Are Central Bankers Inflation Nutters?

---- A Bayesian MCMC Estimator of the Long Memory Parameter in a State Space Model

Fredrik N.G. Andersson*

Department of Economics, Lund University, Sweden

Tel: +46 46 222 46 44

Fax: +46 46 222 41 18

Email: ngf.andersson@nek.lu.se

www.nek.lu.se

Yushu Li

Department of Business and Management Science, the Norwegian School of Economics, Norway

Email: Yushu.li@nhh.no

Abstract: Several central banks have adopted inflation targets. The implementation of these targets is flexible; the central banks aim to meet the target over the long term but allow inflation to deviate from the target in the short-term in order to avoid unnecessary volatility in the real economy. In this paper, we propose modeling the degree of flexibility using an ARFIMA model. Under the assumption that the central bankers control the long-run inflation rates, the fractional integration order captures the flexibility of the inflation targets. A higher integration order is associated with a more flexible target. Several estimators of the fractional integration order have been proposed in the literature. Grassi and Magistris (2011) show that a state-based maximum likelihood estimator is superior to other estimators, but our simulations show that their finding is over-biased for a nearly non-stationary time series. We resolve this issue by using a Bayesian Monte Carlo Markov Chain (MCMC) estimator. Applying this estimator to inflation from six inflation-targeting countries for the period 1999M1 to 2013M3, we find that inflation is integrated of order 0.8 to 0.9 depending on the country. The inflation targets are thus implemented with a high degree of flexibility.

Keywords: *fractional integration, inflation-targeting, state space model*

Fredrik NG Andersson and Yushu Li gratefully acknowledge funding from Swedish Research Council (project number 421-2009-2663)

1 Introduction

Several central banks around the world have switched from a fixed exchange rate regime to an inflation-targeting regime during the last 25 years. The inflation target is either a constant or a range and is calculated using the year-on-year change in the consumer price index (CPI). Facing a trade-off between inflation stability and real economic stability in the short run (Svensson, 1997), most central banks have chosen a flexible inflation target to maintain real economic stability. In the words of the previous Governor of the Bank of England, Mervyn King, central bankers are not “inflation nutters”. Consequently, inflation contains long swings around its mean. Moreover, there is evidence that inflation is covariance non-stationary yet mean-reverting, i.e., fractionally integrated (Hassler and Wolters, 1995; Caggio and Castelnuovo, 2011).

The flexibility of the inflation targets is commonly modeled using a Taylor rule (Clarida, Gali and Gertler, 1998; Cobion and Goldstein, 2012). According to the Taylor rule, the central bankers set the interest rate based on the deviation of inflation from the target and the size of the output gap. Empirical estimation of the Taylor rule is difficult because it requires both an estimate of the long-run equilibrium real interest rate and an estimate of the output gap. The model also assumes that central bankers do not consider other variables when making interest rate decisions (Svensson, 2003).

An alternative approach is to estimate the degree of flexibility by modeling inflation with an ARFIMA model. Here, AR and MA components capture the short-run dynamics of the inflation target and the fractional integration order of the long-run dynamics. Using the fractional integration order under the assumption that the central bank controls the long-run inflation rate, we can estimate how flexibly the inflation targets are implemented. A higher integration order indicates that the central banker is more willing to allow inflation to deviate from its target - thus the inflation target is more flexible, and conversely, the central banker is then more willing to allow inflation to deviate from its target. By analyzing the fractional integration order in an ARFIMA model, it is possible to determine whether the central bankers are “inflation nutters”, or, flexible in their implementation of the inflation target.

Several estimators of ARFIMA models have been proposed. These estimators include the parametric method, which is based on the maximum likelihood function (Fox and Taqqu, 1986; Sowell, 1992; Giraitis and Taqqu, 1999), and the regression-based approach in spectral domain (Geweke and Porter-Hudak, 1983). Additional estimators include the semi-parametric (Robison 1995a, b; Shimotsu and Phillips, 2005), and the wavelet-based semi-parametric (McCoy and Walden 1996; Jensen 2004).

Chan and Palma (1998) established a theoretical foundation to estimate the ARFIMA model with an approximate maximum likelihood estimation (MLE)-based state space model. The authors truncate the infinite AR or MA representations of the ARFIMA model into finite lags and calculate the approximate maximum likelihood using the Kalman filter. Chan and Palma (1998) show that the approximate MLE-based state space model has desirable asymptotic properties and a rapid converging rate. Recently, Grassi and Magistris (2011) conducted a simulation study to compare the state space model-based long memory estimation with several widely applied parametric and semi-parametric methods. Grassi and Magistris (2011) show that compared with the other estimations, the state space model method is robust to the *t* distribution and is missing value, measurement error and level shift.

ARFIMA models are estimated for inflation from six inflation-targeting regions: Canada, the Euro Area, Norway, Sweden, the United Kingdom and the United States, for the period of 1999M1 to 2013M3. Our results show that most of the inflation persistence is caused by the long-run dynamics and that the short-run dynamics exhibits low persistence. The fractional integration order falls within the interval of 0.8 and 0.90 for all regions except Norway, where the integration order is 1.05. However, the estimated integration order for Norway is not significantly greater than one at the 5% significance level. Overall, our results show that none of the central banks are “inflation nutters”.

The rest of the paper is organized as follows: section 2 introduces the state space model-based MLE for long memory series, section 3 combines the state space model with the MCMC algorithm to estimate the fractional difference parameters, section 4 applies empirical examples, and the conclusions can be found in the final sections.

2 State Space Maximum Likelihood Estimator

Consider the ARFIMA(*p,d,q*) model $\phi(B)(1-L)^d Y_t = \theta(B)\varepsilon_t$, $0<d<1$ and $\varepsilon_t \sim i.i.d.N(0,\sigma_\varepsilon^2)$, $0<d<1$ and $\varepsilon_t \sim i.i.d.N(0,\sigma_\varepsilon^2)$. When *p,q* are less than or equal to one, we can obtain a truncated AR or MA representation of the ARFIMA(*p,d,q*) model, and estimate the parameters by approximate *MLE*. It is difficult to write out closed form AR or MA representations and carry out the estimation when *p* and *q* exceed one. However, we can use Hosking’s (1981) method and estimate the parameters in the ARFIMA model recursively:

Step 1: Estimate d^0 by viewing Y_t as pure fractional difference series and then applying the ARIMA(*p,0,q*) process $u_t^0 = (1-L)^{d^0} Y_t$.

Step 2: Use the Box-Jenkins method to identify and estimate ϕ^0 and θ^0 parameters in the ARIMA(*p,0,q*) model $\phi(B)u_t^0 = \theta(B)\varepsilon_t$.

Step 3: Apply the ARIMA(*0,d,0*) process $x_t^0 = \{\theta^0(B)\}^{-1}\phi^0(B)Y_t$, and estimate d^1 in the fractional difference process $(1-L)^{d^1} x_t = \varepsilon_t$.

Step 4: Check for convergence with the convergence rule $d^i - d^{i-1} < 0.005$, and obtain the estimation results d^i, ϕ^i and θ^i.

For simplicity, consider the pure long memory series $(1-L)^d Y_t = \varepsilon_t$. There are three reasons to consider a long memory model in state space form. First, it can streamline the prediction and interpolations through the recursive expression in Kalman filter. Second, it can utilize the skipping approach (Durbin and Koopman, 2001), which controls for missing data by extrapolating the data that is available rather than relying on assumptions. Third, the state equation form addresses indirectly observed process.

To obtain the state space form representation for the long memory series $(1-L)^d Y_t = \varepsilon_t$, Chan and Palma (1998) suggested writing the model in the form of truncated MA and AR expansions: $y_t = \sum_{j=1}^{\infty} \pi_j y_{t-j} + \varepsilon_t$ or $y_t = \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j}$, where π_j and ψ_j can refer to Hosking (1981). This paper use AR representation and $\pi_j = \frac{-(j-d-1)!}{j!(-d-1)!}$. The state space form representation can be expressed as:

$$\begin{cases} y_t = Z\alpha_t & \text{(Measurement equation)} \\ \alpha_{t+1} = T\alpha_t + H\eta_t, \ \ \eta_t \sim NID(0,Q) & \text{(State equation)} \end{cases}.$$

With the truncated lag length setting as m, we have $\alpha_t = \begin{pmatrix} y_t \\ y_{t-1} \\ \dots \\ y_{t-(m-1)} \end{pmatrix}_{m*1}$

$Z_{1*m} = [1, 0, \dots, 0]$, $\mathrm{T}_{m*m} = \begin{pmatrix} \pi_1 & \dots & \pi_m \\ 0 & \mathrm{I}_{m-1} & \end{pmatrix}$, $\mathrm{H}_{m*1}=(1,0,\dots0)$, $\mathrm{Q}_{m*m} = \begin{pmatrix} \sigma_\varepsilon^2 \dots & 0 \\ & 0 \end{pmatrix}$. Based on the truncated state space form representation, we can obtain the approximate likelihood function with the corresponding estimation algorithm order being $O(n)$. Compared with order $O(n^3)$ in exact MLE, the reduced computation order will achieve a more efficient estimation and faster computation time (Chan and Palma, 1998).

The Kalman filter is utilized to calculate the likelihood function. Let I_{t-1} denote the information set at time $t-1$. The optimal predictor of the state α_t and its variance matrix are, respectively: $\alpha_{t|t-1} = E[\alpha_t | I_{t-1}] = \mathrm{T}_t \alpha_{t-1}$, and $\mathrm{P}_{t|t-1} = Var[\alpha_t | I_{t-1}] = \mathrm{T}_t \mathrm{P}_{t-1} \mathrm{T}_{t-1}^{'} + \mathrm{Q}_t$. The corresponding optimal predictor for y_t is then $\mathrm{y}_{t|t-1} = \mathrm{Z}_t \alpha_{t|t-1}$. Once the new observation y_t is available, the optimal predictors $\alpha_{t|t-1}$ and $\mathrm{P}_{t|t-1}$ are updated as: $\alpha_t = \alpha_{t|t-1} + \mathrm{P}_{t-1} \mathrm{Z}_t^{'} \mathrm{F}_t^{-1} (y_t - \mathrm{Z}_t \alpha_{t|t-1}) = \alpha_{t|t-1} + \mathrm{P}_{t-1} \mathrm{Z}_t^{'} \mathrm{F}_t^{-1} \nu_t$, $\mathrm{P}_{t|t-1} = \mathrm{P}_{t|t-1} - \mathrm{P}_{t|t-1} \mathrm{Z}_t^{'} \mathrm{F}_t^{-1} \mathrm{Z}_t \mathrm{P}_{t|t-1}$. When the initial value $\alpha_{1|0}$ and $\mathrm{P}_{1|0}$ are specified, the Kalman filter returns prediction errors $\nu_t = y_t - \mathrm{Z}_t \alpha_{t|t-1}$ and the variance matrix $E(\nu_t \nu_t^{'}) = \mathrm{F}_t = \mathrm{Z}_t \mathrm{P}_{t|t-1} \mathrm{Z}_t^{'} + \mathrm{H}_t$. Finally, by maximizing the log likelihood function $\ln L(y|\theta) = -\frac{NT}{2} \ln(2\pi) - \frac{1}{2} \left\{ \sum_{t=1}^{T} \ln |F_t| + \sum_{t=1}^{T} \nu_t^{'} F_t^{-1} \nu_t \right\}$, the parameters $\theta = (d, \sigma_\varepsilon^2)$ can be estimated. Chan and Palma (1998) established the asymptotic properties of the approximate maximum likelihood estimation, and the simulation shows that the approach is efficient. The most current study on the state space model long memory estimation is the one conducted by

Grassi and Magistris in 2011. However, Chan and Palma (1998) and Grassi and Magistris (2011) only consider stationary series with $0 < d < 0.4$ where $\sigma_{\varepsilon}^{2}=1$ and is assumed to be known.

The range of integration orders considered in their simulations is relatively narrow from an economic point of view. Several economic time series such as exchange rates (Andersson, 2013), inflation (Hassler and Wolters, 1995; Caggio and Castelnuovo, 2011) and interest rates (Tkacz,2001; Coleman and Sirichand, 2012) have been found to be covariance non-stationary yet mean-reverting. We thus expand the simulations (see Tables 1 to 3) to also include nearly non-stationary time series ($d = 0.45, 0.48$), non-stationary though mean-reverting ($d = 0.7, 0.8, 0.9$) and nearly unit root ($d = 0.95, 0.98$). Unlike Chan and Palma (1998) and Grassi and Magistris (2011) we also consider both the case when σ_{ε}^{2} is known (Table 1), and the case when σ_{ε}^{2} is unknown and estimated jointly with *d* (Table 2). In the simulation, the initial value of $\alpha_{1|0}$ is set as 0, and $P_{1|0}$ is the empirical auto-covariance matrix up to lag m, which is set to 10. We concentrate on the case where $T = 170$, which corresponds to the sample size in our empirical analysis. The standard deviation of the shocks is set to $\sigma_{\varepsilon} \in (1, 3, 5)$. The simulation is based on 500 repetitions.

The estimates of the integration order are unbiased for all cases except where *d* is close to 0.5 and the estimates contain a positive bias. The bias is relatively large (between 0.10 and 0.12). In an empirical analysis, this large and positive bias increases the risk of concluding that a series is non-stationary when it is actually stationary.

[Table 1]

[Table 2]

As can also be seen in the tables, the bias is independent of whether σ_ε^2 is known or unknown and of the value of σ_ε^2. The estimates of σ_ε^2 are unbiased irrespective of *d* (see Table 3), and only the estimates of *d* are biased for the series with an integration order close to 0.5. Overall, the state space model based estimation gives out a satisfactory result in most cases except when $d = 0.45, 0.48$.

[Table 3]

3 Bayesian MCMC Estimator

In certain situations, we have some prior knowledge of the series' properties, whether they are covariance-stationary, mean-reverting or non-stationary. Such information can potentially be used to improve the accuracy of the estimator, and in the case of the MLE described in Section 2, it can solve the over-bias problem for the nearly non-stationary series.

The estimation in Section 2 is based on the maximization of the log likelihood function $\ln L(y|\theta) = -\frac{NT}{2}\ln(2\pi) - \frac{1}{2}\left\{\sum_{t=1}^{T}\ln|F_t| + \sum_{t=1}^{T} v_t' F_t^{-1} v_t\right\}$, where we assume θ is fixed but unknown. If we know whether the series is stationary or non-stationary, we can set d as a random variable with the definition domain as $0 < d < 0.5$ or $0.5 < d < 1$ respectively. To estimate the parameters in the fractional difference series using the Bayesian methodology, we can refer to Koop *et al.* (1997), Petris (1997) and the recent literature (Jensen 2004; Ko and Vannuchi 2006 a; Holan *et al.* 2009; Ko *et al.* 2009). Rather than estimating the parameters by maximizing the log likelihood function $\ln L(y|\theta)$, we first construct the posterior distribution $L(\theta|y)$ based on the prior distribution $\pi(\theta)$, and we construct the approximate likelihood function $L(y|\theta)$ by $L(\theta|y) \propto L(y|\theta)\pi(\theta)$.

The prior information $\pi(\theta)$ is chosen as the independent priors for d and σ_ε with $\pi(\theta) = \pi(d)\pi(\sigma)$. For d, where we have prior knowledge that the series is stationary with $0 < d < 0.5$ and non-stationary with $0.5 < d < 1$, we choose a respectively uniform distribution $Unif(0, 0.5)$ and $Unif(0.5,1)$. The prior distribution σ_ε does not depend on d, and this paper uses $Unif(0,10)$. The posterior distributions for d and σ_ε are:

$$p(d|\sigma, y) \propto (2\pi)^{-NT/2} \exp\left\{-\frac{1}{2}\sum_{t=1}^{T}\ln|F_t| - \frac{1}{2}\sum_{t=1}^{T} v_t' F_t^{-1} v_t\right\} \times I_{(0,0.5)}(d);$$

$$p(\sigma|d, y) \propto (2\pi)^{-NT/2} \exp\left\{-\frac{1}{2}\sum_{t=1}^{T}\ln|F_t| - \frac{1}{2}\sum_{t=1}^{T} v_t' F_t^{-1} v_t\right\} \times I_{(0,10)}(\sigma).$$

The estimators for d and σ_ε are simply the posterior mean and $\hat{\sigma} = \int \sigma \, \mathrm{d}P(\sigma|y)$. Because the marginal posterior $P(d|y)$ and $P(\sigma|y)$ result in the integration being analytically intractable, and the posterior distribution for d, σ_ε are conditionally depend on each other, a two-step iterative Metropolis-Hasting method is applied (Scollnik, 1996; Brooks, 1998; Besag, 2004).

Simulation results using the Bayesian approach are presented in Tables 4 to 6. We use the same simulation set-up as in Section 2. Table 4 contains the results when σ_ε is known. Table 5 contains the results when σ_ε i are unknown and estimated jointly with *d*. The results for the Bayesian approach are similar to the result of the MLE for all *d*, except $d \in (0.4, 0.45, 0.48)$. In this case, the bias issue has disappeared. Therefore, with certain prior information, the Bayesian-based method can improve the estimation for the nearly non-stationary series and generate the same accurate results for the other integration orders.

[Table 4]

[Table 5]

Compared with the MLE estimates of σ_{ε}, however, the Bayesian estimates are more biased when $d = 0.48$, which is a result of choosing a prior distribution. But, given the reduction in the bias of *d*, the bias of $\hat{\sigma}_{\varepsilon}$ is acceptable.

[Table 6]

4 Empirical Analysis

The integration order is estimated using an ARFIMA model for six inflation-targeting regions (Canada, the Euro Area, Norway, Sweden, the United Kingdom and the United States) for the period following the introduction of the Euro (i.e., 1999M1 to 2013M3). Inflation is measured as the year-on-year increase in the Harmonized Index of Consumer Prices (HICP) in all regions but Canada and the United States, where the Consumer Price Index (CPI) is used because there is no HICP data. All data are collected from Eurostat[1].

Descriptive statistics are available in Table 7. Table 7 also presents the official inflation targets. Average inflation is fairly close to the targets in all countries, although average inflation has been approximately 0.5 percentage points lower than the target in Norway and Sweden and approximately 0.5 percentage points higher than the 2012-defined United States target.

[TABLE 7]

The estimated parameters in the ARIMA model are presented in Table 8. The variance of the inflation shocks ranges between 0.243 (Euro Area) and 0.578 (the United States), and the shocks are auto-correlated in all countries. The degree of autocorrelation is relatively low. In most countries, the shocks follow an AR(1) model with an AR-parameter between 0.20 and 0.30. The exceptions are Norway and the United States, where the AR-parameters are within the range of

[1] http://epp.eurostat.ec.europa.eu/portal/page/portal/eurostat/home/

0.65 to 0.8. The variance of the inflation shocks are also the highest in Norway and the United States.

[TABLE 8]

Because of the relatively low degree of autocorrelation in the inflation shocks, most of the long swings in inflation are not caused by the inflation shocks but by the central bankers' response to those shocks. The estimated fractional integration orders ranges between 0.824 (the United States) to 1.049 (Norway). The integration order for Norway is explosive, but the integration order is not significantly greater than one, and we cannot also reject mean-reversion for Norway.

Excluding Norway, the estimated integration orders are similar amongst regions and within the range of 0.824 to 0.887. Central bankers in these countries appear to agree upon how flexibly the inflation target should be implemented. The integration orders are, moreover, relatively high and show that the inflation targets are implemented flexibly and that additional issues play an important role in the central banks' policies. Although relatively high, the integration orders are all considerably smaller than 1 at the 5% significance level.

5 Conclusion

In this paper, we model the degree of inflation flexibility using an ARFIMA model in the framework of state space models. We divide the fractional difference series into four groups: pure stationary, nearly non-stationary, pure non-stationary and nearly unit root. We estimate both the difference parameter d and the variance σ_ε. The simulation result indicates that the method calculates quite precise estimation in most cases, other than when d nears 0.5. We argue that in certain situations, we have prior knowledge of whether the series is stationary or non-stationary. This knowledge can improve the estimation when we set the prior distribution for d and σ_ε in the framework of Bayesian inference. We utilized the Metropolis–Hastings algorithm to show that this methodology can improve the estimation to a large extent when $0.4 < d < 0.5$. Because the state space-based estimator works quite well when $0.7 < d < 1$, we use it to estimate inflation-targeting in the empirical example. The result shows that inflation contains long swings and that these swings are caused by the central bankers' preferences rather than the nature of the inflation shocks. All central banks in the study pursue an inflation-targeting policy with a high degree of flexibility.

References:

Andersson ,F.N.G (2013, in press). Exchange Rate Dynamics Revisited: A Panel Data Test of the Fractional Integration Order. *Empirical Economics*, in press

Beran, J. (1993). “Fitting long-memory models by generalized linear regression”, *Biometrika*, Vol.80 (4), pp.817-822.

Beran, J. (1994). *Statistics for Long-Memory Processes*. Chapman & Hall, New York.

Besag, J. (2004). “An introduction to Markov Chain Monte Carlo methods”, In: Mark Johnson, Sanjeev P. Khudanpur, Mari Ostendorf, and Roni Rosenfeld, (eds.), *Mathematical Foundations of Speech and Language Processing*, pp. 247-270. Springer, New York.

Brockwell, P.J. and Davis, R.A. (1991). *Time Series: Theory and Methods*, Spinger-Verlag, New York.

Brooks, S. P. (1998). “Markov chain Monte Carlo method and its application”, *The Statistician*, Vol.47(1), pp.69-100.

Coelman, S. and Sirichand, K. (2012). Fractional Integration and the Volatility of UK Interest Rates. *Economics Letters*, 116(3), 381-384.

Chan, N. H. and Palma, W. (1998). “State space modeling of long-memory processes”, Annals of Statistics, Vol.26(2), pp.719-740.

Durbin, J. and Koopman, S.J. (2001). *Time Series Analysis by State Space Methods*, Oxford University Press, Oxford.

Fox, R. and Taqqu, M. (1986). “Large sample properties of parameter estimates for strongly dependent stationary time series”, *The Annals of Statistics*, Vol.4, pp.517-532.

Granger, C. W. J. (1980). “Long memory relationships and the aggregation of dynamic models”, *Journal of econometrics*, Vol.14, pp.227-238.

Geweke, J. and Porter-Hudak, S. (1983). “The estimation and application of long-memory time series models”, *Journal of Time Series Models*, Vol.4, pp.221-237.

Granger, C. W. J. and Joyeux. R. (1980). “An introduction to long-memory time series models and fractional differencing”, *Journal of Time Series Analysis*, Vol.1, pp.15-29.

Grassi, S. and Magistris, P.S. (2011). “When Long Memory Meets the Kalman Filter: A Comparative Study”, *Computational Statistics & Data Analysis*, in press.

Giraitis, L. and Taqqu, M. (1999). “Whittle Estimator for Finite-variance Non-Gaussian time Series with Long Memory”, *The Annals of Statistics*, Vol.27(1), pp. 178-203.

Holan, S., McElroy, T. and Chakraborty, S. (2009). “A Bayesian Approach to Estimating the Long Memory Parameters”, *Bayesian Analysis*, Vol.4(1), pp. 159-190.

Jensen, M. J. (2004). “Semiparametric Bayesian inference of long-memory stochastic volatility models”, *Journal of Time Series Analysis*, Vol.25(6), pp. 895-922.

Ko, K. and Vannucci, M. (2006a). “Bayesian wavelet analysis of autoregressive fractionally integrated moving-average processes”, *Journal of Statistical Planning and Inference*, Vol.136, pp.3415-3434.

Ko, K., Qu, L., and Vannucci, M. (2009). "Wavelet-based Bayesian estimation of partially linear regression models with long memory errors", *Statistical Sinica*, Vol.19(4), pp.1463-1478.

Koop, G., Ley, E., Osiewalski, J. and Stell, M. (1997). "Bayesian analysis of long memory and persistence using ARFIMA models", *Journal of Econometrics*, vol.76 (1-2), pp.149-169.

Palma, W. (2007). *Long-Memory Time Series: Theory and Methods*, Wiley-Interscience, Hoboken.

Petris, G. (1997). "Bayesian Analysis of Long Memory Time Series", Ph.D. thesis, Duke University.

Robinson, P. M. (1995a). "Log-Periodogram Regression of times series with long range dependence", *The Annals of Statistics*, Vol.23, pp.1048-1072.

Robinson, P. M. (1995b). "Gaussian semiparametric estimation of long range dependence", *Annals of Statistics*, Vol.23, pp.1630-1661.

Robinson, P. M. (2003). *Time Series With Long Memory*, Oxford University Press, Oxford.

Scollnik, D.P.M. (1996). "An Introduction to Markov Chain Monte Carlo Methods and Their Actuarial Applications", *Proceedings of the Casualty Actuarial Society*, Vol.83, pp.114-65.

Shimotsu, K. (2010). "Exact local whittle estimation of fractional integration with unknown mean and time trend", *Econometric Theory*, Vol.26(2), pp.501-540.

Shimotsu, K., and P. C. B. Phillips (2005). "Exact local Whittle estimation of fractional integration", *Annals of Statistics 33*, pp.1890-1933.

Sowell, F. (1992). "Maximum Likelihood Estimation of Stationary Univariate Fractionally Integrated Time Series Models", *Journal of Econometrics*, Vol. 53, pp.165-188.

Steel, Mark F. J. (2008). "Bayesian time series analysis", In: Durlauf, S. N. and Blume, L. E., (eds.), *The New Palgrave dictionary of economic*, Basingstoke, Hampshire; New York.

Tkacz, G. (2001). Estimating the Fractional Order of Integration of Interest Rates using Wavelet OLS Estimator. *Studies in Nonlinear Dynamics and Econometrics*, 5, 1-21.

Table 1 Estimation of *d* based on the state space model when σ_ε is known

d	0.20	0.30	0.40	0.45	0.48	0.70	0.80	0.90	0.95	0.98
$\sigma = 1$										
Bias	0.001	0.014	0.044	0.079	0.124	0.033	0.014	0.003	-0.003	-0.011
RMSE	0.076	0.070	0.090	0.124	0.167	0.079	0.069	0.059	0.056	0.060
$\sigma = 3$										
Bias	0.001	0.010	0.041	0.081	0.131	0.027	0.018	0.002	-0.004	-0.008
RMSE	0.066	0.067	0.093	0.125	0.172	0.077	0.071	0.062	0.059	0.061
$\sigma = 5$										
Bias	-0.003	0.017	0.048	0.077	0.134	0.025	0.018	0.003	-0.004	-0.005
RMSE	0.065	0.075	0.099	0.121	0.171	0.076	0.074	0.062	0.056	0.058

Table 2 Estimation of *d* based on the state space model when σ_ε is unknown

d	0.20	0.30	0.40	0.45	0.48	0.70	0.80	0.90	0.95	0.98
$\sigma = 1$										
Bias	-0.001	0.017	0.043	0.079	0.125	0.029	0.014	0.004	-0.005	-0.009
RMSE	0.077	0.069	0.096	0.121	0.166	0.079	0.065	0.064	0.058	0.057
$\sigma = 3$										
Bias	0.006	0.020	0.041	0.084	0.112	0.030	0.011	0.004	-0.004	-0.007
RMSE	0.077	0.064	0.088	0.123	0.155	0.081	0.067	0.061	0.058	0.058
$\sigma = 5$										
Bias	0.002	0.015	0.041	0.067	0.121	0.036	0.016	0.001	-0.008	-0.009
RMSE	0.056	0.066	0.089	0.100	0.170	0.078	0.070	0.060	0.049	0.050

Table 3 Estimation of σ_ε based on the state space model

d	0.20	0.30	0.04	0.45	0.48	0.70	0.80	0.90	0.95	0.98
$\sigma = 1$										
Bias	-0.001	-0.001	0.004	0.015	0.025	-0.001	-0.001	-0.004	-0.005	0.006
RMSE	0.055	0.054	0.054	0.059	0.064	0.053	0.055	0.055	0.052	0.055
$\sigma = 3$										
Bias	-0.002	0.002	0.023	0.041	0.062	0.010	-0.006	0.011	-0.003	-0.013
RMSE	0.166	0.155	0.167	0.181	0.180	0.153	0.152	0.132	0.135	0.150
$\sigma = 5$										
Bias	-0.010	-0.016	0.019	0.065	0.142	0.002	-0.015	-0.008	-0.037	-0.038
RMSE	0.227	0.234	0.224	0.255	0.314	0.221	0.208	0.225	0.266	0.228

Table 4 Estimation of *d* based on the Bayes model when σ_ε is known

D	0.20	0.30	0.40	0.45	0.48	0.70	0.80	0.90	0.95	0.98
$\sigma=1$										
Bias	0.012	0.020	0.018	0.003	-0.006	0.037	0.018	-0.007	-0.024	-0.037
RMSE	0.061	0.067	0.054	0.042	0.031	0.079	0.067	0.054	0.050	0.051
$\sigma=3$										
Bias	0.014	0.016	0.018	0.005	-0.006	0.035	0.024	-0.006	-0.020	-0.036
RMSE	0.060	0.067	0.055	0.039	0.032	0.077	0.070	0.050	0.046	0.053
$\sigma=5$										
Bias	0.008	0.011	0.018	0.003	-0.004	0.039	0.023	-0.003	-0.024	-0.039
RMSE	0.065	0.064	0.055	0.040	0.026	0.081	0.066	0.052	0.048	0.053

Table 5 Estimation of *d* based on the Bayes model when σ_ε is unknown

D	0.20	0.30	0.40	0.45	0.48	0.70	0.80	0.90	0.95	0.98
$\sigma=1$										
Bias	0.010	0.019	0.019	0.003	-0.007	0.035	0.016	0.001	-0.024	-0.039
RMSE	0.066	0.062	0.057	0.042	0.031	0.80	0.067	0.047	0.046	0.052
$\sigma=3$										
Bias	0.009	0.018	0.016	0.005	-0.004	0.033	0.023	-0.001	-0.020	-0.038
RMSE	0.063	0.067	0.055	0.037	0.030	0.077	0.067	0.051	0.045	0.059
$\sigma=5$										
Bias	0.011	0.016	0.017	0.005	-0.004	0.034	0.020	-0.004	-0.020	-0.037
RMSE	0.050	0.068	0.056	0.038	0.027	0.079	0.069	0.051	0.048	0.050

Table 6 Estimation of σ_ε based on the Bayesian model

D	0.20	0.30	0.04	0.45	0.48	0.70	0.80	0.90	0.95	0.98
$\sigma=1$										
Bias	0.007	0.009	0.025	0.042	0.101	0.018	0.019	0.011	-0.009	0.004
RMSE	0.086	0.069	0.071	0.094	0.183	0.068	0.071	0.064	0.063	0.059
$\sigma=3$										
Bias	0.022	0.035	0.060	0.120	0.350	0.044	0.030	0.018	0.017	0.019
RMSE	0.160	0.162	0.164	0.200	0.500	0.165	0.150	0.170	0.150	0.165
$\sigma=5$										
Bias	0.020	0.061	0.078	0.190	0.557	0.052	0.038	0.037	0.024	0.023
RMSE	0.27	0.230	0.260	0.400	0.092	0.290	0.244	0.269	0.235	0.228

Table 7 Descriptive statistics

	Canada	Euro Area	Norway	Sweden	United Kingdom	United States
Inflation Target	1% - 3%	<2%	2.5%	2.0%	2.0%	2.0%
Average	2.06%	2.08%	1.82%	1.64%	2.18%	2.51%
Std. Dev.	0.95	0.77	1.17	0.87	1.10	1.65

Table 8 Estimation results

	Canada	Euro Area	Norway	Sweden	United Kingdom	United States
d	0.835 (0.041)	0.857 (0.036)	1.048 (0.025)	0.857 (0.040)	0.887 (0.038)	0.824 (0.044)
σ_{ε}	0.482 (0.019)	0.243 (0.009)	0.511 (0.019)	0.346 (0.013)	0.291 (0.011)	0.578 (0.022)
AR(1)	0.220 (0.076)	0.338 (0.081)	0.807 (0.050)	0.201 (0.080)	0.237 (0.076)	0.653 (0.075)
AR(2)	---	0.081 (0.081)	-0.714 (0.076)	---	0.145 (0.076)	-0.221 (0.075)
MA(1)	---	---		---	---	---

ARBEIDSNOTATER
DISCUSSION PAPERS

(A complete list of Discussion Papers, going back to the beginning of our series in 1990, may be found at the Department's Web site, at http://www.nhh.no/Default.aspx?ID=2238.)

NR. NO.	FORFATTER AUTHOR	TITTEL TITLE
2014/1	Ivan Belik Kurt Jörnsten	A New Semi-Lagrangean Relaxation for the K-Cardinality Assignment Problem
2014/2	Diwakar Poudel Leif K. Sandal	Stochastic Optimization for Multispecies Fisheries in the Barents Sea
2014/3	Knut K. Aase	Recursive utility with dependence on past consumption; the continuous-time model
2014/4	Iver Bragelien Joost Impink	Relationship-Specificity, Bargaining Power Growth, and Firm Performance
2014/5	Knut K. Aase	Heterogeniety and limited stock market Participation
2014/6	Thomas P. Tangerås Johannes Mauritzen	Real-time versus day-ahead market power in a hydro-based electricity market
2014/7	Johannes Mauritzen	The effect of oil prices on offshore production: evidence from the Norwegian Continental Shelf
2014/8	Ivan Belik	The Analysis of Split Graphs in Social Networks Based on the K-Cardinality Assignment Problem
2014/9	Knut K. Aase	Recursive utility and jump-diffusions
2014/10	Yushu Li Simon Reese	Wavelet improvement in turning point detection using a Hidden Markov Model
2014/11	Yushu Li Fredrik N.G. Andersson	A simple wavelet-based test for serial correlation in panel data models
2014/12	Yushu Li Jonas Andersson	A Likelihood Ratio and Markov Chain Based Method to Evaluate Density Forecasting
2014/13	Knut K. Aase	Life Insurance and Pension Contracts I: The Time Additive Life Cycle Model

2014/14	Patrick A. Narbel Jan Petter Hansen	Estimating the cost of future global energy supply
2014/15	Mario Guajardo Kurt Jörnsten	Common Mistakes in Computing the Nucleolus
2014/16	Patrick A. Narbel Elisabeth T. Isaksen	A carbon footprint proportional to expenditure - a case for Norway?
2014/17	Patrick A. Narbel	Rethinking how to support intermittent renewables
2014/18	Morten S. Henningsen Torbjørn Hægeland Jarle Møen	Estimating the additionality of R&D subsidies using proposal evaluation data to control for research intentions
2014/19	Knut K. Aase	The Life Cycle Model with Recursive Utility: New insights on pension and life insurance contracts
2014/20	Ivan Belik Kurt Jörnsten	The Coauthorship Network Analysis of the Norwegian School of Economics
2014/21	Søren Bo Nielsen Dirk Schindler Guttorm Schjelderup	Abusive Transfer Pricing and Economic Activity
2014/22	Dirk Schindler Guttorm Schjelderup	Transfer Pricing and Debt Shifting in Multinationals
2014/23	Floris T. Zoutman	The Effect of Capital Taxes on Household's Portfolio Composition and Intertemporal Choice: Evidence from the Dutch 2001 Capital Income Tax Reform
2014/24	Axel Haus Steffen Juranek	Patent Trolls, Litigation, and the Market for Innovation
2014/25	Evelina Gavrilova	A Partner in Crime: Assortative Matching and Bias in the Crime Market
2014/26	Michail Chronopoulos Afzal Siddiqui	When is it Better to Wait for a New Version? Optimal Replacement of an Emerging Technology under Uncertainty
2014/27	Endre Bjørndal Mette Bjørndal Hong Cai	Nodal Pricing in a Coupled Electricity Market

2014/28	Xiaomei Cheng Endre Bjørndal Mette Bjørndal	Cost Efficiency Analysis based on The DEA and StoNED Models: Case of Norwegian Electricity Distribution Companies
2014/29	Mette Bjørndal Victoria Gribkovskaia Kurt Jörnsten	Market Power in a Power Market with Transmission Constraints
2014/30	Endre Bjørndal Mette Bjørndal Victoria Gribkovskaia	Simulation of Congestion Management and Security Constraints in the Nordic Electricity Market
2014/31	Ivan Belik Kurt Jörnsten	The Coauthorship Network Analysis of the BI Norwegian Business School
2014/32	Sander Renes Floris T. Zoutman	When a Price is Enough: Implementation in Optimal Tax Design
2014/33	Floris T. Zoutman Bas Jacobs	Optimal Redistribution and Monitoring of Labor Effort
2014/34	Ivan Belik Kurt Jörnsten	The Comparative Analysis of the NHH and BI Networks
2014/35	Johannes Mauritzen	Sun and Lemons: Getting over Information Asymmetries in the California Solar Market
2014/36	Arnt O. Hopland Sturla F. Kvamsdal	Optimal maintenance scheduling of local public purpose buildings
2014/37	Kurt R. Brekke Armando J. Garcia Pires Dirk Schindler Guttorm Schjelderup	Capital Taxation and Imperfect Competition: ACE vs. CBIT
2014/38	Fredrik N.G. Andersson Yushu Li	Are Central Bankers Inflation Nutters? - A Bayesian MCMC Estimator of the Long Memory Parameter in a State Space Model