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Inventory management of spare parts in an energy company

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Abstract

We address a problem of inventory management of spare parts in the context of a large energy company, producer of oil and gas. Spare parts are critical for assuring operational conditions in offshore platforms. About 200,000 different items are held in several inventory plants. The inventory system implemented at the company corresponds to a min-max system. The control parameters are decided based mainly on the expert judgment of the planners. Also, though the inventory plants can in practice be supplied from each other, the inventory planning is performed separately by the plant planners. This is because of different ownership structures where the studied company has the operative responsibility. The company is pursuing a system in which all planners conform to the same inventory management approach and evaluation, as well as being more cost efficient. Our work focuses on supporting this goal. We apply methods to decide the inventory control parameters for this system under a service level constraint. The methodology we use distinguishes unit-size and lot-size demand cases. We perform computational experiments to find control parameters for a sample of items. After the control parameters are found, we use them to explore the impact of risk pooling among the plants and inaccuracy arising from duplicate item codes.

Key words:

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Inventory of spare parts; risk pooling; duplicates; inventory inaccuracy; OR in the oil and gas sector.

1. Introduction

This research has been motivated in the context of Statoil ASA, a large international energy company. Headquartered in Norway, this company is one of the world's largest net sellers of crude oil and the second largest exporter of gas to Europe. It acts as operator of several offshore platforms with different ownership structures and it holds inventory of about 200,000 spare part items in a number of locations spread in the Scandinavian region. Some of these parts are highly critical to assure safety and production. At the same time, the store of spare parts entails an important binding cost for the company.

Currently, the prevailing inventory control policy is based on the min-max or (s, S) system, whose control parameters are set at each inventory location per separate. Although the (s, S) systems are frequently encountered in practice, the values of the control parameters are usually set in a rather arbitrary fashion (Silver et al. [1], pp. 239). The company in our case does not differ much from this, but it is in process of automating this task, in order to be more uniform across the locations, to speed up the evaluations and to become more efficient.

This article describes the inventory management of spare parts at the company and focuses on two problems. The first is to develop a quantitative evaluation methodology which suggests inventory control parameters for an (s, S) system subject to a service level constraint. The second is to use such methodology in order to analyze the potential impact of two main sources of savings: risk pooling among the inventory locations and correcting inventory inaccuracy arising from duplicate item codes.

A number of articles have focused on spare parts inventories. An overview of related literature on spare parts inventories can be found in Kennedy et al. [2]. Most of the work in spare parts has been motivated by air force organizations and commercial airlines (e.g., [3], [4], [5], [6]). Applications in other contexts include the service support of IBM in the US (Cohen et al. [7]), a chemical plant in Belgium (Vereecke and Verstraeten [8]), a white goods manufacturer in Italy (Kalchschmidt et al. [9]) and a distributor of castors and wheels in Greece (Nenes et al. [10]). In the oil industry, we have found only one recent article, published by Porras and Dekker [11]. Characterized by high service levels (because of safety and production factors), customized equipment specifications with long lead times, and facility networks spread onshore and offshore, we believe the problem of inventory management of spare parts in this industry deserves attention from the research community and our article contributes by addressing a practical problem in this industry. Also, note that despite several decades of inventory research on (s, S) systems, finding optimal control parameters s and S subject to the fill rate service level constraint remains an open problem in the general case; most of the applications utilize approximation techniques. Our article provides insights of how methods from the literature behave under a real-world data set and we use them to analyze the impact of risk pooling and duplicates. While risk pooling is a traditional source of savings on inventory management, the duplicates issue has not been addressed in previous literature.

In Section 2 we provide background on the company under study and its current practice on inventory of spare parts. In Section 3 we outline our problem settings. In Section 4 we describe the solving methodology, considering different demand models. We solve numerical examples and present an analysis in Section 5. Our concluding remarks are presented in Section 6.

2. Background

Statoil ASA is an international energy company headquartered in Norway, with presence in 42 countries. Its activities include extraction of petroleum, refinement, and production of gas and methanol. Several types of equipment are used in these activities. Inventory of spare parts is held to replace equipment, in order to assure operating conditions.

The company has operational responsibility for seven warehouses located along the Norwegian coast, which serve offshore installations. Within this structure of warehouses, there are 24 inventory plants (or *license inventories*). Statoil ASA is the majority owner of these, but there is also a set of other companies owning a share of them. Each of the 24 plants holds its own inventory of spare parts and they are managed per separate. Note that although the same warehouse is responsible for several platforms it keeps separate physical inventories (shelves) for each platform. In the inventory system of the company, there are also other warehouses involved, serving on-shore installations.

The acquisition of spare parts is driven by consumption requirements triggered at the installations. The frequency of the requirement for a given spare part and the number needed are, in general, uncertain. For example, a spare part may not be required in 5 years but it may sometimes be needed a couple of times in few days.

The type of items can be anything from high value items required for production (for example, valves and compressors) to consumable products (smaller tools, such as gaskets, bolts and nuts). Some of these items are highly critical to assure safety and production. Depending on its role, the same part can be *highly critical* in one context and *less or not critical* in another context.

The age of the different equipment ranges from the 70s until now, which means that many different materials with different technological characteristics are stored. Also, the company is a result of the merger of three companies (Statoil, Norsk Hydro and Saga Petroleum), whose inventory practices differed and the goal is to now uniform them.

All information of the spare parts is managed by the enterprise resource planning system SAP. In total, there are about 950,000 codes of items in the system, including warehouses serving onshore and offshore operations. Currently, the items in stock correspond to about 245,000 of those codes. The same type of item can actually appear in the database with more than one code. This may be, for example, because they were obtained from different suppliers. A rough estimation indicates that between 10% and 30% of the item codes correspond to duplicates, and the number of different items actually in stock is around 200,000.

An estimate of the value of the current spare part inventory is in the order of half billion euros. There is the belief that this sum is too high, meaning a significant binding cost for the company. On the other hand, because of the important figures present in the oil and gas business, the shortage of highly critical items could imply severe costs if production breaks down, for example, when a platform must close operations. Some of these spare parts are also highly customized, with requirements specified on frame agreements with suppliers, thus their construction could take relatively long lead times (several months or even more than a year). In consequence, an important trade-off materializes between service level and inventory cost, two measures associated with the objectives of different units within the company.

As a spare part is required and the corresponding inventory plant provides it, replenishment occurs based on a heuristic type managed inventory system with either min or min and max allowable levels. Setting s equal to the min level minus 1 and S to the max level, this system can be described by a continuous review (s, S) policy, i.e., every time the inventory drops to the reorder point s or lower, a new order is placed so as to reach the order-up-to level S. When only the min level is used, an order is placed every time there is a demand event in order to achieve a base-stock level, which is a particular case of the (s, S) known as $(S - 1, S)$ policy. For some items, the company sets a min level equal to zero, which means that no inventory of them is stocked and they are rather ordered on the spot when required for consumption.

Each of the plants determines its own inventory control parameters s and S separately, and is linked to one productive location (and vice-versa). When a spare part is required, it is provided from the corresponding plant. However, when a productive location requires a certain spare part in such number that it is not available at the corresponding plant, the requirement can be fulfilled (if agreed) from another plant that has availability in stock. This is especially realized when the required part is highly critical. This type of fulfilment occurs based on an informal agreement between license inventories, but this is not considered when deciding the control parameters s and S. If no plant has stock, the number of the spare part needed is ordered directly from the supplier (even when the parts are taken from another plant, an order is placed with the supplier so as to raise the inventory position to the order-up-to level). For a given item in a given plant, these parameters are determined in a two-stage process. In the first stage, the supplier proposes the number of spare parts for a particular piece of equipment during the operation phase. The supplier proposes the number only as a function of the procured item (without necessarily knowing the exact context in which it will be used, and without knowing the current number of spare parts in stock). In the second stage, the inventory planner at the plant evaluates and decides the actual s and S values. The evaluation before decision intends to consider, beyond the supplier's recommendation, the current stock of items in inventory, its use, price and specific role.

In practice, it is the expert judgment of the inventory planners at the plants who currently play a key role in the decision making rather than that of structured quantitative methodology. While the experience is essential, two members of the staff may provide different inventory control parameters for the same item (since they may have different bases for their evaluations). Furthermore, each inventory planner decides the control parameters by only taking into account the inventory at her plant, thus the company lacks a more integrated approach that considers the multi-location network of inventory plants.

Our research focuses on two problems. The first is to develop a quantitative evaluation methodology which suggests inventory control parameters for the (s, S) system in the case under study. Finding correct levels can contribute not only to the cost savings, but also to establish a better relation between risk and criticality levels. Secondly, we use the methodology to analyze how risk pooling and the correction of duplicates would impact the inventory costs.

As we mentioned in the introduction, the only previous contribution to recent literature of spare parts in the oil industry is given by Porras and Dekker [11]. Motivated by a case study at a refinery in the Netherlands, they compare different reorder point methods for spare parts inventory control and show that total savings of up to 6.4% in holding costs can be achieved by a better inventory control. Our work differs in several aspects. First, while their problem takes place in a single refinery, ours accounts for multiple locations. Second, we include items with both unit-size and lot-size demand behaviours, thus considering the undershoot that they neglect. Third, while they explore different inventory policies, we focus on the (s, S) policy. We also explore the problem of duplicates, which has not been faced in their case or in any of the other articles on spare parts that we have reviewed.

3. Problem settings

Our first problem is to carry out a test of methods for inventory control parameters of a continuous review (s, S) system, considering different demand models. We first focus on a single-location problem and we then incorporate other locations into the risk pooling analysis.

The problem setting is characterized as follows:

- The goal is minimizing expected carrying cost plus ordering cost, subject to a service level constraint.
- The service level constraint we utilize is a lower bound β such that

$$1 - \frac{\text{average shortage per replenishment cycle}}{\text{average demand per replenishment cycle}} \geq \beta. \quad (1)$$

The left-hand side above is the expression traditionally utilized to compute the *fill rate*, a performance measure defined as the fraction of demand that is fulfilled directly from the stock on hand. As pointed out by Guijarro et al. [12], while this expression is an approximation to the fill rate, it is the most common method to compute it. Based on simulation, their numerical experiments reveal that this traditional approximation underestimates the simulated fill rate and, therefore, in our purpose to set inventory control parameters it would lead to a more conservative approach. It has been agreed in our project that this conservative approach is acceptable. The lower bound β represents the target fill rate, which depends on the item and the context in which it is used. The target values have been predefined by the company, based on the criticality of the items. Each criticality index is associated with a target value. For items appearing with different criticality indexes (because of the different contexts in which they can be used), the company prefers to consider the highest of the associated service levels.

- The future demand for spare parts is stochastic. We use probability distributions to model demand and estimate their parameters by using historical data from the company.

- The demands for different spare parts are independent at item and plant levels.

- The lead time is fixed. The procurement of spare parts is obtained from external suppliers, with whom the conditions of the procurement are fixed beforehand by frame agreements. The frame agreements include the delivery time, together with the price of the items. The company also has knowledge on the setup time of the equipment. These times are reasonably fulfilled, thus supporting the fixed lead time assumption.

- Replenishment is carried out only with new parts acquired from the supplier, thus, in this paper we do not study the possibility of repairing. Alternatively, our approach could cope with that possibility if we were to consider that the parts are sent for repair every time the reorder point is hit and the lead time value is the time that it takes since then until they are back in stock.

Note that the motivation to use the service level constraint instead of a penalization cost on backorders comes from our discussion with managers and planners of Statoil ASA. Minimizing the average carrying and ordering costs, subject to such service level constraint is a popular strategy in practice, since the shortage costs are usually difficult to assign. On the other hand, the service level criterion is generally easier to state and interpret by practitioners. A number of references point out this fact, such as Chen and Krass [13], Bashyam and Fu [14], and Cohen et al. [15] and [16].

4. Demand modelling and solving methodology

Let A be the fixed cost per order, μ the average demand per unit of time, v the unit cost, r the carrying charge and β the target fill rate.

For a given item at a given plant, the methodology to determine s and S levels that we use consists of two main steps. First, we set the difference $S - s$ as the economic order quantity $Q = \sqrt{\frac{2A\mu}{vr}}$, rounded to the closest positive integer.

Second, given that value of $S - s$, we try to obtain the minimum reorder point s such that the fill rate β is satisfied. Then, we get $S = s + Q$.

At the second step we evaluate the fill rate consecutively, starting from $s = 0$. Let us call $\tilde{\beta}_s$ the approximated fill rate achieved for a given reorder point s, namely the left-hand side in constraint (1). If the service level β is not satisfied, i.e., if $\tilde{\beta}_s < \beta$, we try $s = s + 1$ and so on, until we find the lowest value of s such that $\tilde{\beta}_s \geq \beta$.

For purposes of implementation, an upper bound on s could be set as the maximum allowable reorder point. In our experience with spare parts, this iterative evaluation procedure finishes quickly, especially for slow-moving items for which the reorder points are relatively low.

Choosing s and S through this sequence of two steps does not necessarily drive to optimal values (in terms of expected costs). For achieving optimality, the problem should run in both variables simultaneously, but such an exact approach would highly complicate the problem. In fact, despite several decades of inventory research on (s, S) systems, finding optimal control parameters s and S subject to the fill rate service level constraint remains as an open problem in the general case; most of the applications utilize approximation techniques specialized in particular settings. For example, Cohen et al. [16] approach an (s, S) inventory system problem with fixed lead times, two priority demand classes and lost sales of excess demand. They use a renewal approach to arrive at an approximate formulation and then a greedy heuristic to solve the approximated problem. Bashyam and Fu [14] consider a periodic review version of the (s, S) system, with general random lead times that allows for order crossings. Due to analytic intractability, they propose a feasible directions procedure that is simulation based. Schneider and Ringuest [17] propose an analytic approximation for the (s, S) system under periodic review, considering a service level measure defined as the average backlog (demand on backorder) relative to average demand. The replenishment lead time is assumed to be fixed and their solution approach is based on a power approximation method.

The sequential computation of s and $Q = S - s$ as we use and other variations is a common approach in the literature of inventory control (see for example, several decision rules in [1] that work sequentially, [8] and [11]). An important distinction is how to consider the demand behaviour, which affects the tractability of the fill rate. In the literature, there has been large interest on demand modelling and forecasting motivated for the intermittent and slow-moving consumption patterns observed in spare parts inventory (see, for example, [18], [19] and [20]).

In order to calculate the fill rate, we distinguish between unit-size demand items (only demand events for one unit of the same item at a time can occur) and lot-size demand (events for either one or more units of the same item can occur).

For unit-size demand, we test three distributions: Poisson, gamma and gamma with probability mass at zero.

For lot-size demand, we test two distributions: normal and gamma.

We choose these traditional distributions which have proved to be effective in case studies ([9], [10], [11], [21]). They are also easy to communicate to practitioners and to implement in software packages of common use. In order to check how our data fit the distributions, we perform a statistical test. This in practice leads us to rule out the normal distribution for all the items in our database.

4.1. Unit-size demand

In the unit-size demand case it is relatively easy to compute $\tilde{\beta}_s$. Note that the denominator equals the order quantity $Q = S - s$ and the numerator depends on the lead time demand distribution. Next, we outline the computations for three distributions.

4.1.1. Poisson distribution

Let us call X the demand per unit of time and assume it follows a Poisson distribution of parameter $\lambda > 0$. Then $P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$, for all non-negative integer values of k.

The approximated fill rate in this case can be easily calculated as

$$\begin{aligned} \tilde{\beta}_s &= 1 - \frac{\sum_{k=s+1}^{\infty} (k-s)P(X_L = k)}{Q} \\ &= 1 - \frac{\lambda_L - s[1 - P(X_L \leq s)] - \sum_{k=0}^{s} kP(X_L = k)}{Q}, \end{aligned} \tag{2}$$

where L is the lead time, and X_L and $\lambda_L = \lambda L$ are the demand during the lead time and its mean, respectively.

We estimate the parameter λ as the mean computed from the data, i.e., the total demand for the item divided for the total number of periods considered.

4.1.2. Gamma distribution

The density function of a gamma random variable with scale parameter $\alpha > 0$ and shape parameter $k > 0$ is defined as $g(x) = \frac{\alpha^k x^{k-1} e^{-\alpha x}}{\Gamma(k)}$, $0 \leq x < \infty$, where $\Gamma(k) = \int_0^{\infty} v^{k-1} e^{-v} dv$. Its use on inventory control dates from early times (see, e.g., [22] and [23]).

If the lead time demand follows the gamma distribution, the approximated fill rate can be expressed as

$$\begin{aligned} \tilde{\beta}_s &= 1 - \frac{\int_s^{\infty} (x-s)g(x)dx}{Q} \\ &= 1 - \frac{\frac{k}{\alpha}[1 - G_{1,k+1}(\alpha s)] - s[1 - G_{\alpha,k}(s)]}{Q}, \end{aligned} \tag{3}$$

where $G_{\alpha,k}(x)$ is the cumulative distribution function of a gamma with scale parameter α and shape parameter k, i.e., $G_{\alpha,k}(x) = \int_0^x g(u)du$, $x > 0$.

We use the mean μ and the variance σ^2 from the data for estimating the parameters of the gamma distribution in the lead time as $\alpha = \mu/\sigma^2$ and $k = L\mu^2/\sigma^2$.

4.1.3. Gamma distribution with mass probability at zero

For intermittent demand, it might be convenient to use a distribution with a mass at zero, as suggested by Nenes et al. [10] and Dunsmuir and Snyder [21]. We also test this distribution to model the lead time demand. In this case, the approximated fill rate can be expressed as

$$\tilde{\beta}_s = 1 - \frac{[\int_s^\infty (x-s) g_+(x) dx] p}{Q}, \tag{4}$$

where: p is a weight representing the probability of having positive demand in a unit of time, estimated from the data as the quotient between the number of months where demand was positive and the total number of months; and g_+ is the density function of a gamma random variable, for which we estimate scale parameter $\alpha_+ = \mu_+/\sigma_+^2$ and shape parameter $k = L\mu_+^2/\sigma_+^2$, considering only the data for months with positive demand to calculate the mean μ_+ and the variance σ_+^2. Then, the term in brackets at the numerator of equation (4) can be computed in the same way as we did in equation (3).

4.2. Lot-size demand

As far as we know, in the general case of lot-size demand there are no exact methods to compute the left-hand side of constraint (1). In contrast to the unit-size case, the undershoot distribution makes the lot-size case more complex. This is not a crucial limitation, however, because usually the data on inventory problems is not accurate either, but based on estimations; quoting Silver et al. ([1], pp.239), mathematical optimality *does not make sense* when deciding inventory control parameters for most items.

Next, we apply the approximated fill rate expression in Tijms and Groenevelt [24] for two demand distribution cases: normal and gamma. In that article, the difference $S - s$ is assumed to be predetermined (e.g., by the EOQ) and that it is sufficiently large compared with the average demand per unit of time (say, $S - s \geq 1.5\mu$). Stochastic lead times of replenishment orders are considered, provided that the probability of orders crossing in time is negligible. The reorder point s as to satisfy the fill rate service level constraint is approximately determined by solving the equation

$$\int_s^\infty (x-s)^2 \eta(x) dx - \int_s^\infty (x-s)^2 \xi(x) dx = 2\mu(1-\beta)\left(S - s + \frac{\sigma^2+\mu^2}{2\mu}\right), \tag{5}$$

where $\eta(x)$ is the probability density of the total demand in the lead time plus one unit of time, $\xi(x)$ is the probability density of the total demand in the lead time, and μ and σ are the mean and standard deviation of the demand per unit of time, respectively. Experiments by Moors and Strijbosch [25], who propose an exact approach to compute the fill rate when demand behaves as a gamma distribution with integer-valued shape parameter, confirm that the approximation based on equation (5) behaves satisfactorily if the condition $S - s \geq 1.5\mu$ holds.

4.2.1. Normal distribution

We use the mean μ and the variance σ^2 computed from the data as parameters of the normal density function $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$, $-\infty < x < \infty$. In the case of demand normally distributed, the left-hand side of equation (5) corresponds to

$$M(s) = \sigma_\eta^2 J\left(\frac{s-\mu_\eta}{\sigma_\eta}\right) - {\sigma_\xi}^2 J\left(\frac{s-\mu_\xi}{\sigma_\xi}\right), \tag{6}$$

where: μ_η and σ_η are the mean and standard deviation of demand in the lead time plus a unit of time; μ_ξ and σ_ξ are the mean and standard deviation of demand in the lead time; and $J(x) = \int_x^\infty (u-x)^2 \phi(u) du = (1+x^2)[1-\Phi(x)] - x\phi(x)$ is an expression in terms of the standard normal density $\phi(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$ and its cumulative function $\Phi(x) = \int_{-\infty}^x \phi(u) du$.

Then, the approximation of the fill rate that we utilize in the methodology for the normal lot-size demand case is

$$\tilde{\beta}_s \approx 1 - \frac{M(s)}{2\mu\left(S - s + \frac{\sigma^2+\mu^2}{2\mu}\right)}. \tag{7}$$

Alternatively, note that when $S - s$ is assumed predetermined, the right-hand side of equation (5) is the constant value $y_0 = 2\mu(1-\beta)\left(S - s + \frac{\sigma^2+\mu^2}{2\mu}\right)$. Then, the problem of calculating the reorder point s reduces to find a solution to the equation $M(s) - y_0 = 0$. Tijms and Groenevelt [24] discuss approximation methods to find the value of s that solves this equation. Nowadays, it can be quickly solved by using, for example, the method *fzero* in Matlab, which is based on a successive linear interpolation method by Dekker [26]. We rather compute the approximated fill rate $\tilde{\beta}_s$ for each discrete value of s instead of solving the equation, whose solution can be fractional. Under a truncation procedure, however, both alternatives would conduce to the same solution. In fact, we tried both in our first experiments and checked the solutions obtained were in accordance (note that *fzero* needs a parameter as starting point, for which we used 0.9).

4.2.2. Gamma distribution

We now model the demand per unit of time by a gamma distribution. We use the data to estimate its parameters as $\alpha = \mu/\sigma^2$ and $k = \mu^2/\sigma^2$.

The left-hand side of equation (5), for $s > 0$, is now given by

$$\begin{aligned} M(s) \;=\; & \sigma_\eta^2(a_\eta+1)[1-F_{a_\eta+2}(b_\eta s)] \\ & -2su_\eta[1-F_{a_\eta+1}(b_\eta s)] + s^2[1-F_{a_\eta}(b_\eta s)] \\ & -\sigma_\xi^2(a_\xi+1)[1-F_{a_\xi+2}(b_\xi s)] \\ & +2s\mu_\xi[1-F_{a_\xi+1}(b_\xi s)] - s^2[1-F_{a_\xi}(b_\xi s)], \end{aligned} \tag{8}$$

where $a_\eta = \mu_\eta^2/\sigma_\eta^2$ and $b_\eta = \mu_\eta/\sigma_\eta^2$ (analogously for a_ξ and b_ξ) and $F_k(x)$ is the distribution function of a gamma with scale parameter 1 and shape parameter k, i.e., $F_k(x) = \frac{1}{\Gamma(k)}\int_0^x e^{-u}u^{k-1}du$, $x > 0$.

As in the normal distribution case, we use this expression of $M(s)$ in the approximation (7) and evaluate it for consecutive reorder point s, starting from $s = 0$ to up, until the target β is fulfilled.

4.3. Selecting a demand model

Five alternatives to model demand can provide five different outcomes. We illustrate this by an example.

Table 1 presents data on 9 items over a period of 67 months, where: μ and σ are the mean and standard deviation of the demand per month, calculated from the historical data accounting all months in the sample; μ_+ and σ_+ are the mean and standard deviation of the demand per month, calculated from the historical data accounting only the months with positive demand; $N_{D>0}$ is the number of months with demand greater than zero; $N_{D>1}$ is the number of months with demand greater than one; L is the lead time (in months); and β is the target fill rate.

We apply the solving methodology for the five alternatives of demand models. In all cases the results are obtained very quickly, in a matter of a second. The resulting values of s and S in each case are reported in Table 2. For those items whose standard deviation for months with positive demand σ_+ is zero (i.e., items M2, M4 and M7), we do not compute results for the gamma with mass probability at zero, because it requires positive parameters.

Table 1: Illustrative data.

Item	μ	σ	μ_+	σ_+	$N_{D>0}$	$N_{D>1}$	L	β
M1	0.16	0.48	1.22	0.63	9	1	0.33	0.95
M2	0.03	0.17	1.00	0.00	2	0	0.50	0.95
M3	0.04	0.27	1.50	0.50	2	1	0.33	0.95
M4	0.03	0.17	1.00	0.00	2	0	10.20	0.97
M5	0.15	0.55	1.43	1.05	7	1	0.17	0.95
M6	0.28	0.73	1.73	0.86	11	6	6.47	0.97
M7	0.04	0.21	1.00	0.00	3	0	6.67	0.97
M8	0.04	0.27	1.50	0.50	2	1	1.17	0.95
M9	1.73	7.57	29.00	13.00	4	4	0.47	0.95

Table 2: Results of the illustrative data example.

Plant	Unit-size method						Lot-size method			
P1	Poisson		Gamma		Gamma_0		Normal		Gamma	
Item	s	S	s	S	s	S	s	S	s	S
M1	0	4	0	4	0	4	1	5	2	6
M2	0	1	0	1	-	-	1	2	2	3
M3	0	1	0	1	0	1	1	2	3	4
M4	2	3	2	3	-	-	2	3	3	4
M5	0	1	0	1	0	1	1	2	4	5
M6	5	6	8	9	14	15	6	7	9	10
M7	2	3	2	3	-	-	2	3	3	4
M8	1	2	1	2	1	2	1	2	3	4
M9	1	9	10	18	8	16	14	22	65	73

From this example, we emphasize that important differences can arise for some items depending on the demand model used. For instance, the Poisson model conduces to a reorder point $s = 1$ for item M9, which is the one with highest volatility, while the other models conduce to much higher reorder points, in a range from 8 to 65. Note that, by its definition, the Poisson distribution can-not incorporate information on an empirical variance much higher than the empirical mean, which can explain its outcome in comparison to the other distributions in this example.

Likewise, among the two lot-size demand models, the normal distribution conduces to lower reorder points than the gamma distributions for all items, which is expectable since the normal allows negative values.

In other cases, different demand models conduce to equal or similar results. For instance, the three unit-size models produce the same result for items M1, M3 and M5.

In general, the fill rate expressions are not so tractable as to get structural properties relating the outcomes of different distributions. In the particular case when $s^* = 0$, it is easy to check that the service level constraint (1) reduces to $\mu L/Q \leq 1 - \beta$ for each of the three unit-size demand distributions (which explains the result obtained for materials M1, M3 and M5). In the general case, this kind of outcomes is hard to identify without explicitly computing the fill rates for each demand model.

Beyond the particular outcomes of these examples, they point out that the demand model and the data input can significantly affect the results. A natural question is then how to select only one demand model from the five we have proposed to study. The suggestions in the literature are rather fuzzy. Silver et al. [1] refer to a number of distributions and as a rule of thumb they recommend: the normal distribution for items such that the coefficient of variation σ/μ is less than 0.5; if this coefficient is greater than 0.5 it might be desirable to use another distribution such as the gamma; for slow-moving items, the Poisson distribution is a good candidate. Nenes et al. [10] suggest the use of the chi-square goodness-of-fit test to ensure that the Poisson or gamma distribution give acceptable representations of the demand behaviour for some items.

In our case, we first check whether the demand has historically behaved as unit-size or lot-size. If there was at most one period with demand greater than one (i.e., if $N_{D>1} \leq 1$), then we classify the demand as unit-size behaviour; if there were two or more periods with demand greater than one (i.e., if $N_{D>1} \geq 2$), then we classify the demand as lot-size behaviour.

We then perform a chi-square goodness-of-fit test and see if the hypothesis that the data follows a given distribution is rejected or not. Then, we propose a simple decision rule which goes as follows:

1. If the demand has historically behaved as unit-size, then choose one of the three unit-size demand models such that it was non-rejected in the test and it obtained the highest p-value among them.
2. If the demand has historically behaved as lot-size and $S - s \geq 1.5\mu$, then choose one of the two lot-size demand models such that it was non-rejected in the test and it obtained the highest p-value among

them.

3. If neither conditions 1 nor 2 conduced to select a demand model, then leave the item for further (managerial) revision.

In our implementation, we run the five methods for computing s and S values for all items, independent of which of the five demand models is finally recommended. This has been useful for benchmark purposes, as we showed in Table 2.

We have presented the methodology as one for all the items. Note, however, that by discerning between unit-size demand and lot-size demand items and setting $S - s$ by the EOQ formula, we tend to distinguish the most important items and treat them with more accurate calculation than the least important ones. In fact, most of the important items tend to present a unit-size demand behaviour in the historical data base, so our recommendation will be based on the fill rate's traditional approximation (1) rather than the lot-size approximation (7). Including the lot-size demand method is still useful, however, for the rest of the items. Despite them perhaps representing lower carrying cost, its computation still requires time resources and effort from the staff; then, a recommendation under a quantitative methodology provides valuable support.

5. Numerical results and analysis

We present numerical computations, using a database with historical consumption over a period of 67 months. We firstly focus on a single plant problem, involving a sample of 992 items that presented consumption over this period at the main plant (henceforth, plant P1). Afterwards, we incorporate another 19 plants into the analysis. Then, we perform additional runs for the single plant problem in order to discuss how results change with different criticality settings and what is the impact of duplicates.

5.1. Single plant planning

We use the historical demand from the main plant for the data set of 992 items. Our computational implementation is coded in Matlab R2011a on an Intel Core2 Duo 2.27GHz processor with 2GB of RAM, supported by a spreadsheet in Excel. The methodology runs reasonably fast, providing results in about one minute for the whole set of 992 items including the five demand models.

The results of applying the chi-square goodness-of-fit test are summarized in Table 3. A first observation is that the normal distribution is rejected for all the items. There is previous literature pointing out the limitations of the normal distribution to model demand of spare parts (see, e.g., [8], [21], [27] and [28]) and our work with real-world data provides additional evidence of that.

In contrast to the normal, the Poisson distribution and the gamma distribution are not rejected in approximately 90% or more items. The gamma distribution with spike at zero is not rejected for 11.1%. We note this figure is relatively low, because for several items the standard deviation for positive demand registers was zero so we could not use this distribution for them. For only 0.7% of the items all the distributions were rejected.

Table 3: Outcome of the chi-square goodness-of-fit test over a set of 992 items.

Distribution	Non-rejected	Percentage
Poisson	899	90.6%
Gamma	975	98.3%
Gamma mass zero	110	11.1%
Normal	0	0.0%
None	7	0.7%

We then apply our selection criteria for demand models and recommend inventory control parameters. Table 4 presents a summary of the frequency in which each distribution is selected. The Poisson distribution is the one selected for most of the items, with a frequency of 65%. We select the unit-size and lot-size demand behaviour with gamma distribution for 16.7% and 16.6% of the items; and the gamma with mass at zero for unit-size demand in only 0.2% of the items. Note that for only 1.3%, which corresponds to 14 items, our methodology does not recommend any of the above alternatives and rather leaves the items for managerial evaluation. We point out that all these 14 items presented a lot-size demand behaviour and both the normal and gamma distributions were rejected by the test for 12 of them. For the remaining two items, the gamma distribution was not rejected, but the coefficient $(S - s)/\mu$ was less than 1.5, thus not sufficiently large to apply the approximated fill rate of the lot-size gamma demand case.

Table 4: Outcome of the methodology over a set of 992 items.

Distribution	Recommendations	Percentage
Poisson	645	65.0%
Gamma	166	16.7%
Gamma mass zero	2	0.2%
Gamma lot-size	165	16.6%
Normal	0	0.0%
None	14	1.3%
Total	992	100.0%

5.2. Multiple plants planning and risk pooling

A main usefulness of the methodology we have presented in the previous section is that it helps us to evaluate the potential impact of some savings sources. One of these sources is risk pooling among different inventory plants.

The importance of lateral resupply and pooling is illustrated by Muckstadt [3], who gives examples of systems in which pooling exists that require roughly a third of the safety stock required when operating a completely decentralized system. A recent application by Kranenburg and van Houtum [29] reports that under full pooling more than 50% of the no pooling cost can be saved in the case of ASML, a Dutch equipment manufacturer. Before implementation, ASML did not take lateral transshipment into account in the planning phase, i.e., the inventory in each local warehouse was planned separately. Nevertheless, in daily practice lateral transshipment was used; the same practice as realized at the company in our case. When comparing their implementation to such a practice, the savings in yearly cost were about 31%. A similar situation is presented by Kukreja and Schmidt [30], who deal with a large utility company having 29 generating plants. The plants do not consider the effect of being supplied from each other when deciding their own inventory control policies, but in practice they also collaborate. Experimentation by the authors including from two to five plants shows that pooling leads to savings between 31% and 58.78%. Other works in lateral transshipments under emergency situations are reported in Archibald [31], Wong et al. [32] and Alfredsson and Verrijdt [33].

As in the case of Kukreja and Schmidt [30], we consider a single-echelon situation with a multi-location, continuous-review inventory system in which complete pooling is permitted and each location utilizes an (s, S) policy. We carry out computations for twenty plants by aggregating their historical demand. We do not consider lateral transshipment costs and rather focus on the potential of risk pooling for protection against variability. Note that all the plants we are dealing with lie along the Norwegian coast, relatively close to each other. Also, the most extreme emergency cases are those when a main part on an off-shore platform needs to be replaced and its corresponding on-shore plant does not have the part in stock. If the part is available from other plant, the part does not need to be sent from one to another on-shore plant before being sent off-shore, but is just sent straight from the on-shore plant supplying it to the off-shore location.

We focus on 992 items that presented consumption in the main plant P1, for which we also account with demand data on the consumption realizations in the other 19 plants.

We first apply the methodology for determining inventory control parameters s and S in each plant separately.

In order to analyze the potential of risk pooling, we also consider a fictitious *integrated* plant, as if instead of the 20 real plants there would be only one central plant serving the demand from all of them. The data we use for this integrated plant is the aggregated demand data for the 20 real plants.

From the 992 items, we consider only items for which there was at least one realization of demand in at least one of the 19 other plants; in total, these account for 212 items. For fair comparison purposes, from these 212 items we leave out those that in at least one plant the methodology did not recommend inventory control parameter values; in total, we are left with a sample of 175 items. For a given item, we use the same service level bound in the separated planning and the integrated planning.

From the results so obtained, we summarize important measures in Table 5. For each plant, the figures aggregate all the items. κ is the safety stock, calculated as the reorder point s minus the expected demand during the lead time. N_A is the expected number of orders per year, calculated as $\mu/(S-s)$. And $\bar{I}$ is the average inventory on hand, calculated as $\kappa + (S-s)/2$.

The risk pooling approach results in 1% less safety stock, 11% fewer expected number of orders and 4% less average inventory on hand. In order to quantify the impact on inventory costs, we have (internally) calculated the total expected cost as $C_T = C_r + C_A$, where C_r is the expected carrying cost and C_A is the expected ordering cost ($C_r = \bar{I}vr$ and $C_A = \frac{A\mu}{S-s}$). In this way, the reduction of inventory means total annual savings in the order of 13% when comparing the integrated case to the separated planning case. Since we have used the same lower bound on the service level in both cases and

Table 5: Safety stock, expected number of orders and average inventory on hand under separated planning and integrated planning over a set of 175 items in 20 plants.

Plant ID	κ	N_A	$\bar{I}$
P01	1,085	78	1,570
P02	51	11	83
P03	28	5	39
P04	87	10	198
P05	3	2	7
P06	70	7	78
P07	366	19	427
P08	121	7	135
P09	135	7	143
P10	39	8	57
P11	12	5	18
P12	18	1	22
P13	18	2	23
P14	2	1	5
P15	80	10	98
P16	32	6	59
P17	1	0	1
P18	31	6	47
P19	17	5	29
P20	119	4	232
Total separated	2,313	194	3,272
Integrated	2,287	172	3,125
Difference	-1%	-11%	-4%

have limited the analysis to a sample of only 175 items in 20 plants of the pool, the potential savings by an integrated planning in the whole problem appear to be promising.

5.3. Analysis of criticality settings

A natural concern in our study is to compare our recommendation with the inventory control parameters currently used at the case study company. This is part of our internal validation process at the company. Without getting into sensitive information details, we provide some insights in Table 6. We use three different safety settings, depending on the fill rates bounds: *high* *safe* setting, with bounds ranging from 90% to 99%; *medium safe* setting, with bounds ranging from 80% to 89%; *low safe* setting, with bounds ranging from 70% to 79%. The values in the table indicate the number of times that our recommendation of reorder point s^* is greater, equal or lower than the actual reorder point s_a at plant P1. For a fair comparison, we discard 164 items for which the company has preferred to not hold inventory and thus ordering them just when required for consumption (i.e., those where the min level is set to zero). We also discard 14 of the remaining items for which none of the 5 demand models was recommended. In total, we are left with 814 items for this comparison.

Table 6: Comparison of the reorder point s^* obtained by our methodology and the actual s_a under different service level bounds, considering 814 materials of plant P1.

Service level	$s^* > s_a$	$s^* = s_a$	$s^* < s_a$
High safe	373	268	173
Medium safe	157	427	230
Low safe	92	467	255

In the high safe setting, our recommendation is higher than the actual for 46% of the items, while lower for 21% of them. In the medium safe setting, our recommendation is equal to the actual for most of the items, while lower for 28% and higher for 19% of them. In the low safe setting, our recommendation is higher than the actual for only 11% of the items, while lower for 31% of them.

Hence, for high target service levels, our recommendation tends to be higher than the actual. If we assume the actual demand follows the distribution selected, the analysis would reveal that the current control parameters do not meet the target fill rates for most of the items. As we decrease the target fill rate, on the other hand, our recommendation tends to reduce the current reorder points. Similar observations could be made comparing the order-up-to-levels S.

The methodology can thus serve as support for the inventory planners to carry out a quantitative evaluation of the trade-off between service levels and inventory costs. It has, of course, some limitations. For instance, some few items (1.3% in the case of plant P1) could not be modeled by any of the demand distribution tested. It will always be possible to include more probability distributions of common use in the context of spare parts, such as the compound Poisson or negative binomial. Alternatively, a forecasting rather than a modelling demand approach could be considered. The tests of methods applied in our work have proved to be useful for modelling a large majority of the items, while at the same time keeping the computations reasonably simple and quick.

There is also an issue with basing the analysis on historical data when it comes to new items with either short or no historical demand behaviour to analyze. In the other extreme, there are old items with a large amount of historical data but whose use is starting to disappear as they are replaced by more modern equipment. In the next section we pursue the task of overcoming another data issue: the existence of duplicates in the system.

5.4. Duplicates

A duplicate means that the same spare part item has been registered in the database with different codes. In the event of there being few items to manage, it would be relatively easy to identify which items correspond to duplicates and the inventory staff could quickly realize any discrepancies between the information on the system and the actual stock. However, when the number of items is in the order of a hundred thousand and the inventory network is multi-location, the duplicates create a more difficult puzzle to deal with. Evidence from the retail sector (DeHoratius et al. [34]) has pointed out that inventory record inaccuracy is a significant problem in practice, and the elimination of inventory inaccuracy can reduce supply chain costs as well as the out-of-stock level (Fleisch and Tellkamp [35]). In these references, the inaccuracies refer to discrepancy between the recorded inventory quantity and the actual inventory quantity physically available on the shelf. In our case, the discrepancies arise from the duplicated codes, which have been estimated to be roughly between 10% to 30% of the item codes in the system. This may lead to inventory levels higher than desirable when, for example, one item with a given code in the system reaches its reorder point, triggering a replenishment order while the same item with different code is still available. On the other hand, when a stockout of an item occurs, the realized service level could have been better if the item was actually available but with a different code and it was not realized by the staff. Still if in practice these two problems would not occur for all the duplicated items, the existence of duplicate codes in the system also affect the data quality for diverse evaluation purposes.

We perform a simple procedure in order to estimate the impact of the spare parts duplicated in the system. Let us assume there is a 10% of duplicates. Then, our procedure is based on aggregating one out of each ten items in the system. First, we sort the database by prices, from the most expensive item to the cheapest one. Then, we merge the 10th item with the 9th item, the 19th with the 20th and so on, as if they would correspond to the same item. The merging of two items consists of aggregating their historical demand (obtaining the resulting average, standard deviation, etc.) and calculating for the merged item a new lead time, unit cost and order cost as the average of the corresponding values from the two separate items. As for the criticality index, if the two items appeared with different values, we use the highest one. We implemented this procedure and merged 198 items of the 992 items from our database into 99 items. Then, we applied the methodology of Section 4 to compute the inventory control parameters and we compared the results to those obtained in the case where the 198 items were considered per separate. We found that for the aggregated items the average inventory on hand decreased by 18%, the safety stock decreased by 8% and the average number of orders decreased by 10%. These figures translate into savings of 28% of the expected cost per year for those items. We carried out the same experiment a further eight times, varying the position of the merged items. Namely, in one experiment we merged the 9th item with the 8th item and so on, instead of the 10th item with the 9th item; in other experiment we merged every 8th and 7th items, and in the last experiment we merged every 2nd and 1st items. On average, the total expected cost per year when the items were merged was 22% less than when they were not merged.

We performed a similar analysis varying the percentage of merged items from 0% to 30%, increasing it by 5% in each run. This analysis is motivated by the lack of knowledge of exactly how many items are duplicated in the system. If we assume a certain percentage of items are duplicated and that they are corrected by merging them, we can compute the savings compared to the total expected cost when all the original items in the database are considered per separate. As shown in Figure 1, the merger of up to 30% of the items led to savings of almost 12% in total expected cost.

If there are between 10% to 30% of duplicates in the system, by this analysis our computations indicate on average there would be 8% of savings if they would be corrected.

Other than the relative order in the sorted-by-price list, one could add more conditions when merging items, such as their similarity in other attributes. We have performed additional runs, merging two consecutive items

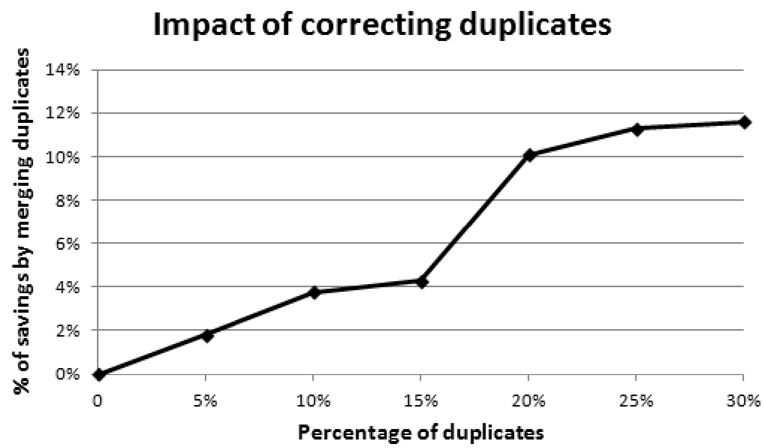

Figure 1: Percentage of savings in total expected cost as a function of the percentage of duplicated items, if the duplicates are corrected.

in the sorted-by-price list only if their unit prices differ in no more than 10% and their lead times differ in no more than 10%. Under these criteria, 238 items (equivalent to 25%) are merged into 119 items, resulting in savings of 17% in comparison to when they are not merged. Likewise, if we would consider the cost of the merged items as basis, the impact of duplicates in cost corresponds to 21%.

6. Concluding remarks

We have studied a problem of spare parts inventory in Statoil ASA, a large energy company. Our attention focused on how to set reorder and order-up-to level parameters for the (s, S) continuous review system, in order to explore the benefits of risk pooling and the correction of duplicates.

We considered five demand models, including unit-size and lot-size cases, and used simple rules and a goodness-of-fit test to choose a model. We implemented them in a methodology which runs quickly and suggested inventory control parameters for a variety of items. Our results provide insights on how these models and corresponding solving methods behave in a real-world data set.

We applied the methodology to explore the potential benefits of risk pooling in the case under study. Our results suggest the risk pooling conduces to reduction of safety stock, expected number of orders and average inventory on hand, which translates into cost savings of 13% while satisfying the required service level bounds. In practice, the implementation of risk pooling is not straight forward, however, because of political issues and the difficulties in configuring a cost sharing structure. One of our future research direction aims at how the different inventory license owners should share costs in the pool if it is implemented. Different owners may have different preferences on how to share or manage an inventory. Some of them may be less willing to collaborate as they have larger shares in other platforms beyond those operated by the company under study. Likewise, some equipment may be more important to one than another company, but the resources (e.g., space) they utilize at the warehouse are the same. Recent literature has introduced the use of game theory to address this cost sharing issue in risk pooling problems (Wong et al. [36], Karsten et al. [37]), opening a promising research line within the literature on spare parts inventories.

Getting correct demand description usually needs historical data. In practice, this presents some difficulties, like the existence of duplicated item codes in the system. By aggregating some of the items based on their similarity in prices, we identified the correction of the duplicates issue as another source of important savings (on average 8% when duplicates go from 10% to 30% of the items). Other difficulties with respect to the data are the scarcity of registers for new items and the lack of clarity when deciding whether a consumption register is actual demand or an internal balance movement. Analyzing the impact of these problems is also part of our future research and we are working together with the information technology units of Statoil ASA on these and other issues of the data processing, as well as on testing the methods presented in this article on a larger set of data.

We have assumed in this article that only new items are considered when replenishing stock. In practice, some of the items can be repaired. The company has set different repair strategies, depending on the type of items and the location where they should be repaired. In future research, we expect to incorporate repairable items and use a multi-echelon modelling approach, as is used in the literature of spare parts related to aircraft (Muckstadt [3], Sherbrooke [4]).

Also, we have focused on the (s, S) policy, as it is the one prevailing at the company today. Comparing the performance of this with other systems is also a relevant question for further research. Note that most of the important items are usually expensive enough to make the EOQ equal to 1. In consequence, the policy with these items corresponds to the well-known

$(S-1, S)$, which is commonly assumed when working with critical spare parts ([3], [4]). Although this policy is a particular case of (s, S), there are special results for the $(S-1, S)$ policy that could be useful to incorporate, such as the exact expression of the fill rate for the Poisson demand model (see, e.g., [3], pp. 52). Also, when the same item appears with different criticality indexes in different contexts, it would be interesting to measure the benefits of a threshold rationing policy, which gives priority to the demand in the highest criticality context (Deshpande et al. [38], Dekker et al. [39]), over the (s, S) policy with round-up service level bounds as considered in our case.

Another point for further work is evaluating the alternative of outsourcing the inventory service with the supplier, which might be convenient for some of the items with short lead times and low criticality.

Although our article has been inspired in the context of Statoil ASA, our contribution extends to a more general context which we would like to summarize in four main points. First, we have used five demand models and defined simple and clear rules to select one of them for each item, rather than setting a demand model *a priori*. As pointed out recently by Syntetos et al. [40], we believe that the development of goodness-of-fit tests for application in inventory control of intermittent demand items is an interesting future research issue. They use the Kolmogorov-Smirnov test (Massey [41]), which is not appropriate when the parameters are estimated from the sample (as it is in our case) and it only applies to continuous distributions. The chi-square goodness-of-fit test, on its hand, may fail on discerning whether a distribution is rejectable or not when the demand values have presented small frequencies in the history. This could be the case of binary demand or so called *clumped* items. Using other distributions, such as the binomial and beta-binomial, could be an alternative approach for these items, as tested by Dolgui and Pashkevich [27].

Second, we have explored the benefits of risk pooling in a problem with a single-echelon, multi-location, continuous-review inventory system in which complete pooling is permitted and each location utilizes an (s, S) policy. As noted by Kukreja and Schmidt [30], this problem has not received special attention from the literature, despite its complexity and relevance in practice.

Third, in addition to the risk pooling, which is a traditional source of savings in inventory management, we have introduced the correction of duplicate codes as another source which, in large inventory systems, may lead to considerable savings as well. To our knowledge, this duplicates issue has not been addressed in previous literature and we believe it is worthy of in-depth exploration in the context of spare parts, as has been done with related issues of record inaccuracy on retail inventories (DeHoratius et al. [34], Fleisch and Tellkamp [35]).

Fourth, our work has highlighted the importance of the inventory management of spare parts in the oil and gas industry, which so far has not been a main protagonist in related literature. Characterized by high service levels, geographical networks with interaction between onshore and offshore facilities and items with customized specifications and long lead times, we believe there are important chances of improving the inventory practices in the industry, with opportunities for achieving great impact on savings while at the same time maintaining or increasing service levels.

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References

[1] Silver EA, Pyke DF, Peterson R. Inventory Management and Production Planning and Scheduling. United States: John Wiley & Sons; third ed.; 1998.

[2] Kennedy WJ, Patterson JW, Fredendall LD. An overview of recent literature on spare parts inventories. International Journal of Production Economics 2002;76:201–215.

[3] Muckstadt JA. Analysis and Algorithms for Service Parts Supply Chain. New York, USA: Springer Series in Operations Research and Financial Engineering; first ed.; 2005.

[4] Sherbrooke CC. Optimal Inventory Modeling of Systems Multi-Echelon Techniques. Boston, USA: Kluwer Academic Publishers; second ed.; 2004.

[5] Eaves AHC, Kingsman BG. Forecasting for the ordering and stock-holding of spare parts. Journal of the Operational Research Society 2004;55:431–437.

[6] Ghobbar AA, Friend CH. Evaluation of forecasting methods for intermittent parts demand in the field of aviation: a predictive model. Computers & Operations Research 2003;30:2097–2114.

[7] Cohen M, Kamesam PV, Kleindorfer P, Lee H. Optimizer: Ibm's multi-echelon inventory system for managing service logistics. Interfaces 1990;20:65–82.

[8] Vereecke A, Verstraeten P. An inventory management model for an inventory consisting of lumpy items, slow movers and fast movers. International Journal of Production Economics 1994;35:379–389.

[9] Kalchschmidt M, Zotteri G, Verganti R. Inventory management in a multi-echelon spare parts supply chain. International Journal of Production Economics 2003;81-82:397–413.

[10] Nenes G, Panagiotidou S, Tagaras G. Inventory management of multiple items with irregular demand: A case study. European Journal of Operational Research 2010;205:313–324.

[11] Porras E, Dekker R. An inventory control system for spare parts at a refinery: An empirical comparison of different re-order point methods. European Journal of Operational Research 2008;184:101–132.

[12] Guijarro E, Cardós M, Babiloni E. On the exact calculation of the fillrate in a periodic review inventory policy under discrete demand patterns. European Journal of Operational Research 2012;218:442–447.

[13] Chen FY, Krass D. Inventory models with minimal service level constraints. European Journal of Operational Research 2001;134:120–140.

[14] Bashyam S, Fu MC. Optimization of (s, s) inventory systems with random lead times and a service level constraint. Management Science 1998;44:S243–S256.

[15] Cohen MA, Kleindorfer PR, Lee HL. Near-optimal service constrained stocking policies for spare parts. Operations Research 1989;37:104–117.

[16] Cohen MA, Kleindorfer PR, Lee HL. Service constrained (s,s) inventory systems with priority demand classes and lost sales. Management Science 1988;34:482–499.

[17] Schneider H, Ringuest JL. Power approximation for computing (s, s) policies using service level. Management Science 1990;36:822–834.

[18] Croston JD. Forecasting and stock control for intermittent demandsauthor. Operational Research Quarterly 1972;23(3):289–303.

[19] Willemain TR, Smart CN, Schwarz HF. A new approach to forecasting intermittent demand for service parts inventories. International Journal of Forecasting 2004;20:375–387.

[20] Syntetos A, Boylan J. On the stock control performance of intermittent demand estimators. International Journal of Production Economics 2006;29(103):36–47.

[21] Dunsmuir WTM, Snyder RD. Control of inventories with intermittent demand. European Journal of Operational Research 1989;40:16–21.

[22] Burgin TA. The gamma distribution and inventory control. Operational Research Quarterly 1975;26(3):507–525.

[23] Snyder RD. Inventory control with the gamma probability distribution. European Journal of Operational Research 1984;17:373–381.

[24] Tijms HC, Groenevelt H. Simple approximations for the reorder point in periodic and continuous review (s, s) inventory systems with service level constraints. European Journal of Operational Research 1984;17:175–190.

[25] Moors JJA, Strijbosch LWG. Exact fill rates for (r, s, s) inventory control with gamma distributed demand. Journal of the Operational Research Society 2002;53:1268–1274.

[26] Dekker TJ. Finding a zero by means of successive linear interpolation. In: Dejon B and Henrici P, editors, Constructive Aspects of the Fundamental Theorem of Algebra 1969;Wiley-Interscience:37–48.

[27] Dolgui A, Pashkevich M. On the performance of binomial and beta-binomial models of demand forecasting for multiple slow-moving inventory items. Computers & Operations Research 2008;35:893–205.

[28] Syntetos A, Keyes M, Babai M. Demand categorisation in a european spare parts logistics network. International Journal of Operations & Production Management 2009;29(3):292–316.

[29] Kranenburg AA, van Houtum GJ. A new partial pooling structure for spare parts networks. European Journal of Operational Research 2009;199:908–921.

[30] Kukreja A, Schmidt CP. A model for lumpy demand parts in a multi-location inventory system with transshipments. Computers & Operations Research 2005;32:2059–2075.

[31] Archibald TW. Modelling replenishment and transshipment decisions in periodic review multilocation inventory systems. Journal of the Operational Research Society 2007;58:948–956.

[32] Wong H, van Houtum GJ, Cattrysse D, van Oudheusden D. Multi-item spare parts systems with lateral transshipment and waiting time constraints. European Journal of Operational Research 2006;171:1071–1093.

[33] Alfredsson P, Verrijdt J. Modeling emergency supply flexibility in a two-echelon inventory system. Management Science 1999;45:1416–1431.

[34] DeHoratius N, Mersereau AJ, Schrage L. Retail inventory management when records are inaccurate. Manufacturing & Service Operations Management 2008;10:257–277.

[35] Fleisch E, Tellkamp C. Inventory inaccuracy and supply chain performance: a simulation study of a retail supply chain. International Journal of Production Economics 2005;95:373–385.

[36] Wong H, Oudheusden DV, Cattrysse D. Cost allocation in spare parts inventory pooling. Transportation Research Part E 2007;43:370–386.

[37] Karsten F, Slikker M, Houtum GV. Spare parts inventory pooling games. BETA Working Paper Eindhoven University of Technology 2009;300.

[38] Deshpande V, Cohen MA, Donohue K. A threshold inventory rationing policy for service-differentiated demand classes. Management Science 2003;49:683–703.

[39] Dekker R, Kleijn MJ, de Rooij PJ. A spare parts stocking policy based on equipment criticality. International Journal of Production Economics 1998;56-57:69–77.

[40] Syntetos AA, Babai MZ, Altay N. Modelling spare parts' demand: An empirical investigation. 8th International Conference of Modeling and Simulation MOSIM'10 Hammamet Tunisia 2010;68.

[41] Massey FJJ. The kolmogorov-smirnov test for goodness of fit. Journal of the American Statistical Association 1951;46:68–78.