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BY Chang Koo Chi & Kyoung Jin Choi

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Chang Koo Chi[1] and Kyoung Jin Choi[†2]

[1]Department of Economics, Norwegian School of Economics
[2]Department of Finance, University of Calgary Haskayne School of Business

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Abstract

This note explores how to evaluate an agent's performance in standard incentive contracts. We show that the MPS criterion proposed by Kim (1995) becomes a tight condition for one performance measurement system to be more informative than another, as long as the first-order approach can be justified. In the one-signal case obeying the monotone likelihood ratio property, the MPS criterion is equivalent to the way of ordering signals developed by Lehmann (1988), establishing a link to statistical decision theory. Our results demonstrate that depending on the agent's potential deviations, ideal performance measures can be different.

JEL Classification: D86.

Keywords: Agency problems, performance measurement, informativeness criterion, signal orderings.

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†Chi: chang-koo.chi@nhh.no; Choi: kjchoi@ucalgary.ca

1. Introduction

In a principal-agent problem, it is typically assumed that there exist contractible signals (e.g., output) conveying partial information about the agent's hidden action. To motivate the agent, the principal designs a compensation scheme based on those signals. This problem, how to pay, is one of central issues in incentive theory and has been extensively studied in a variety of environments. However, it is often costly to write a contract based on all of available signals, especially when the agent is risk-averse. In this case, the principal must choose which signals or performance measures are to be used in the contract. In other words, she has to consider how to evaluate the agent's performance. This second problem is often referred to as performance measurement in literature, and has been overlooked relative to the problem of compensation design. This paper addresses the nature of an ideal performance measure for improvement of contractual efficiency in the standard agency model.

The premise of agency theory is that it is impossible or considerably costly for the principal to perfectly monitor the agent's productive input, leading to the problem of moral hazard. Although the principal can use other informational variables to motivate the agent through an incentive scheme, there is a cost and benefit of inducing desired actions from the risk-averse agent. As a result, incentive contracts reflect the trade-off between risk and incentives, which drives a wedge between the first-best and second-best outcomes. Invoking this conventional wisdom, it is natural to think that an ideal performance measure must be the most informative signal about the agent's action, and the principal can mitigate the problem by writing contractual clauses based on that signal. For this reason, an agency problem is closely related to a statistical decision problem in ranking a set of signals. Early literature (e.g., Holmström (1979), Gjesdal (1982), and Grossman and Hart (1983)) has developed the theory of performance measurement in agency frameworks by applying Blackwell's theorem.

However, as Gjesdal (1982) has first pointed out, there is one subtle but important difference between the two problems: While a signal is used to *estimate* unknown parameters in the decision-making process, it is used to *control* the agent's hidden actions in the agency model. Gjesdal illustrated the difference with one example in which Blackwell's ranking is not valid if the agent's payoff function is not additively separable. Subsequently, Kim (1995) presented a novel approach to ordering signals in agency models based on the property that a more informative performance measure leads to greater variability of likelihood ratios. He then demonstrated that in comparison with the mean-preserving spread (MPS) criterion, the notion of sufficiency results in an excessively restrictive order, and thus the necessary part of Blackwell's theorem does not hold in agency models.

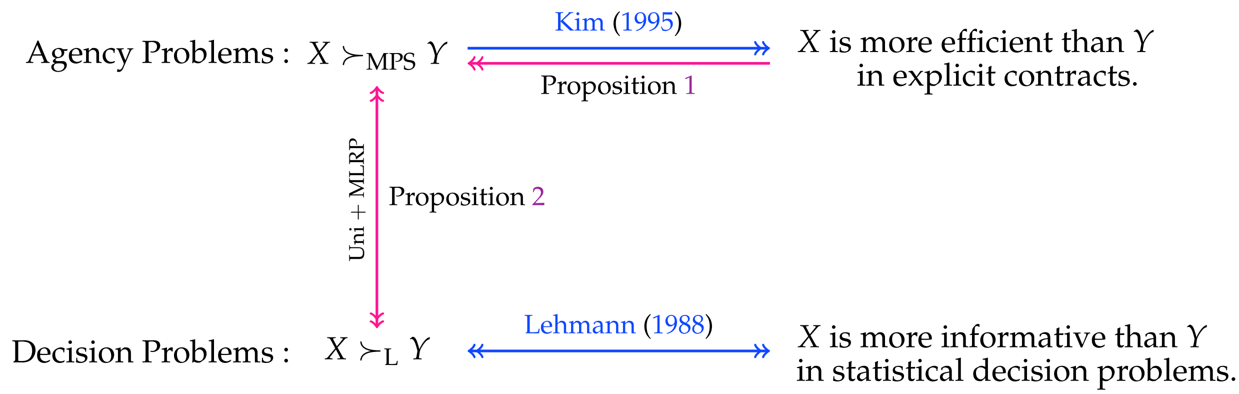


Figure 1: Summary of the Results - $X \succ_{\mathrm{MPS}} Y$ denotes the MPS-domination of system X over system Y, and $X \succ_{\mathrm{L}} Y$ denotes the domination of signal X over signal Y in Lehmann's order.

The previous studies tell us that an ideal performance measure in agency problems differs in kind from an informative signal in decision-making problems, but there are two shortcomings in their analysis. First, in contrast with Blackwell's theorem, the necessary part of the MPS criterion has not been established yet.[1] More precisely, it is unknown whether a better performance measure is fully characterized by the MPS criterion. Second, as Lehmann (1988) has demonstrated earlier, Blackwell's ranking based on the notion of sufficiency requires domination of one signal over another for *all* payoff functions, and thus it is too restrictive to provide a reasonable reference for comparison. It is then natural to ask if the ranking of measurement systems in agency problems can be implied by a more complete signal order than Blackwell's.

This paper makes up for these two shortcomings by complementing the previous results on performance measurement. We first show that provided the first-order approach (FOA) is valid, Kim's MPS criterion becomes a sufficient and necessary condition for one performance measurement system to be more efficient than another system in explicit contracts.[2] For this purpose, we in Section 2 identify one agent's characteristics with a concave utility function from monetary income and a nondecreasing cost function from effort, and then sort out the set of characteristics in which the FOA can be justified. Utilizing the tools of convex analysis, Proposition 1 in Section 3 shows that the MPS criterion

[1] Kim (1995) proved the only sufficient part: if a performance measurement system leads to more variable likelihood ratios than another system in the sense of the mean-preserving spread, then the principal prefers to evaluate the agent's performance based on the former system as long as the first-order approach is justified.

[2] That is, we seek for the informativeness criterion for contractible signals. A recent paper by Chi and Olsen (2018) develops a novel informativeness criterion for non-contractible signals in relational (implicit) contracts.

is necessary if the result holds for all agent's characteristics in this set. We then examine the relation between the MPS criterion and the signal ordering introduced by Lehmann (1988) in statistical decision theory. Proposition 2 in Section 4 demonstrates that the two criterions are equivalent when the ranking is determined for univariate signals satisfying the monotone likelihood ratio property.[3] Therefore, our results provide a full characterization of ideal performance measures used in explicit contracts and restore a link to the existing stochastic order.

The MPS criterion hinges upon the assumption that the FOA is valid, put differently, the set of incentive compatibility (IC) constraints can be replaced by one local IC condition. Under this approach, the agent is willing to deviate *locally* from the intended action, and other global deviations are unprofitable. In this case, the likelihood ratio contains all relevant information to the agent's possible deviations, and hence the informativeness criterion is based on this informational variable. In Section 5, we examine the case in which the FOA is not valid and find that neither the MPS criterion nor Lehmann's order can play a role as the informativeness criterion. This clarifies the structural difference between the agency model and the decision problems under uncertainty, and suggests that depending on the agent's potential deviations, the principal's choice of performance measures should be different.

2. The Agency Model

We consider the standard agency problem in which a risk-neutral principal (she) delegates a single task to a risk-averse agent (he), taking advantage of the agent's expertise. The principal first chooses a set of performance measures between $\mathbf{X} = (X_1, \cdots, X_n)$ and $\mathbf{Y} = (Y_1, \cdots, Y_m)$ with $n, m \geq 1$. Part or all of the measures in $\mathbf{X}$ can be overlapped with those in $\mathbf{Y}$, but all available measures are commonly observable and verifiable.[4] Subsequent to the choice of measures, the principal designs a contract that specifies the agent's wage on the basis of the observed performance, and offers the contract to the agent. If the agent accepts, he chooses effort a from a compact set $A \subset \Re$ for the delegated task by incurring a cost of $\psi(a)$. This move is unobservable to the principal. If the agent rejects the contract, he obtains a reservation payoff of $\underline{V}$.

The agent's effort a generates an expected profit of $B(a)$ for the principal and also influ-

[3]The same result can be found in Dewatripont, Jewitt and Tirole (1999), but the proof is omitted. Furthermore, to our best knowledge, their result relies upon the differentiability of measurement systems with respect to the agent's action. We provide a general proof on the equivalence result that can be applied to a model with discrete actions.

[4]If $\mathbf{Y}$ includes all measures in $\mathbf{X}$, then the principal's choice problem is identical with the one in Holmström (1979).

ences his performance as follows: If measure $\mathbf{X}$ was chosen at the outset, his realized performance $\mathbf{x} = (x_1, \cdots, x_n)$ is drawn from cumulative distribution function (CDF) $F(\mathbf{x}|a) \equiv \Pr(\mathbf{X} \leq \mathbf{x}|a)$; similarly, if $\mathbf{Y}$ was chosen, his performance $\mathbf{y} = (y_1, \cdots, y_m)$ is drawn from $G(\mathbf{y}|a)$. Throughout the paper, we refer to the set of possible distributions as a performance measurement system.[5] Regarding performance measures, we use a capital letter to denote a random variable and a small letter to denote its realization. Also, a bold letter indicates a vector and a normal letter indicates a scalar. Lastly, we denote by $f(\cdot|a)$ and $g(\cdot|a)$ the probability density function.

We make the following assumptions on measurement systems:

(A1) Both F and G are non-degenerate distributions, and the support of each system, denoted $\mathbb{X} \subset \Re^n$ and $\mathbb{Y} \subset \Re^m$, is independent of the agent's choice of effort.

(A2) In a model with continuous effort $A = [\underline{a}, \overline{a}]$, both $f(\cdot|a)$ and $g(\cdot|a)$ are differentiable with respect to a.

(A3) Each system satisfies the monotone likelihood ratio property (MLRP).

The MLRP reflects stochastic complementarity between *each* performance measure and the agent's effort, in the sense that as the agent exerts more effort, each x_i is more likely to take high values.[6] In a model with discrete effort $A = \{a_1, \cdots, a_K\}$ with order $a_k < a_{k+1}$, the property is satisfied if $1 - [f(\mathbf{x}|a_{k-1})/f(\mathbf{x}|a_k)]$ is increasing in $\mathbf{x}$ for all $k = 2, \cdots, K$. In a model with continuous effort satisfying (A2), Milgrom (1981) has shown that system F satisfies the MLRP if and only if $f_a(\mathbf{x}|a)/f(\mathbf{x}|a)$ is increasing in $\mathbf{x}$ for all a where f_a denotes the partial derivative with respect to a.

Like the agent's action, some performance measures may not be continuous. Nevertheless, we can assume without any loss of generality that every X_i and Y_j is a continuous random variable. This is due to Lehmann (1988): Although some measure X_i takes a finite value, we can construct a continuous random variable which is information-equivalent to the measure. The idea is simple. If $X_i \in \{x_L, x_H\}$ is a binary random variable that takes a low value x_L with probability $p(a)$ conditional on effort a, we can define a new random variable X_i^* on the unit interval $[0,1]$ that is uniformly distributed on $[0, p(a)]$ when $X_i = x_L$ and on $[p(a), 1]$ when $X_i = x_H$. Then both X_i^* and X_i contain the same information about a, and thus they are information-equivalent measures.

Given a realized performance $\mathbf{x} \in \mathbb{X}$ and a performance-based pay $w : \mathbb{X} \to [\underline{w}, \infty)$, the agent obtains a payoff of $u(w(\mathbf{x})) - \psi(a)$ after the contract is executed. The first

[5]The system is often referred to as an information system or an information structure in contract theory, and also referred to as an experiment or a signal in statistical decision theory.

[6]The MLRP guarantees that optimal incentive schemes are monotone with the agent's performance, provided that the first-order approach is valid.

term $u(w)$ is an increasing and concave function that represents utility from monetary income, while the second term ψ is a nondecreasing function that represents the cost from effort. Following literature the agent's payoff function is assumed additively separable, which separates his preferences on risky income from his choice of effort.[7] For each measurement system $H = F, G$ and an incentive scheme w based on the system, the agent's expected payoff is $V_H(w,a) = \mathbb{E}[u(w)] - \psi(a)$ and the principal's expected payoff is $B(a) - \mathbb{E}[w]$.

Suppose that the principal decides to adopt measurement system F. Since the principal is assumed risk-neutral, we follow Grossman and Hart (1983) to subdivide the optimal contract problem into two parts: For each effort $a \in A$, the principal first computes the minimum expected cost necessary for implementation of a, and then carries out a cost-benefit analysis to decide the optimal effort. The first part of this procedure is represented by the following problem:

$$\min_{w \geq \underline{w}} \mathbb{E}[w(\mathbf{X})] \equiv \int_{\mathbb{X}} w(\mathbf{x}) f(\mathbf{x}|a) d\mathbf{x} \tag{$\star$}$$

subject to

$$V_F(w,a) \geq \underline{V} \tag{IR}$$
$$V_F(w,a) \geq V_F(w,a') \quad \forall\, a' \in A, \tag{IC}$$

where (IR) is the individual rationality condition under which the agent would accept the offered contract and (IC) are the incentive compatibility conditions under which the agent would make the desired effort.

Let $w_F^a : \mathbb{X} \to \Re_+$ be a solution to the problem ($\star$), that is, an incentive scheme that implements effort a at the least cost under system F. Also, define as $C_F(a) \equiv \mathbb{E}[w_F^a(\mathbf{X})]$ the corresponding value function. $C_F(a)$ can be interpreted as the minimum expected pay to the agent for implementation of effort a. For the other system G, we define w_G^a and $C_G(a)$ in the same manner. For a benchmark, let $\underline{C}(a)$ denote the least cost under perfect information. Classic incentive theory tells us that with moral hazard and risk aversion, the agency cost arises from the trade-off of risk and incentives, and thus under each system $H = F, G$, we have $C_H(a) \geq \underline{C}(a)$ for all a.

In this notation, we say that F is a more efficient performance measurement system than G if $C_F(a) \leq C_G(a)$ for every $a \in A$. To put it into words, a more efficient system enables the principal to reduce the agency cost by writing a contract based on that system, and

[7] This assumption is essential for our subsequent analysis. The additively separable utility function allows us to focus on a deterministic incentive contract. Without this assumption, a randomized incentive scheme may be Pareto-efficient for the parties as is shown by Gjesdal (1982).

thus she can control the agent's hidden action more effectively. Consequently, if F is more efficient than G, the principal can induce higher effort under F because

$$\underset{a\in A}{\operatorname{argmax}}\ B(a) - C_F(a) \ \geq\ \underset{a\in A}{\operatorname{argmax}}\ B(a) - C_G(a)$$

follows from the monotonicity theorem in Milgrom and Shannon (1994). Along with the fact that the constraint (IR) is binding at the optimum under both systems, this in turn ensures existence of a feasible contract $w_F(\mathbf{x})$ under F which Pareto dominates the optimal contract $w^*_G(\mathbf{y})$ under G.

Our discussion suggests that the only first part of the optimal contract problem is relevant to comparison of measurement systems. The standard way to solve the problem ($\star$) is to replace the set of IC constraints with a local stationary condition that prevents the agent's local deviation from effort a,

$$\frac{\partial}{\partial a} V_F(w,a) \ =\ 0, \quad \text{or} \int_{\mathbb{X}} u(w(\mathbf{x})) f_a(\mathbf{x}|a) d\mathbf{x} \ =\ \psi'(a), \tag{L-IC}$$

and then check if the obtained contract from the relaxed problem indeed satisfies the global constraint.[8] Previous studies have proposed a various set of conditions under which this first-order approach (FOA) can be justified given a measurement system.[9] Invoking the existing conditions, we focus on the case where the constraint (L-IC) is only relevant. The other case will be discussed in Section 5.

To state our problem formally, we first let $\theta \equiv (u,\psi)$ and define

$$\Theta \ =\ \Big\{\theta = (u,\psi) \,\Big|\, u \text{ is nondecreasing concave, } \psi \text{ is nondecreasing}\Big\}.$$

Observe that each element of the set Θ summarizes one agent's payoff-relevant characteristics in the agency problem. In addition, define $\Theta^* \subset \Theta$ as the collection of θ such that the first-order approach can be justified under the two measurement systems. The boundary of the set Θ^* depends on the statistical properties of F and G. For example, if both systems satisfy (A3) and the concave increasing set probability (CISP) condition introduced by Conlon (2009), requiring that $\Pr(\mathbf{X} \in E|a)$ is concave in a for every increasing

[8]In a model with discrete effort, the relevant condition to implementation of a_k is the local downward constraint $V_F(w,a_k) = V_F(w,a_{k-1})$.

[9]The related literature takes two different approaches to identifying conditions under which the agent's problem is globally concave in his choice variable. In the one-signal case, Rogerson (1985) developed the so-called convexity of the distribution function condition (CDFC) on F to show that the FOA is justified if F satisfies the CDFC and (A3). On the other hand, Jewitt (1988) relaxed the CDFC but instead imposed a condition on the agent's payoff function, to provide another set of conditions for the FOA. Kirkegaard (2017) found a link between these two approaches in terms of stochastic orders, and proposed a set of conditions that can be applied to multi-tasking models.

set $E \subset \Re^n$, then the FOA is valid for all $\theta \in \Theta$ with a convex function ψ.[10]

Within the set Θ^*, the optimal contract under system F must satisfy the following first-order condition:

$$\frac{1}{u'(w_F^a(\mathbf{x}))} = \lambda_F + \mu_F \cdot L_F^a(\mathbf{x}), \tag{1}$$

where λ_F and μ_F is the Lagrange multiplier for the (IR) and (L-IC) constraint, respectively, and $L_F^a \equiv \frac{\partial}{\partial a}(\log f(\mathbf{x}|a))$ indicates the likelihood ratio. In the agency model with a risk-neutral principal, the two constraints are binding (Jewitt (1988)) so that the two multipliers must be strictly positive by the complementary slackness condition. Consequently, it is immediate from (1) and (A3) that the optimal contract is an increasing function for all $\theta \in \Theta^*$.

Before turning into the next section, it is worthwhile to remark that the principal's objective $B(a)$ is independent of performance measurement systems. Whether the principal writes a contract with variables $\mathbf{X}$ or $\mathbf{Y}$, her expected profit is determined by the agent's productive inputs. Hence the principal's choice of performance measures only indirectly affects her objective through the agent's effort. This can be easily justified in two environments: (i) the realized profit is not observable to the parties at the stage of payment or hard to verify in the same spirit of Baker (1992), and thus it cannot be used in a contract, or (ii) the profit is a key performance indicator and thus is contained in both systems, but F and G have the same marginal distribution on the profit. By abstracting away the direct effect, the rest of the paper is devoted to the problem of comparing performance measurement systems in terms of the incentive effect.

3. The MPS Criterion

In this section, we provide a condition that characterizes a more efficient performance measurement system. As mentioned in the Introduction, Kim (1995) has developed the mean-preserving spread (MPS) criterion that provides a sufficient condition for efficiency of a measurement system under the assumption that the FOA is valid. We first show that the MPS condition is not just sufficient but also necessary if one system is more efficient than another for all agents in the set Θ^*.

From (1), it is natural to think that the condition is related to the likelihood ratio of the two systems. The MPS criterion states that F becomes a more efficient system within the class Θ^* if the distribution of likelihood ratio L_F^a under F dominates the distribution of

[10] In the multi-signal case, a recent paper by Jung and Kim (2015) developed one condition on the distribution of likelihood ratios for justifying the FOA, which requires convexity of the distribution in line with the CDFC, generalizing the CISP condition. Although their condition dispenses with the need for the MLRP, we maintain (A3). The role of (A3) in the current paper will be clear in the next sections.

L_G^a under G for all $a \in A$ in the sense of the second stochastic dominance. To state this MPS condition in a simple fashion, we adopt some stochastic orders commonly used in statistics for comparing the variability of random variables (refer to Shaked and Shanthikumar (2007)). To illustrate, suppose that X and Y are univariate random variables satisfying $\mathbb{E}[\sigma(X)] \geq \mathbb{E}[\sigma(Y)]$ for all convex functions $\sigma : \Re \to \Re$. Then it is said that X dominates Y in the *convex* order, and it is written as $X \geq_{\text{cx}} Y$. If, in addition, the two random variables have the same mean, then $X \geq_{\text{cx}} Y$ is equivalent to $X \leq_{\text{icv}} Y$, where $\geq_{\text{icv}}$ indicate the *increasing concave* order. The stochastic order $\leq_{\text{icv}}$ is then equivalent to the second-order stochastic dominance of X over Y (Rothschild and Stiglitz (1970)). Since the likelihood ratio is a univariate information variable with zero mean for all measurement systems, we can rewrite the MPS condition for efficiency as $L_F^a(\mathbf{X}) \geq_{\text{cx}} L_G^a(\mathbf{Y})$ in the convex order, or equivalently $L_F^a(\mathbf{X}) \leq_{\text{icv}} L_G^a(\mathbf{Y})$.

With this, the first result of the paper can be stated as follows:

Proposition 1 (MPS Criterion)**.** *Performance measurement system F is more efficient than system G within the class Θ^* if and only if the MPS condition is satisfied.*

PROOF OF PROPOSITION 1: We first establish the sufficient part.[11] Suppose that $L_F^a(\mathbf{X}) \geq_{\text{cx}} L_G^a(\mathbf{Y})$ for all a. Dropping the superscript a for notational simplicity, we define a scalar variable $q_H \equiv \lambda_H + \mu_H \cdot L_H$ for each measurement system $H = F, G$. Observe that the defined variable is the expression on the right-hand side of the first-order condition (1). Hence we can write the optimal contract $w(q_H)$ as a function of q_H, and similarly write (1) as $u'(w(q_H))q_H = 1$.

Define function $m : \Re \to \Re$ as

$$m(q_H) \;\equiv\; u(w(q_H))q_H - w(q_H).$$

Taking the derivative of m with respect to q_H two times in a row, we can compute the second derivative as $m''(q_H) = u'(w(q_H))w'(q_H)$.[12] Note that the sign of m'' is nonnegative for all q_H as both $u' \geq 0$ and $w' \geq 0$, which implies that the function m is globally convex in the likelihood ratio L_H (recall that $\mu_H > 0$ for each system H, so q_H is a positive affine transformation of L_H). Since $L_F \geq_{\text{cx}} L_G$ implies $\lambda_G + \mu_G \cdot L_F \geq_{\text{cx}} \lambda_G + \mu_G \cdot L_G$ and the

[11] Kim (1995) proved this part, but we provide a more succinct proof for completeness of the paper. Our proof utilizes the fact that the cost minimization problem ($\star$) is convex in the likelihood ratio.

[12] To see how to obtain the expression of m'', note that its first derivative reduces to

$$m'(q_H) \;=\; u'(w(q_H))w'(q_H)q_H + u(w(q_H)) - w'(q_H) \;=\; u(w(q_H)), \tag{2}$$

where the second equality follows from the first-order condition (1).

function m is convex, we have

$$\mathbb{E}[m(\widehat{q})] \;\geq\; \mathbb{E}[m(q_G)] \;\text{ where }\; \widehat{q} \equiv \lambda_G + \mu_G \cdot L_F. \tag{3}$$

The right-hand side of (3) can be written as

$$\begin{aligned}
\mathbb{E}[m(q_G)] &= \int_{\mathbb{Y}} \Big[u(w(q_G))q_G - w(q_G)\Big] g(\mathbf{y}|a)d\mathbf{y} \\
&= -\int_{\mathbb{Y}} w(q_G)g(\mathbf{y}|a)d\mathbf{y} + \lambda_G \int_{\mathbb{Y}} u(w(q_G))g(\mathbf{y}|a)d\mathbf{y} \\
&\qquad\qquad\qquad\qquad + \mu_G \int_{\mathbb{Y}} u(w(q_G))L_G(\mathbf{y})g(\mathbf{y}|a)d\mathbf{y} \\
&= \mathcal{L}(w_G, \lambda_G, \mu_G) + \lambda_G \psi(a) + \mu_G \psi'(a),
\end{aligned}$$

where $\mathcal{L}(w_G, \lambda_G, \mu_G)$ stands for the Lagrangian associated with the problem ($\star$) under system G evaluated at the optimum. The first two equalities are immediate from the definition of m and q_G, and the last equality follows from the fact that (IR) and (L-IC) are binding at the optimum. Since $\mathcal{L}(w_G, \lambda_G, \mu_G) = -C_G(a)$ by the Kuhn-Tucker theorem, we have $\mathbb{E}[m(q_G)] = -C_G(a) + \lambda_G \psi(a) + \mu_G \psi'(a)$. Substituting this obtained expression into (3) gives us one upper bound for $-C_G(a)$:

$$\underbrace{\mathbb{E}^F[m(\widehat{q})] - \lambda_G \psi(a) - \mu_G \psi'(a)}_{=\mathcal{L}(w(\widehat{q}), \lambda_G, \mu_G)} \geq -C_G(a).$$

Note that the value function under F, $\mathcal{L}(w_F, \lambda_F, \mu_F) = -C^F(a)$, should be larger than the expression on the left-hand side. This establishes sufficiency of the MPS criterion.

To prove the converse, we demonstrate that if the two likelihood ratios are not ranked by the MPS criterion, there exists an agent with $(u, \psi) \in \Theta^*$ for whom system F gives rise to a higher agency cost than system G. This proves that the condition $L_F^a(\mathbf{X}) \geq_{\text{cx}} L_G^a(\mathbf{Y})$ is necessary for F to be more efficient than G for all agents in Θ^*.

To start, suppose to the contrary that the MPS condition does not hold, or equivalently, L_F does not dominate L_G according to the second-order stochastic dominance. This implies that there exists an increasing convex function $\phi : \Re \to \Re$ such that for every constant $\lambda, \mu > 0$, we have

$$\mathbb{E}\left[\phi\Big(\lambda + \mu L_G\Big)\right] \;>\; \mathbb{E}\left[\phi\Big(\lambda + \mu L_F\Big)\right]. \tag{4}$$

Let $u = \phi^*$ denote the Legendre-Fenchel transformation of ϕ (see Luenberger (1969)).

Then

$$u(p) \; = \; \inf_{q} \left\{ \frac{\phi(q)}{q} + \frac{p}{q} \right\} \tag{5}$$

and its corresponding dual function $u^* = \phi^{**} = \phi$ is defined by

$$\phi(q) \; = \; \sup_{p} \left\{ u(p)q - p \right\}. \tag{6}$$

Observe that given the increasing concave function u defined in (5), there exists an increasing function ψ such that (u, ψ) represents one agent's characteristics and validates the FOA under both performance measurement systems. That is, $(u, \psi) \in \Theta^*$. We now show that system F is less efficient than G in contracting with the agent having (u, ψ).

Let $w(q)$ be a solution to problem (6), that is, $w(q)$ satisfies $u'(w(q))q = 1$. In light of the first-order condition (1), this implies that when the parameter q is equal to $q_F \equiv \lambda_F + \mu_F \cdot L_F$, the corresponding solution $w(q_F)$ constitutes the optimal contract w_F under system F. This allows us to write the minimum value of the objective in problem ($\star$) as

$$\begin{aligned} C_F(a) \; &= \; \int_{\mathbb{X}} w(q_F)\, f(\mathbf{x}|a) d\mathbf{x} \\ &\geq \; \int_{\mathbb{X}} u(w(q_F)) \Big(\lambda_G + \mu_G \cdot L_F(\mathbf{x}) \Big) f(\mathbf{x}|a) d\mathbf{x} \\ &\qquad\qquad - \int_{\mathbb{X}} \phi \Big(\lambda_G + \mu_G \cdot L_F(\mathbf{x}) \Big) f(\mathbf{x}|a) d\mathbf{x}, \end{aligned} \tag{7}$$

where the inequality follows from the function u defined in (5):

$$u(p)q - \phi(q) \; \leq \; p \quad \forall\, q \quad \Rightarrow \quad u(w(q_F))q - \phi(q) \; \leq \; w(q_F) \quad \forall\, q.$$

Since (IR) and (L-IC) are binding at the optimum under both systems, we can rewrite the first term of (7) as

$$\begin{aligned} \int_{\mathbb{X}} u(w(q_F)) \Big(\lambda_G + \mu_G \cdot L_F(\mathbf{x}) \Big) f(\mathbf{x}|a) d\mathbf{x} \; &= \; \lambda_G \Big(\psi(a) + \overline{U} \Big) + \mu_G \psi'(a) \\ &= \; \int_{\mathbb{Y}} u(w(q_G)) \Big(\lambda_G + \mu_G \cdot L_G(\mathbf{x}) \Big) g(\mathbf{y}|a) d\mathbf{y} \\ &= \; \int_{\mathbb{Y}} u(w(q_G)) q_G\, g(\mathbf{y}|a) d\mathbf{y}, \end{aligned}$$

where $q_G \equiv \lambda_G + \mu_G \cdot L_G$. For the same reason as above, we have $w(q) = w_G(q)$ at $q = q_G$.

Substituting the last expression into the inequality (7) and using (4) leads us to

$$C_F(a) \; > \; \int_{\mathbb{Y}} \Big\{ u(w(q_G)) q_G - \phi(q_G) \Big\} g(\mathbf{y}|a) d\mathbf{y}$$

Observe that by the dual expression of ϕ in (6), we have $\phi(q_G) = u(p)q_G - p$ at $p = w(q_G)$. Consequently, the curly-bracketed expression on the right-hand side is simply $w(q_G)$, the optimal contract under system G, leading to the desired contradiction:

$$C_F(a) \; > \; C_G(a).$$

This establishes necessity of the MPS criterion for a more efficient information system. In a model with discrete effort, the same proof can be applied with the likelihood ratio $L_F^a(\mathbf{x}) = 1 - (f(\mathbf{x}|a_{k-1})/f(\mathbf{x}|a_k))$ and the local binding IC constraint $V_F(w, a_k) = V_F(w, a_{k-1})$, to establish the equivalence. The proof is now complete. □

To understand the key idea of the MPS criterion, recall that whenever the FOA is valid, the local IC constraint (L-IC) is only relevant so that the agent has no incentive to deviate *locally* from the intended effort. In this case, it is the likelihood ratio that captures the potential local deviation, and thus the optimal contract hinges on this informational variable rather than on the density function itself. If the likelihood ratio is more variable under one measurement system in response to the agent's deviation than under another, the system conveys more accurate information about the hidden action. Therefore, the principal can implement the desired effort at less cost.

4. Lehmann's Order and Equivalence

The source of the moral hazard problem in agency models is the principal's inability of perfectly monitoring the agent's behavior. As a result, the principal faces a trade-off between motivating the agent and sharing risks with him when designing an incentive contract. From this conventional wisdom, it is natural to think that with a more *precise* signal or performance measure about the agent's action, the principal can alleviate the problem of moral hazard and thereby reduce the agency cost. In this aspect, the literature has developed informativeness criterions on the basis of statistical decision theory. However, as Gjesdal (1982) has first pointed out, there is a subtle difference: While a signal is used to estimate the unknown parameter in decision theory, it is used to control the hidden action in incentive theory. The classic example in his paper illustrates this difference by showing that the notion of sufficiency (Blackwell (1951, 1953)) is not suited with agency models when the agent's payoff function is not additively separable. Even with separable payoff functions, Kim (1995) demonstrated the difference by comparing the MPS condition with Blackwell's ranking. However, the notion of sufficiency is powerful in the aspect that it can be applied to ordering signals in every decision problem, but is quite restrictive in that only a small subset of signals can be ordered in terms of sufficiency.

In this section, we compare the MPS condition with a more complete order and thereby clarify the difference of ordering signals. For this purpose, we adopt the following notion of precision developed by Lehmann (1988) which is a tight condition of ordering *univariate* signals satisfying the MLRP in decision-making problems.

Definition 1 (Lehmann (1988))**.** *Let $X \in \mathbb{X}$ and $Y \in \mathbb{Y}$ be two unidimensional signals satisfying the* MLRP*. Then X is more precise about unknown parameter $a \in A$ than signal Y if for each outcome $y \in \mathbb{Y}$, there exists an increasing function $T_y : A \to \mathbb{X}$ such that*

$$F\left(T_y(a)|a\right) \;=\; G(y|a). \tag{P}$$

If (P) *holds between the two signals, we write $X \succ_{\mathrm{L}} Y$.*

As the two signals are continuous random variables, there exists such a function T_y in (P) that it has the identical distribution with signal Y conditional on a. It is therefore its monotone property that is essential for ranking the two signals. To better understand the role of the T-transformation, consider a binary action space: $A = \{a_1, a_2\}$ with $a_1 < a_2$ and select one outcome y from the support $\mathbb{Y}$. Let $p_1 = G(y|a_1)$ and $p_2 = G(y|a_2)$. Given the quantile $p_1 \in [0, 1]$, there is an outcome $x_1 \in \mathbb{X}$ at which the distribution function $F(\cdot|a_1)$ takes the value p_1, and we put $x_1 = T_y(a_1)$. Similarly, to the other quantile p_2, we can find the corresponding outcome $x_2 = T_y(a_2)$ at which $F(x_2|a_2) = p_2$. Observe that the monotone property of T_y implies $x_1 \leq x_2$ which in turn implies

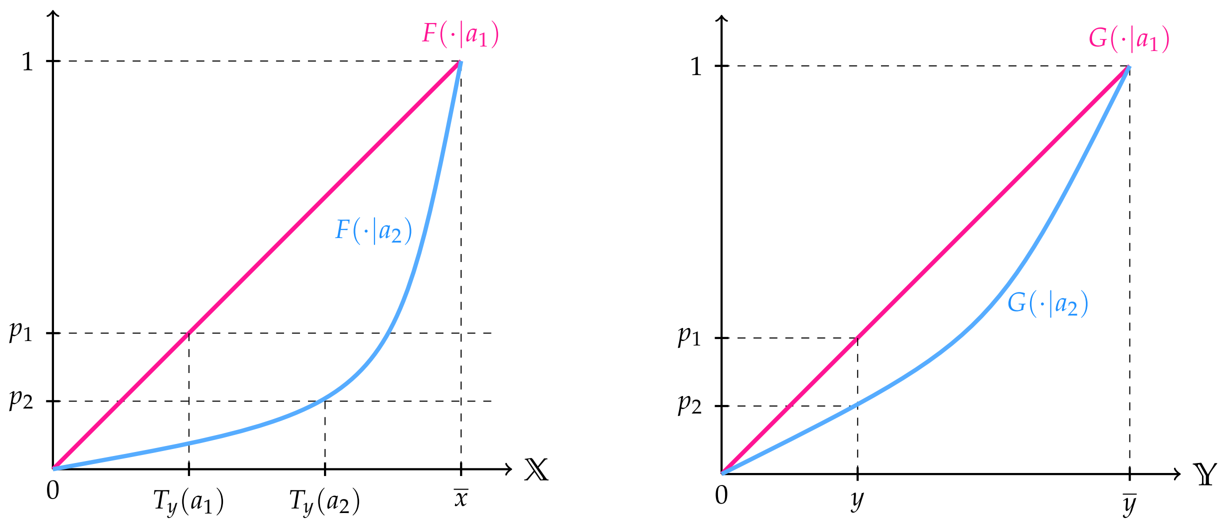
$$F(x_1|a_1) \;=\; G(y|a_1) \quad \text{and} \quad F(x_1|a_2) \;\leq\; G(y|a_2).$$

To put into words, the distribution F assigns more densities to high outcomes than G when the agent chooses high effort, as is displayed in Figure 2. Consequently, the monotone T-transformation in (P) leads to X being more statistically precise than Y.

With this notion of ordering signals, Lehmann (1988) has shown that signal X is more informative than signal Y in statistical decision problems if and only if $X \succ_{\mathrm{L}} Y$. While Blackwell's order based on sufficiency requires domination of one signal over another for all decision problems, Lehmann's order on precision requires its dominance for a subclass of decision problems (which include all important inference problems in statistics). As a result, the notion of precision not just provides a more complete signal ranking, but is easier to check compared with sufficiency.

The next lemma provides a simple characterization for Lehmann's order.

Lemma 1. *Signal X is more precise than signal Y in the sense of Lehmann if and only if for every pair $x \in \mathbb{X}$ and $y \in \mathbb{Y}$, $G(y|a) - F(x|a)$ satisfies the single-crossing property in the parameter a.*



(a) Signal X (b) Signal Y

Figure 2: The role of T-transformation in Lehmann's order

PROOF OF LEMMA 1 : Suppose $X \succ_L Y$ and thus there exists an increasing transformation $T_Y(a)$ such that $Y \stackrel{d}{=} T_Y(a)$ for all a. Choose a pair (x, y) and then assume $G(y|a_0) - F(x|a_0) \geq 0$ for some a_0. Since $F(\cdot|a)$ is nondecreasing in a by the MLRP, and since $G(y|a_0) = F(T_y(a_0)|a_0)$, it must be the case that $x \leq T_y(a_0)$. Therefore, for $a \geq a_0$,

$$G(y|a) - F(x|a) \;\geq\; G(y|a) - F(T_y(a_0)|a) \;\geq\; G(y|a) - F(T_y(a)|a) \;=\; 0,$$

where the last inequality results from $T_y(a) \geq T_y(a_0)$. This establishes that the monotone property of T_y implies the desired single-crossing property.

To prove the converse, construct a transformation $T_y : A \to \mathbb{X}$ for each outcome y such that $G(y|a) = F(T_y(a)|a)$ holds. Continuity of the two distributions ensures the existence of such T_y. We need to show that the constructed T_y increases with a if the single-crossing property in Lemma 1 is satisfied. Given a pair of x and y, suppose that $G(y|a_0) - F(x|a_0) = 0$ for some a_0. Then $T_y(a_0) = x$ follows by construction. Due to the single crossing property, we have for every $a \geq a_0$

$$G(y|a) - F(x|a) \;=\; G(y|a) - F(T_y(a_0)|a) \;\geq\; 0.$$

Since $G(y|a) - F(T_y(a)|a) = 0$ and $F(\cdot|a)$ is a nondecreasing function, the desired property $T_y(a) \geq T_y(a_0)$ follows from the inequality above. $\square$

With the notion of precision in hand, the second result can be stated as follows:

Proposition 2. *Let X and Y be univariate performance measures for the agent satisfying* (A1) $\sim$ (A3). *Then $X \succ_L Y$ if and only if the MPS condition is satisfied.*

PROOF OF PROPOSITION 2 : Suppose $X \succ_L Y$. Writing $G(y|a) = F(T_y(a)|a)$ in (P) in integral forms, we have

$$\int_{t \leq y} g(t|a)dt \; = \; \int_{t \leq T_y(a)} f(t|a)dt.$$

Using the Leibniz integral rule, we take the derivative of both sides with respect to a to obtain

$$\int_{t \leq y} g_a(t|a)dt \; = \; f\left(T_y(a)|a\right) \cdot \frac{\partial}{\partial a} T_y(a) + \int_{t \leq T_y(a)} f_a(t|a)dt. \tag{8}$$

Since the first term on the right-hand side of (8) is nonnegative due to the monotonicity of T_y, we have

$$\int_{t \leq y} L_G^a(t) g(t|a)dt \; \geq \; \int_{t \leq T_y(a)} L_F^a(t) f(t|a)dt,$$

where we used the fact that $g_a(y|a) = L_G^a(y)g(y|a)$. Let $c \equiv L_G^a(y)$ and add $c[1 - G(y|a)] = c[1 - F(T_y(a)|a)]$ to each side of the last inequality. Then we integrate the left-hand side by parts, to obtain

$$\mathbb{E}\left[\min\{L_G^a(Y), c\}\right] \; \geq \; c[1 - F(T_y(a)|a)] + \int_{t \leq T_y(a)} L_F^a(t) f(t|a)dt,$$

Since the likelihood ratio function L_F^a is increasing due to the MLRP, for all $x \in \mathbb{X}$, we have[13]

$$c[1 - F(x|a)] + \int_{t \leq x} L_F^a(t) f(t|a)dt \; \geq \; \mathbb{E}\left[\min\{L_F^a(X), c\}\right]. \tag{9}$$

Putting the last two inequalities together leads to $\mathbb{E}\left[\min\{L_G^a(Y), c\}\right] \geq \mathbb{E}\left[\min\{L_F^a(Y), c\}\right]$. Since every concave function lies in the closed convex hull of the set $\{\min\{x, c\} | c \in \Re\}$ up to constants, we have $L_G^a \geq_{\text{cv}} L_F^a$. Both the likelihood ratios have the same mean, the desired result $L_F^a \geq_{\text{cx}} L_G^a$ follows.

We prove the converse by contradiction. To this end, suppose that $L_F^a \leq_{\text{cv}} L_G^a$ for all a but X is not greater than Y in Lehmann's order. Then it follows from Lemma 1 that for some pair (x, y), there exists an $\epsilon > 0$ such that for all $a^* \in (a, a + \epsilon)$, $G(y|a) = F(x|a)$ but

[13] To see how the inequality in (9) is derived, define the expression on its left-hand side as a function of x:

$$\phi(x) \; \equiv \; c\left[1 - F(x|a)\right] + \int_{t \leq x} L_F^a(t) f(t|a)dt.$$

Observe that the defined function ϕ has the derivative $\phi'(x) = f(x|a)(L_F^a(x) - c_y)$, which changes its sign once from negative to positive. Hence ϕ attains its minimum at x satisfying $\phi'(x) = 0$, or equivalently $L_F^a(x) = c$, and the corresponding extreme value is $\mathbb{E}[\min\{L_F^a(X), c\}]$.

$G(y|a^*) < F(x|a^*)$. Note that

$$G(y|a^*) - G(y|a) \;=\; \int_{t \le y} \left(\frac{g(t|a^*)}{g(t|a)} - 1 \right) g(t|a)dt. \tag{10}$$

Let $c_x = f(x|a^*)/f(x|a) - 1$. By the assumptions we made above, it follows that

$$c_x \left[1 - G(y|a)\right] + G(y|a^*) - G(y|a) \;<\; c_x \left[1 - F(x|a)\right] + F(x|a^*) - F(x|a).$$

Using (10), we can rewrite the last inequality into

$$\begin{aligned} c_x \left[1 - G(y|a)\right] &+ \int_{t \le y} \left(\frac{g(t|a^*)}{g(t|a)} - 1 \right) g(t|a)dt \\ &< \; c_x \left[1 - F(x|a)\right] + \int_{t \le x} \left(\frac{f(t|a^*)}{f(t|a)} - 1 \right) f(t|a)dt. \end{aligned} \tag{11}$$

Divide both sides by ϵ and take $\epsilon \to 0$. Then it follows by the Lebesgue Dominated Convergence Theorem that the right-hand side converges to $\mathbb{E}[\min\{L_F^a(X), c_x\}]$. Moreover, the left-hand side is greater than $\mathbb{E}[\min\{L_G^a(Y), c_x\}]$ for the same reason as (9): See footnote 13. Hence the inequality (11) results in $\mathbb{E}[\min\{L_G^a(Y), c_x\}] < \mathbb{E}[\min\{L_F^a(X), c_x\}]$, which is a contradiction with $L_G^a \geq_{\text{cv}} L_F^a$.

In case of discrete effort, the same proof can be used to establish the equivalence between the MPS condition and Lehmann's order. The proof is now complete. □

Proposition 2 generalizes the characterization results of the MPS condition in Kim (1995), which show that if signal X is sufficient for signal Y, then the MPS condition is met; but the converse is not true. This implies that the MPS condition is a more complete order than the notion of sufficiency, and Kim demonstrated with one counterexample the structural difference between agency problems and statistical decision problems. His example concerns the comparison of the following two unidimensional measurement systems:

$$F \;=\; \begin{bmatrix} 1/2 & 1/3 & 1/5 \\ 1/3 & 1/3 & 1/3 \\ 1/6 & 1/3 & 7/15 \end{bmatrix} \qquad \text{and} \qquad G \;=\; \begin{bmatrix} 5/12 & 1/3 & 1/4 \\ 1/3 & 1/3 & 1/3 \\ 1/4 & 1/3 & 5/12 \end{bmatrix},$$

where for each system, the ij-th element indicates the probability of the agent's performance being $i = L, M, H$ conditional on his choice of effort $a = a_j$ with $j = L, M, H$. It can be shown that the two systems can be ranked by the MPS condition, but not by Blackwell's sufficiency (see Proposition 3 in Kim (1995) for the formal proof). However, as Lehmann (1988) has pointed out, Blackwell's order is too restrictive to hold in many situations in which one signal is intuitively more informative than another. Lemma 1 can be utilized to show that the system F is more precise than G in the example above.

By comparing with Lehmann's order, Proposition 2 recovers the link between the ways of ordering signals in agency models and in decision theory. Proposition 1 and 2 suggest that Lehmann's order is applicable to agency models, in so far as the principal chooses between two univariate performance measures satisfying (A1) $\sim$ (A3) and the FOA can be justified. Compared with the MPS condition, there is no need to compute the likelihood ratio and its distribution for Lehmann's order. Therefore, our results provide a simple and convenient tool for performance measurement.

5. The First-Order Approach and Value of Information

In the previous sections, we proved that the following statements are equivalent under the assumptions (A1) $\sim$ (A3):

- (**Efficiency**) Performance measurement system $\mathbf{X} \sim F$ is more efficient than system $\mathbf{Y} \sim G$ in contracting with a risk-averse agent in the set Θ^*.
- (**MPS condition**) The distribution of the likelihood ratio L_F^a is a mean-preserving spread of the distribution of L_G^a for all $a \in A$.
- (**Lehmann's order**) In the one-signal case, system F is more precise than system G in the sense of Lehmann.

In this section, we investigate the environment where the FOA is not valid, and show with one counterexample that the equivalence between efficiency and the MPS condition, or the equivalence between efficiency and Lehmann's order does not hold. Put it another way, implementing an action through a contract upon performance measure X can be more costly than doing so through a contract upon measure Y, although X is more precise than Y, i.e., L_F^a is more variable than L_G^a.

Our example describes a contracting environment with a single performance measure and discrete effort. Suppose that the agent can choose one level of effort among $A = \{a_L, a_M, a_H\}$ with $a_L < a_M < a_H$. The agent's utility function over income is $u(w) = (3w)^{1/3}$, and reservation utility is $\underline{V} = (4 + 3\sqrt{3} + \sqrt{2})/12$. The cost from effort is

$$\psi(a_L) = 0, \quad \psi(a_M) = \frac{\sqrt{3} + 3\sqrt{2} - 2}{12}, \quad \text{and} \quad \psi(a_H) = \frac{14\sqrt{2} - 7}{24},$$

respectively. Observe that the function u is increasing and concave, and the function ψ is increasing. Hence the given (u, ψ) is an element of the set Θ.

Under performance measurement system G, both parties can observe a discrete signal $y \in \{y_L, y_M, y_H\}$ that is governed by

$$\left[\Pr(y = y_i | a_j)\right]_{i,j=L,M,H} = \begin{bmatrix} 2/3 & 1/3 & 1/12 \\ 1/4 & 1/3 & 1/4 \\ 1/12 & 1/3 & 2/3 \end{bmatrix}.$$

Under system F, the parties can observe a signal $x \in \{x_L, x_M, x_H\}$ that is governed by

$$\left[\Pr(x = x_i | a_j)\right]_{i,j=L,M,H} = \begin{bmatrix} 2/3 & 1/3 & 1/15 \\ 1/4 & 1/3 & 4/15 \\ 1/12 & 1/3 & 2/3 \end{bmatrix}.$$

The two systems are designed such that the two signals involve the same information when the agent chooses a_L or a_M, whereas X conveys more information than Y when the agent chooses a_H. Thus, F is a more informative system. More precisely, each system satisfies the MLRP, and for every pair $(x, y) \in \{x_L, x_M\} \times \{y_L, y_M\}$, $G(y|a) - F(x|a)$ satisfies the single-crossing property in a. Hence it follows from Lemma 1 that X is a more precise measure.

We now determine their ranking according to contractual efficiency. Suppose that the principal desires to induce the agent to choose a_M. To compute the least expected salary for this objective, we reformulate the optimization problem in terms of the expected payoff to the agent $u_i = u(w_i)$ á la Grossman and Hart (1983). The first-order condition under system G is then written as

$$\begin{bmatrix} u_L^2 \\ u_M^2 \\ u_H^2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 3/4 \\ 1 & 1/4 & 1/4 \\ 1 & 3/4 & -1 \end{bmatrix} \cdot \begin{bmatrix} \lambda_G \\ \mu_G^1 \\ \mu_G^2 \end{bmatrix}, \tag{12}$$

where λ_G, μ_G^1, and μ_G^2 indicate the Lagrangian multipliers associated with the IR constraint and the two IC constraints preventing a possible deviation to low and high effort, respectively. Along with the complementary slackness conditions, the system of equations (12) is solved by $(\lambda_G, \mu_G^1, \mu_G^2) = (7/4, 3, 2) > \mathbf{0}$. The positive signs of μ_G^1 and μ_G^2 tell us that the downward and upward pa-ic constraints are binding, and thus the FOA is not justified in this example.[14] Using our notation, $(u, \psi) \notin \Theta^*$. The positive multipliers yield $u_L = 1/2$, $u_M = \sqrt{3}$, and $u_H = \sqrt{2}$. Consequently, the least expected pay for

[14]Note that the constructed performance measurement systems satisfy the MLRP but violate the CDFC.

implementation of a_M under system G amounts to

$$C_G(a_M) \;=\; \frac{1}{3}\left(\frac{1}{3}u_L^3 + \frac{1}{3}u_M^3 + \frac{1}{3}u_H^3\right) \;=\; 0.9055.$$

To compute the least expected salary for inducing a_M under system F, we have to solve

$$\begin{bmatrix} u_L^2 \\ u_M^2 \\ u_H^2 \end{bmatrix} \;=\; \begin{bmatrix} 1 & -1 & 4/5 \\ 1 & 1/4 & 1/5 \\ 1 & 3/4 & -1 \end{bmatrix} \cdot \begin{bmatrix} \lambda_F \\ \mu_F \\ \nu_F \end{bmatrix}, \tag{13}$$

plus the complementary slackness conditions. It can be shown (numerically) that like in the previous case, all of the Lagrange multipliers are strictly positive, and thus the two pa-ic constraints are binding:

$$\begin{aligned} \frac{2}{3}u_L + \frac{1}{4}u_M + \frac{1}{12}u_H \;&=\; \frac{1}{3}u_L + \frac{1}{3}u_M + \frac{1}{3}u_H - \psi(a_M) \\ &=\; \frac{1}{15}u_L + \frac{4}{15}u_M + \frac{2}{3}u_H - \psi(a_H) \;=\; \underline{V}. \end{aligned}$$

Solving these equations gives us a non-monotone expected payoff:

$$\begin{aligned} u_L \;&=\; \frac{49}{96} - \frac{1}{48}\sqrt{3} \;=\; 0.4743 \\ u_M \;&=\; -\frac{7}{192} + \frac{103}{96}\sqrt{3} \;=\; 1.8219 \\ u_H \;&=\; \frac{5}{192} - \frac{5}{96}\sqrt{3} + \sqrt{2} \;=\; 1.3500 \end{aligned}$$

Therefore, the minimum pay under system F is

$$C_F(a_M) \;=\; \frac{1}{3}\left(\frac{1}{3}u_L^3 + \frac{1}{3}u_M^3 + \frac{1}{3}u_H^3\right) \;=\; 0.9572,$$

which is higher than $C_G(a_M)$. This shows that system F is less efficient than G in implementing middle effort, breaking the equivalence between contractual efficiency and Lehmann's order (the MPS condition).

Our example demonstrates that when the FOA is not valid, neither the MPS condition nor the notion of precision is well suited for ordering performance measurement systems in agency models. This fact suggests that the way of ordering systems hinges upon whether the local approach is justified. The reason for the MPS condition is simple. When the FOA is not justified, the local constraint (L-IC) itself does not ensure that the agent is prevented from deviating to distant effort from the target. Put differently, another nonlocal IC constraints are also binding. In this case, the likelihood ratio which concerns only local deviations does not contain all relevant information.

To understand why Lehmann's order does not hold, recall that when the FOA is not valid, the MLRP *per se* does not guarantee monotonicity of the optimal incentive scheme. For implementation of middle effort in our example, the principal must demotivate the agent by paying less for high performance to the extent that the agent feels indifferent between middle and high effort, leading to the non-monotone incentive schemes. On the other hand, Lehmann's order is applicable only to monotone decision problems, i.e., to a strict subset of Θ in agency models where the optimal incentive scheme retains monotonicity.[15] Hence the notion of precision is too weak to rank measurement systems in the large set of agency models, so we need adopt a stronger notion such as sufficiency.[16]

6. Conclusion

In this paper, we investigated the statistical properties of ideal performance measures in standard agency models. The properties depend on whether the first-order approach is valid or not, put differently, whether the agent has an incentive to locally or globally deviate from the intended action. When the local approach is valid, the likelihood ratio involves all information relevant to the agent's potential deviations. Hence the MPS condition proposed by Kim (1995) becomes sufficient and necessary for one measurement system to be more efficient and informative than another. We compared the MPS condition with Lehmann's order based on the notion of precision, and established their equivalence in case of one-signals satisfying the MLRP. This finding highlights a close link between agency and decision problems, and provides a simple tool of finding a more favorable performance measure to the principal. On the other hand, when the local approach is not valid, neither the MPS criterion nor Lehmann's order can play a role as the informativeness criterion. This clarifies the structural difference between the agency model and the decision-making problem under uncertainty, and suggests that depending on the agent's potential deviations, the principal's choice of performance measures should be different.

[15] The scope of payoff functions considered in Lehmann (1988), which is termed the family of quasi-concave functions with increasing peaks by Quah and Strulovici (2009), features the optimal decision rule monotone with signal realizations. Quah and Strulovici (2009) generalized Lehmann's theorem to the family of interval dominance order functions which also features monotonicity of optimal decision rules.

[16] Grossman and Hart (1983) has shown that the sufficient part of Blackwell's theorem holds in agency problems, regardless of whether the FOA is applicable.

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NHH

NORGES HANDELSHØYSKOLE
Norwegian School of Economics

Helleveien 30
NO-5045 Bergen
Norway

T +47 55 95 90 00
E nhh.postmottak@nhh.no
W www.nhh.no