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Bi-level electricity market models

The impact of irrelevant constraints on pool-based electricity market equilibria under strategic bidding.

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Abstract

Advancements in optimization solvers lead to an increased use of complex bi-level problems (BLP) in operations research (OR). For electricity market modelling, BLPs are applied to simulate physical pool-based markets which include transmission constraints. Equilibrium problems with equilibrium constraints (EPEC) are thereby a specific form of BLPs which allow to incorporate several strategically operating market participants in one model. However, there is an inherent risk involved with BLP optimization techniques in general. Irrelevant constraints (IC) can negate the optimality of the solution and thus void the equilibrium. Even though EPECs are used in academia and industry, research on this mathematical phenomena called independency of irrelevant constraints (IIC) is limited and we have no knowledge about the impact of ICs on complex electricity market EPECs.

The aim of this thesis is to verify if such EPECs are IIC and gain insight on how ICs could affect electricity market equilibria. A specifically developed process, based on the mathematical principles of the phenomena, is used to numerically identify ICs. In order to verify how ICs impact optimality under different market settings, several scenarios are applied in a test environment. Focus is put on the impact of objective functions, subsets of ICs, strategic bidding and the effect of ICs on producer bidding behaviour in day-ahead auctions.

It could by shown that the implemented EPEC model is not IIC and that the equilibrium changed, once an IC was rendered active. The introduced three step process proved as reliable approach and four factors were recognised as relevant for the effect on the voided equilibrium. To emphasize practical significance, the thesis also provides a numerical scenario that demonstrates the implications of ICs for electricity market applications and OR. Consequently, the findings of this thesis add to the general understanding of ICs and build a solid foundation for future research on the IIC property.

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Abbreviations

AC	Alternating Current
BLP	Bi-Level Program
CM	Complementary Modelling
DC	Direct Current
EPEC	Equilibrium Problem with Equilibrium Constraints
GenCo	Generating Company
GNE	Generalized Nash Equilibrium
IC	Irrelevant Constraint
IIC	Independent of Irrelevant Constraints
KKT	Karush-Kuhn-Tucker
LLC	Lower-Level Constraint
LLP	Lower-Level Problem
LMP	Locational Marginal Price
LP	Linear Programs
MILP	Mixed Integer Linear Programming
MO	Market Operator
MPEC	Mathematical Problem with Equilibrium Constraint
NPS	Nord Pool Spot
OPF	Optimal Power Flow
OR	Operations Research
SOC	Stepwise Offer Curve
SW	Social Welfare
TP	Total Profit
TSO	Transmission System Operator
ULC	Upper-Level Constraint
ULP	Upper-Level Problem
VBA	Visual Basic for Applications

1. Introduction

Ever since various electricity markets worldwide were restructured and deregulated, market operators (MO) and regulators seek to enhance their efficiency. In the Nordic region, the Nord Pool Spot (NPS) power exchange was established as one of the leading physical markets for electricity [1], [2]. The objective of this electricity exchange is to find a meaningful equilibrium price, under the objective of maximizing the overall social welfare (the sum of consumer and producer surplus), by allocating submitted offers and bids from both consumers and producers [3]. Market knowledge is thereby essential for market participants, MOs and regulating bodies. Thus, electricity market modelling was adapted in academia to develop models that support market participants. Generating companies (GenCo) use such models to reduce their risk exposure and to provide decision support. Regulatory agencies apply them in order to monitor and supervise market performance and efficiency. Due to the physical characteristics and constraints of electricity markets, mathematical models have to combine a detailed representation of the physical system and the economic, rational, modelling of firms' behaviour [4]. Operations research (OR) literature contains a variety of models, distinguishable by mathematical structure, purpose, and context. Consequently, to replicate a market like NPS, a method to characterize meaningful equilibria in pool-based markets with stepwise offer curves (SOC) and incorporating physical network constraints, needs to be applied. Among others, Ruiz, Conejo and Smeers [5] developed a method capable of modelling the behaviour of GenCo's operating in such a pool. However, OR techniques of this structure can be influenced by irrelevant constraints (IC), which could render their results deficient [6]. This phenomenon, or potential flaw in models, demonstrably affects electricity market models [7] but academia has devoted only little effort to research its implications.

1.1 Motivation and purpose

More specifically, equilibrium models, such as the one of [5], are structured as bi-level programming problems (BLP). BLPs are hierarchical optimization problems where an upper-level problem (ULP) is restricted by the solution set of a second lower-level problem (LLP) optimization. Macal and Hurter [6] however proved that under certain conditions BLPs are not independent of irrelevant constraints (IIC). Indicating that, if certain inactive constraints are included in a BLP, the original optimal solution might not be optimal any longer. As shown by Bjørndal, Gribkovskaia and Jörnsten [7], bi-level electricity market models can be considered to be not IIC. In their case, the optimal solution changed once an irrelevant extension of a transmission line was introduced. For this study, the authors applied a model based on a simplified electricity pool with only one strategic GenCo, accompanied by two market followers. Thus, the model did not entirely reflect the economic environment in a pool based electricity market, as those incorporate several strategic market participants. Furthermore, studies show that numerous European electricity markets are still dominated by few GenCo's with relatively strong market power [8], [9]. In OR models, strategic interactions between producers within network constrained electricity markets can be formulated as equilibrium problem with equilibrium constraints (EPEC). Consequently, focusing on EPECs that simulate oligopolistic markets, where few strategically operating market participants provide a majority of supply, reflects praxis. Such models, even though they are relatively new in OR, closely reassemble real market structures and hence are frequently used to analyse market power, investment strategies and market efficiency [4]. Thus, it is of scholar interest, to verify if EPECs used to model electricity markets are affected by ICs. The significance of this research reveals, if the EPEC does not feature the IIC property. This finding would imply that models used in various academic and practical applications may be subject to this error and their solutions are not optimal. In fact, electricity market EPECs have never been analysed in this perspective.

1.2 Research question

Are equilibria in electricity market models, which are based on EPECs and applying SOC, independent of irrelevant constraints? If not, how does the IIC property influence such models and is there a method to numerically identify if a EPEC model is affected?

Although Macal and Hurter [6] emphasize that adding a IC to the LLP can generally destroy global optimality, they note that this is not true for all classes of BLPs. In their publication, the authors highlighted that linear BLPs are thereby particularly difficult to prove IIC. Due to the natural convex characteristic of linear programs (LP) they might appear to be always IIC. Convexity implies that if an LP has an optimal solution, there also exists an optimal solution at an extreme point. This nature of LPs however, does not necessarily guarantee that linear BLPs are IIC [10]. Consequently, to prove if EPEC electricity market models hold the property, a hands-on approach is required. The model of [5] is thereby used as reference model since incorporates relevant market aspects and utilizes advanced OR techniques.

A three node mashed network, like applied by [7], will provide a constrained electricity market for this numerical study. Generally, the bi-level nature of the BLP is composed as follows: the ULP consists of the strategic producers profit maximization objective function, subject to the producers own physical constraints and the LLP; the LLP represents the market clearing mechanism employed by the MO. Its objective function is to maximize social welfare (SW), subject to producer, demand and physical network constraints. Within a network constrained system, the effect of transmission capacity can be shown using locational marginal prices (LMP). LMPs are thereby resulting from transmission capacity to a node, as well as demand and production capacity in that node. If transmission capacity of a line between two nodes is increased, the LMPs on those nodes will alter [11]. This adjustment to the system reflects a change in the LLP, because the model is facing different market clearing conditions. The method of extending transmission capacities, on previously not fully utilized lines, similarly to the approach of [7], will thus be used to verify if the model is IIC. Ultimately, the objective of the thesis is to provide a structured approach for identifying if electricity market EPECs are IIC. The property will then be analysed in detail and in context of a variety of relevant scenarios, whereas the following, not yet researched areas, are highlighted: oligopoly and triopoly EPEC compositions and the influence of strategic bidding using SOCs; different objective functions and their impact; subsets of ICs; implications on scenarios in transmission system planning. As the model focuses on physical short term (day-ahead) electricity markets, the financial side of the market will not be discussed.

1.3 Outline

The thesis is organized as follows: Section 2 presents physical and economic electricity market fundamentals. It includes an introduction in pool based electricity markets and congestion management, as those are essential for the model. Furthermore, the section provides a background to literature on OR and complementary modelling. The concept of EPECs is introduced as a frequently used method for modelling strategic behavior between GenCo's in network constrained electricity markets. Section 3 specifies the problem and its characteristics, as well as the detailed method used to identify the IIC property. Section 4 contains the mathematical model formulation and notation. Analysis and numerical results are displayed in Section 5. Moreover, this section contains observations on several scenarios, which are applied to provide better understanding on the IIC properties' implications. Section 6 closes the thesis by summarizing the main findings and implications on electricity market modelling.

2. Literature and therory

Literature directly related to the problem as formulated above is limited. The IIC property on BLPs was first introduced by [6] but an explicit connection to electricity market modelling was only drawn by [7]. Thus, to understand the approach and the model used to answer the research question, it is required to provide a solid understanding on electricity market modelling and mathematical programming. Electricity markets in general are quite extensive and can be viewed from a variety of different angles. The literature section is therefore limited to subjects relevant for the thesis. The same is true for the OR part and the specifics of electricity market modelling. Consequently, this subsection focuses on OR techniques as applied in the EPEC model and the mathematical concepts required to derive a linear model, which in turn can be solved using modern optimization solvers.

2.1 Electriciy market fundamentals

Electricity as a tradable product is unique in terms of its physical properties. It can be considered as a bundled commodity of energy [Watt/hours] and the associated transportation. Electricity has to be consumed and produced equally as it is non-storable. Furthermore, it depends on a grid where electricity can flow continuously [3]. Hence, it is essential that markets are built around those characteristics, to maintain stability of the electrical system [12]. Considering power production as a supply chain, the primary components required to supply electricity are: generation, transmission, distribution and retail supply. Historically, these components were vertically integrated in electric utilities and thus the markets evolved as strictly regulated monopolies [13]. Throughout the last three decades technological development enabled functional specialisation and liberalization of the markets. In many cases liberalized markets took the shape of a pool-based wholesale market [13],[14]. The Nordic power market is thereby often cited as one of the most successful examples for a restructuring process [1], [15], [16]. In the context of this Nordic wholesale market, generating units no longer depend on state or utility-based centralized producers but on decentralized generation firms. Furthermore the transmission (delivery between areas of supply/demand) and distribution (delivery to end customers) parts of the system are separated from generation and retail supply. To enable this unbundling of historically connected functions, a pool-based wholesale market has to incorporate specific functions in its design.

2.1.1 Power market characteristics and designs

From the viewpoint of standard economic theory, wholesale markets for electricity are inherently incomplete and imperfectly competitive. Parts of this incompleteness are inevitable, because electricity is a flow (or field) of energy that cannot be monitored perfectly and storing energy is not economically viable. Also, flows on transmission lines are continuously constrained by operational limits, physical capacity limitations and environmental factors [12]. However the primary cause for this incompleteness is the notoriously small short-run elasticity of demand, which is not (yet) matched with flexible spot pricing at the retail side. Consequently, demand functions of electricity markets can be considered as rather predictable or certain [17]. Power producers thus optimize the utilization of their generating capacity to best possibly profit from static demand curves. Furthermore, analysis of European electricity markets show, that market power is rather concentrated and few large GenCo's execute their power to influence prices via strategic bidding [8], [18]–[20].

In order to overcome those inefficiencies and counteract monopoly market situations, modern electricity markets combine various elements as shown in Figure 1. In unbundled markets, such as the Nordic power market, a pool-based electricity exchange is responsible for operating the bidding market. As electricity requires simultaneous generation and consumption, balancing markets are required to ensure stability in the grid. A transmission system operator (TSO) therefore manages delivery and dispatch. Physical electricity markets are subsequently supported by financial markets for hedging, trading and financial settlement. Actual bidding of power takes place in two different time frames: day ahead markets to derive prices for a 24 hours schedule before delivery; intraday markets to manage short term (but not immediate) deviations from schedule through flexible bids [12].

	Electricity Exchange			**Transmission System Operator**	
Market Operator	Nasdaq OMX etc.	Nord Pool Spot		Statnet, Fingrid, TenneT etc.	
Products	Financial products	**Physical products (electricity)**		Physical products (electricity)	
		ELSPOT	ELBAS	Balancing (reserve) market	Imbalance power
Time Frame	Well ahead	**Day ahead**	Hours before	Real time	Past time
	Years - weeks	***Tomorrow***	*Intraday, up to one hour before delivery*	*Activation of reserves if required*	*Imbalance settlement after delivery*
Tasks	Trading	Bidding		Delivery	

Figure 1: Elements of an unbundled electricity market according to [21].

In terms of market power and strategic bidding, day ahead markets, such as the NPS market ELSPOT, are especially relevant to study, since market participants are given sufficient time to plan their operations and optimize their bids accordingly. Other physical or financial markets are excluded from this thesis, as they do not provide the same room for analysis.

Physical day ahead markets

In day ahead markets, prices are determined on an hourly basis for the upcoming 24 hours. At the start of each period, the TSO submits available transmission capacity to the MO. This information is then published to all market participants, since transmission capacity significantly influences bidding behaviour and price formation [22]. Market participants then specify their bids in stepwise increasing offer curves for the respective hour. The MO thereby defines the auction principle and how demand and supply are matched. NPS, for example, allows single hourly, block (fill or kill) or flexible bid SOCs and clears the market in order to maximize social welfare (SW), as defined in (1a) [21].

$$Max \sum_{n} \left\{ \int_{0}^{d^a} D^a(x)dx - \int_{0}^{s^a} S^a(y)dy \right\} \tag{1a}$$

Hereby the MO takes consumers' utility, expressed as demand function $D^a(x)$, and deducts producers' cost $S^a(y)$ for every area a in the network. This simplified function is further subject to several constraints such as: volume constraints; area balances and transmission capacities between areas; ramping rates on transmission lines; different bid layouts (e.g. block bids) and the respective hour (not indexed) [23]. As prices are calculated in advance, market clearing is performed for every hour, where submitted bids are matched as shown in Figure 2.

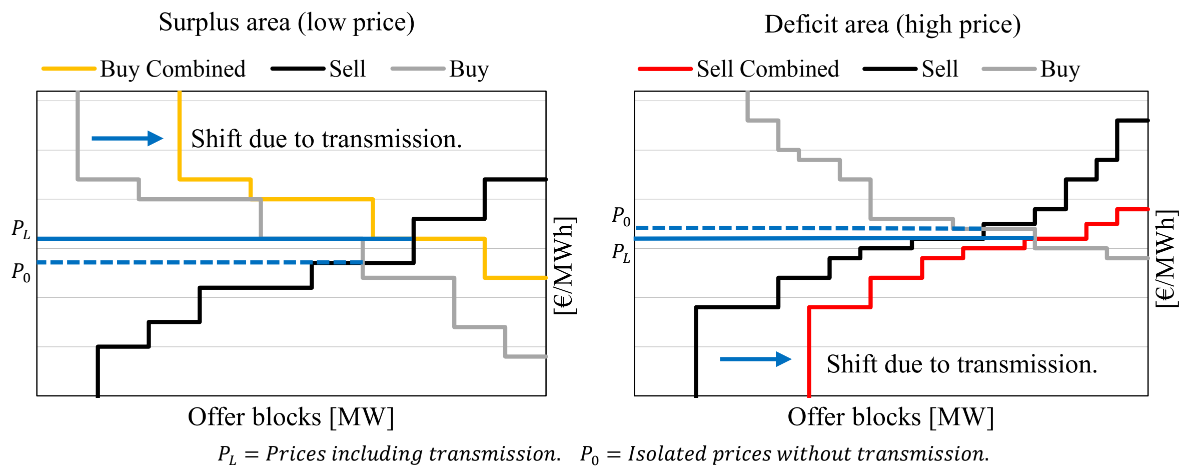

Figure 2: Graphical representation of the market clearing process according to [21], [24].

The displayed scenario explains how bidding curves in two separate areas, a surplus and a deficit area, are aggregated by means of transmission lines between the areas. Consequently, the MO derives an equilibrium where the generated prices are subject to transmission constraints, so that the highest SW can be found. Eventually, both areas will have different prices and once those are found, market participants are invoiced accordingly. The performance of such unbundled markets is highly depending on the auction principle applied by the MO. Since auction principles influence bidding behaviour and revenue distribution among market participants, they have been subject to several studies [25],[26]. Auctions with short-lived bids, where bids are only valid for the respective period, and where available transmission capacity is implicitly included in the auction, are commonly used in both academia and praxis. The auction principles applied by NPS, as well as in [5] and [7], build on such short-lived bid implicit auctions and thus other auction forms are not further discussed.

2.1.2 Transmission and congestion management

In general, the objective of a deregulated electricity market is short and long run efficiency. Short run is thereby the best possible utilization of existing resources and long run relates to grid extensions, to reduce congestion and market inefficiencies. Efficiency can be evaluated as to what extent the theoretically possible optimal power flow (OPF), also called economic dispatch, can be realized. This benchmark refers to the uncongested single period maximal SW equilibrium, given the existing supply and demand curve, subject to thermal and capacity constraints. In praxis, structural differences in networks, by means of marginal generation cost or marginal consumer utility but also by means of available generation capacity in comparison to demand, influence how prices are derived. Sufficient transmission networks are thus a crucial part of efficient electricity market design, as transmission capacity is decisive for calculating area prices [11]. If transmission capacity is limited and the OPF between areas exceeds physical transmission capacities, a network is congested. This opportunity cost of transmission constraints is defined as congestion rent [27]. Thus, MOs seek to reduce network congestion by means of several congestion management mechanisms [12], [17].

For the day ahead market, the MO takes transmission capacities into consideration when defining price areas. These bidding areas are set up so that a uniform price can be derived within the area. Consequently, areas are mostly locations of uniform structures, connected by long lasting high capacity lines. The price derived at such an area is referred to as LMP and reflects the marginal cost of supplying the next increment of demand in this area.

Consequently, LMPs are derived subject to the transmission capacity to and from this area. Definition of such areas is complex and sensitive for the entire market, as it fundamentally changes the markets structure and the SW derived in the optimum. Furthermore, the method of how areas are aggregated can also be modified in order to optimize SW. Academia thereby distinguishes between nodal and zonal pricing, both having their specific advantages [11], [28], [29]. Grid extension is another method to reduce congestion. Thereby, additional transmission lines are implemented in the system. Simulations are then used as decision support and to estimate how additional transmission capacity affects the equilibrium price [4].

Electricity market models that incorporate transmission and network congestion must also be compliant to the physical nature of grids. Transmission grids usually consist of three phase alternating current (AC) high voltage lines. Furthermore, they have to conform to the fundamental laws of electricity flow introduced by Kirchhoff: as electric power is conserved, the flow of power to any point must equal its outflow. If electricity is consumed at a certain node, the voltage level drops at this point. Since total electricity injected must equal energy consumed, any voltage drop must be compensated by a voltage increase at another node. Consequently, the sum of all voltage drops in a closed system must equal zero [3]. However, due to the non-convexity of AC circuit simulations such models are considered complex and not often applied for economic purposes. Direct current (DC) approximation models, introduced by [30], simplify the AC nature of grids and are thus widely used in academia [31]. The most relevant simplifications thereby are: resistance and reactance of power lines do not lead to losses; only real power (not reactive power) is considered; voltage magnitudes equal 1; and voltage angle differences between lines are disregarded [30]. Collectively, a DC model includes power flow equations following Kirchhoff's laws (1b).

$$P_n^G - P_n^D = \sum_{m:(n,m)\in\Theta} P_{nm} - \sum_{m:(m,n)\in\Theta} P_{mn} \quad \forall n,m \in \Theta \tag{1b}$$

Accordingly, the net power flow to a node n must be equal to the net power consumption in that node. Thereby, P_n^G represents generation and P_n^D power consumption in node n. P_{mn} denotes the power flow on a line that is connected to node $n \in \Theta$, in a given direction *nm*.

$$P_{nm} = B_{nm}(\delta_n - \delta_m) \leq P_{nm}^{max} \quad \forall n,m \in \Theta \tag{1c}$$

$$-\pi \leq \delta_n \leq \pi \quad \forall n \in \mathrm{N} \tag{1d}$$

$$\delta_n = 0 \quad n = 1 \tag{1e}$$

The power flow is defined according to (1c) and limited by the maximum capacity of a transmission line P_{nm}^{max}. The admittance of a line, B_{nm}, is a measure of how freely power can flow in a closed circuit and is composed of several parameters, such as the resistance and reactance of a line [32]. It defines the power flow in a line, which is according to (1c), based on Kirchhoff's law, where the sum of voltages in a closed circuit is 0, and δ_n denotes the phase angle at a node. Finally, (1d) ensures angularity and (1e) defines node 1 to be the swing bus.

Considering those physical principles, it is now possible to model electricity markets that are constrained by transmission networks and congestion. Adding the specifics of day ahead markets and how transmission is utilized to derive LMPs, enables to replicate a market cleared by an electricity exchange such as NPS. As demand is assumed to be a function based on marginal utility, the optimization only gains complexity with integrating producers. Adding several strategically acting GenCo's however, requires complex bi-level OR methods that are introduced in the following subsection.

2.2 Optimization models for electricity markets

Electricity market modelling covers a variety of different purposes and applications. The sheer amount of modelling approaches found in literature was structured in a study by [4]. The authors thereby characterize models according to their purpose but also to their mathematical structure, as shown in Table 1. Weron [33] later extended this study to include newly introduced modelling trends, building on statistical modelling and computational intelligence. In terms of market representation, it could be shown that electricity market models differentiate by the means of degree of competition, time scope, uncertainty in supply or demand, inter-period links and transmission constraints. On the mathematical point of view, the models differ depending on the economic purpose they serve. Single firm optimization models, for example, take the constraints of one profit maximizing entity into consideration and solve the model accordingly.

Electricity Market Modeling	Optimization Problem for One Firm	Exogenous Price Demand-price Function
	Market Equilibrium Considering All Firms	Cournot Equilibrium Supply Function Equilibrium
	Simulation Models	**Equilibrium Models** Agent-based Models

Table 1: Electricity Market Modeling according to [4].

For this thesis relevant are equilibrium or simulation models which consider multiple entities and thus require more complex mathematical formulations. In terms of market representation, such models can cover a variety of applications. Answering the research question requires models that simulate oligopolistic competition in pool-based day ahead markets which are operated by an MO, cleared in order to maximize SW, and include transmission constraints. As such advanced OR tools are capable of representing real market scenarios, they have been widely discussed in academia, praised for their capabilities but also criticised for their lacking robustness and multiplicity of their results [4], [33]. Complex optimization techniques, like equilibrium models, can be considered as technical advancement to their less mathematically demanding predecessors, Cournot- and Nash-Equilibrium models [4]. Thus, to understand how equilibrium models are structured, a short introduction to game theory and Nash games is required.

2.2.1 Strategic games and complementary modelling

In OR the denomination "strategic" refers to the capability of a producer or market participant to alter the formation of the market clearing prices [5]. The objective of strategic games is thereby to simulate, how strategic actors operate in a given market environment and derive a corresponding market equilibrium. Nash [34] formulated an equilibrium as a set of strategies that guarantee that no player can improve its objective function by unilaterally changing its strategy. The Nash formulation can be further extended to include a variety of different actors. In case of a Nash-Cournot equilibrium each market participant is characterized by the ability to anticipate its impact on the market and by its knowledge of the inverse demand curve. Such a case would, under perfect competition and optimizing SW, lead to equilibria where the competitive behaviour of all firms results in low market prices and profits but higher production and SW [35],[36]. An example of an even further advanced Nash-game is the generalized Nash equilibrium (GNE). Here, the standard Nash equilibrium is formulated over a variety of players. Consequently, the strategy of each player depends on the strategy of all other players, whereas each player has sufficient knowledge of the market environment. GNEs are however known to be generally difficult to solve, because they present non-square systems with more variables than equations. Accordingly, a GNE can have no, multiple, or infinitely many solutions and thus finding meaningful equilibria involves certain difficulties [37].

Through advancements in the development of optimization solvers and mathematical formulations, it is however possible to reformulate such models and derive meaningful solutions. Complementary modelling (CM) thereby emerged as technique that is increasingly used in energy market modelling. CM is based on the duality theory, which states that there exists a dual problem to every mathematical linear (primal) problem, which is defined with exactly the same input data as the original primal problem [38]. According to [38], a CM is one that solves for a vector of variables x (of dimension n) to meet the conditions of the form $f(x) \geq 0, x \geq 0$, and $f(x)^T x = 0$, where $f(x)$ is a vector-valued function of length n. These conditions are commonly expressed using the perpendicular $\perp$ symbol: $0 \leq f(x) \perp x \geq 0$. As GNE models, in their primary formulation, are highly non-convex, they have to be reformulated and linearized in order to be solved. Thereby, the Karush-Kuhn-Tucker (KKT) optimality conditions for continuous optimization problems are applied. KKT conditions are derived by applying the Lagrangian function on relevant quadratic equations and constraints in the model [39]. Once those conditions are derived, the model can be reformulated and solved via mixed integer linear programming (MILP). Summarized, modelling complex strategic games is based on a process: define the basic problem; derive optimality conditions according to KKT; linearize optimality conditions and reformulate the model as MILP.

2.2.2 Electricity Market Modeling

The basic problem is defined by combining the electricity market and physical elements that should be modelled in one general formulation. However, pool based electricity markets include various actors with different, even opposing, objective functions. Producers aim to maximize their profit, while consumers minimize their cost. MOs thus intend to maximize SW while considering all market participants and the physical structure (transmission constraints and areas) of the market place. Consequently, an equilibrium model capable of representing the entire market must include more than one objective function. Demand is often considered as function (static or dynamic) and thus consumer optimization criteria are simplified or excluded in OR formulations [4]. To start with, the producers' objective function to maximize profits must be included in the model. This maximization is subject to the producers' individual constraints and the market equilibrium conditions, i.e. the market clearing and the objective functions of other strategic producers. The MO's market clearing procedure represents thereby the second optimization criteria, as he intends to optimize SW subject to producer offers, demand and physical market constraints [1].

Objective Function	Actor	Definition
Maximize Profit:		**ULP (1-J)**
Subject To: Capacity constraints Operational and cost limitations	Producer (1-J)	ULC (1-J)
Maximize Social Welfare:		**LLP (1)**
Subject To: Offer limits Energy balance equations Transmission flow constraints	Market Operator (1)	LLC (1)

Table 2: Bi-level nature of pool based electricity markets.

Such problems can thus be formulated as bi-level program (BLP). Briefly explained, a BLP is a problem where the decision variables of an LLP constitute constraints in an ULP. Table 2 schematically represents how this bi-level structure can be set up. As shown, the two problems are interrelated: producers determine in their ULP the optimal offer curve to submit to the MO, whereas the LMPs, which are derived based on those offers and other lower-level constraints (LLC) of the LLP, have a direct impact on the producer profit of the ULP. Thus, the LMPs, a decision variable in the LLP, constitute an upper-level constraint (ULC) of the ULP [1],[5]. The complexity of this BLP is further augmented as several producers (1-J) bid in such a pool, which is operated by one MO. Consequently, all producers share the same LLP whereas they have their individual ULP (except for the LLP ULC). In this formulation, the problem can be interpreted as a multi-leader-common-follower game and modeled as GNE [40].

Mathematical Problems with Equilibrium Constraints (MPEC)

Solving this problem requires to formulate it as MPEC. A MPEC is thereby an optimization problem whose constraints include equilibrium conditions [41]. MPECs are hence related to Stackelberg games, where a leader (producer) anticipates the reaction of one or several followers (MO) [35]. Equilibrium conditions, in the case of electricity market modelling, are found in the LLP, where the MO derives equilibrium LMPs. In order to transform the BLP into a single-level problem, the LLP needs to be replaced by its first order necessary optimality conditions (KKT). In this case, the LLP is non-convex and thus, the KKT conditions are also conditions for optimality. LMPs are thereby a good example how the duality theory is applied in complementary modelling. In the LLP, LMPs are represented as decision variables. Since the LLP is linear, the LMP primal variable (LLP) can be replaced by its dual variable in the ULP formulation. Consequently, if this procedure is applied on all relevant LLP variables, the model is transformed to a single-level problem as the ULP only includes dual variables [5].

Equilibrium Problems with Equilibrium Constraints (EPEC)

The MPEC, as described above, includes only one producer/leader in the Stackelberg game. Pool based electricity markets do however incorporate a number of producers and thus the MPEC has to be formulated as an EPEC. Generally, an EPEC can be interpreted as a multiple-leader Stackelberg game [38]. In such a game, several leaders (indexed by *J*) are incorporated in one market. As they all share the same market conditions, each one of those leaders solves an MPEC. Those MPECs are however interrelated, because both the objective function and the equilibrium conditions within the MPEC depend on the decision variables of all other leaders. The formulated EPEC can then be considered as mathematical representation of a GNE and is suitable to simulate the intended electricity market [35]. Observe, that this formulation again constitutes a BLP and thus, the same method as for deriving the MPEC needs to be applied. Accordingly, the MPECs are replaced by their strong stationary conditions, which are equivalent to the KKT conditions. But, as stated by [42], the nature of MPECs makes it difficult to define second-order sufficient conditions for optimality, which implies that the resulting set of Lagrange multipliers is unbounded and not unique. To solve this mathematical problem, Ruiz, Conejo and Smeers [5] applied exact linearization techniques, which can only be solved due to improvements in mathematical branch-and-cut solvers [43]. The Fortuny-Amat and McCarl [44] decomposition is thereby used in integer programming to accommodate the complementary slackness conditions. Thereby, a KKT condition of the form $0 \leq \mu \perp P \geq 0$ can be reformulated and solved using $\mu \geq 0,\ P \geq 0,$ $\mu \leq \psi M^{\mu},\ P \leq (1-\psi)M^{P}, \psi \in \{0,1\}$, where M^{μ} and M^{P} are large enough parameters to not impose additional bounds on the model. The variable ψ is the binary decision variable that enables a MILP formulation. In summary, the process from BLP to a problem that can be solved using MILP is shown in Figure 3.

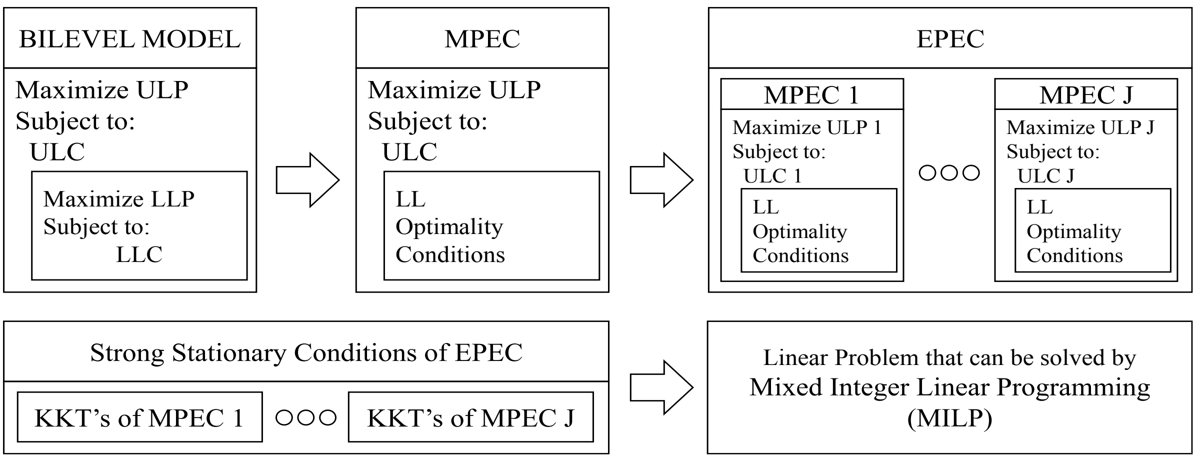

Figure 3: Bi-level to MPEC to EPEC and the method to derive a MILP according [5].

2.2.3 Independency of irrelevant constraints (IIC)

The IIC property is desirable for every mathematical program to have, since it implicates that the model is not dependent on irrelevant constraints. Macal and Hurter [6] mathematically proved that BLPs are, under certain conditions, not IIC. This implies that when inactive constraints are included in the LLP of a BLP, the original equilibrium is no longer optimal. Thus, constraints that seem irrelevant to the optimal solution, in effect, determine the solution to the BLP. Consider therefore the following BLP:

$$\begin{aligned} f_{GS}^{*} \equiv\ & Min_x f(x,y) \\ & Subject\ to\ (x,y) \in F \\ & \text{where y solves} \\ & Min_y g(x,y) \\ & Subject\ to\ G(x,y) \geq 0 \\ & \qquad\qquad\quad S(x,y) \geq 0 \end{aligned} \tag{1f}$$

Let (x^*, y^*) be the solution to BLP (1f) and define a set $\Delta_S \equiv \{(x,y) \in F | S(x,y) \geq 0\}$ for an arbitrary constraint $S(x,y) \geq 0$. If this constraint is now part of a combined set $G \cap S$, where $\Delta_G \equiv \{(x,y) \in F | G(x,y) \geq 0\}$ and $L_G \equiv \{(x,y) | y \in \mathbb{R}(x)\}$ is a set of points feasible on the ULP, the LLP depends on the constraint G. The problem is only then IIC, if its solution (x^*, y^*) is also a solution to the BLP (1f) for every set Δ_S that contains (x^*, y^*) [6].

A more approachable, economic, interpretation of the IIC property can be given in the following example. Consider a central planner in a firm that intends to minimize cost using a BLP and suppose this planner found an optimal solution at the use of 100 resources. In a later stage, due to a production outtake, supply of that resource is limited to 101. If all other input factors in the cost minimizing BLP remain the same, it is fair to assume that the previous solution, using 100 resources, still holds under the new constraint of 101 available resources. However, due to this new IC, the reduced resource availability might allow the ULP of the BLP to see that costs could be reduced even further. Thus, the model has to be solved including this seemingly IC, in order to validate if the model is IIC or not. The significance of the IIC property is therefore found in the implications it has for applied BLPs in real world situations. Macal and Hurter even state that "for any bi-level program that has ever been solved and that is not independent of irrelevant constraints, one can produce an arbitrarily large set of constraints, which taken singly or in combination, negate the optimality of the solution obtained" [6]. Hence, the IIC property is highly relevant for BLP electricity market models but has not been sufficiently researched in this context.

3. Research setting

In order to verify if complex bi-level electricity market models are subject to ICs, the same setting as introduced by [7] is applied. Here, a simplified three node network (see Figure 4) provides the environment to study bidding behaviour of strategically acting GenCo's. Generation units and demand points are located at each node and the nodes are connected by transmission lines. Electricity is supplied by three different producers J1, J2 and J3 whereas demand at each node is indicated by DN. In contrast to the MPEC formulation applied by [7], the advanced EPEC model of [5] is used for this study. This model has the advantage that it is capable of reproducing actual market practise. However, it has not yet been applied in such a context, what renderes the implementation and analysis challenging. In a EPEC, numerous strategically acting producers compete in a network constrained electricity market and submit their bids to an MO. Each producer defines its supply curve in order to maximize its own profit (described as ULP), whereas the producer affects the joint price formation by representing the market clearing within its LLP. To replicate practice used by day-ahead markets in pool based systems, bids are submitted in stepwise blocks. The MO then collects both supply and demand bids and clears the market considering transmission constraints. Thereby, the MO maximizes social welfare through the formation of efficient LMPs. In the model, each computational run represents a one hour period. The outcome is thus the single period equilibrium optimized with respect to the maximization of SW.

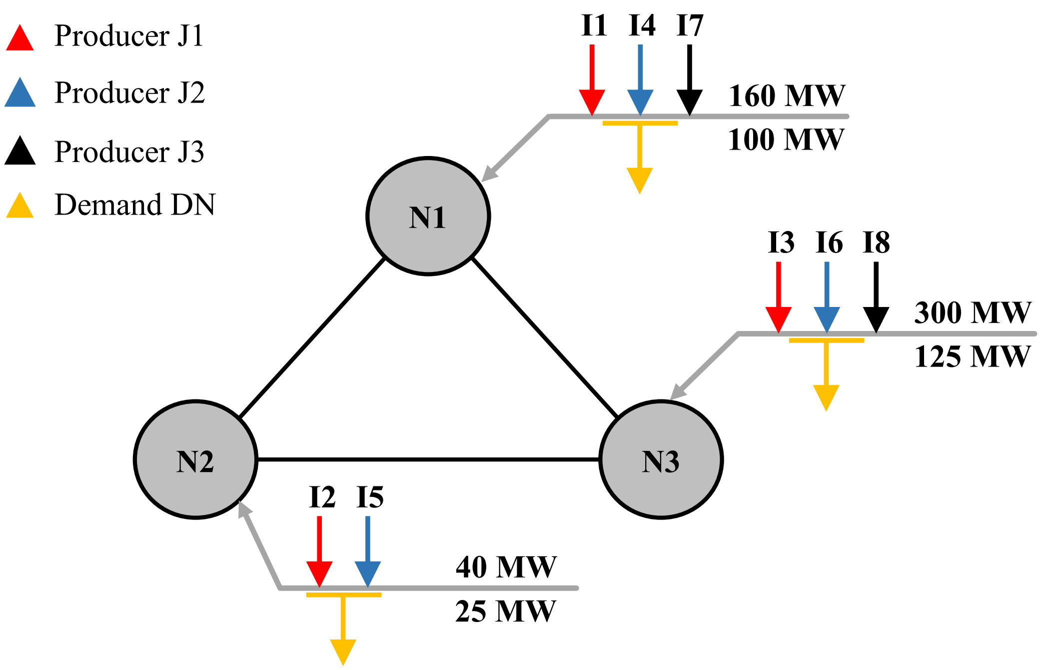

Figure 4: Three node network as applied by [7].

3.1 Problem definition

In order to test this bi-level electricity market model for the IIC property, a practical approach, building upon the findings of Bjørndal, Gribkovskaia and Jörnsten [7], is utilized. In their scenario, and using an MPEC model, the authors could prove that one transmission constraint triggered the IIC property. Specifically, a transmission constraint that was assumed irrelevant for the optimal solution. This constraint however became active (constrained the model), if the transmission line's capacity was marginally higher than the power flow derived in the previously optimal solution. This practical finding confirmed the mathematical description of [6] and showed that the original equilibrium was no longer optimal, if an IC, active for the LLP of the BLP, was included. As noted by [7], it is however uncertain how the IIC property behaves for more complex EPEC models. Consequently, [7] laid the basis for this study, as they found that MPEC models are subject to ICs but left certain points open for further research. It is, for example, unknown how strategic producers altered their bids and if the IIC property can be triggered on subsets of transmission constraints. This research thus focuses on the complex nature of EPEC models and in what way they are affected by the IIC property. The model of Ruiz, Conejo and Smeers [5] is considered for this analysis since it is frequently used in academia and known to be among the most advanced EPEC models [45]–[50].

As per the definition of [6], an irrelevant constraint is an inactive constraint in the LLP of a bi-level model. In the case of electricity markets, the lower-level function represents the market clearing procedure of the MO. Applicable constraints are thereby transmission constraints, energy balance and offer limits. Transmission constraints gain significane due to their implications for market analysis, investment planning and price formation (e.g. LMP). To reliably validate if the model is IIC, a structured approach is required. Following the mathematical definition of [6], a three step procedure could be identified to test for IIC:

1. The bi-level model is solved without active transmission constraints to derive the OPF (unconstrained solution) and to identify reference values for the power flow on lines.
2. An active constraint is set on a line and the model is solved. The output is then a congested solution with the line utilization, or power flow, at the level of the capacity parameter. The power flow on other lines now serves as a proxy value for the next step.
3. The capacity parameter of any line other than the one congested in step 2 is set to a level that is marginally higher than its utilization in step 2. The model is then run for several instances to find the threshold value, when this new, irrelevant, constraint becomes active.

As this procedure is essential for the research, it is summarized in the example of Figure 5.

In step 1, the uncongested solution constitutes an equilibrium by setting the power flow of line 1-2 at 5 MW, 1-3 at 25 MW and line 2-3 at 20 MW respectively. In order to identify a line as IC, another line must be constrained in the first place. This is done by setting the capacity of line 1-3 to 20 MW, which is 5 MW lower than the OPF.

After the numerical run in step 2 a new, constrained, equilibrium is achieved. The constraint on line 1-3 is active as the line's power flow equals its capacity of 20 MW. Moreover, the utilization of the other two lines changed in the new equilibrium. Adding an irrelevant constraint requires now to set the capacity of any other line above the level of their current power flow.

For step 3, the capacity of line 1-2 is thus set at 10 MW and decreased gradually in each computational run (e.g. 10 runs, each 0.8 MW less capacity). The IIC property is only found true if no change in the utilization of line 1-2 occurs, as the capacity is decreased until it matches the constrained power flow of 2.5 MW.

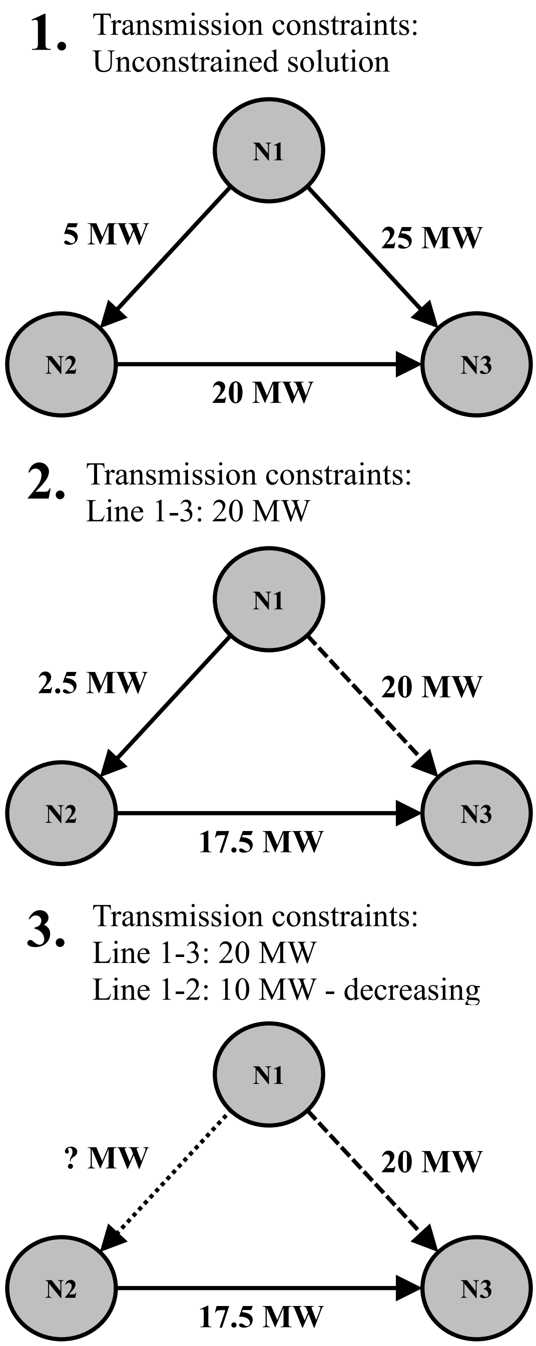

Figure 5: Three step procedure.

The process can have two outcomes: the capacity will reach the constrained (step 2) power flow without prior change and thus the model is IIC; the equilibrium will change as the power flow of line 1-2 is forced equal a given capacity and thus the model not IIC. If e.g. the capacity of line 1-2 is 4.5 MW and the new power flow is found to be 4.5 MW, the seemingly irrelevant constraint of 4.5 MW (compared to step 2) is not irrelevant and the model is found not IIC.

3.2 Parameters

For this numerical analysis, parameters are set equal to those introduced by [7]. Slight adjustments had to be made in terms of adding two bidding steps for both generation and demand. The total supply and demand are equally distributed between the two steps, also referred to as blocks. The model of [5] applies constant marginal cost (GenCo) and marginal utility (consumer) functions to derive the value of each MW offered. This displays a simiplification compared to the downward sloping demand curve as used in [7] but the impact is irrelevant for the course of this thesis. The effect of this alternation is merely that all demand bids are fulfilled, since marginal cost are set lower marginal utility. Using downward sloping demand curves on the other hand leads to unfulfilled demand as the lower marginal utility will, at some point, not rectify offering bids at prices lower than marginal cost. Detailed information on generating unit types used in the system can be found in Table 3. In total, eight generating units are distributed throughout the system. Their location and allocation to producers can be found in Table 4. As the model incorporates an EPEC formulation, at least two strategically operating producers are required to fully utilize the capacity of the model. The model used in [7] sets producer J1 as strategically acting, whereas for this research both J1 and J2 are strategic producers. Note, that producer J3, the marginal cost bidder, holds the largest but also the most expensive generation capacity.

	Block	Unit	T1	T2	T3	T4	T5	T6
Marginal Cost	1	[€/MWh]	1	5	2	20	6	15
	2	[€/MWh]	1	5	2	20	6	15
Capacity	1	[MW]	45	10	40	10	25	100
	2	[MW]	45	10	40	10	25	100
	Total	[MW]	90	20	80	20	50	200

Table 3: Generating unit types.

Producer	Gen. Unit	Type	Location	Capacity	Marginal Cost
				[MW]	[€/MWh]
J1	I1	T1	N1	90	1
J1	I2	T2	N2	20	5
J1	I3	T3	N3	80	2
J2	I4	T2	N1	20	5
J2	I5	T2	N2	20	5
J2	I6	T4	N3	20	20
J3	I7	T5	N1	50	6
J3	I8	T6	N3	200	15

Table 4: Producer, generating unit and node allocation.

Further market parameters, such as demand and marginal utility, are displayed in Table 5. Notice, that marginal utility is set at a level higher or equal to the highest marginal cost in the system. Furthermore, the system features sufficient supply capacity to fulfill demand. This setting leaves room for strategically placed offers by the generators. In terms of market power, it is to note that the two strategic producers share 50% of the generating capacity. As those two players taken together have the capacity to influence the system price, they are from now on referred to as the oligopoly case. Other for the model relevant parameters are the transmission capacity and the admittance of each line. Admittance is fixed to 9.14 for all lines respectively, as it was found that this parameter does not influence the cause of the research. To simplify the model, transmission lines share the same capacity in both directions. Furthermore, transmission tariffs are excluded and losses are neglected in the DC flow model. The parameters for transmission capacity will be individually outlined as they have to be defined individually for each specific scenario. Lastly, the dual variable γ_j^{DT}, which was included by [5] to linearize the model, is considered as parameter and fixed to a value of 5.

		Unit				Total
			J1	**J2**	**J3**	
Production	Capacity	[MW]	190	60	250	500
		%	38%	12%	50%	100%
			D1	**D2**	**D3**	
Consumption	Marginal Utility	[€/MWh]	24	20	30	
	Capacity	[MW]	100	25	125	250
		%	40%	10%	50%	100%

Table 5: Market parameters and market power.

As introduced, the three step process requires alternations in the transmission capacity parameters. All other parameters remain unchanged to eliminate possible noise while identifying the impact of the irrelevant constraint. In general, two main scenarios are used for numerical analysis. The oligopoly scenario, where J1 & J2 are the strategic producers and J3 is the marginal cost bidding follower, and a triopoly scenario, where all three producers are strategic actors. Those scenarios constitute the foundation to apply the model. Altogether, this setting constitutes a research framework that was not yet shown in literature. To increase reliability of the results, parameters were chosen in proximity to the three node network of [7]. In this case, the authors used a single strategic producer, accompanied by two followers. Furthermore, they tested the model for IIC under line 1-2 only and did not analyse why the equilibrium changed or if different objective functions alter the IIC property. Consequently, if the results of their MPEC model can be replicated, it strengthens the results of this thesis.

4. Model and implementation

The model is applied as introduced by [5] and for the cause of comparability, notation and model formulation remained mostly unchanged. Only for some equations, where notational errors could be identified, adjustments to the model formulation were made. As the model was intended to be used to replicate oligopolistic markets, constraint (3v) was added to include a market follower in addition to the two strategic producers J1 and J2.

4.1 Notation and model formulation

Throughout the model formulation and in the further course of this thesis, the notation displayed in Table 6 is applied. Dual variables are identified in the model formulation, together with their originator constraint, followed by a colon. However, some dual variables are not introduced in this manner but listed at the end of Table 6. The reason for this separate indexing is that those dual variables were used in [5] to derive the EPEC from the MPECs but the original constraint was eliminated or linearized in the sequence of the model formulation.

Symbol	Definition
Indices	
j	Producers from 1 to *J*.
i	Generating units from 1 to *I*.
b	Generating blocks from 1 to *B*.
d	Demands from 1 to *D*.
k	Demand blocks from 1 to *K*.
n/m	Buses from 1 to *N/M*.
Parameters	
λ_{ib}^{G}	Marginal cost of block *b* of unit *i*.
λ_{dk}^{D}	Marginal utility of block *k* of demand *d*.
P_{ib}^{Gmax}	Capacity of block *b* of unit *i*.
P_{dk}^{Dmax}	Capacity of block *k* of demand *d*.
B_{nm}	Susceptance of line *n-m*.
P_{nm}^{max}	Transmission capacity of line *n-m*.
γ_{j}^{DT}	Parameterized dual variable.
Variables	
α_{ib}	Price offer for block *b* of unit *i*.
λ_{n}	Locational marginal price (LMP) at node *n*.
P_{ib}^{G}	Power produced in generating block *b* of unit *i*.
P_{dk}^{D}	Power consumed in demand block *k* of demand *d*.
δ_{n}	Voltage/Phase angle of node *n*.
P_{nm}	Power flow through line *n-m*.

Subsets & Identifiers / Matrix Parameters	
$i \in \Omega_j$	Generating units *i* of producer *j*.
$i \notin \Omega_j$	Inverse of generating units *i* of producer *j*.
$i/d \in \Psi_n$	Generating unit *i* / demand *d* located at node *n*.
$m \in \Theta_n$	Nodes *m* connected to node *n*.
$i \in \Omega_j \cap \Psi_n$	Generating units *i* of producer *j* located at node *n*.
Other dual variables: $\beta_{jn}^{\delta min}, \beta_{jn}^{\delta max}, \beta_{jnm}^{Lmax}, \eta_{jib}^{Gmax}, \eta_{jib}^{Gmin}, \eta_{jdk}^{Dmax}, \eta_{jdk}^{Dmin}, \eta_{jnm}^{vmax}, \eta_{jn}^{\xi min}, \eta_{jn}^{\xi max}$	

Table 6: Notation

In order to increase readability, compact notations are used: $\forall i$, or just i in notations $\sum_i f(x)$, substitutes for $\forall i \in I$, unless explicitly denoted by a subset $i \in \Omega_j$.The model is designed as BLP whereas the ULP consists of the profit maximization function of the producers (2a)–(2c) and the LLP defines the market clearing procedure where the MO maximizes social welfare (SW) (2d)–(2j). ULP and LLP are interlinked as the GenCo's determines optimal offer curves subject to their individual constraints in (2a)-(2c) of the ULP, which is then subject to the LLP. In the LLP the MO sets optimal production (2f)-(2g) and transmission (2h)–(2j) quantities and derives the resulting LMPs (2e). The LLP thus directly influences each GenCo's individual profit but all share the same LLP.

$$\textbf{\textit{Maximize}} \sum_{(i\in\Omega_j)b} (\lambda_{(n:i\in\Psi_n)} P_{ib}^G - \lambda_{ib}^G P_{ib}^G) \tag{2a}$$

Subject to: $\quad : \beta_{ji1}^{\alpha}$

$$\alpha_{i1} \geq 0 \quad \forall i \in \Omega_j \qquad : \beta_{jib}^{\alpha} \tag{2b}$$

$$\alpha_{ib} \geq \alpha_{i(b-1)} \quad \forall i \in \Omega_j, b \geq 2 \tag{2c}$$

$$\textbf{\textit{Maximize}} \sum_{dk} \lambda_{dk}^D P_{dk}^D - \sum_{ib} \alpha_{ib} P_{ib}^G \tag{2d}$$

Subject to:

$$\sum_{(d\in\Psi_n)k} P_{dk}^D - \sum_{(i\in\Psi_n)b} P_{ib}^G + \sum_{m\in\Theta_n} B_{nm}(\delta_n - \delta_m) = 0 \quad \forall n \qquad : \lambda_n \tag{2e}$$

$$0 \leq P_{ib}^G \leq P_{ib}^{Gmax} \quad \forall i, b \qquad : \mu_{ib}^{Gmin}, \mu_{ib}^{Gmax} \tag{2f}$$

$$0 \leq P_{dk}^D \leq P_{dk}^{Dmax} \quad \forall d, k \qquad : \mu_{dk}^{Dmin}, \mu_{dk}^{Dmax} \tag{2g}$$

$$B_{nm}(\delta_n - \delta_m) \leq P_{nm}^{max} \quad \forall n, m \in \Theta_n \qquad : \nu_{nm}^{Lmax} \tag{2h}$$

$$-\pi \leq \delta_n \leq \pi \quad \forall n \qquad : \xi_n^{min}, \xi_n^{max} \tag{2i}$$

$$\delta_n = 0 \quad n = 1 \qquad : \xi_{n=1}^1 \tag{2j}$$

To obtain the best offer strategy, each producer determines its block bids according to the profit maximization function (2a). The subscript $i \in \Omega_j$ thereby denotes all generating units belonging to a producer j. Revenues earned by each producer are determined by the LMP obtained as the dual variable λ_n in the LLP. The LMP at node *n* of the relevant generating unit *i* is denoted by $i \in \Psi_n$. Notice that ULP and LLP are linked through several decision variables as e.g. the amount of electricity displaced P_{ib}^G is determined in the LLP which in turn depends on the offer α_{ib} set by the producer. Those offer blocks are set to be positive and stepwise increasing according to (2b)-(2c). For this research, the amount of blocks is set to two. Lastly, the strategic offers do not incorporate financial market contracts and thus exclude contracting obligations such as forwards and futures closed by either producers or consumers.

The MO's market clearing procedure maximizes SW (2d) according to the marginal utility λ_{dk}^D obtained by consumers, in context to the revenue generated by producers. Considering the entire LLP, the MO determines the OPF by taking the producers strategic bids α_{ib} as parameters in its objective function. The LLP thus represents the actual market procedure where the MO collects all bids and then clears the market to derive the OPF. Generation and demand are set by the constraints (2f)-(2g). The model includes a DC representation of Kirchhoff's first and second law. Kirchhoff's junction rule is implemented as the power balance equation for each bus in (2e). The dual variable λ_n corresponds to the LMP at each node. Constraints (2h)-(2j) enforce Kirchhoff's loop rule where δ_n represents the voltage angle at each bus. The power flow through each transmission line is limited by (2h) where B_{nm} is a constant set to 9.14 for all lines. This induces that all lines are assumed to have the same electrical characteristics, and thermal or other losses are neglected. The subscript $m \in \Theta_n$ is thereby used to identify the lines connecting nodes *n* to nodes *m*. The maximal thermal transmission capacity is denoted by P_{nm}^{max}. Voltage angle limits are set by constraint (2i) and node *n = 1* serves as reverence bus (2j).

A characteristic of this model is that both primal and dual variables of the LLP are common for all GenCo's as those variables do not include the index j. Since this market clearing procedure is shared by all producers the model can be considered as a multi-leader-common-follower game [40]. Each producer is thus a leader and the MO is the common follower. As each leader's strategic decisions influence the market, the overall equilibrium is found by concentrating all producers' bi-level models into an EPEC. The thereby derived model can then best be characterized as a "coordination game" that solves a GNE [5].

To solve this EPEC model, [5] applied several steps, of which the final mathematical programming formulation, as it is implemented and used in the further course of this thesis, is presented in the model (3a)-(4m). The first step is to transform the bi-level model into a single level model. Therefore, the LLP is replaced by its first-order necessary optimality conditions. Since the LLP is linear, those conditions are also sufficient conditions of optimality. Taking advantage of this property, [5] replaced the LLP by its optimality conditions which are represented as primal-dual formulation (primal and dual constraints) and the strong duality theorem equality. This formulation applied to a linear program is, according to [51], equivalent to the KKT conditions but provides computational advantages that are utilized by [5]. The resulting single level problem constitutes an MPEC.

The EPEC includes all producers MPECs reformulated as BLPs. Consequently, the BLPs need to be reformulated by replacing each MPEC with its strong stationary conditions. Therefore the MPECs primal constraints (3b) and (3f), dual constraints (3c)-(3e) and the strong duality equality constraint (4j)-(4m) have to be extended through the equalities derived by differentiating the Lagrangian in (3g)-(3u). The non-linear complementary conditions to the EPEC are linearized in (4a)-(4i). Observe that the strong duality equality is non-linear as well and is thus linearized in (4j)-(4m). The Fortuny-Amat and McCarl conversion [44] is thereby used to linearize those complementary conditions, where the parameter values of M^* are chosen to be sufficiently large. It was found that a value of 10,000, applied for all M^*, was adequate for the purpose. As this method is applied to KKT conditions of the form $0 \leq \mu \perp P \geq 0$, the resulting equations are clustered in logical blocks. To linearize the Lagrangian equations containing γ_j^{DT}, [5] treat the dual variable as parameter and fix its value at 5. This approach was sufficiently tested in their paper and found as a valid approach to identify meaningful equilibria. It should be mentioned that the model presented in (3a)-(4m) only contains necessary equations for the MILP formulation. Intermediate steps, such as the formulation of all MPEC constraints, are excluded, since the mathematical proof of this model is not part of the thesis. Please refer to [5] for a detailed introduction to the model. Finally, the formulation of [5] was extended by a market follower constraint (3v) that sets producer J3 as a marginal cost λ_{ib}^{G} bidding actor. This constraint is only active in the oligopoly case.

The MILP formulation can be solved maximizing total profit (TP) of all producers (3a2) or social welfare (SW) in the system (3a1). In this thesis, both objective functions are used to identify if this criteria has an impact on the IIC property.

$$\boldsymbol{Maximize} \sum_{dk} \lambda_{dk}^{D} P_{dk}^{D} - \sum_{ib} \lambda_{ib}^{G} P_{ib}^{G}$$ **SW** (3a1)

$$\boldsymbol{Maximize} \sum_{dk} (\lambda_{dk}^{D} P_{dk}^{D} - \mu_{dk}^{Dmax} P_{dk}^{Dmax}) - \sum_{ib} \lambda_{ib}^{G} P_{ib}^{G} - \sum_{n(m\in\Theta_n)} \nu_{nm}^{Lmax} P_{nm}^{max} - \pi \sum_{n} (\xi_{n}^{max} + \xi_{n}^{min})$$ **TP** (3a2)

Subject to:

MPEC Constraints (primal and dual)

$$\sum_{(d\in\Psi_n)k} P_{dk}^{D} - \sum_{(i\in\Psi_n)b} P_{ib}^{G} + \sum_{m\in\Theta_n} B_{nm}(\delta_m - \delta_n) = 0 \quad \forall n \qquad : \gamma_{jn}^{bal} \qquad (3b)$$

$$\alpha_{ib} - \lambda_{(n:i\in\Psi_n)} + \mu_{ib}^{Gmax} - \mu_{ib}^{Gmin} = 0 \quad \forall i, b \qquad : \gamma_{jib}^{G} \qquad (3c)$$

$$\lambda_{(n:i\in\Psi_n)} - \lambda_{dk}^{D} + \mu_{dk}^{Dmax} - \mu_{dk}^{Dmin} = 0 \quad \forall d, k \qquad : \gamma_{jdk}^{D} \qquad (3d)$$

$$\sum_{m\in\Theta_n} B_{nm}(\lambda_n - \lambda_m) + \sum_{m\in\Theta_n} B_{nm}(\nu_{nm}^{Lmax} - \nu_{mn}^{Lmax}) + \xi_{n}^{max} - \xi_{n}^{min} + \xi_{n=1}^{1} = 0 \quad \forall n \qquad : \gamma_{jn}^{\delta} \qquad (3e)$$

$$\delta_n = 0 \quad n = 1 \qquad : \gamma_{j1}^{\delta} \qquad (3f)$$

EPEC Equalities (Lagrangian)

$$-\sum_{b} P_{(i\in\Omega_j\cap\Psi_n)b}^{G} - \sum_{ib} \gamma_{j(i\in\Psi_n)b}^{G} + \sum_{dk} \gamma_{j(d\in\Psi_n)k}^{D} + \sum_{m\in\Theta_n} B_{nm}(\delta_m - \delta_n) = 0 \quad \forall j, n \qquad (3g)$$

$$-\lambda_{(n:i\in\Psi_n)} + \lambda_{ib}^{G} + \gamma_{j(n:i\in\Psi_n)}^{bal} - \beta_{jib}^{Gmin} + \beta_{jib}^{Gmax} + \gamma_{j}^{DT} \alpha_{ib} = 0 \quad \forall j, i \in \Omega_j, b \qquad (3h)$$

$$-\gamma_{j(n:i\in\Psi_n)}^{bal} - \beta_{jib}^{Gmin} + \beta_{jib}^{Gmax} + \gamma_{j}^{DT} \alpha_{ib} = 0 \quad \forall j, i \notin \Omega_j, b \qquad (3i)$$

$$-\gamma_{j(n:d\in\Psi_n)}^{bal} - \beta_{jdk}^{Dmin} + \beta_{jdk}^{Dmax} - \gamma_{j}^{DT} \lambda_{dk}^{D} = 0 \quad \forall j, d \in \Psi_n, k \qquad (3j)$$

$$\gamma_{jib}^{G} + \gamma_{j}^{DT} P_{ib}^{Gmax} - \gamma_{j}^{DT} \eta_{jib}^{Gmax} = 0 \quad \forall j, i, b \qquad (3k)$$

$$\gamma_{jdk}^{D} + \gamma_{j}^{DT} P_{dk}^{Dmax} - \gamma_{j}^{DT} \eta_{jdk}^{Dmax} = 0 \quad \forall j, d, k \qquad (3l)$$

$$-\gamma_{jib}^{G} - \eta_{jib}^{Gmin} = 0 \quad \forall j, i, b \qquad (3m)$$

$$-\gamma_{jdk}^{D} - \eta_{jdk}^{Dmin} = 0 \quad \forall j, d, k \qquad (3n)$$

$$\gamma_{jib}^{G} + \gamma_j^{DT} P_{ib}^{G} - \beta_{jib}^{\alpha} + \beta_{ji(b+1)}^{\alpha} = 0 \quad \forall j, i \in \Omega_j, b < B \tag{3o}$$

$$\gamma_{jiB}^{G} + \gamma_j^{DT} P_{iB}^{G} - \beta_{jiB}^{\alpha} = 0 \quad \forall j, i \in \Omega_j \tag{3p}$$

$$\sum_{m \in \Theta_n} B_{nm}(\gamma_{jm}^{bal} - \gamma_{jn}^{bal}) + \sum_{m \in \Theta_n} B_{nm}(\beta_{jmn}^{Lmax} - \beta_{jnm}^{Lmax}) + \beta_{jn}^{\delta max} - \beta_{jn}^{\delta min} + \gamma_{j1}^{\delta} = 0 \quad \forall j, n \tag{3q}$$

$$B_{nm}\left(\gamma_{jm}^{\delta} - \gamma_{jn}^{\delta}\right) + \gamma_j^{DT} P_{nm}^{max} - \eta_{jnm}^{vmax} = 0 \quad \forall j, m \in \Theta_n \tag{3r}$$

$$\gamma_{jn}^{\delta} + \gamma_j^{DT} \pi - \eta_{jn}^{\xi max} = 0 \quad \forall j, n \tag{3s}$$

$$-\gamma_{jn}^{\delta} + \gamma_j^{DT} \pi - \eta_{jn}^{\xi min} = 0 \quad \forall j, n \tag{3t}$$

$$\gamma_{jn}^{\delta} = 0 \quad \forall j, n = 1 \tag{3u}$$

$$\alpha_{ib} = \lambda_{ib}^{G} \quad \forall i \in \Omega_{J3}, b \tag{3v}$$

Binary Constraints (linearization)

$$\begin{aligned}
&\alpha_{ib} \geq 0 \quad \forall j, b \\
&\beta_{jib}^{\alpha} \geq 0 \quad \forall j, b \\
&\alpha_{ib} - \alpha_{i(b-1)} \geq 0 \quad \forall j, b \geq 2 \\
&\alpha_{ib} \leq \psi_{jib}^{\alpha} M^{\alpha} \quad \forall j, i \in \Omega_j, b = 1 \\
&\alpha_{ib} - \alpha_{i(b-1)} \leq \psi_{jib}^{\alpha} M^{\alpha} \quad \forall j, i \in \Omega_j, b \geq 2 \\
&\beta_{jib}^{\alpha} \leq \left(1 - \psi_{jib}^{\beta^{\alpha}}\right) M^{\beta^{\alpha}} \quad \forall j, i \in \Omega_j, b = 1 \\
&\beta_{jib}^{\alpha} \leq \left(1 - \psi_{jib}^{\beta^{\alpha}}\right) M^{\beta^{\alpha}} \quad \forall j, i \in \Omega_j, b \geq 2 \\
&\psi_{jib}^{\alpha}, \psi_{jib}^{\beta^{\alpha}} \in \{0,1\}
\end{aligned} \tag{4a}$$

$$\begin{aligned}
&P_{ib}^{G} \geq 0 \quad \forall i, b \\
&P_{dk}^{D} \geq 0 \quad \forall d, k \\
&\beta_{jib}^{Gmin} \geq 0 \quad \forall j, i, b \\
&\beta_{jdk}^{Dmin} \geq 0 \quad \forall j, d, k \\
&P_{ib}^{G} \leq \psi_{jib}^{G^{min}} M^{P} \quad \forall j, i, b \\
&P_{dk}^{D} \leq \psi_{jdk}^{D^{min}} M^{P} \quad \forall j, d, k \\
&\beta_{jib}^{Gmin} \leq \left(1 - \psi_{jib}^{G^{min}}\right) M^{\beta P} \quad \forall j, i, b \\
&\beta_{jdk}^{Dmin} \leq \left(1 - \psi_{jdk}^{D^{min}}\right) M^{\beta P} \quad \forall j, d, k \\
&\psi_{jib}^{G^{min}}, \psi_{jdk}^{D^{min}} \in \{0,1\}
\end{aligned} \tag{4b}$$

$$
\begin{aligned}
& P_{ib}^{Gmax} - P_{ib}^{G} \geq 0 \quad \forall i, b \\
& P_{dk}^{Dmax} - P_{dk}^{D} \geq 0 \quad \forall d, k \\
& \beta_{jib}^{Gmax} \geq 0 \quad \forall j, j, b \\
& \beta_{jdk}^{Dmax} \geq 0 \quad \forall j, d, k \\
& P_{ib}^{Gmax} - P_{ib}^{G} \leq \psi_{jib}^{G^{max}} M^{P} \quad \forall j, i, b \\
& P_{dk}^{Dmax} - P_{dk}^{D} \leq \psi_{jdk}^{D^{max}} M^{P} \quad \forall j, d, k \\
& \beta_{jib}^{Gmax} \leq \left(1 - \psi_{jib}^{G^{max}}\right) M^{\beta P} \quad \forall j, i, b \\
& \beta_{jdk}^{Dmax} \leq \left(1 - \psi_{jdk}^{D^{max}}\right) M^{\beta P} \quad \forall j, d, k \\
& \psi_{jib}^{G^{max}}, \psi_{jdk}^{D^{max}} \in \{0,1\}
\end{aligned}
\tag{4c}
$$

$$
\begin{aligned}
& P_{nm}^{max} - B_{nm}(\delta_m - \delta_n) \geq 0 \quad \forall n, m \in \Theta_n \\
& \beta_{jnm}^{Lmax} \geq 0 \quad \forall j, n, m \in \Theta_n \\
& P_{nm}^{max} - B_{nm}(\delta_m - \delta_n) \leq \psi_{jnm}^{L^{max}} M^{L} \quad \forall j, n, m \in \Theta_n \\
& \beta_{jnm}^{Lmax} \leq \left(1 - \psi_{jnm}^{L^{max}}\right) M^{\beta L} \quad \forall j, n, m \in \Theta_n \\
& \psi_{jnm}^{L^{max}} \in \{0,1\}
\end{aligned}
\tag{4d}
$$

$$
\begin{aligned}
& \pi - \delta_n \geq 0 \quad \forall n \\
& \pi + \delta_n \geq 0 \quad \forall n \\
& \beta_{jn}^{\delta min} \geq 0 \quad \forall j, n \\
& \beta_{jn}^{\delta max} \geq 0 \quad \forall j, n \\
& \pi + \delta_n \leq \psi_{jn}^{\delta min} M^{\delta} \quad \forall j, n \\
& \pi - \delta_n \leq \psi_{jn}^{\delta max} M^{\delta} \quad \forall j, n \\
& \beta_{jn}^{\delta min} \leq \left(1 - \psi_{jn}^{\delta min}\right) M^{\beta\delta} \quad \forall j, n \\
& \beta_{jn}^{\delta max} \leq \left(1 - \psi_{jn}^{\delta max}\right) M^{\beta\delta} \quad \forall j, n \\
& \psi_{jn}^{\delta min}, \psi_{jn}^{\delta max} \in \{0,1\}
\end{aligned}
\tag{4e}
$$

$$
\begin{aligned}
& \nu_{nm}^{Lmax} \geq 0 \quad \forall n, m \in \Theta_n \\
& \eta_{jnm}^{\nu max} \geq 0 \quad \forall j, n, m \in \Theta_n \\
& \nu_{nm}^{Lmax} \leq \psi_{jnm}^{\nu^{max}} M^{\nu} \quad \forall j, n, m \in \Theta_n \\
& \eta_{jnm}^{\nu max} \leq \left(1 - \psi_{jnm}^{\nu^{max}}\right) M^{\eta\nu} \quad \forall j, n, m \in \Theta_n \\
& \psi_{jnm}^{\nu^{max}} \in \{0,1\}
\end{aligned}
\tag{4f}
$$

$$
\begin{aligned}
&\mu_{ib}^{Gmin} \geq 0 \quad \forall i, b \\
&\mu_{dk}^{Dmin} \geq 0 \quad \forall d, k \\
&\eta_{jib}^{Gmin} \geq 0 \quad \forall j, i, b \\
&\eta_{jdk}^{Dmin} \geq 0 \quad \forall j, d, k \\
&\mu_{ib}^{Gmin} \leq \psi_{jib}^{\mu G^{min}} M^{\mu} \quad \forall j, i, b \\
&\mu_{dk}^{Dmin} \leq \psi_{jdk}^{\mu D^{min}} M^{\mu} \quad \forall j, d, k \\
&\eta_{jib}^{Gmin} \leq \left(1 - \psi_{jib}^{\mu G^{min}}\right) M^{\eta\mu} \quad \forall j, i, b \\
&\eta_{jdk}^{Dmin} \leq \left(1 - \psi_{jdk}^{\mu D^{min}}\right) M^{\eta\mu} \quad \forall j, d, k \\
&\psi_{jib}^{\mu G^{min}}, \psi_{jdk}^{\mu D^{min}} \in \{0,1\}
\end{aligned}
\tag{4g}
$$

$$
\begin{aligned}
&\mu_{ib}^{Gmax} \geq 0 \quad \forall i, b \\
&\mu_{dk}^{Dmax} \geq 0 \quad \forall d, k \\
&\eta_{jib}^{Gmax} \geq 0 \quad \forall j, j, b \\
&\eta_{jdk}^{Dmax} \geq 0 \quad \forall j, d, k \\
&\mu_{ib}^{Gmax} \leq \psi_{jib}^{\mu G^{max}} M^{\mu} \quad \forall j, i, b \\
&\mu_{dk}^{Dmax} \leq \psi_{jdk}^{\mu D^{max}} M^{\mu} \quad \forall j, d, k \\
&\eta_{jib}^{Gmax} \leq \left(1 - \psi_{jib}^{\mu G^{max}}\right) M^{\eta\mu} \quad \forall j, i, b \\
&\eta_{jdk}^{Dmax} \leq \left(1 - \psi_{jdk}^{\mu D^{max}}\right) M^{\eta\mu} \quad \forall j, d, k \\
&\psi_{jib}^{\mu G^{max}}, \psi_{jdk}^{\mu D^{max}} \in \{0,1\}
\end{aligned}
\tag{4h}
$$

$$
\begin{aligned}
&\xi_{n}^{min} \geq 0 \quad \forall n \\
&\xi_{n}^{max} \geq 0 \quad \forall n \\
&\eta_{jn}^{\xi min} \geq 0 \quad \forall j, n \\
&\eta_{jn}^{\xi max} \geq 0 \quad \forall j, n \\
&\xi_{n}^{min} \leq \psi_{jn}^{\xi^{min}} M^{\xi} \quad \forall j, n \\
&\xi_{n}^{max} \leq \psi_{jn}^{\xi^{max}} M^{\xi} \quad \forall j, n \\
&\eta_{jn}^{\xi min} \leq \left(1 - \psi_{jn}^{\xi^{min}}\right) M^{\eta\xi} \quad \forall j, n \\
&\xi_{n}^{max} \leq \left(1 - \psi_{jn}^{\xi^{max}}\right) M^{\eta\xi} \quad \forall j, n \\
&\psi_{jn}^{\xi^{min}}, \psi_{jn}^{\xi^{max}} \in \{0,1\}
\end{aligned}
\tag{4i}
$$

$$P_{ib}^{G} \leq \psi_{ib}^{\omega G^{min}} M^{\omega} \quad \forall i, b$$
$$P_{dk}^{D} \leq \psi_{dk}^{\omega D^{min}} M^{\omega} \quad \forall d, k$$
$$\mu_{ib}^{Gmin} \leq \left(1 - \psi_{ib}^{\omega G^{min}}\right) M^{\Omega} \quad \forall i, b \tag{4j}$$
$$\mu_{dk}^{Dmin} \leq \left(1 - \psi_{dk}^{\omega D^{min}}\right) M^{\Omega} \quad \forall d, k$$
$$\psi_{ib}^{\omega G^{min}}, \psi_{dk}^{\omega D^{min}} \in \{0,1\}$$

$$P_{ib}^{Gmax} - P_{ib}^{G} \leq \psi_{ib}^{\omega G^{max}} M^{\omega} \quad \forall i, b$$
$$P_{dk}^{Dmax} - P_{dk}^{D} \leq \psi_{dk}^{\omega D^{max}} M^{\omega} \quad \forall d, k$$
$$\mu_{dk}^{Dmin} \leq \left(1 - \psi_{ib}^{\omega G^{max}}\right) M^{\Omega} \quad \forall i, b \tag{4k}$$
$$\mu_{dk}^{Dmax} \leq \left(1 - \psi_{dk}^{\omega D^{max}}\right) M^{\Omega} \quad \forall d, k$$
$$\psi_{ib}^{\omega G^{max}}, \psi_{dk}^{\omega D^{max}} \in \{0,1\}$$

$$P_{nm}^{max} - B_{nm}(\delta_m - \delta_n) \leq \psi_{nm}^{\nu L^{max}} M^{\omega L} \quad \forall n, m \in \Theta_n$$
$$\nu_{nm}^{Lmax} \leq \left(1 - \psi_{nm}^{\nu L^{max}}\right) M^{\Omega L} \quad \forall n, m \in \Theta_n \tag{4l}$$
$$\psi_{nm}^{\nu L^{max}} \in \{0,1\}$$

$$\pi + \delta_n \leq \psi_{n}^{\xi \delta min} M^{\omega \delta} \quad \forall n$$
$$\pi - \delta_n \leq \psi_{n}^{\xi \delta max} M^{\omega \delta} \quad \forall n$$
$$\xi_{n}^{min} \leq \left(1 - \psi_{jn}^{\xi \delta min}\right) M^{\Omega \delta} \quad \forall n \tag{4m}$$
$$\xi_{n}^{max} \leq \left(1 - \psi_{jn}^{\xi \delta max}\right) M^{\Omega \delta} \quad \forall n$$
$$\psi_{n}^{\delta min}, \psi_{n}^{\delta max} \in \{0,1\}$$

4.2 Implementation

As it is unknown when (and if) the power flow on a given line changes as a result of the IIC property, the transmission capacity needs to be reduced gradually, until the transmission capacity is equal to the constrained utilization. Thus, a considerable number of numerical runs with different input parameters is required. In general, the implementation can be interpreted as a process: defining parameters (P_{nm}^{max}), writing the input files for the optimization solver, running the solver, analysing the results and finally, adjusting the input parameters if necessary. This process is displayed in Figure 6. To optimize the implementation process and accurately document the several hundred numerical runs, an automated implementation workflow was designed. Thereby Microsoft Excel is used to define and manipulate all input parameters given in Subsection 3.2. Furthermore, Visual Basic for Applications (VBA) was used to write an automated script that reads a defined number of instances and forwards them to the optimization solver. The mathematical programming formulation of Section 4.1, equations (3a)-(4m), is coded using AMPL (A Modeling Language for Mathematical Programming) [52], with CPLEX 12.6.0 [43] as solver. The graphical user interface of AMPL IDE 1.0 was used for coding and operation of the solver. After the simulation runs are completed, the VBA script collects the results and imports them into Microsoft Excel. There, they are filled in predefined formats and diagrams to enable fast analysis and interpretation. Depending on the results, parameters are then modified, e.g. lower the transmission capacity P_{nm}^{max} threshold even further, and the process is continued. Finally, the results are stored in Excel to document the research process and ensure tractability throughout the research.

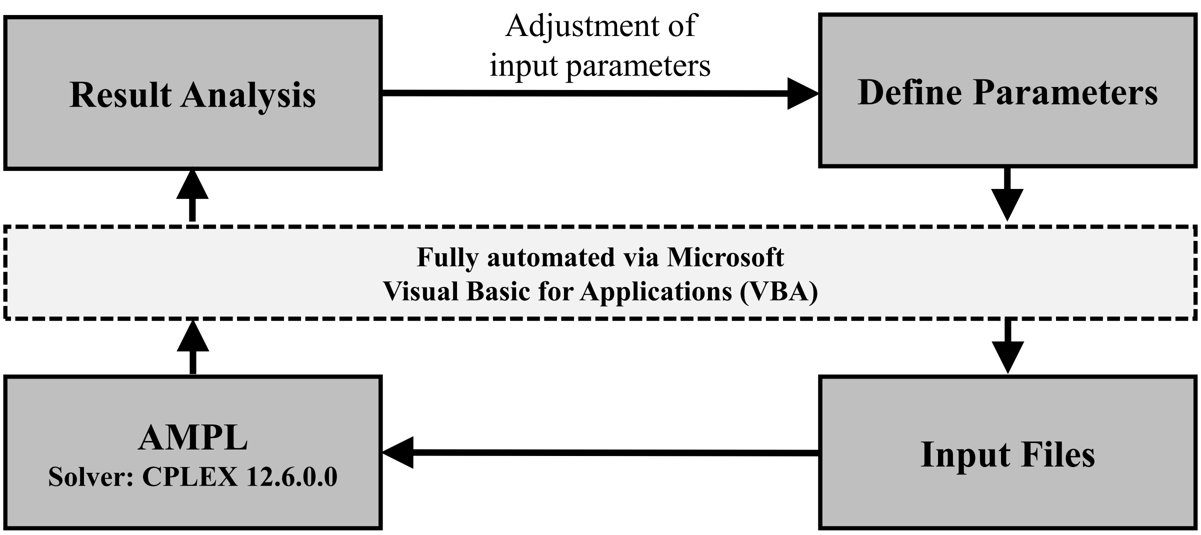

Figure 6: Implementation process.

A detailed visualization of the research implementation can be found in the screenshots presented in Figure 7. The upper left corner displays the input parameters where P_{nm}^{max} can be gradually decreased in each run. In this example P_{1-3}^{max} is decreased through a factor multiple (e.g. 99%) from 30 MW in the 1st to 27.45 MW in the 10th instance. Those input parameters are forwarded to AMPL, shown in the upper right corner. Here, the numerical runs for all 10 instances are conducted. Once the simulation runs are completed, the results are collected and loaded in the analysis page of Microsoft Excel (lower half of Figure 7). This page lists all relevant parameters to analyse the equilibria of all 10 instances (indicated by their P_{1-3}^{max} value at that point). The analysis is further supported by graphs for e.g. income distribution, production per node, generating unit utilization and power flow over transmission lines.

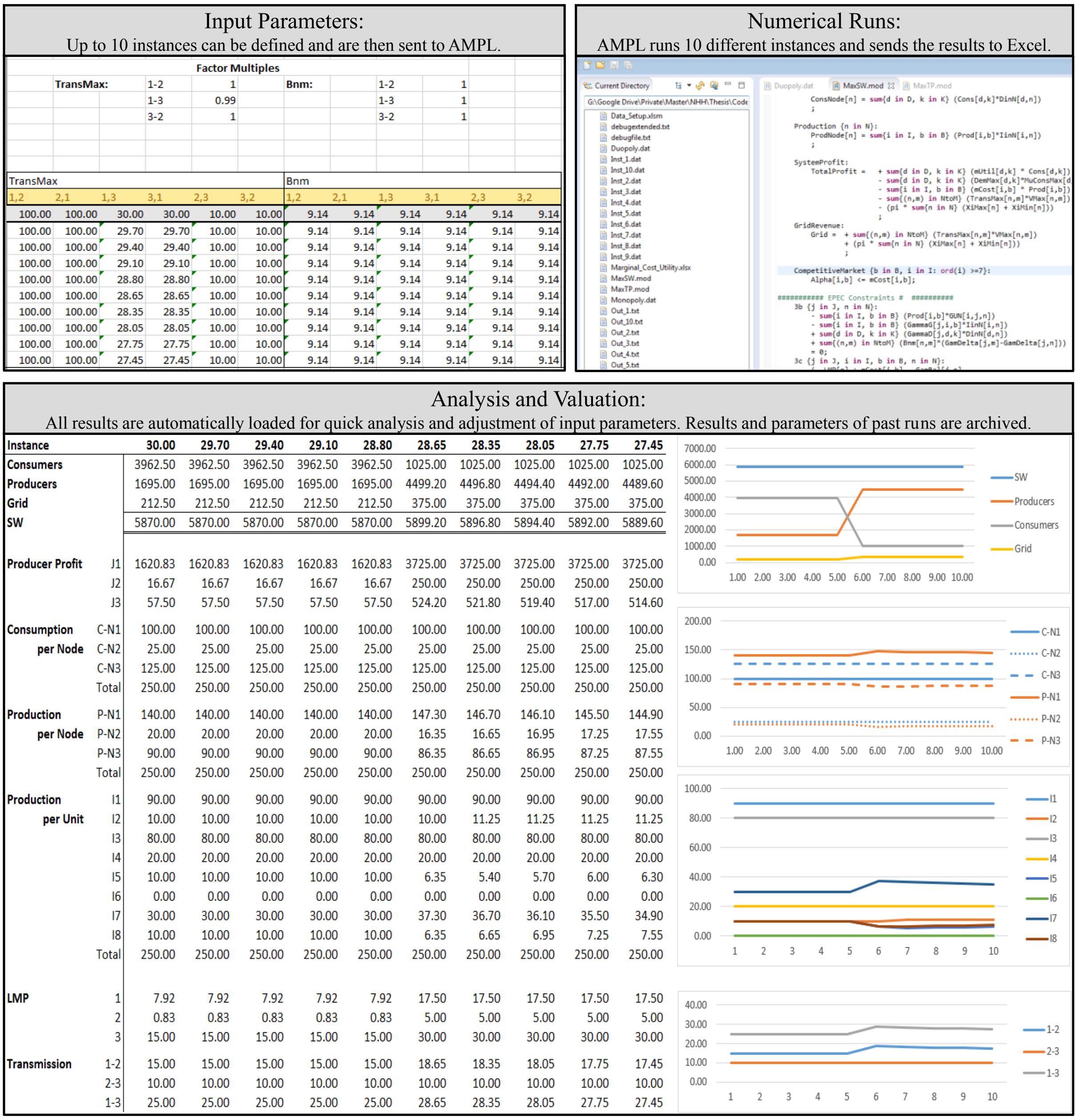

Figure 7: Research implementation screenshots.

5. Numerical results and analysis

As the objective of this thesis is twofold, several numerical runs and analysis on different scenarios have to be undertaken. First, in order to verify if a complex EPEC model like introduced in Section 4 is subject to the IIC property, the three step process introduced in Section 3.1 is applied on the oligopoly scenario. Step 1 of the process requires a solid understanding of the uncongested equilibria. Thus, the analysis begins with the uncongested solution for all scenarios (oligopoly & triopoly) and settings used in the further course of the thesis. Second, in case the model is found to be not IIC, insight on the impact of this mathematical phenomenon on electricity market models should be gained. Hence, numerous tests on different transmission lines, using a variety of capacity parameters, need to be conducted. To cover a greater range of possible impact factors, the IIC analysis is applied to both scenarios using the two different objective functions of maximizing SW, equation (3a1), and maximizing TP, equation (3a2), respectively. Due to the large amount of data, the analysis only highlights results. For further details, please refer to the Appendix.

5.1 Uncongested network

The uncongested solution provides a reference point, because it sets a benchmark for the income distribution according to consumer and producer surplus as well as grid revenue (congestion rent). Results for the oligopoly and triopoly scenarios are presented in Table 7 and split according to the selected objective function SW or TP. Grid revenue is in all cases zero, since power can flow freely and the surplus of other parties is not reduced. In general, findings of both [5] and [7] regarding income distribution for uncongested cases could be confirmed. More competitive markets lead to a surplus redistribution from producers to consumers. But it is to add that this observation depends on the applied objective function.

Scenario	**Oligopoly**		**Triopoly**	
	Objective function			
Distribution	**SW**	**TP**	**SW**	**TP**
Consumer	5150	2900	5150	750
Producer	830	3050	830	4860
Grid	0	0	0	0
Social Welfare	5980	5950	5980	5610
Line				
1-2	5.0	11.7	5.0	1.7
2-3	20.0	16.7	20.0	21.7
1-3	25.0	28.3	25.0	23.3

Table 7: Unconstrained solutions for all scenarios.

Results for maximizing SW were found equal in both scenarios. Optimizing total producer profit (TP) does however lead to significantly different outcomes. To begin with, SW is lower in both cases but in the triopoly scenario, the additional strategic market participant further decreases SW. Moreover, a shift from consumer to producer surplus can be identified. But for the oligopoly case, market power of strategic producers is reduced and consequently SW increased. Thus, the redistribution effect is lowered. Lastly, the power flow $P_{nm} = B_{nm}(\delta_n - \delta_m)$ displayed in Table 7 is the reference P_{nm} to apply the three step process.

5.2 Congestion and IIC

To identify if the model is IIC the same scenario as in [7] is applied. In this scenario, line 1-3 was constrained to $P_{1-3}^{max} = 20\ MW$ and line 1-2 was set to be the IC. The test is carried out under the oligopoly scenario and the objective function is set to maximize SW, because this case replicates the uncongested and congested solutions of [7] in the best possible way. As described in Section 3.1, P_{1-2}^{max} is now gradually reduced from $P_{1-2}^{max} = 5\ MW$ until the point when the IC becomes active. Figure 8 displays P_{nm} of each line whereas the X-axis lists P_{nm}^{max} of the IC line 1-2. Note that the long-dashes indicate that this line is the active constraint (from now on referred to as P_{nm}^{Ξ} – here line 1-3) and the short-dashes line indicate that this line is the currently tested IC (line 1-2). Figure 9 shows the income redistribution according to consumer and producer surplus, as well as grid revenue. Here too, P_{nm}^{max} of the IC constraint is displayed on the X-axis. Those two figures confirm that the EPEC is not IIC and that the irrelevant constraint becomes active in the same manner as found by [7]. Strikingly, this effect appears to take place in an abrupt way, as it spikes at $P_{nm}^{max} = P_{nm} = 4.5\ MW$.

Numerical results for: Oligopoly / Objective: max SW / Congested line: 1-3 / IC line: 1-2

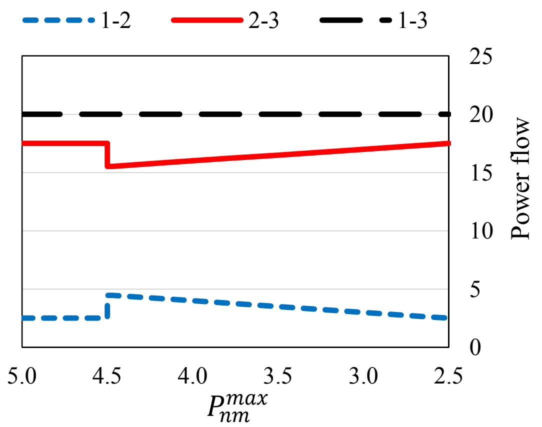

Figure 8: Line utilization with active IC.

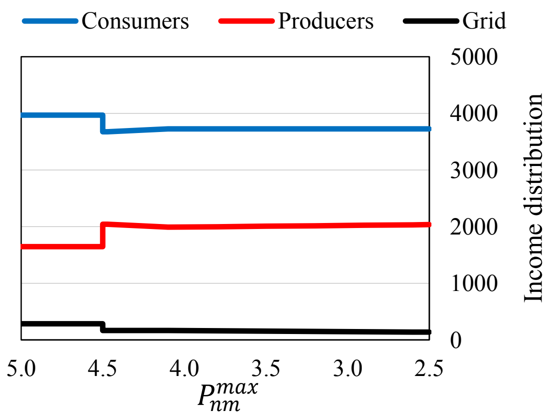

Figure 9: Income distribution with active IC.

The numerical results for the line 12 IIC test are summarized in Table 8. This layout will further be used to study IC lines for their impact on the equilibrium. Similar to the findings in [7], where total producer profit increased by 15.4% after the IC became active, it can be shown that producer profit increased by 18.8%, compared to the constrained solution. SW declined by 20 € and consequently, consumer surplus is reduced. This decrease reflects the same 0.3% reduction found by [7]. Reducing P_{nm}^{max} until $P_{nm}^{max} = P_{nm}^{Const} = P_{nm} = 2.5\ MW$ shows that even though P_{nm}^{max} gradually decreases, the redistributed income remains fairly constant. This is especially interesting for the point when $P_{nm}^{max} = P_{nm}^{Const}$, as it would be expected that the equilibrium is equal to the one found in P_{nm}^{Const}. However, since the IC is now actively constraining the model, the P_{nm}^{Const} constrained equilibrium cannot be reached.

	Constrained		**Irrelevant Constraint**			**Constrained**	**Irrelevant Constraint**	**Delta**
Line	P_{nm}^{Ξ}	P_{nm}^{Const}	P_{nm}^{max}	P_{nm}	Income redistribution			
1-2		2.5	**4.5**	4.5	Consumer	3913	3675	-237
2-3		17.5		15.5	Producer	1718	2041	323
1-3	20.0	20.0	20.0	20.0	Grid	270	165	-105
					SW	**5900**	**5880**	**-20**

Table 8: Numerical result for IC 1-2. The table lists power flow and income distribution for the constrained result (step 2) and the P_{nm}^{max} where the IC became active (bold).

To gain further insight in how the equilibrium changes, once an IC becomes active, producers bidding behaviour needs to be analysed. All accepted bids which were submitted to the MO are listed in Figure 10 (congested case) and Figure 11 (active IC). Producers are thereby indicated by colour. Since bids are submitted for each generating unit *i* belonging to producer *j*, the marginal cost related to that unit are presented in grey dots.

Bidding behaviour: Oligopoly / Objective: max SW / Congested line: 1-3 / IC line: 1-2

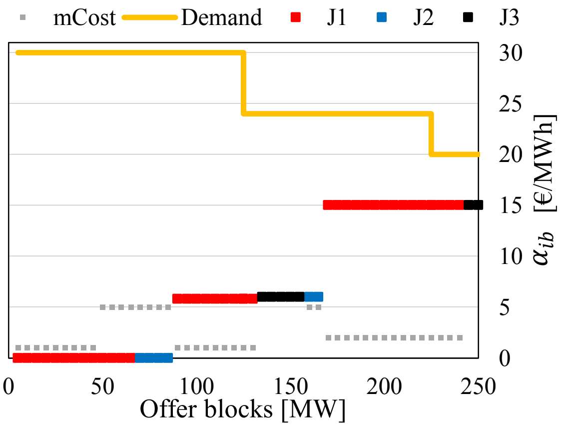

Figure 10: Accepted bids under P_{nm}^{Const}.

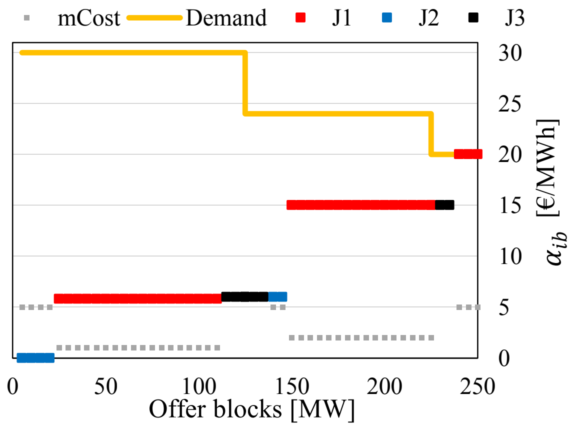

Figure 11: Accepted bids for active IC.

For further investigation of bidding behaviour, it is important to recall the general principle applied when the MO clears the market. In the model, P_{nm} and λ_n (LMP) are treated as dual variables common to all producers and are set by the MO. In order to economically interpret the effect taking place when the IC on P_{nm} becomes active, a simplified interpretation of the model can be considered. Assume that the MO is forced by the active IC to utilize the respective line at P_{nm}^{max} (in this case 1-2 at 4.5 MW). Further note that the equilibrium formation in the model can be interpreted as an iterative process and the objective function is the maximization of SW. Additionally, it needs to be taken into consideration that node 1 is the node with the least expensive generation capacity, node 2 medium-cost and node 3 the most costly node. Since producers are now confronted with a situation where the power flow in line 1-2 is set to a higher value, they revise their offers. As the producers can access more of the less expensive capacity in node 1 and discharge generating units in node 2, producers profit increases. Additionally, since the market follower has no capacity in node 2, there are no lower bids that could counter act their high priced offers and the follower is pushed out of the market. This results in the highest accepted bid being priced equal to marginal utility and higher LMPs. Where the LMPs for node 1 were set at 6 €/MWh, node 2 at 10.5 €/MWh and node 3 at 15 €/MWh in the constrained solution, LMPs are now 6 €/MWh, 20 €/MWh and 15 €/MWh respectively. Also, generation has increased in node 1 by 1.6%, in node 3 by 2.3% and decreased in node 2 by 10.1%. As producer J1 captures now most of the market share (according to bids accepted in Figure 11) he could increase his profit by 8.1% to 1730.9 €. The most benefiting party in this new equilibrium is however producer J2. A strategically placed offer (below marginal cost) allows him to enlarge his market share, heave the LMP in node 2 to 20 €/MWh and hence 2.6 fold its profit from 117.5 € to 309.5 €. The changes in this new equilibrium can thus mostly be attributed to the lower production in node 2 and the fact that producer J2 could, through strategic offering, utilize its capacity in node 1.

To further proceed with the presentation of results, an interims conclusion is required. It could be proven that the EPEC model is not IIC, as a seemingly irrelevant constraints lead to an altered optimal solution. The IC on line 1-2 was rendered active, as soon as a certain threshold value for P_{nm}^{max} was reached, what replicates the findings of [7]. The approach can thus be accepted as valid. Recall that it is not yet researched if every subset of transmission lines in a model can constitute an IC. Also, it is unknown how the trigger value P_{nm}^{max} behaves on those lines. Therefore, a sensitivity analysis on all combinations of lines is presented in the following subsections.

5.2.1 Transmission line analysis

In a first instance, it is of importance to extend the research of [7] and test every line for the IIC property. Therefore, the three step process is applied in the known manner. Results are presented in Table 9, where each bracket represents an individual test case. The first bracket shows the counterpart to the previously tested case, whereas here line 2-3 (and not 1-2) is being verified as the IC. Observe that the line which is fixed for step 1 is identified as P_{nm}^{Ξ} and that the line representing the IC is marked bold in the column labelled P_{nm}^{max}. If the IC is found to have an impact on the system, the effect is displayed on the right side of Table 9, where the redistribution of income is listed. As for the IC test on line 2-3, constrained line P_{1-3}^{Ξ}, it can be shown that the model is in fact not depending on this IC. The line is marked as IIC for P_{2-3}^{max}, since no change in the equilibrium was found until $P_{2-3}^{max} = P_{2-3}^{Const} = P_{2-3} = 17.5\ MW$. Consequently, even though the model in general is not IIC, the specific outcome depends on each line. Hence, the IIC property needs to be considered as subject to subsets of constraints. Otherwise, the model could mistakenly be labeled as IIC, if only a subset of lines is considered for the test. To gain insight into this attribute the next line P_{2-3}^{Ξ} is analyzed.

Line	Constrained P_{nm}^{Ξ}	Constrained P_{nm}^{Const}	Irrelevant Constraint P_{nm}^{max}	Irrelevant Constraint P_{nm}		Constrained (Income redistribution)	Irrelevant Constraint (Income redistribution)	Delta (Income redistribution)
1-2		2.5		2.5	Consumer	3913	3913	0
2-3		17.5	**IIC**	17.5	Producer	1718	1718	0
1-3	20.0	20.0	20.0	20.0	Grid	270	270	0
					SW	**5900**	**5900**	**0**
1-2		12.5	**13.7**	13.7	Consumer	2900	3275	375
2-3	15.0	15.0	15.0	15.0	Producer	3025	2457	-569
1-3		27.5		28.7	Grid	0	206	206
					SW	**5925**	**5937**	**12**
1-2		12.5		13.7	Consumer	2900	3650	750
2-3	15.0	15.0	15.0	15.0	Producer	3025	2062	-963
1-3		27.5	**28.7**	28.7	Grid	0	225	225
					SW	**5925**	**5937**	**12**
1-2	3.0	3.0	3.0	3.0	Consumer	3925	3925	0
2-3		21.0	**IIC**	21.0	Producer	1950	1950	0
1-3		24.0		24.0	Grid	63	63	0
					SW	**5938**	**5938**	**0**
1-2	3.0	3.0	3.0	3.0	Consumer	3925	3925	0
2-3		21.0		21.0	Producer	1950	1950	0
1-3		24.0	**IIC**	24.0	Grid	63	63	0
					SW	**5938**	**5938**	**0**

Table 9: IC analysis for oligopoly scenario under maximization of SW.

Setting line 2-3 as the constrained line ($P_{2-3}^{\Xi} = 15\ MW$) results in a noteworthy scenario for three reasons. First and foremost, consumer surplus benefits from active ICs on line 1-2 and 1-3 (Figure 12 & Figure 13). This is contradictory to the findings of [7] where the IIC property was identified to contribute solely to producer surplus. Furthermore, SW increased by 12 € to 5937 €, raising the question why this equilibrium was not identified beforehand.

Numerical results for: Oligopoly / Objective: max SW / Congested line: 2-3 / IC line: 1-2

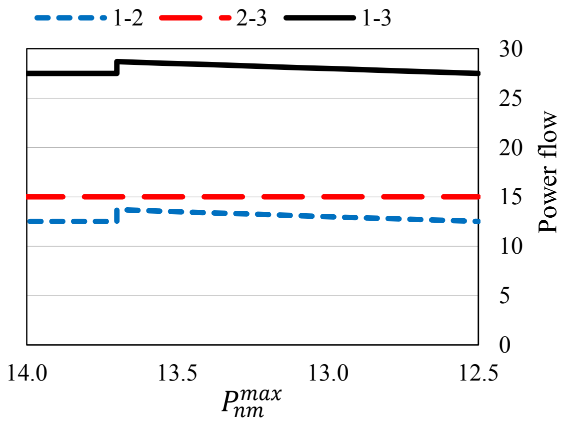

Figure 12: Line utilization IC line 1-2.

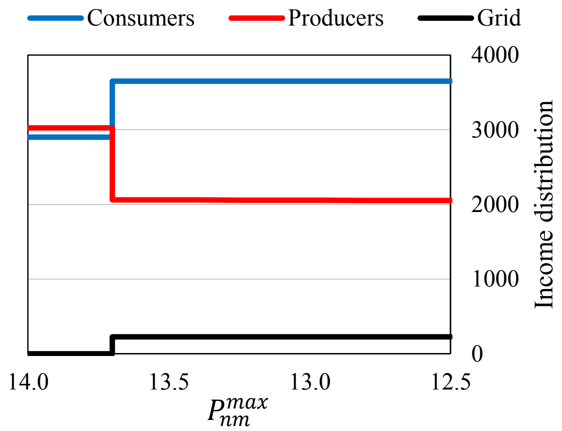

Figure 13: Income distribution IC line 1-2.

Second, the threshold value triggering the IC is mirrored for line 1-2 and 1-3 respectively. Line 1-2 becomes an active IC at 13.7 MW, where the power flow on line 1-3 is set to 28.7 MW. This reflects also the IC threshold value on line 1-3. Third, as shown in the income distribution and delta column, even though the IC lines are mirrored and SW is equal in both cases, the equilibrium is composed differently. To understand how this shift from producer to consumer surplus is assembled, bidding behavior as shown in Figure 14 and Figure 15 needs to be considered. For P_{nm}^{Const} the market follower J3 sets the LMP by bidding at marginal cost.

Bidding behaviour: Oligopoly / Objective: max SW / Congested line: 2-3 / IC line: 1-2

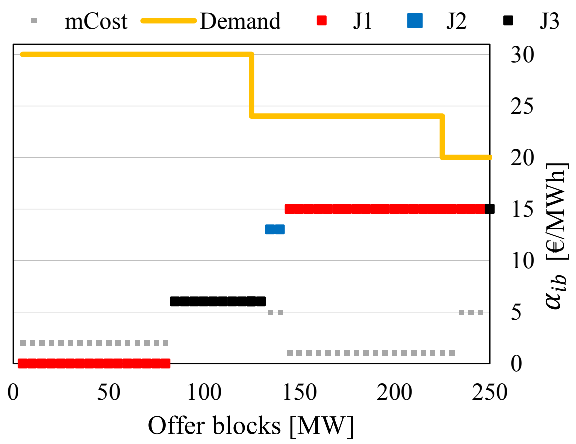

Figure 14: Accepted bids P_{nm}^{Const} line 2-3.

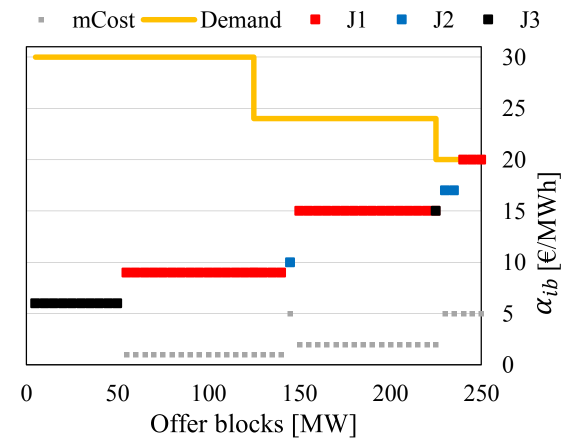

Figure 15: Accepted bids with active IC.

It is interesting to note that once more, node 2 plays an essential role in shifting the equilibrium once an IC is active (either 1-2 or 1-3). Independent of which line is set as active IC, the MO is forced to increase transmission to node 2. Thus, production in node 2 decreases by 4.4% whereas the generating units in node 1 act as substitutes. This leads to higher competition between producers J1 and J2. According to the bidding curve, J2 is awarded with the increased supply from node 1. Subsequently, stronger competition on node 1 reduces the LMP from 15 €/MWh to 10 €/MWh, shifting the surplus heavily in the favor of consumers.

Considering those findings, it can be concluded that the equilibrium in case P_{2-3}^{Ξ} constitutes a saddle point or local optimum. Furthermore, due to the complexity of this EPEC model, and as mentioned by [5], the constrained optimum (under P_{nm}^{Const}) must not necessarily hold as a global optimum. By forcing the IC active, an equilibrium with higher SW was found. Also, it could be verified that the relation between the actors in node 1 and node 2 plays a crucial role in this framework. Especially in the context that producer J3 does not feature any generation capacity in node 2. This can be affirmed by defining line 1-2 as P_{nm}^{Ξ} and testing the other lines for active ICs. As shown in Table 9 both lines are IIC when the power flow on line 1-2 is constrained to 3 MW. It seems to be illogical that the same balancing effect between node 1 and node 2 does not occur between node 2 and node 3 but under consideration of the regional cost structure, this effect becomes quite obvious. Node 3 is known to be the high cost region and the unit with the lowest marginal cost I3 ($\lambda_{I3b}^{G} = 2\ €/MWh$) is already fully utilized. Thus, a shift from node 3 to node 2 would not contribute to a higher SW equilibrium and the lines are IIC. This also explains why line 2-3 was found IIC in the case P_{1-3}^{Ξ} as it reflects the same situation.

It can be proven that the IIC property is applicable for subsets of constraints. However, whether a certain line is found IIC or not depends on the parameters in the model. This contributes as a numerical prove to the general description of Macal and Hurter [6]. As the impact of ICs could now be revealed, a question emerges on how the property affects the new IC equilibrium. This context was not further researched by [6]. Even though the authors could mathematically prove that the IIC phenomenon exists, they did not highlight how certain constraints change the optimal solution. Thus, in order to further understand in what extent the model is influenced by active ICs, a sensitivity analysis was performed.

5.2.2 Sensitivity analysis

Recall that IIC property requires a constraint to be active in the first place. For this analysis, a transmission line P_{nm}^{Ξ} was constrained at a level that was lower than the power flow in the unconstrained solution. Consequently, it is of interest, if the magnitude of P_{nm}^{Ξ} affects the impact of an active IC. Thus, to test for the IIC property's sensitivity, a stepwise reduction of P_{nm}^{Ξ} is applied (indexed as $P_{nm_p}^{\Xi}$). For each new $P_{nm_{p+1}}^{\Xi} < P_{nm_0}^{\Xi}$ the three step process is carried out as aforementioned. Table 10 displays the results for the first applied case P_{1-3}^{Ξ} and line 1-2 set as IC. Note that each bracket represents a $P_{1-3_p}^{\Xi}$ reduced by 2 MW, starting from the base case $P_{1-3_0}^{\Xi} = 20\ MW$.

	Constrained		Irrelevant Constraint			Constrained	Irrelevant Constraint	Delta
Line	$P_{1-3_p}^{\Xi}$	P_{nm}^{Const}	P_{nm}^{max}	P_{nm}	Income redistribution			
1-2		2.5	**4.5**	4.5	Consumer	4025	3675	-350
2-3		17.5		15.5	Producer	1575	2041	466
1-3	20.0	20.0	20.0	20.0	Grid	300	165	-135
					SW	**5900**	**5880**	**-20**
1-2		1.5	**3.5**	3.5	Consumer	3969	3775	-194
2-3		16.5		14.5	Producer	1645	1915	270
1-3	18.0	18.0	18.0	18.0	Grid	257	160	-97
					SW	**5870**	**5850**	**-20**
1-2		0.5	**2.5**	2.5	Consumer	4025	3675	-350
2-3		15.5		13.5	Producer	1575	2035	460
1-3	16.0	16.0	16.0	16.0	Grid	240	111	-129
					SW	**5840**	**5820**	**-20**

Table 10: Sensitivity analysis under P_{1-3}^{Ξ} with IC on line 1-2 applying objective function max SW.

Two characteristics thereby strike out as worth mentioning. First, a reduction of available transmission capacity negatively affects SW. But the striking finding is that the impact of an active IC seems to be constant in the range of -20 € SW. Furthermore, the income redistribution always shifts towards producers but the extent is different for all cases. Second, the IC is activated at $P_{nm}^{max} = P_{nm}^{Const} + 2\ MW$ for every $P_{1-3_p}^{\Xi}$, whereas P_{nm}^{max} refers to the threshold value when the IC becomes active. To identify if such a threshold value can be generalized for the IIC property, line 2-3 is next set as $P_{nm_p}^{\Xi}$ and the same procedure is applied. Results are displayed in Table 11. In this case, both lines 1-2 and 1-3 are tested as IC and are displayed in the same manner as introduced earlier.

Interesting to mention beforehand is that the mirror effect holds for all $P^{\Xi}_{2-3_p}$. Furthermore, it can be confirmed that the found equilibrium remains a local optimum or saddle. Considering the impact of $P^{\Xi}_{2-3_p}$ a contradictory finding to one stated above can be shown. The stronger the constraint of $P^{\Xi}_{2-3_p}$ the higher the impact on SW. However, analyzing the two cases in joint consideration leads to another finding. For this model and test environment, the impact on SW is equal to $(P^{max}_{nm} - P^{Const}_{nm})$*10. However, due to the complex nature of the EPEC model, it cannot be reliably proven if this finding acts as a generally applicable state of the IIC property. The results in Table 11 do however confirm an outcome introduced earlier. Observe, that the power flow through line 1-3 (P_{1-3}) remains constant for all cases $P^{\Xi}_{2-3_p}$ but P_{1-2} increases continuously, as $P^{\Xi}_{2-3_p}$ is reduced. This supports the conclusion that the impact of an active IC depends not only on the model but also on the parameters.

	Constrained		**Irrelevant Constraint**			**Constrained**	**Irrelevant Constraint**	**Delta**
Line	$P^{\Xi}_{2-3_p}$	P^{Const}_{nm}	P^{max}_{nm}	P_{nm}	Income redistribution			
1-2		12.5	**13.7**	13.7	Consumer	2900	3275	375
2-3	15.0	15.0	15.0	15.0	Producer	3025	2457	-569
1-3		27.5		28.7	Grid	0	206	206
					SW	**5925**	**5937**	**12**
1-2		12.5		13.7	Consumer	2900	3650	750
2-3	15.0	15.0	15.0	15.0	Producer	3025	2062	-963
1-3		27.5	**28.7**	28.7	Grid	0	225	225
					SW	**5925**	**5937**	**12**
1-2		13.8	**16.2**	16.2	Consumer	2900	3650	750
2-3	12.5	12.5	12.5	12.5	Producer	2988	2074	-914
1-3		26.3		28.5	Grid	0	188	188
					SW	**5888**	**5912**	**24**
1-2		13.8		16.2	Consumer	2900	3650	750
2-3	12.5	12.5	12.5	12.5	Producer	2988	2074	-913
1-3		26.3	**28.7**	28.7	Grid	0	188	188
					SW	**5888**	**5912**	**24**
1-2		15.0	**18.7**	18.7	Consumer	2900	3275	375
2-3	10.0	10.0	10.0	10.0	Producer	2950	2332	-618
1-3		25.0		28.7	Grid	0	280	280
					SW	**5850**	**5887**	**37**
1-2		15.0		18.7	Consumer	2900	3650	750
2-3	10.0	10.0	10.0	10.0	Producer	2950	2087	-863
1-3		25.0	**28.7**	28.7	Grid	0	150	150
					SW	**5850**	**5887**	**37**

Table 11: Sensitivity analysis P^{Ξ}_{2-3} with IC on line 1-2 & 1-3 applying objective function max SW.

5.3 Analyzing IIC: Oligopoly

All prior tests have been carried out under the oligopoly scenario maximizing SW. It was found that the scenario is characterized by the strategic games between producers J1 & J2. Production in and transmission to node 2 was identified as essential, since the two GenCo's share production units in this node. Additionally, J3 plays a crucial role in limiting market power of J1 & J2, as he provides substantial capacity at marginal cost. The effect of the IIC property is hence depending on this leader-follower composition. If power flow is forced in direction favourable for SW, the MO can utilize J3's marginal cost bids to lower LMPs. This is however not true for all cases, since the results of the first IC scenario P_{1-3}^{Ξ} show that producer profit increases even though the overall maximization function is SW (3a1). Thus, consider the hypothesis that this effect is augmented if the model is set to maximize TP (3a2).

5.3.1 Maximizing Total Profit

For the uncongested case (Table 7) it can be shown that, in comparison to the SW scenarios, producer surplus significantly increased from 830 € to 3050 €. In terms of income distribution, J2 and J3 could gain substantial profits as the optimization function favors bids set at higher levels. Thus, J3 can utilize more of its high cost units and increase its market share. Consequently LMPs increased from 6 €/MWh (SW) to 15 €/MWh (TP). As the power flow on all lines differs from the power flows in the SW solution, new values for P_{nm}^{Ξ} need to be set. For a first test however, the P_{nm}^{Ξ} values were set to the same instances as in the SW case.

Surprisingly, no change in to the P_{nm}^{Const} equilibrium could be identified for all tested lines and P_{1-3}^{Ξ} values and consequently, P_{1-3}^{Ξ} is now IIC. This can be explained by the impact of the changed objective function. Consider that the objective function maximizes profit for all producers, subject to the MO's objective function of finding the OPF that maximizes SW. Thus, the bids submitted by each strategic producer can now be set at a level that fully emphasizes their market power, until the highest producer profit equilibrium is found. Also, note that producer J3 is still considered as follower but his relevance for the equilibrium has changed. His rather expensive marginal cost bids now act as a lever for the strategic actors to build their own bids on. J3 is of course still limiting the strategists' power, however his restricted margin to place bids is now exploited by J1 and J2, as they force the LMPs to be at least at the marginal cost of J3. In fact, the equilibrium found by P_{1-3}^{Ξ} represents a state where the capacity of the strategically significant node 2 is fully utilized. Consequently, line 1-2 is found IIC as there remains no margin to alter power flow from or to node 2. Hence, P_{1-3}^{Ξ} is not necessarily an optimal case in order to validate the impact of an objective function on ICs.

The results for the IC test on P_{2-3}^{Ξ} are summarized in Table 12. Note that the SW P_{nm}^{Const} equilibrium reassembles the TP P_{nm}^{Const} equilibrium. Furthermore, the threshold at which an IC becomes active, is again mirrored for lines 1-2 and 1-3 respectively. Also, the net impact (delta) of an active IC is identical to the impact identified for the maximization of SW. Surprisingly, the changed objective function does affect the new, active IC, equilibrium. Recall that for the SW case under active IC, SW was higher than in the P_{nm}^{Const} optimum and distribution of this additional income was in favor of consumers. Likewise, the active IC TP equilibrium is superior in terms of overall SW. The increased income is however attributed to producers. As shown in Table 12, these findings also hold for a reduced P_{2-3}^{Ξ}. Lastly, even though income redistribution and power flow are equal for both IC lines 1-2 and 1-3, the derived equilibria are local optima, since the share of profit among the producers is different. Consider for example case $P_{2-3}^{\Xi} = 12.5\ MW$ with active IC 1-2: J1 could gain a profit of 2.500 €, J2 of 62 € and J3 of 450€. In the case of active IC 1-3 the profit is distributed differently: J1 2.400 €, J2 162 € and J3 450 € respectively. Identical to the findings of the IIC test on SW, P_{1-3}^{Ξ} could be proven IIC for all cases, whereas the same justification as introduced earlier holds. As this analysis concludes the possible aspects involving the oligopoly scenario, the impact of the market setting in terms of strategically acting producers has to be analyzed.

	Constrained		**Irrelevant Constraint**			**Constrained**	**Irrelevant Constraint**	**Delta**
Line	P_{nm}^{Ξ}	P_{nm}^{Const}	P_{nm}^{max}	P_{nm}	Income redistribution			
1-2		12.5	**13.7**	13.7	Consumer	2900	2900	0
2-3	15.0	15.0	15.0	15.0	Producer	3025	3037	12
1-3		27.5		28.7	Grid	0	0	0
					SW	**5925**	**5937**	**12**
1-2		12.5		13.7	Consumer	2900	2900	0
2-3	15.0	15.0	15.0	15.0	Producer	3025	3037	12
1-3		27.5	**28.7**	28.7	Grid	0	0	0
					SW	**5925**	**5937**	**12**
1-2		13.8	**16.2**	16.2	Consumer	2900	2900	0
2-3	12.5	12.5	12.5	12.5	Producer	2988	3012	24
1-3		26.3		28.7	Grid	0	0	0
					SW	**5888**	**5912**	**24**
1-2		13.8		16.2	Consumer	2900	2900	0
2-3	12.5	12.5	12.5	12.5	Producer	2988	3012	24
1-3		26.3	**28.7**	28.7	Grid	0	0	0
					SW	**5888**	**5912**	**24**

Table 12: Sensitivity analysis P_{2-3}^{Ξ} with IC on line 1-2 & 1-3 applying objective function max TP.

5.4 Analyzing IIC: Triopoly

In contrast to the oligopoly scenario, where only J1 and J2 bid strategically, the triopoly scenario considers all GenCo's as strategic actors. In terms of the mathematical formulation, constraint (3v), which limited J3 to bid at marginal cost, was removed from the model. Since all producers now act as strategically operating individuals, the triopoly scenario can be considered as more complex. Hypothetically, stronger competition among producers should reduce prices and thus lead to increased consumer surplus. But since such a scenario never was researched in this context, detailed analysis is required. The unconstrained solution (Table 7) shows that this first assumption is not necessarily true for this scenario. In case of maximizing SW, the uncongested triopoly solution was found equal to the uncongested oligopoly solution. This unexpected result can be explained through the high marginal cost of producer J3. While maximizing SW, J3's power to influence prices is suppressed, as the global optimum is found at a LMPs equal to or lower than J3's marginal cost. Consequently, the results reassemble the ones obtained in the uncongested oligopoly case. Maximizing TP on the other hand reinforces strategic bids and thus consumer surplus and SW are considerably reduced. This result can be explained through price settings in the system. For some nodes LMPs were found equal to marginal utility, which eliminates consumer surplus.

5.4.1 Maximizing Social Welfare

For IIC tests, the same setting as in Subsection 5.2 was applied. Generally, the results for IC, found while maximizing SW in the oligopoly scenario, differ from the results obtained in the triopoly scenario. The outcomes of all relevant IC tests are summarized in Table 13. Note that P^{Ξ}_{1-2} is again IIC for all lines and thus excluded from the analysis. Following the three step procedure, it can be shown that equilibria obtained in the starting point P^{Const}_{nm} vary from the ones obtained earlier. In contrast to the uncongested case, where the additional strategic producer had limited impact on the equilibrium, the P^{Const}_{nm} solutions now show a redistribution from consumer to producer surplus. Observe that P^{Ξ}_{1-3} is now found IIC for both lines. This can be explained through considering the production in node 2 for P^{Const}_{nm}. Here, node 2 is already fully utilized and consequently, as illustrated in Subsection 5.3.1, the IIC property has no impact. To identify if this effect originates from the changed market setting, the equilibrium needs to be analyzed closely. First, the MO derives LMPs at 20 €/MWh uniformly for all nodes. This represents the highest marginal cost in the system, attributed to unit I6 of J2.

Consequently, J3 managed to place strategic bids that forced J1 and J2 to fully utilize their low cost units in node 2. Furthermore, through exercising market power, J3 pushed J2 to bid at marginal cost and in fact, J2 now adds the highest unit cost to the equilibrium. The changed market setting did consequently influence the IIC property, as it removes the lever to alter power flow to and from node 2.

	Constrained		**Irrelevant Constraint**			**Constrained**	**Irrelevant Constraint**	**Delta**
Line	P^{Ξ}_{nm}	P^{Const}_{nm}	P^{max}_{nm}	P_{nm}		Income redistribution		
1-2		2.5	**IIC**	2.5	Consumer	1650	1650	0
2-3		17.5		17.5	Producer	4263	4263	0
1-3	20.0	20.0	20.0	20.0	Grid	0	0	0
					SW	**5913**	**5913**	**0**
1-2		2.5		2.5	Consumer	1650	1650	0
2-3		17.5	**IIC**	17.5	Producer	4263	4263	0
1-3	20.0	20.0	20.0	20.0	Grid	0	0	0
					SW	**5913**	**5913**	**0**
1-2		10.0	**13.7**	13.7	Consumer	2900	1975	-925
2-3	15.0	15.0	15.0	15.0	Producer	3025	3939	914
1-3		25.0		28.7	Grid	0	41	41
					SW	**5925**	**5955**	**30**
1-2		10.0		13.7	Consumer	2900	3650	750
2-3	15.0	15.0	15.0	15.0	Producer	3025	2080	-945
1-3		25.0	**28.7**	28.7	Grid	0	225	225
					SW	**5925**	**5955**	**30**

Table 13: IC analysis for triopoly scenario under maximization of SW.

Applying P^{Ξ}_{2-3} and testing lines 1-2 and 1-3 leads to similar results obtained in the oligopoly SW scenario. The IC lines are mirrored and SW increases once an IC becomes active. In contrast to the oligopoly scenario, where consumer surplus increased with active ICs, an income redistribution effect, favoring either producers or consumers, can be observed. Interestingly, the SW obtained in both solutions is equal, confirming that this equilibrium constitutes a saddle or local optimum. To further analyze this special case, the numerical results and bidding behavior of producers are displayed in Figure 16 to Figure 19. Observe that a redistribution of income takes place when the IC becomes active at $P^{max}_{nm} = P_{nm} = 13.7\ MW$ but also in the later stage when $P^{max}_{nm} = P_{nm} = 13\ MW$. In this case, when the power flow on line 1-2 was forced to be 13 MW, an equilibrium that favors consumers was found. The multiplicity of an IC equilibrium is further emphasized in the test of P^{Ξ}_{2-3} IC on line 1-3. Here, consumer surplus is enhanced in the first IC instance but in the second case, when P^{max}_{1-3} is again reduced, the redistribution shifts from consumers to producers. In contrast to the oligopoly case, where strategic bids of producer J1 and J2 were placed significantly lower than J3's marginal cost, bids are now located at a higher price level.

Numerical results for: Triopoly / Objective: max SW / Congested line: 2-3 / IC line: 1-2

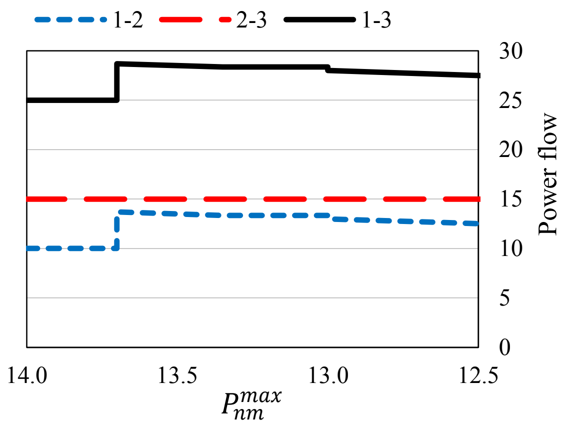

Figure 16: Line utilization IC line 1-2.

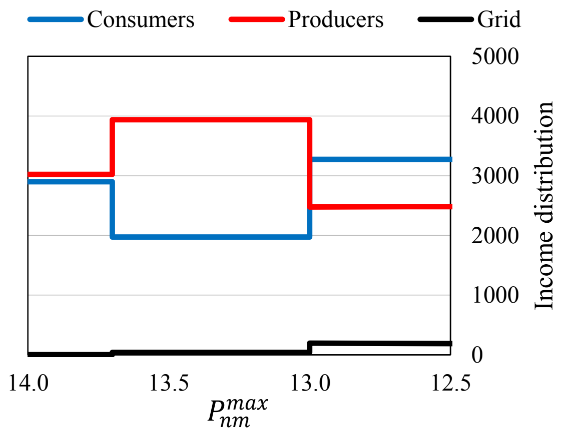

Figure 17: Income distribution IC line 1-2.

Bidding behaviour: Triopoly / Objective: max SW / Congested line: 2-3 / IC line: 1-2

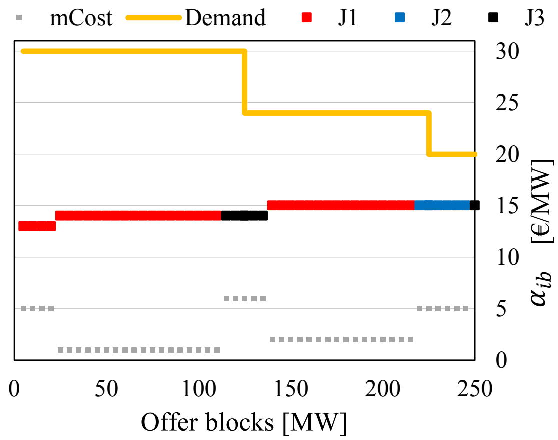

Figure 18: Accepted bids P_{nm}^{Const} line 2-3.

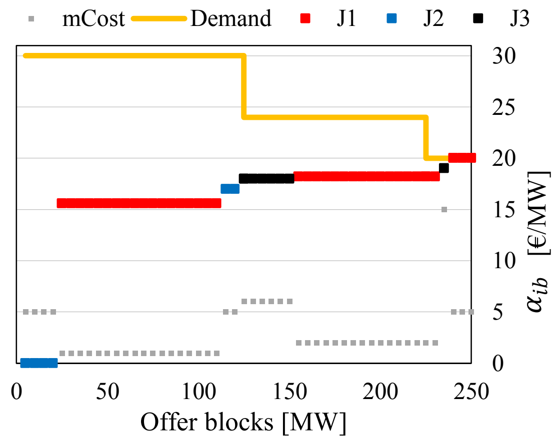

Figure 19: Accepted bids with active IC.

This behavior can be explained by analyzing J3's bids, since he now can bid at a level that maximizes his profit. Considering the active IC case, it can again be shown that once the power flow over line 1-2 is forced to be 13.7 MW, node 2 plays a significant role. In this case J2 is required to place a strategic bid at 0 €/MWh to not be pushed out by J1 and J3. Also, where producer J2 benefited most from the IIC property in the oligopoly case, producer J3 is now able to utilize his units in node 1. Consequently, production in node 1 is increased to fill for the reduced production in node 2. LMPs are raised from 15 €/MWh for all nodes to 18 €/MWh for node 1, 20 €/MWh for node 2 and 19 €/MWh for node 3 and hence producers could increase their profit. Numerically, J1 could increase his profit by 25%, J2 by 37% and J3 by 75%. However, this stage only holds for $P_{nm}^{max} = P_{nm} = 13.7\ MW$. When the power flow is reduced to $P_{nm}^{max} = P_{nm} = 13\ MW$, bidding behavior changes again. Subsequently, LMPs on node 1 and node 3 are reduced to 10 €/MWh and 15 €/MWh respectively (LMP at node 2 remained 20 €/MWh) and thus a shift from producer surplus to consumer surplus is found.

5.4.2 Maximizing Total Profit

Maximizing TP in the triopoly scenario leads to an income redistribution heavily in favour of producers. Already the uncongested case in Table 7 shows that producers gather a significant share of the total SW, which, in comparison to all other scenarios, is also the lowest obtained. In terms of active ICs, it was shown that when applying the oligopoly scenario and setting TP as objective function, producers could strengthen their position in all cases where an active IC was found. Considering IC tests for the triopoly scenario, it is now of interest if the IIC property further enhances the effect it has on producer profit. Table 14 summarizes relevant results for those tests. Observe that under P_{1-3}^{Ξ} the power flow on line 1-2 is zero and that line 2-3 is identified as IIC. However, even though the line is not utilized in the P_{nm}^{Const} equilibrium, it is still found as actively constraining at $P_{nm}^{max} = P_{nm} = 5\ MW$. Here, producer surplus is further increased through reducing grid revenue. Considering P_{2-3}^{Ξ}, an active IC has relatively little impact on the equilibrium. SW increased for 5 € once $P_{nm}^{max} = P_{nm}$ and producer profit could be enlarged by 57 €. This constitutes, in comparison to the oligopoly TP, a bigger impact on producer profit but compared to both SW maximization cases, producer profit could benefit less. Recall that for TP optimization, producer profit is already set at a high level. Therefore, leeway for further reinforcement of TP is limited, even if power flow is forced on a specific value. P_{1-2}^{Ξ} is again found IIC for both lines and thus not included in Table 14.

	Constrained		**Irrelevant Constraint**			**Constrained**	**Irrelevant Constraint**	**Delta**
Line	P_{nm}^{Ξ}	P_{nm}^{Const}	P_{nm}^{max}	P_{nm}	Income redistribution			
1-2		0.0	**5.0**	5.0	Consumer	0	0	0
2-3		20.0		15.0	Producer	5385	5420	35
1-3	20.0	20.0	20.0	20.0	Grid	180	90	-90
					SW	**5565**	**5510**	**-55**
1-2		0.0		0.0	Consumer	0	0	0
2-3		20.0	**IIC**	20.0	Producer	5385	5385	0
1-3	20.0	20.0	20.0	20.0	Grid	180	180	0
					SW	**5565**	**5565**	**0**
1-2		10.0	**13.7**	13.7	Consumer	50	0	-50
2-3	15.0	15.0	15.0	15.0	Producer	5605	5662	57
1-3		25.0		28.7	Grid	270	267	-3
					SW	**5925**	**5930**	**5**
1-2		10.0		13.7	Consumer	50	0	-50
2-3	15.0	15.0	15.0	15.0	Producer	5605	5662	57
1-3		25.0	**28.7**	28.7	Grid	270	267	-3
					SW	**5925**	**5930**	**5**

Table 14: IC analysis for triopoly scenario under maximization of TP.

Through analysing bidding behaviour in this rather extreme TP triopoly case, the reason why consumer surplus is reduced to zero becomes obvious. As shown in both Figure 21 and Figure 20, LMPs are now defined by the highest offers placed, rather than the marginal cost of that unit. Consequently, LMPs for P_{nm}^{Const} are set at 24 €/MWh for node 1, 18 €/MWh for node 2 and 30 €/MWh for node 3. Since the LMP for node 2 is still at a level below marginal utility of 20 €/MWh, consumer surplus can be found at 50 € (for 25 MW purchased in node 2). Producer J1 captures the largest share, as he places strategic bids (below marginal cost) that allow full utilization of his production units and additionally push out his competitors (profit of 4570 €). Producers J2 and J3 managed to split the remaining market share equally and, at similar margins, obtained analogous profits of 510 € (J2) and 525 € (J3). This distribution changes drastically once the power flow through line 1-2 is forced at $P_{nm}^{max} = P_{nm} = 13.7\ MW$. Here again, node 2 constitutes the strategic hub for all producers. J1 manages to keep his share (reduced by 10 MW) since he placed strategic offers for most of his capacity. Producer J3 however could capture market share in node 1 and force producer J2 to utilize his high cost unit I6 ($\lambda_{6b}^{G} = 20\ €/MWh$). Doing so, J3 significantly augmented his profit by 71% to 900 €. Subsequently, J2 could only place high risk bids, at a level of exactly marginal utility, and thus defined LMPs in all nodes at 24 €/MWh for node 1, 20 €/MWh for node 2 and 30 €/MWh for node 3 respectively. The assignment of unit I6 and the loss in market share resulted in a 41% reduced profit of 302 € for J2. Considered holistically, as consumer surplus was reduced to zero, producers could increase their profit. Even though this scenario is rather extreme, it shows quite impressively how the IIC property influences equilibria for results which were supposed to be optimal solutions.

Bidding behaviour: Triopoly / Objective: max TP / Congested line: 2-3 / IC line: 1-2

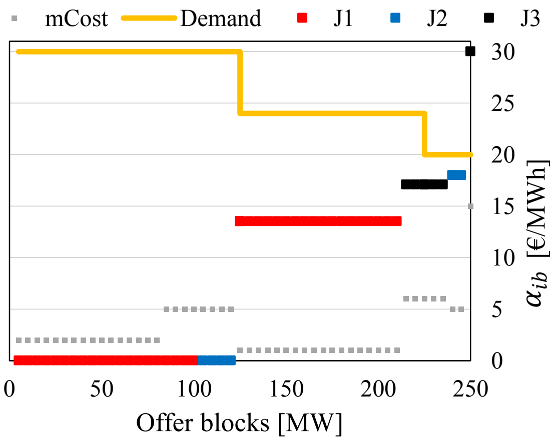

Figure 20: Accepted bids P_{nm}^{Const} line 2-3.

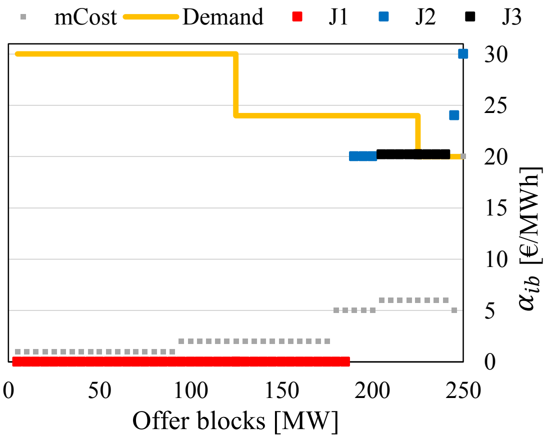

Figure 21: Accepted bids with active IC.

5.5 Applied scenarios and IIC

To emphasize the IIC properties' practical significance for market regulation and transmission extension planning, consider the example displayed in Figure 22. Assume two identical markets (node 1 – node 3 and node 4 – node 6) are connected via one interconnector line 2-6. As this line only features limited capacity (3 MW) an additional interconnector is to be planned. The two markets are exact copies of the three node network used in Section 5. Producers J1 and J2 are considered strategic actors and operate in both markets. Each producer has the exact same generating units in market one (N1 – N3) and market two (N4 – N6). Since the markets are supposed to be separated, they both feature a unique market follower, which are assumed as marginal cost bidding. Those are J3 for market one and J4 for market two. They both hold 50% of the generation capacity in their respective market. All other parameters such as marginal utility, demand and marginal cost are chosen to be equal to the parameters introduced in Subsection 3.2. As of now, only the capacity constraints of lines 1-3 and 4-6 are limiting the system (20 MW).

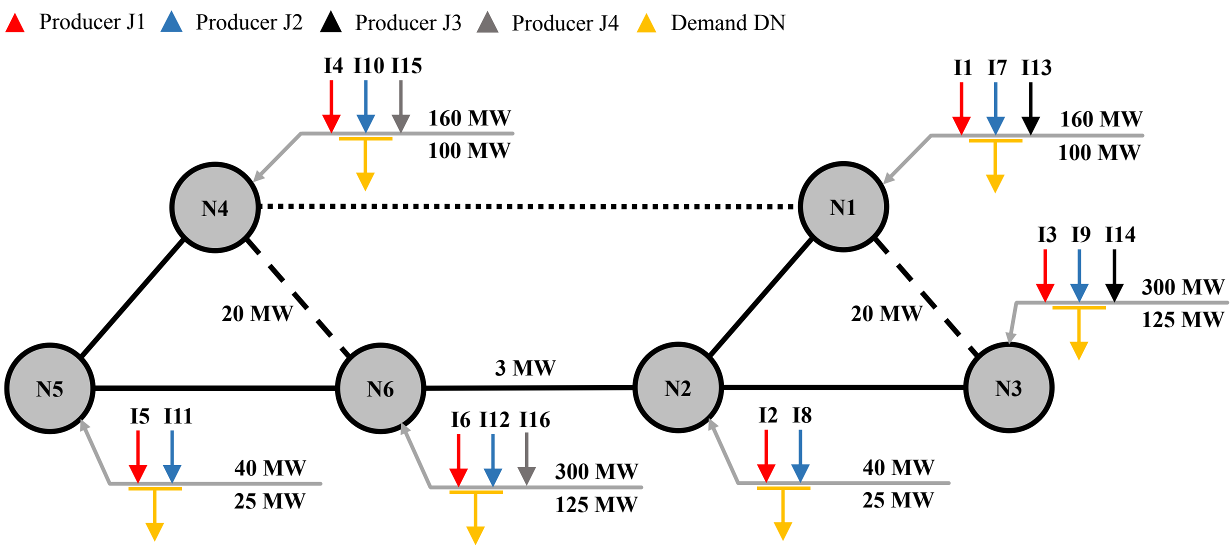

Figure 22: A practical example for a proposed interconnector line between two markets.

To relieve congestion in lines 1-3 and 4-6 and increase SW, a new interconnector between node 1 and node 4 (dotted line) is planned. This interconnector would enable the market flowers to better dispatch their capacity in the respective other market. Thus, market power of the strategic producers J1 and J2 would be reduced and SW increased. The following steps simulate how a regulating body would approach feasibility studies for such a proposed interconnector. Please note that this is just a simplified example emphasising the importance of IC tests when constraints are added to an EPEC electricity market model.

Table 15 demonstrates the first step, where the equilibrium with the existing infrastructure is identified. As shown, producer surplus is slightly above consumer surplus (the objective function is set maximizing TP) and SW was identified at 11,423 €. Producer J1 captures the largest market share, as its marginal cost are amongst the lowest in the system. Observe that power in the existing interconnector is sent from node 6 to node 2, as node 2 is higher priced than node 6. A similar result would be expected when introducing the interconnector 1-4, since the LMP in node 1 is currently 10.50 €/MWh while the LMP in node 4 is at marginal utility of 24 €/MWh.

Income distribution [€]				**Producer profit [€]**		
Consumers		5125.00			J1	4823.50
Producers		6244.00			J2	295.50
Grid		54.00			J3	225.00
SW		11423.00			J4	900.00
LMP per node [€/MWh]						
1	2	3	4	5	6	
10.50	19.50	15.00	24.00	19.50	15.00	
Power flow per line [MWh]						
1-2	2-3	1-3	4-5	5-6	4-6	2-6
21.00	-1.00	20.00	1.50	16.50	18.00	-3.00

Table 15: Numerical results with no additional interconnector.

The next step is to perform an analysis, estimating the impact the new interconnector could have on the system. Therefore, the capacity of the new interconnector is set at a level high enough to not constrain the equilibrium, so that the OPF without congestion can be derived. The results are displayed in Table 16. Adding the new line 1-4 would lead to an equally congested network, as LMPs could be obtained at 15 €/MWh for each node. Consequently, producer profit would be decreased and SW increased to 11,544 €. As intended, the new line facilitates power flow from the low priced node 1 to the high priced node 4, relieves network congestion and reduces market power of the strategic producers.

Income distribution [€]				**Producer profit [€]**			
Consumers		5800.00			J1	4644.00	
Producers		5744.00			J2	200.00	
Grid		0.00			J3	450.00	
SW		11544.00			J4	450.00	
LMP per node [€/MWh]							
1	2	3	4	5	6		
15.00	15.00	15.00	15.00	15.00	15.00		
Power flow per line [MWh]							
1-2	2-3	1-3	4-5	5-6	4-6	2-6	1-4
18.50	1.50	20.00	2.50	17.50	20.00	3.00	1.50

Table 16: Numerical results with OPF, line 1-4 P_{1-4}^{max}: ∞ .

The analysis shows that this new line would contribute to an equilibrium featuring higher SW and consumer surplus, while producer surplus and grid congestion are reduced. Consequently, it is reasonable to assume that such a line could be physically implemented in an electricity market. According to the findings of this thesis, a crucial step would be now to test the line for IIC, before further actions are undertaken. As shown in the analysis of Section 5, a BLP affected by ICs could deliver results that do not reassemble the true, unaffected, optimum. Indeed, line 1-4 is found not IIC, as indicated by the numerical results in Table 17 below.

Income distribution [€]					**Producer profit [€]**		
Consumers		5125.00				J1	4737.25
Producers		6166.00				J2	303.75
Grid		94.50				J3	225.00
SW		11385.50				J4	900.00
LMP per node [€/MWh]							
1	2	3	4	5	6		
10.50	19.50	15.00	24.00	19.50	15.00		
Power flow per line [MWh]							
1-2	2-3	1-3	4-5	5-6	4-6	2-6	1-4
21.00	-1.00	20.00	0.75	15.75	16.50	-3.00	1.50

Table 17: Numerical results with new line 1-4 implemented.

These results were generated by adding the parameters for $P_{1-4}^{max} = 1.5\ MW$, corresponding to the OPF obtained in the solution, when P_{1-4}^{max} was set at a high enough parameter (∞) that did not constrain the model. Doing so, a line 1-4 with the optimal capacity of 1.5 MW is included in the electricity market. Observe that line 1-4 is indeed utilized at the maximum of 1.5 MW. However, the new equilibrium now strengthens producer surplus and congestion rent is increased. SW is, compared to the original solution, even lower than before the line was implemented. In fact, LMPs are found at the level they were before the change to the model. Both strategic producers J1 and J2 could increase profit, while profit for the market followers J3 and J4 remained unchanged. To understand how J1 and J2 could benefit from the new line 1-4, consider the offer curves of all accepted bids in Figure 23 to Figure 25. Interestingly, the offer curves before and after the implementation are almost equal. This explains why power flow and LMPs are relatively equal for both solutions. The bidding curve for $\mathrm{P}_{1-4}^{\mathrm{max}} = \infty$ however, shows the market followers J3 and J4 limiting the LMP at 15 €/MWh for all nodes. In this case, the MO was free to define the power flow on line 1-4 on any level. And as it turns out, deriving a power flow of 1.5 MW on 1-4 would provide a better solution. In this case, producers J1 and J2 are forced to place strategic bids and utilize their rather expensive units in Node 2 (see Table 18) and doing so, negatively impact their profits.

Bidding behaviour: Oligopoly / Objective: max TP / All three P_{1-4}^{max} scenarios

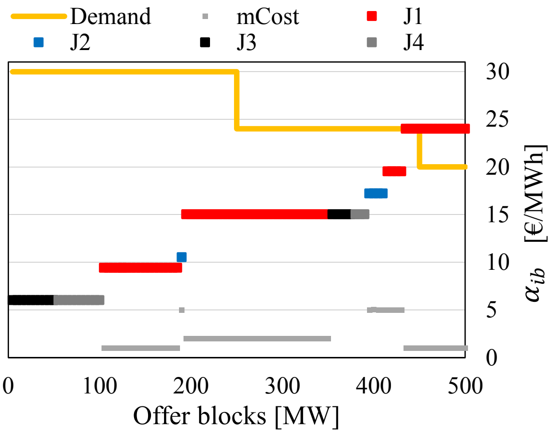

Figure 23: Accepted bids P_{1-4}^{max}: 0 MW.

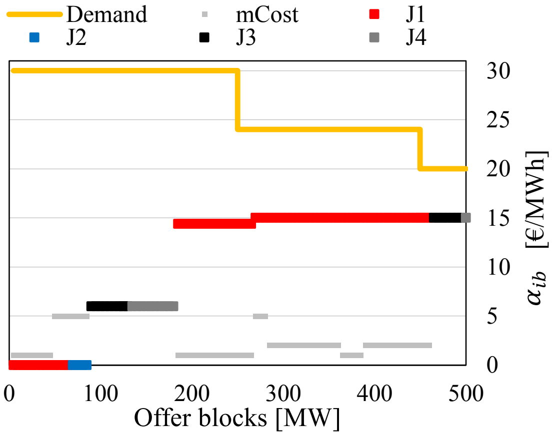

Figure 24: Accepted bids P_{1-4}^{max}: ∞.

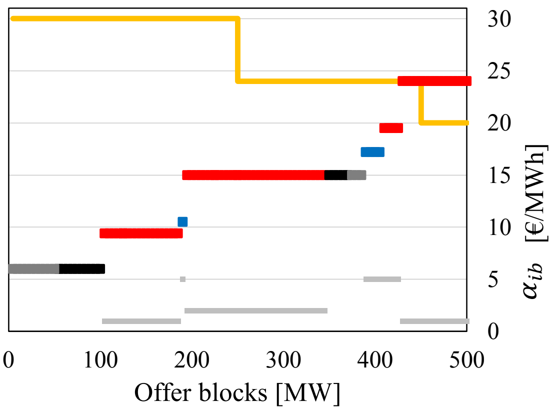

Figure 25: Accepted bids P_{1-4}^{max}: 1.50 MW.

P_{nm}^{max}	0.00	∞	1.50
Node	Production		
N1	141.00	140.00	142.50
N2	0.00	11.00	0.00
N3	106.00	103.50	106.00
N4	119.50	121.00	115.75
N5	40.00	40.00	40.00
N6	93.50	84.50	95.75
	500.00	500.00	500.00

Table 18: Production per node for all P_{1-4}^{max}.

Due to the IIC property, power flow on line 1-4 is now forced to be exactly $\mathrm{P}_{1-4}^{\max} = 1.50$ MW. The strategic producers exploit this property by placing bids that compel the market followers to provide for the required power flow. Comparing the optimal solution $\mathrm{P}_{1-4}^{\max} = \infty$ and $\mathrm{P}_{1-4}^{\max} = 1.50$ MW, J3's and J4's combined production increased by 10% (13.75 MW). Consequently, J1 and J2 can now place their bids at a higher level and raise LMPs to previous values.

This scenario shows that results obtained in EPEC electricity market models with transmission constraints, should not be taken for granted. Due to the IIC property, the OPF derived in a solution is very likely a result of a transmission constraint that is forced active. Implying that every transmission line in a model could potentially constitute an IC and that the models are flawed. For example, the capacity of a line might be set at 20 MW and the optimum could derive a power flow of 20 MW. This power flow might however be a result of one of two causes: one, the optimal utilization is actually 20 MW and the solution is correct; two, the line is an active IC forced at $P_{nm}^{max} = P_{nm} = 20\ MW$ and the model is flawed.

5.6 Computational issues

All simulations were carried out on an Intel i7-3517U quad core processor running at 1.9 GHz and 4 GB of RAM. The code was written in the programming environment of AMPL IDE 1.0, whereas CPLEX 12.6.0 was used as a solver. VBA was used to forward parameters from Microsoft Excel to AMPL and vice versa. Computation times required to solve the model are summarized in Table 19. Findings of [5] are confirmed, as it is shown that computation time increases with congestion and with the number of strategic producers added to the system. With respect to the research question of the thesis, it could be shown that solving a scenario where the model could be proven not IIC is significantly more demanding (IC line P^{Ξ}_{nm}). It takes approximately 1.33 seconds to run the congested oligopoly scenario maximizing SW. On the two lines which were found active ICs, 1-2 and 2-3, calculation time used by the solver increased to 14.64 and 11.67 seconds respectively. Line 1-3 however was found IIC and the solve-time is roughly equal to the congested solution. Similar results were found under the triopoly scenario. Lines not IIC required more computations and thus longer solution times.

	Uncongested	Congested	IC line P^{Ξ}_{nm} 1-2	IC line P^{Ξ}_{nm} 2-3	IC line P^{Ξ}_{nm} 1-3
			Oligopoly		
Max SW	0.46	1.33	14.64	11.67	1.39
Max TP		0.52	0.58	0.63	0.53
			Triopoly		
Max SW	0.46	1.66	1.60	13.25	1.62
Max TP	0.71	0.83	6.82	3.65	0.91

Table 19: Computation time in seconds.

Observe, that maximizing TP is less computationally demanding in almost all cases, except for the uncongested triopoly scenario. Intuitively, it could be assumed that the more complex TP objective function, equation (3a2), requires more computation time. As this is not the case, and in fact conflictive to the results obtained by [5], the applied setting and parameters supposedly support rather obvious TP equilibria which are identified easily. Hence, it is worth mentioning that results cannot be generalized, both for computational time as well as for the IIC property collectively. The outcome heavily depends on the parameters used and the objective function applied.

6. Conclusions

We have only limited knowledge on how complex EPEC models, applied to simulate electricity markets, are influenced by the IIC property. Academia describes this phenomenon mostly in a general, mathematical, approach. Thus, the numerical study implemented for this thesis provides insight on the IIC property and how equilibria are derived under impact of active ICs. Furthermore, the thesis adds value by supplying a detailed analysis of the property's impact on electricity market models. Hence, the thesis contributes to the understanding on BLPs and their dependence on ICs. As such models are often used in OR, the thesis is of academic and practical significance. Identifying IIC flaws in models can help to prevent not necessarily optimal solutions to be carried over into industrial areas.

6.1 Summary of findings

The EPEC model developed by [5], applied in the test environment introduced by [7], is not independent of irrelevant constraints. The mathematical phenomenon forced an inactive, irrelevant, constraint to actively constrain the equilibrium, once a certain threshold value was reached. Those numerical results affirm the general description of [6]. Furthermore, the findings of [7] are confirmed, as producer profit increased by 15.4%, while SW decreased by 0.3%, once the IC line 1-2 actively constrained the model at 4.5 MW. Through the analysis of producer bidding behaviour, it was discovered that accepted bids drastically change once the IC is rendered active. The IIC property can be economically interpreted as forcing the MO to utilize a certain transmission line at the given P_{nm}^{max} value. Producers then alter their bids to match this power flow. Doing so, the derived solution can even result in an equilibrium that is, in terms of SW, superior to the pre-IC equilibrium. The analysis revealed that a conclusion should not be generalized, as four factors determining the property could be identified.

First, the impact of an active IC is depending on the model parameters and settings. As shown, the cost structure and composition of production units in node 2 defined whether a line was found IIC or not. This conclusion is supported by the fact that some subsets of the transmission constraints did not have an impact on the model. So could a specific transmission line constitute an irrelevant constraint but another line would not have that impact on the optimum. The outcome was thereby defined by the market setting and parameters on the relevant nodes. If, for example, a node was already fully utilized, the line was identified as IIC.

Second, different objective functions affect the property in two dimensions: one, they alter the occurrence; two they define the impact of active ICs. While comparing the results for maximizing SW or TP under the same scenario (e.g. oligopoly), it was found that lines, which were not IIC under the SW case, were IIC under the TP case. This can however be attributed to the parameters in the model. In the situation where a line was IIC, a close analysis revealed that an altered power flow was not possible due to the utilization in that specific node. The impact on the equilibrium was on the other hand greatly depending on the optimization criteria. While maximizing TP active ICs generally had less effect on the equilibrium, compared to the SW maximizing cases. This effect can be measured by the change of the optimal condition in terms of SW and income redistribution. While the impact on the total system SW was similar (comparing SW and TP objective function on the same IC line), income redistribution changed depending on the optimization criteria. Such results can be observed in the equilibria for P_{2-3}^{Ξ}. In the SW case, active ICs force a considerable income redistribution to either producers or consumers. For the TP case however, the redistribution only occurred in favor of producers and its magnitude was significantly lower than in the SW case.

Third, market settings and the number of strategically acting producers influence the IIC property. It was shown that under the same test conditions the oligopoly scenario was more likely to be affected by active ICs than the triopoly scenario. Also, the impact of ICs was higher in the oligopoly case. This conclusion needs to be put into perspective while considering the findings summarized above. As highlighted, the additional strategic producer forces the two other producers to fully utilize their inexpensive capacity and thus entirely changes the equilibrium. Consequently, the production units in node 2, identified as essential node for the IIC property, could not be operated otherwise and the IIC property had little or no effect. Furthermore, the impact of active ICs was reduced, as for example TP was already optimized at a high level. Thus, market setting and the number of strategic producers can be considered as a factor that augments the impact of other parameters and the objective function.

Fourth, it was discovered that the IIC property is depending on the originally active constraint's P_{nm}^{Ξ} magnitude. The sensitivity analysis revealed that stronger constraints P_{nm}^{Ξ} can lead to a higher impact of active ICs. Due to the complexity of the setting, it could not be shown, how P_{nm}^{Ξ} affects the IC equilibrium. Also, a connection between P_{nm}^{Ξ} and the value when an IC is rendered active ($P_{nm}^{max} = P_{nm}$) could not be accurately identified.

The specifics of this setting prove a final conclusion challenging, as some equilibria were found in saddles or local optima. This confirms the general perception that EPEC models are characterized by the multiplicity of their optima [5],[35]. Consequently, it can be concluded that **this** EPEC model, **as applied**, is not IIC; subject to the setting in which the model is implemented. Nonetheless, according to [6] it is more than likely that all forms of electricity market EPECs are subject to ICs. From a critical perspective, this conclusion only partially answers the research question. However, the value of this thesis reveals with the insight gained while identifying if the model is IIC. It could be proven that the three step procedure can be utilized as a valid approach to verify if a model is depending on ICs. Once the procedure is applied, as described in Subsection 3.1, the four influence factors identified above need to be taken into consideration. This method is recommended for all electricity market BLPs as such models are subject to an inherent risk involving the IIC property. Consequently, and as shown in this thesis, ICs can for the observer unknowingly constrain the optimum, and the derived equilibrium does not reflect the true, non IC influenced, optimal solution. A hands-on approach is then to apply a variety of scenarios, each with modified parameters, market settings and, if applicable, different objective functions.

6.2 Implications

As indicated above, the implications of this thesis are relevant for OR models used in industrial backgrounds or in an academic environments. Only few papers and articles were devoted to the property in general [6], [10], [53]. This numerical analysis hence adds academic value by highlighting the importance of this unexplored topic. More importantly, BLPs developed in academia are often adopted by business. The findings of this thesis are thus particularly significant for OR and electricity market modelling. As shown in the scenario of subsection 5.5, the implications of ICs are wide reaching. This case roughly reassembles the procedure a centralized planner or a TSO would apply when identifying the potential for new transmission lines. In this example, the new interconnector represented an active IC and altered the optimal solution. This could have two implications: the planned interconnector would not be implemented, even though the non-IC optimal solution would benefit the equilibrium. Or, the interconnector would be built based on the positive non-IC solution but the actual real market equilibrium would then strengthen the positions of producers, instead of relieving congestion. The point is, the solution to a model that is not IIC is not necessarily wrong, the additional constraint yet results in unintended consequences.

Subsequently, policy makers, TSOs, MOs and market participants should recognize the IIC property as possible flaw in BLPs. Otherwise, errors resulting out of active ICs might be carried over to real world applications. It is however uncertain how physical electricity market equilibria behave, if such a not necessarily optimal constraint is added to a system. Therefore, GenCo's should take the IIC property into consideration when applying EPEC models for capacity planning [54], [55] or for the optimization of bidding strategies [47]. In such cases, producers could make investment decisions that are not necessarily optimal, due to flaws in the optimization model. A central planner might also be exposed to such scenarios, for example when opening tender processes for power plants to increase competition. Subtle changes in parameters, perhaps by adding generation capacity in a node, alter the model specifics and possibly force ICs active. This might potentially contribute to producers' profit, as it was shown in Section 5 and in fact lead to unintended results. Regulators should therefore be aware of such issues while using EPEC models to analyse market power [49], [56], [57]. Lastly, MOs and TSOs must take the property into close consideration. In their cases EPEC models are used for transmission extension planning [48], [57] and market analysis [46]. In such scenarios, adding transmission capacity to a real market might mitigate market power of GenCo's or augment strategic bidding.

The strong claim of Macal and Hurter that "for any bilevel program that has ever been solved and that is not independent of irrelevant constraints" [6], adding a IC can negate the optimality of the solution obtained, should thus be put into perspective. Generally, Macal and Hurter could be proven correct. However, as shown in this thesis, the IIC property depends on a variety of factors. It is hence always subject to the BLP, its objective function and its parameters, how irrelevant constraints influence the equilibrium.

In summary, the thesis adds to methodical and theoretical knowledge of OR by several means: To begin with, it provides a structured approach for identifying IC in bi-level electricity market models. Furthermore, it contributes to the understanding of IIC, since not yet researched areas of Macal and Hurter's property are numerically supported. Thereby worth mentioning are the impact of diverse market settings, such as the oligopoly and triopoly scenarios, different objective functions, such as the TP and SW maximization criteria, and the detailed analysis on producer bidding behaviour. Finally, the applied scenario provides a more hands-on approach and is intended to raise awareness of IIC in the scientific and industrial OR community.

6.3 Outlook

The thesis' comprehensive analysis revealed that the phenomenon IIC is a remarkably complex topic. Since this numerical study is limited to particular parameters and transmission network settings, research on ICs can be extended in various ways. In terms of EPEC models incorporating SOCs, like the model applied in the thesis, bidding behaviour was found as an essential driver for the impact of ICs. Recall that for this thesis, bidding block size was limited to two. Consequently, a next step would be to analyse how different block sizes affect optimality. Furthermore, it was found that the number of producers does influence the model. From this research perspective it could be of interest to see how more complex multi-producer simulations are affected by ICs. Likewise, bi-level OR techniques used for market simulations under uncertainty are subject to further studies. Especially since such simulations include stochastic elements which are not yet researched under IIC. The time horizon can be equally compelling. As the EPEC introduced by [5] is set up to derive a single-period equilibrium, an extension of the model to derive a multi-period equilibrium could be aspired. It is however unclear how a producers strategy that covers multiple time periods alters the multiplicity of the optimality condition [5], [58]. The impact of ICs can then be considered ambiguous and a more extensive numerical analysis, or even analytical approach, would be challenging.

In terms of the IIC property, it remains unclear whether or not a correlation between P_{nm}^{Ξ} and the IC threshold value $P_{nm}^{max} = P_{nm}$ exists. Moreover, if the impact of an IC on the solution can be mathematically verified. That is, if given parameters or settings predetermine how an active IC will influence SW and income redistribution. Relevant in this context is also, if the phenomenon is applicable for newly introduced models, such as tri-level models. Those models are used to combine centralized transmission planning with decentralized capacity extension measures and bidding strategies [59], [60]. Lastly, the analysis revealed that producers take an essential part in altering the active IC equilibrium. Their bids changed drastically once power flow was forced over a given line. Hence, the question arises if, and how, producers could benefit from the IIC property. Investigations with this respect would require the use of either MPEC or EPEC models from the perspective of a GenCo, assuming that a central planner or regulating body applies a model subject to ICs. Such scenarios would involve models that closely reassemble the market composition and require detailed knowledge about all market participants' parameters. Problems like those are significant in order to verify the impact of ICs on physical markets and open for future research.

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Appendix: Numerical results

Results for IIC lines are only shown for the Oligopoly max SW case in order to illustrate how the equilibrium remains constant over every instance. As those results do not provide any deeper insight, other IIC line numerical results are excluded from the appendix.

Table 20: Oligopoly / Objective: max SW / Congested line: 1-3 / IC line: 1-2

Instance	**5.0**	**4.7**	**4.5**	**4.2**	**3.9**	**3.6**	**3.3**	**3.0**	**2.7**	**2.5**
Income Distribution [€]										
Consumer	3912.5	3968.8	3675.0	3675.0	3675.0	3675.0	3675.0	3675.0	3675.0	3675.0
Producer	1717.5	1646.3	2040.4	2049.1	2057.8	2066.5	2075.2	2083.9	2092.6	2097.5
Grid	270.0	285.0	164.9	159.2	153.5	147.8	142.1	136.4	130.7	127.5
SW	5900.0	5900.0	5880.3	5883.3	5886.3	5889.3	5892.3	5895.3	5898.3	5900.0
Producer Profit [€]										
J1	1600.0	1600.0	1730.9	1739.9	1748.9	1757.9	1766.9	1775.9	1784.9	1790.0
J2	117.5	117.5	309.5	309.2	308.9	308.6	308.3	308.0	307.7	307.5
J3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Consumption per Node [MWh]										
C-N1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
C-N2	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
C-N3	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Node [MWh]										
P-N1	122.5	122.5	124.5	124.2	123.9	123.6	123.3	123.0	122.7	122.5
P-N2	40.0	40.0	36.1	36.7	37.3	37.9	38.5	39.1	39.7	40.0
P-N3	87.5	87.5	89.5	89.2	88.9	88.6	88.3	88.0	87.7	87.5
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Unit [MWh]										
I1	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0
I2	20.0	20.0	16.1	16.7	17.3	17.9	18.5	19.1	19.7	20.0
I3	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0
I4	7.5	7.5	9.5	9.2	8.9	8.6	8.3	8.0	7.7	7.5
I5	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
I6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I7	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
I8	7.5	7.5	9.5	9.2	8.9	8.6	8.3	8.0	7.7	7.5
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Locational Marginal Price [€/MWh]										
1.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0
2.0	10.5	10.5	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
3.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
Transmission Lines [MWh]										
1-2	2.5	2.5	**4.5**	4.2	3.9	3.6	3.3	3.0	2.7	2.5
2-3	17.5	17.5	**15.5**	15.8	16.1	16.4	16.7	17.0	17.3	17.5
1-3	20.0	20.0	**20.0**	20.0	20.0	20.0	20.0	20.0	20.0	20.0

Table 21: Oligopoly / Objective: max SW / Congested line: 1-3 / IC line: 2-3

Instance	**20.3**	**20.0**	**19.7**	**19.4**	**19.1**	**18.8**	**18.5**	**18.2**	**17.9**	**17.5**
Income Distribution [€]										
Consumer	3912.5	3912.5	3912.5	3912.5	3912.5	3912.5	3912.5	3912.5	3912.5	3912.5
Producer	1717.5	1717.5	1717.5	1717.5	1717.5	1717.5	1717.5	1717.5	1717.5	1717.5
Grid	270.0	270.0	270.0	270.0	270.0	270.0	270.0	270.0	270.0	270.0
SW	5900.0	5900.0	5900.0	5900.0	5900.0	5900.0	5900.0	5900.0	5900.0	5900.0
Producer Profit [€]										
J1	1600.0	1600.0	1600.0	1600.0	1600.0	1600.0	1600.0	1600.0	1600.0	1600.0
J2	117.5	117.5	117.5	117.5	117.5	117.5	117.5	117.5	117.5	117.5
J3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Consumption per Node [MWh]										
C-N1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
C-N2	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
C-N3	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Node [MWh]										
P-N1	122.5	122.5	122.5	122.5	122.5	122.5	122.5	122.5	122.5	122.5
P-N2	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0
P-N3	87.5	87.5	87.5	87.5	87.5	87.5	87.5	87.5	87.5	87.5
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Unit [MWh]										
I1	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0
I2	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
I3	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0
I4	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5
I5	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
I6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I7	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
I8	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Locational Marginal Price [€/MWh]										
1.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0
2.0	10.5	10.5	10.5	10.5	10.5	10.5	10.5	10.5	10.5	10.5
3.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
Transmission Lines [MWh]										
1-2	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
2-3	17.5	17.5	17.5	17.5	17.5	17.5	17.5	17.5	17.5	17.5
1-3	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0

Table 22: Oligopoly / Objective: max SW / Congested line: 2-3 / IC line: 1-2

Instance	**15.0**	**14.9**	**13.7**	**13.6**	**13.4**	**13.3**	**13.1**	**13.0**	**12.8**	**12.5**
Income Distribution [€]										
Consumer	2900.0	2900.0	3650.0	3650.0	3650.0	3650.0	3650.0	3650.0	3650.0	3650.0
Producer	3025.0	3025.0	2062.0	2060.5	2059.0	2057.5	2056.0	2054.5	2053.0	2050.0
Grid	0.0	0.0	225.0	225.0	225.0	225.0	225.0	225.0	225.0	225.0
SW	5925.0	5925.0	5937.0	5935.5	5934.0	5932.5	5931.0	5929.5	5928.0	5925.0
Producer Profit [€]										
J1	2400.0	2400.0	1850.0	1850.0	1850.0	1850.0	1850.0	1850.0	1850.0	1850.0
J2	175.0	175.0	12.0	10.5	9.0	7.5	6.0	4.5	3.0	0.0
J3	450.0	450.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
Consumption per Node [MWh]										
C-N1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
C-N2	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
C-N3	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Node [MWh]										
P-N1	140.0	140.0	142.4	142.1	141.8	141.5	141.2	140.9	140.6	140.0
P-N2	27.5	27.5	26.3	26.5	26.6	26.8	26.9	27.1	27.2	27.5
P-N3	82.5	82.5	81.3	81.5	81.6	81.8	81.9	82.1	82.2	82.5
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Unit [MWh]										
I1	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0
I2	10.0	10.0	16.3	16.5	10.0	10.0	10.0	10.0	10.0	10.0
I3	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0
I4	0.0	0.0	2.4	2.1	1.8	1.5	1.2	0.9	0.6	0.0
I5	17.5	17.5	10.0	10.0	16.6	16.8	16.9	17.1	17.2	17.5
I6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I7	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
I8	2.5	2.5	1.3	1.5	1.6	1.8	1.9	2.1	2.2	2.5
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Locational Marginal Price [€/MWh]										
1.0	15.0	15.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
2.0	15.0	15.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0
3.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
Transmission Lines [MWh]										
1-2	12.5	12.5	**13.7**	13.6	13.4	13.3	13.1	13.0	12.8	12.5
2-3	15.0	15.0	**15.0**	15.0	15.0	15.0	15.0	15.0	15.0	15.0
1-3	27.5	27.5	**28.7**	28.6	28.4	28.3	28.1	28.0	27.8	27.5

Table 23: Oligopoly / Objective: max SW / Congested line: 2-3 / IC line: 1-3

Instance	**30.0**	**29.8**	**29.6**	**28.7**	**28.5**	**28.3**	**28.1**	**27.9**	**27.7**	**27.5**
Income Distribution [€]										
Consumer	2900.0	2900.0	2900.0	3650.0	3650.0	3650.0	3650.0	3650.0	3650.0	3650.0
Producer	3025.0	3025.0	3025.0	2062.0	2059.9	2057.8	2055.7	2053.6	2051.5	2050.0
Grid	0.0	0.0	0.0	225.0	225.0	225.0	225.0	225.0	225.0	225.0
SW	5925.0	5925.0	5925.0	5937.0	5934.9	5932.8	5930.7	5928.6	5926.5	5925.0
Producer Profit [€]										
J1	2475.0	2475.0	2475.0	1850.0	1850.0	1850.0	1850.0	1850.0	1850.0	1850.0
J2	100.0	100.0	100.0	12.0	9.9	7.8	5.7	3.6	1.5	0.0
J3	450.0	450.0	450.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
Consumption per Node [MWh]										
C-N1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
C-N2	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
C-N3	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Node [MWh]										
P-N1	140.0	140.0	140.0	142.4	142.0	141.6	141.1	140.7	140.3	140.0
P-N2	27.5	27.5	27.5	26.3	26.5	26.7	26.9	27.1	27.4	27.5
P-N3	82.5	82.5	82.5	81.3	81.5	81.7	81.9	82.1	82.4	82.5
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Unit [MWh]										
I1	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0
I2	17.5	17.5	17.5	16.3	16.5	16.7	16.9	17.1	17.4	17.5
I3	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0
I4	0.0	0.0	0.0	2.4	2.0	1.6	1.1	0.7	0.3	0.0
I5	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
I6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I7	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
I8	2.5	2.5	2.5	1.3	1.5	1.7	1.9	2.1	2.4	2.5
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Locational Marginal Price [€/MWh]										
1.0	15.0	15.0	15.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
2.0	15.0	15.0	15.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0
3.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
Transmission Lines [MWh]										
1-2	12.5	12.5	12.5	**13.7**	13.5	13.3	13.1	12.9	12.7	12.5
2-3	15.0	15.0	15.0	**15.0**	15.0	15.0	15.0	15.0	15.0	15.0
1-3	27.5	27.5	27.5	**28.7**	28.5	28.3	28.1	27.9	27.7	27.5

Table 24: Oligopoly / Objective: max SW / Congested line: 1-3 / IC line: 1-2 & 2-3

Instance	**25.0**	**24.6**	**24.1**	**23.7**	**23.2**	**22.8**	**22.3**	**21.9**	**21.4**	**21.0**
Income Distribution [€]										
Consumer	3925.0	3925.0	3925.0	3925.0	3925.0	3925.0	3925.0	3925.0	3925.0	3925.0
Producer	1950.0	1950.0	1950.0	1950.0	1950.0	1950.0	1950.0	1950.0	1950.0	1950.0
Grid	63.0	63.0	63.0	63.0	63.0	63.0	63.0	63.0	63.0	63.0
SW	5938.0	5938.0	5938.0	5938.0	5938.0	5938.0	5938.0	5938.0	5938.0	5938.0
Producer Profit [€]										
J1	1630.0	1630.0	1630.0	1630.0	1630.0	1630.0	1630.0	1630.0	1630.0	1630.0
J2	320.0	320.0	320.0	320.0	320.0	320.0	320.0	320.0	320.0	320.0
J3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Consumption per Node [MWh]										
C-N1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
C-N2	22.0	22.0	22.0	22.0	22.0	22.0	22.0	22.0	22.0	22.0
C-N3	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
Total	247.0	247.0	247.0	247.0	247.0	247.0	247.0	247.0	247.0	247.0
Production per Node [MWh]										
P-N1	127.0	127.0	127.0	127.0	127.0	127.0	127.0	127.0	127.0	127.0
P-N2	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0
P-N3	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0
Total	247.0	247.0	247.0	247.0	247.0	247.0	247.0	247.0	247.0	247.0
Production per Unit [MWh]										
I1	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0
I2	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
I3	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0
I4	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
I5	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
I6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I7	17.0	17.0	17.0	17.0	17.0	17.0	17.0	17.0	17.0	17.0
I8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	247.0	247.0	247.0	247.0	247.0	247.0	247.0	247.0	247.0	247.0
Locational Marginal Price [€/MWh]										
1.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0
2.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
3.0	13.0	13.0	13.0	13.0	13.0	13.0	13.0	13.0	13.0	13.0
Transmission Lines [MWh]										
1-2	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0
2-3	21.0	21.0	21.0	21.0	21.0	21.0	21.0	21.0	21.0	21.0
1-3	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0

Table 25: Oligopoly / Objective: max TP / Congested line: 2-3 / IC line: 1-2

Instance	**17.0**	**16.9**	**16.5**	**16.2**	**15.7**	**15.4**	**15.0**	**14.6**	**14.2**	**13.8**
Income Distribution [€]										
Consumer	2900.0	2900.0	2900.0	2900.0	2900.0	2900.0	2900.0	2900.0	2900.0	2900.0
Producer	2987.5	2987.5	2987.5	3012.0	3007.3	3003.5	2987.5	2987.5	2987.5	2987.5
Grid	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SW	5887.5	5887.5	5887.5	5912.0	5907.3	5903.5	5887.5	5887.5	5887.5	5887.5
Producer Profit [€]										
J1	2337.5	2337.5	2337.5	2500.0	2500.0	2500.0	2437.5	2437.5	2437.5	2437.5
J2	200.0	200.0	200.0	62.0	57.3	53.5	100.0	100.0	100.0	100.0
J3	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
Consumption per Node [MWh]										
C-N1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
C-N2	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
C-N3	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Node [MWh]										
P-N1	140.0	140.0	140.0	144.9	144.0	143.2	140.0	140.0	140.0	140.0
P-N2	23.8	23.8	23.8	21.3	21.8	22.1	23.8	23.8	23.8	23.8
P-N3	86.3	86.3	86.3	83.8	84.3	84.6	86.3	86.3	86.3	86.3
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Unit [MWh]										
I1	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0
I2	3.8	3.8	3.8	20.0	20.0	20.0	13.8	13.8	13.8	13.8
I3	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0
I4	0.0	0.0	0.0	4.9	4.0	3.2	0.0	0.0	0.0	0.0
I5	20.0	20.0	20.0	1.3	1.8	2.1	10.0	10.0	10.0	10.0
I6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I7	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
I8	6.3	6.3	6.3	3.8	4.3	4.6	6.3	6.3	6.3	6.3
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Locational Marginal Price [€/MWh]										
1.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
2.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
3.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
Transmission Lines [MWh]										
1-2	13.8	13.8	13.8	**16.2**	15.7	15.4	13.8	13.8	13.8	13.8
2-3	12.5	12.5	12.5	**12.5**	12.5	12.5	12.5	12.5	12.5	12.5
1-3	26.3	26.3	26.3	**28.7**	28.2	27.9	26.3	26.3	26.3	26.3

Table 26: Oligopoly / Objective: max TP / Congested line: 2-3 / IC line: 1-3

Instance	**30.0**	**29.6**	**29.2**	**28.7**	**28.3**	**27.9**	**27.4**	**27.0**	**26.6**	**26.3**
Income Distribution [€]										
Consumer	2900.0	2900.0	2900.0	2900.0	2900.0	2900.0	2900.0	2900.0	2900.0	2900.0
Producer	2987.5	2987.5	2987.5	3012.0	3007.8	3003.6	2999.4	2995.2	2991.0	2987.5
Grid	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SW	5887.5	5887.5	5887.5	5912.0	5907.8	5903.6	5899.4	5895.2	5891.0	5887.5
Producer Profit [€]										
J1	2437.5	2437.5	2437.5	2400.0	2400.0	2400.0	2400.0	2400.0	2400.0	2400.0
J2	100.0	100.0	100.0	162.0	157.8	153.6	149.4	145.2	141.0	137.5
J3	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0	450.0
Consumption per Node [MWh]										
C-N1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
C-N2	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
C-N3	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Node [MWh]										
P-N1	140.0	140.0	140.0	144.9	144.1	143.2	142.4	141.5	140.7	140.0
P-N2	23.8	23.8	23.8	21.3	21.7	22.1	22.6	23.0	23.4	23.8
P-N3	86.3	86.3	86.3	83.8	84.2	84.6	85.1	85.5	85.9	86.3
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Unit [MWh]										
I1	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0
I2	13.8	13.8	13.8	10.0	10.0	10.0	10.0	10.0	10.0	10.0
I3	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0
I4	0.0	0.0	0.0	4.9	4.1	3.2	2.4	1.5	0.7	0.0
I5	10.0	10.0	10.0	11.3	11.7	12.1	12.6	13.0	13.4	13.8
I6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I7	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
I8	6.3	6.3	6.3	3.8	4.2	4.6	5.1	5.5	5.9	6.3
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Locational Marginal Price [€/MWh]										
1.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
2.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
3.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
Transmission Lines [MWh]										
1-2	13.8	13.8	13.8	**16.2**	15.8	15.4	14.9	14.5	14.1	13.8
2-3	12.5	12.5	12.5	**12.5**	12.5	12.5	12.5	12.5	12.5	12.5
1-3	26.3	26.3	26.3	**28.7**	28.3	27.9	27.4	27.0	26.6	26.3

Table 27: Triopoly / Objective: max SW / Congested line: 1-2/ IC line: 2-3

Instance	**25.0**	**24.1**	**23.3**	**22.4**	**21.5**	**20.6**	**19.8**	**18.9**	**18.0**	**17.5**
Income Distribution [€]										
Consumer	1650.0	1650.0	1650.0	1650.0	1650.0	1650.0	1650.0	1650.0	1650.0	1650.0
Producer	4262.5	4262.5	4262.5	4262.5	4262.5	4262.5	4262.5	4262.5	4262.5	4262.5
Grid	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SW	5912.5	5912.5	5912.5	5912.5	5912.5	5912.5	5912.5	5912.5	5912.5	5912.5
Producer Profit [€]										
J1	3450.0	3450.0	3450.0	3450.0	3450.0	3450.0	3450.0	3450.0	3450.0	3450.0
J2	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0
J3	212.5	212.5	212.5	212.5	212.5	212.5	212.5	212.5	212.5	212.5
Consumption per Node [MWh]										
C-N1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
C-N2	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
C-N3	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Node [MWh]										
P-N1	122.5	122.5	122.5	122.5	122.5	122.5	122.5	122.5	122.5	122.5
P-N2	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0
P-N3	87.5	87.5	87.5	87.5	87.5	87.5	87.5	87.5	87.5	87.5
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Unit [MWh]										
I1	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0
I2	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
I3	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0
I4	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
I5	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
I6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I7	12.5	12.5	12.5	12.5	12.5	12.5	12.5	12.5	12.5	12.5
I8	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Locational Marginal Price [€/MWh]										
1.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
2.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
3.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
Transmission Lines [MWh]										
1-2	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
2-3	17.5	17.5	17.5	17.5	17.5	17.5	17.5	17.5	17.5	17.5
1-3	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0

Table 28: Triopoly / Objective: max SW / Congested line: 2-3 / IC line: 1-2

Instance	**14.0**	**13.7**	**13.4**	**13.0**	**12.7**	**12.3**	**12.0**	**11.6**	**11.3**	**10.0**
Income Distribution [€]										
Consumer	2900.0	1975.0	1975.0	3275.0	3275.0	3275.0	3275.0	3275.0	3275.0	3275.0
Producer	3025.0	3938.5	3936.8	2479.0	2481.5	2483.9	2486.4	2488.8	2491.3	2500.0
Grid	0.0	41.1	40.1	195.0	189.8	184.5	179.3	174.0	168.8	150.0
SW	5925.0	5954.6	5951.8	5949.0	5946.2	5943.4	5940.6	5937.8	5935.0	5925.0
Producer Profit [€]										
J1	2500.0	3134.5	3139.8	2105.0	2110.3	2115.5	2120.8	2126.0	2131.3	2150.0
J2	300.0	410.0	410.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
J3	225.0	394.0	387.0	124.0	121.2	118.4	115.6	112.8	110.0	100.0
Consumption per Node [MWh]										
C-N1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
C-N2	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
C-N3	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Node [MWh]										
P-N1	135.0	142.4	141.7	141.0	140.3	139.6	138.9	138.2	137.5	135.0
P-N2	30.0	26.3	26.7	27.0	27.4	27.7	28.1	28.4	28.8	30.0
P-N3	85.0	81.3	81.7	82.0	82.4	82.7	83.1	83.4	83.8	85.0
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Unit [MWh]										
I1	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0
I2	20.0	16.3	16.7	17.0	17.4	17.7	18.1	18.4	18.8	20.0
I3	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0
I4	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
I5	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
I6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I7	25.0	32.4	31.7	31.0	30.3	29.6	28.9	28.2	27.5	25.0
I8	5.0	1.3	1.7	2.0	2.4	2.7	3.1	3.4	3.8	5.0
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Locational Marginal Price [€/MWh]										
1.0	15.0	18.0	18.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
2.0	15.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
3.0	15.0	19.0	19.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
Transmission Lines [MWh]										
1-2	10.0	**13.7**	13.4	13.0	12.7	12.3	12.0	11.6	11.3	10.0
2-3	15.0	**15.0**	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
1-3	25.0	**28.7**	28.4	28.0	27.7	27.3	27.0	26.6	26.3	25.0

Table 29: Triopoly / Objective: max SW / Congested line: 2-3 / IC line: 1-3

Instance	**30.0**	**29.6**	**28.7**	**28.1**	**27.5**	**26.9**	**26.3**	**25.7**	**25.1**	**25.0**
Income Distribution [€]										
Consumer	2900.0	2900.0	3650.0	1025.0	1025.0	1025.0	1025.0	1025.0	1025.0	1025.0
Producer	3025.0	3025.0	2079.6	4362.3	4357.5	4352.7	4347.9	4343.1	4338.3	4337.5
Grid	0.0	0.0	225.0	562.5	562.5	562.5	562.5	562.5	562.5	562.5
SW	5925.0	5925.0	5954.6	5949.8	5945.0	5940.2	5935.4	5930.6	5925.8	5925.0
Producer Profit [€]										
J1	2375.0	2375.0	1850.0	3725.0	3725.0	3725.0	3725.0	3725.0	3725.0	3725.0
J2	200.0	200.0	100.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
J3	450.0	450.0	129.6	387.3	382.5	377.7	372.9	368.1	363.3	362.5
Consumption per Node [MWh]										
C-N1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
C-N2	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
C-N3	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Node [MWh]										
P-N1	140.0	140.0	142.4	141.2	140.0	138.8	137.6	136.4	135.2	135.0
P-N2	27.5	27.5	26.3	26.9	27.5	28.1	28.7	29.3	29.9	30.0
P-N3	82.5	82.5	81.3	81.9	82.5	83.1	83.7	84.3	84.9	85.0
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Unit [MWh]										
I1	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0
I2	7.5	7.5	8.1	10.0	10.0	18.0	10.9	11.6	12.4	12.5
I3	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0
I4	0.0	0.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
I5	20.0	20.0	18.2	16.9	17.5	10.1	17.8	17.7	17.5	17.5
I6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I7	50.0	50.0	32.4	31.2	30.0	28.8	27.6	26.4	25.2	25.0
I8	2.5	2.5	1.3	1.9	2.5	3.1	3.7	4.3	4.9	5.0
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Locational Marginal Price [€/MWh]										
1.0	15.0	15.0	10.0	17.5	17.5	17.5	17.5	17.5	17.5	17.5
2.0	15.0	15.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0
3.0	15.0	15.0	15.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0
Transmission Lines [MWh]										
1-2	12.5	12.5	**13.7**	13.1	12.5	11.9	11.3	10.7	10.1	10.0
2-3	15.0	15.0	**15.0**	15.0	15.0	15.0	15.0	15.0	15.0	15.0
1-3	27.5	27.5	**28.7**	28.1	27.5	26.9	26.3	25.7	25.1	25.0

Table 30: Triopoly / Objective: max TP / Congested line: 1-3 / IC line: 1-2

Instance	**5.5**	**5.0**	**4.3**	**3.7**	**3.0**	**2.4**	**1.7**	**1.0**	**0.4**	**0.0**
Income Distribution [€]										
Consumer	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Producer	5385.0	5420.0	5414.1	5414.9	5437.3	5459.8	5482.2	5504.6	5527.1	5565.0
Grid	180.0	90.0	78.1	66.2	54.4	42.5	30.6	18.7	6.8	0.0
SW	5565.0	5510.0	5492.2	5481.1	5491.7	5502.2	5512.8	5523.4	5533.9	5565.0
Producer Profit [€]										
J1	4530.0	4310.0	4620.0	4620.0	4620.0	4620.0	4620.0	4620.0	4620.0	4620.0
J2	600.0	690.0	380.0	548.6	583.0	617.3	651.6	685.9	720.2	690.0
J3	255.0	420.0	414.1	246.2	234.4	222.5	210.6	198.7	186.8	255.0
Consumption per Node [MWh]										
C-N1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
C-N2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C-N3	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
Total	225.0	225.0	225.0	225.0	225.0	225.0	225.0	225.0	225.0	225.0
Production per Node [MWh]										
P-N1	120.0	125.0	123.0	123.7	123.0	122.4	121.7	121.0	120.4	120.0
P-N2	20.0	10.0	10.0	12.6	14.0	15.3	16.6	17.9	19.2	20.0
P-N3	85.0	90.0	92.0	88.7	88.0	87.4	86.7	86.0	85.4	85.0
Total	225.0	225.0	225.0	225.0	225.0	225.0	225.0	225.0	225.0	225.0
Production per Unit [MWh]										
I1	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0
I2	10.0	0.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
I3	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0
I4	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
I5	10.0	10.0	0.0	2.6	4.0	5.3	6.6	7.9	9.2	10.0
I6	0.0	0.0	0.0	8.7	8.0	7.4	6.7	6.0	5.4	0.0
I7	10.0	15.0	13.0	13.7	13.0	12.4	11.7	11.0	10.4	10.0
I8	5.0	10.0	12.0	0.0	0.0	0.0	0.0	0.0	0.0	5.0
Total	225.0	225.0	225.0	225.0	225.0	225.0	225.0	225.0	225.0	225.0
Locational Marginal Price [€/MWh]										
1.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0
2.0	27.0	36.0	36.0	36.0	36.0	36.0	36.0	36.0	36.0	36.0
3.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0
Transmission Lines [MWh]										
1-2	0.0	**5.0**	4.3	3.7	3.0	2.4	1.7	1.0	0.4	0.0
2-3	20.0	**15.0**	14.3	16.3	17.0	17.6	18.3	19.0	19.6	20.0
1-3	20.0	**20.0**	18.7	20.0	20.0	20.0	20.0	20.0	20.0	20.0

Table 31: Triopoly / Objective: max TP / Congested line: 2-3 / IC line: 1-2 & 1-3

Instance	**14.0**	**13.7**	**13.2**	**12.8**	**12.3**	**11.9**	**11.4**	**11.0**	**10.5**	**10.0**
Income Distribution [€]										
Consumer	50.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Producer	5605.0	5662.5	5656.6	5650.8	5643.4	5634.0	5624.6	5615.2	5605.8	5665.0
Grid	270.0	267.3	266.4	265.5	264.6	263.7	262.8	261.9	261.0	260.0
SW	5925.0	5929.8	5923.0	5916.3	5908.0	5897.7	5887.4	5877.1	5866.8	5925.0
Producer Profit [€]										
J1	4570.0	4460.0	4460.0	4460.0	4566.5	4552.6	4538.7	4524.8	4510.9	4610.0
J2	510.0	302.5	296.6	290.8	176.9	181.4	185.9	190.4	194.9	530.0
J3	525.0	900.0	900.0	900.0	900.0	900.0	900.0	900.0	900.0	525.0
Consumption per Node [MWh]										
C-N1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
C-N2	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
C-N3	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Node [MWh]										
P-N1	135.0	142.3	141.4	140.5	139.6	138.7	137.8	136.9	136.0	135.0
P-N2	30.0	26.4	26.8	27.2	27.7	28.1	28.6	29.0	29.5	30.0
P-N3	85.0	81.4	81.8	82.2	82.7	83.1	83.6	84.0	84.5	85.0
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Production per Unit [MWh]										
I1	90.0	90.0	90.0	90.0	89.6	88.7	87.8	86.9	86.0	90.0
I2	20.0	10.0	10.0	10.0	17.7	18.1	18.6	19.0	19.5	20.0
I3	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0
I4	20.0	2.3	1.4	0.5	0.0	0.0	0.0	0.0	0.0	20.0
I5	10.0	16.4	16.8	17.2	10.0	10.0	10.0	10.0	10.0	10.0
I6	0.0	1.4	1.8	2.2	2.7	3.1	3.6	4.0	4.5	0.0
I7	25.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	25.0
I8	5.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.0
Total	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0	250.0
Locational Marginal Price [€/MWh]										
1.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0
2.0	18.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
3.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0
Transmission Lines [MWh]										
1-2	10.0	**13.7**	13.2	12.8	12.3	11.9	11.4	11.0	10.5	10.0
2-3	15.0	**15.0**	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
1-3	25.0	**28.7**	28.2	27.8	27.3	26.9	26.4	26.0	25.5	25.0