Implications of the Solvency II Regulations for Investment Incentives

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Abstract

We analyze optimal investment incentives for a medium sized insurance company complying with the Solvency II regulations. Assuming that market investments are made independent of other operations, we find that the insurance company has incentives to increase investment in low stress factor equities. If the Solvency II standard formula stress test is a good estimation of the underlying risk, this leads to a reduction in risk taken by the insurance company.

Allowing for reallocation of the market portfolio after reporting capital requirements decreases the risk reduction incentive. This effect may be mitigated by introducing transaction costs. At last we do not impose Solvency II restrictions on investment, but model the effect of supervisory intervention at a future date. This leads to risk reducing incentives for the insurancy company, contingent on the viability of the supervisory intervention threat.

Contents

1 Introduction

Solvency II is a European supervisory framework for insurance and reinsurance companies which came into effect on 1 January 2016. The framework consists of three pillars. Calculation of reserves, Management of risks and governance, and Reporting and disclosure. The framework aims to incentivise (re)insurance companies to identify and manage the risk they face (EIOPA, 2016).

Solvency II divides risk into six modules. Market risk, default (credit) risk, life underwriting risk, health underwriting risk, non-life underwriting risk, and intangible asset risk. The total Solvency Capital Requirement (SCR) is an aggregation of the capital requirements which stems from the different modules, accounting for some diversification effects (EIOPA, 2014).

The Solvency II standard formula for capital risk aggregation aims to be an estimation of a 99,5% Value-at-Risk level. Reinsurance and insurance companies are however free to implement internal 99,5% VaR-models as a substitute for the entire Solvency standard formula, or parts of it. Internal models are subject to validation and approval by supervisory authorities (CEIOPS, 2009b).

This thesis will focus on the SCR market risk module. The market risk module calculates interest rate risk, equity risk, property risk, spread risk, concentration risk and currency risk. Each of these submodules are furthermore divided into subgroups based on certain risk criterias. The capital requirement for each subgroup is calculated based on the market value invested in the subgroup and a predetermined percentage stress factor. In the equity subgroups for instance, the stress factors determines the capital buffers needed as a fraction of market value the insurance company holds in each of the subgroups. The capital requirements from each subgroup are then aggregated into the SCR for the market risk module, accounting for some diversification effects between the different submodules and subgroups.

In addition to the SCR, the Solvency II framework also includes a Minimum Capital Requirement (MCR). The MCR is the *"minimum level of security below which the amount of resources should not fall"* (European Comission, 2016). The MCR has a floor value of 25% of the SCR and a cap of 45% of the SCR. The specific MCR within this corridor is determined by characteristics of the insurance company (CEIOPS, 2009a).

If an insurer's available capital falls below the SCR, then supervisory national authorities will take action in order to restore the insurer's finances. If the financial situation deterioate further, then the supervisory intervention will progressively intensify. If the insurer's available capital falls below the MCR, then the ultimate supervisory action is triggered. This means that the insurer's liabilities will be taken over by another insurance company, or that the company will be liquidated (European Comission, 2016).

In this thesis we will undertake an analysis of the SCR standard formula in a utility maximizing setting. In particular, we study the implications of the SCR standard formula on optimal investment incentives.

The main purpose of our thesis is to answer the question:

How does the Solvency II regulation affect the investment incentives of a medium sized insurance company applying the standard formula stress test?

In order to answer this question we will study; how different stress factors between asset groups in the SCR standard model will affect an insurance company's investment incentives, and their risk. Whether the risk an insurance company face changes inbetween reporting dates. Whether the threat of supervisory intervention will affect the insurance company's investment incentives.

2 Background

2.1 Motivation and scope

The implementation of Solvency II regulations has been the subject of study in a number of economic publications. Since the release of the first Solvency II proposals it has spurred criticism (Doff, 2008; International Monetary Fund, 2011). Ambiguous requirements of the Solvency II standard model stress test is often emphasized (Aria et al., 2010; Pfeiffer and Strassburger, 2008; S Mittnik, 2011). Some have encouraged the use of own models (pwc, 2011). Implementing internal models may however be costly, especially for medium and small sized insurance companies.

In our thesis we have a different approach than the Solvency II studies to date. We analyze the optimal investment decision of an insurance company complying with the Solvency II regulation in a theoretical framework. Although we have not found literature analyzing Solvency II from a theoretical viewpoint, prior regulations has been analyzed in a similar way. The Value-at-risk based risk management is the foundation for the standard formula and many other financial regulations, for instance Basel III (Basel III,2011). This framework has been analyzed thoroughly (see for instance Basak and Shapiro 2001). We will not focus on the Value-at-risk aspect, but rather focus on the specific implementation through the standard formula stress test.

We compare incentives under Solvency II regulations with the incentives of a regular company with only a budget restriction. We will not compare Solvency II regulations nor the standard formula against other regulations.

2.2 Economic Setting

We are considering an economy with a single consumption good, the numeraire. There are discrete time periods denoted by $t = 0, 1, 2, ...$. At time t, the world may be in one of M different states. At all times, the probability of states occuring next period will be uniformly distributed. We assume that at $t = 0$ there is only one state. The economy has an investment opportunity set consisting of $N + 1$ assets, where the return of asset 0 is known at all time periods t and is thus considered riskless. At all dates, and in all states,

the assets' price vector, **P**, is known. The payoff distribution of every asset is assumed to be known one period ahead. The payoffs can be represented by a $M \times (N+1)$ payoff matrix, **X**, where columns denote the $N + 1$ assets and the rows denote the M states. From the price vector **P** and the payoff matrix **X**, every assets return distribution one period ahead can be calculated and is denoted by **R**.

The economy is complete without any transaction costs. We assume no arbitrage opportunities. Any wealth distribution the next period can thus be achieved, only limited by the funds available for investment. The result of this assumption is that we can calculate pure security prices as defined by Arrow and Debreu (1954). A pure security is a contract which yields the owner one unit of a numeraire in one state, and zero units of a numeraire in all other states. In equilibrium, the price of such securities is assumed to be determined by the probability of the state to occur and the aggregate supply in the particular state. By purchasing pure securities, the agent is able to freely choose the state-distribution of final period wealth within his budget.

2.3   The Market Participants

Throughout our thesis we are looking at a medium-sized non-life insurance company. We analyze investment incentives in partial equilibrium, where the insurance company will not affect the asset prices or the risk-free rate. We are assuming one agent in control of the entire market investment portfolio of the insurance company. The agent is given a fixed amount of funds to invest on behalf of the firm. All aspects, other than market investments of the insurance company's operations are fixed, including the SCR from other operations. We assume that the agent is acting within the Solvency II regulations.

Life insurance companies will typically have a strong relationship between investment and operations. In a non-life insurance company it may be more realistic to find the saperation of investment and operation decisions. When we assume separation of these concerns as well as risk aversion, it has several implications. Our thesis will not result in a guidance to optimal investment within an insurance company. Instead we take the regulatory view, as to how the implementation of Solvency II will actually affect

investment incentives.

Shareholders are often believed to be risk neutral when it comes to diversifiable risk. However, as Bickel (2006) shows, the cost of financial distress might induce risk aversion in the company. Bickel (2006) further shows that a risk averse decision maker might induce a risk averse company when the decision maker have positive investments in the company. Apart from risk aversion, we assume aligned incentives between the shareholders and the agent. The agent maximizes the final period expected utility. A similar approach of assuming a risk averse decision maker has been used in previous literature, see for instance Hipp and Plum (2000) or Browne (1995).

In our thesis we are assuming that the risk averse agent confines to the axioms of rational bahavior, first proposed by von Neumann and Morgenstern (1944). If an individual's preferences satisfies the axioms of *completeness, transitivity, continuity, independence* and *dominance*, then these preferences can be represented by an interval scale Pennacchi (2008). The individual will always perform actions which maximizes expected utility. Hence, we are able to determine the agent's optimal behaviour. By assuming a risk averse agent which confines to these axioms, and has a state additive utility function, we are able to propose optimization problems where the agent maximizes

$$E[u(W_T)] \tag{2.1}$$

W_T denotes the final period value of the companys' market portfolio.

Assuming a risk averse and non-satiable agent we can assign the following structure on the agent's utility function.

$$u'() > 0 \qquad u''() \leq 0 \tag{2.2}$$

2.4 Model I - Direct Effect

As previously stated, the Solvency II standard formula consists of different submodules, where assets are categorized into subgroups by several measures reflecting their level of risk. The different subgroups' contribution to the total SCR depends on their particular stress factor. If our agent is facing an exogenous SCR limit which he must comply with, then investments in an asset group with a high stress factor will be restricted. Investments in a high stress factor asset group may then have an extra opportunity cost as it limits investment in another high stress factor asset group. The stress factor may then be important as to how the Solvency II framework might alter the agent's investment incentives, and his risk taking. For now we assume that asset separation reflects the assets' actual risk profile.

The agent is limited to three stocks and a riskless bond. We categorize Asset 2 and Asset 3 as risky. Asset 1 is less risky and will be stressed with a lower stress factor. This represents the standard formula equity submodule, which are divided into two sub-groups by whether they are listed in an OECD country, or not. Whether this separation is a true indicator of risk is a question we do not answer in this thesis. There is how-ever a good possibility that one could achieve high risk levels investing in OECD-listed stocks. Whether this will lead to a shift in investments from non-OECD countries to OECD countries is an important question that ought be investigated further, but it is outside the scope of this thesis.

Limiting investment to only stocks and a riskless bond is a simplification that is coherent with what our first model should analyze. We want to study whether different stress factors will shift investment between asset groups. In addition, we want the model to answer whether or not investment will shift within the risky asset group.

The model should reveal the direct effects of the SCR standard formula, considering the insurance company must report a capital buffer satisfying the SCR. The agent is then constrained to a certain $t = 0$ capital reserve, but he gets his utility from his final period wealth level which is a result of these investments.

Figure 2.1 depicts the insurance companies stressed balance sheet when excluding the market portfolio. M_O is the difference between the stressed value of other assets and
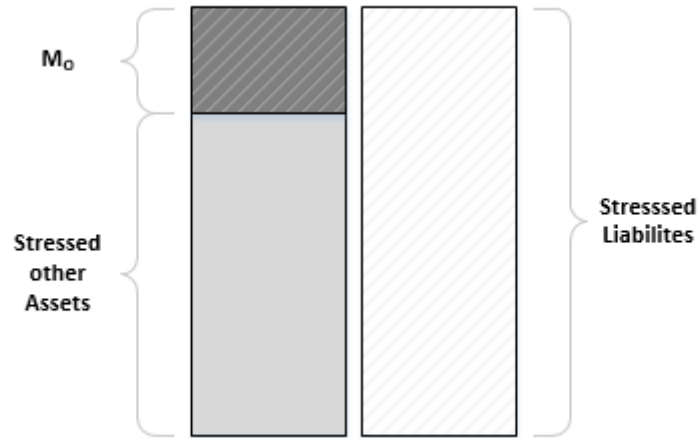
Figure 2.1: Stressed balance sheet - Market Portfolio not included

the value of stressed liabilites. We will assume that this is positive in the sense that the stressed liabilities have a higher value than the stressed other assets. Then the company must cover the amount M_O with the stressed value of the market portfolio.

Figure 2.2 depicts the balance sheet as seen by our agent. The value M_O, stressed other assets and stressed liabilites are assumed to be exogenous to our agent. The initial value of the market portfolio, W_0, is also exogenously given. Our agent can only affect M_e, which is the capital requirement stemming from stressing his market investment. To comply with the Solvency II regulations, the agent must invest such that the value of stressed other assets and the initial wealth W_0 is greater then the SCR, as can be seen in figure 2.2. In this model, the Solvency II regulations will have a direct effect on the agent's investments, as the agent must comply with the following constraint

$$M_o + M_e \leq W_0 \tag{2.3}$$

To calculate M_e we use the Solvency II standard formula for the equity subgroup. The total investment in the two equity subgroups is multiplied with its stress factor value s_r, where r denotes the asset group. If the assets $1, ..., m$ are in asset group r, we denote the total investment in these assets as $x_r = x^1 + x^2 + ... + x^m$. x^j is dollars invested in asset

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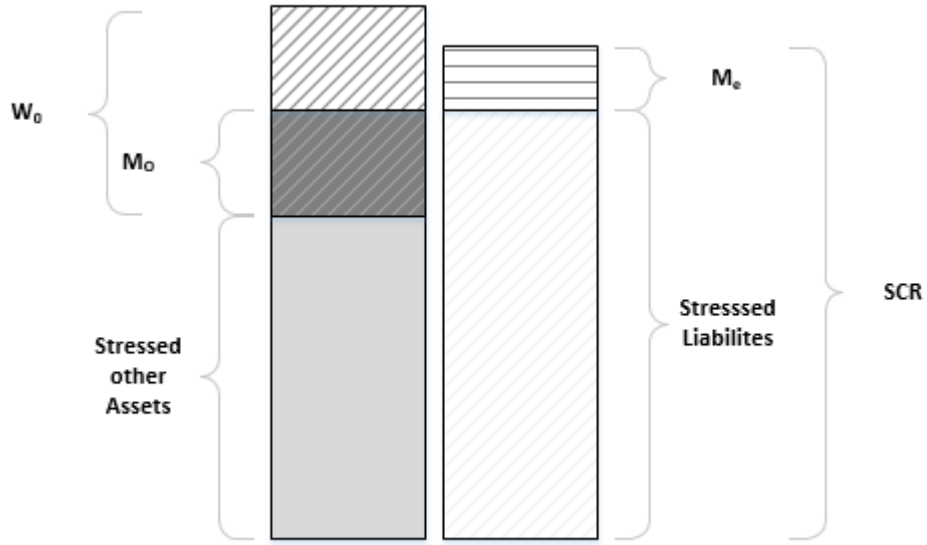
Figure 2.2: Stressed balance sheet - Market Portfolio included

j. Net short positions in assets are neglected when computing x_r in compliance to the Solvency II regulations (EIOPA, 2014). We define the contribution from asset group r as

$$M_{e,r} = s_r \cdot x_r \tag{2.4}$$

Correlation matrix $\mathbf{V}$ exhibits diversification effects between the two subgroups.

$$\mathbf{V} = \begin{bmatrix} 1 & 0.75 \\ 0.75 & 1 \end{bmatrix} \tag{2.5}$$

The diversification effect is taken into account when calculating M_e.

$$M_e = \sqrt{\sum_{r,c} \mathbf{V}_{r,c} \cdot M_{e,r} \cdot M_{e,c}} \tag{2.6}$$

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Rearranging equation 2.3 we get the agent's optimization problem. Denoting the unit vector as $\mathbf{e}$ and the dollars invested as $\mathbf{x}$. $\mathbf{x}^T$ is the transposed of $\mathbf{x}$.

$$\max_{\mathbf{x}} E[u(R\mathbf{x})] \tag{2.7}$$

s.t.

$$\mathbf{x}^T \mathbf{e} \leq W_0 \tag{2.8}$$

$$M_e \leq W_0 - M_o \tag{2.9}$$

The objective function and budget constraint given in equations 2.7 and 2.8 are similar to a regular utility optimizing agent. Constraint 2.9 requires the agent to fullfill the Solvency II capital requirements at $t = 0$.

2.5 Model II - Multiperiod model

Model I directly influences investment by limiting it to comply with the Solvency II regulation. In reality, this effect will only occur at the time of reporting. There is no reporting of investments between reporting dates. In our seccond model we want to investigate whether Solvency II regulations will alter investments if we allow for repositioning of the investment portfolio between the reporting date and the final period. For now we assume that the agent's investment horizon is shorter than the reporting periods, such that he does not account for reporting at future dates. The effect of future Solvency II reporting is accounted for in model III.

From our first model we add a time period. The agent invests his portfolio at $t = 0$ and must comply with the Solvency II regulations. The investment opportunity set still consists of three stocks and a riskless bond. Utility is gained from final period wealth. In addition, the agent will be able to reposition his portfolio at $t = 1$. Final period wealth is measured at $t = 2$. Time periods will be denoted by subscript $t = 0, 1, 2$. The return matrix, $\mathbf{R}_{t+1}$, denotes the return in the period between t and $t + 1$. Dollars invested at time t is given by the vector $\mathbf{x}_t$. The utility function at date t is denoted by $U(W_t, t)$.

The agent will maximize expected final period wealth, $W_T = \mathbf{R_2 x_1}$.

$$\max_{\mathbf{x}} E[U(R_2\mathbf{x}_1, 2)] \tag{2.10}$$

$$\mathbf{x}_0^T \mathbf{e} \leq W_0 \tag{2.11}$$

$$\mathbf{x}_1^T \mathbf{e} \leq R_1 \mathbf{x}_0 \tag{2.12}$$

$$M_e \leq W_0 - M_o \tag{2.13}$$

Equation 2.11 and equation 2.12 are budget constraints, respectively at $t = 0$ and $t = 1$. As in Model I the agent must satisfy the Solvency II regulations at $t = 0$ as depicted in equation 2.13. However, the agent is not limited by any Solvency II regulations on investments done at $t = 1$.

2.6 Model III - Supervisory Intervention

Model I and Model II studies the effect of the Solvency II standard formula on optimal asset allocation when applying the Solvency II framework at $t = 0$. Our third model aims to reveal the effect of supervisory intervention when having having to meet capital requirements in the future. As the insurance company's capital reserves are decreased below the SCR, the supervisory authorities have tools which may limit the insurance companies' investment opportunity set, or directly intervene with the company's operations.

To explore the effect of supervisory intervention we will use a single period model with investments at $t = 0$. We will not have any Solvency II related constraints at the beginning of the period, but the agent will have to comply with a budget constraint. The agent is free to choose from any assets, and we assume a complete economy. The agent is then able to invest directly in the economy's pure securities. θ^s are units of pure security s bought by the investor. The pure security prices are given by the vector $\mathbf{p}$, which implies that $t = 0$ investment costs $\theta \cdot \mathbf{p}$, and that $t = 1$ direct wealth is given by the investment vector θ.

In Model II we redefine final period wealth W_T. In each state, we are now subtracting a punitive monetary effect when the number of pure securities bought is less than a pre-determined wealth level, C_k.

In state s, we define final period wealth as

$$W_T = \theta^s - \sum_{k=1}^{K} A_k \cdot S_k^s \qquad (2.14)$$

Where S_k^s is the amount by which final period wealth in state s is less than a pre-determined wealth level. In this model, the punishment from supervisory intervention comes in the form of fines, as a monetary amount. As we mentioned in the introduction this is not the case in reality. In reality, supervisory intervention comes in the form of involvement in the decision making, and thus putting constraints on the insurance company's investments. At best, this will leave the insurance company's expected utility unchanged. Presumably it will decrease it. We have chosen to model supervisory intervention as a monetary fine, as this changes the insurance company's expected utility in a similar way. When the final period wealth falls below a certain level, this will decrease the agent's utility in this state, just as supervisory intervention presumably will do the same. This punishment effect will progressively increase in each state as the final period wealth decreases further.

Overall, there are K different wealth levels in our model. If the final period wealth falls below another wealth level, the overall increase of the punishment effect will intensify. In the model, A_k is a constant which determines the magnitude of the punishment effect from S on the agent's utility.

Our model should reveal the theoretical effect that supervisory intervention has on an insurance company's investment decision. Implementing equation 2.14 into the objective function, our optimization problem becomes

$$\max_{\theta} \ E\left[u(W_T)\right] \qquad (2.15)$$

subject to

$$\theta^{\mathbf{T}}\mathbf{p} \leq W_0 \qquad (2.16)$$

$$\theta^s \geq C_k - S_k^s \qquad \forall \; s, \; k \qquad\qquad (2.17)$$

In combination with equation 2.15, equation 2.17 ensures that when $\theta^s \leq C_k$, then $S_k^s = C_k - \theta^s$.

3 Solutions and Discussion

3.1 Model I - Direct Effect

3.1 Analytical solution

We first solve the model given in section 2.4. The Lagrangian function is given by

$$\mathscr{L} = E[u(R\mathbf{x})] - \lambda_1(x^T\mathbf{e} - W_0) - \lambda_2(M_e - W_0 + M_o) \tag{3.1}$$

Denoting the total return of asset j as R^j, the first order conditions are given by

$$\frac{\partial\mathscr{L}}{\partial x^j} = E[u'(R\mathbf{x})R^j] - \lambda_1 - \lambda_2\left(\frac{\partial M_e}{\partial x^j}\right) = 0 \tag{3.2}$$

$$\lambda_1\frac{\partial\mathscr{L}}{\partial\lambda_1} = \lambda_1(\mathbf{x}^T\mathbf{e} - W_0) = 0 \tag{3.3}$$

$$\lambda_2\frac{\partial\mathscr{L}}{\partial\lambda_2} = \lambda_2(M_e - W_0 + M_o) = 0 \tag{3.4}$$

It is safe to assume that the budget constraint is binding since we have a non-satiable agent. If we first assume the solvency constraint not to be binding, we have $\lambda_2 = 0$ and condition 3.2 can be restated as

$$\frac{\partial\mathscr{L}}{\partial x^j} = E[u'(Rx)R^j] - \lambda_1 = 0 \tag{3.5}$$

Comparing arbitrarily chosen assets j and i gives the following relationship

$$E[u'(Rx)R_j] = E[u'(Rx)R^i] \tag{3.6}$$

This is the same investment strategy performed by a regular risk averse investor, investing such that the marginal utility weighted return of each asset equals each other. Next we assume that the solvency constraint 2.9 is binding. Optimal investment is now given by

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$$E[u'(R\mathbf{x})R^j] = E[u'(Rx)R^i] - \lambda_2 \left(\frac{\partial M_e}{\partial x^i} - \frac{\partial M_e}{\partial x^j} \right) \qquad (3.7)$$

This leads to proposition 1.

Proposition 1. *Assume a risk inverse decision maker constrained by Solvency II regulations. If there are no net short positions, and other investments are held at a fixed level, the optimal investment in equities with a low stress factor will increase relative to investment in equities with a higher stress factor. This effect decreases as the investment in the low stress factor equity group is increased.*

Proof. See appendix A. ∎

Proposition 1 is expected. We assume that the agent follows the Solvency II regulation, and calculation of capital requirements is constructed such that high stress factor equities will induce a higher capital requirement, and hence investment in this group must be decreased as long as the Solvency II constraint is binding and no other alterations of the portfolio is considered.

If the agent is free to alter his entire portfolio, he may invest more in other asset groups, besides equity, with a lower impact on the total capital requirement. The Solvency II regulations would still induce the effect described in proposition 1. The effect of a volatility decrease in end of period wealth following an increase in low risk asset group investment may induce increasing investment in high risk equities. The sum of these effects may decrease, or increase relative investment in equity group one and two.

Proposition 1 relates to how investment is altered between the high and low stress factor equity groups. Next we consider investment alterations within the same equity group.

Proposition 2. *Independent of whether the Solvency II constraint is binding or not, the investment in equities j,i belonging to the same stress factor group will follow the relationship given in equation 3.6.*

Proof. See appendix A. ∎

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Proposition 2 may contradict reallocation incentives within an asset group following a binding Solvency II constraint. We must however take into consideration that the distribution of final period wealth changes with a binding Solvency II constraint according to proposition 1. If we assume that low stress factor stocks have less volatility, proposition 1 implies that the final period wealth distribution is less volatile. The agent would in this situation be less reluctant to take on the risk in high risk assets belonging to the high stress factor asset group.

3.1 Numerical example

At $t = 1$ we may be in one of four states, with each of the states having equal probability of occurring. The price vector, $\mathbf{P}$, denotes the $t = 0$ price of each asset. Matrix $\mathbf{X}$ denotes the payoffs.

$$
\mathbf{P} = \begin{bmatrix} 0.99 \\ 2.1 \\ 2.72 \\ 2.2 \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} 1 & 2 & 2 & 0 \\ 1 & 1 & 2 & 5 \\ 1 & 3 & 3 & 3 \\ 1 & 3 & 5 & 2 \end{bmatrix}, \quad E[\mathbf{R}] = \begin{bmatrix} 1.01 \\ 1.07 \\ 1.10 \\ 1.14 \end{bmatrix} \tag{3.8}
$$

We have a complete economy and by replicating the pure security payoffs using our four securities, we obtain the following pure security prices.

$$
\mathbf{p} = \begin{bmatrix} 0.36 \\ 0.25 \\ 0.19 \\ 0.18 \end{bmatrix} \tag{3.9}
$$

The agent faces a various amount of an exogenously given capital requirement, M_o. The initial value of his market portfolio, W_0, is set at 10. The agent is risk averse and has the following utility function.

$$
u(x) = \frac{x^{1-\gamma} - 1}{1 - \gamma} \tag{3.10}
$$

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With relative risk aversion $\gamma = 0.4$.

Asset one belongs to equity group one with stress factor $s_1 = 0.39$. Asset two and asset three belongs to equity group two and have a stress factor $s_2 = 0.49$.[1]

We solve the agent's optimization problem using MATLAB and the solver *fmincon* using the *interior-point* algorithm.[2] The problem is then reduced to a sequence of approximate subproblems. An overview of the solution method can be found at MathWorks (2016). An in depth explanation of the *interior-point* algorithm can be found in Byrd, Schnabel and Schultz (1999), Byrd, Gilbert and Nocedal (2000) or Waltz et al. (2006).

Without restrictions on investments, or borrowing at the risk-free rate, the agent reduces the volatility of final period wealth significantly as the exogenous capital requirement M_o is increased. The distribution of final period wealth is shown in figure 3.1a. A greater exogenous capital requirement limits the agent's investment opportunity set, as he is forced to either adjust his equity portfolio or increase investment in the riskless bond. Initially the agent borrows at the risk-free rate and invests evenly between the two asset groups as shown in figure 3.1b. As the investment opportunity set decreases with increasing exogenous capital requirements, the agent invests positive amounts in the risk-free asset and decreases investments in equities. This gives a smoother final period wealth, while at the same time altering the optimal allocation of the equity investment.

As the agent invests more in the riskless bond, he both decreases the relative investment in equity group one, in addition to increasing investment in the risky asset three relative to the less risky asset two. This may seem like a contradiction to proposition 1, but we must take into account that we now do not keep investment in the riskless bond constant. Hence, increasing investment in the riskless bond smoothes final period wealth, which in turn makes the agent willing to increase investments in risky assets relative to low risk assets within the equities.

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[1]Stress factors of equity asset group one and two in the Solvency II standard formula is in reality a varying amount. Their base levels are 39% for asset group one and 49% for asset group two. They are then symmetrically adjusted based on current market conditions (EIOPA, 2014).

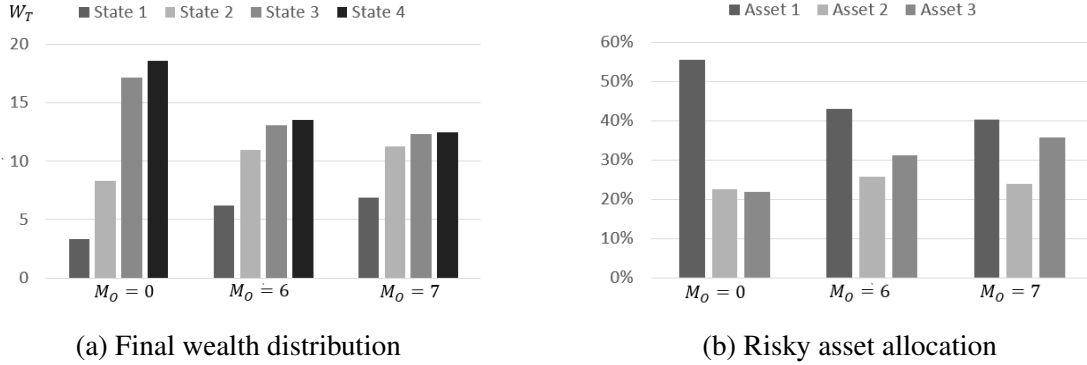[2]The code can be found in appendix B.

(a) Final wealth distribution

(b) Risky asset allocation

Figure 3.1: Model I - Final period wealth and risky asset allocation at various levels of M_O without restriction on riskless asset investment

The agent's equity portfolio reallocation, as shown in figure 3.1b, is a good example of how smoothing of final period wealth mitigates the pure Solvency II effect as stated in proposition 1. To analyze reallocation effects within the equities, keeping the other investments constant, we now limit investment in the riskless bond to a small interval, $0 \leq x_f \leq 1$.

(a) Final wealth distribution

(b) Risky asset allocation
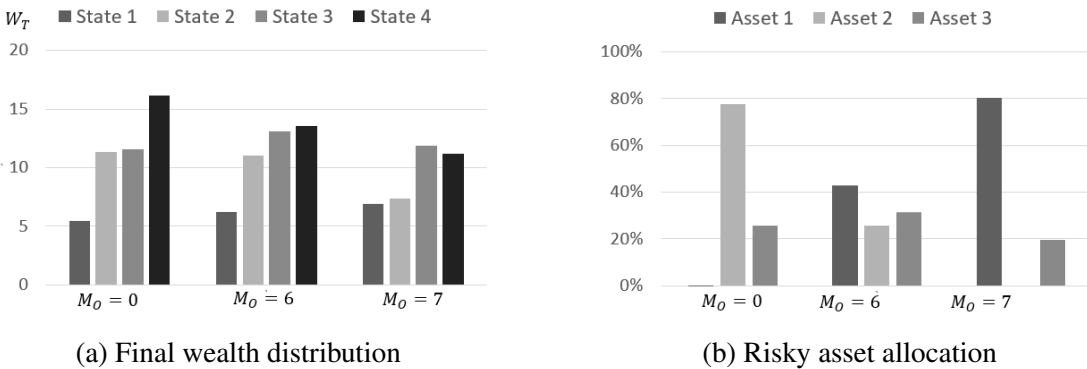
Figure 3.2: Model I - Final period wealth and risky asset allocation at various levels of M_O with restriction on riskless asset investment

As we can see from figure 3.2a, the agent is still smoothing final period wealth. From figure 3.2b we can see the equity portfolio reallocation. As proposition 1 states, keeping the other investments fixed, causes a shift in the equity investment. We see a reallocation from the high stress factor group consisting of asset two and three, to the low stress

factor group consisting of asset one. As the exogenous capital requirement increases, the Solvency II constraint increasingly limits the investment opportunity set, leaving the agent no choice but to increase investment in asset one. Because asset one is indeed less risky, this smoothes final period wealth, leaving the agent less risk averse when choosing the optimal investment in the high stress factor group. The agent then increases investment in asset three compared to asset two, as asset three has a higher expected return.

In our example, the high stress factor equities have a riskier return distribution than asset one with a lower stress factor. In the Solvency II regulation, the assets' stress factor is based on whether they are listed in an OECD country or not. One could argue that this is not a meaningful classification, and that there are many risky stocks listed within the OECD countries, and many low risk stocks listed outside the OECD. As shown in figure 3.2b, if the Solvency II constraint is binding, the agent is less likely to invest in low risk equities within the high stress factor equity group, if this means lowering the expected return. The agent is better off by investing in assets within the low stress factor group, effectively increasing the investment opportunity set by lowering the capital requirement.

As we can see from figure 3.1a and 3.1b, if the agent is free to shift investment away from equities to assets with a lower stress factor, like government bonds, the agent is likely to increase investment in risky equities relative to low risk equities. In both cases, the risk of insolvency is reduced, as the total risk is reduced.

We only analyze the effects of the standard model on equity investments. Although we do not analyze these effects in our model, we would expect to see similar results for other asset groups. Bonds are in the same way as equity divided into subgroups determined by various risk measures in the standard formula. We would expect to see the same dynamics in these asset groups when the agent is constrained by the Solvency II regulations. Investments in high stress factor bond groups are expected to decrease. We would also expect investments to shift to bonds with a higher yield within the high stress factor bond groups.

3.2 Model II - Multiperiod

The model is presented in section 2.5. Remembering that x^i denotes dollars invested in asset i, fraction invested at time t is defined as

$$\omega_t^i = \frac{x_t^i}{\sum_{i=0}^N x_t^i} \tag{3.11}$$

We define the return on the portfolio in the period between t and $t+1$, while holding a specific allocation ω_t, as

$$R_{t+1} = R_{t+1}^0 + \sum_{i=1}^N \omega_t^i (R_{t+1}^i - R_{t+1}^0) \tag{3.12}$$

The agent has an initial wealth, $W_0 > 0$. The information set, $\mathscr{F}_t$, is defined as all information available to the agent at time t. We assume that at time t, the wealth is known by the agent, as well as the portfolio return in the period between $t-1$ and t.

$$\left.\begin{matrix} W_t \\ R_t \end{matrix}\right\} \in \mathscr{F}_t \tag{3.13}$$

To simplify notation we define

$$\frac{\partial}{\partial x} g(x, \ldots) = g_x(x, \ldots), \quad \forall g(.) \tag{3.14}$$

3.2 Analytical solution

To solve the agent's utility maximizing problem we will follow the dynamic programming solution technique provided in for instance Judd (1998) and Pennacchi (2008). We assume first that the Solvency II constraint will never be binding. The agent maximizes

$$\max_{\{\omega\}} E[U(W_T, T)] \tag{3.15}$$

The budget constraints given in equation 2.11 and 2.12 is reflected through the wealth dynamics.

$$W_{t+1} = W_t \left(R_{0,t} + \sum_{i=1}^{N} \omega_{i,t}(R_{i,t} - R_{0,t}) \right) = W_t R_t \tag{3.16}$$

We introduce the indirect utility function.

$$J(W_t, t, \mathscr{F}_t) \triangleq \max_{\{\omega\}_t} E_t[U(W_T, T)] \tag{3.17}$$

In the following, we will implicitly assume that $J(W_t, t)$ is a function of information set $\mathscr{F}_t$. Using the law of iterated expectation

$$J(W_t, t) = \max_{\{\omega\}_t} E_t\left[E_{t+1}[U(W_T, T)]\right] \tag{3.18}$$

Using the *principle of optimality* we get the recursive Bellman equation (Bellman, 1957).

$$J(W_t, t) = \max_{\{\omega\}_t} E_t\left[\max_{\{\omega\}_{t+1}} E_{t+1}[U(W_T, T)]\right] = \max_{\{\omega_t\}} E_t[J(W_{t+1}, t+1)] \tag{3.19}$$

Together with the wealth dynamics, this yields the following first order condition at any time t.

$$E_t[J_{W_{t+1}}(W_{t+1}, t+1)W_t(R_{t+1}^n - R_{t+1}^0)] = 0, \qquad n = 1...N \tag{3.20}$$

This holds if and only if

$$E_t[J_{W_{t+1}}(W_{t+1}, t+1)(R_{t+1}^n - R_{t+1}^0)] = 0, \qquad n = 1...N \tag{3.21}$$

To find our agent's optimal investment strategy, we will start at $t = T$, and use backwards induction. We implement the agent's utility function as defined in equation 3.10, where γ depicts the agent's relative risk aversion.

$$J(W_T, T) = E_T[U(W_T, T)] = U(W_T, T) = \frac{W_T^{1-\gamma} - 1}{1 - \gamma} \tag{3.22}$$

This is trivial since $W_T \in \mathscr{F}_T$. At $t = T - 1$ we get the following indirect utility function.

$$J(W_{T-1}, T-1) = \max_{\{\omega\}_{T-1}} E_{T-1}[J(W_T, T)] = \max_{\{\omega\}_{T-1}} E_{T-1}[U(W_T, T)] \tag{3.23}$$

We now define the portfolio return while keeping the optimal asset allocation ω^*.

$$R_{t+1}^* \equiv R_{t+1}^0 + \sum_{i=1}^{N} \omega_t^{i*}(R_{t+1}^i - R_{t+1}^0) \tag{3.24}$$

This gives us the following indirect utility function at $t = T - 1$.

$$J(W_{T-1}, T-1) = \frac{W_{T-1}^{1-\gamma}}{(1-\gamma)} E_{T-1}\left[R_T^{1-\gamma*}\right] - \frac{1}{1-\gamma} \tag{3.25}$$

Next, we implement the agent's utility function and use the optimality conditions stated in equation 3.21. Denoting excess return as $R_t^{ne} = R_t^n - R_t^0$, we get the following optimal condition at $t = T - 1$,

$$E_{T-1}[U_{W_T}(W_T, T)R_T^n] = E_{T-1}\left[\frac{R_T^{ne}}{(1-\gamma)W_{T-1}^{\gamma}R_T^{\gamma*}}\right] = 0 \tag{3.26}$$

Equation 3.26 holds if and only if

$$E_{T-1}\left[\frac{R_T^{ne}}{R_T^{\gamma*}}\right] = 0 \tag{3.27}$$

We observe that if we compare two arbitrary assets j, i, we get the same relationship as given in equation 3.6. At the second-to-last period, the agent chooses his portfolio exactly the same way as in the single period model. Looking back two time periods, at $t = 0$, we have the following indirect utility function.

$$
\begin{aligned}
J(W_{T-2}, T-2) &= \max_{\{\omega\}_{T-2}} E_{T-2}[J(W_{T-1}, T-1)] \\
&= \max_{\{\omega\}_{T-1}} E_{T-2} \left[\frac{W_{T-1}^{1-\gamma}}{(1-\gamma)} E_{T-1} \left[R_T^{1-\gamma*} \right] - \frac{1}{1-\gamma} \right]
\end{aligned}
\tag{3.28}
$$

For now we assume that the returns are independent and identically distributed (i.i.d). We then have the property that $E_{T-1} \left[\frac{R_T^{ne}}{R_T^{\gamma*}} \right] = E_0 \left[\frac{R_T^{ne}}{R_T^{\gamma*}} \right] = 0$. Implementing the optimality conditions from 3.21, at $t = T - 2$ gives us

$$
E_{T-2} \left[\frac{R_{T-1}^{ne}}{(1-\gamma)W_{T-2}^{1-\gamma}R_{T-1}^{1-\gamma*}} R_T^{1-\gamma*} \right] = 0
\tag{3.29}
$$

We observe from 3.27 that ω_{T-1}^{i*} is independent of W_{T-1}. Thus 3.29 holds if and only if

$$
E_{T-2} \left[\frac{R_{T-1}^{ne}}{R_{T-1}^{\gamma*}} \right] = 0
\tag{3.30}
$$

The implication of this result is that our agent exhibit myopic preferences. He will make investment incentives at $t = 0$ that is independent of the uncertain return distribution of available assets at $t = 1$. As shown by Mossin (1968), this is due to the i.i.d property of the returns and the specific utility function. Since the agent knows the return distribution at later periods, he does not have incentives to hedge against possible changes in the investment opportunity set at future dates. We also observe from equation 3.27 and equation 3.30, that by assuming a constant investment opportunity set, the agent will exhibit partially myopic preferences. The fraction invested in each of the risky assets, relative to the total investment in risky assets, will remain constant at each period.

Assuming i.i.d returns is a strong assumption. Simplifying models may render them unusable, but it may also be expedient if it highlights the effects the model should analyze. The motivation for our multiperiod model is the fact that Solvency II regulations only require reporting of the insurance companies' current financial situation at specific reporting dates. There is no regulation on investments between these dates, and hence our multiperiod model should analyze how the agent's incentives change accordingly.

The analytical results obtained is true if our agent does not face Solvency II regulations. Applying the Solvency II constraint defined in equation 2.13 will alter investment at $t = 0$, as shown in our first model, given that the exogenous capital requirement M_o is high enough. With i.i.d assumptions and the resulting myopic behaviour, we can still evaluate how the Solvency II constraint alters investment as this will alter the agent's investment incentives at $t = 0$ and $t = 1$.

3.2 Numerical example

To obtain a better understanding of how the agent will invest while constrained by the Solvency II regulations, we introduce a numerical example. At $t = 0$ the agent faces the same investment opportunity set as in our numerical example in Model I. Hence, at $t = 1$ he can find himself in one of four different states. We assume a constant investment opportunity set, which implies that at $t = 1$, the agent faces the exact same investment opportunity set as he did at $t = 0$, independent of which state he is in. As in the previous model we assume a uniform probability distribution. At $t = 2$, the agent may be in one of sixteen states. Utility is only obtained over wealth in these states, as stated in equation 2.10. The solvency constraint defined in equation 2.13 will be included with a varying exogenous capital requirement M_o.

As in Model I, the agent has an initial wealth of $W_0 = 10$, and risk aversion coefficient $\gamma = 0.4$. We use the same software, solver and algorithm. We do not restrict borrowing or investment in the riskless bond.

The final period wealth, using the optimal strategy facing varying exogenous capital requirements M_o is shown in figure 3.3. Imposing the Solvency II regulations on the agent reduces wealth in the good states, and increases wealth in the bad states. We observe that

in the state labeled 13, the final period wealth is lower under the restriction of Solvency II regulations than otherwise. If insolvency were to occur in states where the final market portfolio value were below $W_T = 5$, the insurance company in our model would be more likely to go bankrupt when complying with the Solvency II regulations.
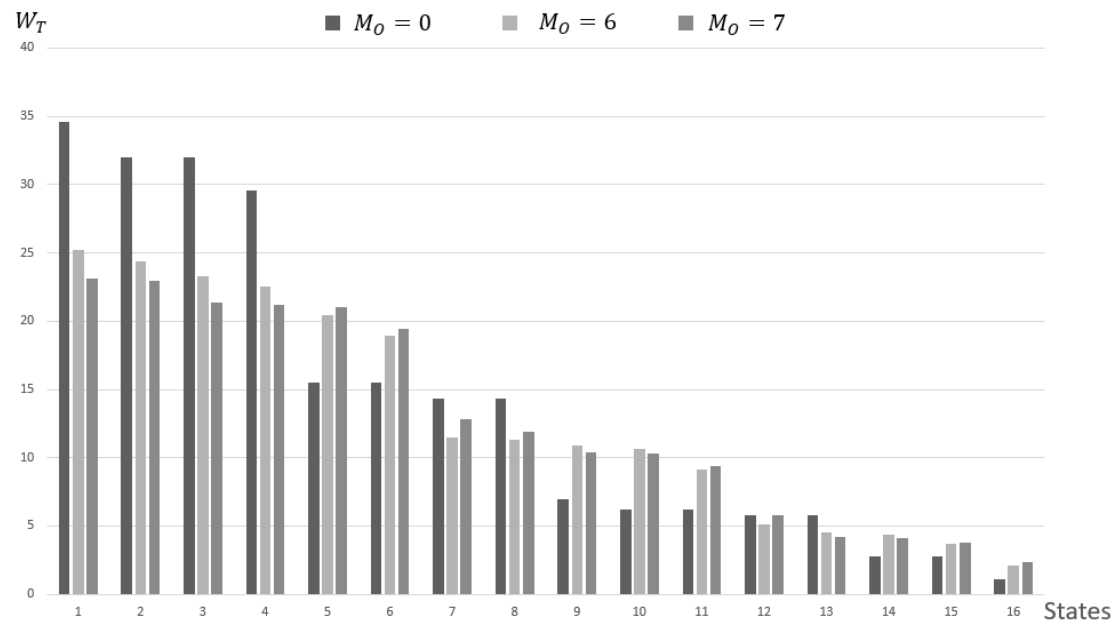

Figure 3.3: Model II - Final period wealth distribution with a varying exogenous capital requirement, M_o. Each distribution is sorted from a high wealth level to a low wealth level.

The observation that Solvency II regulations might increase the possibility of bankruptcy is specific to a certain insolvency wealth level. It is also restricted to our model, including the assumptions made about our investment opportunity set. What we observe from figure 3.3 is that the smoothing effect seen in our first model is reduced.

Figures 3.4a and 3.4b explains why the smoothing effect has decreased. When the agent is not constrained by the Solvency II regulations, he invests a constant fraction in each of the risky assets. Investment in the riskless bond relative to investment in the risky assets remains constant. This is due to the myopic investment behaviour combined with a constant relative risk aversion utility function.

When constrained by the Solvency II regulations, $M_o = 6$, the agent does not invest

constant fractions in the available assets. At $t = 0$, an increased M_o yields a higher investment in the riskless bond and a reallocation of the risky asset portfolio. Since we do not have any restrictions on investment in the riskless bond, we do not see the same effects as stated in proposition 1. What we do observe is that the agent at $t = 1$ invests the same fractions in each of the risky assets in all four states. The fraction invested in each asset, including the riskless bond, is indeed the same at $t = 1$ regardless of M_o. This is not surprising, as the myopic investment behaviour implies that the agent optimizes each period as if it was a single period investment decision. We do however observe that when our agent is bound to report compliance with Solvency II regulations at a single date, he will reallocate his portfolio to resume the optimal investment strategy in the abscence of regulations in the next period. If we were to allow for continuous reallocation, this would imply that the agent reallocates his portfolio as soon as the solvency level is reported, and would lead to an even lower wealth smoothing than what we observe.
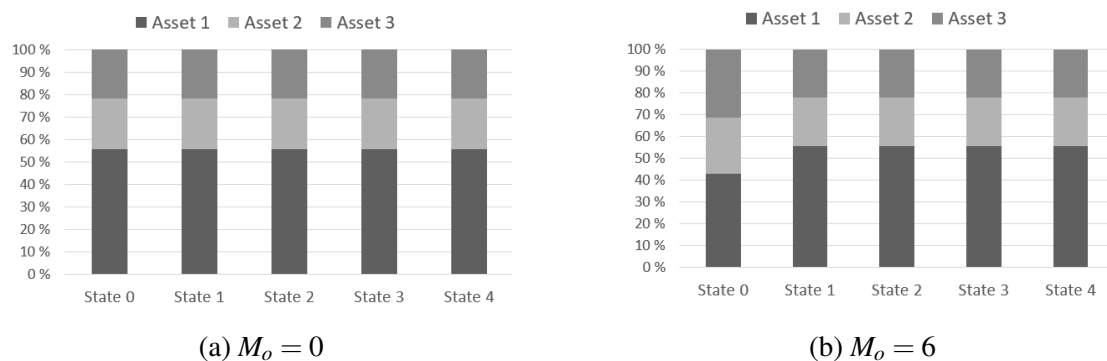

(a) $M_o = 0$ \hspace{4cm} (b) $M_o = 6$

Figure 3.4: Model II - Risky asset allocation with exogenous capital requirement. State 0 is allocation at $t = 0$. State 1 to 4 depicts the possible states at $t = 1$.

Introducing transaction costs to our model would decrease the agent's incentive to real-locate his portfolio between Solvency II reporting dates. This would lead to investment incentives closer to what we observed in our first model, with the corresponding smoothing of final period wealth and decreasing insolvency risk. Whether transaction costs are sufficient to compensate the lack of regulation between reporting dates is beyond the scope of this thesis.

The exact behaviour of our agent is dependent on the myopic behaviour resulting from

our assumptions. We do not observe any hedging incentives against possible changes in future investment opportunity sets. In addition to regular changes in future investment opportunity sets faced by all investors, our agent must account for Solvency II reporting at later dates. In our multiperiod model, the agent only reports at $t = 0$ and we assume that utility is obtained prior to the next reporting date. In reality, an agent would have a longer horizon such that he would be facing Solvency II regulations at later dates. If the market portfolio level is low at these future reporting dates, reallocation of the market portfolio may be insufficient to meet the future capital requirements. We analyze the effect of future capital requirements in our next model.

3.3 Model III - Supervisory Intervention

3.3 Analytical solution

The model is presented in section 2.6. Without binding regulation constraints, the Lagrangien function is given by

$$\mathscr{L} = \sum_{i=1}^{M} (\pi^i u(\theta^i) - \lambda_1(\theta^T \mathbf{p} - W_0)) \tag{3.31}$$

The first order condition is given by

$$\frac{\partial \mathscr{L}}{\partial \theta^i} = \pi^i u'(\theta^i) - p^i \lambda_1 = 0 \quad \Rightarrow \lambda_1 = \frac{\pi^i}{p^i} u'(\theta^i) \tag{3.32}$$

For asset i and asset j we get the following optimal allocation condition

$$\frac{\pi^i}{p^i} u'(\theta^i) = \frac{\pi^j}{p^j} u'(\theta^j) \tag{3.33}$$

Where π^i equals the probability for the state i to occur. This means that in the optimal solution, the utility gain per dollar invested in pure security i equals the utility gain per dollar invested in pure security j.

Consider next a world with only two states; a "good" state, G, with a low pure security price, p^G, and a "bad" state, B, with a high pure security price p^B. We let $\pi^G = \pi^B$. For a regular investor, not constrained by the Solvency II regulations, we get

$$u'(\theta^G) = \frac{p^G}{p^B} u'(\theta^B) \tag{3.34}$$

Hence, we see that $u'(\theta^G) < u'(\theta^B)$, making $\theta^G > \theta^B$ in the optimal allocation.

Say that we add a single wealth level constraint, C, where $\theta^B < C < \theta^G$. Then we need to consider another solution to our optimization problem. For simplicity we let $A_k = 1$, $\forall\, k$.

The optimization problem now becomes

$$\max_{\theta} u(\theta^G - S_G) + u(\theta^B - S_B) \tag{3.35}$$

Subject to

$$\theta^G p^G + \theta^B p^B \leq W_0 \tag{3.36}$$

$$C - S_G \leq \theta^G \tag{3.37}$$

$$C - S_B \leq \theta^B \tag{3.38}$$

Lagrangien:

$$\mathscr{L} = u(\theta^G - S_G) + u(\theta^B - S_B) - \lambda_1(\theta^G p^G + \theta^B p^B - W_0) - \lambda_G(C - S_G - \theta^G) - \lambda_B(C - S_B - \theta^B) \tag{3.39}$$

First order conditions:

$$\frac{\partial \mathscr{L}}{\partial \theta^G} = u'(\theta^G - S_G) - \lambda_1 p^G + \lambda_B = 0 \tag{3.40}$$

$$\frac{\partial \mathscr{L}}{\partial \theta^B} = u'(\theta^B - S_B) - \lambda_1 p^B + \lambda_G = 0 \tag{3.41}$$

Kuhn-Tucker conditions:

$$\lambda_1(\theta^G p^G + \theta^B p^B - W_0) = 0 \tag{3.42}$$

$$\lambda_G(C - S_G - \theta^G) = 0 \tag{3.43}$$

$$\lambda_B(C - S_B - \theta^B) = 0 \tag{3.44}$$

For a non-satiable agent with a utility function strictly increasing in θ, equation 3.36 will always be binding, which makes it safe to assume that $\lambda_1 > 0$. Further we consider the case where equation 3.37 will be non-binding and equation 3.38 will be binding. Thus making $\lambda_G = 0$, while $\lambda_B > 0$.

From equation 3.40 and 3.41 we then get

$$\lambda_1 = \frac{u'(\theta^G)}{p^G} \tag{3.45}$$

$$\lambda_1 = \frac{u'(\theta^B - S_B) + \lambda_B}{p^B} \tag{3.46}$$

By combining equation 3.45 and equation 3.46 we get

$$u'(\theta^G) = \frac{p^G}{p^B}(u'(\theta^B - S_B) + \lambda_B) \tag{3.47}$$

By definition we know that $S_B \geq 0$, $\lambda_B > 0$. Equation 3.47 thus states that $u'(\theta^G)$ is an increasing function of both S_B and λ_B. This means that by imposing the wealth level constraint, we are *decreasing* the optimal θ^G, thus *increasing* θ^B.

Next we will use a numerical example to study whether the same effect is apparent when adding more states, and more wealth levels.

3.3 Numerical example

At the end of the period we may be in one of a possible hundred states, each having equal probability of occurring. The pure security price vector denotes the price at $t = 0$, and has the distribution seen in figure 3.5.
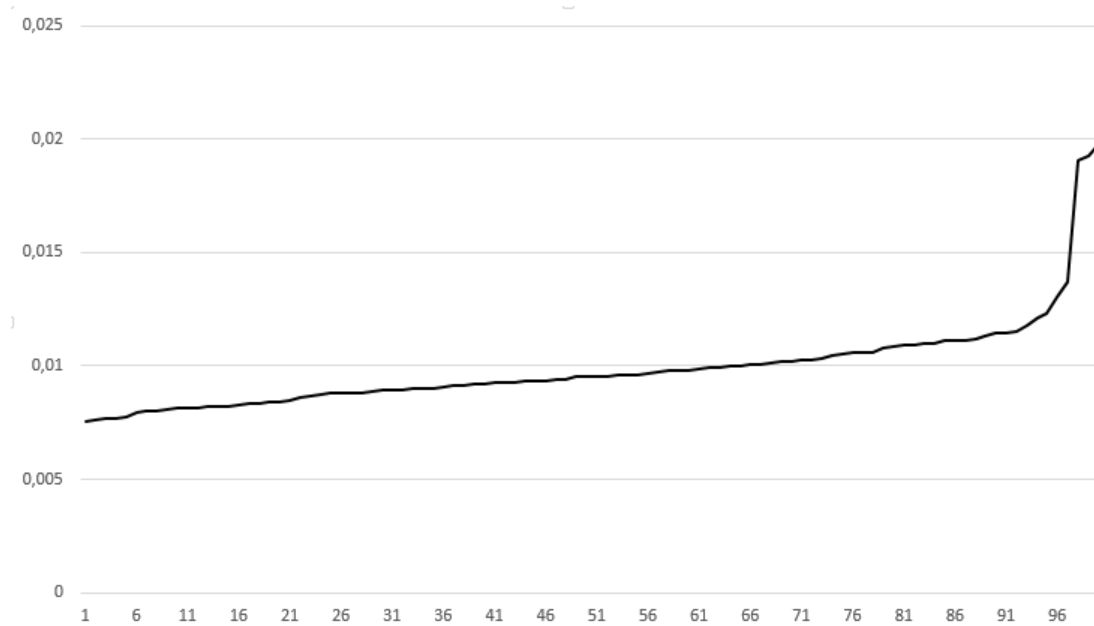
Figure 3.5: Distribution of security prices at $t = 0$ used in Model III.

The risk-free rate is defined by

$$R^f = \frac{1}{\sum_{s=1}^{100} p^s} = 1.01 \tag{3.48}$$

Other parameters in our example have the following values:

$$\mathbf{C} = \begin{bmatrix} 450 \\ 500 \\ 1000 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 0.7 \\ 0.1 \\ 0.2 \end{bmatrix}, \quad W_0 = 1000 \tag{3.49}$$

There are three wealth levels, denoted by $\mathbf{C}$. The highest and lowest wealth levels are supposed to represent the SCR and the MCR. If the final period wealth falls below

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the highest wealth level in a state, the intended SCR-punishment comes into effect. If the final period wealth goes below the seccond wealth level, the marginal punishment effect increases, as the wealth moves closer to the MCR. When the final period wealth moves below the lowest wealth level in a state, the marginal punishment effect increases greatly.

The agent has the same power utility function as shown in equation 3.10, with relative risk aversion, $\gamma = 0.4$.

The problem is solved using AMPL and the BARON solver. BARON uses a *Branch-and-Reduce* algorithm based on the *Branch-and-Bound* (B&B) algorithm. The B&B algorithm divides the feasible set *S* into several subsets, and by comparing the *branches* generated by the division with a known bound, it can implicitely search large subsets of *S*. This increases the efficiency of the algorithm as well as providing a global solution to the problem. The bound of which the subsets are compared is altered as more branches are searched. A general description of the B&B algorithm can be found in Clausen (1999). For a detailed view of the BARON solvers *Branch-and-Reduce* algorithm, see Sahinidis (2003).

In figure 3.6 we see that supervisory intervention gives the agent an incentive to smooth his wealth distribution. In effect, this leads to less wealth in the good states, and more wealth in the bad states. The number of states where the final period wealth is below the SCR have decreased. The final period wealth is greater or equal the MCR in all states.

Given the framework of our model; if the probability of insolvency were to increase in the states where final period wealth is below the SCR and/or the MCR, then we have shown that the insurance company will be less likely to go bankrupt given the threat of supervisory intervention. The fact that the company faces consequences at a future date, when not maintaining certain capital levels, is in itself a risk reducing property of the Solvency II regulations.

It is likely that the risk reducing effect analyzed in Model III will occur to some degree even if the final period wealth is greater then the SCR. As we have seen in Model I and Model II, the reporting of available capital requires the insurance company to
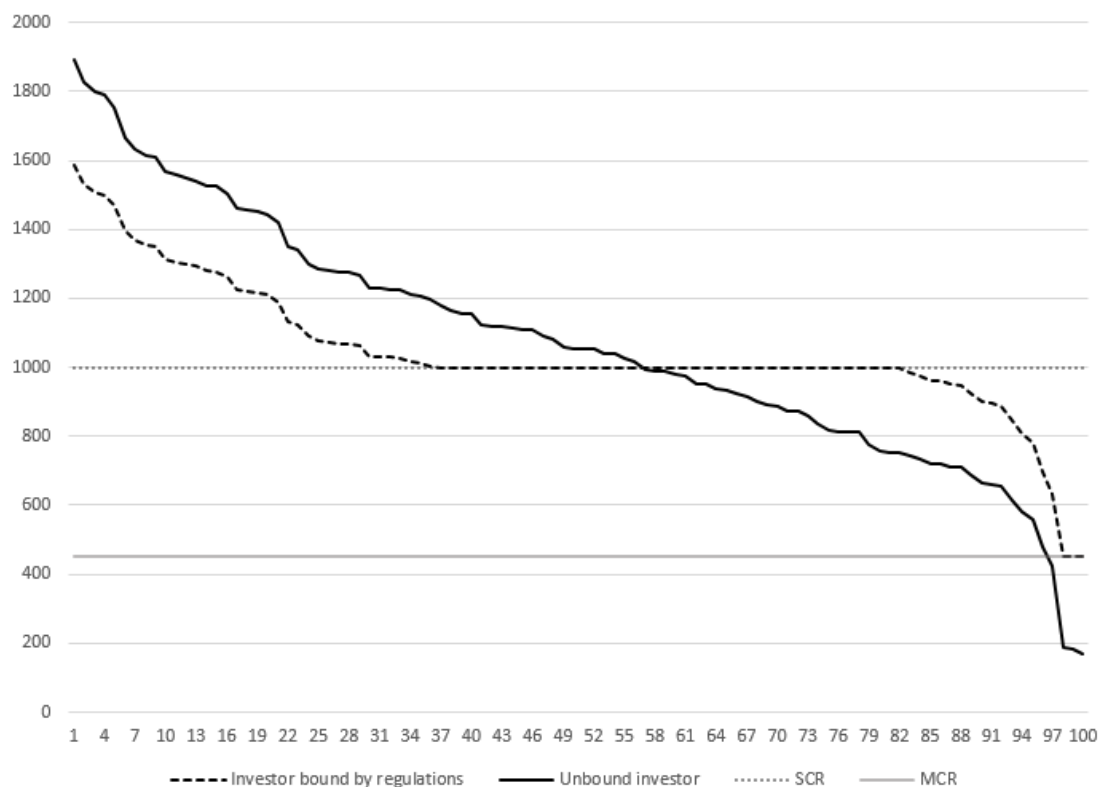
Figure 3.6: Model III - Optimal wealth allocation over the states, with and without supervisory intervention for given wealth levels.

shift its investments if the Solvency II regulations are binding. This is equivalent to a reduction in the investment opportunity set, and is a form of supervisory punishment. An interesting expansion of our models would be to analyze the agent's incentives over several periods, with several Solvency II reporting dates. This is however beyond the scope of this thesis.

The implication of the result we obtained in Model III is that the seemingly naive Solvency II standard formula may still induce risk reducing incentives for insurance companies. In our first model, the final period wealth volatility was reduced, and especially the wealth in the worst states was improved. This result is contingent on the assumption that equity stress factors is a real representation of the underlying risk. As discussed, the separation between OECD listed stocks an non-OECD listed stocks may not be a good risk measurement. Combined with our findings in Model II, where the

investor reallocates to a regular portfolio as soon as possible, this could indicate that the Solvency II standard formula is a poor tool for risk management. In Model III however, we show that even without any application of the standard formula, it is optimal for the investor to reduce risk when the threat of supervisory intervention is present.

4 Conclusion

We analyze the effects of the Solvency II regulations on investment incentives for a medium sized insurance company. We assume an agent in control of the insurance company's market portfolio, with no control over other operations. We assume a complete economy with one or two time periods. The outcome of the agent's investments is uncertain, with a discrete statespace at later periods.

In our first model we examine whether the agent obtains incentives to reallocate his equity investments. We specifically look at the effect of differences in equity stress factors within the Solvency II standard formula. We find that the agent has incentives to invest more in low stress factor equities if bound by the Solvency II regulations. Assuming that the Solvency II stress factors are a good measurement of the underlying risk, this will reduce the risk taken by the agent. If we allow for reallocation between equities and a riskless asset, we cannot answer whether the result of these incentives will increase investments in the less risky equities or not.

In Model II, the agent is allowed to reallocate his portfolio after complying with the Solvency II regulations at time zero. The agent maximizes expected utility from his wealth level at the end of the second period. Assuming a myopic agent, this leads to a portfolio reallocation at time one. Although the agent complies with the Solvency II regulations at the initial investment date, he reallocates his portfolio such that the investments is equal to the investments of a regular investor at time one. His final period wealth distribution is less volatile than for a regular investor, but the effect has decreased compared to Model I. The risk reduction is still contingent on the assumption that assets with a higher stress factor has a higher underlying risk.

In our last model, the agent is only constrained by a budget. The agent gains utility over the end-of-period wealth, but may be confronted with a monetary fine if his wealth level is below a set number of thresholds. Model III replicates the effect of supervisory intervention from regulatory authorities in the case where available capital has decreased below the capital requirements. When confronted with the possibility of a reduced investment set and/or supervisory intervention, the agent will reduce his risk compared to a regular investor.

Even if the standard formula risk measurements is a poor representation of actual risk,

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the regulation may still induce risk reducing incentives. Supervisory authorities must impose a viable threat of supervisory intervention when the wealth level drops below certain levels. If this is the case, insurance companies will have risk reducing incentives following the Solvency II regulations.

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A Proofs

Proof of proposition 1. Assume that the solvency constraint is strictly binding, implying $\lambda_2 > 0$. Contribution to M_e from asset group r is denoted $M_{e,r}$. Using the fact that we only have two equity groups and a correlation between them of 0.75 as given in **V** we can write

$$M_e = \left(\sum_{r,c} \mathbf{V}^{r,c} \cdot M_{e,r} \cdot M_{e,c} \right)^{\frac{1}{2}} = \left(M_{e,1}^2 + 1.5 \cdot M_{e,1} \cdot M_{e,2} + M_{e,2}^2 \right)^{\frac{1}{2}} \tag{A.1}$$

Continuing from 3.2 and choosing two equities H,L we get

$$E[u'(R\mathbf{x})R^L] = E[u'(R\mathbf{x})R^H] - \lambda_2 \left(\frac{\partial M_e}{\partial x^H} - \frac{\partial M_e}{\partial x^L} \right) \tag{A.2}$$

We introduce the following notation

$$\tau = \frac{\partial M_e}{\partial x^H} - \frac{\partial M_e}{\partial x^L} \tag{A.3}$$

Assume two linearly independent stocks. Stock L is in asset group one and stock H in asset group two. We denote indicator function $1_{\{x^j > 0\}}$ to be one if $x^j > 0$, and zero otherwise. This ensures that short positions do not affect M_e. Stress factor for an asset group k is denoted s_k. If we have two asset groups, $1, 2$, we have the following condition for arbitrary asset j in asset group 1

$$\frac{\partial M_e}{\partial x^j} = \frac{\partial M_e}{\partial M_{e,1}} \frac{\partial M_{e,1}}{\partial x^j} = \frac{2M_{e,1} + 1.5M_{e,2}}{2\sqrt{M_e}} \cdot s_1 \cdot 1_{\{x^j > 0\}} \tag{A.4}$$

We can then evaluate τ

$$\tau = \frac{\partial M_e}{\partial x^H} - \frac{\partial M_e}{\partial x^L} = \frac{2M_{e,2} + 1.5M_{e,1}}{2\sqrt{M_e}} \cdot s_2 \cdot 1_{\{x^H > 0\}} - \frac{2M_{e,1} + 1.5M_{e,2}}{2\sqrt{M_e}} \cdot s_1 \cdot 1_{\{x^L > 0\}} \tag{A.5}$$

We observe that τ increases in $s_2 - s_1$, and decreases in $M_{e,1} - M_{e,2}$. The following conditions apply to A.5

$$M_{e,1} = M_{e,2}, \quad s_1 < s_2 \quad \Rightarrow \quad \tau > 0$$
$$M_{e,1} < M_{e,2}, \quad s_1 = s_2 \quad \Rightarrow \quad \tau > 0 \tag{A.6}$$

Since we have $\lambda_2 > 0$, a positive τ yields the following condition to the optimal solution under the Solvency constraint

$$E[u'(Rx)R^L] = E[u'(Rx)R^H] - k, \quad k > 0 \tag{A.7}$$

We now have to establish whether this leads to a relative increase in asset L or H. Define R^{-LH} to be the total returns excluding assets L, H. Vector $\mathbf{x}^{-LH}$ denotes in a similar way investments excluding assets L, H. If we have an optimal solution not constrained by Solvency II regulations, with investment vector $\mathbf{x}^*$ and corresponding wealth in state s $W_s^* = \mathbf{R}_s \mathbf{x}^*$, we have the following condition.

$$E[u'(W_s^*)R^H] - k = E[u'(W_s^*)R^L], \quad k = 0$$
$$E[u'(W_s^*)(R^j - R^i)] = 0 \tag{A.8}$$

Now assume $k > 0$ and linear independence between R^H, R^L. The agent must then reallocate investments such that a new wealth distribution W is reached. The reallocation must be such that

$$E[u'(W)R^L] < E[u'(W)R^H]$$
$$\Rightarrow E[u'(W)(R^L - R^H)] < 0 = E[u'(W^*)(R^L - R^H)] \tag{A.9}$$

We denote the adjusted investment vector as $\mathbf{x}$. We only adjust investment in asset L, H. Investment in asset L is given by $x^L = x^{L*} + \lambda$, and investment in asset H is $x^H = x^{H*} - \lambda$, where λ is a constant. The other investments stays the same, $\mathbf{x}^{-LH} = \mathbf{x}^{-LH*}$. Wealth in state s after reallocation is now given by

$$W_s = R_s^L(x^L + \lambda) + R_s^H(x^H - \lambda) + R_s^{-LH}x^{-LH} = W_s^* + \lambda(R_s^L - R_s^H) \tag{A.10}$$

Combining A.10 and A.9

$$E[u'(W^* + \lambda(R^L - R^H))(R^L - R^H)] < E[u'(W^*)(R^L - R^H)] \qquad \text{(A.11)}$$

A.11 is true if the following is true

$$\begin{aligned} &\text{If } R^L - R^H > 0, \text{ then } u'(W) < u'(W^*) \\ &\text{If } R^L - R^H < 0, \text{ then } u'(W) > u'(W^*) \end{aligned} \qquad \text{(A.12)}$$

Since we have that $u''(.) < 0$, we see that this is true if and only if $\lambda > 0$. Compared to the optimal solution without a binding Solvency II constraint, the optimal reallocation is to increase investment in asset L relative to asset H.

∎

Proof of proposition 2. Continuing from A.5 assuming assets H, L are both in asset group k. If $x^H \geq 0, x^L \geq 0$ it is straightforward to see that

$$\frac{\partial M_e}{\partial x^H} - \frac{\partial M_e}{\partial x^L} = 0 \qquad \text{(A.13)}$$

Inserted into A.2 we get the same relationship as in 3.6. ∎

B Code

The code of our optimization programs can be found at `https://github.com/EirikBerglund/`
`InvestmentOptimization`. We will include short versions of our code here, which
contains the most important calculations. The online repository contains a more de-
tailed code which is thoroughly commented. It also contains code for construction of
returns and pure security prices for Model III, which are not included here.

B.1   MATLAB code for Model I and Model II

Listing B.1: procedure.m

```
debug =0;

M =  4;
N =  4;

MultiPeriod  =  1;

mFactor  =  5;

ConstantRiskless  =  0;

pi  =  ones (M, 1) * (1 /M) ;

X =  [1  2  2  0;...
      1  1  2  5;...
      1  3  3  3;...
      1  3  5  2];

P =  [0.99;2.1;2.72;2.2];

SCRLevels  =  [0.39;0.49];
```

```matlab
gamma = 2;

W0 = 10;

MKTo = 0;

R = [sum(X(:,1)/(P(1)*M)); sum(X(:,2)/(P(2)*M));...
     sum(X(:,3))/(P(3)*M); sum(X(:,4)/(P(4)*M))];

pure = eye(M);

p = zeros(M,1);

for m=1:M
    b = linsolve(X,pure(:,m));
    p(m) = b'*P;
end

f = @(x) objective(x,X,gamma,MultiPeriod);

g = @(x) constraint(x,P,W0,MKTo,SCRLevels,X,...
    ConstantRiskless,MultiPeriod);

options = optimoptions(@fmincon,'Algorithm','interior-point');

x0 = ones(N*mFactor,1)./(N*mFactor);

if ~debug
    [x,fval,exitflag,output] = fmincon(f,x0,[],[],[],[],[],[],...
        g,options);
    prevRes = x;
end
```

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Listing B.2: objective.m

```
function [ util ] = objective( x,X,gamma,MultiPeriod )

util = 0;

if MultiPeriod
    for m=1:4
        util = util + powerUtility(X(m,:)*x(5:8),gamma);
        util = util + powerUtility(X(m,:)*x(9:12),gamma);
        util = util + powerUtility(X(m,:)*x(13:16),gamma);
        util = util + powerUtility(X(m,:)*x(17:20),gamma);
    end
else
    for m=1:size(x,1)
        util = util + powerUtility(X(m,:)*x,gamma);
    end
end

util = -util;

end
```

Listing B.3: constraint.m

```
function [ c,ceq ] = constraint(x,P,W0, MKTo,SCRLevels, X, ...
    ConstantRiskless, MultiPeriod)

c(1) = x(1:4)'*P-W0;

MKTeq = getEquityStress(x(1:4),P,SCRLevels);
c(2) = MKTeq + MKTo - W0;

if ConstantRiskless
    c(3) = x(1)-1;
```

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```
    c(4) = -x(1);
end


if MultiPeriod
    c(5) = x(5:8)'*P-X(1,:)*x(1:4);
    c(6) = x(9:12)'*P-X(2,:)*x(1:4);
    c(7) = x(13:16)'*P-X(3,:)*x(1:4);
    c(8) = x(17:20)'*P-X(4,:)*x(1:4);
end


ceq = [];


end
```

Listing B.4: powerUtility.m

```
function util = powerUtility(c,gamma)


if gamma==1
    util = log(c);
else
    util = (c^(1-gamma)-1)/(1-gamma);
end


end
```

Listing B.5: getEquityStress.m

```
function [ MKTeq ] = getEquityStress( x,P,SCRLevels )


for i=1:size(x,1)
    x(i) = x(i)*P(i);
end


if x(2)>0
```

```
    MKTeqI(1) = x(2)*SCRLevels(1);
else
    MKTeqI(1)=0;
end

if x(3) > 0 && x(4) > 0
    x2 = x(3)+x(4);
elseif x(3) > 0
    x2 = x(3);
elseif x(4) > 0
    x2 = x(4);
else
    x2 = 0;
end

MKTeqI(2) =x2*SCRLevels(2);

MKTeq=sqrt(MKTeqI(1)^2+MKTeqI(2)^2 + 2*0.75*MKTeqI(1)*MKTeqI(2));

end
```

B.2 AMPL code for Model III

Listing B.6: eco.mod

```
set S;
set K;

param restrict := 0;

param Ck {j in K};              #Endret fra {i in K}

param fact {j in K};    #Endret fra {i in K}
```

```
param p {i in S};

param gamma = 0.4;

param W0 = 1000;

var x {i in S} >= 1;

var SCR {i in S, j in K} >= 0;

#var Limit {i in S} binary;

maximize util : sum{i in S} ((x[i]
    - sum{j in K} (SCR[i,j]*fact[j]*restrict))^(1-gamma)-1)/(1-gamma);

subject to budget_constraint :
        sum{i in S} p[i]*x[i] <= W0;

subject to solvency_constraint {i in S, j in K} :
        x[i]*restrict >= Ck[j]*restrict - SCR[i,j]
;
```

Listing B.7: eco.dat

```
set S :=
1 ... 100 # For abbreviation denoted like this.
          # In reality a list from 1-100.
          # Can be found in online repository.
;

set K :=
1
2
3
```

```
;

param Ck :=
1        450
2        500
3        1000
;

param fact :=
1        0.7
2        0.1
3        0.2
;

param p :=
1        0.007534636
2        0.007642496
3        0.007694041
4        0.007709167
5        0.007773043
6        0.007930923
7        0.007992133
8        0.008028653
9        0.008044244
10       0.008127291
11       0.008148957
12       0.008162823
13       0.00818022
14       0.00821016
15       0.008218946
16       0.008265166
17       0.008355046
```

18	0.008371893
19	0.008380633
20	0.00840327
21	0.008455444
22	0.00862785
23	0.008653758
24	0.00876327
25	0.008804303
26	0.008814989
27	0.008828097
28	0.008829185
29	0.008851998
30	0.008956606
31	0.008959865
32	0.008963326
33	0.008974475
34	0.009013858
35	0.009021203
36	0.009051824
37	0.009106428
38	0.009157838
39	0.009175984
40	0.009183913
41	0.009289151
42	0.009293367
43	0.009295552
44	0.009322373
45	0.009333157
46	0.00933795
47	0.009387392
48	0.009423383
49	0.009502663

50	0.009522212
51	0.009522512
52	0.009528323
53	0.00956778
54	0.009571654
55	0.009625582
56	0.009667326
57	0.009755966
58	0.009770609
59	0.009772655
60	0.009798595
61	0.009831702
62	0.009919384
63	0.009923956
64	0.009983651
65	0.010008255
66	0.010034156
67	0.010069688
68	0.010136475
69	0.01018723
70	0.010194507
71	0.010263502
72	0.010268917
73	0.010330165
74	0.01045567
75	0.010545365
76	0.010557973
77	0.010559943
78	0.010561367
79	0.010768855
80	0.010867167
81	0.010892966

82	0.01089811
83	0.010962771
84	0.011000285
85	0.011091918
86	0.011094781
87	0.011141464
88	0.011158129
89	0.011293622
90	0.011443332
91	0.011479463
92	0.011526382
93	0.01180796
94	0.012081567
95	0.012289875
96	0.013029537
97	0.013704887
98	0.019035419
99	0.019242641
100	0.019799813

;

Listing B.8: eco.run

```
reset;

model eco.mod;
data eco.dat;

option solver baron;
option baron_options 'trace=baron.log';

solve;

display x > model3.txt;
```