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COOPERATIVE GAME THEORY IN LOCATION-ROUTING

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This thesis was written as a part of the Double Degree programme between NHH MSc in Economics and Business Administration and LSM Master in Business Engineering. Neither the institutions, the supervisors, nor the censors are – through the approval of this thesis – responsible for neither the theories and methods used, nor results and conclusions drawn in this work.

LOUVAIN SCHOOL OF MANAGEMENT
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Abstract

Cooperative game theory in location-routing

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Horizontal collaboration in transportation is attracting increased interest from academics and practitioners. This approach can bring significant savings to the companies and help reduce their carbon footprints. Game theory acts as a useful tool to study horizontal collaboration. The thesis focuses on studying cooperative game theory in location-routing, which to our knowledge has never been done before. Location-routing consists of locating facilities while taking into account routing aspects. The interdependence of location and routing has been acknowledged in the literature. The integrated approach of location-routing can lead to more efficient logistic planning. Location-routing has varied applications ranging from package delivery to spare parts distribution.

In the thesis, we first define this new cooperative game. We study its properties analytically using tools from the operations research literature, and principles and methods from cooperative game theory. We find that this game, while being monotone and subadditive, can have an empty core and be non-convex. We also study this game numerically. 6,000 instances of the game are generated and played. The cost is reduced by 43% on average and emissions by 31%. Emptiness of the core is observed in 44% of the instances.

In collaboration, allocating the costs of the coalition is a major issue. For each instance of the numerical experiment we compute the values obtained by several allocation methods, like the Shapley value and the Equal Profit Method. We then analyze how the cost sharing techniques fare in this new game. Their performance is examined through several criteria like stability, fairness and transparency of the method. There is no method which is best on all the criteria and their performance can vary depending on the properties of the instance. However, we find all cost sharing techniques perform well regarding stability for the location-routing game.

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List of Abbreviations

3PL	**T**hird-**P**arty **L**ogistics
DC	**D**istribution **C**enter
EPM	**E**qual **P**rofit **M**ethod
FTL	**F**ull **T**ruck**L**oad
LP	**L**inear **P**rogram
LR	**L**ocation-**R**outing
LRG	**L**ocation-**R**outing **G**ame
LRP	**L**ocation-**R**outing **P**roblem
LTL	**L**ess than **T**ruck**L**oad
NP	**N**ondeterministic **P**olynomial time
TSP	**T**raveling **S**alesman **P**roblem
TU	**T**ransferable **U**tility
VRG	**V**ehicle-**R**outing **G**ame
VRP	**V**ehicle-**R**outing **P**roblem

Chapter 1

Introduction

The purpose of the dissertation is to study collaborative game theory applied to location-routing. To our knowledge, this is the first work investigating this topic. We define a new cooperative game, the location-routing game, and derive its properties.

The location-routing game can be used to model horizontal collaboration in logistics. Horizontal cooperation in logistics is defined as "active cooperation between two or more firms that operate on the same level of the supply chain and perform a comparable logistics function" (Cruijssen, 2006, p.12). In a study from the European Commission (2015), more than one hundred practitioners active in supply chain management were interviewed. From the interviews, horizontal collaboration is identified as the "major next step" to optimize costs and carbon emissions in supply chains. It is estimated one quarter of trucks ride empty in Europe (European Commission, 2014) and the fill rate of non-empty trucks is below 60% (CO^3 project, 2014). Collaboration between companies can significantly enhance the efficiency of logistics assets, thereby reducing costs and carbon emissions (Cruijssen, 2006). Initiatives to promote horizontal collaboration in logistics have appeared. In 2008, the world's first company specialized in the organization and the management of horizontal alliances in logistics was founded (TRI-VIZOR, 2016). TRI-VIZOR has been elected Belgium's spin-off of the year in 2012 and has won several Supply Chain awards. The company also received the Belgian Business Award for the Environment in 2011, a competition organized by the Federation of Enterprises in Belgium. It acted as a recognition of the role of horizontal collaboration to improve supply chains' sustainability. In parallel, the European Union launched the CO^3 project which aims to advertise horizontal collaboration in logistics (CO^3 project, 2014). Additionally, this topic has become an important field in the transportation literature (J. Li, Cai, & Zeng, 2016). It has particularly attracted the attention of researchers in the last ten years (Guajardo & Rönnqvist, 2016). In the literature, horizontal collaboration is analyzed through concepts from cooperative game theory.

Location-routing is also an interesting topic. In location-routing, decisions on the location of facilities and on routing are made at the same time. This approach can bring significant savings compared to the traditional method where the focus is first on the location problem and then on the routing problem (Salhi & Rand, 1989). Location-routing is used in varied supply chains. The applications range from package delivery to spare parts distribution. As on horizontal collaboration, active research on the location-routing problem has been conducted.

The dissertation extends the literature on horizontal collaboration to the location-routing problem. The facility location problem and the vehicle-routing problem under collaboration have already been studied. However, to apply cooperative game theory to a problem where decisions on the location of facilities and on routing are integrated is completely new. In the dissertation, we also discuss applications of the location-routing game. A case involving cosmetic companies is presented. Furthermore, we study concepts ranging from the core to the convexity of the game for the location-routing game and some of its extensions. We also elaborate an experiment where 6,000 different instances of the game are played. From our analytical and experimental studies, we find that emptiness of the core can occur. In other words, there might not exist a allocation which ensure stability of the coalition. However, when there is collaboration, significant reductions in costs and carbon emissions can be realized. We also find vehicle and facility capacities strongly influence the convexity of the game. Additionally, the values taken by these two parameters sometimes play a significant role in making the core of the game to be empty.

In collaboration, allocating the gains or the costs of the alliance is a crucial problem (Guajardo & Rönnqvist, 2016). This allocation problem can be solved using tools from game theory. In 2012, the Nobel Memorial Prize in Economic Science was awarded to Alvin E. Roth and Lloyd S. Shapley for their work which encompasses a theory of stable allocations. Several allocation methods exist. In the dissertation, we first look at the theoretical properties of these allocations. Second, we further analyze the behavior of the methods taking into account the results of the experiment and the properties of the game.

In short, the dissertation answers to the two following research questions. The first question asks what are the properties of this newly-defined game. The second question is what are the most adequate allocation methods for the game? The dissertation is organized as follows. A literature review on concepts related to the location-routing game is made in chapter 2. We look at the location-routing problem, horizontal cooperation in logistics, cooperative game theory and cost-allocation methods. In chapter 3, we define the game, study applications of this game and analyze its properties. We also look at how these properties change for two extensions of the location-routing game. The experiment and its results are presented in chapter 4 and the conclusion is in chapter 5.

Chapter 2

Literature review of the related concepts

2.1 The location-routing problem

The location-routing problem (LRP) integrates the decisions on the location of facilities and on the vehicle-routes supplying the customers (Prodhon & Prins, 2014). The integrated approach is an important characteristic (Nagy & Salhi, 2007). The decisions on the locations of facilities belong to a strategic level and the decisions on the routing to an operational level. Therefore, the two problems have often been considered independently (Salhi & Rand, 1989). The idea of combining these two problems first appeared more than fifty years ago (Prodhon & Prins, 2014), as in von Boventer (1961) and Maranzana (1964). Some practitioners were already aware that integrating these two problems can bring benefits (Rand, 1976). Indeed Salhi and Rand (1989) show that neglecting the routing problem while solving the location problem does not necessarily lead to the optimal solution. Since this crucial remark, research on the location-routing problem has been really active (Nagy & Salhi, 2007; Prodhon & Prins, 2014).

The location-routing problem belongs to the class of NP-hard problems (Schittekat & Sörensen, 2009). Indeed, both the facility location and the vehicle routing problems are NP-hard (Schittekat & Sörensen, 2009). These two problems are subproblems of the location-routing problem. If the facilities to be open are already determined, the LRP becomes a vehicle routing problem. If the capacity of the vehicles is only sufficient for the demand of one customer, no routes are possible and the LRP becomes a facility location problem.

The difficulty of this problem is also a reason research on the LRP has taken off only recently. Fifty years ago, to integrate the location and routing decisions required optimization techniques and computers that were not yet developed enough (Prodhon & Prins, 2014; Rand, 1976).

Some applications of the location-routing problem The location-routing problem has applications in varied sectors. Schittekat and Sörensen (2009) use a LRP in a case study about automobile spare parts distribution. Chan and Baker (2005) and Murty and

Djang (1999) study applications of the LRP in the military. The LRP is also used in the distribution of perishable food products, as in Govindan, Jafarian, Khodaverdi, and Devika (2014). We can also cite applications in waste management (Caballero, González, Guerrero, Molina, & Paralera, 2007) and blood banking systems (Or & Pierskalla, 1979).

Most articles about the LRP deal with cost minimization (Drexl & Schneider, 2015). Other objectives or combinations of objectives can also be considered. For example, Caballero et al. (2007) take also into account social rejection for the installation of incineration plants. Govindan et al. (2014) minimize a combination of the total cost and the greenhouse gases emissions of the supply chain studied.

It is also worthwhile to remember that the facilities can denote any object to be located (Drexl & Schneider, 2014). The same applies for the customers who can be simple clients of a company or any location that must be visited in a tour. For instance, Ahn, de Weck, Geng, and Klabjan (2012) define a space exploration problem as a LRP. The facilities become potential landing positions and the customers are interesting exploration locations.

Other examples of applications and reviews on the location-routing problem can be found in Drexl and Schneider (2014, 2015), Min, Jayaraman, and Srivastava (1998) and Prodhon and Prins (2014).

2.1.1 Class of the location-routing problem studied

In this section, we begin with what we call the *standard* location-routing problem. Location-routing problems can be classified as in Nagy and Salhi (2007). Using this classification, we define the type of LRP studied. We take into account the difficulty of the problem while choosing the parameters.

1. *Hierarchical structure*. The customers are directly served from central depots by means of vehicle tours. In other words, this is a one-echelon problem as are most location-routing problems studied in the literature.

2. *Type of input data* (deterministic or stochastic). The data is known in advance and is therefore deterministic.

3. *Planning period*. For a multi-period LRP, even instances of small size cannot be solved exactly (Albareda-Sambola, Fernández, & Nickel, 2012). Therefore, we investigate the static case, i.e. with a single period.

4. *Solution method* (exact or heuristic). We decide to keep it simple to ease the interpretation of the results. Therefore, we choose to solve the problem with an exact solution method.

5. *Objective function*. Cost minimization is the objective. This is the most frequent objective in the literature.

6. *Solution space* (discrete, network or continuous). We take a discrete solution space as in most of the LRP literature. The sites for the facilities have to be chosen among a specified subset of vertexes.

7. *Number of depots* (simple or multiple). We will consider both cases in the dissertation. In the first place, we define the standard LRP as a LRP in which multiple depots can be open.

8. *Number and types of vehicles*. A homogeneous fleet, where all the vehicles have the same capacity, is considered. The number of vehicles is not known in advance.

9. *Route structure*. We use the standard case where vehicles follow a route starting from the facility, going through a number of customers nodes and going back to the same facility. Each customer must also be served by one and only one vehicle. In some other types of LRP, vehicles visit edges rather than nodes or the routes contain several facilities.

2.1.2 Mathematical formulation

The problem can be formulated with three indexes as in Tuzun and Burke (1999), Prins, Prodhon, and Calvo (2006) and Prodhon (2006). However alternative formulations also exist and are for instance listed in Laporte, Nobert, and Taillefer (1988) and Prodhon (2006).

Mathematical model with three indexes

Sets

- V: set of all feasible sites and customers (also referred to as nodes);
- G: subset of feasible sites of candidate facilities;
- $I = V \backslash G$: subset of customers to be served;
- K: set of vehicles available for routing from the facilities;

Model parameters

- c_{ij}: average annual cost of traveling from node i to node j, $i \in V$, $j \in V$;
- c: annual cost of acquiring a vehicle;

- f_g: annual cost of establishing and operating a facility at site g, $g \in G$;
- d_i: average number of units demanded by customer i, $i \in I$;
- q: capacity of one vehicle;

Decision variables

$$X_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ goes from node } i \text{ to node } j, \quad i \in V, j \in V, k \in K, i \neq j \\ 0 & \text{otherwise} \end{cases}$$

$$Z_g = \begin{cases} 1 & \text{if a facility is established at site } g, \quad g \in G \\ 0 & \text{otherwise} \end{cases}$$

$$z = \quad \min \quad \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} X_{ijk} + \sum_{k \in K} \left(c \sum_{g \in G} \sum_{j \in I} X_{gjk} \right) + \sum_{g \in G} f_g Z_g$$

subject to

$$\sum_{k \in K} \sum_{i \in V} X_{ijk} = 1 \quad \forall j \in I \tag{2.1}$$

$$\sum_{j \in I} \sum_{i \in V} d_j X_{ijk} \leq q \quad \forall k \in K \tag{2.2}$$

$$\sum_{i \in V} X_{ipk} - \sum_{j \in V} X_{pjk} = 0 \quad \forall p \in V, \forall k \in K \tag{2.3}$$

$$\sum_{g \in G} \sum_{j \in I} X_{gjk} \leq 1 \quad \forall k \in K \tag{2.4}$$

$$\sum_{i \in S} \sum_{j \in S} X_{ijk} \leq |S| - 1 \quad \forall S \subseteq I, \forall k \in K \tag{2.5}$$

$$\sum_{k \in K} X_{gmk} = 0 \quad \forall g, m \in G \tag{2.6}$$

$$\sum_{j \in I} X_{gjk} - Z_g \leq 0 \quad \forall k \in K, \forall g \in G \tag{2.7}$$

$$X_{ijk} = 0 \text{ or } 1 \quad \forall i, j \in V, \forall k \in K \tag{2.8}$$

$$Z_g = 0 \text{ or } 1 \quad \forall g \in G \tag{2.9}$$

The objective function sums the traveling costs, the costs of acquiring the vehicles and the costs of the depots. The constraints (2.1) state that each customer must be served by one vehicle. The constraints (2.2) concern the respect of the vehicles capacities. The constraints (2.3) are the flow conservation constraints. The constraints (2.4) express the fact that each vehicle route cannot contain more than one depot. The constraints (2.3) and (2.4) also make mandatory the fact that each vehicle has to come back to the same depot it departed from. Indeed, if a vehicle stops at a depot different of the one it originates from, the flow conservation constraints (2.3) are not satisfied. The constraints (2.5) are the sub-tour elimination constraints. They forbid that a route is constituted of customers only. The constraints (2.6) express the fact that no link can connect any two depots. We can note that if the average annual cost of traveling between two nodes respect the triangle inequality, this equation becomes unnecessary. Indeed, in that case, there is no incentive to go through a second depot. The triangle inequality means that going directly from one node to another is as cheap or cheaper than going through any intermediate node. The constraints (2.7) ensure that a depot can be used if and only if this depot is open. The constraints (2.8) and (2.9) ensure the variables are binary.

Mathematical model with routes known

In this model, we consider only the subsets of customers for which the sum of the demands is lower than the capacity of one vehicle. We assume that the set of minimum routes for each such customer subset is known before the resolution of the problem. We can for instance formulate a Traveling Salesman problem (TSP) for each subset of customers and for each facility to know the costs ot such minimum routes. Once this is done, we can formulate the standard LRP as follows.

This integer model follows the same strategy as the one formulated by Göthe-Lundgren, Jörnsten, and Värbrand (1996) for the basic vehicle-routing problem. We can compare this formulation to the one of a set-covering problem.

Sets

- I: set of customers to be served;
- R: set of minimum cost routes for each customer subset;
- G: set of feasible sites of candidate facilities;

Model parameters

- a_{irg}: parameter equal to 1 if customer i is covered by a route r leaving from facility g, 0 otherwise, $i \in I$, $r \in R$, $g \in G$;

- c_{rg}: annual cost of a minimum cost route r leaving from facility g, $r \in R$, $g \in G$;
- f_g: annual cost of establishing and operating a facility at site g, $g \in G$;

Decision variables

$$X_{rg} = \begin{cases} 1 & \text{if the route } r \text{ leaving from facility } g \text{ is used,} \quad r \in R, g \in G \\ 0 & \text{otherwise} \end{cases}$$

$$Z_g = \begin{cases} 1 & \text{if a facility is established at site } g, \quad g \in G \\ 0 & \text{otherwise} \end{cases}$$

$$z = \quad \min \quad \sum_{g \in G} \left(f_g Z_g + \sum_{r \in R} c_{rg} X_{rg} \right)$$

subject to

$$\sum_{r \in R} \sum_{g \in G} a_{irg} X_{rg} = 1 \quad \forall i \in I \tag{2.10}$$

$$X_{rg} \leq Z_g \quad \forall r \in R, \forall g \in G \tag{2.11}$$

$$X_{rg} = 0 \text{ or } 1 \quad \forall r \in R, \forall g \in G \tag{2.12}$$

$$Z_g = 0 \text{ or } 1 \quad \forall g \in G \tag{2.13}$$

The constraints (2.10) state that each customer must be served only once. The constraints (2.11) assure that a route can leave a facility only if this facility is open. The constraints (2.12) and (2.13) concern the nature of the variables.

2.2 Horizontal cooperation in logistics

2.2.1 A topic gaining increased interest in the literature and from practitioners

Modern game theory and the study of cooperative games of several players began with the seminal work of von Neumann and Morgenstern (1944). The emphasis is on the coalitions and their outcomes and the interrelationships between the coalitions (von Neumann & Morgenstern, 1944).

However, as pointed by Cachon and Netessine (2004), the focus of the economics literature has long been on non-cooperative game theory. Contrary to cooperative theory, non-cooperative game theory assumes the absence of coalitions, i.e. there is no collaboration nor communication between the participants (Nash, 1951). The distinction between the two theories is sometimes a source of confusion (Van Damme & Furth, 2002). It is not the case that cooperative game players compulsorily cooperate (Van Damme & Furth, 2002). Players in non-cooperative games can also be willing to cooperate (Van Damme & Furth, 2002). The distinction between the theories is more on the level of complexity of the game (Fiestras-Janeiro, García-Jurado, & Mosquera, 2011). More precisely, "non-cooperative models assume that all the possibilities for cooperation have been included as formal moves in the game, while cooperative models are *incomplete* and allow players to act outside of the detailed rules that have been specified" (Van Damme & Furth, 2002, pp.51-52).

Interest on using cooperative game theory to study supply chains increased only recently. Indeed, both Cachon and Netessine (2004) and Cruijssen, Dullaert, and Fleuren (2007) mention that literature about this topic is scarce. This was last decade and since then numerous articles have been published. J. Li et al. (2016) mention that "substantial research on collaborative transportation has been conducted" (J. Li et al., 2016, p.84). In their literature review, Guajardo and Rönnqvist (2016) confirm this growth in the number of papers about collaborative transportation. A famous example of horizontal collaboration in logistics from the literature is the award-winning article of Frisk, Göthe-Lundgren, Jörnsten, and Rönnqvist (2010). Frisk et al. (2010) study a case in collaborative forest transportation in Sweden. D'Amours and Rönnqvist (2013) present the follow-up of the case. Three companies among the eight considered decided to collaborate. Savings were between 5 and 15% each month.

Collaborative transportation gained interest in practice, too. Cruijssen, Cools, and Dullaert (2007) mention an increase in the number of partnerships in horizontal logistics. The fact that negotiations in supply chain relationships are widespread is likely amplifying this trend (Cachon & Netessine, 2004). In 2007, Cruijssen, Cools, and Dullaert (2007) were already aware of at least 30 alliances in Belgium and the Netherlands, "the logistics centers of gravity in Europe." Since 2007, new alliances in logistics have been created. We can cite alliances between Procter & Gamble and Tupperware for the transportation of their products from Belgium to Greece, between PepsiCo and Nestlé in Belgium, between Mars and three other retailer suppliers in France, between a multinational active in lightweight plastic applications and a producer of heavy metal components for their logistics operations in Czech Republic and Germany, etc. (CO^3 project, 2014). The CO^3

project, CO^3 standing for Collaboration Concepts for Co-modality, is a project funded by the European Commission which promotes horizontal collaboration in logistics in Europe. In the framework of this project, collaboration between companies caused reductions of the logistic cost and of the companies'carbon footprint estimated at 10-20 % and 20-30% respectively (European Commission, 2015). Horizontal collaboration in logistics is not limited to large companies and can be applied by small and medium-sized enterprises (SMEs). For example, two Belgian chocolate producers collaborate to transport their products to the United Kingdom (POM Antwerp & CO^3 project, 2012). They managed to divide the number of trucks used by two.

2.2.2 Opportunities and hurdles

Cruijssen (2006) and Cruijssen, Cools, and Dullaert (2007) study the opportunities and impediments of horizontal collaboration in logistics. They adapt concepts from the extended vertical cooperation literature to horizontal cooperation in logistics. Their non-exhaustive list includes three group of opportunities: (1) cost and productivity, (2) customer service, and (3) market position. (1) Cost can decrease and productivity can increase thanks to economies of scale (less empty hauling, better use of the facilities, etc.).

For example, Procter & Gamble and Tupperware both transport large volumes from Belgium to Greece (Muylaert & Stofferis, 2014). Muylaert and Stofferis (2014) explain that the products transported by the two companies have opposite characteristics. The trucks from Tupperware are full in volume but half empty in weight and trucks of Procter & Gamble have the opposite situation. Thanks to collaboration and synergies, costs decreased by 17%. Also, 200 tons of CO_2 are saved every year.

Cruijssen (2006) and Cruijssen, Cools, and Dullaert (2007) mention that an increase in the frequency of deliveries or graphical coverage can also increase customer service (2). Four competitors in the cosmetics industry, Colgate-Palmolive, Henkel, GlaxoSmithKline and Sara Lee, collaborate in France to deliver their products to the retailers (Eyers, 2010). Deliveries are now made almost daily instead of one or three times a week. The companies went from less than truckload (LTL) to full truckload (FTL) shipping, dividing the number of trucks received by two and reducing their annual CO_2 emissions by 240 tons. The distribution service level increased by five to ten points of percentage to reach almost 99%.

Finally, Cruijssen (2006) and Cruijssen, Cools, and Dullaert (2007) write that alliances in horizontal logistics can enhance the market position of the companies (3). Companies can for instance increase their bargaining power vis-à-vis the suppliers or be able to supply bigger customers. They also mention other opportunities of horizontal collaboration like overcoming legal barriers or accessing better technologies.

Opportunities can make alliances very attractive and the literature tends to focus more on successful partnerships (Cruijssen, 2006). However, the failure rate is not negligible. Zineldin and Bredenlöw (2003) project a failure rate of 70 % for the strategic alliances. Even worse, this failure rate does not tend to decrease (Hughes & Weiss, 2007). Cruijssen (2006) and Cruijssen, Cools, and Dullaert (2007) list some barriers to overcome for successful horizontal collaboration in logistics. The impediments can be related to four areas: "(1) partners, (2) determining and dividing the gains, (3) negotiation, and (4) coordination and Information & Communication Technology (ICT)." Regarding partners, opportunism (Williamson, 1985), i.e an act of guile from one of the partners, is a famous example. In horizontal collaboration in logistics, if the partners are competitors, there might be a temptation to steal the clients of the other company. As rightly noted by Jacobs, Van Lent, Verstrepen, and Bogen (2014), human and psychological aspects play a key role in collaboration and a lack of trust has the potential to break a project. This even if significant savings can be realized. Finding associates or determining a trusted third-party are other frequent difficulties (Cruijssen, 2006). To precisely assess the gains or savings resulting from collaboration and to allocate the costs are other important hurdles (Cruijssen, 2006). We discuss more this cost allocation problem in section 2.4. Regarding negotiation, Cruijssen (2006) mentions unequal bargaining positions, high indispensable ICT costs and high coordinating costs as possible causes of failures of the alliances in horizontal logistics.

We can also cite legal aspects as a potential hurdle for horizontal collaboration in logistics. Jacobs et al. (2014) and Verstrepen and Van den Bossche (2013) mention that fears of antitrust penalties and the absence of a standard legal framework have stopped several projects of horizontal collaboration in logistics. This is therefore a fundamental issue. There are numerous factors for a "horizontal community" to be legal with regards to the European competition legislation (Verstrepen & Van den Bossche, 2013). Among them, the size of the participants and of the collaboration, whether it involves competitors, whether the aim of the collaboration has been precisely defined, etc. (Verstrepen & Van den Bossche, 2013). The companies can overcome this obstacle if they take sufficient precautions. Jacobs et al. (2014) write in their case study that Nestlé and PepsiCo were "very apprehensive" with matters related to antitrust laws. Agreements were closely analyzed by the legal departments of both companies and by an external lawyer. Given that detailed cost information could not be shared between companies, they mandated neutral trustees. To solve this potential legal problem, the European Commission decided to encourage the creation of a legal framework (European Commission, 2015). This was one of the missions of the CO^3 project and this framework has now been created (European Commission, 2015).

2.2.3 Transferable utility games

The transferable utility games, or TU games, are critical within cooperative game theory (Fiestras-Janeiro et al., 2011). When cooperative games are mentioned in the literature, the articles most often mean TU cooperative games (Fiestras-Janeiro et al., 2011). It is also this convention we will use in the rest of the dissertation. Transferable utility games are models with the assumption that collaboration gains can be freely allocated among players (Roth, 1988). Roth (1988) lists three assumptions for a cooperative game to be a TU game. First, all outcomes can be expressed in "utility money", i.e. some medium of exchange. Additionally, the players which are not members of the coalition do not need to be taken into account while assessing the outcome of the game. The third assumption is that there is no cost to enforce the costs or profit sharing agreements. We can challenge these assumptions in several ways. First, some synergies cannot be expressed in "utility money." Collaboration in supply chain management can sometimes bring a positive impact on the public image of the company (Mentzer, 2004). For instance, a decrease of the companies' carbon footprint through collaboration has likely a positive impact on their image. However, it is unlikely this outcome will be correctly evaluated. Indeed, it is hard to assess the value of intangible assets (Eccles, Newquist, & Schatz, 2007). Regarding the second assumption, a company not involved in an alliance could threaten the other companies (Cruijssen, 2006). Third, the cost of enforcing the agreements is a frequent impediment of collaboration (Cruijssen, 2006). We will keep in mind the limitations of the model while analyzing the location-routing game.

2.2.4 The facility location game and the vehicle-routing game

As we mentioned in section 2.1, the location-routing problem is closely linked to the facility location problem and to the vehicle-routing problem. The two related TU cooperative games are the facility location game and the vehicle-routing game. We do a brief overview of these two cooperative games in the following paragraphs. We compare some properties of the location-routing game with those of the appropriate subclass of these two games in chapter 3.

Facility location game Location games begin with the famous work of Hotelling (1929). This game was in a competitive setting, however. Considering the cost-sharing cooperative location games, Fragnelli and Gagliardo (2013) trace back the first works to the 1980's. Granot (1987) studies a game based on the problem of locating one facility in a tree network. Tamir (1993) applies game theory concepts to a covering problem.

Resulting from the multiple variants of the facility location problem, several facility location games exist (Curiel, 2008). For example, Puerto, García-Jurado, and Fernández (2001) define a single facility location game in a continuous solution space.

Vehicle-routing game Fishburn and Pollak (1983) define the cost allocation problem for fixed routes. Based on their work, the routing game is introduced by Potters, Curiel, and Tijs (1992). The costs are split among the visited sites. The properties of the game are further developed by Göthe-Lundgren et al. (1996) and Derks and Kuipers (1997).

2.3 Notations and useful concepts used in game theory

The notations we use in the thesis are similar to the ones used in the literature (Fiestras-Janeiro et al., 2011; Frisk et al., 2010; Guajardo, Jörnsten, & Rönnqvist, 2015).

We define the *grand coalition* $N = \{1, \ldots, n\}$ as the set of companies or *players* and K as the set of all non-empty subsets of N. It is assumed that players can form and collaborate in a *coalition* S, with $S \in K$.

The *characteristic function* is a function from K to $\mathbb{R}$ that assigns to each coalition $S \in K$ a cost $C(S)$, with $C(\emptyset) = 0$. A transferable utility game is therefore represented as a pair (N, C). The notation (N, C) refers to cost sharing games and the notation (N, v) to profit sharing games (Guajardo et al., 2015).

A *preimputation* or cost-allocation vector is a "vector $x = (x_1, \ldots, x_n)$ that assigns to each player $j \in N$ a quantity x_j such that $\sum_{j \in N} x_j = C(N)$" (Guajardo & Jörnsten, 2015, p.931). Each member of the grand coalition has therefore to pay a share of the cost according to the allocation x ($x_j \in \mathbb{R}$, $\forall j \in N$).

The core is another interesting concept and allows to know if it is possible to allocate the costs of the grand coalition among the players while keeping the coalition stable, i.e no smaller coalition has an incentive to be formed.

The core of the game (N, C) is defined by Gillies (1959) as the set of preimputations $x \in \mathbb{R}^n$ satisfying the two following conditions:

1. $\sum_{j \in S} x_j \geq C(S) \quad \forall S \subset N$
2. $\sum_{j \in N} x_j = C(N)$

The core of a game is denoted as Core(N). The first condition guarantees that the coalition is stable and the second condition that the total cost is allocated.

2.4 Cost allocations methods

Logistics service providers in Flanders and the Netherlands mention that establishing a fair allocation of the costs is "essential" (Cruijssen, 2006). Lambert, Emmelhainz, and Gardner (1996), Gibson, Rutner, and Keller (2002) and Zineldin and Bredenlöw (2003) also stress the importance of an appropriate distribution of the costs in horizontal collaboration. Numerous methods to allocate costs exist, some specific to certain games (Guajardo & Rönnqvist, 2016). Guajardo and Rönnqvist (2016) list more than 40 different cost-allocation measures employed in collaborative logistics. We define some of the most iconic methods in this section.

2.4.1 Shapley value

One of the most frequently used methods to allocate costs among players is the Shapley value (Guajardo & Rönnqvist, 2016). The Shapley value has been the focus of a constant interest in the cooperative game theory literature since its definition (Roth, 1988).

The Shapley value (Shapley, 1953) is defined as follows.

$$x_j = \sum_{S \subseteq N : j \in S} \left(\frac{(n-|S|)!(|S|-1)!}{n!} \cdot (C(S) - C(S - \{j\}) \right) \quad \forall j \in N$$

$C(S) - C(S - \{j\}$ is the marginal cost of a player to join a coalition S. The Shapley value can be interpreted as "the expected marginal contribution" of a player if the players enter the coalitions one at a time in a random order (Frisk et al., 2010; Roth, 1988).

The Shapley value is the only allocation method that satisfies all the four following axioms (Shapley, 1953).

- Efficiency:

$$\sum_{i \in N} x_i = C(N)$$

  The total cost is allocated among the players.

- Dummy property: the player i is a dummy player if $C(S \cup i) = C(S) \quad \forall S \subseteq N \backslash \{i\}$. The dummy property is then that:

$$x_i = 0 \quad \forall \text{ dummy players } i \in N$$

- Symmetry: "$i, j \in N$ are said to be symmetric in C if, for every subset $S \subset N \backslash \{i, j\}$, it holds that $C(S \cup \{i\}) = C(S \cup \{j\})$" (Fiestras-Janeiro et al., 2011, p.4).

The symmetry property is then that:

$$x_i = x_j \quad \forall i, j \in N \text{ which are symmetric in } C$$

Players who are treated equivalently by the characteristic function should have the same allocations (Roth, 1988). In other words, the names of the players do not matter in determining the Shapley value (Roth, 1988). This is why this property is sometimes referred to as the anonymity property.

- Additivity: For any characteristic functions C_1 and C_2 with $[C_1 + C_2](S) = C_1(S) + C_2(S) \quad \forall S \subset N$:
$$x(C_1 + C_2)_i = x(C_1)_i + x(C_2)_i \quad \forall i \in N$$

Another advantage of the Shapley value is that it is unique (Shapley, 1953). If all the costs for all the coalitions are known, it is also easy to compute.

As mentioned by Engevall, Göthe-Lundgren, and Värbrand (1998), the main drawback of the Shapley value is that it is not always in the core, even if the core is non-empty.

2.4.2 Proportional method

In practice, this proportional method based on stand-alone costs is the most frequently used cost allocation method (Liu, Wu, & Xu, 2010). Using the notations from section 2.3, it is defined as follows.

$$x_j = \alpha_j \cdot C(N) \quad \forall j \in N$$

with $\sum_{j \in N} \alpha_j = 1$.

α_j can be determined based on different factors. It can be based on the stand-alone costs with $\alpha_j = \dfrac{C(\{j\})}{\sum_{j \in N} C(\{j\})}$ or be egalitarian with $\alpha_j = \dfrac{1}{N}$. For the VRP, Engevall, Göthe-Lundgren, and Värbrand (2004) use, among others, a proportional method based on the demand quantities d_j. In that case, $\alpha_j = \dfrac{d_j}{\sum_{j \in N} d_j}$.

The proportional method is easy to compute and to understand. However, this cost-allocation method can assign to some players a cost higher than their stand-alone cost (Liu et al., 2010). Players could then be tempted to leave the grand coalition. The proportional methods have a poor record regarding stability in the literature (Guajardo & Rönnqvist, 2016).

The proportional method is often used in the literature (Guajardo & Rönnqvist, 2016) to be compared to other methods.

The proportional methods satisfy additivity (Moulin, 1987). However, the dummy player property does not apply (Roth, 1988). The symmetry property is also not fulfilled for the demand proportional. Of course, it is valid for the egalitarian method by definition. For every player, all the allocations are then the same. For the cost-proportional method, if two players have exactly the same characteristic functions, their allocations will also be the same. However, this property is not valid for the demand proportional method. For this method, the characteristic function does not influence the allocation of a player. Two players with exactly the same characteristic function but with different demands do not have the same allocations. This assuming that the demand does not have an impact on the cost of the coalition. For instance, for the location-routing problem, this is the case if the vehicle and facility capacity constraints are not binding. Therefore, even if the core is non-empty, the vector of cost-allocations is not always in the core.

2.4.3 Equal Profit Method

The Equal Profit Method (EPM) is defined by Frisk et al. (2010). They noticed that a method offering similar relative profits for all the participants would be advantageous during negotiations between companies. We adapt this method to a cost allocation game by considering the relative savings instead.

In order to find the cost allocations x_i, a linear problem must be solved.

$$\min \quad f$$

s.t.

$$f \geq \frac{x_i}{C(\{i\})} - \frac{x_j}{C(\{j\})} \quad \forall i,j \in N \tag{2.14}$$

$$\sum_{j \in S} x_j \leq C(S) \quad \forall S \subset N \tag{2.15}$$

$$\sum_{j \in N} x_j = C(N) \tag{2.16}$$

$$x_j \geq 0 \quad \forall j \in N \tag{2.17}$$

$$f \in \mathbb{R} \tag{2.18}$$

Constraints (2.14) and the objective function are such that f is equal to the highest difference in relative savings of any two participants. Indeed, the relative savings of one participant are equal to $\frac{C(\{i\}) - x_i}{C(\{i\})} = 1 - \frac{x_i}{C(\{i\})}$. Constraints (2.15) impose that the

TABLE 2.1: Counter-example to prove that the Equal Profit Method does not fulfill the additivity property.

(A) Characteristic function for each game.

Coalition	C_1	C_2	$C_1 + C_2$
{1}	581	618	1199
{2}	687	621	1308
{3}	676	770	1446
{1,2}	864	723	1587
{1,3}	824	893	1717
{2,3}	942	896	1838
{1,2,3}	1082	971	2053

(B) Cost allocations for each game. $\Delta = x(C_1) + x(C_2) - x(C_1 + C_2)$.

Player	$x(C_1)$	$x(C_2)$	$x(C_1) + x(C_2)$	$x(C_1 + C_2)$	Δ
1	323.3755	298.6949	622.0704	622.7035	-0.6331
2	382.3735	300.1448	682.5183	679.3129	3.2054
3	376.2510	372.1603	748.4113	750.9836	-2.5722

allocation is stable. Efficiency of the cost allocation is guaranteed by (2.16). Constraints (2.17) and (2.18) are linked to the nature of the variables.

To our knowledge, there does not exist any axiomatization of the Equal Profit Method. In this paragraph, we investigate the behavior of this method regarding several axioms. First, symmetry is fulfilled for the EPM. Two players treated exactly in the same way by the characteristic function receive the same cost allocation. The constraints (2.14) would be exactly the same when $i \neq j$ and would become $f \geq 0$ when $i = j$. Besides, the dummy player property does not apply for the EPM. If a player has a stand-alone cost equal to 0, there is a division by 0 in the constraints (2.14). Additionally, the EPM does not satisfy the additivity property. We present a counter-example to prove that the EPM does not fulfill the additivity property in table 2.1. We can indeed notice that $x(C_1 + C_2) \neq x(C_1) + x(C_2)$. In this counter-example, the core is non-empty and $f = 0$ for each of these games.

For the EPM the stability and efficiency conditions are fulfilled, this cost-allocation method is "core-guaranteed" when the core is non-empty. This is the main advantage of this method. In the case where the core is empty, the EPM must be adapted. Frisk et al. (2010) suggest to use the "epsilon-core" as defined by Shapley and Shubik (1966). This means that constraints (2.15) would then become:

$$\sum_{j \in S} x_j \leq C(S) + \epsilon \quad \forall S \subset N \tag{2.19}$$

with $\epsilon \geq 0$.

The strength of the incentive for some companies to start their new coalition would then be represented by this epsilon. The value of this epsilon is computed using a separate LP model.

$$\min \quad \epsilon$$

s.t.

$$\sum_{j \in S} x_j \leq C(S) + \epsilon \quad \forall S \subset N \tag{2.20}$$

$$\sum_{j \in N} x_j = C(N) \tag{2.21}$$

Frisk et al. (2010) also mention an alternative approach for the case the core is empty. That is to simply search the maximum number of players for which the core would be non-empty. However, they write that no method has yet been found for the selection of this subgroup of players. For the location-routing problem, the instances studied will be of a small size since this problem is NP-hard. Thus, this subset can thus be determined manually for a reasonable number of companies.

2.4.4 Nucleolus

The nucleolus is an allocation method defined by Schmeidler (1969). It is also one of the most frequently used cost allocation methods (Guajardo & Jörnsten, 2015). Its original formulation as a profit game is adapted to a cost-sharing game (N, C) in Guajardo and Rönnqvist (2016). Their notations are taken in the following paragraph.

First, the excess of a coalition is defined. Let the excess of coalition S with the cost-allocation vector x in game (N, C) be $\varepsilon(x, C, S) = C(S) - \sum_{j \in N} x_j$. The excess $\varepsilon(x, C, S)$ is used to evaluate the savings of a coalition S with a particular allocation x. The excess vector is then defined as $e(x, C) = \{\varepsilon(x, C, S_1), \ldots, \varepsilon(x, C, S_p)\}$ with $S_i \in K$ (the set of all non-empty subsets of N) and therefore $p = 2^n - 1$. A mapping θ is then defined and is such that for an excess vector $e \in \mathbb{R}^p$, $\theta(e) = y$. $y \in \mathbb{R}^p$ is the vector with its elements arranged in a non-decreasing order.

"A vector $y \in \mathbb{R}^p$ is lexicographically greater than $\bar{y}$ (written $y \succeq \bar{y}$) if either $y = \bar{y}$ or there exists $h \in \{1, \ldots, p\}$ such that $y_h > \bar{y}_h$ and $y_i = \bar{y}_i \ \forall i < h$" (Guajardo & Rönnqvist, 2016, p.11).

The nucleolus of a cost-sharing game (N, C) is defined as

$$\mathcal{N} = \{x \in \mathcal{X} : \theta(e(x, C)) \succeq \theta(e(y, C)) \, \forall y \in \mathcal{X}\}$$

$\mathcal{X}$ being the set of all payoff vectors.

An interesting property of the nucleolus is that it is unique, as shown by Schmeidler (1969). He also shows that the nucleolus always belongs to the core when it is non-empty.

Additionally, Potters (1991) and Snijders (1995) present axiomatizations of the nucleolus and find it is efficient, individually rational, and it fulfills the symmetry and the dummy-player properties. The nucleolus is not additive, however (Engevall et al., 1998).

Moreover, the computation of the nucleolus of a game is more complex than the methods based on formulas (Guajardo & Rönnqvist, 2016). Mistakes in computing the nucleolus are not rare (Guajardo & Jörnsten, 2015). Guajardo and Jörnsten (2015) also outline the algorithm of Kopelowitz (1967) and Fromen (1997) to compute the nucleolus, which we will use.

Briefly, a sequence of linear program (LPs) is solved. At the first step, the LP maximizes the minimum excess ε among all the coalitions. The first LP gives the first element of the vector $\theta(e(x, C))$, or elements if several coalitions have the same minimum excess. At the next step, we solve the dual of the LP to find the coalition(s) corresponding to the minimum excess of the LP. We then add constraints to the LP so that all the excesses for the coalitions already in the vector $\theta(e(x, C))$ remain the same. We then solve the current version of the LP and proceed to the next step. The minimum excess will decrease at each step. The algorithm stops when each coalition is assigned a minimum excess. In other words, this means we know the vector $\theta(e(x, C))$. The cost-allocations found at the last LP form the nucleolus of the game.

As mentioned by Guajardo and Jörnsten (2015), their procedure computes the *pre-nucleolus*. The pre-nucleolus is equal to the nucleolus if the core is non-empty, otherwise it is necessary to add individual rationality constraints (Guajardo & Jörnsten, 2015). The individual rationality constraints are $x_j \leq C(\{j\}) \quad \forall j \in N$.

2.4.5 What is the best allocation method?

In theoretical studies, there is no closed answer to the question of which allocation method to prefer (Tijs & Driessen, 1986). Tijs and Driessen (1986) mention several factors influencing the choice of the best allocation method in a concrete situation. They cite: "the idea of the participants with respect to fairness, the power feelings of the participants, the difficulty of understanding and calculating a cost allocation proposal, and many other factors" (Tijs & Driessen, 1986, p.1015).

We summarize the behavior of the cost-allocation methods regarding some properties in table 2.2. We are aware that the easiness to compute and to understand the methods are more subjective properties. Nonetheless, we can confidently state that some methods fare better than others. Regarding the easiness to understand or intuitiveness

TABLE 2.2: Summary of some properties of the studied cost-allocation methods.

	Shapley	**Nucleolus**	**EPM**	**Proportional**
Efficiency	✓	✓	✓	✓
Dummy player property	✓	✓		
Symmetry	✓	✓	✓	✓[1]
Additivity	✓			✓
Stability (when the core exists)		✓	✓	
Easiness to calculate				✓
Intuitiveness			✓	✓

[1] For the egalitarian method and the cost proportional but not for the demand proportional.

of the allocation method, we can consider the Shapley value as superior to the nucleolus, whose intuitive meaning may not appear easily (Chardaire, 1998). The concepts behind the proportional methods and the Equal Profit Method are even easier to understand, however. The difficulty of calculating the allocations is another important issue. This issue has been widely discussed in the literature (Chardaire, 1998). Hardness results for the Shapley value and the nucleolus vary depending on the type of the game, the number of players, etc. (Chardaire, 1998). All the Shapley value, the nucleolus and the EPM require to know the costs of all the coalitions. On the other hand, the proportional methods only demand the cost of the grand coalition and, depending on the method, the stand-alone cost or the demands of the players.

Chapter 3

The location-routing game

3.1 A cooperative location-routing game

3.1.1 Definition of the game

The location-routing game (LRG) is here defined as a cost sharing game and is consequently denoted as (N, C). The value $C(S)$ of the characteristic cost function of the game (N, C) is the total cost occurring if a coalition $S \subseteq N$ of players is formed. Per definition, $C(\emptyset) = 0$.

Each value $C(S)$, $S \neq \emptyset$, is computed by solving a standard LRP. For each coalition of players, almost all the sets and parameters of the LRP remain the same. Only the set of customers I to be served changes with each coalition S.

The set of customers for a coalition S is constituted of all the customers assigned to the companies of the coalition. I_n is the subset of customers of player $n \in N$ and in the model each customer i is the client of only one company. Consequently $\bigcap_{n \in N} I_n = \emptyset$. The set of customers used in the LRP to compute $C(S)$ is thus $I = \bigcup_{n \in S} I_n$.

The fact that for instance two Fast Moving Consumer Goods companies supply the same store can be represented in the model. In the mathematical model, the store is represented by two distinct customers located in the same position. The reason we do not merge the two clients is because the set of customers changes for each coalition. Therefore, the two customers are not always in the same coalition.

3.1.2 Applications of the location-routing game

The location-routing game has applications similar to the facility location games. However, as we mentioned earlier, neglecting the routing problem while locating facilities may lead to suboptimal solutions (Salhi & Rand, 1989). Consequently, the solutions of the facility location problems and of the facility location games can be misleading. First, because the wrong facilities could be open. Second, because the routing costs go from a separate cost, i.e not included in the game, to a common cost. The characteristic function changes and unstable coalitions can become stable and vice versa.

There are, however, certain cases where not defining a location-routing game may not be an issue. First, if it is obvious for the practitioners that the routing costs will not play a role in the location of the facilities. This statement would depend on a lot of factors like the location and costs of facilities, the relative importance of the transportation costs, etc. Second, if integrating the routing costs in the characteristic function would not change the outcome of the game and the total cost for each of the players. For instance, if the routing costs are approximately the same for each player and that they would not change the relative savings (for the EPM) or the proportion of the stand-alone costs (for the cost proportional method) etc.

The location-routing game could be used to model horizontal collaboration between retailers, in package delivery, in city logistics or in any supply chain.

A real-life example An example of horizontal collaboration including location and routing aspects is the successful alliance between Colgate-Palmolive, GlaxoSmithKline, Henkel and Sara Lee for their logistics operations in France. The history of the collaboration is presented in figure 3.1. The alliance started in September 2005 with Colgate-Palmolive, Henkel and GlaxoSmithKline (GSK). They began to collaborate for the routing aspects of delivering their products to the distribution centers (DCs) of four retailers (Auchan, Carrefour, Match and Monoprix). The trucks were going through Henkel's depot in Lieusaint, Colgate-Palmolive's depot in Crépy-en-Valois and GSK's depot in Orléans and then to the DCs of the retailers. In January 2006, Henkel moved its activities to its new site in Château-Thierry. In June 2006, all the three companies decided to use the warehouse in Château-Thierry. The companies were likely to realize the potential of synergies caused by increased collaboration. First, they decreased their facility costs, going from four to only one facility. The arrangement of the facility is displayed in figure 3.2. Second, they decreased their transportation costs and CO_2 emissions. As we can notice in figure 3.3, the multi-pick system caused the trucks to ride significant distances before supplying the retailers' DCs. The distance between Orléans and Château-Thierry is around 200 km.

A proof of the success of the collaboration is that other companies are willing to join. In April 2008, Sara Lee joined the alliance and the effective deployment with four companies began fourteen months later. However, Sara Lee does not have its depot in Château-Thierry. The reason is that their contract with their third-party logistics is still valid. They only enjoy the synergies caused by routing collaboration. A small truck daily brings their products from their depot in Soissons to the one in Château-Thierry, 40 km each way. The medium-sized company Eugène Perma, which sells hair care products, joined the alliance in January 2011 (Camman, Guieu, Livolsi, & Monnet, 2013). They proceed as Sara Lee. A truck rides the 60 km from their manufacturing facilities in Reims to Château-Thierry.

The third-party logistics (3PL) FM Logistic plays a key role in the alliance. Besides their role as logistics provider, they act as a controller. They use the term *pooling* as another name for routing collaboration. Several retailers' warehouses are supplied by the same lorry with products from the four manufacturers (FM Logistic, 2013).

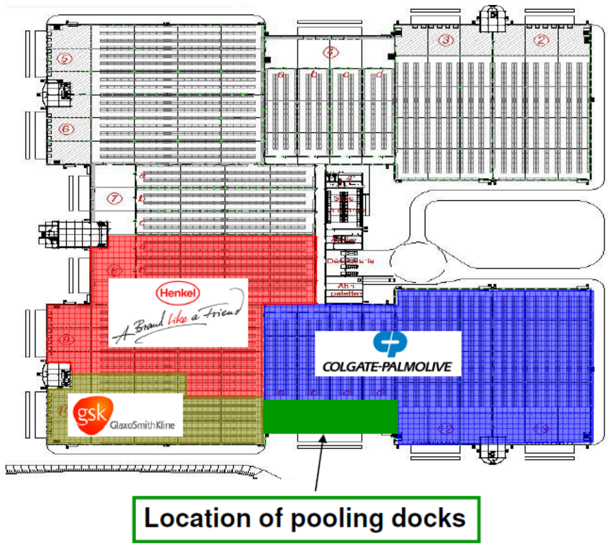
As we mentioned in chapter 2, this alliance brought significant improvements in customer service and a decrease of CO_2 emissions. Figures on the cost savings are not provided. The companies only mention that their logistic costs are "controlled" while improving customer service (Eyers, 2010). However, Camman et al. (2013) write that the common willingness of the companies to reduce transportation costs is the main reason of the alliance.

In this example, companies went from a collaboration focused only on routing to a collaboration considering location of the facilities and routing. We would model the first situation as a vehicle-routing game and the second situation as a location-routing game. We suppose in this example the first phase of the collaboration helped to build trust between the partners. This trust was then a good asset to facilitate long-term decision making, like on the location of facilities.

A similar path of collaboration could be followed by other alliances focused only on routing.

Realized on basis of data from Camman et al. (2013) and Eyers (2010).

FIGURE 3.1: Time-line of the collaborative approach between the cosmetics manufacturers.



Source: Eyers (2010).

FIGURE 3.2: Arrangement of the warehouse in Château-Thierry for the collaboration between the cosmetics manufacturers.

3.1.3 Properties of the game

Monotonicity From Engevall et al. (2004), we have the definitions 3.1 and 3.2.

Definition 3.1. The game (N, C) is called *monotone* if the characteristic function is monotone, i.e.

$$C(S) \leq C(T) \quad \forall S \subset T \subset N \tag{3.1}$$

Proposition 3.1. The location-routing game (N, C) is monotone.

Proof. Assuming that the triangle inequality for the average annual cost of traveling between two different nodes is satisfied, the characteristic function is not going to decrease when more customers are added. □

Subadditivity

Definition 3.2. The game (N, C) is called *subadditive* or *proper* if the characteristic function is subadditive, i.e.

$$C(S \cup T) \leq C(S) + C(T) \quad \forall S, T \subset N, S \cap T = \{\emptyset\} \tag{3.2}$$

Subadditivity is a crucial property. Players will try forming coalitions only if the characteristic function is subadditive.

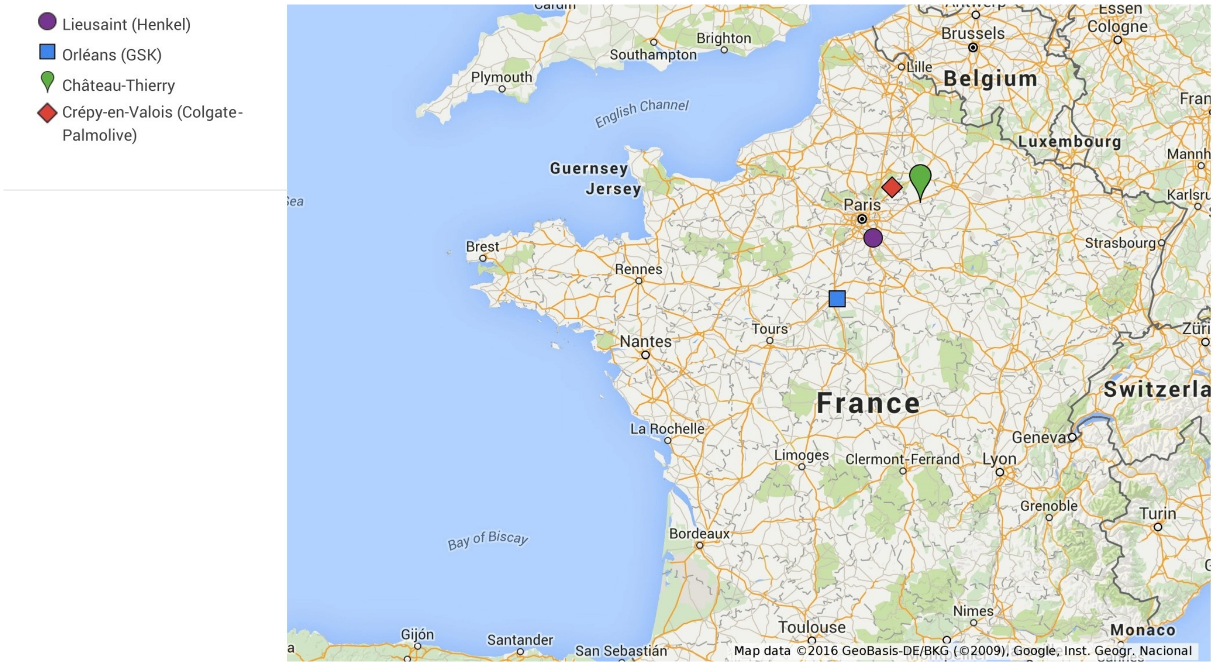


FIGURE 3.3: Location of the facilities of the cosmetics manufacturers.

Proposition 3.2. The location-routing game (N, C) is subadditive.

Proof. Let the location-routing problem defined previously with two sets of players S and T and $S \cap T = \{\emptyset\}$. To each set of players corresponds a set of customers I_S or I_T. If we consider the problem with a set of players $S \cup T$, the solution where the customers of set I_T/I_S are served using the same depots, routes and vehicles as beforehand is a feasible solution with the set of customers $I_S \cup I_T$. The fact that some depots coincide would not be a problem since there is no capacity constraint on the depots. This solution is therefore an upper bound on the optimal solution. □

Core In Göthe-Lundgren et al. (1996), they show that the core of the basic vehicle routing game can be empty. By adapting their example, we prove the same for the core of the basic LRP.

The location of the three customers (1, 2 and 3) and of the feasible sites of candidate facilities (A, B and C) is shown in figure 3.4. The figure also contains the transportation costs. The customers are the players of the game. The annual costs of establishing and operating a facility are equal to one for each of the sites. The demand for each customer is of one unit. The capacity for each vehicle is of two units. The cost of using one vehicle is set to 0.

Calculating the characteristic cost function for the singletons, we obtain $C(\{1\}) = C(\{2\}) = C(\{3\}) = 3$. The routing cost is equal to two and the location cost to one. The facility opened is one of the two adjacent to the customer. For the two-customer coalitions, $C(\{1,2\}) = C(\{1,3\}) = C(\{2,3\}) = 4.7$ (routing cost of 3.7 and location cost

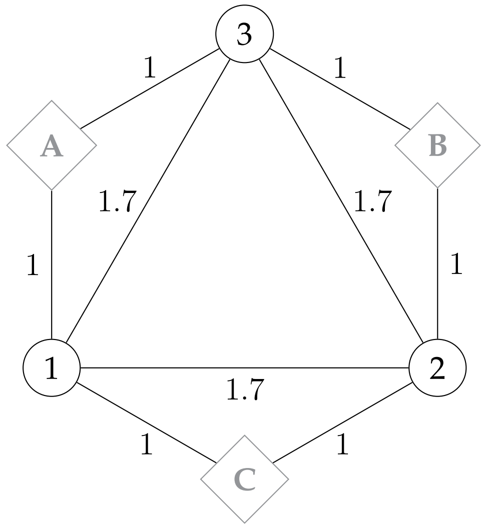


FIGURE 3.4: Example with three customers indicating the cost of traveling between the nodes, the location of the customers $\{1,2,3\}$ and the feasible sites of candidate facilities $\{A,B,C\}$.

of 1). For the grand coalition, $C(\{1,2,3\}) = 7.7$. Two depots are open (location costs $= 2$). For instance, depots A and C. In this case, one route is A–1–3–A and another route is C–2–C. The routing costs are therefore 5.7.

We can notice there is an incentive for a 2-customer coalition. Two customers supplied from one depot with one vehicle minimizes their costs. Therefore the core is empty. More remarkably, as noticed by Göthe-Lundgren et al. (1996) for the vehicle-routing game, there is no reason for why a particular pair of players should cooperate. Indeed any pair of players could do. This means that there is no stable coalition at all in this example.

Balance From Curiel (2008), we have the two following definitions:

Definition 3.3. A collection $\mathcal{B}$ of non-empty subsets of N is called a *balanced collection* if for all coalitions $S \in \mathcal{B}$ there exist positive numbers λ_S such that

$$\sum_{S \in \mathcal{B}} \lambda_S = N \tag{3.3}$$

The numbers λ_S are called the weights of the elements of $\mathcal{B}$.

Definition 3.4. A cooperative game is called a *balanced game* if

$$\sum_{S \in \mathcal{B}} \lambda_S C(S) = C(N) \tag{3.4}$$

for every balanced collection $\mathcal{B}$ with weights $\{\lambda_S\}_{S \in \mathcal{B}}$.

The Bondareva-Shapley theorem (Shapley, 1967) states that a cooperative game is balanced if and only if its core is non-empty.

Since the core of this game can be empty, it is not always balanced.

Convexity From Engevall et al. (2004), we have the following definition:

Definition 3.5. A cost game is *convex* if its characteristic function is *concave* (or *submodular*), i.e.,

$$C(S \cup T) + C(S \cap T) \leq C(S) + C(T) \quad \forall S, T \subseteq N \tag{3.5}$$

or equivalently,

$$C(S \cup i) - C(S) \geq C(T \cup i) - C(T) \quad \forall i \in N, S \subseteq T \subseteq N \backslash \{i\} \tag{3.6}$$

Let's first look at the convexity of games related to the LR game.

Regarding the facility location game, its characteristic function can be concave or convex (G. Li, Li, Shu, & Xu, 2012). For extensions of the game, conditions on the characteristic function to be concave exist. For example, the game defined by J. Li et al. (2016) is a facility location game taking into account the decay of perishable products. They show that two conditions linked to the decay losses and the variable costs make the characteristic function concave.

Regarding routing problems, concavity of the characteristic function has not been studied in depth. Engevall et al. (2004) provide an example to show that the basic vehicle routing game (VRG) is not convex. They use the theorem from Shapley (1971) stating that the core of a convex game is not empty. By contraposition, it is equivalent to state that if the core of a game is empty, the game is non-convex. Therefore, since the core of the VRG can be empty, the VRG is non-convex (Engevall et al., 2004).

Proposition 3.3. The location-routing game is non-convex.

Proof. Since we just proved the core of the game can be empty, this means that this game is non-convex. □

Indeed, the game presented in figure 3.4 is non-convex. For instance, let's take $S = \{1\}, T = \{1, 2\}$ and $i = 3$. Using equation (3.6), we then have that $4.7 - 3 \not\geq 7.7 - 4.7$.

3.2 Extensions of the standard location-routing game

3.2.1 LRG with a limited number of depots

Applications

Some types of facilities can cause nuisance and social rejection. A city taking this into account could decide to impose a limit on the number of a certain type of facilities.

Modification of the mathematical formulation

If we add the constraint that only l depot(s) can be used, we have to modify the models from section 2.1.2. The parameter l is therefore added and defined as the maximum number of depots that can be used. We also add the following constraint to the model:

$$\sum_{g \in G} Z_g \leq l \tag{3.7}$$

Properties of the game

Proposition 3.4. This game is not subadditive

Proof. We take the counter-example of figure 3.5. We consider that each player has one customer ($N = I$). Only one depot can be used ($l = 1$). The annual costs of establishing and operating a facility are equal to one for each of the sites. The demand for each customer is of one unit. The capacity for each vehicle is of two units. Assume the cost of using one vehicle is 0.

With the set S of customers ($S = \{1, 2\}$), we open facility A and the routing costs are equal to 3. Thus, the total costs are equal to 4. The same costs are obtained for the set T of customers ($T = \{3, 4\}$). We then open factory B. $C(S) = C(T) = 4$

However, for the set of customers $S \cup T$, whatever the site we open, $C(S \cup T) = 10 + 1 = 11$.

Therefore,

$$C(S \cup T) \nleq C(S) + C(T)$$

□

Since the game is not subadditive, the core is empty. There is no allocation which satisfy $\sum_{j \in S} x_j \leq C(S)$, $\sum_{j \in T} x_j \leq C(T)$ and $\sum_{j \in S \cup T} x_j \leq C(S \cup T)$. Thus it is non-convex and not balanced.

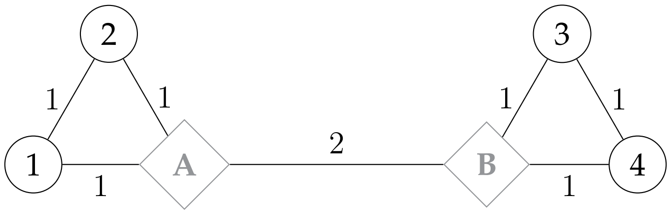


FIGURE 3.5: Example with four customers indicating the cost of traveling between the nodes, the location of the customers $\{1, 2, 3, 4\}$ and the feasible sites of candidate facilities $\{A, B\}$.

3.2.2 LRG with capacitated facilities

Applications

It is the most common case in reality. We can use the assumption of uncapacitated depots only when the capacity of each depot is above the total demand.

Modification of the mathematical formulation

If in the definition of the problem, we add that the depots have also a limited capacity, we only use the model with three indexes from section 2.1.2. We have to modify this model as follows.

The parameter w_g is added and is defined as the capacity of depot g, $g \in G$. The variable F_{ig} is also added.

$$F_{ig} = \begin{cases} 1 & \text{if customer } i \text{ is assigned to depot } g, \quad i \in I, g \in G \\ 0 & \text{otherwise} \end{cases}$$

We add the constraints (3.8) and replace (2.7) by (3.9).

$$\sum_{u \in V} X_{guk} + \sum_{u \in V} X_{uik} \leq 1 + F_{ig} \quad \forall g \in G, \forall i \in I, \forall k \in K \tag{3.8}$$

$$\sum_{i \in I} d_i F_{ig} \leq w_g Z_g \quad \forall g \in G \tag{3.9}$$

The constraints (3.8) state that each client served by a vehicle from a certain depot must be assigned to this depot. $\sum_{u \in V} X_{guk}$ is equal to 1 if the vehicle k departs from the depot g. $\sum_{u \in V} X_{uik}$ is equal to 1 if the vehicle k supplies the customer i. Therefore, F_{ig} must be equal to 1 and the other cases is equal to 0. These assignment constraints are similarly defined in Prodhon (2006) and Wu, Low, and Bai (2002). The constraints

(3.9) ensure that the capacity of the depot must be respected. These constraints (3.9) also forbid that a depot can be used without being open.

Properties of the game

Proposition 3.5. This game is not subadditive

Proof. The cost of traveling between the nodes, the location of the customers and feasible sites of candidate facilities are represented in figure 3.5. We consider that each player has one customer ($N = I$). The annual costs of establishing and operating facilities are equal to 1 for site A and 6 for site B. The depot have now a maximum capacity equal to one unit. The demand for each customer is of one unit. The capacity for each vehicle is of two units. The cost of using one vehicle is set to 0.

With the set S of customers ($S = \{1, 2\}$), we open facility A and the routing costs are equal to 3. Thus, $C(S) = 4$. With the set T of customers ($T = \{3, 4\}$), we also open facility A and the routing costs are equal to 7. Consequently, $C(T) = 8$

However, for the set of customers $S \cup T$, we open both facilities. The routing costs are equal to 6. $C(S \cup T) = 13$

Therefore,

$$C(S \cup T) \nleq C(S) + C(T)$$

□

Since the game is not subadditive, its core can be empty and the game is therefore non-convex and not balanced.

Chapter 4

Experimental results

4.1 Metrics of the cost allocations' performance for the LRG

In this section, we discuss performance measures for the cost allocation methods. We look at the indicators used for the facility location game and the routing games. This to try to determine which measures would be the most adequate for the location-routing game. We base our analysis on the characteristics of the LRG, of the LRP and of course on those of the varied cost allocation methods. We studied these characteristics in chapters 2 and 3.

4.1.1 Metrics for facility location games

Goemans and Skutella (2004) discuss fair cost allocations for the facility location game. They take the example of towns building sport complexes but not willing to pay more than their fair share of the total cost, "whatever that means." They insist on the stability condition but do not provide with any measure to determine which allocation to choose in the core. In the case where the core is empty, they use a relaxed version of the core.

Verdonck, Beullens, Caris, Ramaekers, and Janssens (in press 2016) analyze the cost allocation techniques for the cooperative carrier facility location problem. This game belongs to the class of facility location games which is the most comparable to the location-routing game. It is indeed the first article where the players are the carriers and not the customers (Verdonck et al., in press 2016). This article states that for reasons related to transparency, fairness and mathematical complexity, it is useful to also apply cost allocation methods that do not come from game theory (like the EPM).

For other classes of facility location games, Harks and von Falkenhausen (2014) mention budget-balance (or efficiency) and stability. The other articles do not explicitly mention efficiency. We guess they consider as implicit that a cost-allocation method should split the total cost among the players.

4.1.2 Metrics for collaborative transportation

Stability and fairness are the main criteria used in collaborative transportation (Guajardo & Rönnqvist, 2016). Different relaxed versions of the core are used when the core is empty. The stability constraints or the efficiency constraint can be relaxed by an absolute or relative term depending on the definition of the relaxed core (Guajardo & Rönnqvist, 2016). Regarding fairness, the way fairness is measured varies across the literature (Guajardo & Rönnqvist, 2016). Frisk et al. (2010) want relative profits as equal as possible for the different players. Therefore they develop the Equal Profit Method. Other authors have different definition of fairness and thus define their own allocation method (Guajardo & Rönnqvist, 2016). Özener and Ergun (2008) consider the difference between the cooperative and the original lane costs. A lane is a "shipment delivery from an origin to a destination with a full truckload" (Özener & Ergun, 2008). Other examples are provided in Guajardo and Rönnqvist (2016).

According to the CO^3 project, an initiative from the European Commission, the two most important criteria are again stability and fairness (Tseng, Yan, & Cruijssen, 2013). Fairness means here that the allocation method must fulfill at least four properties: symmetry, efficiency, dummy and additivity. Therefore, they strongly advertise the Shapley value, the only allocation method that fulfills all these properties (Shapley, 1953). However, they show flexibility in some cases. In a case study in retail collaboration in France, they suggest to keep the EPM (Guinouet, Jordans, & Cruijssen, 2012). The first reason is that the EPM is already stable. The second reason is that the allocations obtained are close to the Shapley values.

4.1.3 Metrics for the LRG

From the comparison with the games linked to the LRP, we can already get some insights. Stability of the cost-allocation is one of the most important criteria. Since the core of the LRP can be empty, we will use the epsilon-core when necessary. Regarding fairness, there is no agreement in the literature neither for the location game nor for collaborative transportation. In our opinion, a minimum difference in relative savings as in Frisk et al. (2010), is a good measure for fairness for the LRP. Besides, we take into account the complexity of the computations. Finally, the transparency of the methods for the companies is a factor to keep in mind.

4.2 Design of the experiment

4.2.1 Instances

For the standard LRP, Drexl and Schneider (2015) list eight batches of instances used in scientific articles and PhD dissertations. However, these sets of instances were created in order to test algorithms. The purpose here is completely different. It is rather to discuss cost-allocation methods applied to the location-routing problem. The number of nodes for theses instances often goes beyond 50. Given the difficulty of this problem, solving such complicated instances would involve a large amount of computing time. Indeed a particular instance has to be solved for each combination of players. This since the computation of each value of the characteristic cost function requires a location-routing problem to be solved.

Therefore, given the difficulty of the problem, the combinatorial explosion of the number of coalitions and the computational capabilities of the computer used, instances with three potential sites for the facilities and three companies having each two customers are considered. With three players, the number of coalitions remains reasonable ($2^{|N|} - 1 = 2^3 - 1 = 7$). In practice, horizontal collaboration in logistics also tends to involve a few partners (Lozano, Moreno, Adenso-Díaz, & Algaba, 2013). This is due to the complexity and the costs of coordinating a large coalition (Lozano et al., 2013). Indeed, the alliances we mentioned in the dissertation contained only two to five partners.

We tested several sizes for the instances. An instance of the LRP with 21 nodes (18 customers and three candidate facilities) had still a gap of 98% after 30 minutes of running time. To have an instance with a sensible computing time, we decreased the number of customers per client. We then considered to randomly split the ones used in the literature. However, the major disadvantage is that the newly created instances do not have realistic properties. The depots are in most of the cases really far from the customers, the facility costs often represent 90% of the total cost, etc. Rather than modifying all these parameters, we decide to start from scratch and create new sets of instances. We can therefore more easily test the impact of the characteristics of the parameters.

4.2.2 Data generation

We follow a procedure analogous to the ones of Albareda-Sambola, Díaz, and Fernández (2005), Tuzun and Burke (1999) and Prodhon (2006) to create the instances. Our choices are made such as to attain an optimum between the extent and the quality of the analysis.

Six customers and three depots are located in a square 100×100. Each of the coordinates x and y of the clients and depots follows a uniform distribution between 0 and 100. Each company is randomly assigned two customers.

The demands are chosen such that the companies are not too different from each other. The demand for each customer follows a uniform distribution between 10 and 20.

The vehicle capacity must be higher than the demand of two customers. Otherwise, we would have a basic facility location problem since no tour between customers would be possible. Three different values are chosen for the vehicle capacity: 40 (maximum demand of two customers), 80 (intermediate value) and 120 (maximum total demand). The fleet is homogeneous.

Regarding the capacities of the facilities, two values are tested. The first one corresponds to the uncapacitated LR game. The capacities of the depots are such that the facility capacity constraints will not be binding. In the second case, capacities on the facilities are introduced. All the facilities have the same capacity. The capacities are equal to 80. The capacity of one facility is high enough for two players but not always for three players. Other values could have been considered. However, if the capacities are too low this is equivalent to a constraint that all the depots must be open. The problem would then become a vehicle-routing problem with several depots.

Regarding the costs, we first conducted an experiment with 450 instances with parameters chosen in an extended interval. From this first experiment, we had some insights on how the proportion of each type of costs evolves given the values of the parameters.

If the costs for the facilities are too heterogeneous in the same instance, the choice of the facilities becomes obvious. The decision on the facilities is implicitly made before solving the problem. If the choice of facilities is obvious, this means that the problem should be modeled as a vehicle-routing problem. This would then be out of the scope of the thesis. Indeed, the particularity of location-routing is that there is a trade-off between the facility costs and the traveling and vehicle costs.

Therefore, in a same instance, the difference between the costs of two facilities will never be higher than 150. From the first experiment, we noticed that when the delta is above 150, the same facility is chosen for every coalition. The facility costs for facility A is randomly chosen between 250 and 600. The cost for each of the two other feasible sites is the cost of facility A plus a value randomly selected in the interval $[-75; 75]$.

The transportation costs between two nodes are the Euclidean distances between these two nodes.[1] The vehicle costs are between 10 and 200.

A recap chart is presented in table 4.1.

[1] $c_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad \forall i, j \in V, i \neq j$

A remark on the units There is a reason no units are mentioned for the parameters. It is a way to extend even more the reach of the analysis. The same solutions will be found regardless the units chosen. The demands and the capacities could be expressed in number of items, in tons, in number of pallets, etc. The same reasoning applies for the costs which could be expressed in thousands of euros, in millions of kroner, etc.

TABLE 4.1: Procedure followed for the selection of the parameters for each instance.

Parameters selected among specific values		**Homogeneity of the parameters in an instance**
Vehicle capacity	40, 70 or 100	Homogeneous
Facility capacity	80 or 120	Homogeneous

Parameters selected in an interval		**Homogeneity of the parameters in an instance**
x coordinates	[0;100]	
y coordinates	[0;100]	
Customer demand	[10;20]	Heterogeneous
Facility cost [A]	[250;600]	
Facility cost [B]	Facility cost [A] + [-75;75]	
Facility cost [C]	Facility cost [A] + [-75;75]	
Vehicle cost	[10;200]	Homogeneous

4.2.3 Solving the instances

To have meaningful results, we generated a large number of instances. 6,000 instances were created. For each instance, the values of the characteristic function are calculated for the seven coalitions. This represents 42,000 runs of the LRP. Cost allocations are then computed. This represents again several runs per instance for the nucleolus and the EPM. The model implementations are coded and solved using AMPL/CPLEX 12.6.2.0.

The detailed results of the experiment are available in appendix. The first sheet of the Excel file, "Results LRP", contains the results for each of the 42,000 LRP solved. This gives us the characteristic functions of the 6,000 LRG. The sheet also comprises the values of the parameters for each LRP. We also check if each instance of the game is convex. In the second sheet, "Properties of the game", we compute the savings in absolute value and in percentage. Additionally, the sheet mentions the proportion of the total cost coming from the traveling cost, vehicle cost and facility cost. It also mentions if the core of the game is empty or non-empty. In the third sheet, "Cost-allocations", all the cost-allocations are given for each player, for each instance and for each method. We also write if the allocation vector is in the core for each method (except for the nucleolus and the EPM which are always in the core). If the allocation-vector is not in the core, we compute by how much a constraint should be relaxed in the columns "Epsilon core". This epsilon core is compared to the total cost of the grand coalition. Furthermore, we also give the value of the objective function f for the EPM. For the proportional methods, we give the proportion of the cost of the grand coalition allocated to the player, the α.

A comment on the LRG with a limited number of depots We studied the LRG with a limited number of depots in section 3.2.1. However, we have for the uncapacitated version of the LRG a number of depots always equal to one. For the capacitated LRG, the capacity constraints are binding for each instance. Therefore, to add a constraint on the maximum number of depots would either not change our results, either make the problem infeasible.

4.3 Detailed results for one instance

To better understand the workings of the game, we now deeply analyze the results of a random instance.

In table 4.2, data about the coordinates of the customers and candidate facility sites can be found. Company 1 has the customers 1 & 2, company 2 the customers 3 & 4 and company 5 the customers 5 & 6. The vehicle capacity is 70, and the cost per vehicle is 24. The capacity per depot is equal to 120. The values of the characteristic function and the origins of the costs for each coalition are in table 4.3. Remarkably, the cost of the grand coalition compared to the sum of the stand-alone costs is 54.6% lower. The facilities opened and the routes used for each coalition are represented in figure 4.1. The coordinates of the depots and of the customers come table 4.2. from We notice that a different facility is sometimes open depending on the coalition of players. The model makes the trade-off between facility costs, traveling costs and vehicle costs.

We now analyze the cost allocations from table 4.4. In table 4.4, we also write by how much the cost for the player decreases. Given that we know the characteristic function, we also check if the allocation vectors are in the core.

The core of this game is defined by the set of preimputations $x \in \mathbb{R}^3$ satisfying the following inequalities.

$$x_1 \leq 461.5$$

$$x_2 \leq 514.7$$

$$x_3 \leq 564.5$$

$$x_1 + x_2 \leq 553.2$$

$$x_1 + x_3 \leq 627.8$$

$$x_2 + x_3 \leq 581.8$$

$$x_1 + x_2 + x_3 = 699.1$$

The core is non-empty and therefore the nucleolus and the EPM are elements of the core, as we saw in section 2.4. The Shapley value, the demand and cost proportional allocation methods are also in the core for this instance. The Equal Profit Method manages to give the same relative savings for each player.

This game is also non-convex. The conditions from definition 3.5 are not fulfilled. For instance, for $S = \{2\}$, $T = \{1, 2\}$ and $i = 3$, we have that the following equation is not valid.

$$C\{2,3\} - C\{2\} \geq C\{1,2,3\} - C\{1,2\}$$

Indeed,

$$581.8 - 514.7 \leq 699.1 - 553.2$$

$$67.1 \leq 145.9$$

TABLE 4.2: Data for one of the instances studied.

Company (player)	Facility/ Customer	x coordinates	y coordinates	Distance to									Demand	Facility cost
				A	B	C	1	2	3	4	5	6		
	A	70	41	–	37.8	45.0	73.8	37.0	53.2	18.2	30.0	67.6	–	395
	B	100	64	37.8	–	82.0	111.5	74.8	78.0	52.0	30.8	78.0	–	426
	C	43	5	45.0	82.0	–	35.2	13.2	62.6	40.0	71.3	90.3	–	352
Company 1	1	8	1	73.8	111.5	35.2	–	37.1	64.5	61.5	93.5	95.1	20	–
	2	41	18	37.0	74.8	13.2	37.1	–	49.8	28.2	60.4	77.1	11	–
Company 2	3	22	64	53.2	78.0	62.6	64.5	49.8	–	36.1	48.5	30.6	18	–
	4	52	44	18.2	52.0	40.0	61.5	28.2	36.1	–	32.5	55.5	16	–
Company 3	5	70	71	30.0	30.8	71.3	93.5	60.4	48.5	32.5	–	47.9	18	–
	6	28	94	67.6	78.0	90.3	95.1	77.1	30.6	55.5	47.9	–	14	–

4.4 General characteristics of the solutions

4.4.1 Cost origins

For the grand coalition, the facility costs go from 16 to 85% of the total costs, the vehicle costs from 1 to 53% and the traveling costs from 8 to 67%. The histograms of the three costs are in figure 4.2. We can notice the experiment covers a lot of different cases.

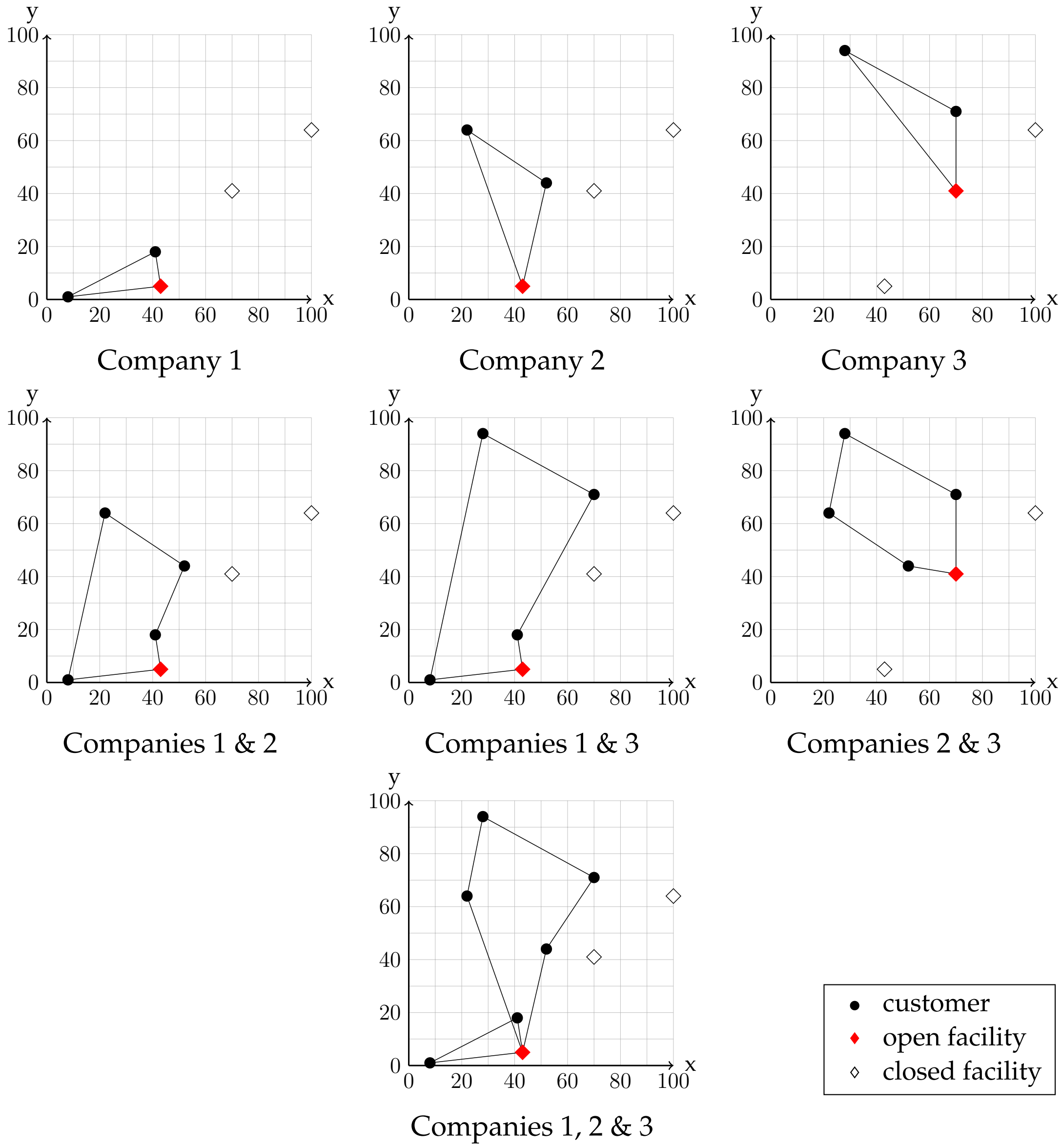


FIGURE 4.1: Representation of the solutions for the seven coalitions of companies.

TABLE 4.3: Origins of the costs for the seven coalitions for the instances studied.

Coalition	Total cost	Traveling cost	%	Vehicle cost	%	Facility cost	%
{1}	461.5	85.5	18.5	24	5.2	352	76.3
{2}	514.7	138.7	26.9	24	4.7	352	68.4
{3}	564.5	145.5	25.8	24	4.3	395	70.0
{1,2}	553.2	177.2	32.0	24	4.3	352	63.6
{1,3}	627.8	251.8	40.1	24	3.8	352	56.1
{2,3}	581.8	162.8	28.0	24	4.1	395	67.9
{1,2,3}	699.1	299.1	42.8	48	6.9	352	50.4

TABLE 4.4: Allocation of costs using different methods and savings with respect to the stand-alone costs.

Company	C({j})	Shapley	%	Nucleolus	%	EPM	%	Cost Proportional	%	Demand Proportional	%
1	461.5	209.9	54.5	238.8	48.2	209.4	54.6	210.9	54.3	230.6	50.0
2	514.7	213.5	58.5	192.8	62.5	233.5	54.6	235.3	54.3	245.0	52.4
3	564.5	275.7	51.2	267.4	52.6	256.1	54.6	252.9	55.2	223.4	60.4
Sum	1540.7	699.1		699.1		699.1		699.1		699.1	
In the core		Yes		Yes		Yes		Yes		Yes	

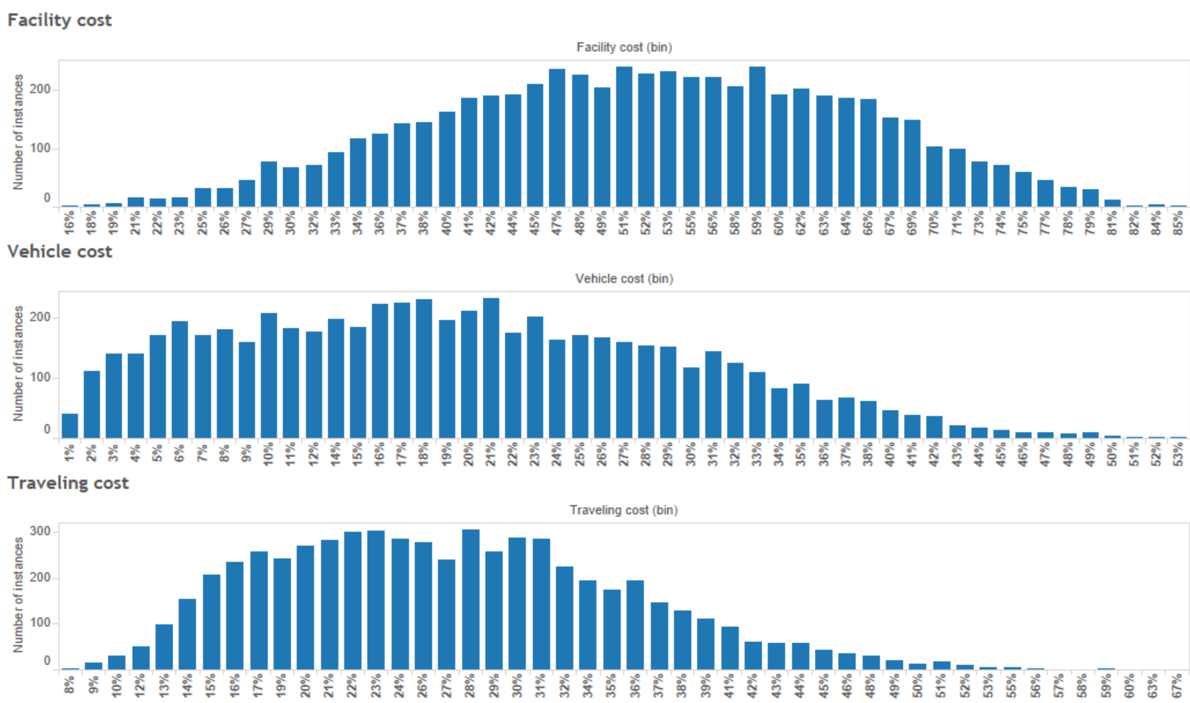


FIGURE 4.2: Histograms of the facility, vehicle and traveling costs.

4.4.2 Savings

The savings for the companies can be substantial and are presented in figure 4.3. Compared to the sum of their stand-alone costs, the total cost for the grand-coalition is between 10 and 66 % inferior. The average and median of the relative savings are both equal to around 43 %. The main source of the savings is the fact that only one or two facilities, instead of three, are necessary.

We can also estimate by how much the CO_2 emissions would decrease. As a proxy for the emissions, we take the total traveling cost. As we can notice in figure 4.4, it decreases on average by 31%. The median is also 31%. However, horizontal collaboration is not always a synonym of decrease in carbon emissions. The total traveling cost increases in 3% of the instances. The LRG, which follows an integrated approach, considers the routing and location decisions. Indeed, the model minimizes the total cost, which includes traveling, facility and vehicle costs.

To reduce the likelihood of an increase in emissions, we could change the objective function to include a component linked to carbon emissions. Alternatively, a constraint on the maximum CO_2 emissions could be added.

Please note that we only took into consideration the CO_2 emissions due to transportation. This underestimates the decline in total CO_2 emissions. In the experiment, the total number of depots decreases by two or one units. We suppose that this has a positive impact on the total CO_2 emissions. For instance, this could decrease the total amount of energy spent on air conditioning and lighting. In the alliance between cosmetics manufacturers we mentioned in section 3.1.2, they moved to the most recent, and likely most energy efficient, warehouse.

The potential decrease in carbon footprint depends on a lot of factors. For the footprint due to transportation, we are aware that the proxy is not perfect. For example, we could have considered that the CO_2 emissions depend on the load of the truck. However, this factor depends a lot on the type of product transported. For instance, the trucks of Tupperware were full in volume but not in weight. The difference in CO_2 emissions between empty and non-empty trucks is then less than for Procter&Gamble. Other example, if the product requires the trucks and warehouses to be refrigerated, this will have a large impact on fuel consumption. This list is non-exhaustive but we can notice numerous intervene. Therefore, since we do not know the nature of the product nor the precise characteristics of the trucks and warehouses, we keep the traveling cost as a proxy for CO_2 emissions.

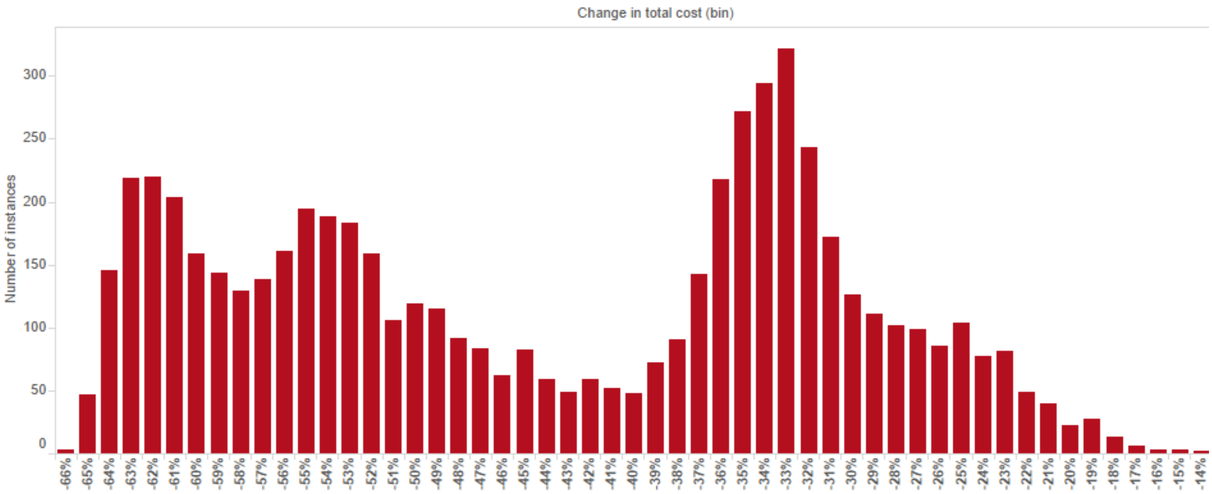


FIGURE 4.3: Histograms of the changes in percentages of the total costs.

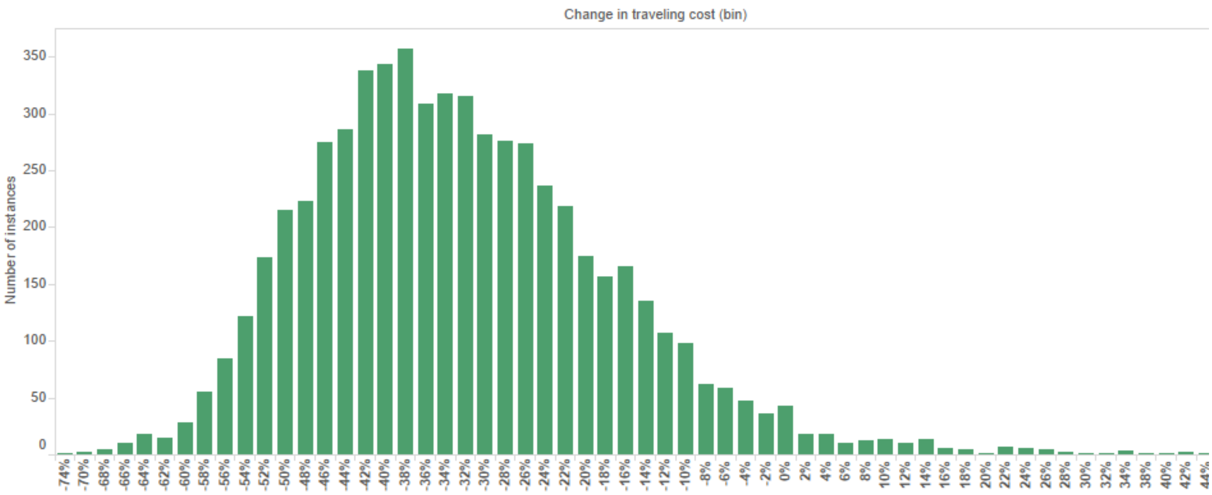


FIGURE 4.4: Histograms of the changes in percentages of the total traveling costs.

4.4.3 Subadditivity

No collaboration could occur in instances that do not have the subadditivity property. In the experiment, all the 6,000 games are subadditive. It is no wonder for the uncapacitated LRGs given our findings in chapter 3. For the capacitated instances, subadditivity does not occur only in really specific situations. This is the case in the example we studied in section 3.2.2. In the experiment, the costs caused by the fact an additional facility is required are lower than the synergies of increased collaboration.

4.4.4 Emptiness of the core

Among the 6,000 instances, 2,651 (44 %) have an empty core. All these instances have facilities with capacity 80 and a total demand above 80. However, nine instances have these characteristics but have a non-empty core. Also, we do not say that instances with a facility capacity above the total demand cannot have an empty core. As shown in chapter 3, this can happen but we do not observe it in the experiment.

4.4.5 Convexity

The cost game is convex in only 149 instances. Among these instances, 125 are uncapacitated. 116 have a vehicle capacity of 100. We already knew from chapter 3 that the LRG is not always convex. However, we did not know how frequent non-convexity occurs. We find here that non-convexity is widespread for the LRG.

To evaluate if a game is convex, we compare the marginal cost of adding a player to a coalition to the marginal cost of adding the same player to a coalition that includes the former coalition. If the second marginal cost is higher in at least one case, the game is non-convex. In the definition of a convex game (definition 3.5), this condition was expressed as follows:

$$C(S \cup i) - C(S) \geq C(T \cup i) - C(T) \quad \forall i \in N, S \subseteq T \subseteq N \backslash \{i\}$$

With three players, the left-hand side is the marginal cost of adding a player to a one-player coalition. The right-hand side is the marginal cost of adding the same player to a two-player coalition.

We now analyze in which cases non-convexity happens for the LRP. Non-convexity is mainly due to the fact that one cost exists on the right-hand side of the equation but not on the left-hand side.

First, we look at the facility cost. Between one and two players, the change in the facility cost is likely to be small. One facility has enough capacity to supply the total demand. The maximum demand is indeed 80 and the minimum capacity is 80. The model will therefore tend to keep only one facility but may decide to change the location of this facility. With three players, demand can become larger than the facility capacity. In that case, an extra facility is necessary. The right-hand side of the equation is then likely to be higher than the left-hand side. In the experiment, we indeed observe that among the 149 convex instances 124 are uncapacitated.

Regarding the vehicle capacities, it is more complex. The vehicle capacities can play a significant role in making a game non-convex if two conditions are fulfilled. First, if going from one to two players does not require an extra vehicle. Second, if going from two to three players does. With a vehicle capacity of 100, the two conditions are not fulfilled in most of the cases. (The average demand per customer is 15. The average total demand for three players is $6 \cdot 15 = 90$.) If the capacity is 70, the two conditions are likely fulfilled. If the capacity is 40, the scenario where both conditions are fulfilled is still likely. However, in some cases, the second but not the first condition is satisfied. In other words, the extra vehicle cost exists for both sides of the equation. Therefore it cancels out. In the experiment, all the instances with a vehicle capacity of 70 are non-convex but one. All but 15 instances are non-convex if the capacity is 40. Among the convex instances, 133 out of 149 have a vehicle capacity of 100. This confirms our insight.

Finally, we look at the traveling cost. A lot of different cases can happen. The traveling cost can be higher on the left-hand side than on the right-hand side and vice-versa. Even when only one facility is open and one vehicle used, transportation cost can be non-concave. (A convex cost game has a concave characteristic function.) It then depends on the location of the customers. There are synergies on routing but these synergies do not always increase with the number of players. If facilities or vehicles are added, the traveling costs can even decrease when more players are added. In other words, it may be non-monotone.

The main factors influencing convexity of an instance are the capacities of the depots and the vehicles. However, non-concavity for one cost does not mean convexity for the game. Particularly when one cost is only slightly higher on the right-hand side than on the left-hand side of the equation. Convexity results from a combination of numerous factors. A slight change in one parameter can break the property for the instance.

4.5 Analysis of the cost allocations

4.5.1 Stability of the allocations

In this subsection, we only consider the instances that have a non-empty core. The EPM and the nucleolus are always in the core per definition. If the game is convex, the

Shapley value is always in the core and is the barycenter of the core (Shapley, 1971). This last property can be useful when there is a large margin of error in the evaluation of the characteristic function. This then reduces the risk the allocation becomes non-stable with the real characteristic function. As we mentioned in the previous section, convexity occurs in only a small number of cases for the LRG. Fortunately, the Shapley value can also be in the core if the game is non-convex. The Shapley value is in the core in 99.7 % of the cases (3,340 out of 3,349 instances). The demand proportional in 96.6 % (3,235 out of 3,349) and the cost proportional in 98.1 % (3,285 out of 3,349) of the cases. This is a first interesting finding, the proportional and the Shapley methods are here almost always stable. The Shapley value is however slightly better than the proportional methods. There is still a risk the coalition disintegrates but this risk seems to be lower than what is found for other games. In the literature, stability is often presented as a weakness of the proportional methods (Guajardo & Rönnqvist, 2016). For a game considering only collaboration on routing, Özener and Ergun (2008) have for instance a value of 75% for the proportional methods.

Size of the relaxation of the rationality constraints When the allocations are not stable, it is useful to evaluate the size of the deviation. A small one is better and can be neglected by the companies. In practice we can indeed confidently suppose there is a margin of error in the companies' cost estimations. A slightly unstable cost-allocation can therefore be *perceived* as stable by the practitioners. The size of the project and the resources of the companies are factors impacting this misrepresentation. How close are the unstable cost allocations to be stable (when the core is non-empty)? We use the concept of the epsilon core to quantify this gap. Given the cost-allocations, we calculate the size of the epsilon. In other words, by how much should at least one stability constraint be relaxed. This epsilon is also the cumulated gain for the companies that leave the grand coalition. For the Shapley value, this epsilon is on average 1.1 % of the total cost, with a maximum of 2.8 %. For the cost proportional method, the average is 2.0 % and the maximum is 10.3 %. For the demand proportional, the average is 2.2 % and the maximum is 8.3 %. These percentages can represent large amounts of money or be almost negligible depending on the project. If small deviations of the stability conditions are acceptable, the Shapley value is in our case better than the proportional methods regarding stability. If even a small deviation is likely to be noticed, both the Shapley value and the proportional methods are not completely satisfying.

Individual rationality When the cost allocations are not stable, we can check if at least the individual rationality constraints are satisfied. The individual rationality constraints are always satisfied for the Shapley value. This is a theoretical property of the Shapley value which is not satisfied by the proportional methods. In the experiment, the demand proportional does not respect it in one instance and the cost proportional in two instances. Regarding this property, we therefore conclude there is no significant difference between the Shapley value and the proportional methods.

4.5.2 Behavior when the core is empty

If the core is empty, this means that no cost-allocation would be stable. Therefore, the grand coalition will never form. Why then study the cases where the core is empty? A regulatory body, taking into account the welfare loss, might force players to collaborate. For instance, Button (2003) studies if regulation is needed when the core is empty in the airline industry. A possible example in logistics would be that the state imposes some public institutions to cooperate. This to maximize the synergies. The grand coalition can also be formed by mistake. Tseng et al. (2013) write that companies focus mainly on individual rationality constraints. The other constraints are sometimes neglected in the short run. Of course, unstable coalitions do not tend to last in the long run (Tseng et al., 2013).

For the LRP, emptiness of the core is frequent. We here check the behavior of the cost allocations. We assume the grand coalition is formed for whatever reason.

We would then advise to use the EPM or the nucleolus. Indeed, for these methods the epsilon is minimum. This epsilon is the same as the one found to compute the epsilon-core. This epsilon is on average equal to 9.7% for the EPM and the nucleolus.

To take the Shapley value or the proportional methods risks increasing even more the risk of dislocation of the coalition. The epsilon represents on average 10.9% of the total cost for the Shapley value, 13.1 % for the demand proportional and 14.4% for the cost proportional.

4.5.3 Similarities between the allocations obtained

Euclidean distances between the vectors Since it is difficult to compute the nucleolus, it is interesting to know which measures could the best approximate it. Engevall et al. (2004) propose to evaluate this ability by using the Euclidean distance between two vectors y and v: $\sqrt{\sum_{i \in N}(y_i - v_i)^2}$. In table 4.5, we notice that the Euclidean distances between the Shapley value, the EPM and the nucleolus are approximately the same. Given that the average total cost for the grand coalition is 1091.3, we can conclude that the allocations are relatively similar. When the core is empty, the nucleolus and the EPM give exactly the same cost allocations.

TABLE 4.5: Averages of the Euclidean distances between the cost allocations vectors.

	Nucleolus	**EPM**	**Cost proportional**	**Demand proportional**
Shapley	22.05	23.32	64.96	81.18
Nucleolus	–	19.61	70.61	79.58
EPM	–	–	60.56	75.86
Cost proportional	–	–	–	92.35

Correlations We here investigate how the allocations methods relate to each other. The correlations between the allocation methods for the uncapacitated and capacitated cases are in table 4.6 and table 4.7. We only observe minor differences between the two tables. We can notice the Shapley value, the nucleolus and the EPM are strongly correlated with each other. The linear relationships of these methods with the proportional methods are lower. This is valid for both the capacitated and the uncapacitated cases. Also, the demand proportional is the less well correlated with the stand-alone cost. This is without surprise since the stand-alone cost is not used in the computations of the demand proportional. This contrary to the other methods.

TABLE 4.6: Correlations between the allocation methods and the stand-alone cost for the uncapacitated case.

	Shapley	**Nucleolus**	**EPM**	**Cost Proportional**	**Demand Proportional**	**Stand-alone cost**
Shapley	1.00	0.98	0.98	0.85	0.81	0.73
Nucleolus	0.98	1.00	0.94	0.82	0.82	0.69
EPM	0.98	0.94	1.00	0.89	0.85	0.73
Cost Proportional	0.85	0.82	0.89	1.00	0.77	0.64
Demand Proportional	0.81	0.82	0.85	0.77	1.00	0.59
Stand-alone cost	0.73	0.69	0.73	0.64	0.59	1.00

TABLE 4.7: Correlations between the allocation methods and the stand-alone cost for the capacitated case.

	Shapley	**Nucleolus**	**EPM**	**Cost Proportional**	**Demand Proportional**	**Stand-alone cost**
Shapley	1.00	0.98	0.98	0.85	0.79	0.78
Nucleolus	0.98	1.00	0.95	0.82	0.80	0.75
EPM	0.98	0.95	1.00	0.89	0.84	0.79
Cost Proportional	0.85	0.82	0.89	1.00	0.77	0.70
Demand Proportional	0.79	0.80	0.84	0.77	1.00	0.65
Stand-alone cost	0.78	0.75	0.79	0.70	0.65	1.00

4.5.4 Allocation methods and parameters

In Naber, de Ree, Spliet, and van den Heuvel (2015), they investigate the robustness of the allocation methods for a vehicle routing game when the characteristics of the customers vary. We are also interested in the link between the characteristics of the game and the cost allocations. We consider the correlations between the parameters and the different allocation methods. This is a useful indicator to evaluate how dependent is an allocation method to the modification of a parameter.

We are aware of the limitations of the correlations. The main ones are that the correlations do not consider non-linear relationships. Therefore, we plotted the data and did not notice non-linear relationships between the parameters and the allocation methods.

We compute a proxy for the concentration of the customers and facilities: the average of the traveling costs of the instance [2]. We also compute the expected vehicle cost, which is an estimation of the real future cost based on known parameters. It is the total demand divided by the vehicle capacity and multiplied by the cost per vehicle.

In table 4.8 and table 4.9, we find the correlations between the parameters of the game and the allocation methods for the uncapacitated and the capacitated case. We also include the origins of the costs in the table, even if they are calculated after the simulations.

The correlations between the stand-alone cost and the parameters are also mentioned. They serve as a way to double check the results from our integer model. Given the values of the parameters, a lot of correlations are close to 0. For instance, there is no reason to have a link between the vehicle capacity and the stand-alone cost. Indeed, the vehicle capacity constraint is never binding when there is only one player.

As expected the α demand (demand of the player divided by total demand) is positively correlated with the demand proportional method. The same applies for the cost proportional method.

Interesting is the fact that the expected vehicle cost is strongly correlated ($\rho \approx 0.7$) with all the allocation methods. This means that if this expected vehicle cost increases, it is very likely that the player's allocation will increase, too. We can also note that the allocations costs are completely uncorrelated with the total demand. An increase of the total demand can therefore have an unforeseen impact on the cost allocated to the company. The same is also true for the player's demand, except of course for the demand proportional allocation method. We also notice that all the cost-sharing technique follow a rational behavior. If the constraint on the vehicle capacity is relaxed, the allocations tend to decrease. If the costs increase the allocations tend to increase, etc.

[2] $\sum_{i \in V} \sum_{j \in V : j \neq i} \frac{c_{ij}}{|V|(|V|-1)}$

TABLE 4.8: Correlations between the allocation methods and the parameters of the game for the uncapacitated case.

	Shapley	**Nucleolus**	**EPM**	**Cost Proportional**	**Demand Proportional**	**Stand-alone cost**
Vehicle capacity	-0.54	-0.53	-0.57	-0.52	-0.51	0.00
Vehicle cost	0.47	0.47	0.50	0.46	0.45	0.43
Expected vehicle cost	0.72	0.72	0.77	0.71	0.69	0.33
Average traveling cost	0.20	0.20	0.21	0.20	0.19	0.18
Total demand	0.08	0.08	0.09	0.08	0.08	0.01
Demand of the player	0.07	0.09	0.05	0.03	0.43	0.00
Average facility cost	0.45	0.45	0.48	0.44	0.43	0.81
α demand	0.03	0.05	0.00	-0.02	0.47	0.00
α cost	0.08	0.02	0.06	0.44	-0.02	0.08
Total traveling cost	0.52	0.51	0.55	0.50	0.49	0.12
Total vehicle cost	0.74	0.73	0.78	0.72	0.71	0.33
Total facility cost	0.48	0.48	0.51	0.47	0.46	0.82

TABLE 4.9: Correlations between the allocation methods and the parameters of the game for the capacitated case.

	Shapley	**Nucleolus**	**EPM**	**Cost Proportional**	**Demand Proportional**	**Stand-alone cost**
Vehicle capacity	-0.23	-0.23	-0.23	-0.22	-0.21	-0.01
Vehicle cost	0.41	0.41	0.41	0.39	0.38	0.45
Expected vehicle cost	0.52	0.51	0.52	0.49	0.47	0.34
Average traveling cost	0.12	0.12	0.12	0.11	0.11	0.14
Total demand	0.32	0.32	0.32	0.31	0.30	0.01
Demand of the player	0.20	0.22	0.22	0.17	0.56	0.00
Average facility cost	0.62	0.62	0.62	0.58	0.57	0.82
α demand	0.02	0.04	0.04	-0.01	0.47	0.00
α cost	0.05	0.01	0.02	0.42	-0.01	0.08
Total traveling cost	0.24	0.24	0.24	0.23	0.22	0.11
Total vehicle cost	0.56	0.55	0.56	0.52	0.51	0.40
Total facility cost	0.80	0.80	0.80	0.76	0.73	0.67

4.5.5 Computation times of the allocation methods

The computation times for all the allocation methods are small. The number of players is indeed only equal to three. For the proportional methods and the Shapley value, the computation times are almost non-existent. For the nucleolus and the EPM, they are measured in tenths of seconds.

However, with more players we suppose the computation times will increase faster for the nucleolus and the EPM. This has been noticed by Naber et al. (2015) for a variant of the vehicle-routing game.

Chapter 5

Conclusion

In this dissertation we studied for the first time collaborative game theory in location-routing. Thanks to principles in game theory and models from the Operations Research literature, we defined the location-routing game. We then answered the question of what the properties of this newly defined game are. The next question regarded how the companies should allocate the costs of the coalition for the LRG.

We defined the location-routing game as a transferable utility cooperative game. The characteristic function is computed through solving several LRPs. We chose the characteristics of the location-routing problem and we provided with two different integer models. This location-routing problem can indeed be modeled in several ways. We also defined two extensions of the location-routing game.

Regarding the properties of the game, we found that this game is monotone and subadditive. Therefore, there is potential for collaboration. However, subadditivity does not always hold in the extensions of the game. We presented one counter-example where subadditivity is not fulfilled when depots have limited capacities. Nevertheless, subadditivity held in all the instances of the experiment. This suggests that non-subadditivity for the LRG is rare in reality. The core can be empty in the location-routing game and particularly so when some conditions on the capacities of the depots and of the vehicles are fulfilled. In the experiment, around 55% of the instances have a non-empty core. In these instances, it is therefore possible for the players to form a stable coalition. We also studied the convexity of the game. We found the LRG is not convex in almost 98% of the cases. The capacity of depots and vehicles play a role in making a game non-convex.

This game enables to make significant savings. Indeed, they reached on average 43% of the total cost. Collaboration in LR also decrease carbon emissions in the majority of the cases. On average this reduction amounts to around 30 %. This supports the fact collaboration can increase the sustainability of supply chains. However, we found that in 3% of the cases CO_2 emissions can increase. Given the integrated approach of the LRP, the model sometimes increases traveling costs in order to minimize the total cost. This effect can be mitigated by modifying the integer model.

Regarding the cost-allocation methods, we computed for each instance the Shapley value, the Equal Profit Method (EPM), the nucleolus, the demand proportional and the cost proportional. Stability of the cost allocation is a crucial characteristic. We first only considered instances in which the core is non-empty. As expected, the Equal Profit Method and the nucleolus are always stable. Regarding the Shapley value, in theory it is not always in the core. However, for the LRG it is in the core in 99.7% of the instances. An interesting finding is that the proportional methods are stable in more than 96.6% of the cases. Usually it is the main weakness of these methods. An analysis on the behavior of cost allocations when the core is empty was also performed. Again, the nucleolus and the EPM are preferred regarding stability. They are then followed by the Shapley value and finally the proportional methods.

We also investigated how close the allocations found by the different methods are. The EPM, the Shapley value and the nucleolus tend to evolve together. The link is much weaker with the proportional methods. We also looked at the robustness of the cost allocation methods regarding a change in a parameter. On most parameters, all the cost-sharing techniques have similar results.

Fairness of the cost-allocation is another key characteristic for the ideal methods. There is no widely accepted definition of fairness in game theory. In the literature, the Shapley value is sometimes considered as the fairest method because it fulfills a certain set of properties. Others consider that the EPM or the nucleolus are fairer.

A more important criterion is in our opinion the transparency of the method. It is indeed crucial to take human aspects into account while considering collaboration. In game theory the focus is on stability of the coalitions. However, even stable alliances sometimes fail because there is not enough trust between partners. Collaboration with a method that does not fulfill all the theoretical properties is better than no collaboration at all. If the cost-sharing technique is not accepted by the companies, this might undermine the stability of the coalition. On transparency, the proportional methods and the EPM are the best performers.

To sum up, the proportional methods are intuitive and have here a good record regarding stability. Additionally, they are easily computable given they do not require all the values of the characteristic function. The EPM and the Shapley value are also strong candidates. When the core is non-empty, they always give stable allocations and are easy to compute if the number of players is not too high. However, they require the complete characteristic function. We found the nucleolus performs always equally or worse than the EPM on all the criteria. The only exception is when there is a risk that a dummy player joins the coalition. In conclusion, there is no definitive answer on which cost-sharing technique is the best for the LRG. The choice of the ideal allocation method depends on the complexity of the problem and on the expectations of the companies.

Limitations of the findings Regarding the experiment, the limitations are linked to the parameters. Other choices could have been made on the values of the parameters. The results might then be different. However, the experiment has already covered a large variety of cases.

The location-routing game was defined as a cooperative game with transferable utility. We mentioned the theoretical assumptions of a TU game and provided with several counter-examples. For instance, collaboration can positively affect the value of intangible assets. Before collaboration starts, it is indeed difficult to estimate the positive impact caused by a decrease of the company's carbon footprint. Therefore, this effect is not included in the characteristic function. If this factor is taken into account, collaboration can occur even if the core of the game is empty. The opposite situation can also happen. The core may be non-empty but it is possible the players do not collaborate. This can be due to legal aspects, a lack of trust between partners, an impression the allocations are not fair, etc. In that case, neutral trustees can help solve this problem.

Further research It would be interesting to investigate extensions of the location-routing game by modifying the class of the LRP we defined in section 2.1.1. The location-routing game used in a dynamic setting could be one extension of the LRG. The location-routing game deals with decisions on the strategic and operational levels. This dynamic setting would create a link between these decision levels.

The number of nodes could also be increased. Further research could focus on what happens with the LRG when the problem is too complex and heuristics must be used.

Additionally, to study the location-routing game with more players would be a useful contribution.

Regarding the theoretical properties of the LRG, it would be beneficial to find the mathematical conditions for the core to be empty. Similar proofs have been seen for related games. For the uncapacitated facility location game, Chardaire (1998) mentions the difficulty (NP-completeness) of determining if the core of the game is non-empty. Goemans and Skutella (2004) show for the facility location problem that the core is non-empty if and only if there is no integrality gap for the relaxation of this problem. J. Li et al. (2016) adapt the proof for their own problem.

The same is proved for the basic vehicle-routing problem by Göthe-Lundgren et al. (1996). Tamir (1989) shows that the core of a traveling salesman game (TSG) is always non-empty when $|N| \leq 4$. However, they also show that the core of a TSG can be empty in some other cases. Sun and Karwan (2015) prove that if a TSG is defined on a route with no-integrality gap, the core is non-empty.

Therefore, we suppose there might be a link between emptiness of the core and an integrality gap of the relaxation of the problem for the LRG.

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