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Discussion paper

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BY

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Abstract

Road tolls are a well established way of dealing with problems of congestion. Over recent years, the literature has expanded to take account of how congestion charges might interact with imperfections in other markets. In this paper, we consider the case where congestion occurs within a complex road network, with congestion on multiple links. To derive a truly optimal toll, account must be taken of the entire network. As a case study, we take a stylised version of the road network in Bergen, Norway.

1 Introduction

Transportation infrastructure plays a vital role in overcoming spatial separation. Every day, millions of commuters around the world undertake their journey to work, many of them by road. A combination of population growth, urbanisation and increasing economic prosperity has led to significant congestion problems in many parts of the world, and in particular in the urban areas. This negative

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externalities has implications for the efficiency of the economy, people's well-being and the environment. In the U.S., Schrank et al. (2012) estimated the cost of congestion in 2012 to be \$121 bn, some \$818 per commuter.

Given the seriousness of the problem, policy makers have considered a number of solutions. One approach is to expand capacity by building new roads. This approach may be undesirable due to, for instance, its high costs or the environmental impact. Another problem with this approach is that it may in fact generate so-called induced demand (Small and Verhoef, 2007, p. 176). A potentially attractive alternative to expanding supply is to manage demand with the implementation of congestion pricing.

The basic theory of congestion pricing is well understood, see for instance Rouwendal and Verhoef (2006); Lindsey and Verhoef (2001). When only one congested link is considered, under some simplifying assumptions it is trivial to derive the optimal congestion charge, and to show that this optimises social welfare. However, several authors have highlighted the fact that the real-world implementation of congestion pricing regimes can be more complicated due to, for example, pollution externalities (Newbery, 1988), road accidents (Small and Kazimi, 1995), public transport subsidies (Glaister and Lewis, 1978) or distortions in other markets (Parry and Bento, 2001).

One other theme which has been examined is the effect of a congestion charge on one link when there are other congested links on the network. If congestion exists on several links on a network, then a set of optimal prices needs to be set to reach a socially optimal solution. Simply pricing one link may result in undesired outcomes. This topic has been studied by Verhoef (2002a); Choe and Clarke (2000); Braid (1996); Verhoef et al. (1996); Liu and McDonald (1998), among others. These studies show that the optimal price on a particular link, given the existence of unpriced congestion on other links, should be set lower

than the first-best optimal price (Choe and Clarke, 2000).

In this paper, we construct a complex, multi-link model based on the road network of the city of Bergen, Norway. The model is used to illustrate how road pricing could be used to manage traffic demand in a network which is more complex than the sort usually considered in the literature. Of particular interest will be the difference between a do-nothing scenario, a partially-tolled situation and a full set of optimum tolls. The scenarios will be compared based on their impact on traffic flows, congestion, social welfare and the environment.

2 Theory and models

A brief presentation of the theory of optimal road pricing will be given here. For a review, see Rouwendal and Verhoef (2006). Assume that the number of trips across a particular road link in a network, F_r , is a function of the generalised cost of traversing the link, c_r . The cost can be decomposed into two components. Part of the cost is determined by the length of the link, d_r . The other part of the cost is caused by congestion. Congestion occurs when the flow of traffic, F_r , exceeds the capacity of the road, ω_r .

We restrict our attention here to the journey to work and a static congestion problem. A commuter will make the commute to work if the reward for doing so outweighs the cost. In this case, the reward is the wage rate, ψ . So for a commuter, the journey is profitable if $\psi > c_r$. However, for society, this outcome is suboptimal, because the individual addition of one commuter to the road increases the cost of travel for all other road users. Only if the excess of the wage over the total marginal cost is positive should the commute take place i.e. $\psi > c_r + F_r \frac{\partial c_r}{\partial F_r}$. In order to move to this socially optimal outcome, a congestion charge equal to $F_r \frac{\partial c_r}{\partial F_r}$ should be imposed.

This result holds as long as the underlying assumptions hold. As mentioned

in the introduction, the literature has explored how departures from the first-best solution may occur. See for example McArthur et al. (2012) for a discussion of some of these issues. In this paper, we turn our attention to the case where there are other market imperfections in the system, namely unpriced congestion on other routes. We will present a model which allows us to find a system-wide optimal set of prices.

2.1 Trip distribution

To begin, we must specify how demand and congestion is determined. A number of simplifying assumptions underpin the model. Such simplifying assumption allow us to consider a complex network, which is the main focus of this paper. A short-term model situation is considered. In this situation, the locations of workers and firms is assumed to be fixed. Furthermore, workers and jobs are assumed to be homogeneous, and wages are fixed and equal. To further simplify matters, we assume that only one mode of transport is available. This may be thought of as a short-term response, before travellers have had time to change their mode of transport.

In spatial interaction modelling, there are four aspects which can be considered: 1) trip generation, 2) trip distribution, 3) mode choice and 4) route choice. In this paper we consider only trip distribution and route choice. Trip generation is assumed to be exogenous to the model, and only one mode of transport is assumed to exist. We will return in the concluding remarks to the possibility of relaxing these assumptions. We begin by considering the trip distribution problem. The trip distribution should have the property that longer commutes are chosen less frequently than short commutes. One model which fits this description is the doubly-constrained gravity model.

$$f_{ij} = a_i b_j e^{-\beta C_{ij}} \quad (1)$$

$$a_i = \frac{O_i}{\sum_j b_j D_j e^{-\beta C_{ij}}} \quad (2)$$

$$b_j = \frac{D_j}{\sum_i a_i O_i e^{-\beta C_{ij}}} \quad (3)$$

In this model, f_{ij} is the number of commuters travelling from origin i to destination j , O_i is the number of trips originating from i , and D_j is the number of trips terminating in destination j . The quantities a and b are balancing factors, with a ensuring that the rows of the trip distribution matrix sum to give O and while b ensures the columns sum to give D . The idea of a gravity model for spatial interaction is inspired by Newtonian physics, however variants of this model may also be derived from entropy maximisation (Wilson, 1967) or from random utility theory (Anas, 1983). For example, the doubly-constrained gravity model can be shown to be identical to the multinomial logit model (Anas, 1983).

In the gravity model, a measure of spatial separation is included. While it is common to include distance here, we choose to include a generalised cost measure. This allows the costs accruing from distance, time and tolls to be aggregated. McArthur et al. (2013) show how commuters may react differently to costs related to time and monetary expenses, and provide guidance on how such costs can be aggregated. If i and j are directly connected, the C_{ij} is simply the cost of travelling on that road. If they are not directly connected, then C_{ij} is the sum of the costs incurred on the roads connecting i and j .

When the matrix of generalised costs is fixed, the solution to the gravity model is well known; see for example Sen and Smith (1995). However, in our

model C_{ij} is a function of f_{ij} . This considerably increases the complexity of computing the trip distribution matrix.

2.2 Route choice

As has already been alluded to, the cost of commuting along a stretch of road, r , can be split into three components: 1) a fixed cost per km, 2) any costs imposed by congestion and 3) any road tolls levied. This can be expressed as:

$$c_r = \gamma d_r + \alpha T_r + \delta_r R_r \quad (4)$$

where γ is the cost per km, d_r is the length of road r , α is the value of travel time, T_r is the travel time, δ_r is the dummy variable representing if a toll can be charged on road r and R_r is any toll which may apply. The cost of a journey from origin i to destination j can be obtained by summing over all of the roads which link i and j .

The time taken to travel on a given link must be defined. The travel time is defined as a function of the length of the link, d_r , the flow of traffic on the road, F_r , and the capacity of the road, ω_r i.e. $T_r = f(d_r, F_r, \omega_r)$. We would like a smooth function which is twice differentiable, and which has the properties that $\frac{\partial T_r}{\partial F_r} > 0$ and $\frac{\partial^2 T_r}{\partial F_r^2} > 0$. A simple speed-density relationship such as that given by Noland (1997) fits this description. See Castillo and Benitez (1995) for a history of the speed-density relationship and a discussion of functional form.

$$T_r = d_r \left[T_r^0 + T_r^1 \left(\frac{F_r}{\omega_r} \right)^\varepsilon \right] \quad (5)$$

All variables here are defined as before. The quantity T_r^0 represents the time taken to travel 1 km in the absence of congestion. The values T_r^1 and ε control how quickly journey times rise as the ratio of use to capacity changes. A value

of 0.15 is used for T^1 and a value of 4 for ε in Noland (1997). We allow the values of T_r^0 and T_r^1 to vary by road, to account for differences in speed limits. We set $T_r^1 = 0.15T_r^0$, which implies that congestion affects travelling time more on roads with lower speed limits (Liu and McDonald, 1998). This results in the travel time function given below.

$$T_r = d_r T_r^0 \left[1 + 0.15 \left(\frac{F_r}{\omega_r} \right)^4 \right] \quad (6)$$

In Equation (6), d_r , T_r^0 and ω_r are constants. The flows on road r , F_r are calculated based on the flows calculated in Equations (1)-(3). When the traffic flows are known, it is relatively straightforward to calculate the traffic on any individual link in the road network, as shown in Equation (7).

$$F_r = \sum_{i,j} \lambda_{ij}^r f_{ij} \quad (7)$$

In this equation, λ_{ij}^r is a dummy variable which is equal to 1 if commuters travelling from i to j use road r , and 0 otherwise. To summarise, commuters will choose the least costly path conditional on congestion. For any commuter travelling from k to l , this may be expressed more formally as:

$$\min_{\lambda_{kl}^r} \sum_r c_r \lambda_{kl}^r \quad (8)$$

Subject to

1. Selected roads form a path from origin k to destination l
2. $c_r = \gamma d_r + \alpha T_r + \delta_r R_r$ For all $r = 1, \dots, m$
3. $T_r = d_r T_r^0 \left[1 + 0.15 \left(\frac{F_r}{\omega_r} \right)^4 \right]$ For all $r = 1, \dots, m$
4. $F_r = \sum_{i,j} \lambda_{ij}^r f_{ij}$ For all $r = 1, \dots, m$

5. $f_{ij} = a_i b_j e^{-\beta C_{ij}}$ For all $i = 1, \dots, n$ and $j = 1, \dots, n$
6. $C_{ij} = \sum_r \lambda_{ij}^r c_r$ For all $i = 1, \dots, n$ and $j = 1, \dots, n$

An important point to note is that C_{ij} is the minimum generalised cost for all other commuters who travel from origin i to destination j . It can be seen from the optimisation problem that the optimal decision for a particular commuter (λ_{kl}^r) depends on the decisions of other commuters (λ_{ij}^r). This problem can therefore be considered a game between commuters.

2.3 Welfare maximisation

The goal of the planner is assumed to be the maximisation of social welfare by setting an optimal set of congestion prices. We choose to use the change in consumer surplus as the basis of our welfare measure, as in Verhoef (2002b). The maximisation problem to be solved by the planner is given in Equation (9).

$$\max_{R_r \geq 0} \sum_{i,j} \int_0^{f_{ij}} D_{ij}(x_{ij}) dx_{ij} - \sum_r (c_r - \delta_r R_r) F_r \quad (9)$$

In this equation, $D_{ij}(x)$ is the inverse demand function. The expression therefore equates to subtracting the total cost from the total benefit. For a discussion of Equation (9), see Verhoef (2002b)¹.

3 The road network

The road network considered in this paper is based on the city of Bergen, Norway, and its surrounding area. The geography of the west coast of Norway provides an interesting case study. The presence of fjords and islands necessitates the use of bridges and ferries, as well as involving significant deviations

¹Although the second expression in the objective function looks different from Verhoef's model, it is only due to the difference in definition of c_r . In this paper, c_r includes the road toll but in Verhoef's model it does not.

from the Euclidean distance. This creates bottlenecks in the network which may result in congestion. The city of Bergen itself is the second largest in Norway, and has a population of around 270 000. The city has suffered from traffic problems for many years (Ieromonachou et al., 2006; Odeck and Bråthen, 2002). This has led to queuing in the city, as well as contributing to rather serious air quality problems in the city (McArthur and Osland, 2013).

A total of 25 zones were defined in the Bergen area, based on post code areas. Not all of these zones are directly connected to one another. This results in journeys between certain pairs of zones passing through other zones on the way. In total, there are 35 road links in the simplified network which we consider.

The zones vary in size, both with respect to geographical area and population. The number of people living in each zone, and the number working in each zone is available at the level of post code area. The zones range from the densely populated central business district (CBD) of Bergen, to the suburbs and to settlements in the periphery. This gives zones of a variety of different sizes, and involves a wide range of travel distances. The spatial configuration of the zones along with the connectivity between them is illustrated in Figure 1.

In the network, we distinguish between roads which are congested, and those which are not. The majority of the roads in the network are uncongested, as the capacity exceeds the demand by a considerable margin. It is the roads in the city centre which experience congestion, and it is therefore these roads which will be the focus of this paper.

It is worth noting that in this network, the roads are neither perfect substitutes nor perfect complements. Due to the complexity of the network, a commuter travelling from a given origin to a given destination may have several alternative routes. Whether a road is a substitute for or a complement to another road, will depend on the start and end points of the commuters journey.

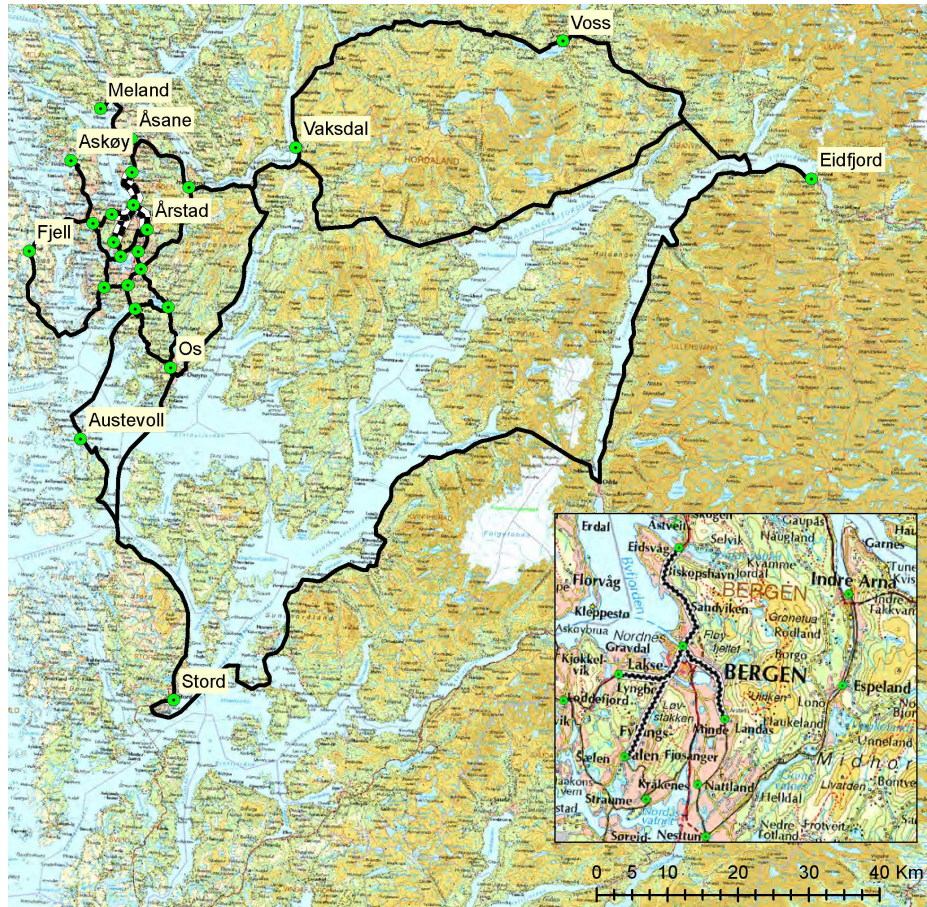


Figure 1: A map of Bergen with the transportation network we consider. Selected zone names are displayed. The inset map shows the centre of bergen, along with the four roads which are assumed to be congested.

4 Results

The aim here is to solve the model to determine the set of optimal congestion charges for the road network in Bergen. Solving the model is not a trivial problem, since and changes in travelling costs on even one link may potentially change the entire trip distribution matrix. This change in traffic flows causes changes in travel times, due to changes in congestion. This in turn causes further changes in the trip distribution matrix. We can calculate the change in consumer surplus for each commuter when moving from one trip distribution matrix to another. The change in social welfare is defined as the sum of these changes in consumer surplus plus the change in revenue generated by the change in the congestion charge.

The first step in deriving the optimal price in the model is to construct demand curves for each origin-destination pair in the network. For any given congestion charge, this is achieved by systematically varying the price and recording the demand. This process continues until a sufficient number of points to allow the construction of a demand curve have been obtained.

Numerical integration methods are used to find the change in consumer surplus on each link when moving from one congestion charge to another. Social welfare is calculated by summing over all commuters and adding in the proceeds of the charge.

The Newton-Raphson method was employed to find the optimal price. Two initial prices were chosen, and the marginal social welfare was calculated at these prices. Comparing these two values leads to a new estimated price. One of the initial estimates is replaced with this new estimate, and the marginal social welfare is estimated again. The process is repeated until convergence is achieved.

4.1 The optimal charge for a single link

As a starting point, we consider the more traditional case where a congestion charge will be applied to only one of the roads in the network. We selected the most congested road to have a charge applied, and calculate the change in social welfare. It is useful to examine a plot of the social welfare function. To do this, the marginal social welfare is calculated at different prices. For example, for $R = 10$, a point either side of this is evaluated. For example $R = 9.9$ and $R = 10.1$. The change in social welfare between $R = 9.9$ and $R = 10.1$ can then be calculated. From this, it is straightforward to calculate the marginal social welfare. It is assumed that the social welfare function is linear for sufficiently small intervals. The cumulative social welfare at different prices is presented in Figure 2.

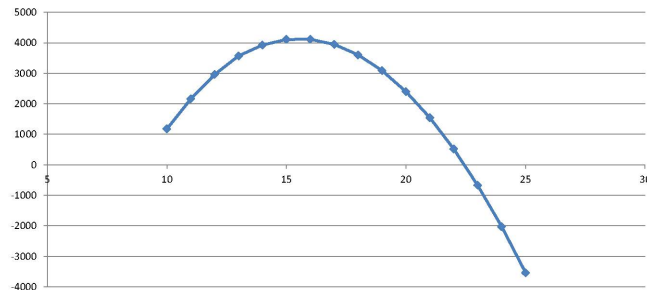


Figure 2: Cumulative social welfare.

As can be seen from Figure 2, the function is smooth and well-behaved, indicating that it is appropriate to apply classical numerical techniques. From the plot, it is apparent that the optimal price for this road is around 16 NOK (around £1.77, €2.09 or \$2.71). The Newton-Raphson method provides a more accurate estimate of 16.04 NOK.

4.2 A full set of optimum charges

We now turn our attention to the more complex and interesting problem of finding four optimal congestion charges. An iterative algorithm is used to find the set of optimal charges. To begin with, the charge on three of the congested roads is set to zero. The optimal price for the first road is found as in Section 4.1. Once this has been found, the price is fixed, and the optimum price is found for the second road. This process is then repeated for all of the roads until the prices converge. The prices converge rapidly, and are stable after the second iteration. Four iterations were carried out to ensure that convergence had been achieved. The four optimal prices obtained are around 10, 11, 17 and 18 NOK. As expected, the prices are higher for the more congested roads. Note that optimal charge when considering only one link is not the optimal charge when the system is considered in its entirety.

It is not possible to plot the social welfare function as in Figure 2, since the optimisation takes place in 4 dimensions. However, some insight can be gained by taking a partial approach. In Figure 3, the social welfare function is plotted for each the roads assuming that the others have their charge fixed at its optimal level.

Figure 3 shows that for all roads, the social welfare functions are smooth and well-behaved. It also shows that they all behave slightly differently. Each has its own optimum charge, and the sensitivity of welfare to changes in the charge also varies. Note that road 1 shows the greatest potential for improvements in welfare. Road 3 shows some potential for welfare improvement, and is less sensitive to charging a sub-optimal amount.

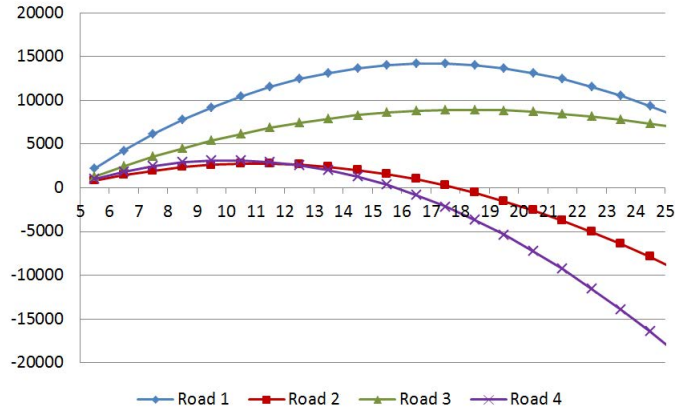


Figure 3: Cumulative social welfare for four different roads.

4.3 A single price for all roads

We can also consider a case where due to political or technical constraints, it is not possible to differentiate the charge between different roads. Given this constraint, a new optimum charge must be found. The same procedure as that used in Section 4.1 is applied, except that the price applies to all four congested roads. When this is done, the optimum price is found to be just under 16 NOK.

4.4 Comparison of results

We have seen that moving from no charging, to partial charging to full charging of congestion improves welfare. There are however other effects which we may consider. The main way in which congestion charging improves social welfare is by reducing the time spend travelling, and particularly the time spent queuing. The aggregate travelling time per day is presented in Table 1. The time spent queuing is also presented.

Table 1 shows that, as expected, an increase in the level of congestion charging decreases the aggregate time spend travelling. The time spent in queues is reduced particularly dramatically, by some 36% where full charging is imple-

| | Road 1 | Road 2 | Road 3 | Road 4 | Total | Queuing |
|------------------|--------|--------|--------|--------|---------|---------|
| No charging | 18 857 | 14 230 | 11 724 | 16 882 | 255 027 | 7 202 |
| Partial charging | 16 131 | 14 146 | 11 857 | 16 805 | 252 486 | 6 121 |
| Full charging | 16 091 | 12 884 | 9 989 | 15 484 | 246 987 | 4 591 |

Table 1: Aggregate travel time in hours per day, for the entire road network.

mented compared to a situation with no charging. Looking at the travelling time on each of the congested roads under the different charging regimes reveals an interesting pattern. Moving from the situation with no charging to the situation where road one has a charge implemented reduces the total time travelled on road 1. This is as expected. The charge also has the effect of lowering the demand for and congestion on roads 2 and 4. However, the demand for travelling on road 3 increases, along with the congestion. Implementing charging on all four roads leads to a reduction in travelling on all of these roads, with a corresponding drop in congestion.

The point which is highlighted here is that the road charges can be used to modify people’s route choice decisions. Hence, charging on only one road may have the undesirable consequence of making people choose another road which also suffers from congestion. This problem can be handled by charging on all congested roads.

For policy makers, achieving a more efficient use of the road network may be the main aim. Congestion charging may also be used as a means of financing new roads (Ieromonachou et al., 2006). However, another aim which policy makers may have in mind is improving urban air quality (Johansson, 1997). In our modelling framework, we have assumed that there are no other externalities. Here, we consider how atmospheric emissions would be affected by different congestion charging schemes.

In order to do this, some assumptions must be made regarding how travelling distances translate into emissions. We base our assumptions on Hagman et al. (2011), who have carried out emissions calculations for cars in Bergen. All cars

are assumed to run on petrol. Hagman et al. (2011) provide emissions factors for driving in a queue, in the city and on a country road. Our road network is therefore split into countryside and urban areas. Distances spent in a queue are also calculated. For mono-nitrogen oxides, NO_x , we assume an emission of 0.088 g/km, 0.067 g/km and 0.034 g/km for queuing, city and countryside driving respectively. For particulate matter, PM, we assume 0.0009 g/km, 0.0006 g/km and 0.0006 g/km. For carbon dioxide, CO_2 , we assume 322 g/km, 187 g/km and 133 g/km.

As well as calculating the quantity of emissions, we also consider a valuation of these emissions. Magnussen et al. (2010) provide estimated unit prices of the pollutants we consider for Bergen, and for the countryside. For emissions in urban areas, we assume a costs for NO_x , PM and CO_2 of 200 NOK/kg, 2,900 NOK/kg and 0.2 NOK/kg. For non-urban areas we assume 50, 0 and 0.2 NOK/kg. The estimated environmental costs are presented in Table 2.

| | Roads 1-4 | All roads | Total |
|------------------|-----------|-----------|-----------|
| No charges | 47 104 | 3 093 944 | 3 141 048 |
| Partial charging | 45 422 | 3 104 130 | 3 149 551 |
| Full charging | 5 401 | 3 130 845 | 3 136 246 |

Table 2: Estimated environmental cost per day under different charging regimes. Value are in 2008 NOK. 10 NOK is around £1.11, €1.30 or \$ 1.69

As expected, the application of a congestion charge to a road has the effect of lowering the emissions from that particular road. Since this reduction in congestion comes in part from making some road users choose alternative routes, it is perhaps unsurprising that the emissions from traffic on the other roads increases. However, the direction of the net effect may be somewhat surprising. When partial charging is introduced, the overall level of emissions increases. This is due to some people diverting from the charged road onto other congested routes, and on to longer but less expensive routes. Drivers queuing on the other congested routes have a higher level of emissions per km.

The overall level of emissions drops when moving to a scenario with full charging. The fall in emissions on the congested routes is more than enough to offset the rise in the non-congested roads. The reduction in traffic on the charged-roads has the effect of reducing the time spent in queues, which is the least efficient form of driving. With regard to the environment then, the situation with full optimal charging is the best, followed by no charging, with partial charging giving the worst outcome. To arrive at optimal charges which take into account congestion and environmental impacts, any environmental externalities should be incorporated into the social welfare function, and all roads in the network should be subject to a charge.

5 Concluding remarks

In this paper, we have considered the case of the optimal congestion charge for a road which is part of a network, where other roads in the network may also be congested. To do this, a static congestion problem was set, and a short-run situation considered. Within this framework, three congestion charging regimes were considered: a do-nothing scenario where no roads are charged, a situation where one congested road has a charge levied and one where all congested roads are charged.

The results of the model confirmed existing findings in the literature showing that in order to maximise social welfare, the transportation network in its entirety must be considered. Charging on only the most congested road can have the undesired effect of pushing people on to other congested roads. A socially optimal outcome can be achieved by deriving and implementing a set of optimal prices.

Although in our model we considered a situation where congestion was the only externalities, we also estimated the likely emissions associated with the

traffic in our model, and estimated the cost of these emissions. We showed the expected result that emissions on the roads subject to a charge fell. More surprising perhaps is that we showed an increase in the system-wide emissions for a situation with partial charging. This was due to people substituting the charged-road with other congested roads and with longer but uncongested roads.

The model highlights the complexity which can arise when considering real-world networks. While a road pricing regime may seem optimal when considering only one link, or only one set of links, the wider consequences may lead to undesirable outcomes. This is a particularly important point in the Norwegian case. Many of Norway's public services are provided by its 429 municipalities. This is quite a fine subdivision given that Norway has only around five million inhabitants. Having so many municipalities may hinder cooperation on road pricing, making it difficult to take a system-wide approach to the issue. It would be better if municipalities were defined according to labour-market areas.

An important point to note is that we have considered a short-run situation. In this short-run, we have assumed that the location of firms and workers remains unchanged. In addition, we have assumed that motorists do not change their mode of transport. Future research should look at the relaxation of these assumptions, in the context of a complex network. It may be the case that the short- and long-run optimal prices are different. In such a case, understanding the dynamic aspect of the adjustment process is important. It is also important that policy makers understand the long-term consequences of their actions.

6 Notation

| | |
|----------------------|---|
| n | Number of nodes in the network |
| m | Number of links in the network |
| F_r | Flow on road r |
| c_r | Generalised cost of traversing a particular link in a network |
| d_r | Length of a particular link |
| ω_r | Capacity of a particular link |
| ψ | The wage rate |
| f_{ij} | Number of commuters travelling from origin i to destination j |
| O_i | Number of trips originating from node i |
| D_j | Number of trips terminating in node destination j |
| a, b | Balancing factors in gravity model |
| C_{ij} | Generalised travel cost from origin i to destination j |
| γ | Cost per km |
| α | Value of travel time |
| T_r | Travel time on road r |
| δ_r | Dummy variable which is equal to 1 if a toll can be charged on link r , and 0 otherwise |
| R_r | Toll on road r |
| T_r^0 | The time taken to travel 1 km in the absence of congestion |
| T_r^1, ε | Congestion sensitivity of travel time |
| λ_{ij}^r | Dummy variable which is equal to 1 if commuters travelling from i to j use road r , and 0 otherwise |
| $D_{ij}(x)$ | Inverse demand function for commuting from origin i to destination j |

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