Pricing in Non-Convex Electricity Markets

Veronica Araoz Castillo

Department of Finance and Management Science

Norwegian School of Economics NHH

A thesis submitted for the degree of

PhD in Finance and Management Science

Abstract

Deregulation of the electricity markets has brought several interesting topics to the research agenda. Switching from a monopoly based industry to the free market industry has not been straight forward. The competitive segments of the deregulated electricity markets, the wholesale market and the retail market, have evolved in different ways across the globe, and consequently there are different market designs and different pricing mechanisms.

If we assume that a neoclassical economic model applies to the electricity market, then the monotonically increasing supply curves of all generators would be aggregated to create the industry supply curve. Similarly, the monotonically decreasing demand curves of all consumers would be aggregated to create the industry demand curve. The competitive equilibrium price for electricity would be set at the level where the two curves intersect.

It is important to highlight that convexity is a property that economic models require for a competitive equilibrium. However electricity treated as a commodity has several characteristics that do not fit into the neoclassical economic model. First of all, electricity cannot be stored; therefore the total production should always match total consumption. In addition, electricity generators have several operational requirements to avoid problems with the technology used, such as minimum and/or maximum output, start-up and shut down costs, and minimum ramp rates. These requirements generate non-convexities in the production function. Moreover, generators are committed to production in indivisible units, which also creates non-convexities. On the demand side, it has been seen that the demand function is extremely inelastic, which may contribute to high spikes in prices.

Non-convex cost structures can be a challenge for the price discovery process,

since the supply and demand curves may not intersect; or if they intersect, the price found may not be high enough to cover the total cost of production.

In this dissertation, I review previous work on setting prices for the day-ahead electricity market in a power pool auction. In addition, I propose an alternative pricing mechanism using a Semi Lagrangean relaxation technique. This technique provides a price that can be applied in the electricity pool markets. In this type of markets a central system operator decides who produces and how much they should produce.

The proposed pricing approach not only accommodates the non-convexities that the electricity market has, but also provides a shadow value that represents the price of the day-ahead electricity. At this price, the demand is fulfilled at minimum cost and all generators are covering their fixed costs.

The Semi-Lagrangean technique is applied to examples from the literature. First to a simple case, and then it is expanded so as to obtain prices not only for electricity but for capacity as well. In addition, the technique is applied to a network model to obtain different prices at the nodes, as well as to obtain prices in the congested lines. The prices obtained are high enough to cover the producers total costs, and follow the optimal resource allocation at a minimum cost. The prices found are an alternative solution to the price discovery problem in non-convexities economies. This approach sends better signals to market participants since it does not require that the system operator offers side-payments or up-lifts. The technique is also tried in a power exchange with block bids.

To Gaute and my family who are the most important piece in my life.

Acknowledgements

This thesis would not have been possible without the support and guidance of my supervisor Kurt Jörnsten, who heard my discussions, read my long e-mails, and helped me obtained knowledge with his encouragement, assistance and moral support. I owe thanks to my second supervisor Kenneth Fjell, who read my drafts and highlighted issues to address.

I am grateful to all the department involved with Energy and Electricity markets, Mette Bjørndal, Endre Bjørndal, Gunnar Eskeland, Leif Sandal, and special thanks to Linda Rud with whom I had very interesting discussions. Thanks to all the administrative staff for helping with funds applications, and special tanks to Vigdis who has made available her support in a number of ways. I am thankful to my colleagues and friends with whom I shared lunches, conferences, small and long talks.

Lastly, many thanks to my family, particularly to Gaute for his patient and unconditional support.

Veronica Araoz.

Contents

CONTENTS

List of Figures

List of Tables

1

Introduction

Pricing in a linear divisible world is straight forward with the use of linear mathematical programming and its dual theory. However the problem begins to complicate when some of the variables are indivisible, such as the number of generators to be engaged into production. The problem becomes then a mixed integer programming problem (MIP), and in certain circumstances the dual prices are not longer an optimal price.

In an economic view, the optimal price is set-up where the demand function meets the supply function. This is under certain assumption of constant returns of scale and convexities, which is similar to a linear mathematical problem. Electricity has several characteristics that do not fulfil the requirements of a classical economical model. Therefore in this dissertation, I study the problem of pricing in a non-convex electricity market. Non-convexities arise in the electricity problem not only in the indivisibility of the generating unit, but also in the cost function and in some of the technical and operative constraints that the generating unit has to satisfied. The contribution of this thesis is to present an alternative pricing approach for the day-ahead electricity market mainly based on an pool based market. The technique implemented is a Semi-Lagrangean Relaxation, to obtain not only prices for electricity but also for capacity and in the case of a network set up for nodal prices and congestion prices.

The thesis is organised in seven sections:

1. INTRODUCTION

The aim of Section 2 is to provide the reader with a background on electricity and economics, as well as to give the reader an overview on how the industry established prices before deregulation and after deregulation. In addition a synopsis of the trading mechanisms is presented along with a discussion on why the electricity market is complicated and what are the challenges with deregulation.

In section 3 the pricing approaches after deregulation are presented. The deregulation of the market seems to have split USA and Europe in the way to approach and solve the pricing problem. The approaches and issues are summarised in this section.

The main objective of this thesis is to present a new approach to price in electricity market based on a new technique built using a Lagrangean Relaxation. Therefore section 4 presents the mathematical background on mathematical programming, primal and dual problems, along with a review on Lagrange relaxation. Moreover it illustrates how a Walrasian equilibrium in a Pool market may be complicated to achieve. In addition, it shows how the Unit Commitment and Dispatch Problem is written mathematically. Finally, it points out where the non-convexities arise in the Electricity Market.

Section 5 presents the Semi-Lagrangean approach and how it is implemented in a Unit Commitment and Dispatch Problem (UCDP). The different approaches to obtain a possible price to the market are described. First, the general approach applied to the UCDP with a given demand and with a price sensitive demand is presented. Then, prices in a second price auction are discussed, followed by two extensions of the model to obtain prices for capacity and prices in a network framework.

From the problems presented in Section 5, Section 6 shows the numerical results by solving the different models using examples from the literature. Finally, section 7 concludes and discusses further research.

Section 8 contains the appendix with numerical results for some of the examples presented.

2

Deregulation of Electricity Industry

Deregulation of electricity markets started at the beginning of 1980's in South America, and since then many countries and regions around the world have followed the trend. Different market designs and market structures have been developed due to the fact that each region, or country, will have different characteristics, natural resources,technologies, and politics (Baldick, Helman, Hobbs, and O'Neill, 2005). In other words, there is not a single unique model that fits all countries. In addition to this, several approaches and rules to discover the electricity price in a competitive market structure have been implemented, and some of them have been discarded because of its negative consequences. Therefore, it can be said that the establishment of an electricity market is an ongoing process.

Deregulation is also known in some countries as liberalisation, privatisation or restructure of the electricity market. In this work we use the term deregulation to mean the split of the four elements of the supply chain in electricity: generation, transmission, distribution and retail entities. The split has triggered the existence of a competitive market for generation and retail and a natural monopoly for transmission and distribution. This is illustrated in Figure 2.1[1].

The electricity market has peculiar characteristics that introduce a higher level of complexity to a market environment. These complexities were not a challenge under a monopoly environment where all the decisions were centrally made. However, in a competitive market structure, these complexities can originate difficulties to find equilibrium in an economic sense; that is,

[1] Figure obtained from https://aepretailenergy.com/residential/get-started/aep-ohio/understanding-your-bill

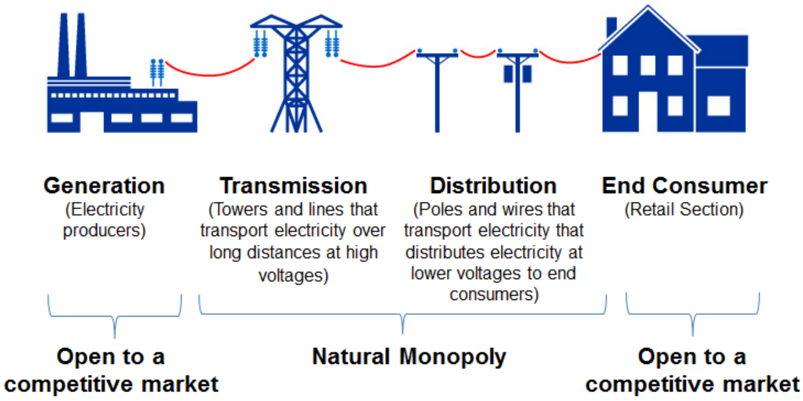

Figure 2.1: Elements of the Electricity Industry

to find prices at which all agents in the market maximise their surplus (i.e. consumers maximise benefits and producers maximise profits) and the aggregate quantity supplied equals the aggregated quantity demanded.

Setting the right price in the electricity market is essential because it affects the subsequent markets linked to the spot market. In some places, the spot electricity price is used to set transmission charges, as well as create contracts such as forwards, futures and contracts for differences.

2.1 Electricity and Economics

If a neoclassical economic model is assumed for electricity markets, then the monotonically increasing supply curves of all generators would be aggregated to create the industry supply curve, and similarly the monotonically decreasing demand curves of all consumers would be aggregated to create the industry demand curve. The competitive equilibrium price for electricity, (P^*), would be set at the level where the two curves intersect as shown in Figure 2.2. It is important to highlight that convexity in production and benefit functions is one of the most important assumptions in the standard neoclassical economic model (Mas-Collel, Whinston, and Green, 1995). In this set-up, marginal cost defines the market clearing prices. At this point, social welfare is maximised, and it is depicted by the triangle ABC which represents consumer's utility minus producer's costs.

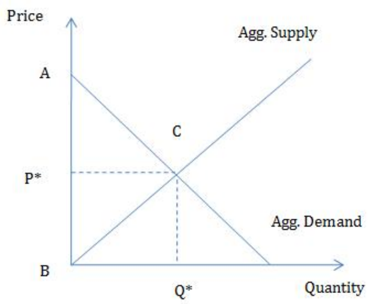

Figure 2.2: Neoclassical Economic Model

The electricity treated as a commodity, however, has several characteristics that do not fit into the neoclassical economic view. First of all, the electricity cannot be stored; therefore the total production should always match total consumption. In addition, electricity has several security requirements, such as minimum and/or maximum output, start up costs, minimum ramp rates and shut down costs, that generate non-convexities in the production function. Moreover, generators are engaged into producing electricity in non-divisible units, which also creates non-convexities. On the demand side it has been seen that the demand function is extremely inelastic, which adds also complexity to the problem of finding a price in this market.

Figure 2.3 shows an incremental energy cost function and its corresponding energy supply function for a given generator with start up costs (Hao, Angelidis, Singh, and Papalexopoulos, 1998). The total cost function is non-linear, therefore marginal pricing will not always provide linear market clearing price that will support an equilibrium. It could also be that the supply and demand functions do not intersect.

In summary, non-convexities in either of the participants' functions may not yield to an equilibrium in the economic sense since linear prices are not guaranteed and the prices obtained may not maximise the market participants' surplus. In many market structures the solutions obtained are suboptimal solutions.

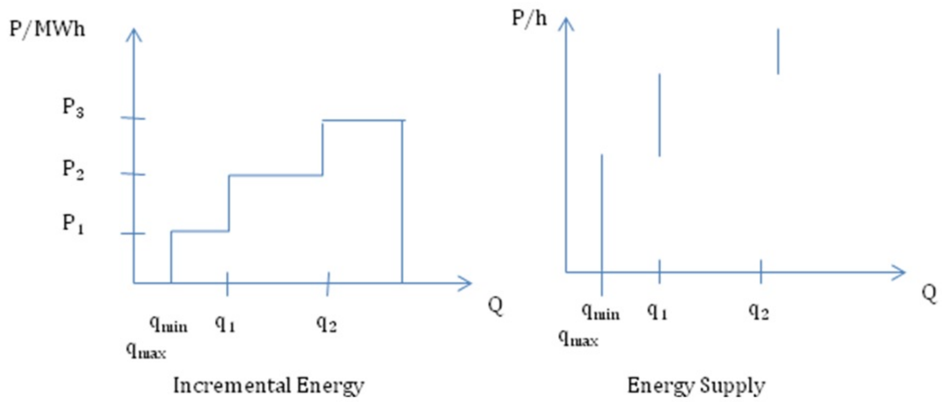

Figure 2.3: Incremental energy cost and energy supply functions for a given generator

2.2 Prior to Deregulation

Before deregulation took place, generating units were owned by a single entity that minimised the total cost of producing electricity so as to cover a given demand. This entity was a regulated utility that accepted the price set by a government regulation. This regulation could be based on a rate-of-return, a cost-plus or any other scheme decided by the government.

Some researchers pointed out the difficulties to set a completely deregulated spot electricity market before the deregulation of the market (Westfield, 1988). The author shows concern about addressing the technical characteristics that the electricity market may present. Indeed, many deregulated electricity structures have kept a centralized scheme with a unit commitment and dispatch problem solved by a System Operator or Market Operator.

2.3 Deregulation

Deregulation was intended to increase efficiency in the electricity market. This means to have a market that provides a price which in turn sends the correct signals to the market participants about their investment decision. That is, to promote investment in the most efficient technologies. An efficient market will allocate the resources in the most economical way, that would imply that the electricity will be provided at the least cost mix of inputs, and all the resources are allocated economically Stoft (2002). In many countries, deregulation meant the creation of a tough and competitive generation market that would provide large efficiency gains and cost savings to consumers.

Under deregulation, price is not set any longer by a government rule; instead, it is set by the market open competition. As mentioned earlier, different countries have adopted different market structures, and within them the trading mechanisms have evolved in different ways. So far, there have been two main generic trading models: bilateral contracts and centralised markets. Within centralised markets there are power pools and power exchanges. These mechanisms are described in the following section.

2.3.1 Trading Mechanism

- **Bilateral Trading**. Under this scheme, the market participants arrange contracts amongst them with specific terms and conditions and the price is unique to each transaction. For a review of a bilateral trading model the reader can find Bower and Bunn (2000) interesting.

- **Centralised Markets**. This scheme are modelled as an auction with a central entity who receives bids from all the market participants (sellers and buyers) and then it sets the winners as well as the quantities that should be traded and the price that clears the market (Motto, Galiana, Conejo, and Huneault, 2001). There are different approaches to settle the prices and quantities. The structures mainly used are a pool based power structure and a power exchange structure. It seems that the USA has implemented the former, while in Europe the latter is found in most markets.

  a) **Power Pools**. In this type of market, the system operator receives information from all the generators regarding their cost structure and operating limitations. The information is provided in a bid format that usually contains fixed costs, start up costs, production range, minimum and maximum down times, and ramp rate. The system operator in turn, as a central scheduler, solves a problem known as the Unite Commitment and Dispatch Problem (UCDP) and specifies which generators will be engaged, how much energy each of them will produce and also what the energy price will be. There are also several approaches to settle the price and quantities, which are reviewed in the next section.

b) **Power Exchanges**. In contrast to power pools, market participants in power exchanges determine their own production and consumption, that is, they are self-committed given the price observed. The UCDP is not used and therefore, the market operator does gather neither cost nor benefit functions nor operating constraints. Price is set at the point where supply and demand intersects. For a review on power exchanges in Europe please refer to Madlener and Kaufmann (2002).

The design of an electricity market is not fixed, but an evolving task. There is no right or wrong format, since each country will apply or create the one that suits best its endowments. Some advantages and disadvantages of the pools versus exchanges can be found in Wilson (1998) and Stoft (2002)

2.3.2 Why the electricity market is a complicated market?

This section aims to highlight some of the characteristics in the electricity market that make the design complicated.

The electricity prices are usually calculated in a day-ahead basis. In addition to energy, generating units should provide ancillary services to the market. Ancillary services keep the market reliable by covering last minute differences in the energy quantities demanded and supplied. The different ancillary services, such as reserves, along with energy production create multiple interdependent products that are offered in to the market on different time periods (O'Neil, Helman, and Sotkiewicz, 2001). Therefore, one problem is what to include in the price: energy only; energy and capacity; capacity energy and ancillary services.

The centralised market structure is based on an auction format, and again, each country or region has implemented different type of auctions as well as settlement rules. Some countries implemented initially pay-as-bid formats and later discovered that it could create high peaks in the prices. Other countries implemented uniform pricing, while others have a mix of uniform pricing and an additional side payment to compensate any loss for committing.

There are different trading methods and pricing criteria that yield to different quantity and financial settlements. This gives a further complication into the market design when it comes to deciding market rules and bid formats. There can be single bids, multiple bids, block bids and in addition the market can set specific rules regarding what to include in the bids. Some rules may require that bids reflect marginal costs, or total costs, or that generators submit separately variable costs and start-up costs along with security and physical constraints, or that bids should be strictly convex.

Power auctions face more challenges than standard auction models. There are more issues and difficulties involved in designing efficient energy auctions, these can be found in Elmaghraby et al. (2004) and O'Neill (2009). In Elmaghraby et al. (2004), the authors suggests that a market in electricity may need three components, namely a commodity price, a capacity price and production quantities. In O'Neill (2009) the author addresses the market design process for market power mitigation, optimal dispatch, pricing and the role of price signals. In Baldick et al. (2005) the authors address several design issues connected to the integrated or centralised spot market scheme with a clear view into the US markets.

In addition, the mechanisms to set prices could vary also if the system is considered as a network or not, if the transmission constraints are included in the model or not. This leads to different pricing approaches such as nodal pricing and zonal pricing. This will be covered further in Section 3.5

2.3.3 Challenges with deregulation

After deregulation, one of the market designs was to set the electricity market as an auction. An auction is efficient when it maximises the total surplus from trade given the bids submitted by the participants. In addition, the price obtained sends clear signals to the participants for investments and supports an efficient production and consumption decisions amongst the participants. Moreover, it provides a competitive short term resource allocation.

In a competitive market, the participants are price takers and their decision to take part in the market and how much to produce or consume should be based on their independent profit/benefit expectations as well as on the market price and conditions. In theory, the market price obtained from the market should be a sufficient incentive not only to equate supply and demand, but also to satisfy the security of the system, without the need of a central intervention. However, this is still a challenge for the electricity markets. Prices for electricity market will generally need to be higher than that obtained from the auction, not only to cover the operational costs of the generators, but also to send the right investment signal to the market participants. In fact, one feature in the electricity wholesale market that seems persistent across market designs is the ”missing money problem”. This problem arises when the generators cannot recover their investment cost through energy payments only. The wholesale spot price for electricity is not high enough for the generators to cover their investment costs (Joskow, 2008).

3

Pricing approaches after deregulation

Before deregulation took place, generating units were owned by a single entity that minimised the total cost of producing electricity so as to cover a given demand. This entity was a regulated utility that accepted the price set by a government regulation. This regulation could be based on a rate-of-return, a cost-plus or any other scheme decided by the government. After deregulation different approaches were set in place to allow the new market to set prices.

In this section I will review the literature and explain some of the pricing approaches suggested in different market designs, such as a pool and an exchange. I will start first by describing in general how the USA markets differ from the European markets and then show how marginal pricing can be a problematic approach. I end the section by describing concepts for pricing in a network model.

3.1 USA and Europe

Both market structures for trading, Power Pools and Power Exchanges, have some disadvantages. Power Pools have long-run inefficiencies due to the wrong signals in the market for investment in capacity. This is mainly caused by the side payments and uplifts usually present in a Power Pool. The drawbacks of granting uplifts to the generators that incur in losses are mainly two. The first one is wrong signals to the market in terms of what technology to invest into; the second one, is that some generators may be tempted to bid cost curves higher than they actually are. This is due to the learning process where they will learn that if they are scheduled and incur in losses, they will be compensated. Power Exchanges, on the other hand, present short-run inefficiencies in terms of finding the least cost schedule. If one or the other is the best option in a deregulated electricity market for economic efficiency remains an open question that requires further research.

The problem seems to be in that either we minimised costs and obtain a right efficient schedules, or maximised social welfare which give the right pricing signals. The first problem is found mainly in power pools, and the prices obtained are somewhat deviated from a linear pricing. The second case is seen more in Power Exchanges, where a suboptimal solutions in term of welfare may be implemented.

Power Pools are more present in the USA (PJM, New England, New York, ERCOT, Mid West SO), while the Power Exchanges are more used in Europe. Figure 3.1 shows an overview of the USA Electric power markets, while Figure 3.2 shows the restructuring state or deregulation state in the country.

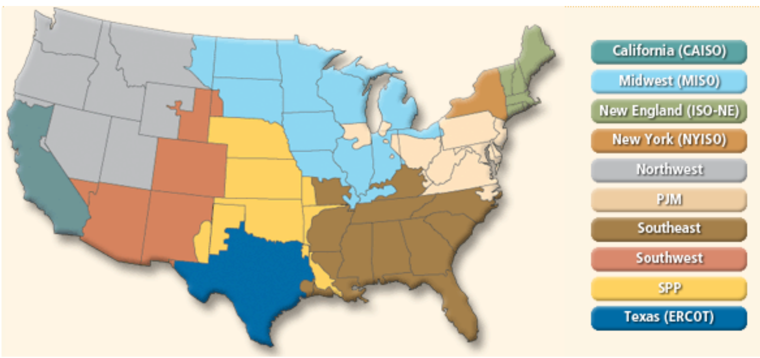

Figure 3.1: USA Electric Power Market Overview
Source: FERC.gov

The four largest power exchanges in Europe are: Nordpool for the Nordic market covering Norway, Sweden, Finland, Denmark and Estonia; APEX that covers France, German, Austria and Switzerland; OMEL for Spain and Portugal; and APX in the UK. Other countries have also their own power exchange, like Italy (IPEX) and Belgium (BELPEX). Figure 3.3 shows in colour (not grey) the different exchanges in Europe.

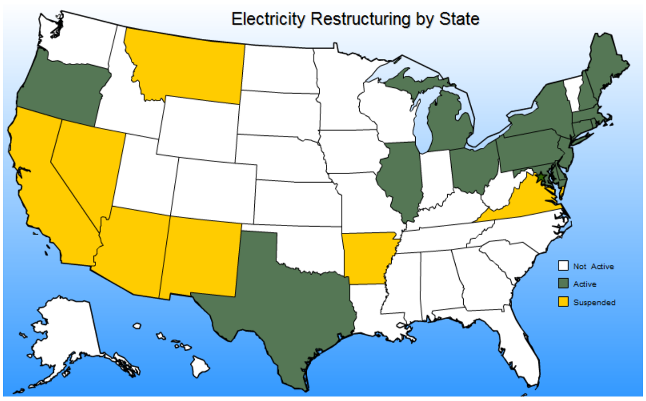

Figure 3.2: USA electricity restructuring
Source: Energy Information Administration

Figure 3.3: European Power Exchanges

3.2 Approaches and issues

3.2.1 Marginal Cost Pricing

Consider a market where there are three producers, each with their own capacity constraints and cost structure as shown in Table 3.1. Let us assume for now that there are no variable costs and therefore no marginal costs; then, the producer's total costs will be the same, whether one unit is produced or the generator runs at full capacity. In the same Table 3.1, the cost of producing represents the total fixed costs (F) for operating the generator.

Table 3.1: Cost Structure

Producer (i)	Capacity (k_i)	Cost of producing (F_i)	Average cost at full capacity
A	16	800	50
B	9	630	70
C	5	550	110

If a producer decides not to sell any units, then no costs are incurred. However, if at least one unit is generated, then there will be costs of producing the unit. The average cost at full capacity for each producer will be given by the cost incurred divided by its capacity (F_i/k_i). Therefore, producer A will be interested in producing as long as the price is higher than or equal to 50, while producer B will enter the market if the price reaches 70 or more, and finally producer C will participate if the price is higher than or equal to 110, as shown in Figure 3.4.

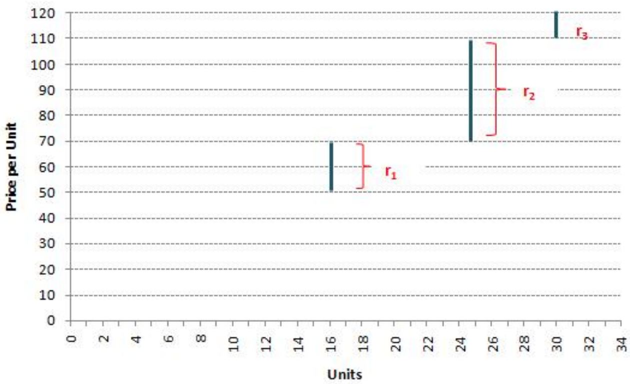

Figure 3.4: Total Cost Function

From Figure 3.4 we can see that the total cost function is not linear, and there will be competitive market clearing prices only if the market demand is 16, 25, or 30. That is, if demand is, for example, 16, then any price within the open interval (50, 70) represented by r_1 will clear the market. Similarly for a demand value of 25, prices within the range r_2 will also clear the market and for demand of 30, range r_3 will contain the equilibrium prices. However, when the demand is not contained in any of these three values, there will be no intersection of demand and supply curves and there will be no market equilibrium; a competitive market clearing price is non-existent (Van Boening and Wilcox, 1996).

If variable costs are considered, then the price will be set by the marginal producer. This will not always lead to optimal prices since some units may not cover their total production costs[1]. Take the same producers, A, B and C with the added information about their variable costs (V) as shown in Table 3.2.

Table 3.2: Cost Structure with Variable Costs

Producer (i)	Capacity (k_i)	Fixed Costs (F_i)	Cost per unit produced (V_i)	Average cost at full capacity[2]
A	16	800	60	110
B	9	630	150	220
C	5	550	225	335

If the level of demand is such that the marginal producer is not engaged at its full capacity, then the marginal producer will not recover its costs, since the price will be set equal to its marginal costs. This price will not necessarily cover the total costs of producing for all the generators. For example, given the capacities and marginal costs from Table 3.2, and given a demand level of 20 units, producer A and B would be engaged in the production. Producer A will be engaged at full capacity while Producer B will be committed to cover the remaining demand, i.e. 4 units. Since Producer B is the marginal producer, it will set the price equal to its marginal cost which is 150. At this price, Producer A will obtain a profit of 640 = (150-60) * 16 - 800, but the marginal generator in this case will not cover its total costs of $1230 = (630 + 150 * 4)$ and will incur losses for 630 = (4 * 150 - 1230), as shown in Table 3.3.

Table 3.3: Example with demand value set to 20

Demand=20			
Price = Marginal Producer = B = Marginal Cost = 150			
Producer	Revenue	Costs	Profit
A	2400	1760	640
B	600	1230	-630

[1] The total production costs is calculated as $(F_i + V_i * q_i)$, where q_i is the amount produced and it is less than or equal to k_i.

[2] The average cost at full capacity is given by the total cost of producing, divided by the total production, that is $V_i + F_i/k_i$.

Similarly, if the demand level is 28, Producer C in Table 3.4 will be the marginal producer, and will set the price to 225. Producers A and B will be committed at full capacity, and Producer C will cover the remaining demand of three units. The first two producers, A and B, will recover their total costs $(F_i + q_i * V_i)$, but Producer C will not. Its total costs are larger than the revenue it makes, i.e. $(F_i + q_i * V_i) > P * q_i$, where q_i is the quantity produced and P is the price.

Table 3.4: Example with demand value set to 28

Demand=28			
Price = Marginal Producer = B = Marginal Cost = 225			
Producer	Revenue	Costs	Profit
A	3600	1760	1840
B	2025	1980	45
C	675	1225	-550

Figure 3.5 provides us with a glimpse of non-convexities in this example. The red dotted line illustrates the value function of minimising the cost of production for this specific example, considering that the plants can be call into production at divisible units. In reality however, the plants are either engaged or not. This mean that the problems needs integrality constraint to accommodate the fact that the plants have to be added in discrete units. If that is the case, then the total cost function is shown by the blue continuous line. The non-convexities in the cost function create difficulties in finding a price.

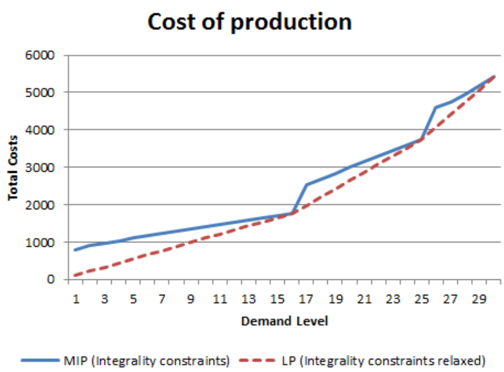

Figure 3.5: Total Cost of production for Linear and Integer Cases

As more generators come to play with different cost structures along their production plan, the marginal cost pricing presents spikes. That is the case in an illustration presented in Gribik, Hogan, and Pope (2007), where the plants have two different variable costs depending on the section they are operating. Table 3.5 contains the characteristics of that example, and Figure 3.6 depicts the behaviour of marginal cost pricing if the least cost generating units are engaged in production.

Table 3.5: Generators with piece-wise variable costs

	Plants		
	A	B	C
Fixed Costs	0	6000	8000
Variable Costs1 (>=0 ; <= 100MW)	65	40	25
Variable Costs2 (>100; <=200MW)	110	90	35

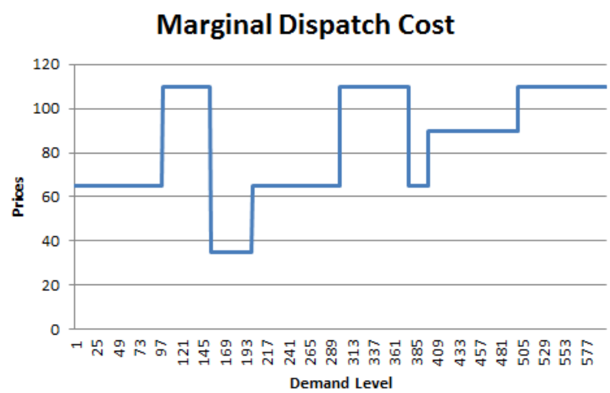

Figure 3.6: Marginal Cost Pricing with generators that have linear piece-wise variable costs

In summary, marginal pricing will not always cover the costs incurred by the generators who are profit maximizers under the deregulated market. The price will not properly coordinate the participants to the pool and consequently, marginal pricing is not enough to guarantee efficient self-committed schedules nor a linear equilibrium market clearing price (Toczylowski and Zoltowska, 2009).

3.3 Under Power Pool Structure

In 1997, some authors (Jacobs, 1997; Johnson, Oren, and Svoboda, 1997) raised certain issues regarding the problems of implementing a Pool in the Electricity Market. One of the problems mentioned is the possibility of multiple solutions, where the impact to the total cost could be negligible, but the effect on the profitability of some market participants could be large. That is, two sets of prices that are equally efficient would likely generate different schedules and consequently different profit levels to the participants. An additional problem highlighted by the authors is the risk of distorting the incentives for investment. Other authors have shown concern about how well the deregulated market under Unit Commitment and Dispatch Problem (UCDP) will provide feasible solutions, minimise costs and avoid gaming opportunities at the same time (Yan and Stern, 2002). They show with straightforward examples that the solution in a deregulated system may not necessarily be the minimum cost solution obtained from the old regulated structure.

Markets organized with a power pool structure have also evolved in different ways across the globe. They can operate with different formats, and will diverge from each other in how the bids are form, what type of networks are considered in the optimisation problem, as well as how the quantities and prices are determined. In the next pages, I will briefly present the pricing mechanisms studied and proposed recently.

3.3.1 Uplifts and side payments

In cases where the solution to maximising social welfare is not aligned with the solution obtained by each agent's maximisation problem, the market operator offers an uplift or side payment to incentive the market participants to follow the schedule obtained by solving the maximum social welfare problem. This measure could help the marginal generators from Tables 3.3 and 3.4 to recover their losses.

In other words, when the market operator solves the UCDP and sets both, the market price and schedules to follow, to the market participants, it can happen that some generators may incur monetary losses. The market operator then compensates these generators with an uplift or side payment so they are willing to produce the scheduled amount.

Motto, Galiana, Conejo, and Huneault (2001) model a power exchange and apply a market rule to make the offer functions from the generators and distributors convex; such a rule avoids having irrational and chaotic behaviour. However it is not clear how can a profit optimality rule can be ensured since the examples given assume that the primal and dual problem obtain the same solution. They highlight the fact that an alternative pricing approach will be needed in the presence of duality, but do not suggest one. The prices are updated by a Newtown algorithm.

Galiana, Motto, and Bouffard (2003) propose the implementation of a generalised uplift function that modifies the market participants' profit function by adding an uplift function. The consumers contribute to the payment of the uplift through the increase in the system price, which is increased by modifying the offered generation costs. Rules are set for the generators to contribute in any loss of profit of generators that would incur in losses under a UCDP. Uplifts differ by market participant and can be negative or positive and its sum across all the market participants is zero.

By augmenting the number of commodities to be priced, O'Neill, Sotkiewicz, Hobbs, Rothkopf, and Stewart (2005) reach a discriminatory multipart prices. The commodity price is highly volatile. In their model the auctioneer solves the UCDP by means of a mixed integer programming which contains all bids from the market participants. The bids contain cost (or benefit) structure and technical limitations. The auctioneer then inserts the optimal solution for the integer constraints as equality and runs the UCDP to find the dual prices for electricity, capacity and start-up. The generators receive payment for the electricity produced, their capacity and start-up. The last two are considered as the uplift and it can be positive or negative. That is, it is possible that the generator has to pay in order to be schedule if the uplift is negative. This could be problematic. Furthermore, it is not clear where the payment for uplifts will come from since the total uplift sum can be negative.

The model presented by Motto and Galiana (2002) also suggests augmented pricing, however that refers to changes in the parameters of the generators' cost functions, as the work in Galiana, Motto, and Bouffard (2003). Under this approach, the consumers will pay the uplifts in a pro-rata basis. The model assumes a quadratic function and it adds a disincentive function into the generators profit maximisation problem. The sum of the disincentive function across all agents is equal to zero, which guarantees that no external money is required in the pool.

By applying an iterative optimisation method that is founded on a cutting plane algorithm, Bouffard and Galiana (2005) expand on the work presented earlier by Galiana, Motto, and Bouffard (2003); Motto and Galiana (2002), and arrive to single uniform linear prices. The uplift is given by changes in bid offering parameters but in addition it takes into account intertemporal couplings. Under this scheme, consumers and producers share the profit and losses of the suboptimal solution.

Discriminatory multi-part prices are also suggested by Hogan and Ring (2003). The authors suggest a minimum uplift that is paid to the generator conditional on accepting the scheduled obtained by the market operator. The uplifts are never negative and electricity prices are more stable. The uplift is the minimum amount that would make the generator indifferent between accepting the solution from the market operator and responding optimally to the commodity price given. It has been stated that in a linear case, the commodity prices obtained under this approach are equal to the linear programming prices, while the minimum uplifts are equal to the size of the duality gap (Bjørndal and Jörnsten, 2008).

In the models presented by Bouffard and Galiana (2005); Galiana, Motto, and Bouffard (2003); Motto and Galiana (2002); Motto, Galiana, Conejo, and Huneault (2001) it is not clear, however, how the market participants will obtain the information needed about the uplifts before they run their maximisation problems.

Gribik, Hogan, and Pope (2007) present an uplift comparison across three different pricing models. The first model is a restrictive model where the UCDP is solved after setting the integer variables to its optimal values, as suggested by O'Neill, Sotkiewicz, Hobbs, Rothkopf, and Stewart (2005). The second model is a dispatchable model where the total cost function is approximated to closely related convex function and in addition treats the unit commitment with continuous variables within the integer bounds, that is a relaxed problem of the UCDP. The last model is a convex hull model which uses an approximation of the aggregate cost function by using its convex hull. The Semi-Lagragean approach suggested in this work is also applied to the example in Gribik, Hogan, and Pope (2007) in section 6.1.3.

Recently Van Vyve (2011) proposed a new approach and used the numeric example in Gribik, Hogan, and Pope (2007) to illustrate his approach. The author solves the UCDP first, and then, he solves a linear problem that includes the financing of the uplift by the committed plants. According to the author, the maximum of such contribution is minimised, instead of minimising the total uplift. In his approach, a new variable is created for each of bid. This variable represents the difference between the market price and the actual price at which the bid is taken. If the actual price is higher than the market price, then there is a compensation or uplift; and if the opposite happens (i.e. the actual price is lower than the market price), then the generator contributes to the financing of the uplift. There is a maximum bound for the contribution to the uplift that the generator has to make.

There are three main problems with applying uplifts. First, uplifts and side payments can have impact on the signalling to the market. That is, they may send the wrong signal to the generators about which technology should be expanded; second, it is not clear where the extra money for the uplifts will come from; and last, they also present a risk for strategic bidding, that is bidding higher than the real cost from the generators, since they know that they would be compensated for any monetary loss (Elmaghraby, O'Neil, Rothkopf, and Stewart, 2004; Stoft, 2002). It seems that under side payment mechanisms the market can be inefficient in the long run.

3.3.2 Other

A **Bender decomposition** approach is used by Bjørndal and Jörnsten (2008) to obtain more stable prices than in models such as those presented by Hogan and Ring (2003); O'Neill, Sotkiewicz, Hobbs, Rothkopf, and Stewart (2005) . In fact the prices obtained are supported by a non-linear function and are non-discriminatory. The authors use the optimal solution from the UCDP to reformulate the problem with additional supporting inequalities. How the uplifts are collected and paid remained unclear.

Average cost prices. Under the assumption of perfect information, Muratore (2008) develops a model where the generators are consolidated in technologies (with the same cost, operational characteristics and owner) and the price for electricity is in fact the maximum average price of the technologies engaged to produce electricity.

A similar technique was implemented in the early stages of the UK market and some problems were found due to the behaviour of the market participants who were able to strategically choose the parameters submitted in their cost functions, which in turn set the prices much higher than they should had been (Madrigal and Quintana, 2001). However, irrespective of the participants' behaviour, the prices under this scheme can reach unreasonable high prices due to the discontinuities of the total supply function. That is, if the demand is located just above a discontinuity, the prices increase will be very high for an extra unit.

Non-linear and discriminatory prices. A model where the price for consumers is different from the price for generators is suggested by Madrigal and Quintana (2001). The authors highlight how under disequilibrium situations, it is difficult to select a final schedule as well as setting the price. In this model, the cost that is not recovered by the generators with the dual prices is shared in equal proportion with the generators and the consumers. In order to do so, the price is adjusted by decrements or increments using a formula in which the suppliers' price increment is amortised in accordance to the positive profits under the dual optimal prices. The prices are higher than the dual value but much smaller than the average cost model.

Relaxing complementarity constraints in the mixed integer programming formulation is an alternative proposed recently by Conejo and Ruiz (2011), where the complementarity conditions are relaxed and included into the objective function. The authors proposal include non-negative profit conditions for the producers, that are linearised using binary expansion. The final problem is in fact minimisation of the duality gap, subject to primal and dual constraints. This approach does not require uplift payments, and all the generators engaged have non-negative profits.

3.4 Under Power Exchange Structures

In a Power Exchange structure, market participants can submit bids for the day-ahead following the rules of the specific exchange. The exchange then applies a market clearing process for each of the time periods. Different exchanges allow different type of bids, for example hourly bids, flexible hourly bids, block bids and linked block bids (Nordpool),(Ravn, 2010). Some of the reasons to incorporate the last two types of bids are to represent non-convexities, such as minimum up and down times. For instance a thermal unit may prefer to run continuously over a time period and prefer to bid in a block format. Exchanges with different bid formats may also have different processes to clear the market and settlement rules.

The problem with block bids is that an equilibrium price may not exist. Figure 3.7 shows a continuous hourly sell bid represented by the section $q_1 - q_2$ and a block bid ranging from $q_2 - q_3$. A block bid is an all or nothing decision. If the block bid is accepted then the corresponding price would be less than what the generator's bid (p_1), as shown in Figure 3.8. If the block bid is rejected, then the corresponding price is higher than the generator's bid (p_2), which seems not logical since the generator would be losing some profits, as shown in Figure 3.9. The paradox of rejecting a block bid even though it was a profitable bid is widely discussed by Meeus, Verhaegen, and Belmans (2009) and Ravn (2010). Meeus, Verhaegen, and Belmans (2005) offers a pricing mechanism for the electricity markets with block bids.

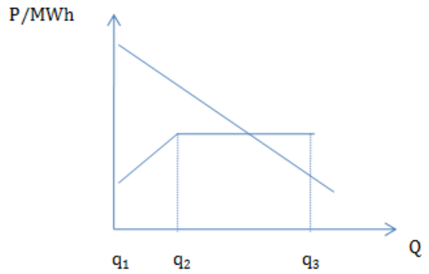

Figure 3.7: Bids Received

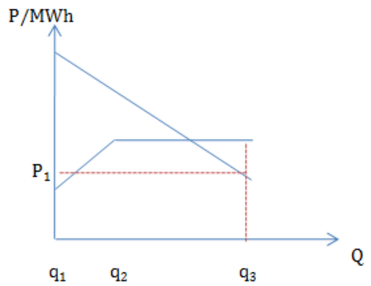

Figure 3.8: Block Bid Accepted

3.5 Pricing with Network models

The electricity industry has four main sectors: generation, transmission, distribution and retail. The power plants are not always placed where the demand is needed; therefore transmission and distribution cables are needed to transport the electricity to the end users. The electricity injected into the grid will take all available paths between the generator and the end user. The

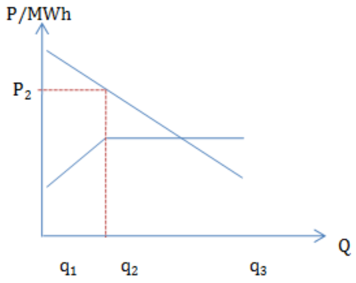

Figure 3.9: Block Bids Rejected

flow will not follow one specific path, but a path that will depend on the relative resistance between two lines. This can have nasty consequences, since the actions of one participant in one node will definitely affect other participants and can also modify the prices in the grid.

Most of the electricity markets have chosen to have either a uniform marginal pricing, a nodal marginal pricing or a zonal marginal pricing (Ding and Fuller, 2005). The first one ignores the idea of a network and reduces the problem to the case where there are no transmission constraints and no losses. This approach provides a single price for all nodes on the network. In theory, nodal pricing is the best scheme to reflect the real cost of electricity and its transmission, since it accounts for the physical laws of electricity flow in the network as well as losses. The prices across the grid will be the same as long as there is no congestion in the lines. As soon as one line is congested, then the prices in the nodes will be different (Hsu, 1997). Like the uniform marginal pricing, the zonal marginal pricing also ignores the physical laws and the power flow in the grid but within a given area. The prices obtained are the same within any given zone. Because of the complexity in calculating nodal marginal prices, many models in the literature simplify their approach by not taking into account network constraints and power flow (Doorman and Nygreen, 2003).

3.5.1 The physical laws

Assume a three nodes network that is interconnected by three transmission lines as depicted in Figure 3.10, where Node A has two generators (G1, and G2), and Node B has a third generator (G3), and demand is situated at Node C.

If there are no restrictions on the line capacities, the prices at Node A, B and C will be the same (Schweppe, Caraminis, Tabors, and Bohn, 1988). The power injected at Node A will flow not only through line AC, but also through line AB and then BC to be withdrawn at Node C. Similarly the power injected at Node B will follow two routes: BA-AC and BC. Ignoring for now the effect of losses and assuming as well that all lines have equal impedance and length, when generators at Node A inject power, it will follow two paths. Path AB-BC has two times the impedance of the second path, namely AC. The flow divides inversely to impedance, so when generators at Node A supply to node C, the flow across the path AB-BC is half that of the path AC. That is, two thirds will flow on path AC and one third will flow on path AB-BC. Similarly, when the generator at node B produces energy, the impedance in the path BA-AC has two times the impedance of path BC. So we have that two thirds of the power will flow through path BC and one third will flow through path BA-AC.

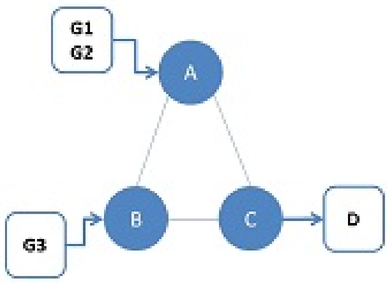

Figure 3.10: Three nodes network

As an example, assume that the load is 75MW and that generators at Node A can produce up to 60 MW for a price of 10, while the generator at Node B can produce up to 30 at a price of 15. Then, the electricity will flow as shown in Figure 3.11. The power will flow through the lines according to their reactance. In a DC model, like the one assumed in this work, the reactance of a line is assumed to be proportional to its resistance. The corresponding flows in Line AB, BC, and AC are 15, 30 and 45 respectively.

The cheapest generator is engaged first and the price is set by the marginal generator at 15. Since there is no congestion the prices are the same across the three nodes. However, if line AB is capacitated at 10, for example, then the line will be congested, and the total flow in

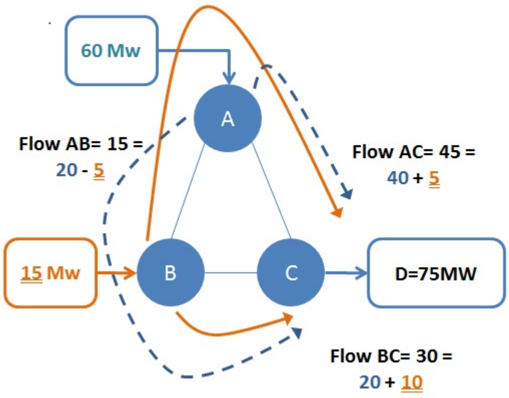

Figure 3.11: Power Flow

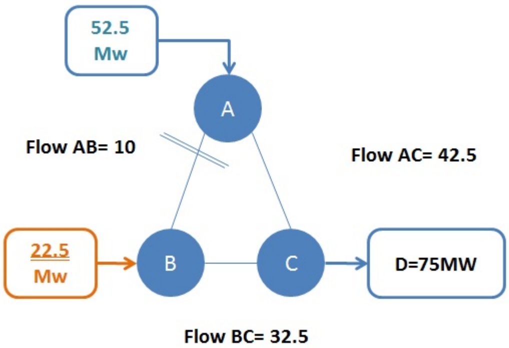

Figure 3.12: Congested Network

line AB will be limited. This will affect the prices in all three nodes, since more espensive generators may need to be engaged to reduce the power flows. In the congested area, the price will increase to the marginal cost of the local generator (Green, 2007). Indeed, if the capacity of Line AB is 10, then generators at A will produce less electricity (52.5MW), while generators at Node B will produce more (22.5MW), as shown in Figure 3.12. The prices at Node A, B and C will be different: 10, 15 and 12.5 respectively. A clear presentation of the marginal nodal pricing is covered in (Hsu, 1997).

4

Mathematical problems

The approach to obtain prices for the electricity market suggested in this dissertation is based on mathematical programming, dual theory, and the Unit Commitment and Dispatch Problem (UCDP). Therefore, in this section I will cover briefly some fundamentals on linear programming (LP), the primal and dual problems, after highlighting the connexions between economics and mathematical programming. Then, I will explain how the use of dual prices are only optimal when there is no duality gap. Next, I will present the UCDP as a mathematical problem without transmission constraints and in a network with transmission constraints. A brief presentation on block bidding, and a discussion on the problems that power exchanges face are also included. Finally, I will highlight what constraints in the model generates the non-convexities.

4.1 Mathematical Programming and Economics

There is a link between the theory of economic equilibrium and mathematical programming. Back in 1990, Scarf (1990) reminded us on their common features. In a decentralised economy, equilibrium prices will equilibrate the demand and supply for the different commodities. It is assumed that the goal of a supplier is to choose resources and production plans so as to maximise profit, while the goal of a consumer is to maximise their benefit or consumption satisfaction. The objective in a mathematical programming problem is to optimise, maximise or minimise, a function of several variables subject to a set of constraints.

If the economy is in equilibrium, and a new activity appears, then by using microeconomic analysis we can decide if the new activity should be used or not in the economy. Economic theory states that the new activity should be included in the economy if by doing so, all consumers can be made better off. This will happen if and only if the new activity makes a positive profit at the current equilibrium prices.

The most common tool to solve a linear mathematical programming problem (LP) is the simplex method. Scarf describes how each step in the simplex method can have an economic interpretation aligned to the economic theory, since the latter searches for decentralised prices that equilibrate the supply and demand for factors of production. He states "...at each step of the simplex method a trial solution to the linear program is proposed. To test for optimality of this solution, we find those prices that yield a profit of zero for the activities in use, and use them to calculate the profitability of the remaining activities. The trial solution is optimal if none of the remaining activities make a positive profit; if one of them is profitable, we simple increase the level of its use from zero, making compensating changes in the previous activity levels until one of them falls to zero. The algorithm continues until a trial solution is found that passes the pricing test for optimality".

Indeed, as we will cover in the next section, the solutions from the primal and dual problems have useful economic interpretations in terms of obtaining prices for the economy. However, the linear programming problems, as well as the classic economic theory are based on assumptions of convexities and constant returns to scale. Economies of scale, specially increasing returns to scale, are an important characteristic in modern economics. As mentioned before, the electricity market has several non-convexities due to the indivisibilities presented in the production possibility sets, due to increasing returns to scale, amongst others. In that respect, another type of programming has to be set in place: integer programming (IP).

Scarf (1960) also argues that in an integer program, the test for optimality is not applicable. In a linear program, given a feasible solution, we can find the prices that will grant a zero profit. If at these prices all the remaining activities make a loss or break even, then the feasible solution found is an optimal solution. However, in several IP problems, there is not such a vector of prices where the activities used in the optimal solution earn zero profits, while the activities not used earn negative profits. In addition, if a new activity, that can be added only at integer level, is presented in an IP problem at equilibrium, the profitability of the new activity is not a necessary nor a sufficient condition to decide to include it into the economy. An improvement in the objective function may require a mix of profitable and unprofitable activities. In a later paper, Scarf (1994) clarifies how the problem may arise. Even though a new activity can make a positive profit at the old prices, it is not scheduled for use at any discrete level to achieve a Pareto improvement.

Pricing mechanisms in a linear programming problem are straight forward. However, when the problem is integer or non-linear, the pricing mechanisms become challenging.

4.2 Primal and Dual problems

Every linear programming (LP) problem is linked to another linear programming problem, a dual problem. When one of them is solved, we implicitly solve the other one. If we have the following primal problem

$$\begin{aligned} &\max_x \sum_{j=1}^{n} c_j x_j \\ &\text{subject to} \\ &\sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad i = 1,2,\ldots,m \\ &x_j \geq 0 \quad j = 1,2,\ldots,n \\ &x_j \in \mathbb{R} \end{aligned} \tag{4.1}$$

then the dual problem is written as

$$\begin{aligned} &\min_y \sum_{i=1}^{m} b_i y_i \\ &\text{subject to} \\ &\sum_{i=1}^{m} y_i a_{ij} \geq c_j \quad j = 1,2,\ldots,n \\ &y_i \geq 0 \quad i = 1,2,\ldots,m \\ &y_i \in \mathbb{R} \end{aligned} \tag{4.2}$$

The constraints in one problem, correspond to the variables in the other problem. The constraint vector **b**, that contains $[b_1, b_2, ..., b_m]$, from the primal problem above, produces the objective function of the dual problem and vice versa. The inequality constraints are reversed, that is, if the primal has " $\leq$ " inequalities, the dual will have " $\geq$ " inequalities.

In a Linear programming (LP) problem with constraints, the shadow price of the ith constraint is the amount by which the optimal value of the problem is improved when the right hand side of that constraint is increased by one. This will hold if the current basis of the problem remains optimal. The shadow prices are also known as dual variables, or marginal prices. They can be often interpreted as the price of the resource availability of that constraint. That is, they provide the marginal value of additional amounts of the ith resource when all resources are allocated optimally. We could say the the primal is a resource allocation problem, while the dual represents the resource valuation problem.

Weak Duality

Given any feasible solution to the primal problem, and any feasible solution to the dual, the value function for the dual problem will be equal or greater than the value function for the primal problem. That is, if x' and y' are feasible solutions to the primal and dual problem respectively, then by weak duality we can write that $\sum_{j=1}^{n} c_j x'_j \leq \sum_{i=1}^{m} b_i y'_i$.

Given that $y'_i \geq 0$, when we multiply the ith primal constraints in problem 4.1 by y'_i and adding, we obtain the inequality

$$\sum_{i=1}^{m} \sum_{j=1}^{n} y'_i a_{ij} x'_j \leq \sum_{i=1}^{m} b_i y'_i.$$

Similarly noting that $x'_j \geq 0$ and multiplying the jth dual constraint from the dual problem 4.2 by x'_j and adding, we get the inequality

$$\sum_{j=1}^{n} \sum_{i=1}^{m} x'_j a_{ij} y'_i \geq \sum_{i=j}^{n} c_j x'_j.$$

Taking these two results together, we have that

$$\sum_{j=1}^{n} c_j x'_j \leq \sum_{i=1}^{m} x'_j a_{ij} y'_i \leq \sum_{i=1}^{m} b_i y'_i.$$

That is the objective function value for the primal problem is $\leq$ the objective function value for the dual problem, i.e. $\sum_{j=1}^{n} c_j x'_j \leq \sum_{i=1}^{m} b_i y'_i$.

Optimality Property

The optimality property states that if there is a feasible solution to the primal problem and the dual problem such as the objective function values are the same, then the solutions are optimal to each problem. In other words, if the primal problem has a feasible solution given by x_j^* for $j = 1,2,...,n$, and the dual problem has also a feasible solution given by y_i^* for $i = 1,2,...,m$ such as $\sum_{j=1}^{n} c_j x_j^* = \sum_{i=1}^{m} b_i y_i^*$, then x_j^* is an optimal solution for the primal problem and y_i^* is an optimal solution for the dual problem.

Strong Duality

The concept of strong duality states that if the primal problem has an optimal solution, then the dual problem also has an optimal solution, and the objective value function is the same. In other words, if the primal problem has an optimal solution given by $x^* = (x_1^*, x_2^*, ..., x_n^*)$, then the dual problem has also an optimal solution $y^* = (y_1^*, y_2^*, ..., y_m^*)$ such as $\sum_{j=1}^{n} c_j x_j^* = \sum_{i=1}^{m} b_i y_i^*$.

4.2.1 Lagrangean relaxation

Duality theory is based on a Lagrangean function. The Lagrangean formulation involves creating a new problem by relaxing some constraints of the initial problem and adding them to the objective function with the use of Lagrangean multipliers (λ_i). These multipliers act as a penalisation for any infeasibility with respect to the relaxed constraints.

The Lagrangean relaxation has been widely used to solve linear problems, but it can be also applied for solving non-linear problems as well as non-convex problems. The relaxation is based on duality theory and is called Lagrangean Duality. From the Lagrangean Function a dual function and a dual problem can be formulated to obtain a solution to the initial problem.

Let us consider the the following primal problem

$$
\begin{aligned}
&\min_{x} f(\mathbf{x}) \\
&\text{subject to} \\
&g_i(\mathbf{x}) \leq 0, \quad i = 1,2,\ldots,m
\end{aligned}
\tag{4.3}
$$

where $\mathbf{x} = [x_1, x_2, ..., x_n]^T$. The Lagrangean formulation is written as:

$$L(\mathbf{x},\lambda) = f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i g_i(\mathbf{x}) \tag{4.4}$$

The dual function provides a lower bound for the primal problem 4.3, and it is written as:

$$\begin{aligned} h(\lambda) &= \min_x(\mathbf{x},(\lambda)) \\ &= \min_x f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i g_i(\mathbf{x}) \end{aligned} \tag{4.5}$$

Given that the solution to the dual function provides a lower bound for $f(\mathbf{x})$, the greatest lower bound should occur at the maximum value of $h(\lambda)$. The dual problem is then to find the maximum lower bound:

$$\max_{(\lambda)} h((\lambda)) \tag{4.6}$$

The ideal situation is to have $L(\mathbf{x}^*,\lambda^*) = F(\mathbf{x}^*)$, when there is no duality gap. The difference between the objective value from solving the dual problem $L(\mathbf{x}^*,\lambda^*)$, and the objective value of the original problem $F(\mathbf{x}^*)$ is called duality gap. Figure 4.1 shows a geometric interpretation of the dual problem and the multipliers. We can see that the red line contains the multiplier that makes the dual problem reach the same optimal value as the primal problem, that is $p*$. In this case there is no duality gap and there is strong duality.

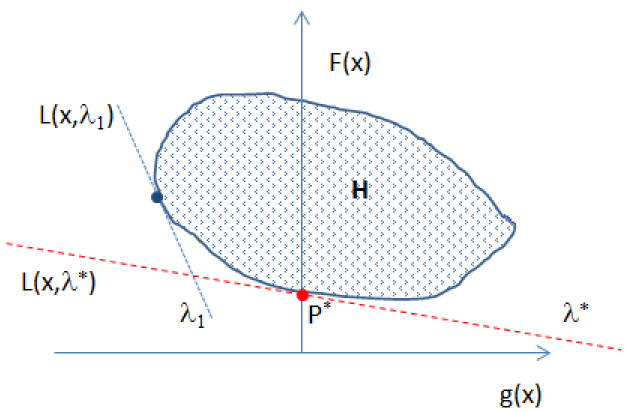

Figure 4.1: Geometric Interpretation of Duality

4.2.2 Non-Linear Cases

As mentioned before, finding prices in integer and non-linear problems is challenging. In an integer programming (IP) problem, the dual values do not necessarily have an economical interpretation as in the LP problems. In a IP problem the last constraints from problems 4.1 and 4.2 are modified to $x_j \in \mathbb{Z}$ and $y_i \in \mathbb{Z}$ respectively, where $\mathbb{Z}$ is a set of integer values.

If we relax the integrality constraint, the problem is known as a relaxed IP, and the tightest lower bound for the relaxed IP problem is given by the dual solution. The solution for the IP problem will not necessarily give the same objective value as the relaxed IP problem, which creates a duality gap.

Figure 4.2 shows a graphical representation of the duality gap when the set H has non-convexities.

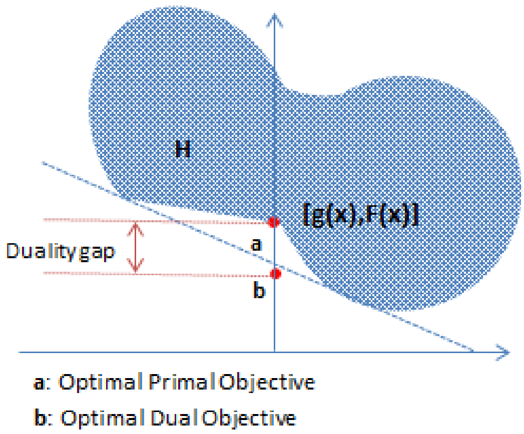

Figure 4.2: Geometric Interpretation of Duality Gap

Duality theory for integer programming problems, mixed integer programming problems, and non-linear problems has been studied by several authors. The studies could be divided in two, those that propose algorithms to solve the problem and obtain dual prices (Balinski and Baumol, 1968; Bertsekas, 1995; Geoffrion, 1971, 1974; Gould, 1969; Wolsey, 1981); and those that try to make an economic interpretation of the dual values (Alcaly and Klevorick, 1966; Gomory and Baumol, 1960; Sen and Genc, 2008).

Wolsey (1981) studied different methods to solve IP problems and the dual optimal price function that they generate. He uses the price function as a set of prices is used in a LP problem. This approach is complex, and there is still the need to find dual prices that can have some economic interpretation. Williams (1996) examines methods to produce a dual for an IP problem, for instance Gomoroy and Boumal Prices, Lagrangean Duals and surrogate duals. He concludes that mixed integer linear problems (MILP) do not have dual prices with the same characteristics and interpretations as the dual prices from a linear problem (LP).

Recent work related with electricity markets have tried to make a use an interpretation of the dual values for electricity prices Conejo and Ruiz (2011); Fuller (2008); O'Neill et al. (2005), but the issue remains under research. Finding appropriate dual prices, or shadow prices for MILP is still a challenge due to the fact that the objective function can be discontinuous and/or non-convex.

4.3 The Walrasian Equilibrium and a Pool Market

Prior to deregulation, there was an entity that would solve the UCDP and would stablish the quantities that each generator should produce. After deregulation each generating company as well as the consumers (or distribution companies) would act as profit maximizers. The result of an independent maximimising problem of the market participants is not necessary the same as the solution that a centralised market would obtain. This section is a summary of the work presented by Motto, Galiana, Conejo, and Huneault (2002b)

Assume for now strictly convex cost functions and convex feasible sets. Assume further that λ_{qi} is the price at node i and q_{gi} is the amount of electricity generated by producer i. Its production is an amount between its minimum capacity $\underline{q}_{gi}$ and its maximum capacity $\bar{q}_{gi}$. The cost function of each generators will depend on the quantity produced, and it is expressed as $C_i(q_{gi})$. Then, each generating company will maximise their profit by solving the following problem

$$\begin{aligned}&\max(\lambda_{qi}q_{gi} - C_i(q_{gi}))\\&\text{subject to}\\&\underline{q}_{gi} \leq q_{gi} \leq \bar{q}_{gi}\\&q_{gi} \in \mathbb{R}.\end{aligned} \tag{4.7}$$

From the consumer point (or from the distribution companies), let us assume the following: strictly concave feasible benefit function $B_i(q_{di})$ that depends on the amount of electricity consumed (q_{di}) by the ith consumer, as well as a compact set. Given that λ_{qi} is the price at node i, the maximising problem that the consumers or distribution companies face is

$$\max B_i(q_{di}) - \lambda_{qi}q_{di} \tag{4.8}$$

A Walrasian equilibrium $(\lambda^*_{qi}, q^*_{gi}, q^*_{di})$ will exist for all nodes (i) in the network (N) if and only if there is a vector of prices λ that can balance the electrity production and at the same time every agent maximises their surpluses. In other words, there is a Walrasian equilibrium if the following holds:

a) For all $i \in N$, q^*_{gi} solves problem 4.7 given λ^*_{qi} is maximising the producer's surpluses;

b) For all $i \in N$, q^*_{di} solves problem 4.8 given λ^*_{qi} is maximising the consumer's surpluses; and

c) $q^*_{gi} = q^*_{di}$

If we look a centralised electricity pool, the system operator faces the following maximisation social welfare problem:

$$\begin{aligned}&\max \sum_{i\in N} B_i(q_{di}) - \sum_{i\in N} C_i(q_{gi})\\&\text{subject to}\\&q^*_{gi} = q^*_{di}\\&\underline{q}_{gi} \leq q_{gi} \leq \bar{q}_{gi} \quad foralli \in N\end{aligned} \tag{4.9}$$

The corresponding Lagrangean Function is written as

$$\mathcal{L} = \sum_{i \in N} \left[B_i(q_{di}) - C_i(q_{gi}) + \lambda^*_{qi}(q^*_{gi} - q^*_{di}) \right] \tag{4.10}$$

and the dual function is

$$\mathcal{H} = \sum_{i \in N} max(\lambda_{qi} q_{gi} - C_i(q_{gi}))) + \sum_{i \in N} \max(B_i(q_{di}) - \lambda_{qi} q_{di}) \tag{4.11}$$

Only when the optimal solution to problem 4.9 maximises the surplus for every independent market participant, problem 4.7 and problem 4.8 can be the same as problem 4.9. If that is the case, then we can value the dual function at the optimality point composed by λ^*_{qi} , q^*_{gi}, q^*_{di} and obtain

$$\begin{aligned} \mathcal{H} &= \sum_{i \in N} (\lambda^*_{qi} q^*_{gi} - C_i(q^*_{gi}))) + (B_i(q^*_{di}) - \lambda^*_{qi} q^*_{di}) \\ &= B_i(q^*_{di}) - C_i(q^*_{gi} + \lambda^*_{qi}(q^*_{gi} - q^*_{di}) \end{aligned} \tag{4.12}$$

Given that $q_{gi} = q_{di}$ as stated in problem 4.9 , the dual function $\mathcal{H}$ is simplified into

$$\mathcal{H} = B_i(q^*_{di}) - C_i(q^*_{gi}) \tag{4.13}$$

This implies that there is no duality gap, and as a consequence, the dual prices can be use as optimal prices to the market.

4.4 Unit Commitment and Dispatch Problem UCDP

The Unit Commitment and Dispatch Problem (UCDP) is a well-known problem. Its objective is not only to find the least cost generators that should be committed to produce electricity but also to find the quantities they should produce in order to fulfil the demand, subject to a set of operational and security constraints.

In other words, the goal is to minimise the total production cost of electricity across all the generators, during a specific time period, where the total electricity produced across all generators during a specific time must be equal to the demand for that specific time period, and for each time periods considered. The production from each generator should be within its feasible set. The feasible set contains technical and physical constraints, such as allowable power outputs, ramping limits, minimum up and maximum down time constraints.

Although the dispatch problem is continuous, the unit commitment problem involve discrete variables, which makes the problem a Mixed Integer Problem (MIP).

4.4.1 A Mixed Integer Programming Problem (MIP)

Assume that there are k different technologies than can produce electricity. Each technology in turn has p plants that can be committed in the production. The cost function for each technology depends on two variables: the number of plants committed (z) and the amount of electricity produced by each plant(q). Assuming that the plants within a technology have the same costs values, the cost function for each technology i is given by $C_i(q_i, z_i) = \sum_j^{p_i} FC_i z_{ij} + \sum_j^{p_i} VC_i q_{ij}$, where FC represents the fixed costs incurred if plant j within technology i is committed to production, and VC is the variable cost per unit q produced by plant j within technology i.

Then, the system operator faces the following problem[1] for a given time period:

$$\min_{q,z} \sum_{i=1}^{k} \sum_{j=1}^{p_i} FC_i z_{ij} + \sum_{i=1}^{k} \sum_{j=1}^{p_i} VC_i q_{ij} \tag{4.14}$$

subject to

$$\sum_{i=1}^{k} \sum_{j=1}^{p_i} q_i = D \tag{4.15}$$

$$q_{ij} \geq \underline{x}_{ij} z_{ij} \quad i = 1, \ldots, k; j = 1, \ldots, p_i \tag{4.16}$$

$$q_{ij} \leq \bar{x}_{ij} z_{ij} \quad i = 1, \ldots, k; j = 1, \ldots, p_i \tag{4.17}$$

$$z_{ij} \in (0, 1), \tag{4.18}$$

Equation 4.15 is the balance constraint where the total amount produced across the different technologies and plants ($\sum_{i=1}^{k} \sum_{j=1}^{p_i} q_i$) should equal the demand (D). Constraint 4.16 and 4.17 limit the electricity generated to the production possibility set for plant j within technology i. That is, the amount produced by a given plant within a technology (q_{ij}) should be greater or

[1] For simplicity, we focus on a stylised model leaving out the time dimension as well as ramp rates, and other security constraints. In addition, the model focuses on one time period with an inelastic demand.

equal to its minimum requirement ($\underline{x}$), and cannot exceed its capacity or maximum output ($\bar{x}$). The last constraint 4.18 specifies that the plants are either engaged or not, making this problem a Mixed Integer Problem (MIP).

For simplicity we will speak about technologies, knowing that within them there are plants, but we will refer to them as technology i that belongs to the set of all technologies K $(i = 1...k)$.

4.4.1.1 Demand function

The problem presented in the previous section can include a demand function. Let us define $D^{(-1)}(QDemand)$ as the inverse demand function for the market, and $Price$ the price associated to that function. We can write the problem as follows:

$$\max_{q,z} \int_0^{QDemand} D^{(-1)}(x)d(x) - \sum_{i=1}^{k}\sum_{j=1}^{p_i} FC_i z_{ij} - \sum_{i=1}^{k}\sum_{j=1}^{p_i} VC_i q_{ij} \tag{4.19}$$

subject to

$$\sum_{i=1}^{k}\sum_{j=1}^{p_i} q_i = QDemand \tag{4.20}$$

$$Price = D^{(-1)}(QDemand) \tag{4.21}$$

$$q_{ij} \geq \underline{x}_{ij} z_{ij} \quad i = 1, \dots, k; j = 1, \dots, p_i \tag{4.22}$$

$$q_{ij} \leq \bar{x}_{ij} z_{ij} \quad i = 1, \dots, k; j = 1, \dots, p_i \tag{4.23}$$

$$z_{ij} \in (0, 1), \tag{4.24}$$

where the objective function now includes the demand side. The objective function now maximizes the usual economic welfare function. Constraints 4.20 and 4.21 tell us that at a given price $Price$, the total amount demanded will be equal to the total amount supplied.

4.4.2 UCDP in a Network

A network model will include more constraints into the model. Not only the transmission line capacity constraint is added, but also the node rule equations (or Kirchhoff's junction rule), the loop rule equations, and the energy balance constraint (Bjørndal, Jörnsten, and Rud, 2008). The node rule constraint, refers to the rule that all that comes into a node should be equal to that of what goes out of it; the loop rule, states that the arithmetical sum of the potential differences across all components around any loop should be zero; and the energy balance constraint guarantees that all the energy that is produced is consumed.

For notational and mathematical simplicity we have assumed a lossless model, in one period of time with an inelastic given demand. Following the notation from 4.4.1, let us assume that there are k technologies with different variable costs (VC) incurred per unit q produced, different fixed costs (FC) incurred per plant z committed to production, different minimum output $(\underline{x})$ and different capacities $(\bar{x})$. Assume also that each technology has a maximum number of plants that can engage into production Z; then, the SO will solve the following cost minimisation UCDP:

$$\begin{aligned}
\min_{q_i,z_i} \quad & \sum_{i=1}^{k} VC_i q_i^S + \sum_{i=1}^{k} FC_i z_i && (4.25)\\
\text{subject to} \quad & && \\
q_r &= \sum_{i\in G_r} q_i^s - q_r^d, \quad r=1,\ldots,n && (4.26)\\
q_r &= \sum_{r\neq s} q_{rs}, \quad r=1,\ldots,n-1 && (4.27)\\
\sum_{rs\in L_l} q_{rs} &= 0, \quad l=1,\ldots,m-n+1 && (4.28)\\
\sum_r q_r &= 0, && (4.29)\\
q_{rs} &\leq LineCap_{rs}, \quad r\neq s, r=1,\ldots,n-1, j=1,\ldots,n-1 && (4.30)\\
q_i^S &\geq \underline{x} z_i, \quad i=1,\ldots,k && (4.31)\\
q_i^S &\leq \bar{x} z_i, \quad i=1,\ldots,k && (4.32)\\
z_i &\in Z_i, \quad i=1,\ldots,k && (4.33)
\end{aligned}$$

where q_r is the net injection at node r , which is in turn given by the total energy produced by generators at that node $(\sum_{i\in G_r} q_i^s)$ minus the quantity demanded at that node q_r^d . Equation 4.27 represents the node rule, while equation 4.28 and 4.29 represent the loop flow and energy conservation rule respectively. Equation 4.30 establishes that the power flow running between node r and s should be less than or equal to the capacity of the line joining those nodes $LineCap_{rs}$.

Let us use the three-node network presented in Section 3.5.1, and reproduce the three nodes network presented before in Figure 4.3.

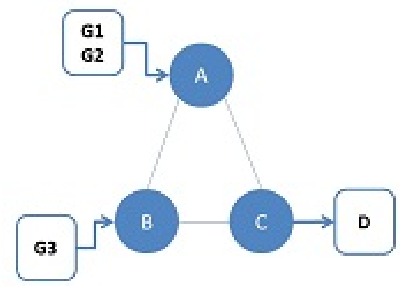

Figure 4.3: Three nodes network

For this type of network with two generators at Node A, one generator at Node B and all the demand at Node C, the model can be written as follows:

$$\underset{q_i,z_i}{\text{minimise}} \quad \textstyle\sum_{i=1}^{3} VC_i q_i^S + \sum_{i=1}^{3} FC_i z_i \tag{4.34}$$

$$\text{subject to} \tag{4.35}$$

$$\sum_{i\in G_A} q_i^s - q_A^d = q_{AB} + q_{AC} \tag{4.36}$$

$$\sum_{i\in G_B} q_i^s - q_B^d = q_{BC} - q_{AB} \tag{4.37}$$

$$\sum_{i\in G_C} q_i^s - q_C^d = -q_{BC} - q_{AC} \tag{4.38}$$

$$q_{AC} = q_{AB} + q_{BC} \tag{4.39}$$

$$\sum_{g\in G_A} q_i^s - q_A^d + \sum_{g\in G_B} q_i^s - q_B^d + \sum_{g\in G_C} q_i^s - q_C^d = 0 \tag{4.40}$$

$$q_{AB} \leq LineCap_{AB} \tag{4.41}$$

$$q_{BC} \leq LineCap_{BC} \tag{4.42}$$

$$q_{CD} \leq LineCap_{CD} \tag{4.43}$$

$$q_i^S \leq \bar{x} z_i, \quad i = 1,2,3 \tag{4.44}$$

$$q_i^S \geq \underline{x} z_i, \quad i = 1,2,3 \tag{4.45}$$

$$z_i \in Z_i, \quad i = 1,2,3 \tag{4.46}$$

where in turn the flows in each transmission are given by the following equations:

$$q_{AB} = \frac{1}{3}\left(\sum_{i \in G_A} q_i^s - q_A^d\right) - \frac{1}{3}\left(\sum_{i \in G_B} q_i^s - q_B^d\right) \tag{4.47}$$

$$q_{AC} = \frac{2}{3}\left(\sum_{i \in G_A} q_i^s - q_A^d\right) + \frac{1}{3}\left(\sum_{i \in G_B} q_i^s - q_B^d\right) \tag{4.48}$$

$$q_{BC} = \frac{1}{3}\left(\sum_{i \in G_A} q_i^s - q_A^d\right) + \frac{2}{3}\left(\sum_{i \in G_B} q_i^s - q_B^d\right) \tag{4.49}$$

4.5 Lagrangean Relaxation in UCDP

The technique that has been widely used to solve the Unit Commitment and Dispatch Problem is the Lagrange Relaxation (Sioshansi, O'Neill, and Oren, 2008), where the balance constraint is relaxed and inserted into the objective function, as explained previously in section 4.2.1. The Lagrange multiplier could be interpreted as the price for the commodity if and only if there is no duality gap. In the cases where a duality gap exists, the Lagrange multiplier does not reflect an optimal price, but just a dual variable. This is because some generators will not be willing to produce if the price is set to the dual variable (Madrigal and Quintana, 2000; Motto and Galiana, 2002; Motto, Galiana, Conejo, and Huneault, 2001).

There is extensive literature on the use of this tool in the electricity market, the interested reader is referred to Sheble and Fahd (1994) and Baldick (1995) for further information about its applications.

4.5.1 Dual prices from Lagrangean Relaxation:Criticism

Implementing prices from the Lagrange Relaxation in an auction has several problems and disadvantages (Dekrajangpetch, Sheble, and Conejo, 1999). Some of them are:

- Difficulties to find an optimal price in the presence of a duality gap.
- Multiple solutions in the presence of degeneracy in the balance constraint.
- Heuristics applied to find the Lagrange multiplier. Some authors argue that the sub-gradient optimization approach is an unstable technique.

- Revenue deficiency. If prices are set equal to the dual variable then there is a cost that will not be recovered by some participants. It has been proofed that the cost that is not recovered equals the duality gap (Madrigal and Quintana, 2001).

4.6 Power Exchanges and Surrogate relaxation

In Europe the most common format for the electricity market is a Power Exchange. These exchanges apply different heuristics to clear their market, and the heuristic results are usually competitive for simple orders but not for block orders Meeus, Verhaegen, and Belmans (2005). The bids can be simple bids, or block orders. The block orders are a "all or nothing" condition, and makes the problem move from a linear problem to a discrete one with integer constraints. A block order covers more than one time period and it is indivisible. In addition it usually has an average price limit.

Let consider the problem with supply (S) offers, and demand (D) bids, where they can submit single bid/offers, or block bid/offers. For the single bids/offers let: P^S_{ih} and Q^S_{ih} be the price and quantity of a single offer i respectively; and P^D_{jh} and Q^D_{jh} be the price and quantity of a single bid j respectively. For the block bids/offers: let P^S_k, Q_{kh} and H_k represent the average price and quantity of block offer k to be delivered in time period h for H_k periods; and similarly for the demand side let P^D_l, Q_{lh} and H_l represent the average price and quantity of block bid l to be delivered in time period h for H_l periods. Let us also use q_{ih}, q_{jh}, q_{kh}, q_{lh} as the quantities accepted of order i, j, and block orders k, l at a specific time period h respectively. Finally, in order to write the model addressing the block constraints for the demand side and supply side, let z_k and z_l be binary variables that will be set to 1 if the block order k, l has been accepted, other wise its value is zero.

The problem is then a mixed integer linear problem (MILP) than can be written as follows:

$$\max_{q_{ih}, q_{jh}, q_{kh}, q_{lh}, z_k, z_l} \quad \sum_h (\sum_j q_{jh} P^D jh + \sum_l q_{lh} P^D lh - \sum_i q_{ih} P^S ih - \sum_k q_{kh} P^S kh) \tag{4.50}$$

subject to

$$\sum_j q_{jh} + \sum_l q_{lh} = \sum_i q_{ih} + \sum_k q_{kh} \quad \forall h \tag{4.51}$$

$$q_{ih} \leq Q_{ih} \quad \forall i, h \tag{4.52}$$

$$q_{jh} \leq Q_{jh} \quad \forall j, h \tag{4.53}$$

$$q_{kh} = Q_{kh} z_k \quad \forall k, h \tag{4.54}$$

$$q_{lh} = Q_{lh} z_l \quad \forall l, h \tag{4.55}$$

The first constraint is the balance constraint where demand equals supply, while the second and third constraint specify that the amount accepted should be less than or equal to the quantities bid/offered. The last two constraints are the block constraints that translates the ”all or nothing” characteristic of the block bids/offers.

Surrogate Relaxation

Lagrangean relaxation and finding the Lagrangean multipliers is a well researched and developed field. However, there are other types of relaxation that haven not been explored as much as the Lagrangean relaxation. That is the case of the surrogate relaxation. This type of relaxation provides a better objective bound than the Lagrangean relaxation Karwan and Rardin (1984).

The surrogate relaxation consist of adding the constraints to be relaxed in just on constraint. Each constraint relaxed will have a multiplier assigned. So, if we have an integer programming problem written as

$$\begin{aligned} &\max_x cx \\ &\text{subject to} \\ &Ax \leq b \\ &Dx \leq f \\ &x \in \mathbb{Z} \end{aligned} \tag{4.56}$$

and we would like to surrogate relaxed constraints in $Dx \leq d$, then the problem becomes

$$
\begin{aligned}
&\max_x cx \\
&\text{subject to} \\
&Ax \leq b \\
&\lambda Dx \leq \lambda f \\
&x \in \mathbb{Z}
\end{aligned}
\tag{4.57}
$$

In the surrogate relaxation, the multipliers λ work as a weighting of the constraints. If the value for a multiplier is large, the constraint is satisfied at an expense of another one. If the value for the multiplier is zero, it means that the constraint is dropped. As with the Lagrangean relaxation, we want to find those multipliers that give us the tightest bound and gets us the value function as close as possible to the value function of the initial problem. In fact, the bounds from a surrogate relaxation are tighter than those from the Lagrangean relaxation.

For the problem 4.56, the Lagrangean relaxation for the constraint $Dx \leq d$ is written as:

$$
\begin{aligned}
&\max_x cx - \mu(Dx - f) \\
&\text{subject to} \\
&Ax \leq b \\
&x \in \mathbb{Z}
\end{aligned}
\tag{4.58}
$$

So, while for the Lagrangean relaxation we have that $max\{cx : Dx <= f, x \in conv(Ax <= b, x \in \mathbb{Z})\}$, for the surrogate relaxation we have $max\{cx : x \in conv(Ax <= b, \lambda Dx <= \lambda f, x \in \mathbb{Z})\}$. The latter is a tighter bound than the former.

For the problem 4.50 we could apply the surrogate relaxation to the balance constraint to obtain the following problem

$$\max_{q_{ih},q_{jh},q_{kh},q_{lh},z_k,z_l} \quad \sum_h \Big(\sum_j q_{jh} P^D jh + \sum_l q_{lh} P^D lh - \sum_i q_{ih} P^S ih - \sum_k q_{kh} P^S kh\Big) \tag{4.59}$$

subject to

$$\sum_h \lambda_h \Big(\sum_j q_{jh} + \sum_l q_{lh}\Big) = \sum_h \lambda_h \Big(\sum_i q_{ih} + \sum_k q_{kh}\Big) \tag{4.60}$$

$$q_{ih} \leq Q_{ih} \quad \forall i,h \tag{4.61}$$

$$q_{jh} \leq Q_{jh} \quad \forall j,h \tag{4.62}$$

$$q_{kh} = Q_{kh} z_k \quad \forall k,h \tag{4.63}$$

$$q_{lh} = Q_{lh} z_l \quad \forall l,h \tag{4.64}$$

4.7 Sources of non-convexities in the Electricity Market

Non-convexities appear in the Electricity market in each generator's cost function due to the start-up costs, or fixed costs. The minimum and maximum output constraints also add non-convexities to the problem (equations 4.16,4.17,4.31,4.32), as well as ramping rates, minimum down time and up-time requirements. In addition, the market presents non-convexities due to the indivisibilities of the units to be engaged (equation 4.18,4.33), and in the european markets they are represented by block bids (such as equations 4.54 and 4.55).

The consequences of these convexities are that that marginal prices are not always efficient, the prices obtained will be not linear, and if marginal prices are set in place, they will be sending the wrong signals to the market participants. Moreover with Mixed Integer Programming (MIP) problems such as the UCDP presented here, the dual variables cannot have a price interpretation anymore, as in a Linear Program (LP) problem.

5

Semi-Lagrangean Approach

Beltran, Tadonki, and Vial (2006) developed an approach that they called Semi-Lagrangean. They base their approach on the Lagrangean Relaxation, but it has a twist. Besides adding the relaxed constraint to the objective function, the approach leaves the relaxed constraint as an inequality constraint in the sub-problem. To clarify this, assume that A, b, and c in the following primal problem are non-negative:

$$\begin{aligned} z^* &= \min_x c^T x \\ s.t. \quad & Ax = b \\ & x \in S = X \cap \mathbb{N}^n \end{aligned} \tag{5.1}$$

In addition, assume that $X \subset \mathbb{R}^n$. As explained in section 4.2.1, the Lagrangean Relaxation consists of relaxing the linear constraint and solving the dual problem:

$$\begin{aligned} z_{LR} &= \max_\lambda \mathcal{L}_{LR}(\lambda) \\ &= \max_\lambda \{b^T \lambda + \min_x \{(c - A^T \lambda)^T x | x \in S\}\} \end{aligned} \tag{5.2}$$

The optimal solution of the Lagrangean dual will provide a lower bound for the original problem, therefore $z_{LR} \leq z^*$.

The Semi-Lagrangean relaxation can be written as:

$$\begin{aligned} z_{SLR} &= \max_\lambda \mathcal{L}_{SLR}(\lambda) \\ &= \max_\lambda \{b^T \lambda + \min_x \{(c - A^T \lambda)^T x | Ax \leq b; x \in S\}\} \end{aligned} \tag{5.3}$$

The Semi-Lagrangean problem is more constrained than the Lagrangean problem, and therefore we have that $z_L R \leq z_{SLR} \leq z^*$, which clearly makes the integrality gap smaller than in the LR.

If we consider an extreme positive value for λ, and rewrite $\mathcal{L}_{SLR}(\lambda)$ as

$$\mathcal{L}_{SLR}(\lambda) = min_x\{c^T x + (b - A^T x)^T \lambda | Ax \leq b; x \in S\} \tag{5.4}$$

then we can see that a very large penalty on the constraint $(b - A^T x)$ will make us choose x such that $Ax \geq b$. However, given that x is constrained by $Ax \geq b$, the optimal solution to the Semi-Lagrangean relaxation problem, with a large value for the multiplier, will meet the original constraint $Ax = b$.

The proof to show that the Semi-Lagrangean has no duality gap is straight forward by noting that the constraint set for the relaxation is in fact $Conv(Ax \leq b; x \in S) \cap Conv(Ax \geq b)$; that is, the solution to the Semi-Lagrangean fulfils all the constraints in the original problem. In other words, there is no duality gap.

5.1 Approach applied to UCDP

If we take problem in section 4.4.1, and apply the Semi-Lagrangean relaxation to the balance constraint (equation 4.15), then we modify the objective function (4.14) and the balance constraint (equation 4.15) to { $\min_{q,z} VC_i q_i + FC_i z_i + \lambda(\sum_i q_i - D) | \sum_i q_i \leq D$ } with $\lambda \leq 0$. Given that $\lambda \leq 0$, the problem can be rewritten as:

$$\min_{q,z} \sum_{i=1}^{k} \sum_{j=1}^{p_i} FC_i z_{ij} + \sum_{i=1}^{k} \sum_{j=1}^{p_i} VC_i q_{ij} + \lambda \left(D - \sum_{i=1}^{k} \sum_{j=1}^{p_i} q_i \right) \tag{5.5}$$

subject to

$$\sum_{i=1}^{k} \sum_{j=1}^{p_i} q_i \leq D \tag{5.6}$$

$$q_{ij} \geq \underline{x}_{ij} z_{ij} \quad i = 1, \dots, k; j = 1, \dots, p_i \tag{5.7}$$

$$q_{ij} \leq \bar{x}_{ij} z_{ij} \quad i = 1, \dots, k; j = 1, \dots, p_i \tag{5.8}$$

$$z_{ij} \in (0, 1), \tag{5.9}$$

The value for λ will provide us with a new set of prices that can be used as a price value. The prices will be high enough to cover the generator's costs and will send the right signal to the market participants. This technique is applied to an example from the literature in section 6.1.[1]

5.1.1 Approach applied to UCDP with a price sensitive demand function

Expanding the formulation in problem 4.19, we write the semilagrangean formulation as:

$$\begin{aligned} \min_{q,z} \quad & \sum_{i=1}^{k}\sum_{j=1}^{p_i} FC_i z_{ij} + \sum_{i=1}^{k}\sum_{j=1}^{p_i} VC_i q_{ij} - \int_0^{qd} D^{(-1)}(x)d(x) && (5.10)\\ & +Price*(QDemand - \sum_{j=1}^{p_i} VC_i q_{ij}) \\ \text{subject to} & && (5.11) \end{aligned}$$

$$\begin{aligned} \textstyle\sum_{i=1}^{k}\sum_{j=1}^{p_i} q_i \quad & <= QDemand && (5.12)\\ Price \quad & = D^{(-1)}(qd) && (5.13)\\ q_{ij} \quad & \geq \underline{x}_{ij} z_{ij} \quad i=1,\ldots,k; j=1,\ldots,p_i && (5.14)\\ q_{ij} \quad & \leq \bar{x}_{ij} z_{ij} \quad i=1,\ldots,k; j=1,\ldots,p_i && (5.15)\\ z_{ij} \quad & \in (0,1), && (5.16) \end{aligned}$$

This time, *Price* will provide the price at which supply will equal demand.

5.2 A second-price auction

Auctions have played a key role in the development of electricity markets. It is believed that auction formats are efficient for allocating scarce resources (Leautier, 2001). There are different types of auctions and as different countries create their own market design they also decide what type of auction mechanism is implemented in the market, if any. The two most known auctions in electricity markets are pay-as-bid and market clearing price. Each auction method has its advantages and disadvantages (Yan and Stern, 2002). California has experienced the

[1] This approach was recently published in EJOR (Araoz and Jörnsten, 2011).

faults of the pay-as-bid auction and there are several articles on that topic as well. Research on auction theory for the electricity market is a wide area and there are several reviews in the literature (Yamamoto and Tezuka, 2007).

In order to generate a pricing mechanism based on the Semi-Lagrangean market price that permits only the optimal mix of plants obtained from the UCDP to produce, we propose that the system operator might use a secondary auction in which all plant combinations that can produce the required amount D can submit their bid to the ISO taking into account the current market price. This price is calculated by the Semi-Lagreangean approach. That is, the system operator could announce the Semi-Lagrangean price and could set rules for a second price auction. These rules include that only generators that are willing to produce (i.e. they generate a profit) at the Semi-Lagrangean price can produce if they can entirely fulfill the demand, otherwise they have to join a coalition with other generators in order to produce the total amount required. Consequently, the second price auction would contain a set of single generators and/or coalitions that are able to produce the total required amount.

The bidder with the lowest costs will win the auction and will be committed to produce the agreed amount. However, even though the winner coalition has the minimum cost amongst the bidders, the cost that will be considered for calculating its profit or payment is that of the second best bidder. That is, the minimum cost coalition will win with a total cost slightly smaller than that of the second best coalition.

Let π_{WC} and $\pi_{2ndBest}$ represent the profit calculated under Semi-Lagrangean Prices for the winning coalition and the second best coalition respectively. Let also TC_{WC} and $TC_{2ndBest}$ represent the Total Cost (or bid) from the winning coalition and second best coalition respectively. Then we have that the price for the commodity to announce in the market is given by: $\frac{\pi_{WC}-\pi_{2ndBest}+TC_{WC}}{D} = \frac{TC_{2ndBest}}{D} = CommodityPrice = P^*$.

The winner coalition will be paid at the commodity price obtained from the second auction. This can be expressed simply as $\pi_{WC} = D(P* - TC_{WC})$. The rents for the coalition will be shared using a game theory algorithm. However that is out of the scope of this work and may be a further topic of research. The Commodity Price P^* will cause only the generators that can fulfil the demand at minimum cost to be committed.[1]

5.3 Extended models

5.3.1 Extended prices for capacity

In this section we expand the Semi-Lagrangean model presented in section 5.1 so as to obtain a price for the capacity engaged. The model developed by Pirnia and Fuller (2010) and Fuller (2008) is combined with the Semi-Lagrangean approach. Those models set the sum of the capacities of all plants engaged in production equal to the sum of the capacities of the optimal plants (Z^*) obtained from the UCDP. The UCDP is solved first by using a MIP. The capacity price is obtained by relaxing the sum of capacities constraint. In other words, constraint 5.8 in the previous problem will be relaxed with the Semi-Lagrangean approach. In addition, we include two more constraints to the problem. The problem is the following:

$$\min_{q,z} \quad \sum_{i=1}^{k} VC_i q_i + \sum_{i=1}^{k} FC_i z_i \tag{5.17}$$
$$+EP(D - \sum_{i=1}^{k} q_i) + CP(\sum_{i}^{K} \bar{x}_i Z_i^* - \sum_{i}^{k} \bar{x}_i z_i)$$

subject to

$$\sum_{i=1}^{k} q_i \leq D \tag{5.18}$$
$$q_i \geq \underline{x}_i z_i \quad i = 1, \ldots, k; \tag{5.19}$$
$$q_i \leq \bar{x}_i z_i \quad i = 1, \ldots, k; \tag{5.20}$$
$$\sum_{i}^{k} \bar{x}_i z_i \leq \sum_{i}^{k} \bar{x}_i Z_i^* \tag{5.21}$$
$$z_i \leq Z_i^* \quad i = 1, \ldots, k; \tag{5.22}$$
$$z_i \leq q_i \quad i = 1, \ldots, k; \tag{5.23}$$

where EP is the electricity price and CP is the capacity price. Each technology will receive EP for every unit of electricity produced and also CP for the the capacity of the plants engaged to

[1] This approach was presented at EEM10 (Araoz and Jörnsten, 2010).

produce electricity. Constraint 5.22 will ensure that the plants engaged are those from the MIP optimal solution, while constraint 5.23 guarantees that the plants engaged will be producing. The capacity payment is effectively an extra payment to the energy produced, which is paid at the Semi-Lagrangean price.

Assume that the dual variables for constraints 5.18 to 5.23 are α,β,γ,δ,φ,θ respectively. Then, by setting the first derivative with respect to q_i and z_i of the Lagrangean function to zero, we obtain the price for the commodity (electricity) and the price for capacity, *EP* and *CP* respectively.

$$EP = \min(VC_i - \alpha + \gamma_i + \theta_i)$$

$$CP = \min(\frac{FC_i - \varphi - \theta}{\bar{x}_i} - \delta - \gamma)$$

The final prices will be the minimum *EP* price and *CP* price across the different technologies. Section 6.3 presents some results of this problem.

5.3.2 Extended prices for transmission

The Nodal model presented in section 4.4.2 gives us the optimal dispatch in order to minimise the cost of electricity production by respecting not only the usual constraints, but also the physical constraints of the power flow in the transmission network and line capacity. The outputs from the nodal model are the optimal quantities to be produced and the optimal number of plants to be engaged (q_i^*, z_i^*). We benefit from this information, so we modify constraint 4.46 in the model to $z_i \leq z_i^*, i = 1,2,3$. We also relax constraints 4.36, 4.37, and 4.38 using the Semi-Lagrangean approach to find the prices for each of the nodes. Mathematically the model is written as follows:

$$\underset{q_g, z_g}{\text{minimise}} \quad \sum_{i=1}^{3} VC_i q_i^S + \sum_{i=1}^{3} FC_i z_i \tag{5.24}$$
$$+PNode_A(q_A^d - \sum_{i \in G_A} q_i^s + q_{AB} + q_{AC})$$
$$+PNode_B(q_B^d - \sum_{i \in G_B} q_i^s + q_{BC} - q_{AB})$$
$$+PNode_C(q_C^d - \sum_{i \in G_C} q_i^s - q_{BC} - q_{AC})$$

subject to

$$\sum_{i \in G_A} q_i^s - q_{AB} - q_{AC} \leq q_A^d \tag{5.25}$$
$$\sum_{i \in G_B} q_i^s - q_{BC} + q_{AB} \leq q_B^d \tag{5.26}$$
$$\sum_{i \in G_C} q_i^s + q_{BC} + q_{AC} \leq q_C^d \tag{5.27}$$
$$q_{AC} = q_{AB} + q_{BC} \tag{5.28}$$
$$\sum_{i \in G_A} q_i^s + \sum_{i \in G_B} q_i^s + \sum_{i \in G_C} q_i^s = q_A^d + q_B^d + q_C^d \tag{5.29}$$
$$q_{AB} \leq LineCap_{AB} \tag{5.30}$$
$$q_{BC} \leq LineCap_{BC} \tag{5.31}$$
$$q_{CD} \leq LineCap_{CD} \tag{5.32}$$
$$q_i^S \leq \bar{x} z_g, \quad i = 1,2,3 \tag{5.33}$$
$$q_i^S \geq \underline{x} z_i, \quad i = 1,2,3 \tag{5.34}$$
$$z_i \leq Z_i^{*MIP}, \quad i = 1,2,3 \tag{5.35}$$

We will obtain the prices for each node while the total production mix will be the same as for the MIP $q_g^{*SLP} = q_g^{*MIP}$.

In order to obtain the transmission price, we first verify that the prices are stable by setting the transmission capacity constraint to equality constraint and making sure that the prices are the same. Given that the power flow in that line is binding, if congested, setting it to an equality constraint should not, in theory, change the value of the quantities produced, the plants engaged or the prices obtained. After confirming that the prices are stable, we add to the objective function the following term $+PLine_{AB}(LineCap_{AB} - q_{AB})$ in order to obtain a price for the transmission in Line AB.

Results are discussed in section 6.4.

5.4 In a power exchange

Pricing in a power exchange with block bids is not an easy task. Most of the exchanges solve a linear problem and then apply heuristics to obtain prices to announce in the market. The block bids add complexity to the problem due to its discrete constraints. The Semi-Lagrangean approach can be applied in order to obtain prices for each of the time periods analysed. Let us use the problem 4.50 presented in section 4.6 but with an inelastic given demand. Then, by applying the Semi-Lagrangean relaxation to the balance constraint we obtain that the price for time period h will be given by $Price_h$ from the following problem:

$$\min_{q_{ih},q_{kh},z_k} \quad \sum_h (\sum_i q_{ih} P^S ih + \sum_k q_{kh} P^S kh + Price_h (Demand - \sum_i q_{ih} - \sum_k q_{kh})) \tag{5.36}$$

subject to

$$\sum_i q_{ih} + \sum_k q_{kh} \quad <= \quad Demand \quad \forall h \tag{5.37}$$

$$q_{ih} \quad \leq \quad Q_{ih} \quad \forall i,h \tag{5.38}$$

$$q_{jh} \quad \leq \quad Q_{jh} \quad \forall j,h \tag{5.39}$$

$$q_{kh} \quad = \quad Q_{kh} z_k \quad \forall k,h \tag{5.40}$$

$$q_{lh} \quad = \quad Q_{lh} z_l \quad \forall l,h \tag{5.41}$$

The different relaxation can also be used in combination. That is, not only to apply the Lagrangean relaxation to the problem, but also to implement a surrogate relaxation in the same problem. A even more interesting idea would be to combine the Semi-Lagrangean approach with the surrogate relaxation such as to solve the problem in an exchange power market. Let us use the two period example shown by Meeus et al. (2005) where at Period 1, a single bid is presented producing up to 60MWh at a price of 10. While, at Period 2 a single bid of 60MWh but at a price of 40 is put in the market. In addition, there is a block bid consisting of 100MWh at each period at an average price of 30. The demand will be considered inelastic at 100MWh for the first period and 150MWh for the second period. There can be several approaches to implement a combination of the relaxation techniques.

Option A)

- Write the balance constraints for each time period as two inequalities
- Semi-Lagrange one of the inequalities and include it into the objective function
- The remaining inequality in the subproblem can then be surrogate relax

$$\min_{q,z} \quad 10q_1 + 40q_2 + 30q_3 + 30q_4 + PriceT1(100 - q_1 - q_3) + PriceT2(150 - q_2 - q_4) \tag{5.42}$$

subject to

$$\lambda_1(q_1 + q_3) + \lambda_2(q_2 + q_4) <= 100\lambda_1 + 150Lambda\lambda_2 \tag{5.43}$$

$$q_1 \leq 60 \tag{5.44}$$

$$q_2 \leq 60 \tag{5.45}$$

$$q_3 = 100z_3 \tag{5.46}$$

$$q_4 = 100z_4 \tag{5.47}$$

$$q_3 = q_4 \tag{5.48}$$

Option B)

- Write the problem as a surrogate problem by relaxing the balance constraint
- Semi-Lagrange the surrogate problem

$$\min_{q,z} \quad 10q_1 + 40q_2 + 30q_3 + 30q_4 + Price(100\lambda_1 + 150\lambda_2 - \lambda_1(q_1+q_3) - \lambda_2(q2+q4)) \tag{5.49}$$

subject to

$$\lambda_1(q_1+q_3) + \lambda_2(q_2+q_4) \quad <= \quad 100\lambda_1 + 150\lambda_2 \tag{5.50}$$

$$q_1 \leq 60 \tag{5.51}$$

$$q_2 \leq 60 \tag{5.52}$$

$$q_3 = 100z_3 \tag{5.53}$$

$$q_4 = 100z_4 \tag{5.54}$$

$$q_3 = q_4 \tag{5.55}$$

Option C)

- Write the problem as a surrogate problem by relaxing the balance constraint
- Keep the demand constraints for each period in the model and Semi-Lagrange them

$$\min_{qz} \quad 10q_1 + 40q_2 + 30q_3 + 30q_4 + Price_1(100 - q_1 - q_3) + Price_2(150 - q2 - q4) \tag{5.56}$$

subject to

$$\lambda_1(q_1+q_3) + \lambda_2(q_2+q_4) \quad <= \quad 100\lambda_1 + 150\lambda_2 \tag{5.57}$$

$$q_1 + q_3 \leq 100 \tag{5.58}$$

$$q_2 + q_4 \leq 150 \tag{5.59}$$

$$q_1 \leq 60 \tag{5.60}$$

$$q_2 \leq 60 \tag{5.61}$$

$$q_3 = 100z_3 \tag{5.62}$$

$$q_4 = 100z_4 \tag{5.63}$$

$$q_3 = q_4 \tag{5.64}$$

There is however not clear or easy search algorithms to find the optimal surrogate multiplier. Therefore the last three problems are left to future research. The interested reader is referred to (Duan and Sun, 2006) and (Karwan and Rardin, 1984).

6

Computational Examples

In this section the results of applying the Semi-Lagrangean approach to the different models presented in the previous section are illustrated with an example taken from the literature (Bjørndal and Jörnsten, 2008; Scarf, 1994). The information for the technologies and plants within the technologies is summarised in Table 6.1.

Table 6.1: Example from the literature

	Technologies(i)		
	Smokestack (SK)	High Tech (HT)	Medium Tech (MT)
Variable Costs(VC_i)	3	2	7
Fixed Costs(FC_i)	53	30	0
Capacity($\bar{x}_i$)	16	7	6
Minimum Output($\underline{x}_i$)	0	0	0
Avg. Cost at full capacity	6.3125	6.2857	7
Plants available	6	5	5

AMPL software was used to develop the algorithms to solve the problems presented in this section. The solvers used were CPLEX or MINLP when needed.

6.1 Semi-Lagrangean Relaxation

The Semi-Lagrangean approach builds upon the well-known Lagrangean relaxation, but with the difference that when there is an equality constraint, the constraint is divided in two inequalities, namely a ”greater than or equal to” inequality and a ”less than or equal to” inequality. The former is relaxed and added to the objective function, while the latter is left as an inequality constraint in the sub-problem.

Let us assume that the demand level is set to 44 units. The solution to the UCDP using mixed integer programming would show that the minimum cost is achieved by engaging 1 SK technology at full capacity, and 4 HT technologies at full capacity. The total cost is 277 and the corresponding dual price, or Lagrangean multiplier for the balance constraint, is 3. However, at this price we would have that SK Technology would incur a loss of 53 while HT would experience a loss of 92. This is indeed not desirable for the producers, who would not be willing to produce any units. By relaxing the MIP and solving it, we obtain a price of 6.3125, leaving only HT with a small profit. However the problem with this solution is that the number of plants engaged is not feasible, since it would require 0.5625 SK plants, which in the real world is not possible.

As mentioned, we believe that the prices to be announced to the market should be at least greater than or equal to the relaxed mixed integer problem. Therefore we use the dual variable for the balance constraint (6.3125) from the relaxed problem as a guide for the search of the commodity price. By applying the Semi-Lagrangean approach along side with a subgradient algorithm, we obtain a price for the commodity equal to 6.3333, which is clearly high enough to cover the costs of the two relevant producers and has the same minimum cost as the MIP solution. It is important to remember that the number of plants engaged comes in non-divisible numbers, therefore a price below will engaged a non-discrete number of the plants, which, as mentioned before, is not possible.

By solving the Semi-Lagrangean problem we obtain a single price for electricity, which is not linear, and it is more stable than the prices obtained from the UCDP. It is important to highlight that the prices from the Semi-Lagrangean approach are greater than or equal to those from the LP relaxation of the UCDP, as shown in Figure 6.1. The LP relaxation provides dual prices that are equal to the average cost at full capacity from the most expensive technologies engaged. Given that the LP relaxation does not have any integer constraints, the extra plant committed can be added partially, and the dual price for the balance constraint is equal to the average cost at full capacity of the marginal technology. This is true until the capacity for that technology is fully utilised. Therefore we can see from Figure 6.1 that the LP relaxation price is equal to 6.2857 in the demand range 1-35[1], then it is equal to 6.3125 for demand range 36-131[2] and finally it is set to 7 when the demand is higher than 131[3].

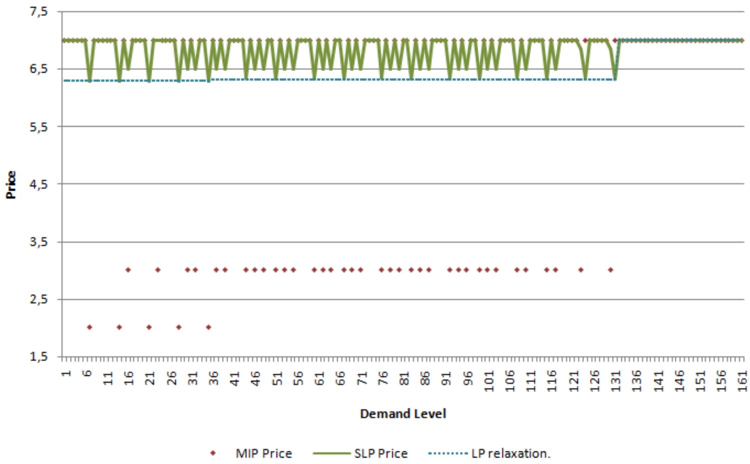

Figure 6.1: MIP, SLP and LP prices

However, the real world requires integrality constraints, and even though the dual prices from the MIP cannot be considered as a price, we will use that name so as to compare them with the LP relax and the SLP Price. The dual value from the balance constraint in the MIP problem is equal to the marginal cost of the last technology engaged in the production, i.e. the marginal producer. Although MT Technologies have the highest average cost at full capacity, they also have the lowest average cost per unit produced below full capacity. Therefore, whenever MT has the lowest average cost per unit produced across the different demand ranges, it will be selected in the optimal solution and the dual value will be set to its marginal cost which is 7. For some demand levels the optimal solution does not include MT Technologies; the optimal solution will try to engage the cheapest options at its full capacity. That is, first HT and then SK Technologies. In the cases where HT is the only Technology involved in the production, then the dual price will be equal to 2. This is the case for demand levels multiples of 7, and up to 35, given that the capacity of each plant is 7 and there are only 5 plants. Finally, when SK is engaged in combination with HT only to satisfy the demand, the dual value for the balance

[1]HT Technology is the cheapest option; therefore it will be committed until all its plants are producing at full capacity. That is, until all the 5 HT plants are producing at its full capacity of 7.

[2]The next cheapest technology after HT Technology is SK Technology, which can generate a total of 96 units when all its 6 plants are producing at its full capacity of 16.

[3] The most expensive technology is MT, which is chosen at last when all the other technologies are producing at full capacity. MT will continue producing until all its 5 plants are committed at their full capacity of 6

constraint will be set to 3, which is the marginal cost for SK.

If we consider the dual variables from the MIP problem as prices, we can observe that the profits can be negative, whereas with the Semi-Lagrangean prices the profits are never negative as shown in Figures 6.2 - 6.4.

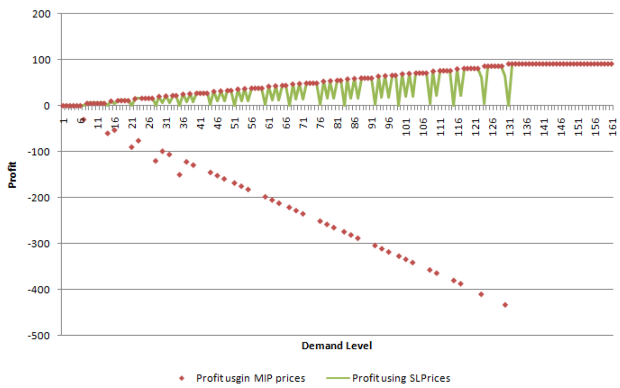

Figure 6.2: Global profit with MIP and SLP prices

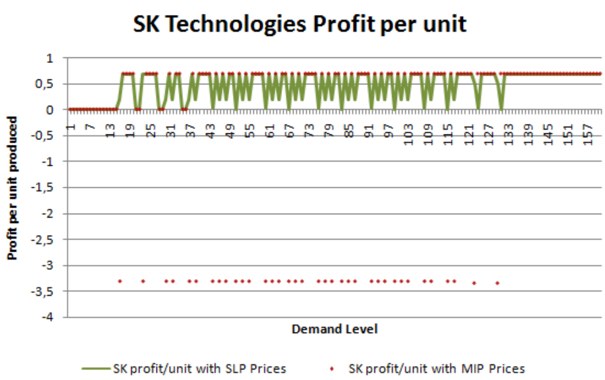

Figure 6.3: Profit for SK Technology with SLP and MIP prices

After demand level 131, the price from the LP relaxed problem, the MIP problem, and the SLP problem is the same. It is set by the MT Technology at 7 which makes the MT Technology earn zero profits. However, for SK and HT Technologies the profits obtained will vary depending on which technology sets the price. This can be seen clearer on Figure 6.3 and Figure 6.4. We can observe also that with SLP prices the technologies get nonnegative profits, whereas with the MIP prices the technologies can incur losses. It is in these cases where some market designs

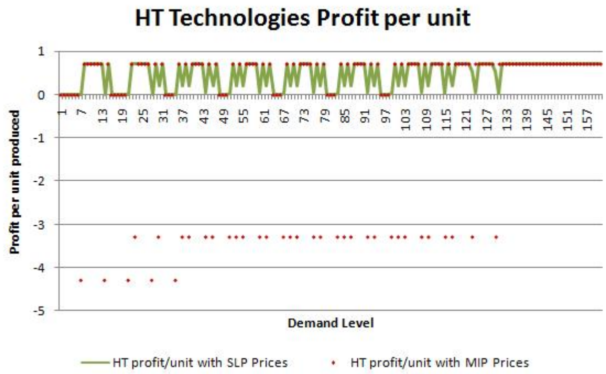

Figure 6.4: Profit for HT Technology with SLP and MIP prices

provide side payments, also known as uplifts, to those technologies that should be producing but if they follow the schedule they incur in losses.

6.1.1 Effect of minimum output

In the example presented, the non-convexities are introduced in the problem by the fixed costs and the integrality constraints. A common feature in electricity generation is that some generators may have a minimum output to start producing. A minimum output requirement generates an additional non-convexity to the problem, which will yield to different levels of Semi-Lagrangean prices. In fact, this requirement has a greater impact in the prices than the fixed costs, since it creates spikes in the Semi-Lagrangean (SL) Prices obtained.

The minimum output requirement makes a technology unavailable for certain levels of production. Let us modify the initial example for the Med Tech plants (MT) to have a minimum output ($\underline{x}_{MT} = 2$). Figure 6.5 graphs the values for a demand range from 2-90, and from it we can observe how the SL Prices show a more non-steady trend compared to the case without minimum output requirement.

This example was also used as a test for the approach suggested by Ruiz et al. (2012). Their approach uses a primal-dual formulation, where the duality gap is minimised by relaxing complementarity constraints, and adding a constraint to ensure that the generators engaged into

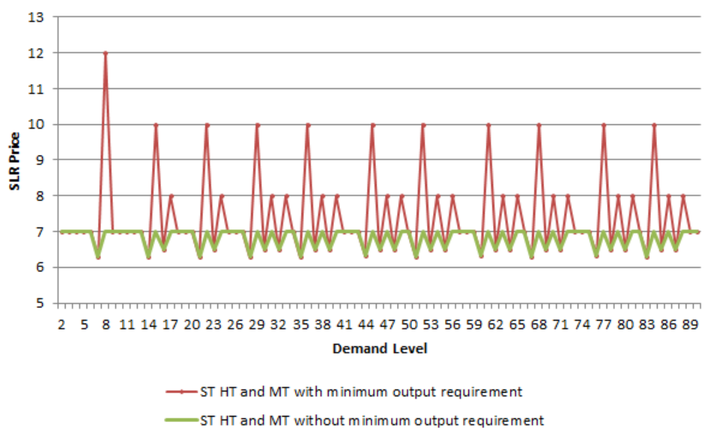

Figure 6.5: SL Prices with and without minimum output requirement

production are profitable. The results from their approach and the SL Prices are shown in Figure 6.6.

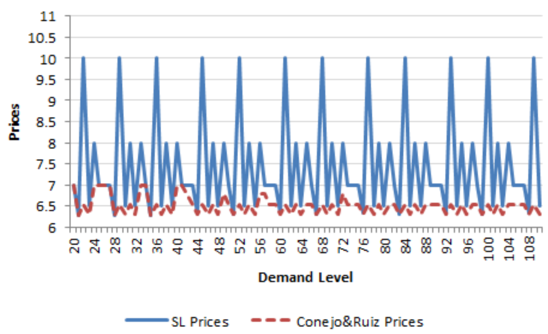

Figure 6.6: Comparison of SL Prices with minimum output requirement and Conejo and Ruiz results

We can also imagine that the Medium Tech could have closer average costs to the other two technologies (SK and HT), and still with a minimum output requirement. Let us modify the variable costs for MT as shown in Table 6.2.

Then, we obtained the Semi-Lagrangean prices shown in Figure 6.7. The prices are steadier when the average price at full capacity is closer to each other, as in Case B. It is important to notice that despite the fact that the average cost is similar, the total average cost is indeed different and therefore the MIP finds a quick solution. When the average cost at full capacity

Table 6.2: Different Average Costs for MT

	Case A MT	Case B MT	Case C MT
Variable Costs(VC_i)	7	6.3	6.4
Fixed Costs(FC_i)	0	0	0
Capacity($\bar{x}_i$)	6	6	6
Minimum Output($\underline{x}_i$)	2	2	2
Avg. Cost at full capacity	7	6.3	6.4
Plants available	5	5	5

starts to vary, the SL price starts to jump more (Case A and Case C).

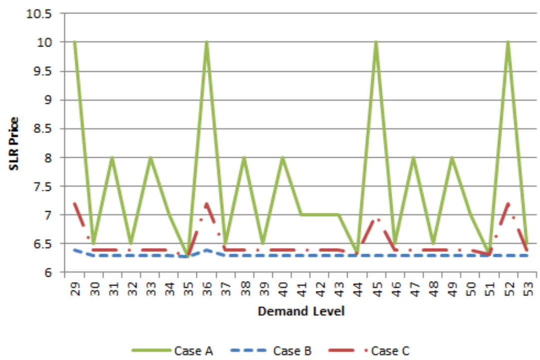

Figure 6.7: SL Prices without minimum output requirement and different average costs

6.1.2 Price demand function

A linear demand function is introduced into the problem $Price = Intercept - M * QDemand$, where $Price$ in turn will be also given by the Semi-Lagrangean formulation.

Four slightly different linear demand functions are used, and the results obtained are summarised in Table 6.3.

The optimal dispatch and commitment from this solution are aligned to the results from the MIP solution. Moreover, the new Semi-Lagrangean prices obtained here are very close to those obtained in section 6.1. The last column in Table 6.4 shows the values for the approach

Table 6.3: Semi-Lagrangean prices with a demand function

Case	Intercept	M	q_i^*	z_i^*	New SLPrice	$QDemand = \sum q_i$
A	26	0.3	[48 , 14, 1.18]	[3,2,1]	**7.044**	63.1861
B	26.5	0.3	[31.6667, 35, 0.00]	[2,5,0]	**6.5**	66.6667
C	27	0.3	[64 , 0, 2.20]	[4,0,1]	**7.138**	66.2044
D	28.5	0.29	[48, 28, 0.00]	[3,4,0]	**6.46**	76

with an inelastic demand. The new prices are very close to those obtained previously.

Table 6.4: Semi-Lagrangean prices comparison

Case	New SLPrice	$QDemand = \sum q_i$	Old SLP
A	**7.044**	63.1861	**7.0000** at demand=63
B	**6.5**	66.6667	**7.0000** at demand=66 and
			6.3125 at demand=67
C	**7.138**	66.2044	**7.0000** at demand=66
D	**6.46**	76	**6.3334** at demand=76

6.1.3 Piece-wise variable costs

Let us use the information presented in Table 3.5 in section 3.2.1, where three generators (A, B, and C) are available and they have piece-wise variable cost functions as depicted in Figure 6.8. The average costs at full capacity for the first section of the cost functions, are 65, 100 and 105 for Generator A, B and C respectively; while for the second section of the cost function, the average costs are 87.5, 95 and 70 respectively.

In a linear problem, Generator A will be increasingly committed to production up to demand level 100. After that, Generator A will be fixed in 100 units while Generator C will be engaged increasingly in the production from demand level 101 to 300. At that point, capacity for Generator C is reached, while Generator A continue to produce 100 units. The two Generators A and C are on, C is at its full capacity and cannot add more into production. But Generator A has 100 available. The extra cost of adding 1 unit from Generator A is 110, but from Generator

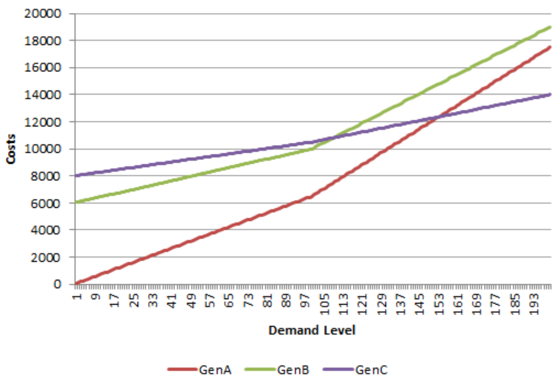

Figure 6.8: Total cost function per generator from Gribik (2007) example

B is only 90. Therefore for demand level 300 up to 500, Generator B is committed to produce its 200 units. Generator B is finally engaged and will produce increasingly from demand level 301 to 500, where it reaches also its capacity. The last 100 units will be covered by Generator A, until it too, reaches its capacity.

The corresponding dual values for the LP problem then are 65 (for demand range: 0-100); 70 (for demand range: 101-300); 95 (for demand range: 301-500); and 110 (for demand range: 501-600)). At these prices all the generators make non-negative profits. The prices are related to the marginal cost for generating unit A, and the average cost at full capacity of units C and B.

In an MIP set up, with binary variables for engaging the generators (0,1), we have that Generator A will be engaged first producing from demand 0-153. Then, it will be disengaged when the demand level reaches 154, since it is cheaper to produce only with C. This we could see from the total cost function graph Figure 6.8, where the total cost function of C intersects the cost function of A at 153.33. Generator C will increasingly produce until its capacity is reached at demand level 200, and it will keep producing all its capacity from demand level 200 to 600. At demand level 201, Generator A is called into production again, since it is cheaper than B in the range of 201 to 377. When the demand reaches 378, it is cheaper to to involve Generator B producing 100 units, and setting A to produce 78. Generator B will have a fix output of 100 from demand level 378 to 400. While Generator A continues to increase production up to 100 units B will be fixed. In the region of demand level 400 to 500, Generator A produces only up to 100, and Generator B an increasing amount until it reaches its capacity at demand level 500.

From demand 501, Generator A starts adding one unit at a time.

The dual variables for this MIP problem are the marginal cost of the marginal generator at that specific demand level. That is, the prices are either 65 or 110 if the first generator is the marginal one; 90 if Generator B is the marginal one; or 35, if Generator C is the marginal producer. The dual variables for the MIP problem are not linear, and in some areas the price obtained is below the LP dual price. Whenever the dual price in the MIP is less than the dual price in the LP, the marginal generator will be making a loss. From demand 154-299, and 378-500, the dual MIP price will be below to the average cost of producing at full capacity for some of the plants. For instance, the average cost for Generator C of producing at full capacity 200 units is 100. The dual price from demand level 154 to 200 is 35, and from 201 to 299 is 65. At those prices, Generator C cannot cover its fixed costs. Table 6.5 shows the marginal producer and the corresponding dual MIP for reference.

Table 6.5: Marginal Unit and Dual Price for the MIP

Demand	Marginal Unit	Dual MIP
1-99	A	65
100-153	A	110
154-200	C	35
201-299	A	65
300-377	A	110
378-399	A	65
400-500	B	90
501-600	A	110

In contrast to the LP solution, if we set the prices at the dual value, we will have generators making losses. The losses are due to the fixed costs that are not covered completely. Comparing the costs from the MIP solution and the LP solution we have integrality duality gaps at certain levels of demand. This is due to the integrality constraints in the problem which give us a different generation mix at a higher portfolio cost.

The Semi-Lagrangean approach was applied to this example and the prices obtained are shown in Figure 6.9. These price will support the minimum cost optimal solution from the MIP and at the same time will make all the generators committed to production earn non-negative profits, as shown in Figure 6.10.

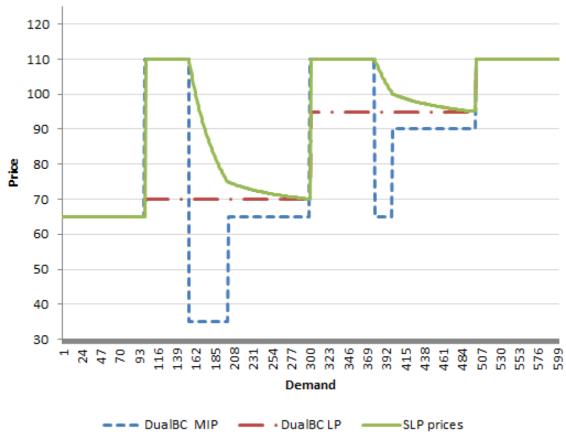

Figure 6.9: SL Prices to Gribik (2007) example

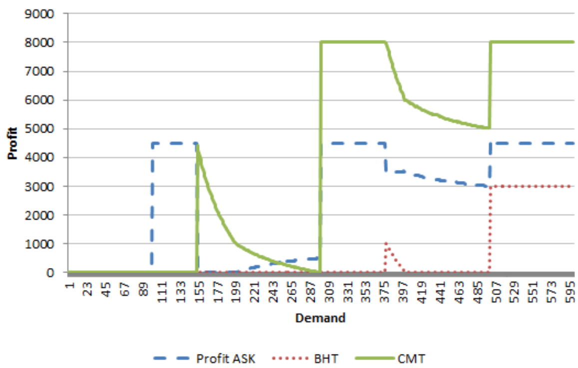

Figure 6.10: Profit per technology with SL Prices

This example is also used by Van Vyve (2011), who presented the graph in Figure 6.11 with the prices obtained using his approach.

The values obtained from the Semi-Lagrangean approach (SLP prices) are not far from the prices obtained by Van Vyve (2011). However, at demand ranges (154-300) and (378-500) the former approach (SLP prices) go down to the LP dual prices much smoother than the latter approach.

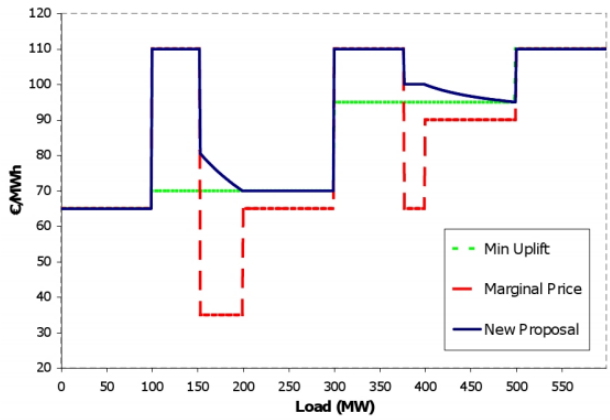

Figure 6.11: Prices from Van Vyve (2001)

6.2 Second-Price Auction

The Semi-Lagrangean approach was applied to a single period market where there are only two technologies available, Smokestack and High Tech from Table 6.1. The UCDP can raise problems in terms of inequality, since there could be a solution where two generators yield the same solution. In order to avoid this problem and test the Semi-Lagrangean (SL) Prices, we consider two further cases. In one case we differentiate the technologies by changing the variable cost, and in another case by changing the fixed costs across the same technology. Tables 6.6 and 6.7 show the variable cost α and and fixed costs β used in each case.

Table 6.6: Case A - Variable Cost across same technologies varies by .01

Generator	α_i	Generator	α_i
SK1	2.01	HT1	3.00
SK2	2.02	HT2	3.01
SK3	2.03	HT3	3.02
SK4	2.04	HT4	3.03
SK5	2.05	HT5	3.04
SK6	2.06		

The results for the Semi-Lagrangean approach as well as the final commodity price obtained through the secondary auction are shown in Table 6.8 for a demand range from 22- 67.

Table 6.7: Case B - Fixed Cost across same technologies varies by .5

Generator	β_i	Generator	β_i
SK1	53.00	HT1	30.00
SK2	53.00	HT2	30.50
SK3	54.00	HT3	31.00
SK4	54.50	HT4	31.50
SK5	55.00	HT5	32.00
SK6	55.50		

The Semi-Lagrangean prices as well as the final commodity price for the cases where the fixed cost or variable cost were modified, as per Table 6.6 and Table 6.7, did not vary significantly from the prices shown in Table 6.8.

If we consider demand level 33, $D = 33$, the Semi-Lagrangean price is $P = 14$, and the optimal combination of technologies and output so as to minimise costs, indicate that only 5 High Tech plants should be involved. The total profit that the Technology would achieve is 246.

The second auction is started with the announced Semi-Lagrangean Price, $P = 14$, and the participants will be only those generators that can fulfil the whole demand requirement, or if that is not possible the generators will have to find other generators with whom to create coalitions that can entirely fulfil the production needed. For the second price auction there will be different alternative viable producer combinations, such as:

- 3 Smokestack plants with a total generation cost = 258
- 2 Smokestack plants and 1 High tech plant with a total generation cost = 228
- 1 Smokestack plant and 3 High tech plants with a total generation cost = 221
- 5 High tech plants with a total generation cost = 216

In this example the winner coalition is that of 5 High Tech plants, as stated above. The second best coalition has a total cost of 221, with a profit of 241. Then, the commodity price is 6.6970

Table 6.8: Semi-Lagrangean prices and commodity prices from a second price auction

Demand	SLR Price	Commodity Price	Demand	SLR Price	Commodity Price
22	10.0000	6.5000	45	10.0000	6.4000
23	6.5000	6.3043	46	6.5000	6.4565
24	23.0000	7.1250	47	10.0000	6.5745
25	12.5000	6.9600	48	6.5000	6.5000
26	9.0000	6.8077	49	12.0000	6.4490
27	7.2500	6.6667	50	7.5000	6.3800
28	6.2847	6.2857	51	6.3125	6.2941
29	10.0000	6.4483	52	10.0000	6.3846
30	6.5000	6.3000	53	6.5000	6.4340
31	10.0000	6.8387	54	10.0000	6.3889
32	6.5000	6.3125	55	6.5000	6.3091
33	14.0000	6.6970	56	19.0000	6.5536
34	8.0000	6.5882	57	11.0000	6.4912
35	6.2857	6.2857	58	8.3333	6.4310
36	10.0000	6.4167	59	7.0000	6.3729
37	6.5000	6.4865	60	6.3333	6.3000
38	10.0000	6.4211	61	10.0000	6.3770
39	6.5000	6.3077	62	6.5000	6.4194
40	19.0000	6.6500	63	10.0000	6.5079
41	11.0000	6.5610	64	6.5000	6.4531
42	8.3333	6.4762	65	12.0000	6.4154
43	7.0000	6.3953	66	7.5000	6.3636
44	6.3333	6.2955	67	6.3125	6.2985

$= \frac{\pi_{WC} - \pi_{2ndBest} + TC_{WC}}{D} = \frac{TC_{2ndBest}}{D} = CommodityPrice = P^*.$

At this price, the second best coalition would be making zero profits and would not be interested in producing. Therefore the best production coalition with minimum cost will be engaged. Its final profit, or excess profit, would be 5. It is important to highlight that for the second price auction, only coalitions fulfilling the whole demand requirement will participate in the auction.

Figure 6.12 compares the commodity prices and uplift under the proposed approach, the approach proposed by Hogan and Ring (2003) and the approach proposed by O'Neill et al. (2005). For the Commodity Price obtained from the Semi-Lagrangean techniques through the secondary auction there is no uplift. Figure 6.13 shows in addition prices for a wider range of demand level, comparing only the results obtained by Hogan and Ring (2003) and obtained through the Semi-Lagrangean relaxation second auction approach.

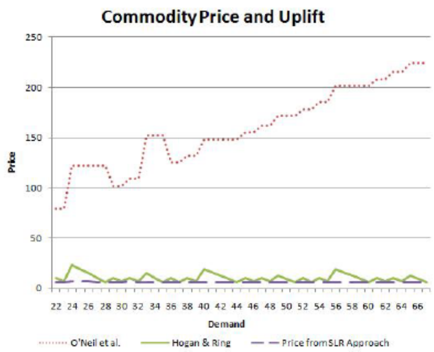

Figure 6.12: Commodity price and uplift

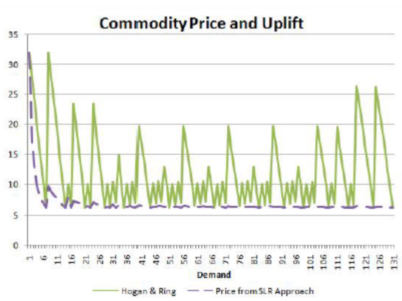

Figure 6.13: Commodity price and uplift with SLR and Hogan and Ring Approach

The results for the case with three technologies involved are shown in the Appendix.

This scheme, however, may present some problems, since in some cases the price obtained is below the marginal LP price. For example, at demand level 23. This issue can be further analysed and leave to future research. It is possible that the approach to obtain the second best coalition is not the best one, since it uses a very similar mix of generators and units as the optimal solution. An approach to delete from the problem the units that have been used in the optimal solution has to be further research so as to apply this approach.

6.3 Commodity and Capacity Prices

The new Semi-Lagrangean problem with sum of capacity constraint is not any easier to solve. We apply a sub-gradient algorithm to find the results. This approach is sensitive to the starting point of the multipliers, so we chose three different options to see the effect in prices. Case A sets the starting point for the multiplier for electricity (EP price) to zero; while case B sets it equal to the dual value from the balance constraint from the traditional UCDP solved using a MIP; and case C fixes the starting point to the dual value for the balance constraint from the relaxed MIP unit commitment and dispatch problem, i.e. a linear programming (LP) problem.

In all the cases, each technology recovers their fixed costs; the marginal generator earns zero or infra marginal profits and the other two technologies have non-zero profit, as can be seen in Figure 6.14. Although it is not easy to see from the graph, the graph shows that non of the technologies make any loss.

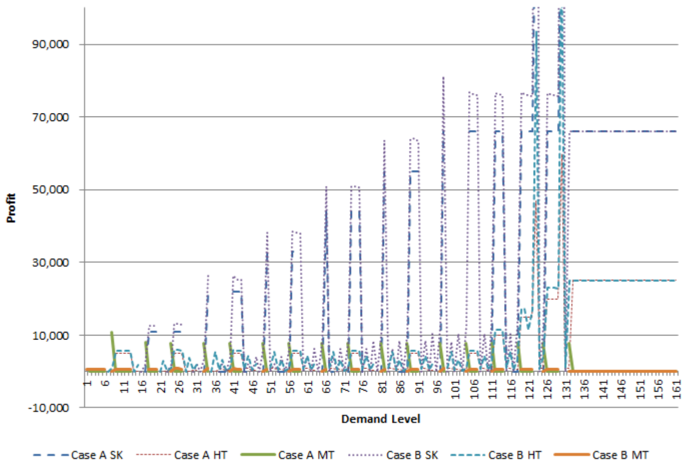

Figure 6.14: Profits from SL Approach

Overall the technologies receive almost the same profit if the multiplier starting point is either set to the LP or MIP dual value. The excess revenue obtained with this approach can be used as a guide to future investment and in this way, this procedure can help to find the ”missing money”. The plants called for production recover their operational costs and the efficient portfolio is chosen.

Prices for the commodity are the most volatile (and the lowest) when the starting point for the EP multiplier is set to zero; if the starting point for the EP is set to the LP dual value, the commodity price is less volatile and they are close to the LP prices. However, while the latter gives very small prices for the capacity, the former gives the highest price for capacity. If we decide to use the prices obtained when the starting point for the multiplier is set to the MIP value instead of the LP value, then we have that the electricity prices are smaller, but the prices for capacity increases. Figure 6.15 and Figure 6.16 show these trends.

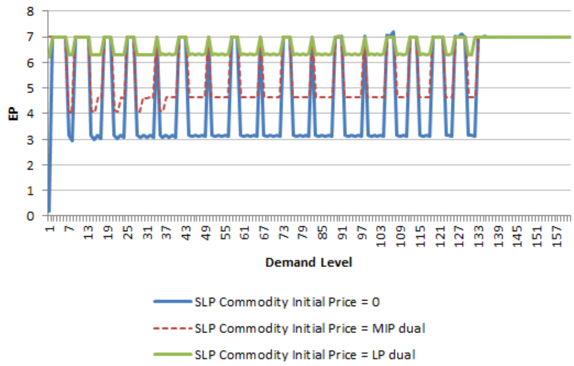

Figure 6.15: SL Price for commodity

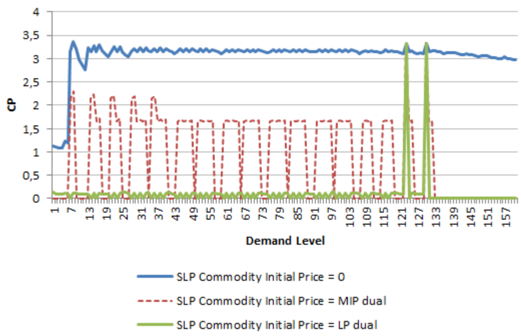

Figure 6.16: SL Price for capacity

Figure 6.17 depicts the behaviour of the prices for case B, where the starting point for the EP multiplier is set to the MIP dual value; while Figure 6.18 shows the behaviour for case C, where the starting point is the LP dual value of the balance constraint. The prices for commodity and capacity are inversely interrelated, as expected. By comparing Figure 6.17 and Figure 6.18, we can see that if electricity prices are ”lower”, then the capacity prices are ”higher” as in Figure

6.17; but if prices for electricity are "higher", then the capacity prices will be "lower", as in Figure 6.18.

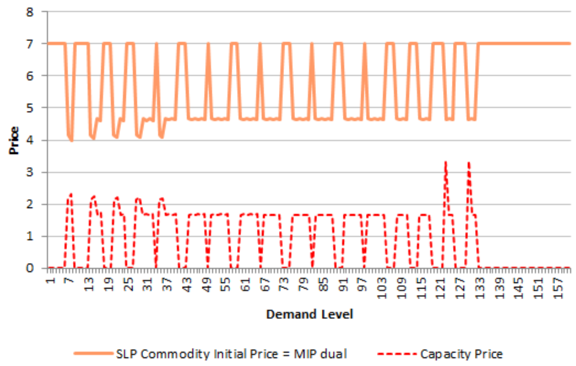

Figure 6.17: SL Price for commodity and capacity Initial Point=MIP

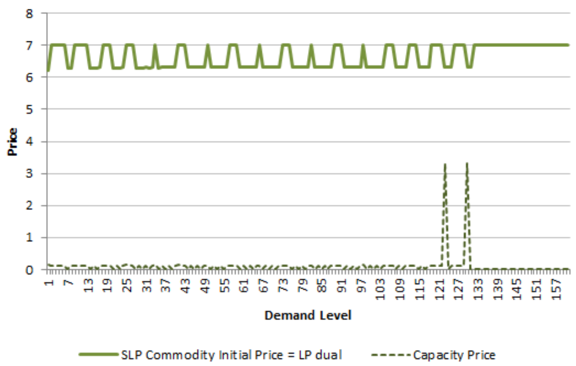

Figure 6.18: SL Price for commodity and capacity Initial Point=LP

The commodity and capacity prices present jumps (or downs for capacity) every time the MT technology has to be engaged to produce more than one unit. In the case of non-storable goods, such as electricity, having capacity limits on the production can generate extreme fluctuations in prices. Moreover, as discussed somewhere else, one of the main conflicts with marginal cost pricing (MCP) is the need for revenues. The approach presented here can help to lessen these two problems by providing more stable prices than in the MCP and providing with the technologies engaged zero or non-negative profits.

Take the case of demand level 123. If marginal pricing was to be followed, and we order the generator according to their marginal cost, then High Tech would provide all its capacity (35 units) and Smokestack would cover the remaining units (88), as summarised in Table 6.9. If the marginal generator sets the commodity price equal to its marginal costs, then the commodity price would be 3. This price is clearly not high enough to cover the generators costs. Smokestack would incurred a loss of 291.5, while High Tech would experienced a loss of 115. The total cost of that portfolio generation would be 775.5.

The dual price of the balance constraint for a LP problem is 6.3125, which could be a candidate for a commodity price, since the marginal generator earns zero profit and the other generator gains non-zero profits. However due to the integrality constraint in the plants engaged, the LP solution is infeasible. With the integrality condition, this portfolio in fact would cost 802. Therefore, if we want to achieve an efficient least cost generation portfolio we need to commit only 4 plants of High Tech at full capacity producing a total of 28 units, and 6 Smokestack plants producing a total of 95 units. The total cost of production would be 779. The dual for the balance constraint for the MIP is 3, which again is not enough to cover the cost. We need higher prices that can send the right signal to the market.

By applying the expanded Semi-Lagrangean approach, and setting the EP multiplier starting point to the LP dual value of the balance constraint (Case C), we obtain that the price for the commodity would be 6.3125 and for capacity 3.3125. These prices are clearly higher than the MIP, as shown in Table 6.10.

Table 6.9: Results for Demand Level 123

	q^*			z^*			Costs		
Problem	SK	HT	MT	SK	HT	MT	SK	HT	MT
LP	88	35	0	5.5	5	0	555.5	222	0
MIP	95	28	0	6	4	0	603	176	0
SL	95	28	0	6	4	0	603	176	0

From Figure 6.16 , we can see how the capacity price jumps when the SK technology is used below its full capacity, at demand level 123 and 130. At these prices the generators will receive 6.3125 for each unit of electricity produced and 3.3125 for the total capacity of the plants engaged in production. Smokestack total revenue would be 917.7 (95*6.3125 + 16*6*3.3125), while High Tech would have total revenue of 269.5 (28*6.3125 +4*7*3.3125). This will create a total profit for Smokestack and High Tech of 314 and 93 respectively. These prices will provide generators with profits to expand their capacity. Table 6.10 shows this, as well as the prices for Case B, where the starting point is the MIP dual value of the balance constraint. In any case the optimal minimum cost efficient portfolio mix from the MIP is achieved with the Semi-Lagrangean (SL) approach. If Case B is chosen, then the prices for electricity are much closer to those from a MIP solution, and the technologies will receive a higher extra payment for the capacity of the plants engaged than the payment they would receive if Case A was chosen. Case A has smaller volatility than case B for the electricity price, and a very small extra payment for the capacity of the technologies engaged.

Table 6.10: Commodity and Capacity Price for Case B and C

Case B		Profit			Total
EP initial val.=MIP shadow val.		SK	HT	MT	Profit
Commodity Price	4.66				
Capacity Price	4.66	155.5	46.66	0	222.2
Case C		Profit			Total
EP initial val.=LP shadow val.		SK	HT	MT	Profit
Commodity Price	6.3125				
Capacity Price	3.3125	269.5	917.7	0	1187.2

It is important to mention that whenever Medium Tech is engaged in production with more than one unit, then the price for commodity will be set high enough to cover the production cost of this technology. At this price the other two technologies will receive positive revenue for investing in extra capacity.

6.4 SL Prices in a Network

The example presented at the beginning of this section was implemented into a nodal model. The information for each node, generation plants within them, their characteristics and demand, as well as line capacities are given in Table 6.11.

Table 6.11: Example from the literature

	Nodes(i)			
	Node A		Node B	Node C
	Generator1	Generator2	Generator3	Demand
Demand(i)	0	0	0	$q_c^d 4$
Variable Costs(VC_g)	3	2	7	
Fixed Costs(FC_g)	53	30	0	
Capacity($\bar{x}_g$)	16	7	6	
Minimum Output($\underline{x}_g$)	0	0	2	
Average Cost at full capacity	6.3125	6.2857	7	
Plants available (Z_g)	6	5	5	

LineAB Capacity	15

In a linear world without capacity constraints, and without losses, we would have the cheapest generators produce until the demand is covered. For demand levels within 50 and 75, it means that all plants of Generator 2 will be involved and plants of Generator 1 will be engaged as needed. The optimisation problem is a linear program and it is convex, so the dual values can be used. The price at all the nodes will be given by Generator 1, setting it to 6.3125. However, given that there is a constraint in Line AB, the prices would be set to 6.65625 at node C, 6.3125 at node A, while for Node B it would be 7. The dual for the congested line is 1.03125.

The linear solution is not feasible since the problem requires indivisibilities in the number of plants engaged, converting the problem into a Mixed Integer Problem (MIP). As mentioned earlier, whenever there is no congestion the price in the grid across all nodes would be the same. In the MIP solution this means that all nodes will have a price of 7. However, whenever the line is congested, that is when line AB is fully utilised, then the prices at node A will be 3, node B will be 7 and at the demand node it would be set to 5. At these prices the generators at Node A cannot recover their costs. The dual values in the MIP are no longer operational.

By implementing the Semi-Lagrangean approach in the problem, we obtain prices of 6.3125 for Node A, 7 for Node B and at Node C the electricity is priced at 6.6563, for cases where the line is congested, as shown in Table 6.12. For cases where the line is not congested, the Semi-Lagrangean prices are the same as for the MIP prices, as Table 6.13 presents. At these prices the generators will cover their production costs and the minimum cost optimal mix will also be achieved, while at the same time the physical laws in the transmission flows and line capacity are respected.

Table 6.12: Semi-Lagrangean prices for demand level with congested line and line capacity inequalities

	Prices at Node			
Demand	A	B	C	Flow in Line AB
51	6.3127	7	6.6563	15
56	6.3125	7	6.6563	15
57	6.3126	7	6.6566	15
61	6.3127	7	6.6563	15
65	6.3130	7	6.6565	15
74	6.3125	7	6.6564	15
75	6.3127	7	6.6563	15

The prices remain stable after running the semi-Lagrangean problem with equality constraint for the line capacity AB as shown in Table 6.14. That is, constraint 4.41 from section 4.4.2 is modified to an equality constraint. Given that the line capacity constraint is an equality constraint, we can then also apply the semi-Lagrangean approach in order to obtain the congestion charge. Then, as mentioned before, we add to the objective function the following term $+PLine_{AB}(LineCap_{AB} - q_{AB})$, leaving the line capacity constraint as a ”less than or equal to” inequality constraint as in the initial problem.

The prices obtained are presented in Table 6.15. They are stable for each of the nodes, in the sense that prices from Table 6.14 and 6.14 are very similar, and very close to the LP solution. The prices for the congested line, or congestion charge, range from 1.6909 to 1.1820, which are close to the LP dual value of 1.03125. If the network is solved as a linear problem the dual

Table 6.13: Semi-Lagrangean prices for demand level with non-congested line and line capacity inequalities

	Prices at Node			
Demand	A	B	C	Flow in Line AB
50	7	7	7	14.00
52	7	7	7	14.67
53	7	7	7	14.33
54	7	7	7	14.00
55	7	7	7	13.67
58	7	7	7	14.67
59	7	7	7	14.33
60	7	7	7	14.00
62	7	7	7	14.67
63	7	7	7	14.33
64	7	7	7	14.00
66	7	7	7	14.67
67	7	7	7	14.33
68	7	7	7	14.00
69	7	7	7	13.67
70	7	7	7	13.33
71	7	7	7	13.00
72	7	7	7	12.67
73	7	7	7	12.33

at Node 3 is 6.65625, and the dual for the capacity constraint in line AB is 1.03125. Running the model as a MIP then the dual for capacity constraint is 6, while the price at node 3 is 5. We can see how the SL prices are very close to those from the LP solution.

Eventhough the line is not congested, we could calculate what the capacity charges would be if the line were to be congested at its optimal used. From Tables 6.12 and 6.13, we have the flow running in the link joining Node A and Node B for the optimal MIP solution. By setting the capacity constraint in Line AB to an equality to these values, we can then apply the Semi-Lagrangean relaxation to the constraint in order to obtain congestion prices for the whole demand range from 50-75. The results are shown in Table 6.16, and we can see that prices from Node 1 are always around 6.3125, prices at Node 2 are 7, and prices at the demand node

Table 6.14: Semi-Lagrangean prices for demand level with congested line and line capacity equality constraint

	Prices at Node			
Demand	A	B	C	Flow in Line AB
51	6.3143	7	6.6571	15
56	6.3125	7	6.6563	15
57	6.3135	7	6.6566	15
61	6.3129	7	6.6564	15
65	6.3130	7	6.6564	15
74	6.3125	7	6.6568	15
75	6.3129	7	6.6564	15

Table 6.15: Semi-Lagrangean prices for demand level and and congested LineAB and line capacity equality constraint

	Prices at Node			
Demand	A	B	C	Price for Congested Line AB
51	6.3128	7	6.6564	1.6909
56	6.3125	7	6.6580	1.5498
57	6.3131	7	6.6566	1.5248
61	6.3131	7	6.6565	1.4338
65	6.3132	7	6.6566	1.3525
74	6.3125	7	6.6569	1.1968
75	6.3126	7	6.6563	1.1819

are around 6.656. For the capacity charges there will be a range of values that vary depending on the flow in the line. These prices are very close to those of an LP solution.

6.5 SL Prices in a Power Exchange

The Semi-Lagrangean approach is applied also to a power exchange format with block bids. The example used is not the same as the previous sections, but it is taken also from the literature (Meeus, Verhaegen, and Belmans, 2005) and summarised in Table 6.17. The Semi-Lagrangean will provide prices for each of the time periods.

Table 6.16: Semi-Lagrangean prices and capacity charge

		Prices at Node			
Demand	Flow in Line AB	A	B	C	Capacity Charge Line AB
50	14.00	6.31387	7	6.65694	1.61618
51	15.00	6.31279	7	6.65639	1.69094
52	14.67	6.31397	7	6.65699	1.62748
53	14.33	6.31464	7	6.65732	1.56618
54	14.00	6.31443	7	6.65721	1.50659
55	13.67	6.31253	7	6.65627	1.44878
56	15.00	6.3125	7	6.65795	1.54985
57	15.00	6.31312	7	6.65656	1.5248
58	14.67	6.31262	7	6.65631	1.46947
59	14.33	6.31252	7	6.65626	1.41582
60	14.00	6.31287	7	6.65643	1.36339
61	15.00	6.31307	7	6.65654	1.4338
62	14.67	6.31272	7	6.65636	1.38325
63	14.33	6.31253	7	6.65627	1.33399
64	14.00	6.31322	7	6.65661	1.28576
65	15.00	6.31321	7	6.65661	1.35259
66	14.67	6.31329	7	6.65665	1.30574
67	14.33	6.31269	7	6.65635	1.26042
68	14.00	6.31322	7	6.65661	1.21585
69	13.67	6.31316	7	6.65658	1.17255
70	13.33	6.31311	7	6.65655	1.13026
71	13.00	6.31307	7	6.65653	1.08899
72	12.67	6.31294	7	6.65647	1.04873
73	12.33	6.31269	7	6.65635	1.00945
74	15.00	6.3125	7	6.65689	1.19683

The prices obtained are sensitive to the starting point of the multipliers, therefore several starting points were chosen. They key ones were the prices obtained from a simple LP problem and a MIP. The results are presented in Figure 6.19.

At these prices the generators engaged will not have losses, as in the previous examples, and

Table 6.17: Example from Meeus, 2005

	Time Period	
	T1	T2
Simple Bid Q	60 Mwh	60 Mwh
Simple Bid Price	10	40
Block Bid Q	100 MWh	
Block Bid Avg. Price	30	
Demand	100	150

Starting Point												
T1	0	10	10	30	30	40	10	0	0	10	30	0
T2	0	40	10	40	30	40	50	40	30	0	0	50
SL Prices												
T1	19.90	14.34	21.68	30.79	33.03	40.00	10.41	10.78	13.01	23.04	40.17	6.08
T2	46.76	49.45	45.97	41.95	40.89	40.00	50.93	50.76	49.77	45.31	43.05	55.47
Average	33.33	31.90	33.83	36.37	36.96	40.00	30.67	30.77	31.39	34.18	41.61	30.77
Bidders would receive Profit												
Block	6666	6379	6765	7274	7391	8000	6134	6155	6279	6836	8323	6155
Simple Bid in T2	2338	2473	2299	2098	2044	2000	2546	2538	2489	2266	2153	2773
Demand side would pay												
Demand	9004	8852	9064	9371	9436	10000	8680	8693	8767	9101	10475	8928
Profit for generators												
Block	666	379	765	1,274	1,391	2,000	134	155	279	836	2,323	155
Simple Bid in T2	338	473	299	98	44	-	546	538	489	266	153	773

Figure 6.19: Results from different starting points

will be committed to the schedule.

However, there is more research to do in this subject. For example how to decide which price should be implemented in the market. This approach is sensitive to the starting point; therefore combining another type of relaxation in the problem such as surrogate relaxation could add some stability to the Semi-Lagrangean approach. This could be topic for further research.

7

Conclusions and further research

Finding prices in a non-linear environment is a challenge. By matching the economic interpretation of supply and demand intersection with a mathematical problem of maximising welfare, we will obtain marginal pricing. In an electricity industry this means that the marginal generating unit will set the optimal price in the market equal to its marginal cost. These prices in turn are a supporting function of the optimal solution reached by the economic dispatch problem, in a linear convex world. However, in a non-linear environment, marginal prices are not longer an optimal set of prices. If the marginal prices are applied, not all the generators will cover their costs and they would not be willing to produce.

Pricing in the electricity market has been a challenge due to its several non-convexities. The non-convexities presented in the electricity market such as start up costs, minimum output requirements, minimum up and minimum down time requirements, create difficulties to find linear prices that can support the solution obtained from solving the unit commitment and dispatch problem. This issue has been studied in the last years with a numerous pricing approaches proposed. This work reviews different approaches to find optimal prices in the non-convex electricity market. But in addition, it suggests an alternative pricing approach by applying the Semi-Lagrangean Relaxation technique to several models.

The Semi-Lagrangean Relaxation technique consists of relaxing an equality constraint and adding it into the objective function with a penalty term, i.e. a multiplier. The relaxed constraint is also incorporated in the main problem as a weak constraint, as an inequality. This technique is applied to different models presented in this work. The first model consists of a model with inelastic demand, then a second model incorporates a sensitive demand. A secondary auction is also coupled with the model, but this procedure may have some issues regarding the final prices, since in some cases the price goes below the prices obtained in a linear programming formulation. This is in fact an issue for further research. The model incorporates also a minimum output requirement, and it has been highlighted the impact that such constraint has on the commodity prices. The minimum output requirement is a stronger or harsher constraint in the non-convex problem that makes the prices spike.

In addition, a model to obtain prices for capacity and transmission is developed. Many previous models ignore transmission constraints and the physical laws that govern power flow. This work contributes to the growing literature on pricing mechanism in the electricity sector by expanding the model to incorporate transmission constraints and power flow. The technique was applied to a three-node network and the prices obtained are in fact the dual values of the linear programming version of the problem. This model also provides prices for the congested lines.

The Semi-Lagrangean prices obtained for each model are high enough to cover the generators' costs. The prices obtained are high enough to make the market participants willing to generate the amounts of electricity scheduled by the system operator, since they will recover their costs. There is no need to implement any up-lift or side payments. We believe that these prices send the right signals to the market participants and at the same time fulfil the demand at the minimum cost.

Further research

Future research can be done in the second price auction, since the scheme presented some problems. In some cases the price obtained was below the marginal LP price. This issue can be further analysed. It is possible that the approach to obtain the second best coalition is not the best one, since it uses a very similar mix of generators and units as the optimal solution. An approach to delete from the problem the units that have been used in the optimal solution has to be further research so as to apply this approach.

The model can be further improved by including additional non-convexities into the model, as well as a multi-period time horizon model. In the network model, a bigger network could be developed and try the Semi-Lagrangean approach on it. For example, a five node with more alternative producers, or to try to reproduce the Norwegian market, or a zonal pricing approach.

Finding the best surrogate multiplier is topic of further research also, as well as the combination of the Semi-Lagrangean relaxation and surrogate Relaxation to find prices for exchange power markets that have block orders.

7. CONCLUSIONS AND FURTHER RESEARCH

8

Appendix

Demand	MIP price	SLP price	Demand	MIP price	SLP price	Demand	MIP price	SLP price	Demand	MIP price	SLP price	Demand	MIP price	SLP price	Demand	MIP price	SLP price
1	7	7	31	7	7	61	7	7	91	7	7	121	7	7	151	7	7
2	7	7	32	3	6.5	62	3	6.5	92	3	6.3333	122	7	7	152	7	7
3	7	7	33	7	7	63	7	7	93	7	7	123	3	6.8333	153	7	7
4	7	7	34	7	7	64	3	6.5	94	3	6.5	124	7	6.3333	154	7	7
5	7	7	35	2	6.2857	65	7	7	95	7	7	125	7	7	155	7	7
6	7	7	36	7	7	66	7	7	96	3	6.5	126	7	7	156	7	7
7	2	6.2857	37	3	6.5	67	3	6.3125	97	7	7	127	7	7	157	7	7
8	7	7	38	7	7	68	7	7	98	7	7	128	7	7	158	7	7
9	7	7	39	3	6.5	69	3	6.5	99	3	6.3125	129	7	7	159	7	7
10	7	7	40	7	7	70	7	7	100	7	7	130	3	6.8333	160	7	7
11	7	7	41	7	7	71	3	6.5	101	3	6.5	131	7	6.3125	161	7	7
12	7	7	42	7	7	72	7	7	102	7	7	132	7	7			
13	7	7	43	7	7	73	7	7	103	3	6.5	133	7	7			
14	2	6.2857	44	3	6.3333	74	7	7	104	7	7	134	7	7			
15	7	7	45	7	7	75	7	7	105	7	7	135	7	7			
16	3	6.5	46	3	6.5	76	3	6.3334	106	7	7	136	7	7			
17	7	7	47	7	7	77	7	7	107	7	7	137	7	7			
18	7	7	48	3	6.5	78	3	6.5	108	3	6.3333	138	7	7			
19	7	7	49	7	7	79	7	7	109	7	7	139	7	7			
20	7	7	50	7	7	80	3	6.5	110	3	6.5	140	7	7			
21	2	6.2857	51	3	6.3125	81	7	7	111	7	7	141	7	7			
22	7	7	52	7	7	82	7	7	112	7	7	142	7	7			
23	3	7	53	3	6.5	83	3	6.3125	113	7	7	143	7	7			
24	7	7	54	7	7	84	7	7	114	7	7	144	7	7			
25	7	7	55	3	6.5	85	3	6.5	115	3	6.3125	145	7	7			
26	7	7	56	7	7	86	7	7	116	7	7	**146**	7	7			
27	7	7	57	7	7	87	3	6.5	117	3	6.5	147	7	7			
28	2	6.2857	58	7	7	88	7	7	**118**	7	7	148	7	7			
29	7	7	59	7	7	89	7	7	119	7	7	149	7	7			
30	3	6.5	60	3	6.3334	90	7	7	120	7	7	150	7	7			

Figure 8.1: Semi-Lagrangean Prices

LP Solution

Demand	Shadow price for balance constraint	Total Cost	Optimal Quantity (q*)			Optimal Plants (z*)		
			SK	HT	MT	SK	HT	MT
1	6.28571	6.2857	0	0.14	0	0	1	0
2	6.28571	12.5714	0	0.29	0	0	2	0
3	6.28571	18.8571	0	0.43	0	0	3	0
4	6.28571	25.1429	0	0.57	0	0	4	0
5	6.28571	31.4286	0	0.71	0	0	5	0
6	6.28571	37.7143	0	0.86	0	0	6	0
7	6.28571	44.0000	0	1	0	0	7	0
8	6.28571	50.2857	0	1.14	0	0	8	0
9	6.28571	56.5714	0	1.29	0	0	9	0
10	6.28571	62.8571	0	1.43	0	0	10	0
11	6.28571	69.1429	0	1.57	0	0	11	0
12	6.28571	75.4286	0	1.71	0	0	12	0
13	6.28571	81.7143	0	1.86	0	0	13	0
14	6.28571	88.0000	0	2	0	0	14	0
15	6.28571	94.2857	0	2.14	0	0	15	0
16	6.28571	100.5710	0	2.29	0	0	16	0
17	6.28571	106.8570	0	2.43	0	0	17	0
18	6.28571	113.1430	0	2.57	0	0	18	0
19	6.28571	119.4290	0	2.71	0	0	19	0
20	6.28571	125.7140	0	2.86	0	0	20	0
21	6.28571	132.0000	0	3	0	0	21	0
22	6.28571	138.2860	0	3.14	0	0	22	0
23	6.28571	144.5710	0	3.29	0	0	23	0
24	6.28571	150.8570	0	3.43	0	0	24	0
25	6.28571	157.1430	0	3.57	0	0	25	0
26	6.28571	163.4290	0	3.71	0	0	26	0
27	6.28571	169.7140	0	3.86	0	0	27	0
28	6.28571	176.0000	0	4	0	0	28	0
29	6.28571	182.2860	0	4.14	0	0	29	0
30	6.28571	188.5710	0	4.29	0	0	30	0

LP Solution

Demand	Shadow price for balance constraint	Total Cost	Optimal Quantity (q*)			Optimal Plants (z*)		
			SK	HT	MT	SK	HT	MT
31	6.28571	194.8570	0	4.43	0	0	31	0
32	6.28571	201.1430	0	4.57	0	0	32	0
33	6.28571	207.4290	0	4.71	0	0	33	0
34	6.28571	213.7140	0	4.86	0	0	34	0
35	6.28571	220.0000	0	5	0	0	35	0
36	6.3125	226.3120	0.06	5	0	1	35	0
37	6.3125	232.6250	0.13	5	0	2	35	0
38	6.3125	238.9380	0.19	5	0	3	35	0
39	6.3125	245.2500	0.25	5	0	4	35	0
40	6.3125	251.5620	0.31	5	0	5	35	0
41	6.3125	257.8750	0.38	5	0	6	35	0
42	6.3125	264.1880	0.44	5	0	7	35	0
43	6.3125	270.5000	0.5	5	0	8	35	0
44	6.3125	276.8120	0.56	5	0	9	35	0
45	6.3125	283.1250	0.63	5	0	10	35	0
46	6.3125	289.4380	0.69	5	0	11	35	0
47	6.3125	295.7500	0.75	5	0	12	35	0
48	6.3125	302.0620	0.81	5	0	13	35	0
49	6.3125	308.3750	0.88	5	0	14	35	0
50	6.3125	314.6880	0.94	5	0	15	35	0
51	6.3125	321.0000	1	5	0	16	35	0
52	6.3125	327.3120	1.06	5	0	17	35	0
53	6.3125	333.6250	1.13	5	0	18	35	0
54	6.3125	339.9380	1.19	5	0	19	35	0
55	6.3125	346.2500	1.25	5	0	20	35	0
56	6.3125	352.5620	1.31	5	0	21	35	0
57	6.3125	358.8750	1.38	5	0	22	35	0
58	6.3125	365.1880	1.44	5	0	23	35	0
59	6.3125	371.5000	1.5	5	0	24	35	0
60	6.3125	377.8120	1.56	5	0	25	35	0

LP Solution

Demand	Shadow price for balance constraint	Total Cost	Optimal Quantity (q*)			Optimal Plants (z*)		
			SK	HT	MT	SK	HT	MT
61	6.3125	384.1250	1.63	5	0	26	35	0
62	6.3125	390.4380	1.69	5	0	27	35	0
63	6.3125	396.7500	1.75	5	0	28	35	0
64	6.3125	403.0620	1.81	5	0	29	35	0
65	6.3125	409.3750	1.88	5	0	30	35	0
66	6.3125	415.6880	1.94	5	0	31	35	0
67	6.3125	422.0000	2	5	0	32	35	0
68	6.3125	428.3120	2.06	5	0	33	35	0
69	6.3125	434.6250	2.13	5	0	34	35	0
70	6.3125	440.9380	2.19	5	0	35	35	0
71	6.3125	447.2500	2.25	5	0	36	35	0
72	6.3125	453.5620	2.31	5	0	37	35	0
73	6.3125	459.8750	2.38	5	0	38	35	0
74	6.3125	466.1880	2.44	5	0	39	35	0
75	6.3125	472.5000	2.5	5	0	40	35	0
76	6.3125	478.8120	2.56	5	0	41	35	0
77	6.3125	485.1250	2.63	5	0	42	35	0
78	6.3125	491.4380	2.69	5	0	43	35	0
79	6.3125	497.7500	2.75	5	0	44	35	0
80	6.3125	504.0620	2.81	5	0	45	35	0
81	6.3125	510.3750	2.88	5	0	46	35	0
82	6.3125	516.6880	2.94	5	0	47	35	0
83	6.3125	523.0000	3	5	0	48	35	0
84	6.3125	529.3120	3.06	5	0	49	35	0
85	6.3125	535.6250	3.13	5	0	50	35	0
86	6.3125	541.9380	3.19	5	0	51	35	0
87	6.3125	548.2500	3.25	5	0	52	35	0
88	6.3125	554.5620	3.31	5	0	53	35	0
89	6.3125	560.8750	3.38	5	0	54	35	0
90	6.3125	567.1880	3.44	5	0	55	35	0

Figure 8.2: LP Results

LP Solution

Demand	Shadow price for balance constraint	Total Cost	Optimal Quantity (q*) SK	HT	MT	Optimal Plants (z*) SK	HT	MT
91	6.3125	573.5000	3.5	5	0	56	35	0
92	6.3125	579.8120	3.56	5	0	57	35	0
93	6.3125	586.1250	3.63	5	0	58	35	0
94	6.3125	592.4380	3.69	5	0	59	35	0
95	6.3125	598.7500	3.75	5	0	60	35	0
96	6.3125	605.0620	3.81	5	0	61	35	0
97	6.3125	611.3750	3.88	5	0	62	35	0
98	6.3125	617.6880	3.94	5	0	63	35	0
99	6.3125	624.0000	4	5	0	64	35	0
100	6.3125	630.3120	4.06	5	0	65	35	0
101	6.3125	636.6250	4.13	5	0	66	35	0
102	6.3125	642.9380	4.19	5	0	67	35	0
103	6.3125	649.2500	4.25	5	0	68	35	0
104	6.3125	655.5620	4.31	5	0	69	35	0
105	6.3125	661.8750	4.38	5	0	70	35	0
106	6.3125	668.1880	4.44	5	0	71	35	0
107	6.3125	674.5000	4.5	5	0	72	35	0
108	6.3125	680.8120	4.56	5	0	73	35	0
109	6.3125	687.1250	4.63	5	0	74	35	0
110	6.3125	693.4380	4.69	5	0	75	35	0
111	6.3125	699.7500	4.75	5	0	76	35	0
112	6.3125	706.0620	4.81	5	0	77	35	0
113	6.3125	712.3750	4.88	5	0	78	35	0
114	6.3125	718.6880	4.94	5	0	79	35	0
115	6.3125	725.0000	5	5	0	80	35	0
116	6.3125	731.3120	5.06	5	0	81	35	0
117	6.3125	737.6250	5.13	5	0	82	35	0
118	6.3125	743.9380	5.19	5	0	83	35	0
119	6.3125	750.2500	5.25	5	0	84	35	0
120	6.3125	756.5620	5.31	5	0	85	35	0

LP Solution

Demand	Shadow price for balance constraint	Total Cost	Optimal Quantity (q*) SK	HT	MT	Optimal Plants (z*) SK	HT	MT
121	6.3125	762.8750	5.38	5	0	86	35	0
122	6.3125	769.1880	5.44	5	0	87	35	0
123	6.3125	775.5000	5.5	5	0	88	35	0
124	6.3125	781.8120	5.56	5	0	89	35	0
125	6.3125	788.1250	5.63	5	0	90	35	0
126	6.3125	794.4380	5.69	5	0	91	35	0
127	6.3125	800.7500	5.75	5	0	92	35	0
128	6.3125	807.0620	5.81	5	0	93	35	0
129	6.3125	813.3750	5.88	5	0	94	35	0
130	6.3125	819.6880	5.94	5	0	95	35	0
131	6.3125	826.0000	6	5	0	96	35	0
132	7	833.0000	6	5	0.17	96	35	1
133	7	840.0000	6	5	0.33	96	35	2
134	7	847.0000	6	5	0.5	96	35	3
135	7	854.0000	6	5	0.67	96	35	4
136	7	861.0000	6	5	0.83	96	35	5
137	7	868.0000	6	5	1	96	35	6
138	7	875.0000	6	5	1.17	96	35	7
139	7	882.0000	6	5	1.33	96	35	8
140	7	889.0000	6	5	1.5	96	35	9
141	7	896.0000	6	5	1.67	96	35	10
142	7	903.0000	6	5	1.83	96	35	11
143	7	910.0000	6	5	2	96	35	12
144	7	917.0000	6	5	2.17	96	35	13
145	7	924.0000	6	5	2.33	96	35	14
146	7	931.0000	6	5	2.5	96	35	15
147	7	938.0000	6	5	2.67	96	35	16
148	7	945.0000	6	5	2.83	96	35	17
149	7	952.0000	6	5	3	96	35	18
150	7	959.0000	6	5	3.17	96	35	19

LP Solution

Demand	Shadow price for balance constraint	Total Cost	Optimal Quantity (q*) SK	HT	MT	Optimal Plants (z*) SK	HT	MT
151	7	966.0000	6	5	3.33	96	35	20
152	7	973.0000	6	5	3.5	96	35	21
153	7	980.0000	6	5	3.67	96	35	22
154	7	987.0000	6	5	3.83	96	35	23
155	7	994.0000	6	5	4	96	35	24
156	7	1,001.0000	6	5	4.17	96	35	25
157	7	1,008.0000	6	5	4.33	96	35	26
158	7	1,015.0000	6	5	4.5	96	35	27
159	7	1,022.0000	6	5	4.67	96	35	28
160	7	1,029.0000	6	5	4.83	96	35	29
161	7	1,036.0000	6	5	5	96	35	30

Figure 8.3: LP Results

MIP Solution

Demand	Shadow price for balance constraint	Total Cost	Optimal Quantity (q*)			Optimal Plants (z*)		
			SK	HT	MT	SK	HT	MT
1	2	7.0000	0	0	1	0	0	1
2	7	14.0000	0	0	1	0	0	2
3	7	21.0000	0	0	1	0	0	3
4	7	28.0000	0	0	1	0	0	4
5	7	35.0000	0	0	1	0	0	5
6	7	42.0000	0	0	1	0	0	6
7	2	44.0000	0	1	0	0	7	0
8	2	51.0000	0	1	1	0	7	1
9	7	58.0000	0	1	1	0	7	2
10	7	65.0000	0	1	1	0	7	3
11	7	72.0000	0	1	1	0	7	4
12	7	79.0000	0	1	1	0	7	5
13	7	86.0000	0	1	1	0	7	6
14	2	88.0000	0	2	0	0	14	0
15	2	95.0000	0	2	1	0	14	1
16	3	101.0000	1	0	0	16	0	0
17	3	108.0000	1	0	1	16	0	1
18	7	115.0000	1	0	1	16	0	2
19	7	122.0000	1	0	1	16	0	3
20	7	129.0000	1	0	1	16	0	4
21	2	132.0000	0	3	0	0	21	0
22	2	139.0000	0	3	1	0	21	1
23	3	145.0000	1	1	0	16	7	0
24	3	152.0000	1	1	1	16	7	1
25	7	159.0000	1	1	1	16	7	2
26	7	166.0000	1	1	1	16	7	3
27	7	173.0000	1	1	1	16	7	4
28	2	176.0000	0	4	0	0	28	0
29	2	183.0000	0	4	1	0	28	1
30	3	189.0000	1	2	0	16	14	0

MIP Solution

Demand	Shadow price for balance constraint	Total Cost	Optimal Quantity (q*)			Optimal Plants (z*)		
			SK	HT	MT	SK	HT	MT
31	3	196.0000	1	2	1	16	14	1
32	3	202.0000	2	0	0	32	0	0
33	3	209.0000	2	0	1	32	0	1
34	7	216.0000	2	0	1	32	0	2
35	2	220.0000	0	5	0	0	35	0
36	2	227.0000	0	5	1	0	35	1
37	3	233.0000	1	3	0	16	21	0
38	3	240.0000	1	3	1	16	21	1
39	3	246.0000	2	2	0	32	7	0
40	3	253.0000	2	1	1	32	7	1
41	7	260.0000	2	1	1	32	7	2
42	7	267.0000	2	1	1	32	7	3
43	7	274.0000	2	1	1	32	7	4
44	3	277.0000	1	4	0	16	28	0
45	3	284.0000	1	4	1	16	28	1
46	3	290.0000	2	3	0	32	14	0
47	3	297.0000	2	2	1	32	14	1
48	3	303.0000	3	0	0	48	0	0
49	3	310.0000	3	0	1	48	0	1
50	7	317.0000	3	0	1	48	0	2
51	3	321.0000	1	5	0	16	35	0
52	3	328.0000	1	5	1	16	35	1
53	3	334.0000	2	3	0	32	21	0
54	3	341.0000	2	3	1	32	21	1
55	3	347.0000	3	2	0	48	7	0
56	3	354.0000	3	1	1	48	7	1
57	7	361.0000	3	1	1	48	7	2
58	7	368.0000	3	1	1	48	7	3
59	7	375.0000	3	1	1	48	7	4
60	3	378.0000	2	4	0	32	28	0

MIP Solution

Demand	Shadow price for balance constraint	Total Cost	Optimal Quantity (q*)			Optimal Plants (z*)		
			SK	HT	MT	SK	HT	MT
61	3	385.0000	2	4	1	32	28	1
62	3	391.0000	3	3	0	48	14	0
63	3	398.0000	3	2	1	48	14	1
64	3	404.0000	4	0	0	64	0	0
65	3	411.0000	4	0	1	64	0	1
66	7	418.0000	4	0	1	64	0	2
67	3	422.0000	2	5	0	32	35	0
68	3	429.0000	2	5	1	32	35	1
69	3	435.0000	3	3	0	48	21	0
70	3	442.0000	3	3	1	48	21	1
71	3	448.0000	4	1	0	64	7	0
72	3	455.0000	4	1	1	64	7	1
73	7	462.0000	4	1	1	64	7	2
74	7	469.0000	4	1	1	64	7	3
75	7	476.0000	4	1	1	64	7	4
76	3	479.0000	3	4	0	48	28	0
77	3	486.0000	3	4	1	48	28	1
78	3	492.0000	4	3	0	64	14	0
79	3	499.0000	4	2	1	64	14	1
80	3	505.0000	5	0	0	80	0	0
81	3	512.0000	5	0	1	80	0	1
82	7	519.0000	5	0	1	80	0	2
83	3	523.0000	3	5	0	48	35	0
84	3	530.0000	3	5	1	48	35	1
85	3	536.0000	4	3	0	64	21	0
86	3	543.0000	4	3	1	64	21	1
87	3	549.0000	5	1	0	80	7	0
88	3	556.0000	5	1	1	80	7	1
89	7	563.0000	5	1	1	80	7	2
90	7	570.0000	5	1	1	80	7	3

Figure 8.4: MIP Results

MIP Solution

Demand	Shadow price for balance constraint	Total Cost	Optimal Quantity (q*)			Optimal Plants (z*)		
			SK	HT	MT	SK	HT	MT
91	7	577.0000	5	1	1	80	7	4
92	3	580.0000	4	4	0	64	28	0
93	3	587.0000	4	4	1	64	28	1
94	3	593.0000	5	2	0	80	14	0
95	3	600.0000	5	2	1	80	14	1
96	3	606.0000	6	0	0	96	0	0
97	3	613.0000	6	0	1	96	0	1
98	7	620.0000	6	0	1	96	0	2
99	3	624.0000	4	5	0	64	35	0
100	3	631.0000	4	5	1	64	35	1
101	3	637.0000	5	3	0	80	21	0
102	3	644.0000	5	3	1	80	21	1
103	3	650.0000	6	1	0	96	7	0
104	3	657.0000	6	1	1	96	7	1
105	7	664.0000	6	1	1	96	7	2
106	7	671.0000	6	1	1	96	7	3
107	7	678.0000	6	1	1	96	7	4
108	3	681.0000	5	4	0	80	28	0
109	3	688.0000	5	4	1	80	28	1
110	3	694.0000	6	2	0	96	14	0
111	3	701.0000	6	2	1	96	14	1
112	7	708.0000	6	2	1	96	14	2
113	7	715.0000	6	2	1	96	14	3
114	7	722.0000	6	2	1	96	14	4
115	3	725.0000	5	5	0	80	35	0
116	3	732.0000	5	5	1	80	35	1
117	3	738.0000	6	3	0	96	21	0
118	3	745.0000	6	3	1	96	21	1
119	7	752.0000	6	3	1	96	21	2
120	7	759.0000	6	3	1	96	21	3

MIP Solution

Demand	Shadow price for balance constraint	Total Cost	Optimal Quantity (q*)			Optimal Plants (z*)		
			SK	HT	MT	SK	HT	MT
121	7	766.0000	6	3	1	96	21	4
122	7	773.0000	6	3	1	96	21	5
123	3	779.0000	6	4	0	95	28	0
124	3	782.0000	6	4	0	96	28	0
125	3	789.0000	6	4	1	96	28	1
126	7	796.0000	6	4	1	96	28	2
127	7	803.0000	6	4	1	96	28	3
128	7	810.0000	6	4	1	96	28	4
129	7	817.0000	6	4	1	96	28	5
130	3	823.0000	6	5	0	95	35	0
131	3	826.0000	6	5	0	96	35	0
132	3	833.0000	6	5	1	96	35	1
133	7	840.0000	6	5	1	96	35	2
134	7	847.0000	6	5	1	96	35	3
135	7	854.0000	6	5	1	96	35	4
136	7	861.0000	6	5	1	96	35	5
137	7	868.0000	6	5	1	96	35	6
138	7	875.0000	6	5	2	96	35	7
139	7	882.0000	6	5	2	96	35	8
140	7	889.0000	6	5	2	96	35	9
141	7	896.0000	6	5	2	96	35	10
142	7	903.0000	6	5	2	96	35	11
143	7	910.0000	6	5	2	96	35	12
144	7	917.0000	6	5	3	96	35	13
145	7	924.0000	6	5	3	96	35	14
146	7	931.0000	6	5	3	96	35	15
147	7	938.0000	6	5	3	96	35	16
148	7	945.0000	6	5	3	96	35	17
149	7	952.0000	6	5	3	96	35	18
150	7	959.0000	6	5	4	96	35	19

MIP Solution

Demand	Shadow price for balance constraint	Total Cost	Optimal Quantity (q*)			Optimal Plants (z*)		
			SK	HT	MT	SK	HT	MT
151	7	966.0000	6	5	4	96	35	20
152	7	973.0000	6	5	4	96	35	21
153	7	980.0000	6	5	4	96	35	22
154	7	987.0000	6	5	4	96	35	23
155	7	994.0000	6	5	4	96	35	24
156	7	1,001.0000	6	5	5	96	35	25
157	7	1,008.0000	6	5	5	96	35	26
158	7	1,015.0000	6	5	5	96	35	27
159	7	1,022.0000	6	5	5	96	35	28
160	7	1,029.0000	6	5	5	96	35	29
161	7	1,036.0000	6	5	5	96	35	30

Figure 8.5: MIP Results

D	Semi-Lagrangean prices… .. with min. output req.	…without min. output req.	D	Semi-Lagrangean .. with min. output req.	…without min. output req.	D	Semi-Lagrangean .. with min. output req.	…without min. output req.	D	Semi-Lagrangean .. with min. output req.	…without min. output req.	D	Semi-Lagrangean prices… .. with min. output req.	…without min. output req.
1	32	7	31	8	7	61	10	7	91	7	7	121	7	7
2	7	7	32	6.5	6.5	62	6.5	6.5	92	6.33333	6.33333	122	7	7
3	7	7	33	8	7	63	8	7	93	10	7	123	6.83333	6.83333
4	7	7	34	7	7	64	6.5	6.5	94	6.5	6.5	124	6.33333	6.33333
5	7	7	35	6.28571	6.28571	65	8	7	95	8	7	125	11	7
6	7	7	36	10	7	66	7	7	96	6.5	6.5	126	7	7
7	6.28571	6.28571	37	6.5	6.5	67	6.3125	6.3125	97	8	7	127	7	7
8	12	7	38	8	7	68	10	7	98	7	7	128	7	7
9	7	7	39	6.5	6.5	69	6.5	6.5	99	6.3125	6.3125	129	7	7
10	7	7	40	8	7	70	8	7	100	10	7	130	6.83333	6.83333
11	7	7	41	7	7	71	6.5	6.5	101	6.5	6.5	131	6.3125	6.3125
12	7	7	42	7	7	72	8	7	102	8	7	132	11	7
13	7	7	43	7	7	73	7	7	103	6.5	6.5	133	7	7
14	6.28571	6.28571	44	6.33333	6.33333	74	7	7	104	8	7	134	7	7
15	10	7	45	10	7	75	7	7	105	7	7	135	7	7
16	6.5	6.5	46	6.5	6.5	76	6.33333	6.33333	106	7	7	136	7	7
17	8	7	47	8	7	77	10	7	107	7	7	137	7	7
18	7	7	48	6.5	6.5	78	6.5	6.5	108	6.33333	6.33333	138	7	7
19	7	7	49	8	7	79	8	7	109	10	7	139	7	7
20	7	7	50	7	7	80	6.5	6.5	110	6.5	6.5	140	7	7
21	6.28571	6.28571	51	6.3125	6.3125	81	8	7	111	8	7	141	7	7
22	10	7	52	10	7	82	7	7	112	7	7	142	7	7
23	6.5	6.5	53	6.5	6.5	83	6.3125	6.3125	113	7	7	143	7	7
24	8	7	54	8	7	84	10	7	114	7	7	144	7	7
25	7	7	55	6.5	6.5	85	6.5	6.5	115	6.3125	6.3125	145	7	7
26	7	7	56	8	7	86	8	7	116	10	7	146	7	7
27	7	7	57	7	7	87	6.5	6.5	117	6.5	6.5	147	7	7
28	6.28571	6.28571	58	7	7	88	8	7	118	8	7	148	7	7
29	10	7	59	7	7	89	7	7	119	7	7	149	7	7
30	6.5	6.5	60	6.33333	6.33333	90	7	7	120	7	7	150	7	7

Figure 8.6: Semi-Lagrangean Prices with and without minimimum output requirement

Demand	SLR Price	Commodity Price	Demand	SLR Price	Commodity Price	Demand	SLR Price	Commodity Price	Demand	SLR Price	Commodity Price	Demand	SLR Price	Commodity Price	Demand	SLR Price	Commodity Price
1	7.0000	7.0000	31	7.0000	6.3226	61	7.0000	6.3770	91	7.0000	7.0000	121	7.0000	7.0000	151	7.0000	7.0000
2	7.0000	7.0000	32	6.5000	6.3125	62	6.5000	6.3065	92	6.3333	6.3333	122	7.0000	7.0000	152	7.0000	7.0000
3	7.0000	7.0000	33	7.0000	6.3333	63	7.0000	6.3175	93	7.0000	7.0000	123	6.8333	6.8333	153	7.0000	7.0000
4	7.0000	7.0000	34	7.0000	6.3529	64	6.5000	6.3438	94	6.5000	6.5000	124	6.3333	6.3333	154	7.0000	7.0000
5	7.0000	7.0000	35	6.2857	6.2857	65	7.0000	6.3538	95	7.0000	7.0000	125	7.0000	7.0000	155	7.0000	7.0000
6	7.0000	7.0000	36	7.0000	6.3889	66	7.0000	6.3485	96	6.5000	6.5000	126	7.0000	7.0000	156	7.0000	7.0000
7	6.2857	6.2857	37	6.5000	6.4054	67	6.3125	6.3125	97	7.0000	7.0000	127	7.0000	7.0000	157	7.0000	7.0000
8	7.0000	6.3750	38	7.0000	6.3947	68	7.0000	6.3529	98	7.0000	7.0000	128	7.0000	7.0000	158	7.0000	7.0000
9	7.0000	6.4444	39	6.5000	6.3077	69	6.5000	6.3768	99	6.3125	6.3125	129	7.0000	7.0000	159	7.0000	7.0000
10	7.0000	6.5000	40	7.0000	6.3250	70	7.0000	6.3857	100	7.0000	7.0000	130	6.8333	6.8333	160	7.0000	7.0000
11	7.0000	6.5455	41	7.0000	6.3415	71	6.5000	6.4085	101	6.5000	6.5000	131	6.3125	6.3125	161	7.0000	7.0000
12	7.0000	6.5833	42	7.0000	6.3571	72	7.0000	6.4167	102	7.0000	7.0000	132	7.0000	7.0000			
13	7.0000	6.6154	43	7.0000	6.3721	73	7.0000	6.4247	103	6.5000	6.5000	133	7.0000	7.0000			
14	6.2857	6.2857	44	6.3333	6.3333	74	7.0000	6.4324	104	7.0000	7.0000	134	7.0000	7.0000			
15	7.0000	6.3333	45	7.0000	6.4000	75	7.0000	6.4400	105	7.0000	7.0000	135	7.0000	7.0000			
16	6.5000	6.3125	46	6.5000	6.3043	76	6.3333	6.3333	106	7.0000	7.0000	136	7.0000	7.0000			
17	7.0000	6.3529	47	7.0000	6.3191	77	7.0000	6.5065	107	7.0000	7.0000	137	7.0000	7.0000			
18	7.0000	6.3889	48	6.5000	6.3125	78	6.5000	6.5000	108	6.3333	6.3333	138	7.0000	7.0000			
19	7.0000	6.4211	49	7.0000	6.3265	79	7.0000	7.0000	109	7.0000	7.0000	139	7.0000	7.0000			
20	7.0000	6.4500	50	7.0000	6.3400	80	6.5000	6.5000	110	6.5000	6.5000	140	7.0000	7.0000			
21	6.2857	6.2857	51	6.3125	6.3125	81	7.0000	7.0000	111	7.0000	7.0000	141	7.0000	7.0000			
22	7.0000	6.4545	52	7.0000	6.3654	82	7.0000	7.0000	112	7.0000	7.0000	142	7.0000	7.0000			
23	6.5000	6.3043	53	6.5000	6.3774	83	6.3125	6.3125	113	7.0000	7.0000	143	7.0000	7.0000			
24	7.0000	6.3333	54	7.0000	6.3704	84	7.0000	7.0000	114	7.0000	7.0000	144	7.0000	7.0000			
25	7.0000	6.3600	55	6.5000	6.3091	85	6.5000	6.5000	115	6.3125	6.3125	145	7.0000	7.0000			
26	7.0000	6.3846	56	7.0000	6.3214	86	7.0000	7.0000	116	7.0000	7.0000	146	7.0000	7.0000			
27	7.0000	6.4074	57	7.0000	6.3333	87	6.5000	6.5000	117	6.5000	6.5000	147	7.0000	7.0000			
28	6.2857	6.2857	58	7.0000	6.3448	88	7.0000	7.0000	118	7.0000	7.0000	148	7.0000	7.0000			
29	7.0000	6.4483	59	7.0000	6.3559	89	7.0000	7.0000	119	7.0000	7.0000	149	7.0000	7.0000			
30	6.5000	6.3000	60	6.3333	6.3333	90	7.0000	7.0000	120	7.0000	7.0000	150	7.0000	7.0000			

Figure 8.7: Results from Second Price Auction with Three Technologies

Bibliography

AMPL: A Modeling Language for Mathematical Programming. Duxbury Press, November 2002. ISBN 0534388094. URL http://www.amazon.com/exec/obidos/redirect?tag=citeulike07-20&path=ASIN/0534388094.

Energy Information Administration. Electricity restructuring by state. URL http://205.254.135.24/cneaf/electricity/page/restructuring/restructureelect.html.

R.E. Alcaly and A.K. Klevorick. A note on the dual prices of integer programs. *Econometrica*, 34(1):pp. 206–214, 1966. ISSN 00129682. URL http://www.jstor.org/stable/1909864.

V. Araoz and K. Jörnsten. Alternative pricing approach for the lumpy electricity markets. In *7th International Conference on the European Energy Markets*, 2010.

V. Araoz and K. Jörnsten. Semi-lagrangean approach for price discovery in markets with non-convexities. *European Journal of Operational Research*, 214(2):411–417, 2011.

A. G. Bakirtzis and C. E. Zoumas. Lambda of lagrangian relaxation solution to unit commitment problem. *Iee Proceedings-Generation Transmission and Distribution*, 147(2):131–136, 2000.

R. Baldick. The generalized unit commitment problem. *IEEE Transactions on Power Systems*, 10(1):465–475, 1995.

R. Baldick, U. Helman, B. F. Hobbs, and R. P. O'Neill. Design of efficient generation markets. *Proceedings of the IEEE*, 93(11):1998–2012, 2005.

M. L. Balinski and W. J. Baumol. The dual in nonlinear programming and its economic interpretation. *The Review of Economic Studies*, 35(3):pp. 237–256, 1968. ISSN 00346527. URL http://www.jstor.org/stable/2296660.

C. Beltran, C. Tadonki, and J. P. Vial. Solving the p-median problem with a semi-lagrangian relaxation. *Computational Optimization and Applications*, 35(2):239–260, 2006.

D.P. Bertsekas. *Nonlinear programming*. Athena Scientific, 1995.

M. Bjørndal and K. Jörnsten. Equilibrium prices supported by dual price functions in markets with non-convexities. *European Journal of Operational Research*, 190(3):768–789, 2008.

M. Bjørndal, K. Jörnsten, and L. Rud. *Capacity Charges: A Price Adjustment Process for Managing Congestion in Electricity Transmission Networks*, pages 267–292. Energy Systems. Springer, 2008.

F. Bouffard and F.D. Galiana. *Generalized Uplifts in Pool Based Electricity Markets*. Springer, 2005.

J. Bower and D. W. Bunn. Model-based comparisons of pool and bilateral markets for electricity. *Energy Journal*, 21(3):1–29, 2000. 49 INT ASSOC ENERGY ECONOMICS CLEVELAND 338ZA.

S. Boyd and L. Vandenberghe. *Convex Optimization*. Cambridge University Press, 2004.

FERC Federal Energy Regulatory Commission. Usa electricity power market overview. URL `http://www.ferc.gov/market-oversight/mkt-electric/overview.asp`.

A.J. Conejo and C. Ruiz. Pricing non-convexities in an electricity pool, 2011. Workshop on Pricing in Deregulated Electricity Markets, Voss, Norway.

S. Dekrajangpetch, G. B. Sheble, and A. J. Conejo. Auction implementation problems using lagrangian relaxation. *IEEE Transactions on Power Systems*, 14(1):82–88, 1999.

F. Ding and J. D. Fuller. Nodal, uniform, or zonal pricing: Distribution of economic surplus. *IEEE Transactions on Power Systems*, 20(2):875–882, 2005. Ding, F Fuller, JD.

G. Doorman and B. Nygreen. Market price calculations in restructured electricity markets. *Annals of Operations Research*, 124(1-4):49–67, 2003. 12 KLUWER ACADEMIC PUBL 745MC.

L. Duan and X. Sun. *Nonlinear Integer Programming*. Springer, Dordrecht, 2006.

W. Elmaghraby, R. O'Neil, M. Rothkopf, and W. Stewart. Pricing and efficiency in "lumpy" energy markets. *The Electricity Journal*, 17(5):54–64, 2004.

L.A.F.M. Ferreira. On the duality gap for thermal unit commitment problems. *IEEE International Symposium on Circuits and Systems*, 4:2204 – 2207, 1993.

J.D. Fuller. Market equilibrium models with continuous and binary variables. 2008.

F. D. Galiana, A. L. Motto, and F. Bouffard. Reconciling social welfare, agent profits, and consumer payments in electricity pools. *IEEE Transactions on Power Systems*, 18(2):452–459, 2003.

A.M. Geoffrion. Duality in nonlinear programming: A simplified applications-oriented development. *Society for Industrial and Applied Mathematics*, 13(1):1–37, 1971.

A.M Geoffrion. *Lagrangean relaxation for integer programming*, volume 2 of *Mathematical Programming Studies*. Springer Berlin Heidelberg, 1974.

Nemhauser G.L. and Wolsey L. *Integer and combinatorial optimization*. Wiley-Blackwell, 1999.

R.E Gomory and W.J. Baumol. Integer programming and pricing. *Econometrica*, 28:521–550, 1960.

F. J. Gould. Extensions of lagrange multipliers in nonlinear programming. *SIAM Journal on Applied Mathematics*, 17(6):pp. 1280–1297, 1969. ISSN 00361399. URL `http://www.jstor.org/stable/2099209`.

R. Green. Nodal pricing of electricity: how much does it cost to get it wrong? *Journal of Regulatory Economics*, 31(2):125–149, 2007. Green, Richard.

P.R. Gribik, W.W. Hogan, and S.L. Pope. Market-clearing electricity prices and energy uplift. 2007. URL `http://www.hks.harvard.edu/fs/whogan`.

S. Y. Hao, G. A. Angelidis, H. Singh, and A. D. Papalexopoulos. Consumer payment minimization in power pool auctions. *IEEE Transactions on Power Systems*, 13(3):986–991, 1998.

W.W. Hogan and B.J. Ring. On minimum-uplift pricing for electricity markets. *Working Paper*, 2003.

M. Hsu. An introduction to the pricing of electric power transmission. *Utilities Policy*, 6(3): 257–270, 1997.

J. M. Jacobs. Artificial power markets and unintended consequences. *IEEE Transactions on Power Systems*, 12(2):968–972, 1997.

R.B. Johnson, S.S. Oren, and A.J. Svoboda. Equity and efficiency of unit commitment in competitive electricity markets. *Utilities Policy*, 6(1):9–19, 1997.

P.L. Joskow. Capacity mechanisms in imperfect electricity markets: Need and design. *Utilities Policy*, 16(3):159–170, 2008.

M. H. Karwan and Rardin R.L. Some relationships between lagrangean and surrogate duality in integer programming. *Mathematical Programming*, 17:230–334, 1979.

M.H. Karwan and R.L. Rardin. Surrogate dual multiplier search procedures in integer programming. *Operations Research*, 32(1):52–69, 1984.

T. O. Leautier. Electricity auctions. *Journal of Economics and Management Strategy*, 10(3): 331–358, 2001.

J. Lundgren, M. Rönnqvist, and P. Värbrand. *Optimization*. Studentlitteratur, 2010.

R. Madlener and M. Kaufmann. Power exchange spot market trading in europe: theoretical considerations and empirical evidence, 2002.

M. Madrigal and V. H. Quintana. Existence and determination of competitive equilibrium in unit commitment power pool auctions: Price setting and scheduling alternatives. *IEEE Transactions on Power Systems*, 16(3):380–388, 2001.

M. Madrigal and V.H. Quintana. Using optimization models and techniques to implement electricity auctions. In IEEE Power Engineering Society, editor, *IEEE Power Engineering Society Winter Meeting 2000*, pages 897–902. IEEE, 2000.

A. Mas-Collel, M.D. Whinston, and J.R. Green. *Microeconomic Theory*. Oxford University Press, 1995.

L. Meeus, K. Verhaegen, and R. Belmans. Pricing in electric energy auctions with block orders. 2005.

L. Meeus, K. Verhaegen, and R. Belmans. Block order restrictions in combinatorial electric energy auctions. *European Journal of Operational Research*, 196(3):1202–1206, 2009. Meeus, Leonardo Verhaegen, Karolien Belmans, Ronnie.

A. L. Motto and F. D. Galiana. Equilibrium of auction markets with unit commitment: the need for augmented pricing. *IEEE Transactions on Power Systems*, 17(3):798–805, 2002.

A. L. Motto, F. D. Galiana, A. J. Conejo, and J. M. Arroyo. Network-constrained multiperiod auction for a pool-based electricity market. *IEEE Transactions on Power Systems*, 17(3): 646–653, 2002a.

A.L. Motto, F.D. Galiana, A.J. Conejo, and M. Huneault. *Decentralized nodal-price self dispatch and unit commitment*, chapter 15, pages 271–292. Kluwer, 2001.

A.L. Motto, F.D. Galiana, A.J. Conejo, and M. Huneault. On walrasian equilibrium for pool-based electricity markets. *IEEE Transactions on Power Systems*, 17(3):774–781, 2002b.

G. Muratore. Equilibria in markets with non-convexities and a solution to the missing money phenomenon in energy markets, 2008.

R.P. O'Neil, U. Helman, and P.M. Sotkiewicz. *Regulatory Evolution Market Design and Unit commitment*, chapter 2, pages 15–38. Kluwer, 2001.

R.P. O'Neill. Nonconvex electricity market design. In *Power Energy Society General Meeting, 2009. PES '09. IEEE*, pages 1 –7, july 2009. doi: 10.1109/PES.2009.5275398.

R.P. O'Neill, P.M. Sotkiewicz, B.F. Hobbs, M.H. Rothkopf, and W.R. Stewart. Efficient market-clearing prices in markets with nonconvexities. *European Journal of Operational Research*, 164(1):269–285, 2005.

G. Pepermans and B. Willems. Cost recovery in congested electricity networks. *ZEITSCHRIFT FR ENERGIEWIRTSCHAFT*, 34(3):195–208, 2010.

M. Pirnia and J.D. Fuller. A mixed integer nonlinear program for electric generation expansion with energy and capacity pricing. *Power and Energy Society General Meeting*, July 2010:1 – 7, 2010.

H. Ravn. Day-ahead market clearing methods nord pool spot, 2010.

C. Ruiz, A. J. Conejo, and S. A. Gabriel. Pricing non-convexities in an electricity pool. *IEEE Transactions on Power Systems*, To be published. DOI: 10.1109/TPWRS.2012.2184562, 2012.

H. E. Scarf. Mathematical programming and economic theory. *Operations Research*, 38(3): 377–385, 1990.

H. E. Scarf. The allocation of resources in the presence of indivisibilities. *Journal of Economic Perspectives*, 8(4):111–128, 1994.

F.C. Schweppe, M.C. Caraminis, R.D. Tabors, and R.E. Bohn. *Spot pricing of electricity*. Kluwer Academic Publishers, Boston, 1988.

S. Sen and T.S. Genc. Start-up prices and shadow prices for resource allocation models with indivisibilities. 2008.

G. B. Sheble and G. N. Fahd. Unit commitment literature synopsis. *IEEE Transactions on Power Systems*, 9(1):128–135, 1994.

R. Sioshansi, R. O'Neill, and S. S. Oren. Economic consequences of alternative solution methods for centralized unit commitment in day-ahead electricity markets. *IEEE Transactions on Power Systems*, 23(2):344–352, 2008. Sioshansi, Ramteen O'Neill, Richard Oren, Shmuel S.

S. E. Stoft. *Power System Economics: Designing Markets for Electricity*. Chichester, Wiley, 2002.

E. Toczylowski and I. Zoltowska. A new pricing scheme for a multi-period pool-based electricity auction. *European Journal of Operational Research*, 197(3):1051–1062, 2009. Toczylowski, Eugeniusz Zoltowska, Izabela.

M. V. Van Boening and N. T. Wilcox. Avoidable cost: Ride a double auction roller coaster. *American Economic Review*, 86(3):461–477, 1996.

M. Van Vyve. Linear prices for non-convex electricity markets: models and algorithms. 2011. URL `http://www.ulouvain.be/cps/ucl/doc/core/documents/coredp2011_50web.pdf`.

F. M. Westfield. Electric power: Competition or centralization? *Energy Policy*, 16(4):378–383, 1988.

H. Williams. Duality in mathematics and linear and integer programming. *Journal of Optimization Theory and Applications*, 90:257–278, 1996. ISSN 0022-3239. URL `http://dx.doi.org/10.1007/BF02189998`. 10.1007/BF02189998.

R.B. Wilson. *Designing Competitive Electricity Markets*, chapter 11, pages 159–184. Kluwer Academic Publishers, 1998.

D.A. Wismer and R. Chattergy. *Introduction to nonlinear optimization: a problem solving approach*. System Science and Engineering. 1983.

L. A. Wolsey. Integer programming duality: Price functions and sensitivity analysis. *Mathematical Programming*, 20(2):173–195, 1981.

Y. Yamamoto and T. Tezuka. A large generator's bids in an electricity supply auction: Uniform pricing versus pay-as-bid pricing. *Electrical Engineering in Japan*, 160(4):41–48, 2007. Yamamoto, Yoshihiro Tezuka, Tetsuo.

J.H. Yan and G.A. Stern. Simultaneous optimal auction and unit commitment for deregulated electricity markets. *The Electricity Journal*, pages 72–80, 2002.