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Valuation of Extension Options on Offshore Rig Contracts on the NCS

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Abstract

Offshore rigs are a necessary tool in exploration and development projects in the oil and gas industry. The offshore rig market is unique in the way that the majority of the rig fleet is owned by independent rig firms and leased to oil and gas operators. Despite the substantial economic scope of this sector, the research on the offshore rig market has been limited.

The purpose of this paper is to investigate a special aspect in the contracts between a rig owner and an oil operator - *an option to extend the initial drilling period*. This option gives the operator flexibility and a potential financial upside. By using option theory, we wish to examine the financial value these options represent and how this value is affected by whether the market is *tight* or *soft*. We also wish to investigate what challenges the use of extension options present.

The findings in our analysis proves that a one-well-option embedded in drilling contracts represents a substantial financial value. The financial value of the option will vary among different types of rigs and different market conditions. In addition, our research suggests that options offer invaluable flexibility, but also present challenges of unpredictability surrounding future expected rig rates and rig commitment.

Acknowledgements

This thesis is written as a conducting part of our Master in Finance at Norwegian School of Economics (NHH). When writing our thesis, we wanted to contribute to the sparse selection of existing research on the offshore rig market. From internships this summer, we gained insight into the oil and gas industry and were inspired to further develop our understanding of the market.

We would like to extend a special thanks to those who were critical in making this thesis possible. Thank you to our supervisor, Svein-Arne Persson, for his excellent feedback and for helping us find the relevant theoretical fundament. Thank you to Oddmund Føre and Jo Friedmann from Rystad Energy for giving us access to relevant and excessive data on drilling contracts, as well as Wendy DiBenedetto at RigLogix for helping us to navigate their impressive database. In addition, we would like to thank Roar Ådland for introducing us to relevant persons and literature, and Julio Cesar Goez for helping us with Python. Last but not least, thank you to Transocean, Odfjell, Aker BP and Lundin Norway for their invaluable insight into to the offshore rig market.

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1 Introduction

When drilling exploration- and development (E&D) wells an oil operator will generally hire a rig from a rig owner. The terms for this operation are stated in the contract between the parties. An important component that is often included in drilling contracts, is an *extension option*. These options are one-sided provisions which allow operators to extend the drilling contracts. Rig owners are not explicitly compensated for including these options. However, it is reasonable to assume that the options are implicitly priced in the market, as they unquestionably provide operators with value in the form of flexibility and a potential financial upside. We want to investigate the explicit financial value such options represent. The purpose of this thesis is to evaluate the following questions:

- *What is the financial value of an option to extend drilling operations?*
- *How is the option value affected by whether the market is soft or tight?*
- *What challenges does the use of options present in the market?*

To answer these questions, we will first simulate the rig market rates. These simulations will be the foundation of our option valuation. Further, we will construct a real option model based on the optionality to extend the initial contract. The option model will be used to determine a theoretical financial value of the options. The results will be further investigated in a scenario analysis.

The thesis is structured in six chapters. The next chapter, 1.1 and 1.2, will give a brief introduction to the rig market and a presentation of the contents in a drilling contract. In chapter 2, a literature review will be presented, before giving an overview of our dataset in chapter 3. In chapter 4, we discuss the underlying theories and methods for solving the valuation of the option. Finally, chapter 5 shows the results of our analysis, while chapter 6 summarizes and concludes our work.

1.1 The Offshore Rig Market

The offshore market for rigs is specialized in offering drilling rigs and drilling operations offshore. This thesis will focus on the offshore rig market on the Norwegian Continental Shelf (NCS). Osmundsen (2012) describes the Norwegian offshore rig market as a specialized market of rigs, with high day rates protected by high barriers to entry. Based on these observations, Osmundsen argues that the Norwegian market can be segmented as a unique market within the global rig market. According to Pareto Securities Equity Research (2012), the use of options is also more extensive on the NCS compared to the rest of the offshore rig market.

Within the segment of offshore rigs there are two main categories; jackups and floaters. Jackups operate at shallower sea, as they have a series of retractable legs that must be affixed to the ocean floor. They will typically operate at a water depth of maximum 150m (RigLogix, 2018). Floaters are not limited to 150m water depth, as they do not rely on standing legs (Deutsche Bank, 2013). The category of floaters can be further divided into semisubmersibles and drillships. This thesis will focus on jackups and semisubmersibles, since as of today drillships are not present on the NCS (RigCube, 2018). Figure 1 depicts the three abovementioned rigs and their operating water depths.

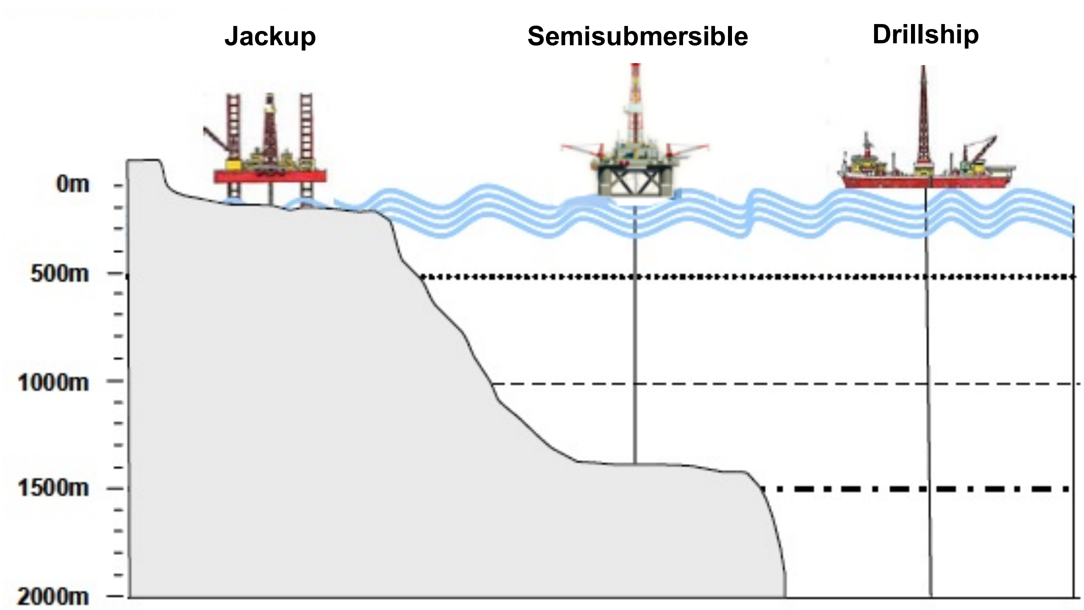

Figure 1 - Illustration of three types of rigs and their water depth
Source: Deutsche Bank

Semisubmersibles are on average more expensive to build and operate than jackups (Kaiser & Snyder, 2013). These differences in costs are reflected in the average day rate for the two types of rigs.

Offshore rigs are in general owned by independent rig companies and leased to oil and gas operators for use in their E&D projects (Corts, 2008). The largest cost component in such projects is the leasing cost of rigs. As this cost is so large it could be an argument for oil companies to own their own rigs. However, it is more efficient for operators to lease rigs. This is because investment in E&D shifts in response to changes in the oil price (Kaiser, 2014). Further, different oil fields will require rigs of different specifications, again making leasing a better option (Kellogg, 2011).

When leasing a rig, the operator is charged a daily fee, called the day rate. The day rate is set in USD. Amongst other, the day rates vary depending on the location, the type of rig and the market conditions (Corts, 2008). Day rates are said to be the primary descriptor of the rig market (Kaiser & Snyder, 2013). Essentially, this means that day rates are highly affected by changes in supply and demand.

In the short to medium term, supply in the rig market is essentially inelastic (Carter & Ghiselin, 2003). The rig owners add capacity to their fleet through newbuilding. However, it takes years to build and deliver a new rig. Further, the rig market is extremely capital intensive and requires enormous investments (Carter & Ghiselin, 2003). These effects create massive entry barriers and effectively limits newcomers. On the NCS this is especially true, as the market in addition is highly specialized and regulated (Osmundsen, 2012).

The demand for offshore rigs is more elastic, and highly affected by oil prices. In times of declining oil prices, oil companies will cut investments in E&D (Kaiser, 2014). To exemplify, a lower oil price yields lower expected profitability, thus decreasing investment in E&D. Hence, rig demand declines. As demand declines, rig owners will bid aggressively to secure contracts and thus rig rates will decrease. Conversely, higher oil prices increase rig demand and rig rates.

The negotiating power of both rig owners and oil operators are closely related to rig utilization. Utilization is the ratio of available rigs under contract and therefore a measure of spare capacity in the market (Kaiser & Snyder, 2013). In a *tight* market with high oil prices, high rig rates and high rig utilization, the rig owners are able to negotiate favorable terms for themselves (Skjerpen, Storrøsten, Rosendahl & Osmundsen, 2015). Conversely, in a *soft* market oil prices, rig rates and rig utilization are low, and the rig companies bid aggressively to win work. This leads to the rig companies' negotiating power being weakened and oil companies are able to negotiate contracts in their favor.

The current market situation in the Norwegian offshore rig market has been positively affected by recovering oil prices (RigCube, 2018). However, due to a recent volatile development in oil prices the future demand for rigs is uncertain.

1.2 Drilling Contracts and Options

A drilling contract serves as a rule book between the rig owner and the operator. Amongst other, it specifies the equipment, personnel, materials and services each party is to provide. Further, it defines the scope of work, and addresses the day rate the operator must pay during the leasing period, known as the *firm period*. The firm period is defined as the initial contract period (Moomjian, 1989).

There are mainly two different types of rig contracts; day rate- and fixed price contracts. In this thesis we will only consider day rate contracts, as these are the only one in use on the NCS (Osmundsen, Toft & Dragvik, 2006). Day rate and other specifications will be set in negotiation between the rig owner and the operator. The operator defines the rig specifications needed for a specific job. Then, the operator sends out a request for quotation to different rig owners. The relevant rig owners will participate in a tender process. Amongst other, the offers are considered based on price, technical specification and rig availability (Moomjian, 1989).

Drilling contracts often include options to extend drilling operations (RigCube, 2018). Options are a tool to increase flexibility for operators, as offshore drilling in general is a very unpredictable activity. Therefore, oil companies are dependent on doing preliminary exploration of a field before moving forward with further exploration and development. Options give operators a possibility to not commit unnecessary capital to a project before the preliminary results are available.

A drilling contract may include one or more options (RigCube, 2018). The day rate of the extended option period(s) will be stated in the drilling contract. In general, the day rate for the extended option period is the same as the day rate for the firm period. We observed this when examining contracts with options on the NCS in RigCube (2018). The deadline for when the operator will have to choose whether to exercise the option or not will be stated in the drilling contract. A rational operator will exercise the option if they have a need for the rig and the option is in-the-money. The option will be in-the-money if the operator gets a lower day rate by exercising the option, than what the operator could get in the market. Vice versa, the option will be out-of-the-money if the operator could lease another rig cheaper in the market.

What makes an extension option in the rig market unique, is that the *exercise deadline* will be at least half a year before the *start of the option period*. In other words, the operator will need to decide on whether to exercise the option on beforehand. This dynamic is illustrated in figure 2.

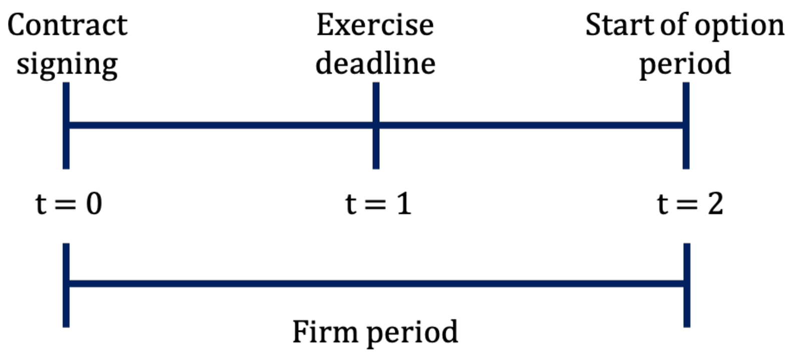

Figure 2 - Illustration of the three crucial time points for the extension option

At *t=0* the drilling contract will be signed, and the firm period begins. The deadline for exercising the option is at *t=1*. At *t=2* the firm period ends, and the potential option period starts. See figure 2 for illustration of the three crucial time points.

The period from the exercise deadline to the start of the option period, *t=1* to *t=2*, is not standardized and will vary with how soft or tight the market is. The same applies to the firm period, *t=0* to *t=2*. In general, both these periods are longer in times of a tight market. In a tight market, oil companies will be incentivized to secure rig capacity and at the same time rig owners are in a position to demand longer contracts (Kaiser & Snyder, 2013). This leads to longer firm periods, thus increasing the period from *t=0* to *t=2*. In addition, as the rig owners are in a position of power, they will increase the period from the exercise deadline to the start of the option period, *t=1* to *t=2*. In the event the option is not exercised, rig owners want to increase the period they can market their rig before the contract ends. This is to avoid having their rig left idle.

In a soft market, oil companies will not need to secure rig capacity, and would rather enter into contracts with shorter firm periods (Corts & Singh, 2004). In addition, as the oil companies are in the position of power, they would want to minimize the time period from the exercise deadline to the start of the option period, $t=1$ to $t=2$. This is to decrease the uncertainty surrounding whether or not the option will be in-the-money at the start of the option period. In addition, as a soft market represents a beneficial negotiating position for operators, they will often be in a position to renegotiate the day rate of the options to favorable terms. This essentially means that the operator is in a position to renegotiate the day rates on options that are out-of-the-money at the time of exercise.

An option represents value for the operator in form of flexibility and a potential financial upside. As the option is without an explicit cost for the operator, it is obvious that contracts with options are preferred among the operators. For rig owners the only upside an option can hold, is that their rig could be committed longer. Thereby, the fact that options often are included in rig contracts, essentially reflects the general negotiating power of both rig owners and operators. This power structure is further emphasized by the fact that the operators often can renegotiate the rates on their options.

As already established, the offshore rig market is heavily affected by changes in the oil prices. Changes in oil prices also affect the exercising of options. Rystad Energy (2018) reports that the exercising of options is positively correlated with how tight the market is. In the same article they investigate how the recent market development could point to a recovery in the rig market. The average ratio for the global offshore rig market was 63% for the period 2014 through 2017 and has improved greatly to about 90% so far this year (Rystad, 2018).

2 Literature Review

In this section we will provide the reader with relevant literature with respect to the thesis. The articles by Carter and Ghiselin (2003), Kaiser (2014), and Kaiser and Snyder (2013) give an overview of supply and demand in the rig market. Osmundsen, Rosendahl and Skjerpen (2015) also contribute to explaining the dynamics of the rig market. In their analysis of the jackup market in the Gulf of Mexico they prove that high rig demand is related to high oil prices and that high rig utilization leads to high rig rates. They also find that longer firm periods are associated with higher rig rates. Skjerpen, Storrøsten, Rosendahl and Osmundsen (2018) provide a similar approach for floaters on the NCS.

The literature on rig contracts mostly discusses how a contract is formed and standardization of the contract's content. Moomjian (1989), addresses the fundamental objectives of drilling contracts and discusses how a standardized approach could achieve equitable contract terms in both soft and tight markets. Moomjian also published three articles in 1999, where he considered the risk allocation between the rig owner and the operator in drilling contracts. He discusses how negotiating power changes between the operator and rig owner in a tight and soft market. A similar approach has been done by Osmundsen (2009) for contracts on the NCS.

The literature on real option valuation is based on the fundament that an investment decision could be viewed as a call option on an underlying real asset, and dates back to Myers (1977). Many practitioners in the maritime sector have used real option theory to value options. The method is widely used in the offshore shipping market. In general, the shipping- and rig market share similar characteristics and are attributed to a common field of maritime economics. Conceptually, the framework is therefore applicable to this thesis.

Within shipping, Tvedt (1997) used real option theory to value contracts in the VLCC (Very Large Crude Carrier) market, with different duration, as well as forwards and options written on the contracts. Further, Sødal, Koekebakker and Aadland (2008) examined real option theory to model the flexibility for a carrier to switch between the dry bulk and wet bulk market.

A fundament of real option theory is to say something about how the underlying prices develop. A stochastic process is a tool to address this. Paddock, Siegel and Smith (1988) created one of the earliest analogies of the Black & Scholes option pricing model. Here, they used a Brownian Motion as the stochastic process to model offshore petroleum leases. Further, Bjerksund and Ekern (1995) applied a mean reverting model, called the *Ornstein-Uhlenbeck* process (O-U), to their analysis of freight rates in shipping. Tvedt (1997) introduced a similar model in logarithmic form, the *Geometric Mean Reversion* process (GMR). He discussed both the *O-U* and *GMR* process when he simulated charter rates in the VLCC market. His findings suggest that the *GMR* process is more appropriate than the *O-U* process, as the *O-U* process could simulate negative rates. The *GMR* process avoids negative rates as it is in logarithmic form.

To our knowledge, there has not been published a real option valuation on options to extend drilling operations, nor have rig rates been modeled using a stochastic process. However, real option valuation has been used to study other dynamics and behaviors in the rig market. For instance, Brennan and Schwartz (1985) included a real option variable to evaluate the binary decision to enter or leave the rig market permanently. In addition, Corts (2008) examined the idling decisions of a rig owner with a real option approach.

3 Data

Rystad Energy has provided us with the monthly market rates for both jackups and semisubmersibles on the NCS for the period from June 2000 through December 2018. This corresponds to 223 monthly observations.

The monthly market rate is collected from Rystad Energy's database, RigCube. Through RigCube we also got access to information on all contracts signed on the NCS in the period of June 2000 through December 2018, including contracts with options. In RigCube, data is gathered from publicly available sources and through direct dialogue with rig owners, operators and brokers. As Rystad Energy has strong connections in the oil and gas industry, RigCube contains disclosed rates which contributes with valuable insight to the development in the market rates.

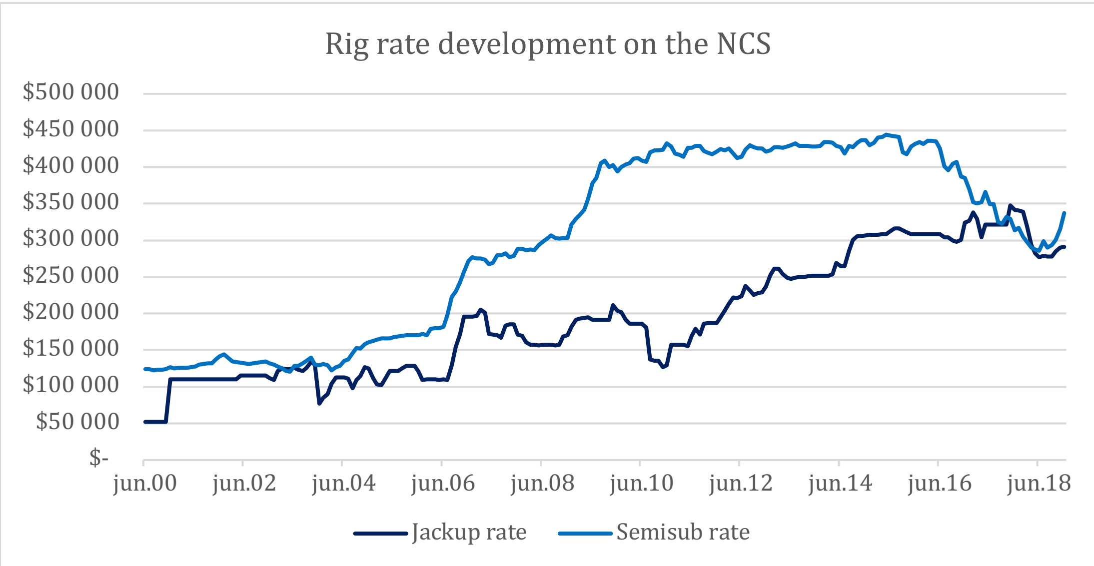

Figure 3 - Rig market rate development on the NCS June 2000 - December 2018
Source: RigCube

Figure 3 depicts the development in market rates for jackups and semisubmersibles from June 2000 to December 2018 on the NCS. The higher cost required to build and operate a semisubmersible, translates into higher average day rates. See figure 3. Over the period 2000-2018 the semisubmersible market rate is consistently higher than the jackup market rate. However, the jackup market rate has been equal to or higher than the semisubmersible market rate under the recession of 2002 and after the oil crisis in 2014.

Table 1 - Descriptive Statistics

	Jackup	Semisubmersible
Median	$185,617	$306,542
Mean	$193,978	$297,898
Minimum	$52,000	$120,575
Maximum	$347,640	$444,245
Standard Deviation (Yearly)	26,25%	7,66%
Standard Deviation (Monthly)	7,58%	2,21%
N. of observations	223	223

Descriptive statistics for the market rates at the NCS are given in table 1. The table illustrates evidence that jackups has a higher standard deviation than semisubmersibles. This is consistent with the development we observe in figure 3. As expected, the mean and median values are higher for semisubmersibles than for jackups.

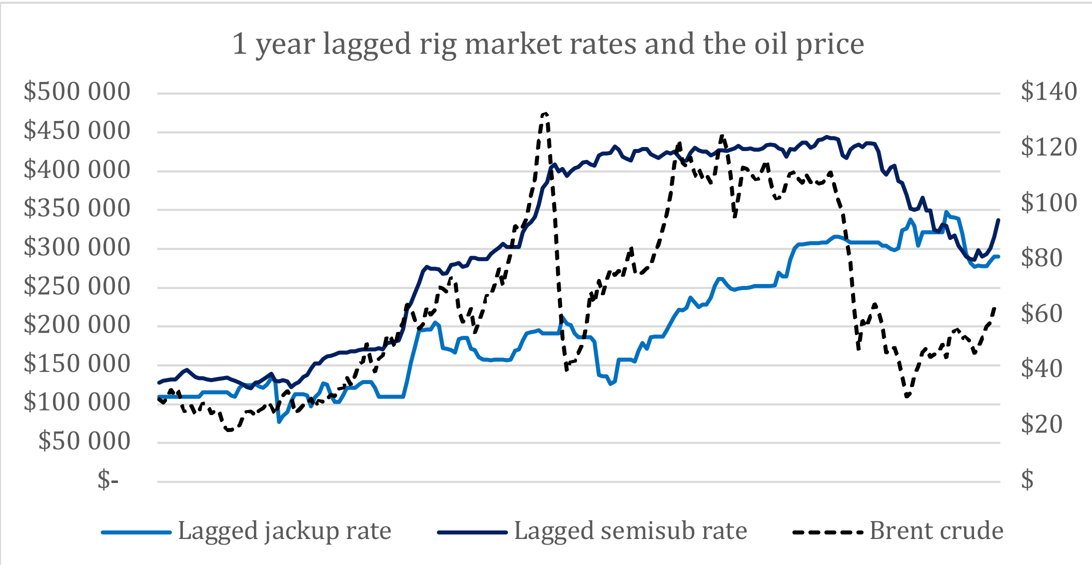

Figure 4 - 1 Year Lagged Rig Market Rates on the NCS and the Oil Price (Brent Crude)
Source: RigCube

Figure 4 depicts the one year lagged market rates for jackups and semisubmersibles on the NCS, plotted against the Brent Crude oil price. We observe that the development in rig rates follow the oil price rather closely. However, it is evident that the oil price experiences larger shocks and higher volatility.

The reasoning behind lagging the rates is that the rig market reacts to changes in oil prices gradually, rather than instantaneously. As mentioned in chapter 1.1, rig rates are highly affected by changes in supply and demand. In times of declining oil prices, one of the first budget cuts is investments in E&D projects (Corts, 2008). This effect decreases the demand and by that the rig rates. However, an operator cannot immediately stop all investment in E&D, as they are often committed to running contracts. Supply is more inelastic than demand. Therefore, rig owners react slower to changes in the oil price.

Table 2 - Correlation Between 1 Year Lagged Rig Rate and Oil Price

	Jackup	Semisubmersible
Correlation between 1 year lagged rig rate and oil price	0.45	0.78

Table 2 displays a positive correlation coefficient between both lagged market rates and the oil price. The semisubmersible rate has a higher correlation coefficient than the jackup rate. This is presumably due to the slight uptick we observe after 2014 in the jackup rates. See figure 3.

In the further analysis we will use inflation adjusted rates. Rates are adjusted with June 2000 as the base. The reasoning for this is to eliminate trends that are due to inflation rather than real rate development.

4 Theory and Method

In this section we will present the theories and methods used to find the financial value of the options embedded in the rig contracts. Firstly, we will introduce the relevant option theory. Secondly, we will present the stochastic process and finally our model to value the extension options.

4.1 Option Theory

An option is defined as a right, but not an obligation, to buy or sell an underlying asset or instrument, at a specified strike prior to or on a specified date (Hull, 2012). In option theory, it is common to distinguish between financial and real options. Financial options are often used as a product for hedging or speculating. A real option, on the other hand, concerns a strategic decision for a real asset and is a tool for investment decisions under uncertainty (Myers, 1977).

The existence of real options is a consequence of uncertainty (Hull, 2012). As stated in 1.1, there is extensive uncertainty associated with E&D projects. Extension options provide operators with flexibility in the face of this uncertainty. In addition, the development in future rig rates is uncertain. Therefore, extension options will also act as a financial option, i.e. it is possible for operators to profit financially from having options embedded in their contracts.

There are several types of options (Hull, 2012). In our analysis, we will categorize the extension option as a European call option. A European option can only be exercised once, and the exercise date is set at the time of contract signing. A call option gives the holder a right, but not an obligation, to buy an underlying asset at specified price (Hull, 2012). Both these features are consistent with the extension option.

The extension option contains elements from both compound and barriers options. A compound option has two strikes and two exercises (McDonald, 2013). The extension option also has two strikes, but only one exercise. A barrier option has a payoff depending on whether the price of the underlying asset reaches a specified barrier (McDonald, 2013). The payoff of the extension option is also dependent on the underlying rig rates exceeding the contracted option rate at the time of exercise.

4.2 Stochastic Process

Any variable whose value changes over time in a random or uncertain way may conveniently be modelled as a *stochastic process.* A stochastic process is defined as the probabilistic evolution of the changing value through time and could be used to simulate the future outcome and development of a chosen variable (Hull, 2012). The choice of stochastic model will affect the valuation of a chosen variable (Smith & Mccardle, 1999). Therefore, to determine which stochastic process that will be the best fit for modelling the rig rates, we must determine the dynamics affecting the rates. In this section, we will discuss three relevant stochastic processes: *Geometric Brownian Motion, Ornstein-Uhlenbeck* and *Geometric Mean Reversion*. These stochastic processes are widely used to model the oil price and offshore freight rates.

Geometric Brownian Motion

One of the most common models to value options is the Black & Scholes (1973) model. They present a method to calculate the value of a European call option on a stock. The model assumes that the stock price follows a stochastic process that is described as a *Geometric Brownian Motion* (GBM). The differential equation for *GBM is*

$$dX_t = \mu X_t dt + \sigma X_t dZ_t. \tag{4.1}$$

In the model, X_t is the value of the stochastic process at time t. Further, μ is the expected rate of growth. σ represents volatility and is measured as standard deviation. Z_t is the Brownian motion. A Brownian motion (also referred to as a Wiener process) is characterized as an *independent increment* and is a *Markov process* (Smith & Mccardle, 1999). By independent increment we mean that the probability distribution for the process is independent of previous data. A Markov process implies that only current information is useful when forecasting future development (Dixit & Pindyck, 1994). Finally, Z_t has a standard normal distribution with zero mean and a standard deviation equal to one; dZt ~ N[0, 1] (Hull, 2012).

GBM is often used when modelling the oil price. However, Schwartz (1997) argues that the oil price would not be correctly modelled with *GBM*, as the historical oil price seems to eventually return back towards a mean price. Therefore, Schwartz (1997) suggests that assuming an underlying *mean reversion* would be a better fit. To make use of a mean reverting model is further supported by literature in the shipping market, where this is the preferred model (Sødal et al., 2008).

In line with this, as the market rates for rigs are correlated and indirectly affected by movements in oil prices, it is natural to assume that mean reversion is also present in the rig rates. Further, in discussion with both oil and rig companies we are left with the understanding that there exists a long-term equilibrium rate that the market rate fluctuates around. Typically, this rate will reflect a price where both operational and capital costs are covered for the rig owner.

In a tight market, the day rates will increase beyond the long-term equilibrium. With increasing rates, demand will fall as drilling operations will be more expensive, and rates will revert back towards the equilibrium. On the other hand, in a soft market, the day rates will decrease below the long-term equilibrium. With decreasing rates, rig owners will be hesitant to contract out their rigs. If the market rate falls below operational costs, rigs will eventually be laid up or scrapped and thus supply will decrease. Decreasing supply will increase the market rates towards the equilibrium.

Ornstein-Uhlenbeck Process

The *Ornstein-Uhlenbeck* (O-U) process is a mean reversion model that has been commonly used when modelling the oil price (Dixit & Pindyck, 1994). The *O-U* differential equation is

$$dX_t = k(\alpha - X_t)dt + \sigma dZ_t. \tag{4.2}$$

In the model α represents the long-term equilibrium rate. The constant, k, is the speed of mean reversion. A higher k will make the variable X_t move back to α at a faster frequency, and vice versa. The second term in the *O-U* process consists of the volatility, σ, and the Brownian motion, Z_t. Different from *GBM* is that the *O-U* process takes absolute measures of volatility.

An issue with the *O-U* process is that it allows for negative rates (Tvedt, 1997). Negative rates will not be possible in the rig market as rig owners will scrap or lay up their rigs when rates drop below operational costs. This indicates that the rig rates are downward restricted at a level above zero.

Geometric Mean Reversion

Tvedt (1997) presents a different version of the *O-U* process that effectively handles the issue of negative rates. This process is called the *Geometric Mean Reversion* (GMR) process. The differential equation of the process is

$$dX_t = k(\alpha - lnX_t)X_t dt + \sigma X_t dZ_t. \tag{4.3}$$

Similar as for the *O-U* process, k is the mean reversion parameter. The log of the process, X_t, is reverted towards the level given by α. The Brownian motion, Z_t, is the same as in the previous equations and σ represents volatility.

We find the *GMR* process to be the best fit for modelling the rig rates and proceed with this process as the foundation of our analysis.[1] We use the discrete version of the *GMR* process to estimate our coefficients. The discrete version is

$$lnX_t = \beta_0 + \beta_1 lnX_{t-1}dt + \varepsilon_t. \tag{4.4}$$

We run an Ordinary Least Squares (OLS) regression in Stata on equation (4.4) using the inflation adjusted monthly market rates from June 2000 – December 2018. The log of X_t is the dependent variable and the log of X_{t-1} the independent variable. The output from the regression is given in table 3 and 4.

Table 3 - Estimated OLS Values for Jackups

Parameter	Estimated Coefficients	Standard Error	t-value
β_0	0.66117	0.1994951	3.35
β_1	0.944049	0.0168292	56.10

Standard deviation of regression: 0.07685
R^2: 0.9347

Table 4 - Estimated OLS Values for Semisubmersibles

Parameter	Estimated Coefficients	Standard Error	t-value
β_0	0.0858027	0.0575757	1.49
β_1	0.9931565	0.0046908	211.72

Standard deviation of regression: 0.02623
R^2: 0.9951

The output of the regressions will be used to estimate the parameters of the *GMR* process. But firstly, we will need to test whether the estimated coefficients are reliable.

[1] See appendix 4 for comparison with the *O-U* process

Tests of the Estimated Coefficients

We perform two tests to check whether the OLS regression is providing us with reliable estimated coefficients; *Augmented Dickey-Fuller* and *Breusch-Godfrey*. See table 5.

Table 5 - Statistical Tests of OLS regression for Jackups and Semisubmersibles

Tests	Jackup	Semisubmersible
Augmented Dickey-Fuller	t = -3.325 p = 0.0138	t = -1.459 p = 0.5537
Breusch-Godfrey	$chi^2 = 1.416$ p = 0.2341	$chi^2 = 7.8$ p = 0.000

Critical values for DF-test: 1% level: -3.469, 5% level: -2.882, 10%level: -2.572

Critical values for BG-test with 1 degree-of-freedom: 1% level: 6.63, 5% level: 3.84, 10%level: 2.71

A mean reverting process is by definition stationary, as stationarity is defined by constant mean and variance (Wooldridge, 2016). Since *GMR* is a stationary process, i.e. does not have unit root, we perform an Augmented Dickey-Fuller test. The Augmented Dickey-Fuller test has a null hypothesis of unit root. The null hypothesis is rejected if the t-stat is lower than the critical value (Wooldridge, 2016). From table 5 we read that jackup rates are stationary at 1.38% significance level. However, we cannot reject the null hypothesis of unit root for semisubmersibles.

Dixit and Pindyck (1994) argues that the Augmented Dickey-Fuller test is a weak test for short time series (30 years or less). This is because the test will often fail to reject the null hypothesis of unit root, even if the series is in fact mean reverting. The reason is that any mean reversion is typically very slow and is therefore hard to discern in a short time series. In our case, we argue that the semisubmersible rates are in fact mean reverting, but that it is hard to detect in the time series of only 18 years. Furthermore, by adding a drift component to the Augmented Dickey-Fuller test, we can reject the null hypothesis of unit root at a 10% significance level.[2]

An important assumption for OLS on time series is that there should be no autocorrelation in the residuals. To test for this, we use the Breusch-Godfrey test with the null hypothesis of no autocorrelation (Wooldridge, 2016). The results presented in table 5 illustrates that we reject the null hypothesis for semisubmersibles, but not for jackups. This suggests that the error terms of semisubmersible rates are not fully independent from the previous error terms, i.e. they experience autocorrelation. However, this is not of large concern, as autocorrelation does not affect the consistency of the OLS estimates (Wooldridge, 2016). This means that as the sample

[2] See appendix 2 for explanation of the results of the Dickey-Fuller test with drift

size increases, the estimates converge to the true value of the parameter being estimated. As we have a rather large sample size, it is reasonable to believe that our estimates are accurate representations of the true value of the estimator.

Another consequence of autocorrelation is that, in most cases, OLS will give an overestimated R-squared (Wooldridge, 2016). R-squared, also referred to as the goodness of fit, is the ratio of the explained variation compared to the total variation. The R-squared obtained from the regression of semisubmersible rates was estimated to be 0.9951. Hence, the reader must be informed that due to failure of rejecting the null hypothesis of no autocorrelation, this goodness of fit measure is presumably overestimated.

Parameters

From the regression output presented in table 3 and 4 we calculate the parameters of the *GMR* model. The parameters are calculated from the equations

$$\beta_0 = (\alpha - \frac{\sigma^2}{2k})(1 - e^{-k}), \tag{4.5}$$

$$\beta_1 = e^{-k}, \tag{4.6}$$

where σ is the standard deviation of the regression.

We rearrange equation (4.5) and (4.6) and get

$$\alpha = \frac{\beta_0}{1-e^{-k}} + \frac{\sigma^2}{2k}, \tag{4.5*}$$

$$k = -ln\beta_0. \tag{4.6*}$$

The calculated parameters are stated in table 6.

Table 6 - Calculated *GMR* Parameters

	α – Long-Term Equilibrium Rate	k – Mean Reversion Speed	σ – Volatility (Monthly)
Jackup	$161,522	0.05757721	0.07685
Semisubmersible	$293,002	0.00686702	0.02623

From table 6 we observe that the development in jackup rates has a higher mean reversion speed and volatility, compared to semisubmersibles. This is consistent with the historical data presented in chapter 3. As depicted in figure 3, the development in the historical jackup rates fluctuates around the mean to a much larger degree than the semisubmersible rates. Further, we observe that the estimated volatility parameter for both jackup and semisubmersible are close to identical to the monthly standard deviations presented in table 1.

When evaluating the *GMR* parameters it is natural to consider risk, as any risk-adverse investor in any market will demand a premium to bear risk. A potential approach to risk adjustment is to adjust α.[3] However, we will follow Tvedt's (1997) approach of not risk adjusting.

4.3 Option Model

Our option model is based on the option pricing framework developed by Black & Scholes (1973). As previously mentioned, the option to extend a drilling contract can be classified as a European call option, with features from both compound and barrier options.

When an operator hires a rig, the day rate is fixed during the contract period. On the time of contract signing, *t=0*, a fixed day rate for the option is also agreed upon. In our model we have assumed that the day rate on the option is the same as the day rate of the firm period, X_0. Our reasoning for this assumption is that it is common practice in the market to set the option day rate equal to the day rate in the firm period (RigCube, 2018). On the time of exercise, *t=1*, a market rate, X_1, will be observable in the market. When deciding whether to exercise the option or not the main consideration will be the price difference $\boldsymbol{P_1 = X_1 - X_0}$.

To assess whether the embedded extension option is in-the-money at *t=1*, a dummy variable, D, is created.

$$D = 1 \; if \; (P_1 * OP) + RC \; \geq 0, or \; 0 \; if \; not. \tag{4.10}$$

P_1 is multiplied with the option period, OP, in order to calculate the aggregated price difference over the length of the contract. Other than the difference between the option day rate and the market rate, P_1, and the length of the option period, OP, a replacement cost, RC, is also taken into consideration. RC affects the decision to exercise the option positively, as there will be a considerable cost associated with turning to the market to rent a new rig. In other words, there

[3] See appendix 3 for Bjeksund and Ekern's (1995) approach to risk adjustment

will be a cost associated with not exercising the option. The replacement cost will typically be an estimate of the cost associated with delaying drilling operations and drawing up a new contract with a different rig owner.

Given that the value of the option is equal to or greater than 0, the dummy variable will be 1 and the option will be exercised. In our option valuation model, we assume that operators always have ongoing drilling projects. Thus, any extension option that is in-the-money will be exercised.

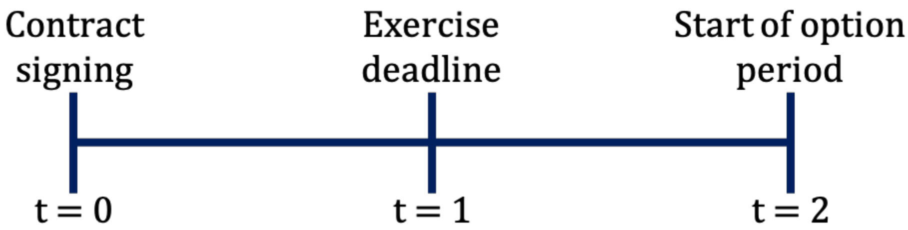

Figure 5 - Illustration of the three crucial time points for the extension option

To calculate the value of the option we will use two different approaches: Valuing the option at the *exercise deadline* and at the *start of the option period*. Finally, we will derive the value of the option at the time of *contract signing.* See figure 5 for illustration.

The value of the option at exercise deadline, *t=1*, is

$$Vopt_{t=1} = D * ((P_1 * OP) + RC). \tag{4.11}$$

Equation (4.11) is equal to (4.10), only changing the fact that it calculates the value rather than assesses whether the option is in-the-money or not.

The value of the option at start of the option period, *t=2*, is

$$Vopt_{t=2} = D * \left((P_2 * OP) + RC\right). \tag{4.12}$$

At the start of the option period a new market rate will be observable in the market. This rate is denoted X_2 and the price difference is $\boldsymbol{P_2 = X_2 - X_0}$.

$Vopt_{t=1}$ is the value of the option at the exercise deadline. $Vopt_{t=2}$ is the value of the option when the option period starts. However, the true value of the option is the value at contract signing, *t=0*. Therefore, we will use $Vopt_{t=0}$ when valuing our options. Both $Vopt_{t=1}$ and $Vopt_{t=2}$ are calculated as an average of exercised options, i.e. not considering options that are out-of-the-money at *t=1*. $Vopt_{t=0}$ on the other hand, represents the value of an average option at *t=0*, before knowing whether the option will be in- or out-of-the-money at *t=1*.

The expected payoff of the extension option can be expressed as

$$Vopt_{t=0} = E[e^{-rT} \max((P_2 * OP) + RC, 0)\ \mathrm{I}\,(\mathrm{D} = 1)]. \tag{4.13}$$

From equation (4.13) we observe that the payoff, $(P_2 * OP) + RC$, is dependent on the option being in-the-money at the time of exercise, $D = 1$, i.e. if the option is not exercised there will be no possible payoff.

r is the risk-free rate. When discounting the option value from *t=2* to *t=0*, we will use the 3-year Norwegian government bond as the risk-free rate. As of December 11th, 2018, this rate is 1.22% (Norges Bank, 2018).

4.4 Python Code

Using Python, we have coded a program based on our option model. The Python code will run 100,000 replications of equation (4.10), (4.11) and (4.12) and calculate the option value for each simulation. To simulate the market rates, the program includes the *GMR* process.

For each option, an outcome of the market rates will be simulated. At *t=1,* Python will consider whether the option is in-the-money or not, by comparing the simulated rate with the option day rate. If the option is not exercised the simulation will stop here. If the option is exercised, on the other hand, Python will calculate $Vopt_{t=1}$ and continue to simulate the market rates from *t=1* to *t=2*. Python will then calculate $Vopt_{t=2}$. When all simulations are finished, Python will calculate the ratio of exercised options and $Vopt_{t=0}$. $Vopt_{t=0}$ is calculated as the aggregated value of all exercised options at *t=2,* divided by the number of simulations, i.e. 100,000.[4]

The program we have coded is presented in appendix 1.

[4] $Vopt_{t=0}$ is discounted outside Python, i.e. the code provides option values that are not discounted

Presented below in figure 6 and 7, is the observed market rate plotted against simulated rates for jackups and semisubmersibles, respectively.

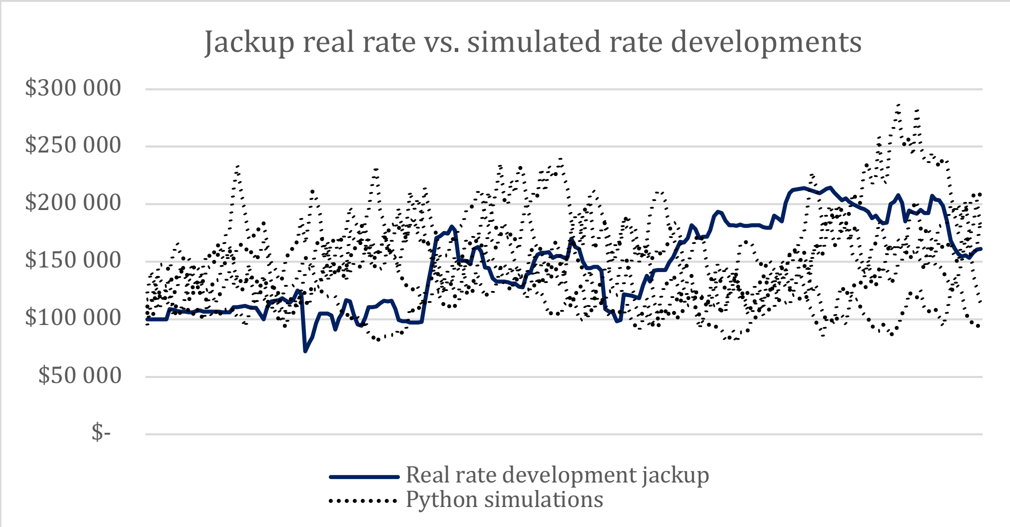

Figure 6 – Real rate vs. simulated rate developments for jackups

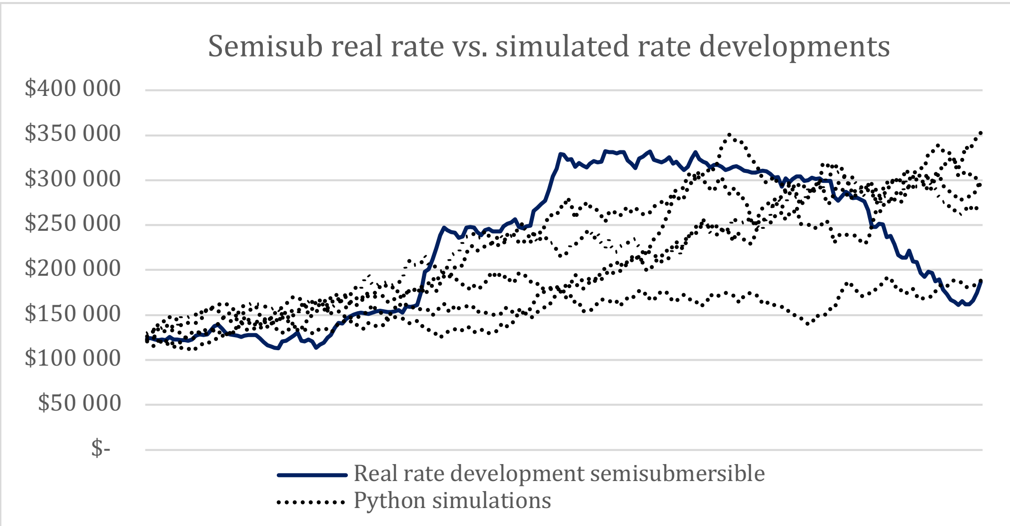

Figure 7 – Real vs. simulated rate development for semisubmersibles

5 Analysis

In our analysis we will first present and discuss a base case for each of the two types of rigs. The base case will represent a neutral standoff in the rig market. In a neutral market, rig rates will be at their long-term equilibrium, and market power will neither lay with the rig- nor oil companies. After a discussion of the base case we will continue with a sensitivity analysis, before we conduct a scenario analysis to investigate how a tight and soft market affect the option value. Finally, we will discuss what challenges extension options present in the market.

5.1 Base Case

Table 7 summarizes the input variables used in the option model. The table also presents the calculated value of the extension option in a neutral market. The explanation and reasoning behind the input variables are provided below table 7. We will also present a discussion of the calculated option value.

Table 7 - Base Case Input Variables and Option Value for Jackups and Semisubmersibles

Input Variables	Jackup	Semisubmersible
α – Long-Term Equilibrium	\$161,522	\$293,002
k – Mean Reversion	0.057577207	0.006867024
σ – Volatility (Monthly)	7.6485%	2.2623%
X_0 – Option Day Rate	\$161,522	\$293,002
OP – Option Period	30 days	30 days
RC – Replacement Cost	\$300,000	\$600,000
Option Value	\$300,000	\$640,000

<u>*GMR* parameters – α, k, σ</u>

The long-term equilibrium rate, α, mean reversion speed, k, and volatility, σ, are the three parameters we calculated using the *GMR* process. These parameters will be held fixed throughout our analysis.

<u>Option day rate – X_0</u>

In the base case, the option day rate, X_0, will be set equal to the long-term equilibrium rate, as this is our best indication of a neutral market. The option day rate will be adjusted in the scenario analysis to better fit the scenarios.

Option period – OP

The option period is set to the drilling time of an average exploration well; 30 days. Our reasoning for choosing this length, is both because jackups and semisubmersibles are mostly involved in E&D drilling, but also because options often are well-based. By well-based, we mean options that extend over the period it takes to drill a given number of wells, usually one. An option period of 30 days also agrees well with the option periods observed on real contracts from RigCube (2018). The option period will be equal for both jackups and semisubmersibles and will be held fixed in all scenarios.

Replacement Cost – RC

As presented in chapter 4.3, the replacement cost can be identified as the cost associated with the time and effort necessary to replace a rig, and the cost of delaying drilling operations. In general, it is very difficult to say anything definitive as to what this cost would be. We have therefore chosen to focus on establishing what premium operators are willing to pay, in excess of the market rate, to avoid having to switch rigs. In discussion with oil and rig companies we have arrived at a markup of $20,000 for semisubmersibles and $10,000 for jackups. As these markups are on the day rate, $20,000 and $10,000 is what operators are willing to pay extra each day in the option period. As the option period is set to 30 days, the replacement cost for jackups and semisubmersibles, respectively, will be calculated to $300,000 ($10,000 x 30 days) and $600,000 ($ 20,000 x 30 days).

Option value - $Vopt_{t=0}$

The explicit financial value of the options in a neutral market is estimated to be $300,000 and $640,000 for jackups and semisubmersibles, respectively.[5] These values represent the financial gain an operator can expect when entering into an option agreement in a neutral rig market. As observed from table 7, the option values are about two times higher than the initial contract rate, X_0, for both jackups and semisubmersibles. This implies that the option value will amount to 7 and 8% of the value of the option contract, for jackups and semisubmersibles respectively. We argue that this is a substantial value, both in absolute terms and as a share of the option contract value.

[5] See appendix 4 for comparison with option values computed based on the *O-U* process

5.2 Sensitivity Analysis of the Base Case Values

The sensitivity analysis that follows will show how the different input variables in the base case affect the value of the option. We will investigate the sensitivity of each input variable, by holding the other variables constant.

Long-term equilibrium rate - α

The long-term equilibrium rate, α, was estimated to be \$161,522 for jackup and \$293,002 for semisubmersibles. Figure 8 illustrates how the option value changes when this parameter is changed by +/- 15 percent.

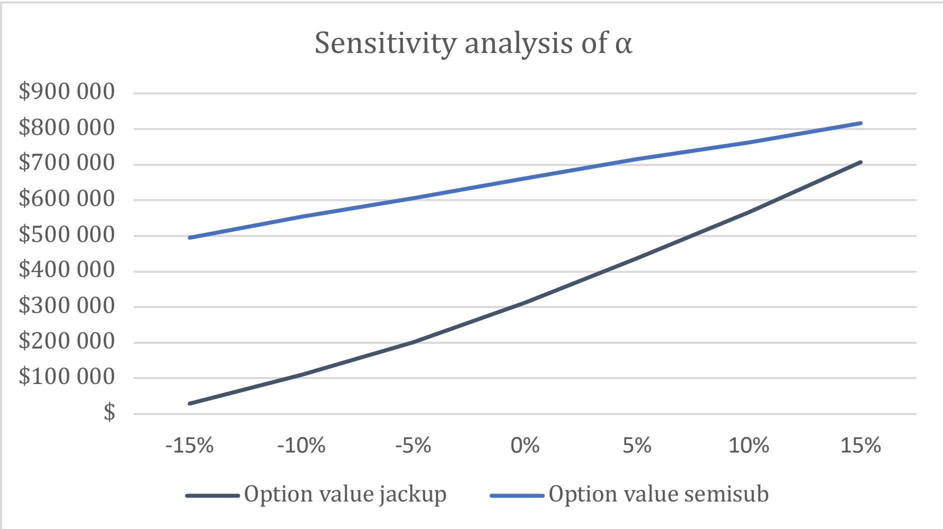

Figure 8 - Sensitivity analysis of α

From figure 8, we observe that an increase in the long-term equilibrium rate will increase the value of the option. Technically, this is due to mean reversion. When the long-term equilibrium rate is increased, while the option day rate is held fixed, rates will revert up towards the mean. This will positively affect the ratio of exercised options and also how deep in-the-money the options are.

This observed effect is also consistent with option theory. The long-term equilibrium rate can be compared to the price of the underlying asset in an option contract, and the option day rate represents the strike price. The value of an option will increase when the price of the underlying asset increases compared to the strike price (Hull, 2012).

From figure 8, we also observe that the option value for jackups is more sensitive to changes in the long-term equilibrium rate, compared to the option value for semisubmersibles. This is because jackup rates experience a larger degree of mean reversion.

<u>Mean reversion – *k*</u>

The mean reversion parameter, $k,$ was estimated to 0.057577207 and 0.006867024 for jackups and semisubmersibles, respectively. In figure 9, we see how the option value changes when this parameter is changed by +/- 15 percent.

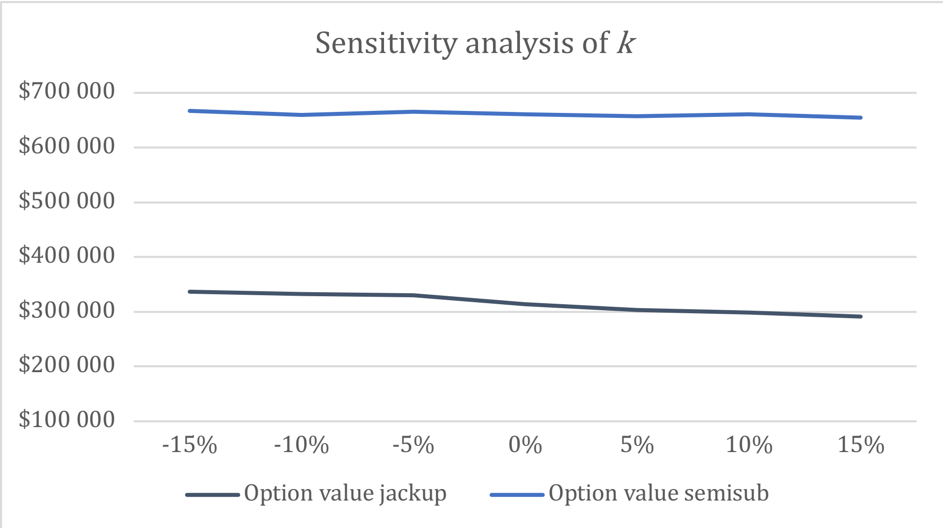

Figure 9 - Sensitivity analysis of k

From figure 9, we observe that the option value will decrease slightly with an increasing *k*. An increased mean reversion will affect the option value negatively as it will keep the rate development closer to the equilibrium, and by that decrease differences between the option day rate and the market rate.

Further, we observe that the change in the option value is slightly greater for jackups than for semisubmersibles. This is due to the differences in the size of the mean reversion parameter between the two rig types.

<u>Volatility – σ</u>

The monthly volatility, σ, was estimated to be 7,685% and 2,623% for jackups and semisubmersibles, respectively. Figure 10 shows how the option value changes as the volatility is changed by +/- 15 percent.

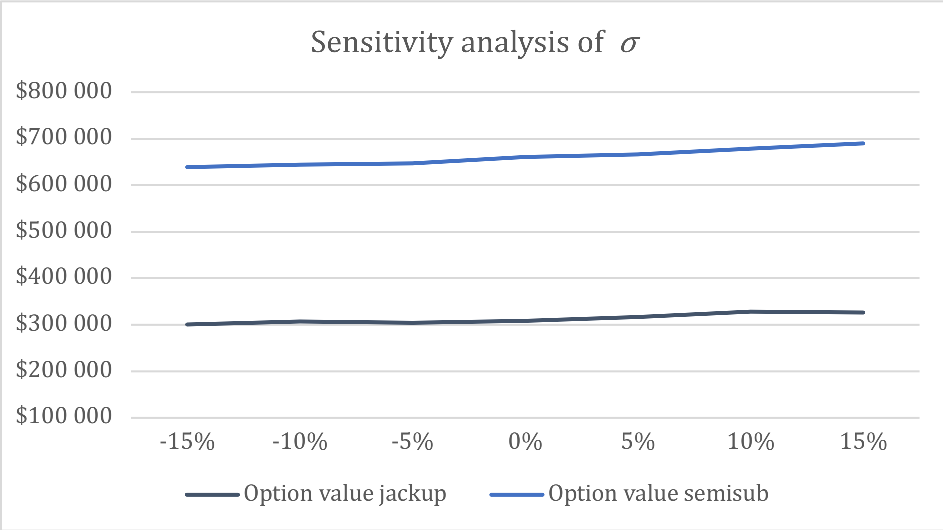

Figure 10 - Sensitivity analysis of σ

Figure 10 shows that the option value for both jackup and semisubmersible increases slightly as the volatility increases. This is supported by option theory, as the option value by definition is positively correlated with volatility. An option is more valuable in an uncertain market.

Replacement cost – RC

We have assumed that the replacement cost is $300,000 and $600,000 for jackups and semisubmersibles, respectively. Figure 11 illustrates how the option value changes as the replacement cost is changed by +/- 15 percent.

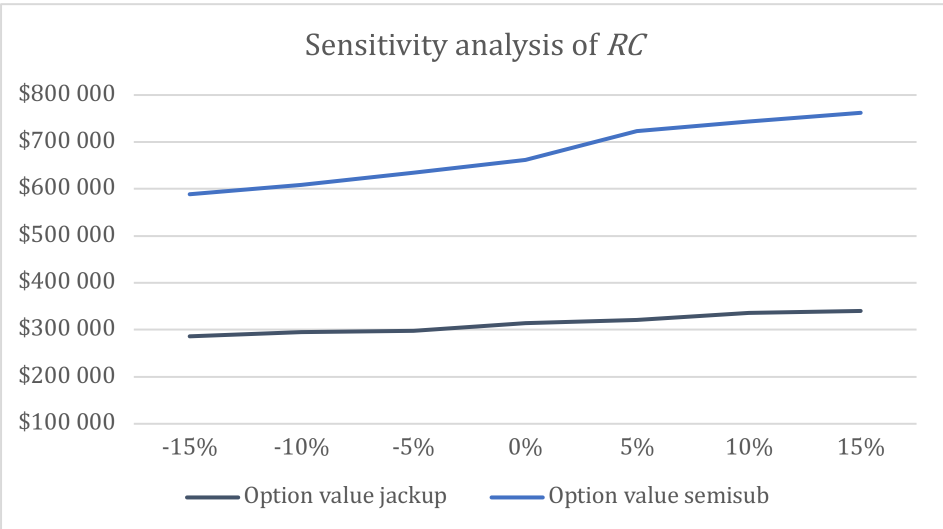

Figure 11 - Sensitivity analysis of RC

We observe that an increased replacement cost, increases the option value. This is obvious, as increasing the replacement cost will increase the rate of exercised options. In addition, the options that were already in-the-money will be deeper in-the-money.

Option period – OP

We have assumed that the option period is 30 days for both jackups and semisubmersibles. Figure 12 shows how the option value changes as the option period is changed by +/- 15 percent.

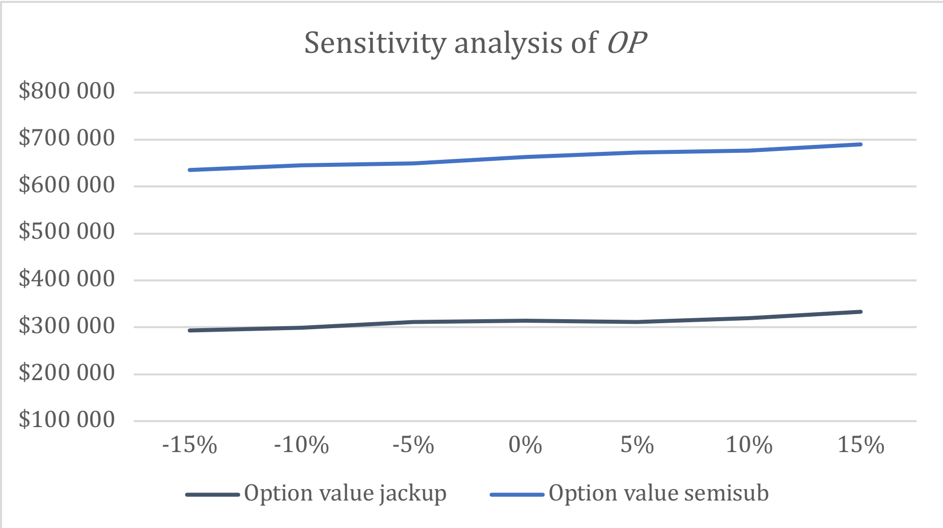

Figure 12 - Sensitivity analysis of OP

An increased option period is positively correlated with the option value. This is because all exercised options will be more valuable when the option period is increased.

Option day rate – X_0

We have assumed that the option day rate is $161,522 and $293,002 for jackups and semisubmersibles, respectively. Figure 13 shows how the option value changes as the option day rate is changed by +/- 15 percent.

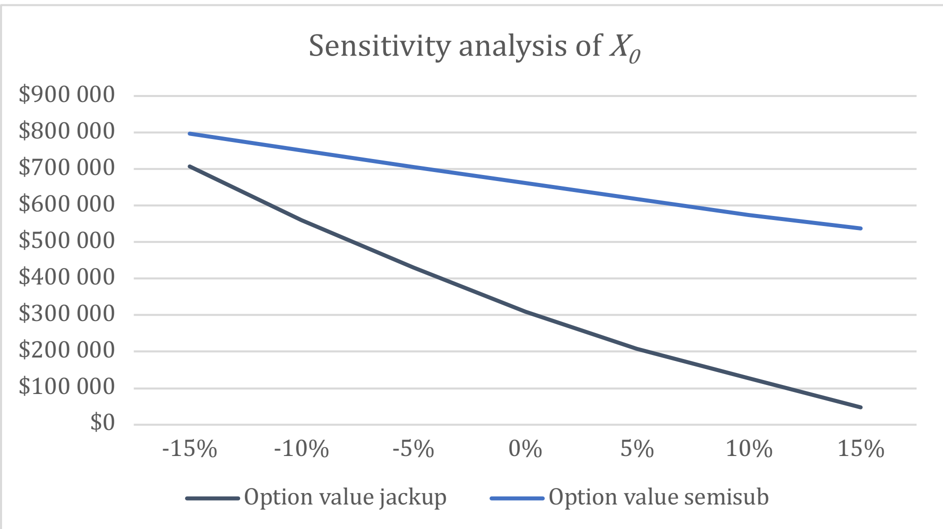

Figure 13 - Sensitivity analysis of X_0

From figure 13 we can observe that the option value is negatively correlated with increases in the option day rate for both jackups and semisubmersibles. Technically, this is due to mean reversion. When the option day rate is increased, while the long-term equilibrium rate is held fixed, rates will revert down towards the mean. This will negatively affect the rate of exercised options and also how deep in-the-money the options are.

This effect is also consistent with option theory. The option day rate can be compared to the strike price in an option contract, and the long-term equilibrium price represents the price of the underlying asset. Given that the price of the underlying asset is held fixed, out of two identical call options, the one with the lowest strike price will always be the most valuable (Hull, 2012).

The steeper development for the option value of jackups is due to higher mean reversion compared to that of semisubmersibles.

Together with changes in α, changes in X_0 has the biggest effect on the option values for both jackups and semisubmersibles. We also observe that when changing the other input variables, with +/- 15%, it has limited effect on the option value.

Effect of Potential Risk Adjustment

As already established in chapter 4.2, we did not risk adjust the stochastic process. However, for cautionary reasons, we want to investigate the effect a potential risk adjustment would have had on option values. We do this by conducting a sensitivity analysis.

When performing this analysis, we will change both the long-term equilibrium rate, α, and the option day rate, X_0. We change α because a risk adjustment would involve adjusting the long-term equilibrium rate. In addition, since the option day rate is set equal to the long-term equilibrium rate in the base case, we must also adjust the option day rate, X_0.

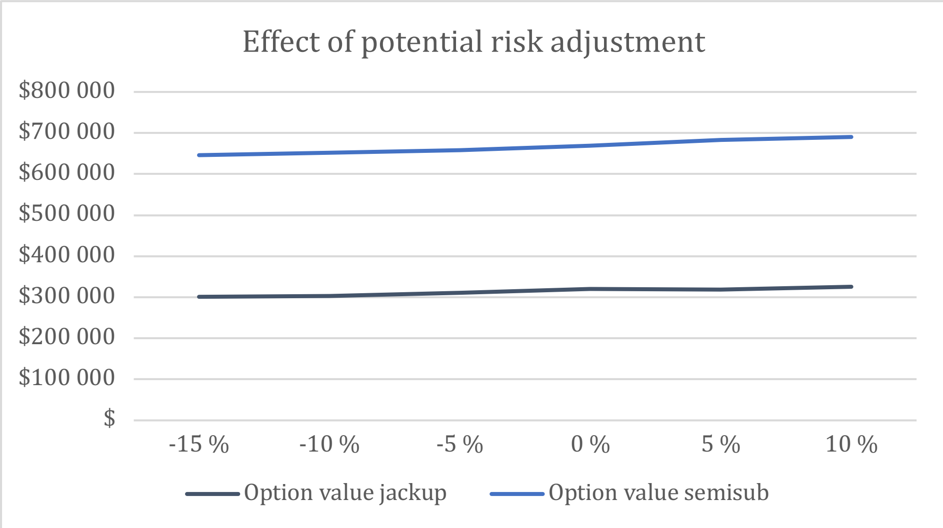

Figure 14 - Sensitivity analysis of potential risk adjustment

As we see in figure 14, a potential risk adjustment will have a limited effect on the option value for both rig types. This indicates that our results, i.e. the calculated option values, would not have been heavily affected by a risk adjustment.

5.3 Scenario Analysis

The base case represented a neutral standoff in the market. Moving forward, to get a more comprehensive understanding of how dynamics in the rig market affect option value, we will conduct a scenario analysis. To do this we will analyze the option value in two different market scenarios; a soft and a tight market.

As previously mentioned, a soft rig market is characterized by low oil prices, low rig utilization and low rig rates. In addition, oil companies will be in the strongest negotiating position. Conversely, a tight rig market is characterized by high oil prices, high rig utilization and high rig rates. Further, rig owners will be in the strongest negotiating position. To mimic the effects of a soft and tight rig market, both the option day rates and the length between the three crucial time points will be changed between the two scenarios. The option day rate will be changed to mimic the effect of higher rig rates and the contract length will be changed to mimic the changes in power structure.

The option day rate for the two different scenarios was chosen from investigating historic rig market rates in relation to historic rig utilization. The chosen rates have been verified by the oil and rig companies we have been in contact with. For jackups the option day rates for a tight and soft market were set to be $250,000 and $100,000, respectively. For semisubmersibles the option day rates for a tight and soft market were set to $400,000 and $150,000.

In our option model, the option day rate will be changed in the different scenarios, by changing the value of X_0.[6]

In the option model, changes in the power structure between oil and rig companies will be expressed through decreasing or increasing the length of the *firm period*, *t=0* to *t=2*, as well as changing the time from the *exercise deadline* to the *start of the option period*, *t=1* to *t=2*. See figure 15. As explained in chapter 1.2, both the length of the firm period, and the period between the exercise deadline and the start of the option period, will be shorter in a soft market and longer in a tight market.

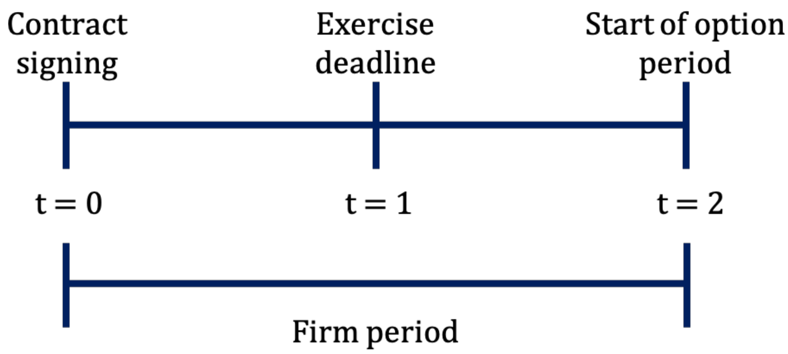

Figure 15 - Illustration of the three crucial time points for the extension option

The lengths of the different time periods were set in discussion with oil and rig companies. In the scenario of a tight market, we have arrived at a firm period, *t=0* to *t=2*, of 3 years. The deadline for exercising the option will be 18 months into the firm period and 18 months before the start of the option period. In the scenario of a soft market, we have arrived at a firm period, *t=0* to *t=2*, of 1 year. The deadline for exercising the option will be 6 months into the firm period and 6 months before the start of the option period. In comparison, the firm period was set to 2 years in a neutral market, and the exercise deadline 12 months into the firm period.

The changes in length between the different scenarios will be the same for the two different types of rigs. In the option model, we will change the length by increasing or decreasing the number of monthly observations between the different time points. As we have monthly observations, we will have 6 and 18 observations between *t=0* to *t=1* and *t=1* to *t=2*, for the soft and tight market respectively.[6] See figure 16.

[6] See appendix 1 for how the code differs between the different scenarios

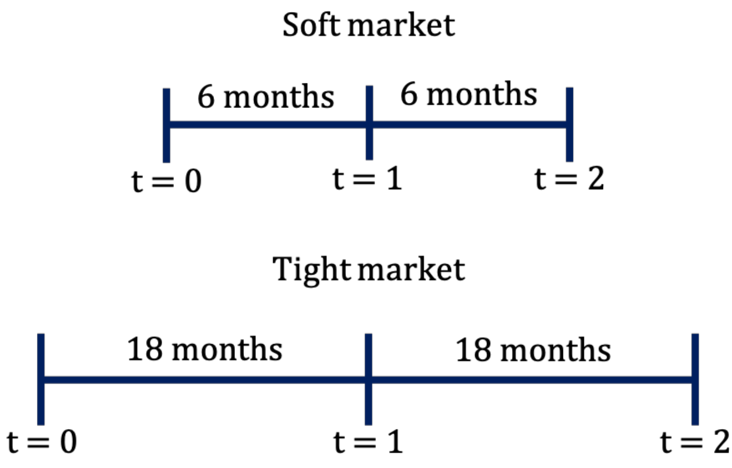

Figure 16 - Illustration of the different contract lengths

Other than changing the option day rate and the contract lengths, the other input variables will be equal to the input variables in the base case.

Discussion of Option Values

The option values and ratio of exercised options for jackups and semisubmersibles are stated in table 8 and 9.[7]

Table 8 - Option Values for Jackup

Scenarios	Firm Period	Option Day Rate	Option Value at t=1	Option Value at t=2	Ratio of Exercised Options	**Option Value at t=0**
Soft Market	1 year	$100,000	$760,000	$1,110,000	92%	**$990,000**
Neutral Market	2 years	$161,522	$890,000	$540,000	58%	**$300,000**
Tight Market	3 years	$250,000	$810,000	($1,520,000)	10%	**($150,000)**

As presented in table 8, the option value for jackups is $990,000 and - $150,000 in the scenario of a soft and a tight market, respectively. We can also observe that the ratio of exercised options decreases when the market moves from soft to tight.

[7] See appendix 4 for comparison with option values computed based on the *O-U* process

Table 9 - Option Values for Semisubmersible

Scenarios	Firm Period	Option Day Rate	Option Value at t=1	Option Value at t=2	Ratio of Exercised Options	**Option Value at t=0**
Soft Market	1 year	$150,000	$700,000	$820,000	98%	**$810,000**
Neutral Market	2 years	$293,002	$860,000	$840,000	76%	**$640,000**
Tight Market	3 years	$400,00	$1,100,000	$550,000	54%	**$290,000**

As presented in table 9, the option value for semisubmersibles is $810,000 and $290,000 in the scenario of a soft and a tight market, respectively. We can again observe that the ratio of exercised options decreases when the market tightens.

We observe that the option value for both rig types decreases when the market tightens. In other words, as the market gets better the option is less valuable. This is intuitive, as we have assumed that the rig rates are mean reverting. We would expect the rates to increase to the long-term equilibrium in a soft market. This will increase the value of the option, as the average rate movement will create a positive difference between the option day rate and the market rate. On the other hand, in a tight market, rates will gradually move towards a lower value and create a negative difference between the day rate on the option and the market rate. Hence, decreasing the value of the option. For illustration see figure 17 and 18.

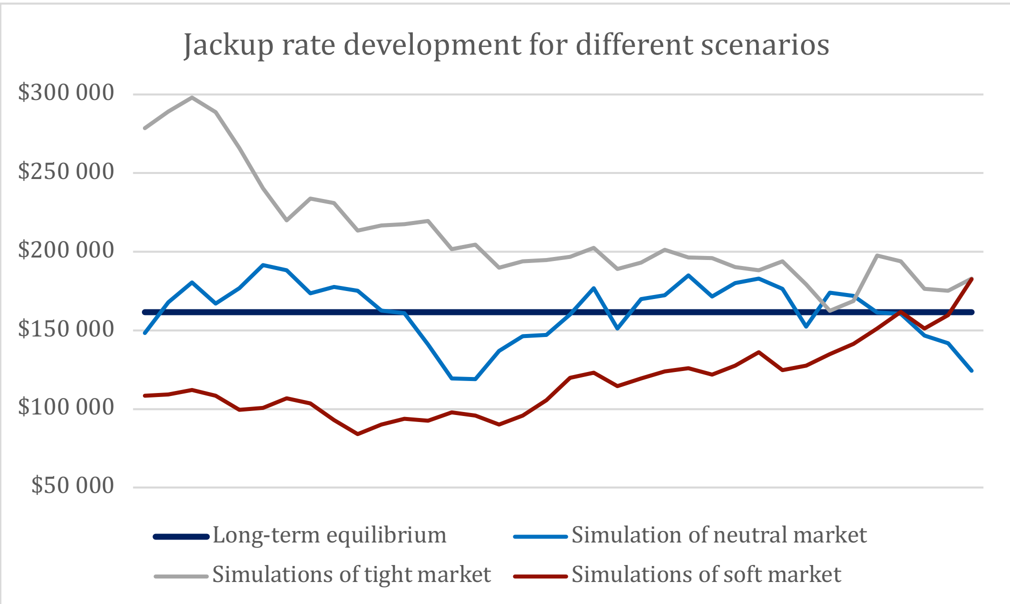

Figure 17 - Simulated rate development for different scenarios for jackup

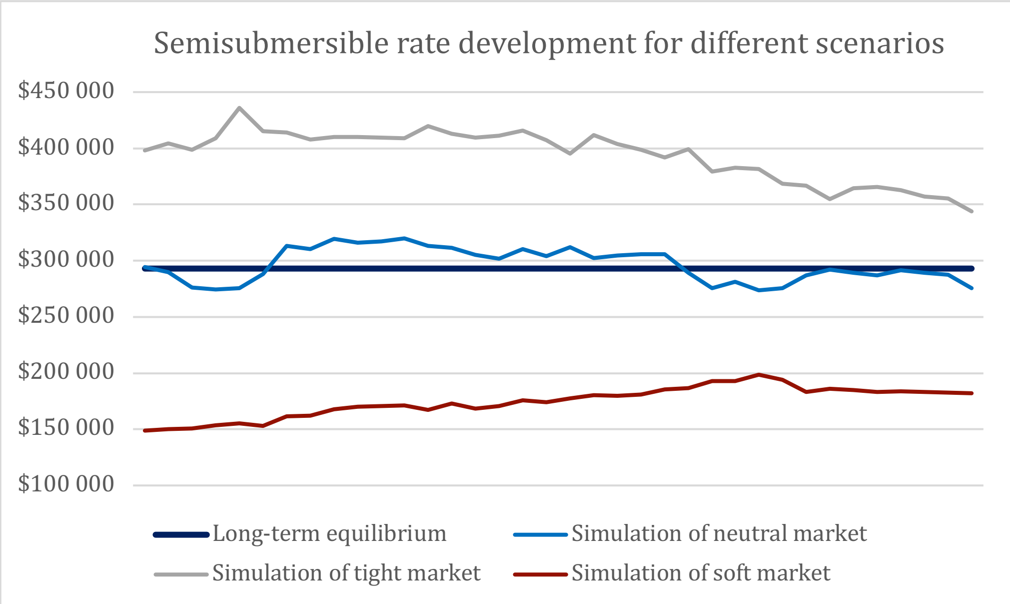

Figure 18 –Simulated rate development for different scenarios for semisubmersible

The effect of mean reversion is also observable in the ratio of exercised options. In a tight market, fewer options are in-the-money at the exercise deadline and the ratio of exercised options fall.

The findings presented above agree well with our understanding of the rig market. In a tight market, rig owners will have the upper hand in terms of power. Intuitively, they will give away less value.

Another interesting observation is the difference in the option value from the time of exercise, $t=1$, to the start of the option period, $t=2$. In a soft market, the value of the average exercised option increases from the exercise deadline, $t=1$, to the start of the option period, $t=2$. Conversely, in a tight market the option value decreases over this time horizon. From table 8 and 9 we observe that this applies to both jackups and semisubmersibles. This effect is anticipated because as time increases the rates revert more towards the long-term equilibrium rate. Hence, when moving from $t=1$ to $t=2$, option value will increase in a soft market and decrease in a tight market.

Discussion of Differences Between the Rig Types

Further, the analysis will focus on the differences in option value between jackups and semisubmersibles in the different scenarios. From table 8 and 9 we observe that the option values for semisubmersibles experience a more stable development between the different scenarios, compared to the option values for jackups. The same is true for the development of the ratio of exercised options.

In the scenario of a soft market, we observe that the option value for jackups, \$990,000, is higher than the option value for semisubmersibles, \$820,000. However, the option value for jackups falls dramatically, compared to the option value for semisubmersibles when the market tightens. In a tight market scenario, the option value for jackups is -\$150,000 and \$290,000 for semisubmersibles.

The larger change in option value for jackups is due to a higher rate of mean reversion. This implies that the jackup rates experience a faster movement towards the long-term equilibrium rate. This increases the option value in a soft market but decreases it in a tight market, compared to semisubmersibles. The higher rate of mean reversion is observable in the differences between figure 17 and 18. We can also observe that the jackup rate experience a more volatile development.

In a tight market, we observe that we get a negative option value for jackups. This is because an option that is in-the-money at $t=1$, not necessarily is in the money at $t=2$. To be in-the-money at $t=1$, the rate must have experienced a steady or positive development from $t=0$ to $t=1$ in a tight market. This is the opposite development of what we would expect from rates that are mean reverting. Hence, we observe that only 10% of the simulations will generate options that are in-the-money at $t=1$. See table 8. From $t=1$ to $t=2$, many of the options that were in-the-money at $t=1$ will experience to revert downwards towards the long-term equilibrium and fall out-of-the-money. When this happens to a large enough share of the exercised options, the option value will become negative. See figure 19 for illustration.

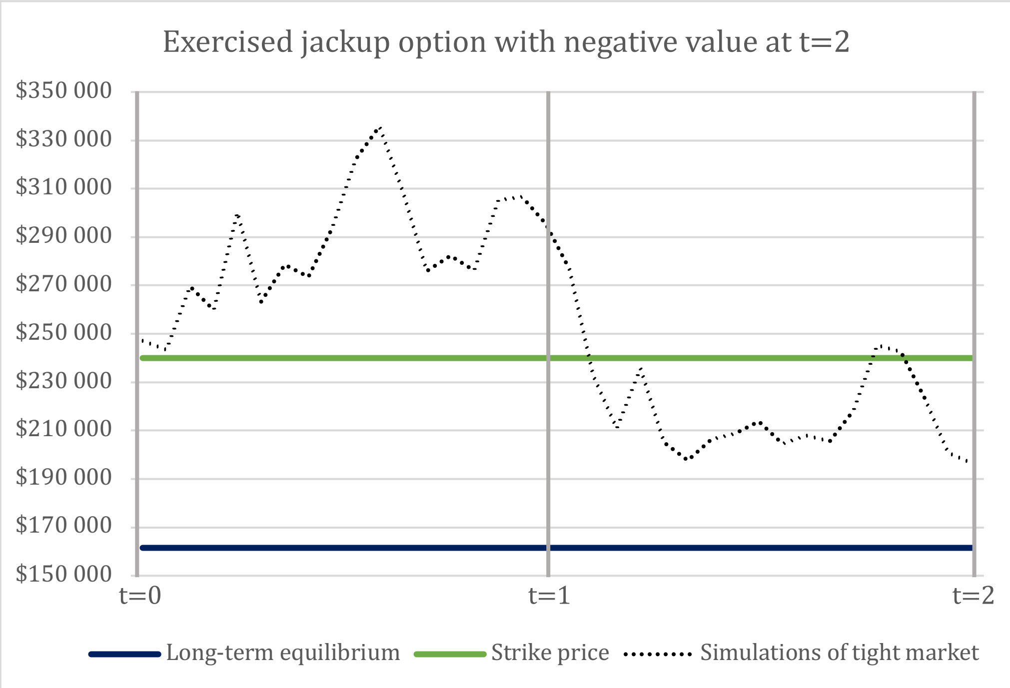

Figure 19 - Example of simulated rate that causes negative option value

We observe from figure 19, that the option is in-the-money at *t=1* and therefore exercised. However, at *t=2* the option is out-of-the-money and generates a negative option value.

5.4 How is the Option Value Affected by Two Strikes?

We want to investigate how the option value is affected by changing from two to one strikes. By two strikes, we mean the exercise deadline and the start of the option period, *t=1* and *t=2*. When adjusting to one strike the exercise deadline will be set to the start of the option period, i.e. the exercise deadline is at *t=2*. See figure 20.

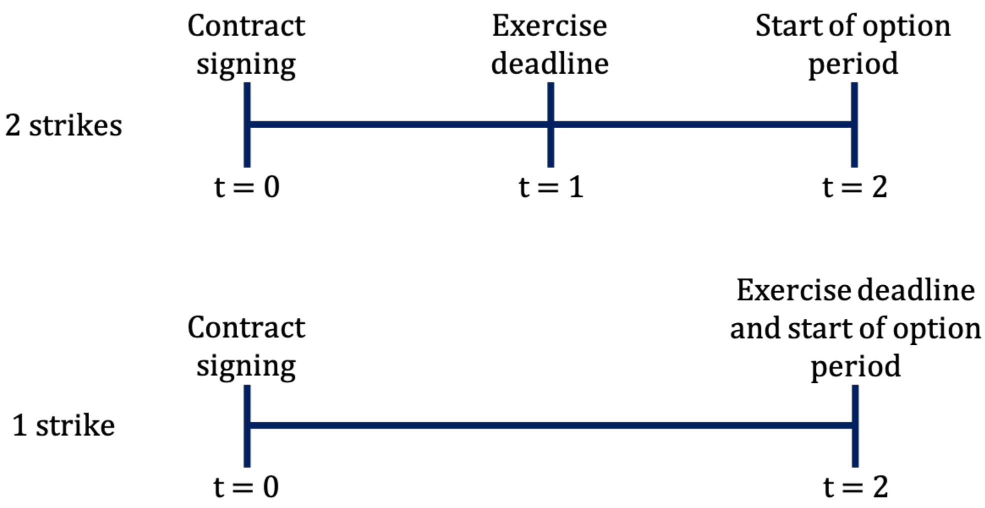

Figure 20 - 1 vs. 2 strikes

The differences in option values are displayed in table 10 and 11, for jackups and semisubmersibles respectively.

Table 10 – Differences in Option Values Between 1 and 2 Strikes for Jackups

Scenarios	Option Value 1 Strike	Option Value 2 Strikes
Soft Market	$1,100,000	$1,000,000
Neutral Market	$530,000	$300,000
Tight Market	$29,000	($150,000)

Table 11 – Differences in Option Values Between 1 and 2 Strikes for Semisubmersibles

Scenarios	Option Value 1 Strike	Option Value 2 Strikes
Soft Market	$820,000	$810,000
Neutral Market	$750,000	$640,000
Tight Market	$520,000	$290,000

As we can observe from table 10 and 11, the option value is higher in all scenarios for both rig types with one strike, compared to two. This is because the operator will know when exercising the option, whether the option is in-the-money or not at *t=2*. This implies that options that fall out-of-the-money from the time of the initial exercise, *t=1*, to the start of the option period, *t=2*, will not be exercised. In addition, options that are out-of-the-money at the initial time of exercise, *t=1*, but in-the-money at the start of the option period, *t=2*, will be exercised.

Presented in table 12 and 13 are the differences in the ratio of exercised options between one and two strikes, for jackups and semisubmersibles respectively.

Table 12 – Differences in Ratio of Exercised Options Between 1 and 2 Strikes for Jackup

Scenarios	Ratio of Exercised Options 1 Strike	Ratio of Exercised Options 2 Strikes
Soft Market	94%	92%
Neutral Market	55%	58%
Tight Market	5%	10%

Table 13 – Differences in Ratio of Exercised Options Between 1 and 2 Strikes for Semisubmersible

Scenarios	Ratio of Exercised Options 1 Strike	Ratio of Exercised Options 2 Strikes
Soft Market	99%	98%
Neutral Market	71%	76%
Tight Market	43%	54%

As we can observe, the ratio of exercised options decreases in the tight and neutral market when changing from two to one strikes for both rig types. However, the ratio of exercised options increases in a soft market for both rig types. This is intuitive, when assuming that rates are mean reverting.

In a soft market, increasing the time between the contract signing and the exercise date, i.e. moving the exercise date form *t=1* to *t=2*, will revert rates even more towards the higher equilibrium rate before the exercise date. This will increase the number of options that are in-the-money at the time of exercise. For the exercising of options in a tight market, the effect will be the opposite.

5.5 Other Dynamics of the Rig Market and How They Affect Option Value

In the process of making our option model we made some strict assumptions. We will now ease these assumptions to evaluate how they affect the option value.

Renegotiating Power

When we calculated the value of the option, we assumed that the day rate on the embedded option was set at the time of contract signing. In addition, our model assumes that the agreed upon rate is not negotiable at a later time. However, in reality operators are often in a position where they can renegotiate the option day rate up to the time of exercise. As established earlier, operators' power increase with a softer market. Intuitively, this is also true for operators' renegotiating power.

In practice, operators' renegotiating power affects the number of options that are in-the-money positively, and thereby increases option value. If an option is out-of-the-money, operators that want to extend drilling operations will renegotiate the option day rate down to the market rate.

In our option pricing model, the renegotiating power would have had the same effect; more options would be in-the-money. This would affect the ratio of exercised options and also positively affect the option value.

The Value of Postponing a Choice

In the option model, we calculated the value of the options as a financial gain, solely. When making this decision, we overlooked the value of postponing a choice for an operator. As explained in chapter 1.2, E&D projects are associated with a high degree of uncertainty. This indicates that there is a significant value associated with increased flexibility.

In reality, the value of the option is higher than just the financial gain, as there will be value associated with having more flexibility. This flexibility will however be difficult to price and is therefore not accounted for in our option model. To give an estimate as to the value of postponing a choice, an elaborate study would have been required for each specific oil field, as there is no oil field alike and a huge specter of different uncertainties.

In our option model the inclusion of the value of flexibility would have affected the option value positively. It is reasonable to believe that the value of flexibility would have been of substantial size.

Empirical Observations on the Exercising of Options

As mentioned, Rystad Energy (2018) identifies that the exercising of options is positively correlated with high oil prices and high demand in the rig market, i.e. a tight market. These findings indicate that not all options that are in-the-money will be exercised in a soft market. This is intuitive, as the operator will limit their E&D spending in times of declining oil prices.

In practice, these dynamics will affect the value of optionality in a soft market negatively. This is because options that are in-the-money will go unexercised. This effect is not captured in our model, as we have assumed that operators always have ongoing drilling operations, and always wish to exercise an option when it is in-the-money.

Multiple Options

Operators are often given a number of sequential options in their contracts, cf. chapter 1.2. Multiple options will not affect the financial value of an individual option, but it will give the operator increased value through increased flexibility.

Inflation Adjusted Rig Rates

As established in chapter 3, we have used inflation adjusted rig rates to estimate the *GMR* parameters. Using inflation adjusted rates will mainly affect the calculated long-term equilibrium rate, and by that our chosen option day rate in the base case scenario. However, as we saw under the sensitivity analysis of the potential risk adjustment, when changing the long-term equilibrium rate and option day rate equally the effect on option value was minimal.

5.6 Challenges Associated with the Use of Options

Options provide operators with flexibility in the face of uncertainty, and also a possible financial upside. However, options also present challenges in the rig market.

For rig owners, the use of extension options create a problem as rig owners cannot predict future rig rates and therefore cannot accurately set the day rate on their options. In addition, it is extensive uncertainty associated with committing a rig to an option period for rig owners. In a situation where an option is not exercised, it is often not enough time to get the rig committed to a new contract, before the other contract ends. This creates a problem, as it might result in the rig being left idle. If the rig is left idle for a longer period, it will be stacked.

The uncertainty associated with rig commitment can also be a problem for operators. In discussion with Aker BP, we understood that operators feel that the use of options create artificially high rig utilization. This is because the rig cannot be marketed in the period before the exercise deadline and is therefore perceived as committed. In a situation with multiple embedded options, the period of uncertainty around whether the rig is committed or not increases. Artificially high rig utilization creates a problem, as it drives up market rates.

All of the abovementioned challenges are problematic as they decrease the efficiency in the use of a scarce and valuable resource. It is especially problematic that the use of options can lead to unpredictable rig commitment, causing higher rig rates and possibly idle rigs.

In the next part we want to investigate the extension option in the framework of the prisoner's dilemma. Finally, we will discuss how the market itself tries to limit the negative effects of using options.

Prisoner's Dilemma

In many ways, extension options present a version of the prisoner's dilemma. It would be better for all rig owners if they agreed on not including options in their contracts. However, for the individual, options are beneficial as they can be part of securing contracts. For operators, not demanding options would go a long way in limiting artificially high rig utilization and by that higher rig rates. However, for the individual in the market it is beneficial to secure drilling capacity and increase flexibility.

In the prisoner's dilemma the possibility of individual gain is reinforced by the possibility of individual loss. If a rig owner chooses not to offer options in their contracts, it is likely that they will lose contracts to other rig owners. Similar, if an operator turns down the possibility for options, the operator will lose flexibility. This can lead to higher E&D costs compared to competitors.

In fear of losing compared to their competitors, all participants choose to continue with the practice of options. In game theory this is known as the Nash-equilibrium (Nash, J.K, 1950). This equilibrium represents the only outcome where no participants have an incentive to deviate from their initial strategy.

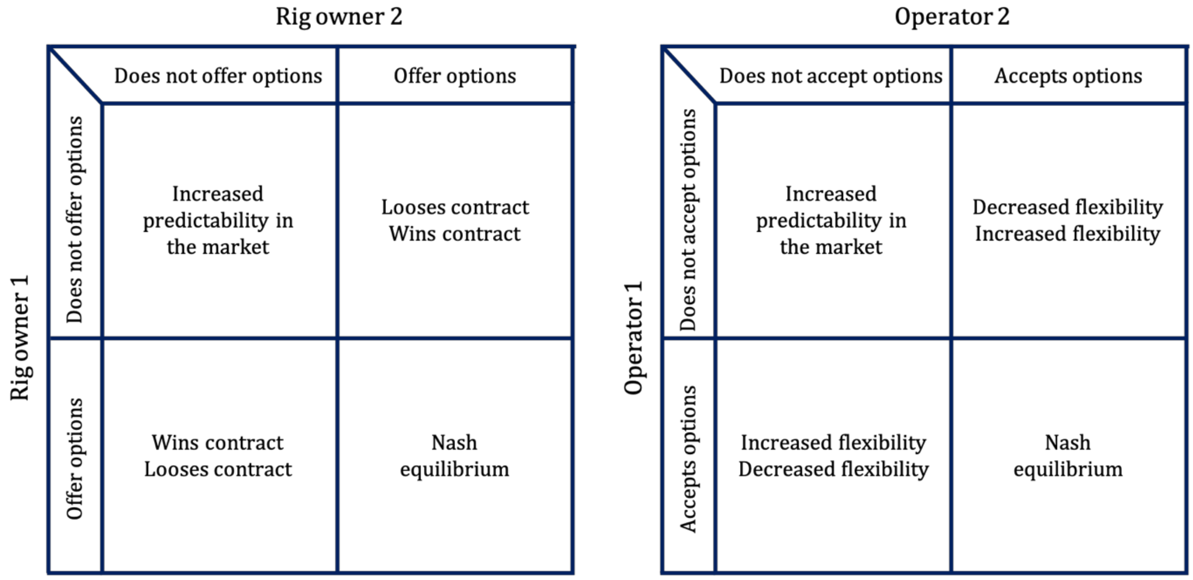

Figure 21 – The prisoner's dilemma in the rig market

The prisoner's dilemma in the rig market, for both rig owners and operators, is illustrated in figure 21. As already established, the equilibrium will be the situation where all participants in the market make use of options. In figure 21, these outcomes are presented in the lower right corner. However, a better situation for all participants would be a scenario without options as we know them today. In the figure, these outcomes are presented in the top left corner. What prevents participants to move away from options is the fear that their competitors will not follow. Then, they end up in a situation where they themselves will not use options, while their competitors still do. In figure 21, these outcomes are presented in the top right and bottom left corners.

In a neutral market, the option value for jackups and semisubmersibles was calculated to be \$310,000 and \$660,000, respectively. This represents the Nash equilibrium and equals a financial loss for rig owners and a financial gain for operators.

What is the Market Doing to Solve these Challenges?

Looking beyond strict game theory, we examine how the rig market itself tries to limit the problem of uncertainty surrounding future rig rate development and rig commitment caused by the use of options.

To minimize the problem of uncertainty surrounding future rig rates, a solution is to move away from options with day rates set at the time of contract signing. In conversation with rig and oil companies, we have understood that there is an increasing trend towards using options with market–based rates. Here, the option day rate is set by two independent brokers. At an agreed upon time, the two independent brokers will make their best predictions of future market rates and the option day rate will be set as an average of the two estimates.

Options with market-based rates will help minimize the problem of uncertainty, because the day rate will be set equal to the market's predictions. If the market's predictions are correct, the explicit financial value of the options would be zero, as there would be no difference between the option day rate and the market rate when the option period starts.

Other than the uncertainty associated with future rig rates, there is also considerable unpredictability surrounding rig commitment. However, rig owners and operators are trying to minimize this problem by narrowing the spread between real and perceived rig commitment. Firstly, operators are making an effort to signalize whether or not they are exercising an option as early as possible. In conversation with Aker BP, we learned that they are doing so to create a more transparent market where rig commitment is easier observed.

Secondly, operators are signing exclusive contracts on rigs. Exclusive contracts on specific rigs implies that a rig is bound to an operator for as long as the operator has work for the rig. If an operator at any time has less than 1 year of work for the rig, it can be marketed for other contracts. These precise contract conditions decrease the uncertainty regarding rig commitment by increasing transparency for both the rig owner and operator.

Given the uncertainty surrounding drilling activities, extension options are highly valuable and also an important mechanism to maintain exploration activity. However, as we explored the use of options also present challenges. Moving forward, we would expect to see a shift towards options with market-based rates. In addition, we would expect the rig market to remain focused on making rig commitment more predictable. We believe that these changes could lead to a more efficient rig market.

6 Conclusion

This thesis provides insight into the financial value associated with options to extend a drilling contract. The main conclusion of the thesis can be summarized as follows: *The option value is substantial and decreases as the market gets tighter*. This conclusion applies to both jackup and semisubmersible rigs at the NCS.

The financial value of the options was estimated to be \$300 000 and \$640 000 for jackup and semisubmersibles, respectively. This value is deemed to be of substantial size.

There are differences in the development of the option value between the two types of rigs. The option values for jackups are affected by a more volatile rate development than that of semisubmersibles. In times of soft market, an option on a jackup is more valuable than an option on a semisubmersible. However, in a tight market the opposite is true. Contrary to options on jackups, options on semisubmersibles will be of value in all market scenarios.

Options offer valuable flexibility to operators. However, options also present challenges. To minimize these challenges, we propose a change towards options with market-based rates and continued focus on making rig commitment as transparent as possible.

The purpose of our work has been to provide insight into the complex nature of options to extend drilling operations in the offshore rig market. We do believe our thesis provides valuable insight into this unexplored topic. In further research, it could be interesting to expand our option model to be a better fit with reality. It could be expanded to include renegotiating power and maybe the value of postponing a choice. It could also be interesting to conduct a similar analysis to ours for different markets or for different time periods.

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8 Appendices

Appendix 1 – Python Code

Base Case

```
1   import math
2   import numpy as np
3
4   def general_model(contract_length, replacement_cost, x0, volatility, mean_reversion, median, nsimul = 100000):
5       count = 0
6
7       valueList = []
8       dummyList = []
9
10      xt = calc_xt(x0, volatility, mean_reversion, median)
11
12      for i in range(1, nsimul):
13
14          xt = calc_xt(x0, volatility, mean_reversion, median)
15
16          # Dummy >= 0 --> Option will be excersised
17          dummy = ((xt[11] - xt[0]) * contract_length) + replacement_cost    # Option value at t=1
18          value2 = ((xt[23] - xt[0]) * contract_length) + replacement_cost   # Option value at t=2
19
20          if dummy >= 0:
21              count += 1
22              valueList.append(value2)
23              dummyList.append(dummy)
24
25      return "Value at t=2", np.mean(valueList), "Value at t=1", np.mean(dummyList), \
26             "Value all options", sum(valueList)/nsimul, "Count", count
27
28  # End general_model
29
30
31  def calc_xt(x0, volatility, mean_reversion, median):
32      xt = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]
33      xt[0] = x0  # Value at t=0
34
35      # Variables for normal Gaussian distribution, mean and standard deviation
36      mu, sigma = 0, 1
37
38      for i in range(1, len(xt)):
39          z = np.random.normal(mu, sigma) # Creates random variable
40
41          xt[i] = xt[i - 1] + (mean_reversion * (median - math.log(xt[i - 1])) * xt[i - 1]) \
42                  + xt[i - 1] * volatility * z  # GMR
43
44      return xt  # Returns list of 24 variables xt[0],xt[1],xt[2]...xt[23]
45  # End calc_xt
46
47  # Main program starts here
48
49  # scnenario = [contract length, replacement cost, x0, volatility, mean reversion, long term equilibrium]
50  scenarios = [[30, 300000, 161522, 0.07685, 0.057577207, 11.99239617],  # Jackup neutral market
51               [30, 600000, 293002, 0.02623, 0.006867024, 12.58793424]]  # Semisub neutral market
52
53  for scn in scenarios:
54      print(general_model(scn[0], scn[1], scn[2], scn[3], scn[4], scn[5]))  # Print all scenarios
```

Figure 22 - Python code neutral market

Between the different scenarios, line 17, 18 and 32 will be changed to accommodate for the shorter/longer contract period. Other than that, *x0* will also be changed between the scenarios. *x0* represents the option day rate, X_0. See line 50 and 51. As we can observe, *x0* is set equal to the long-term equilibrium rate in the base case, $161,522 and $293,002.

In line 32, *xt* is a vector of 24 zeros, i.e. an empty vector with 24 places. These 24 empty places will be filled with simulated monthly market rates. 24 represents the 2-year firm period in a neutral market. The rates are simulated using the *GMR* process. See line 41 and 42.

The simulated rates are used in line 17 and 18 to calculate the dummy and option value. The dummy takes the simulated rate from a year into the firm period, *xt[11]*, and subtract the option day rate, *x[0]*. The difference is multiplied with the length of the option period, and the replacement cost is added. Python starts counting at zero and *xt[11]* is therefore a year into to the firm period. $Vopt_{t=1}$ is calculated based on the dummy variable.

In *value2*, $Vopt_{t=2}$ is calculated. As we can observe in line 18 the calculation of the option value is based on the difference between the simulated rate at *t=2* and the option day rate, *xt[23] – x[0]*. $Vopt_{t=0}$ is calculated based on *value2*. See line 26 and "Value all options".

Soft Market

```
1   import math
2   import numpy as np
3
4   def general_model(contract_length, replacement_cost, x0, volatility, mean_reversion, median, nsimul = 100000):
5       count = 0
6
7       valueList = []
8       dummyList = []
9
10      xt = calc_xt(x0, volatility, mean_reversion, median)
11
12      for i in range(1, nsimul):
13
14          xt = calc_xt(x0, volatility, mean_reversion, median)
15
16          # Dummy >= 0 --> Option will be excersised
17          dummy = ((xt[5] - xt[0]) * contract_length) + replacement_cost    # Option value at t=1
18          value2 = ((xt[11] - xt[0]) * contract_length) + replacement_cost   # Option value at t=2
19
20          if dummy >= 0:
21              count += 1
22              valueList.append(value2)
23              dummyList.append(dummy)
24
25      return "Value at t=2", np.mean(valueList), "Value at t=1", np.mean(dummyList), \
26             "Value all options", sum(valueList)/nsimul, "Count", count
27
28  # End general_model
29
30
31  def calc_xt(x0, volatility, mean_reversion, median):
32      xt = [0,0,0,0,0,0,0,0,0,0,0,0]
33      xt[0] = x0  # Value at t=0
34
35      # Variables for normal Gaussian distribution, mean and standard deviation
36      mu, sigma = 0, 1
37
38      for i in range(1, len(xt)):
39          z = np.random.normal(mu, sigma) # Creates random variable
40
41          xt[i] = xt[i - 1] + (mean_reversion * (median - math.log(xt[i - 1])) * xt[i - 1]) \
42                  + xt[i - 1] * volatility * z  # GMR
43
44      return xt  # Returns list of 12 variables xt[0],xt[1],xt[2]...xt[11]
45  # End calc_xt
46
47
48  # Main program starts here
49
50  # scenario = [contract length, replacement cost, x0, volatility, mean reversion, median]
51  scenarios = [[30, 300000, 100000, 0.07685, 0.057577207, 11.99239617],  # Jackup soft market
52               [30, 600000, 150000, 0.02623, 0.006867024, 12.58793424]]  # Semisub soft market
53
54  for scn in scenarios:
55      print(general_model(scn[0], scn[1], scn[2], scn[3], scn[4], scn[5]))  # Print all scenarios
```

Figure 23 - Python code of soft market

As we can observe from figure 23, line 32 is changed to fit the firm period in a soft market, 12 months. The change is also incorporated in line 17 and 18. Finally the option day rate is changed in line 51 and 52.

Tight Market

```python
1  import math
2  import numpy as np
3
4  def general_model(contract_length, replacement_cost, x0, volatility, mean_reversion, median, nsimul = 100000):
5      count = 0
6
7      valueList = []
8      dummyList = []
9
10     xt = calc_xt(x0, volatility, mean_reversion, median)
11
12     for i in range(1, nsimul):
13
14         xt = calc_xt(x0, volatility, mean_reversion, median)
15
16         # Dummy >= 0 --> Option will be excersised
17         dummy = ((xt[17] - xt[0]) * contract_length) + replacement_cost    # Option value at t=1
18         value2 = ((xt[35] - xt[0]) * contract_length) + replacement_cost   # Option value at t=2
19
20         if dummy >= 0:
21             count += 1
22             valueList.append(value2)
23             dummyList.append(dummy)
24
25     return "Value at t=2", np.mean(valueList), "Value at t=1", np.mean(dummyList), \
26            "Value all options", sum(valueList)/nsimul, "Count", count
27
28 # End general_model
29
30
31 def calc_xt(x0, volatility, mean_reversion, median):
32     xt = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]
33     xt[0] = x0  # Value at t=0
34
35     # Variables for normal Gaussian distribution, mean and standard deviation
36     mu, sigma = 0, 1
37
38     for i in range(1, len(xt)):
39         z = np.random.normal(mu, sigma) # Creates random variable
40
41         xt[i] = xt[i - 1] + (mean_reversion * (median - math.log(xt[i - 1])) * xt[i - 1]) \
42                 + xt[i - 1] * volatility * z  # GMR
43
44     return xt  # Returns list of 36 variables xt[0],xt[1],xt[2]...xt[35]
45 # End calc_xt
46
47
48 # Main program starts here
49
50 # scnenario = [contract length, replacement cost, x0, volatility, mean reversion, median]
51 scenarios = [[30, 300000, 250000, 0.07685, 0.057577207, 11.99239617],  # Jackup tight market
52              [30, 600000, 400000, 0.02623, 0.006867024, 12.58793424]]  # Semisub tight market
53
54 for scn in scenarios:
55     print(general_model(scn[0], scn[1], scn[2], scn[3], scn[4], scn[5]))  # Print all scenarios
```

Figure 24 - Python code of tight market

As we can observe from figure 24, line 32 is changed to fit the firm period in a tight market, 36 months. The change is also incorporated in line 17 and 18. Finally the option day rate is changed in line 51 and 52.

Appendix 2 – Augmented Dickey-Fuller Test with Drift

We performed an Augmented Dickey-Fuller test to investigate if the collected time series were stationary. For the market rates of semisubmersibles, we were not able to reject the null hypothesis of unit roots in the original test. From the graph presented in chapter 3, we observed a clear increasement in the semisubmersibles rates from 2000 to 2018. Hence, we argue that we can use an Augmented Dickey Fuller test with *drift.* The drift term can be included when a time series is clearly increasing or decreasing (Stata, 2018). In this case, the null hypothesis is a random walk with nonzero drift.

```
. dfuller log_SS, drift

Dickey-Fuller test for unit root                  Number of obs   =       222

                                  ——————— Z(t) has t-distribution ———————
                  Test          1% Critical      5% Critical      10% Critical
                Statistic          Value            Value             Value
──────────────────────────────────────────────────────────────────────────────
 Z(t)             -1.459          -2.343           -1.652            -1.285
──────────────────────────────────────────────────────────────────────────────
p-value for Z(t) = 0.0730
```

Figure 25 – Output from STATA: Augmented Dickey-Fuller test with drift

The output of the test is presented in figure 25. We observe that the null hypothesis can be rejected at a 10% significance level. Hence, we argue that the rates of semisubmersibles also are stationary when fluctuating around a trend.

Appendix 3 – Bjerksund and Ekern's (1995) Approach to Risk Adjusting an Ornstein-Uhlenbeck Process

Bjerksund & Ekern (1995) proved that an *O-U* process can be risk adjusted by finding the risk adjusted long-term equilibrium, α^*. The equation for α^* is

$$\alpha^* = \alpha - \frac{\sigma\lambda}{k}, \tag{4.8}$$

where λ is market price of rate risk. The market price on risk is calculated using Hull's (2012) approach

$$\lambda = \frac{\rho}{\sigma_m}(\mu_m - r), \tag{4.9}$$

where ρ is the correlation between the market and the risky asset, α. Further, μ_m and σ_m is the market return and volatility, respectively. r is the risk-free rate.

In this method market risk is first allocated to the asset. This is done by calculating the Sharpe ratio of the market and multiplying it with the correlation coefficient, ρ. Then, when calculating the risk adjusted long-term equilibrium, α^*, the allocated market risk, λ, is scaled with the asset specific risk, σ.

We tried using Bjerksund & Ekern's (1995) approach, but with minimal effect. When using this approach, the impact on α was close to none. We have identified a few factors that might have affected our risk adjustment. Firstly, the *O-U* process takes absolute standard deviations, while the *GMR* process takes percentage standard deviations. So, to calculate α^* we had to modify Bjerksund & Ekern's (1995) approach. This was done by multiplying σ with α, to get an absolute measurement of the standard deviation.

Secondly, finding correct estimates of the parameters to calculate λ turned out to be difficult. When calculating λ we choose S&P 500 as the broad stock index representing the market. Not only was it difficult to find estimates of the return and volatility for the exact time period, June 2000 – December 2018, but it was also a large difference in the estimates between different sources. When conducting our risk adjustment, we choose the yield of the 20-year US Treasury bill from June 2000 as the risk-free rate. By having different estimates for S&P500 we got unprecise estimates of λ. In addition, the calculations of λ were in general very small.

Appendix 4 – Option Values Using the Ornstein-Uhlenbeck Process

The discrete version of the *O-U* process is used to estimate our coefficients and is

$$X_t - X_{t-1} = \alpha k - kX_{t-1} + \epsilon_t. \tag{8.1}$$

The coefficients obtained from the OLS regression are presented in table 14 and 15.

Table 14 – Estimated OLS Values for Jackup

Parameter	Estimated Coefficients	Standard Error	t-value
β_0	5349.112	2079.335	2.57
β_1	-0.0332085	0.0136987	-2.42

Standard deviation of regression: 8258
R^2: 0.0260

Table 15 – Estimated OLS Values for Semisubmersible

Parameter	Estimated Coefficients	Standard Error	t-value
β_0	1638.967	1166.188	1.41
β_1	-0.0059622	0.0048498	-1.23

Standard deviation of regression: 5634
R^2: 0.0068

From the output in table 14 and 15 we can calculate the parameters in the *O-U* model. The parameters are calculated from the equations

$$k = -\beta_1, \tag{8.2}$$

$$\alpha = \frac{\beta_0}{k}. \tag{8.3}$$

σ is the standard deviation of the regression.

The calculated *O-U* parameters are presented in table 16.

Table 16 – Calculated *O-U* Parameters

	α – Long-Term Equilibrium Rate	k – Mean Reversion Speed	σ – Volatility (Monthly)
Jackup	\$161,077	0.0332085	8258
Semisubmersible	\$274,893	0.0059622	5634

From table 16 we can observe that the calculated parameters using the *O-U* process is close to the parameters we calculated using the *GMR* process. For jackup, the long-term equilibrium rate, α, is close to identical between the two approaches, \$161,077 vs. \$161,522. The mean reversion speed, k, is calculated to be lower when using the *O-U* process than the *GMR* process, 0.0332085 vs. 0.057577207.

Similar differences in the calculated parameters between the *O-U* and *GMR* process are observable for semisubmersibles. The long-term equilibrium rate, α, is rather similar between the two different approaches, \$274,893 vs. \$293,002, but higher when calculated using *GMR*. Further, the mean reversion speed, k, is higher for *GMR* than for *O-U*, 0.006867024 vs. 0.0059622.

The volatility, σ, in the *O-U* process is calculated in absolute terms rather than percent as in the *GMR* process. This makes it difficult to compare the parameters between the different approaches. However, the volatility measure from *O-U* only represents 5 and 2% of the long-term equilibrium rate in the *O-U* process. In the *GMR* process the volatility was calculated to 7.685 and 2.623% for jackups and semisubmersibles, respectively. This could indicate that the volatility is calculated to be lower with the *O-U* process.

Presented in table 17 and 18 are the option values for jackups and semisubmersibles calculated using both the *O-U* and *GMR* process.

Table 17 – Option Values for Jackup *O-U* vs. *GMR*

Scenarios	Option Value *O-U*	Option Value *GMR*	Difference
Soft Market	\$840,000	\$1,000,000	(\$160,000)
Neutral Market	\$360,000	\$310,000	\$50,000
Tight Market	(\$120,000)	(\$150,000)	\$30,000

Table 18 - Option Values for Semisubmersible *O-U* vs. *GMR*

Scenarios	Option Value *O-U*	Option Value *GMR*	Difference
Soft Market	\$830,000	\$810,000	\$20,000
Neutral Market	\$610,000	\$640,000	(\$30,000)
Tight Market	\$150,000	\$290,000	(\$140,000)

As we can observe from table 17 and 18, equal to the *GMR* process, the option value decreases as the market gets tighter also for the *O-U* process. Further, we can observe that the option values do in general not differ a lot between the different stochastic processes. However, in the scenario of a soft market for jackups and a tight market for semisubmersibles the difference between the option values is larger.

In the scenario of a soft market for jackups, the larger difference in option value is due to differences in the calculated mean reversion parameters. As the mean reversion speed is higher for *GMR*, rates will revert faster towards the higher mean. Thus, increasing option value.

In the scenario of a tight market for semisubmersibles, the larger difference in option value is due to differences in the calculated long-term equilibrium rate. As the long-term equilibrium rate is lower for *O-U*, rates will revert more drastically compared to for *GMR*. As evident from the calculated option values, the negative effect of a lower long-term equilibrium rate is larger than the positive effect of lower mean reversion, in a tight market.

Using the *O-U* approach we reject the null hypothesis of no autocorrelation for both jackups and semisubmersibles, compared to the *GMR* process where only the semisubmersible rates experienced autocorrelation. In addition, the goodness-of-fit measure, R-squared, was much lower for *O-U* than for *GMR*. These findings further underscore our decision to choose the *GMR* process over the *O-U* process.