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Abstract

We consider an electricity market with two sequential market clearings, for instance representing a day-ahead and a real-time market. When the first market is cleared, there is uncertainty with respect to generation and/or load, while this uncertainty is resolved when the second market is cleared. We compare the outcomes of a stochastic market clearing model, i.e. a market clearing model taking into account both markets and the uncertainty, to a myopic market model where the first market is cleared based only on given bids, and not taking into account neither the uncertainty nor the bids in the second market. While the stochastic market clearing gives a solution with a higher total social welfare, it poses several challenges for market design. The stochastic dispatch may lead to a dispatch where the prices deviate from the bid curves in the first market. This can lead to incentives for self-scheduling, require producers to produce above marginal cost and consumers to pay above their marginal value in the first market. Our analysis show that the wind producer has an incentive to deviate from the system optimal plan in both the myopic and stochastic model, and this incentive is particularly strong under the myopic model. We also discuss how the total social welfare of the market outcome under stochastic market clearing depends on the quality of the information that the system operator will base the market clearing on. In particular, we show that the wind producer has an incentive to misreport the probability distribution for wind.

Keywords: market design, electricity, stochastic programming

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1. Introduction

An increased share of power production from non-dispatchable energy sources cause challenges for the energy systems and calls for a revision of electricity market designs. One option for making electricity markets more robust with respect to uncertainty in production and consumption levels is to use a stochastic dispatch when clearing the markets. In the stochastic dispatch, the market clearing is performed under explicit consideration of possible market clearings in subsequent markets. We consider an electricity market with two sequential market clearings. The first is in the day-ahead market, while the second is the real-time market. The real-time market is primarily used to resolve any deviations between the day-ahead market clearing and the real demand and supply. Such deviations may occur due to the uncertainty with regards to demand levels and production from non-dispatchable energy sources when the day-ahead market is cleared. Traditionally, these markets are cleared in sequence, and the day-ahead market is cleared only based on the supply and demand bids, not considering what will happen in the real-time market. With an increased share of intermittent generation it is timely to look at alternative designs, where these two are more tightly linked. In order to improve the day-ahead market clearing it is beneficial to let the day-ahead market models foresee the possible outcomes of the uncertain demand and supply in the real-time market. Given that we are able to predict the possible future outcomes of supply and demand it is possible to use a stochastic model to do the market clearing. This is beneficial because the cost of deviations from planned production and consumption is usually lower when the potential deviations are considered in the initial planning phase.

Real-time flexibility comes at a cost, i.e., extra costs will be incurred if a flexible producer or consumer has to deviate from his initial plan in real-time. The flexibility costs, as discussed by Khazaei et al. (2014) and Bjørndal et al. (2013b), are incurred because real-time changes have to be made at short notice. Such costs could be caused by, e.g., decreased component life due to frequent changes in generation levels, limitations on ramp-rates, or limited ability to re-optimize plans on short notice. See NETL (2012) for a discussion of different sources of flexibility costs.

Numerous authors have developed stochastic market clearing models and showed that they yield better plans, in terms of expected social surplus, than deterministic market clearing models. Examples include Bouffard et al.

(2005a,b); Bouffard and Galiana (2008); Ruiz et al. (2009b,a); Papavasileiou et al. (2011); Papavasileiou and Oren (2012); Khazaei et al. (2014).

The pricing issue under stochastic market clearing was first discussed by Kaye et al. (1990), who argued that day-ahead prices should be set equal to the expected values of real-time prices. Wong and Fuller (2007) propose several pricing schemes for their stochastic model, and they show that real-time pricing yields cost recovery in expectation and prices at, or above marginal cost for every generator that supplies energy. Morales et al. (2012) formulate a two-stage stochastic programming problem for clearing the market in an electricity pool with high wind power production. They also prove that their pricing scheme gives, in expectation, revenue adequacy for the system operator and cost recovery for the generators. A similar model is used in Pritchard et al. (2010) where also the load (in addition to wind production) is uncertain in the first-stage of the model. They prove that their pricing scheme is revenue-adequate in expectation. Morales et al. (2014) propose an improved version of the conventional deterministic market clearing model, in which the system operator controls the intermittent generator's bid in the day-ahead market in order to optimize the system as a whole. The procedure is solved using a bilevel optimization model, and yields an expected social surplus that is smaller or equal to the surplus under stochastic market clearing. Since their approach clears the day-ahead market in a deterministic manner, the merit order for all generators, except the intermittent generator, will be preserved. Zavala et al. (2015) proves, in a model similar to that of Pritchard et al. (2010), a bound for the difference between day-ahead prices and expected real-time prices. The bound depends on the parameters describing the flexibility costs, i.e., they show that day-ahead prices will converge to expected real-time prices when flexibility costs are small enough.

Our contribution is a thorough discussion of challenges and potential improvements from a market design based on stochastic dispatch, where we discuss the obtained market outcomes and compare these to the results from a conventional market clearing. Our model is similar to the energy-only models in Pritchard et al. (2010) and Morales et al. (2014). We discuss incentive challenges for the participants in the market, and we include a discussion of individual rationality as well as an analysis highlighting how misrepresentations of the probability distribution for wind can alter the total social welfare in the network and the distribution of surplus between the participants in the market. A stochastic model will find the most efficient market solutions (for the given representation of the uncertainty), and as such increase the total social welfare in the network. The solutions from the stochastic model can, however, lead to non-intuitive results that will pose severe challenges for the market design and operation of the network. Since the stochastic model will consider both the day-ahead market and the real-time market when choosing the market clearing in the day-ahead market, the solution may require the market participants to accept prices that deviate from their marginal values. That is, a consumer may have to consume more or less than its marginal benefit suggests, and a producer may have to produce more or less than its marginal cost suggests.

In Section 2 we present the mathematical formulation used in our models, before we define the two market clearing models in Section 3. We specify in detail our numerical example in Section 4 and discuss the assumptions we have made for our case studies. In this section we also present the results from our numerical analysis. The implications for market design is discussed in Section 5. We end the paper with conclusions in Section 6.

2. Mathematical models

We provide two alternative models for dispatch in the day-ahead market. The first model is a deterministic myopic model clearing the day-ahead market based on the demand bids and the supply bids only. Later, when uncertainty is resolved and generation and load is known, imbalances are cleared in the real-time market. The second model is a stochastic dispatch model, where the clearing of the day-ahead market is done integrated with the clearing of the real-time market. In this case, each possible outcome of the realized generation and the corresponding cost in the real-time market is taken into account, when deciding the dispatch for the day-ahead. For both models there are separate supply bids into the real-time markets, including flexibility costs. There will then be a trade-off between the day-ahead dispatch cost and the resulting expected regulation cost, which is considered in an integrated manner.

We have chosen an approach for stochastic dispatch in line with Pritchard et al. (2010) and Morales et al. (2014), where a day-ahead price and volume is announced after the day-ahead clearing. We will give the details on the underlying assumptions and the mathematical formulation for the two models in the rest of this section. First we will introduce notation and give some more detail on the flexibility costs in the real-time market.

2.1. Generation and load

Our model framework is similar to that of Pritchard et al. (2010). We consider a collection of offers $i \in I$, where each offer can represent either generation (positive values) or load (negative values). For each $i \in I$ we require a solution (x_i, X_i), where x_i is the solution for the first-stage dispatch, and X_i is a vector of stochastic variables representing the solution for the second-stage dispatch. The first-stage dispatch corresponds to the market clearing in the day-ahead market, while the second-stage dispatch is the results from the real-time market clearing. The set of feasible solutions for the first stage is denoted C_i^1, while the set of feasible solutions for the second stage will depend on the realized scenario $\omega \in \Omega$ as well as the decision x_i from the first stage. We denote this set as $C_i^2(\omega, x_i)$. A feasible solution (x_i, X_i) to both stages must satisfy

$$\begin{aligned} x_i &\in C_i^1 && \forall i \in I \\ X_{i\omega} &\in C_i^2(\omega, x_i) && \forall i \in I,\ \omega \in \Omega. \end{aligned}$$

Our set up differs somewhat from that of Pritchard et al. (2010), since we will also study alternatives to the integrated stochastic model, where the two stages are resolved in a sequential manner, as in Morales et al. (2014).

The models have a system perspective, i.e. as if the dispatches were performed centrally in an energy-only mandatory dispatch. We do not consider unit commitment, intertemporal constraints (water values are assumed equal), other types of ramping constraints, etc. These may be represented indirectly by the flexibility costs, however they are not considered explicitly. We also assume that all possible outcomes are modelled by our scenarios (which is clearly unrealistic), and we do not consider out-of-sample effects of the day-ahead market clearing. When discussing up- and down-regulation we will use the convention from the Norwegian market. Up-regulation then refers to a change in production or consumption that increases the net supply situation in the system. Down-regulation on the other hand, decreases the net supply situation in the system (i.e. generation is decreased and / or consumption is increased).

Our focus is on deviations from the day-ahead scheduling and how we model the cost- and benefit curves of flexible producers and consumers. That is, the regulation costs refer to the costs of changing production and / or consumption in the real-time market. If the consumers increase the quantity consumed in real time, it is not as valuable as if it was planned. If they reduce it, they would ask for more than the day-ahead willingness to pay. If the generators must increase their production beyond the planned level it is more costly, and if they reduce production form the planned level, they will not save all the incurred marginal costs. Hence, the flexibility costs represent real costs incurred by the participants in the market.

2.2. Objective function

The objective function for our models is minimization of total costs in the system. This includes the sum of costs from the day-ahead market and the regulation costs incurred in the real-time market. An illustration of the components in the objective function is provided in Figure 1. The figure on the left illustrates a supply function for a generator, while the figure on the right illustrates a demand function for a consumer. In addition, the two figures illustrates the flexibility costs incurred in the real-time market when there is a deviation from the day-ahead market clearing. The day-ahead clearing is given by volume x_i, whilst examples of up- and down-regulation volumes are given by $X^d_{i\omega_1}$, $X^u_{i\omega_1}$, $X^d_{i\omega_2}$ and $X^u_{i\omega_2}$. The functions for up- and down-regulation costs have the parameters, respectively, a^u_i, b^u_i and a^d_i, and b^d_i.

We use linear functions to represent the cost and benefit functions for the participants in the market. Each offer $i \in I$ is associated with a day-ahead cost- and benefit function with non-negative parameters a_i and b_i, given by

$$\hat{c}_i(x_i) = a_i x_i + 0.5 b_i x_i^2.$$

For the supply side, this cost function is based on an assumption of a linear marginal cost function: $a_i + b_i x_i$. In this expression, the parameter a_i represents a constant marginal cost, while the parameter b_i represents the slope of the marginal cost curve. The second stage cost- and benefit function parameters will typically differ from those in the first stage, due to reduced flexibility at this stage. We assume that this can be represented, for any flexible generator, with parameters a^u_i and b^u_i for up-regulation and a^d_i and b^d_i for down-regulation, where $a^d_i \leq a_i \leq a^u_i$ and $\min\{b^u_i, b^d_i\} \geq b_i$.

To represent the demand side, and keep the formulation compact, we use the same set of formulas. If $x_i < 0$, then $-a_i x_i$ will represent the benefit from consuming an amount $-x_i$. The inverse linear demand curve is given as $a_i + b_i x_i$. Since x_i will take negative values, this corresponds to a downward sloping demand curve. For both generators and consumers, the slopes of their

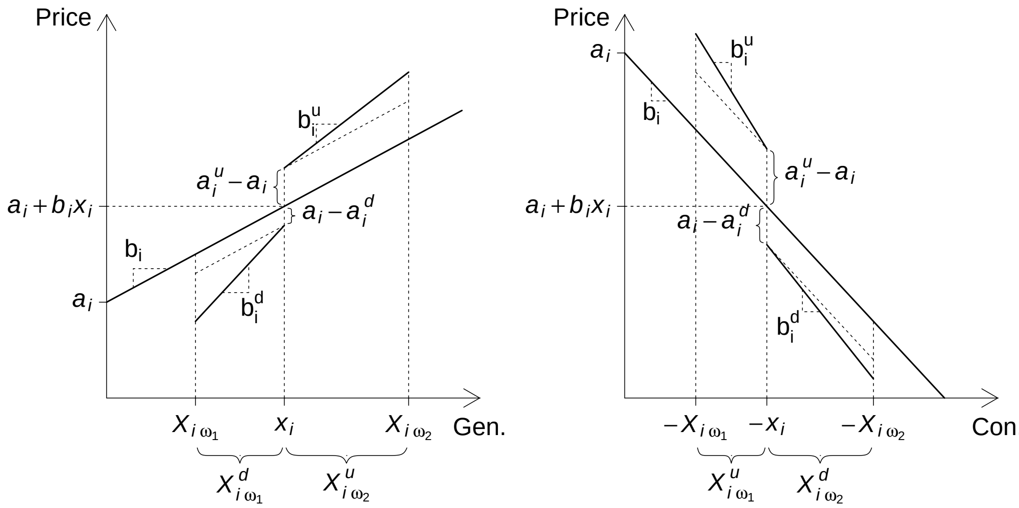

Figure 1: Illustration of the cost- and benfit functions used in our models. The figure on the left illustrates a supply function for a generator, while the figure on the right illustrates a demand function for a consumer. In addition, the two figures illustrates the flexibility costs incurred in the real-time market when there is a deviation from the day-ahead market clearing.

cost functions for changes in dispatch in the real-time market is steeper than their cost functions in the day-ahead market. The parameter b_i represents the slope of a linear demand curve. Similarly as for the supply side, we assume that any flexible consumer can be represented with parameters a_i^u and b_i^u for up-regulation and a_i^d and b_i^d for down-regulation, where $a_i^d \leq a_i \leq a_i^u$ and $\min\{b_i^u, b_i^d\} \geq b_i$.

With reference to Figure 1 we can formulate the total cost after the second-stage regulation as:

$$c_i(x_i, X_{i\omega}) = \hat{c}_i(X_{i\omega}) + \tilde{c}_i(x_i, X_{i\omega}),$$

where $\hat{c}_i(X_{i\omega})$ is the total cost of the final schedule evaluated at the day-ahead cost parameters, and $\tilde{c}_i(X_{i\omega}, x_i)$ is the additional cost caused by inflexibility in the real-time market. The flexibility cost associated with the first-stage quantity x_i and the revised quantity $X_{i\omega}$ in scenario ω is

$$\tilde{c}_i(x_i, X_{i\omega}) = (a_i^u - a_i)X_{i\omega}^u + 0.5(b_i^u - b_i)(X_{i\omega}^u)^2 + (a_i - a_i^d)X_{i\omega}^d + 0.5(b_i^d - b_i)(X_{i\omega}^d)^2,$$

where $X_{i\omega}^u = \max\{X_{i\omega} - x_i, 0\}$ and $X_{i\omega}^d = \max\{x_i - X_{i\omega}, 0\}$.

This formulation allows for a number of assumptions for flexibility cost for both consumers and generators. Figure 2 shows three examples of how the initial schedules may be adjusted, as well as the effect on cost and benefit. The leftmost diagram illustrates an example where $a_i = 0$ and $b_i > 0$, i.e., a generator with an increasing marginal cost starting from zero. His day-ahead schedule is x_i, and in scenario ω this quantity is up-regulated to $X_{i\omega}$. The slope of the up-regulation cost curve is given by the parameter $b_i^u > b_i$. The area of the light gray triangle equals $\hat{c}_i(X_{i\omega}) = 0.5b_i(X_{i\omega})^2$, i.e., the cost of the final schedule given the day-ahead cost function, and the area of the dark gray triangle equals the flexibility cost $\tilde{c}_i(x_i, X_{i\omega}) = 0.5(b_i^u - b_i)(X_{i\omega}^u)^2$. The middle diagram illustrates a generator with a constant day-ahead marginal cost a_i, and a marginal cost $a_i^d < a_i$ for down-regulation. The total cost after down-regulation is $a_iX_{i\omega} + (a_i - a_i^d)X_{i\omega}^d$, where the last part $(a_i - a_i^d)X_{i\omega}^d$ is the non-avoidable cost that remains after the initial scheduled quantity has been reduced by $X_{i\omega}^d$. The rightmost diagram illustrates a consumer with a first-stage demand function with intercept and slope parameters equal to a_i and b_i. Consumption quantities are negative, so the second-stage increase in consumption is equivalent to down-regulation. Again, the light gray area represents the benefit of the final schedule evaluated at the day-ahead parameters, i.e., equal to $-(a_iX_{i\omega} + 0.5b_i(X_{i\omega})^2)$, and the cross-hatched triangle equals the flexibility cost $\tilde{c}_i(x_i, X_{i\omega}) = 0.5(b_i^d - b_i)(X_{i\omega}^d)^2$.

2.3. Network flow equations

The generator and load entities are linked to a set of nodes N. For a particular offer $i \in I$ we denote by $\nu(i) \in N$ the node where generator / consumer i is located. We then consider the network as a directed graph where the nodes are connected by a set of transmission lines L. For a given flow vector $f = (f_l)_{l\in L}$, we let $\tau_n(f)$ denote the net inflow of power in node n from the transmission network. We define $\nu_0(l)$ as the starting point and $\nu_1(l)$ as the end point of line l, and $f_l > 0$ implies that power is flowing from $\nu_0(l)$ to $\nu_1(l)$. We assume, as in Pritchard et al. (2010), that lines are lossless, and this implies that:

$$\tau_n(f) = \sum_{l:\nu_1(l)=n} f_l - \sum_{l:\nu_0(l)=n} f_l.$$

See Pritchard et al. (2010) for a discussion of how the network model can be generalized to incorporate line losses. We will associate the day-ahead

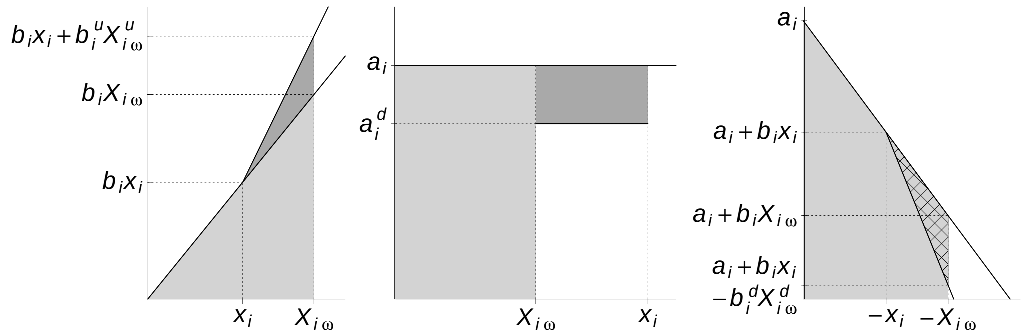

Figure 2: Calculation of cost and benefit for suppliers and consumers. The two figures to the left shows how the flexibility cost are for two suppliers with different supply functions. The light gray area illustrates the cost of the final dispatch with the original cost function (not including flexibility costs), while the dark gray area is the flexibility cost due to up- or down-regulation. The figure to the right illustrates the same for the consumers. The light gray area is the consumer benefit with the original demand parameters, while the chequered area shows the loss in consumer surplus due to flexibility costs.

schedule x with a flow vector f. The production and consumption quantities given by x must be consistent with the flow f, and in a lossless system this implies that

$$\tau_n(f) + \sum_{i \in I(n)} x_i = 0$$

for all $n \in N$. These constraints give an energy balance in the system.

Similarly we associate the final schedule X_ω with the flow vector F_ω, and consistency implies that

$$\tau_n(F_\omega) - \tau_n(f) + \sum_{i \in I(n)} (X_{i\omega} - x_i) = 0$$

for all $n \in N$. The energy balance in the network is also guaranteed by including this constraint. It is written on difference form, as in Pritchard et al. (2010), in order to provide dual prices that can be used to price real-time *deviations* from the day-ahead schedule in a meaningful way.

Additional network constraints for the first and second stage are given by:

$$\begin{aligned} & f \in U^1 \\ & F_\omega \in U^2 \qquad\qquad \forall \omega \in \Omega \end{aligned}$$

The sets U^1 and U^2 can represent capacity constraints for individual lines, loop flow constraints, or other relevant network constraints, as discussed in Bjørndal et al. (2013). Note that we need not have $U^1 = U^2$, since the representation of the network can differ in the two market clearing stages. In some day-ahead markets, such as Nord Pool Spot, the network flow model is a simplified locational price model, like zonal pricing and market coupling. In Bjørndal et al. (2016) a discussion of the effect of flow constraints in the day-ahead market is provided. However, we will not focus on congestion management in this paper, so the network flow constraints in the day-ahead and real-time market clearing are not so important. There will not be any binding capacity constraints in our examples, i.e., they satisfy $U^1 = U^2 = \mathbb{R}^{|L|}$.

3. Market clearing

We consider a situation where the electricity market consists of a day-ahead market and a real-time market. In the following we present two different dispatch models where the connection between the market clearings in these two markets is handled differently. The markets are cleared sequentially in two stages. In the first stage, the day-ahead market is cleared with uncertainty regarding load and / or generation levels in the real-time market (the second stage). In the second stage, the real-time market is cleared after all uncertainty is resolved. In the stochastic market clearing model, the first-stage is solved taking into account the uncertainty in the second-stage and the connection between the costs and benefits in the different stages. In the myopic market model, however, the day-ahead market is cleared based only on given bids, not taking into account neither the uncertainty nor the bids in the real-time market.

In the myopic model, the following problem describes the market clearing in the day-ahead market (first stage):

$$
\begin{aligned}
\min_{x,f} \quad & \sum_{i \in I} \hat{c}_i(x_i) && && \text{(1a)}\\
\text{s.t.} \quad & && && \\
& x_i \in C_i^1 && \forall i \in I && \text{(1b)}\\
& \tau_n(f) + \sum_{i \in I(n)} x_i = 0 && \forall n \in N \qquad [\lambda_n^{da}] && \text{(1c)}\\
& f \in U^1 && && \text{(1d)}
\end{aligned}
$$

where λ_n^{da} is the shadow price for the nodal balance constraints. In the real-time market (second stage), for every scenario $\omega \in \Omega$, the market clearing is found by solving

$$
\begin{aligned}
\min_{X_\omega, F_\omega} \quad & \sum_{i \in I} \Big(\hat{c}_i(X_{i\omega}) + \tilde{c}_i(x_i, X_{i\omega}) \Big) && && \text{(2a)}\\
\text{s.t.} \quad & && && \\
& X_{i\omega} \in C_i^2(\omega, x_i) && \forall i \in I && \text{(2b)}\\
& \tau_n(F_\omega) - \tau_n(f) + \sum_{i \in I(n)} (X_{i\omega} - x_i) = 0 && \forall n \in N \qquad [\lambda_{n\omega}^{rt}] && \text{(2c)}\\
& F_\omega \in U^2, && && \text{(2d)}
\end{aligned}
$$

where (x, f) is an optimal solution to (1). The resulting expected welfare from the two stages will be

$$
\mathbb{E}\left[\sum_{i \in I} \Big(\hat{c}_i(X_{i\omega}) + \tilde{c}_i(x_i, X_{i\omega}) \Big) \right]. \tag{3}
$$

We will refer to (1) and (2), solved sequentially, as the *myopic* market clearing model. In the myopic model, x and f will be fixed to the solution from the first-stage when solving the second stage.

In the *stochastic* market clearing model, the two markets are considered in an integrated manner. This means that the consequences for the real-time market clearing in the different scenarios is considered by the model when the day-ahead market is cleared. The objective function of this model is then to minimize the costs from the day-ahead market and the expected costs from the real-time market. This corresponds to the resulting expected welfare from the myopic model (see (3)), but in the myopic model the contributions from the day-ahead and real-time markets are calculated independent of each other.

$$\min_{x,f,X,F} \; \mathbb{E}\left[\sum_{i\in I}\Big(\hat{c}_i(X_i) + \tilde{c}_i(x_i, X_i)\Big)\right] \tag{4a}$$

s.t.

$$x_i \in C_i^1 \qquad \forall i \in I \tag{4b}$$

$$X_{i\omega} \in C_i^2(\omega, x_i) \qquad \forall i \in I,\ \omega \in \Omega \tag{4c}$$

$$\tau_n(f) + \sum_{i\in I(n)} x_i = 0 \qquad \forall n \in N \qquad [\lambda_n^{da}] \tag{4d}$$

$$\tau_n(F_\omega) - \tau_n(f) + \sum_{i\in I(n)} (X_{i\omega} - x_i) = 0 \quad \forall n \in N,\ \omega \in \Omega \quad [p_\omega \lambda_{n\omega}^{rt}] \tag{4e}$$

$$f \in U^1 \tag{4f}$$

$$F_\omega \in U^2 \qquad \forall \omega \in \Omega \tag{4g}$$

The main difference between the two model variants is the information available when the day-ahead market is cleared. In the stochastic dispatch model, the day-ahead part of the optimization problem takes into account the possible outcomes of the uncertain parameters and the corresponding consequences for the market clearing in the regulation market. Due to the sequential clearing of the markets in the myopic model, the market clearing in the day-ahead market will be independent of the uncertain parameters (except for the influence on bids).

Under myopic market clearing, we calculate prices in the day-ahead and real-time markets by using the dual variables from constraints (1c) and (2c), respectively. The dual variable λ_n^{da} from constraint (1c) can be interpreted as the marginal cost of a deterministic load at node n, both for the day-ahead and the real-time market. The dual variable $\lambda_{n\omega}^{rt}$ from constraint (2c) can then be interpreted as the marginal cost of the changed load in node n in scenario ω. Similarly, under stochastic market clearing, the day-ahead and real-time prices are given by the dual variables of the constraints (4d) and (4e), respectively. For a further discussion on these dual variables we refer to Pritchard et al. (2010).

4. Numerical example

For a discussion of the stochastic dispatch solutions and their economic interpretations, we use a simple example with an uncongested network consisting of 3 nodes, as illustrated in Figure 3. The example is motivated by the day-ahead market in Nord Pool Spot (the Nordic Power Exchange), which is an energy only market that includes both inflexible and flexible load (for instance due to large power-intensive industries), as well as different characteristics of the marginal cost curves of the producers (due to a combination of hydro power, wind power, thermal power and nuclear power). For a more detailed description of market clearing in Nord Pool Spot, see Bjørndal et al. (2013).

4.1. Data and parameters

In the example, there are 5 generators of various types, and their day-ahead marginal cost curves are given by the solid lines in the diagrams in Figure 3. We assume that wind power (Node 1) has a marginal cost of zero, up to the capacity limit, and will thus always be dispatched in the day-ahead market in the myopic market clearing model. For hydro power (Nodes 1 and 2), marginal cost equals water values, and we assume that they increase linearly with the quantity produced. Moreover, we assume that they are not affected by the market clearing model. Nuclear (Node 3) has a low and constant marginal cost, while thermal (Node 3) has a higher constant marginal cost. The demand curves are given by the dashed lines, and we assume that only Node 2 has elastic demand. In the example, there is no load shedding, so the value of the inelastic demand is constant and not represented in the objective functions. The only source of uncertainty is the wind generator in Node 1. Three wind scenarios and their probabilities are described in Figure 4. The marginal cost curve for Node 1 in Figure 3 is based on Scenario 2, where the wind generator can produce up to 7000 MWh/h. Expected wind power is 9650 MWh/h.

The cost parameters of the real-time market are described in Table 1. We assume that the wind generator's capacity is uncertain when the day-ahead market bid is submitted. The wind generator may regulate the quantity up or down without any extra costs, but must respect the realized capacity constraints given by the scenarios in Figure 4. Thus, the wind power generator is partly flexible. The two hydro generators, as well as the thermal generator in Node 3 and the load in Node 2, are also assumed to be

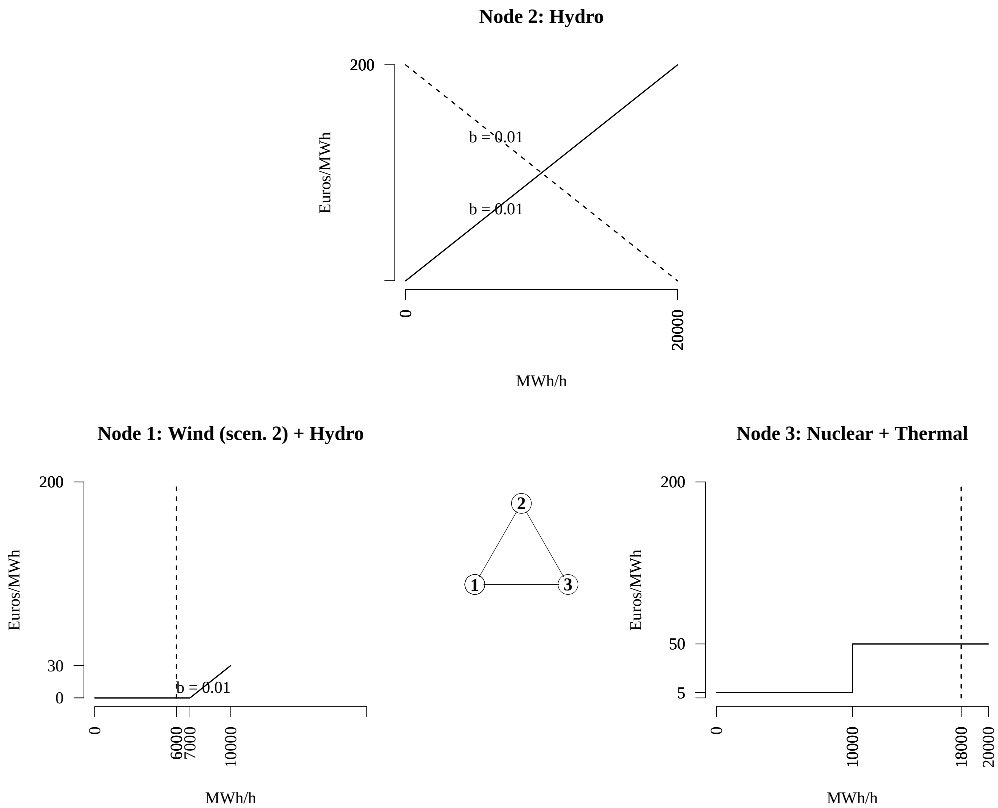

Figure 3: Example parameters. The figure shows the three nodes in the network along with the supply (solid lines) and demand (dashed lines) curves.

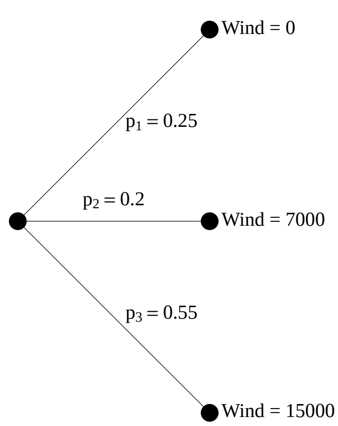

Figure 4: Illustration of the three wind scenarios.

Entity	Node	Flexible?	real-time cost
Wind	1	Partly	0 €/MWh
Hydro	1	Yes	$b^u = b^d = 1.1c$
Hydro	2	Yes	$b^u = b^d = 1.1c$
Nucl.	3	No	N/A
Therm.	3	Yes	$a^u - a = a - a^d = 5$ €/MWh
Load	1	No	N/A
Load	2	Yes	$b^u = b^d = 1.5b$
Load	3	No	N/A

Table 1: Flexible entities and their regulation costs. The number in parentheses gives the index number for the entity in the node.

flexible, however with some additional cost compared to being scheduled in the the day-ahead market. The up- and down-regulation costs of the hydro generators and the elastic load is represented by increasing the slopes of the corresponding bid curves compared to the day-ahead market. We increase the slope for the hydro generators with 10%, i.e. $b^u_{hydro1} = b^d_{hydro1} = b^u_{hydro2} = b^d_{hydro2} = 0.01 \cdot 1.1 = 0.011$, and for the flexible load with 50%, i.e. $b^u_{load2} = b^d_{load2} = 0.01 \cdot 1.5 = 0.015$. For the flexible thermal generator, the up/down-regulation cost is included by increasing/decreasing the intercept of the real-time market bid curve relative to the corresponding intercept for the day-ahead market, i.e., $a^u_{thermal} = a_{thermal} + 5 = 55$ € /MWh, and $a^d_{thermal} = a_{thermal} - 5 = 45$ € /MWh. Thus, like Pritchard et al. (2010), we assume symmetric up- and down-regulation costs for all market participants.

4.2. Results from the two dispatch models: Stochastic and Myopic

4.2.1. Day-ahead part

In Figure 5 the day-ahead part of the stochastic dispatch model is illustrated. Since the network is uncongested, the price found from the nodal balance constraint in equation (4d), is equal for all three nodes(68.1 € /MWh). We note that the wind production is planned at 11505 MWh/h, which is well above the expected wind power available in real time. Moreover, in Node 3, the flexible thermal producer is dispatched at 5877 MWh/h (seen as the total generation in Node 3 of 15877 MWh/h less the 10000 MWh/h produced by the nuclear generator) while, with a day-ahead price of 68.1 €/MWh, his bid curve suggests that he would want to produce at full capacity. In order to achieve such a solution in the day-ahead market, we cannot rely on a market mechanism that only uses price as the signal - quantities would have to be included as well. Note that this is also the case with traditional market design with stepwise bid functions, but in these cases the producers would be indifferent within each step. If only price signals were used to clear the day-ahead market, the thermal producer would want to produce more electricity. The solution that would result from a traditional market clearing (found with the Myopic Model) is illustrated in Figure 6. The day-ahead market price is now changed to 50 €/MWh, i.e., the marginal cost of the thermal generator. The day-ahead market quantity of the thermal generator has been increased to 9495 MWh, but it is still below full capacity. In the myopic model however, the thermal generator is dispatched at his marginal cost and will be indifferent with respect to the production level, thus there is no longer an incentive for self-scheduling.

The difference between the results from the day-ahead dispatch in the stochastic model (Figure 5) and the day-ahead market clearing in the myopic model (Figure 6) is due to the difference in information structure between the two models. The stochastic model will take into consideration how the day-ahead market clearing influences the regulation market clearing given the possible realizations of the uncertain parameters, whilst the myopic model will only consider the day-ahead market. Since the myopic model does not include the flexibility costs and the need to deal with wind uncertainty when clearing the day-ahead market, it will find the most efficient market clearing for the day-ahead market. The stochastic model, however, will choose a different dispatch in the day-ahead market to prepare for the uncertainty that it foresees in the regulation market. This means that it will choose a dispatch that provides flexibility in the regulation market. In our example, this can be seen from difference in how the flexible thermal producer is dispatched in the myopic and stochastic model. Under stochastic market clearing the thermal producer is dispatched at a level where he can be used for both up- and down-regulation in the real-time market, while in the myopic model he is dispatched at almost full capacity in the day-ahead market.

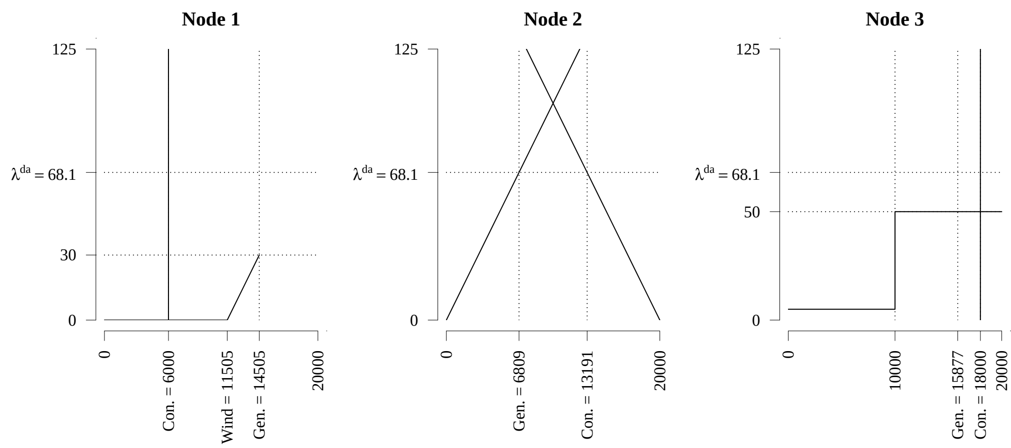

Figure 5: Day-ahead market schedule under stochastic market clearing.

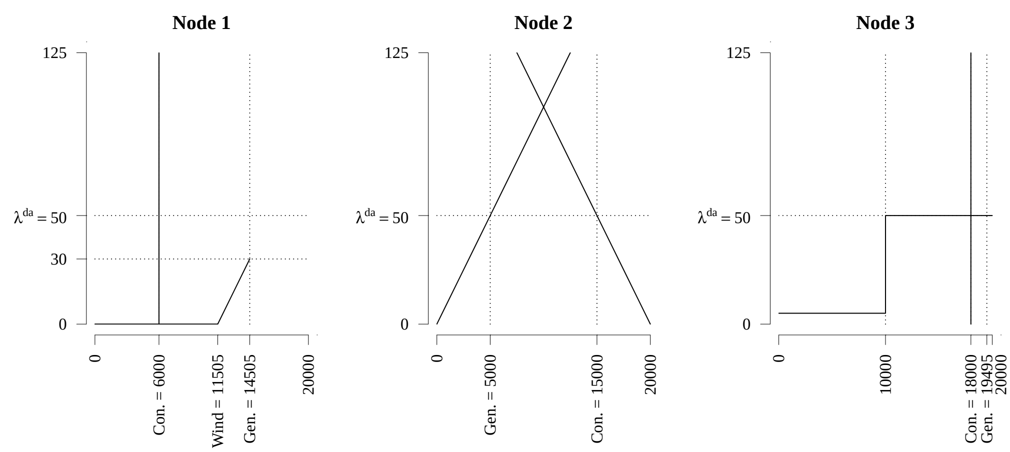

Figure 6: Day-ahead market schedule under standard (myopic) market clearing.

4.2.2. Real-time part

Based on the market clearing decided in the day-ahead market, there will be different needs, possibilities and costs associated with clearing the regulation market. Figure 7 shows the market clearing for the stochastic model in the three different scenarios, while Figure 8 shows the same for the myopic model. Starting with the stochastic model, we see that in Scenario 1, the reduction (from the day-ahead market dispatch) in wind production is offset by the increased production from all flexible producers that are not already dispatched at full capacity in the day-ahead market, as well as reduction of the flexible load in Node 2. In the myopic model the thermal producer is dispatched at almost full capacity in the day-ahead market and cannot be used to offset the lost wind production in Node 1. This must be compensated by larger increases in hydro generation in Node 2, as well as a larger decrease in the flexible load in Node 2. This effect is strongest in scenario 1 where the wind production is reduced drastically without the possibility to compensate fully with increased production from the thermal producer.

In Scenario 2, where the wind production is at the medium level (7000 MWh/h), the stochastic model adjusts the day-ahead market clearing by increasing the production of the thermal generator to full capacity, while the quantities for the other flexible entities are changed in the same direction, but with smaller amounts, as in Scenario 1. In the myopic model, almost the entire capacity of the thermal generator has been dispatched in the day-ahead market, so most of the reduction in production by the wind power plant must be compensated by the other flexible entities.

Finally, in Scenario 3 we see that the thermal producer has an unchanged production level in the regulation market in the stochastic model. In order to incorporate the high wind production (15000 MWh/h) in this scenario, the hydro power producer in Node 2 reduces his production and the load in Node 2 is increased. In the myopic model, the thermal producer is regulated down from 9495 MWh/h to 6788 MWh/h. In addition, the hydro power producer in Node 2 is regulated in the same manner as for the stochastic solution. The same is true for the load in Node 2.

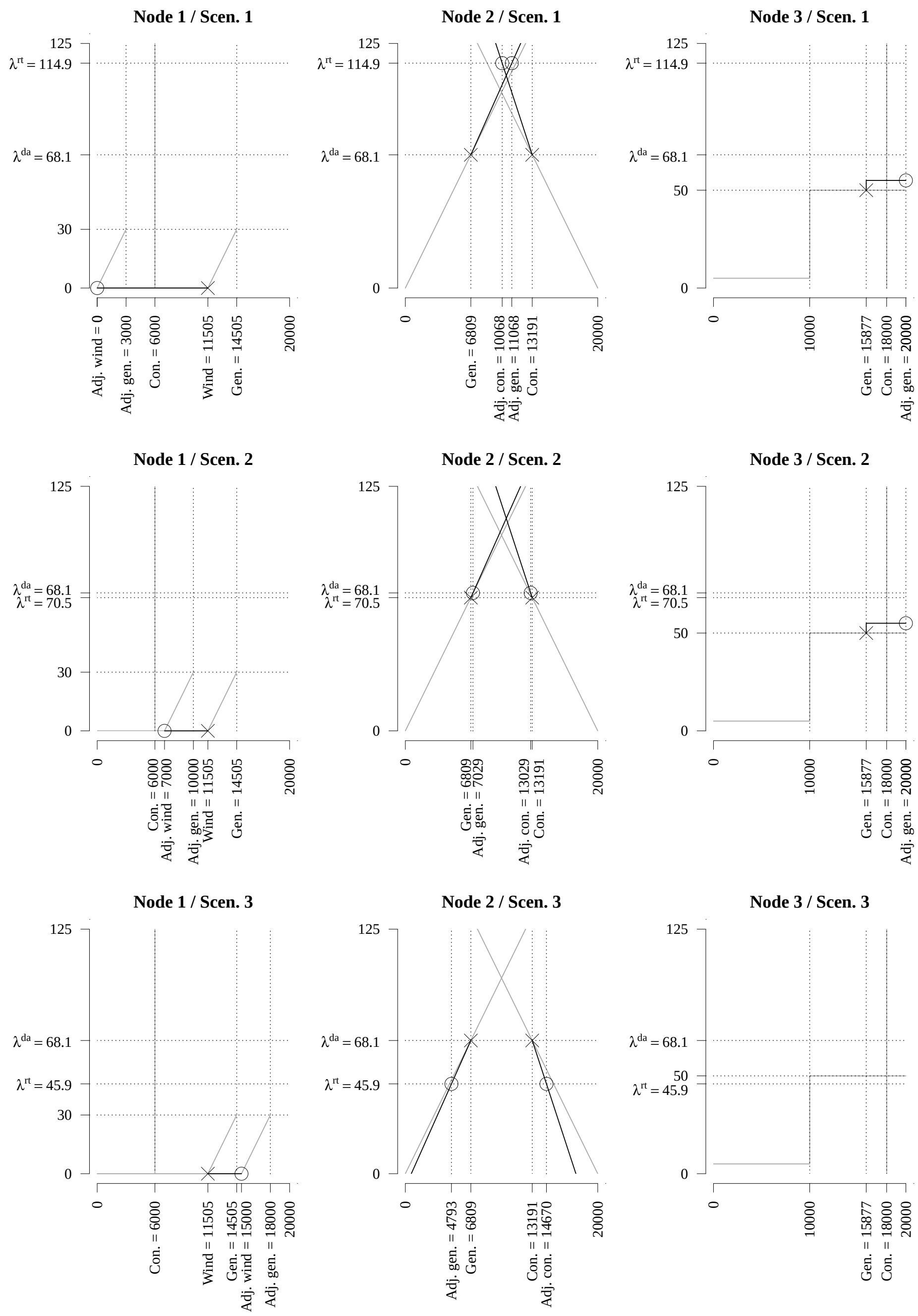

Figure 7: Adjusted (stage 2) schedule under stochastic market clearing. The crosses show the day-ahead dispatch while the circles show the dispatch in the real-time market.

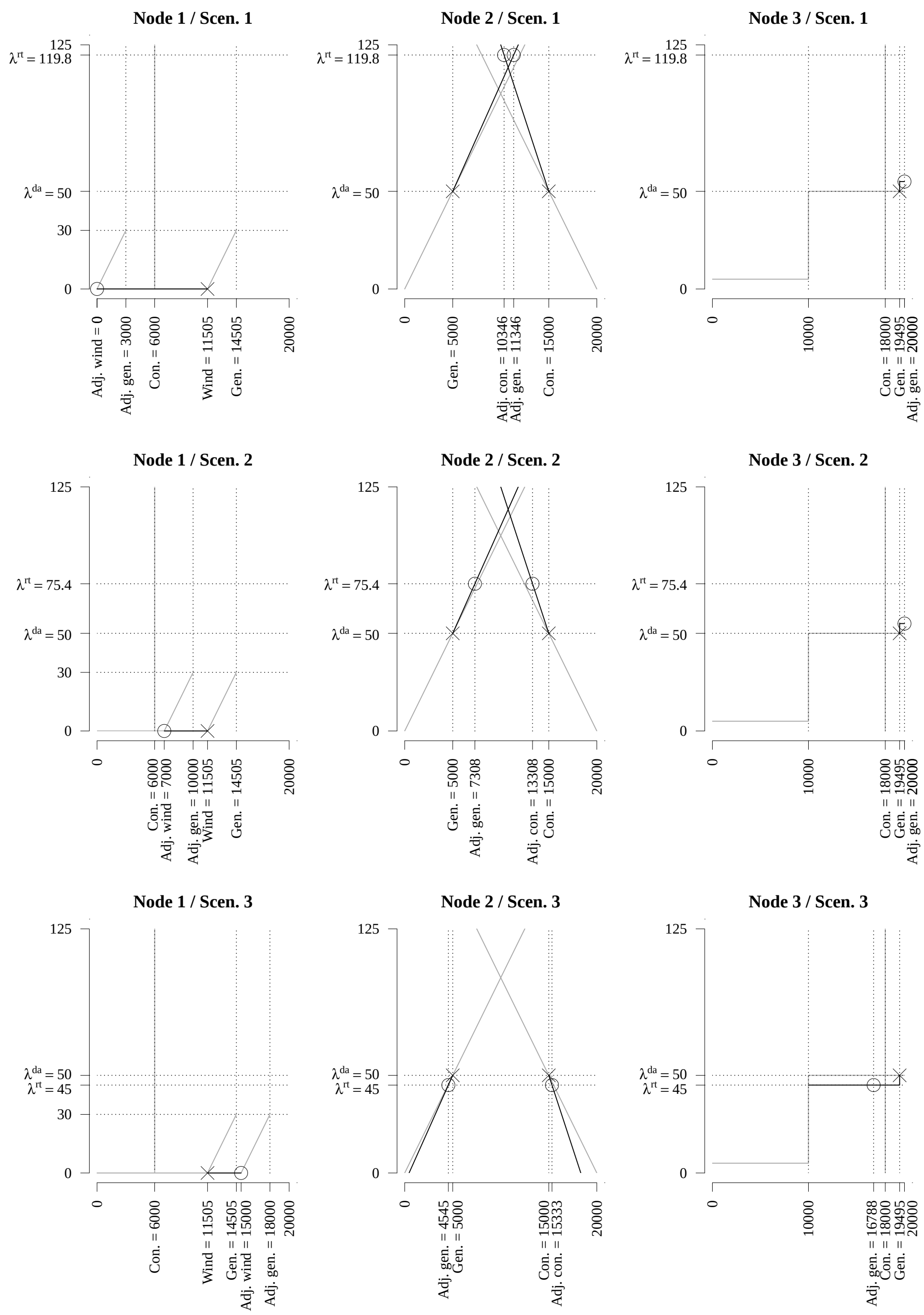

Figure 8: Adjusted (stage 2) schedule under standard (myopic) market clearing. The crosses show the day-ahead dispatch while the circles show the dispatch in the real-time market.

Tables 2 and 3 shows the results from the two models. As expected, we can see that the social surplus in the day-ahead market is higher in the myopic model (1180.3' €) than in the stochastic model (1147.5' €), while the total expected surplus is higher in the stochastic model (980.9' €) than in the myopic model (971.7' €). This difference in results is due to the lower flexibility costs in the regulation market in the stochastic model. The difference is illustrated by how the two model variants dispatch the thermal producer. In the myopic model the thermal producer is dispatched at almost full capacity in the day-ahead market, while in the stochastic model the thermal producer is dispatched at a much lower level (5877 MWh/h instead of 9495 MWh/h of a capacity of 10000 MWh/h). Due to the relatively low flexibility costs of the thermal producer, he can be used both for absorbing high levels of non-dispatchable production (in Scenario 3) as well as substituting for low production levels (in Scenario 1 and 2). In the myopic model, the thermal producer is dispatched almost at his production limit in the day-ahead market. This strongly limits his ability to up-regulate in the low-winds scenarios, and the myopic model must then rely on the more expensive hydro-power producers to compensate. The stochastic model however chooses to dispatch the thermal producer at a level where both up- and down-regulation can be used in the different scenarios.

Table 2: Summary of results for stochastic model. Surpluses in 1000 €s. Δx is used to indicate the difference between the real-time market and the day-ahead quantities, while $-\Delta c$ denotes the increase in the total surplus.

		Day-ahead			No wind (0.25)			Medium wind (0.2)			High wind (0.55)			
Entity	Node	λ	x	$-c$	λ	Δx	$-\Delta c$	λ	Δx	$-\Delta c$	λ	Δx	$-\Delta c$	$E[-c-\Delta c]$
Wind	1	68.1	11505	783.4	114.9	-11505	-1322.4	70.5	-4505	-317.7	45.9	3495	160.4	477.5
Hydro	1	68.1	3000	159.3										159.3
Hydro	2	68.1	6809	231.8	114.9	4259	99.8	70.5	221	0.3	45.9	-2016	22.4	269.1
Nucl.	3	68.1	10000	630.9										630.9
Therm.	3	68.1	5877	106.3	114.9	4123	247.1	70.5	4123	64.0				180.9
Load	1	68.1	6000	-408.5										-408.5
Load	2	68.1	13191	870.0	114.9	-3123	73.2	70.5	-162	0.2	45.9	1479	16.4	897.4
Load	3	68.1	18000	-1225.6										-1225.6
Grid				-0.0			-0.0			-0.0			0.0	-0.0
Total				1147.5			-902.3			-253.3			199.2	980.9

5. Market design implications

One way of achieving the day-ahead market outcome suggested by the stochastic dispatch is to assume a central planner that can dictate the production and consumption levels. Theoretically, rules could also be put in

Table 3: Summary of results for myopic model. Surpluses in 1000 €s.

		Day-ahead			No wind (0.25)			Medium wind (0.2)			High wind (0.55)			
Entity	Node	λ	x	$-c$	λ	Δx	$-\Delta c$	λ	Δx	$-\Delta c$	λ	Δx	$-\Delta c$	$E[-c-\Delta c]$
Wind	1	50.0	11505	575.3	119.8	-11505	-1378.4	75.4	-4505	-339.6	45.0	3495	157.3	249.2
Hydro	1	50.0	3000	105.0										105.0
Hydro	2	50.0	5000	125.0	119.8	6346	221.5	75.4	2308	29.3	45.0	-455	1.1	186.9
Nucl.	3	50.0	10000	450.0										450.0
Therm.	3	50.0	9495	-0.0	119.8	505	32.7	75.4	505	10.3	45.0	-2707	0.0	10.2
Load	1	50.0	6000	-300.0										-300.0
Load	2	50.0	15000	1125.0	119.8	-4654	162.4	75.4	-1692	21.5	45.0	333	0.8	1170.4
Load	3	50.0	18000	-900.0										-900.0
Grid				0.0			-0.0			-0.0			-0.0	0.0
Total				1180.3			-961.7			-278.6			159.2	971.7

place that would distribute the surplus and risk in the network between the participants. There are, however, several challenges with using the central planner approach in the network. One major challenge is the question of individual rationality: How would the participants in the markets behave given that they know the central planner will use a stochastic dispatch when clearing the markets? In Section 5.1 we will discuss how the wind producer may have incentives to strategically alter their behavior when they know that a stochastic dispatch solution will be used to clear the markets. Another main question related to the stochastic dispatch is where the information regarding uncertainty is coming from. When studying the issue with a stochastic model from a system perspective, we include the distribution of possible wind production for the producer in our problem. Hence, we implicitly assume that the wind producer truthfully reports his distribution to the system operator, who will use this information to determine the optimal bid in the day-ahead market. Is this assumption valid, or will the wind producer have an incentive to deviate from the true distribution? The distributions that are used to represent the uncertain demand and supply will highly influence the stochastic dispatch, and as such, both the surplus and risk for each of the market participants. In Section 5.2 we will investigate how varying assumptions regarding the probability distribution for production from the wind producer in Node 1 can influence the results in the markets.

5.1. Rationality in bidding behavior

A market design for the electricity market based on a stochastic model face several challenges. One of these is the individual rationality of the participants in the market. When a system perspective is used to find the optimal production decisions in the network, the resulting prices may lead to

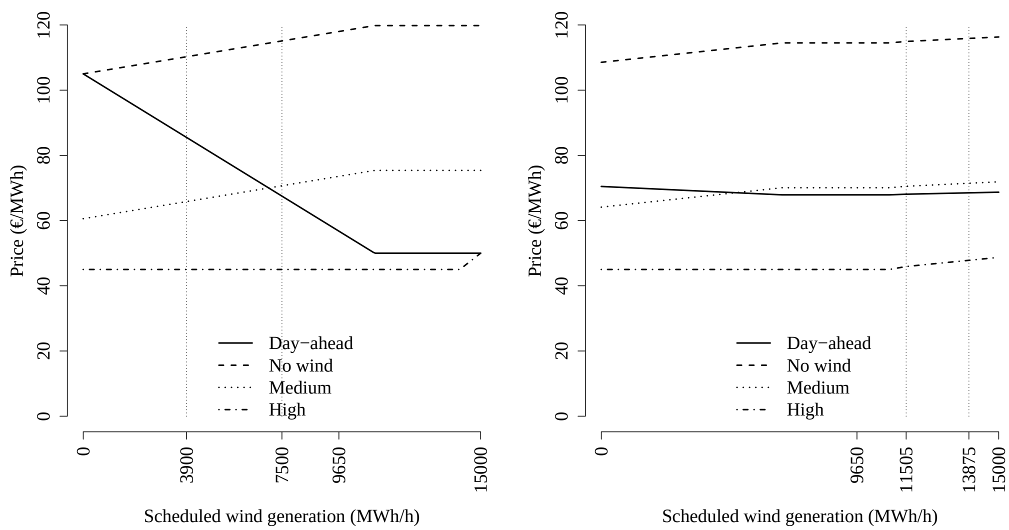

Figure 9: Effect on prices of varying the day-ahead market quantity for the wind generator in myopic (left) and stochastic (right) model.

small (or even negative) profits for the participants in the day-ahead market. Although Pritchard et al. (2010) show that the participants in a market using stochastic dispatch will have revenue adequacy on expectation, the participants may not have a positive revenue in all scenarios. In a recent paper by Morales et al. (2014), a small example that illustrates the impact of a stochastic model on a flexibility provider in an electricity network that uses a stochastic model for market clearing is provided. In the example, the flexibility provider ends up with a very small expected profit and with a large probability of negative profits. It is natural to then question how this producer would behave given this market design. When the producer knows that a stochastic dispatch will be used, would he change his bidding curves?

In the following, we will investigate these issues by studying a simple example. We will use an MPEC model where the upper level is the determination of the day-ahead market quantity for the wind producer, while the lower level problem is the stochastic dispatch model used by the system operator. We solve the MPEC by using enumeration over a discrete number of possible production quantities for the wind producer. This solution procedure allows us to draw the profit functions for each market participant as a function of the wind quantity included in the day-ahead market clearing.

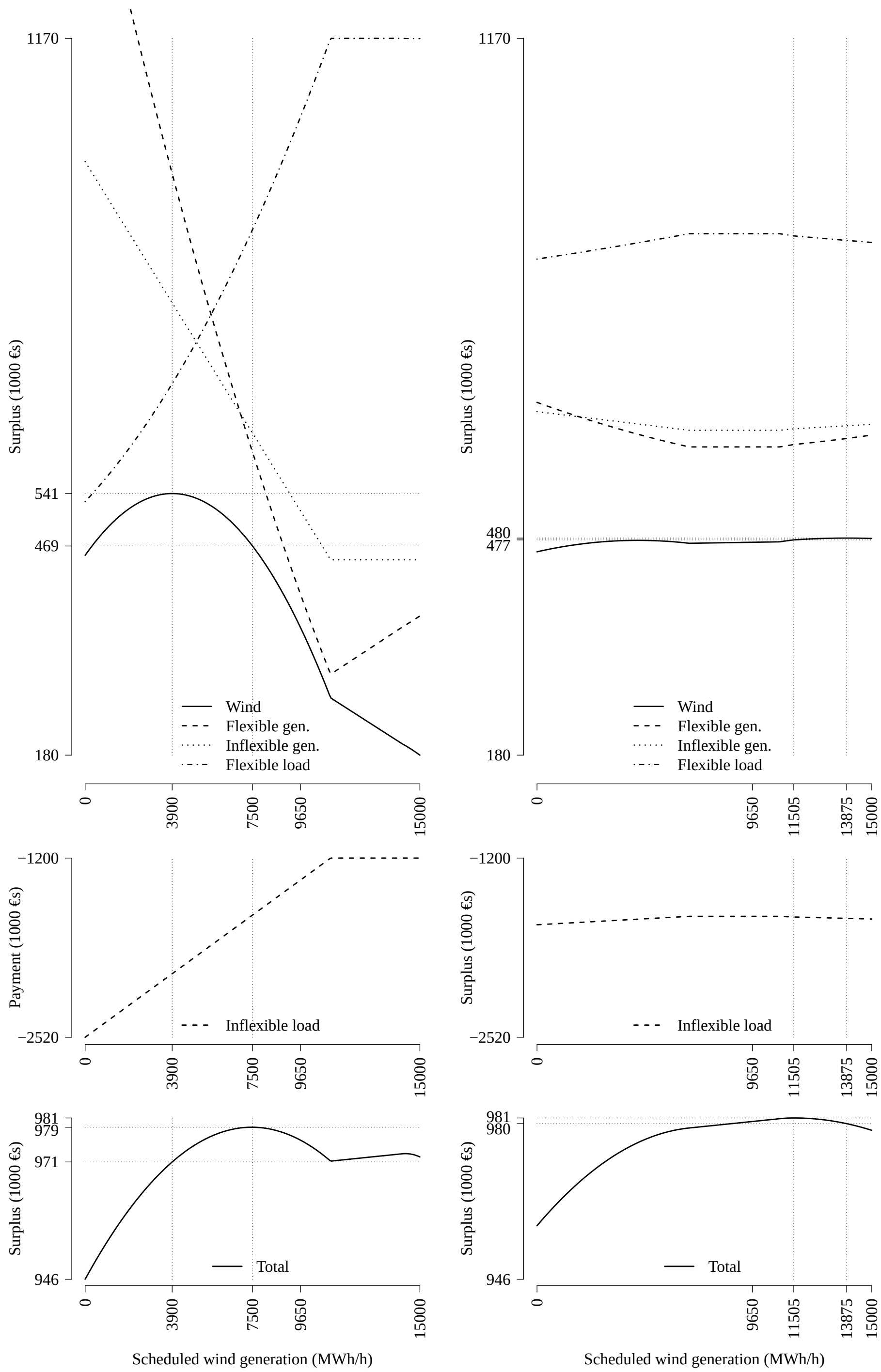

Figure 10: Effect on surpluses of varying the quantity of wind included in the day-ahead market clearing in myopic (left) and stochastic (right) model.

This approach is different from the one used in Morales et al. (2014) where the optimal day-ahead bid conditional on the system optimum is found. In our example, we focus on the individual rationality of the wind producer, not on the system as a whole.

The results from our analysis for both the myopic model and the stochastic model is shown in Figures 9 and 10. As we can see from both figures, the wind producer will optimize his profits with a day-ahead market quantity that is different from the system optimum. The system optimum is achieved with a production quantity of 7500 MWh/h in the myopic model and 11505 MWh/h in the stochastic model. The system optimal expected surplus is, as expected, higher with stochastic market clearing (981' €) than with myopic market clearing (979' €). If we look at the surplus for the wind producer however, we see that in the case of the myopic model the wind producer would benefit from lowering the production quantity to 3900 MWh/h, whilst in the stochastic model he would benefit from increasing the quantity to 13875 MWh/h. This means that the incentives of the wind producer and the system are not aligned, and that the wind producer would like to deviate from the day-ahead market dispatch that has been chosen by the TSO. The figures also clearly illustrates a difference between the stochastic dispatch and the myopic model in that the consequences of deviating from the system optimal wind quantities in the day-ahead market is smaller for the stochastic dispatch model. This is as expected given that the stochastic dispatch model will have more information available when the dispatch in the day-ahead market is decided upon. The stochastic model will realize the consequences of the changed wind production on the flexibility costs in the regulation market and adjust accordingly. The myopic model on the other hand, will not consider the possibility of actual wind production deviating from the quantity cleared in the day-ahead market.

5.2. Information requirements

An important assumption for the stochastic dispatch model (and for stochastic programming models in general) is the assumption of a known, joint probability distribution. Such a distribution must be assumed available to the decision maker, either from objectively available sources or based on subjective analysis. Given that the stochastic dispatch will depend on this joint probability distribution, it is very important for the market and the market participants that this probability distribution is determined in a fair manner. The credibility of the market will depend directly on the

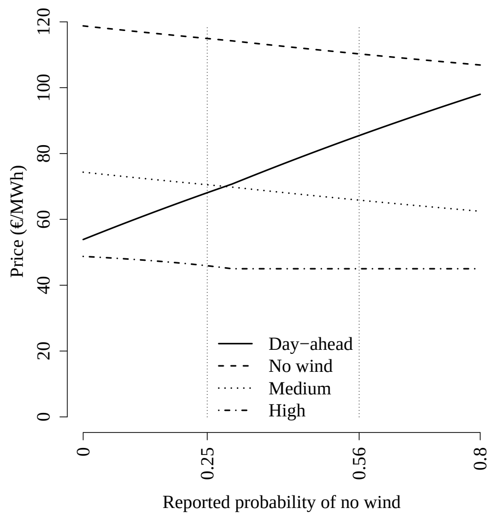

Figure 11: Effect on prices of varying the reported probabilities.

credibility of the process for estimating the probability distribution. If the probability distribution is not correct it will impact the distribution of surplus between the participants, the proofs of revenue adequacy in the system (Pritchard et al. (2010)) will no longer hold, and the risk for the participants in the market will increase. It is then vital to determine where this probability distribution will come from, who will estimate it and how it should be represented in the stochastic dispatch model.

In the following we assume that the wind producer reports his estimation of the probability distribution for wind production, and we examine whether or not he will have an incentive to misrepresent this probability distribution. We will perform this analysis by varying the probabilities for the extreme scenarios ("No wind" and "High wind"). The probability of the middle scenario (wind production of 7000 MWh/h) will be held constant at 0.2, while the probabilities of "No wind" and "High wind" will be varied between 0 and 0.8 (a decrease in one of the probabilities will be offset by an identical increase in the other). After the stochastic dispatch is solved with this new probability distribution, we will then recalculate expected surpluses with the original probability distribution which we assume is the correct one.

The results, which are illustrated in Figures 11 and 12 show that the wind producer will have an incentive to misreport the probability distribution for

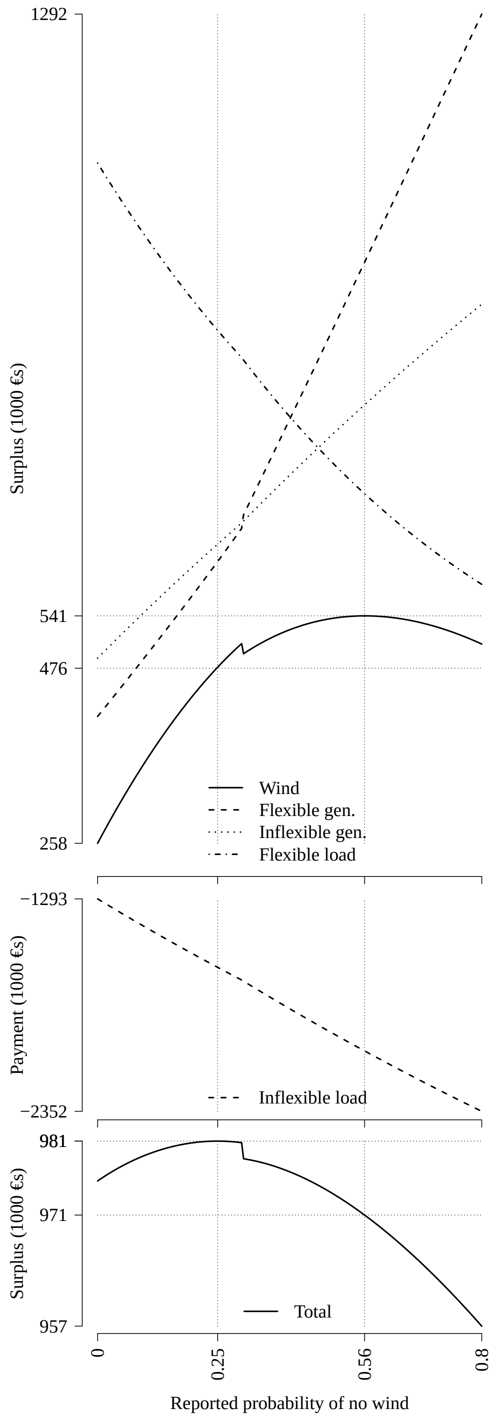

Figure 12: Effect on surpluses of varying the reported probabilities.

the wind production. In our example, he will have an incentive to report a lower probability of high wind production. This way, his expected profits will increase while his risk exposure (in terms of uncertainty of profits) will decrease. For other parameter sets we have seen different results, but the problem of the wind producer not being incentive compatible with the total system will be valid for most parameter sets (it will naturally, theoretically, be possible to construct a parameter set such that the results of the wind producers are aligned with the total system). In our example, the wind producer will alter the probability of the "No wind" scenario from 0.25 to 0.56, and the probability of the "High wind" scenario from 0.55 to 0.24. This example illustrates both the impact on the surplus of the different participants in the system of a misrepresented probability distribution for wind production, as well as the potential for gaming with this information for the participants. Given that there is no "true" probability distribution for wind production ahead in time, such estimations will always be subject for discussions. This poses a serious challenge for designing market mechanisms where both the distribution of surplus between the participants in the system, as well as the risk exposure of the participants must be handled in a fair and efficient manner.

6. Conclusions

We have presented a discussion and analysis of a stochastic dispatch model for electricity networks with two sequential markets. The examples illustrate both the potential gains from using a stochastic dispatch as well as some of the challenges. Generally, if the representation of the underlying uncertainty is correct, the stochastic dispatch model will always give a higher expected surplus in the network than the conventional myopic market model, as illustrated by our example. Under both types of market clearing, however, there is a discrepancy between what is optimal for the system, with respect to scheduled wind power production, and what is optimal for the wind power producer. The consequences of this discrepancy, in terms of reduction in social surplus, is more severe in the myopic model, than with stochastic market clearing. We have also demonstrated that the wind power producer may exercise market power by manipulating the information about the probability distribution of wind, if he is responsible for supplying this information.

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