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Teams in Relational Contracts

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Abstract

We analyze relational contracting between a principal and a team of agents where only aggregate output is observable. We deduce optimal team incentive contracts under different set of assumptions, and show that the principal can use team size and team composition as instruments in order to improve incentives. In particular, the principal can strengthen the agents' incentives by composing teams that utilize stochastic dependencies between the agents' outputs. We also show that more agents in the team may under certain conditions increase each team member's effort incentives, in particular if outputs are negatively correlated.

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1 Introduction

Incentive contracts within firms, between a principal and her agents, are often based on performance measures that are hard to verify by a third party (see e.g. MacLeod and Parent, 1999, and Gibbs et al, 2004). The quality or value of the agents' performance may be observable to the principal, but cannot easily be assessed by a court of law. The parties must then rely on self-enforcing relational contracts. Through repeated interactions the parties can make it costly for each other to breach the contract, by letting breach ruin future trade. But relational contracts cannot fully solve the principal's incentive problem, since the agents' monetary incentives (bonuses) are limited by the value of the future relationship. If bonuses are too large (or too small), the principal (or agents) may deviate by not paying as promised, and thereby undermining the relational contract. The principal must thus provide as efficient incentives as possible, under the constraint that the feasible bonuses are limited.

The literature has studied this problem under the assumption that the agents' *individual* outputs are non-verifiable, but still observable for the contracting parties (as in Levin, 2002 and 2003). However, agents often work in teams in which only aggregate output is observable, while individual outputs are non-observable.¹ While a team's aggregate output may be easier to verify than individual outputs, there is still a range of situations in which a team's output is non-verifiable. Teams are also, like individuals, exposed to discretionary bonuses and subjective performance evaluation, which by definition cannot be externally enforced.²

In this paper, we thus analyze a relational contract between a principal and a team of agents where only the team's aggregate output is observable. We

¹A majority of firms in the US and UK report some use of teamwork in which groups of employees share the same goals or objectives, and the incidence of team work has been increasing over time (see Lazear and Shaw, 2007 and Bandiera et al, 2012, and the references therein).

²As an example, firms often promise fixed bonus pools to a team of workers before they allocate discretionary individual rewards within the team. However, the size of the team's bonus pool may also be discretionary, or non-contractible, and firms will need relational contracts in order to commit to actual pay the total team bonus as promised, see Glover and Xue (2014) and Deb et al (2015) and the references therein. Another example is corporate bonuses or division bonuses, which by definition are group-based, and often discretionarily determined by the board at the end of the year.

show that when the maximum team bonus is limited by the relational contract, the principal can use team size and team composition as instruments in order to improve incentives. In particular, the principal can strengthen the agents' incentives by composing teams that utilize stochastic dependencies between the agents' outputs.³ These outputs will often be positively correlated, for instance if team members are exposed to the same business cycles. In other situations, agents' outputs are negatively correlated, for instance when specialists with different expertise are differently exposed to business cycles, or meet different sets of demands from customers or superiors. In this paper we investigate how the principal can use information about correlation between the workers' individual output in order to implement optimal team based incentives. Moreover, we investigate how adding more agents to a team affect incentives. In particular, we ask: can a larger team do better (in terms of output per agent) than a smaller team? That is: can a larger team yield higher-powered incentives?

We first show that as long as the monotone likelihood ratio property (MLRP) holds, the optimal team incentive scheme is simple: Each agent is paid a bonus for aggregate output above a threshold. However, when MLRP does not hold (which may well be the case under correlated outputs), then it may be optimal to reward the team for e.g. low and high output, but not for intermediate ones. Moreover, we show that if individual outputs are stochastically independent, more agents in the team always reduces effort. However, once outputs are correlated (positively or negatively), the $1/n$ free-rider problem does not generally hold. More agents in the team may under certain conditions increase each team members' effort incentives, in particular if outputs are negatively correlated.

The general mechanism that lies behind these results is as follows: In any relational contract there is a maximal self-enforcing bonus; its magnitude is bounded by the value of the future relationship. For this given bonus, incentives will be maximal when the bonus is awarded for all outcomes where the marginal effect of effort on the probability of those outcomes is positive. Correlation among individual outputs will affect the distribution of team output, and hence also these marginal effort effects; their signs as

³Even if individual outputs are unobservable, the parties may know how and to what extent they are correlated.

well as their magnitudes. Thus it will affect the set of outputs for which the bonus is awarded, as well as the individual effort incentives generated by this bonus. Whenever the latter is strengthened by adding another agent to the team, the bigger team will do better. We show that this is often more likely under negative than under positive correlation.

These effects, and in particular the effects of correlated outputs on the team's efficiency, turn out to be quite transparent and very striking in the case of normally distributed outputs. The MLRP then holds for team output, and hence the bonus is optimally awarded for output above a threshold. Moreover, the individual marginal effect of effort on the probability of obtaining the bonus is inversely proportional to the standard deviation of the team's output. Hence, since the standard deviation is reduced (increased) when more agents are added under negative (positive) correlation, a larger team provides stronger incentives and thus performs better if and only if individual outputs are negatively correlated. Under normally distributed outputs, this result is thus related to the fact that by including more agents in the team, we may obtain a more precise performance measure. This is beneficial not because a more precise measure reduces risk (since all agents are risk neutral by assumption), but because it strengthens, for any given bonus level, the incentives for each team member to provide effort. The analysis of the normal case reveals that for sufficiently small variance, the standard first order approach (used by e.g. Levin, 2003) is not valid, but we show that a threshold bonus is nevertheless optimal, and we characterize its properties.

However, the results of the normal case may well not hold for other distributions; in particular we may have MLRP satisfied for individual outputs, but not for aggregate output. This will affect the shape of optimal incentive schemes, and will generally also affect how optimal schemes and associated efforts are influenced by correlations between individual outputs. We therefore also analyze a setting with discrete (binary) outputs, and characterize conditions under which a larger team will do better, but also show that a hurdle scheme may well not be optimal in this setting.

Our results have several implications. First, the canonical $1/n$ free-rider problem does not generally hold. This may inform practitioners and empirical researchers: Under correlated outputs, larger teams may actually do

better than smaller ones. Second, threshold schemes are not necessarily optimal under correlated outputs, and may in fact lead to perverse incentives under certain conditions. Empirical researchers who observe team incentives schemes that fail, may wrongfully infer that it is due to a free-rider problem. Third, the positive incentive effects of negative correlation relates to questions concerning optimal team composition. One can conjecture that negative correlations are more associated with heterogeneous teams than homogenous teams, and also more associated with task-related diversity (functional expertise, education, organizational tenure) than with bio-demographic diversity (age, gender, ethnicity). There is no reason to believe that e.g. men and women’s outputs are negatively correlated. However, workers with different functional expertise may be differently exposed to common shocks, and meet different sets of demands. This can give rise to negative output correlations. Interestingly, a comprehensive meta-study by Horwitz and Horwitz (2007), investigating 35 papers on the topic, finds no relationship between bio-demographic diversity and performance, but a strong positive relationship between team performance and task-related diversity.⁴ An explanation is that task-related diversity creates positive complementarity effects. We point to an alternative explanation, namely that diversity may create negative correlations that increases each agents’ marginal incentives for effort. The team members “must step forward when others fail”. Diversity and heterogeneity among team members can thus yield considerable efficiency improvements.

The rest of the paper is organized as follows. In Section 2 we discuss related literature, while in Section 3 we introduce the model. Section 4 analyzes the case of normally distributed outputs, while Section 5 considers discrete outputs. Section 6 concludes.

2 Related literature

We study optimal incentive contracts for $n > 1$ agents when both individual outputs are unobservable *and* aggregate output is non-verifiable. Non-

⁴Hamilton et al (2003) provide one of a very few empirical studies on teams within the economics literature. They find that more heterogeneous teams (with respect to ability) are more productive (average ability held constant).

verifiable output calls for relational contracts, and relational contracts between a principal and a team of agents where only aggregate output is observable has (to our best knowledge) not yet been studied. Levin (2002) considers a multilateral relational contract between a principal and $n > 1$ agents, but where individual outputs are observable and stochastically independent. He shows among other things that a tournament scheme is optimal. The few relational contracting papers on team incentives also consider situations in which individual outputs are observable. Here, team incentives turns out optimal due to repeated interaction between agents (Kvaløy and Olsen, 2006; Rayo, 2007) or complementarity in production (Kvaløy and Olsen, 2008, Baldenius et al. 2015). Recent papers also consider relational team incentive contracts where individual bonuses are based on subjective measures (Glover and Xue, 2014, and Deb et al, 2015)

Although we focus on the multiagent case, our paper is indebted to the seminal literature on *bilateral* relational contracts, starting with Klein and Leffler (1981), Shapiro and Stiglitz (1984) and Bull (1987).⁵ MacLeod and Malcomson (1989) provides a general treatment of the symmetric information case, while (the now influential paper by) Levin (2003) generalizes the case of asymmetric information. Our threshold scheme, in which each agent is paid a maximal bonus for aggregate output above a threshold and a minimal (no) bonus otherwise, parallels Levin’s (2003) characterization for the single-agent case. However, while Levin (like most other papers in this literature) uses the standard first order approach (FOA), we characterize the optimal bonus scheme in our application (the normal case) also when FOA is not valid, and we show that it is in fact a threshold scheme. Technically this hinges on MLRP being valid for team output. In fact, the analysis shows that in the single-agent case, MLRP will generally ensure that a threshold bonus is optimal (whether FOA is valid or not).

Our paper also relates to the literature on optimal team incentives. This literature is twofold. One strand, starting with Alchian and Demsetz (1972), assume, like us, that individual output is unobservable, but (unlike us) that aggregate output is verifiable. The main focus is then on the free-rider

⁵While the formal literature starts with Klein and Leffler, the concept of relational contracts had was first defined and explored by legal sholars (Macaulay, 1963, Macneil, 1974,1978)

problem, and how it can be solved or mitigated with legally enforceable contracts.⁶ Such contracts are not feasible here.

Another strand of the literature studies team incentives when individual output is observable. The idea is that the principal, by tying compensation to the joint performance of a team of agents, can foster cooperation (e.g. Itoh; 1991, 1992, 1993; Holmström and Milgrom, 1990; Macho-Stadler and Perez-Castrillo 1993), exploit peer effects (e.g. Kandel and Lazear, 1992; Che and Yoo, 2001), or help mitigate multitask problems (Corts, 2007, Mukherjee and Vasconcelos 2011, Ishihara 2016). While we do not consider observable individual output, our paper is related to this literature in the sense that we also exploit dependencies between the agents in order to improve efficiency, see in particular Rajan and Reichelstein (2006) who show (in a model where total team output is verifiable) that correlation between subjective performance measures within teams may benefit the principal.

Our focus on stochastic dependencies also relates to the literature on relative performance evaluation. By tying compensation to an agent's relative performance, the principal can improve efficiency by filtering out common noise and thereby expose them to less risk (Holmström, 1982; and Mookherjee, 1984).⁷ We show that correlation may improve efficiency even in the absence of risk considerations. In this respect, the correlation effects we demonstrate (in the normal case application) relates to insights from the finance literature, starting with Diamond (1984) who show that correlated signals/shocks may reduce output variance and thus reduce entrepreneurs' moral hazard opportunities towards investors.

Although the literature on team incentives generally recognizes team size as an important determinant for team performance, questions concerning optimal team size has received limited attention. Most notable are the con-

⁶Holmström (1982) formalized Alchian and Demsetz argument and showed that a budget breaker who holds claim on the team's output can assure first best effort incentives. A literature has followed that both extend and refine Holmström's insights (e.g. Rasmussen, 1987, and McAfee and McMillan (1991), including papers showing that the budget breaking requirement is too strong, and that first-best incentives can be achieved without requiring that all agents are made residual claimant (Legros and Matsushima, 1991; Legros and Matthews, 1993).

⁷Fleckinger (2012) provides a more general treatment of stochastic dependencies and RPE, and shows that greater correlation in outcomes does not necessarily call for RPE schemes.

tributions within the accounting literature, see Ziv (2000), Huddart and Liang (2005) and Liang et al. (2008) who show that team size can effect monitoring activities within teams, as well as how teams respond to exogenous shocks.

3 The setting

We analyze an ongoing economic relationship between a principal and n (symmetric) agents. The agents constitute a team. All parties are risk neutral. Each period, each agent i exerts effort e_i incurring a private cost $c(e_i)$. Costs are strictly increasing and convex in effort, i.e., $c'(e_i) > 0$, $c''(e_i) > 0$ and $c(0) = c'(0) = 0$. Each agent's effort generates a stochastic contribution (output) x_i to the team's total output $y = \Sigma x_i$. Agents are symmetric, and each agent's output has a probability distribution depending only on the agent's own effort, and represented by a CDF $F(x_i, e_i)$. We focus here on team effects generated by stochastic dependencies among agents' contributions, and thus assume a simple linear "production structure", but allow individual outputs to be stochastically dependent. Expected outputs are given by $\bar{x}(e_i) = E(x_i | e_i) = \int x_i dF(x_i, e_i)$ and total surplus per agent is $W(e_i) = \bar{x}(e_i) - c(e_i)$. First best is then achieved when $\bar{x}'(e_i^{FB}) - c'(e_i^{FB}) = 0$. Outputs are stochastically independent (given efforts) across time.

The parties cannot contract on effort provision. We assume that effort e_i is hidden and only observed by agent i . With respect to output, we assume that only total output $y = \Sigma x_i$ is observable, and moreover non-verifiable by a third party. Hence, the parties cannot write a legally enforceable contract on output provision, but have to rely on self-enforcing relational contracts.

Each period, the principal and the agents then face the following contracting situation. First, the principal offers a contract saying that agent i receives a non-contingent fixed salary α_i plus a bonus $b_i(y)$, $i = 1 \dots n$ conditional on total output $y = \Sigma x_i$ from the n agents.⁸ Second, the agents simultaneously choose efforts, and value realization $y = \Sigma x_i$ is revealed. Third, the parties observe y and the fixed salary α_i is paid. Then the parties choose whether

⁸We thus assume stationary contracts, which have been shown to be optimal in settings like this (Levin 2002, 2003).

or not to honor the contingent bonus contract $b_i(y)$.

Conditional on efforts, agent i 's expected wage in the contract is then $w_i = E(b_i(y) | e_1 \dots e_n) + \alpha_i$, while the principal expects $\Pi = E(y | e_1 \dots e_n) - \Sigma w_i = \Sigma_i E(x_i | e_i) - \Sigma w_i$. If the contract is expected to be honored, agent i chooses effort e_i to maximize his payoff, i.e.

$$e_i = \arg \max_{e'_i} (E(b_i(y) | e'_i, e_{-i}) - c(e'_i)) \quad (\text{IC})$$

The parties have outside (reservation) values normalized to zero. In the repeated game we consider, like Levin (2002), a multilateral punishment structure where any deviation by the principal triggers punishment from all agents. The principal honors the contract only if all agents honored the contract in the previous period. The agents honor the contract only if the principal honored the contract with all agents in the previous period. Thus, if the principal reneges on the relational contract, all agents take their outside option forever after. And vice versa: if one (or all) of the agents renege, take her outside option forever after.⁹ A natural explanation for this is that the agents interpret a unilateral contract breach (i.e. the principal deviates from the contract with only one or some of the agents) as evidence that the principal is not trustworthy (see discussions in Bewley 1999, Levin 2002).

Now, (given that (IC) holds) the principal will honor the contract with all agents $i = 1, 2, \dots, n$ if

$$-\Sigma_i b_i(y) + \frac{\delta}{1 - \delta} (E(y | e_1 \dots e_n) - \Sigma w_i) \geq 0 \quad (\text{EP})$$

where δ is a common discount factor. The LHS of the inequality shows the principal's expected present value from honoring the contract, which involves paying out the promised bonuses and then receiving the expected value from relational contracting in all future periods. The RHS shows the expected present value from reneging, which implies breaking up the relational contract and receive the reservation value in all future periods.

⁹See Miller and Watson (2013) on alternative strategies and "disagreement play" in repeated games.

Agent i will honor the contract if

$$b_i(y) + \frac{\delta}{1-\delta}(w_i - c(e_i)) \geq 0 \quad (\text{EA})$$

where similarly the LHS shows the agent's expected present value from honoring the contract, while the RHS shows the expected present value from reneging.

Following established procedures (e.g. Levin 2002) we have the following:

Lemma 1 *For given efforts $e = (e_1 \dots e_n)$ there is a wage scheme that satisfies (IC, EP, EA) and hence implements e , iff there are bonuses $b_i(y)$ and fixed salaries α_i with $b_i(y) \geq 0$, $i = 1, \dots, n$, such that (IC) and condition (EC) below holds:*

$$\sum_i b_i(y) \leq \frac{\delta}{1-\delta} \sum_i W(e_i) \quad (\text{EC})$$

To see sufficiency, set the fixed wages α_i such that each agent's payoff in the contract equals his reservation payoff, i.e. $\alpha_i + E(b_i(y)|e) - c(e_i) = 0$. Then EA holds since $b_i(y) \geq 0$. Moreover, the principal's payoff in the contract will be $\Pi = \sum_i W(e_i)$ i.e. the surplus generated by the contract. Then EC implies that EP holds. Necessity follows by standard arguments.

Unless otherwise explicitly noted, we will follow the standard assumption in the literature and assume that the first order approach (FOA) is valid, and hence that each agent's optimal effort choice is given by the first-order condition (FOC):

$$\frac{\partial}{\partial e_i} E(b_i(y)|e_1 \dots e_n) = c'(e_i) \quad (1)$$

We will refer to this as a 'modified' IC constraint.

The optimal contract now maximizes total surplus ($\sum_i W(e_i) = \sum_i (E(x_i|e_i) - c(e_i))$) subject to EC and the 'modified' IC constraint (1). To state our first result, let $G(y; e_1 \dots e_n)$ denote the CDF for team output $y = \sum x_i$. We will consider discrete as well as continuous outputs, and will let $g(y; e_1 \dots e_n)$ denote the probability of outcome y in the former case, and the density at outcome y in the latter. We will further say that the 'monotone likelihood ratio property' (MLRP) holds for aggregate output y if $\frac{g_{e_i}(y, e)}{g(y, e)}$ is increasing in y . Then we have the following:

Proposition 1 *The optimal symmetric scheme pays a maximal bonus to each agent for all outputs y for which $\frac{\partial}{\partial e_i}g(y; e_1\dots e_n) > 0$. If MLRP holds, then this entails paying the bonus for output above a threshold ($y > y_0$) and no bonus otherwise.*

The maximal symmetric bonus is by EC $b_i(y) = b(y) = \frac{\delta}{1-\delta}W(e_i)$ when efforts e_i are equal for all i . The result under MLRP parallels that of Levin (2003) for the single agent case. The threshold property comes from the fact that incentives should be maximal (minimal) where the likelihood ratio is positive (negative). Since this ratio is monotone increasing under MLRP, there is a threshold y_0 where it shifts from being negative to positive, and hence incentives should optimally shift from being minimal to maximal at that point.

Letting Y_+^n be the set of outcomes for which $\frac{\partial}{\partial e_i}g(y; e_1\dots e_n) > 0$ under equilibrium efforts (and given that FOA is valid), then these efforts are given by the IC constraint (1), where now the marginal incentive for effort is $b\frac{\partial}{\partial e_i}P(y \in Y_+^n | e_1\dots e_n)$. For given bonus of magnitude b , the marginal incentive for effort is here determined by the marginal effect of effort on the probability of obtaining the bonus. This bonus is in turn determined by EC, and thus we have in equilibrium

$$c'(e_i) = b\frac{\partial}{\partial e_i}P(y \in Y_+^n | e_1\dots e_n) \quad \text{and} \quad b = \frac{\delta}{1-\delta}W(e_i)$$

This equilibrium will depend on team size (n) and team composition—in particular the type of stochastic dependencies among members' contributions—via the term $\frac{\partial}{\partial e_i}P(y \in Y_+^n | e_1\dots e_n)$, i.e. via the marginal effect of individual effort (ME) on the probability of obtaining the bonus under the optimal scheme. Any variation—in team size or composition—that makes this marginal effect of effort stronger, will improve team efficiency in the sense that it will allow higher individual efforts to be implemented.

In particular, to analyse variations in team size n for the optimal scheme in Proposition 1, define

$$m^n(e_i) = \frac{\partial}{\partial e_i}P(y^n \in Y_+^n | e)$$

where we have emphasized that team output $y^n = \sum_1^n x_i$ depends on n , and

where the partial on the RHS is evaluated at $e = (e_1 \dots e_n)$ with all individual efforts equal (due to symmetry). Comparing teams of size n and $n + 1$, it is clear that if $e_i = e_1^n$ is optimal for team size n , and

$$m^{n+1}(e_1^n) > m^n(e_1^n),$$

then the larger team (of size $n + 1$) can implement higher individual efforts, and will thus be more efficient. It turns out that this can not occur if the agents' contributions/outputs are independent, and thus we have the following.

Proposition 2 *For stochastically independent outputs we have $m^{n+1}(e_i) \leq m^n(e_i)$ for all e_i ; hence incentives for effort will then decrease with increasing team size.*

This tells us immediately that for increasing team size to be beneficial in this setting, individual contributions must be stochastically dependent. To this we now turn.

4 Normally distributed outputs

We will now consider normally distributed outputs. As in several other areas, e.g. tournaments (Lazear-Rosen 1981) or multi-tasking (Holmstrom-Milgrom 1991), this assumption greatly simplifies the analysis, and can be highly relevant for applications. So we now consider the case where *outputs are (multi)normally distributed and correlated*. We assume also that covariances are independent of efforts. Given this assumption, and (by symmetry) each x_i being $N(e_i, s^2)$, then total output $y = \sum x_i$ is also normal with expectation $Ey = \sum e_i$ and variance

$$s_n^2 = \text{var}(y) = \sum_i \text{var}(x_i) + \sum_{i \neq j} \text{cov}(x_i, x_j) = ns^2 + s^2 \sum_{i \neq j} \text{corr}(x_i, x_j)$$

Letting $g(y, e_1 \dots e_n)$ denote the density of y , it follows from the form of the normal density that the likelihood ratio is linear and given by $\frac{g_{e_i}(y, e_1 \dots e_n)}{g(y, e_1 \dots e_n)} = (y - \sum e_i)/s_n$. As shown in Proposition 1, the optimal bonus is maximal (minimal) for outcomes where the likelihood ratio is positive (negative),

and hence has a threshold $y_0 = \Sigma e_i^*$ in equilibrium. Applying the normal distribution, it then follows (as shown below, see (5)) that the marginal return to effort for each agent in equilibrium is given by

$$b \frac{\partial}{\partial e_i} P(y > y_0) = b \int_{y > y_0} g_{e_i}(y; e^*) dy = \frac{b}{M s_n}, \quad M = \sqrt{2\pi} \quad (2)$$

The marginal return to effort is thus inversely proportional to the standard deviation of total output in this setting. This implies that a team composition that reduces this standard deviation, and thus increases the precision of the available performance measure (total output) will improve incentives and thus be beneficial here.¹⁰

The IC condition (1) for each agent's (symmetric) equilibrium effort is now $c'(e_i) = \frac{b}{s_n} \frac{1}{M}$, and it then follows that the maximal effort per agent that can be sustained, is given by

$$c'(e_i^*) s_n M = b = \frac{\delta}{1 - \delta} W(e_i^*) \quad (3)$$

Consider now a variation in team size. When all agents' outputs are fully symmetric in the sense that all correlations as well as all variances are equal across agents, i.e. $var(x_i) = s^2$ and $corr(x_i, x_j) = \rho$ for all i, j , then the variance in total output will be

$$s_n^2 = n s^2 + s^2 \sum_{i \neq j} corr(x_i, x_j) = n s^2 (1 + \rho(n - 1))$$

If $\rho \geq 0$ the variance will increase with n , and this will be detrimental for efficiency. Optimal n should therefore be smaller with larger ρ . Moreover, the standard deviation of total output (s_n) increases rapidly with n when $\rho \geq 0$ (at least of order \sqrt{n}), hence the effort per agent that can be sustained will then decrease rapidly with n . Large teams are therefore very inefficient if all agents' outputs are non-negatively correlated.

For negative correlations the situation is quite different. If $\rho < 0$ one can in principle reduce the variance to (almost) zero by including sufficiently many agents. The model then indicates that adding more and more agents to the

¹⁰There is however a caveat. For sufficiently small standard deviation, the first-order approach is no longer valid, and the analysis must be modified. See the following section.

team is beneficial, at least as long as $1 + \rho(n - 1) > 0$ and the conditions for FOA to be valid are fulfilled.¹¹ (We show below that for this to be the case, the variance of the performance measure, here s_n^2 , cannot be too small.)

Note that assuming symmetric pairwise negative correlations among n stochastic variables only makes sense if the sum has non-negative variance, and hence $1 + \rho(n - 1) \geq 0$.¹² Given $\rho < 0$, there can thus only be a maximum number n of such variables (agents). And given $n > 2$, we must have $\rho > -\frac{1}{n-1}$. Note also that for given negative $\rho > -\frac{1}{2}$, the variance is first increasing, then decreasing in n (it is maximal for $n = \frac{1}{2}(1 - \frac{1}{\rho})$). Hence the optimal team size in this setting is either very small ($n = 2$) or 'very large' (includes all).

Proposition 3 *For normally distributed outputs, efficiency decreases rapidly with team size if outputs are non-negatively correlated. For symmetric agents with negatively correlated outputs, efficiency first decreases (for $n > 2$) and then increases with increasing team size, hence efficiency is maximal either for a small or for a large team (within the feasible range).*

The assumption of equal pairwise correlations among all involved agents is admittedly somewhat special, but illustrates in a simple way the forces at play when the team size varies. In reality there might be positive as well as negative correlations among agents. A procedure to pick agents for least variance would then be for each n , to pick those n that yield the smallest variance.

4.1 Optimal schemes when FOA fails

We will now first examine under what conditions the FOA is valid for the normal model analyzed here, and second derive optimal bonus schemes when FOA fails in this setting. A recent literature has examined such issues for static moral hazard with contractible outputs, see Kadan and Swinkels (2013), Ewerhart (2014) and Kirkegaard (2014), but not (to our best knowledge)

¹¹This is related to results by Hwang (2014), who analyzes conditions under which additional signals (information) will be valuable in a single-agent relational contract.

¹²Indeed, $1 + \rho(n - 1) > 0$ is the condition for the covariance matrix to be positive definit, and hence for the multinormal model to be well specified.

for moral hazard in relational contracting, neither for single-agent nor multi-agent settings.

So consider y normally distributed with expectation $Ey = \Sigma e_i$ and a variance that will be denoted by $s^2 = \text{var}(y)$ in this section (to simplify notation). As already noted, this distribution satisfies MLRP. Given that FOA holds, and the principal seeks to implement effort e_i^* from each agent, the optimal bonus $b(y)$ has a threshold at $y_0 = \Sigma e_i^* = ne_i^*$. Agent i 's expected payoff, given own effort e_i and efforts $e_j^* = e_i^*$ from the other agents, is then

$$\begin{aligned} & b \Pr(y > y_0 | e_i) - c(e_i) \\ &= b \Pr(y - \Sigma_{j \neq i} e_j^* - e_i > e_i^* - e_i) - c(e_i) \\ &= b(1 - H(e_i^* - e_i)) - c(e_i) \end{aligned}$$

where $H()$ is the CDF for an $N(0, s^2)$ distribution. The FOC for the agent's choice is

$$bh(e_i^* - e_i) - c'(e_i) = 0 \quad (4)$$

where $h()$ is the density; $h() = H'()$. The FOA is valid if the agent's optimal choice is e_i^* and is given by this first-order condition, i.e. if

$$bh(0) - c'(e_i^*) = 0 \quad (5)$$

and no other effort $e_i \neq e_i^*$ yields a higher payoff for the agent. We note in passing that $h(0) = 1/\sqrt{2\pi\text{var}(y)}$, verifying the formula (2) above.

Due to the shape of the normal density, the agent's payoff is generally not concave.¹³ The payoff is locally concave at $e_i = e_i^*$ (since $h'(0) = 0$), hence e_i^* is a local maximum, but there may be other local maxima (other solutions to FOC) for $e_i < e_i^*$. The situation is illustrated in Figure 1, which depicts the agent's marginal revenue ($bh(e_i^* - e_i)$) and marginal cost for two values of the variance $s^2 = \text{var}(y)$. If the variance is sufficiently small there is a local maximum at some $e_i < e_i^*$ (satisfying the FOC), and the figure indicates (comparing areas under MC and MR) that this local maximum dominates that at e_i^* .

(See Figure 1 in the appendix)

¹³The second derivative is $-bh'(e_i^* - e_i) - c''(e_i)$, where $h'(e_i^* - e_i) < 0$ for $e_i < e_i^*$.

This indicates that the FOA is valid here only if the variance of the performance measure (y) is not too small, and is confirmed in the following proposition.¹⁴ (The first part of the proposition also follows from a general result by Hwang 2016 on the validity of FOA in the single agent case.) Moreover, this proposition confirms that a symmetric solution with equal efforts across agents is then indeed optimal.

Proposition 4 *For the normal case $y \sim N(\Sigma_i e_i, s^2)$ the first order approach is valid if the variance of output s^2 is sufficiently large, but not valid if s^2 is sufficiently small. In the former case, symmetric efforts is indeed optimal.*

For negatively correlated agents, the variance in the performance measure y can be made quite small by including many agents in the team. We saw that this was beneficial for incentives and consequently for efficiency as long as the analysis building on FOA was valid. But for sufficiently small variance FOA is not valid, so this immediately raises the question of what a team can achieve under such circumstances. In the following we will show that a threshold bonus is nevertheless always optimal for the team model with normally distributed outputs, and moreover characterize its properties.

The EC constraint for symmetric efforts is $0 \leq b(y) \leq \frac{\delta}{1-\delta}W(e_i)$. To provide incentives, the bonus cannot be maximal for all outputs y , hence the expected bonus payment for an agent must be less than the maximal bonus, i.e. $E(b(y)|e_i, e_{-i}) < \frac{\delta}{1-\delta}W(e_i)$. On the other hand, the agent's expected payoff from exerting effort must be non-negative; $E(b(y)|e_i, e_{-i}) - c(e_i) \geq E(b(y)|e_i = 0, e_{-i}) \geq 0$, so in any symmetric equilibrium we must have $c(e_i) < \frac{\delta}{1-\delta}W(e_i)$. It follows from this that the effort e_u^* and associated surplus $W(e_u^*)$ defined by

$$c(e_u^*) = \frac{\delta}{1-\delta}W(e_u^*) \tag{6}$$

constitute upper bounds for, respectively, the effort and surplus (per agent) that can be achieved in a relational contract.¹⁵ Note also that this upper bound can be achieved if there is no uncertainty, i.e. if (team) effort can be

¹⁴This is similar to results on the validity of FOA in tournaments, see Gurtler (2011).

¹⁵We assume that δ is small enough so that e_u^* is below first best effort.

observed without noise; namely by paying the maximal bonus $b = c(e_u^*)$ to each agent conditional on total effort being at least ne_u^* .

We will now show that the optimal bonus is a threshold bonus which induces effort that converges to the upper bound as the variance in the performance measure goes to zero. The scheme is a simple modification of the threshold bonus scheme identified in the FOA analysis, and consists of a relaxation of the threshold combined with an increase of the bonus relative to the latter scheme.

To show this let, for any (symmetric) bonus $b = b(y)$, agent i 's performance-related payoff (utility) be denoted

$$u(b; e) = u(b; e_i, e_{-i}) = \int b(y)g(y, e)dy - c(e_i)$$

As a first step we show that if a non-threshold and a threshold bonus yield the same payoff to the agent (for given effort), the latter bonus yields the strongest marginal incentives.

Lemma 2 *If MLRP holds for (general) $g(y; e)$, we have:*

- i) If $\tilde{b}(y)$ is not a threshold bonus, then for a threshold scheme $b_h(y)$ with $u(\tilde{b}; e) = u(b_h; e)$ it holds: $u_{e_i}(b_h; e) > u_{e_i}(\tilde{b}; e)$.*
- ii) If $\tilde{b}(y)$ is not a threshold bonus, and e^* is the associated equilibrium, then there is a threshold scheme $b_h(y)$ with $u(\tilde{b}; e^*) = u(b_h; e^*)$, $u_{e_i}(b_h; e^*) > u_{e_i}(\tilde{b}; e^*) = 0$ and $u(b_h; e_i, e_{-i}^*) \leq u(b_h; e_i^*, e_{-i}^*)$ for all $e_i < e_i^*$.*

Remark Note that *in a single-agent case, statement (ii) in the lemma implies that a threshold scheme must be optimal whenever MLRP holds.* For should some other scheme be optimal, then (ii) shows that there is a threshold scheme that will induce higher effort by the agent ($e_i > e_i^*$). This means that the assumptions traditionally invoked to ensure validity of FOA, such as convexity of the distribution function (CDF) in addition to MLRP (as in e.g. Levin 2003), are much stronger than necessary to ensure that a threshold bonus is optimal in a relational contract with moral hazard.¹⁶ On the other hand, Hwang (2016) has recently shown that a weaker condition than

¹⁶This result is in some respects similar to results in Poblete and Spulber 2012, showing that simpler assumptions than CDF and MLRP are sufficient for a debt-type contract to be optimal in the static principal-agent model under risk neutrality and limited liability.

CDF is sufficient for FOA to be valid in the single-agent case (irrespective of whether MLRP holds or not).

Using the lemma above, we can show that a threshold bonus will be optimal in the team model with normally distributed output considered in this section.

Proposition 5 *For the team model with normally distributed output $y \sim N(\Sigma_i e_i, s^2)$, the optimal symmetric bonus is a threshold bonus.*

When FOA is valid, the optimal threshold is the output at which the likelihood ratio is zero, which is the output $y_0 = \Sigma e_i^*$ in the normally distributed case. The problem with this scheme is that for sufficiently small s the agent's payoff is non-concave. In particular, for $c'()$ convex ($c''' \geq 0$), the payoff has two local maxima¹⁷, at e_i^* and at $e_i^0 < e_i^*$, respectively, and e_i^0 then gives the highest payoff for small s , so the agent will deviate from the supposed equilibrium effort e_i^* . Now, this can be rectified by setting a lower threshold $y'_0 < y_0 = \Sigma e_i^*$, i.e. making it easier to obtain the bonus, and at the same time increase the bonus level. We find that this is indeed optimal.

Proposition 6 *For $y \sim N(\Sigma_i e_i, s^2)$ we have: Given convex marginal costs ($c''' \geq 0$), there is a critical $s_c > 0$ for the standard deviation of output such that for $s \geq s_c$ FOA is valid and the optimal threshold y_0 is the output at which the likelihood ratio is zero, thus $y_0 = \Sigma e_i^*$. For $s < s_c$ the optimal threshold is an output $y'_0 = \Sigma e_i^* - \tau$, (at which the likelihood ratio is negative), and the optimal scheme is given by (20 - 22) in the appendix, with all relations holding with equality. Effort e_i^* is strictly higher when s is lower, and $e_i^* \rightarrow e_u^*$ defined by (6) as $s \rightarrow 0$.*

It may be noted that for the set of variances $s^2 = \text{var}(y)$ sufficiently large to make FOA valid, the largest effort per agent that can be implemented must satisfy $2c(e_i^*) \leq \frac{\delta}{1-\delta} W(e_i^*)$, and hence be considerably smaller than the upper bound e_u^* defined in (6). This is so because the agent obtains the bonus (b) with probability $\frac{1}{2}$ in equilibrium in the FOA scheme, hence

¹⁷It follows from the shape of the density $h()$ that for $c'()$ convex ($c''' \geq 0$), the FOC (4) for effort can yield at most two local maxima.

we must have $b_{\frac{1}{2}} \geq c(e_i^*)$ in that setting. This illustrates that a lower output variance can yield considerable benefits in relational contracting. The benefits are not associated with risk reduction (since all agents are risk neutral by assumption), nor with sharper competition, since in the team setting there is none. The benefits arise because a lower variance strengthens individual incentives for effort, for a given bonus level. Since the bonuses in the relational contract are discretionary and hence must be kept within bounds, the added effort incentives coming from a lower variance are valuable. And the value added may be considerable, as we have seen.

In this subsection we have taken the output variance ($s^2 = \text{var}(y)$) as an exogenous parameter. We know that this variance can be substantially reduced if a team can be put together, consisting of several agents whose individual outputs are negatively correlated. Under normally distributed outputs, an expansion of the team will thus enhance efficiency exactly when it leads a lower variance for the team's output, i.e. a more precise performance measure. The enhanced precision is thus the decisive factor, but this is related to properties of the normal distribution.

5 Discrete outputs

In the normal case analyzed so far, the precision of the performance measure was a decisive factor, in the sense that higher precision unambiguously lead to stronger individual incentives. The analysis revealed that this partly hinges on the fact that a hurdle scheme was optimal for any team composition, a fact which technically follows from the property that MLRP always holds in the normal case. This may well not hold for other distributions; in particular we may have MLRP satisfied for individual outputs, but not for aggregate output. This will affect the shape of optimal incentive schemes, and will generally also affect how optimal schemes and associated efforts are influenced by correlations between individual outputs. To this we now turn.

To handle teams with correlated individual contributions (outputs) in a relatively general setting, we consider discrete outputs. Moreover, we assume binary individual outcomes, so an agent's contribution is $x_i \in \{G, B\}$, with $G > B \geq 0$. Without loss of generality we will normalize and set $G = 1$ and

$B = 0$. In this setting we can also identify an agent's effort with his/her probability of a good outcome: if $p(e_i)$ is this probability as a function of "natural effort", with $p(e_i) \in [p_0, \bar{p}] \subseteq [0, 1]$, $p'(e_i) > 0$, redefine effort to be $p_i = p(e_i)$ with cost $c(p^{-1}(p_i))$. We assume that this cost function is also strictly convex with zero marginal cost at $p_i = p_0$.

We allow for correlations between team members contributions. Following Fleckinger (2012), the joint distribution for two agents' ($i \neq j$) outcomes can be written as

$$\begin{aligned} P(1, 1) &= p_i p_j + \gamma(p_i, p_j), & P(1, 0) &= p_i(1 - p_j) - \gamma(p_i, p_j) \\ P(0, 1) &= (1 - p_i)p_j - \gamma(p_i, p_j), & P(0, 0) &= (1 - p_i)(1 - p_j) + \gamma(p_i, p_j), \end{aligned} \quad (7)$$

where $P(k, l)$ is the probability of $x_i = k, x_j = l$. Given our normalization with $x_i \in \{1, 0\}$, the function $\gamma(\cdot)$ is simply the covariance between the two agent's outcomes, i.e. $\gamma(p_i, p_j) = \text{cov}(x_i, x_j)$. To have a manageable and yet interesting model we follow Gupta-Tao (2010) and others and assume that for any n , the random variables $x_1 \dots x_n$ have no second- or higher-order interactions (see appendix for details). It then follows that the joint distribution of $x_1 \dots x_n$, and hence the distribution of total team output $y^n = \sum_1^n x_i$, is determined by "efforts" $p_1 \dots p_n$ and covariances $\gamma(p_i, p_j), i \neq j$. Specifically we have:

$$P(y^n = k) = P(y^{n-1} = k - 1)p_n + P(y^{n-1} = k)(1 - p_n) + \sum_{j=1}^{n-1} \gamma(p_n, p_j) a_{n,k}^j \quad (8)$$

where the coefficients $a_{n,k}^j$ depend on (p_1, \dots, p_{n-1}) and can be determined inductively (see appendix), and we define $P(y^{n'} = r) = 0$ for $r = -1, n' + 1$. For independent variables ($\gamma \equiv 0$) this is a standard binomial formula, conditioning on one agent's success or failure (here agent n). The last term in the formula adjusts for stochastic dependencies.

Differentiating this (wrt p_n , say) and using symmetry – including symmetric derivatives and $p_i = p_1$, all i – we obtain

$$\frac{\partial}{\partial p_1} P(y^n = k) = P(y^{n-1} = k - 1) - P(y^{n-1} = k) + \sum_{j=1}^{n-1} \gamma_1(p_1, p_1) a_{n,k}^j \quad (9)$$

The marginal effect of an agent's effort on the probability that the team achieves $y^n = k$ will thus now depend both on the covariance level γ (via

the first two terms on the RHS) and on its derivative γ_1 . These influences imply, among other things, that optimal bonus schemes may well not be of the simple threshold type, and that a larger team may provide stronger incentives than a smaller one. To illustrate this, we consider the case where *the covariance level γ is independent of efforts, thus $\gamma = \text{const.}$* ¹⁸

Note first that under this assumption we have from (8) that all output probabilities, and hence an agent's expected bonus payments, are linear in the agent's effort (p_n), and hence that *FOA will certainly be valid* (given strictly convex effort costs).

From (9) we have now, since $\gamma_1 \equiv 0$:

$$\frac{\partial}{\partial p_1} P(y^n = k) = P(y^{n-1} = k-1) - P(y^{n-1} = k), \quad (10)$$

and thus

$$\frac{\partial}{\partial p_1} P(y^n \geq k) = P(y^{n-1} = k-1) \quad (11)$$

In a team of n agents where a bonus is offered for team output $y^n \geq k$, the marginal effect of an agent's effort to obtain the bonus is thus determined by the probability that the ensemble of the other $n-1$ agents achieves exactly the output $y^{n-1} = k-1$.

For *independent* contributions ($\gamma = 0$) it turns out that $\frac{\partial}{\partial p_1} P(y^n = k)$ is positive iff $k > np_1$ (and $k \geq 1$). This implies that to implement (symmetric) individual effort $p_1 \in (\frac{k-1}{n}, \frac{k}{n}]$ it is then optimal to reward for all team outcomes with $y^n \geq k$. The optimal scheme under stochastic independence is thus a threshold scheme, with a threshold adapted to the effort that is to be implemented. We find the following.

Proposition 7 *For stochastically independent contributions the optimal bonus scheme is a threshold scheme for the team's output, and we have*

$$m^n(p_1) = \frac{\partial}{\partial p_1} P(y^n \geq k) = P(y^{n-1} = k-1), \quad p_1 \in ((k-1)/n, k/n], \quad (12)$$

for $k = 1, \dots, n-1$. *Moreover, a larger team will always provide weaker incentives than a smaller one, and thus be less efficient. Specifically we*

¹⁸If the covariance depends on effort, the analysis becomes much more complicated, but does not (for our purpose) add new insights. The analysis is available from the authors.

have $m^{n+1}(p_1) \leq m^n(p_1)$ with strict inequality for all p_1 except $p_1 = \frac{k}{n}$, $k = 1, \dots, n - 1$.

Consider now correlated outputs ($\gamma \neq 0$). To build intuition we consider first small teams. For $n = 2$ and effort $p_1 \geq \frac{1}{2}$ the optimal bonus scheme here is a scheme with threshold $y^2 \geq 2$, and thus with marginal effect of effort (ME)

$$m^2 = \frac{\partial}{\partial p_1} P(y^2 = 2) = P(y^1 = 1) + 0 = p_1$$

This scheme is optimal because we have $\frac{\partial}{\partial p_1} P(y^2 = k) \leq 0, k = 0, 1$ for $p_1 \geq \frac{1}{2}$. The formula for m^2 shows that the marginal effect of individual effort to achieve a team outcome with 2 successes is given by $P(y^1 = 1)$, i.e. by the probability that the other agent achieves a success. This is trivially true for independent outcomes, where the probability of two successes is $p_1 p_2$, and thus the marginal effect of individual effort is given by the other agent's success probability. When $\gamma_1 \equiv 0$ the same formula also holds for correlated projects.

Consider now, for $n = 3$ a bonus scheme with threshold $y^3 \geq 2$. From the formula (11) above it follows that the ME for this scheme is given by

$$\frac{\partial}{\partial p_1} P(y^3 \geq 2) = P(y^2 = 1) = 2p_1(1 - p_1) - 2\gamma,$$

where the second equality follows from (7). Comparing the ME's for the two team sizes, we see that the difference is

$$P(y^2 = 1) - P(y^1 = 1) = 2p_1(1 - p_1) - 2\gamma - p_1$$

For $\gamma \geq 0$ the difference is negative for all $p_1 \geq \frac{1}{2}$, but for $\gamma < 0$ the difference is *positive* for a range of p_1 's exceeding $\frac{1}{2}$. Thus, with negatively correlated outputs, the larger team will provide stronger incentives for a range of effort levels exceeding $p_1 = \frac{1}{2}$. This is due to the fact that under negative correlation and for these efforts, the probability that two agents produce exactly one unit of output is higher than the probability that a single agent does so. The marginal incentives for individual effort are then larger in a team of 3 agents than in a team of 2 agents.

Consider now $n > 3$. Since threshold bonus schemes are optimal for $\gamma = 0$, and the marginal effects of effort $\frac{\partial}{\partial p_1} P(y^n = k)$ depend continuously (in

fact linearly) on γ (see formulas (9) and (8)), such threshold schemes will also be optimal for $|\gamma|$ small. In this bonus regime we can show that for given team size n , marginal incentives will decrease with increasing γ for all efforts p_1 in an interval $I_n \approx (\frac{1}{n}, 1 - \frac{1}{n})$. Thus, except for very small and very high (perhaps infeasible) effort p_1 , marginal incentives will be higher when individual contributions are negatively correlated compared to non-negatively correlated, at least for a range of γ with $|\gamma|$ small. In this range, negative covariance will then improve incentives and hence efficiency in the team, while positive covariance will reduce the team's efficiency.

Proposition 8 *For given $n \geq 3$, a threshold bonus scheme is optimal for $|\gamma|$ small. In this scheme, the marginal incentive for effort –and hence the team's efficiency– will be decreasing in γ for $p_1 \in (\pi_1^n, \pi_{n-1}^n)$ and increasing in γ for $p_1 < \pi_1^n$ or $p_1 > \pi_{n-1}^n$, where $\pi_1^n \rightarrow \frac{1}{n}, \pi_{n-1}^n \rightarrow 1 - \frac{1}{n}$ as $\gamma \rightarrow 0$.*

This means that if the team's optimal effort under zero correlation entails $p_1 \in I_n \approx (\frac{1}{n}, 1 - \frac{1}{n})$, then a stronger positive (negative) covariance will reduce (improve) the team's efficiency for some range of γ including $\gamma = 0$. Note that the optimal effort p_1 will certainly be contained in I_n if the feasible range for effort (measured as the probability of success) is within this interval, i.e. $p(e_i) \in [p_0, \bar{p}] \subseteq I_n$.

Comparing team sizes, we have the following result.

Proposition 9 *Comparing n and $n + 1$ for $n \geq 3$, then for $|\gamma|$ small we have (i) the larger team provides weaker incentives ($m^{n+1} \leq m^n$) for all p_1 if $\gamma > 0$, and (ii) the larger team provides stronger incentives ($m^{n+1} > m^n$) for some set of p_1 's if $\gamma < 0$. The set includes neighborhoods of all $p_1 = \frac{k}{n}$, $k = 1, \dots, n - 1$.*

These results show that if the covariance level is independent of efforts and relatively small in absolute value, then a larger team can never be more efficient if the covariance is positive, but it may well be more efficient if the covariance is negative.

So far this analysis has demonstrated that threshold schemes are optimal when covariances are small (and effort independent), and that a larger team

may then provide stronger incentives if and only if outputs are negatively correlated. But what if covariances are positive and large? We will now demonstrate that a larger team may provide stronger incentives also when outputs are positively correlated, provided covariances are sufficiently large. Moreover, the optimal bonus scheme may then well not be a threshold scheme.

To see this, compare teams of sizes $n = 3$ and $n = 2$. Consider a bonus scheme for the larger team that awards for exactly 1 or 3 units of output ($y^3 = 1, 3$). From (10) and (7) we see that the ME for this scheme is

$$\frac{\partial}{\partial p_1} P(y^3 = 1, 3) = P(y^2 = 0) - P(y^2 = 1) + P(y^2 = 2) = 1 - 4p_1(1 - p_1) + 4\gamma$$

For $\gamma = \text{const}$, the optimal bonus scheme for the smaller team is independent of γ , and hence has hurdle $y^2 \geq 1$ for $p_1 < \frac{1}{2}$ and hurdle $y^2 \geq 2$ for $p_1 \geq \frac{1}{2}$. Comparing the ME's for the two teams we have

$$\frac{\partial}{\partial p_1} P(y^3 = 1, 3) - m^2 = 1 - 4p_1(1 - p_1) + 4\gamma - \max\{p_1, 1 - p_1\} \quad (13)$$

This reveals that, provided $\gamma > \frac{1}{8}$, this difference will be positive for effort level $p_1 = \frac{1}{2}$, and thus for a range of efforts around this level. Thus we see that the larger team may also provide stronger incentives for positively correlated individual outputs, but only if the covariance level exceeds a lower positive bound.¹⁹ In fact, for this case of $\gamma = \text{const}$, one can verify that the larger team provides stronger incentives under positive correlation only if the covariance exceeds this lower bound and the bonus scheme for the larger team rewards for exactly 1 or 3 units of output. The larger team can in this case never provide stronger incentives under positive correlation if hurdle schemes are optimal for both teams.

To see why a hurdle scheme may well not be optimal under positive correlation, consider

$$\frac{\partial}{\partial p_1} P(y^3 = k) = P(y^2 = k - 1) - P(y^2 = k), \quad k = 1, 2$$

With positive and relatively high correlation, a team of 2 agents is more likely to achieve $k = 2$ successes than only one success, and then more effort by a third agent will only reduce the probability that the 3-agent team

¹⁹It may be noted that $\gamma > \frac{1}{8}$ for $p_1 = \frac{1}{2}$ implies a correlation coefficient of at least 0.5.

will achieve 2 successes, so $\frac{\partial}{\partial p_1}P(y^3 = 2) < 0$. A team of two agents is then also more likely to have 2 failures ($y^2 = 0$) than 1 failure ($y^2 = 1$), hence $\frac{\partial}{\partial p_1}P(y^3 = 1) > 0$. If the 3-agent team is paid a bonus for 1 success, the marginal incentive for effort is then positive, while it is negative if the team is paid for 2 successes. A hurdle scheme is then clearly not optimal. Technically, MLRP does not hold under these conditions. We have the following:

Proposition 10 *When individual contributions are correlated, the optimal bonus scheme for the team may well not be a hurdle scheme. In particular, for $n = 3$ and $\gamma = \text{const}$, the optimal scheme awards for team output $y^3 \in \{1, 3\}$ if the covariance is positive and sufficiently large ($\gamma > \frac{1}{9}$).*

This non-monotonic incentive scheme may look peculiar. Since the team is rewarded for $y^3 = 1$ but not for $y^3 = 2$, the agents are in some sense rewarded for failure. But the intuition is simple; under correlated outputs, low effort may yield a high probability for an intermediate result ($y^3 = 2$), and should thus not be rewarded. However, non-monotonic incentive schemes are rarely observed in practise. But the fact that they may be optimal, indicates that the more standard hurdle schemes can give rise to perverse incentives if the hurdle is not accurately placed.

6 Conclusion

In relational contracts, the agents' incentives, i.e. the size of the bonuses, are limited by the value of the future relationship. If bonuses are too large (or too small), the principal (or agents) may deviate by not paying as promised, and thereby undermine the relational contract. For a given maximum bonus, the principal must thus look for other ways to strengthen the agents' incentives. In this paper, we show that when the principal contracts with a team of agents, and the maximum bonuses are limited by the relational contract, the principal can strengthen the agents' effort incentives by composing teams that utilize stochastic dependencies between the agents' outputs.

We have shown that efficiency decreases with team size when individual contributions are stochastically independent. This is due to the well known

$1/n$ free-rider problem. However, efficiency may *increase* with team size if outputs are stochastically dependent, and particularly when individual contributions are negatively correlated. Hence, correlation – and in particular negative correlation – between team members’ contributions may enhance team performance. We have also shown that correlation may affect the type of incentive scheme that is optimal for the team. Hurdle schemes may or may not be optimal, depending on the stochastic dependencies. In particular we point out that under correlated outputs, it may be optimal to reward the team for e.g. low and high outputs, but not for intermediate ones.

Stochastic dependencies relates to questions concerning optimal team composition. In the management literature a central question is whether teams should be homogenous or heterogeneous with respect to tasks as well as bio-demographic characteristics (e.g. Horwitz and Horwitz, 2007). One can conjecture that negative correlations are more associated with heterogeneous teams than homogenous teams, and also more associated with task-related diversity than with bio-demographic diversity. Our model can thus contribute to explain why heterogeneity among team members and task-related diversity can yield considerable efficiency improvements.

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APPENDIX

Proof of Proposition 1. Maximizing total surplus ($\sum_i W(e_i) \equiv \sum_i (E(x_i | e_i) - c(e_i))$) subject to EC and the 'modified' IC constraint (1) yields

$$\mu_i \frac{\partial}{\partial e_i} g(y; e_1 \dots e_n) - \lambda(y) \leq 0, \quad b_i(y) \geq 0,$$

where the inequalities hold with complementary slackness, and $\mu_i > 0$, $\lambda(y) \geq 0$ are Lagrange multipliers. It is clear that $b_i(y) > 0$ iff $\frac{\partial}{\partial e_i} g(y; e_1 \dots e_n) > 0$, and then EC will bind ($\lambda(y) > 0$).

Given MLRP and symmetry we have $\frac{\partial}{\partial e_i} g(y; e_1 \dots e_n) > 0$ iff $y > y_0$, and thus EC binding with all bonuses equal and maximal for $y > y_0$. On the other hand, for $y < y_0$ we have $\frac{\partial}{\partial e_i} g(y; e_1 \dots e_n) < 0$ by MLRP and hence $b_i(y) = 0$ for all i .

Proof of Proposition 2. Consider the case of continuous outputs (the discrete case is similar), and let $g^n(y | e^n)$ be the density for a team of size n under efforts $e^n = (e_1 \dots e_n)$. By stochastic independence we then have

$$g^{n+1}(y | e^{n+1}) = \int_{-\infty}^{\infty} g^n(y - x | e^n) f(x | e_{n+1}) dx.$$

(Densities are zero outside bounded supports.) So for variations in any of $e_1 \dots e_n$ we have

$$g_{e_i}^{n+1}(y | e^{n+1}) = \int_{-\infty}^{\infty} g_{e_i}^n(y - x | e^n) f(x | e_{n+1}) dx$$

Consider $i = 1$, and let $Y_+^{n+1} = \{y : g_{e_1}^{n+1}(y | e^{n+1}) > 0\}$. Then

$$\begin{aligned} m^{n+1}(e_1) &= \int_{Y_+^{n+1}} g_{e_1}^{n+1}(y | e^{n+1}) dy = \int_{Y_+^{n+1}} \int_{-\infty}^{\infty} g_{e_1}^n(y - x | e^n) f(x | e_{n+1}) dx dy \\ &= \int_{-\infty}^{\infty} f(x | e_{n+1}) \int_{Y_+^{n+1}} g_{e_1}^n(y - x | e^n) dy dx \\ &= \int_{-\infty}^{\infty} f(x | e_{n+1}) \int_{Y(x)} g_{e_1}^n(y' | e^n) dy' dx \end{aligned}$$

where $Y(x) = \{y' : y' = y - x, y \in Y_+^{n+1}\}$

Given that $Y_+^n = \{y : g_{e_1}^n(y | e^n) > 0\}$, we have, for any set Y :

$$\int_Y g_{e_1}^n(y | e^n) dy \leq \int_{Y_+^n} g_{e_1}^n(y | e^n) dy = m^n(e_1)$$

Combined with the expression for $m^{n+1}(e_1)$ above this yields

$$m^{n+1}(e_1) \leq \int_{-\infty}^{\infty} f(x | e_{n+1}) m^n(e_1) dx = m^n(e_1)$$

By symmetry this is true for any e_i , and this proves the proposition.

Proof of Proposition 4. It is obvious from the shape of $h(\cdot)$ that the FOC for effort has a single solution for s sufficiently large, and hence that FOA is then valid. (See also Hwang 2016, p129.)

To see that the optimal solution then must be symmetric, note first that the normal density can be written as $g(y; l(e_1 \dots e_n))$, with $l(e_1 \dots e_n) = \sum_i e_i$. Assume the solution is asymmetric; say that $e_i < e_j$. Let $b_0 = (b_i + b_j)/2$ and consider

$$\begin{aligned} \int b_0(y)g_l(y; l(e_1 \dots e_n))dy &= \frac{1}{2} \int b_i(y)g_l(y; l(e_1 \dots e_n))dy + \frac{1}{2} \int b_j(y)g_l(y; l(e_1 \dots e_n))dy \\ &= \frac{1}{2}c'(e_i) + \frac{1}{2}c'(e_j) \geq c'\left(\frac{e_i + e_j}{2}\right) \end{aligned}$$

Hence the bonus $b_0(y)$ to each of i and j is feasible and would induce effort at least $\frac{e_i + e_j}{2} = e_0$ from each. Thus a slightly lower bonus to each is feasible and will induce effort e_0 from each. This yields higher value since the objective is concave.

Now consider s small. If FOA is valid, the agent's optimal payoff is $b\frac{1}{2} - c(e_i^*)$. This must be no less than the payoff for $e_i = 0$, which is positive, thus we have $c(e_i^*) < b\frac{1}{2} \leq \frac{\delta}{1-\delta}W(e_i^*)\frac{1}{2}$. There is a critical $\delta^F > 0$ such that these inequalities do not hold for $e_i^* = e_i^{FB}$ and $\delta < \delta^F$, hence first best effort can not be obtained for $\delta < \delta^F$. Given such a δ , if FOA is valid for all $s > 0$, then $b \rightarrow 0$ as $s \rightarrow 0$ (since $h(0) \sim \frac{1}{s}$), and hence, since EC binds, $e_i^* \rightarrow 0$. But this is a contradiction, since when FOA is valid, effort e_i^* should increase when s is reduced. This is so because if bonus b_s implements effort e_i^* for some $s > 0$, then b_s implements (by FOC) a higher effort for $s' < s$, yielding slack in EC, and hence room for a higher bonus to increase effort further. This shows that FOA cannot be valid for all $s > 0$.

Proof of Lemma 2. For given e , admissible bonuses satisfy $0 \leq b(y) \leq \frac{\delta}{1-\delta}W(e_i) \equiv B$. Let y_0 be the hurdle (threshold) for $b_h(y)$. Then

$$0 = u(b_h; e) - u(\tilde{b}; e) = \int_{\underline{y}}^{y_0} (-\tilde{b}(y))g(y, e) + \int_{y_0}^{\bar{y}} (B - \tilde{b}(y))g(y, e) \quad (14)$$

This yields

$$\begin{aligned}
u_{e_i}(b_h; e) - u_{e_i}(\tilde{b}; e) &= \int_{\underline{y}}^{y_0} (-\tilde{b}(y)) \frac{g_{e_i}(y, e)}{g(y, e)} g(y, e) dy + \int_{y_0}^{\bar{y}} (B - \tilde{b}(y)) \frac{g_{e_i}(y, e)}{g(y, e)} g(y, e) dy \\
&> \frac{g_{e_i}(y_0, e)}{g(y_0, e)} \left[\int_{\underline{y}}^{y_0} (-\tilde{b}(y)) g(y, e) dy + \int_{y_0}^{\bar{y}} (B - \tilde{b}(y)) g(y, e) dy \right] = 0
\end{aligned}$$

where the inequality follows from MLRP, and the last equality from $u(\tilde{b}; e) = u(b_h; e)$. This proves statement (i)

(ii) Note that, for given $e = e^*$ the RHS of (14) is strictly decreasing in y_0 , and hence there is a unique y_0 satisfying the equation. Let $b_h(y)$ be the associated hurdle scheme. To simplify notation, write $u(\tilde{b}; e) = \tilde{u}(e)$ and $u(b_h; e) = u(e)$. Then from (i) we now have $\tilde{u}(e^*) = u(e^*)$ and $u_{e_i}(e^*) > \tilde{u}_{e_i}(e^*)$, where $\tilde{u}_{e_i}(e^*) = 0$ since e^* is an equilibrium for bonus $\tilde{b}(y)$.

Now assume, to get a contradiction, that there is $e'_i < e^*_i$ with $u(e'_i, e^*_{-i}) > u(e^*_i, e^*_{-i})$. Then for $\xi(e_i) = u(e_i, e^*_{-i}) - \tilde{u}(e_i, e^*_{-i})$ we have $\xi(e'_i) > \xi(e^*_i) = 0$ and $\xi'(e^*_i) = u_{e_i}(e^*_i, e^*_{-i}) > 0$. Hence by continuity there must be some $e''_i \in (e'_i, e^*_i)$ such that $\xi(e''_i) = 0$ and $\xi'(e''_i) \leq 0$. At e''_i we thus have $u(e''_i, e^*_{-i}) = \tilde{u}(e''_i, e^*_{-i})$ and $u_{e_i}(e''_i, e^*_{-i}) \leq u_{e_i}(e^*_i, e^*_{-i})$. But this contradicts statement (i) in the lemma. This proves (ii) and thus the lemma.

Proof of Proposition 5. Suppose the optimal bonus $\tilde{b}(y)$ is not a hurdle (threshold) bonus, and let $e^* > 0$ be the associated efforts. So $u_{e_i}(\tilde{b}; e^*) = 0$ by FOC. Let $b = \frac{\delta}{1-\delta} W(e^*_i)$, and let b_h be a symmetric hurdle scheme (with $0 \leq b_h(y) \leq b$), with the same utility as \tilde{b} ; i.e. $u(\tilde{b}; e^*) = u(b_h; e^*)$, and hence $u_{e_i}(b_h; e^*) > u_{e_i}(\tilde{b}; e^*) = 0$ by Lemma 2. Let y_0 be the threshold for b_h . The idea of the proof is to modify this threshold (to $y_0 - \tau_0$) such that e^* gets to be an equilibrium for the modified threshold bonus

To show this, note that for a bonus with threshold $y'_0 = y_0 - \tau$ an agent's expected bonus payment is $b \Pr(y > y'_0 | e)$, and that for $y \sim N(\Sigma e_i, s^2)$ the agent's expected payoff (excluding the fixed salary) can be written, for $e_{-i} = e^*_{-i}$ as

$$u(\tau, e_i, e^*_{-i}) = b(1 - H(y_0^* - \tau - e_i)) - c(e_i), \quad y_0^* = y_0 - (n-1)e_i^*,$$

where $H(\cdot)$ is the CDF for $N(0, s^2)$. For $\tau = 0$ the threshold is that of b_h (i.e. y_0) and we have by Lemma 2

$$u(0, e_i, e^*_{-i}) \leq u(0, e_i^*, e^*_{-i}) \quad \text{for all } e_i < e_i^*, \quad (15)$$

and $0 < u_{e_i}(0, e_i^*, e_{-i}^*) = bh(y_0^* - e_i^*) - c'(e_i^*)$, where $h() = H'()$ is the normal density. Now define $\tau_0 > 0$ such that

$$u_{e_i}(\tau_0, e_i^*, e_{-i}^*) = h(y_0^* - \tau_0 - e_i^*) - c'(e_i^*) = 0 \quad \text{and} \quad y_0^* - \tau_0 - e_i^* < 0 \quad (16)$$

This is feasible because by the shape of $h()$, if $h(x) > C > 0$, then there is $\tau_0 > 0$ such that $h(x - \tau_0) = C$ and $x - \tau_0 < 0$. Note that this implies $h(y_0^* - \tau_0 - e_i) < h(y_0^* - \tau_0 - e_i^*)$ and thus $u_{e_i}(\tau_0, e_i, e_{-i}^*) < 0$ for $e_i > e_i^*$. No deviation to $e_i > e_i^*$ can therefore be profitable.

Next, if $2(y_0^* - \tau_0) > e_i^*$ define $e_i' \in (0, e_i^*)$ by

$$y_0^* - \tau_0 - e_i' = -(y_0^* - \tau_0 - e_i^*) > 0 \quad (17)$$

and note that this implies (by the shape of $h()$):

$$h(y_0^* - \tau_0 - e_i) > h(y_0^* - \tau_0 - e_i^*) \quad \text{for} \quad e_i \in (e_i', e_i^*) \quad (18)$$

This in turn implies, since $h(y_0^* - \tau_0 - e_i^*) = c'(e_i^*) > c'(e_i)$ for $e_i < e_i^*$, that we have $u_{e_i}(\tau_0, e_i, e_{-i}^*) > u_{e_i}(\tau_0, e_i^*, e_{-i}^*) = 0$ and hence

$$u(\tau_0, e_i, e_{-i}^*) < u(\tau_0, e_i^*, e_{-i}^*) \quad \text{for} \quad e_i \in [e_i', e_i^*) \quad (19)$$

If $2(y_0^* - \tau_0) \leq e_i^*$ define $e_i' = 0$, and it is then straightforward to see that (18) and hence (19) holds for that case as well. In that case the proof is then complete since (19) implies that no deviations to $e_i < e_i^*$ can be profitable.

For the case $e_i' > 0$, define, for $e_i < e_i'$ and $\tau \in [0, \tau_0]$ the payoff difference

$$\Delta(\tau, e_i) = u(\tau, e_i^*, e_{-i}^*) - u(\tau, e_i, e_{-i}^*)$$

By (15) we know that for $\tau = 0$ we have $\Delta(0, e_i) \geq 0$ for all $e_i \leq e_i' < e_i^*$.

Let now $\tau \in (0, \tau_0)$, and consider

$$\frac{\partial \Delta(\tau, e_i)}{\partial \tau} = bh(y_0^* - \tau - e_i^*) - bh(y_0^* - \tau - e_i)$$

For $\tau < \tau_0$ and $e_i < e_i'$ we have $y_0^* - \tau - e_i > y_0^* - \tau_0 - e_i' > 0$ (see (17)) and hence $h(y_0^* - \tau - e_i) < h(y_0^* - \tau_0 - e_i')$. Thus we have

$$\begin{aligned} \frac{\partial \Delta(\tau, e_i)}{\partial \tau} \frac{1}{b} &> h(y_0^* - \tau - e_i^*) - h(y_0^* - \tau - e_i') \\ &= h(y_0^* - \tau_0 - e_i^* + (\tau_0 - \tau)) - h(y_0^* - \tau_0 - e_i' + (\tau_0 - \tau)) \end{aligned}$$

Note that by (17) the last difference can be written as $h(-x+z) - h(x+z)$ with $x, z > 0$, and this difference is thus positive (by the shape of $h(\cdot)$). Since $\frac{\partial \Delta(\tau, e_i)}{\partial \tau} > 0$ we then have, for $e_i \leq e'_i$:

$$\Delta(\tau_0, e_i) = u(\tau_0, e_i^*, e_{-i}^*) - u(\tau_0, e_i, e_{-i}^*) > u(0, e_i^*, e_{-i}^*) - u(0, e_i, e_{-i}^*)$$

It now follows from (15) that $u(\tau_0, e_i^*, e_{-i}^*) > u(\tau_0, e_i, e_{-i}^*)$ for $e_i \leq e'_i$. This completes the proof that e^* is a (symmetric) equilibrium for the modified bonus with threshold $y_0 - \tau_0$.

Proof of Proposition 6. As noted in the text, an agent's payoff has two local maxima, at e_i^* and at $e_i^0 < e_i^*$, respectively, and e_i^0 gives the highest payoff for sufficiently small s . The critical s is where the two local maxima yield the same payoff; i.e. $b(1 - H(0; s)) - c(e_i^*) = b(1 - H(e_i^* - e_i^0; s)) - c(e_i^0)$, where $\Pr(y > y_0 | e_i, e_{-i}^*) = 1 - H(e_i^* - e_i; s)$ and $H(\cdot; s)$ is the CDF for an $N(0, s^2)$ variable. In addition they both satisfy FOC, so $bh(e_i^* - e_i^0; s) = c'(e_i^0)$ and $bh(0; s) = c'(e_i^*)$.

For s below this critical level, the agent's payoff is higher at e_i^0 . This can be rectified by setting a lower threshold $y'_0 < y_0 = ne_i^*$, and at the same time increase the bonus level. For $y'_0 = y_0 - \tau$ we have

$$\Pr(y > y'_0 | e_{-i}^*, e_i) = 1 - H(e_i^* - e_i - \tau; s)$$

We can then choose τ and the bonus b such that e_i^* satisfies FOC and yields a payoff at least as high as the other local maximum e_i^0 , i.e. such that we have

$$b(1 - H(-\tau; s)) - c(e_i^*) \geq b(1 - H(e_i^* - e_i^0 - \tau; s)) - c(e_i^0) \quad (20)$$

and

$$bh(-\tau; s) - c'(e_i^*) = 0 = bh(e_i^* - e_i^0 - \tau; s) - c'(e_i^0) \quad (21)$$

The smaller τ is, the smaller is the required bonus to satisfy FOC for e_i^* . The minimal such τ yields equality between the payoffs. Now, this scheme can at most allow a bonus

$$b \leq \frac{\delta}{1 - \delta} W(e_i^*) \quad (22)$$

Hence, we see that the highest effort e_i that can be implemented by this

scheme is the effort e_i^* defined by the conditions (20 - 22), where all hold with equality. We now show that this is indeed the optimal scheme for s below the critical level where FOA ceases to be valid.

We have $H(x; s) = \Phi(\frac{x}{s})$, and $h(x; s) = \phi(\frac{x}{s})\frac{1}{s}$ where $\Phi()$ is the N(0,1) CDF and $\phi()$ its density. The relations (20 - 22) can then be written as

$$b(1 - \Phi(\frac{-\tau}{s})) - c(e_i^*) \geq b(1 - \Phi(\frac{e_i^* - e_i^0 - \tau}{s})) - c(e_i^0) \quad (23)$$

$$b\phi(\frac{-\tau}{s})\frac{1}{s} - c'(e_i^*) = 0 = b\phi(\frac{e_i^* - e_i^0 - \tau}{s})\frac{1}{s} - c'(e_i^0) \quad (24)$$

$$b \leq \frac{\delta}{1 - \delta} W(e_i^*) \quad (25)$$

For $c''' \geq 0$, so $c'(e_i)$ is convex, there can at most be two local maxima (e_i^* and e_i^0) for the agent's payoff. Note that for the minimal $s = s_c$ for which the FOA is valid, all relations (20 - 22) hold with equality, and $\tau = 0$. Denote the associated effort and bonus by $e_i^* = e_c^*$ and $b = b_c$, respectively. For $s < s_c$ the optimal threshold must be some $y'_0 \neq ne_i^*$, thus $y'_0 = ne_i^* - \tau$, $\tau \neq 0$. We show below that $\tau > 0$, as assumed in the text, is optimal.

First we show that *for an optimal $\tau > 0$ all constraints must bind*. To see this, define Δ as the difference in payoffs between e_i^* and e_i^0 , i.e. from (23);

$$\Delta = b(\Phi(\frac{e_i^* - e_i^0 - \tau}{s}) - \Phi(\frac{-\tau}{s})) - (c(e_i^*) - c(e_i^0)), \quad (26)$$

and note that Δ is increasing in b and in τ . This is so because (by the envelope property) $\frac{d\Delta}{db} = \Phi(\frac{e_i^* - e_i^0 - \tau}{s}) - \Phi(\frac{-\tau}{s}) > 0$ and $\frac{d\Delta}{d\tau} s = c'(e_i^*) - c'(e_i^0) > 0$. But then, if the EC constraint (25) does not bind, we can increase b without violating the payoff constraint (23), since $\frac{d\Delta}{db} > 0$. The higher bonus will induce higher effort e_i^* (by FOC), hence EC must bind in optimum.

If the payoff constraint (23) does not bind, then by reducing τ , keeping b fixed, effort e_i^* will increase (by FOC), and the EC constraint (25) will be relaxed. The payoff constraint (23) must therefore also bind in optimum.

Now we show that $\tau < 0$ *cannot be optimal*. Suppose it is, i.e. that for some $s < s_c$ a hurdle $y'_0 = y_0 - \tau'$ with $\tau' < 0$ is optimal. The optimal bonus b and effort e_i^* must satisfy FOC. Note that the FOC for e_i^* will also be satisfied

for $\tau'' = -\tau' > 0$, because $\phi(\frac{-\tau}{s}) = \phi(\frac{\tau}{s})$. Then, since $\frac{d\Delta}{d\tau} > 0$, the payoff difference Δ will be strictly higher for $\tau = \tau'' > 0$ than for $\tau' < 0$. But then e_i^* is a strict optimum for the agent ($\Delta > 0$) for $\tau = \tau'' > 0$, and in such a case it is, as we have seen above, possible to implement an even higher effort by, say, increasing the bonus somewhat. A hurdle $y'_0 = y_0 - \tau'$ with $\tau' < 0$ can thus not be optimal.

We now show that *effort e_i^* is higher when s is lower*. To this end fix $s_a < s_c$, and let the optimal effort, bonus and hurdle parameter be $e_i^* = e_a^*$, $b = b_a$ and $\tau = \tau_a$, respectively. Then $\Delta = 0$ and EC (25) binds. We first show that for $s < s_a$ effort $e_i^* = e_a^*$ can be implemented with $b = b_a$, and a suitable choice of τ . Indeed, fix $e_i^* = e_a^*$ and $b = b_a$, and let $\tau(s)$ and $e_i^0(s)$ be defined by the FOCs (24) for e_i^* and e_i^0 , respectively. For $s = s_a$ we have $\tau = \tau_a$ and all relations hold with equality. We show below (see (27)) that the payoff difference $\Delta = \Delta(\tau(s), e_i^0(s))$ satisfies $\frac{d\Delta}{ds} < 0$ (keeping $e_i^* = e_a^*$ and $b = b_a$ fixed). This implies that $e_i^* = e_a^*$ can be implemented with $b = b_a$ and $\tau = \tau(s)$ when $s < s_a$, and that the associated payoff difference is then strictly positive ($\Delta > 0$). But in such a case we can, as shown above, implement a strictly higher effort $e_i^* > e_a^*$. This shows that for $s < s_a$ optimal effort is $e_i^* > e_a^*$, as was to be shown.

Finally we show that *in the limit we have $e_i^* \rightarrow e_u^*$ as $s \rightarrow 0$* . For suppose that (at least along a subsequence) $e_i^* \rightarrow e_l^* < e_u^*$ as $s \rightarrow 0$. Note that we then must have $\frac{\tau}{s} \rightarrow \infty$ as $s \rightarrow 0$. For if not, then $b \rightarrow 0$ by FOC for e_i^* in (24), which implies a negative payoff at e_i^* . For the same reason we must also have $\frac{e_i^* - e_i^0 - \tau}{s} \rightarrow \infty$. Then we must have $e_i^0 \rightarrow e_l^0 = 0$ as $s \rightarrow 0$, for otherwise the payoff at e_i^0 would converge to $-c(e_l^0) < 0$. This is impossible, since the payoff at e_i^0 exceeds that at $e_i = 0$, and hence must be non-negative.

Taking limits in the first relation (23) with equality, we then get $\lim b \cdot 1 - c(e_l^*) = 0$, and hence from the last equation (for b) that $c(e_l^*) = \frac{\delta}{1-\delta} W(e_l^*)$. This cannot hold for $e_l^* < e_u^*$, hence we must have $e_l^* = e_u^*$.

It remains to prove $\frac{d\Delta}{ds} < 0$, where Δ is given by (26), $\tau = \tau(s)$ and $e_i^0 = e_i^0(s)$ are given by the FOCs in (24), and b and e_i^* are kept fixed ($e_i^* = e_a^*$, $b = b_a$).

In fact, we will show that

$$\frac{d\Delta}{ds} = (c'(e_i^*) - c'(e_i^0))\left(-\frac{s}{\tau}\right) - c'(e_i^0)\frac{e_i^* - e_i^0}{s} < 0 \quad (27)$$

To this end, using the FOC (24) we find, for the payoff at e_i^0 :

$$\frac{d}{ds} \left(b(1 - \Phi(\frac{e_i^* - e_i^0 - \tau}{s})) - c(e_i^0) \right) = c'(e_i^0) \left(\frac{d\tau}{ds} + \frac{e_i^* - e_i^0 - \tau}{s} \right)$$

Similarly, for the payoff at e_i^* we find

$$\frac{d}{ds} \left(b(1 - \Phi(\frac{-\tau}{s})) - c(e_i^*) \right) = c'(e_i^*) \left(\frac{d\tau}{ds} - \frac{\tau}{s} \right)$$

Hence

$$\frac{d\Delta}{ds} = (c'(e_i^*) - c'(e_i^0))\left(\frac{d\tau}{ds} - \frac{\tau}{s}\right) - c'(e_i^0)\frac{e_i^* - e_i^0}{s}$$

From the FOCs (24) and the fact that $\phi'(z) = -z\phi(z)$ we obtain by differentiation $(\frac{d\tau}{ds} - \frac{\tau}{s}) = -\frac{s}{\tau}$. This proves (27), and thus completes the proof.

Proof of Proposition 7. From (10) with $\gamma = 0$ we have

$$\frac{\partial}{\partial p_1} P(y^n = k) = \left[\binom{n-1}{k-1} (1 - p_1) - \binom{n-1}{k} p_1 \right] p_1^{k-1} (1 - p_1)^{n-1-k}$$

The square bracket equals $\binom{n-1}{k-1} (k - np_1) \frac{1}{k}$, and hence $\frac{\partial}{\partial p_1} P(y^n = k) > 0$ for $k > np_1$, so it is optimal to award the bonus for all such outcomes. This verifies (12).

Consider now $p_1 \in \left(\frac{k}{n+1}, \frac{k+1}{n+1} \right]$ for $k = 0, \dots, n$. For team size $n+1$ we there have

$$m^{n+1}(p_1) = \frac{\partial}{\partial p_1} P(y^{n+1} \geq k+1) = P(y^n = k), \quad p_1 \in \left(\frac{k}{n+1}, \frac{k+1}{n+1} \right]. \quad (28)$$

Note that $\frac{k}{n+1} < \frac{k}{n} < \frac{k+1}{n+1}$. Consider first $p_1 \in \left(\frac{k}{n}, \frac{k+1}{n+1} \right]$. There we have

$$m^n(p_1) = \frac{\partial}{\partial p_1} P(y^n \geq k+1) = P(y^{n-1} = k)$$

hence

$$m^{n+1}(p_1) - m^n(p_1) = \left[\binom{n}{k} (1 - p_1) - \binom{n-1}{k} \right] p_1^k (1 - p_1)^{n-1-k} < 0$$

where the inequality follows from the square bracket being equal to $\binom{n-1}{k} \frac{k - np_1}{n-k}$, and $p_1 > \frac{k}{n}$.

Consider next $p_1 \in \left(\frac{k}{n+1}, \frac{k}{n}\right]$, where we have $m^n(p_1) = \frac{\partial}{\partial p_1} P(y^n \geq k) = P(y^{n-1} = k-1)$, and hence

$$m^{n+1}(p_1) - m^n(p_1) = \left[\binom{n}{k} p_1 - \binom{n-1}{k-1}\right] p_1^{k-1} (1-p_1)^{n-k}$$

The square bracket equals $\left[\frac{n}{k} p_1 - 1\right] \binom{n-1}{k-1} \leq 0$, and the inequality is strict except at $p_1 = \frac{k}{n}$. This completes the proof.

Verification of (8) Let here $P(x_1 \dots x_n)$ denote the joint probability that n outputs take values $x_1 \dots x_n$, with $x_i \in \{0, 1\}$. No second- or higher-order additive interaction entails that we have (Gupta-Tao 2010)

$$\frac{P(x_1, x_2, \dots, x_n)}{P(x_1)P(x_2) \dots P(x_n)} = \sum_{1 \leq i, j \leq n} \frac{P(x_i, x_j)}{P(x_i)P(x_j)} - \frac{n(n-1)}{2} + 1$$

Assuming this holds for all $n > 1$, Gupta-Tao (2010) showed that formula (8) then follows, with coefficients $a_{n,r}^j(p_1 \dots p_{n-1})$ given by an inductive formula, see their Theorem 2.2.3 p.67. (Gupta-Tao considered the case of constant pairwise correlations, i.e. $\gamma(p_i, p_j) = \phi \sqrt{p_i(1-p_i)p_j(1-p_j)}$ but their arguments do not depend on this specification.) For $p_1 = \dots = p_n$ the coefficients $a_{n,r}^j$ are for $n > 2$ given by

$$a_{n,r}^j = p_1 a_{n-1,r-1}^{j-1} + (1-p_1) a_{n-1,r}^{j-1} \quad \text{if } j = n-1 \quad (29)$$

$$a_{n,r}^j = p_1 a_{n-1,r-1}^j + (1-p_1) a_{n-1,r}^j \quad \text{if } j = 1, 2, \dots, n-2 \quad (30)$$

where $a_{n,r}^j = 0$ if $r < 0$ or $r > n$, while for $n = 2$ we have

$$a_{2,0}^1 = 1, \quad a_{2,1}^1 = -2, \quad a_{2,2}^1 = 1$$

This verifies (8), based on Gupta-Tao (2010).

For later use, we note here that for symmetric p_i 's the coefficients $a_{n,r}^j$ enter (8) via the sums $\sum_{j=1}^{n-1} a_{n,r}^j \equiv A_r^n$. From (29-30) we find, for $n = 3$ (see also Gupta-Tao 2010, p. 64):

$$A_0^3 = 2(1-p_1), \quad A_1^3 = 2(3p_1-2), \quad A_2^3 = 2(1-3p_1), \quad A_3^3 = 2p_1 \quad (31)$$

From (9) we then obtain, for $n = 3$ (when $\gamma_1 \equiv 0$)

$$\frac{\partial}{\partial p_1} P(y^3 = 3) = p_1^2 + \gamma$$

$$\begin{aligned}\frac{\partial}{\partial p_1}P(y^3 = 2) &= p_1(2 - 3p_1) - 3\gamma \\ \frac{\partial}{\partial p_1}P(y^3 = 1) &= (1 - p_1)(1 - 3p_1) + 3\gamma \\ \frac{\partial}{\partial p_1}P(y^3 = 0) &= -(1 - p_1)^2 - \gamma\end{aligned}$$

Proof of Proposition 8. It follows from (8) that all probabilities $P(y^n = r)$ are linear in γ , and for symmetric (equal) p_i 's can be written as

$$P(y^n = r) = B_r^n(p_1) + \gamma C_r^n(p_1), \quad (32)$$

where $B_r^n(p_1)$ is a standard binomial probability for iid variables (corresponding to $\gamma = 0$). For $n = 2$ it follows directly from (7) that $C_0^2 = C_2^2 = 1$, $C_1^2 = -2$. For $n \geq 3$ we obtain the following result, which we prove below.

Lemma A. (i) For $C_r^n(p_1)$ defined in (32) we have for $n = 3$; $C_r^3(p_1) = \frac{3}{2}A_r^3(p_1)$, and for $n \geq 4$:

$$\begin{aligned}C_r^n(p_1) &= \xi_n p_1^{r-2} (1 - p_1)^{n-2-r} \mu_r^n(p_1) & 2 \leq r \leq n - 2 \\ C_r^n(p_1) &= \xi_n p_1^{r-2} \mu_r^n(p_1) & r = n - 1, n \\ C_r^n(p_1) &= \xi_n (1 - p_1)^{n-2-r} \mu_r^n(p_1) & r = 0, 1\end{aligned}$$

where $\xi_n = \frac{1}{2}n(n - 1)$ and $\mu_r^n(p_1)$ is given as follows: $\mu_0^n(p_1) = \mu_n^n(p_1) = 1$, and

$$\mu_r^n(p_1) = \binom{n}{r} p_1^2 - 2 \binom{n-1}{r-1} p_1 + \binom{n-2}{r-2}, \quad 2 \leq r \leq n - 2 \quad (33)$$

$$\mu_1^n(p_1) = np_1 - 2, \quad \mu_{n-1}^n(p_1) = n(1 - p_1) - 2 \quad (34)$$

(ii) Moreover, for $n \geq 2$ we have $C_r^n(p_1) < c_r^n < 0$ for $p_1 \in \left[\frac{r}{n+1}, \frac{r+1}{n+1}\right]$, all $r = 1, \dots, n - 1$, and $C_r^n(p_1) > c_r^n > 0$ for $p_1 \in \left[\frac{r}{n+1}, \frac{r+1}{n+1}\right]$, $r = 0, n$.

Now, for $\gamma = 0$ the optimal bonus scheme is a hurdle scheme with $m^n(p_1) = \frac{\partial}{\partial p_1}P(y^n \geq k)$ for $p_1 \in \left(\frac{k-1}{n}, \frac{k}{n}\right)$, see (12). The model is continuous in γ , hence for $|\gamma|$ small, such a hurdle scheme is still optimal, and thus we have

$$m^n(p_1; \gamma) = \frac{\partial}{\partial p_1}P(y^n \geq k) = P(y^{n-1} = k - 1) = B_{k-1}^{n-1}(p_1) + \gamma C_{k-1}^{n-1}(p_1) \quad (35)$$

for $p_1 \in (\pi_{k-1}^n, \pi_k^n)$, where $\pi_r^n = \pi_r^n(\gamma) \rightarrow \frac{r}{n}$ as $\gamma \rightarrow 0$. (The second equality in (35) follows from (10) and the third from (32).)

From Lemma A(ii) we have

$$\begin{aligned} C_{k-1}^{n-1}(p_1) &< c_{k-1}^{n-1} < 0 \quad \text{for } p_1 \in \left[\frac{k-1}{n}, \frac{k}{n}\right], \text{ all } k = 2, \dots, n-1 \\ C_{k-1}^{n-1}(p_1) &> c_{k-1}^{n-1} > 0 \quad \text{for } p_1 \in \left[\frac{k-1}{n}, \frac{k}{n}\right], k = 1, n \end{aligned}$$

This implies that for $|\gamma|$ small, $m^n(p_1; \gamma)$ is strictly decreasing in γ for $p_1 \in (\pi_1^n, \pi_{n-1}^n)$, and strictly increasing in γ for $p_1 < \pi_1^n$ or $p_1 > \pi_{n-1}^n$, where $\pi_1^n \rightarrow \frac{1}{n}$ and $\pi_{n-1}^n \rightarrow \frac{n-1}{n}$ as $\gamma \rightarrow 0$. This completes the proof.

Proof of Lemma A(i). Since all probabilities are linear in γ , it follows from (8) and (32) that we have, under symmetry (all p_i equal):

$$C_r^n(p_1) = p_1 C_{r-1}^{n-1}(p_1) + (1-p_1) C_r^{n-1}(p_1) + \sum_{j=1}^{n-1} a_{n,r}^j(p_1)$$

with $C_r^l(p_1) = 0$ if $r < 0$ or $r > l$. From this relation and (29 - 30) we obtain, by straightforward induction, the following:

$$A_r^n(p_1) \equiv \sum_{j=1}^{n-1} a_{n,r}^j(p_1) = (n-1)a_{n,r}^{n-1}(p_1) \quad \text{and} \quad C_r^n(p_1) = \xi_n a_{n,r}^{n-1}(p_1), \quad \xi_n = n(n-1)/2 \quad (36)$$

and, for $n \geq 4$,

$$\begin{aligned} a_{n,r}^{n-1}(p_1) &= p_1^{r-2} (1-p_1)^{n-2-r} \mu_r^n(p_1) & 2 \leq r \leq n-2 \\ a_{n,r}^{n-1}(p_1) &= p_1^{r-2} \mu_r^n(p_1) & r = n-1, n \\ a_{n,r}^{n-1}(p_1) &= (1-p_1)^{n-2-r} \mu_r^n(p_1) & r = 0, 1 \end{aligned}$$

where $\mu_r^n(p_1)$ is a polynomial of degree 2, except for $r = 1, n-1$, where $\mu_r^n(p_1)$ is linear and given by (34), and for $r = 0, n$, where $\mu_r^n(p_1) = 1$; and moreover,

$$\mu_2^n(p_1) = p_1 \mu_1^{n-1}(p_1) + \mu_2^{n-1}(p_1) \quad (37)$$

$$\mu_r^n(p_1) = \mu_{r-1}^{n-1}(p_1) + \mu_r^{n-1}(p_1), \quad 3 \leq r \leq n-2 \quad (38)$$

Given these relations and the formula for $C_r^n(p_1)$ in (36), it only remains to verify (33) to complete the proof of the lemma. Given that (33) holds for $n-1$, it follows from (37) and (34) that we have, for $r = 2$

$$\begin{aligned}\mu_2^n(p_1) &= p_1((n-1)p_1 - 2) + \binom{n-1}{2}p_1^2 - 2\binom{n-2}{2-1}p_1 + \binom{n-3}{2-2} \\ &= \frac{n(n-1)}{2}p_1^2 - 2(n-1)p_1 + 1\end{aligned}$$

where the last equality follows by collecting terms. This verifies (33) for $r = 2$.

Similarly we have from (38) and (33), for $3 \leq r \leq n-2$:

$$\begin{aligned}\mu_r^n(p_1) &= \left(\binom{n-1}{r-1} + \binom{n-1}{r}\right)p_1^2 - 2\left(\binom{n-2}{r-2} + \binom{n-2}{r-1}\right)p_1 + \binom{n-3}{r-3} + \binom{n-3}{r-2} \\ &= \binom{n}{r}p_1^2 - 2\binom{n-1}{r-1}p_1 + \binom{n-2}{r-2},\end{aligned}$$

where the last equality follows from $\binom{l}{k-1} + \binom{l}{k} = \binom{l+1}{k}$. This verifies (33) for $3 \leq r \leq n-2$, and completes the proof of Lemma A (i).

Proof Lemma A(ii). For $n = 2, 3$ the statement is verified directly. For $n \geq 4$ we now *claim* that there are numbers κ_r^n , such that $\mu_r^n(p_1) < \kappa_r^n < 0$ for $p_1 \in \left[\frac{r}{n+1}, \frac{r+1}{n+1}\right]$, all $r = 1, \dots, n-1$, and $\mu_r^n(p_1) > \kappa_r^n > 0$ for $p_1 \in \left[\frac{r}{n+1}, \frac{r+1}{n+1}\right]$, $r = 0, n$. The statement in (ii) then follows from the expressions for $C_r^n(p_1)$ in part (i) of the lemma.

First note that, since $\mu_0^n(p_1) = \mu_n^n(p_1) = 1$, the claim is trivially true for $r = 0, n$

For $r = 1$ and $p_1 \in \left[\frac{r}{n+1}, \frac{r+1}{n+1}\right]$ we have

$$\mu_1^n(p_1) \leq \mu_1^n\left(\frac{2}{n+1}\right) = n\frac{2}{n+1} - 2 = -\frac{2}{n+1} \equiv \kappa_1^n < 0,$$

and for $r = n-1$ and $p_1 \in \left[\frac{r}{n+1}, \frac{r+1}{n+1}\right]$

$$\mu_{n-1}^n(p_1) \leq \mu_{n-1}^n\left(\frac{n-1}{n+1}\right) = n\left(1 - \frac{n-1}{n+1}\right) - 2 = -\frac{2}{n+1} \equiv \kappa_{n-1}^n < 0$$

For $2 \leq r \leq n-2$, $\mu_r^n(p_1)$ is a strictly convex quadratic function, and hence bounded above for all $p_1 \in \left[\frac{r}{n+1}, \frac{r+1}{n+1}\right]$ by its largest value at the endpoints. The claim regarding $\mu_r^n(p_1) < \kappa_r^n < 0$ on $\left[\frac{r}{n+1}, \frac{r+1}{n+1}\right]$ is then true if the endpoint values are negative. Checking this we find that $\mu_r^n\left(\frac{r}{n+1}\right)$ is proportional to

$$n(n-1)p_1^2 - 2r(n-1)p_1 + r(r-1) = \frac{r}{(n+1)^2} \left((3+n)r - (n+1)^2 \right)$$

The last parenthesis is increasing in r and strictly negative (equal to -4) for $r = n-1$.

Next, checking $p_1 = \frac{r+1}{n+1}$ we see that $\mu_r^n(\frac{r+1}{n+1})$ is proportional to

$$n(n-1)p_1^2 - 2r(n-1)p_1 + r(r-1) = \frac{(n-r)}{(n+1)^2}(n-1-(3+n)r)$$

The last parenthesis is decreasing in r and is strictly negative for $r = 2$. This completes the proof of the claim, and hence the lemma.

Proof of Proposition 9. Since all probabilities are linear in γ , so are m^{n+1} and m^n , and hence also the difference

$$\tilde{\Delta}^n(p_1; \gamma) \equiv m^{n+1}(p_1; \gamma) - m^n(p_1; \gamma) = \Delta_0^n(p_1) + \gamma \Delta^n(p_1) \quad (39)$$

For $\gamma = 0$ we know that the difference (given by $\Delta_0^n(p_1)$) is strictly negative except at $p_1 = \frac{k}{n}$, $k = 1, \dots, n-1$, where it is zero. We will now show that $\Delta^n(p_1) < 0$ at $p_1 = \frac{k}{n}$, $k = 1, \dots, n-1$. It then follows that for $|\gamma|$ small, we will have $\tilde{\Delta}^n(p_1; \gamma) > 0$ in a neighborhood of $p_1 = \frac{k}{n}$ if $\gamma < 0$, but $\tilde{\Delta}^n(p_1; \gamma) < 0$ for all p_1 if $\gamma > 0$. The statement in the proposition follows from this.

So consider $\Delta^n(p_1)$ defined by (39), evaluated at $p_1 = \frac{k}{n}$. For $\gamma = 0$ the optimal bonus scheme for team size n is a hurdle scheme where the hurdle shifts from k (for $p_1 \leq \frac{k}{n}$) to $k+1$ (for $p_1 > \frac{k}{n}$). For $|\gamma|$ small, the optimal scheme will also be a hurdle scheme, but it will have hurdle k or $k+1$ at $p_1 = \frac{k}{n}$ depending on the sign of

$$\frac{\partial}{\partial p_1} P(y^n = k) = P(y^{n-1} = k-1) - P(y^{n-1} = k) = \gamma(C_{k-1}^{n-1}(p_1) - C_k^{n-1}(p_1)), \quad p_1 = \frac{k}{n} \quad (40)$$

(Here we have used (32), and the fact that $B_{k-1}^{n-1}(p_1) = B_k^{n-1}(p_1)$ at $p_1 = \frac{k}{n}$.) The hurdle for the team of size n will be k iff the expression in (40) is positive, otherwise it will be $k+1$.

For the team of size $n+1$ the optimal scheme has hurdle $k+1$ at $p_1 = \frac{k}{n}$ for $|\gamma|$ small. This holds for $|\gamma|$ small because it is true for $\gamma = 0$, and because $\frac{\partial}{\partial p_1} P(y^{n+1} = k)$ is strictly negative at $p_1 = \frac{k}{n}$ when $\gamma = 0$. (For $\gamma = 0$ the optimal scheme has hurdle $k+1$ for $p_1 \in (\frac{k}{n+1}, \frac{k+1}{n+1})$.)

At $p_1 = \frac{k}{n}$ we thus have

$$m^{n+1}(p_1; \gamma) - m^n(p_1; \gamma) = \frac{\partial}{\partial p_1} P(y^{n+1} \geq k+1)$$

$$-\left(\frac{\partial}{\partial p_1}P(y^n \geq k+1) + \left[\frac{\partial}{\partial p_1}P(y^n = k)\right]^+\right),$$

where we have used the notation $[x]^+ = \max\{x, 0\}$. Using this relation, (32), (35) and (39), plus the fact that $\Delta_0^n(p_1) = 0$ at $p_1 = \frac{k}{n}$, we obtain

$$\tilde{\Delta}^n(p_1; \gamma) = \gamma(C_k^n(p_1) - C_k^{n-1}(p_1)) - [\gamma(C_{k-1}^{n-1}(p_1) - C_k^{n-1}(p_1))]^+, \quad p_1 = \frac{k}{n} \quad (41)$$

Claim: at $p_1 = \frac{k}{n}$ we have, for $k = 1, 2, \dots, n-1$

$$C_k^n(p_1) - C_k^{n-1}(p_1) < 0$$

$$C_k^n(p_1) - C_k^{n-1}(p_1) - (C_{k-1}^{n-1}(p_1) - C_k^{n-1}(p_1)) \equiv D_k^n < 0$$

We prove this claim below. It follows from the claim that for $\gamma > 0$ we have $\tilde{\Delta}^n(p_1; \gamma) \leq \gamma(C_k^n(p_1) - C_k^{n-1}(p_1)) < 0$ at $p_1 = \frac{k}{n}$. For $\gamma < 0$ and $p_1 = \frac{k}{n}$ we have $\tilde{\Delta}^n(p_1; \gamma) = \gamma(C_k^n(p_1) - C_k^{n-1}(p_1)) > 0$ if $C_{k-1}^{n-1}(p_1) - C_k^{n-1}(p_1) \geq 0$, and $\tilde{\Delta}^n(p_1; \gamma) = \gamma D_k^n > 0$ if $C_{k-1}^{n-1}(p_1) - C_k^{n-1}(p_1) < 0$. This verifies the statements in the paragraph following (39), and hence proves the proposition.

It remains to verify the claim. Using Lemma A we find, for $2 \leq r \leq n-3$:

$$C_r^n(p_1) - C_r^{n-1}(p_1) \propto \xi_n(1-p_1)\mu_r^n(p_1) - \xi_{n-1}\mu_r^{n-1}(p_1) = \frac{-r\Gamma(n)}{n^2\Gamma(n-r)\Gamma(r)}, \quad p_1 = \frac{r}{n}$$

where \propto denotes "proportional to", and $\Gamma(l) = (l-1)!$ for $l = 1, 2, \dots$. This proves the first claim for $2 \leq k \leq n-3$ (details available from the authors).

Using Lemma A again we find, for $3 \leq r \leq n-2$:

$$D_r^n = C_r^n(p_1) - C_{r-1}^{n-1}(p_1) \propto \xi_n p_1 \mu_r^n(p_1) - \xi_{n-1} \mu_{r-1}^{n-1}(p_1) = \frac{-(n-r)\Gamma(n)}{n^2\Gamma(n-r)\Gamma(r)}, \quad p_1 = \frac{r}{n}$$

This proves the second claim for $3 \leq k \leq n-2$ (details available from the authors). The claims can similarly be verified for $k = 1, 2, n-1$ by use of Lemma A.

Proof of Proposition 10. From the formulas following (31) we see that with $\gamma_1 = 0$ and $\gamma > 0$ we have $\frac{\partial}{\partial p_1}P(y^3 = 3) > 0$, $\frac{\partial}{\partial p_1}P(y^3 = 0) < 0$. Moreover, for $\gamma > \frac{1}{9}$ we then also have $\frac{\partial}{\partial p_1}P(y^3 = 1) > 0$, $\frac{\partial}{\partial p_1}P(y^3 = 2) < 0$

for all $p_1 \in (0, 1)$. A bonus scheme that awards for $y^3 \in \{1, 3\}$ is then optimal.

FIGURES

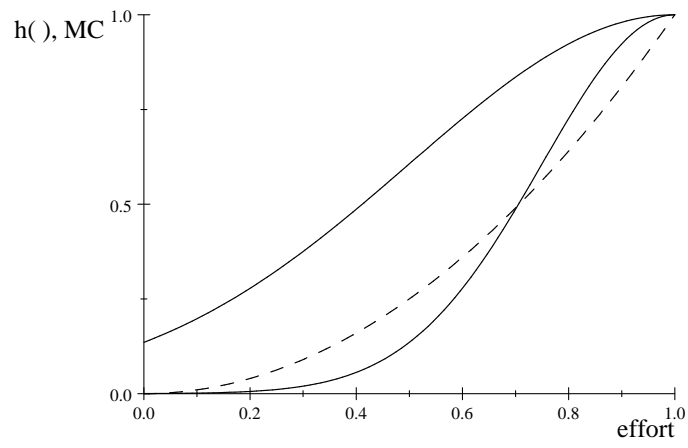


Figure 1. Illustration of FOC