Cashier Fraud - Solution

Analysis

The available data table is extended here by computing the average return for each cashier (right hand column). Next we compute the average of these for all cashiers except A, named the control group. This is taken as (an estimate of) expected return amount (i.e. ability) for an arbitrarily chosen cashier assuming non-fraudulent behaviour of all others except A. Finally we compute the standard deviation of these averages to represent the variation in ability among such cashiers.

Cashier	Return NOK	No. of returns	Average
A	400 611	804	498.27
B	18 574	390	47.63
C	70 730	774	91.38
D	88 473	938	94.32
E	49 201	769	63.98
F	55 142	873	63.16
G	47 919	1 306	36.69
H	34 915	758	46.06
I	6 030	115	52.43
J	69 758	1 574	44.32
K	5 604	141	39.74
L	52 882	387	136.65
M	3 312	72	46.00
N	27 613	713	38.73
O	109 812	1 064	103.21
P	41 892	720	58.18
All B to P:		Average =	64.17
		Standard deviation =	29.19

Average considerations

From this we see that cashier A clearly is an outlier with an average return of NOK 498.27 compared with NOK 64.17 in the control group. If cashier A with her 804 returns had behaved as those in the control group, her total "expected" return would be 804 x NOK 64.17 = NOK 51 592. The average calculations support the following:

Defendants total returns	400 611
Expected return	51 592
Deviation (unexplained)	349 019

Conclusion:
 The data has made plausible an embezzlement of about NOK 350 000.

<u>Note</u>. The inherent assumption regarding the control group of cashiers B to P may be questioned, e.g. is cashier L an outlier too? We should accept that abilities may depart from a normal distribution, and inclusion of L is reasonable in this context, since the defender may question the exclusion and disrepute the statistician as witness in the courtroom..

 Favour consideration based on normal individual variations

It may be the case that the abilities of the defendant were different from the others at the outset. An estimate of most likely individual variations in return amounts is +/- 23 400, computed from the standard deviations of amount per return among cashiers of NOK 29.19 in the control group scaled up to the 804 returns of the defendant.
To give favour to the defendant according to a normal distribution of abilities, amounts to reducing the unexpected deviation above by $k * 23\,400$, where k is dependent on how much favour to be given. The following table shows some cases:

k	Favour	From bottom	Reduction	Deviation (reduced)
0	No	50 %	0	350 000
1	Moderate	16 %	23 400	326 600
2	Large	2.5 %	46 800	303 200
3	Extreme	0.13 %	70 200	279 800

Here the right hand column is an estimate of embezzled amount for various degree of favour. Extreme favour (only about 1 of 1000 cashiers comparable to the control group have abilities below this level) corresponds to an estimate of about NOK 280 000.

Alternative favour considerations could be made by taking the extremes in the control group itself as benchmark, but this gives less than extreme favour according to the normal consideration:

i) Computation based on cashier (L) in the low end of control group (who also is somewhat atypical) gives 400 000 - 804 x 136.65 = 290 000.

ii) Computation based on return amount in per cent of own sales turnover
 (NOK 8 005 000) gives:

Normal return in control group: 8 005 000 x 0.54 % = 43 000
Extreme return (cashier L):   8 005 000 x 1 %   = 80 000
Deviation: 400 000 - 80 000 = 320 000

<u>Conclusion</u>
Different favour considerations gives a deviation from natural variation ranging from NOK 280 000 to NOK 350 000 (no favour). How much favour to be given is up to the court to decide, also taking the considerations below into account.

<u>Note</u>
The issue of the case is variation and not confidence interval. Surprisingly, when presented to an audience of academic statisticians, many tried to link it to traditional textbook theory about confidence intervals.

The abilities of the defendant
Date for successive time periods may document the normal abilities of the defendant, and also indicate when the embezzlement started and how it developed.

The information about number of returns and average return amounts in the 7 successive was

	2001	2002				2003	
No. of return	155	118	106	102	109	126	88
Amount	235	299	299	616	641	819	693

Notice that the number of returns decreased after her first period of employment, indicating increased abilities. Nevertheless the average amounts show an increased tendency over time, with a dramatic jump in the middle of year 1992. Notice however that the average return amount is already higher in the first period than for the other cashiers. This could indicate that she is in some sense different from the others at the outset. This point is however weakened if the data from 1991 is split into two periods: The 10 first workdays had an average of NOK 40 (55 returns) increasing to NOK 343 (100 return) in the next 30 workdays. The difference is statistically significant, that is, not likely due to chance alone. The return amount the first two weeks were in line with the other cashiers. This may raise the suspicion that the embezzlement started earlier than conceded. However some caution must be taken: The special work situation of a newly employed may cause special behaviour not reflecting normal working capacity.

I see no other support for assuming that the defendant may have had abilities different from the others at the outset. For instance the number of returns compared to own sales turnover and number of items entered is in line with the others. Five cashiers in the control group have in fact more returns compared to the number of items entered.

Conclusion
There is no basis for assuming that the defendant at the outset had a capacity different from the other cashiers. This means that there is no compelling argument for giving the defendant favour beyond the observed individual variation among the cashiers in general expressed in section 7.2.

The Court Proceedings
An experienced judge administered the court, having two laymen.

The size of fraud settled on by the court would be crucial to the defendant, since the amount qualifying for more severe punishment is somewhere in the range of NOK 200 000 - 300 000.

Prior to the proceedings the prosecutor was not sure how the court would react to statistical arguments, they could just as well become confused by them. Consequently some effort was also spent on the high returns which was individually checked.

Testimony was given by the store owner and by the store manager, who also provided compelling bar charts of some key statistics (of skyscrapers among bungalows, according to a local newspaper). The statistician then gave a brief summary of his report.

Based on the evidence and testimony the prosecutor advocated a fraud of NOK 300 000 as proved beyond reasonable doubt. The defender did not dispute the statistics at all. However the defendant confirmed a question by defence that she occasionally entered and corrected some high amounts as to cover up, by making such entries more likely. The prosecutor said afterwards that he at this point feared that this possibility could have destroyed his case, despite the compelling statistical evidence. The store manager however denied that correct high returns existed at all and this closed the argument.

Although we have no definite knowledge, we have reasons to believe that the judges understood and valued the statistical arguments, despite the fact that the defendant was given the added benefit of doubt by settling on a verdict of NOK 250 000.

Afterthoughts
The story told is a true story about a context and a mission for a statistical expert witness, and a true story about how the mission was carried out. The mission provided an opportunity to reflect on the use of statistics as evidence, and on the importance of understanding data in a context. With the available data a more sophisticated analysis was possible, for instance by looking at the data in a time series context. However, by introducing modes of analysis that are harder to explain to the court, you may increase the risk that the statistical arguments are dismissed altogether.

The store supervisor, the police investigator, the prosecutor and the defence lawyer could all make use of the same data in their context, but pose different questions and use different modes of analysis, depending on the available supplementary information.

For the store supervisor the graphics below would be sufficient to raise suspicion and go ahead. The statistical expert may regard this as too persuasive to be used in court.
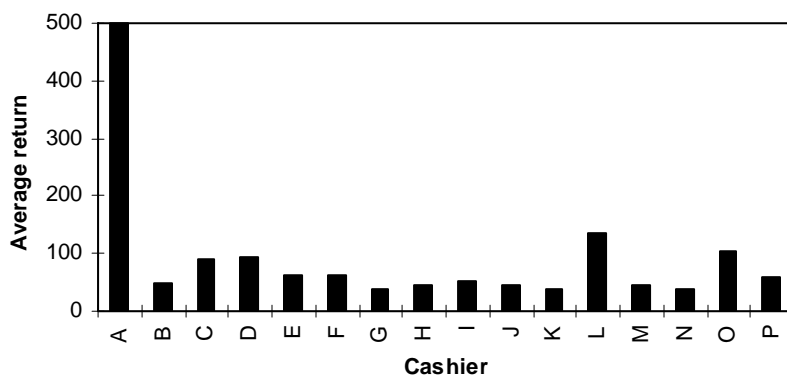
An interesting and important question is whether and how things will be different, prior to court proceedings and in court, if the accused had admitted nothing. This has to do with the limits of statistics as proof. One could argue that occurrence of an unlikely event proves nothing. Take the lottery analogue: Someone will always win the top price. It does not make sense for the lucky one or the unlucky ones to compute a probability afterwards to advocate that the outcome should not happen.

In the given context the case is about variation and not about confidence intervals and not about outlier detection either. In a presentation to a statistical audience the author was met by several comments linked to these misconceptions. It is maybe a symptom of our profession that we are likely to phrase statistical problems in our inference terms, whether it is relevant or not. It is my sincere opinion that we should talk a lot more about variation and less about formal inference.