93 - 25.61) + 20 * (27.42 - 25.61) + 20 * (26.38 - 25.61) + 20 * (25.74 - 25.61) + 12 * (25.40 - 25.61) + 8 * (26.07 - 25.61) + 16 * (26.89 - 25.61) + 20 * (25.70 - 25.61) + 20 * (26.05 - 25.61) + 16 * (27.31 - 25.61) + 8 * (26.84 - 25.61) = 372.92\ EUR$$

Since $V_2 = 372.92\ EUR > V_1 = 207.2\ EUR$, we will give up option 1 and choose option 2 to re-nominate our hours. The specific volume for each hour has been demonstrated in the previous table.

The re-nomination process gives us the second chances to adjust our observation and maximize our profit under certain limitation. The total performance of our nomination depends both on the first forecast also on finding the optimal value at the re-nomination. When the first prediction is not ideal enough, the re-nomination will help bidders to find optimal value to minimize risk and maximize values.

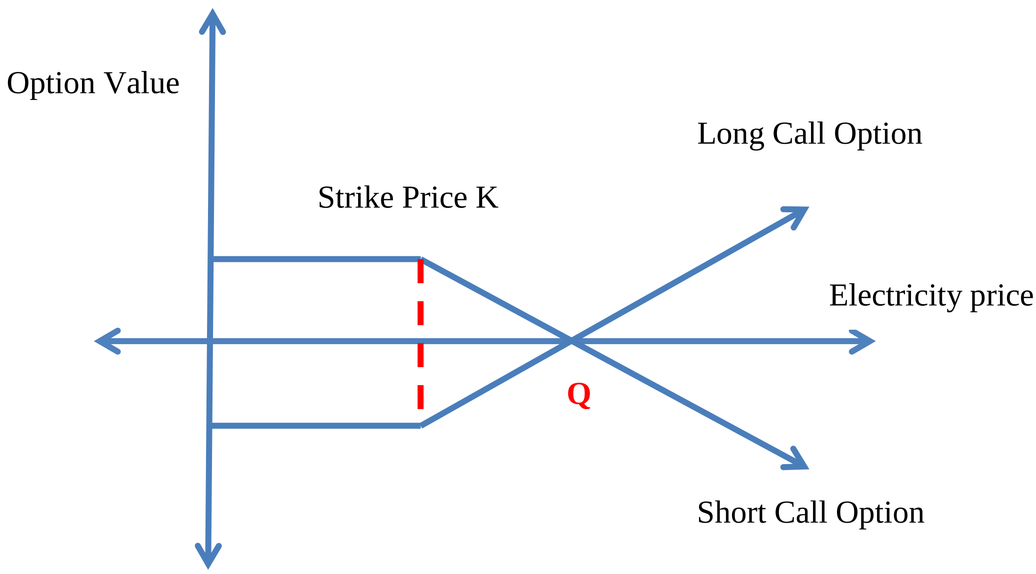
For some bidders re-nomination reduce their risks of the market price uncertainty and the inaccuracy of the first prediction by using different models. For risk adventures, they prefer to build a perfect model to help them forecast the right pattern and make good decisions. But for some small players they may lack of the relevant knowledge, information, and resource to build the right model. Even they build the right model, there could be situation the model goes wrong or extreme situation occurs. Better performance in the first prediction may bring more possibility of large profits. But the truth is not all the prediction can outperform others. Re-nomination make sure the VPP owners have the chances to gain some certain profits by adjust their plans in time.

6. Risk management for Dong Energy

Until now, we have analysed the evaluation and exercise of VPP for a small player. VPP products give the small players both the right and the flexibility to utilize those financial options. The risks the biggest player Dong Energy face in front of issuing VPP products is what we are going to discuss in this sector. By observing the challenge of the biggest player, it will help participants and regulators to better understand the electricity market and thus build a dynamic wholesale market.

First, let us have a look of the risks of both parties. Smaller players have the rights to exercise VPP at a fixed electricity price. So in the VPP auction relations smaller players are longing call options. On the contrary, Dong Energy is shorting call options. The risks for smaller players are limited but the benefits are unlimited. For Dong Energy, it is opposite. The risks of shorting options are unlimited while the benefits are limited, as you can see from the bellowing graph.

Graph 13 Long Call option and Short Call option



The graph above gives basic description of risks and benefits for different parts of call options. For a single VPP option, for example VPP-Q2-2014, the strike price K=25.61 EUR/MWh. And the option premium, which is the value paid to Dong Energy for getting the call option, is 8.13 EUR/MWh [Dong Energy].

$$Q = \text{Strike price} + \text{Option premium} = 25.61 + 8.13 = 33.74 \text{ EUR/MWh}$$

- When electricity price$\leq 25.61;$ VPP owner will not exercise their call options, Dong energy will earn the whole option premium 8.13EUR/MWh.
- When $25.61 \leq$ electricity price $\leq 33.74;$ VPP owner will exercise their call options.
- When electricity price $\geq 33.74;$ VPP owners will definitely exercise their options and Dong energy will out of money and have losses.

$$\text{Dong energy's profits} = 8.13\text{EUR/MWh} - (\text{electricity price} - \text{strike price}).$$

The larger the electricity price is than the strike price, the more loss Dong Energy is going to take. It is very crucial for Dong Energy to understand the impact of price change in electricity on the option value and how to hedge the potential risk.

6.1 Delta

Delta is the relationship between the option price and the underlying price. More formally, the delta is described as:

$$\text{Delta} = \frac{Change\ in\ option\ price}{Change\ in\ uncerlying\ price}$$

The Delta can be obtained approximately from the Black Scholes formula as the value of $\mathrm{N}(d_1)$ for call options. By given the value of $\mathrm{N}(d_1)$, we can know the approximate value of change in option price by applying the formula:

$$\text{Change in option price} = \text{Delta} * \text{change in underlying price}$$
$$= \mathrm{N}(d_1) * \text{change in underlying price}$$

Therefore, for 1 EUR change in the price of the electricity in DK1 area, we should expect:

$$\text{Change in hourly option } V_i = \mathrm{N}(d_1)_i$$

Delta is important as a risk measure. The delta defines the sensitivity of the option price to a change in the price of the underlying. Dealers can use delta to construct hedges to offset the risk they have assumed by entering into other transactions. Dong Energy is the seller of the call options. The risks will increase as the prices of electricity increase. To have a clear understand in the Delta will help Dong Energy to make decision early to engage into other financial activities and hedge risks.

For example, on 17st Feb, 2014 we observed $N(d_1)$ of 24 hours on 1st April, 2014 based on our predicted forward price and volatilities. The strike price was given.

Table 8 N (d1) for different hours on 1st April, 2014

	N(d1)
00 - 01	0.35
01 - 02	0.35
02 - 03	0.27
03 - 04	0.23
04 - 05	0.15
05 - 06	0.11
06 - 07	0.32
07 - 08	0.36
08 - 09	0.47
09 - 10	0.47
10 - 11	0.51
11 - 12	0.51
12 - 13	0.45
13 - 14	0.37
14 - 15	0.35
15 - 16	0.33
16 - 17	0.35
17 - 18	0.36
18 - 19	0.36
19 - 20	0.36
20 - 21	0.35
21 - 22	0.33
22 - 23	0.33
23 - 00	0.28

We can see different hours have different delta approximation. Delta in peak hours normally higher than delta in off peak hours, which demonstrated that option prices are more sensitive in peak hours than off peak hours in one day horizon.

Assuming that based on new information, we predict that the underlying electricity hourly price for 1st April, 2014 will change. The price change will be demonstrated in the following table:

Table 9 Prices change for underlying product. EUR/MWh

	Forecasted Price	New-forecasted Price	**Price Change**
00 - 01	21.92	23.96	2.05
01 - 02	21.95	23.53	1.57
02 - 03	20.55	22.98	2.43
03 - 04	19.83	22.79	2.96
04 - 05	18.22	22.81	4.60
05 - 06	17.40	24.24	6.83
06 - 07	21.43	27.12	5.69
07 - 08	22.15	32.92	10.77
08 - 09	23.97	40.00	16.04
09 - 10	24.12	33.55	9.43
10 - 11	24.68	28.53	3.85
11 - 12	24.78	27.72	2.94
12 - 13	23.75	28.05	4.30
13 - 14	22.33	27.37	5.03
14 - 15	22.04	27.05	5.00
15 - 16	21.74	26.87	5.12
16 - 17	21.92	26.35	4.43
17 - 18	22.11	26.48	4.38
18 - 19	22.10	27.97	5.87
19 - 20	22.10	28.65	6.55
20 - 21	21.99	27.51	5.53
21 - 22	21.59	27.46	5.87
22 - 23	21.61	25.77	4.16
23 - 00	20.73	24.30	3.56

Assuming that for the moment the delta tells us precisely the movement in the option for a movement in the underlying. We already know the price movement of electricity from incoming new information. By combing Delta and the price change for each hour, we can get the total value change for VPP on 1st April, 2014.

$$V_{change} = \sum_{i=1}^{24} N(d_1)_i * P_{change_1} = 46.08\ EUR/MW$$

Dong Energy is shorting VPP on that day. The prediction tells that the price for each hour is going to increase to a different degree, the hourly option value for that specific hour will decrease according to the multiply value from delta $N(d_1)_i$ and P_{change_1}.

From the above analysis, we get the conclusion that. The option value for Dong Energy will decrease 46.08 EUR/MW in total on 1st April, 2014.

6.2 Dynamic hedging

Dynamic hedging is a hedging technique which is used to provide insurance for a portfolio of investments by limiting exposure to delta or gamma. The hedge should be frequently adjusted in terms with the changes in the underlying price in order to offset the deltas of an actual position with those of a simulated one. For example, the dealer may establish a delta hedge of a non-linear position such as an exotic option, or with a linear position such as a spot transaction. The delta of these two different positions will cancel out each other, and the whole position is said to be hedged. Dynamic hedging is commonly used by dealers who hold vast number of short options on a underlying and want to offset the position by taking long position on the same underlying since such long options are not always available at the market. By doing this, the return will be insured and risks will be limited [Investment and Finance].

From delta analysis, we can tell how sensitive the option price for the change of underlying price. For Dong energy the risks of losing money is very huge when the price of electricity increases. How to hedge risks and minimize loss is a big challenge for Dong. In order to hedge risks from the short position in call, Dong Energy should also engage into a long position, thus the values will be offset.

First we calculate the total delta for the 24 hours on 1st April, 2014:

$$Delta_{Short} = \sum_{i=1}^{24} N(d_1)_i = 8.28$$

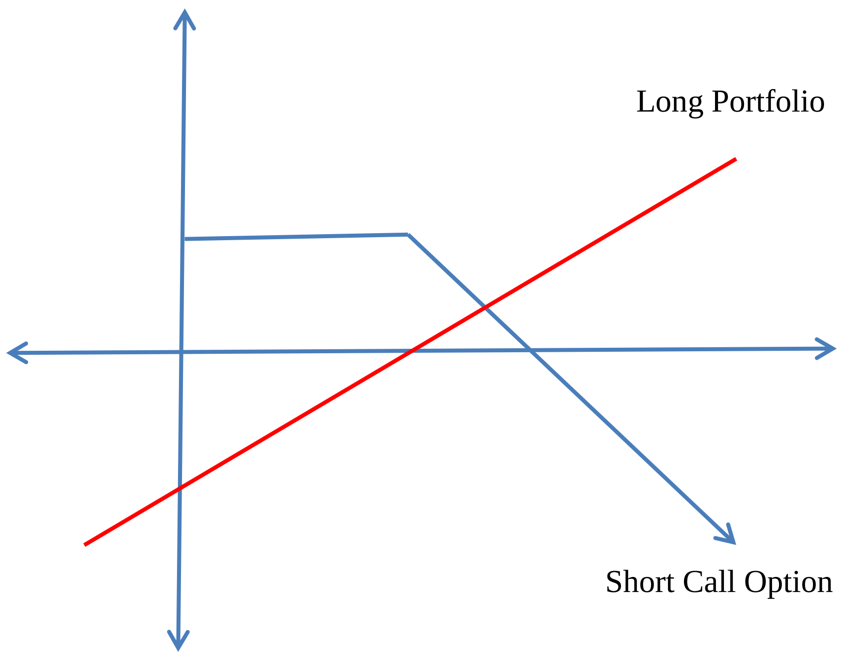
$$Delta_{Long} = Delta_{Short}$$

So Dong Energy should design a portfolio which has a short position. The underlying product of this portfolio will be the electricity and the total delta of the portfolio will just equal the total delta of the VPP for 1st April, 2014. Assuming that there are different financial tools in the electricity market for participants to hedge their risks, and the products

Dong Energy would like to buy are available. The portfolio can be call options, futures, forwards or other assets. The total value of the delta will be zero.

Assuming Dong Energy built a linear portfolio which will bring Dong 8.28 EUR for every 1EUR increase in electricity price. And the portfolio is combined of different asset with underlying product as electricity. The benefits from the asset return will hedge the loss in shorting a call option. The total return of Dong Energy will be guaranteed. By doing these, the risks will be hedged and it also makes sure Dong will have all the positive returns.

Graph 14 Delta hedge: Long portfolio and Short call option



The delta is not just influenced by the underlying price. The delta is always changeable when the option moves towards expiration even if the underlying price will not change. If the underlying price does not move, a call delta will move towards 1 if the call is in the money or 0 if the call is out of the money as the call moves toward the expiration day. So the delta is constantly changing, which mean the delta hedging is a dynamic process.

The strategies of dynamic hedging and how to build the linear or non-linear portfolio can be discussed in the further research in order to find the optimal hedging strategy for Dong Energy, but it is beyond our research in this thesis.

7. Conclusion

VPP evaluation is a complicated process. Basically it requires a very competitive and deregulated market which reflected all the necessary information. In reality, our market cannot be as perfect as we assumed. We choose the Nordic electricity market as the research object because that it is a mature and liberalized market with comparable complete information. By using Black 76 to evaluate VPP (a series of European call options) gives the small players a clear picture of the value it will bring to join the VPP auctions. It also helps the small players to find the bottom line of the products they would like to pay. And it helps regulators to adjust the right price for VPP products and build a more competitive market as well.

Black 76 is a great tool for pricing European call options. It is easy to implement and simple to understand. According to our analysis, we found it is quite beneficial to join the VPP auctions for small players. Not just because the value it hold for itself as a strip of options. Another important part is because during the second nomination VPP gives it owners the flexibility to adjust plans and find the optimal return. That means the risks has been minimized in the second round based on the incoming market information. For small players it is extremely important for that their prediction performance may underperform the big competitors because of the lack of resource, capital or related information. The flexibility in the second round will helps small players to adjust their plan and hedge the risks from inaccuracy of previous prediction. The flexibility also explained the result from our first prediction by using black 76, that the first prediction will always be lower than the total value, the value added from the second nomination. How to evaluate the value from re-nomination depends on the industry characteristics and the prediction ability of individual small players.

In our evaluation we made some assumptions under Black 76. We built a price pattern model to get hourly forward price. There are different methods to predict hourly forward price. The disadvantage of our analysis is the inaccuracy from hourly forward price prediction may influence the total price evaluation of VPP. In the future research this can be improved by comparing different forecasting methods and find the optimal one. Another potential problem is the prediction of day-ahead spot price for making the decision to exercise or not.

The performance from the prediction will influence directly on our decision making. We use multiple linear regression models to forecast the day-ahead spot price. This model ignored other important factors such as weather and transmission capacity which will definitely matter for the coming electricity price. We should consider quantifying these factors and including them in our further analysis. But as we mentioned before, the whole purpose of this thesis is to demonstrate that Black 76 can be used to evaluate VPP as an effective tool for small players and the importance of flexibility value added to the total value of VPP.

As for Dong Energy, the risks of issuing VPP and short the call option is huge. It is important for Dong to transfer and hedge potential risks by engaging to other products or assets in electricity financial market. By understanding delta, Dong Energy will have a better understanding of how sensitive the option value in total is for the change of electricity price. Dong Energy can use dynamic hedging to build a portfolio in a long position with electricity as underlying product and have the same delta as the VPP. Since the hedging is a dynamic process which requires Dong Energy to adjust its portfolio continuously. This also requires the electricity market to be dynamic and competitive. The further research for the VPP can start from the perspective of Dong Energy and try to find the strategies to build dynamic hedging. And these hedging strategies can also be replicated by small players in their daily operation for similar situation.

References

Azadeh, A., Ghaderi, S. F., Pourvalikhan Nokhandan, B., & Sheikhalishahi, M. (2012). A new genetic algorithm approach for optimizing bidding strategy viewpoint of profit maximization of a generation company. *Expert Systems with Applications*, *39*(1), 1565-1574.

Ausubel, L. M., & Cramton, P. (2010). Virtual power plant auctions. *Utilities Policy*, *18*(4), 201-208.

Benth, F. E., Koekkebakker, S., & Ollmar, F. (2007). Extracting and applying smooth forward curves from average-based commodity contracts with seasonal variation. *The Journal of Derivatives*, *15*(1), 52-66.

Black, F. (1976). The pricing of commodity contracts. *Journal of financial economics*, *3*(1), 167-179.

Bjerksund, P., F. Carlsen, and G. Stensland (i) (2006). Valuation of Power forwards and futures. Lecture note.

Bjerksund, P., F. Carlsen, and G. Stensland (ii) (2006). Valuation of Power Options, Lecture note.

Bjerksund, P., Myksvoll, B., & Stensland, G. (2008). Exercising flexible load contracts: Two simple strategies. *Applied Stochastic Models in Business and Industry*, *24*(2), 93-107.

Bjerksund, P., Stensland, G., & Vagstad, F. (2011). Gas Storage Valuation: Price Modelling v. Optimization Methods. *Energy Journal*, 32(1).

Bjørndal M.(2012). *Electricity Market Developments in the Nord Pool Area*, Working paper at Harvard Electricity Policy Group Meeting.

Caro, Grégoire, et al.(2011). *A quantitative analysis of the electricity market in Switzerland. Energy Market (EEM),* 8th International Conference on the European.

Corchero García, C. (2011). Short-term bidding strategies for a generation company in the Iberian Electricity Market.

Deng, S. J., & Oren, S. S. (2006). Electricity derivatives and risk management. *Energy*, *31*(6), 940-953.

Dong Energy, Denmark, accessed 7th February 2014, http://www.dongenergyvpp.com/Pages/index.aspx .

EDF, Electricité de France, accessed 3th February2014, http://capacityauctions.edf.com/the-edf-group/capacity-auctions/overview-114023.html .

Fourer, R., Gay, D. M., & Kernighan, B. W. (1990). A modeling language for mathematical programming. *Management Science*, *36*(5), 519-554.

Fleten, S. E., & Lemming, J. (2003). Constructing forward price curves in electricity markets. *Energy Economics*, *25*(5), 409-424.

Hill, R. C., Griffiths, W. E., & Lim, G. C. (2008). *Principles of econometrics*(Vol. 5). Hoboken, NJ: Wiley.

ICIS, Electrabel, Belgium, accessed 27th ,May 2014, http://www.icis.com/energy/

Investment and Finance, accessed 27th May, 2014, http://www.investment-and-finance.net/index.html

KYOS Energy Consulting BV.(2010).Pricing of DONG VPP for Q1-2011 and Yr-2011.Working paper. http://www.kyos.com/files/uploads/publications/Dong_VPP_report_KYOS.pdf

Maurer, L., & Barroso, L. A. (2011). *Electricity auctions: an overview of efficient practices.* World Bank Publications.

Nord Pool Spot, Norway, accessed 2rd April 2014, http://www.nordpoolspot.com/ .

NASDAQ OMX, Nordic countries' financial market, accessed 13th February 2014, http://www.nasdaqomx.com/commodities .

Pardo M. (2005).Pricing daily options in the German power market. PHD thesis in Universidad Pontificia Comillas.

Schultz, C. (2005). *Virtual capacity and competition* (No. 1487). CESifo working papers..

Appendix 1

Number of auctions	Quarter-Year	Option Price(EUR/MW)	Electricity Set Price(EUR/MWh)
1st	Q1-2006	69,351 €	13.26 €
2nd	Q1-2007	52,503 €	20.63 €
3rd	Q2-2007	24,600 €	17.62 €
4th	Q3-2007	34,503 €	18.00 €
5th	Q4-2007	36,300 €	20.97 €
6th	Q1-2008	48,948 €	38.94 €
7th	Q2-2008	23,415 €	42.31 €
8th	Q3-2008	50,340 €	46.94 €
9th	Q4-2008	66,300 €	54.39 €
10th	Q1-2009	44,700 €	31.72 €
11th	Q2-2009	19,053 €	26.13 €
12th	Q3-2009	33,000 €	24.73 €
13th	Q4-2009	29,346 €	27.94 €
14th	Q1-2010	26,943 €	26.72 €
15th	Q2-2010	38,853 €	26.52 €
16th	Q3-2010	43,098 €	31.58 €
17th	Q4-2010	43,161 €	35.46 €
18th	Q1-2011	34,050 €	37.50 €
19th	Q2-2011	31,521 €	37.82 €
20th	Q3-2011	42,900 €	38.35 €
21st	Q4-2011	46,665 €	37.65 €
22nd	Q1-2012	31,128 €	35.69 €
23rd	Q2-2012	22,956 €	32.98 €
24th	Q3-2012	19,299 €	30.56 €
25th	Q4-2012	35,406 €	31.31 €
26th	Q1-2013	26,622 €	32.08 €
27th	Q2-2013	23,847 €	27.33 €
28th	Q3-2013	23,490 €	27.10 €
29th	Q4-2013	33,015 €	23.90 €
30th	Q1-2014	33,711 €	25.39 €
31th	Q2-2014	17,760 €	25.61 €

Source: data collected from Dong energy’s website

Appendix 2

Matlab calculation

```
load ('C:\Users\Administrator\Desktop\VPP\HourlyPrices.txt');
n=30;
Hourly_Prices=zeros(24,30);
Previous_Week_Prices=zeros(24,30);
Previous_Day_Prices=zeros(24,30);
for j=1:24
        Hourly_Prices(j,:)=HourlyPrices(j,:);
        Previous_Day_Prices(j,:)=HourlyPrices(j+24,:);
        Previous_Week_Prices(j,:)=HourlyPrices(j+48,:);
end
S=1:24;
L=1:24;
L1=1:24;
L2=1:24;
L3=1:24;
Y=1:24;
Y1=1:24;
Y2=1:24;
mat_1=[1:3;1:3;1:3];
mat_2=[1;2;3];
coefficient=[1;2;3];
A=1:24;
B=1:24;
```

```
C=1:24;
for i=1:24
S(i)=sum(Hourly_Prices(i,:));
L(i)=sum(Previous_Day_Prices(i,:));
L1(i)=sum(Previous_Day_Prices(i,:).*Previous_Day_Prices(i,:));
L2(i)=sum(Previous_Day_Prices(i,:).*Previous_Week_Prices(i,:));
L3(i)=sum(Previous_Day_Prices(i,:).*Hourly_Prices(i,:));
Y(i)=sum(Previous_Week_Prices(i,:));
Y1(i)=sum(Previous_Week_Prices(i,:).*Previous_Week_Prices(i,:));
Y2(i)=sum(Previous_Week_Prices(i,:).*Hourly_Prices(i,:));
mat_1=[n L(i) Y(i); L(i) L1(i) L2(i); Y(i) L2(i) Y1(i)];
mat_2=[S(i); L3(i); Y2(i)];
coefficient=mat_1\mat_2;
A(i)=coefficient(1,1);
B(i)=coefficient(2,1);
C(i)=coefficient(3,1);
end
load ('C:\Users\Administrator\Desktop\VPP\X.txt');
X1=X(1,:);
X2=X(2,:);
Y=1:24;
for n=1:1:24
Y(n)=A(n)+B(n)*X1(n)+C(n)*X2(n);
end
x=1:1:24;
load ('C:\Users\Administrator\Desktop\VPP\Strikeprice.txt');
```

```
S= Strikeprice(1,:);

F= Y;

plot(x,F,'b--',x,S,'r'),
xlabel('Time[Hour]'),ylabel('Price[EUR/MWh]'),legend('Forcasted','Strike')
```

Appendix 3

AMPL calculation

```
#----------------------

#Model file:mv.mod

#----------------------

set T;#number of hours

param s{T};# spot price of each hour

param p{T}; # vpp strike price of each hour

var q{T}>=0,<=20; #VPP capacity of each hour

maximize v:

sum{t in T} q[t]*(s[t]-p[t]);

subject to

capacity1 {t in 1..23}: q[t+1]+8>=q[t];

capacity2 {t in 1..23}: q[t+1]<=q[t]+8;

# ----------------------

#Data file: mv.dat

#-----------------------

set T :=1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24;

param p :=

        1 25.61

        2 25.61

        3 25.61

        4 25.61

        5 25.61

        6 25.61
```

7 25.61

8 25.61

9 25.61

10 25.61

11 25.61

12 25.61

13 25.61

14 25.61

15 25.61

16 25.61

17 25.61

18 25.61

19 25.61

20 25.61

21 25.61

22 25.61

23 25.61

24 25.61;

param s :=

1 25.00

2 23.91

3 19.07

4 21.44

5 21.03

6 22.49

7 28.65

8 28.52

9 28.93

10 28.30

11 27.93

12 27.42

13 26.38

14 25.74

15 25.40

16 25.21

17 22.78

18 26.07

19 26.89

20 25.70

21 26.05

22 27.31

23 26.84

24 20.32;