Discussion paper

Congestion Management in a Stochastic Dispatch Model for Electricity Markets

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Abstract

We consider an electricity market organized with two settlements: one for a pre-delivery (day-ahead) market and one for real time, where uncertainty regarding production from non-dispatchable energy sources as well as variable load is resolved in the latter stage. We formulate two models to study the efficiency of this market design. In the myopic model, the day-ahead market is cleared independently of the real-time market, while in the integrated stochastic dispatch model the possible outcomes of the real-time market clearing are considered when the day-ahead market is cleared. We focus on how changes in the design of the electricity market influence the efficiency of the dispatch, measured by expected total cost or social welfare. In particular, we examine how relaxing network flow constraints and, for the stochastic dispatch model, even the balancing constraints in the day-ahead part of the dispatch models affects the overall efficiency of the system. This allows the dispatch to be infeasible day-ahead, while these infeasibilities will be handled in the real-time market. For the stochastic dispatch model we find that relaxing the network flows and balancing constraints in the day-ahead part of the market provides additional flexibility that can be valuable to the system. In our examples with high up-regulation cost we find a value of "overbooking" that lead to lower total costs. In the myopic model the results are more ambiguous, however, leaving too many constraints to be resolved in the real-time market only, can lead to infeasibilities or high regulation cost.

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1. Introduction

During the last decade many power systems around the world, have seen large changes in the generation mix, with a move towards renewable power sources, like wind, solar and small scale hydro power. Common to these generation sources are that the availability in real time is highly uncertain until close to real-time operation. On the other hand, some conventional generation, like nuclear and base load thermal power, need to be planned in good time before delivery in order to ensure minimum operating cost. An important question thus is how to design markets to benefit from the early planning of conventional sources, while at the same time dealing with the uncertainty of the renewables.

Electricity systems must balance supply and demand at every instance in time, and in doing so, keeping within system limits. In practice, electricity markets are often organized in sequential markets, from long term contract markets between generators and consumer representatives to real-time arrangements, where system operators deal with instantaneous frequency control. Some of the real-time tools of the system operators are market-based, others are part of the regulated system operation, and the costs are socialized through network tariffs or similar.

Organized trade of physical electricity is often accomplished by at least a day-ahead market, and a real-time market, which is cleared close to the delivery hour. Often, this is supplemented by intraday markets, taking place between day-ahead and real time, where market agents can reposition their obligations. An example from the Nordic market operated by Nord Pool Spot is given in Figure 1.

The day-ahead market, Elspot, is cleared 12-36 hours before the delivery hour. Elspot balances supply and demand bids by setting simplified locational marginal prices, based on a zonal pricing or market splitting approach, implying that actual network constraints are only partially taken into account in the day-ahead market dispatch by aggregate transfer capacities between relatively large geographical regions (for a more detailed description see for instance Bjørndal et al. (2014)). Figure 2 shows the price regions of the Nordic day-ahead market.
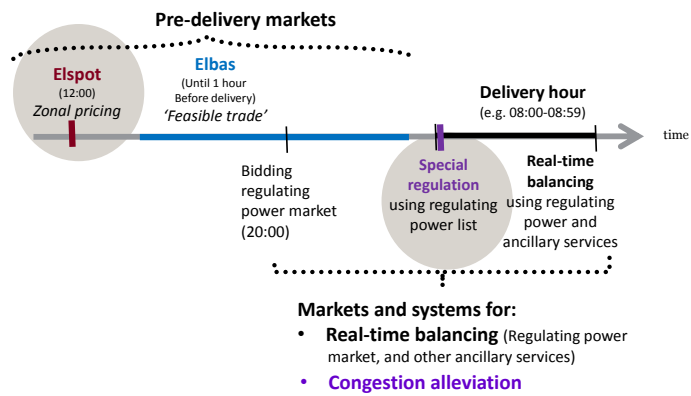
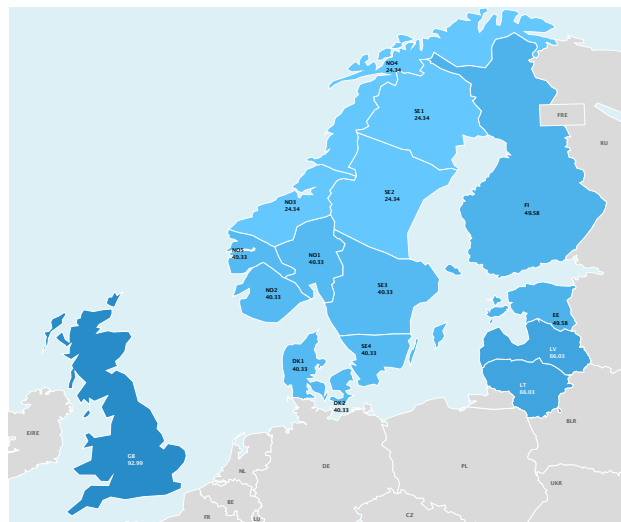Figure 1: Illustration of the Nordic power market.

Figure 2: Illustration of the price regions of the Nordic day-ahead market. Source: Nord-Pool Spot.

After Elspot is cleared, the intraday market, Elbas, opens for continuous trading of "Feasible flows", i.e. transfers that do not violate the aggregate transfer capacities between regions. Elbas remains open for trading until one hour before delivery. At the same time, on the evening the day before delivery, generators and large consumers provide bids for up- and down-regulation to the regulation power market as well as other ancillary services markets operated by the system operators. These bids are used for special regulation, to alleviate congestion that remains after the day-ahead market clearing, and for real-time balancing. As is clearly illustrated from the Nordic example, the constraints that are taken into account in the different sequential markets may differ. In the Nordic and European power markets, only a few aggregate transfer limits between large regions are part of the day-ahead price process, while all relevant constraints must be complied with in real time. In contrast, in many US power markets, nodal pricing is used both for real-time and for day-ahead markets, representing much more detailed constraints also at the day-ahead stage of the market.

With more uncertainty due to renewables, more emphasis has been put on intraday and real-time markets. Holttinen (2005) discusses for instance the value for wind power generators in the Nordic power market to bid closer to real time in order to avoid regulation costs. Weber (2010) considers how the intraday markets in Europe can be adapted to account for the integration of large amounts of renewable generation in the years to come, whether the non-dispatchable renewables are balancing responsible or not. Mauritzen (2015) discusses further the interaction of subsidies for renewables and intraday markets, with data from Danish wind power production and the Elbas intraday market. In a European context, day-ahead markets are to a large extent integrated, however, intraday and real-time markets (or other short-term arrangements) are still much more nationally oriented.

Fabbri et al. (2005) illustrate how the forecasting errors of wind power may be reduced closer to the delivery hour. On the other hand, for conventional generators, there may be a cost connected to resetting plans close to the delivery hour. This may be due to a requirement to operate on a non-optimal scale, and the need for using more expensive units. NETL (2012) gives an overview over constraints that may be challenged and extra costs that may be incurred due to short-term deviations from initial plans in conventional power generation. Thus, there may be a trade-off between delaying dispatch until the uncertainty regarding production from non-dispatchable sources and variable load is reduced, and the increased flexibility costs con-
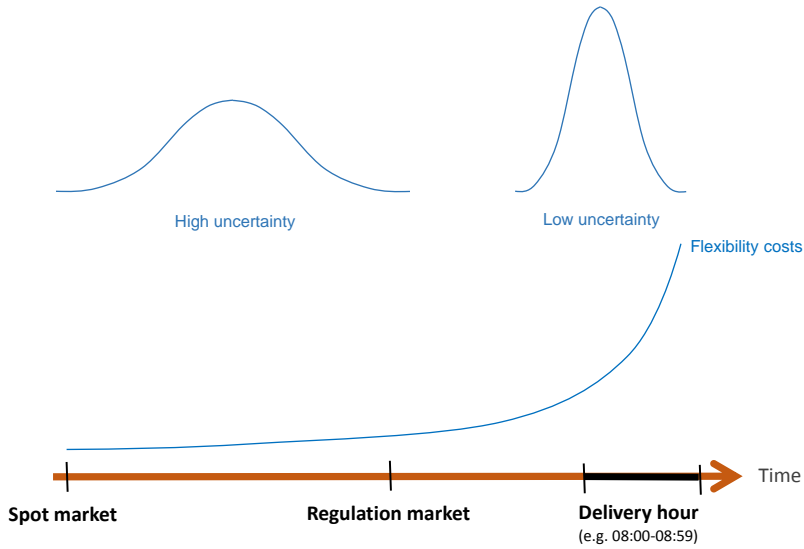
Figure 3: Illustration of the relation between uncertainty and flexibility costs.

nected to changes in production and consumption with short notice. This trade-off is illustrated in Figure 3, where early market clearing leads to planning of cheap base load under high uncertainty, while late clearing has little uncertainty but higher production costs.

In general, the increased share of renewables may lead to revisions of market clearing procedures and the timing of different sub-markets. It may however also require a more fundamental rethinking of the market clearing algorithms used, for instance if it makes sense to take explicitly into account uncertainty at a later stage when clearing day-ahead or other pre-delivery markets. This may be accomplished by using market clearing models based on stochastic programming.

Numerous authors have developed stochastic market clearing models and showed that they yield better plans, in terms of expected social surplus, than deterministic market clearing models. Examples include Bouffard et al. (2005*a*,*b*); Bouffard and Galiana (2008); Ruiz, Philbrick, Zak, Cheung and Sauer (2009); Ruiz, Philbrick and Sauer (2009); Papavasileiou et al. (2011); Papavasileiou and Oren (2012); Khazaei et al. (2014). Pricing issues are discussed by Kaye et al. (1990); Wong and Fuller (2007); Pritchard et al. (2010); Morales et al. (2012, 2014); Zavala et al. (2015). Pritchard et al.

(2010) proposes a stochastic market clearing model for a system where load and/or generation may be uncertain. They prove that their pricing scheme is revenue-adequate in expectation. Morales et al. (2014), focusing on a system with uncertain intermittent power generation, handles uncertainty by proposing an improved version of the conventional deterministic market clearing model, in which the system operator controls the intermittent generator's bid in the day-ahead market in order to optimize the system as a whole. The procedure is solved using a bi-level optimization model, and yields an expected social surplus that is smaller or equal to the surplus under stochastic market clearing. Bjørndal et al. (2016) discuss functional organization and informational requirements related to implementation of stochastic market clearing.

In this paper, we consider a sequential energy-only electricity market, consisting of a day-ahead and a real-time market. We focus on the interaction of the two when production is uncertain, and when congestion management methods differ between the two sub-markets. We study the effects of using a stochastic dispatch model, following Pritchard et al. (2010), and in particular, how relaxing network and/or energy balance constraints in the day-ahead clearing affects the overall expected costs in the electricity market. We use both the stochastic and a myopic or sequential market clearing model similar to Morales et al. (2014) to provide further insights into the effects of relaxing day-ahead network constraints. Pricing issues are discussed in a companion paper (Pritchard et al., 2016).

The rest of the paper is organized as follows. In Section 2, a mathematical formulation is presented, before a discussion of our model setup and different dispatch models is provided in Section 3. We then present two numerical examples in Section 4 before conclusions are drawn in Section 5.

2. Mathematical model

2.1. Generation and load

Our model framework is similar to that of Pritchard et al. (2010). We consider a collection of offers $i \in I$, where each offer can represent either generation (positive values) or load (negative values). For each $i \in I$ we require a solution (x_i, X_i), where x_i is the solution for the first-stage dispatch, and X_i is a vector of stochastic variables representing the solution for the second-stage dispatch. The first-stage dispatch corresponds to the market clearing in the day-ahead market, while the second-stage dispatch is the

results from the real-time market clearing. The set of feasible solutions for the first stage is denoted C_i^1, while the set of feasible solutions for the second stage will depend on the realized scenario $\omega \in \Omega$ as well as the decision x_i from the first stage. We denote this set as $C_i^2(\omega, x_i)$. A feasible solution (x_i, X_i) to both stages must satisfy

$$x_i \in C_i^1 \qquad\qquad \forall i \in I$$
$$X_{i\omega} \in C_i^2(\omega, x_i) \qquad\qquad \forall i \in I, \ \omega \in \Omega.$$

When considering different dispatch models, we take on a system perspective, i.e. as if the dispatches were performed centrally in an energy only mandatory dispatch. We do not consider unit commitment, intertemporal constraints (water values are assumed to be the same in all models), other types of ramping constraints, etc. These may be represented indirectly by the flexibility costs, however they are not considered explicitly. We also assume that all possible outcomes are modelled by our scenarios (which is clearly unrealistic), and do not consider out-of-sample effects of the day-ahead market clearing. When discussing up- and down-regulation we will use the convention from the Norwegian market. Up-regulation then refers to a change in production or consumption that increases the net supply situation in the system. Down-regulation, on the other hand, decreases the net supply situation in the system (i.e. generation is decreased and / or consumption is increased).

Our focus is on deviations from the day-ahead scheduling, and the cost and benefit curves of flexible producers and consumers are modelled. That is, the regulation costs refer to the costs of changing production and / or consumption in the real-time market. If the consumers increase the quantity consumed in real time, it is not as valuable as if it was planned in the day-ahead market. If they reduce it, they would ask for more than the day-ahead willingness to pay. If the generators must increase their production beyond the planned level, it is more costly than the day-ahead marginal cost, and if they reduce production from the planned level, they will not save all day-ahead marginal cost. That is, the flexibility costs modelled are a representation of real costs incurred by the participants in the market.

2.2. Objective function

The objective function for our models is minimization of total costs in the system. This includes the sum of costs from the day-ahead market and

the regulation costs incurred in the real-time market. Consumption benefit is represented as negative cost. An illustration of the components in the objective function is provided in Figure 4. The figure on the left illustrates a supply function for a generator, while the figure on the right illustrates a demand function for a consumer. In addition, the two figures illustrate the flexibility costs incurred in the real-time market when there is a deviation from the day-ahead market clearing. The day-ahead clearing is given by volume x_i, whilst examples of up- and down-regulation volumes are given by $X^d_{i\omega_1}$, $X^u_{i\omega_1}$, $X^d_{i\omega_2}$ and $X^u_{i\omega_2}$.

We use linear functions to represent the cost and benefit functions for the participants in the market. Each offer $i \in I$ is associated with a day-ahead cost and benefit function with non-negative parameters a_i and b_i, given by

$$c_i(x_i) = a_i x_i + 0.5 b_i x_i^2.$$

For the supply side, this cost function is based on an assumption of a linear marginal cost function: $a_i + b_i x_i$. The second stage cost and benefit function parameters will typically differ from those in the first stage, due to reduced flexibility at this stage. We assume that this can be represented, for any flexible generator, with parameters a^u_i and b^u_i for up-regulation and a^d_i and b^d_i for down-regulation, where $a^d_i \leq a_i \leq a^u_i$ and $\min\{b^u_i, b^d_i\} \geq b_i$ (refer the supply function illustrated in the left-hand diagram in Figure 4).

To represent the demand side, and keep the formulation compact, we use $x_i < 0$ to represent consumed quantities. The inverse linear demand curve is given as $a_i + b_i x_i$. Since x_i will take negative values, this corresponds to a downward sloping demand curve. For both generators and consumers, the slopes of the cost and benefit functions for changes in dispatch in the real-time market are steeper than the corresponding functions in the day-ahead market. Similarly as for the supply side, we assume that any flexible consumer can be represented with parameters a^u_i and b^u_i for up-regulation and a^d_i and b^d_i for down-regulation, where $a^d_i \leq a_i \leq a^u_i$ and $\min\{b^u_i, b^d_i\} \geq b_i$ (refer the demand function illustrated in the right-hand diagram in Figure 4).

With reference to Figure 4 we can formulate the total cost after the second-stage regulation as:

$$c_i(X_{i\omega}) + \tilde{c}_i(x_i, X_{i\omega}),$$

where $c_i(X_{i\omega})$ is the total cost of the final schedule evaluated at the day-ahead
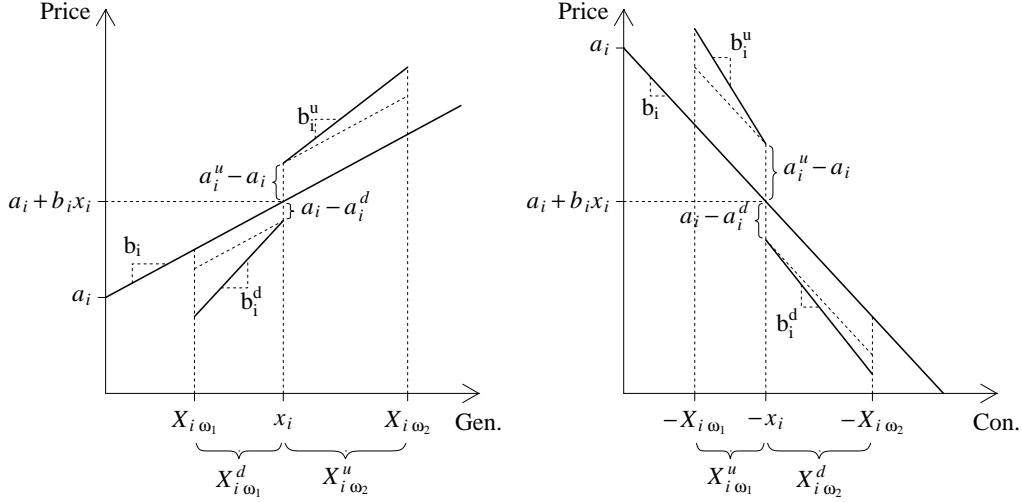
Figure 4: Illustration of the cost and benfit functions used in our models. The figure on the left illustrates a supply function for a generator, while the figure on the right illustrates a demand function for a consumer. In addition, the two figures illustrates the flexibility costs incurred in the real-time market when there is a deviation from the day-ahead market clearing.

cost parameters, and $\tilde{c}_i(X_{i\omega}, x_i)$ is the additional cost caused by inflexibility in the real-time market. The flexibility cost associated with the first-stage quantity x_i and the revised quantity $X_{i\omega}$ in scenario ω is

$$\tilde{c}_i(x_i, X_{i\omega}) = (a_i^u - a_i)X_{i\omega}^u + 0.5(b_i^u - b_i)(X_{i\omega}^u)^2 + (a_i - a_i^d)X_{i\omega}^d + 0.5(b_i^d - b_i)(X_{i\omega}^d)^2,$$

where $X_{i\omega}^u = \max\{X_{i\omega} - x_i, 0\}$ and $X_{i\omega}^d = \max\{x_i - X_{i\omega}, 0\}$.

This formulation allows for many different assumptions about cost and benefit curves for consumers and generators, both day-ahead and real-time. Figure 5 shows three examples of how the initial schedules may be adjusted, as well as the effect on cost and benefit. The leftmost diagram illustrates an example where $a_i = 0$ and $b_i > 0$, i.e., a generator with an increasing marginal cost starting from zero. The day-ahead schedule is x_i, and in scenario ω this quantity is up-regulated to $X_{i\omega}$. The slope of the up-regulation cost curve is given by the parameter $b_i^u > b_i$. The area of the light gray triangle equals $c_i(X_{i\omega}) = 0.5b_i(X_{i\omega})^2$, i.e., the cost of the final schedule given by the day-ahead cost function, and the area of the dark gray triangle equals the flexibility cost $\tilde{c}_i(x_i, X_{i\omega}) = 0.5(b_i^u - b_i)(X_{i\omega}^u)^2$. The middle diagram illustrates a generator with a constant day-ahead marginal cost
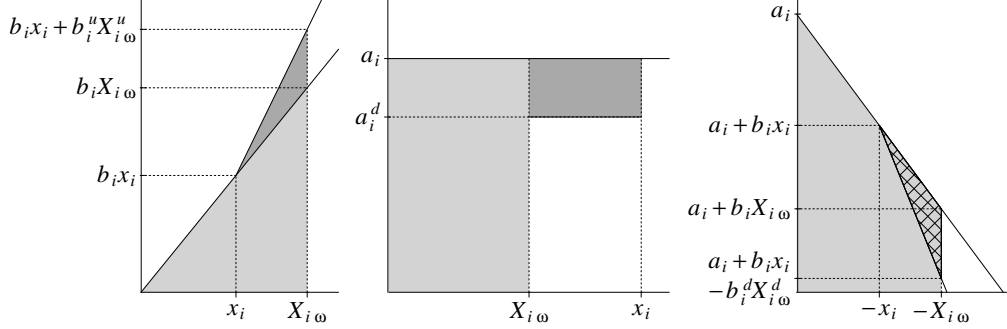
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Figure 5: Calculation of cost and benefit for suppliers and consumers. The two figures to the left shows how the flexibility cost are for two suppliers with different supply functions. The light grey area illustrates the cost of the final dispatch with the original cost function (not including flexibility costs), while the dark grey area is the flexibility cost due to up- or down-regulation. The figure to the right illustrates the same for the consumers. The light grey area is the consumer benefit with the original demand parameters, while the chequered area shows the loss in consumer surplus due to flexibility costs.

a_i, and a marginal cost $a_i^d < a_i$ for down-regulation. The total cost after down-regulation is $a_i X_{i\omega} + (a_i - a_i^d) X_{i\omega}^d$, where the last part $(a_i - a_i^d) X_{i\omega}^d$ is the non-avoidable cost that remains after the initial scheduled quantity has been reduced by $X_{i\omega}^d$. The rightmost diagram illustrates a consumer with a first-stage demand function with intercept and slope parameters equal to a_i and b_i. Consumption quantities are negative, so the second-stage increase in consumption is equivalent to down-regulation. Again, the light gray area represents the benefit of the final schedule evaluated at the day-ahead parameters, i.e., equal to $-(a_i X_{i\omega} + 0.5 b_i (X_{i\omega})^2)$, and the cross-hatched triangle equals the flexibility cost $\tilde{c}_i(x_i, X_{i\omega}) = 0.5(b_i^d - b_i)(X_{i\omega}^d)^2$.

2.3. Network flow equations

The generator and load entities are linked to a set of nodes N. For a particular offer $i \in I$ we denote by $\nu(i) \in N$ the node where generator / consumer i is located. We then consider the network as a directed graph where the nodes are connected by a set of transmission lines L. For a given flow vector $f = (f_l)_{l \in L}$, we let $\tau_n(f)$ denote the net inflow of power in node n from the transmission network. We define $\nu_0(l)$ as the starting point and $\nu_1(l)$ as the end point of line l, and $f_l > 0$ implies that power is flowing from $\nu_0(l)$ to $\nu_1(l)$. We assume, as in Pritchard et al. (2010), that lines are lossless, and this implies that:

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$$\tau_n(f) = \sum_{l:\nu_1(l)=n} f_l - \sum_{l:\nu_0(l)=n} f_l. \tag{1}$$

See Pritchard et al. (2010) for a discussion of how the network model can be generalized to incorporate line losses. We will associate the day-ahead schedule x with a flow vector f. The production and consumption quantities given by x must be consistent with the flow f, and in a lossless system this implies that

$$\tau_n(f) + \sum_{i \in I(n)} x_i = 0 \qquad \forall n \in N. \tag{2}$$

Similarly we associate the final schedule X_ω with the flow vector F_ω, and consistency implies that

$$\tau_n(F_\omega) + \sum_{i \in I(n)} X_{i\omega} = 0, \qquad \forall n \in N. \tag{3}$$

The energy balance in the network is guaranteed by Equations (2) and (3). Additional network constraints for the first and second stage are given by:

$$f \in U^1 \tag{4}$$
$$F_\omega \in U^2 \qquad \forall \omega \in \Omega \tag{5}$$

The sets U^1 and U^2 can represent capacity constraints for individual lines, loop flow constraints, or other relevant network constraints. Note that we may have $U^1 \neq U^2$, since the representation of the network can differ in the day-ahead and real-time stages.

We assume, throughout the paper, that U^2 represents the network constraints in a DC load flow model without losses. Then the flow vector $g \in U^2$ is equivalent to

$$g_l = y_l \left(\theta_{\nu_0(l)} - \theta_{\nu_1(l)} \right) \qquad \forall l \in L \tag{6a}$$
$$\theta_1 = 0 \tag{6b}$$
$$- cap_l \leq g_l \leq cap_l \qquad \forall l \in L, \tag{6c}$$

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where (6a) relates the flow, g_l, over line l to the voltage angle difference between the end nodes of the line, and where y_l is a parameter that represents the electrical characteristics of line l in the approximate DC representation of the network (for instance the admittance). Constraint (6b) sets one of the voltage angles equal to zero in order to obtain a unique solution. The inequalities in (6c) represents the thermal constraints on the line flow. One could also add further constraints to the description of U^2, such as the security constraints that are discussed in, e.g., Bjørndal et al. (2014).

3. Dispatch models

3.1. Market clearing

We consider a situation where the electricity market consists of a planned or day-ahead market and a real-time market at or very close to delivery. At the day-ahead stage some load and/or generation levels in real time are uncertain. In real time all uncertainty is resolved. In the following, we present two different dispatch models, termed *stochastic* and *myopic*, where the connection between the two markets is handled differently. In the stochastic market clearing model, the first stage is solved taking into account the uncertainty in the second stage and the connection between the costs and benefits in the two stages. In the myopic market model, however, the day-ahead market is cleared based only on given bids, not taking into account neither the uncertainty nor the bids in the real-time market.

The *myopic* model, corresponding to the conventional dispatch model in (Morales et al., 2014), solves the following problem in the day-ahead market (first stage):

$$\min_{x,f} \ \sum_{i \in I} c_i(x_i) \tag{7a}$$

$$\text{s.t.}$$

$$x_i \in C_i^1 \qquad\qquad \forall i \in I \tag{7b}$$

$$\tau_n(f) + \sum_{i \in I(n)} x_i = 0 \qquad\qquad \forall n \in N \qquad [\pi_n] \tag{7c}$$

$$f \in U^1 \tag{7d}$$

where τ_n is the shadow price for the nodal balance constraints. In the real-time market (second stage), for every scenario $\omega \in \Omega$, the market clearing is found by solving

$$\min_{X_\omega, F_\omega} \sum_{i \in I} \Big(c_i(X_{i\omega}) + \tilde{c}_i(x_i, X_{i\omega}) \Big) \tag{8a}$$

s.t.

$$X_{i\omega} \in C_i^2(\omega, x_i) \qquad \forall i \in I \tag{8b}$$

$$\tau_n(F_\omega) + \sum_{i \in I(n)} X_{i\omega} = 0 \qquad \forall n \in N \qquad [\lambda_{n\omega}] \tag{8c}$$

$$F_\omega \in U^2, \tag{8d}$$

where (x, f) is fixed to an optimal solution to (7), and $\lambda_{n\omega}$ is the shadow price of the balance constraint of node n in scenario ω. The resulting expected welfare from the two stages will be

$$\mathbb{E}\left[\sum_{i \in I} \Big(c_i(X_{i\omega}) + \tilde{c}_i(x_i, X_{i\omega}) \Big) \right]. \tag{9}$$

In the *stochastic* market clearing model given by (10), the two markets are considered in an integrated manner. This means that the model considers the consequences for the real-time market clearing in the different scenarios when the day-ahead market is cleared. The objective function of this model is analogous to (9).

$$\min_{x,f,X,F} \; \mathbb{E}\left[\sum_{i \in I} \Big(c_i(X_i) + \tilde{c}_i(x_i, X_i) \Big) \right] \tag{10a}$$

s.t.

$$x_i \in C_i^1 \qquad\qquad\qquad \forall i \in I \tag{10b}$$

$$X_{i\omega} \in C_i^2(\omega, x_i) \qquad\qquad \forall i \in I, \; \omega \in \Omega \tag{10c}$$

$$\tau_n(f) + \sum_{i \in I(n)} x_i = 0 \qquad \forall n \in N \qquad\qquad [\pi_n] \tag{10d}$$

$$\tau_n(F_\omega) + \sum_{i \in I(n)} X_{i\omega} = 0 \qquad \forall n \in N, \; \omega \in \Omega \quad [p_\omega \lambda_{n\omega}] \tag{10e}$$

$$f \in U^1 \tag{10f}$$

$$F_\omega \in U^2 \qquad\qquad\qquad \forall \omega \in \Omega \tag{10g}$$

To make the real-time shadow prices comparable, the shadow price of the nodal balance constraint (10e) for node n in scenario ω is $p_\omega \lambda_{n\omega}$, where p_ω is the probability of scenario ω.[1]

3.2. Effect of network constraints in the day-ahead market

In both market clearing models, we distinguish between the set of flow constraints in the two stages, i.e., U^1 and U^2. A key issue in this paper is the effect of different assumptions about U^1. One alternative is to set $U^1 = U^2$, i.e., include a full network representation also in the day-ahead stage. We refer to this alternative as the *nodal* model. In European electricity markets, the day-ahead market is currently cleared with a simplified network representation, based on a partitioning of the network nodes into zones. Let $z \in Z$ represent the set of price zones, N_z the set of nodes belonging to zone z, and $L(x,z) = \{l \in L : \nu_0(l) \in N_x, \; \nu_1(l) \in N_z\}$ the set of network lines where the starting node belongs to zone x and the end node to zone z. The day-head flow constraints $f \in U^1$ in a *zonal* model can then be expressed as

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[1]The energy balance equation (10e) is different from that in (Pritchard et al., 2010), where a net formulation is used in order to use the real-time shadow prices for market settlement. We do not discuss pricing, and it will be more convenient to use the gross formulation here. Pricing will be discussed in a companion paper (Pritchard et al., 2016).

Table 1: The alternative network constraint formulations for the day-ahead market.

Model	Network constraints
Nodal	$U^1_{nodal} = U^2$
Balanced	$U^1_{bal} = \mathbb{R}^{|L|}$
Unconstrained	$U^1_{unc} = \mathbb{R}^{|L|}$ and nodal balance equations removed
Zonal	$U^1_{zonal} = \{f \in \mathbb{R}^{|L|} : (11) \text{ is satisfied}\}$

$$-zcap_{zx} \le \sum_{l \in L(x,z)} f_l - \sum_{l \in L(z,x)} f_l \le zcap_{xz} \qquad \forall (x,z) \in Z \times Z. \qquad (11)$$

Another alternative is to leave out the network constraints altogether, i.e., to set $U^1 = \mathbb{R}^{|L|}$. For the stochastic model, we also consider leaving out the nodal balance equations (10d). This gives us two variants of a less constrained model. We define the *unconstrained* model to be without neither balancing constraints nor network constraints, while the *balanced* model includes balancing constraints, but no network constraints. Consequently, the unconstrained model allows for over- or under-booking in the day-ahead part of the electricity market, and is thus similar to the Newsvendor model.

In total, we then have four different alternatives for the day-ahead market clearing model, and we denote the corresponding sets of feasible flows as U^1_{nodal}, U^1_{zonal}, U^1_{bal}, and U^1_{unc}, respectively. The four alternatives are summarized in Table 1.

Since the balanced and the unconstrained models are obtained from the nodal or zonal models by successively relaxing constraints, we can rank the optimal value v of each of the problem instances for the stochastic clearing model:

$$\min[v^{stoch}_{nodal}, v^{stoch}_{zonal}] \ge v^{stoch}_{bal} \ge v^{stoch}_{unc}$$

It is not possible to determine a general ranking of the optimal value for the nodal and zonal model since both may be a relaxation of the other, depending on how the node aggregation and zonal capacities are determined. If the zonal network constraints are obtained by simply aggregating the network constraints in the nodal model, i.e., if $zcap_{xz} = \sum_{l \in L(x,z)} cap_l$ for any pair of zones x and z, then we would have $U^1_{nodal} \subseteq U^1_{zonal}$ and $v^{stoch}_{nodal} \ge v^{stoch}_{zonal}$.

In practice, the system operators often set the interzonal capacities based on a number of considerations such as loop flow and security of supply, and not by simply summing the line capacities. In that case, we cannot determine any general relationship between U_{nodal}^1 and U_{zonal}^1.

For the myopic market clearing model it is not possible to foresee how the different alternatives for the day-ahead market clearing model will perform relative to each other. When clearing the day-ahead market, the myopic model will maximize the net benefits in this market without considering the effects on the real-time market. Depending on the flexibility costs and network configurations, this may make either of the models arbitrarily bad or good. Given that the representation of the uncertainty is the same as for the stochastic clearing model, and that $U_{myopic}^1 = U_{stoch}^1$, the upper limit on the expected performance of the myopic model will be equal to the results from the stochastic market clearing model.

4. Numerical examples

4.1. Example 1

In our first example we demonstrate that the specification of the day-ahead market clearing via U^1 can be important. In the three-node network, illustrated in Figure 6, the load is located in Node 1 and the generators in Nodes 2 and 3. We consider two equiprobable scenarios $\omega \in \{1, 2\}$. In this stylized example, all cost parameters are equal to zero, except $a_1^u = a_2^u = 1$ and $a_3^u = 0.25$. This means that we have asymmetric flexibility costs where it is costly to up-regulate, while down-regulation is free. The load and generation quantity vectors in the two scenarios are given by $X_1 = (30, 0, -30)$ and $X_2 = (0, 60, -60)$. Since the real-time quantities are given, the only decisions to be taken are the day-ahead quantities x.
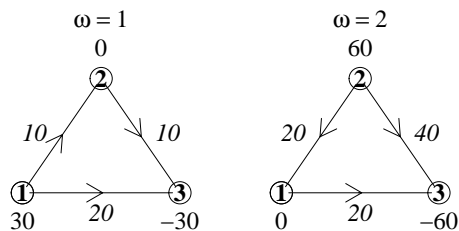

Figure 6: Real-time schedules for the scenarios in Example 1.

All lines in the network have identical impedances. Given a DC approximation of the network model, the real-time quantities will result in the flows shown in italics in the figure. We assume further that line $(2, 3)$ has a thermal capacity limit of 40, equal to the flow over this line in scenario 2. There are no other constraints on line flows. Further, we assume that there are no feasibility constraints on the day-ahead schedule other than the requirement that it must be possible to adjust the schedule to obtain the real-time solution in the respective scenarios. In the case where we require the day-ahead schedule to respect the network constraints, the stochastic market clearing model can be written as:

$$
\min \ 0.5 \cdot 1 \cdot \left([30 - x_1]^+ + [0 - x_1]^+ + [0 - x_2]^+ + [60 - x_2]^+\right) \tag{12}
$$
$$
+ \, 0.5 \cdot 0.25 \cdot \left([-30 - x_3]^+ + [-60 - x_3]^+\right)
$$
$$
\text{s.t.}
$$
$$
x_1 = f_{13} + f_{12}
$$
$$
x_2 = -f_{12} + f_{23}
$$
$$
x_3 = -f_{13} - f_{23} \tag{13}
$$
$$
f_{12} + f_{23} - f_{13} = 0
$$
$$
-40 \le f_{23} \le 40
$$

where x_1, x_2 and x_3 are the day-ahead quantities. The constraints in (13) can be rewritten, eliminating the flow variables, as

$$
x_1 + x_2 + x_3 = 0, \tag{14}
$$
$$
-40 \le \frac{x_2 - x_3}{3} \le 40, \tag{15}
$$

where (14) represents the energy balance, while (15) is the thermal capacity constraint for line $(1, 3)$. In Figure 7, the grey plane corresponds to the set of solutions satisfying the balance constraint (14), while the solutions in the dotted part of the plane also satisfy (15).

Since the real-time quantities are given, the objective function (12) is the expected flexibility costs caused by differences between the day-ahead schedule and the real-time schedules in the various scenarios. The general expression for the expected flexibility cost is
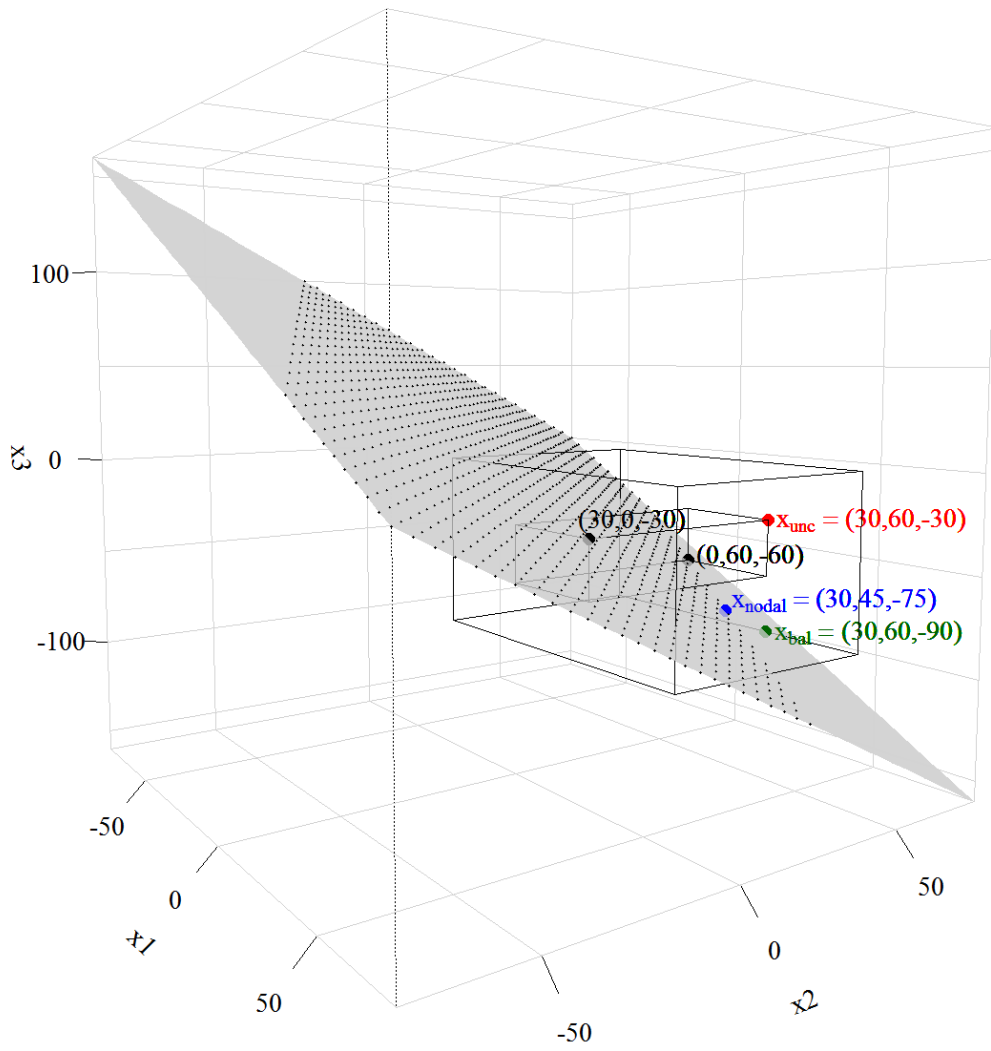
Figure 7: Day-ahead schedules in Example 1.

$$\mathbb{E}\left[\sum_{i \in I}\left(\tilde{c}_i(x_i, X_i)\right)\right], \tag{16}$$

where

$$\tilde{c}_i(x_i, X_{i\omega}) = (a_i^u - a_i)X_{i\omega}^u + 0.5(b_i^u - b_i)(X_{i\omega}^u)^2 + (a_i - a_i^d)X_{i\omega}^d + 0.5(b_i^d - b_i)(X_{i\omega}^d)^2$$

for scenario ω and offer i. If there are no network or energy balance constraints on the day-ahead schedule of the stochastic dispatch model, the day-ahead plan for the different generators and loads can be determined independently of each other. In the unconstrained case then (Table 1), we can always find an optimal day-ahead schedule that satisfies, for all $i \in I$,

$$\min_\omega X_{i\omega} \leq x_i \leq \max_\omega X_{i\omega}, \tag{17}$$

since choosing x_i outside this interval will lead to up- or down-regulation in all scenarios. For Example 1, this corresponds to the smallest box in Figure 7. Since down-regulation is costless, the value of (12) is maximal at the corner point $(30, 60, -30)$, i.e., all the offers are scheduled at their maximal respective quantities, and no up-regulation is necessary. The optimal value is zero.

Imposing the balance constraint (14) means that the value chosen for x_i will also affect the scheduled quantities for the other offers $j \neq i$. In this case an optimal solution can be found in the interval given by, for all $i \in I$,

$$\min\left[\min_\omega X_{i\omega}, -\sum_{j \neq i}\max_\omega X_{j\omega}\right] \leq x_i \leq \max\left[\max_\omega X_{i\omega}, -\sum_{j \neq i}\min_\omega X_{j\omega}\right]. \tag{18}$$

For a supplier the interpretation of (18) is that x_i is limited from below by (1) the minimal own production across all real-time scenarios and (2) the minimal residual demand. Similarly, x_i is limited from above by (1) the maximal own production across all real-time scenarios and (2) the maximal residual demand. Similar interpretations may be given for demand offers.

For Example 1, (18) corresponds to the larger box in Figure 7. The optimal day-ahead schedule when the balance constraints are imposed is $(30, 60, -90)$, which lies in the intersection of the balance plane and the border of the box. The load in node 1 has to be up-regulated in both scenarios

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(that is, the load is reduced to increase net supply in the network), resulting in an expected cost of

$$0.5 \cdot 60 \cdot 0.25 + 0.5 \cdot 30 \cdot 0.25 = 11.25.$$

The model chooses this solution because up-regulation of load has the lowest flexibility cost.

Next, we impose the capacity constraint (15), and this will further constrain the optimal day-ahead schedule to lie within the dotted surface in Figure 7. The optimal schedule is now $(30, 45, -75)$. The load in node 1 will have to be up-regulated in both scenarios, by 45 and 15, respectively, and the generation in node 3 will have to be up-regulated by 15 in scenario 2. The total expected cost is now

$$0.5 \cdot 45 \cdot 0.25 + 0.5 \cdot (15 \cdot 1 + 15 \cdot 0.25) = 15.$$

This stylized example illustrates the potential benefits from relaxing the network constraints in the day-ahead market clearing problem. In the next example, we will extend the analysis with a more realistic example.

4.2. Example 2

The network configuration as well as the various generators and loads are described in Figure 8. The example is motivated by the bid curves that can be observed in Nord Pool Spot, with a combination of hydro, wind, thermal and nuclear power generation. For a more detailed description of bid curves on Nord Pool spot, see Bjørndal et al. (2014).

The three nodes in our network are connected by three identical lines, each with a transmission capacity of 5000 MWh/h. There are 5 generators of various types. Their respective cost curves and flexibility costs are provided in Table 4.2. In Node 1, there is an inelastic load of 15000 MWh/h, with a benefit curve given by the dashed lines. We assume that this load can be shed, and that the VOLL (Value of Lost Load) is 2000 €/MWh. Moreover, there is a wind and a thermal power producer, and the only uncertainty in the system comes from the capacity of the wind generator. This uncertainty is represented by three scenarios that are described in Figure 9. The thermal generator has a capacity of 5000 MWh/h. In order to illustrate the cost curves in our network, we have used the wind capacity from Scenario 2 in Figure 8. Node 2 has a nuclear generator with a capacity of 10000 MWh/h,
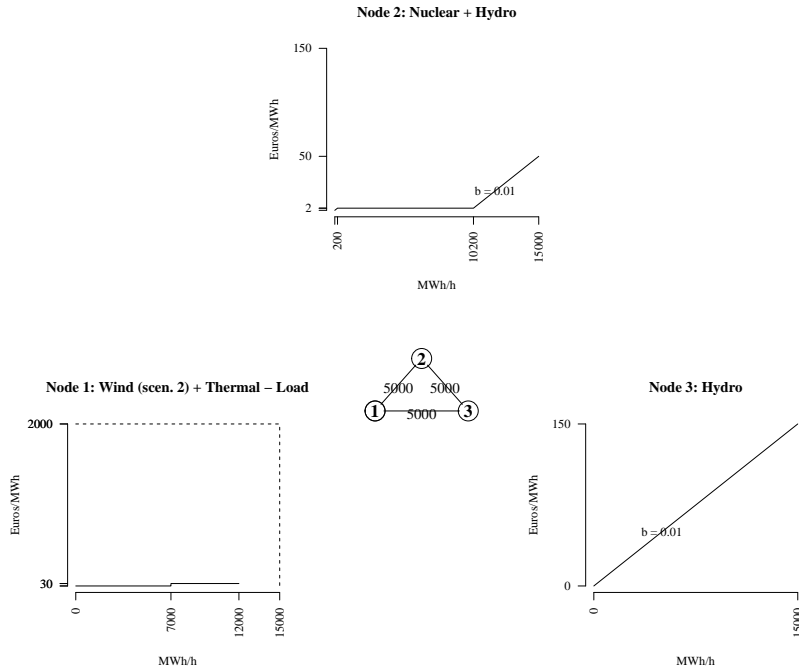
Figure 8: Cost parameters and capacities for Example 2.

as well as a hydro generator with a capacity of 5000 MWh/h. In Node 3 there is a hydro generator with a capacity of 15000 MWh/h.

The wind generator capacity is not known when the day-ahead market bids are submitted. We assume that the wind generator may regulate up or down without any additional costs[2], but the final quantities must respect the realized capacity constraints given by the scenarios in Figure 9. There are three flexible generators in the system (in addition to the wind producer): the two hydro generators and the thermal generator. Up- and down-regulation by hydro generators is made costly by increasing the slopes of the corresponding cost curves. In the example we increase the up-regulation slope for the hydro generators by a factor of 10, giving the new slope parameters $b^u_{hydro2} = b^u_{hydro3} = 0.01 \cdot 10 = 0.1$, while down-regulation by these generators

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[2]In the computations, we add a negligible down-regulation cost in order to break ties in the unconstrained model.
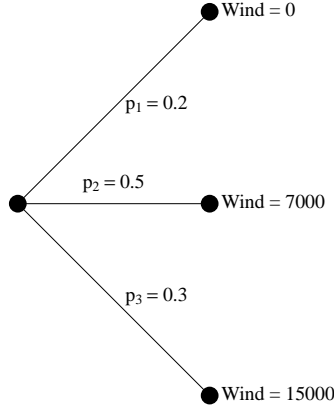
Figure 9: Wind scenario data for Example 2.

can be done without extra costs. For the flexible thermal generator, up-regulation is made costly by increasing the intercept of the real-time market cost curve relative to the corresponding intercept for the day-ahead market, i.e., $a_{therm}^u = a_{therm} + 6 = 36$ €/MWh, while down-regulation can be done without extra costs. Hence, the flexible generators all have asymmetric flexibility cost parameters, where up-regulation is costly. The load is also flexible, with the value of down- or up-regulation given by the VOLL constant of 2000 €/MWh.[1]

Table 2: Cost parameters and flexibility in Example 2.

Entity	Node	Intercept (a)	Slope (b)	Flexible?	Flex. cost up	Flex. cost down
Wind	1	0	0	Partly	$a^u = a$	$a^d = a$
Therm.	1	30	0	Yes	$a^u = a + 6$	$a^d = a$
Load	1	2000	0	Yes[1]	$a^u = a$	$a^d = a$
Nucl.	2	2	0	No	-	-
Hydro	2	0	0.01	Yes	$b^u = 10b$	$b^d = b$
Hydro	3	0	0.01	Yes	$b^u = 10b$	$b^d = b$

Table 3 shows the optimal solution of the stochastic dispatch model given

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[1]Load can be shed at VOLL = a = 2000 Euros / MWh. If day-ahead load shedding takes place, then real-time down-regulation (increased consumption) is limited to shed load quantity.

by (10) for different constraints in the day-ahead problem, see Table 3. We split the value of the objective function (10a) in two parts, where c represents cost and benefit evaluated at the day-ahead parameters, and $\tilde{c}$ represents the extra (flexibility) cost due to more expensive real-time adjustments. We have also adjusted the objective values by removing the contributions from the inelastic load, i.e., 15000 MWh/h valued at a price of 2000 €/MWh, from all the numbers, in order to make them easier to compare (this term would be identical in all model runs). In addition to the different model formulations that we discussed in Section 3.2, we also show the wait-and-see value, i.e., the expected optimal value with perfect information.

The results show that the unconstrained model gives a cost value that is 114.9 % of the wait-and-see value, while the corresponding values for the balanced and nodal models are 117.4 % and 127.4 %, respectively. Hence, the relaxation of the balance constraint and the network capacities will improve the solution in this case. The zonal network constraints can be tighter or looser than the corresponding nodal constraints. When the interzonal capacity is set at 10000 MWh/h, i.e., equal to the sum of the individual line capacities, the zonal model is a relaxation of the nodal model, and we see that the objective function value is slightly better, at 124.4 % of the wait-and-see value. However, if the interzonal capacity is set too tight, e.g., at 5000 MWh/h, the value of the zonal model becomes much worse than the nodal model, at 352.8 % of the wait-and-see value. These results are in line with the discussion in Section 3.2.

Table 3: Optimal expected cost with stochastic market clearing in Example 2.

	$\mathbb{E}[c]$		$\mathbb{E}[\tilde{c}]$		$\mathbb{E}[c + \tilde{c}]$	
Model	€	Relative	€	Relative	€	Relative
Wait-and-see	66360	100.0 %	0	0.0 %	66360	100.0 %
Unconstrained	76250	114.9 %	0	0.0 %	76250	114.9 %
Balanced	76322	115.0 %	1600	2.4 %	77922	117.4 %
Nodal	82325	124.1 %	2190	3.3 %	84515	127.4 %
Zonal ($cap_{\{1\},\{2,3\}} = 5000$)	116977	176.3 %	117168	176.6 %	234144	352.8 %
Zonal ($cap_{\{1\},\{2,3\}} = 10000$)	79810	120.3 %	2769	4.2 %	82578	124.4 %

Table 4 shows the optimal schedules for the nodal and unconstrained models, respectively. While the nodal model provides a balanced day-ahead schedule, the unconstrained day-ahead schedule has an excess supply of 1500

MWh/h, i.e., overbooking of generation.[3] Since the real-time schedule has to be balanced, there is a net down-regulation of 1500 MWh/h in each of the scenarios. The value in the overbooking comes from the flexibility with respect to which generators the model choose to down-regulate in the real-time market. For the nodal model, the table shows that the real-time adjustments for the nodal model involves costly up-regulation by one of the hydro generators in the low-wind and medium-wind scenario. The unconstrained model schedules both of the hydro generators at higher quantities and down-regulates them when necessary, thus avoiding costly up-regulation.

Table 4: Optimal schedules with stochastic market clearing.

Entity	Node	Nodal model				Unconstrained model			
		Day-ahead schedule	Real-time adj.			Day-ahead schedule	Real-time adj.		
			Low	Medium	High		Low	Medium	High
Wind	1	153	-153	6847	9849	0		7000	10000
Therm.	1	5000		-5000	-5000	5000		-5000	-5000
Load	1	-15000				-15000			
Nucl.	2	4998				5000			
Hydro	2	155	-153	245	-155	1500	-1500		-1500
Hydro	3	4694	306	-2092	-4694	5000		-3500	-5000
Total		0	0	0	0	1500	-1500	-1500	-1500

We next investigate how the myopic model, which is more similar to the market clearing methods used in many markets today, will react to different formulations of the day-ahead constraints. Note that it is not obvious how the wind generator's production possibilities should be represented in the day-ahead stage of the myopic model, i.e., how we should represent C^1_{wind}. In the stochastic market clearing models we let $C^1_{wind} = [0, 15000]$,i.e., the support of the probability distribution shown in Figure 9. Since the wind generator has a marginal cost of 0 €/MWh, bidding a capacity of up to 15000 MWh/h would lead the myopic schedule to include as much wind as possible.

Figure 10 illustrates solutions for the myopic model with different values of the day-ahead wind bid from 0 MWh/h to 15000 MWh/h. The left part of

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[3]The day-ahead quantity of the wind generator may be set anywhere in the interval from 0 MWh/h to 10000 MWh/h without affecting the objective function value, and we have chosen the lower end of the interval by adding a negligible down-regulation cost for wind when computing the schedule.

the figure shows results for the myopic model with nodal network constraints in the day-ahead stage, and the right part of the figure shows results for the myopic model with only balance constraints in the day-ahead stage.[4] The upper diagrams show the generation and load quantities in the day-ahead schedules, and the middle diagrams show the physical flows that would result from the day-ahead schedules. We see that, for low values of the wind bid, the day-ahead schedule will consist of mostly nuclear power, as well as a small amount of hydro power. This will result in a flow from Node 2 to Node 1 of more than 5000 MWh/h for wind bid values of less than 7400 MWh/h. The hydro generators are flexible and can be regulated down in real time in scenarios with little wind, but the quantity of the nuclear generator cannot be adjusted. Any day-ahead schedule with more than 7500 MWh/h of nuclear power production will cause an infeasible real-time schedule, since 2/3 of this generation will flow from Node 2 to Node 1, and since there are no generators that can create a counterflow in order to make the schedule feasible. This will happen for all wind bid values of less than 7100 MWh/h. The nodal model avoids the infeasibility problems, since the network constraints are represented in the model used in the day-ahead stage of the market clearing. Still, the expected cost of the day-ahead and real-time schedules depend to a large degree on how the wind bid is represented in the day-ahead stage, and this is illustrated by the lower diagrams in Figure 10. The diagrams show total expected cost, i.e., $\mathbb{E}[c + \tilde{c}]$, and we have split c into VOLL and generation cost. We see that the nodal model has (approximately) the same optimal wind bid as the optimal wind in the stochastic market clearing model with nodal constraints, i.e., 153 MWh/h. Note that this solution is equivalent to the one proposed by (Morales et al., 2014), where the optimal wind bid is found by solving a bi-level optimization problem. The best solution in the balanced model is to set the wind bid equal to 9600 MWh/h, which yields expected cost equal to 320' €, most of which, 224' €, is made up of extra flexibility costs related to real-time regulation. Below the wind bid value of 9600, load shedding is necessary, and VOLL makes up an increasing part of total cost.

Figure 11 illustrates the solutions for myopic market clearing with zonal

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[4]We have not shown any results for an unconstrained myopic model, as this would require that we make an explicit decision about day-ahead over- or underbooking. In the stochastic model, any over- or underbooking is endogenously determined by the model.
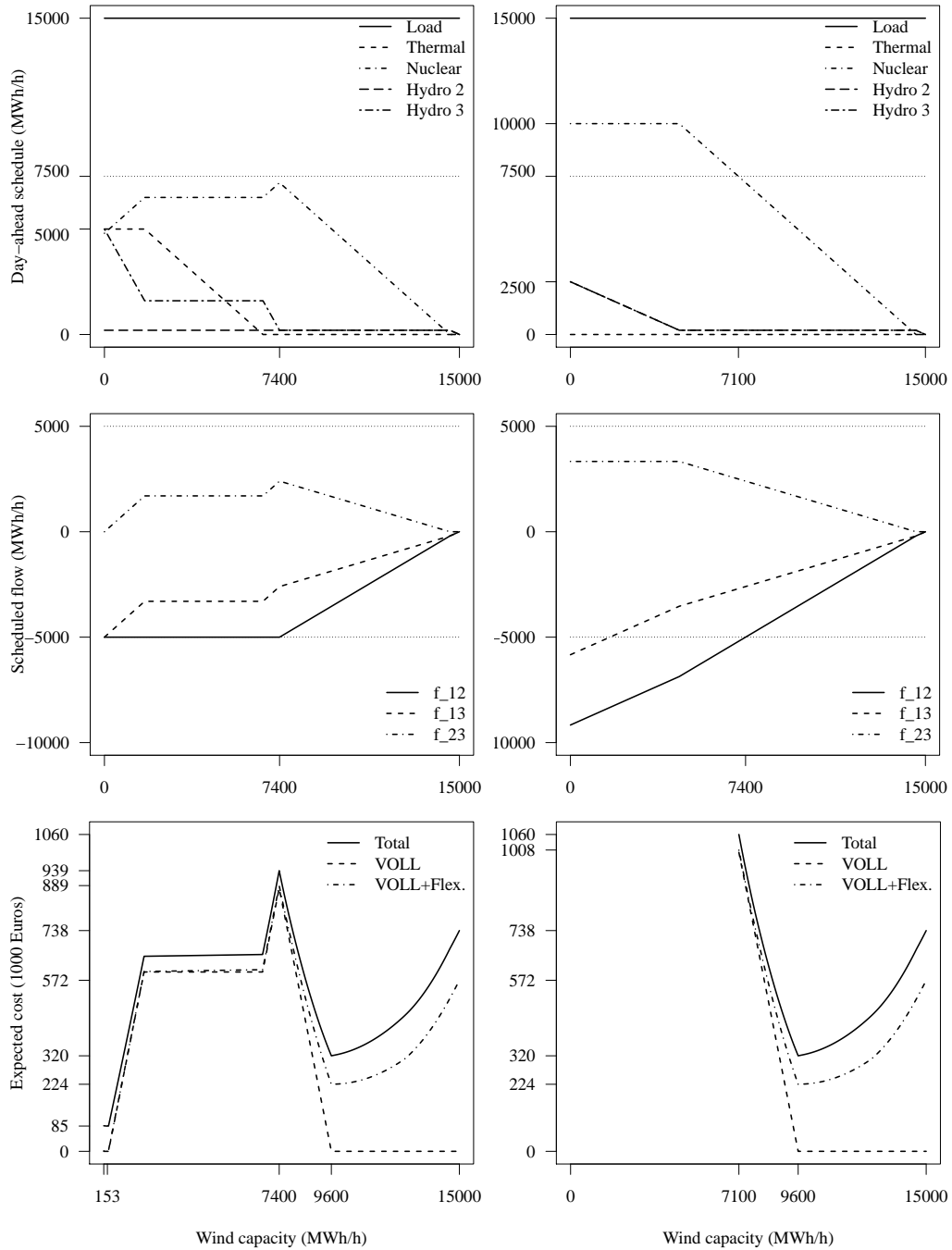
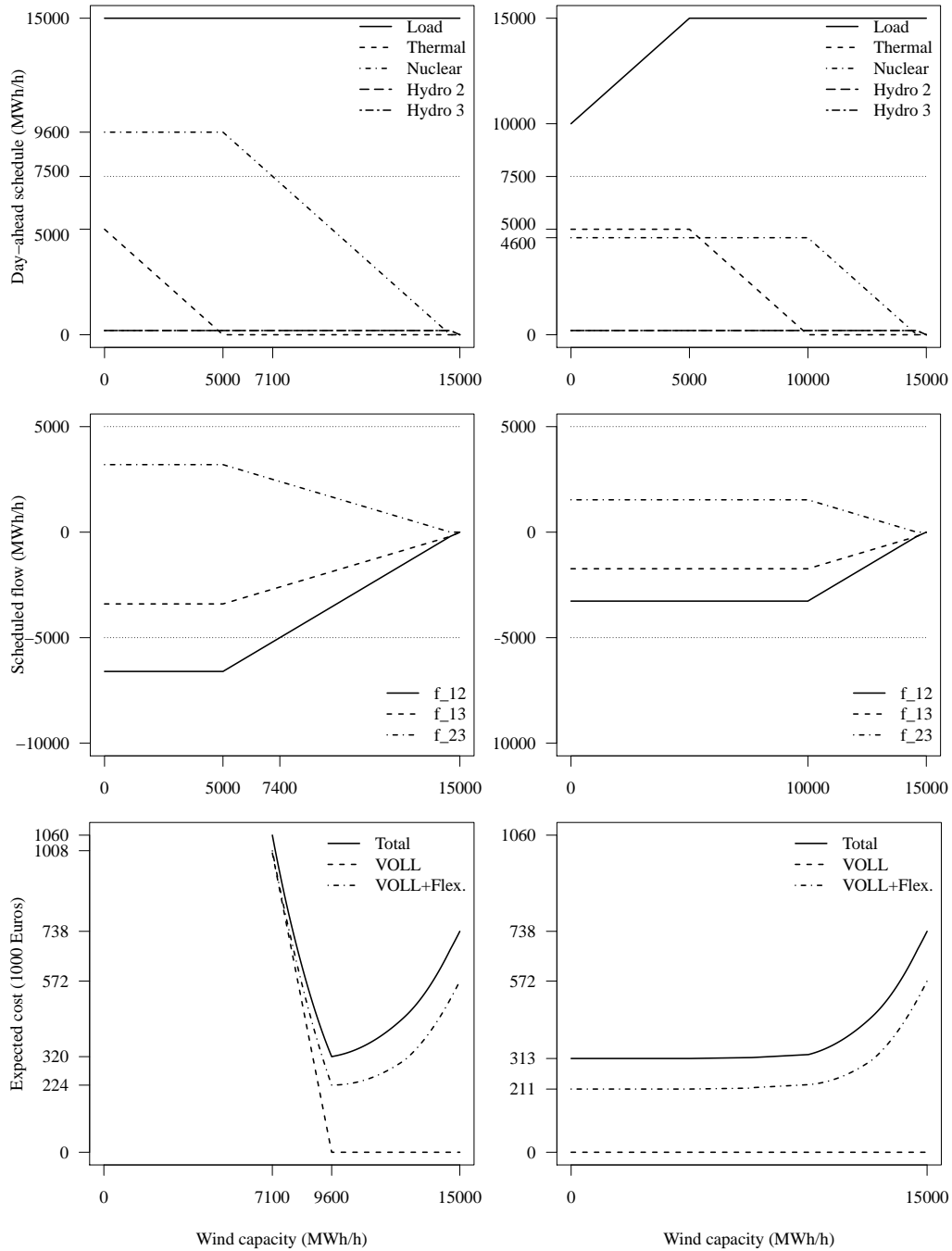Figure 10: Myopic model with nodal (left) or balance (right) constraints, Example 2.

Figure 11: Myopic model with zonal network constraints, Example 2. The interzonal transfer capacity $cap_{\{1\},\{2,3\}}$ is set equal to 10000 (left) or 5000 (right).

network constraints in the day-ahead stage. The left and right parts of the figure correspond to an interzonal capacity of 10000 MWh/h and 5000 MWh/h, respectively. For the case where the interzonal capacity is set at 10000 MWh/h, we see that, although the nuclear generator is never scheduled at its full capacity of 10000 MWh/h, it is still scheduled above 7500 MWh/h for low values of the wind bid, and it is therefore not possible to find a feasible real-time schedule in any scenario. When the interzonal capacity constraint is reduced to 5000 MWh/h, however, we see that the infeasibility issue is avoided. In this case, any wind bid between 0 MWh/h and 5000 MWh/h in the day-ahead stage results in an expected cost of 313' €. Tightening the zonal capacity constraint resolves the infeasibility issue, but the resulting schedule is inferior to the nodal model in terms of cost.

Example 2 illustrates that how energy balance and network constraints are modelled affects the optimal solution and the overall welfare, both in the stochastic and the myopic dispatch models. In the stochastic model, from an optimization point of view, there is no need to keep these constraints in the day-ahead stage of the problem, and leaving out the energy balance may give a Newsvendor structure on the solution, with over- or under-booking of generation, depending on the relative cost for up- and down-regulation. For the myopic model, the constraints in the day-ahead part of the market clearing may make a big difference to the solutions. Leaving too many constraints to be resolved only in the real-time market may lead to infeasible flows or high cost. Moreover, the results of the myopic market dispatch depend crucially on the wind power bids to the day-ahead market, and the optimal bid is not necessarily the expected wind power availability.

5. Conclusions

We have presented an analysis of an electricity market with two settlements, i.e. a day-ahead market and a real-time market. When the day-ahead market is cleared, there can be uncertainty regarding production from non-dispatchable energy sources as well as variable load. We have formulated two main models to study the efficiency of this market design. The first model is a myopic model, where the day-ahead market is cleared independently of the real-time market. The second model is a stochastic dispatch model where the possible outcomes of the real-time market clearing are considered when the day-ahead market is cleared. We have studied how changes in the design of the electricity market influence the efficiency and feasibility of the final

dispatches. In particular, we have studied effects of different constraints for the day-ahead part of the market clearing models.

For the stochastic dispatch model, we show analytically that the overall efficiency may be improved by relaxing constraints on network flows and energy balance in the day-ahead part of the model. We have illustrated potential savings with two examples, both with asymmetric regulation cost, where up-regulation is expensive. In this case, relaxing the energy balance constraints allows for over-booking that leads to lower total system costs. The value of over-booking depends on network configuration, structure and size of flexibility costs, and the uncertainty faced by the markets. For other cost parameters it is possible to show similar values from under-booking. If we are to include energy balance and network flow constraints in the day-ahead part of the dispatch, the type of network constraints matter. European-style zonal constraints may yield solutions that are more or less efficient than nodal pricing depending on how the capacities of the zonal system are set.

For the myopic dispatch model, bids from the stochastic resources to the day-ahead market clearing will not be determined endogenously by the market clearing model, but must be provided in the day-ahead stage. The second example, where only wind power is stochastic, shows that the efficiency and real-time feasibility of the dispatch is influenced by the assumed wind bids to the day-ahead market, as well as by the network flow constraints in the day-ahead market clearing. In particular, we see that removing day-ahead constraints can lead to plans that are infeasible in some real-time scenarios, and that regulation cost may be very high.

In a companion paper (Pritchard et al., 2016), we investigate the pricing of power in the proposed stochastic dispatch model. Other important aspects to investigate further include revenue distribution and incentives in markets using stochastic dispatch models, as well at how robust the optimal strategies are with respect to out of sample realizations of the uncertain factors.

Acknowledgement

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