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Jump-Diffusion Models for Option Pricing versus the Black Scholes Model

Håkon Båtnes Storeng

Supervisor: Professor Svein-Arne Persson

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Abstract

In general, the daily logarithmic returns of individual stocks are not normally distributed. This poses a challenge when trying to compute the most accurate option prices. This thesis investigates three different models for option pricing, The Black Scholes Model (1973), the Merton Jump-Diffusion Model (1975) and the Kou Double-Exponential Jump-Diffusion Model (2002).

The jump-diffusion models do not make the same assumption as the Black Scholes model regarding the behavior of the underlying assets' returns; the assumption of normally distributed logarithmic returns. This could make the models more able to produce accurate results.

Both the Merton Jump-Diffusion Model and the Kou Double-Exponential Jump-Diffusion Model shows promising results, especially when looking at how they are able to reproduce the leptokurtic feature and to some extent the "volatility smile". However, because the observed implied volatility surface is skewed and tends to flatten out for longer maturities, the two models abilities to produce accurate results are reduced.

And while visual study reveals some difference between the models, the results are not significant.

Acknowledgements

The writing of this thesis has been a challenging, yet rewarding process. From first learning about the jump-diffusion models to actually calibrating and implementing them has been an educational process.

I would like to thank Professor Svein-Arne Persson for always keeping his door open and being available for questions and discussions regarding the thesis. His feedback has been very constructive and helpful throughout the whole writing process.

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1. Introduction

Since the introduction of the Black Scholes model in 1973, the model has been widely used by both academics and traders and taught in numerous finance courses at universities worldwide. As with many economic models, assumptions are made to the Black Scholes model in order to make it tractable. One of these assumptions is that the asset's price follows a geometric Brownian motion, and as a consequence, its return is normally distributed.

1.2 Motivation

Much research has been conducted to modify the Black Scholes model based on Brownian motion in order to incorporate two empirical features of financial markets:

1) The leptokurtic features. In other words, the return distribution has a higher peak and two heavier tails than those of the normal distribution.
2) The volatility smile. More precisely, if the Black-Scholes model is correct, then the implied volatility should be constant. However, it is widely recognized that the implied volatility curve resembles a "smile", meaning that it is a convex curve of the strike price.

In order to incorporate these two features, several models have been developed in the wake of the Black Scholes model. Among these is the Merton Jump-Diffusion Model (1975), denoted Merton from now on, which can be seen as a foundation for the jump-diffusion models, and the Kou Double-Exponential Jump-Diffusion Model (2002), denoted Kou, as a new creation.

The goal of this thesis is to give an in-depth study on how these models perform when multiple strike prices and maturities are considered. This will be done by looking at the degree of mispricing across the entire strike range and for different maturities.

1.3 Thesis structure

The models will be described and tested against the most traded call option as of May the 5th, 2014. This is the call option on the Bank of America Corporation stock. A comparison of the accuracy between the Black Scholes-model and the jump-diffusion models will be carried out as a measure of which model produces the most accurate results. Visual study as well as hypothesis testing for differences in pricing errors will be conducted in order to answer this question.

2. What is an option?

This chapter is from Hull (2008)

Options are traded both on exchanges and in the over-the-counter market[1]. There are two types of options:

- *Call* options. A call option gives the holder the right, but not the obligation, to buy the underlying asset at a certain date at a certain price.
- *Put* options. A put option gives the holder the right, but not the obligation, to sell the underlying asset at a certain date at a certain price.

The price in the contract is known as the *exercise price* or *strike price*; the date in the contract is known as the *expiration date* or *maturity*.

American options can be exercised at any time up to the expiration date. *European options* can be exercised only on the expiration date itself. Most of the options traded on exchanges are American. In the exchange-traded equity option market, one contract is usually an agreement to buy or sell 100 shares.

The underlying asset can be basically anything of financial value. It could be a stock, gold, crude oil or even an option to buy an option.

2.1 Factors affecting option prices

There are six factors affecting the price of a stock option:

1. The current stock price, S_0.
2. The strike price, K.
3. The time to expiration, T.
4. The volatility of the stock price, σ.

[1] A decentralized market, without a central physical location, where market participants trade with one another through various communication modes such as the telephone, email and proprietary electronic trading systems.

5. The risk-free interest rate, *r*.
6. The dividends expected during the life of the option.

In this section I will consider what happens to option prices when one of these factors change, holding the other factors constant.

For the rest of the thesis, I will only consider call options.

2.1.1 Stock price and Strike price

If a call option is exercised at some future time, the payoff will be the amount by which the stock price exceeds the strike price. Call options therefore become more valuable as the stock price increases and less valuable as the strike price increases.

2.1.2 Time to expiration

American call options become more valuable (or at least do not decrease in value) as the time to expiration increases. Consider two American options that differ only as far as the expiration date is concerned. The owner of the long-life option has all the exercise opportunities open to the owner of the short-life option – and more. The long-life option must therefore always be worth as least as much as the short life option.

2.1.3 Volatility

The volatility of a stock is a measure of our uncertainty about the returns provided by the stock. Stocks typically have volatility between 15% and 60%. As volatility increases, the chance that the stock performs very well or very badly, increases. For the owner of a stock, these two outcomes tend to offset each other. However, this is not so for the owner of a call. The owner of a call benefits from price increases but has limited downside risk in the event of price decreases because the most the owner can lose is the price of the option.

2.1.4 Risk-free Interest Rate

The risk-free interest rate affects the price of an option in a less clear-cut way. As interest rates in the economy increase, the expected return required by investors from the stock tends to increase. In addition, the present value of any future cash flow received by the holder of the option decreases. The combined impact of these two effects is to increase the value of call options.

2.1.5 Amount of future dividends

Because the options considered in this thesis are on stocks that do not pay dividends during the life of the option, I will not describe how it affects the value of the call option. For interested readers I refer to Hull (2008)

3. Deriving the price of an option

The price of an option is partly derived from supply and demand and partly from theoretical models. As mentioned in the introduction, this thesis will look at three different models for option pricing, with the Black Scholes model being the most commonly used and easiest to implement. The models will be described in more detail, starting with the Black Scholes model.

3.1 The Black Scholes option pricing model

As a starting point, the assumptions of the model will be presented. Section (3.3) will describe the model in more detail.

3.2 Assumptions

Assumptions of the model:

1. The stock price follows a geometric Brownian motion.
2. The short selling of securities with full use of proceeds is permitted.
3. There are no transaction costs or taxes. All securities are perfectly divisible.
4. There are no dividends during the life of the derivative.
5. There are no riskless arbitrage opportunities.
6. Security trading is continuous.
7. The risk-free rate of interest, r, is constant and the same for all securities.

Assumption 1) is described more in detail below.

3.2.1 The Stock Price following a geometric Brownian motion

This section is from Osseiran (2010)

In the Black Scholes model, the price of the underlying asset is modeled as a lognormal random variable. The stochastic differential equation (SDE) governing the dynamics of the price under the risk-neutral probability measure[2], is given by

$$dS(t) = rS(t)dt + \sigma S(t)dW(t) \tag{1}$$

where r is the risk-free rate, and σ the volatility of the underlying asset. Like a typical SDE, this equation consists of a deterministic part and a random part. The part $dS(t) = rS(t)dt$ is a deterministic, ordinary differential equation, which can be written as $\frac{dS(t)}{dt} = rS(t)$. The addition of the term $\sigma S(t)dW(t)$ introduces randomness into the equation, making it *stochastic*[3]. The random part contains the term $W(t)$, which is Brownian motion; it is a random process that is normally distributed with mean zero and variance t. The assumption of a log-normal price implies that the log prices are normally distributed. The log is another way of expressing returns, so in a different way this is saying that if the price is log-normally distributed, the returns of the underlying asset are normally distributed.

3.3 The formula for option pricing

The Black Scholes formula for the price of a European call option on a non-dividend-paying stock at time 0, is:

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2) \tag{2}$$

[2] The risk-neutral probability measure is important in finance. Most commonly, it is used in the valuation of financial derivatives. Under the risk-neutral measure, the future expected value of the financial derivatives is discounted at the risk-free rate.

[3] Any variable whose value changes over time in an uncertain way is said to follow a stochastic process.

where

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

The function *N(x)* is the cumulative probability distribution function for a standardized normal distribution. In other words, it is the probability that a variable with a standard normal distribution, denoted as $\phi(0,1)$, will be less than x. It is illustrated in the figure below

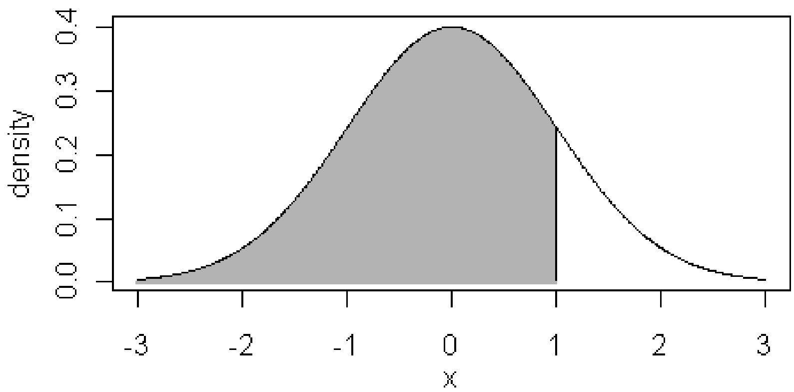

Figure 3.1. *The figure illustrates* $\mathbf{Pr(x \leq 1). \; x \sim \phi(0,1)}$

Where the shaded area represents the probability of $x \leq 1$.

The remaining variables are given in section (2). The variable *c* is the European call option price.

The expression $N(d_2)$ is the probability that the option will be exercised in a risk-neutral world, so that $KN(d_2)$ is the strike price times the probability that the strike price will be paid.

The expression $S_0N(d_1)e^{rT}$ is the expected value in a risk-neutral world of a variable that is equal to S_T if $S_T > K$ and zero otherwise.

When the Black Scholes model is used in practice the interest rate r is set equal to the zero-coupon risk-free interest rate at maturity T.

Since it is never optimal to exercise an American call option on a non-dividend-paying stock early, expression (2) is the value of an American call option on a non-dividend-paying stock.

The only problem in implementing expression (2) is the calculation of $N(d_1)$ and $N(d_2)$. However, this is hardly a challenge, as the only thing one needs is a table for the probabilities of the normal distribution.

As this section shows, the Black Scholes model is a very simple model to implement. However, the assumptions made to the model, especially assumption 1 should make it less likely to produce the observed prices in the market. Because of this, two alternative models, which do not make the same assumptions of the stock price behavior, will be introduced.

4. Expanding the Black Scholes Model

This chapter is based Burger and Kliaras (2013), Kou (2002) and Matsuda (2004)

One of the first approaches of expanding the Black Scholes model was the Merton Jump Diffusion model (Merton) by Robert C. Merton in 1976, which was also involved in the process of developing the Black-Scholes model.

The reason for this new approach was to make the model more realistic by allowing the underlying asset's price to "jump".

Over the years, several kinds of jump diffusion models have been developed based on this model.

4.1 Jump-diffusion models

Jump diffusion models always contain two parts, a jump part and a diffusion part. A common Brownian motion determines the diffusion part and a Poisson process[4] determines the jump part.

4.1.2 SDE under the Physical and Risk-Neutral Probability Measure

In jump-diffusion models, a general expression for the asset price, $S(t)$, under the physical probability measure P[5], is given by the following stochastic differential equation

$$\frac{dS(t)}{S(t-)} = \mu dt + \sigma W(t) + d\left(\sum_{i=1}^{N(t)} (V_i - 1)\right) \tag{3}$$

[4] In probability theory, a Poisson process is a stochastic process that counts the number of events and the time that these events occur in a given time interval. The time between each pair of consecutive events has an expoential distribution with parameter λ and each of these inter-arrival times are assumed to be independent of other inter-arrival times.

[5] Also called actual measure. The physical probability measure is used in computations in the actual world. The most common applications are seen in statistical estimations from historical data and the hedging of portfolios.

Solving the SDE gives the dynamics of the asset price under the physical probability measure

$$S(t) = S(0)exp\left\{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W(t)\right\}\prod_{i=1}^{N(t)} V_i \tag{4}$$

Here $N(t)$ is a Poisson process with rate λ, $W(t)$ is a standard Brownian motion and μ is the drift rate. $\{V_i\}$ is a sequence of independent identically distributed (i.i.d) nonnegative random variables.

In the Merton model, $\log(V_i) = Y_i$ is the absolute asset price jump size and is normally distributed.

In the Kou mode, $\log(V_i) = Y_i$ is the absolute asset price jump size and is double-exponentially distributed.

In the models, all sources of randomness, $N(t)$, $W(t)$ and Y_i are assumed independent.

Compared to equation (1), which is under the risk-neutral measure, μ has taken the place of r, and a Poisson process is added. The drift rate is the expected return on the stock per year. In contrast to the SDE under the risk-neutral measure, the drift component has not been adjusted for the market price of risk.

An arbitrage[6] free option-pricing model is specified under a risk-neutral probability measure. In asset pricing, the condition of no arbitrage is equivalent to the existence of a risk-neutral measure. It arises from a key property of the Black Scholes SDE. This property is that the equation does not involve any variables that are affected by the risk preferences of investors. The SDE would not be independent of risk preferences if it involved the expected return, μ, of the stock. This is because the value of μ depends on risk preferences, Hull (2008).

The corresponding SDE under the risk-neutral probability measure is

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[6] A trading strategy that takes advantage of two or more securities being misprices relative to each other.

$$\frac{dS_t}{S(t-)} = (r - \lambda^* k^*)dt + \sigma W^*(t) + \, d\left(\sum_{i=1}^{N^*(t)} (V_i^* - 1)\right) \qquad (5)$$

Here $W^*(t)$ is a standard Brownian motion under a risk-neutral probability measure. $N^*(t)$ is a Poisson process under a risk-neutral probability measure.

Where $\lambda^* k^* dt$ is the expected relative price change $E[dS_t/S_t]$ from the jump part $dN^*(t)$ in the time interval dt. This is the expected part of the jump. This is why the instantaneous expected return under the risk-neutral probability measure, rdt, is adjusted by $-\lambda^* k^* dt$ in the drift term of the jump-diffusion process to make the jump part an unpredictable innovation.

Solving the SDE gives the dynamics of the asset price under a risk-neutral probability measure

$$S(t) = S(0)exp\left\{\left(r - \lambda^* k^* - \frac{1}{2}\sigma^2\right) t + \sigma W^*(t)\right\} \prod_{i=1}^{N^*(t)} V_i^* \qquad (6)$$

Further explanation on how the Merton model and the Kou model, model the asset price will be given in their corresponding chapters.

5. The Merton Jump-Diffusion Model

I will first present the difference in the assumptions of the model compared to the Black Scholes model and then present the expression for the valuation of the options.

5.1 Assumptions

The model shares all the assumptions of the Black Scholes model, except for how the asset price is modeled (see Merton 1975, pp.1-5).

5.1.2 Modeling the asset price

This section is based on Matsuda (2004) and Merton (1975)

As with any jump-diffusion model, changes in the asset's price in the Merton model consists of a diffusion component modeled by a Brownian motion and a jump component modeled by a Poisson process. The asset price jumps are assumed to be independently and identically distributed.

The probability of a jump occurring during a time interval of length dt, can be expressed as

- Pr $\{the\ event\ does\ not\ occur\ in\ the\ time\ interval\ dt\} = 1 - \lambda^* dt$
- Pr $\{the\ event\ occurs\ once\ in\ the\ time\ intervall\ dt)\} = \lambda^* dt$
- Pr $\{the\ event\ occurs\ more\ than\ once\ in\ the\ time\ interval\ dt\} = 0$

The relative price jump size, or in other words the percentage change in the asset price caused by jumps, is

$$\frac{dS_t}{S(t-)} = \frac{y_t^* S_t - S_t}{S_t} = y_t^* - 1 \qquad (7)$$

$\log(V_i^*) - 1 = (y_t^* - 1)$, which is consistent with equation (5).

The absolute price jump size y_t^* is a nonnegative random variable drawn from a lognormal distribution, i.e. $\ln(y_t) \sim i.i.d.N(\mu, \delta^2)$. The density of the distribution is given by $f_{Y^*}(y^*) = \frac{1}{\delta\sqrt{2\pi}} exp\left\{-\frac{(y^*-\mu)^2}{2\delta}\right\}$, where μ and δ are the mean and standard deviation of y^*.

This in turn implies that $E[y_t^*] = e^{\mu+\frac{1}{2}\delta^2}$.

The relative price jump size $(y_t^* - 1)$ is log normally distributed with the mean $E[y_t^* - 1] = e^{\mu+\frac{1}{2}\delta^2} - 1 = k^*$.

The dynamics of the asset price, which incorporates the above properties, is given by equation (6).

5.2 The formula for option pricing

There are no closed form solutions for the option price in the Merton model. However, Merton developed a solution where he specified the distribution of Y_i as above, and with this, derived a solution for the price of the option.

Assuming that the jumps are log-normally distributed as above, the following expression for the price of a European call option is given in Merton (1975). For simplicity, the superscript * is dropped.

$$V_c(S,\tau) = \sum_{n=0}^{\infty}\left[\frac{e^{-\lambda'\tau}(\lambda'\tau)^n}{n!}\right] BS_C(S, \tau = T-t, K, v_n^2, r_n) \quad (8)$$

The n^{th} term corresponds to the scenario where n jumps occur during the life of the option.

$\lambda' = \lambda(1+k)$

$v_n^2 = \sigma^2 + \frac{n}{\tau}\delta^2$

$r_n = r - \lambda k + n\frac{\ln(1+k)}{\tau}$

δ^2 is the variance of the jump diffusion and k is the mean of the relative asset price jump size.

$\frac{e^{-\lambda' \tau}(\lambda' \tau)^n}{n!}$ is the Poisson probability that the asset price jumps n times during the interval of length τ.

Thus, the option price can be interpreted as the weighted average of the Black-Scholes price on the condition that underlying assets' price jumps n times during the life of the option, with the weights being the probability that the assets' price jumps n times during the life of the option.

While the MJD model is fairly straightforward and easy to implement, even in Excel, the Kou double-exponential model is more complex and will be described more in detail in the next chapter.

6. The Kou double-exponential Jump-Diffusion Model

This chapter is from Kou (2002) and Kou and Wang (2003)

6.1 Assumptions

As in the case of the Merton Jump-Diffusion Model, the only difference in the assumptions of the model compared to the Black Scholes model is the stochastic differential equation for the movement of the underlying asset's returns.

6.1.2 Modeling the asset price

As with the previous model, the stock price consists of two parts. The first part is a continuous part driven by a normal geometric Brownian motion and the second part is the jump part with a logarithm of jump size, which is double exponentially distributed. The number of jumps is determined by the event times of a Poisson process.

The expression for the stock price is given by equation (3), which is under the physical probability measure.

Given that $\log(V_i) = Y_i$ is double-exponentially distributed with the probability density function

$$f_Y(y) = \rho\eta_1 e^{-\eta_1} 1_{\{y\geq 0\}} + q\eta_2 e^{\eta_2 y} 1_{\{y<0\}}, \text{ where } \eta_1 > 1, \eta_2 > 0$$

Where $\rho, q \geq 0, p + q = 1$ are constants and represent the physical probabilities of upwards and downward jumps. In other words,

$$\log(V_i) = Y_i = \begin{cases} \xi^+ \ \textit{with probability } \rho \\ -\xi^- \ \textit{with probability } q \end{cases} \tag{9}$$

ξ^+ and ξ^- are exponential random variables which are equal in distribution with means $1/\eta_1$ and $1/\eta_2$. The means $1/\eta_1$ and $1/\eta_2$ are also constant in the model.

Further, the Brownian motion and the jump process are assumed to be one-dimensional.

Note that,

$$E(Y_i) = \frac{\rho}{\eta_1} - \frac{q}{\eta_2}$$

$$Var(Y_i) = \rho q(\frac{1}{\eta_1} + \frac{1}{\eta_2})^2 + (\frac{\rho}{\eta_1{}^2} + \frac{q}{\eta_2{}^2})$$

$$E(V_i) = E(e^{Y_i}) = q\frac{\eta_2}{\eta_2+1} + \rho\frac{\eta_1}{\eta_1+1}, \qquad \eta_1 > 1, \ \eta_2 > 0$$

The requirement $\eta_1 > 1$ is needed to ensure that $E(V) < \infty$ and $E(S(t)) < \infty$. This essentially means that the average upward jump cannot exceed 100%, which is quite reasonable, because this is not observed in the stock marked.

In the next section, the leptokurtic feature of the jump size distribution, which is inherited by the return distribution, will be illustrated.

6.2 The Leptokurtic Feature

Using equation (4), the return over a time interval dt is given by

$$\frac{dS(t)}{S(t-)} = \frac{S(t+dt)}{S(t-)} - 1$$

$$= \exp\left\{\left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma\big(W(t+dt) - W(t)\big) + \sum_{i=N(t)+1}^{N(t+dt)} Y_i\right\} - 1$$

where the summation over an empty set is taken to be zero. If the time interval Δt is small, as in the case of daily observations, the return can be approximated in distribution, ignoring the terms with orders higher than Δt and using the expansion $e^x \approx 1 + x + \frac{x^2}{2}$, by

$$\frac{dS(t)}{S(t-)} \approx \mu dt + \sigma Z\sqrt{dt} + B \cdot Y_i \tag{10}$$

where Z and B are standard normal and Bernoulli[7] random variables, respectively, with $P(B = 1) = \lambda dt$ and $P(B = 0) = 1 - \lambda \mathrm{dt}$, and Y_i is given by equation (9).

The density [8] g of the right-hand side of (10), being an approximation for the return $\mathrm{d}S(t)/S(t-)$, is plotted in figure (6.1) along with the normal density with the same mean and variance.

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[7] A Bernoulli variable is a variable that takes the value of 1 in case of success and 0 in case of failure.

[8] $g(x) = \frac{1-\lambda \mathrm{dt}}{\sigma\sqrt{\mathrm{dt}}}\varphi\left(\frac{x-\mu \mathrm{dt}}{\sigma\sqrt{\mathrm{dt}}}\right) + \lambda \mathrm{dt}\left\{\rho\eta_1 e^{(\sigma^2\eta_1{}^2\mathrm{dt})/2} e^{-(x-\mu \mathrm{dt})\eta_1}\Phi\left(\frac{x-\mu \mathrm{dt}-\sigma^2\eta_1 \mathrm{dt}}{\sigma\sqrt{\mathrm{dt}}}\right) + q\eta_2 e^{(\sigma^2\eta_2{}^2\mathrm{dt})/2} e^{(x-\mu \mathrm{dt})\eta_2} \times \Phi\left(-\frac{-x-\mu \mathrm{dt}+\sigma^2\eta_2 \mathrm{dt}}{\sigma\sqrt{\mathrm{dt}}}\right)\right\}$

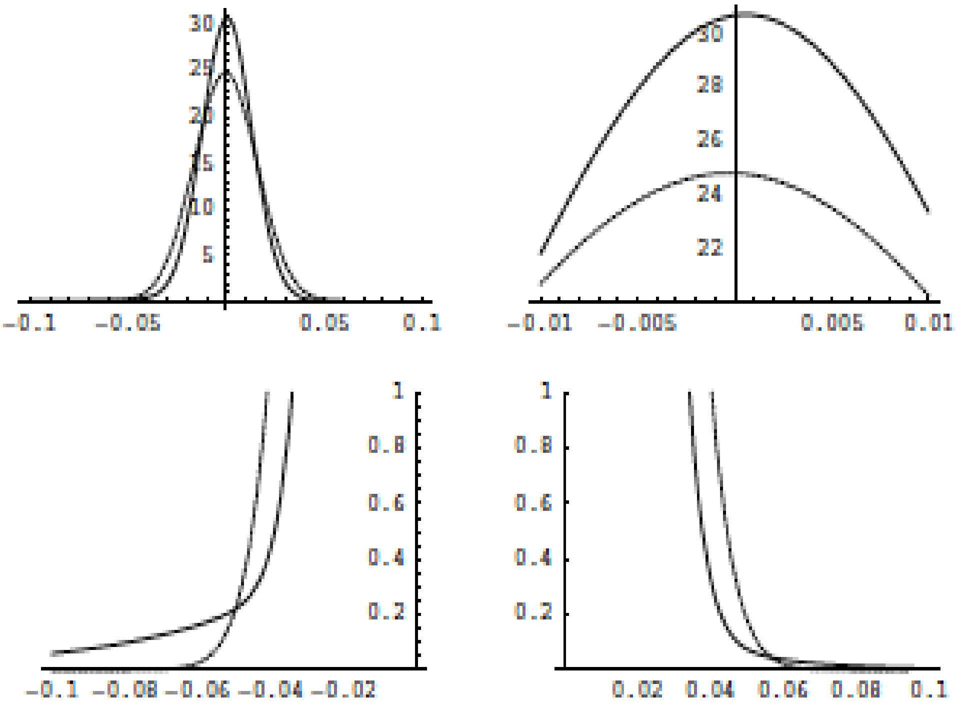

Figure (6.1). *The first panel compares the overall shapes of the density g and the normal density with the same mean and variance, the second one details the shapes around the peak area, and the last two show the left and right tails. The dotted line is used for the normal density, and the solid line is used for the model.*

The parameters are $dt = 1\ day = \frac{1}{250}\ year$*,* $\sigma = 20\%$ *per year,* $\mu = 15\%$ *per year,* $\lambda = 10$ *per year,* $\rho = 0.30$, $\frac{1}{\eta_1} = 2\%$, $\frac{1}{\eta_2} = 4\%$.

The leptokurtic feature is quite evident. The peak of the density *g* is about 31, whereas that of the normal density is about 25. The density *g* has heavier tails than the normal density, especially for the left tail, which could reach well below -10%, while the normal density is basically confined with -6%. An increase in either $1/\eta_i$ or λ would make the higher peaks and heavier tails even more pronounced.

6.3 SDE under risk-neutral probability

Kou and Wang (2003) describe how making use of the rational expectations argument with a HARA[9] type utility function for the representative agent, enables them to state the SDE under a risk-neutral probability measure. They follow the arguments of Lucas (1978) and N&L (1990).

The argument is that one can choose a particular risk-neutral measure, so that the equilibrium price of an option is given by the expectation under this risk-neutral measure of the discounted payoff. Under this risk neutral probability measure, the asset price, $S(t)$, still follows a double exponential jump-diffusion process. Here the SDE under the risk-neutral probability measure is given by equation (5), with ζ^* taking the place of k^*.

$$\zeta^* := E^*[V_i^*] - 1 = \frac{\rho^*\eta_1^*}{\eta_1^*-1} + \frac{q^*\eta_2^*}{\eta_2^*+1} - 1,$$

which is the expected relative jump size in the Kou model under a risk-neutral probability measure.

Because the focus is on option pricing, to simplify the notation, the superscript * is dropped when showing the expression for the European call price under a risk-neutral probability measure in the next section.

[9] In finance, economics and decision theory, hyperbolic absolute risk aversion (HARA) refers to a type of risk aversion that is particularly convenient to model mathematically and to obtailn empirical predictions from.

6.4 The formula for option pricing

Kou (2002) gives the expression for the price of a European call option under a risk-neutral probability as

$$V_c(0) = S(0)\psi\left(r + \frac{1}{2}\sigma^2 - \lambda\zeta, \sigma, \tilde{\lambda}, \widetilde{\eta_1}, \widetilde{\eta_2}, ; \log\left(\frac{K}{S(0)}\right), T \ \right) - Ke^{-rT}\psi\left(r - \frac{1}{2}\sigma^2 - \lambda\zeta, \sigma, \lambda, \rho, \eta_1, \eta_2; \log\left(\frac{K}{S(o)}\right), T\right) \quad (11)$$

Where:

$\tilde{\rho} = \frac{\rho}{1+\zeta} \cdot \frac{\eta_1}{\eta_1 - 1}, \ \widetilde{\eta_1} =, \eta_1 - 1$

$\widetilde{\eta_2} = \eta_2 + 1, \ \tilde{\lambda} = \lambda(\zeta + 1)$

As (11) shows, it resembles the Black-Scholes formula for a call option, with ψ taking the place of N. In this thesis expression (11) will be evaluated with an online calculator that makes use of the fast Fourier transform[10]. Tested against the results from Kou (2002), the calculator gave identical values.

Having described the models of interest, it is now time to look at the data set and check whether a normal distribution fits the returns of the stocks.

[10] Fast Fourier transforms are widely used for many applications in engineering, science and mathematics. A fast Fourier transform (FFT) is an algorithm to compute the discrete Fourier transform (DFT) and its inverse.

7. Description of the data set

The option with the highest open interest[11] as of May the 5th, 2014 is the call option on the Bank of America Corporation stock. As it is highly liquid, this is the reason behind looking at this option.

The data was downloaded from Bloomberg and ranges from 07/28/1980-04/30/2014.

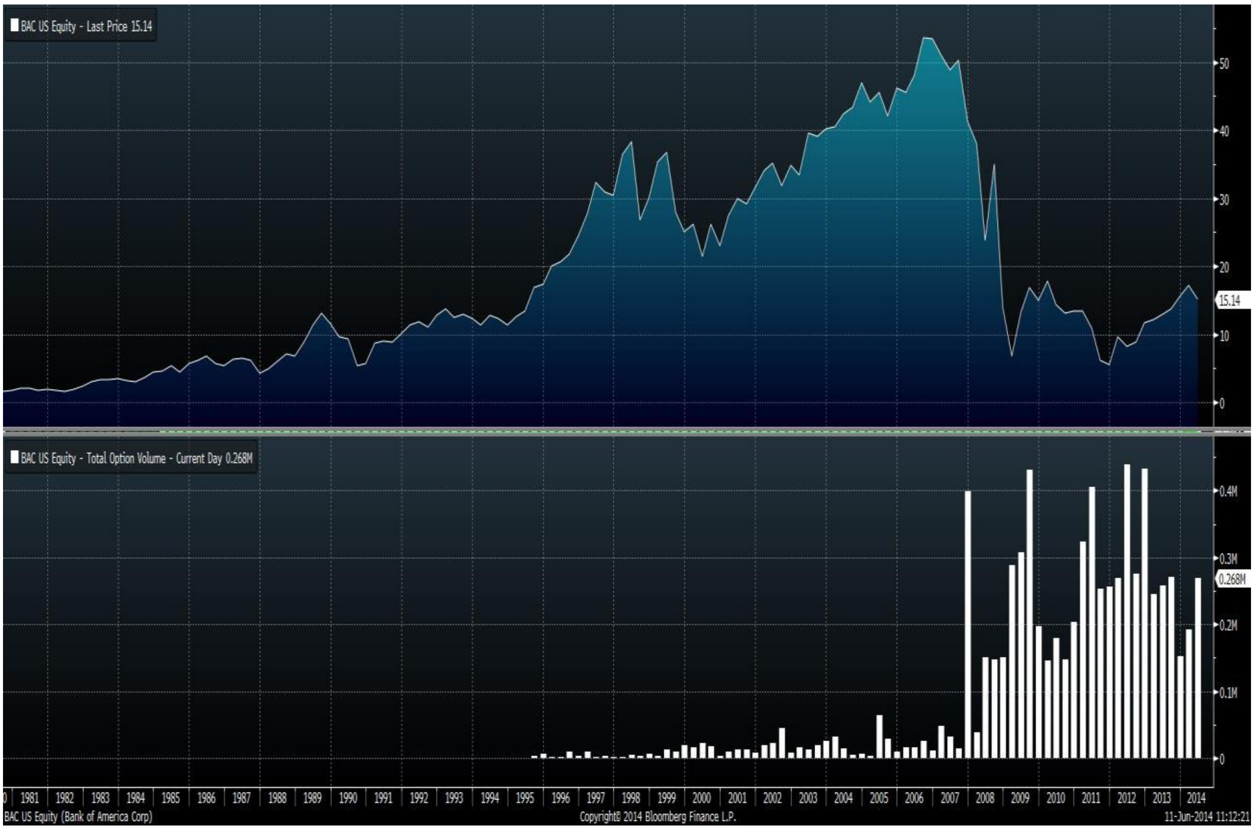

Figure 7.1. *Historical stock price of the BAC stock along with the total volume of options on the stock. Time period 07/28/1980 – 04/30/2014. The graph was downloaded from the Bloomberg database.*

Figure (7.1) shows that during the time period 1980-2014 there has been both steep increases and decreases in the stock price. This is a key contributor to the fat tails observed in the histogram below. The aim is to incorporate these fat tails in the pricing of the options in order to produce more accurate results.

[11] The total number of options that are not closed or delivered on a particular day.

Moreover, the volume of options on the stock has greatly increased since 2007. This might be due to the recent financial crisis, as options can be used to manage risk associated with stocks.

The option prices used for comparisons were downloaded at the Bloomberg-database at Norges Handelshøyskole May 5, 2014.

8. Checking for normality in the daily log-returns

There are several ways to check for normality in stock returns.

Ghasemi and Zahediasl (2011) suggest both visual inspections and numerical tests when checking for normality in a data set.

For visual inspections, a histogram of the daily log-returns along with a superimposed normal distribution, as well as a probability plot of the daily log-returns, provide a visual study and can serve as a starting point for the analysis.

For numerical tests, the authors argue that the Shapiro-Wilk test should be the numerical test of choice. However, ties[12] in the data set can affect the test. For this reason, the Skewness/Kurtosis test, which is not affected by ties, will be used.

8.1 Visual inspection

In this section the histogram and the probability plot of the daily log-returns from *07/28/1980 - 04/30/2014* will be presented.

[12] If there are identical values in the data, these are called ties.

8.1.1 Histogram of the daily log-returns

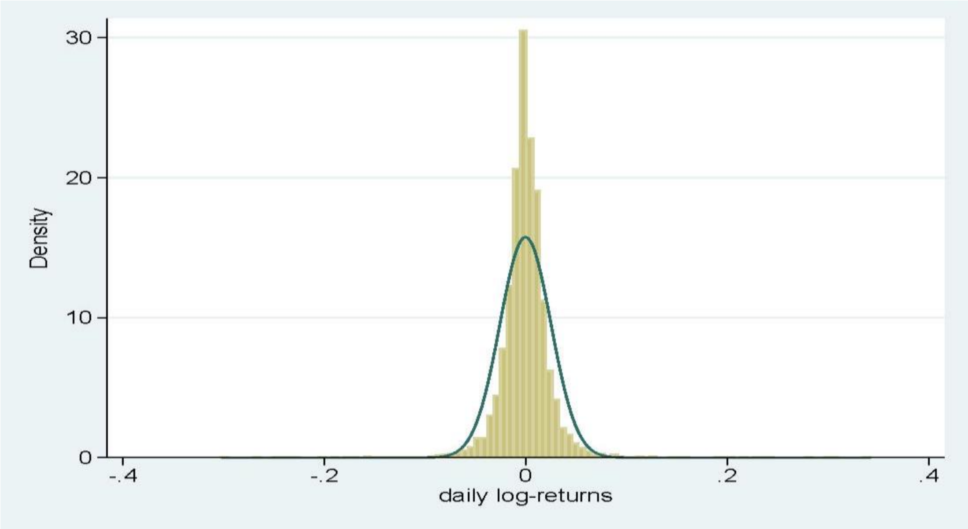

Figure 8.1. *Histogram of the daily log returns for the BAC stock along with a superimposed normal distribution. Time period 07/28/1980 – 04/30/2014.*

Looking at the figure, it is quite evident that a normal distribution does not fit the data very well. If the returns follow a geometric Brownian motion, the histogram should fit the blue line pretty well. As the figure shows, this is not the case and the assumption of geometric Brownian motion does not seem to hold.

The figure points to the existence of a significant number of large changes, especially apparent by the two tails. The perceived leptokurtosis is evident in the high peaks, reaffirming the remarks made about non-normally distributed data.

The high peaks indicate that there is a higher frequency of values near the mean than that of the normal distribution.

It is hard to see from the histogram, but as the summary statistics of the data in a later paragraph shows, the smallest return was -30%, while the highest was 34%. This is well outside of the range of the superimposed normal distribution.

8.1.2 Probability plot of the daily log-returns

The normal probability plot (Chamber 1983) is a graphical technique for assessing whether or not a data set is approximately normally distributed.

Data is plotted against a theoretical normal distribution in such a way that the data points should form an approximately straight line. Departures from this straight line indicate departure from normality.

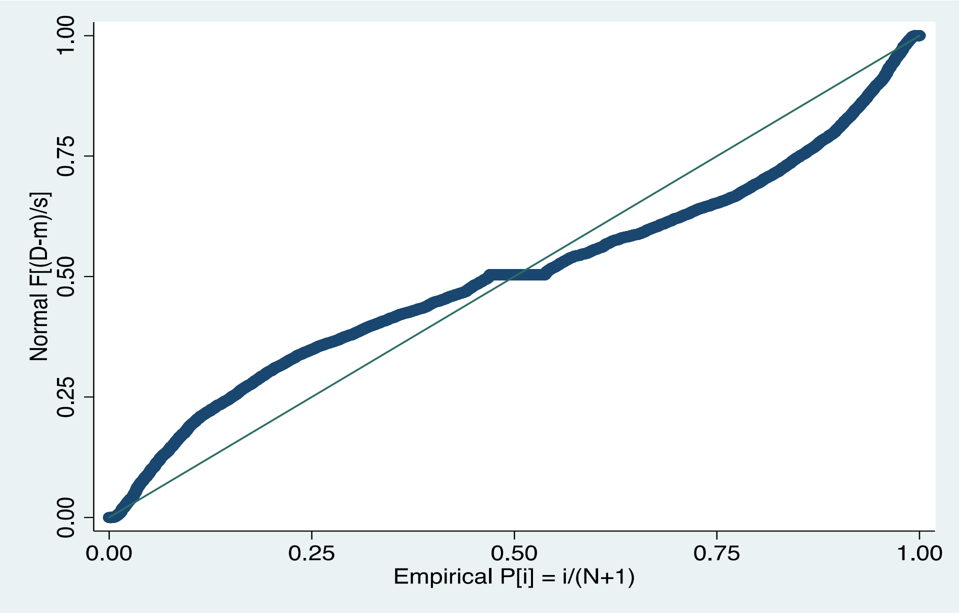

Figure 8.2. *Probability plot of the daily log-returns. Time period 07/28/1980 – 04/30/2014.*

If the data is normally distributed, the thick line should follow the normal distribution more closely. The "S-shape" indicates leptokurtosis in the data set.

8.2 The Skewness/Kurtosis test for normality

The Skewness/Kurtosis test is one of three general normality tests designed to detect all departures from normality. The normal distribution has a skewness of zero and a kurtosis of three. The test is based on the difference between the data`s skewness and zero and the data`s kurtosis and three. The test rejects the hypothesis of normality when the p-value is less than or equal to 0,05. Failing the normality test allows a statement with 95% confidence that the data does not fit the normal distribution.

Passing the normality test only allows a statement of the absence of departure from normality.

Below is the output from the Skewness/Kurtosis test run in Stata13

Skewness/Kurtosis tests for Normality

				joint	
Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	adj chi2(2)	Prob>chi2
BAC	8.5e+03	0.0000	0.0000	.	.

Table 8.1. *Stata output form the Skewness/kurtosis test.*

The test rejects the hypothesis of normality, reaffirming the remarks made from the histogram.

Thus, so far it seems that there is little evidence of normally distributed returns in the BAC stock.

8.4 Summary statistics

	BAC			
	Percentiles	Smallest		
1%	-.0666914	-.3020961		
5%	-.0334833	-.2698774		
10%	-.0224066	-.2447746	Obs	8512
25%	-.0101101	-.2406014	Sum of Wgt.	8512
50%	0		Mean	-.0002423
		Largest	Std. Dev.	.0253163
75%	.0096619	.2789156		
90%	.021819	.2977577	Variance	.0006409
95%	.0323478	.3041631	Skewness	.347219
99%	.0648486	.3420588	Kurtosis	31.36884

Table 8.2. *Summary statistics for the BAC stock for the entire data set. Output from Stata13*

As the table shows, the returns show a fairly high standard deviation, but not more than is expected from a stock. The daily volatility corresponds to a yearly volatility of 39,95%[13].

It also shows some evidence of asymmetry by the presence of positive skewness.

The large value of kurtosis shows that the series displays evidence of fat tails and acute peaks.

[13] $\sigma_{annual} = \sigma_{daily} \times \sqrt{252}$

8.5 Implied volatility smile from the observed option

As mentioned in the introduction, the observed volatility in the option market is not constant; the observed volatility plotted against the strike price looks rather more like a smile.

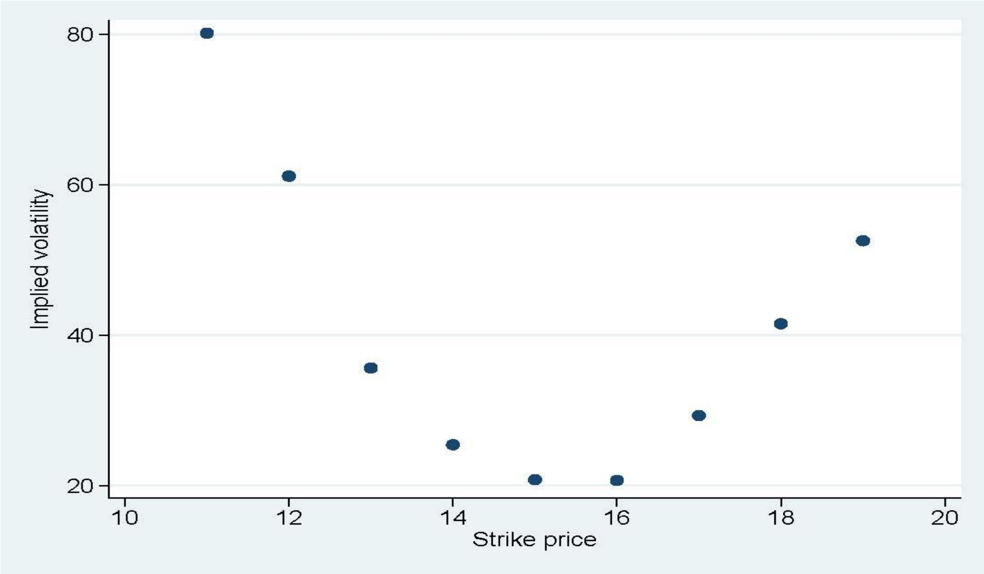

Figure 8.3. *Implied volatility for the BAC call option, with strikes ranging from $11-$19, with* $\boldsymbol{S_0} = \$\mathbf{15,25}, \boldsymbol{rf} = \mathbf{0,13}\%$, $\boldsymbol{time\ to\ maturity} = \mathbf{12}\ \boldsymbol{days}$. *The figure was made in Stata13 by plotting the implied volatilities of the option against their corresponding strike prices. All scatter- and line plots were made in Stata13*

As the figure shows, the scatter plot of the implied volatility for different strike prices resembles a smile.

The smile indicates that deep out-of-the-money-options[14] and deep-in-the-money-options[15] are more volatile than at-the-money-options[16].

This is a key feature of option prices and should be taken into account when trying to compute the most accurate option prices. It should be noted that the smile is skewed. In-the-money-options have higher implied volatilities than out-of-the-money-options.

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[14] For a long call this indicates $K > S_t$
[15] For a long call this indicates $K < S_t$
[16] For a long call this indicates $K = S_t$

8.6 Volatility surface of the observed implied volatilities

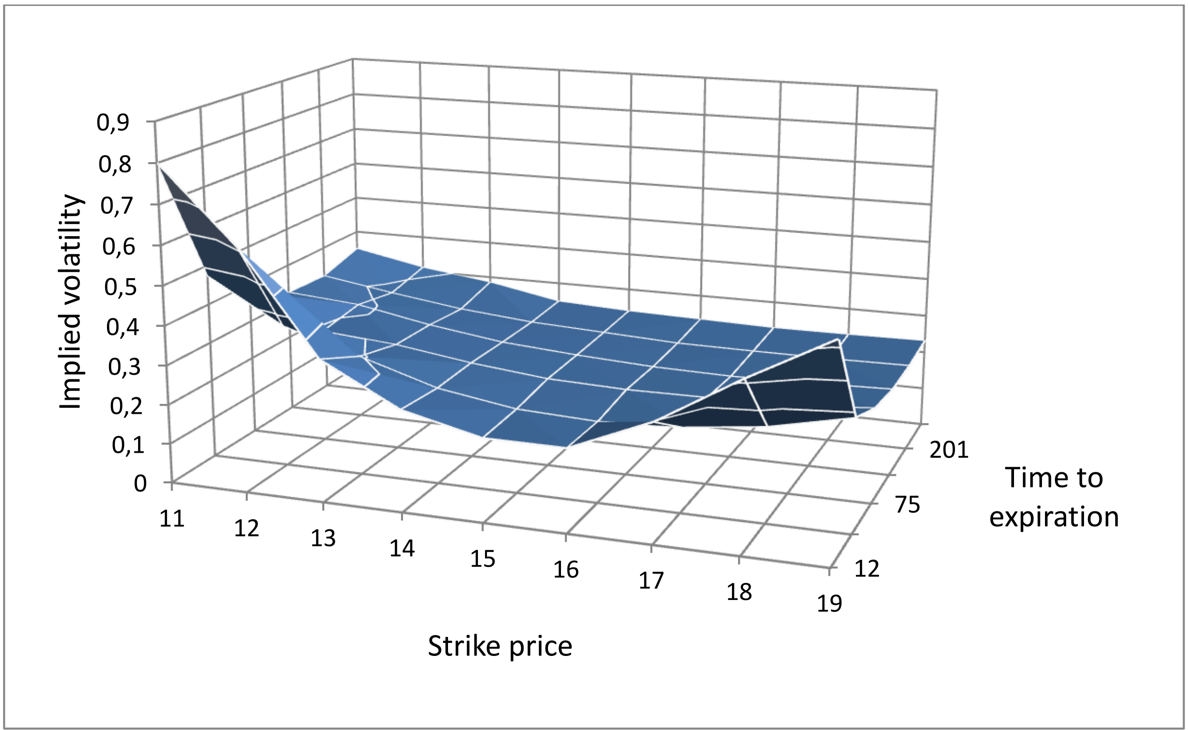

Figure 8.4. *Volatility surface of the BAC call option. The figure was made in Excel 2010 with the 3D-surface graph function.*

The figure shows that the observed volatility surface flattens out as time to maturity increases. This corresponds to the findings of Tehranchi (2010). The author found that for longer maturities the surface tends to flatten in a rather precise manner.

Both the Merton jump-diffusion model and the Kou double-exponential jump-diffusion model are able to capture the leptokurtic feature and the volatility smile, while the Black Scholes model is not able to capture any of the two. This should make the jump-diffusion models more able to compute accurate option prices compared to the Black and Scholes-model.

9. Comparing Black and Scholes values with market values

For different strike prices and maturities, the market values of the BAC call option and the values from the Black Scholes model will be presented in this chapter. A mean squared error[17] will be computed to measure the accuracy of the model compared to the observed prices. For better visual comparisons, ratios of the model price divided by the market price will be presented.

The volatility used in the model is the 1 year historical volatility[18], assumed constant at 22,025%, which is the same volatility used in the Bloomberg database. The risk-free rate is stated in Bloomberg along with the option prices, and varies with the different maturities, so this rate will be used for the calculations. The spot price of the stock is the stock price as of May the 5th, 2014, 10:33, quoted at $15,25. The time measure is calendar days, as this corresponds to the input parameters of the option calculator for theoretical prices in the Bloomberg database. This time measure will be used for all the models.

The prices are for options of the American type; however there were no implied cumulative dividends for any of the maturities, so the American and European call option prices should be the same, as stated in section (3.3). This means that even though the models in this thesis are for European options, they should give correct prices for the American options under investigation.

[17] In statistics, the mean squared error (MSE) of an estimator measures the average of the squares of the ”errors”, that is, the difference between the estimator and what is estimated.

[18] Calculated as the standard deviation of the daily log returns from 04/29/13-04/29/14 times $\sqrt{252}$

9.1 Graphical comparison of prices, ratios and MSE

In this section a graphical comparison of the prices as well as a visual study of the ratios and MSE will be presented.

9.1.1 Comparison of prices

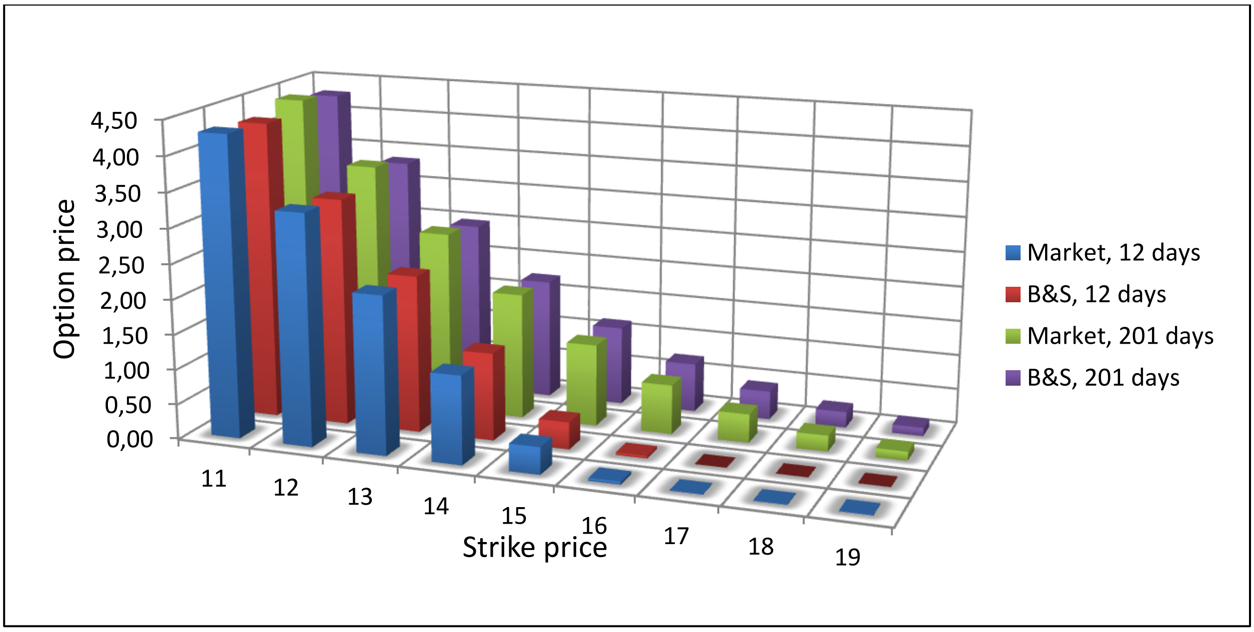

Figure 9.1. *Comparison between the market prices and the prices from the Black Scholes model. Time to maturity= 12 and 201 days*

As the figure shows, and as expected from the empirical evidence of the volatility smile, the model performs well for near at-the-money-options, but moving away-from-the-money, the Black Scholes model undervalues the options. The model fails to capture the increased volatility as the exercise price moves away from the spot price of the stock.

9.1.2 Comparison of ratios

For a better visual comparison between the models, a ratio where the model price is divided by the corresponding market price is presented in the figure below. If the ratio = 1, the prices are identical, a ratio > 1 indicates overestimation while a ratio < 1 indicates underestimation compared to market prices.

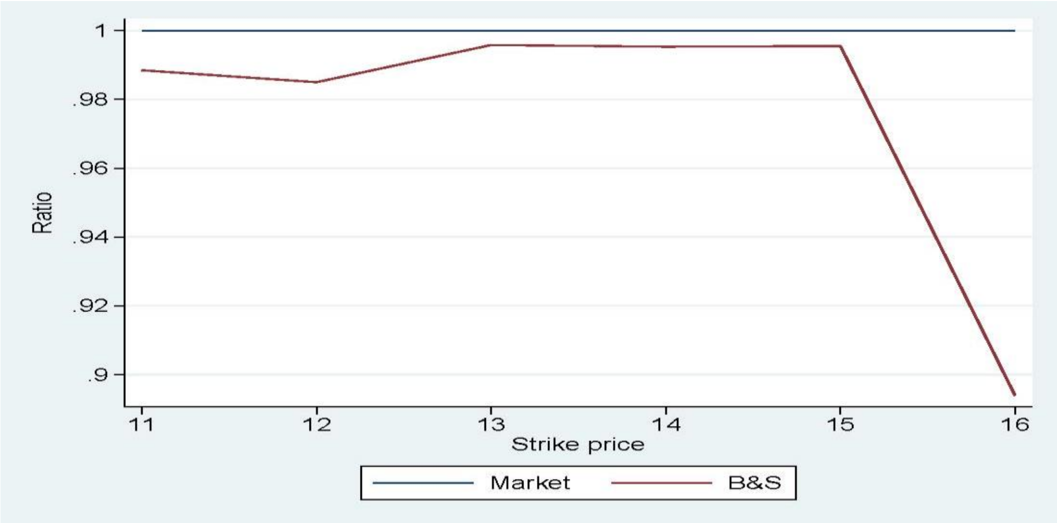

Figure 9.2. *The figure shows the ratios calculated as* $\frac{\textit{Model price}}{\textit{Market price}}$. *The ratios are for a maturity of 12 days.*

The strike range 17-19 was held outside of the graph because of extreme values. Instead the ratios are shown in table (9.1)

Strike	Ratio
17	0,06852940000
18	0,00027545600
19	0,00000027087

Table 9.1. *Ratios, strike range 17-19.*

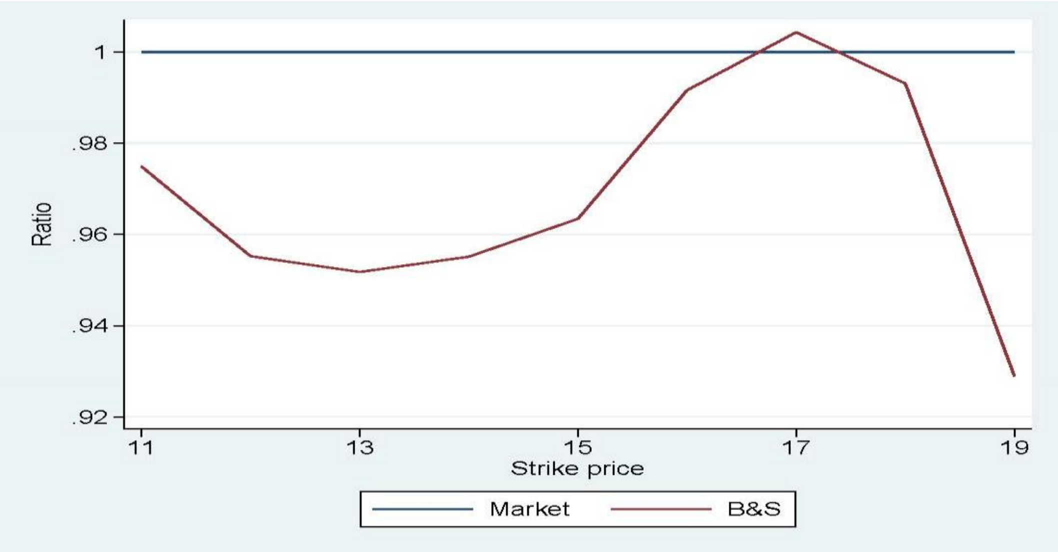

Figure 9.3. *Ratios for a maturity of 201 days.*

The ratios show that the Black Scholes model consistently underestimates the value of the option for maturities of 12 and 201 days, the exception being for a strike price of 17 and a maturity of 201 days.

9.1.3 Visual study of the MSE

Strike	Market prices	B&S	squared errors
11	**4,300000191**	**4,250476656**	**0,00245258**
12	**3,299999952**	**3,250519989**	**0,002448267**
13	**2,25999999**	**2,250567924**	**8,89639E-05**
14	**1,25999999**	**1,25412416**	**3,45254E-05**
15	**0,389999986**	**0,388259114**	**3,03063E-06**
16	**0,039999999**	**0,035756852**	**1,80043E-05**
17	**0,01**	**0,000685294**	**8,67638E-05**
18	**0,01**	**2,75456E-06**	**9,99449E-05**
19	**0,01**	**2,70873E-09**	**9,99999E-05**
MSE			**0,000592453**

Table 9.2. *MSE of the Black Scholes model for the entire strike range for a maturity of 12 days.*

For exercise prices of 13, 14 and $15 the observed prices and the theoretical prices are nearly identical. Moving out-of-the-money, the model fails to reproduce the market prices because of the increased implied volatility.

Just looking at the squared errors, it is tempting to say that the model performs well for out-of-the-money-options. However, the reason for the small squared errors is that the prices are very low, making the values of the differences between the observed prices and the theoretical prices smaller than with higher prices. Figure (9.2), (9.3) and table (9.1) helps remedy this problem.

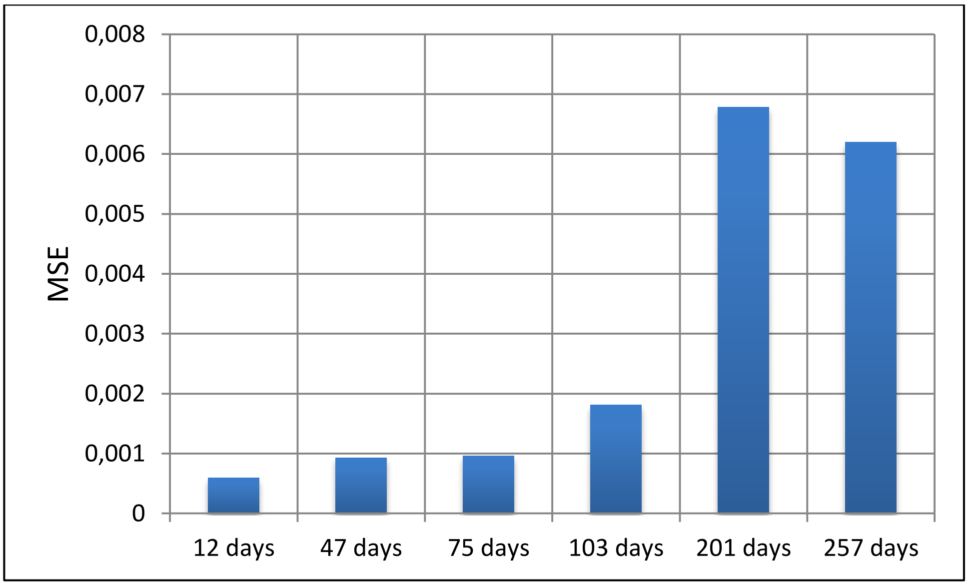

Figure 9.4. *Mean squared error for the different maturities.*

An increase in time to expiration seems to give a higher estimate of the MSE, indicating that as the time to expiration increases, the accuracy of the model decreases.

To give a better view of how the model performs under different strike prices, I will split the MSE into MSE for options in-the-money, out-of-the-money and at-the-money.

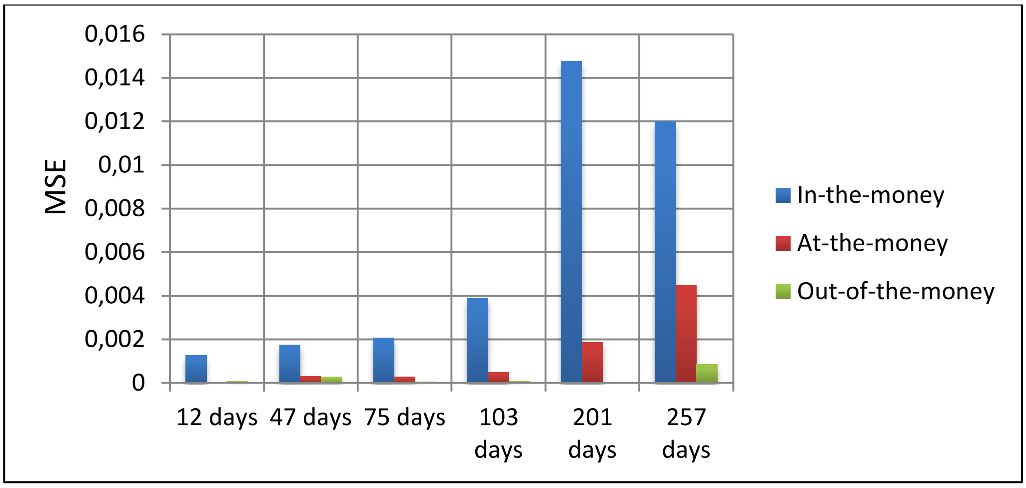

Figure 9.5. *MSE split into in-the-money, at-the-money and out-of-the-money. It should be noted that at-the-money-options are not strictly at-the-money, but close. The spot price is $15,25 while the strike price is $15.*

Figure (9.5) confirms the notion about bias in the MSE. The MSE for out-of-the-money-options is lower for shorter times to expirations than for longer expirations, even though the model actually performs better for out-of-the-money options for longer expirations. This can be confirmed by looking at the ratios presented in figures (9.2) and (9.3).

Because of the fact that the surface of the observed implied volatilities is skewed and flattens out as time to expiration, the model is more able to produce accurate results for out-of-the-money-options as time to expiration increases, whereas for in-the-money-options the model struggles for the entire maturity range.

A table of the squared errors for different strike prices and maturities is shown in the appendix.

9.2 Volatility surface from the implied volatilities from the theoretical prices

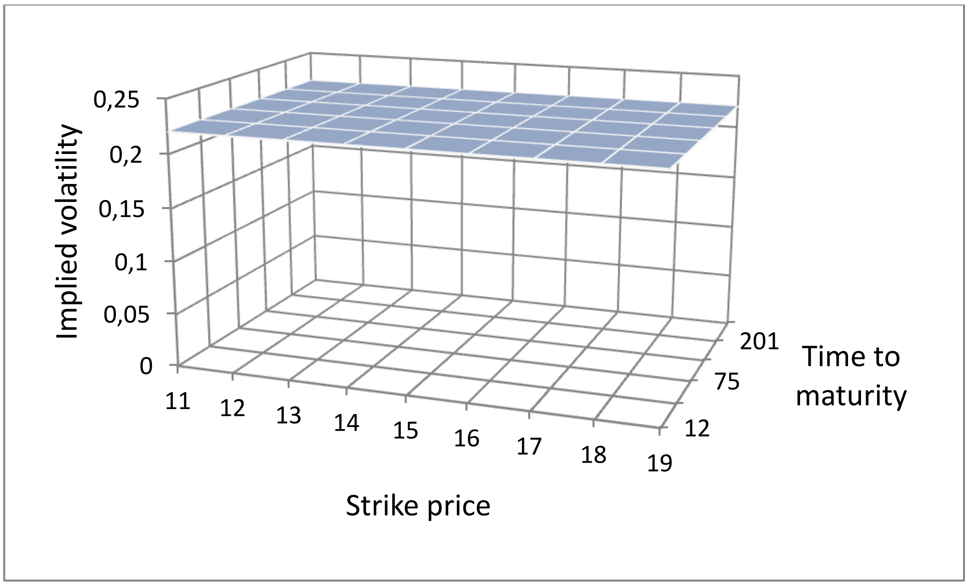

Figure 9.6. *Volatility surface of the implied volatilities from the Black Scholes model.*

Comparing figure (9.6) to figure (8.4), visual inspection reveals a big difference in the surfaces. While figure (8.4) clearly shows that for different strike prices and maturities, the volatility changes, the surface of the implied volatilities of the Black Scholes prices is flat. Keeping in mind that the model assumes constant volatility, this is no surprise.

9.3 Density of the stock returns used in the model

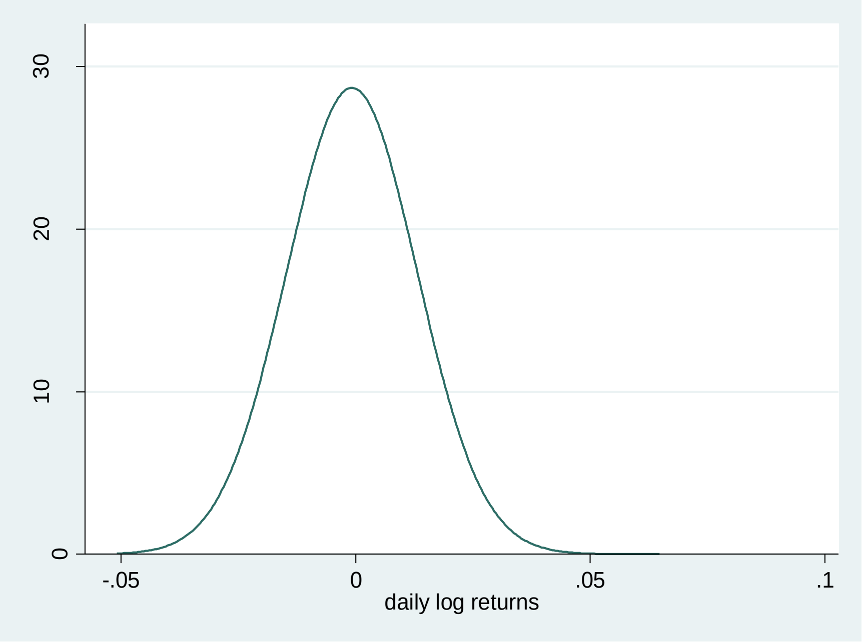

Figure 9.7. *Implied density plot used in the Black Scholes model. The plot is calculated with the same mean and standard deviation as the 1-year historical daily log-returns.*

The plot indicates that the returns are lognormal and follows equation (3), given $dN(t) = 0$

As illustrated, the real log-returns have higher peaks and longer tails than the implied density used in the model. This could also be a factor in reducing the accuracy of the model.

9.4 Summary

To summarize, the Black and Scholes model is accurate for at-the-money-options with short time to maturity, and fairly accurate for out-of-the-money options with longer time to expiration. Despite it not being able to capture the implied volatility smile or the leptokurtic feature apparent in the historical returns of the BAC stock, the model performs well.

10. Market prices versus option prices via the Merton Jump-diffusion model

After showing that the Black Scholes model can produce accurate results, despite the mentioned shortcomings, the simple Jump-diffusion model should be able to produce more accurate values.

As in the case of the Black Scholes model, the volatility used is the historical volatility. The risk-free rate is the same as before. However, within the Merton model, the volatility and the risk-free rate will vary with the number of jumps and their magnitudes.

Matlab 2009 was used for the calculations of the option prices. The code can be found in the appendix.

10.1 Calibrating the model

There are more ways than one to calibrate the model. Ramezani and Zeng (2006) use maximum likelihood estimation to obtain parameter estimates for both the Merton Jump-Diffusion model and the Kou double-exponential Jump-Diffusion model. The details on maximum likelihood estimation for jump-diffusion processes can be found in Sorensen (1988).

Other methods for the estimation of jump-diffusion processes, including the generalized method of moments, the simulated moment estimation, and MCMC methods, among others can be found in Aït-Sahalia and Hansen (2004).

The above methods are computationally extensive and will not be utilized in this thesis. Instead, I will suggest a method where jumps are defined as a percentage increase or decrease in the daily log-returns. The method is described in the next section.

10.2 Determining the number of jumps and their magnitudes

To calculate the number of jumps, different limits of the returns compared to the average return will be set. If the average return of the period is 0,01%, the limit could be set at plus, minus 7% daily logarithmic return.

After determining the limits, the mean size of the jumps and the standard deviation of the jumps can be calculated.

Because there is some degree of freedom in choosing the number of jumps and their means, the number of jumps, their means and standard deviations will be calibrated with the aim of minimizing the mean squared error for the entire range of strike prices and maturities.

Two different time periods will be used in the calculation. A 1-year period, ranging from 04/29/13-04/29/14 and the entire data set, ranging from 07/28/1980-04/30/2014.

10.2.1 Different limits and time periods for calculation of the jumps

Limit	λ	μ	δ^2	δ
+/- 4%	**2**	**0,013781**	**0,003347**	**0,057857**
+/- 3,5%	3	-0,00774	0,002659	0,05157
+/- 3%	9	-0,00995	0,001476	0,038417
+/- 2,5%	20	-0,01418	0,000912	0,030203
+/- 2%	**39**	**-0,00346**	**0,000796**	**0,02822**

Table 10. *The table shows how the number of jumps, the mean of the jumps and the volatility of the jumps vary with different limits of the returns. Time period 04/29/13 – 04/29/14.*

Limit	λ	μ	δ^2	δ
+/- 10%	1,98	0,007005	0,030911	0,175815
+/- 8%	3,26	0,00358087	0,021816689	0,147704734
+/- 6%	6,33	-0,001779052	0,013507971	0,116223797
+/- 4%	**16,67**	**-0,000926954**	**0,006578313**	**0,0811068**
+/- 2%	**59,1**	**-0,000775789**	**0,002423898**	**0,049233099**

Table 10.2. *The table shows how the values change when expanding the time period to include the whole data set.*

Including the whole data set means including some big historical events in the stock market. Two events that should be mentioned includes the stock market crash in 1987, also known as Black Monday, and the recent financial crisis. Including these time periods means including more negative returns to the calculations of the jumps and their magnitudes.

In order to get the same number of jumps as in the time period 04/29/13 – 04/29/14, the limit has to be increased. A limit of 10% gives the same number of jumps as the 4% limit for the 1-year period. However, the mean of the jumps is noticeably smaller, while the standard deviation has increased.

When calibrating the model, the jumps calculated with the whole data set consistently yielded higher MSE than the jumps from the 1-year period. The number of jumps that gave the least mean squared errors was 2, with mean 0,013781 and a standard deviation of 0,057857.

The jump intensity and their corresponding means and standard deviations were based on the above results. Further calibration, with the aim of reducing the MSE, gave a final jump intensity of 2 jumps per year, with a standard deviation of 3% and a mean of 0,1%.

The mean of the jumps might look low, but as it is an average of both positive and negative jumps, and the number of positive and negative jumps is close, they almost cancel each other out.

10.2 Graphical comparison of prices, ratios and MSE

In this section a graphical comparison of the prices, ratios and MSE will be presented.

10.2.1 Comparison of prices

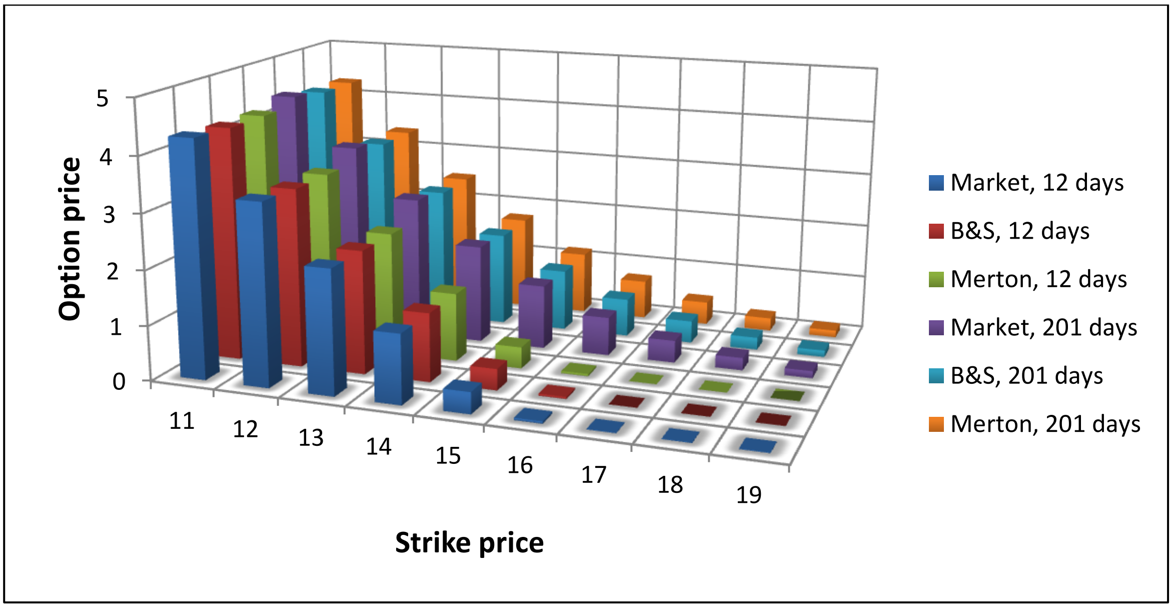

Figure 10.1. *Prices form the Merton-model compared to the observed market prices and the Black-Scholes prices. Time to maturity = 12 and 201 days.*

The differences between the market prices and the Merton prices are very similar to the results from chapter 9.

In order to keep the Merton model from overpricing the options by too high a degree, the jump intensity was set at 2 jumps per year. Because of the low intensity and the low mean and standard deviation of the jumps, the prices of the Black Scholes model and the Merton model are very similar.

10.2.2 Comparison of ratios

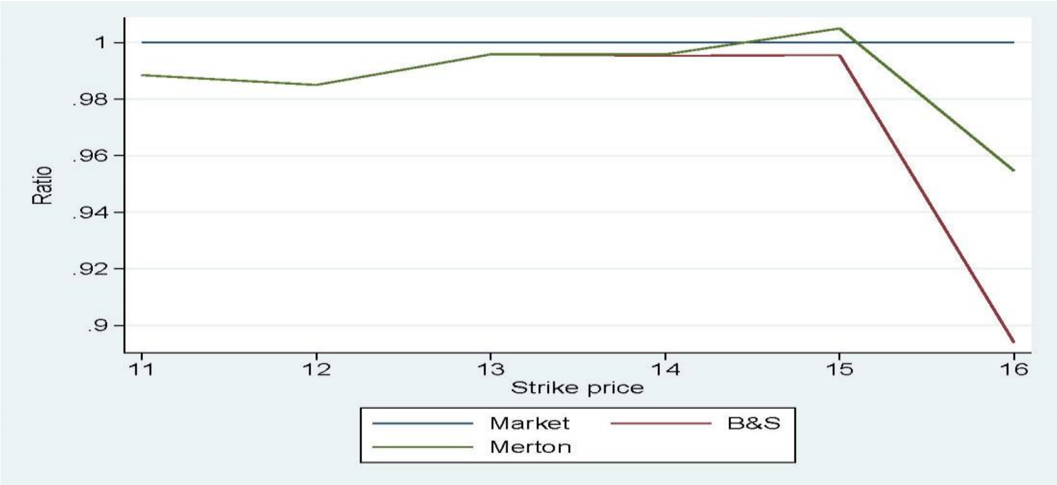

Figure 10.2. *Ratios for the B&S model and the Merton model. Time to maturity = 12 days.*

	Ratio	
Strike	**B&S**	**Merton**
17	0,068529400	0,09583082
18	0,000275456	0,00114749
19	0,000000271	0,00001356

Table 10.1. *Ratios for strike range 17-19.*

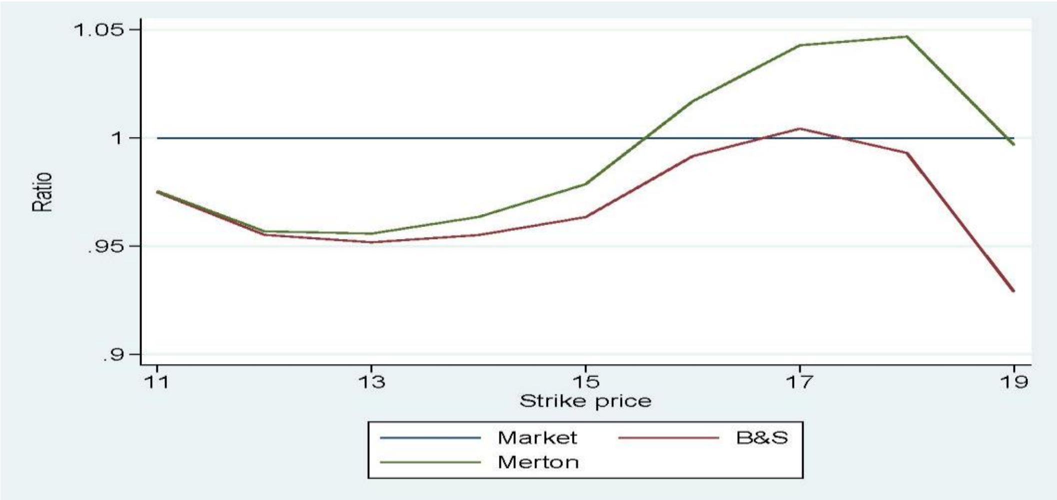

Figure 10.3. *Ratios for time to maturity = 201 days.*

Compared to the Black Scholes model, the Merton Jump-Diffusion model overestimates at-the-money, while underestimating both in- and out-of-the-money for a maturity of 12 days. When increasing the time to maturity the model still underestimates while in-the-money, but now also at-the-money. Moving out-of-the-money, the model overestimates the options value.

10.2.3 Comparison of MSE

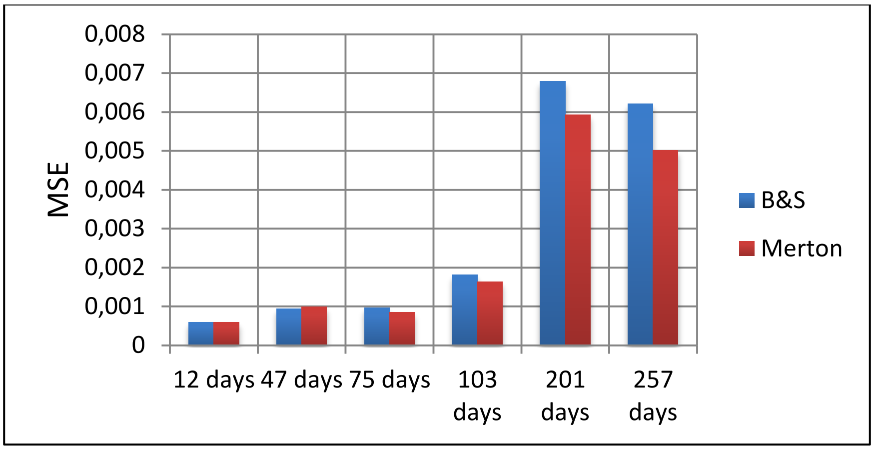

Figure 10.4. *MSE for the* *entire strike range* *for different maturities for the MJD model, compared with the MSE from the Black and Scholes model.*

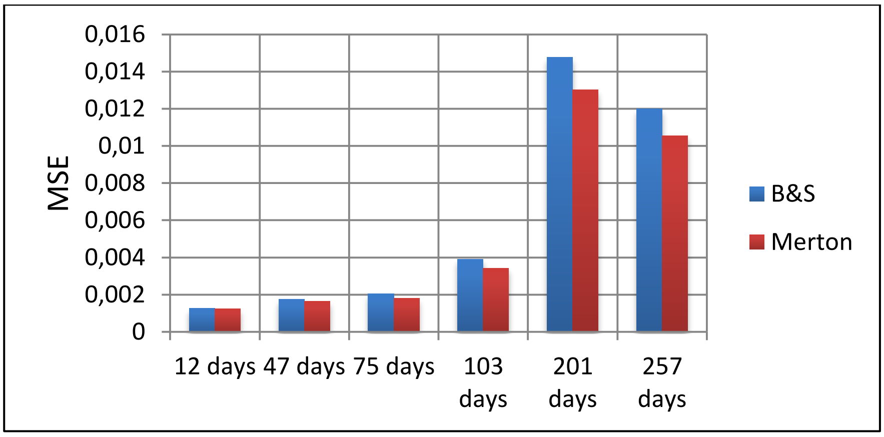

Figure 10.5. *MSE for* *in-the-money-options.*

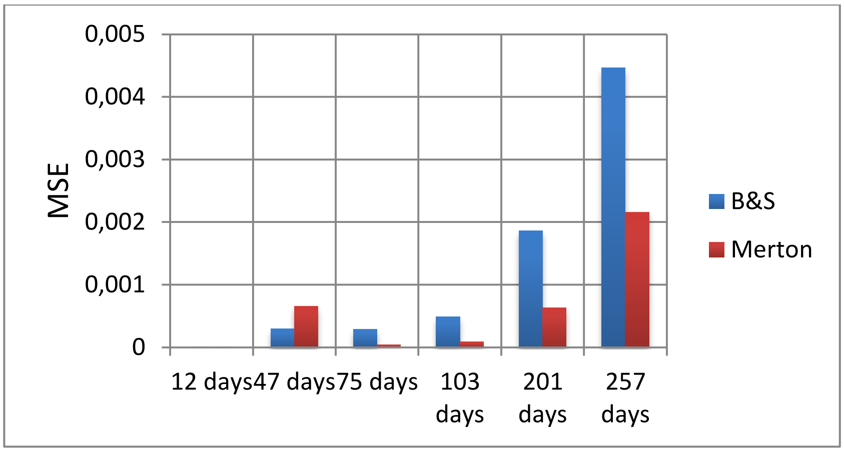

Figure 10.6. *MSE for <u>at-the-money-options</u>.*

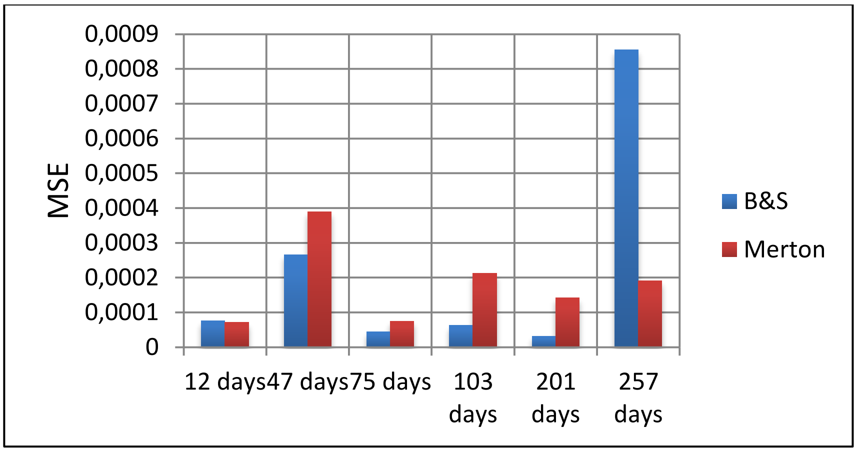

Figure 10.7. *MSE for <u>out-of-the-money-options</u>.*

Figure (10.4) shows that the Merton model performs the best overall when looking at the MSE for the entire range of maturities and strike prices, except for a maturity of 47 days.

However, when the MSE is split into in-the-money-, at-the-money- and out-of-the-money-options, the results differ.

For in-the-money-options, the Merton model has the lowest MSE for all the different maturities. The Merton model is more able to capture the increase in the implied volatility as the strike price decreases.

For at-the-money-options, the Black Scholes model performs the best for the shorter maturities, 12 and 47 days. When the maturities increase, the Merton model again gives the most accuracy. This is because as the time to expiration increases, so does the implied volatility of the at-the-money-options, as shown by table (A2.1 and A2.2) in the appendix.

For out-of-the-money-options, the implied volatility is fairly flat for the different maturities. This makes the Black Scholes model the most accurate when comparing its corresponding MSE and ratios with the Merton model. Because of the limited increase in implied volatility as the option moves out-of-the-money, the Merton model overestimates the values.

10.3 Volatility surface of the implied volatilities from the theoretical prices

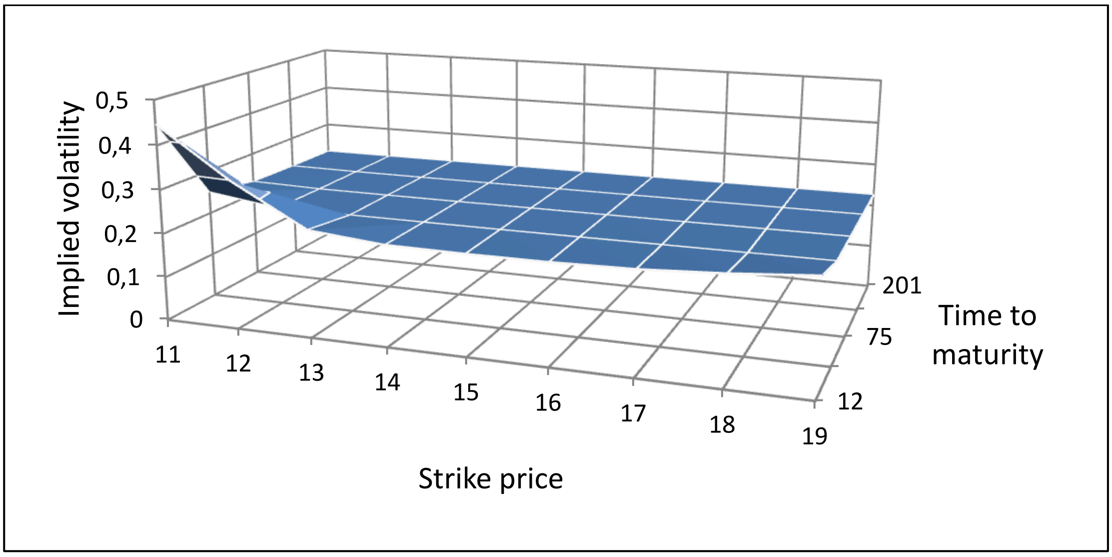

Figure 10.8. *Volatility surface of the implied volatilities of the prices from the Merton model.*

Comparing figure (10.8) to figure (9.6) there is a clear difference. While the volatility surface of the Black Scholes model is flat, the surface from the Merton model is somewhat familiar to the surface from the observed implied volatilities.

The peak of the implied volatility of the model is 45,1%, whereas the real implied volatilities peak at 80,17% for a maturity of 12 days. The low peak is due to the low impact of the jumps.

Comparing figure (10.8) with figure (8.4), the surface from the Merton model flattens out much quicker than the observed surface. Because the jump intensity and impact is set at low levels, the models ability to incorporate more risk in the valuation is reduced.

Still, the model shows that it is able to incorporate the volatility smile compared to the Black Scholes model, where the surface is flat.

To show that the model is able to capture the leptokurtic feature apparent in the daily log-returns, the stock price will be simulated using a downloaded Excel spreadsheet that is based on hull (2000).

The stochastic differential equation used to model the stock price is the same as equation (3), given $\log(V_i) = Y_i$ and Y_i is normally distributed.

Input parameters:

$\mu = 0{,}001$, $\sigma = 20{,}025\%$, $\lambda = 2$, $k = 0{,}3456\%$, $\delta = 3\%$, $\mathrm{T} - \mathrm{t} = 1$,
$S_0 = 12{,}38$, which is the stock price as of 04/29/2013.

The simulated returns are compared with the observed historical daily log-returns via histograms of their corresponding densities.

10.3 Density of the asset returns simulated with jump-diffusion compared to the density of the observed returns

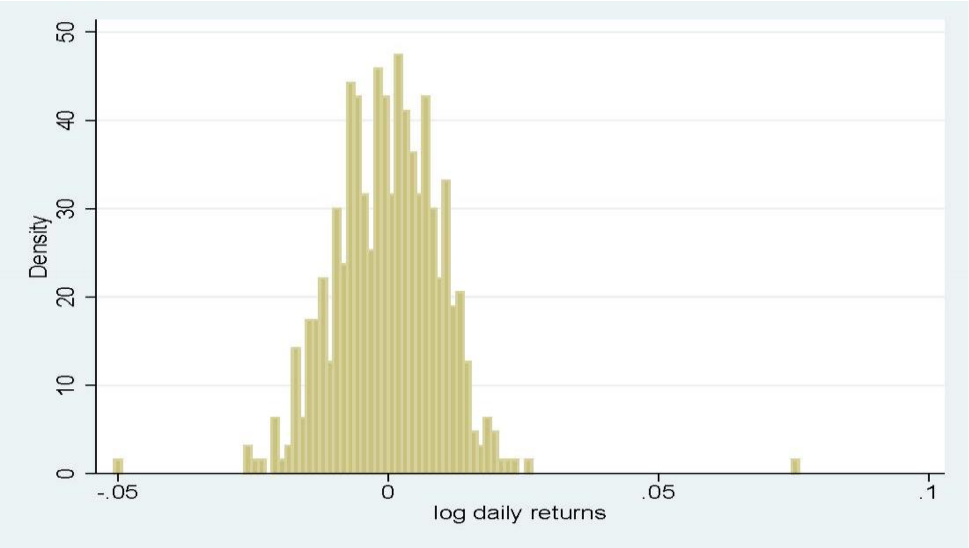

Figure 10.8. *Density of the* *simulated* *log-returns used in the Merton model.*

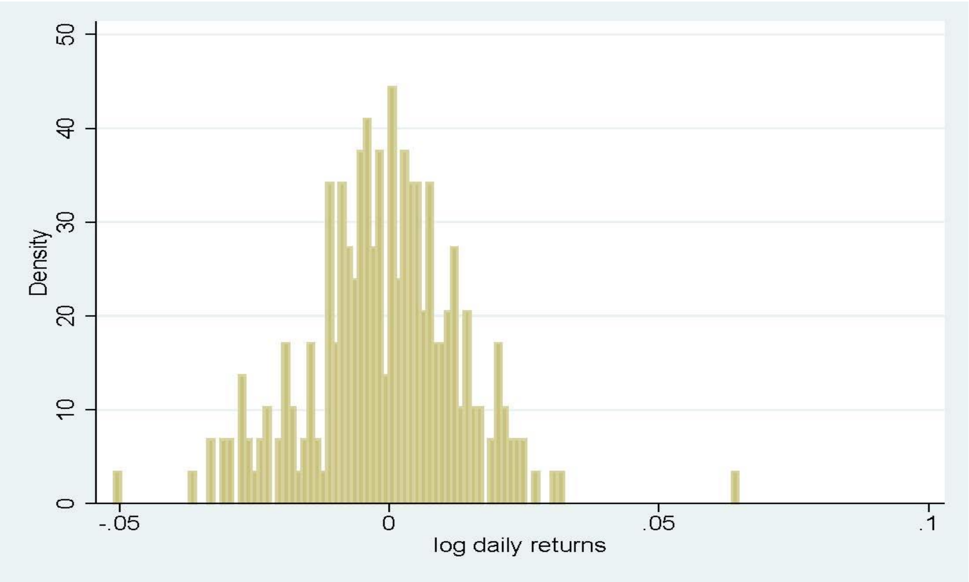

Figure 10.9. *Density of the* *observed* *1-year historical daily log-returns.*

As the two figures show, the density from the simulated log-returns from the Merton model is similar to the density of the real returns. Both histograms peak at a density of 45-50 and contain “extreme” negative and positive returns.

10.4 Summary

To summarize, visual study reveals that the model does not perform as well as one might have thought. While the Merton model consistently performs the best when looking at the MSE for the entire strike range, the results are not so convincing when dividing the MSE into different groups of moneyness.

Even though the model is able to reproduce the volatility smile of the observed market prices to some extent and the leptokurtic feature in the stock’s log-returns, the results from the implementation of the model are not very different from the Black Scholes model. The fact that the jumps are assumed log-normal might be reducing the accuracy of the model. There is no distinction between negative and positive jumps in the Merton model; only a mean of the jumps is made use of.

The next step is to investigate whether the Kou-model, with double exponential distributed jumps, where the jump component is split into positive and negative jumps, performs better than the previous two models.

11. Market prices versus option prices via the Kou-model

To incorporate the Kou-model, additional parameters have to be determined.

These parameters include the mean of the positive jump size, the mean of the negative jump size and the probability of a positive jump.

The advantage of the Kou-model is that it allows you to divide the jump component into positive and negative jumps. This gives you more freedom when calibrating the model to fit the observed market prices.

11.1 Calibrating the model

When calibrating the Kou model the problem was making sure the model did not overprice the options too much when moving out of the money. Because the implied volatility of the out-of-the-money options shows no significant increase, the jump-diffusion models tend to overestimate the values.

The choice of parameters was a case of trial and error. The goal was to determine the jumps and their magnitudes in such a way that they consistently performed better than the previous models for the entire strike range and maturities.

Increasing the average negative jump size, while keeping the positive jump size constant, gave better results when moving out-of-the-money, but still not as good as the Black Scholes model.

The probability of an upward jump was set at 40%, as this gave the most accurate results and the jump intensity was set at 1 jump per year for the same reason. The average positive jump size was set at 2%, while the average negative jump size was set at 3,33%.

The additional parameters of the model are as follows:

$\rho = 0.4, \frac{1}{\eta_1} = 2\%, \ \frac{1}{\eta_2} = 3{,}33\%$

According to the online calculator

Ita_1 corresponds to η_1, which is 1/0,02 = 50

Ita_2 corresponds to η_2, which is 1/0,0333 = 30,03003

11.2 Graphical comparison of prices, ratios and MSE

11.2.1 Comparison of prices

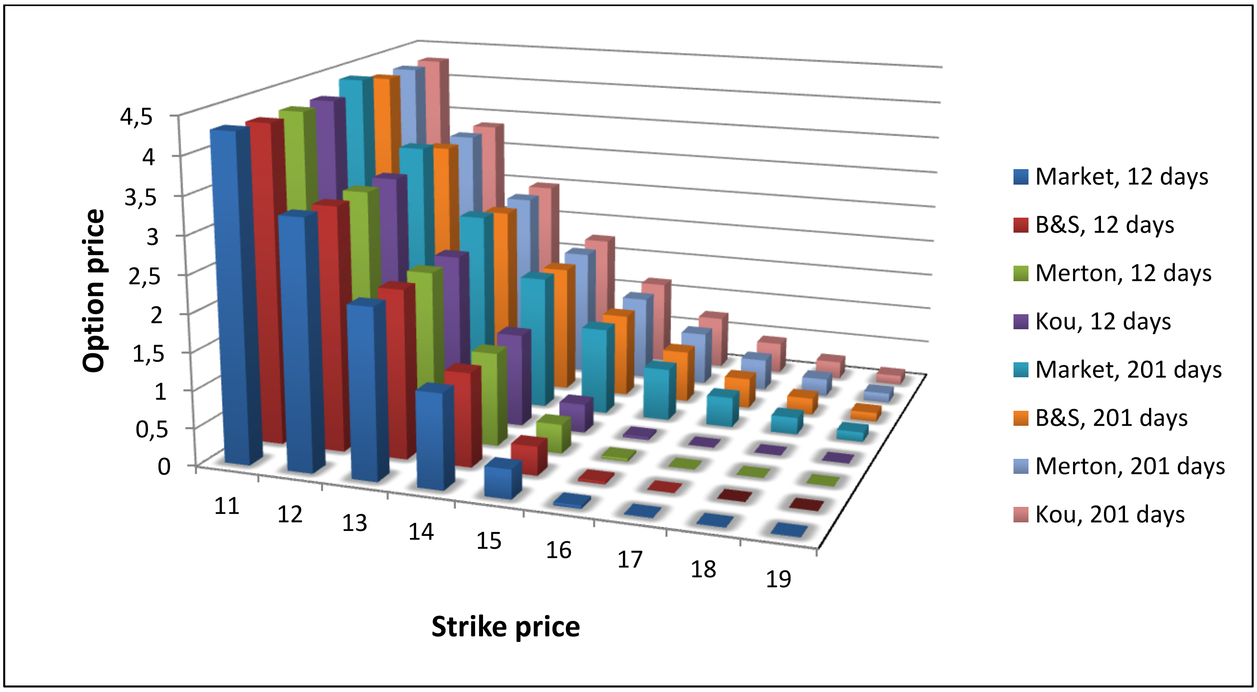

Figure 11.1. *Observed market prices, prices from the B&S model, prices from the Merton model and prices from the Kou model. Time to maturity = 12 and 201 days.*

As with the previous model, the results are fairly similar. The difference in the prices is not large. However, the Kou-model is slightly more accurate than the Merton model when looking at the entire strike range, which is slightly more accurate than the Black Scholes model

11.2.2 Graphical comparison of ratios

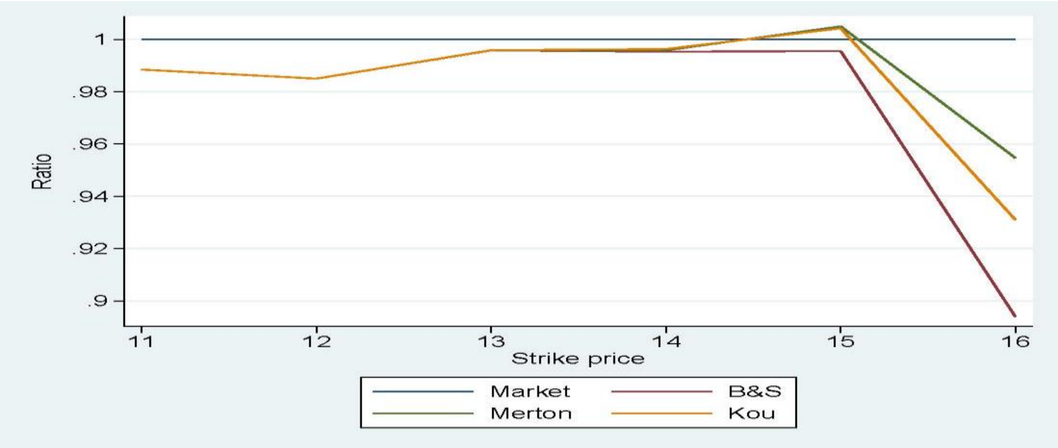

Figure 11.2. *Ratios for the different models. Time to maturity = 12 days.*

Ratio			
Strike	B&S	Merton	Kou
17	0,0685294000	0,095830820	0,083430635
18	0,0002754560	0,001147490	0,001414448
19	0,0000002709	0,000013564	0,000067165

Table 11.1. *Ratios for strike range 17-19*

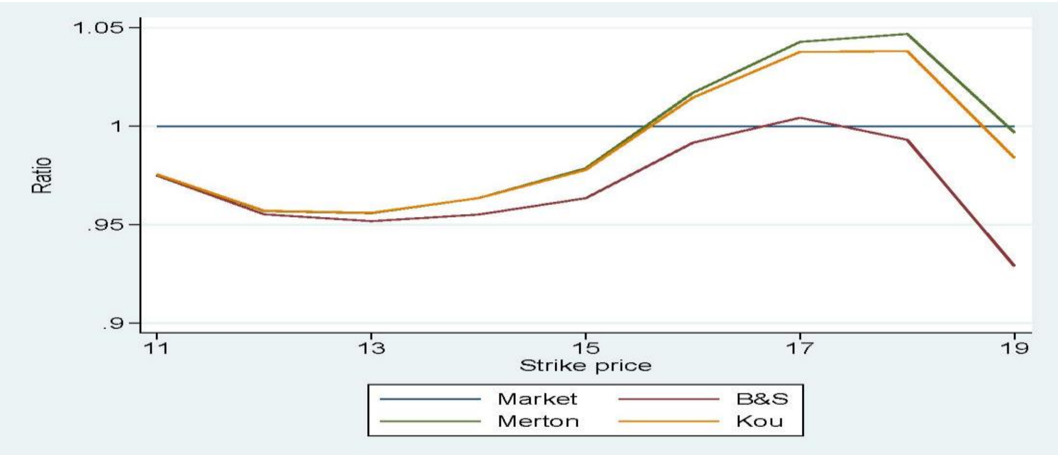

Figure 11.3. *Ratios for time to maturity = 201 days.*

Looking at figures (11.2) and (11.3) there is a difference in how the models perform when considering different maturities.

For the short maturity of 12 days, all of the models underestimate the value of the option both in- and out-of-the-money, while the jump-diffusion models overestimate the value at-the-money. The Black Scholes model consistently underestimates.

For longer maturities (201 days), the models underestimate the value of the option when in-the-money and at-the-money. When moving out-of-the-money, the jump-diffusion models start to overestimate the value, while the Black Scholes model underestimates except for the exercise price of 17 where it is very close to the market value. At-the-money the jump-diffusion models perform very similarly and are more accurate than the Black Scholes model.

11.2.3 Graphical comparison of MSE

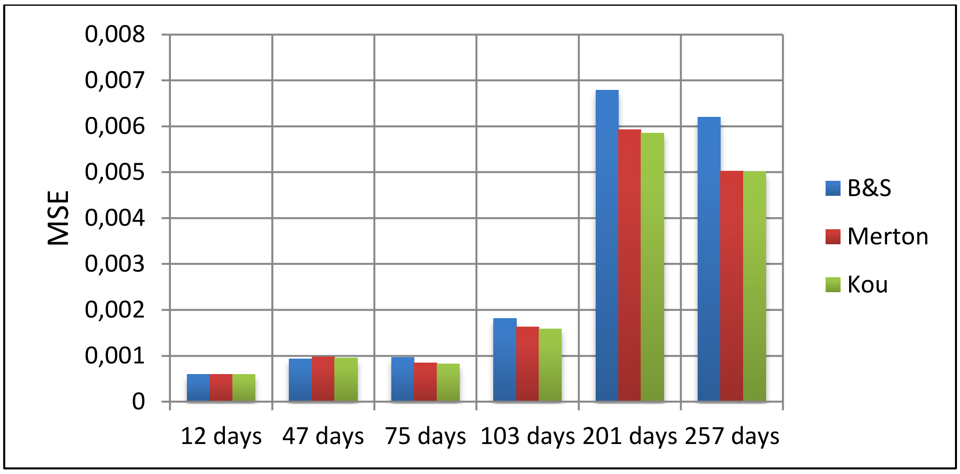

Figure 11.4. *Overall MSE for the entire strike range for the three different models. A table of the MSE is shown in the appendix.*

For all maturities, except 47 days, the Kou-model performs the best when the entire strike range is considered. It should still be noted that the differences are not large.

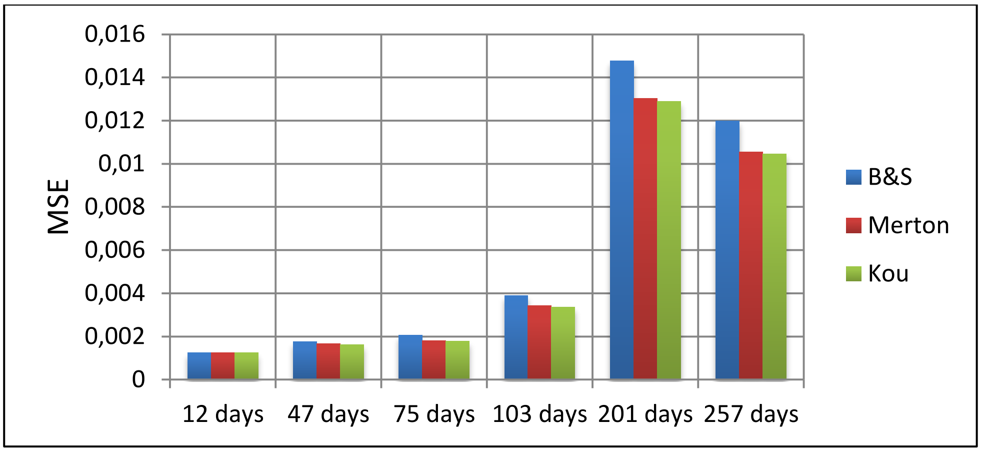

Figure 11.5. *MSE for in-the-money-options*.

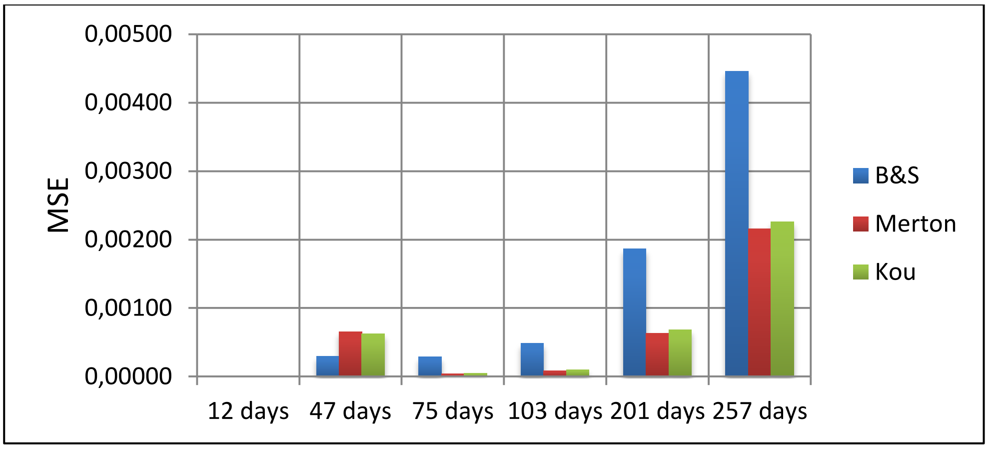

Figure 11.6. *MSE for at-the-money-options*.

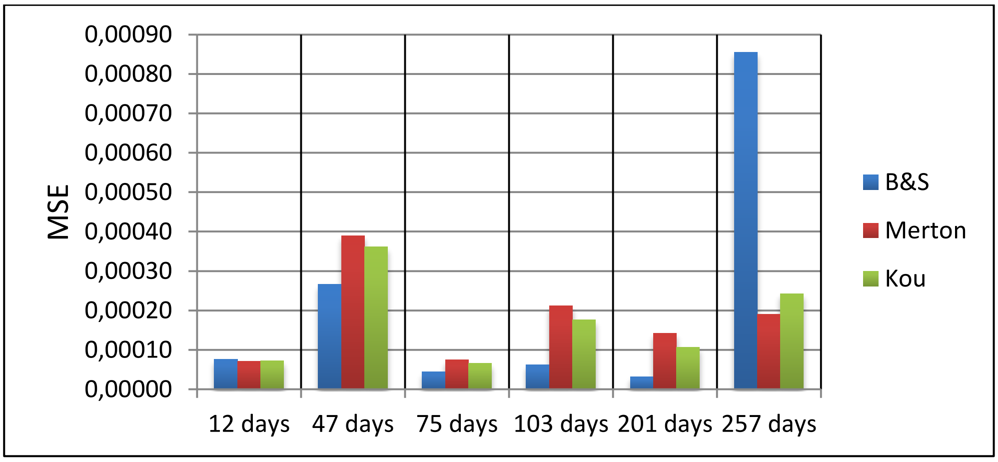

Figure 11.7. *MSE for out-of-the-money-options*.

Looking at the MSE for the different types of moneyness gives conflicting results.

The Kou model performs the best out of the three when in-the-money. At-the-money, the Merton model performs the best for longer maturities. When out-of-the-money, the Black Scholes model is the most accurate overall, while the Kou-model beats the Merton model.

Even though the Kou model produces less accurate results than the Black Scholes model when out-of-the-money, the results show that the Kou-model is more able to produce accurate results when in- and out-of-the-money than the MJD model

11.3 Volatility surface of the theoretical prices.

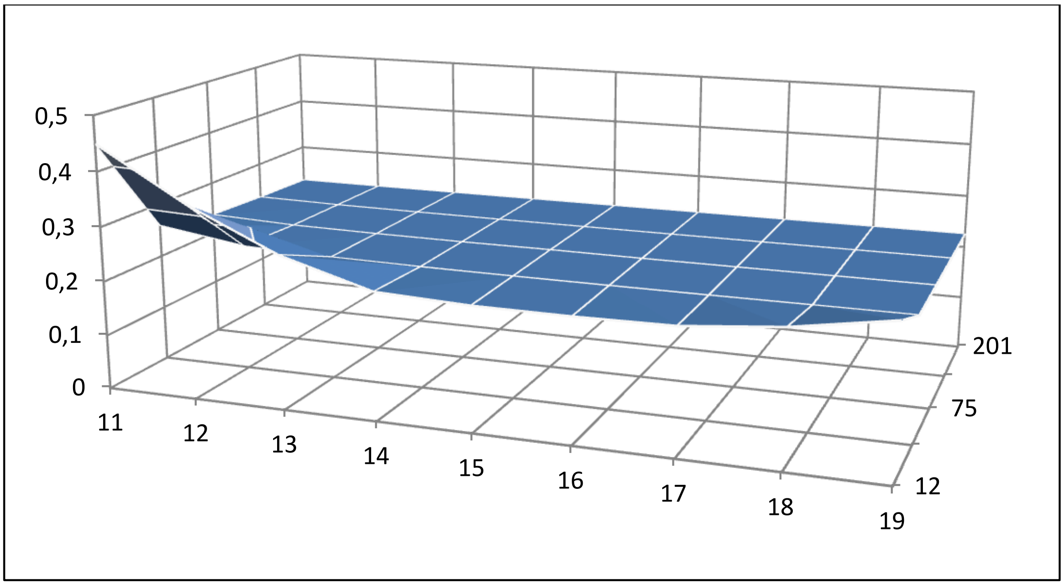

Figure 11.8. *Volatility surface of the implied volatilities from the Kou model.*

Comparing figure (11.8) with figure (10.9), the peak of the implied volatility of the Kou-model is very similar to the peak of the Merton model. While the peak is very similar, the implied volatility of the out-of-the-money-options is marginally lower for the Kou model.

The fact that the model lets you distinguish between positive and negative jumps enables the user to calibrate the model to better fit the implied volatilities in the market.

11.4 Summary

To summarize, by visual inspection the Kou model performs the best overall among the three models considered. Even though the differences between the models are not large, when the entire strike range is considered, the Kou model shows the most accuracy. This is as expected, as the Kou model has additional parameters to help fit the model to the observed market prices.

Because of the additional parameters that have to be determined, the Kou model is more complex to incorporate compared to the very easy and non-complex Black Scholes model and the fairly straightforward Merton model.

Doing the same thorough analysis, as done with the call option on the BAC stock, for several call options is beyond the scope of this thesis. Instead, the volatility surfaces of the observed implied volatilities for four other call options are displayed below. Because the main factor in reducing the accuracy of the models for several strike prices and maturities is the skew in the observed implied volatility and the tendency of the implied volatility to flatten out as time to expiration increases, this problem will apply to options sharing the same implied volatility pattern.

11.4.1 Volatility surface of four additional stocks

The surfaces were downloaded from the Bloomberg database.

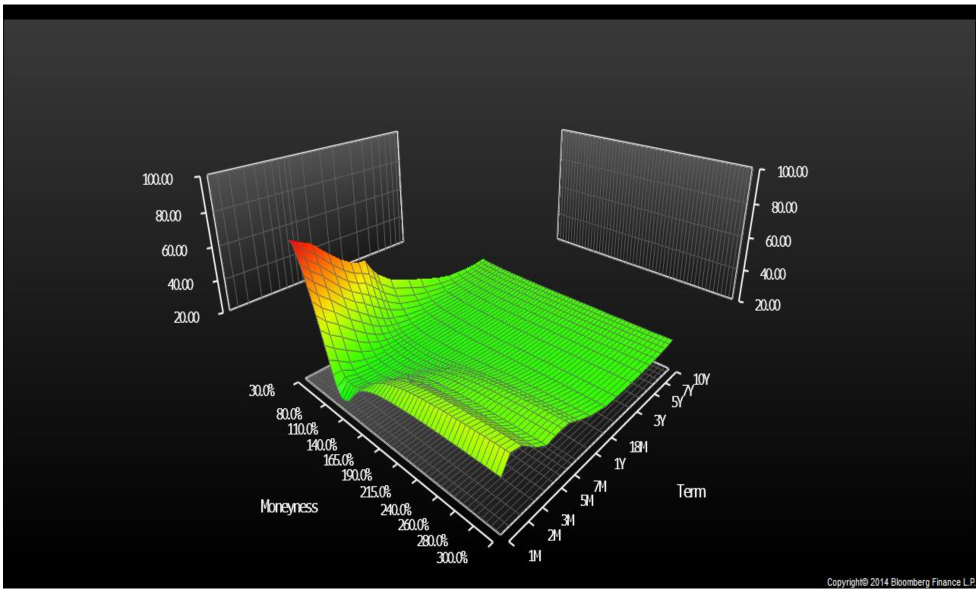

Figure 11.9. *Volatility surface of the implied volatilities for the call option on the Apple Inc. stock.*

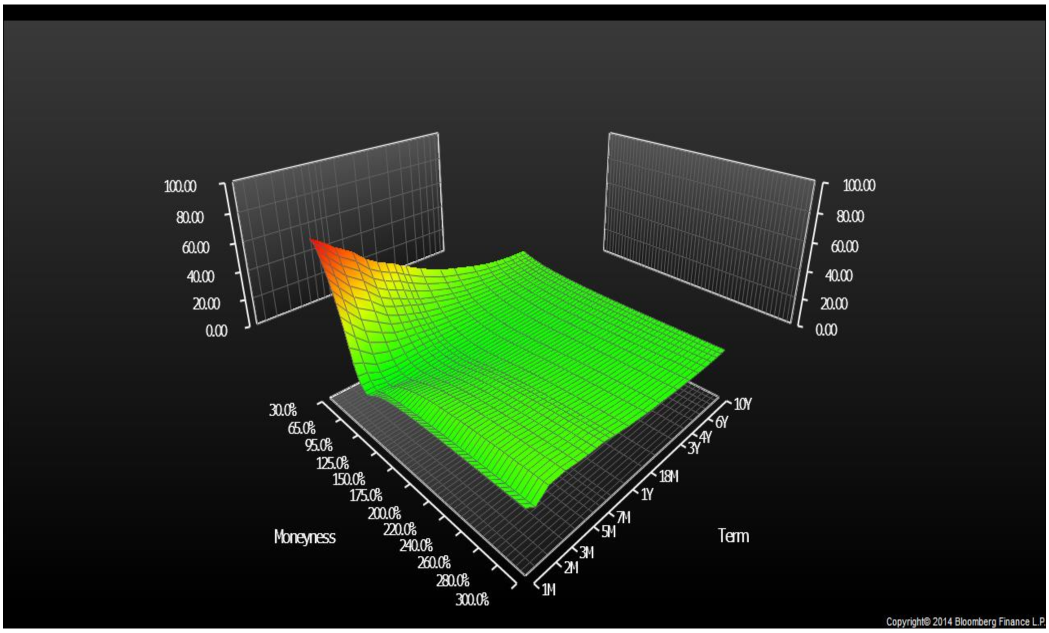

Figure 11.10. *Volatility surface of the implied volatilities for the call option in the Intel Corp. stock.*

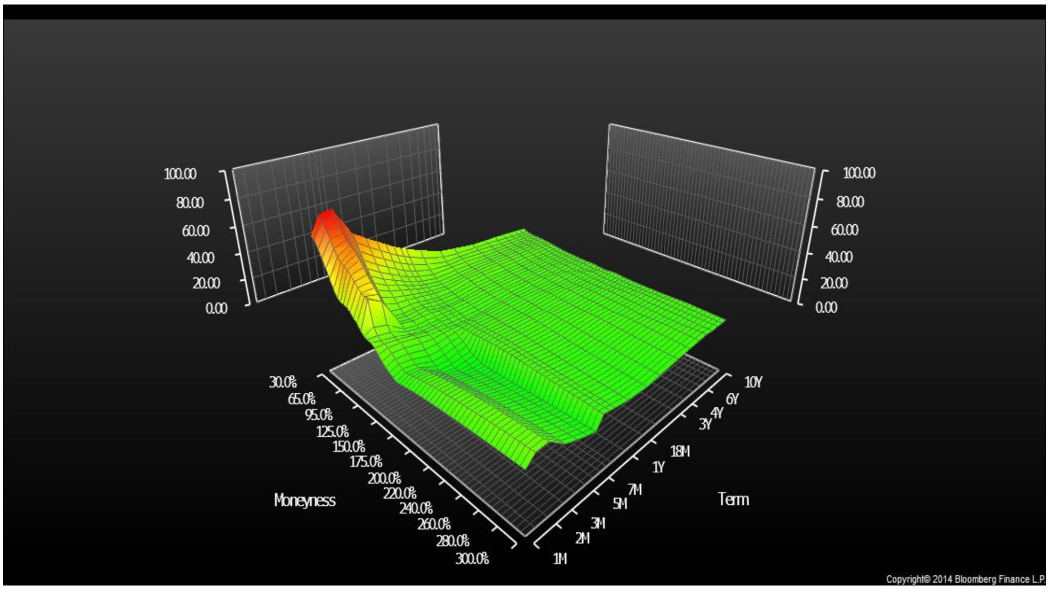

Figure 11.11. *Volatility surface of the implied volatilities for the call option on the CBS Corp. stock.*

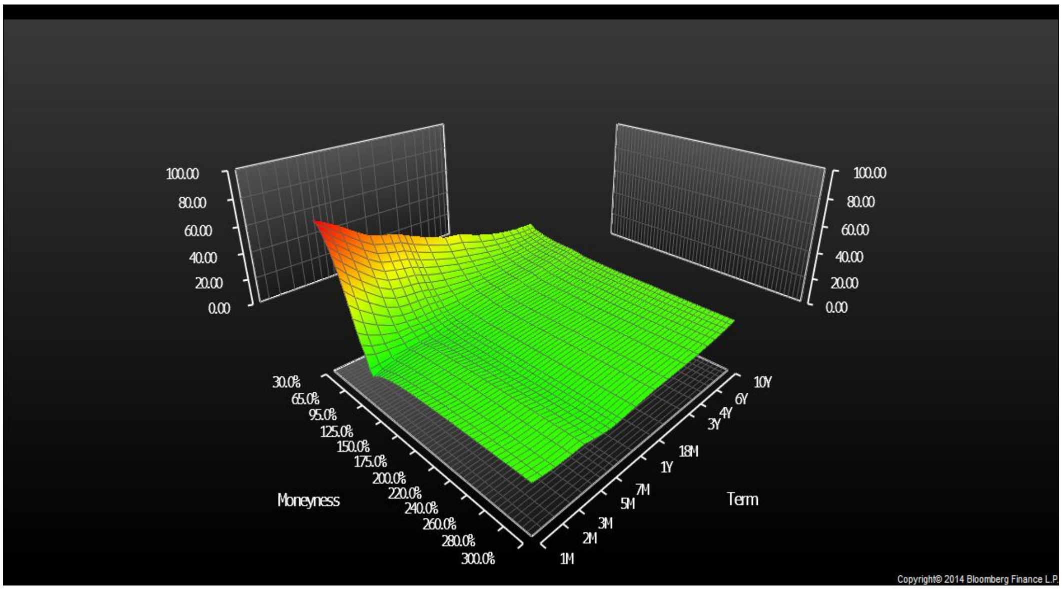

Figure 11.12. *Volatility surface of the implied volatility of the Weyerhaeuser Co. stock.*

Looking at the surfaces, the same tendency observed in the volatility surface of the call option on the BAC stock, holds for the four additional options considered. This means that the same problem regarding accuracy for several strike prices and maturities will occur when computing values for the options in question.

12. Concluding remarks

12.1 Two-sample mean-comparison tests

Visual inspection reveals some difference between the models; however, the differences in values are very small. To help with the conclusion, two-sample mean-comparison tests are carried out to investigate if there is a significant difference between the MSE of the models. The hypothesises were computed with Stata13

The test is a two-sided t-test for independent, unequal, unpaired samples with different variances.

The null hypothesis states that there is no difference between the means of the samples, while the alternative hypothesis states that there is a difference between the means.

Difference in means for MSE for out-of-the-money-options, in-the-money-options and the full strike range for all the maturities were tested at a significance level of 0,01. At-the-money-options were not tested, as a visual inspection is sufficient.

The results are presented on the next pages.

12.2.1 Differences in MSE, entire strike range

Entire strike range	P-value
MSE Compared	**Ha: diff !=0**
B&S-Merton, 12d	**0.9954**
B&S-Kou, 12d	**0.9945**
Merton-Kou, 12d	**0.9991**
B&S-Merton, 47d	**0.9044**
B&S-Kou, 47d	**0.9592**
Merton-Kou, 47d	**0.9438**
B&S-Merton, 75d	**0.8043**
B&S-Kou, 75d	**0.7673**
Merton-Kou, 75d	**0.9614**
B&S-Merton, 103d	**0.8582**
B&S-Kou, 103d	**0.8249**
Merton-Kou, 103d	**0.9651**
B&S-Merton, 201d	**0.8329**
B&S-Kou, 201d	**0.8195**
Merton-Kou, 201d	**0.9864**
B&S-Merton, 257d	**0.1326**
B&S-Kou, 257d	**0.6772**
Merton-Kou, 257d	**0.1316**

Table 12.1. *P-values from the two-sided two-sample t-test for differences in means for the entire strike range. Hypothesis is rejected for p-values < 0,01*

For all maturities, when considering the entire strike range, the t-test cannot reject the null hypothesis of no differences in the MSE for the different models.

12.2.2 Differences in MSE, in-the-money

In-the-money	P-value
MSE Compared	Ha: diff !=0
B&S-Merton, 12d	0.9986
B&S-Kou, 12d	0.9965
Merton-Kou, 12d	0.9979
B&S-Merton, 47d	0.8302
B&S-Kou, 47d	0.7836
Merton-Kou, 47d	0.9531
B&S-Merton, 75d	0.4278
B&S-Kou, 75d	0.3473
Merton-Kou, 75d	0.8884
B&S-Merton, 103d	0.7402
B&S-Kou, 103d	0.7032
Merton-Kou, 103d	0.9628
B&S-Merton, 201d	0.7527
B&S-Kou, 201d	0.7338
Merton-Kou, 201d	0.9817
B&S-Merton, 257d	0.2021
B&S-Kou, 257d	0.6154
Merton-Kou, 257d	0.2014

Table 12.2. *P-values for in-the-money-options.*

As with the entire strike range, the null cannot be rejected.

12.2.3 Differences in MSE, out-of-the-money

Out-of-the-money	P-value
MSE Compared	**Ha: diff !=0**
B&S-Merton, 12d	**0.8736**
B&S-Kou, 12d	**0.9127**
Merton-Kou, 12d	**0.9605**
B&S-Merton, 47d	**0.7581**
B&S-Kou, 47d	**0.7988**
Merton-Kou, 47d	**0.9513**
B&S-Merton, 75d	**0.5810**
B&S-Kou, 75d	**0.6581**
Merton-Kou, 75d	**0.8764**
B&S-Merton, 103d	**0.3361**
B&S-Kou, 103d	**0.3931**
Merton-Kou, 103d	**0.8403**
B&S-Merton, 201d	**0.1717**
B&S-Kou, 201d	**0.2201**
Merton-Kou, 201d	**0.6685**
B&S-Merton, 257d	**0.1379**
B&S-Kou, 257d	**0.1173**
Merton-Kou, 257d	**0.1349**

Table 12.3. *P-values for out-of-the-money-options.*

12.2 Which model is the most accurate?

When considering the entire strike as whole as well as different maturities, the Kou model performs the best. However, as the strike range is split up into different types of moneyness, the results are not as clear. Because of the skew in the observed implied volatilities and the tendency of the observed implied volatility smile to flatten out as time to maturity is increased, the models struggle to produce accurate results for the entire strike range and maturities.

Even though visual study reveals some differences between the models, the two-sided t-test cannot reject the null hypothesis of no difference in MSE across the models.

Because the differences are small, and test statistics shows no significant difference between the models, there is no model that stands out as the most accurate.

It should be noted that the models have only been computed on one option, the call option on the BAC stock. Concluding that one model is better than the other based on one option would not be a very robust result. Even though the implied volatility surfaces of four additional options showed the same pattern as the BAC stock, there could be other options where the differences in the models accuracy are more pronounced.

The fact that the Black Scholes model has been around for more than 40 years, and is still going strong, is no surprise after looking at degree of accuracy. Despite the mentioned advantages of the jump-diffusion models compared to the Black Scholes model, the values from the models do not differ greatly.

12.2 Shortcoming of Jump-Diffusion Models

The main problem with jump-diffusion models is that they cannot capture the volatility clustering effects, which can by captured by other models such as stochastic volatility models. Kou (2002)

Because the implied volatility tends to flatten out as time to maturity increases, the models struggle to produce accurate results for the entire maturity range. In this thesis the jump intensity and magnitude had to be set at low levels to enable the models to perform well for the entire strike and maturity range.

If only short maturities and specific types of moneyness were considered, the jump intensity and magnitude could be set at higher levels, making the models more able to reproduce the observed implied volatilities.

The problem with hedging with the jump-diffusion models should also be noted. Due to the jump part, the market is incomplete, and the conventional riskless hedging arguments are not applicable here, in contrast to the Black Scholes model.

12.3 Are the Jump-Diffusion models used much in practice?

This section is based on Leib (2000)

These days, at least a few option market-making practitioners value the jump-diffusion models highly. Trent Cutler of the San Francisco-based equity options market-making firm Cutler Group LP, regularly goes to great lengths to train new employees to look at options in jump-diffusion terms. "*We probably spend as much time on this as anybody"*, "*and we actually get some pretty interesting outcomes, perhaps even a pricing advantage, although it's hard to quantify how much this has been worth to us."*

Cutler uses his jump-diffusion models as a way to explain initially aberrant-looking pricing. "*We'll look at something that appears out of line, put in an extra jump, and see how much that affects the pricing. Then we'll look out into the real world and see what the market is focusing on. Often we can explain away an odd-looking pricing phenomenon. Other times, we spot an opportunity. We think the model works well in terms of non-continuous movements and systematic changes in volatility over time and stock prices"*.

Other practitioners are well aware of jump-diffusion modeling, but use it rarely. Emanuel Derman, head of risk management at Goldman Sachs, has seen the model used at his firm in specific merger-arbitrage situations, "*for which it was quite useful",* he says, but seldom in other situations.

Mike Kelly, a quantitatively oriented trader at Onyx Capital Management LLC, does not use jump diffusion in his pricing and hedging decisions, but *"finds it theoretically appealing"*

The article is from 2000, but shows how some of the practitioners in the market think about the jump-diffusion models.

12.3 Suggestions for further research

Models that incorporate both jump-diffusion and stochastic volatility (for example the Bates model) would be an interesting study. Looking at the results from this thesis, models that count for volatility clustering could very well outperform the pure jump-diffusion models.

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Data sources:

All data was downloaded from the Bloomberg database at Norges Handelshøyskole, spring 2014.

Appendix

Appendix 1: Market prices, Model prices and pricing errors

	T-t = 12 days, rf=0,13%				T-t = 103 days, rf=0,24%		
Strike	**Market prices**	**B&S**	**squared errors**	**Strike**	**Market prices**	**B&S**	**squared errors**
11	4,300000191	4,250476656	0,00245258	11	4,300000191	4,258795337	0,00169784
12	3,299999952	3,250519989	0,002448267	12	3,349999905	3,270358144	0,00634281
13	2,25999999	2,250567924	8,89639E-05	13	2,390000105	2,325092062	0,004213054
14	1,25999999	1,25412416	3,45254E-05	14	1,549999952	1,492296824	0,003329651
15	0,389999986	0,388259114	3,03063E-06	15	0,870000005	0,847930461	0,000487065
16	0,039999999	0,035756852	1,80043E-05	16	0,419999987	0,422508506	6,29267E-06
17	0,01	0,000685294	8,67638E-05	17	0,170000002	0,184633206	0,000214131
18	0,01	2,75456E-06	9,99449E-05	18	0,07	0,071233998	1,52275E-06
19	0,01	2,70873E-09	9,99999E-05	19	0,029999999	0,024515223	3,00828E-05
MSE			**0,000592453**	**MSE**			**0,001813605**
	T-t = 47 days, rf=0,17%				**T-t = 201 days, rf=0,35%**		
11	4,300000191	4,252445658	0,002261434	11	4,400000095	4,289658979	0,012175162
12	3,299999952	3,253053681	0,002203952	12	3,5	3,343316823	0,024549618
13	2,299999952	2,262174393	0,001430773	13	2,599999905	2,474597693	0,015725715
14	1,370000005	1,336474145	0,001123983	14	1,809999943	1,728770444	0,006598232
15	0,600000024	0,617274736	0,000298416	15	1,179999948	1,136843453	0,001862483
16	0,180000007	0,209405878	0,000864705	16	0,709999979	0,704043011	3,54855E-05
17	0,050000001	0,050996343	9,92697E-07	17	0,409999996	0,411770455	3,13452E-06
18	0,02	0,008992818	0,000121158	18	0,230000004	0,228408756	2,53207E-06
19	0,01	0,001177948	7,78286E-05	19	0,129999995	0,12075822	8,54104E-05
MSE			**0,000931471**	**MSE**			**0,006781975**
	T-t = 75 days, rf=0,21%				**T-t = 257 days, rf=0,42%**		
11	4,300000191	4,25500839	0,002024262	10	5,400000095	5,288718429	0,012383609
12	3,299999952	3,25907482	0,001674866	11	4,449999809	4,319703251	0,016977193
13	2,339999914	2,2891544	0,002585266	12	3,5	3,399055648	0,010189762
14	1,460000038	1,41604322	0,001932202	14	1,940000057	1,848398575	0,008390831
15	0,75999999	0,74310278	0,000285516	15	1,340000033	1,273210467	0,004460846
16	0,319999993	0,32351388	1,23E-05	16	0,879999995	0,839085449	0,001674
17	0,119999997	0,11626718	1,39E-05	17	0,560000002	0,530506657	0,000869857
18	0,039999999	0,03476213	2,74E-05	19	0,209999993	0,189896765	0,00040414
19	0,02	0,00877446	0,000126013	20	0,129999995	0,108325409	0,000469788
MSE			**0,000964649**	**MSE**			**0,006202225**

A1.1 *Market prices, Black Scholes prices and corresponding pricing errors.*

T-t = 12 days, rf=0,13%				T-t = 103 days, rf=0,24%			
Strike	**Market prices**	**Merton**	**squared errors**	**Strike**	**Market prices**	**Merton**	**squared errors**
11	4,300000191	4,250476656	0,00245258	11	4,300000191	4,259066577	0,001675561
12	3,299999952	3,250520002	0,002448265	12	3,349999905	3,271908362	0,006098289
13	2,25999999	2,250577037	8,8792E-05	13	2,390000105	2,329977103	0,003602761
14	1,25999999	1,254744234	2,7623E-05	14	1,549999952	1,501857765	0,00231767
15	0,389999986	0,391948852	3,79808E-06	15	0,870000005	0,860660684	8,72229E-05
16	0,039999999	0,038186452	3,28895E-06	16	0,419999987	0,434884684	0,000221554
17	0,01	0,000958308	8,17522E-05	17	0,170000002	0,193921663	0,000572246
18	0,01	0,000011475	9,97706E-05	18	0,07	0,076860077	4,70607E-05
19	0,01	0,000000136	9,99973E-05	19	0,029999999	0,027365896	6,9385E-06
MSE			**0,000589541**	**MSE**			**0,001625478**
T-t = 47 days, rf=0,17%				**T-t = 201 days, rf=0,35%**			
11	4,300000191	4,252448821	0,002261133	11	4,400000095	4,291876022	0,011690815
12	3,299999952	3,253166951	0,00219333	12	3,5	3,348953366	0,022815086
13	2,299999952	2,263379682	0,001341044	13	2,599999905	2,485076308	0,013207433
14	1,370000005	1,341233482	0,000827513	14	1,809999943	1,743944952	0,004363262
15	0,600000024	0,625635369	0,000657171	15	1,179999948	1,154802652	0,000634904
16	0,180000007	0,216968972	0,001366704	16	0,709999979	0,722069757	0,00014568
17	0,050000001	0,054962025	2,46217E-05	17	0,409999996	0,427569443	0,000308685
18	0,02	0,01032032	9,36962E-05	18	0,230000004	0,240781212	0,000116234
19	0,01	0,001485209	7,25017E-05	19	0,129999995	0,129578168	1,77938E-07
MSE			**0,000981968**	**MSE**			**0,005920253**
T-t = 75 days, rf=0,21%				**T-t = 257 days, rf=0,42%**			
11	4,300000191	4,255070481	0,002018679	10	5,400000095	5,290027971	0,012093868
12	3,299999952	3,259721207	0,001622377	11	4,449999809	4,323476321	0,016008193
13	2,339999914	2,29217249	0,002287463	12	3,5	3,406997883	0,008649394
14	1,460000038	1,423444099	0,001336337	14	1,940000057	1,866043888	0,005469515
15	0,75999999	0,753871284	3,7561E-05	15	1,340000033	1,293540954	0,002158446
16	0,319999993	0,333742886	0,000188867	16	0,879999995	0,859656869	0,000413843
17	0,119999997	0,123109194	9,66711E-06	17	0,560000002	0,549209762	0,000116429
18	0,039999999	0,038186309	3,28947E-06	19	0,209999993	0,201914902	6,53687E-05
19	0,02	0,010121308	9,75886E-05	20	0,129999995	0,117047339	0,000167771
MSE			**0,000844648**	**MSE**			**0,00501587**

A1.2. *Market prices, Merton prices and corresponding pricing errors.*

	T-t = 12 days, rf=0,13%				T-t = 103 days, rf=0,24%		
Strike	**Market prices**	**Kou**	**squared errors**	**Strike**	**Market prices**	**Kou**	**squared errors**
11	4,300000191	4,250477542	0,002452493	11	4,300000191	4,259298577	0,001656621
12	3,299999952	3,250532874	0,002446992	12	3,349999905	3,272558463	0,005997177
13	2,25999999	2,250718223	8,61512E-05	13	2,390000105	2,330881021	0,003495066
14	1,25999999	1,255392437	2,12295E-05	14	1,549999952	1,502232401	0,002281739
15	0,389999986	0,391662075	2,76254E-06	15	0,870000005	0,859881276	0,000102389
16	0,039999999	0,037239728	7,61909E-06	16	0,419999987	0,433209257	0,000174485
17	0,01	0,000834306	8,40099E-05	17	0,170000002	0,192130041	0,000489739
18	0,01	1,41445E-05	9,97173E-05	18	0,07	0,075515679	3,04227E-05
19	0,01	6,7165E-07	9,99866E-05	19	0,029999999	0,026577482	1,17136E-05
MSE			**0,000588996**	**MSE**			**0,00158215**
	T-t = 47 days, rf=0,17%				**T-t = 201 days, rf=0,35%**		
11	4,300000191	4,252476706	0,002258482	11	4,400000095	4,292452181	0,011566554
12	3,299999952	3,253392193	0,002172283	12	3,5	3,34976467	0,022570654
13	2,299999952	2,264133844	0,001286378	13	2,599999905	2,485705596	0,013063189
14	1,370000005	1,341977384	0,000785267	14	1,809999943	1,74388374	0,004371352
15	0,600000024	0,625044521	0,000627227	15	1,179999948	1,153814061	0,000685701
16	0,180000007	0,215414055	0,001254155	16	0,709999979	0,720321429	0,000106532
17	0,050000001	0,053793255	1,43888E-05	17	0,409999996	0,425478659	0,000239589
18	0,02	0,009850687	0,000103009	18	0,230000004	0,238767777	7,68738E-05
19	0,01	0,001366605	7,45355E-05	19	0,129999995	0,127908592	4,37397E-06
MSE			**0,000952858**	**MSE**			**0,005853869**
	T-t = 75 days, rf=0,21%				**T-t = 257 days, rf=0,42%**		
11	4,300000191	4,255181317	0,002008731	10	5,400000095	5,290399207	0,012012355
12	3,299999952	3,260193393	0,001584562	11	4,449999809	4,324135831	0,015841741
13	2,339999914	2,293044535	0,002204808	12	3,5	3,407758073	0,008508573
14	1,460000038	1,423993661	0,001296459	14	1,940000057	1,865808659	0,005504364
15	0,75999999	0,753173126	4,66061E-05	15	1,340000033	1,292459515	0,002260101
16	0,319999993	0,332103518	0,000146495	16	0,879999995	0,857870327	0,000489722
17	0,119999997	0,121533427	2,35141E-06	17	0,560000002	0,547043356	0,000167875
18	0,039999999	0,037215345	7,7543E-06	19	0,209999993	0,199947208	0,000101058
19	0,02	0,009684637	0,000106407	20	0,129999995	0,115443074	0,000211904
MSE			**0,000822686**	**MSE**			**0,005010855**

A1.3. *Market prices, Kou prices and corresponding pricing errors.*

Appendix 2: Market prices downloaded from the Bloomberg database

Strike	Ticker	Bid	Ask	Implied volatility
17 May 14 (12d); CSize 100; R .13				
11	BAC 5/17/14 C11	**4.19999980926514**	**4.30000019073486**	**80.1665725708008**
12	BAC 5/17/14 C12	**3.20000004768372**	**3.29999995231628**	**61.1501159667969**
13	BAC 5/17/14 C13	**2.22000002861023**	**2.25999999046326**	**35.6352767944336**
14	BAC 5/17/14 C14	**1.25**	**1.25999999046326**	**25.4367771148682**
15	BAC 5/17/14 C15	**0.379999995231628**	**0.389999985694885**	**20.7883567810059**
16	BAC 5/17/14 C16	**0.0299999993294477**	**0.0399999991059303**	**20.6928234100342**
17	BAC 5/17/14 C17	**0**	**0.00999999977648258**	**29.3124141693115**
18	BAC 5/17/14 C18	**0**	**0.00999999977648258**	**41.5293655395508**
19	BAC 5/17/14 C19	**0**	**0.00999999977648258**	**52.5624465942383**
21 Jun 14 (47d); CSize 100; R .17				
11	BAC 6/21/14 C11	**4.19999980926514**	**4.30000019073486**	**47.4573364257813**
12	BAC 6/21/14 C12	**3.20000004768372**	**3.29999995231628**	**36.4077796936035**
13	BAC 6/21/14 C13	**2.27999997138977**	**2.29999995231628**	**30.3966617584229**
14	BAC 6/21/14 C14	**1.35000002384186**	**1.37000000476837**	**24.3809242248535**
15	BAC 6/21/14 C15	**0.589999973773956**	**0.600000023841858**	**21.2564964294434**
16	BAC 6/21/14 C16	**0.170000001788139**	**0.180000007152557**	**20.3647289276123**
17	BAC 6/21/14 C17	**0.0399999991059303**	**0.0500000007450581**	**21.3786182403564**
18	BAC 6/21/14 C18	**0.00999999977648258**	**0.0199999995529652**	**23.7993659973145**
19	BAC 6/21/14 C19	**0**	**0.00999999977648258**	**28.2082328796387**
19 Jul 14 (75d); CSize 100; R .21				
11	BAC 7/19/14 C11	**4.19999980926514**	**4.30000019073486**	**38.0483665466309**
12	BAC 7/19/14 C12	**3.20000004768372**	**3.29999995231628**	**29.1665782928467**
13	BAC 7/19/14 C13	**2.3199999332428**	**2.33999991416931**	**27.5755920410156**
14	BAC 7/19/14 C14	**1.45000004768372**	**1.46000003814697**	**24.7540264129639**
15	BAC 7/19/14 C15	**0.75**	**0.759999990463257**	**22.6824684143066**
16	BAC 7/19/14 C16	**0.310000002384186**	**0.319999992847443**	**21.9960918426514**
17	BAC 7/19/14 C17	**0.109999999403954**	**0.119999997317791**	**22.1214752197266**
18	BAC 7/19/14 C18	**0.0299999993294477**	**0.0399999991059303**	**22.1474876403809**
19	BAC 7/19/14 C19	**0.00999999977648258**	**0.0199999995529652**	**23.8816699981689**

A2.1. *Bid/Ask prices and implied volatility, maturity 12-75 days.*

16 Aug 14 (103d); CSize 100; R .24				
11	BAC 8/16/14 C11	**4.19999980926514**	**4.30000019073486**	**32.4560585021973**
12	BAC 8/16/14 C12	**3.25**	**3.34999990463257**	**30.868537902832**
13	BAC 8/16/14 C13	**2.36999988555908**	**2.39000010490417**	**27.1897773742676**
14	BAC 8/16/14 C14	**1.52999997138977**	**1.54999995231628**	**24.3936500549316**
15	BAC 8/16/14 C15	**0.850000023841858**	**0.870000004768372**	**22.9086570739746**
16	BAC 8/16/14 C16	**0.409999996423721**	**0.419999986886978**	**21.8072986602783**
17	BAC 8/16/14 C17	**0.159999996423721**	**0.170000001788139**	**21.3678321838379**
18	BAC 8/16/14 C18	**0.0599999986588955**	**0.0700000002980232**	**21.7096176147461**
19	BAC 8/16/14 C19	**0.0199999995529652**	**0.0299999993294477**	**22.2276477813721**
22 Nov 14 (201d); CSize 100; R .35				
11	BAC 11/22/14 C11	**4.30000019073486**	**4.40000009536743**	**31.9812068939209**
12	BAC 11/22/14 C12	**3.34999990463257**	**3.5**	**28.8561248779297**
13	BAC 11/22/14 C13	**2.5**	**2.59999990463257**	**26.0846004486084**
14	BAC 11/22/14 C14	**1.76999998092651**	**1.80999994277954**	**24.2515850067139**
15	BAC 11/22/14 C15	**1.1599999666214**	**1.17999994754791**	**23.2817916870117**
16	BAC 11/22/14 C16	**0.699999988079071**	**0.709999978542328**	**22.2943553924561**
17	BAC 11/22/14 C17	**0.389999985694885**	**0.409999996423721**	**22.0991287231445**
18	BAC 11/22/14 C18	**0.209999993443489**	**0.230000004172325**	**22.0545501708984**
19	BAC 11/22/14 C19	**0.109999999403954**	**0.129999995231628**	**22.2601699829102**
17 Jan 15 (257d); CSize 100; R .42				
10	BAC 1/17/15 C10	**5.30000019073486**	**5.40000009536743**	**35.0234222412109**
11	BAC 1/17/15 C11	**4.34999990463257**	**4.44999980926514**	**31.1445198059082**
12	BAC 1/17/15 C12	**3.45000004768372**	**3.5**	**28.5300788879395**
14	BAC 1/17/15 C14	**1.91999995708466**	**1.94000005722046**	**24.7938423156738**
15	BAC 1/17/15 C15	**1.33000004291534**	**1.3400000333786**	**23.8353443145752**
16	BAC 1/17/15 C16	**0.870000004768372**	**0.879999995231628**	**23.3510360717773**
17	BAC 1/17/15 C17	**0.550000011920929**	**0.560000002384186**	**22.8449268341064**
19	BAC 1/17/15 C19	**0.200000002980232**	**0.209999993443489**	**22.9211540222168**
20	BAC 1/17/15	**0.119999997317791**	**0.129999995231628**	**23.145450592041**

A2.2. *Bid/Ask prices and implied volatility, maturity 103-257 days.*

Appendix 3: Matlab code for option pricing in the Merton Jump-Diffusion Model

```
function c = CPMertonJD
% function c = CPMertonJD calculates the analytical price
% of a European call under jump diffusion.

  So = ;       % Initial Stock price
  X =;       % Strike
  T =;         % Time to maturity
  delta =;  % std of lognormal jump process
  nu =;        % mean of lognormal jump process
  lambda =;   % intensity of jumps
  vola =;   % vola of stock price
  r =;     % interest rate

  % The price is gven as a series of terms,
  % compute the first N terms for the result
  N =;
  K = exp(nu + 0.5*delta^2) - 1;
  c = 0;
  for n = 0 : N
    sigma_n = sqrt(vola^2 + n*delta^2/T);
    r_n = r - lambda*K + n*log(1+K)/T;
    d1 = (log(So/X) + (r_n +
0.5*sigma_n^2)*T)/(sigma_n*sqrt(T));
    d2 = d1 - sigma_n*sqrt(T);
    f_n = So * normcdf(d1,0,1) - X * exp(-r_n*T) *
normcdf(d2,0,1);
    c = c + exp(-lambda*(1+K)*T) * (lambda*(1+K)*T)^n *
f_n/factorial(n);
  end
end
```