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Decision Support

Duopolistic competition under risk aversion and uncertainty

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ABSTRACT

A monopolist typically defers entry into an industry as both price uncertainty and the level of risk aversion increase. By contrast, the presence of a rival typically hastens entry under risk neutrality. Here, we examine these two opposing effects in a duopoly setting. We demonstrate that the value of a firm and its entry decision behave differently with risk aversion and uncertainty depending on the type of competition. Interestingly, if the leader's role is defined endogenously, then higher uncertainty makes her relatively better off, whereas with the roles exogenously defined, the impact of uncertainty is ambiguous.

1. Introduction

Due to the deregulation of many sectors of the economy, e.g., energy and telecommunications, investment decisions should consider not only market uncertainty but also competition. Simultaneously, more stringent environmental regulation is likely to promote investment in alternative energy technologies. For example, BrightSource Energy, a company based in California, has signed the world's two largest deals to build new solar-power capacity and will begin constructing a series of 14 solar power plants that will collectively supply more than 2.6 gigawatts (GW) of electricity (The Economist, 2009). While solar-thermal power stations have several advantages over solar-photovoltaic projects, as they are typically built on a much larger scale and historically have lower costs, thin-film solar-cell modules are rapidly falling in price and, in some situations, can generate electricity more cheaply than solar-thermal power. As a result, competition from photovoltaic systems for large-scale power generation is expected to be significant. Apart from facing competition and uncertain energy prices, firms undertaking investments in such alternative energy technologies are also likely to be more risk averse since they face technical risk that cannot be diversified. Indeed, with such projects, the underlying commodities are often not freely traded in order to allow the construction of a replicating portfolio. Consequently, the assumption of hedging via spanning assets breaks down, and risk-neutral valuation is not possible.

Although canonical real options theory finds particular application in such sectors as it facilitates the analysis of capital budgeting, its treatment of such decision-making problems has mainly been in a monopoly or a perfect competition setting, while recent work considering a duopolistic setting has assumed risk neutrality. In this paper, we extend the traditional real options approach to strategic decision making under uncertainty by examining how duopolistic competition affects the entry of a risk-averse firm. We consider two identical firms that are risk averse and hold an option each to invest in a project that yields stochastic revenues. Since the two firms operate in the same industry, investment decisions of one firm impact the revenues of both firms. We analyse two settings: pre-emptive and non-pre-emptive competition. In the former, both firms have the incentive to invest in order to obtain a leader advantage, while in the latter, the role of the leader is assigned exogenously. Although in the absence of asymmetries, pre-emptive competition may be more common, nevertheless, non-pre-emptive competition may also arise in many cases, e.g., a particular technology or company may receive governmental support and, thus, has a competitive advantage over less favoured ones. Recently, both regulators and government have promoted EDF in its pursuit to build offshore wind plants over rivals even though its installed wind capacity by 2010 was only 10% of France's overall level (Bloomberg, 2012). Thus, these two settings capture extremes in terms of government involvement in industry. For each setting, we analyse the impact of uncertainty and risk

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aversion on the optimal investment timing decisions of the two competing firms and examine the degree to which the presence of a competitor impacts the entry of a risk-averse firm. Hence, the contribution of this paper is threefold. First, we develop a theoretical framework for analysing investment under uncertainty and risk aversion for pre-emptive and non-pre-emptive duopolies in order to derive closed-form expressions (where possible) for the optimal investment thresholds. Second, we quantify the degree to which competition impacts the strategic investment decisions of a risk-averse firm. Finally, we provide managerial insights for investment decisions and relative firm values under each setting based on analytical and numerical results.

We proceed by discussing some related work in Section 2 and formulate the problems in Section 3. In Section 4, we solve the problems and analyse the impact of uncertainty and risk aversion on the optimal investment timing decisions of the two competing firms in each setting. In Section 5, we provide numerical examples for each case in order to examine the effects of volatility and risk aversion on the optimal investment timing decisions and quantify the degree to which the entry of a risk-averse firm is affected by the presence of a rival. We also illustrate the interaction between risk aversion and uncertainty and present managerial insights to enable more informed investment decisions. Section 6 concludes by summarising the results and offering directions for future research.

2. Related work

The majority of real options models tackle the problem of optimal investment timing without considering competition (McDonald & Siegel, 1985, 1986), while the ones that do, assume risk neutrality (Dixit & Pindyck, 1994). In the area of competition, Smets (1993) first combined real options valuation techniques with game theory concepts, thus developing a continuous-time model of strategic real option exercise under product market competition, assuming that entry is irreversible, demand is stochastic, and simultaneous investment occurs only when the role of the leader is defined exogenously. Huisman and Kort (1999) examine how the deterministic duopoly framework of Fudenberg and Tirole (1985) is affected when uncertainty is introduced. According to Fudenberg and Tirole (1985), under high first-mover advantages, a pre-emption equilibrium occurs with dispersed adoption timings since it is essential for each firm to move quickly and pre-empt investment by its rivals. The introduction of uncertainty creates an opposing force since now there is a positive option value of waiting that increases with uncertainty, thereby increasing the required investment threshold. Indeed, although in deterministic models, high first-mover advantage leads to a pre-emptive equilibrium, in the stochastic case, increasing output-price uncertainty raises the required entry threshold for both firms as it increases the value of waiting. Finally, if first-mover advantages are lower but sufficiently large for the pre-emptive equilibrium to result in the deterministic model, then sufficiently high uncertainty results in simultaneous investment equilibrium (Huisman & Kort, 1999; Thijssen, Huisman, & Kort, 2012). Consequently, the introduction of uncertainty reduces the number of scenarios in which the pre-emptive equilibrium prevails.

Weeds (2002) considers irreversible investment in competing research projects with uncertain returns under a winner-takes-all patent system. The technological success of the project is probabilistic, while the economic value of the patent to be won evolves stochastically over time. Her results indicate that in a pre-emptive leader–follower equilibrium, firms invest sequentially and option values are reduced by competition. However, a symmetric outcome may also occur in which investment is more delayed than the single-firm counterpart. Comparing this with the optimal cooperative investment pattern, investment is found to be more delayed when firms act non-cooperatively as each holds back from investing in the fear of starting a patent race.

Paxson and Pinto (2005) extend the traditional real options approach by presenting a rivalry model in which the profits per unit and the number of units sold are both stochastic variables but captured via an aggregate variable. They examine a pre-emptive setting (where both firms fight for the leader's position) and a non-pre-emptive setting (where the role of the leader is defined exogenously). Their results indicate that the triggers of both the leader and the follower increase in both settings as the correlation between the profits per unit and the quantity of units increases since then the aggregate volatility involving the number of units and the profits per unit also increases. Furthermore, they illustrate how a marginal increase in the number of units sold while the active leader is alone in the market increases the value of the active leader by more than the value of her investment opportunity. Since the extra benefit from delaying investment is less than that from the active project, the non-pre-emptive leader's incentive to invest increases, thereby reducing the discrepancy between the pre-emptive leader's and non-pre-emptive leader's entry thresholds. Finally, they illustrate how increasing first-mover advantages create an incentive for the pre-emptive leader to enter the market sooner since then the entry of the follower is less damaging.

Unlike earlier studies concerning investment strategies in the electricity market, Takashima, Goto, Kimura, and Madarame (2008) assess the effect of competition on market entry and the strategies of firms with asymmetric technologies. They analyse the entry strategies into the electricity market of two firms that have power plants under price uncertainty and consider firms with either a thermal power plant or a nuclear power plant. Among other results, they show that for a nuclear power plant, the entry threshold of the leader is higher compared to a liquified natural gas thermal power plant since the latter has mothballing options that facilitate investment. Also, compared to the firm with a coal power plant or an oil thermal power plant, a firm with a nuclear power plant tends to be the leader because variable and construction costs for a nuclear power plant are lower compared to those of a coal power plant, while the oil thermal power plant may have lower construction costs but has variable costs that are twice as much as those of the nuclear power plant. An extension to asymmetric information over revenues finds that an equilibrium may not occur (Graham, 2011).

Huisman and Kort (2009) model not only the timing but also the size of the investment. They consider a monopoly setting as well as a duopoly setting and compare the results with the standard models in which the firms do not have capacity choice. They show how at a given level of demand, the leader can deter the entry of the follower temporarily by installing a capacity level such that the follower's entry is profitable at higher levels of demand. Moreover, the leader can choose the deterrence strategy only up to a certain high level of demand, above which it is optimal for the follower to enter at the same time as the leader. Similarly, if the demand is low, then it is not optimal for the leader to choose the deterrence strategy as this would result in negative profits. The region in which the leader can choose either one of the two strategies decreases with uncertainty, thereby increasing the range of demand in which the leader chooses the deterrence strategy. Finally, discretion over capacity becomes more valuable at high levels of uncertainty since then, unlike the model with fixed capacity, the optimal strategy for the leader is to invest at a lower demand level and install higher capacity, while the follower's optimal strategy is to invest at a higher demand level and install greater capacity.

Extending the traditional approach that considers only two competing firms, Bouis, Huisman, and Kort (2009) analyse investments in new markets where more than two identical competitors are present. In the setting including three firms, they find that if entry of the third firm is delayed, then the second firm has an incentive to invest earlier because this firm can enjoy the duopoly market structure for a longer time. This reduces the investment incentive for the first firm, which now faces a shorter period in which it can enjoy monopoly profits, and, thus, it invests later. This finding is denoted as the accordion effect and is also observed when the number of competing firms is greater. Indeed, with more than three firms competing, exogenous demand changes affect the timing of entry of the first, third, fifth, etc., investor in the same qualitative way, while the entry of the second, fourth, sixth, etc., investor is affected in exactly the opposite qualitative way. In other words, if a delay is observed for the "odd" investors, then the "even" investors will invest sooner. Also, in an oligopolistic framework, Roques and Savva (2009) explore the effect of a price cap on capacity expansion under uncertainty. However, they assume that no new firm will enter the industry.

Each of these papers assumes a risk-neutral decision maker, and, as a result, the implications of risk aversion, which may be relevant for reasons of market incompleteness or the presence of undiversifiable risk, are not addressed. We contribute to this line of work by developing a utility-based framework in order to examine how optimal investment decisions under uncertainty are affected by competition and risk aversion. This is relevant, for example, for a knowledge-based sector in which firms compete to launch a new product whose technical risk cannot be diversified. In order to describe the preferences of the two firms, we use a constant relative risk aversion (CRRA) utility function and determine the optimal strategies that maximise the expected utility of their future profits in both pre-emptive and non-pre-emptive settings. Such a utility-based framework was first introduced to real options by Hugonnier and Morellec (2007), who extend the work of Dixit and Pindyck (1994) and McDonald and Siegel (1986) by illustrating how risk aversion affects investment under uncertainty when the decision maker faces incomplete markets. Instead of using contingent claims, they use an optimal stopping time approach to allow for the decision maker's risk aversion to be incorporated via a CRRA utility function. Their framework is based on a closed-form expression for the expected discounted utility of stochastic cash flows derived by Karatzas and Shreve (1999). This approach is extended by Chronopoulos, De Reyck, and Siddiqui (2011) in order to examine the impact of operational flexibility, i.e., the ability to suspend and resume the project at any time, on optimal investment policies and option values assuming a risk-averse decision maker. They find that although risk aversion deters investment and hastens abandonment, the inclusion of embedded options, e.g., to suspend the project with subsequent resumption, may delay suspension. A similarly counterintuitive result appears in Miao and Wang (2007) in the context of a real-estate developer that hastens its sale of a plot of land due to risk aversion in order to lock in the lump-sum value of the property. Finally, Henderson and Hobson (2002) extend the real options approach to pricing and hedging assets by taking the perspective of a risk-averse decision maker facing incomplete markets, while Henderson (2007) investigates the impact of risk aversion and incompleteness on investment timing and option value by a risk-averse decision maker with an exponential utility function who can choose at any time to undertake an irreversible investment project and receive a risky payoff.

Our work does not address embedded options or hedging but, rather, focuses on the interaction between competition and risk aversion. We find that the entry of the leader and the follower is delayed due to risk aversion in both pre-emptive and non-pre-emptive settings and that, relative to the monopolist, the non-pre-emptive leader is hurt less from the follower's entry than the pre-emptive leader since the former has the flexibility to delay entry into the market. Interestingly, the loss in the pre-emptive leader's value, relative to that of a monopolist, due to the follower's entry is not affected by risk aversion; by contrast, the non-pre-emptive leader is relatively better off with greater risk aversion. Furthermore, we show that higher uncertainty reduces the loss in value of the pre-emptive leader relative to the monopolist by delaying the entry of the follower, thereby allowing the pre-emptive leader to enjoy monopoly profits for longer time. Although we would expect this result to extend to the non-pre-emptive setting, we find that, depending on the discrepancy in market share, the non-pre-emptive leader may actually become worse off with increasing uncertainty. This seemingly counterintuitive result holds because a high discrepancy in market share makes the increase in option value less profound as it increases the first-mover advantage and, at the same time, increases the impact of the follower's entry, thereby making the non-pre-emptive leader worse off. Conversely, if the discrepancy between the market share of the leader and the follower is low, then the impact of uncertainty on the leader's option value is more profound and offsets the loss in value due to the follower's entry.

3. Problem formulation

3.1. Assumptions and notation

Assume that each firm incurs an investment cost, K, in order to start a project that produces output forever. Time is continuous and denoted by t, and the revenue received from the project at time $t \geqslant 0$ is $R_t = P_t D(Q_t)$ (\$/annum). Here, Q_t denotes the number of firms in the industry, i.e., $Q_t = 0, 1, 2$, and $D(Q_t)$ is a strictly decreasing function reflecting the quantity demanded from each firm per annum. We assume that the price per unit of the project's output, P_t, follows a geometric Brownian motion (GBM):

$$dP_t = \mu P_t dt + \sigma P_t dZ_t, \quad P_0 > 0 \tag{1}$$

where $\mu \geqslant 0$ is the growth rate of $P_t, \sigma \geqslant 0$ is the volatility of P_t, and dZ_t is the increment of the standard Brownian motion. Also, we denote by $r \geqslant 0$ the risk-free rate and by $\rho \geqslant \mu$ the subjective discount rate. Let τ_i^j be the random time at which firm $j, j = \ell, f$ (denoting leader or follower, respectively), enters the industry given market structure $i = m, p, n$ (denoting monopoly, pre-emptive duopoly, or non-pre-emptive duopoly, respectively), i.e.,

$$\tau_i^j \equiv \min\left\{t \geqslant 0 : P_t \geqslant P_{\tau_i^j}\right\} \tag{2}$$

where $P_{\tau_i^j}$ is the corresponding output price. Finally, we denote by $F_{\tau_i^j}(P_0)$ the expected value of firm j's investment opportunity under market structure i that is exercised at time τ_i^j and by $V_i^j(P_0)$ the expected NPV of firm j given the initial output price, P_0.

In order to account for risk aversion, we assume that the preferences of both firms are described by an identical increasing and concave utility function, $U(\cdot)$. In our analysis, we apply a CRRA utility function, which is defined in (3), and assume that the risk aversion parameter $\gamma \in [0, 1)$. Although our analysis can accommodate a wide range of utility functions, such as hyperbolic absolute risk aversion (HARA) and constant absolute risk aversion (CARA), in this paper we focus on CRRA and leave the analysis related to other utility functions for further research.

$$U(P_t) = \begin{cases} \frac{P_t^{1-\gamma}}{1-\gamma} & \text{if } \gamma \geqslant 0 \ \& \ \gamma \neq 1 \\ \ln(P_t) & \text{if } \gamma = 1 \end{cases} \tag{3}$$

3.2. Monopoly

We begin by formulating the problem for the case of monopoly, where a single firm starts a perpetually operating project at a random time τ_m^j. Up to time τ_m^j, the monopolist invests K in a risk-free bond and earns an instantaneous cash flow of rK per time unit with utility $U(rK)$ discounted at her subjective rate of time preference, ρ. At τ_m^j, when the output price is $P_{\tau_m^j}$, the monopolist swaps this risk-free cash flow for a risky one, $P_tD(1)$, with utility $U(P_tD(1))$ as illustrated in Fig. 1.

The conditional expected utility of the cash flows discounted to time $t=0$ is:

$$\mathbb{E}_{P_0}\left[\int_0^{\tau_m^j} e^{-\rho t}U(rK)dt + \int_{\tau_m^j}^{\infty} e^{-\rho t}U(P_tD(1))dt\right]$$
$$= \int_0^{\infty} e^{-\rho t}U(rK)dt + \mathbb{E}_{P_0}\int_{\tau_m^j}^{\infty} e^{-\rho t}[U(P_tD(1)) - U(rK)]dt \qquad (4)$$

where $\mathbb{E}_{P_0}$ denotes the expectation operator, which is conditional on the initial value of the price process and reflects the randomness of both τ_1 and P_t. Due to the law of iterated expectations and the strong Markov property

$$\mathbb{E}_{P_0}\int_{\tau_m^j}^{\infty} e^{-\rho t}[U(P_tD(1)) - U(rK)]dt = \mathbb{E}_{P_0}\left[e^{-\rho\tau_m^j}\right]V_m^j\left(P_{\tau_m^j}\right) \qquad (5)$$

where

$$V_m^j\left(P_{\tau_m^j}\right) = \mathbb{E}_{P_{\tau_m^j}}\left[\int_0^{\infty} e^{-\rho t}[U(P_tD(1)) - U(rK)]dt\right] \qquad (6)$$

is the expected utility of the project's cash flows discounted to τ_m^j, and the monopolist's objective is to maximise the discounted expected utility of the project's cash flows, i.e., $\mathbb{E}_{P_0}\left[e^{-\rho\tau_m^j}\right]V_m^j\left(P_{\tau_m^j}\right)$.

3.3. Duopoly

3.3.1. Pre-emptive duopoly

We extend the previous framework by adding one more firm to the industry. As there are two firms in the industry fighting for the leader's position, each one of them runs the risk of pre-emption, which reduces the value of waiting. The firm that enters the market first is the leader, and the firm that enters second is the follower as shown in Fig. 2.

Consequently, the conditional expected utility of all future cash flows of the follower discounted to $t=\tau_p^\ell$ is:

$$\mathbb{E}_{P_{\tau_p^\ell}}\left[\int_{\tau_p^\ell}^{\tau_p^f} e^{-\rho t}U(rK)dt + \int_{\tau_p^f}^{\infty} e^{-\rho t}U(P_tD(2))dt\right]$$
$$= \int_{\tau_p^\ell}^{\infty} e^{-\rho t}U(rK)dt + \mathbb{E}_{P_{\tau_p^\ell}}\left[e^{-\rho\left(\tau_p^f-\tau_p^\ell\right)}\right]V_p^f\left(P_{\tau_p^f}\right) \qquad (7)$$

where

$$V_p^f\left(P_{\tau_p^f}\right) = \mathbb{E}_{P_{\tau_p^f}}\left[\int_0^{\infty} e^{-\rho t}[U(P_tD(2)) - U(rK)]dt\right] \qquad (8)$$

is the expected utility of the project's cash flows discounted to τ_p^f, and, like the monopoly case, the scope of the pre-emptive follower is to maximise the discounted to τ_p^ℓ expected utility of the project's cash flows, i.e., $\mathbb{E}_{P_{\tau_p^\ell}}\left[e^{-\rho\left(\tau_p^f-\tau_p^\ell\right)}\right]V_p^f\left(P_{\tau_p^f}\right)$.

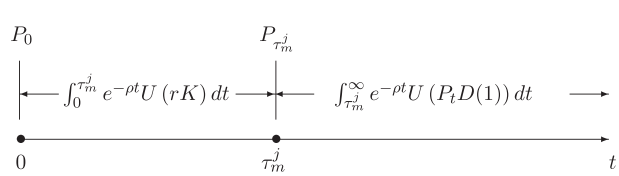

Fig. 1. Discounted utility from investment under risk aversion for a monopoly.

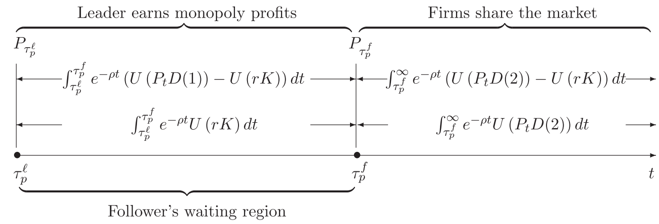

Fig. 2. Discounted utility from investment under risk aversion for a pre-emptive duopoly.

Next, the conditional expected utility of all future cash flows of the leader discounted to $t=\tau_p^\ell$ is:

$$V_p^\ell\left(P_{\tau_p^\ell}\right) = \mathbb{E}_{P_{\tau_p^\ell}}\left[\int_{\tau_p^\ell}^{\tau_p^f} e^{-\rho t}[U(P_tD(1)) - U(rK)]dt + \int_{\tau_p^f}^{\infty} e^{-\rho t}[U(P_tD(2))\right.$$
$$\left. -U(rK)]dt\right] = V_m^j\left(P_{\tau_p^\ell}\right) + \mathbb{E}_{P_{\tau_p^\ell}}\left[e^{-\rho\left(\tau_p^f-\tau_p^\ell\right)}\right]\mathbb{E}_{P_{\tau_p^f}}$$
$$\times\left[\int_0^{\infty} e^{-\rho t}[U(P_tD(2)) - U(P_tD(1))]dt\right] \qquad (9)$$

Notice that up to time τ_p^f, the leader enjoys monopolistic profits as in (6), while after the entry of the follower the two firms share the market, as illustrated in Fig. 2. This implies that, although up to time τ_p^f the leader is alone in the market, her value function does not correspond to that of a monopolist since the future entry of the follower reduces the expected utility of the leader's profits. This reduction is reflected by the second term on the right-hand side of (9), which is negative since $D(2) < D(1)$.

3.3.2. Non-pre-emptive duopoly

Here, the roles of the leader and the follower are defined exogenously. Consequently, the future cash flows of the leader are discounted to time $t=0$. Since the follower will consider entry into the market only when the leader has already invested, the future cash flows of the follower are discounted to τ_n^ℓ, as illustrated in Fig. 3.

The conditional expected utility of the follower's cash flows is the same as in the pre-emptive case but discounted to $t=\tau_n^\ell$, i.e.,

$$\int_{\tau_n^\ell}^{\infty} e^{-\rho t}U(rK)dt + \mathbb{E}_{P_{\tau_n^\ell}}\left[e^{-\rho\left(\tau_n^f-\tau_n^\ell\right)}\right]V_n^f\left(P_{\tau_n^f}\right) \qquad (10)$$

where $V_n^f(\cdot) = V_p^f(\cdot)$ and the objective of the follower is to maximise $\mathbb{E}_{P_{\tau_n^\ell}}\left[e^{-\rho\left(\tau_n^f-\tau_n^\ell\right)}\right]V_n^f\left(P_{\tau_n^f}\right)$.

The leader now knows that she has the right to enter the market first and, therefore, does not run the risk of pre-emption. As a result, the expected utility of the leader's future cash flows discounted to $t=0$ is:

$$\mathbb{E}_{P_0}\left[\int_0^{\tau_n^\ell} e^{-\rho t}U(rK)dt + \int_{\tau_n^\ell}^{\tau_n^f} e^{-\rho t}U(P_tD(1))dt + \int_{\tau_n^f}^{\infty} e^{-\rho t}U(P_tD(2))dt\right]$$
$$= \int_0^{\infty} e^{-\rho t}U(rK)dt + \mathbb{E}_{P_0}\left[e^{-\rho\tau_n^\ell}\right]V_p^\ell\left(P_{\tau_n^\ell}\right) \qquad (11)$$

where $V_p^\ell(\cdot)$ is defined as in (9). Here, the objective of the leader is to maximise $\mathbb{E}_{P_0}\left[e^{-\rho\tau_n^\ell}\right]V_p^\ell\left(P_{\tau_n^\ell}\right)$.

4. Analytical results

4.1. Monopoly

In this case, there is a single firm in the market that contemplates investment without the fear of pre-emption from the entry

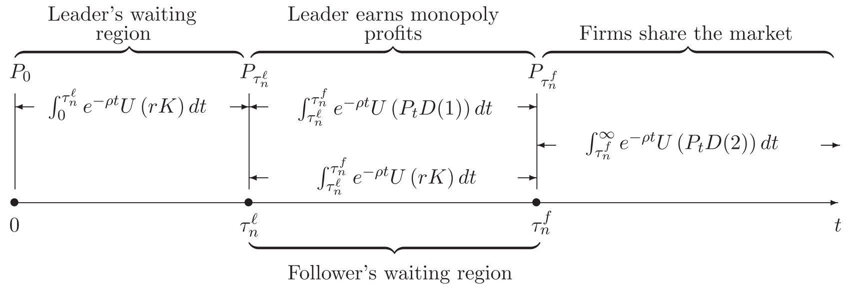

Fig. 3. Discounted utility from investment under risk aversion for a non-pre-emptive duopoly.

of a competitor. Consequently, the firm has the option to delay investment until the output price hits the optimal threshold, $P_{\tau_m^{j*}}$, that will trigger investment. Hence, for $P_0 \leqslant P_{\tau_m^{j*}}$, (12) indicates the value of the monopolist's investment opportunity:

$$F_{\tau_m^j}(P_0) = \sup_{\tau_m^j \in \mathcal{S}} \mathbb{E}_{P_0}\left[\int_{\tau_m^j}^{\infty} e^{-\rho t}[U(P_t D(1)) - U(rK)]dt\right]$$
$$= \sup_{\tau_m^j \in \mathcal{S}} \mathbb{E}_{P_0}\left[e^{-\rho \tau_m^j}\right] V_m^j\left(P_{\tau_m^j}\right) \tag{12}$$

Here, $\mathcal{S}$ denotes the collection of admissible stopping times of the filtration generated by the price process. Using Theorem 9.18 of Karatzas and Shreve (1999) for the CRRA utility function in (3), we find that the expression in (6) can be simplified using the following:

$$\mathbb{E}_{P_0}\int_0^{\infty} e^{-\rho t} U(P_t) dt = \mathcal{A} U(P_0) \tag{13}$$

where $\mathcal{A} = \frac{\beta_1 \beta_2}{\rho(1-\beta_1-\gamma)(1-\beta_2-\gamma)} > 0$, and $\beta_1 > 1, \beta_2 < 0$ are the solutions for x to the following quadratic equation:

$$\frac{1}{2}\sigma^2 x(x-1) + \mu x - \rho = 0 \tag{14}$$

By using the fact that the expected discount factor is $\mathbb{E}_{P_0}\left[e^{-\rho \tau_m^j}\right] = \left(\frac{P_0}{P_{\tau_m^j}}\right)^{\beta_1}$ (Dixit & Pindyck, 1994, p. 315) and applying the strong Markov property along with the law of iterated expectations, (12) can be written as follows:

$$F_{\tau_m^j}(P_0) = \max_{P_{\tau_m^j} \geqslant P_0} \left(\frac{P_0}{P_{\tau_m^j}}\right)^{\beta_1} V_m^j\left(P_{\tau_m^j}\right) \tag{15}$$

Solving the unconstrained optimisation problem (15), we obtain the optimal investment threshold, $P_{\tau_m^{j*}}$, for the monopolist:

$$P_{\tau_m^{j*}} = \frac{rK}{D(1)}\left(\frac{\beta_2 + \gamma - 1}{\beta_2}\right)^{\frac{1}{1-\gamma}} \tag{16}$$

Although the investment threshold is usually expressed in terms of β_1, it is more expedient to use β_2 here (note that $\beta_1\beta_2 = -\frac{2\rho}{\sigma^2}$). According to (16), uncertainty and risk aversion drive a wedge between the optimal investment threshold and the amortised investment cost. Indeed, it can be shown that higher risk aversion increases the required investment threshold by decreasing the expected utility of the investment's payoff, while increased uncertainty increases the investment threshold by increasing the value of waiting (Chronopoulos et al., 2011). All proofs are in the appendix.

Proposition 4.1. *Uncertainty and risk aversion increase the optimal investment threshold.*

4.2. Pre-emptive duopoly

We solve this dynamic game backward by first assuming that the leader has just entered the market. The value of the follower at $\tau_p^\ell < \tau_p^f$ is indicated in (17):

$$F_{\tau_p^f}\left(P_{\tau_p^\ell}\right) = \sup_{\tau_p^f \geqslant \tau_p^\ell} \mathbb{E}_{P_{\tau_p^\ell}}\left[e^{-\rho\left(\tau_p^f - \tau_p^\ell\right)}\right] V_p^f\left(P_{\tau_p^f}\right)$$
$$= \max_{P_{\tau_p^f} \geqslant P_{\tau_p^\ell}} \left(\frac{P_{\tau_p^\ell}}{P_{\tau_p^f}}\right)^{\beta_1} V_p^f\left(P_{\tau_p^f}\right) \tag{17}$$

Solving the unconstrained optimisation problem described by (17), we obtain the optimal threshold, $P_{\tau_p^{f*}}$, that triggers the entry of the follower:

$$P_{\tau_p^{f*}} = \frac{rK}{D(2)}\left(\frac{\beta_2 + \gamma - 1}{\beta_2}\right)^{\frac{1}{1-\gamma}} \tag{18}$$

Notice that since $D(2) < D(1)$, we have $P_{\tau_p^{f*}} > P_{\tau_m^{j*}}$, i.e., the optimal entry threshold of the pre-emptive follower is higher than that of the monopolist. Intuitively, this happens because the follower requires compensation for losing the first-mover advantage. After the critical threshold, $P_{\tau_p^{f*}}$, is hit, the value of the follower is the discounted expected utility of the project's cash flows, as indicated by (8).

Assuming that the follower chooses the optimal policy, the value function of the leader for $P_{\tau_p^\ell} \leqslant P_t < P_{\tau_p^{f*}}$, i.e., when the leader is alone in the market, is:

$$V_p^\ell(P_t) = \mathbb{E}_{P_t}\left[\int_0^{\tau_p^{f*}} e^{-\rho t}(U(P_t D(1)) - U(rK))dt + \int_{\tau_p^{f*}}^{\infty} e^{-\rho t}(U(P_t D(2))\right.$$
$$\left. - U(rK))dt\right] = \mathcal{A} U(P_t D(1)) - \frac{U(rK)}{\rho} + \left(\frac{P_t}{P_{\tau_p^{f*}}}\right)^{\beta_1} \mathcal{A} U\left(P_{\tau_p^{f*}}\right)$$
$$\times \left[D(2)^{1-\gamma} - D(1)^{1-\gamma}\right] \tag{19}$$

For $P_t \geqslant P_{\tau_p^{f*}}$, the two firms share the market and, as a result, the value function of the leader is the same as the follower's.

As we show in Proposition 4.2, the pre-emptive leader's value function is concave and, under a large discrepancy in market share, there exists a finite output price at which the pre-emptive leader's value function is maximised. Otherwise, the pre-emptive leader's value function is strictly increasing. Intuitively, a higher output price simultaneously increases the expected discounted utility of cash flows and facilitates the follower's entry. With a higher loss in market share, the impact of the latter effect dominates.

Proposition 4.2. *The value function of the pre-emptive leader is concave, and its maximum value is obtained prior to the entry of the pre-emptive follower provided that:*

$$D(2) < D(1)\left(\frac{\beta_1+\gamma-1}{\beta_1}\right)^{\frac{1}{1-\gamma}} \tag{20}$$

In order to determine the leader's optimal investment threshold, we need to consider the strategic interactions between the leader and the follower. Let $P_{\tau_p^{\ell^*}}$ denote the threshold price at which a firm is indifferent between becoming a leader or a follower. Recall that in the pre-emptive setting, both firms want to enter first in order to obtain the leader's advantage. However, for $P_t < P_{\tau_p^{\ell^*}}$, the follower has not entered the market, and a firm would be better off being the follower since then $V_p^\ell(P_t) < F_{\tau_p^f}(P_t)$, while for $P_t > P_{\tau_p^{\ell^*}}$, a firm is better off being a leader since then $V_p^\ell(P_t) > F_{\tau_p^f}(P_t)$. Hence, it must be the case that $V_p^\ell\left(P_{\tau_p^{\ell^*}}\right) = F_{\tau_p^f}\left(P_{\tau_p^{\ell^*}}\right)$ for entry, a condition that is found numerically by solving the following equation:

$$\mathcal{A}U\left(P_{\tau_p^{\ell^*}}D(1)\right) - \frac{U(rK)}{\rho} + \left(\frac{P_{\tau_p^{\ell^*}}}{P_{\tau_p^{f^*}}}\right)^{\beta_1}\mathcal{A}U\left(P_{\tau_p^{f^*}}\right)[D(2)^{1-\gamma}-D(1)^{1-\gamma}]$$
$$= \left(\frac{P_{\tau_p^{\ell^*}}}{P_{\tau_p^{f^*}}}\right)^{\beta_1}\left[\mathcal{A}U\left(P_{\tau_p^{f^*}}D(2)\right) - \frac{U(rK)}{\rho}\right] \tag{21}$$

Solving (21) for $P_{\tau_p^{\ell^*}}$, we obtain the entry threshold of the leader that denotes the output price at which a firm is indifferent between becoming a leader or a follower. Indeed, as we show in Proposition 4.3, the optimal entry threshold of the pre-emptive leader is lower than that of the monopolist. This happens because the risk of pre-emption deprives the leader of the option to postpone investment, thereby lowering the required investment threshold.

Proposition 4.3. *The pre-emptive leader's optimal entry threshold is lower than that of the monopolist.*

Although increased risk aversion raises the required investment threshold by decreasing the expected utility of the investment's payoff, the loss in the value of the leader due to the entry of the follower, evaluated at $P_{\tau_p^{\ell^*}}$, relative to that of the monopolist is not affected by risk aversion. Intuitively, the value of the leader at $P_{\tau_p^{\ell^*}}$ equals the value of the follower's investment opportunity. Since both the follower and the monopolist hold a single option each to enter the market, increased risk aversion poses a proportional decrease in the option value of the follower relative to the monopolist.

Proposition 4.4. *The loss in the pre-emptive leader's value relative to the monopolist's value of investment opportunity at the pre-emptive leader's optimal entry threshold price is unaffected by risk aversion.*

We next investigate how this ratio changes with uncertainty. In Fig. 4, the horizontal lines represent the utility of the instantaneous revenues the leader receives over time under low uncertainty, σ, and under high uncertainty, σ'. As we will illustrate numerically, increased uncertainty raises the required entry threshold of the follower by more than that of the leader. This results in the increase of the expected utility of the leader's profits, represented by the shaded area of Fig. 4, since, under higher uncertainty, she enjoys monopoly profits for a longer time and the loss in the leader's expected utility due to the entry of the follower at a higher price is not significant enough to offset it. In fact, this result is enhanced when the discrepancy in market share is large, since the greater $D(1)$ is, the greater the pre-emptive leader's incentive to invest will be since then the first-mover advantages are greater. Notice also that as greater uncertainty raises the required entry threshold of the follower, the leader's instantaneous revenues cannot drop below the level corresponding to σ for $t \geqslant \tau_p^{f'}$.

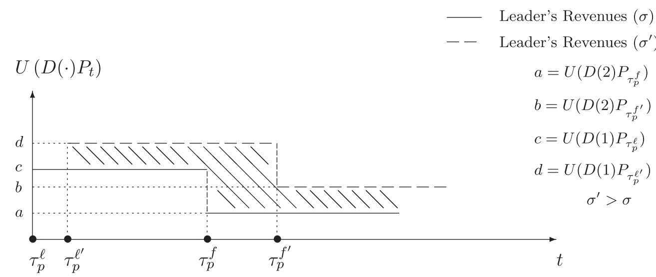

Fig. 4. Incremental change in pre-emptive leader's instantaneous revenues due to increased uncertainty.

Proposition 4.5. *The loss in the pre-emptive leader's value relative the monopolist's at the pre-emptive leader's optimal entry threshold price diminishes with increasing uncertainty.*

4.3. Non-pre-emptive duopoly

In the non-pre-emptive setting, the roles of the leader and the follower are defined exogenously, and, as a result, both firms have the option to delay their entry into the market as the risk of pre-emption is eliminated. The follower's value function and entry threshold are unchanged from the pre-emptive case since she will still enter the market considering that the leader is already there. Hence, the follower's value of investment opportunity at τ_n^ℓ is:

$$F_{\tau_n^f}\left(P_{\tau_n^\ell}\right) = \max_{P_{\tau_n^f} \geqslant P_{\tau_n^\ell}}\left(\frac{P_{\tau_n^\ell}}{P_{\tau_n^f}}\right)^{\beta_1} V_n^f\left(P_{\tau_n^f}\right) \tag{22}$$

Since the non-pre-emptive leader has discretion over investment timing, her value of investment opportunity is described by:

$$F_{\tau_n^\ell}(P_0) = \max_{P_{\tau_n^\ell} \geqslant P_0}\left(\frac{P_0}{P_{\tau_n^\ell}}\right)^{\beta_1}\left[\mathcal{A}U\left(P_{\tau_n^\ell}D(1)\right) - \frac{U(rK)}{\rho} + \left(\frac{P_{\tau_n^\ell}}{P_{\tau_n^{f^*}}}\right)^{\beta_1}\mathcal{A}U\left(P_{\tau_n^{f^*}}\right)\right.$$
$$\left.\times\left[D(2)^{1-\gamma} - D(1)^{1-\gamma}\right]\right] \tag{23}$$

The solution to the optimisation problem (23) yields the optimal entry threshold of the non-pre-emptive leader:

$$P_{\tau_n^{\ell^*}} = \frac{rK}{D(1)}\left(\frac{\beta_2+\gamma-1}{\beta_2}\right)^{\frac{1}{1-\gamma}} \tag{24}$$

Notice that by delaying entry, the leader suffers from forgoing cash flows but benefits from temporarily delaying the entry of the follower. At the same time, allowing the project to start at a higher output price yields a higher expected NPV, but the leader enjoys monopoly revenues for less time. As it is shown in the appendix, the marginal benefit and marginal cost corresponding to the entry of the follower cancel.

Proposition 4.6. *The optimal entry threshold of the non-pre-emptive leader is the same as that of the monopolist.*

After the leader has entered the market and prior to the entry of the follower, i.e., for $P_{\tau_n^\ell} \leqslant P_t < P_{\tau_n^{f^*}}$, the leader receives monopolistic profits with expected utility described by (25):

$$\mathcal{A}U(P_tD(1)) - \frac{U(rK)}{\rho} + \left(\frac{P_t}{P_{\tau_n^{f^*}}}\right)^{\beta_1}\mathcal{A}U\left(P_{\tau_n^{f^*}}\right)[D(2)^{1-\gamma} - D(1)^{1-\gamma}] \tag{25}$$

After the follower's entry, i.e., for $t \geqslant \tau_n^f$, the two firms share the industry, thereby making equal profits, and their value is simply the discounted expected utility of the project's cash flows.

In the non-pre-emptive framework, the value of the leader would be the same as the monopolist's if it were not for the potential entry of the follower that reduces the expected utility of the leader's profits. However, the reduction in the leader's value of investment opportunity due to the potential entry of the follower decreases with risk aversion. This happens because risk aversion delays the entry of the follower, thereby reducing the expected loss in the option value of the leader. Consequently, the relative discrepancy between the leader's value of investment opportunity and the monopolist's diminishes with increasing risk aversion, thereby reducing the relative loss in the value of the non-pre-emptive leader.

Proposition 4.7. *The loss in the value of the investment opportunity for the non-pre-emptive leader relative to that of a monopolist at the pre-emptive leader's optimal entry threshold price decreases with risk aversion.*

According to Proposition 4.8, depending on the discrepancy in market share, uncertainty may increase or decrease the relative loss in the value of the investment opportunity for the non-pre-emptive leader relative to that of a monopolist. Notice that the value of the non-pre-emptive leader consists of the value of the monopolistic investment opportunity and the expected loss in project value due to the entry of the follower. Both of these components increase with uncertainty; however, for the latter, the impact of uncertainty becomes more profound as the discrepancy in market share increases. A version of this result for a risk-neutral duopoly with sequential capacity investments is given in Siddiqui and Takashima (2012).

Proposition 4.8. *The loss in the non-pre-emptive leader's value of investment opportunity relative to the monopolist's at the pre-emptive leader's optimal entry threshold price increases with uncertainty if:*

$$\left(\frac{D(1)}{D(2)}\right)^{\beta_1} > e, \; e \simeq 2.718 \tag{26}$$

In Fig. 5, the instantaneous revenues of the leader are represented by the solid line for low uncertainty, σ, and by the broken line for high uncertainty, σ'. Here, unlike in the pre-emptive setting, the leader has the option to delay entry into the market. Notice that a high discrepancy in market share implies a greater first-mover advantage but leads to a greater loss in the value of the leader upon the entry of the follower, which becomes more profound with higher uncertainty. However, increased uncertainty also raises the value of the leader's investment opportunity, thereby creating an opposing effect. According to Proposition 4.8, if the discrepancy in market share is low, then the loss in the non-pre-emptive leader's value relative to the monopolist's decreases with uncertainty. This happens because although the incentive to delay investment is more profound and increases the opportunity cost, represented by the shaded area between τ_n^{ℓ} and $\tau_n^{\ell'}$ in Fig. 5, the higher expected revenues offset the expected loss in market share due to the follower's entry. The opposite result is observed if the discrepancy in market share is high, since then the impact of uncertainty on the value of the investment opportunity is less pronounced. Consequently, the increase in the leader's expected revenues from delaying investment is not significant enough to offset the expected loss due to the entry of the follower. Hence, as the impact of uncertainty on the loss in project value is more profound, the non-pre-emptive leader becomes worse off.

5. Numerical results

5.1. Pre-emptive duopoly

In order to examine the impact of risk aversion and uncertainty on the entry of the pre-emptive leader and follower, we assume the following parameter values: $\gamma \in [0,1)$, $\sigma \in [0.1, 0.5]$, $\mu = 0.01$, $r = \rho = 0.04$, $K = \$100$, $D(0) = 0$, $D(1) = 1.5$ or 3, and $D(2) = 1$. Although it would be interesting to calibrate these to real data, here, we are primarily concerned with illustrating analytical insights via hypothetical parameters. Fig. 6 illustrates the impact of uncertainty on the value of the pre-emptive leader and follower under risk aversion. First, we observe that the leader's entry threshold is lower than the monopolist's, which illustrates Proposition 4.3. This happens due to pre-emption since the leader does not have the option to defer investment, and, as a result, the risk of pre-emption reduces the required investment threshold. On the other hand, the required investment threshold of the pre-emptive follower is higher than that of the monopolist since the former requires compensation for losing the first-mover advantage. According to the graph on the right, uncertainty increases the value of waiting, thereby raising the required investment threshold and delaying the entry of the follower. This, in turn, increases the time interval in which the leader enjoys monopoly profits and diminishes the relative discrepancy between the value of the pre-emptive leader and that of the monopolist.

Fig. 7 illustrates the impact of risk aversion on the value of the pre-emptive leader and follower. According to the graph on the right, increased risk aversion reduces the expected utility of the investment's payoff for both the leader and the monopolist, thereby raising their required investment thresholds. Furthermore, it seems that the impact of risk aversion on the pre-emptive leader's value is greater than on the follower's value. Consequently, the two curves intersect at a higher output price, thereby indicating that the output price at which a firm is indifferent between becoming a leader or a follower increases with higher risk aversion.

5.2. Non-pre-emptive duopoly

In the non-pre-emptive duopoly, the roles of the leader and the follower are pre-assigned, and, as a result, both firms have the

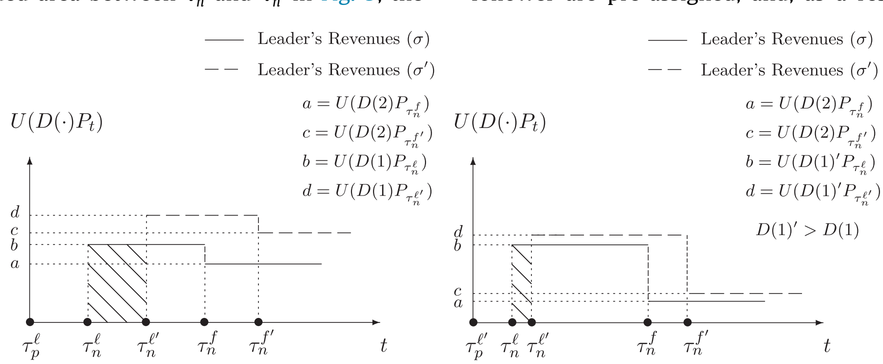

Fig. 5. Incremental change in non-pre-emptive leader's instantaneous revenues due to increased uncertainty under low (left) and high (right) discrepancy in market share.

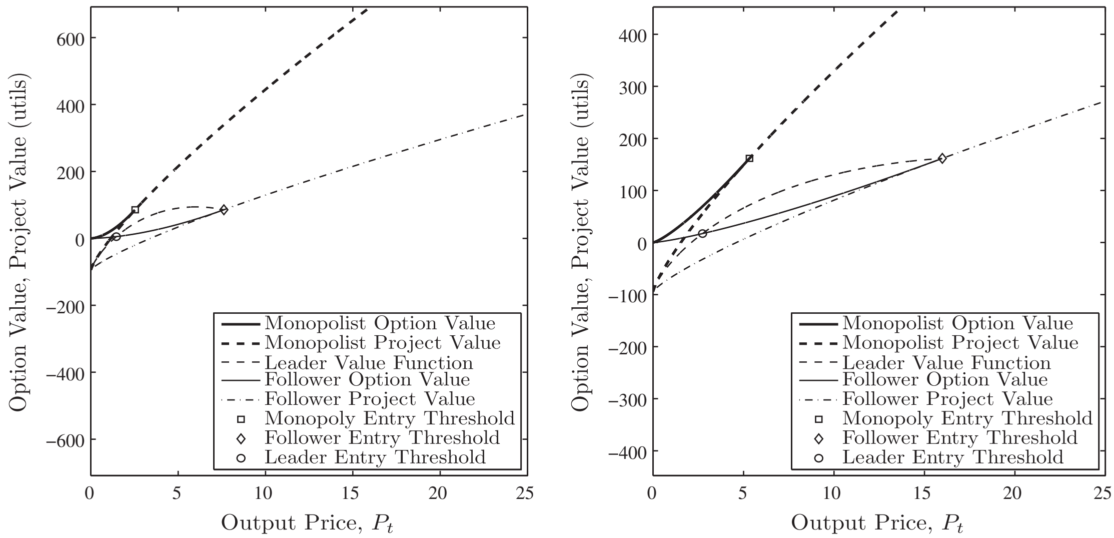

Fig. 6. Project and investment opportunity value of monopolist, pre-emptive leader, and follower for $\sigma = 0.2$ (left) and $\sigma = 0.4$ (right) under risk aversion ($\gamma = 0.2$) for $D(1) = 3$.

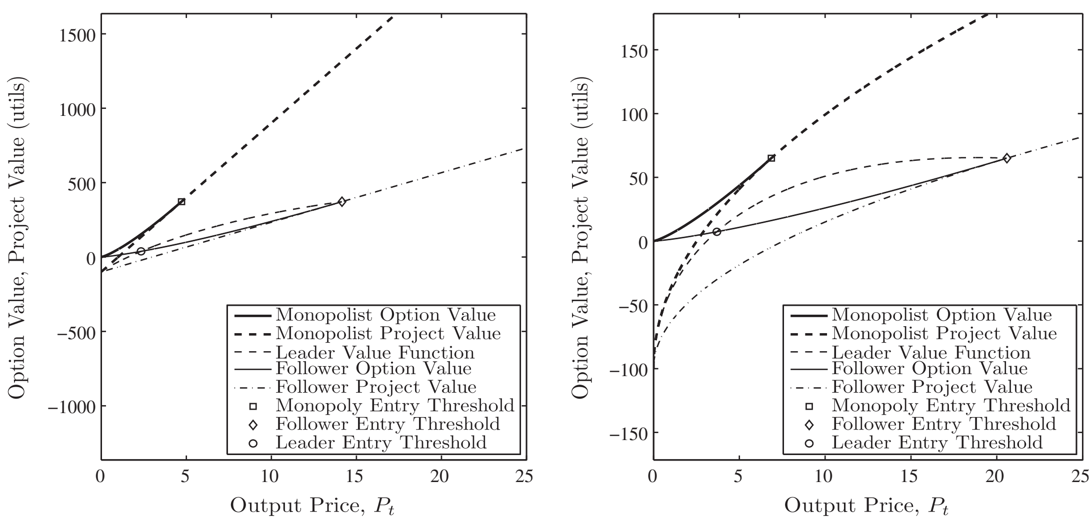

Fig. 7. Investment opportunity and project value of monopolist, pre-emptive leader, and follower under risk neutrality (left) and risk aversion ($\gamma = 0.5$) (right) for $\sigma = 0.4$ and $D(1) = 3$.

option to postpone their entry into the market. According to Fig. 8, the optimal entry threshold of the non-pre-emptive follower is the same as in the pre-emptive case since the follower will still enter the market considering that the leader has already invested. Notice also that the optimal entry threshold of the non-pre-emptive leader is the same as the monopolist's, and, as a result, the required investment threshold of the non-pre-emptive leader is higher than that in the pre-emptive setting. This illustrates Proposition 4.3. Although the optimal entry threshold is the same for the monopolist and non-pre-emptive leader, the investment opportunity value of the latter is lower than that of the former since the potential entry of the follower reduces the expected utility of the leader's profits. As the graph on the right illustrates, increased uncertainty raises the value of waiting, which, in turn, postpones investment in all cases, thereby increasing the required investment thresholds.

Fig. 9 illustrates the impact of risk aversion on the optimal entry thresholds of the monopolist and the non-pre-emptive leader and follower. As indicated in the graphs, higher risk aversion reduces the expected utility of the investment's payoff in all cases, thereby raising the required investment thresholds.

5.3. Sensitivity analysis

As the left panel in Fig. 10 illustrates, all entry thresholds increase with volatility as greater uncertainty implies greater value of waiting and are higher with risk aversion as it delays investment both for the leader and the follower by decreasing the expected utility of the project's cash flows, which illustrates Proposition 4.1. Proposition 4.6 is illustrated by the fact that the leader's optimal investment threshold is the same as the monopolist's. Also, higher first-mover advantages represented by greater $D(1)$ result in the decrease of the required entry thresholds of the pre-emptive and non-pre-emptive leader as illustrated in the graph on the right.

In order to compare the pre-emptive and non-pre-emptive leader's values to the monopolist's, we evaluate both at the

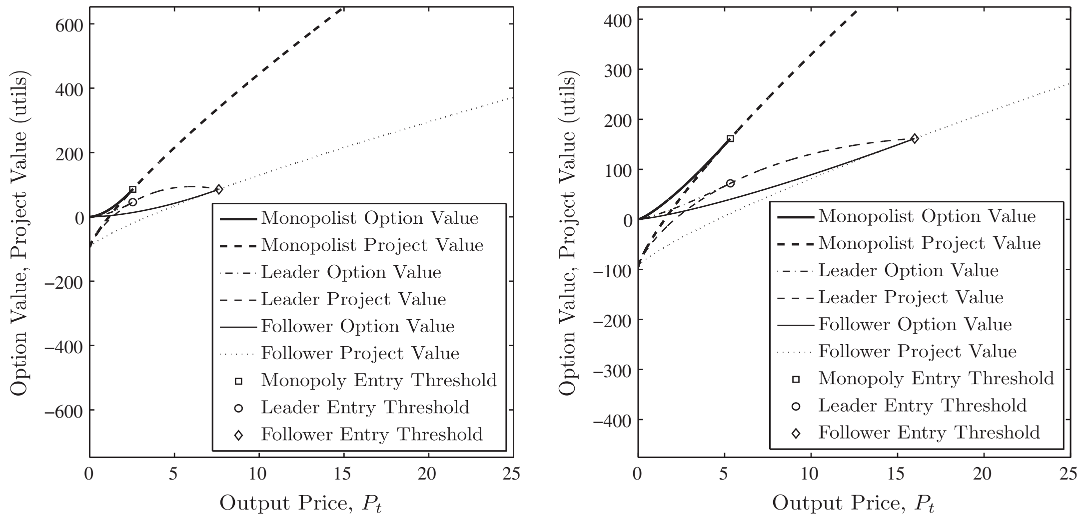

Fig. 8. Project and investment opportunity value for non-pre-emptive leader and follower for $\sigma = 0.2$ (left) and $\sigma = 0.4$ (right) under risk aversion ($\gamma = 0.2$) for $D(1) = 3$.

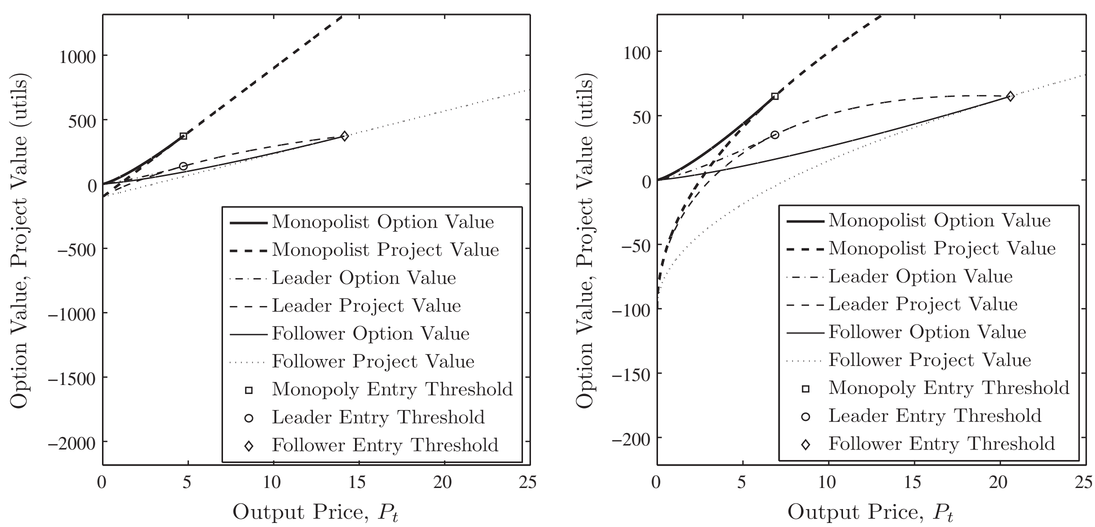

Fig. 9. Project and investment opportunity value for non-pre-emptive leader and follower under risk neutrality (left) and risk aversion ($\gamma = 0.5$) (right) for $\sigma = 0.4$ and $D(1) = 3$.

pre-emptive leader's optimal entry threshold, i.e., at $P_{\tau_p^{\ell*}}$. According to the graph on the left in Fig. 11, increased uncertainty diminishes the relative loss in the pre-emptive leader's value function, i.e.,

$$\frac{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right) - V_p^{\ell}\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)} \tag{27}$$

thereby reducing the discrepancy between the pre-emptive leader's value and the monopolist's value of investment opportunity. This happens because uncertainty postpones the entry of the follower, thus allowing the pre-emptive leader to enjoy monopoly profits longer, which illustrates Proposition 4.5. Notice that the impact of uncertainty is more profound when the discrepancy in market share is low since then the expected loss due to the follower's entry is smaller.

Uncertainty decreases the relative loss in the non-pre-emptive leader's value of investment opportunity, i.e.,

$$\frac{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right) - F_{\tau_n^{\ell}}\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)} \tag{28}$$

if the discrepancy in market share is small, i.e., $\left(\frac{D(1)}{D(2)}\right)^{\beta_1} < e$, as in the graph on the left. Intuitively, this happens because under low discrepancy in market share, the increase in the non-pre-emptive leader's value of investment opportunity due to increased uncertainty is greater than the expected loss due to the entry of the follower. However, if the discrepancy is high, then the increase in option value is less profound with higher uncertainty due to higher first-mover advantages and, as a result, cannot offset the expected loss from the follower's entry, which is now greater. This illustrates Proposition 4.8.

Furthermore, risk aversion does not affect the relative loss in the value of the leader for the pre-emptive duopoly setting, as in Proposition 4.4, but it makes the loss in value relatively less for the leader in a non-pre-emptive duopoly setting due to delayed entry of the follower, which illustrates Proposition 4.7. Notice that at $P_{\tau_p^{\ell*}}$, the value function of the pre-emptive leader is the same as the option value of the pre-emptive follower. As a result, the impact of risk aversion on the value of the pre-emptive leader at $P_{\tau_p^{\ell*}}$ is the same as that on the value of the follower's investment opportunity at the same output price. Since the follower's investment opportunity value differs from the monopolist's only with respect to the market share, risk aversion

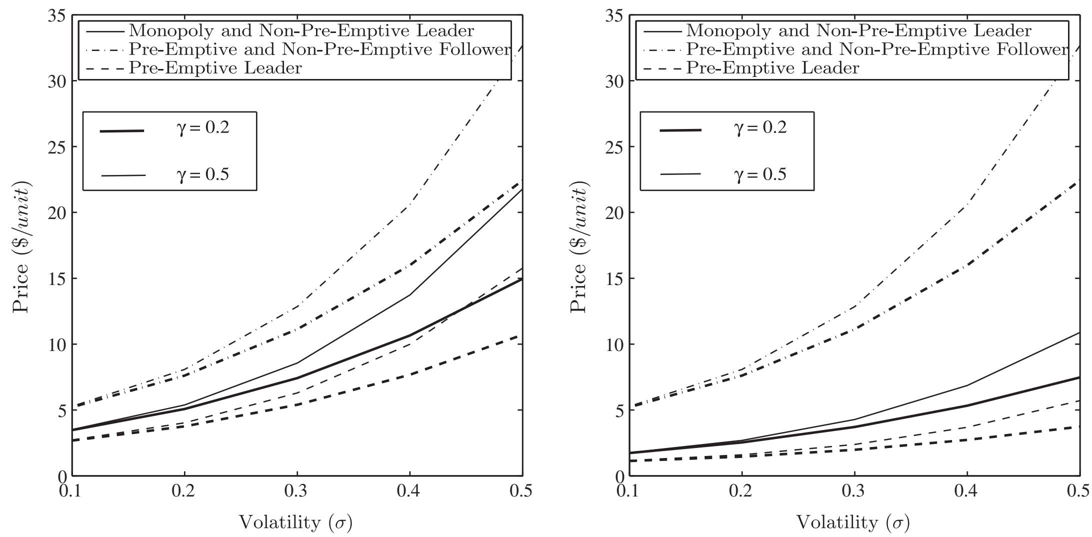

Fig. 10. Optimal entry thresholds for $D(1) = 1.5$ (left) and $D(1) = 3$ (right).

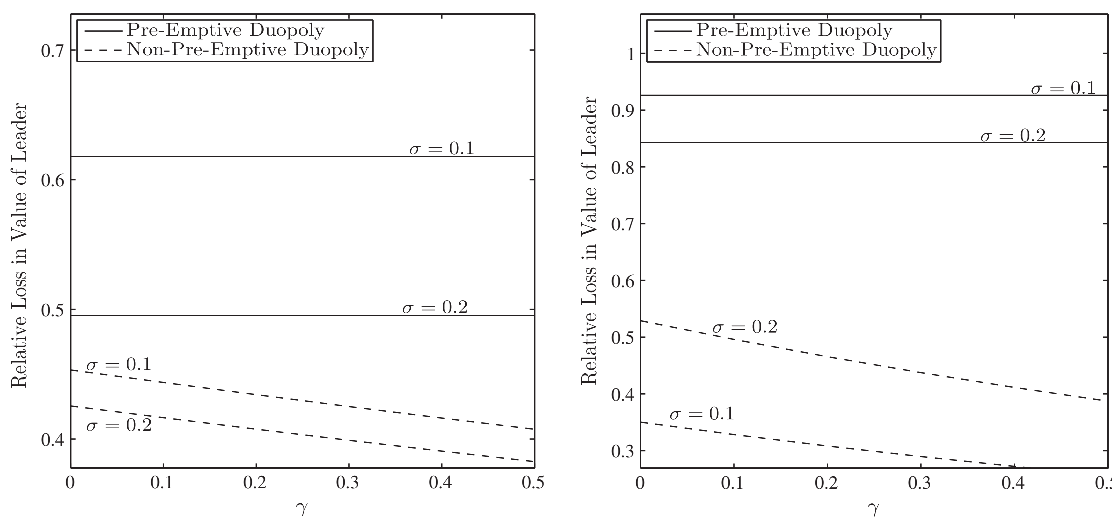

Fig. 11. Relative loss in value of the pre-emptive and non-pre-emptive leader for $D(1) = 1.5$ (left) and $D(1) = 3$ (right).

impacts the values of the follower and the monopolist proportionally.

6. Conclusions

In this paper, we develop a utility-based framework in order to examine the impact of risk aversion and uncertainty on the optimal investment timing decisions of a firm that faces competition. The analysis is motivated both by the increasing competition resulting from the deregulation of many sectors of the economy such as energy and telecommunications, as well as the fact that attitudes towards the undiversifiable risk arising from the development of new products may impact investment decisions of a firm. The combination of these two factors creates the need to incorporate risk aversion into the real options framework in order to analyse strategic aspects of decision making under uncertainty. In the context of our motivating example concerning renewable energy technologies, the nature of industry competition and the degree of risk aversion due to the presence of technical risk could shape the pathway to decarbonisation in unexpected ways.

We find that, under the fear of pre-emption, higher uncertainty reduces the relative loss in the value of the leader due to competition by delaying the entry of the follower. However, in the non-pre-emptive setting, the impact of uncertainty is ambiguous and depends on the discrepancy in market share. If the discrepancy is high, then the non-pre-emptive leader's relative loss in value increases with uncertainty since the impact of the follower's entry is more profound and offsets the increase in the leader's value of investment opportunity. By contrast, under low discrepancy in market share, higher uncertainty makes the non-pre-emptive leader better off as the increase in the value of investment opportunity is greater than the expected loss in value due to competition. Interestingly, the relative loss in the pre-emptive leader's value is not affected by risk aversion, while the non-pre-emptive leader becomes better off with greater risk aversion as it delays the entry of the follower. Hence, regulators may have to think more carefully about energy policy and market design in guiding an energy transition due to the subtle effects of such salient features.

This work considers the case where the two competing firms exhibit the same level of risk aversion. Consequently, a potential extension is to relax this assumption and to investigate a case with different levels of risk aversion for each firm. Directions for future research may also include the application of a different stochastic process, e.g., arithmetic Brownian motion, or the study of other aspects of the real options literature, such as the time to build or capacity sizing, under the same framework. Finally, we may allow for operational flexibility via suspension and resumption options, as this may reduce the incentive to delay investment under high risk aversion, as indicated by Chronopoulos et al. (2011).

Acknowledgement

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Appendix A

Proposition 4.1. *Uncertainty and risk aversion increase the optimal investment threshold.*

Proof. See Propositions 4.2 and 4.3 in Chronopoulos et al. (2011). □

Proposition 4.2. *The value function of the pre-emptive leader is concave and its maximum value is obtained prior to the entry of the pre-emptive follower provided that:*

$$D(2) < D(1)\left(\frac{\beta_1+\gamma-1}{\beta_1}\right)^{\frac{1}{1-\gamma}} \tag{29}$$

Proof. The value of the pre-emptive leader is:

$$V_p^\ell(P_t) = \mathcal{A}U(P_tD(1)) - \frac{U(rK)}{\rho} + \left(\frac{P_t}{P_{\tau_p^{f*}}}\right)^{\beta_1}\mathcal{A}U(P_{\tau_p^{f*}})\left[D(2)^{1-\gamma} - D(1)^{1-\gamma}\right] \tag{30}$$

Differentiating (30) with respect to P_t we have:

$$\frac{\partial V_p^\ell(P_t)}{\partial P_t} = \mathcal{A}D(1)^{1-\gamma}P_t^{-\gamma} + \beta_1\left(\frac{P_t}{P_{\tau_p^{f*}}}\right)^{\beta_1} \times \frac{1}{P_t}\mathcal{A}U(P_{\tau_p^{f*}})\left[D(2)^{1-\gamma} - D(1)^{1-\gamma}\right] \tag{31}$$

Hence,

$$\frac{\partial V_p^\ell(P_t)}{\partial P_t} = 0 \Rightarrow P_t = P_{\tau_p^{f*}}\left[\frac{\beta_1}{1-\gamma}\left[1-\left(\frac{D(2)}{D(1)}\right)^{1-\gamma}\right]\right]^{\frac{1}{1-\beta_1-\gamma}} \tag{32}$$

Notice that $\frac{1}{1-\beta_1-\gamma} < 0$. Hence, for (32) to be valid we must have:

$$1-\left(\frac{D(2)}{D(1)}\right)^{1-\gamma} > 0 \iff \left(\frac{D(2)}{D(1)}\right)^{1-\gamma} < 1 \iff D(2) < D(1) \tag{33}$$

which is true. In order to show that the value of the pre-emptive leader obtains a maximum, we partially differentiate (31) with respect to P_t.

$$\frac{\partial^2 V_p^\ell(P_t)}{\partial P_t^2} = \frac{-\gamma\mathcal{A}D(1)^{1-\gamma}}{P_t^{\gamma+1}} + \beta_1(\beta_1-1)P_t^{\beta_1-2}\left(\frac{1}{P_{\tau_p^{f*}}}\right)^{\beta_1}\mathcal{A}U\left(P_{\tau_p^{f*}}\right) \times \left[D(2)^{1-\gamma} - D(1)^{1-\gamma}\right] \tag{34}$$

As both terms in (34) are negative, we have $\frac{\partial^2 V_p^\ell(P_t)}{\partial P_t^2} < 0\ \forall P_t \in \left[P_{\tau_p^\ell}, P_{\tau_p^{f*}}\right)$. Finally, we will derive the condition under which the output price at which $V_p^\ell(P_t)$ becomes maximised is lower than the optimal entry threshold of the follower:

$$\left[\frac{\beta_1}{1-\gamma}\left[1-\left(\frac{D(2)}{D(1)}\right)^{1-\gamma}\right]\right]^{\frac{1}{\beta_1+\gamma-1}} > 1 \iff D(2) < \left(\frac{\beta_1+\gamma-1}{\beta_1}\right)^{\frac{1}{1-\gamma}}D(1) \tag{35}$$

Notice that $\frac{\beta_1+\gamma-1}{\beta_1} < 1$ implies that, in order for the value function of the pre-emptive leader to decrease prior to the entry of the follower, the discrepancy in market share must be significantly large. □

Proposition 4.3. *The pre-emptive leader's entry threshold is lower than that of the monopolist.*

Proof. First, notice that the follower's value of investment opportunity is:

$$\begin{aligned}
F_{\tau_p^f}(P_t) &= \left(\frac{P_t}{P_{\tau_p^{f*}}}\right)^{\beta_1} V_p^f\left(P_{\tau_p^{f*}}\right) \Rightarrow \frac{\partial F_{\tau_p^f}(P_t)}{\partial P_t} \\
&= \beta_1 P_t^{\beta_1-1}\left(\frac{1}{P_{\tau_p^{f*}}}\right)^{\beta_1} V_p^f\left(P_{\tau_p^{f*}}\right) > 0, \quad \forall P_t \in \left[P_{\tau_p^\ell}, P_{\tau_p^{f*}}\right) \\
&\Rightarrow \frac{\partial^2 F_{\tau_p^f}(P_t)}{\partial P_t^2} = \beta_1(\beta_1-1)P_t^{\beta_1-2}\left(\frac{1}{P_{\tau_p^{f*}}}\right)^{\beta_1} V_p^f\left(P_{\tau_p^{f*}}\right) \\
&> 0, \quad \forall P_t \in \left[P_{\tau_p^\ell}, P_{\tau_p^{f*}}\right)
\end{aligned} \tag{36}$$

Thus, the value of the follower's investment opportunity is convex and strictly increasing from zero. Second, from Proposition 4.2, we know that the pre-emptive leader's value function is strictly concave in P_t starting from a negative value. Consequently, for $P_t < P_{\tau_p^{f*}}$ the two value functions intersect at most once. In order to show that the pre-emptive leader's entry threshold is lower than that of the monopolist, we will evaluate the pre-emptive leader's value and the pre-emptive follower's value of investment opportunity at the monopolist's entry threshold. The objective is to prove that at the monopolist's optimal entry threshold, the value of the pre-emptive leader is greater than the value of the pre-emptive follower's investment opportunity, i.e.,

$$\begin{aligned}
&\mathcal{A}U\left(P_{\tau_m^{j*}}D(1)\right) - \frac{U(rK)}{\rho} \\
&+ \left(\frac{P_{\tau_m^{j*}}}{P_{\tau_p^{f*}}}\right)^{\beta_1}\left[\mathcal{A}U\left(P_{\tau_p^{f*}}D(2)\right) - \mathcal{A}U\left(P_{\tau_p^{f*}}D(1)\right)\right] \\
&> \left(\frac{P_{\tau_m^{j*}}}{P_{\tau_p^{f*}}}\right)^{\beta_1}\left[\mathcal{A}U\left(P_{\tau_p^{f*}}D(2)\right) - \frac{U(rK)}{\rho}\right]
\end{aligned} \tag{37}$$

Substituting for $P_{\tau_m^{j*}}$ and $P_{\tau_p^{f*}}$ we have:

$$(1-\gamma)\left(\frac{D(1)}{D(2)}\right)^{\beta_1} - \beta_1\left(\frac{D(1)}{D(2)}\right)^{1-\gamma} > 1-\beta_1-\gamma \tag{38}$$

The last inequality can be written as $b-a+ax^b-bx^a>0$ where $a=1-\gamma<1$, $b=\beta_1>1$, and $x=\frac{D(1)}{D(2)}>1$. Since $b-a=\beta_1+\gamma-1>0$, in order to show (38), we need to show that $ax^b-bx^a>0$. For this reason, let $f(x)=ax^b-bx^a$, $x>1$. Notice that since $f'(x)>0$ and $f''(x)>0, f(x)$ is increasing and convex. Also, $f'(x)=0 \Rightarrow x=1$ and $f(1)=0$. Thus, $f(x)>f(1)=0 \Rightarrow ax^b-bx^a>0,\ \forall\, x>1$. Therefore, at the entry threshold of the monopolist, the value function of the pre-emptive leader is greater than the follower's value of investment opportunity. Notice also that $P_t \to 0 \Rightarrow V_p^\ell(P_t) < F_{\tau_p^f}(P_t)$. Since, according to Proposition 4.2, the maximum that the value of the pre-emptive leader can obtain in $\left[P_{\tau_p^\ell}, P_{\tau_p^{f*}}\right)$ is global, this implies that, $\forall P_t : P_t < P_{\tau_m^{j*}}, \exists$ at most one price $P_{\tau_p^{\ell*}} : F_{\tau_p^f} = V_p^\ell$. Hence, we conclude that $P_{\tau_p^{\ell*}} < P_{\tau_m^{j*}}$. □

Proposition 4.4. *The loss in the pre-emptive leader's value relative to the monopolist's value of investment opportunity at the pre-emptive leader's optimal entry threshold price is unaffected by risk aversion.*

Proof. In order to show that the relative loss in value is unaffected by risk aversion, we consider the following ratio:

$$\frac{V_p^\ell\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)} \tag{39}$$

Notice that $F_{\tau_m^j}(\cdot)$ is given by (12), which we re-write here for $P_0 = P_{\tau_p^{\ell*}}$:

$$F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right) = \left(\frac{P_{\tau_p^{\ell*}}}{P_{\tau_m^{j*}}}\right)^{\beta_1}\left[\mathcal{A}U\left(P_{\tau_m^{j*}}D(1)\right) - \frac{U(rK)}{\rho}\right] \tag{40}$$

Similarly, the expression for $V_p^\ell(\cdot)$ evaluated at $P_{\tau_p^{\ell*}}$ is given by:

$$V_p^\ell\left(P_{\tau_p^{\ell*}}\right) = \mathcal{A}U\left(P_{\tau_p^{\ell*}}D(1)\right) - \frac{U(rK)}{\rho} + \left(\frac{P_{\tau_p^{\ell*}}}{P_{\tau_p^{f*}}}\right)^{\beta_1}\mathcal{A}U\left(P_{\tau_p^{f*}}\right)\left(D(2)^{1-\gamma} - D(1)^{1-\gamma}\right) \tag{41}$$

Notice also that for $P_{\tau_p^{\ell*}}$, the equality $V_p^\ell(P_{\tau_p^{\ell*}}) = F_{\tau_p^f}(P_{\tau_p^{\ell*}})$ holds, i.e.:

$$\mathcal{A}U\left(P_{\tau_p^{\ell*}}D(1)\right) - \frac{U(rK)}{\rho} + \left(\frac{P_{\tau_p^{\ell*}}}{P_{\tau_p^{f*}}}\right)^{\beta_1}\mathcal{A}U\left(P_{\tau_p^{f*}}\right)\left(D(2)^{1-\gamma} - D(1)^{1-\gamma}\right) = \left(\frac{P_{\tau_p^{\ell*}}}{P_{\tau_p^{f*}}}\right)^{\beta_1}\left[\mathcal{A}U\left(P_{\tau_p^{f*}}D(2)\right) - \frac{U(rK)}{\rho}\right] \tag{42}$$

Substituting the expressions for $P_{\tau_p^{f*}}$ and $P_{\tau_m^{j*}}$ from (16) and (18) into (40) and (42), respectively, we have:

$$F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right) = \left(\frac{P_{\tau_p^{\ell*}}}{P_{\tau_m^{j*}}}\right)^{\beta_1}\left[\frac{1-\gamma}{\beta_1+\gamma-1}\right]\frac{U(rK)}{\rho} \tag{43}$$

and

$$V_p^\ell\left(P_{\tau_p^{\ell*}}\right) = \left(\frac{P_{\tau_p^{\ell*}}}{P_{\tau_p^{f*}}}\right)^{\beta_1}\left[\frac{1-\gamma}{\beta_1+\gamma-1}\right]\frac{U(rK)}{\rho} \tag{44}$$

By cancelling the $P_{\tau_p^{\ell*}}$ term and substituting for $P_{\tau_m^{j*}}$ and $P_{\tau_p^{f*}}$, we have:

$$\frac{V_p^\ell\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)} = \left(\frac{D(2)}{D(1)}\right)^{\beta_1} \tag{45}$$

As a result, the relative loss in the value of the pre-emptive leader is constant and, for this reason, is unaffected by risk aversion. □

Proposition 4.5. *The loss in the pre-emptive leader's value relative to the monopolist's at the pre-emptive leader's optimal entry threshold price diminishes with increasing uncertainty.*

Proof. According to (45), the relative value of the pre-emptive leader compared to that of a monopolist is:

$$\frac{V_p^\ell\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)} = \frac{V_p^\ell\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)} = \left(\frac{D(2)}{D(1)}\right)^{\beta_1} \tag{46}$$

Partially differentiating (46) with respect to σ, we have:

$$\frac{\partial}{\partial\sigma}\left(\frac{D(2)}{D(1)}\right)^{\beta_1} = \left(\frac{D(2)}{D(1)}\right)^{\beta_1}\ln\left(\frac{D(2)}{D(1)}\right)\frac{\partial\beta_1}{\partial\sigma} \tag{47}$$

Notice that since $\frac{\partial\beta_1}{\partial\sigma}<0$ and $\ln\left(\frac{D(2)}{D(1)}\right)<0$, we have:

$$\frac{\partial}{\partial\sigma}\left[\frac{V_p^\ell\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)}\right] > 0 \tag{48}$$

This implies that with increasing uncertainty, the loss in the value of the pre-emptive leader relative to the monopolist's diminishes. □

Proposition 4.6. *The optimal entry threshold of the non-pre-emptive leader is the same as that of the monopolist.*

Proof. Given the initial output price, P_0, and assuming that the follower has chosen the optimal policy, the non-pre-emptive leader's entry problem is described by (49):

$$F_{\tau_n^\ell}(P_0) = \sup_{\tau_n^\ell \geqslant P_0}\mathbb{E}_{P_0}\left[e^{-\rho\tau_n^\ell}\right]V_p^\ell\left(P_{\tau_n^\ell}\right) \tag{49}$$

where

$$V_p^\ell\left(P_{\tau_n^\ell}\right) = V_m^j\left(P_{\tau_n^\ell}\right) + \mathbb{E}_{P_{\tau_n^\ell}}\left[e^{-\rho\left(\tau_p^{f*}-\tau_n^\ell\right)}\right]\mathbb{E}_{P_{\tau_p^{f*}}}\left[\int_0^\infty e^{-\rho t}[U(P_tD(2)) - U(P_tD(1))]dt\right] \tag{50}$$

Hence, (49) may be rewritten as:

$$F_{\tau_n^\ell}(P_0) = \max_{P_{\tau_n^\ell}\geqslant P_0}\left(\frac{P_0}{P_{\tau_n^\ell}}\right)^{\beta_1}\left[V_m^j\left(P_{\tau_n^\ell}\right) + \left(\frac{P_{\tau_n^\ell}}{P_{\tau_n^{f*}}}\right)^{\beta_1}\mathbb{E}_{P_{\tau_p^{f*}}}\left[\int_0^\infty e^{-\rho t}[U(P_tD(2)) - U(P_tD(1))\right]dt\right]\right] \tag{51}$$

The FONC for the optimisation problem (52) is:

$$\begin{aligned}\frac{\partial F_{\tau_n^\ell}(P_0)}{\partial P_{\tau_n^\ell}} = {}& \beta_1\left(\frac{P_0}{P_{\tau_n^\ell}}\right)^{\beta_1}\left(-\frac{1}{P_{\tau_n^\ell}}\right)V_m^j\left(P_{\tau_n^\ell}\right) + \left(\frac{P_0}{P_{\tau_n^\ell}}\right)^{\beta_1}\frac{\partial}{\partial P_{\tau_n^\ell}}V_m^j\left(P_{\tau_n^\ell}\right)\\ &+\beta_1\left(\frac{P_0}{P_{\tau_n^\ell}}\right)^{\beta_1}\left(-\frac{1}{P_{\tau_n^\ell}}\right)\left(\frac{P_{\tau_n^\ell}}{P_{\tau_n^{f*}}}\right)^{\beta_1}\mathbb{E}_{P_{\tau_p^{f*}}}\left[\int_0^\infty e^{-\rho t}[U(P_tD(2))\right.\\ &\left.-U(P_tD(1))]dt\right] + \beta_1\left(\frac{P_0}{P_{\tau_n^\ell}}\right)^{\beta_1}\left(\frac{P_{\tau_n^\ell}}{P_{\tau_n^{f*}}}\right)^{\beta_1}\left(\frac{1}{P_{\tau_n^\ell}}\right)\mathbb{E}_{P_{\tau_p^{f*}}}\\ &\times\left[\int_0^\infty e^{-\rho t}[U(P_tD(2)) - U(P_tD(1))]dt\right]\end{aligned} \tag{52}$$

The third term reflects the marginal benefit from delaying investment, which postpones the entry of the follower, while the fourth term reflects the marginal cost from enjoying monopoly profits for less time. Since the last two lines in (52) cancel, we have:

$$\frac{\partial F_{\tau_n^\ell}(P_0)}{\partial P_{\tau_n^\ell}} = \beta_1 \left(\frac{P_0}{P_{\tau_n^\ell}}\right)^{\beta_1} \left(-\frac{1}{P_{\tau_n^\ell}}\right) V_m^j\left(P_{\tau_n^\ell}\right) + \left(\frac{P_0}{P_{\tau_n^\ell}}\right)^{\beta_1} \times \frac{\partial}{\partial P_{\tau_n^\ell}} V_m^j\left(P_{\tau_n^\ell}\right) \tag{53}$$

The monopolist's optimisation problem is:

$$F_{\tau_m^j}(P_0) = \sup_{\tau_m^j \in \mathcal{S}} \mathbb{E}_{P_0}\left[e^{-\rho \tau_m^j}\right] V_m^j\left(P_{\tau_m^j}\right) \tag{54}$$

and the corresponding FONC is:

$$\frac{\partial F_{\tau_m^j}(P_0)}{\partial P_{\tau_m^j}} = \beta_1 \left(\frac{P_0}{P_{\tau_m^j}}\right)^{\beta_1} \left(-\frac{1}{P_{\tau_m^j}}\right) V_m^j\left(P_{\tau_m^j}\right) + \left(\frac{P_0}{P_{\tau_m^j}}\right)^{\beta_1} \times \frac{\partial V_m^j\left(P_{\tau_m^j}\right)}{\partial P_{\tau_m^j}} \tag{55}$$

Comparing (53) and (55), we conclude that the optimal entry threshold of the non-pre-emptive leader is the same as that of the monopolist, which for the special case of a CRRA utility function is:

$$P_{\tau_n^{\ell*}} = \frac{rK}{D(1)} \left(\frac{\beta_2 + \gamma - 1}{\beta_2}\right)^{\frac{1}{1-\gamma}} \quad \square \tag{56}$$

Proposition 4.7. *The loss in the value of the investment opportunity for the non-pre-emptive leader relative to that of a monopolist at the pre-emptive leader's optimal entry threshold price decreases with risk aversion.*

Proof. The relative loss in the non-pre-emptive leader's value is:

$$\frac{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right) - F_{\tau_n^\ell}\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)} = 1 - \frac{F_{\tau_n^\ell}\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)} \tag{57}$$

Recall that prior to investment, the non-pre-emptive leader's value of investment opportunity at $P_{\tau_p^{\ell*}}$ is:

$$F_{\tau_n^\ell}\left(P_{\tau_p^{\ell*}}\right) = \left(\frac{P_{\tau_p^{\ell*}}}{P_{\tau_n^{\ell*}}}\right)^{\beta_1} \left[\mathcal{A}U\left(P_{\tau_n^{\ell*}} D(1)\right) - \frac{U(rK)}{\rho} + \left(\frac{P_{\tau_n^{\ell*}}}{P_{\tau_f^{\ell*}}}\right)^{\beta_1} \times \left[\mathcal{A}\frac{P_{\tau_f^{\ell*}}^{1-\gamma}}{1-\gamma}\left(D(2)^{1-\gamma} - D(1)^{1-\gamma}\right)\right]\right] \tag{58}$$

Notice that the expression of the monopolist's value of investment opportunity, $F_{\tau_m^j}(\cdot)$, evaluated at $P_{\tau_p^{\ell*}}$ is given by (59):

$$F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right) = \left(\frac{P_{\tau_p^{\ell*}}}{P_{\tau_m^{j*}}}\right)^{\beta_1} \left[\mathcal{A}U\left(P_{\tau_m^{j*}} D(1)\right) - \frac{U(rK)}{\rho}\right] \tag{59}$$

Hence,

$$1 - \frac{F_{\tau_n^\ell}\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)} = -\frac{\left(\frac{P_{\tau_p^{\ell*}}}{P_{\tau_n^{\ell*}}}\right)^{\beta_1}\left(\frac{P_{\tau_n^{\ell*}}}{P_{\tau_f^{\ell*}}}\right)^{\beta_1}\left[\mathcal{A}P_{\tau_f^{\ell*}}^{1-\gamma}\left(D(2)^{1-\gamma} - D(1)^{1-\gamma}\right)\right]}{\left(\frac{P_{\tau_p^{\ell*}}}{P_{\tau_m^{j*}}}\right)^{\beta_1}\left[\mathcal{A}\left(P_{\tau_m^{j*}} D(1)\right)^{1-\gamma} - \frac{(rK)^{1-\gamma}}{\rho}\right]} = \left(\frac{D(2)}{D(1)}\right)^{\beta_1} \frac{\beta_1}{1-\gamma}\left[\left(\frac{D(1)}{D(2)}\right)^{1-\gamma} - 1\right] \tag{60}$$

Partially differentiating (57) with respect to γ yields:

$$\frac{\partial}{\partial \gamma}\left[1 - \frac{F_{\tau_n^\ell}\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_n^{\ell*}}\right)}\right] = \left(\frac{D(2)}{D(1)}\right)^{\beta_1} \times \frac{\beta_1}{1-\gamma}\left\{\frac{\left(\frac{D(1)}{D(2)}\right)^{1-\gamma} - 1}{1-\gamma} - \left(\frac{D(1)}{D(2)}\right)^{1-\gamma} \ln\frac{D(1)}{D(2)}\right\} \tag{61}$$

According to (61),

$$\frac{\partial}{\partial \gamma}\left[1 - \frac{F_{\tau_n^\ell}\left(P_{\tau_n^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_n^{\ell*}}\right)}\right] \leqslant 0 \iff \left(\frac{D(1)}{D(2)}\right)^{1-\gamma} - 1 - \left(\frac{D(1)}{D(2)}\right)^{1-\gamma} \ln\left(\frac{D(1)}{D(2)}\right)^{1-\gamma} \leqslant 0 \tag{62}$$

Setting $x = \left(\frac{D(1)}{D(2)}\right)^{1-\gamma} > 0 \Rightarrow x - 1 - x\ln x \leqslant 0 \iff -\ln\frac{1}{x} \geqslant 1 - \frac{1}{x}$. Setting $\frac{1}{x} = y \Rightarrow -\ln y \geqslant 1 - y \iff \ln y \leqslant y - 1$ which is true. $\square$

Proposition 4.8. *The loss in the non-pre-emptive leader's value of investment opportunity relative to the monopolist's at the pre-emptive leader's optimal entry threshold price increases with uncertainty if:*

$$\left(\frac{D(1)}{D(2)}\right)^{\beta_1} > e$$

Proof. According to (60), the relative loss in option value of the non-pre-emptive leader is:

$$1 - \frac{F_{\tau_n^\ell}\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)} = \left(\frac{D(2)}{D(1)}\right)^{\beta_1} \frac{\beta_1}{1-\gamma}\left[\left(\frac{D(1)}{D(2)}\right)^{1-\gamma} - 1\right] \tag{63}$$

Partially differentiating with respect to σ we have:

$$\frac{\partial}{\partial \sigma}\left[1 - \frac{F_{\tau_n^\ell}\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)}\right] = \left[\left(\frac{D(1)}{D(2)}\right)^{1-\gamma} - 1\right]\left(\frac{D(2)}{D(1)}\right)^{\beta_1}\left\{\frac{\frac{\partial \beta_1}{\partial \sigma}}{1-\gamma} + \frac{\partial \beta_1}{\partial \sigma} \ln \times \left(\frac{D(2)}{D(1)}\right)\frac{\beta_1}{1-\gamma}\right\} \tag{64}$$

Hence,

$$\frac{\partial}{\partial \sigma}\left[1 - \frac{F_{\tau_n^\ell}\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)}\right] > 0 \iff \frac{\frac{\partial \beta_1}{\partial \sigma}}{1-\gamma} + \frac{\partial \beta_1}{\partial \sigma} \ln\frac{D(2)}{D(1)}\frac{\beta_1}{1-\gamma} > 0 \iff \left(\frac{D(1)}{D(2)}\right)^{\beta_1} > e \quad \square \tag{65}$$

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