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# Strategic Technology Switching under Risk Aversion and Uncertainty

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Strategic Technology Switching under Risk Aversion and Uncertainty

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**Abstract**

Sequential investment opportunities or the presence of a rival typically hasten investment under risk neutrality. By contrast, greater price uncertainty or risk aversion increase the incentive to postpone investment in the absence of competition. We analyse how price and technological uncertainty, reflected in the random arrival of innovations, interact with attitudes towards risk to impact both the optimal technology adoption strategy and the optimal investment policy within each strategy, under a proprietary and a non-proprietary duopoly. Results indicate that technological uncertainty increases the follower's investment incentive and delays the entry of the non-proprietary leader, yet it does not affect the proprietary leader's optimal investment policy. Additionally, we show that technological uncertainty decreases the relative loss in the value of the leader due to the follower's entry, while the corresponding impact of risk aversion is ambiguous. Interestingly, we also find that a higher first-mover advantage with respect to a new technology does not affect the leader's entry, and that technological uncertainty may turn a pre-emption game into a war of attrition, where the second-mover gets the higher payoff.

*Keywords:* investment analysis, real options, competition, risk aversion

## 1. Introduction

Emerging technologies are subject to frequent upgrades, that become available at random points in time, and the firm that adopts them first can capture a greater market share (Lieberman & Montgomery, 1988; Zachary *et al.*, 2015). Hence, firms investing in emerging technologies must take into account both strategic interactions and the sequential nature of such investments. Furthermore, emerging technologies typically entail technical risk that cannot be diversified, and, therefore, firms are likely to exhibit risk aversion. Indeed, the underlying commodities of such projects are typically not freely traded, thus preventing the construction of a replicating portfolio. Consequently, risk-neutral valuation may not be possible as the assumption of hedging via spanning assets breaks down. Although various models have been developed in order to analyse sequential investment under price and technological uncertainty, most of these either ignore strategic interactions (Grenadier & Weiss, 1997; Doraszelski, 2001; Chronopoulos & Siddiqui, 2015) or assume risk neutrality (Huisman & Kort, 2003, 2004; Weeds, 2002). Consequently, how strategic interactions impact sequential investment decisions and how price and technological uncertainty interact with risk aversion to impact the optimal investment policy remain important open research questions.

Incorporating such features in an analytical framework for sequential investment is crucial as these are pertinent to various industries, e.g., computer software, telecommunications, pharmaceutical, etc. For example, firms producing brand-name drugs enjoy high revenues so long as their patents are protected. In the early 1980s, a drug which soothes both pain and inflammation was a costly patented product. Today, Boots, a British chemist, sells generic tablets for just 2.5 pence per pill (Wall Street Journal, 2013a). In the area of telecommunications, Apple's iPhone sales declined prior to the introduction of iPhone 4s in 2012, while, at the same time, Samsung's Galaxy $S_3$, the closest rival to Apple's market leading iPhone, took close to 18% of the market (Financial Times, 2012). The legal debate between Apple and Samsung reflects a highly competitive environment in which firms can potentially profit from adopting other firms' patented technologies. Of course, there are various other competitive advantages that a firm may have, that may not be related to the adoption of patented technologies, however, their analysis is beyond of the scope of this paper. For example, Samsung is more vertically integrated than Apple, and, thus, can bring products to the market more quickly (Wall Street Journal, 2013b).

We consider the case of duopolistic competition, where two identical firms invest sequentially in technological innovations facing price and technological uncertainty. Within this context, we analyse the case of proprietary and non-proprietary duopoly. The former occurs when a firm controls the innovation process, and, therefore, does not face the threat of pre-emption. By contrast, the latter occurs when the innovation process is exogenous to both firms, and, therefore, they fight for the leader's position. Hence, we contribute to the existing literature by first developing a utility-based framework for sequential investment in order to analyse how price and technological uncertainty interact with risk aversion to impact investment under duopolistic competition. Second, we derive analytical expressions, where possible, for the optimal entry threshold of the leader and the follower. Thus, for each firm, we determine both the optimal technology adoption strategy, and, within each strategy, the optimal investment rule. Finally, we provide managerial insights for investment decisions based on analytical and numerical results.

We proceed by discussing some related work in Section 2 and introduce assumptions and notation in Section 3. We begin the analysis with the benchmark case of monopoly in Section 4. In Section 5, we assume that firms adopt each technology that becomes available (compulsive strategy) and analyse the case of proprietary and non-proprietary duopoly in Sections 5.1 and 5.2, respectively. In Section 5.3, we also consider how pre-emption may lead to a war of attrition. In Section

6, we assume that a firm may wait for a new technology to become available before deciding to either skip an old technology and invest directly in the new one (leapfrog strategy) or to adopt the old technology first and then the new one (laggard strategy). In Section 7, we provide numerical results for each case and illustrate how attitudes towards risk interact with price and technological uncertainty to impact not only the optimal technology adoption strategy but also the optimal investment rule. Section 8 concludes the paper and offers directions for further research.

## 2. Related Work

Real options models often address the problem of optimal investment timing without considering strategic interactions (McDonald & Siegel, 1985 and 1986), while the ones that do, either ignore the sequential nature of investment opportunities (Pawlina & Kort, 2006; Siddiqui & Takashima, 2012) or attitudes towards risk (Huisman & Kort, 2015). In the area of competition, Spatt & Sterbenz (1985) analyse how the degree of rivalry impacts the learning process and the decision to invest, and find that increasing the number of players hastens investment and that the investment decision resembles the standard NPV rule. Via a deterministic model, Fudenberg & Tirole (1985) show that a high first-mover advantage results in a pre-emption equilibrium with dispersed adoption timings, as it increases a firm's incentive to pre-empt investment by its rivals. Smets (1993) first developed a continuous-time model of strategic real options under product market competition, stochastic demand and irreversibility. Extending the framework of Fudenberg & Tirole (1985), Huisman & Kort (1999) find that uncertainty creates a positive option value of waiting that raises the required investment threshold. Specifically, in deterministic models a high first-mover advantage leads to a pre-emptive equilibrium, yet, in stochastic models, higher uncertainty may turn a pre-emptive into a simultaneous investment equilibrium.

In the same line of work, Lambrecht & Perraudin (2003) incorporate incomplete information into an equilibrium model in which firms invest strategically. Paxson & Pinto (2005) develop a rivalry model that allows for price and quantity uncertainty, and, among other results, they find that an increase in the correlation between the profits per unit and the quantity of units produced raises their aggregate volatility, and, in turn, the investment trigger of both the leader and the follower. Takashima *et al.* (2008) assess the effect of competition on the investment decision of firms with asymmetric technologies under price uncertainty. They show how mothballing options facilitate investment, thereby offering a competitive advantage to a thermal power plant over a nuclear power plant. By contrast, lower variable and construction costs favour coal- and oil-thermal power plants. Bouis *et al.* (2009) analyse investment in markets with more than two identical competitors. In the setting including three firms, they find that, if the entry of the third firm is delayed, then the second firm has an incentive to invest earlier so that it can enjoy the duopoly market structure for a longer time. This increases the incentive for the first firm to delay investment, as it faces a shorter period in which it can enjoy monopoly profits. In the same line of work, Armada *et al.* (2011) introduce a setting with several competitors who arrive according to a Poisson process. Also, Graham (2011) finds that an equilibrium may not exist when allowing for asymmetric information over revenues, Thijssen *et al.* (2012) present an analytical model that deals with the coordination problem in pre-emptive competition, and Siddiqui & Takashima (2012) explore the extent to which sequential decision making offsets the impact of competition. Lavrutich (2017) investigates entry and exit decisions under capacity sizing and duopolistic competition, and finds that the follower can strategically set capacity such that the leader has an incentive to exit. A detailed overview of game-theoretic real options models is offered by Azevedo & Paxson (2014).

Examples of early work in the area of investment under technological uncertainty include Balcer & Lippman (1984), who analyse the optimal timing of technology adoption taking into account the expected flow of technological progress. A model for sequential investment in technological innovations is developed by Grenadier and Weiss (1997), who assume that a firm may either adopt each technology that becomes available (compulsive), or wait for a new technology to arrive before adopting either the new (leapfrog) or the old technology (laggard), or purchase only an early innovation (buy and hold). Their results indicate that a firm may adopt an available technology even though more valuable innovations may occur in the future, while decisions on technology adoption are path dependent. Assuming that innovations follow a Poisson process, Farzin *et al.* (1998) investigate the impact of technological uncertainty on the optimal timing of technology adoption, yet ignore price uncertainty. Doraszelski (2001) revisits the analytical framework of Farzin *et al.* (1998) and shows that, compared to the net present value (NPV) approach, a firm will defer technology adoption when it takes the option value of waiting into account. Weeds (1999) analyses the decision to invest in a research project and finds that increasing technological and economic uncertainty postpone investment, while technological uncertainty may accelerate abandonment when the profitability of the project declines. Also, Chronopoulos & Siddiqui (2015) find that uncertainty over the arrival of innovations facilitates the adoption of an existing technology. Lukas *et al.* (2017) consider an uncertain technological lifetime and show how optimal capacity is related to a product's life-cycle. Although the aforementioned papers offer a comprehensive analysis of investment under technological uncertainty, they assume a risk-neutral decision-maker and ignore the implications of strategic interactions.

Allowing for economic and technological uncertainty, Weeds (2002) analyses strategic investment in competing research projects and identifies the existence of non-cooperative and cooperative games. The former involve i. a pre-emptive competition where firms invest sequentially and ii. a symmetric outcome in which investment is more delayed than in the case of monopoly. The latter involves sequential investment, yet compared to the non-cooperative (pre-emptive leader-follower) game, the investment triggers are higher. Also, compared to the optimal cooperative investment pattern, investment is found to be more delayed when firms act non-cooperatively as each refrains from investing in the fear of starting a patent race. Miltersen & Schwartz (2004) analyse how competition in the development and marketing of a product impacts investment in R&D. They find that competition not only increases production and reduces prices, but also shortens the development stage and raises the probability of a successful outcome. Huisman & Kort (2004) study a dynamic duopoly in which firms compete in the adoption of new technologies under price and technological uncertainty. Their results indicate that taking into account the likely arrival of a new technology could turn a pre-emption game into one where the second mover gets the highest payoff. Leippold & Stromberg (2017) extend Huisman & Kort (2004) by allowing for market incompleteness and find that undiversifiable risk may accelerate technology adoption.

Other examples of analytical models for investment under uncertainty that allow for risk aversion include Henderson & Hobson (2002), who extend Merton (1969) by taking the perspective of a risk-averse decision-maker facing incomplete markets. More specifically, they introduce a second asset into the framework of Merton (1969) on which no trading is allowed and address the question of how to price and hedge this random payoff. Alvarez & Stenbacka (2004) implement attitudes towards risk via a hyperbolic absolute risk aversion (HARA) utility function and develop an analytical framework for optimal regime-switching. They show that if the decision-maker is risk seeking, then increasing price uncertainty does not necessarily decelerate investment. A similar result is indicated in Henderson (2007), who shows that idiosyncratic risk raises the incentive to accelerates investment and lock in the investment payoff. Hugonier & Morellec (2013) use the framework of Karatzas & Shreve (1999) in order to determine the analytical expression for the expected utility of a perpetual stream of cash flows that follows a geometric Brownian motion (GBM). Thus, they express the investment policy as the solution to an optimal stopping-time problem and find that greater risk aversion lowers the expected utility of the project and reduces the probability of investment. By contrast, Chronopoulos *et al.* (2011) show that operational flexibility in the form of suspension and resumption options mitigates the impact of risk aversion and increases the incentive to invest. An extension of the utility-based framework to allowing for Markov-regime switching is described in Chronopoulos & Lumbreras (2017). Although these papers address the impact of risk aversion on investment and operational decisions under price uncertainty, they ignore the implications of both technological uncertainty and competition.

We consider two identical firms that invest sequentially in technological innovations and determine how competition interacts with price and technological uncertainty to affect the technology adoption strategy of each firm. The random arrival of innovations is modelled via a Poisson process, while price uncertainty is modelled via a geometric Brownian motion (GBM). We analyse three strategies, i.e., compulsive, leapfrog, and laggard, and determine the feasibility of each strategy under different levels of risk aversion, price, and technological uncertainty. Results indicate that, technological uncertainty has a non-monotonic impact on the required investment threshold of the follower and the non-proprietary leader, yet it does not impact the proprietary leader's optimal investment policy. Furthermore, we find that technological uncertainty decreases the leader's relative loss in value due to the presence of a rival. We also show how technological uncertainty may induce a firm to skip an existing technology in order to invest in a more efficient one. Finally, by comparing a compulsive with a leapfrog/laggard strategy under proprietary duopoly, we find that the latter strategy may dominate, provided that the rate of innovation and the output price is high.

## 3. Assumptions and Notation

Given a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$, we introduce technological uncertainty by assuming that innovations follow a Poisson process $\{M_t, t \geq 0\}$, where $t$ is continuous and denotes time. We assume that the output price $\{E_t, t \geq 0\}$ is independent of the process $\{M_t, t \geq 0\}$, and evolves according to a GBM, as in (1), where $\mu$ is the annual growth rate, $\sigma$ is the annual volatility, and $dZ_t$ is the increment of the standard Brownian motion. Also, $\rho > \mu$ denotes the subjective discount rate and $r$ is the risk-free rate.

$$dE_t \quad = \quad \mu E_t dt + \sigma E_t dZ_t, \quad E_0 \equiv E > 0 \tag{1}$$

The firms' risk preferences are described by a HARA utility function, which is indicated in (2). Note that standard economic theory assumes that decision-makers are typically risk averse and that risk-seeking behaviour is less plausible (Pratt, 1964). Nevertheless, we assume that $\gamma \in [0.7, 1.3]$, and, thus, examine the implications of both risk-averse $\gamma \in [0.7, 1]$ and risk-seeking behaviour $\gamma \in [1, 1.3]$ to enable comparisons with both Hugonnier & Morellec (2013) and Chronopoulos & Lumbreras (2017). Nevertheless, our analysis can accommodate a wide range of utility functions, e.g., hyperbolic absolute risk aversion (HARA), constant absolute risk aversion (CARA), and constant relative risk aversion (CRRA) utility functions.

$$U(E) \quad = \quad \frac{E^{\gamma}}{\gamma}, \quad \gamma > 0 \tag{2}$$

We let $a = p, n$ denoting proprietary and non-proprietary duopoly, respectively, and $b = m, \ell, f$ denoting the monopolist, the leader and the follower, where the leader is the first firm to enter the market in the case of competition. Also, we assume that each firm holds perpetual options to invest in two technologies, each with an infinite lifetime. There is no operating cost associated with each technology, while the investment cost is $I_i$, $i = 1, 2$ $(I_1 \leq I_2)$ and the corresponding output is $D_i$, where $D_{\underline{i}}$ or $D_{\overline{i}}$ indicates that there is one or two firms in the market, receptively. Hence, $D_i$ is decreasing in number of active firms and increasing in $i$. Thus, depending on the number of firms in the industry, a firm's option to invest in technology $i$ while operating technology $i-1$ is denoted by $F^{ab}_{i-1,i}(\cdot)$, and the expected utility from operating technology $i$ inclusive of embedded options is denoted by $\Phi_i^{ab}(\cdot)$. The time of investment and the optimal investment threshold are denoted by $\tau_{i-1,i}^{ab}$ and $\varepsilon_{i-1,i}^{ab}$, respectively.

## 4. Benchmark Case: Monopoly

First, We consider the benchmark case where a firm holds a single investment opportunity. This has already been analysed in Hugonier & Morellec (2013), but we present the analysis here for ease of exposition and to allow for comparisons. The key insight is to decompose all the cash flows of the project into disjoint time intervals. Hence, we assume that the firm has initially placed the amount of capital required for investment in a certificate of deposit and earns a risk-free rate $r$. Thus, the firm earns the instantaneous utility $U(rI_1)$. At time $\tau_{0,1}^m$, the firm swaps this risk-free cash flow in return for the instantaneous utility $U(E\underline{D_1})$, as shown in Figure 1.

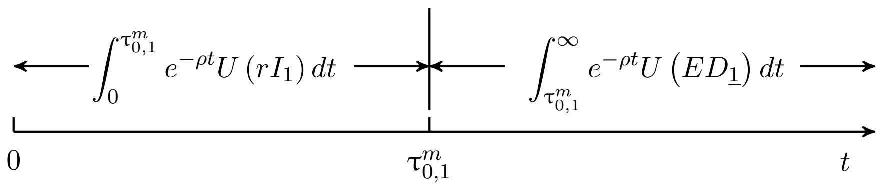


Figure 1: State-transition diagram for a monopolist

The time-zero expected discounted utility of all the cash flows of the project is described in (3), where $\mathbb{E}_E[\cdot]$ denotes the expectation operator that is conditional on the initial output price, $E$.

$$\mathbb{E}_E\left[\int_0^{\tau_{0,1}^m} e^{-\rho t} U(rI_1)\,dt + \int_{\tau_{0,1}^m}^{\infty} e^{-\rho t} U\left(E\underline{D_1}\right) dt\right] \tag{3}$$

By decomposing the first integral, we can rewrite (3) as in (4).

$$\int_0^{\infty} e^{-\rho t} U(rI_1)\,dt + \mathbb{E}_E\left[\int_{\tau_{0,1}^m}^{\infty} e^{-\rho t}\left[U\left(E\underline{D_1}\right) - U(rI_1)\right] dt\right] \tag{4}$$

Notice that the first term in (4) is deterministic, as it does not depend on the investment threshold. Therefore, the optimisation objective is reflected in the second term and can be written as in (5) using the law of iterated expectations and the strong Markov property of the GBM. The latter states that the values of the process $\{E_t, t \geq 0\}$ after time $\tau_{0,1}^m$ are independent of the values of the process before time $\tau_{0,1}^m$ and depend only on the value of the process at time $\tau_{0,1}^m$.

$$F_{0,1}^m(E) \quad = \sup_{\tau_{0,1}^m \in \mathcal{S}} \mathbb{E}_E\left[e^{-\rho \tau_{0,1}^m}\right] \mathbb{E}_{E_{0,1}^m}\left[\int_0^{\infty} e^{-\rho t}\left[U\left(E\underline{D_1}\right) - U(rI_1)\right] dt\right] \tag{5}$$

Note that the stochastic discount factor is $\mathbb{E}_E\left[e^{-\rho\tau}\right] = \left(\frac{E}{E_\tau}\right)^{\beta_1}$ (Dixit & Pindyck, 1994), $\beta_1 > 0, \beta_2 < 0$ are the roots of the quadratic $\frac{1}{2}\sigma^2\beta(\beta-1)+\mu\beta-\rho = 0$, and $\mathcal{S}$ is the set of stopping times generated by the filtration of the process $\{E_t, t \geq 0\}$. Using Theorem 9.18 of Karatzas & Shreeve

(1999), the maximised expected value of the option to invest can be expressed as in (6)

$$F_{0,1}^{m}(E) = \max_{E_{0,1}^{m} > E} \left( \frac{E}{E_{0,1}^{m}} \right)^{\beta_1} \Phi_1^{m}\left(E_{0,1}^{m}\right) \tag{6}$$

where

$$\Phi_1^{m}(E) = \Upsilon U\left(E_{0,1}^{m} D_{\underline{1}}\right) - \frac{U(rI_1)}{\rho}, \quad \Upsilon = \frac{\beta_1\beta_2}{\rho(\beta_1-\gamma)(\beta_2-\gamma)} \tag{7}$$

Solving this unconstrained optimisation problem, we obtain the optimal investment threshold that is indicated in (8) (all proofs can be found in the appendix). Note that, while it is common to formulate the investment threshold in terms of $\beta_1$, it is more expedient to use $\beta_2$ in our case, due to the relationship $\beta_1\beta_2 = -2\rho \,/\, \sigma^2$. Also, the second-order sufficiency condition (SOSC) requires the objective function to be concave at $\varepsilon_{0,1}^{m}$, which is shown in Chronopoulos & Lumbreras (2017).

$$\varepsilon_{0,1}^{m} = rI_1 \left[ \frac{\beta_2 - \gamma}{\beta_2 D_{\underline{1}}^{\gamma}} \right]^{\frac{1}{\gamma}} \quad \text{and} \quad A_{0,1}^{m} = \left( \frac{1}{\varepsilon_{0,1}^{m}} \right)^{\beta_1} \left[ \Upsilon U\left(\varepsilon_{0,1}^{m} D_{\underline{1}}\right) - \frac{U(rI_1)}{\rho} \right] \tag{8}$$

## 5. Compulsive strategy

### *5.1. Proprietary Duopoly*

*Follower*

We extend the benchmark case by assuming that there are two firms in the market competing in the adoption of technological innovations. First, we consider the optimal investment policy of the follower. As illustrated in Figure 2, the follower is initially in state $(0,1)$ and holds the option to invest in the first technology and move to state 1. Once an innovation takes place, the follower moves to state $(1,2)$, where she has the option to invest in the second technology and move to state 2. We denote a transition due to an innovation (investment) by a dashed (solid) line. Note that the follower will always adopt each technology after the leader, and, therefore, for ease of exposition, we will use the notation $\overline{1}$ and $\overline{2}$ to indicate the presence of two firms only where it is necessary to avoid confusion.

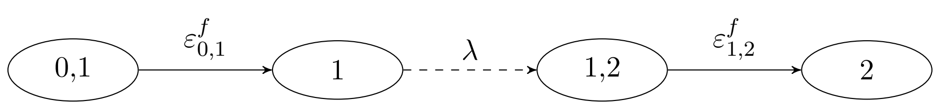


Figure 2: State-transition diagram for the proprietary follower under a compulsive strategy

Similar to the benchmark case, we now assume that the amount of capital required for the adoption of each technology is exchanged immediately at investment for the risky cash flows of the project, as in Figure 3. For example, at time $\tau_{0,1}^{f}$ the follower gets the capital required for investing in the first technology, $I_1$, and exchanges it for the risky cash flows of the project. Analogously to (4) and (5), this results in the instantaneous utility $U\left(ED_{\overline{1}}\right) - U(rI_1)$, which accrues from $\tau_{0,1}^{f}$

until $\tau^f_{1,2}$. Next, at $\tau^f_{1,2}$ the follower gets the capital required for investing in the second technology and exchanges it immediately for the risky cash flows it generates.

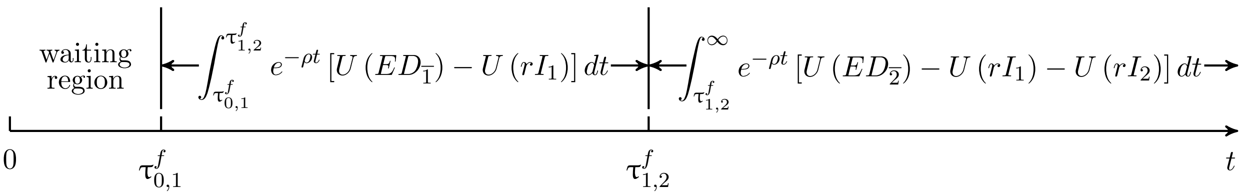


Figure 3: State-transition diagram under a compulsive strategy

The follower's objective is to maximise the time-zero discounted expected utility of all the cash flows of the project, which is described in (9). The first (second) integral in (9) indicates the expected utility of the cash flows from operating the first (second) technology.

$$\mathbb{E}_E\left[\int_{\tau^f_{0,1}}^{\tau^f_{1,2}} e^{-\rho t}\left[U\left(ED_{\overline{1}}\right)-U\left(rI_1\right)\right]dt+\int_{\tau^f_{1,2}}^{\infty} e^{-\rho t}\left[U\left(ED_{\overline{2}}\right)-U\left(rI_1\right)-U\left(rI_2\right)\right]dt\right] \tag{9}$$

By decomposing the first integral, we can rewrite (9) as in (10).

$$\begin{aligned}\mathbb{E}_E\left[e^{-\rho\tau^f_{0,1}}\right]\Bigg[\mathbb{E}_{E^f_{0,1}}\int_0^{\infty} e^{-\rho t}\left[U\left(ED_{\overline{1}}\right)-U\left(rI_1\right)\right]dt+\mathbb{E}_{E^f_{0,1}}\left[e^{-\rho\left(\tau^f_{1,2}-\tau^f_{0,1}\right)}\right]\\ \times\mathbb{E}_{E^f_{1,2}}\int_0^{\infty} e^{-\rho t}\left[\left(D_{\overline{2}}^{\gamma}-D_{\overline{1}}^{\gamma}\right)U\left(E\right)-U\left(rI_2\right)\right]dt\Bigg]\end{aligned} \tag{10}$$

We determine the follower's value function in each state using backward induction. Therefore, we first assume that the follower is already operating the first technology and holds a single embedded option to adopt the second one. The expected utility of the project's cash flows is indicated in (11), where the first term is the expected utility from operating the first technology and the second term is the maximised expected value of the option to adopt the second one.

$$\mathbb{E}_E\left[\int_0^{\infty} e^{-\rho t}\left[U\left(E_tD_{\overline{1}}\right)-U\left(rI_1\right)\right]dt\right]+A^f_{1,2}E^{\beta_1} \tag{11}$$

Notice that the first term does not depend on the investment threshold, and, therefore, the optimisation objective is reflected in the second term. The latter is described in (12).

$$\begin{aligned}A^f_{1,2}E^{\beta_1} &= \max_{E^f_{1,2}>E}\left(\frac{E}{E^f_{1,2}}\right)^{\beta_1}\mathbb{E}_{E^f_{1,2}}\int_0^{\infty} e^{-\rho t}\left[\left(D_{\overline{2}}^{\gamma}-D_{\overline{1}}^{\gamma}\right)U\left(E_t\right)-U\left(rI_2\right)\right]dt\\ &= \max_{E^f_{1,2}>E}\left(\frac{E}{E^f_{1,2}}\right)^{\beta_1}\left[\Upsilon\left(D_{\overline{2}}^{\gamma}-D_{\overline{1}}^{\gamma}\right)U\left(E^f_{1,2}\right)-U\left(rI_2\right)\right]dt\end{aligned} \tag{12}$$

Solving this unconstrained optimisation problem, we obtain the expression of the optimal investment threshold that is indicated in (13).

$$\varepsilon^f_{1,2} = rI_2\left[\frac{\beta_2-\gamma}{\beta_2\left(D_{\overline{2}}^{\gamma}-D_{\overline{1}}^{\gamma}\right)}\right]^{\frac{1}{\gamma}} \tag{13}$$

Equivalently, we can express the value function of the follower in state $(1,2)$ as in (14). The first two terms on the top part reflect the expected utility of the cash flows from operating the first technology, while the third term represents the option to adopt the second one. The bottom part represents the expected utility of the profits from operating the second technology $\Phi_2^f(E) = \Upsilon U\left(ED_{\overline{2}}\right) - \frac{U(rI_2)+U(rI_1)}{\rho}$.

$$F_{1,2}^f(E) \quad = \quad \begin{cases} \Upsilon U\left(ED_{\overline{1}}\right) - \frac{U(rI_1)}{\rho} + A_{1,2}^f E^{\beta_1} & , E < \varepsilon_{1,2}^f \\ \Phi_2^f(E) & , E \geq \varepsilon_{1,2}^f \end{cases} \tag{14}$$

In state 1, the follower is operating the first technology and holds an embedded option to invest in the second one, that has yet to become available. The dynamics of the follower's value function are described in (15), where the first two terms on the right-hand side represent the instantaneous utility of the profits from operating the first technology and the second term is the expected utility of the project in the continuation region. As the second term indicates, with probability $\lambda dt$ the second technology will arrive and the follower will receive the value function, $F_{1,2}^f(E)$, whereas, with probability $1-\lambda dt$, no innovation will occur and the follower will continue to hold the value function, $\Phi_1^f(E)$.

$$\Phi_1^f(E) = \left[U\left(ED_{\overline{1}}\right) - U\left(rI_1\right)\right] dt + \left(1-\rho dt\right) \mathbb{E}_E \left[\lambda dt F_{1,2}^f(E+dE) + \left(1-\lambda dt\right) \Phi_1^f(E+dE)\right] \tag{15}$$

By expanding the right-hand side of (15) using Itô's lemma, we can rewrite (15) as in (16), where $\Lambda = \frac{\Upsilon}{\lambda\Upsilon+1}$ and $A_1^f > 0$, $B_1^f < 0$, are determined analytically by applying value-matching and smooth-pasting conditions to the two branches of (16). The first two terms on the top part represent the expected utility of the revenues and cost, respectively. The third term is the option to invest in the second technology, adjusted via the last term since the second technology is not available yet. The first three terms on the bottom part, represent the expected utility of operating the second technology, while the fourth term represents the likelihood of the price dropping in the waiting region prior to the arrival of an innovation.

$$\Phi_1^f(E) \quad = \quad \begin{cases} \Upsilon U\left(ED_{\overline{1}}\right) - \frac{U(rI_1)}{\rho} + A_{1,2}^f E^{\beta_1} + A_1^f E^{\delta_1} & , E < \varepsilon_{1,2}^f \\ \Lambda\left[\lambda \Upsilon U\left(ED_{\overline{2}}\right) + U\left(ED_{\overline{1}}\right)\right] - \frac{\lambda U(rI_2)}{(\lambda+\rho)\rho} - \frac{U(rI_1)}{\rho} + B_1^f E^{\delta_2} & , E \geq \varepsilon_{1,2}^f \end{cases} \tag{16}$$

Finally, the follower's value function in state $(0,1)$ is indicated in (17). By applying value-matching and smooth-pasting conditions between the two branches of (17), we can solve for the optimal investment threshold, $\varepsilon_{0,1}^f$, and the endogenous constant, $A_{0,1}^f$, numerically.

$$F_{0,1}^f(E) \quad = \quad \begin{cases} A_{0,1}^f E^{\beta_1} & , E < \varepsilon_{0,1}^f \\ \Phi_1^f(E) & , E \geq \varepsilon_{0,1}^f \end{cases} \tag{17}$$

*Leader*

Next, we consider the optimal investment policy of the leader. Notice that once the leader invests in the first technology, thereby moving from state $(0,\underline{1})$ to state $\underline{1}$, she receives monopoly profits until the follower enters. Once the follower adopts the first technology, both firms share the market in state $\overline{1}$. The same process is then repeated with respect to the second technology.

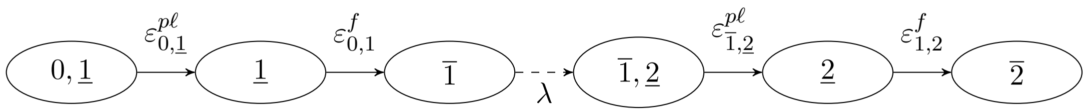


Figure 4: State-transition diagram for the proprietary leader under a compulsive strategy

Assuming that the follower chooses the optimal policy, the value function of the proprietary leader in state $\underline{2}$ is described in (18). The first two terms on the right-hand side reflect the monopoly profits from operating the second technology and the third term is the expected reduction in the proprietary leader's profits due to the follower's entry. The endogenous constant $A^{p\ell}_{\underline{2}}$ is obtained by value-matching (18) with the bottom part of (14) at $\varepsilon^{f}_{\overline{1},2}$ and is indicated in (A–5).

$$\Phi^{p\ell}_{\underline{2}}(E) \quad = \quad \Upsilon U\left(ED_{\underline{2}}\right) - \frac{U\left(rI_1\right) + U\left(rI_2\right)}{\rho} + A^{p\ell}_{\underline{2}} E^{\beta_1}, \quad E < \varepsilon^{f}_{\overline{1},2} \tag{18}$$

Next, the value function of the proprietary leader in state $\left(\overline{1},\underline{2}\right)$ is described in (19). The first two terms on the top part reflect the expected utility from operating the first technology and the third term is the embedded option to invest in the second one.

$$F^{p\ell}_{\overline{1},\underline{2}}(E) \quad = \quad \begin{cases} \Upsilon U\left(ED_{\overline{1}}\right) - \frac{U(rI_1)}{\rho} + A^{p\ell}_{\overline{1},\underline{2}} E^{\beta_1} & , E < \varepsilon^{p\ell}_{\overline{1},\underline{2}} \\ \Phi^{p\ell}_{\underline{2}}(E) & , E \geq \varepsilon^{p\ell}_{\overline{1},\underline{2}} \end{cases} \tag{19}$$

The endogenous constant, $A^{p\ell}_{\overline{1},\underline{2}}$, and the optimal investment threshold, $\varepsilon^{p\ell}_{\overline{1},\underline{2}}$, can be obtained analytically via value-matching and smooth-pasting conditions and are indicated in (20).

$$\varepsilon^{p\ell}_{\overline{1},\underline{2}} = rI_2 \left[\frac{\beta_2 - \gamma}{\beta_2\left(D^{\gamma}_{\underline{2}} - D^{\gamma}_{\overline{1}}\right)}\right]^{\frac{1}{\gamma}} \quad \text{and} \quad A^{p\ell}_{\overline{1},\underline{2}} = \left(\frac{1}{\varepsilon^{p\ell}_{\overline{1},\underline{2}}}\right)^{\beta_1} \left[\Phi^{p\ell}_{\underline{2}}\left(\varepsilon^{p\ell}_{\overline{1},\underline{2}}\right) - \Upsilon U\left(\varepsilon^{p\ell}_{\overline{1},\underline{2}} D_{\overline{1}}\right) + \frac{U\left(rI_1\right)}{\rho}\right] \tag{20}$$

Corollary 1 indicates the necessary condition for a trade-off between the two technologies. Note that, by setting $\gamma = 1$, we can retrieve the condition under risk neutrality, as in Chronopoulos & Siddiqui (2015).

**Corollary 1.** *A trade-off between the two technologies exists if* $\frac{D^{\gamma}_{1}}{I^{\gamma}_{1}} > \frac{D^{\gamma}_{2}}{I^{\gamma}_{1} + I^{\gamma}_{2}}$.

Using Corollary 1, we can show that the leader will not invest in the second technology before the follower adopts the first one, as indicated in Proposition 1. Intuitively, the second technology is considerably more costly and cannot be adopted when the output price is below the follower's required investment threshold for the first technology.

**Proposition 1.** $\varepsilon^{p\ell}_{\underline{1},\underline{2}} > \varepsilon^{f}_{0,1}$.

Contrary to Chronopoulos *et al.* (2014), the leader's required investment threshold in the second technology is lower than that of the monopolist, as shown in Proposition 2. Intuitively, the entry of the follower reduces the monopoly profits of the leader with respect to the first technology. In turn, this raises the value of the leader's option to invest in the second technology and lowers the required adoption threshold, thereby extending the corresponding period of monopoly profits.

**Proposition 2.** $\varepsilon^{p\ell}_{\overline{1},\underline{2}} < \varepsilon^{m}_{1,2}$.

The proprietary leader's value function in state $\overline{1}$ is indicated in (21), where $A^{p\ell}_{\overline{1}}$ and $C^{p\ell}_{\overline{1}}$ are determined by value matching and smooth pasting the two branches, while $B^{p\ell}_{\overline{1}}$ is obtained by value matching (21) with the top branch of (19) at $\varepsilon^{pf}_{0,\overline{1}}$. The first two (three) terms in the top (bottom) part in (21) reflect the expected utility of the profits under a low (high) output price. The third term in the top branch is the option to invest in the second technology adjusted via the fourth term due to technological uncertainty. The fourth term in the bottom branch reflects the reduction in the expected utility of the leader's profits due to the follower's entry adjusted for technological uncertainty via the fifth term. The last term reflects the likelihood of the price dropping in the waiting region.

$$\Phi^{p\ell}_{\overline{1}}(E) = \begin{cases} \Upsilon U\left(ED_{\overline{1}}\right) - \frac{U(rI_1)}{\rho} + A^{p\ell}_{\overline{1},\underline{2}}E^{\beta_1} + A^{p\ell}_{\overline{1}}E^{\delta_1} & , E < \varepsilon^{p\ell}_{\overline{1},\underline{2}} \\ \Lambda\left[\lambda\Upsilon U\left(ED_{\underline{2}}\right) + U\left(ED_{\overline{1}}\right)\right] - \frac{U(rI_1)}{\rho} - \frac{\lambda U(rI_2)}{\rho(\rho+\lambda)} & \\ +A^{p\ell}_{\underline{2}}E^{\beta_1} + B^{p\ell}_{\overline{1}}E^{\delta_1} + C^{p\ell}_{\overline{1}}E^{\delta_2} & , E \geq \varepsilon^{p\ell}_{\overline{1},\underline{2}} \end{cases} \tag{21}$$

Next, the value function of the proprietary leader in state $\underline{1}$ is indicated in (22), where $A^{p\ell}_{\underline{1}} < 0$ is obtained by value matching (22) with the top branch in (21) at $\varepsilon^{f}_{0,1}$ and is described in (A–6). The first two terms in (22) reflect the expected utility from operating the first technology, and, the third term, is the expected reduction in the proprietary leader's profits due to the follower's entry.

$$\Phi^{p\ell}_{\underline{1}}(E) = \Upsilon U\left(ED_{\underline{1}}\right) - \frac{U\left(rI_1\right)}{\rho} + A^{p\ell}_{\underline{1}}E^{\beta_1}, \quad E < \varepsilon^{f}_{0,1} \tag{22}$$

In state $(0,\underline{1})$, the proprietary leader holds the option to invest in the first technology with an embedded option to invest in the second, that has yet to become available. The expression of $F^{p\ell}_{0,\underline{1}}(E)$ is described in (23), where the top part is the value of the option to invest and the bottom part is the expected utility of the active project inclusive of the embedded option to invest in the second technology.

$$F^{p\ell}_{0,\underline{1}}(E) = \begin{cases} A^{p\ell}_{0,\underline{1}}E^{\beta_1} & , E < \varepsilon^{p\ell}_{0,\underline{1}} \\ \Phi^{p\ell}_{\underline{1}}(E) & , E \geq \varepsilon^{p\ell}_{0,\underline{1}} \end{cases} \tag{23}$$

The expression of $\varepsilon^{p\ell}_{0,\underline{1}}$ and $A^{p\ell}_{0,\underline{1}}$ is indicated in (24). Notice that, as shown in Proposition 3, the leader's decision to adopt the first technology is independent of technological uncertainty.

$$\varepsilon^{p\ell}_{0,\underline{1}} = \frac{rI_1}{D_{\underline{1}}}\left(\frac{\beta_2-\gamma}{\beta_2}\right)^{\frac{1}{\gamma}} \quad \text{and} \quad A^{p\ell}_{0,\underline{1}} = \left(\frac{1}{\varepsilon^{p\ell}_{0,\underline{1}}}\right)^{\beta_1}\Phi^{p\ell}_{\underline{1}}\left(\varepsilon^{p\ell}_{0,\underline{1}}\right) \tag{24}$$

**Proposition 3.** *The proprietary leader's required investment threshold for the first technology is independent of $\lambda$.*

*5.2. Non-Proprietary Duopoly*

With two firms in the market fighting for the leader's position, each one of them runs the risk of pre-emption. Note that, under a compulsive strategy, the follower will invest in each technology after the leader has already adopted it. Consequently, the value function of the follower in each state is the same as in Section 5.1. In order to determine the non-proprietary leader's optimal investment threshold for the second technology we consider the strategic interactions between the leader and the follower. Note that the leader's value function in state $\underline{2}$ is described in (18). If $E < \varepsilon^{n\ell}_{\overline{1},\underline{2}}$, then a firm is better off being the follower, since $F^{f}_{1,2}(E) > \Phi^{n\ell}_{\underline{2}}(E)$. By contrast, if $E > \varepsilon^{n\ell}_{\overline{1},\underline{2}}$, then a firm is better off being a leader, since $F^{f}_{1,2}(E) < \Phi^{n\ell}_{\underline{2}}(E)$. Consequently, the point of indifference between being a leader and a follower is determined numerically by solving (25).

$$F^{f}_{1,2}\left(x\right) \quad = \quad \Phi^{n\ell}_{\underline{2}}\left(x\right) \tag{25}$$

We consider two possible scenarios: i. $\varepsilon^{f}_{0,1} > x$ and ii. $\varepsilon^{f}_{0,1} < x$. In the former case, the follower invests in the first technology after the leader can pre-empt the second one, which implies that the leader does not face the risk of pre-emption. By contrast, in the latter case, the follower's optimal investment threshold is lower than that of the non-proprietary leader, which implies that the leader now faces the threat of pre-emption. Consequently, the leader's optimal investment threshold in the second technology is $\varepsilon^{n\ell}_{\overline{1},\underline{2}} = \max\left\{\varepsilon^{f}_{0,1}, x\right\}$, as shown in Proposition 4.

**Proposition 4.** *The optimal investment threshold of the non-proprietary leader for the first technology is $\varepsilon^{n\ell}_{\overline{1},\underline{2}} = \max\left\{\varepsilon^{f}_{0,1}, x\right\}$, where $x$ satisfies the condition $F^{f}_{1,2}\left(x\right) = \Phi^{n\ell}_{\underline{2}}\left(x\right)$.*

Following the same reasoning as in (25), the leader's pre-emption threshold in the first technology is determined by solving (26). Note that the value function of the leader while operating the first technology is indicated (22). However, $A^{p\ell}_{\underline{1}}$ is now determined by value matching (22) with either (16) or (21) depending on whether $\varepsilon^{f}_{0,1} > x$ or $\varepsilon^{f}_{0,1} < x$.

$$F^{f}_{0,1}\left(\varepsilon^{n\ell}_{0,\underline{1}}\right) \quad = \quad \Phi^{n\ell}_{\underline{1}}\left(\varepsilon^{n\ell}_{0,\underline{1}}\right) \tag{26}$$

### 5.3. War of Attrition

Due to the competitive advantage created by ignoring the first technology, a firm may choose to invest in the second one directly. Due to the difference in investment strategies, it is not possible to compare the two firms directly. Therefore, we take the perspective of each firm separately and analyse their value functions assuming initially that it is possible for each firm to assume both roles, i.e., leader and follower. Then, we can conclude which role is viable for each firm (Takashima *et al.*, 2008). Within this context, we denote as follower the firm that gets pre-empted in the adoption of the first technology, and, therefore, has a greater incentive to adopt the second one directly. The follower's value function when investing in the second technology directly is described in (27). The top part is the value of the option to invest and the bottom part is the expected utility of the active project.

$$F_{0,2}^{f}(E) \quad = \quad \begin{cases} A_{0,2}^{f} E^{\beta_1} & , E < \varepsilon_{0,2}^{f} \\ \Upsilon U\left(ED_{\overline{2}}\right) - \frac{U(rI_2)}{\rho} & , E \geq \varepsilon_{0,2}^{f} \end{cases} \tag{27}$$

The expression of $A_{0,2}^{f}$ and $\varepsilon_{0,2}^{f}$ is obtained through value-matching and smooth-pasting conditions and are indicated in (28).

$$\varepsilon_{0,2}^{f} = \frac{rI_2}{D_{\overline{2}}}\left(\frac{\beta_2-\gamma}{\beta_2}\right)^{\frac{1}{\gamma}} \quad \text{and} \quad A_{0,2}^{f} = \left(\frac{1}{\varepsilon_{0,2}^{f}}\right)^{\beta_1}\left[\Upsilon U\left(\varepsilon_{0,2}^{f} D_{\overline{2}}\right) - \frac{U\left(rI_2\right)}{\rho}\right] \tag{28}$$

The corresponding leader's value function is denoted by $\widehat{\Phi}_{\underline{2}}^{n\ell}(\cdot)$ and is described in (29). Note that $\widehat{A}_{\underline{2}}^{n\ell}$ is determined by value matching (29) with the bottom part of (27) at $\varepsilon_{0,2}^{f}$. The pre-emptive leader's threshold, $\widehat{\varepsilon}_{0,\underline{2}}^{n\ell}$, satisfies the condition $F_{0,2}^{f}\left(E\right) = \widehat{\Phi}_{\underline{2}}^{n\ell}\left(E\right)$.

$$\widehat{\Phi}_{\underline{2}}^{n\ell}(E) \quad = \quad \Upsilon U\left(ED_{\underline{2}}\right) - \frac{U\left(rI_2\right)}{\rho} + \widehat{A}_{\underline{2}}^{n\ell} E^{\beta_1} \quad , \widehat{\varepsilon}_{0,\underline{2}}^{n\ell} < E \leq \varepsilon_{0,2}^{f} \tag{29}$$

Similarly, the pre-emptive leader's expected utility from operating the second technology after having already adopted the first one is denoted by $\widetilde{\Phi}_{\underline{2}}^{n\ell}(\cdot)$ and is described in (30). Note that $\widetilde{A}_{\underline{2}}^{n\ell}$ is determined by value matching (30) with the bottom part of (14) at $\varepsilon_{1,2}^{f}$.

$$\widetilde{\Phi}_{\underline{2}}^{n\ell}(E) \quad = \quad \Upsilon U\left(ED_{\underline{2}}\right) - \frac{U\left(rI_1\right) + U\left(rI_2\right)}{\rho} + \widetilde{A}_{\underline{2}}^{n\ell} E^{\beta_1} \quad , \widetilde{\varepsilon}_{0,\underline{2}}^{n\ell} < E \leq \varepsilon_{1,2}^{f} \tag{30}$$

The pre-emptive leader's threshold, $\widetilde{\varepsilon}_{0,\underline{2}}^{n\ell}$, satisfies the condition $F_{1,2}^{f}\left(E\right) = \widetilde{\Phi}_{\underline{2}}^{n\ell}\left(E\right)$. Consequently, skipping the first technology is a feasible strategy provided that $\widetilde{\varepsilon}_{0,\underline{2}}^{n\ell} > \widehat{\varepsilon}_{0,\underline{2}}^{n\ell}$. Intuitively, the follower can invest in the second technology first provided the pre-emption threshold of the compulsive leader is greater than the threshold of directly adopting the second technology. This does not however imply that it is optimal skip one technology, only that it is a feasible strategy.

## 6. Leapfrog and Laggard Strategy

Assuming that the leader has proprietary rights on each technology, she may decide to ignore a technology temporarily in order to wait for a new one to arrive before deciding which one to invest in. If the leader ignores the first technology, then only the second one will be commercialised, and, therefore, the follower will never invest in the first one. The value function of the follower is indicated in (27).

Given the followers optimal response, the proprietary leader can choose whether to adopt a leapfrog or a laggard strategy as illustrated in Figure 5. Instead of moving from $(0, \underline{1})$ to $\underline{1}$, the leader moves to state $(0, \underline{1} \vee \underline{2})$, and then, either invests in the first technology, holding the option to switch to the second one, i.e., state $(\underline{1}, \underline{2})$, or $(\vee)$ invests directly in the second technology, thereby moving to state $\underline{2}$.

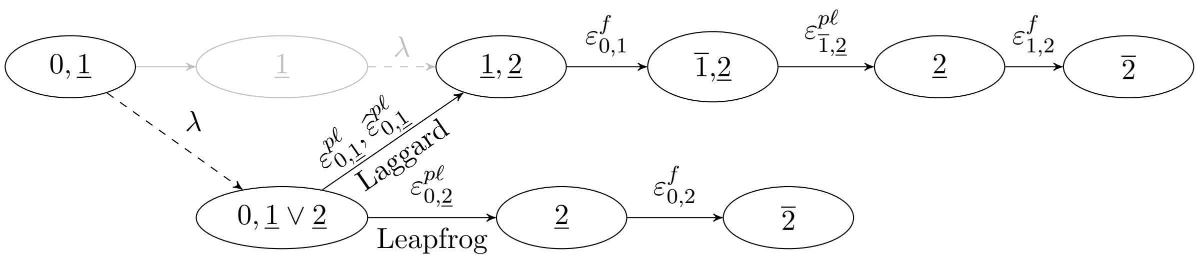


Figure 5: Proprietary duopoly under a leapfrog/laggard strategy

Notice that the value function of the proprietary leader in states $\left(\overline{1}, \underline{2}\right)$, $\underline{2}$, and $\overline{2}$ following a laggard strategy is the same as in Section 5.1, while her value function in state $\underline{2}$ following a leapfrog strategy is indicated in the bottom part of (27). Hence, we proceed directly to state $(\underline{1}, \underline{2})$, where the leader operates the first technology and earns monopoly profits until the follower enters at $\varepsilon^{f}_{0,1}$. The leader's value function is described in (31), where the third term on the right-hand side reflects the expected reduction in the leader's profits due to the followers entry. The endogenous constant, $A^{p\ell}_{\underline{1},\underline{2}}$, is obtained by value matching (31) with the bottom part of (19) at $\varepsilon^{f}_{0,1}$.

$$F^{p\ell}_{\underline{1},\underline{2}}(E) \quad = \quad \Upsilon U\left(ED_{\underline{1}}\right) - \frac{U\left(rI_1\right)}{\rho} + A^{p\ell}_{\underline{1},\underline{2}} E^{\beta_1} \tag{31}$$

Due to the presence of the second technology, there exist two waiting regions: i. $E \leq \varepsilon^{p\ell}_{0,\underline{1}}$ and ii. $\widehat{\varepsilon}^{p\ell}_{0,\underline{1}} \leq E \leq \widehat{\varepsilon}^{p\ell}_{0,\underline{2}}$ (Décamps *et al.*, 2006). Hence, the value function in state $(0, \underline{1} \vee \underline{2})$ is described in (32), where $B^{p\ell}_{0,\underline{1}\vee\underline{2}}$, $C^{p\ell}_{0,\underline{1}\vee\underline{2}}$, $\widehat{\varepsilon}^{p\ell}_{0,\underline{1}}$, and $\widehat{\varepsilon}^{p\ell}_{0,\underline{2}}$ are obtained numerically via value-matching and smooth-pasting conditions between the bottom three branched, and $\Phi^{p\ell}_{\underline{2}}(E)$ is indicated in (18). Notice that, if $E < \varepsilon^{p\ell}_{0,\underline{1}}$, then the firm will wait until $E = \varepsilon^{p\ell}_{0,\underline{1}}$ and then invest in the first technology. By contrast, if $\widehat{\varepsilon}^{p\ell}_{0,\underline{1}} \leq E \leq \widehat{\varepsilon}^{p\ell}_{0,\underline{2}}$, then the firm will either invest directly in the second technology if $E \uparrow \widehat{\varepsilon}^{p\ell}_{0,\underline{2}}$, or it will invest in the first one and hold the option to switch to the second if $E \downarrow \widehat{\varepsilon}^{p\ell}_{0,\underline{1}}$.

$$F^{p\ell}_{0,\underline{1}\vee\underline{2}}(E) = \begin{cases} A^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_1} & , E < \varepsilon^{p\ell}_{0,\underline{1}} \\ F^{p\ell}_{\underline{1},\underline{2}}(E) & , \varepsilon^{p\ell}_{0,\underline{1}} \leq E \leq \widehat{\varepsilon}^{p\ell}_{0,\underline{1}} \\ B^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_2} + C^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_1} & , \widehat{\varepsilon}^{p\ell}_{0,\underline{1}} < E < \widehat{\varepsilon}^{p\ell}_{0,\underline{2}} \\ \Phi^{p\ell}_{\underline{2}}(E) & , \widehat{\varepsilon}^{p\ell}_{0,\underline{2}} \leq E \end{cases} \tag{32}$$

Finally, in state $(0, \underline{1})$ either the second technology will become available with probability $\lambda dt$ and the proprietary leader will receive the value function $F^{p\ell}_{0,\underline{1}\vee\underline{2}}(E)$, or no innovation will take place with probability $1 - \lambda dt$ and the leader will continue to hold the value function $F^{p\ell}_{0,\underline{1}}(E)$.

$$\widehat{F}^{p\ell}_{0,\underline{1}}(E) = (1 - \rho dt)\,\mathbb{E}_E\left[\lambda dt F^{p\ell}_{0,\underline{1}\vee\underline{2}}(E + dE) + (1 - \lambda dt)\, F^{p\ell}_{0,\underline{1}}(E + dE)\right] \tag{33}$$

The expression of the value function in state $(0, \underline{1})$ is indicated in (34), where $D^{p\ell}_{\underline{1}}$, $G^{p\ell}_{\underline{1}}$, $H^{p\ell}_{\underline{1}}$, $J^{p\ell}_{\underline{1}}$, $K^{p\ell}_{\underline{1}}$, $L^{p\ell}_{\underline{1}}$, and $M^{p\ell}_{\underline{1}}$ are determined numerically via the value-matching and smooth-pasting conditions between the branches of (34). Notice that, (34) has five branches, this is a consequence of the value function, $\widehat{\Phi}^{p\ell}_{\underline{2}}(E)$ changing to $\widehat{\Phi}^{pb}_{\overline{2}}(E)$ when the follower enters the market.

$$\widehat{F}^{p\ell}_{0,\underline{1}}(E) = \begin{cases} A^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_1} + D^{p\ell}_{\underline{1}}E^{\delta_1} & , E < \varepsilon^{p\ell}_{0,\underline{1}} \\ \lambda\Lambda\Upsilon U\left(D_{\underline{1}}E\right) - \frac{\lambda U(rI_1)}{\rho(\rho+\lambda)} + A^{p\ell}_{\underline{1},\underline{2}}E^{\beta_1} + G^{p\ell}_{\underline{1}}E^{\delta_1} + H^{p\ell}_{\underline{1}}E^{\delta_2} & , \varepsilon^{p\ell}_{0,\underline{1}} \leq E \leq \widehat{\varepsilon}^{p\ell}_{0,\underline{1}} \\ B^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_2} + C^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_1} + J^{p\ell}_{\underline{1}}E^{\delta_1} + K^{p\ell}_{\underline{1}}E^{\delta_2} & , \widehat{\varepsilon}^{p\ell}_{0,\underline{1}} < E < \widehat{\varepsilon}^{p\ell}_{0,\underline{2}} \\ \lambda\Lambda\Upsilon U\left(D_{\underline{2}}E\right) - \frac{\lambda U(rI_2)}{\rho(\rho+\lambda)} + A^{p\ell}_{\underline{2}}E^{\beta_1} + L^{p\ell}_{\underline{1}}E^{\delta_1} + M^{p\ell}_{\underline{1}}E^{\delta_2} & , \widehat{\varepsilon}^{p\ell}_{0,\underline{2}} \leq E < \varepsilon^{f}_{0,\overline{2}} \\ \lambda\Lambda\Upsilon U\left(D_{\overline{2}}E\right) - \frac{\lambda U(rI_2)}{\rho(\rho+\lambda)} & , \varepsilon^{f}_{0,\overline{2}} \leq E \end{cases} \tag{34}$$

## 7. Numerical Results

*Proprietary duopoly with compulsive firms*

For the numerical results, the parameter values are $\mu = 0.01$, $\rho = r = 0.08$, $\sigma \in [0.1, 0.25]$, $I_1 = 500$, $I_2 = 1500$, $D_{\overline{1}} = 8$, $D_{\overline{2}} = 15$, $D_{\underline{1}} = 11$, $D_{\underline{2}} = 19$, and $\lambda \in \mathbb{R}^+$. These values ensure that there is a trade-off between the two technologies, as shown in Corollary 1. Figure 6 illustrates the value function of the leader and the follower for the case of investment in the first (left panel) and the second (right panel) technology under a compulsive strategy. According to the right panel, the proprietary leader has the option to delay investment, and, therefore, adopts the second technology at $E = 19.76$. By contrast, the non-proprietary leader faces the risk of pre-emption and adopts the second technology at $E = 14.58$. Indeed, for $E < 14.58$ the option value of the follower is greater than the project value of the leader, while for $E > 14.58$ the opposite is observed. Consequently, $E = 14.58$ indicates the point of indifference between being the leader or the follower. For $14.58 < E \leq 32.12$, the leader enjoys monopoly profits, however, once the follower invests in the second technology at 32.12, then both firms share the market. The left panel illustrates the value function of the leader and the follower when contemplating investment in the first technology, while holding an embedded option to invest in the second one, that has yet to become available. Notice that, upon adoption of the first technology, the value function of the proprietary leader (thin curve) is not the same as that of the follower (thick curve), since the leader holds the option to invest in the second technology first. Consequently, unlike state $\overline{2}$, the value function of the proprietary leader value matches with her own value function in state $\overline{1}$ at $E = 7.88$ and not with the follower's value function.

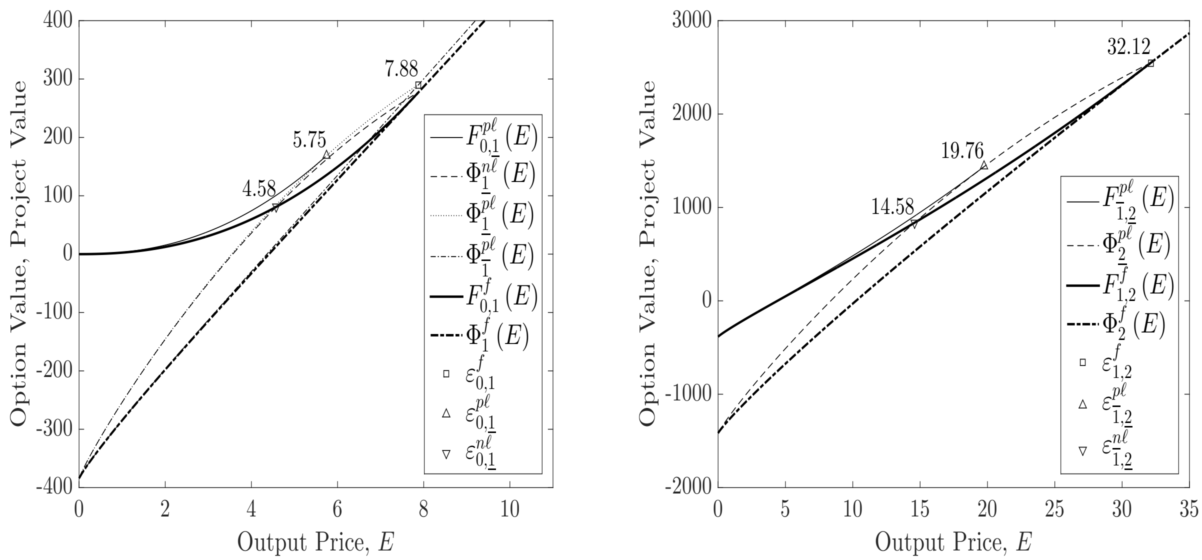


Figure 6: Option and project value of the leader and the follower in the first (left panel) and the second technology (right panel) under a proprietary and a non-proprietary duopoly for $\lambda = 0.1$, $\gamma = 0.9$, and $\sigma = 0.2$.

Figure 7 illustrates the impact of technological uncertainty and risk aversion on the required investment threshold of the proprietary leader (left panel) and the follower (right panel) for $\sigma = 0.18, 0.20$. Notice that price uncertainty increases the required investment threshold of both the leader and the follower by raising the opportunity cost of investing, thereby increasing the value of waiting. Interestingly, while the impact of technological uncertainty on the required investment threshold of the follower is non-monotonic, the proprietary leader's decision to invest is not affected by technological uncertainty. The former result is in line with Chronopoulos & Siddiqui (2015), who show that greater $\lambda$ increases a firm's incentive to adopt the currently available technology in order to have a shot at the yet unreleased version. The latter result happens because the the follower invests in the first technology before the leader invests in the second one, as shown in Proposition 1. Consequently, the leader's adoption of the first technology does not affect her prospective monopoly profits in the second one, thus resulting in a myopic strategy, as indicated in Proposition 3.

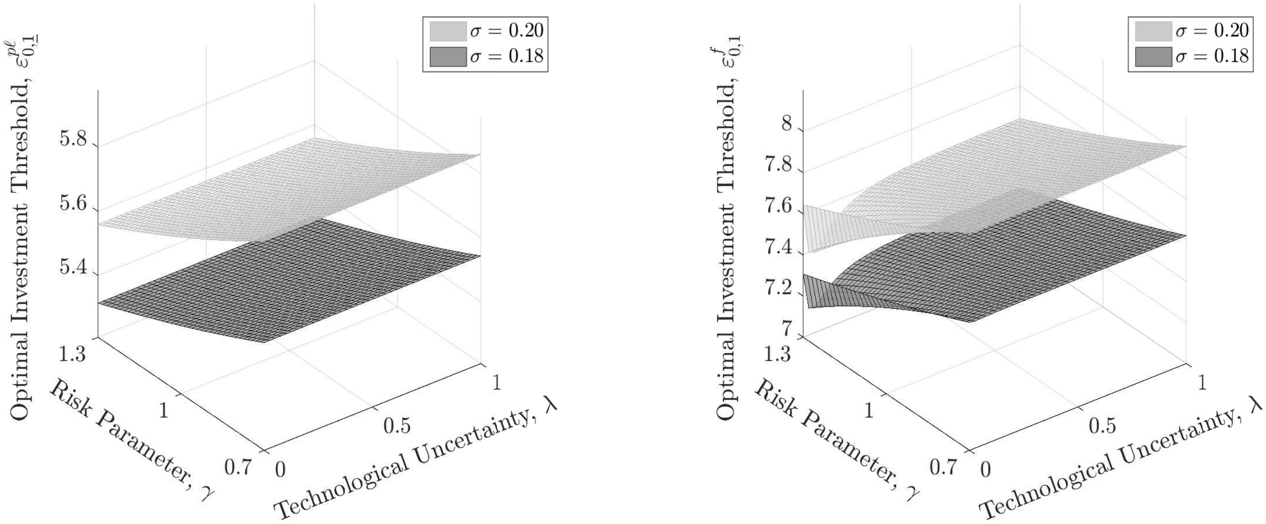


Figure 7: Impact of $\lambda$ and $\gamma$ on the optimal investment threshold of the proprietary leader (left panel) and the follower (right panel).

The right panel of Figure 8 illustrates the impact of $\lambda$ and $\gamma$ on the required investment threshold of the non-proprietary leader for $\sigma = 0.18, 0.20$. Notice that the impact of $\gamma$ and $\sigma$ is the same as in Figure 7, however greater $\lambda$ induces later adoption for the leader. Intuitively, this happens because earlier entry of the follower reduces the period of monopoly profits for the non-proprietary leader, thereby decreasing the attractiveness of the first technology for the leader. In the left panel, we increase the first-mover advantage, which increases the attractiveness of being a leader, thereby lowering the point of indifference between being a leader and a follower.

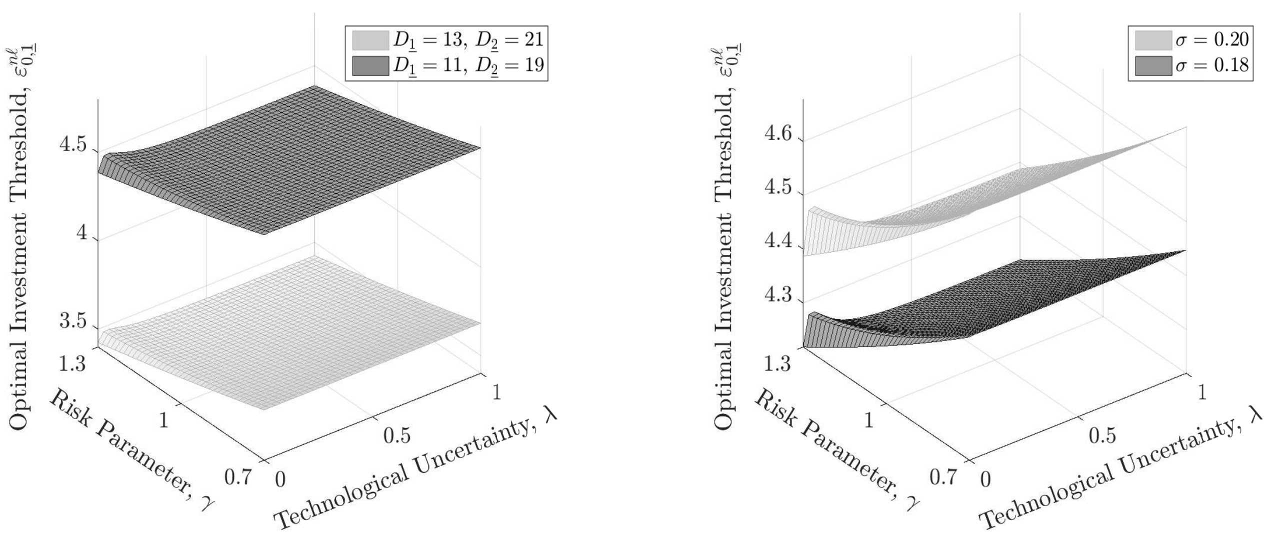


Figure 8: Impact of first-mover advantage on $\varepsilon^{n\ell}_{0,\underline{1}}$ (left panel) and impact of $\lambda$ and $\gamma$ on $\varepsilon^{n\ell}_{0,\underline{1}}$ for $\sigma = 0.18, 0.2$ (right panel).

Figure 9 illustrates the impact of greater first-mover advantage on the required investment threshold of the proprietary (left panel) and non-proprietary leader (right panel). In this numerical example $\varepsilon^{f}_{0,1} < x$, which implies that the follower has entered the market before the indifference threshold is reached, thus, the leader cannot postpone adoption as shown in Proposition 4. As both panels illustrate, the compulsive leader's required investment threshold in the first technology is not affected by the first-mover advantage in the second one. More specifically, since the follower does not adjust her entry in response to higher first mover advantage, the period of monopoly profits in the first technology is unchanged. Consequently, the leader has no incentive to change her adoption time. By contrast, a greater first-mover advantage in the first technology accelerates investment. Also, compared to the left panel, the threat of pre-emption increases the investment incentive, as illustrated in the right panel.

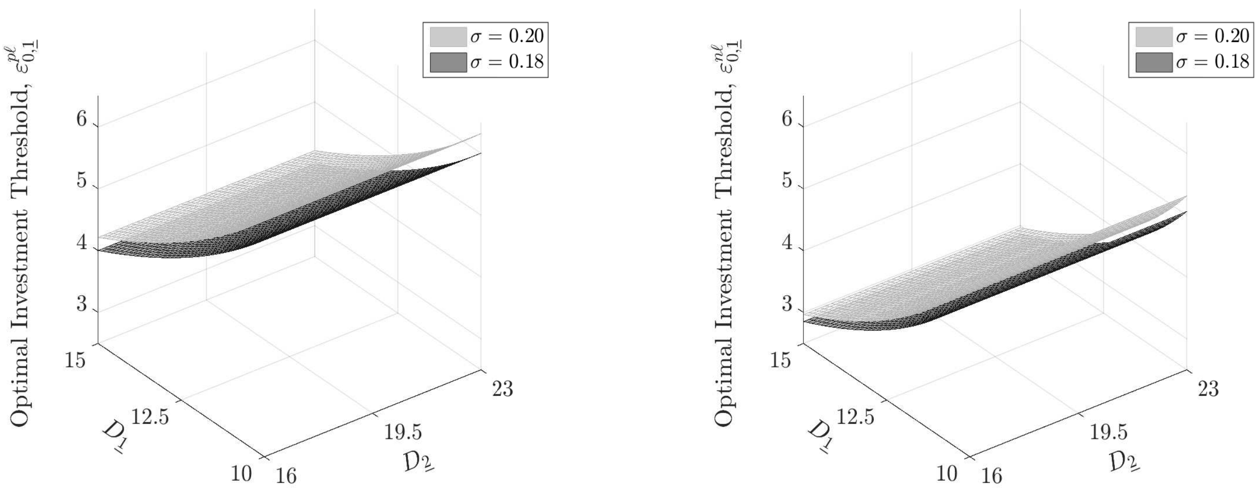


Figure 9: Impact of $D_{\underline{1}}$ and $D_{\underline{2}}$ on the optimal investment threshold of the proprietary (left panel) and non-proprietary leader (right panel).

The impact of $\gamma$ and $\sigma$ on the relative loss in the value of the proprietary and non-proprietary leader is indicated in the left- and the right-hand side expression of (35), respectively, and is illustrated in Figure 10.

$$\frac{A^{m}_{0,\underline{1}}{\varepsilon^{n\ell}_{0,\underline{1}}}^{\beta_1} - A^{p\ell}_{0,\underline{1}}{\varepsilon^{n\ell}_{0,\underline{1}}}^{\beta_1}}{A^{m}_{0,\underline{1}}{\varepsilon^{n\ell}_{0,\underline{1}}}^{\beta_1}} \quad \text{and} \quad \frac{A^{m}_{0,\underline{1}}{\varepsilon^{n\ell}_{0,\underline{1}}}^{\beta_1} - \Phi^{n\ell}_{\underline{1}}(\varepsilon^{n\ell}_{0,\underline{1}})}{A^{m}_{0,\underline{1}}{\varepsilon^{n\ell}_{0,\underline{1}}}^{\beta_1}} \tag{35}$$

In line with Chronopoulos *et al.* (2014), the left panel in Figure 10 illustrates that the relative loss in the value of the proprietary leader increases (decreases) with greater price uncertainty when the first-mover advantage is high (low). Intuitively, this happens because, under low discrepancy in market share, the increase in the proprietary leader's value of investment opportunity due to the follower's late investment is greater than the expected loss due to the entry of the follower. However, when the discrepancy is high the period of time with monopoly profits in the second project is more pronounced causing the relative loss to increase. Also, as the right panel illustrates, greater price uncertainty and a lower first-mover advantage decreases the relative loss in the value of the non-proprietary leader.

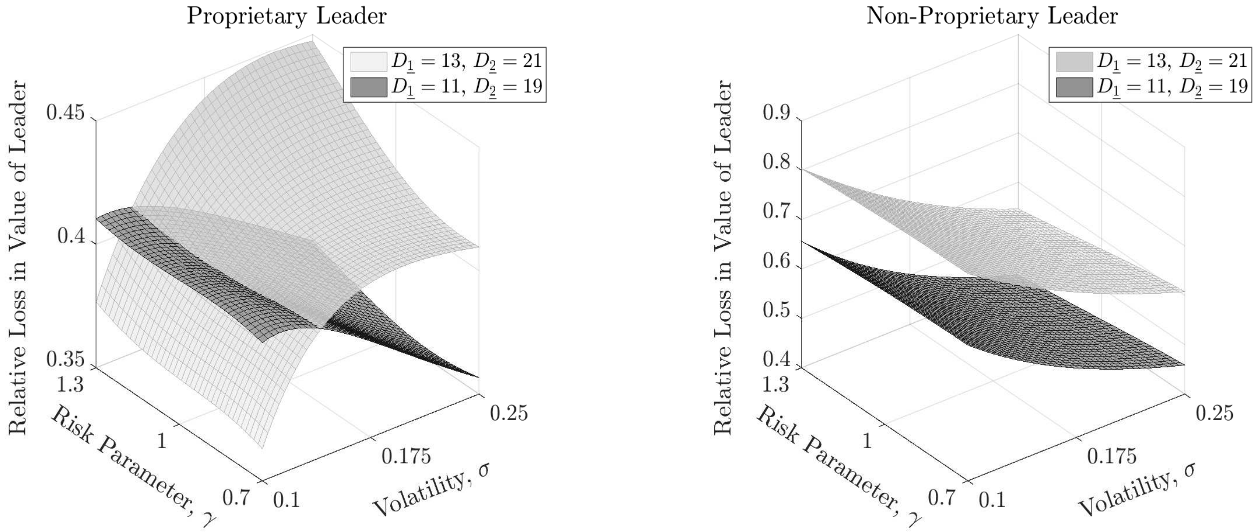


Figure 10: Relative loss in the value of the proprietary (left panel) and non-proprietary leader (right panel) versus $\gamma$ and $\sigma$ for $\lambda = 0.1$.

The impact of $\gamma$ and $\lambda$ on the relative loss in value for the proprietary (left panel) and non-proprietary leader (right panel) is illustrated in Figure 11. As both panels illustrate, a higher innovation rate lowers the relative loss in the value of the leader by raising the expected utility of the embedded option to adopt a more efficient technology. Interestingly, risk aversion has an ambiguous effect on the relative loss in the value of the proprietary leader (left panel). More specifically, under a low (high) rate of technological innovation, greater risk aversion decreases (increases) the relative loss in the value of the leader. This happens because greater risk aversion postpones the entry of the follower, thereby allowing the leader to enjoy monopoly profits for a longer time. However, when $\lambda$ is high, the second technology is more likely to become available, which in turn, gives the leader greater incentive to invest than the monopolist, as shown in Proposition 2. Consequently, the impact of greater risk aversion is mitigated by higher technological uncertainty.

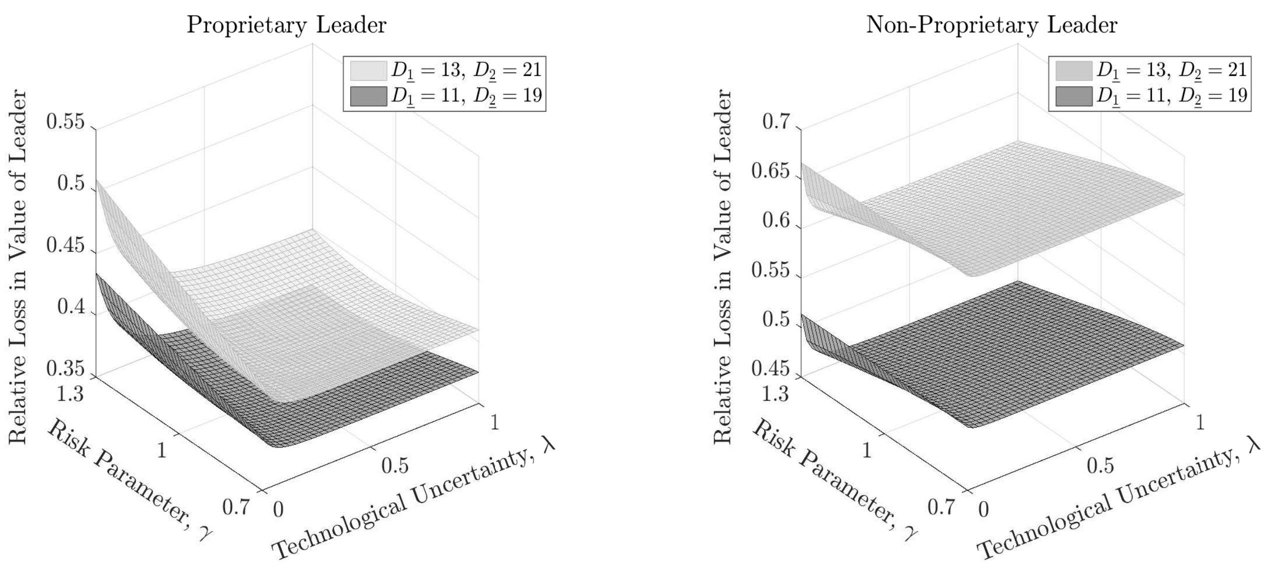


Figure 11: Relative loss in the value of the proprietary (left panel) and non-proprietary leader (right panel) versus $\gamma$ and $\lambda$.

*War of attrition.*

Figure 13 illustrates (36) which is the relative value of the war of attrition and the compulsive strategy under a low (left panel) and a high (right panel) output price without technological uncertainty.

$$\frac{\widehat{\Phi}^{n\ell}_{\underline{2}}(E)}{F^{f}_{0,1}(E)} \tag{36}$$

Note that we take the perspective of each firm separately and analyse their value functions assuming each firm can assume both roles, i.e., leader and follower, as shown in Figure 12. The top panel indicates that the pre-emption threshold for the second technology when the first one has already been adopted is 14.58. However, as the bottom panel illustrates, direct pre-emption of the second technology requires a threshold of 8.31. Consequently, the competitive advantage from ignoring the first technology is enough to enable the pre-emption of the second one.

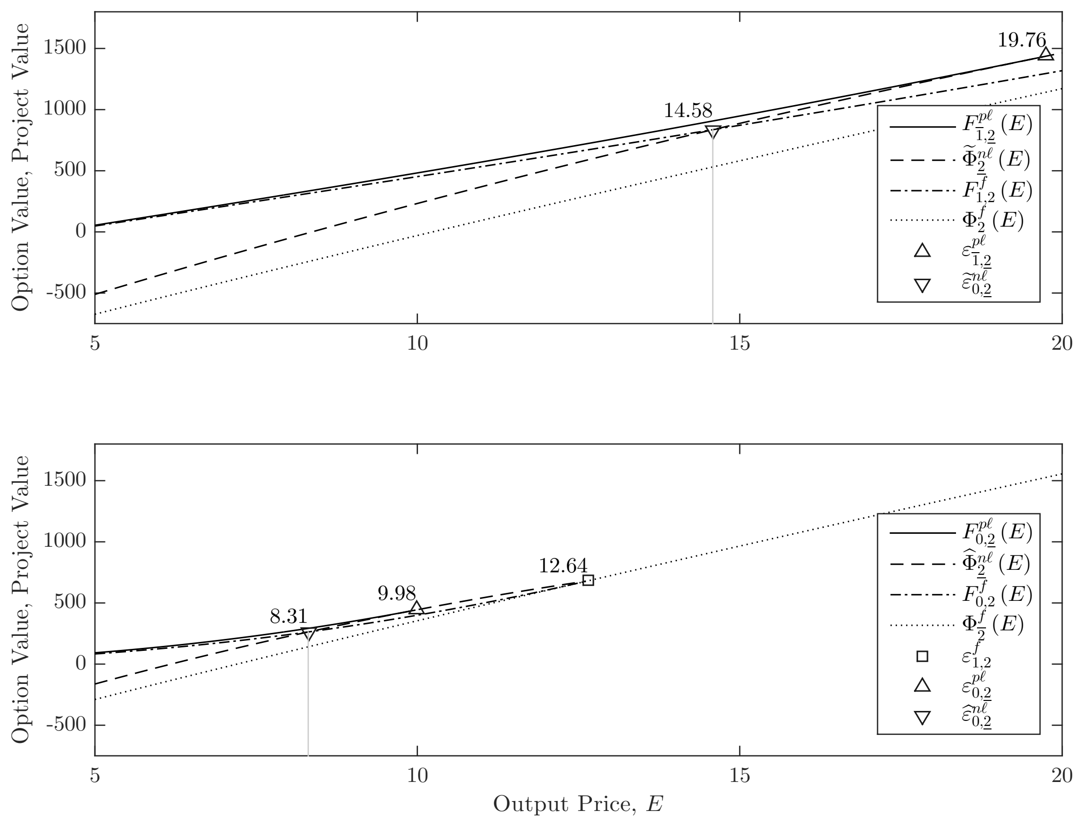


Figure 12: Value function of the leader and the follower, when the follower adopts a leapfrog strategy under a non-proprietary duopoly with $\lambda = 0.1$, $\gamma = 0.9$ and $\sigma = 0.2$.

Although skipping the first technology in order to pre-empt the second may be feasible, it is not necessarily the optimal strategy. Indeed, if output price is low, then it is always better to be a compulsive follower as the left panel of Figure 13 illustrates. However, if the output price is high, then increasing volatility makes it optimal to skip the first technology in order to pre-empt the second one (right panel).

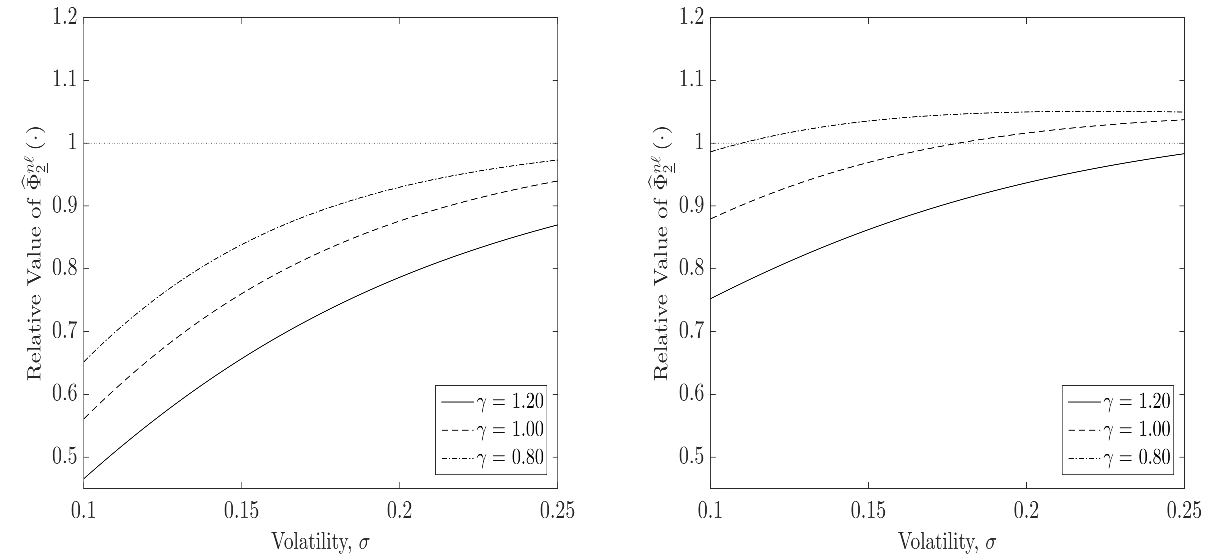


Figure 13: Relative value of a war of attrition strategy compared to the compulsive strategy for the follower evaluated at $E = \widehat{\varepsilon}_{0,\underline{2}}^{n\ell}$ (left panel) and $E = \varepsilon_{0,\underline{2}}^{p\ell}$ (right panel).

*Proprietary duopoly under a leapfrog/laggard strategy.*

The left panel in Figure 14 illustrates the value function of the proprietary leader under a leapfrog/ laggard strategy for $\gamma = 0.9, 1.0$. Notice that if $\gamma = 0.9$ and $E < 5.75$, then the leader will wait until $E = 5.75$ to adopt the first technology and enjoy monopoly profits until the follower enters at $E = 7.90$. By contrast, if $E \in [13.27, 15.13]$, then the leader will either adopt a laggard strategy if $E \downarrow 13.27$ or a leapfrog strategy if $E \uparrow 15.13$. Also, lower risk aversion raises the expected utility of the project and lowers the investment thresholds. As the right panel illustrates, greater price uncertainty raises all investment threshold, yet decreases the likelihood of a laggard strategy by narrowing the intermediate waiting region (Siddiqui & Fleten, 2010).

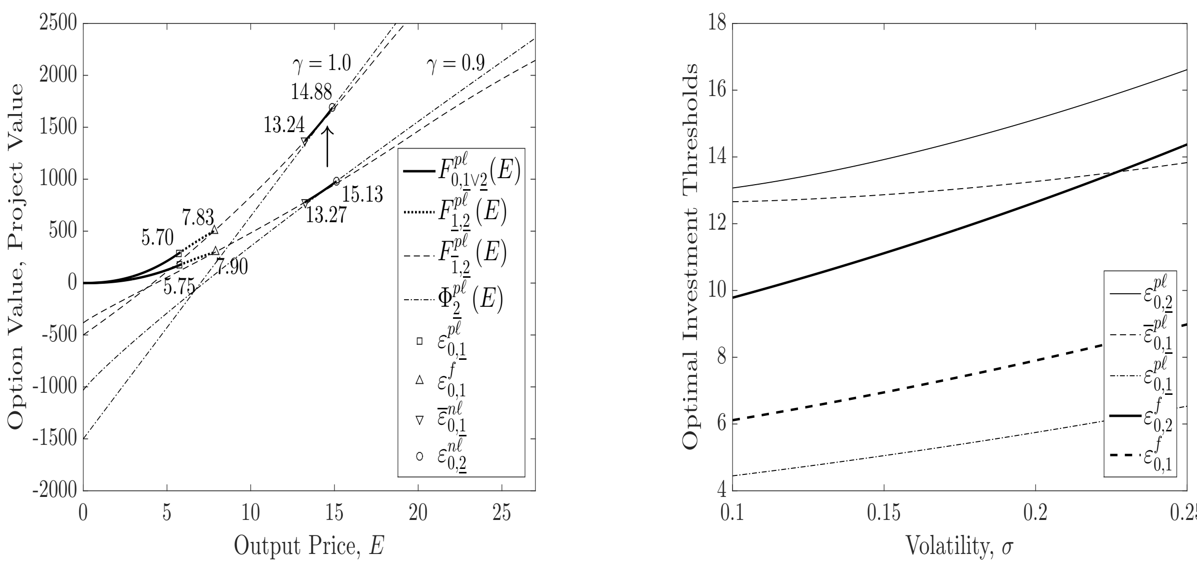


Figure 14: Value function of the proprietary leader for $\sigma = 0.2$ (left panel) and optimal investment thresholds for $\gamma = 0.9$ (right panel), under proprietary duopoly with a leapfrog/laggard strategy.

Figure 15 illustrates the relative value of the compulsive and leapfrog/laggard strategy for the proprietary leader. More specifically, the left panel illustrates the left-hand side expression in (37), which compares the first branch of (34) with (22) when the output price is low, i.e., $E = \varepsilon^{p\ell}_{0,\underline{1}}$. Similarly, the right panel illustrates the right-hand side expression that compares the bottom part of (34) with the top part of (21) when the output price is high, i.e., $E = \varepsilon^{p\ell}_{0,\underline{2}}$.

$$\frac{A^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_1} + D^{p\ell}_{\underline{1}}E^{\delta_1}}{\Upsilon U\left(ED_{\underline{1}}\right) - \frac{U(rI_1)}{\rho} + A^{p\ell}_{\underline{1}}E^{\beta_1}} \quad \text{and} \quad \frac{B^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_2} + C^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_1} + J^{p\ell}_{\underline{1}}E^{\delta_1} + K^{p\ell}_{\underline{1}}E^{\delta_2}}{\Upsilon U\left(ED_{\overline{1}}\right) - \frac{U(rI_1)}{\rho} + A^{f}_{1,2}E^{\beta_1} + A^{f}_{1}E^{\delta_1}} \tag{37}$$

As the right panel illustrates, the compulsive strategy dominates when the output price is low. This happens because a firm must wait longer to invest in the more capital intensive technology and the associated payoff does not offset the foregone revenues from ignoring the existing one. In fact, greater risk aversion promotes the adoption of a compulsive strategy and makes the leapfrog/laggard strategy relative less attractive. Interestingly, unlike Chronopoulos & Siddiqui (2015), the same result holds even at a high output price, as long as the discrepancy in market share is large. However, as the right panel illustrates, a leapfrog strategy may dominate, under a high output price and a low discrepancy in market share.

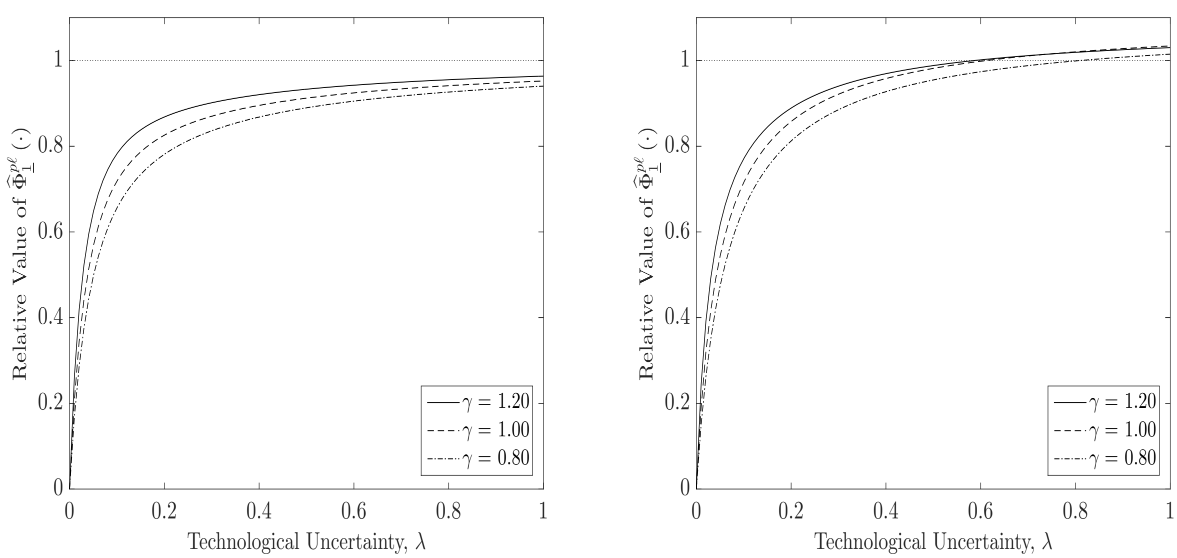


Figure 15: Relative value of compulsive and leapfrog/laggard strategy for the proprietary leader at $E = \varepsilon^{p\ell}_{0,\underline{1}}$ (left panel) and $E = \varepsilon^{p\ell}_{0,\underline{2}}$ where $D_{\underline{1}} = 9$, $D_{\underline{2}} = 16$ (right panel) for $\sigma = 0.2$.

## 8. Conclusions

We analyse how attitudes towards risk interact with price and technological uncertainty to impact the optimal investment decision of firms within the context of duopolistic competition. The analysis is motivated by three main features of the modern economic environment: i. increasing competition due to the deregulation of many sectors of the economy, such as energy and telecommunications; ii.

market incompleteness and associated attitudes towards idiosyncratic risk that impact investment decisions; and iii. the sequential nature of investment decisions in emerging technologies, e.g., energy, and the R&D-based sector of the economy. We incorporate these features into a utility-based framework for investment under uncertainty by assuming that two identical firms compete in the sequential adoption of technological innovations. More specifically, we assume that the firms compete in the adoption of two technologies, of which the first one is available, while the arrival of the second, more efficient one is subject to technological uncertainty.

Results indicate that insights from traditional real options models do not extend naturally to a competitive setting with various interacting uncertainties. Indeed, we find that technological uncertainty increases the follower's incentive to adopt the existing technology. This is in line with Chronopoulos & Siddiqui (2015), who address the problem of sequential investment in technological innovations ignoring, however, the implications of strategic interactions. Interestingly, under a proprietary duopoly, the leader's required investment threshold for both technologies is independent of technological uncertainty. In addition, the required investment threshold of the proprietary leader for the new technology is lower than that of the monopolist. Furthermore, we find that although greater price uncertainty lowers the relative loss in the value of the non-proprietary leader, in the case of proprietary duopoly, the impact of price uncertainty on the relative loss in the leader's option value depends on the discrepancy in market share. Also, a higher innovation rate lowers the relative loss in the value of the proprietary and non-proprietary leader. With respect to the technology adoption strategy, we show how ignoring the existing technology in order to adopt the second one directly may lead to a game, where neither firm wants to be the leader. By comparing a compulsive to a leapfrog/laggard strategy under a proprietary duopoly we find that the former strategy always dominates under a low output price. However, the latter strategy may dominate when the discrepancy in market share is small, provided that both the rate of innovation and the output price is high.

Extensions in the same line of work may include the flexibility to choose not only the time of investment but also the size of the project. In line with Huisman & Kort (2015), this will also enable the analysis of how different types of strategic interactions impact social welfare in terms of the time of investment and the amount of installed capacity. Additionally, other types of uncertainties may also be relevant within the context of strategic interactions. For example, regulatory risk regarding the availability of subsidies for specific technologies may impact strategic interactions, significantly. Other strategies may also be analysed as in Grenadier and Weiss (1997), or asymmetries can be included to analyse proprietary duopoly as in Takashima *et al.* (2008). Finally, to determine the robustness of the analytical and numerical results, it may be interesting to apply an alternative stochastic process such as a mean reverting GBM as well as other utility functions, e.g., Epstein-Zin utility or preferences in accordance with Prospect theory.

## APPENDIX

### Compulsive Strategy

*Follower*

The expected utility from operating the second technology is given in (A–1)

$$\Phi_2^f(E) = \Upsilon U\left(ED_{\overline{2}}\right) - \frac{U\left(rI_1\right) + U\left(rI_2\right)}{\rho} \tag{A–1}$$

and the value function of the follower in state $(1,2)$ is indicated in (A–2).

$$F_{1,2}^f\left(E\right) = \begin{cases} \left[U\left(ED_{\overline{2}}\right) - U\left(rI_1\right)\right]dt + (1-\rho dt)\mathbb{E}_E\left[F_{1,2}^f(E+dE)\right] & , E < \varepsilon_{1,2}^f \\ \Phi_2^f(E) & , E \geq \varepsilon_{1,2}^f \end{cases} \tag{A–2}$$

By expanding the top branch on the right–hand side of (A–2) using Itô's lemma, we obtain the solution that is indicated in (A–3).

$$F_{1,2}^f(E) = \Upsilon U\left(ED_{\overline{2}}\right) - \frac{U\left(rI_1\right)}{\rho} + A_{1,2}^f E^{\beta_1} + C_{1,2}^f E^{\beta_2} \tag{A–3}$$

Notice that $\beta_2 < 0 \Rightarrow C_{1,2}^f E^{\beta_2} \to \infty$ as $E \to 0$. Hence, we must have $C_{1,2}^f = 0$. The endogenous constant, $A_{1,2}^f$, and the required investment threshold, $\varepsilon_{1,2}^f$, are obtained via the value-matching and smooth-pasting conditions. Thus, the value function in state $(1,2)$ is described in (14).

Next, by expanding the right-hand side of (15) using Itô's lemma we obtain the differential equation (A–4), where $\mathcal{L} = \frac{1}{2}\sigma^2 E^2 \frac{\mathbf{d}^2}{\mathbf{d}E^2} + \mu E \frac{\mathbf{d}}{\mathbf{d}E}$ denotes the differential generator. By solving (A–4) for each expression of $F_{1,2}^f(E)$ that is indicated in (14) yields (16).

$$\left[\mathcal{L} - (\rho + \lambda)\right]\Phi_1^f(E) + \lambda F_{1,2}^f(E) + U\left(D_{\overline{1}}E\right) - U\left(\rho I_1\right) = 0 \tag{A–4}$$

*Leader*

In a state $\underline{2}$, the value function of the leader described in (18) will value match with the bottom part of the leader's value function (14), because for $E \geq \varepsilon_{1,2}^f$ the two firms will share the market. Hence, $A_{\underline{2}}^{p\ell}$ is described in (A–5).

$$A_{\underline{2}}^{p\ell} = \left(\frac{1}{\varepsilon_{1,2}^f}\right)^{\beta_1} \Upsilon U\left(\varepsilon_{1,2}^f\right)\left[D_{\overline{2}}^{\gamma} - D_{\underline{2}}^{\gamma}\right] \tag{A–5}$$

By contrast, in state $\underline{1}$, $A_{\underline{1}}^{p\ell}$ is obtained by value matching (22) with the top branch in (21) at $\varepsilon_{0,1}^f$. Hence, the endogenous constant $A_{\underline{1}}^{p\ell}$ is indicated in (A–6).

$$A_{\underline{1}}^{p\ell} = \left(\frac{1}{\varepsilon_{0,1}^f}\right)^{\beta_1}\left[\Upsilon U\left(\varepsilon_{0,1}^f\right)\left[D_{\overline{1}}^{\gamma} - D_{\underline{1}}^{\gamma}\right] + A_{\overline{1},\underline{2}}^{p\ell}\varepsilon_{0,1}^{f\,\beta_1} + A_{\overline{1}}^{p\ell}\varepsilon_{0,1}^{f\,\delta_1}\right] \tag{A–6}$$

$\square$

**Corollary** 1 *There is a trade-off between the two technologies if* $\frac{D_1^\gamma}{I_1^\gamma} > \frac{D_2^\gamma}{I_1^\gamma + I_2^\gamma}$.

**Proof:** Let $\varepsilon$ denote the indifference point between the two projects, i.e., $\Phi_1^{ab}(\varepsilon) = \Phi_2^{ab}(\varepsilon)$.

$$\begin{aligned}
\Phi_1^{ab}(\varepsilon) = \Phi_2^{ab}(\varepsilon) \quad &\Leftrightarrow \quad \Upsilon U(D_2\varepsilon) - \frac{U(rI_2) + U(rI_1)}{\rho} = \Upsilon U(D_1\varepsilon) - \frac{U(rI_1)}{\rho} \\
&\Leftrightarrow \quad \varepsilon = \left(\frac{\gamma U(rI_2)}{\Upsilon\rho\left(D_2^\gamma - D_1^\gamma\right)}\right)^{\frac{1}{\gamma}}
\end{aligned} \tag{A–7}$$

A trade-off between the technologies requires that $\Phi_1^{ab}(\varepsilon) > 0$.

$$\Phi_1^{ab}(\varepsilon) > 0 \Rightarrow \Upsilon U(D_1\varepsilon) - \frac{U(rI_1)}{\rho} > 0 \Rightarrow \frac{D_1^\gamma}{I_1^\gamma} > \frac{D_2^\gamma}{I_1^\gamma + I_2^\gamma} \tag{A–8}$$

$\square$

**Proposition** 1 $\varepsilon_{\overline{1},\underline{2}}^{p\ell} > \varepsilon_{0,1}^{f}$.

**Proof:** From Chronopoulos and Siddiqui (2014), we know that $\varepsilon_{0,1}^{f}$ is described in (A–9).

$$\varepsilon_{0,1}^{f} = \frac{rI_1}{D_{\overline{1}}}\left(\frac{\beta_2 - \gamma}{\beta_2}\right)^{\frac{1}{\gamma}} \tag{A–9}$$

Also, the expression for $\varepsilon_{\overline{1},\underline{2}}^{p\ell}$ is indicated in in (A–10).

$$\varepsilon_{\overline{1},\underline{2}}^{p\ell} = rI_2\left[\frac{\beta_2 - \gamma}{\beta_2\left(D_{\underline{2}}^\gamma - D_{\overline{1}}^\gamma\right)}\right]^{\frac{1}{\gamma}} \tag{A–10}$$

Consequently:

$$\begin{aligned}
\varepsilon_{\overline{1},\underline{2}}^{p\ell} > \varepsilon_{0,1}^{f} \quad &\Leftrightarrow \quad rI_2\left(\frac{\beta_2 - \gamma}{\beta_2}\right)^{\frac{1}{\gamma}}\left(\frac{1}{D_{\underline{2}}^\gamma - D_{\underline{1}}^\gamma}\right)^{\frac{1}{\gamma}} > \frac{rI_1}{D_{\overline{1}}}\left(\frac{\beta_2 - \gamma}{\beta_2}\right)^{\frac{1}{\gamma}} \\
&\Leftrightarrow \quad D_{\overline{1}}^\gamma I_2^\gamma > I_1^\gamma\left(D_{\underline{2}}^\gamma - D_{\underline{1}}^\gamma\right)
\end{aligned} \tag{A–11}$$

Finally, we have $\frac{D_1^\gamma}{I_1^\gamma} > \frac{D_2^\gamma}{I_1^\gamma + I_2^\gamma}$, which is required in order to have trade-off between the first and the second technology according to Corollary 1. $\square$

**Proposition** 2 $\varepsilon_{\overline{1},\underline{2}}^{p\ell} < \varepsilon_{1,2}^{m}$.

**Proof:** Based on the analytical expression of $\varepsilon_{\overline{1},\underline{2}}^{p\ell}$ and $\varepsilon_{1,2}^{m}$, we have:

$$\varepsilon_{\overline{1},\underline{2}}^{p\ell} = rI_2\left[\frac{\beta_2 - \gamma}{\beta_2\left(D_{\underline{2}}^\gamma - D_{\overline{1}}^\gamma\right)}\right]^{\frac{1}{\gamma}} < rI_2\left[\frac{\beta_2 - \gamma}{\beta_2\left(D_{\underline{2}}^\gamma - D_{\underline{1}}^\gamma\right)}\right]^{\frac{1}{\gamma}} = \varepsilon_{1,2}^{m} \quad \Leftrightarrow \quad D_{\underline{1}} > D_{\overline{1}} \tag{A–12}$$

$\square$

**Proposition 3** *The proprietary leader's required investment threshold for the first technology is independent of* $\lambda$*.*

**Proof:** We can alternatively express the first branch in (23) as in (A–13) to investigate the impact of $\lambda$.

$$F_{0,\underline{1}}^{p\ell}(E) = \max_{E_{0,\underline{1}}^{p\ell}>E} \left(\frac{E}{E_{0,\underline{1}}^{p\ell}}\right)^{\beta_1} \Phi_{\underline{1}}^{p\ell}\left(E_{0,\underline{1}}^{p\ell}\right) \tag{A–13}$$

The optimal investment rule is found by applying the first-order necessary conditions to (A–13) with respect to $E_{0,\underline{1}}^{p\ell}$ and is outlined in (A–14), where the marginal benefit (MB) of delaying the investment is equal to the marginal cost (MC).

$$\gamma \Upsilon U\left(D_{\underline{1}}\right) \varepsilon_{0,\underline{1}}^{p\ell\ \gamma-1} + \frac{\beta_1 U\left(rI_1\right)}{\varepsilon_{0,\underline{1}}^{p\ell}\rho} - \beta_1 A_{\underline{1}}^{p\ell} \varepsilon_{0,\underline{1}}^{p\ell\ \beta_1-1} = \beta_1 \Upsilon U\left(D_{\underline{1}}\right) \varepsilon_{0,\underline{1}}^{p\ell\ \gamma-1} - \beta_1 A_{\underline{1}}^{p\ell} \varepsilon_{0,\underline{1}}^{p\ell\ \beta_1-1} \tag{A–14}$$

The first term on the left-hand side reflects the extra benefit from allowing the project to start at a higher price threshold and the second term is the increase in MB form postponing the investment cost. Similarly, the first term on the right-hand side represent opportunity cost of forgone cash flows. The endogenous constant $A_{\underline{1}}^{p\ell}$ is negative since it adjusts for the possible entry of a competitor. Thus, the third and second term on the left- and right-hand side represent the MB of waiting for a higher investment threshold, which is completely offset by the MC on the right-hand side, since the leader enjoys less monopoly profits by waiting. These opposing forces cancel each other, because the follower will enter before the second technology arrives. Consequently, the leader's investment threshold in the first technology does not impact her possible monopoly profits in the second technology, and, thus, it is optimal for the leader to adopt a myopic investment strategy. □

**Proposition 4** *The optimal investment threshold of the non-proprietary leader for the first technology is* $\varepsilon_{\overline{1},\underline{2}}^{n\ell} = \max\left\{\varepsilon_{0,1}^{f}, x\right\}$*, where* $x$ *satisfies the condition* $F_{1,2}^{f}(x) = \Phi_{\underline{2}}^{n\ell}(x)$*.*

**Proof:** Ideally, the leader would invest at the threshold that maximises her expected utility, i.e., $\varepsilon_{\overline{1},\underline{2}}^{p\ell}$. However, the threat of pre-emption lowers the adoption threshold to $\varepsilon_{\overline{1},\underline{2}}^{n\ell}$. Let $x$ denote the price threshold at which the a firm is indifferent between being the leader or the follower, i.e., $F_{1,2}^{f}(x) = \Phi_{\underline{2}}^{n\ell}(x)$. Given that the follower adopts a compulsive strategy, there are two possible scenarios: i. $\varepsilon_{0,1}^{f} > x$ and ii. $\varepsilon_{0,1}^{f} < x$. In the former, the threat of pre-emption is eliminated, however, in the latter the threat still exists. If $\varepsilon_{0,1}^{f} > x$, then the leader will invest at $\varepsilon_{0,1}^{f}$, since $F_{\overline{1},\underline{2}}^{n\ell}\left(\varepsilon_{0,1}^{f}\right) > F_{\overline{1},\underline{2}}^{n\ell}(x)$. By contrast, if $\varepsilon_{0,1}^{f} < x$, then the leader will have to pre-empt the first technology at $x$. Consequently, $\varepsilon_{\overline{1},\underline{2}}^{n\ell} = \max\left\{\varepsilon_{0,1}^{f}, x\right\}$. □

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